Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

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Abstract: I derive a general condition on consumer behavior ensuring that, in a simple textbook model of demand-determined output, any path of aggregate inflation and output that is implementable via interest rate policy is also implementable through time-varying uniform lump-sum transfers ("stimulus checks") alone. The condition is satisfied in popular models of non-Ricardian consumer behavior (e.g., HANK, OLG). Across these models, the transfer-only policy that closes a given demand shortfall is well-characterized by a small number of measurable sufficient statistics. My results extend to environments with investment if transfers are supplemented by a second standard fiscal tool—bonus depreciation.

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1 Introduction

The prescription of standard New Keynesian theory is to conduct stabilization policy through changes in short-term interest rates. In recent years, much policy and academic interest has centered on the question of whether—and if so, how—alternative policy tools could be used to replicate monetary stimulus when nominal interest rates are constrained by a zero or effective lower bound (ELB).[1] Prior work has in particular identified tax policy, often labeled *unconventional* fiscal policy, as an attractive option (Correia et al., 2008, 2013): time-varying tax rates manipulate intertemporal prices just like monetary policy and thus can replicate any desired monetary allocation.

In this paper I ask whether *conventional* fiscal policy—that is, fiscal instruments that are already part of the standard stabilization policy toolkit—are similarly sufficient to replicate any given monetary policy. The setting for much of my analysis is a textbook business-cycle model with nominal rigidities and without capital, extended to allow for more general, non-Ricardian household consumption behavior. The conventional fiscal stabilization tool that I consider are uniform, deficit-financed transfers ("stimulus checks"), a policy instrument used in all recent U.S. recessions. My first contribution is to identify a general sufficient condition under which any time paths of aggregate output and inflation that are implementable via interest rate policy are also implementable solely by adjusting the time path of such uniform lump-sum taxes and transfers. It follows in particular that, under my condition, standard output and inflation targeting rules remain uniquely implementable even with a binding ELB. Next I argue that this theoretical result is also practically relevant: I show that my sufficient condition holds in popular models of non-Ricardian consumption behavior, including heterogeneous-agent (HANK) and overlapping-generations (OLG) models. Furthermore, for all of these models, the required stimulus check policy that closes any given demand shortfall is well-characterized by a very small number of empirically measurable "sufficient statistics." At conventional values for these sufficient statistics, stimulus check policies of the magnitudes already observed in practice suffice to stabilize the economy in the face even of relatively sizable shortfalls in private spending. Finally, I show that all of these conclusions extend to richer models with investment as long as stimulus checks are supplemented by a second, similarly conventional fiscal tool: bonus depreciation stimulus.

[1] A notable early example is Bernanke (2002). Important recent contributions include Correia et al. (2008), Correia et al. (2013), Galí (2020), and Reis & Tenreyro (2022).

Environment & equivalence result. My model economy features a policymaker with access to two instruments: nominal interest rates and uniform, lump-sum taxes and transfers. My objective is to characterize the space of allocations implementable through manipulation of these two instruments. All results apply to (linearized) perfect foresight transition paths, or equivalently to the model's first-order perturbation solution with aggregate risk. Key for me are properties of the two matrices $\mathcal{C}_{i_b}$ and $\mathcal{C}_\tau$, whose (t,s)th entries are, respectively, the derivatives of partial equilibrium consumption demand at time t with respect to a) a change in the time-s rate of interest on bonds and b) a uniform lump-sum transfer paid out at time s. Note that $\mathcal{C}_\tau$ is a matrix of intertemporal marginal propensities to consume (iMPCs), as studied first in Auclert et al. (2018).

In this environment I establish that, if $\mathcal{C}_\tau$ is invertible—a condition that I will refer to as *strong Ricardian non-equivalence*—, then any sequence of aggregate output and inflation that can be attained via interest rate policy is also implementable by only adjusting the time profile of uniform lump-sum transfers. The proof begins with the household consumption-savings problem. A feasible monetary policy is a path of nominal interest rates together with a path of lump-sum taxes or transfers that ensures a balanced government budget. Through the household problem, this policy induces some path of net excess consumption demand. Can a transfer-only policy—that is, a policy that only changes the time profile of taxes and transfers, again subject to budget balance—engineer the same path of net excess demand? For Ricardian households, the answer is no: for them, only the net present value of transfers matters, so any budget-feasible transfer policy leaves spending unchanged. Mathematically, this is reflected in $\mathcal{C}_\tau$ being rank-1. If instead the *timing* of transfers matters (in the strong sense that $\mathcal{C}_\tau$ is invertible), then there does exist some path of transfers and taxes alone that perturbs consumption demand in exactly the same way as the baseline monetary policy. Since this monetary policy was by assumption budget-balanced, the equivalent transfer policy is feasible as well. The argument is then completed by showing that, in my environment, two policies that generate the same partial equilibrium net excess consumption demand paths must be accommodated in general equilibrium through the same market-clearing adjustments in prices (inflation, wages, ...) and quantities (output, hours worked, ...). Though revenue-equivalent in net present value terms, the two policies do invariably induce different short-run government debt dynamics: while interest rate policy can in principle have aggregate effects even if outstanding debt is fixed, uniform stimulus checks work *only* because they change the time path of government bonds held by the private sector.

Under the conditions of my equivalence result, transfer payments can serve as a perfect substitute for interest rate policy in the eyes of a conventional "dual mandate" policymaker. Formally, my results imply that systematic policy *rules* like the well-known (flexible) inflation forecast target criteria (Woodford, 2011) continue to be implementable even if nominal rates are constrained by a binding lower bound. In particular, this conclusion holds completely independently of the menu of non-policy disturbances hitting the economy.

Practical relevance & policy characterization. I next discuss the practical relevance of the theoretical equivalence result. I first ask what assumptions on economic primitives are required to ensure the high-level condition of "strong Ricardian non-equivalence". My main finding here is that this condition holds *generically* in standard analytical models of non-Ricardian consumer behavior, notably including perpetual-youth overlapping generation models (Blanchard, 1985), spender-saver models (Campbell & Mankiw, 1989; Bilbiie, 2008), and models with bonds in the consumer utility function (Michaillat & Saez, 2018). I also numerically verify the condition in several HANK models. Intuitively, in all of these settings, time-varying paths of taxes and transfers will re-shuffle demand over time, e.g., by moving households away to or towards borrowing constraints (HANK), or by redistributing across cohorts (OLG). This however still leaves a second question: even if $\mathcal{C}_\tau$ is technically invertible, the inverse $\mathcal{C}_\tau^{-1}$ may be ill-behaved, and so stabilization through stimulus checks may require excessively large and erratic fluctuations in transfers and government debt.

My next finding is that—across all of these models of non-Ricardian consumer behavior—the transfer policy that is needed to close any given shortfall in aggregate demand is very well-characterized by a small number of measurable sufficient statistics. Mathematically, to characterize this transfer policy, I require the full inverse matrix $\mathcal{C}_\tau^{-1}$—a potentially high-dimensional and complicated object. My main result is that this large matrix can be obtained very accurately from just three measurable numbers: the economy's average MPC ω; the slope θ of how consumers spend a lump-sum income gain over time; and the rate of interest $\bar{r}$. If ω is close to $\bar{r}$ (as in standard permanent-income models of consumption), then $\mathcal{C}_\tau^{-1}$ is ill-behaved, and so the required transfer payments and movements in government debt diverge. For empirically relevant levels of the statistics $\{\omega, \theta, \bar{r}\}$, in contrast, even moderate increases in transfers suffice for aggregate cyclical stabilization. I provide an illustration in a calibrated quantitative HANK model. Here, to close a transitory demand shortfall of one per cent, the policymaker could equivalently (i) cut rates by a cumulative 150 basis points or (ii) increase transfers by around \$600 per household, similar in scope to the stimulus check policy enacted as part of the 2008 Economic Stimulus Act.

Extensions & limitations. As the final step in my argument, I extend the model environment to also feature capital. Conventional monetary policy now operates through two levers: by directly affecting (i) the household consumption-savings decision and (ii) the firm investment decision. Transfer policy is still enough to replicate arbitrary stimulus to consumer demand. For firm investment, it is straightforward to show that a second, similarly conventional fiscal instrument will suffice: bonus depreciation stimulus. I thus conclude that, even in this extended environment, two entirely *conventional* fiscal tools suffice to replicate any desired monetary stimulus.

While most of my analysis is concerned with aggregates, I also briefly discuss important limitations to my equivalence result in the presence of microeconomic heterogeneity. First, I emphasize that—for my equivalence result to apply to HANK-type models—I require particular (though standard) assumptions on wage-setting. These assumptions are consistent with arbitrary *levels* of wealth effects in labor supply, but not with cross-sectional *dispersion* in those wealth effects. I review the empirical evidence on such dispersion and conclude that the scope for this channel to materially affect the equivalence result is limited. Second, my macro-equivalent interest rate and lump-sum transfer policies are generally *not* equivalent household-by-household: nominal interest rate cuts mostly act by directly stimulating consumption at the top of the liquid wealth distribution, whereas the macro-equivalent transfer stimulus almost exclusively acts at the bottom. Thus, compared to a given interest rate policy, the macro-equivalent transfer delivers the same macroeconomic stimulus at smaller cross-sectional consumption dispersion. The normative implications of this positive observation are studied in McKay & Wolf (2022a).

Related literature. The paper relates and contributes to several strands of literature.

First, the analysis is motivated by the recent experience of limits to conventional monetary policy space, caused in particular by: a binding ELB on nominal rates due to arbitrage between bonds and money; adverse effects of further rate cuts on bank profitability (Brunnermeier & Koby, 2018); rates on most outstanding mortgages being close to the ELB (Berger et al., 2018); and durables spending adjustments already having been pulled forward in time (McKay & Wieland, 2019). Prior work has argued that *unconventional* fiscal instruments can be used to substitute for monetary stimulus if needed (Correia et al., 2008, 2013); I instead clarify the conditions under which *conventional* fiscal stimulus tools can do the same.[2]

[2]Another related policy tool that has received both academic as well as substantial policy interest are helicopter drops of money—that is, *money-financed* transfer payments (e.g., see Bernanke, 2002; Galí, 2020;

Second, I relate to an important prior literature on the stimulative effects of transfer payments in the absence of Ricardian equivalence. Blanchard (1985), Woodford (1990) and Bilbiie et al. (2013) all emphasize that, if private planning horizons are finite (due to death) or borrowing constraints bind, then public debt can in principle stimulate spending through its role as private liquidity. Bilbiie et al. (2021) furthermore show that redistribution from savers to spenders in a two-type model can perfectly mimic monetary demand stimulus. Relative to this line of work, my contribution is to: (i) identify a general condition—applicable across all of these environments as well as more general ones—under which *uniform, time-varying* taxes and transfers are stimulative in the precise sense that they can perfectly replicate monetary policy; and (ii) show that the required paths of taxes and transfers can in fact be characterized as a function of a small number of empirically measurable sufficient statistics. For measurement of these sufficient statistics I am fortunate to rely on important recent empirical contributions (notably Parker et al., 2013; Fagereng et al., 2018).

Third, my proof of policy equivalence relies heavily on equilibrium characterizations in sequence space (Boppart et al., 2018; Auclert et al., 2019). So far, the sequence-space set-up has been used to analytically characterize general equilibrium effects (Auclert & Rognlie, 2018; Auclert et al., 2018) or to construct general equilibrium counterfactuals for unobserved shocks (Wolf, 2020). I instead use the same observations to sidestep constraints on policy space. Echoing classical general equilibrium theory (Arrow & Debreu, 1954), the sequence-space perspective reveals that two policies are equivalent if they induce the same net excess demand paths. As such, my equivalence results are conceptually distinct from Correia et al. (2013) and Farhi et al. (2014)—there, equivalence comes from identical wedges in optimality conditions. The advantage of my approach is that it can be applied readily to conventional fiscal instruments (like stimulus checks); the obvious challenge is that characterization of the equivalent policy requires additional arguments (my analysis of $\mathcal{C}_\tau^{-1}$).

Outlook. The rest of the paper proceeds as follows. Section 2 sets up the baseline model, and Section 3 presents the main theoretical result on the set of implementable allocations. Section 4 then establishes that my equivalent transfer-only stabilization policies can robustly be characterized as a function of a small number of measurable sufficient statistics. I discuss the role of micro heterogeneity in Section 5, and finally extend my results to a model with investment in Section 6. Section 7 concludes.

Reis & Tenreyro, 2022). My results reveal that, under natural assumptions on consumer spending behavior, even *deficit-financed*, *uniform* transfer payments are sufficient to replicate monetary stimulus.

2 Environment

I begin with a description of the environment in Section 2.1. My assumptions on consumption behavior will be purposefully general, requiring only the existence of an aggregate consumption function. Section 2.2 discusses particular models that fit into this general environment.

2.1 Model outline

Time is discrete and runs forever, $t = 0, 1, \ldots$. The model economy is populated by households, unions, firms, and a government, and is initially at its deterministic steady state. I study linearized perfect foresight transition paths.[3] The overall set-up is kept deliberately close to the textbook New Keynesian business-cycle framework (Woodford, 2011; Galí, 2015).

At time $t = 0$, the policymaker announces paths for her policy instruments. My objective is to characterize the set of allocations that she can implement. The realization of a variable x at time t along the equilibrium perfect foresight transition path will be denoted x_t, while the entire time path will be denoted $\boldsymbol{x} = \{x_t\}_{t=0}^{\infty}$. Hats denote (log-)deviations from the deterministic steady state and bars denote steady-state values.

The aggregate consumption function. Households consume and supply labor, with total consumption and hours worked denoted by c_t and ℓ_t, respectively. Due to frictions in the labor market, hours worked are taken as given by households and set by optimizing labor unions, to be discussed in detail later.[4] Given paths of income, households decide on their consumption and savings. Rather than specifying the details of this consumption-savings problem, I here simply summarize its solution in the form of an aggregate consumption function. Section 2.2 will consider several particular models of household consumption behavior that fit into this general framework.

Before stating and discussing the aggregate consumption function I begin with the household budget constraint. Total income of the household sector consists of: labor earnings $(1 - \tau_\ell) w_t \ell_t$, where w_t is the real wage and τ_ℓ is the (assumed fixed) labor tax rate; uniform lump-sum transfer receipts τ_t; and dividends d_t. Households can invest in nominally risk-free

[3] My results can thus equivalently be interpreted as applying to the first-order perturbation solution of an analogous model with aggregate risk (e.g. Boppart et al., 2018; Auclert et al., 2019).

[4] I allow for unions in the interest of generality and empirical relevance. For all models except for the quantitative heterogeneous-household model of Section 4, the alternative standard case of frictionless labor supply will correspond to the flexible-wage limit of my union model. I further discuss the role of labor supply in HANK models in Section 5.1.

liquid bonds with nominal returns $i_{b,t}$. The real return to saving is affected by the inflation rate π_t. The period-t budget constraint of the aggregated household sector is thus

$$c_t + b_t = (1-\tau_\ell)w_t\ell_t + \frac{1+i_{b,t-1}}{1+\pi_t}b_{t-1} + \tau_t + d_t \tag{1}$$

where b_t denotes real bond holdings. Given any sequence of household income and asset returns, optimal household behavior yields time paths of aggregate consumption demand $\boldsymbol{c}$ and asset supply $\boldsymbol{b}$. I summarize optimal household consumption behavior in the form of an aggregate consumption function $\mathcal{C}(\bullet)$ (Farhi & Werning, 2019; Auclert et al., 2018):

$$\boldsymbol{c} \quad = \quad \mathcal{C}(\underbrace{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}}_{\text{eq'm aggregates}}; \underbrace{\boldsymbol{\tau}, \boldsymbol{i}_b}_{\text{policy}}) \tag{2}$$

By definition, the aggregate consumption function evaluated at steady state satisfies

$$\bar{\boldsymbol{c}} \quad = \quad \mathcal{C}\left(\bar{\boldsymbol{w}}, \bar{\boldsymbol{\ell}}, \bar{\boldsymbol{\pi}}, \bar{\boldsymbol{d}}; \bar{\boldsymbol{\tau}}, \bar{\boldsymbol{i}}_b\right)$$

In my linearized environment, policy equivalence will be fully governed by the properties of $\mathcal{C}(\bullet)$ around this deterministic steady state. Linearizing (2), we can write

$$\widehat{\boldsymbol{c}} \quad = \quad \mathcal{C}_w\widehat{\boldsymbol{w}} + \mathcal{C}_\ell\widehat{\boldsymbol{\ell}} + \mathcal{C}_\pi\widehat{\boldsymbol{\pi}} + \mathcal{C}_d\widehat{\boldsymbol{d}} + \mathcal{C}_\tau\widehat{\boldsymbol{\tau}} + \mathcal{C}_{i_b}\widehat{\boldsymbol{i}}_b \tag{3}$$

where, for each $q \in \{w, \ell, \pi, d, \tau, i_b\}$, I have defined

$$\mathcal{C}_q \quad \equiv \quad \frac{\partial \mathcal{C}(\bullet)}{\partial \boldsymbol{q}}, \tag{4}$$

with the derivative evaluated at the deterministic steady state. The (t,s)th entry of each of those infinite-dimensional linear maps is the response of aggregate consumption demand at time t to a marginal change in input q at time s. The linear map $\mathcal{C}_\tau$—which indicates how aggregate consumption demand will respond to changes in lump-sum transfers—will play a central role in characterizing the allocations implementable by the policymaker through stimulus check policy. To build further intuition I will in Section 2.2 provide closed-form characterizations of this map in various familiar models of household consumption decisions.

Unions & firms. I summarize the production and wage bargaining block through three key relations. First, a unit continuum of firms produces the final output good using a labor-only production technology:

$$y_t = y(\ell_t)$$

Price-setting is subject to the usual Rotemberg adjustment costs, giving a textbook New Keynesian Phillips Curve (NKPC) in prices (Galí, 2015):

$$\pi_t \quad = \quad \kappa_p \quad \times \quad \underbrace{(\chi_p(w_t, \ell_t) - 1)}_{\text{deviation from price target}} \quad + \quad \beta\pi_{t+1} \tag{5}$$

where $\chi_p(\bullet)$ gives the deviation from the price target and κ_p is the slope of the price-NKPC. Finally, wage bargaining is similarly subject to adjustment costs and so induces a general wage-NKPC, linking wage inflation to the static labor optimality wedge (Erceg et al., 2000):

$$\pi_t^w \quad = \quad \kappa_w \quad \times \quad \underbrace{(\chi_w(w_t, \ell_t, c_t) - 1)}_{\text{deviation from wage target}} \quad + \quad \beta\pi_{t+1}^w \tag{6}$$

where $1 + \pi_t^w = \frac{w_t}{w_{t-1}}(1 + \pi_t)$ denotes wage inflation, $\chi_w(\bullet)$ gives the deviation from the wage target, and κ_w is the slope of the wage-NKPC. I discuss the derivation of (6) for particular assumptions on household preferences in Appendices B.2 to B.6.

Policy. The government flow budget constraint is

$$\frac{1 + i_{b,t-1}}{1 + \pi_t} b_{t-1} + \tau_t = \tau_\ell w_t \ell_t + b_t \tag{7}$$

The policymaker sets nominal interest rates $i_{b,t}$ and uniform lump-sum taxes and transfers τ_t subject to the flow budget constraint (7) and the requirement that $\lim_{t\to\infty} b_t = \bar{b}$.

The remainder of this paper will study the implications of constraints on this policy toolkit. I will focus on two particular kinds of restrictions: *transfer-only* policies and *interest rate-only* policies.

Definition 1. *A* transfer-only *policy is a policy that sets $i_{b,t} = \bar{i}_b$ for all t.*

In a transfer-only policy, the policymaker is forced to keep nominal interest rates fixed, perhaps due to a binding effective lower bound. In light of the ELB's empirical relevance, most discussion in this paper will center on the extent to which a restriction to transfer-only policies meaningfully constrains the policymaker. I note that, for such a policy, the direct mapping from policy instrument to consumer spending is fully governed by the matrix $\mathcal{C}_\tau$.

Definition 2. *An* interest rate-only *policy is a policy that sets, for all* $t = 0, 1, \ldots,$

$$\tau_t = \tau_\ell w_t \ell_t + (1 - \frac{1 + i_{b,t-1}}{1 + \pi_t})\bar{b}. \tag{8}$$

Definition 2 gives the natural opposite to a transfer-only policy: in an interest rate-only policy, the policymaker is free to adjust the path of nominal rates, but forced to passively adjust lump-sum transfers/taxes to balance the budget period-by-period. This policy thus operates by manipulating intertemporal prices, without any time variation in the total amount of government debt in the hands of households.[5] For such a policy, the direct mapping from policy instrument to consumer spending is governed by the following map:

$$\tilde{\mathcal{C}}_{i_b} \quad \equiv \quad \mathcal{C}_{i_b} \quad - \quad \bar{b} \times \mathcal{C}_{\tau,\bullet,-1}, \tag{9}$$

Intuitively, $\tilde{\mathcal{C}}_{i_b}$ combines the direct effect of the nominal rate change itself ($\mathcal{C}_{i_b}$) together with the implied movement in taxes required to balance the budget (second term).[6]

Equilibrium. I am now in a position to define a perfect foresight transition equilibrium in this economy. As usual, I throughout restrict attention to equilibria in which all sequences of policies and macroeconomic aggregates are bounded (Woodford, 2011).

Definition 3. *An equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$ *and a set of macroeconomic aggregates* $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ *such that:*

1. *Consumption is consistent with the aggregate consumption function* (2).
2. *Wage inflation* π_t^w *together with* $\{\ell_t, c_t, w_t\}_{t=0}^{\infty}$ *are consistent with the wage-NKPC* (6).
3. *The paths* $\{\pi_t, w_t, \ell_t\}_{t=0}^{\infty}$ *are consistent with the price-NKPC* (5)*, and dividends are given as* $d_t = y_t - w_t \ell_t$.
4. *The output market clears:* $y_t = c_t$ *for all* $t \geq 0$*, the government budget constraint* (7) *holds at all* t*, and* $\lim_{t\to\infty} \widehat{b}_t = 0$*. The bond market then clears by Walras' law.*

An allocation of macroeconomic aggregates $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ *is said to be implementable if it can be supported as an equilibrium sequence.*

[5] More generally, an interest rate policy is a policy that freely sets nominal rates $i_{b,t}$ and then ensures budget balance in net present value terms (i.e., $\lim_{t\to\infty} \widehat{b}_t = 0$) through some fixed tax-and-transfer adjustment rule. (8) is simply a particularly transparent example of such a financing rule.

[6] Note that, if $\bar{b} = 0$, then—to first order—no financing is required, so $\tilde{\mathcal{C}}_{i_b} = \mathcal{C}_{i_b}$.

Note that Definition 3 specifies policy directly as a path of policy instruments $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$. As is well-known, policies of this sort generically do not induce unique equilibria (Sargent & Wallace, 1975). To address this challenge, I will later also discuss equilibria induced by policy *rules* for interest rates and transfers. I call an allocation $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ uniquely implementable if it is the only equilibrium sequence consistent with those rules.

2.2 Detailed models of household consumption behavior

I will now give examples of several canonical models of household behavior that are consistent with the general aggregate consumption function (2). The purpose of this discussion is twofold: first, to illustrate the generality of the model set-up in Section 2.1; and second, to already give some intuition for the shape and properties of the key matrix $\mathcal{C}_\tau$ (which will feature prominently in Sections 3 and 4).

Analytical models. All analytical results in the remainder of the paper will rely on the following four tractable models of the consumption-savings problem.

1. **Permanent-income consumers.** The classical representative-agent literature on monetary policy transmission assumes standard permanent-income consumers (Galí, 2015; Woodford, 2011). Preferences of the household often take the particular form

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{10}$$

   The consumption-savings problem is then simply to choose sequences $\{\boldsymbol{c}, \boldsymbol{b}\}$ to maximize (10) subject to the budget constraint (1), and the solution to this problem is the aggregate consumption function (2). Studying the solution to this problem (with $\bar{r} \equiv 1/\beta - 1$ denoting the steady-state real rate), it is straightforward to show that

$$\mathcal{C}_\tau^R = \begin{pmatrix} \frac{\bar{r}}{1+\bar{r}} & \frac{\bar{r}}{(1+\bar{r})^2} & \frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ \frac{\bar{r}}{1+\bar{r}} & \frac{\bar{r}}{(1+\bar{r})^2} & \frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ \frac{\bar{r}}{1+\bar{r}} & \frac{\bar{r}}{(1+\bar{r})^2} & \frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{11}$$

   Following receipt of lump-sum income, permanent-income households consume the annuity value of that receipt in each period, giving the expression for $\mathcal{C}_\tau^R$. In particular, any sequence of transfers with zero net present value does not affect household consumption at all. For future reference, it will be useful to note that $\mathcal{C}_\tau$ in this simple model is a function of only one object: the steady-state real interest rate $\bar{r}$.

2. **Spenders and savers.** A simple extension of the canonical permanent-income model adds a margin $\mu \in (0,1)$ of spenders—households that hold no wealth and immediately consume any income. Combining this mechanical spending behavior with the consumption function of savers, we again get an aggregate consumption function (2). It is straightforward to see that, in this case, we have

$$\mathcal{C}_\tau^H = (1-\mu) \times \mathcal{C}_\tau^R + \mu \times I \tag{12}$$

where I denotes the identity matrix. For future reference, it will be useful to note that the average marginal propensity to consume (MPC) ω in this economy is given as

$$\omega = (1-\mu) \times \frac{\bar{r}}{1+\bar{r}} + \mu \times 1$$

Solving for μ and plugging into (12), we see that here we can obtain $\mathcal{C}_\tau^H$ as a function of only two objects: the steady-state real interest rate $\bar{r}$ and the economy's average MPC ω.

3. **Overlapping generations of households.** A canonical model of non-Ricardian household consumption behavior is one with overlapping generations of households (OLG) (e.g., see Blanchard, 1985). I consider a simple perpetual-youth departure from the permanent-income model (10), with households surviving from period to period at rate $\theta \in (0,1]$, bonds paying out as fair annuities, and newborns receiving the wealth of dying households. Detailed model equations (which closely follow Angeletos & Huo (2021)) are presented in Appendix B.4, fully characterizing the model-implied aggregate consumption function (2). Importantly, I in Appendix C.1 I furthermore show that $\mathcal{C}_\tau$ in this model satisfies

$$\mathcal{C}_\tau^{OLG} \approx \omega \times \begin{pmatrix} 1 & \frac{\theta}{1+\bar{r}} & (\frac{\theta}{1+\bar{r}})^2 & \dots \\ \theta & 1 & \frac{\theta}{1+\bar{r}} & \dots \\ \theta^2 & \theta & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{13}$$

where the sense of the approximation $\approx$ is made precise in Appendix C.1, and the MPC ω satisfies $\omega = 1 - \frac{\theta}{1+\bar{r}}$. (13) reveals that the matrix $\mathcal{C}_\tau^{OLG}$ has a very simple shape. On the one hand, because households die over time, spending following lump-sum income receipt is front-loaded, decaying at rate θ (i.e., entries below the main diagonal of $\mathcal{C}_\tau^{OLG}$). On the other hand, household exit also implies that future income receipt only partially affects spending today, with the strength of anticipation effects (i.e., above the main diagonal of $\mathcal{C}_\tau^{OLG}$) also governed by θ—that is, there is a single "slope" θ governing how household spending reacts to lump-sum income receipt over time. Overall we see that $\mathcal{C}_\tau$ is yet again a simple object. Relative to the spender-saver model, the main difference is that the OLG set-up not only generates elevated MPCs ($\omega > \frac{\bar{r}}{1+\bar{r}}$), but also a gradual time profile of spending (i.e., the off-diagonal entries of $\mathcal{C}_\tau$ are not flat, unlike for $\mathcal{C}_\tau^H$).

4. **Bonds in the household utility function.** My fourth and final analytical model is one with bonds in the household utility function. Models of this sort have recently become popular as an analytical alternative to full heterogeneous-household models; intuitively, the reduced-form preference for wealth reflects the precautionary savings motive present in proper incomplete-markets models (Kaplan & Violante, 2018; Michaillat & Saez, 2018). Household preferences here are given as

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{14}$$

where $\{\alpha, \eta\}$ are additional parameters. The representative household chooses sequences $\{\boldsymbol{c}, \boldsymbol{b}\}$ to maximize (14) subject to the budget constraint (1), again inducing an aggregate consumption function (2). As I show in Appendix C.1, we in this model have that

$$\mathcal{C}_\tau^{BiU} \quad \approx \quad \omega \times \begin{pmatrix} 1 & \beta\theta & (\beta\theta)^2 & \dots \\ \theta & 1 & \beta\theta & \dots \\ \theta^2 & \theta & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{15}$$

where the MPC ω and the slope coefficient θ are now complicated functions of model primitives. Comparing (15) and (13), we see that the analytical OLG and bond-in-utility models induce similar mappings from income to consumer spending. In the bond-in-utility model, upon income receipt, households optimally spend their income gradually, ensuring wealth returns to target over time; similarly, they gradually increase spending prior to a future income receipt to smooth out both consumption and wealth. Interestingly, as in the OLG model, both the gradual spending time profile as well as anticipation effects are governed by a common parameter (the "slope" θ), so $\mathcal{C}_\tau$ is yet again low-dimensional.

A QUANTITATIVE "HANK"-TYPE MODEL. The set-up of Section 2.1 is similarly consistent with quantitative heterogeneous-agents models of the consumption-savings problem. In a canonical "HANK" model, a unit continuum of households $i \in [0, 1]$ has preferences

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_{it}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \right] \tag{16}$$

where expectations are now taken over idiosyncratic household productivity e_{it}, with $\int_0^1 e_{it} di = 1$ for all t. The individual household budget constraint is now

$$c_{it} + b_{it} = (1 - \tau_\ell) w_t e_{it} \ell_{it} + \frac{1 + i_{b,t-1}}{1 + \pi_t} b_{it-1} + \tau_t + d_{it}, \quad b_{it} \geq \underline{b} \tag{17}$$

where I have additionally allowed dividends d_{it} to be type-specific, and household borrowing is now subject to an-hoc (tight) borrowing constraint $\underline{b}$. I assume that unions demand the same hours worked from all households, so that $\ell_{it} = \ell_t$ for all $i \in [0, 1]$; the corresponding union wage bargaining problem is presented in detail Appendix B.6. I will return to the role of these assumptions on household labor supply in Section 5.1.

The household consumption-savings decision is now to choose sequences of consumption and savings, $\{\boldsymbol{c}_i, \boldsymbol{b}_i\}$ to maximize (16) subject to (17). Aggregating across households, we yet again obtain an aggregate consumption function (2). This consumption function—and in particular the derivative matrix $\mathcal{C}_\tau$—now does not admit any closed-form characterization. However, these objects can be recovered straightforwardly using the computational methods developed by Boppart et al. (2018) and Auclert et al. (2019).

OUTLOOK. The remainder of the paper will proceed in two main steps. First, in Section 3, I will identify a general sufficient condition on $\mathcal{C}_\tau$ ensuring that transfer policies can replicate arbitrary monetary stimulus. This analysis will only require the general set-up of Section 2.1, and so in particular will apply to all of the specific models reviewed just now. Second, in Section 4, I will then use those particular models and their $\mathcal{C}_\tau$'s to (i) *verify* my sufficient condition and (ii) explicitly *characterize* the macro-equivalent stimulus check policy that closes any given shortfall in aggregate spending. In this characterization, the three objects discussed in this section—the MPC ω, the spending "slope" θ, and the steady-state interest rate $\bar{r}$—will play a key role. As it will turn out, even in rich HANK environments, these three objects will suffice to characterize my equivalent stimulus check policies.

3 Interest rate cuts vs. stimulus checks

This section presents my main theoretical result on the set of implementable allocations. Section 3.1 begins with the general sufficient condition for policy equivalence, and Section 3.2 discusses the implications of this result for macroeconomic policy *rules*.

3.1 A sufficient condition for aggregate policy equivalence

I begin with a preliminary definition: a property of the consumption derivative map $\mathcal{C}_\tau$ that I refer to as *strong Ricardian non-equivalence.* This property will turn out to be a general sufficient condition for my core equivalence result.

Definition 4. *A consumption function $\mathcal{C}(\bullet)$ exhibits strong Ricardian non-equivalence if the linear map $\mathcal{C}_\tau$ is invertible. I denote its inverse by $\mathcal{C}_\tau^{-1}$.*

Under the Barro (1974) definition of Ricardian equivalence, the time path of (lump-sum) taxes used to finance any given fiscal expenditure is completely irrelevant for consumption—only the present value matters. We already saw this in the expression for $\mathcal{C}_\tau$ in the permanent-income model (see (11)): there $\mathcal{C}_\tau$ is rank-1, and so in particular it is not invertible. With non-Ricardian households, on the other hand, the timing of transfers also begins to matter, increasing the rank of $\mathcal{C}_\tau$; strong Ricardian non-equivalence corresponds to the limit case of invertibility. Section 4.1 will establish that the various analytical models of non-Ricardian consumption discussed in Section 2.2 indeed generically satisfy this property.

The equivalence result. I am now in a position to state the policy equivalence result.

Proposition 1. *Consider the model of Section 2.1, and let $\widehat{\boldsymbol{c}}$ be a path of household consumption with zero net present value, i.e., $\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_t = 0$. Suppose that*

$$\widehat{\boldsymbol{c}} \;\in\; image(\mathcal{C}_\tau) \quad \Leftrightarrow \quad \widehat{\boldsymbol{c}} \;\in\; image(\tilde{\mathcal{C}}_{i_b}) \tag{18}$$

Then the two policy instruments τ and i_b are macro-equivalent: any aggregate allocation that is implementable with transfer-only policy is also implementable with interest rate-only policy, and vice-versa.

An easy-to-interpret sufficient condition ensuring the direction "$\Leftarrow$" in the equivalence condition (18) is the notion of strong Ricardian non-equivalence: if $\mathcal{C}_\tau$ is invertible, then the space of aggregate allocations implementable using transfer-only policies is at least as large that implementable using interest rate-only policies.

Proof sketch. Key to the proof of Proposition 1 is the insight that both interest rate as well as transfer policies only directly perturb the model's equilibrium conditions in two places: first, the left-hand side of the output market-clearing condition $\mathcal{C}(\bullet) = \boldsymbol{y}(\boldsymbol{\ell})$; and second, the sequence of government budget constraints (7).

In partial equilibrium—i.e., prior to general equilibrium price and quantity adjustments—a feasible monetary policy is simply a budget-neutral perturbation of relative intertemporal prices, inducing a path of net excess consumption demand

$$\widehat{\boldsymbol{c}}_{i_b}^{PE} \quad \equiv \quad \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b$$

Note that, since my definition of an interest rate policy includes its financing, this demand path necessarily has zero net present value. By (18), we can find *some* transfer sequence $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ that induces the exact same perturbation of net excess demand. Intuitively, the equivalent transfer-only policy $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ twists the household intertemporal spending profile by changing the amount of government bonds held by households, thereby affecting the severity of liquidity constraints (in HANK-type models) or redistributing across cohorts (in an OLG environment). Since the initial monetary policy was consistent with fiscal budget balance, and so since $\widehat{\boldsymbol{c}}_{i_b}^{PE}$ has zero net present value, it then follows from the household budget constraint (1) that the equivalent transfer also necessarily has zero net present value,

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_{t,i_b}^{PE} \quad = \quad \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t(\widehat{\boldsymbol{i}}_b) \quad = \quad 0$$

Thus, prior to any general equilibrium feedback, $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ is also consistent with budget balance, in the sense that the induced debt path $\widehat{\boldsymbol{b}}$ satisfies $\lim_{t\to\infty} \widehat{b}_t = 0$. Finally note that the argument also works in reverse: any transfer-only policy that is consistent with the government budget constraint (7) (i.e., $\lim_{t\to\infty} \widehat{b}_t = 0$) necessarily induces a perturbation of consumption demand with zero net present value, and so by (18) we can always find an equivalent interest rate-only policy. To summarize, this first step leverages the fact that both policies equally flexibly manipulate the same lever: the consumption-savings decision of households.

Given that the transfer and nominal interest rate policies are both budget-feasible and both perturb the output market-clearing condition by exactly the same amounts time period by time period, it then follows via the implicit function theorem that they must also induce the same general equilibrium paths of inflation, hours worked, wages, and dividends. The intuition is simple: if, for example, the excess demand path induced by some monetary policy is accommodated in general equilibrium through increases in inflation and hours worked, then the same inflation and hours worked paths are also consistent with agent optimality and market-clearing after the equivalent transfer policy. This proof strategy is a simple application of the Arrow & Debreu (1954) approach to general equilibrium characterization using state-by-state (here t-by-t) net excess demand functions. Finally, the proof also reveals that the two policies are *revenue-equivalent* in general equilibrium: e.g., if increases in economic activity and inflation lead to an additional budget surplus after a rate cut, then the exact same budget surplus also opens up after the equivalent transfer stimulus.[7]

Discussion of assumptions. The first step of my argument relied on two assumptions: (i) the existence of an aggregate consumption function $\mathcal{C}(\bullet)$, and (ii) the restriction that both policies operate *only* by manipulating that function. Many macroeconomic models induce such consumption functions—e.g., ranging from the models reviewed in Section 2.2 to those with behavioral biases (like Laibson et al., 2020; Lian, 2021)—, and so strong Ricardian non-equivalence is a useful, quite widely applicable sufficient condition. By (ii), however, my conclusions do not immediately extend to models in which monetary policy acts through multiple levers, e.g., firm investment. I discuss such extensions in Section 6.

The second (general equilibrium) step of my proof requires that two policies with identical effects on partial equilibrium net excess consumer demand are accommodated in the same way in general equilibrium. The shape of the wage-NKPC (6) plays an important role in ensuring that this is indeed the case. While this shape is entirely standard for representative-agent models and for those in particular nests the limiting special case of flexible labor supply, it requires additional assumptions in the case of cross-sectional household heterogeneity (as in HANK models). I will return to this point in Section 5.1.

3.2 Implications for policy practice & policy rules

The equivalence result in Proposition 1 was phrased in terms of perfect foresight transition paths, or equivalently in terms of impulse responses to policy shocks in a linearized model

[7] This notion of revenue equivalence is perfectly analogous to the discussion of interest rate policy and distortionary taxation (i.e., unconventional fiscal policy) in Correia et al. (2013).

with aggregate risk. By the results in McKay & Wolf (2022b), such equivalence in terms of responses to policy *shocks* also implies equivalence in terms of policy *rules*: in response to any set of non-policy shocks (e.g., supply or demand), the aggregate outcomes implied by any given nominal interest rate rule can equivalently be implemented using a transfer-only rule. This subsection fleshes out the details on this observation. Throughout I will be relying on the perfect-foresight notation adopted in this paper; I discuss the interpretation of my equivalent policy rules from a recursive aggregate-risk perspective in Appendix C.6.

Adding non-policy shocks. To state my results on the equivalence of policy rules, I extend the baseline model to feature a rich menu of non-policy shocks: wedges $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$ to the aggregate consumption function (2) as well as the Phillips curves (5) - (6), corresponding to simple reduced-form representations of canonical demand and supply shocks. Given such shock paths, I now ask whether the space of allocations that the policymaker can implement through commitment to policy rules is affected by constraints on nominal rates $i_{b,t}$ (e.g., a binding ELB). Under the conditions of Proposition 1, the answer turns out to be "no".

Policy rule equivalence. I will present my equivalence results for two particular kinds of interest rate policy rules: implicit targeting rules and explicit instrument rules (Giannoni & Woodford, 2002). As I will argue below, implicit targeting rules in particular are a relevant description of actual stabilization policy practice.

A classical *implicit* targeting rule specifies a relationship between policy targets. For a standard dual-mandate policymaker, and written in the perfect-foresight notation adopted throughout this paper, such a rule takes the general form

$$\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} \;=\; \mathbf{0} \tag{19}$$

(19) specifies a relationship between inflation and output along the perfect-foresight transition path. It nests as special cases strict inflation targeting ($\widehat{\pi}_t = 0$ and so $\mathcal{B}_\pi = I$, $\mathcal{B}_y = \mathbf{0}$), strict output targeting ($\widehat{y}_t = 0$ and so $\mathcal{B}_\pi = \mathbf{0}$, $\mathcal{B}_y = I$), as well as the canonical implicit targeting rule of a dual-mandate policymaker,

$$\widehat{\pi}_t + \lambda(\widehat{y}_t - \widehat{y}_{t-1}) \;=\; 0, \quad t = 0, 1, 2, \dots$$

where λ is a function of policymaker preferences and model primitives (e.g., see Woodford, 2011, for a derivation). Strong Ricardian non-equivalence is sufficient to ensure that, if a rule of the general form (19) can be (uniquely) implemented using an interest rate-only policy (i.e., with a policy as in Definition 2), then the same is true for a transfer-only policy.

Corollary 1. *Suppose that the implicit targeting rule* (19) *implemented through an interest rate-only policy induces a unique equilibrium. Then, under the conditions of Proposition 1 and strong Ricardian non-equivalence, the rule* (19) *implemented through a transfer-only policy also induces a unique equilibrium featuring the same aggregate allocation.*

When implementing the targeting rule (19) through interest rate policy, the policymaker in the background sets nominal interest rates so that aggregate demand is consistent with output and inflation sequences satisfying (19). By my high-level assumption of strong Ricardian non-equivalence, she can engineer that exact same required time path of aggregate excess demand through transfers—she simply needs to set transfers equal to

$$\widehat{\boldsymbol{\tau}} \;=\; \mathcal{C}_\tau^{-1} \;\times\; \text{demand target}$$

The proof of Corollary 1 formalizes the argument. I note that those results have an important implication for policymaking practice: they imply that, even if nominal interest rates are constrained (e.g., due to a binding effective or zero lower bound), standard (flexible) inflation targeting rules of the general form (19) can still be implemented—the only difference is that the policymaker now does so through a different instrument.[8]

The same logic extends to *explicit* instrument rules—that is, rules that explicitly specify the value of the policy instrument as a function of observables. Again written in linearized perfect-foresight notation, typical rules of this sort take the general form

$$\widehat{\boldsymbol{i}}_b \;=\; \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} \tag{20}$$

(20) here specifies a mapping from inflation and output into nominal interest rates along the perfect-foresight transition path. For example, a simple Taylor rule would take the form

$$\widehat{i}_{b,t} \;=\; \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t, \quad t = 0, 1, 2, \ldots$$

[8] Bernanke (2015) succinctly summarizes the salience of this implicit targeting perspective for actual Federal Reserve policymaking:

> "The Fed has a rule. The Fed's rule is that we will go for a 2 percent inflation rate. We will go for the natural rate of unemployment. We put equal weight on those two things. We will give you information about our projections about our interest rates. That is a rule and that is a framework that should clarify exactly what the Fed is doing."

My main result simply states that, under my assumptions, this rule can equivalently be implemented using a different policy instrument, giving exactly the same equilibrium outcomes of inflation and output.

and so $\mathcal{B}_\pi = \phi_\pi \times I$, $\mathcal{B}_y = \phi_y \times I$. With interest rates set according to (20), taxes under my definition of an interest rate-only policy rule adjust in the background to ensure a balanced government budget (recall Definition 2). In particular, for my environment in Section 2.1, taxes by (8) follow

$$\widehat{\boldsymbol{\tau}} \;=\; \tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) - \bar{b}\widehat{\boldsymbol{i}}_{b,-1} + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}} \tag{21}$$

As before, strong Ricardian non-equivalence is sufficient to ensure that the equilibrium dynamics induced by a rule of the form (20)-(21) can equivalently be implemented uniquely through an explicit transfer-only policy rule.

Corollary 2. *Suppose that, given a sequence of shocks $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$, the explicit interest rate rule* (20)-(21) *induces a unique equilibrium. Then, under the conditions of Proposition 1 and strong Ricardian non-equivalence, the transfer-only policy rule*

$$\widehat{\boldsymbol{\tau}} \;=\; \underbrace{\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}}_{\text{deficit response}} \;+\; \underbrace{\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}\right)}_{\text{active demand management}} \tag{22}$$

together with $\widehat{\boldsymbol{i}}_b = \boldsymbol{0}$ uniquely implements the exact same aggregate allocation.

(22) is an explicit instrument rule for taxes and transfers. Just like (20) did for interest rates, (22) is a rule that gives the time path of taxes and transfers as a function of time paths of inflation and output (as well as the deficit). Intuitively, the rules (20) and (22) are equivalent because they both imply the same mapping from macroeconomic aggregates—output and inflation—into aggregate demand. The only difference is the instrument that is used to achieve that mapping.[9] Appendix C.6 further elaborates on how the perfect-foresight relationship (22) can be interpreted as a policy rule from a recursive perspective.

Taking stock. The analysis in this section has established my first main result: under the sufficient condition of strong Ricardian non-equivalence, constraints on monetary policy do not affect the space of (aggregate) allocations that are implementable by the policymaker. Relative to prior work (e.g., Correia et al., 2008, 2013), a key difference is that my conclusion relies only on a *conventional* fiscal policy tool: deficit-financed (uniform) stimulus checks.

While appealing, the generality of my sufficient condition for policy equivalence also invites obvious further questions. First, the condition on $\mathcal{C}_\tau$ is not phrased in terms of model

[9] Of course it is important to note that policy rules that are "simple" in interest rate space—e.g., in the sense that $\mathcal{B}_\pi$ and $\mathcal{B}_y$ are diagonal, as in simple Taylor rules—need not be simple in transfer space. Intuitively, nothing guarantees that $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$ in (22) is a diagonal matrix. I discuss this further in Appendix C.6.

primitives. Do standard models of non-Ricardian consumption behavior—like those reviewed in Section 2.2—actually imply that $\mathcal{C}_\tau$ is invertible? Second, even if so, the inverse $\mathcal{C}_\tau^{-1}$ may be ill-behaved, and so the policymaker may require implausibly large or erratic sequences of transfers for stabilization. Section 4 deals with these questions.

4 Characterizing the equivalent policy

The purpose of this section is to ascertain the practical relevance of my theoretical equivalence result. By the analysis of Section 3, this requires a characterization of $\mathcal{C}_\tau^{-1}$—the potentially complicated infinite-dimensional object that governs the mapping from any possible shortfall in consumer spending $\widehat{\boldsymbol{c}}^{PE}$ to the transfer policy $\widehat{\boldsymbol{\tau}}$ that would offset it:[10]

$$\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}^{PE}) \;\equiv\; \mathcal{C}_\tau^{-1} \times \widehat{\boldsymbol{c}}^{PE}$$

I will establish two main results about $\mathcal{C}_\tau^{-1}$. First, for the various models of the consumption-savings problem reviewed in Section 2.2 (and including in particular the heterogeneous-agent model), I will show that the required inverse $\mathcal{C}_\tau^{-1}$ exists and is either exactly or approximately pinned down by a small number of measurable "sufficient statistics": the average MPC ω; the spending profile "slope" θ; and the steady-state interest rate $\bar{r}$. Second, for empirically relevant values of these three sufficient statistics, I will find that the inverse is well-behaved—smooth and moderately sized transfer payments suffice for aggregate stabilization in the face of typical shortfalls in aggregate spending $\widehat{\boldsymbol{c}}^{PE}$.

The remainder of the section establishes these conclusions in two steps. In Section 4.1, I first provide explicit formulas for $\mathcal{C}_\tau^{-1}$ in my analytical models. Sections 4.2 and 4.3 then show that these simple formulas continue to characterize $\mathcal{C}_\tau^{-1}$ very well even in state-of-the-art heterogeneous-agent models of the consumption-savings problem.

4.1 Policy equivalence in analytical models

My sufficient condition of strong Ricardian non-equivalence turns out to be satisfied in all of the analytical models of non-Ricardian consumption behavior studied in Section 2.2. That is,

[10] I focus on expected shortfalls in demand since they are arguably the relevant target for policy (see my discussion after Corollary 1). To instead map a given path of *interest rates* into the equivalent transfer (as is needed for explicit rules, see Corollary 2), I would also require $\tilde{\mathcal{C}}_{i_b}$. In the models I consider, the only additional parameter required to characterize $\tilde{\mathcal{C}}_{i_b}$ is the elasticity of intertemporal substitution, so all results extend straightforwardly (see Appendix C.2). In richer models $\tilde{\mathcal{C}}_{i_b}$ may be more complicated; however, *whatever* its shape, invertibility of $\mathcal{C}_\tau$ suffices to ensure that transfers can function as a perfect substitute.

for all of these, households are non-Ricardian in the strong sense that, through manipulation of the time paths of transfers, it is possible to generate *any* desired time path of net excess demand. In this section I first establish this result and furthermore argue that the potentially complicated inverse $\mathcal{C}_\tau^{-1}$ actually takes a simple form. The latter result will pave the way for my general sufficient statistics formula presented in Section 4.3.

Proposition 2. *Consider the analytical models of the household consumption-savings problem discussed in Section 2.2. In all models of except for the permanent-income consumer, $\mathcal{C}_\tau$ is invertible. In particular:*[11]

1. *In the spender-saver model, $\mathcal{C}_\tau^H$ is invertible if there is a non-zero margin of spenders, i.e., $\mu > 0$.*
2. *In the overlapping generations model, $\mathcal{C}_\tau^{OLG}$ is invertible if households die with positive probability, i.e., $\theta < 1$.*
3. *In the bond-in-utility model, $\mathcal{C}_\tau^{BiU}$ is invertible if households value wealth and the marginal utility of wealth is diminishing, i.e., $\alpha, \eta > 0$.*

As already discussed, in the standard permanent-income model, only the present value of transfers matters, so $\mathcal{C}_\tau^R$ has rank-1 and thus is not invertible. For the spender-saver model, invertibility is straightforward to establish—in fact it can be seen quite easily from the model-implied aggregate Euler equation (e.g., as in Bilbiie et al., 2013, 2021): through spenders, transfers appear as a wedge in the aggregated date-t Euler equation, thus suggesting that taxes and transfers can manipulate spending over time just like conventional interest rate policy. The explicit expression for $(\mathcal{C}_\tau^H)^{-1}$, here for simplicity displayed for the special case $\bar{r} = 0$, translates this classical Euler equation perspective to my matrix set-up:

$$(\mathcal{C}_\tau^H)^{-1} \;=\; \frac{1}{\omega} \times I \tag{23}$$

By (23), to engineer a dollar of extra spending at some date t, the policymaker simply needs to increase transfers at date t by $\frac{1}{\omega}$ dollars.[12] Intuitively, the policymaker here leverages the

[11] The proof of Proposition 2 also establishes invertibility for linear combinations of these models—like the hybrid OLG and spender-saver model that underlies my sufficient statistics formula in Section 4.3.

[12] If $\bar{r} > 0$, then savers also increase their spending by an amount equal to the annuity value of the transfer, and in fact they do so forever. To offset this effect, the off-diagonal elements of $(\mathcal{C}_\tau^H)^{-1}$ in that case are small and negative. I present the full expression in Appendix C.1.

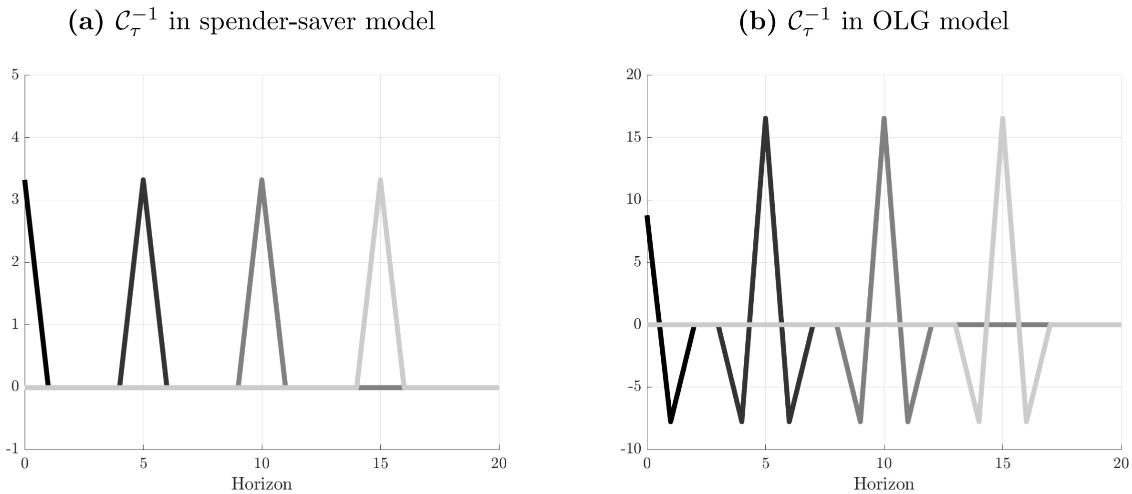

Figure 1: $\mathcal{C}_\tau^{-1}$ in analytical models of the consumption-savings problem: spender-saver (left panel) and overlapping generations (right panel), with $\bar{r} = 0$ and $\omega = 0.3$ for both. The lines correspond to columns $\{1, 6, 11, 16\}$ of $\mathcal{C}_\tau^{-1}$, with lighter shades of grey indicating farther-out columns.

presence of mechanical spenders to adjust net excess demand over time in whatever way she desires. A visual illustration is provided in the left panel of Figure 1.

The overlapping-generations and bond-in-utility models then offer an important refinement of this exclusively static spender-saver intuition. For the OLG model, and assuming again for simplicity that $\bar{r} = 0$, the inverse can be shown to satisfy[13]

$$(\mathcal{C}_\tau^{OLG})^{-1} = \frac{1}{\omega} \times \begin{pmatrix} \frac{1-\theta(1-\theta)}{1-\theta} & -\frac{\theta}{1-\theta} & 0 & 0 & \cdots \\ -\frac{\theta}{1-\theta} & \frac{1+\theta^2}{1-\theta} & -\frac{\theta}{1-\theta} & 0 & \cdots \\ 0 & -\frac{\theta}{1-\theta} & \frac{1+\theta^2}{1-\theta} & -\frac{\theta}{1-\theta} & \cdots \\ 0 & 0 & -\frac{\theta}{1-\theta} & \frac{1+\theta^2}{1-\theta} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{24}$$

Recall from Section 2.2 that the main difference between the spender-saver and OLG models was the time pattern of household spending: in OLG, households *gradually* spend before and after any income receipt, with both gradual spending as well as anticipation effects governed by the single parameter θ. (24) and (23) differ precisely because of these dynamic effects: to engineer a dollar of demand at some date t, the policymaker needs to give money at date t (still scaling in ω^{-1}) but now also take away money at $t-1$ and $t+1$, to prevent leakage across

[13] The general expression for arbitrary $\bar{r}$ is presented in Appendix C.1.

periods. Importantly, this tridiagonal shape, also illustrated in the right panel of Figure 1, follows only because spending decays and builds up over time at a constant rate θ. A second important feature of (24) is that the expression for $(\mathcal{C}_\tau^{OLG})^{-1}$ is still low-dimensional: the inverse is governed by the average MPC ω and the slope parameter θ.[14]

Finally—and unsurprisingly in light of the discussion in Section 2.2—the intuition from the OLG model extends with very little change to the bond-in-utility model. As I show in Appendix C.1, $(\mathcal{C}_\tau^{BiU})^{-1}$ has the exact same tridiagonal shape as that displayed in the right panel of Figure 1 for the OLG model, with both gradual spending as well as anticipation effects again governed by a common coefficient θ.

Taking stock. The analysis in this section constitutes the first step to ascertaining the practical relevance of my policy equivalence result. I have shown that, in popular analytical models of non-Ricardian consumption behavior, my sufficient condition for policy equivalence not only holds, but in fact the all-important inverse $\mathcal{C}_\tau^{-1}$ is also low-dimensional and has a simple, intuitive shape. This result is very promising because we already know from prior work (e.g., Auclert et al., 2018) that consumption behavior even in rich heterogeneous-agent environments looks similar to that in the simpler analytical models studied here. Sections 4.2 and 4.3 will leverage this observation and turn it into a simple, empirically measurable sufficient statistics formula that provides a robust characterization of all of $\mathcal{C}_\tau^{-1}$.

4.2 A quantitative HANK model

The quantitative analysis throughout the remainder of this section will rely on the most popular and empirically relevant model of non-Ricardian consumption behavior: a heterogeneous-household incomplete-markets model, as popularized in the recent HANK literature (Kaplan et al., 2018; Kaplan & Violante, 2018). This section presents the parameterization and discusses the model-implied consumption behavior, while Section 4.3 shows my main result: $\mathcal{C}_\tau^{-1}$ exists, is well-behaved, and is in fact (approximately) low-dimensional.

Parameterization. I consider a calibrated version of the model of Section 2.1, with the aggregate consumption function coming from the heterogeneous-household consumption-savings problem described in Section 2.2. I only present a very brief overview of the (standard) parameterization here, and relegate further details to Appendix B.6.

[14] The two coefficients are in fact related as $\omega = 1 - \theta$, so $(\mathcal{C}_\tau^{OLG})^{-1}$ in this case turns out to be one-dimensional. With general $\bar{r}$ the expression becomes two-dimensional, as shown in Appendix C.1.

Households face the same income process as in Kaplan et al. (2018), and can self-insure by saving, but not borrowing. I calibrate total liquid bond holdings to the amount of liquid wealth in the U.S. economy; corporate wealth, instead, is perfectly illiquid, with households receiving dividend payments as a function of their labor productivity. The economy is closed with a simple constant-returns-to-scale production function as well as conventional degrees of nominal wage and price stickiness. This general equilibrium closure is of course irrelevant for the main results of this section (which only concern partial equilibrium consumer behavior, i.e. $\mathcal{C}_\tau$); rather, the model closure will only start to matter once I report general equilibrium experiments (as I do at the end of Section 4.3 as well as in Section 5).

Properties of $\mathcal{C}_\tau$. In an important contribution, Auclert et al. (2018) establish the following two observations concerning the properties of $\mathcal{C}_\tau$ in quantitative heterogeneous-agent models. First, they argue that the model-implied first column of $\mathcal{C}_\tau$—that is, the response of consumption demand over time to a lump-sum income gain today—is consistent with empirical evidence on consumer spending behavior (e.g., Parker et al., 2013; Fagereng et al., 2018). This finding reaffirms my claim that, for the purposes of my analysis, this heterogeneous-agent model is a quantitatively and empirically relevant one. Second, they show that a mixture of spender-saver and bond-in-utility models is similarly rich enough to match the available evidence on the first column of $\mathcal{C}_\tau$; furthermore, when done so, this mixture model tends to approximately agree with HANK on the *entirety* of the matrix $\mathcal{C}_\tau$.

I combine these two observations with my results in Sections 2.2 and 4.1. The analysis there had revealed that, in analytically tractable models, a small number of measurable statistics—the objects $\{\omega, \theta, \bar{r}\}$—suffice to characterize $\mathcal{C}_\tau^{-1}$. But since these models are known to look similar to HANK (at least as far $\mathcal{C}_\tau$ is concerned), it follows that the very same statistics may even characterize the potentially complicated, infinite-dimensional object $\mathcal{C}_\tau^{-1}$ in HANK. The analysis in the next section confirms these conjectures.

4.3 A simple sufficient statistics approximation

This section presents my three-coefficient "sufficient statistics" approximation to $\mathcal{C}_\tau^{-1}$ and documents its high accuracy in quantitative heterogeneous-agent models. The key takeaway will be that, for empirically supported values of my sufficient statistics, the stimulus check policies required to deal with ordinary cyclical fluctuations are robustly moderate in size and smooth over time, suggesting that stimulus checks are indeed a practically relevant tool for aggregate stabilization.

Approximating $\mathcal{C}_\tau^{-1}$. Guided by the insights from the analysis in Sections 4.1 and 4.2, I propose to approximate $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ through a small number of measurable objects: the average MPC ω, the spending slope θ, and the rate of interest $\bar{r}$. I do so in two steps.

1. Using only the slope θ and and the interest rate $\bar{r}$, I construct an approximation of $\mathcal{C}_\tau$ in the shape implied by overlapping-generations or bond-in-utility models (i.e., (13)).[15]

2. I combine the matrix from the first step with a spender-saver spending matrix (see (12)), with the weights chosen to hit my desired third statistic—the target impact MPC ω. This second step is necessary to disentangle ω and θ (which in OLG or spender-saver models are necessarily tied together as $\omega = 1 - \frac{\theta}{1+\bar{r}}$). The end result is my approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})$, and from there I simply compute the inverse $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$.

Further details are provided in Appendix C.3, including in particular explicit formulas for $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$. Overall, this two-step approximation yields a matrix $\mathcal{C}_\tau(\omega, \theta, \bar{r})$ and inverse $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ that is nothing but a simple combination of the intuitions reviewed in Sections 2.2 and 4.1: consumer spending decays and anticipation effects build up at constant rates over time (the slope coefficient θ), with a margin of mechanical spenders added to disentangle this dynamic rate θ from the level of the average MPC ω.

Accuracy. As it turns out, this three-dimensional approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ provides a highly accurate description of consumption behavior in HANK models. While the analysis in this section will only look at the preferred calibration of my heterogeneous-agent model, I in Appendix C.4 document the robustness of my conclusions by also looking at several alternative, materially different HANK model parameterizations.

Figure 2 begins by displaying several individual columns of $\mathcal{C}_\tau^{-1}$ taken from (i) the full heterogeneous-agent model (shades of grey) and (ii) the sufficient statistics approximation (orange dashed), with the three sufficient statistics $\{\omega, \theta, \bar{r}\}$ set to values that agree with the heterogeneous-agent model: $\omega = 0.30$, $\theta = 0.82$, and $\bar{r} = 0.01$—all values that are consistent with empirical evidence (e.g., Fagereng et al., 2018).[16] My first observation is that the orange

[15]Note that I can do so using θ and $\bar{r}$ alone precisely because the impact MPC is then—by the household budget constraint—necessarily given as $1 - \frac{\theta}{1+\bar{r}}$. I also note here that my sufficient statistics approximation actually does not *exactly* impose the shape displayed in (13), but rather I consider a slight modification that ensures that aggregate budget constraints are always satisfied. I thank an anonymous referee of alerting me of this important point. See Appendix C.3 for details.

[16]To be precise, I set $\bar{r}$ to its value in the heterogeneous-agent model, $\omega = \mathcal{C}_\tau(1,1)$, and finally θ is set to ensure that $\mathcal{C}_\tau(\omega, \theta, \bar{r})(2,1) = \mathcal{C}_\tau(2,1)$.

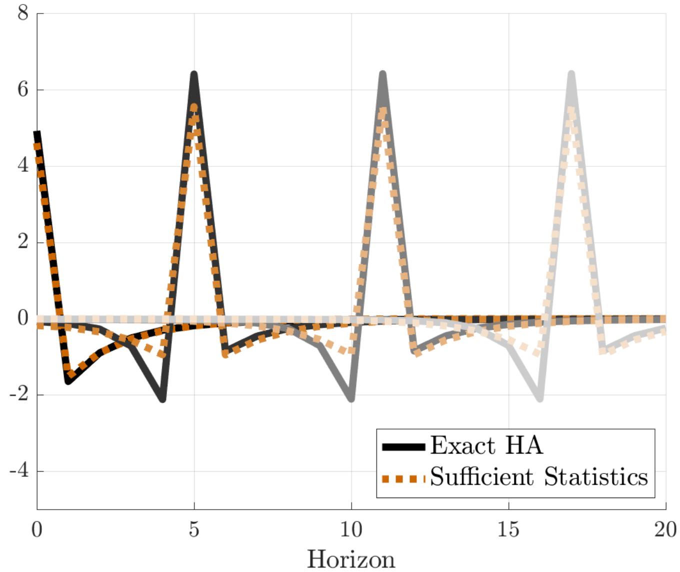

Figure 2: Entries of $\mathcal{C}_\tau^{-1}$ in the quantitative heterogeneous-agent model (shades of grey) and in the sufficient statistics approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ (shades of orange, dashed). Here $\bar{r}$ is set as in the heterogeneous-agent model (to 1 per cent), and $\{\omega, \theta\}$ are set to match $\mathcal{C}_\tau(1,1)$ and $\mathcal{C}_\tau(2,1)$. The lines correspond to columns $\{1, 6, 11, 16\}$, with lighter shades indicating farther-out columns.

dashed lines indeed simply combine the intuitions from the spender-saver and overlapping-generations (or bond-in-utility) models presented in the two panels of Figure 1: to engineer a dollar of excess demand at date t, the policymaker would need to pay out transfers somewhat in excess of ω^{-1} at date t (at around \$6 here), and then take money away in adjacent periods to prevent the cross-period leakage in spending. The second observation is that this very simple logic also describes the full heterogeneous-agent model, with the orange lines qualitatively and quantitatively close to the grey lines throughout.

Why does the three-coefficient formula so accurately capture consumption behavior even in the rich heterogeneous-household model? Intuitively, in that model, some households are up against the borrowing constraint (so they act like spenders), while others will be up against it in the future with some probability, so they engage in precautionary savings (which in reduced-form looks like a direct preference over wealth holdings, see Kaplan & Violante, 2018). By my results in Section 4.1, three simple (and measurable!) parameters are already enough to capture these spending dynamics with a high degree of accuracy.

Next, Figure 3 shows several different "typical" target paths of net excess demand that a policymaker may wish to implement (grey) together with the sequences of transfers and

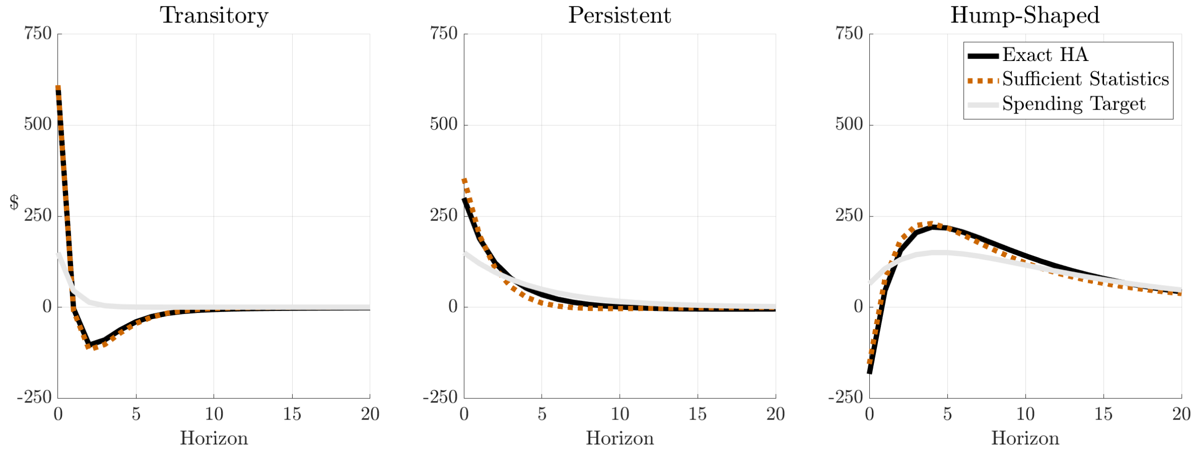

Figure 3: Three desired net excess demand paths $\widehat{\boldsymbol{c}}^{PE}$ (grey) and the required sequences of uniform lump-sum taxes and transfers, $\mathcal{C}_\tau^{-1} \times \widehat{\boldsymbol{c}}^{PE}$, as implied by the full heterogeneous-agent model (black) and my sufficient statistics formula (orange dashed).

taxes that do so (black and orange dashed). More precisely, given the three distinct paths of desired net excess demand $\widehat{\boldsymbol{c}}_\tau^{PE}$ (grey), the figure plots

$$\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}_\tau^{PE}) \;\equiv\; \mathcal{C}_\tau^{-1} \;\times\; \widehat{\boldsymbol{c}}_\tau^{PE}$$

where $\mathcal{C}_\tau^{-1}$ is either taken from the actual heterogeneous-agent model (black) or from my simple sufficient statistics approximation (orange dashed). The three time paths of desired spending that I consider all have a peak of one per cent of steady-state consumption, but they are quite distinct in shape: short-lived in the left panel, persistent in the middle panel, and hump-shaped in the right panel, thus capturing a range of policy-relevant scenarios. As expected in light of Figure 2, I find that, for all three time paths, the actual required transfer sequence and the sufficient statistics prediction are extremely close. I will return to the *magnitudes* of the required stimulus check policies in the next paragraph.

Appendix C.4 repeats the exercises of Figures 2 and 3 for two alternative calibrations of my heterogeneous-agent model: one with much lower household wealth and thus higher MPCs, and the other one with high wealth and so low MPCs. The matrix $\mathcal{C}_\tau^{-1}$ and so the time paths $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}_\tau^{PE})$ materially differ across all of these models, but crucially my three-parameter sufficient statistics approximation remains extremely accurate for all of them.[17]

[17] In Appendix C.7 I furthermore consider model extensions with behavioral households (e.g., cognitive discounting), consistent with recent evidence on the strength of anticipation effects (Ganong & Noel, 2019). I show that in this case a simple *four*-parameter extension of my formula provides an accurate description

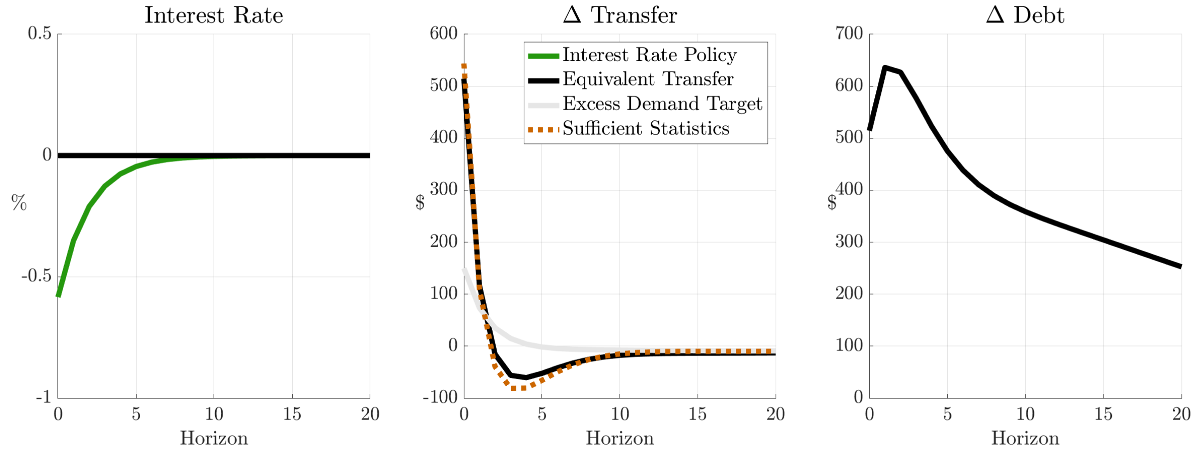

Figure 4: Impulse responses of nominal interest rates, transfers, and debt to interest rate (green) and transfer (black) policies in the calibrated HANK model, chosen to generate the net excess demand path displayed in the middle panel (grey). For transfers and debt I show the *differences* between their paths under the two policies as the black line.

Practical policy implications. So far I have emphasized that my sufficient statistics formula yields an approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ that accurately reflects consumer behavior in rich HANK models, no matter the parameterization. A second important point is that, for *empirically relevant* values of $\{\omega, \theta, \bar{r}\}$, the entries of this inverse are moderate in size. We can see this in Figure 2, with the diagonal entries of $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ somewhat larger than ω^{-1}, and the off-diagonal entries relatively small and quickly converging to zero away from the main diagonal. Given that empirical evidence suggests elevated average MPCs ω, it follows that even moderately sized transfer stimulus suffices to close meaningful aggregate spending shortfalls. We also see this in Figure 3, which reveals that stimulus checks of the magnitudes observed in practice (e.g., around \$600, as in 2008), are predicted to close aggregate demand shortfalls of around one per cent.[18]

Figure 4 provides a final illustration by studying stabilization policy in response to a contractionary demand shock $\boldsymbol{\varepsilon}^c$—that is, a shock that temporarily depresses partial equilibrium consumer spending, here with a peak effect of -1 per cent. By the classical divine coincidence logic, it is possible to perfectly stabilize inflation and output in the face of such a contractionary shock. To do so, policy needs to increase consumer spending to offset the 1 per cent contraction in demand (= \$150 per household), shown in the middle (grey). The

of consumer behavior, with the fourth parameter governing the strength of behavioral frictions.

[18] Appendix C.3 elaborates further: I provide explicit expressions for the entries of $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ and furthermore repeat the exercises in Figure 3 for a *range* of credible values of my sufficient statistics.

usual policy prescription would be to do so through a cut in interest rates (left panel, green); in my heterogeneous-agent model, this requires a relatively short-lived nominal interest rate cut with a cumulative total of around 150 basis points.[19] The black lines indicate how to achieve the exact same perfect stabilization instead using lump-sum tax-and-transfer policy alone: transfers initially go up (here by around \$600), before then being financed through higher taxes down the line. Unsurprisingly in light of Figures 2 and 3, this time path is again predicted almost perfectly by my sufficient statistics formula (orange, dashed). Finally, the right panel shows that the moderate stimulus check policy brings with it a moderate and transitory increase in government debt. Cyclical stabilization through stimulus check policy thus requires no implausibly large or erratic fluctuations in taxes, deficits, or aggregate government debt. In particular, stimulus check policies of the magnitude seen in the 2008 Economic Stimulus Act already suffice to deliver meaningful stimulus and replicate sizable interest rate cuts, thus allowing classical aggregate demand management even when monetary policy is constrained by a lower bound.

5 The role of microeconomic heterogeneity

My analysis so far has been concerned exclusively with policy instrument equivalence at the aggregate level. Microeconomic heterogeneity played a role only to the extent that popular heterogeneous-agent consumption models are a natural (and empirically relevant) candidate to satisfy my sufficient condition of strong Ricardian non-equivalence.

This section sheds further light on the scope and the limitations of the policy equivalence result in the face of microeconomic heterogeneity. First, in Section 5.1, I gauge the extent to which realistic microeconomic heterogeneity in labor supply decisions can break the aggregate equivalence result. I argue that this is possible in theory, but unlikely to matter much in practice. Second, in Section 5.2, I emphasize that equivalence at the aggregate level does not necessarily imply equivalence household-by-household.

5.1 Wealth effects and household labor supply

To understand how household heterogeneity and wealth effects in labor supply can in principle challenge my equivalence result, it will be useful to recall the proof sketch of Proposition 1.

[19] This implied direct mapping from interest rates to consumer net excess demand is broadly consistent with recent empirical evidence (e.g., see Table 4 in Crawley & Kuchler, 2021).

As the first step of the argument, I consider nominal interest rate and transfer policies that induce identical paths of spending. This however is in general not enough to ensure equivalence in general equilibrium—the two policies also need to induce identical responses of household labor supply. In my environment, the specific wage-NKPC (6)—which importantly depends only on aggregate consumption c_t—ensures that interest rate and stimulus check policies with identical direct effects on consumer spending indeed also induce identical labor supply responses, as required.

I conclude from this discussion that it is not wealth effects in labor supply *per se* that threaten the equivalence result; rather, a potential challenge are *heterogeneous* wealth effects, which would allow two policies with identical effects on total spending to potentially lead to different responses of labor supply. I here use two experiments to argue that the scope for such heterogeneity to materially threaten the equivalence result is likely to be limited.

An alternative union bargaining protocol. Some form of nominal wage rigidity is widely argued to be necessary to match business-cycle dynamics in general (Christiano et al., 2005; Smets & Wouters, 2007) and consumption responses to macro shocks in particular (Auclert et al., 2020; Broer et al., 2020); importantly, it is also consistent with microeconomic evidence (Grigsby et al., 2019). The derivation of my wage-NKPC (6) relies on one particular union bargaining protocol that only responds to changes in aggregate consumption, thus ensuring that my two candidate interest rate and stimulus check policies also lead to identical responses of labor supply. The exact same bargaining protocol has also been used in other recent contributions to the HANK literature (Auclert et al., 2021; Aggarwal et al., 2022; McKay & Wolf, 2022a). An alternative union bargaining protocol (used e.g. in Auclert et al., 2018) instead has the union respond to a weighted average of individual household marginal utilities (rather than marginal utility at the average, as in my protocol). Appendix C.5 extends my HANK model environment to such an alternative protocol.

The main takeaway from my analysis in Appendix C.5 is that the equivalence result continues to hold *almost exactly* even under the alternative protocol. Intuitively, equivalence is now not exact because the interest rate and transfer policies that induce identical responses of consumer spending will generically *not* induce identical changes in the weighted average of consumer marginal utilities that enters the union problem. As a result, the adjustments in labor supply—$\widehat{\boldsymbol{\ell}}_{i_b}^{PE}$ and $\widehat{\boldsymbol{\ell}}_{\tau}^{PE}$—are not the same, and equivalence fails. However, marginal utility at the average and average marginal utility still co-move closely, so $\widehat{\boldsymbol{\ell}}_{i_b}^{PE}$ and $\widehat{\boldsymbol{\ell}}_{\tau}^{PE}$ remain quite similar, and so aggregate outcomes are still nearly identical under the two policies.

Matching empirical evidence on labor supply responses. Empirical evidence on household labor supply suggests that *marginal propensities to earn* (MPE)—that is, the response of earned income to a one-time, unexpected lump-sum transfer—are moderate in size, ranging from around 1% - 3% (Cesarini et al., 2017; Golosov et al., 2021), and somewhat increasing in household income, roughly doubling from the lowest to the highest income quartile (Golosov et al., 2021). Standard heterogeneous-household models with flexible labor supply struggle with both observations (Auclert et al., 2020): MPEs are predicted to be of the same order of magnitude as MPCs, and MPEs tend to be highest for high-MPC households. My solution is to adjust household preferences: I consider a hybrid of standard separable and Greenwood et al. (1988) GHH preferences (as originally proposed by Auclert et al., 2020) to match average MPEs, and then allow for preference heterogeneity across households to also match the cross-sectional MPE gradient. To make the heterogeneity particularly stark I consider a two-type model of savers (with bonds in their utility function) and spenders, with their MPEs matched to the first and fourth quartiles of income reported in Golosov et al.. Full results are again reported in Appendix C.5.

In the model with empirically relevant and heterogeneous MPEs, I find that the policy equivalence result again holds almost exactly. The intuition is simply that heterogeneity in MPEs is small relative to the *level* of the average MPC. For example, with an MPC of 30% (an empirically relevant number for stimulus checks of the size studied in Section 4) and MPEs of 2% for spenders and 4% for savers (in line with Golosov et al.), the direct demand stimulus associated with either transfers or the equivalent rate cut is an order of magnitude larger than the difference in labor supply response across households. Aggregate equilibrium dynamics are thus still dominated by the demand effects at the heart of Proposition 1.

5.2 Non-equivalence at the household level

My equivalence result applies to macroeconomic aggregates, but it does not necessarily hold household-by-household. I in this section first provide a brief general discussion of this point and then present a quantitative illustration taken from the HANK model of Section 4.

Household-level consumption functions. Consider the heterogeneous-household environment described in Section 2.2. Along a perfect foresight transition path, consumption of an individual household i is given as

$$\boldsymbol{c}_i \quad = \quad \mathcal{C}_i(\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}; \boldsymbol{\tau}, \boldsymbol{i}_b)$$

where the individual consumption function $\mathcal{C}_i(\bullet)$ is the solution to individual i's consumption-savings problem, indexed by that individual's initial asset holdings and productivity. Now consider two macro-equivalent interest rate and stimulus check policies $\widehat{\boldsymbol{i}}_b$ and $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$, constructed as in the proof of Proposition 1, and let $\Delta_i(\widehat{\boldsymbol{i}}_b)$ denote the difference in household i's consumption under the two policies; that is, let

$$\Delta_i(\widehat{\boldsymbol{i}}_b) \quad \equiv \quad \widehat{\boldsymbol{c}}_{i,i_b} - \widehat{\boldsymbol{c}}_{i,\tau(i_b)}$$

where i_b and τ subscripts indicate transition paths corresponding to interest rate and transfer policy, respectively. Since by construction both policies induce the same general equilibrium price and quantity responses (notably $\{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}\}$), we find that this difference satisfies

$$\Delta_i(\widehat{\boldsymbol{i}}_b) \quad = \quad \tilde{\mathcal{C}}_{i,i_b}\widehat{\boldsymbol{i}}_b \;-\; \mathcal{C}_{i,\tau}\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$$

where $\tilde{\mathcal{C}}_{i,i_b}$ and $\mathcal{C}_{i,\tau}$ are defined like $\tilde{\mathcal{C}}_{i_b}$ and $\mathcal{C}_\tau$, just now for each i. In words, the two policies may result in differences in consumption household-by-household because of potentially differential *direct* effects on consumer spending. Intuitively, while the two policies by design induce the same total direct stimulus (i.e., $\tilde{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b = \mathcal{C}_\tau\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ and so $\int_0^1 \Delta_i(\widehat{\boldsymbol{i}}_b)di = \mathbf{0}$), they may do so by affecting consumption at different points in the cross-section of households (i.e., we can have $\tilde{\mathcal{C}}_{i,i_b}\widehat{\boldsymbol{i}}_b \neq \mathcal{C}_{i,\tau}\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ and so $\Delta_i(\widehat{\boldsymbol{i}}_b) \neq \mathbf{0}$ for individual i).

Distributional outcomes in a HANK model. I provide a numerical illustration of this non-equivalence at the household level by returning to the quantitative HANK model of Section 4. In this environment I compute the evolution of consumption along the household wealth distribution in response to the macro-equivalent nominal interest rate and stimulus check policies displayed in Figure 4.

I find that, while macro-equivalent, interest rate and transfer stimulus policies can have materially different effects in the cross-section of households. Figure 5 presents the results, splitting the impact consumption response by household liquid wealth percentile (x-axis) into (a) the *direct* effects of the policy instrument (green and blue)—defined as the response of consumption demand to the policy instruments $\{\boldsymbol{i}_b, \boldsymbol{\tau}\}$ alone, fixing all non-policy variables at their steady state values forever—and (b) the residual *indirect* effects (shaded purple) coming from general equilibrium feedback. As discussed above, the two policies by construction induce the exact same indirect effects household-by-household. The direct effects, on the other hand, are only guaranteed to integrate to the same number across the entire population,

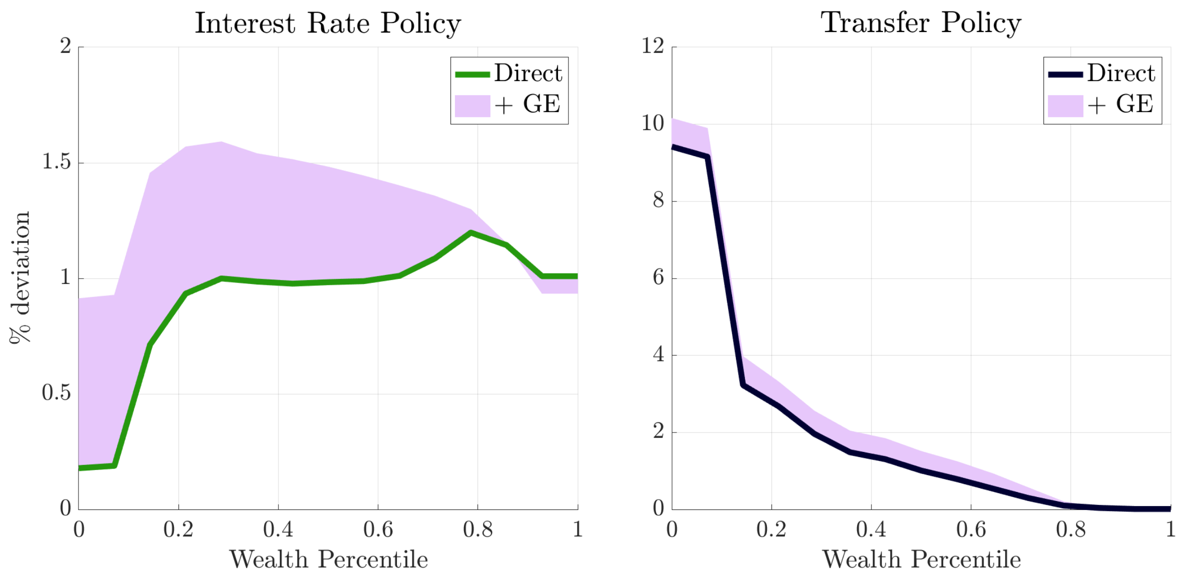

Figure 5: Left panel: The green line shows the time-0 *direct* household spending response by liquid wealth percentile to the nominal interest rate policy of Figure 4. The shaded purple area gives the corresponding *indirect* general equilibrium effects. Right panel: Analogous figure for the equivalent uniform transfer stimulus (dark blue and shaded purple).

but need not agree household-by-household. Figure 5 reveals that the direct effects are indeed quite heterogeneous. On the one hand, nominal interest rate cuts work mostly by directly stimulating the consumption of the rich (green). This is not surprising: wealthy households substitute intertemporally, while poor households are close to their borrowing constraint and so do not. Thus, at the low end of the liquid wealth distribution, monetary policy operates largely through the shaded purple indirect effects (Kaplan et al., 2018).[20] On the other hand, the equivalent lump-sum transfer policy (blue) mostly acts at the bottom: it relaxes borrowing constraints and so stimulates consumption of the poor, while the Ricardian rich households barely respond (since the policy has zero net present value).

Putting the pieces together we see that, relative to a given nominal interest rate cut, stimulus checks that deliver the exact same aggregate stabilization do so at strictly smaller cross-sectional consumption dispersion. In Figure 5, monetary stimulus—once all general equilibrium effects are taken into account—is roughly distributionally neutral (green line

[20]There is, of course, a second effect: lower interest rates redistribute from bondholders (rich households) to the government. Since taxes and transfers are uniform, the lower interest rate expenses are then passed on to all households. These redistributive effects, however, only matter at longer horizons; in the short term, consumption of rich households increases because of intertemporal substitution, and lower government interest rate expenses are largely absorbed by a reduction in outstanding debt (see (B.17)).

plus purple area), while transfer stimulus leads to a substantial compression of cross-sectional consumption inequality (blue line plus purple area).[21] The normative implications of this positive observation are explored in detail in McKay & Wolf (2022a).

6 Extension to investment

As the final step in my argument, I argue that the policy equivalence result extends straightforwardly to a richer environment with investment if stimulus checks are complemented by a second, similarly standard fiscal tool: bonus depreciation stimulus.

6.1 A brief sketch of the environment

I augment the model of Section 2.1 to allow for productive capital. The firm block in this extended environment closely follows the tradition of standard business-cycle modeling (e.g. Smets & Wouters, 2007; Justiniano et al., 2010). I provide a brief sketch of this familiar model here, and relegate further details to Appendix B.7.

Production. A unit continuum of identical, perfectly competitive firms $j \in [0,1]$ produces a homogeneous intermediate good, sold at real relative price p_t^I. The problem of firm j along the perfect foresight transition path is to

$$\max_{\{d_{jt},\ell_{jt},k_{jt},b^f_{jt}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \left(\prod_{q=0}^{t-1} \frac{1+\pi_{q-1}}{1+i_{b,q}} \right) d_{jt}$$

subject to the flow budget constraint

$$d_{jt} = p_t^I y(\ell_{jt}, k_{jt-1}) - w_t \ell_{jt} - \underbrace{[k_{jt} - (1-\delta)k_{jt-1}]}_{\equiv i_{jt}} + \tau_{f,t}(\{i_{jt-q}\}_{q=0}^{t}) - b^f_{jt} + \frac{1+i_{b,t-1}}{1+\pi_t} b^f_{jt-1} \quad (25)$$

as well as constraints on equity and debt issuance

$$d_{jt} \geq \underline{d}, \quad b^f_{jt} \geq \underline{b}^f$$

[21]The relatively flat cross-sectional incidence profile seen in the left panel of Figure 5 is consistent with empirical evidence (see McKay & Wolf, 2022a, and the references therein). My specification of the incidence of dividends (see Appendix B.6) is one of the model features required to deliver this result.

To summarize, intermediate goods producers hire labor on spot markets, invest, pay out dividends, and save in liquid bonds, perhaps subject to financing constraints.[22] My only twist to this entirely familiar model block is that I allow for a general fiscal investment stimulus policy $\boldsymbol{\tau}_f(\bullet)$, mapping investment today into future payments to the firm. Importantly, this general set-up nests the popular bonus depreciation stimulus policy, in which investment today reduces tax liabilities in the future (Zwick & Mahon, 2017; Koby & Wolf, 2020). I will index time-t investment stimulus policies by a single parameter $\tau_{f,t}$.

Proceeding exactly as in Section 2.1, we can define an aggregate investment function $\mathcal{I}(\bullet)$; I relegate a discussion of the arguments of this function to Appendix B.7, as it is not essential here. For my purposes, the only important consideration is that fiscal stimulus is one of those arguments, with the direct effects of such stimulus summarized by the following derivative matrix:

$$\mathcal{I}_{\tau_f} \quad \equiv \quad \frac{\partial \mathcal{I}(\bullet)}{\partial \boldsymbol{\tau}_f}$$

Statements about the degree to which conventional fiscal instruments can be used to replicate monetary stimulus will be statements about the properties of $\mathcal{I}_{\tau_f}$ (and of course $\mathcal{C}_\tau$, as before).

Rest of the economy. The intermediate good is sold to monopolistically competitive retailers subject to nominal rigidities, summarized again with a general price-NKPC:

$$\pi_t \quad = \quad \kappa_p \; \times \; \left(\chi_p(p_t^I) - 1\right) + \beta \pi_{t+1} \tag{26}$$

The remainder of the model is unchanged. The extension of the equilibrium definition in Definition 3 is then straightforward, and provided in Appendix B.7.

6.2 Policy equivalence with conventional fiscal instruments

In this extended model, monetary policy operates through two channels: first, as before, it affects household consumption demand, and second, it changes firm investment and so labor hiring as well as intermediate goods production. Thus, transfer stimulus policy alone is now insufficient to replicate the effects of (infeasible) conventional monetary policy—exactly as in Correia et al. (2013), an additional instrument is needed. In a straightforward generalization of Proposition 1, Proposition 3 shows that invertibility of $\mathcal{C}_\tau$ and $\mathcal{I}_{\tau_f}$ is sufficient to leave the space of implementable output-inflation allocations unchanged.

[22]Capital or investment adjustment costs could be added without affecting any of the subsequent results.

Proposition 3. *Consider the extended model of Section 6.1. Suppose that $\mathcal{C}_\tau$ and $\mathcal{I}_{\tau_f}$ are both invertible, and consider an allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$ that is implementable using an interest rate-only policy. Then it is similarly implementable through time-varying uniform transfer and bonus depreciation policies alone.*

Transfer stimulus now only replicates the consumption channel of monetary policy transmission. If additionally $\mathcal{I}_{\tau_f}$ is invertible, then the general form of investment stimulus considered in (25) suffices to replicate the investment channel, and so leave the set of implementable aggregate allocations unchanged. The final step of my argument is to ascertain that bonus depreciation stimulus can indeed perturb firm investment demand over time as required in Proposition 3. Unlike the stimulus check case, however, this logic is now entirely straightforward and in particular can closely follow the work of Correia et al. (2013). For a particularly transparent example, suppose that intermediate goods firms are not subject to any financial frictions, as in most conventional business-cycle models. Then bonus depreciation is equivalent to a standard investment subsidy (e.g., see the discussion in Winberry, 2021; Koby & Wolf, 2020), and so the firm budget constraint becomes

$$d_{jt} = p_t^I y(\ell_{jt}, k_{jt}) - w_t \ell_{jt} - \underbrace{[k_{jt} - (1-\delta)k_{jt-1}]}_{\equiv i_{jt}} + \tau_{f,t} i_t$$

Bonus depreciation stimulus is thus equivalent to an investment subsidy, and so the discussion of Correia et al. (2013) applies completely unchanged: interest rates $i_{b,t}$ and the subsidy $\tau_{f,t}$ both enter investment optimality conditions as wedges, and so the investment channel of monetary policy can be replicated simply by matching those wedges.[23]

Taking stock. Taken together, the results in this section as well as in Section 4 give the headline practical takeaway of my paper: even in quite large-scale quantitative structural models of the business cycle, an entirely conventional mix of fiscal instruments—transfer stimulus payments together with bonus depreciation tax stimulus—suffice to replicate any desired monetary allocation. The investment side of the argument is straightforward: bonus depreciation as the standard fiscal tool is sufficiently close to a simple investment subsidy that I was able to adapt the original results of Correia et al. with very little change. On the

[23]Binding financial frictions are unlikely to change this overall conclusion: constrained firms would now both respond to intertemporal investment incentives as well as to the cash flow shock itself, thus presumably even increasing the potency of bonus depreciation stimulus (Koby & Wolf, 2020). I leave a more detailed discussion of the shape of $\mathcal{I}_{\tau_f}$ in this case to future work.

consumption side, on the other hand, an entirely different argument was needed—one that I provided in Sections 3 and 4, constituting the main contribution of the paper.

7 Conclusion

Over the past decade, much academic and applied policy interest has centered on the question of how to replicate monetary stimulus when nominal interest rates are constrained.

The central contribution of this paper is to show that, in business-cycle models that are entirely standard except for the presence of non-Ricardian consumers, a *conventional* mix of fiscal instruments—in particular including uniform, deficit-financed stimulus checks—suffices to replicate the aggregate effects of an arbitrary monetary policy. The core insight is a formalization of the notion that interest rate and stimulus check policies can manipulate consumer demand "equally flexibly." This model property—together with a net excess demand approach to equilibrium characterization à la Arrow & Debreu—establishes policy equivalence. My second main result is an explicit characterization of the transfer policy needed to close any given shortfall in aggregate demand; in particular I show that, even in state-of-the-art quantitative heterogeneous-household models, this transfer policy is well-characterized by a very small number of already measured sufficient statistics.

I leave several important extensions for future work. First, it would be interesting to compare interest rate and stimulus check policies in a full Ramsey problem. Steps in this direction are taken in McKay & Wolf (2022a). Second, to the extent that the linear map $\mathcal{C}_\tau$ changes over the business cycle, the required transfer stimulus will also depend on the aggregate state of the economy. Future empirical work should try to better measure that state dependence. And third, the policy equivalence logic presented here applies not only to uniform transfer stimulus; for example, analogous equivalence results could also be established for policies that redistribute across households or through transfers targeted at sub-populations. If (large) swings in the amount of outstanding government debt are intrinsically undesirable, then such alternative equivalent, budget-neutral policies become attractive.

A Appendix

A.1 Proof of Proposition 1

Linearizing the government budget constraint (7), we find

$$\bar{b}\widehat{i}_{b,t-1} - (1+\bar{r})\bar{b}\widehat{\pi}_t + (1+\bar{r})\widehat{b}_{t-1} + \widehat{\tau}_t = \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + \widehat{b}_t \tag{A.1}$$

Using (A.1), I will decompose total transfers into two parts: an endogenous "general equilibrium" component related to labor tax revenue and inflation debt servicing costs,

$$\widehat{\tau}_t^e \equiv \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}\widehat{\pi}_t \tag{A.2}$$

and an exogenous "policy" component

$$\widehat{\tau}_t^x \equiv \widehat{\tau}_t - \widehat{\tau}_t^e \tag{A.3}$$

I now present a constructive proof of Proposition 1: leveraging (18) I will show how to construct a transfer-only policy replicating any interest rate-only policy, and vice-versa. The decomposition in (A.2) and (A.3) will prove useful in this constructive proof.

1. An interest rate-only policy is a tuple $\{i_{b,t}, \tau_t\}_{t=0}^{\infty}$ with

   $$\widehat{\tau}_t = \widehat{\tau}_t^e - \bar{b}\widehat{i}_{b,t-1}$$

   so that $\widehat{\tau}_t^x = -\bar{b}\widehat{i}_{b,t-1}$. By (18), there exists a path of transfers $\{\tau_t^*\}_{t=0}^{\infty}$ such that

   $$\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^* = \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b + \mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x \tag{A.4}$$

   Since the interest rate-only policy by construction has zero net present value, it follows that the transfer-only policy $\widehat{\boldsymbol{\tau}}^*$—which induces the exact same (zero-NPV) consumption sequence—also has zero NPV:

   $$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t^* = 0 \tag{A.5}$$

   Now consider the transfer-only policy tuple $\{\bar{i}_b, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. I will verify that, at the initial $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$, all markets still clear and all agents still behave optimally. First, by (A.5) and the definition of $\widehat{\tau}_t^e$, we still have that $\lim_{t\to\infty}\widehat{b}_t = 0$. Second, by construction of $\widehat{\tau}_t^*$ in (A.4), the path $\widehat{\mathbf{c}}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; \bar{i}_b, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

2. A transfer-only policy is a tuple $\{\bar{i}_b, \tau_t\}_{t=0}^{\infty}$ with

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t^x = 0$$

By (18), there exists a path of interest rates $\{i_{b,t}^*\}_{t=0}^{\infty}$ with $\widehat{\tau}_t^* = -\widehat{\bar{b}i}_{b,t-1}^*$ such that

$$\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x = \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b^* = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b^* + \mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^* \tag{A.6}$$

and where by construction $\{\widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$ has zero NPV:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t^* + \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\bar{b}i}_{b,t-1}^* = 0 \tag{A.7}$$

Now consider the interest rate-only policy tuple $\{i_{b,t}^*, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. As before I will verify that, at the initial $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$, all markets still clear and all agents still behave optimally. First, by (A.7) and the definition of $\widehat{\tau}_t^e$, we have that $\widehat{b}_t = 0$ for all t, so indeed the policy is a valid interest rate-only policy. Second, by construction of $\widehat{i}_{b,t}^*$ in (A.6), the path $\widehat{\mathbf{c}}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; i_{b,t}^*, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

$\square$

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Online Appendix for:
Interest Rate Cuts vs. Stimulus Payments:
An Equivalence Result

This online appendix contains supplemental material for the article "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result". I provide (i) details for the various structural models used in the paper, and (ii) various supplementary theoretical results. The end of this appendix contains proofs and auxiliary lemmas.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded "B."—"D." refer to the main article.

Contents

B Model details

This appendix contains supplementary model details. I begin in Appendix B.1 by discussing in more detail the price-NKPC (5). Appendices B.2 to B.5 derive the optimality conditions for household consumption behavior for the various analytical models of Section 2.2 (and also justify the form of the wage-NKPC (6)). Appendix B.6 does the same for my quantitative heterogeneous-agent model, and also discusses the model calibration for Sections 4 and 5. Finally Appendix B.7 presents the extended model with investment.

B.1 Sticky-price retailers

To derive the price-NKPC (5), let $1-\alpha$ denote the elasticity of output with respect to total labor input at the steady state, $1-\theta_p \in (0,1)$ denote the probability of a price re-set, and ϵ_p the substitutability between different retail varieties in aggregation to the final good. We can then follow the derivations in Galí (2015) to arrive at the following log-linearized price-NKPC:

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}})}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}\left(\widehat{w}_t + \alpha\widehat{\ell}_t\right) + \beta\widehat{\pi}_{t+1} \tag{B.1}$$

(B.1) is the log-linearized version of (5).

B.2 Permanent-income consumers

The solution to the household consumption-savings problem is fully characterized by the log-linearized aggregate budget constraint and Euler equation:

$$\begin{aligned} \bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t &= (1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}(\widehat{b}_{t-1} + \widehat{i}_{b,t-1} - \widehat{\pi}_t) + \bar{\tau}\widehat{\tau}_t + \bar{d}\widehat{d}_t && \text{(B.2)} \\ \widehat{c}_t &= \widehat{c}_{t+1} - \frac{1}{\gamma}\left(\widehat{i}_{b,t} - \widehat{\pi}_{t+1}\right) && \text{(B.3)} \end{aligned}$$

Unions order aggregate consumption and employment streams according to the preferences of the representative household (10). We can then follow the same steps as in Erceg et al. (2000) to arrive at a standard log-linearized wage-NKPC:

$$\widehat{\pi}^w_t = \kappa_w \times \underbrace{\left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma\widehat{c}_t)\right]}_{\text{wedge in labor supply}} + \beta\widehat{\pi}^w_{t+1} \tag{B.4}$$

where κ_w is a function of model primitives, satisfying

$$\kappa_w = \frac{(1 - \frac{1}{1+\bar{r}}\phi_w)(1 - \phi_w)}{\phi_w(\varepsilon_w \frac{1}{\varphi} + 1)},$$

with ϕ_w indicating the degree of wage stickiness, and ε_w indicating the elasticity of substitution between different types of labor. As in the price-NKPC case, (B.4) is the log-linearized version of (6), with the static labor supply optimality wedge function $\chi_w(\bullet)$ given as

$$\chi_w(w_t, \ell_t, c_t) = \frac{\psi \ell_t^{\frac{1}{\varphi}}}{w_t c_t^{-\gamma}} \tag{B.5}$$

B.3 Spender-saver model

All wealth in the economy is held by savers. For simplicity I assume that steady-state transfers to spenders and savers are set so that both have identical steady-state consumption. Optimal consumption behavior of savers (indexed by R) is then described by

$$\bar{c}\widehat{c}_t^R + \bar{b}^R \widehat{b}_t^R = (1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}^R(\widehat{b}_{t-1}^R + \widehat{i}_{b,t-1} - \widehat{\pi}_t) + \bar{\tau}^R\widehat{\tau}_t + \bar{d}\widehat{d}_t \tag{B.6}$$

$$\widehat{c}_t^R = \widehat{c}_{t+1}^R - \frac{1}{\gamma}\left(\widehat{i}_{b,t} - \widehat{\pi}_{t+1}\right) \tag{B.7}$$

where $\bar{b}^R = \bar{b}/(1-\mu)$, while for spenders (indexed by H) we simply have

$$\bar{c}\widehat{c}_t^H = (1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + \bar{\tau}^H\widehat{\tau}_t + \bar{d}\widehat{d}_t \tag{B.8}$$

The union orders consumption and employment streams according to a weighted average of the preferences of savers and spenders. Since spenders and savers have identical steady-state consumption and hours worked (and identical preferences), it follows that to first order union bargaining again gives rise to the wage-NKPC (B.4).[24]

[24] To see this, it is instructive to consider the flexible-price limit (as for example done in Bilbiie et al. (2021)). For an individual household $i \in \{R, H\}$ we have the usual optimality condition

$$\psi \ell_{it}^{\frac{1}{\varphi}} = w_t c_{it}^{-\gamma}$$

Log-linearizing, summing across the two types of households, and leveraging the fact that $\bar{\ell}_i = \bar{\ell}$ and $\bar{c}_i = \bar{c}$, we find that

$$\frac{1}{\varphi}\widehat{\ell}_t = \widehat{w}_t - \gamma\widehat{c}_t$$

Unions thus bargain with the same static labor supply target as before, so we again get (B.4).

B.4 Overlapping generations of households

At each point in time there is a unit continuum of households $i \in [0,1]$, each discounting the future at rate β and surviving from period to period at rate θ. Preferences of an individual household i are thus

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \left\{ \frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_{it}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \right] \tag{B.9}$$

All households supply common hours worked (as intermediated by the sticky-wage union), so $\ell_{it} = \ell_t$. Households invest in fair annuities, and so their date-t budget constraint is (see Angeletos & Huo, 2021, for a detailed discussion)

$$b_{it} = \frac{1}{\theta} \left[(1-\tau_\ell) w_t \ell_t + \frac{1 + i_{b,t-1}}{1+\pi_t} b_{it-1} + \tau_t + d_t - c_{it} \right] - \frac{1-\theta}{\theta} \bar{b} \frac{1+\pi_{t+1}}{1+i_t} \tag{B.10}$$

Proceeding exactly as in Angeletos & Huo (2021, Appendix E), we find that total consumption (aggregated across all households) is governed by the aggregate demand relation

$$\begin{aligned} \widehat{c}_t = (1-\beta\theta) \left[\frac{1}{\beta} \widehat{b}_{t-1} + \frac{\bar{b}}{\bar{y}} (\widehat{i}_{t-1} - \widehat{\pi}_t) + \sum_{k=0}^{\infty} (\beta\theta)^k ((1-\tau_\ell) \bar{w}\bar{\ell} (\widehat{w}_{t+k} + \widehat{\ell}_{t+k}) + \bar{\tau}\widehat{\tau}_{t+k} + \bar{d}\widehat{d}_{t+k}) \right] \\ - \left[\frac{1}{\gamma} \beta\theta - (1-\beta\theta)\beta \frac{\bar{b}}{\bar{y}} \right] \sum_{k=0}^{\infty} (\beta\theta)^k \left(\widehat{i}_{t+k} - \widehat{\pi}_{t+k+1} \right) \end{aligned}$$

Combining this with the linearized aggregate household budget constraint we obtain the following aggregate Euler equation:

$$\begin{aligned} [1-(1-\beta\theta)\theta]\, \widehat{c}_t = \beta\theta \widehat{c}_{t+1} + (1-\beta\theta)(1-\theta) \left(\frac{1}{\beta} \widehat{b}_{t-1} + \frac{\bar{b}}{\bar{y}} (\widehat{i}_{t-1} - \widehat{\pi}_t) \right) + (1-\beta\theta)(1-\theta)((1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) \\ + \bar{\tau}\widehat{\tau}_t + \bar{d}\widehat{d}_t) - \left[\frac{1}{\gamma}\beta\theta - (1-\beta\theta)(1-\theta)\beta\frac{\bar{b}}{\bar{y}} \right] \left(\widehat{i}_t - \widehat{\pi}_{t+1} \right) \end{aligned} \tag{B.11}$$

This Euler equation together with the linearized aggregate household budget constraint fully characterize the evolution of aggregate consumption.

The union orders consumption and employment streams according to the preferences of its member households. The fact that all cohorts share the same steady-state consumption and hours worked implies that optimal union bargaining to first order again yet again gives rise to the wage-NKPC (B.4), analogously to the spender-saver model (see Footnote 24).

B.5 Bonds in household utility

The solution to the household consumption-savings problem is now fully characterized by the following log-linearized aggregate budget constraint and Euler equation:

$$\bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t = (1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}(\widehat{b}_{t-1} + \widehat{i}_{b,t-1} - \widehat{\pi}_t) + \bar{\tau}\widehat{\tau}_t + \bar{d}\widehat{d}_t \quad \text{(B.12)}$$

$$\widehat{c}_t = \beta(1+\bar{r})\widehat{c}_{t+1} + \frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]\widehat{b}_t - \frac{1}{\gamma}\beta(1+\bar{r})\left(\widehat{i}_{b,t} - \widehat{\pi}_{t+1}\right) \quad \text{(B.13)}$$

where $\beta(1+\bar{r}) < 1$ as long as $\alpha, \eta > 0$. Since there is a single representative household with separable preferences over consumption, wealth, and hours worked (see (14)), the static labor supply wedge is yet again given as (B.5) and so as before union bargaining gives (B.4).

B.6 Heterogeneous households

This section completes the description of the quantitative heterogeneous-agent model studied in Section 4. I first discuss my assumptions on union bargaining (which are revisited in Section 5.1) and then describe the model calibration.

Union bargaining. I assume that unions order aggregate consumption and employment streams according to "as-if" representative-agent preferences:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \quad \text{(B.14)}$$

Given these union preferences, we can yet again follow exactly the same steps as in Erceg et al. (2000) to arrive at (B.4).

Note that, if unions instead maximized an equal-weighted average of household utility (as in Auclert et al., 2018),

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 \left\{ \frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_{it}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} di = \sum_{t=0}^{\infty} \beta^t \int_0^1 \left\{ \frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} di \quad \text{(B.15)}$$

then a weighted average of household marginal consumption utilities—rather than marginal consumption utility evaluated at the aggregate consumption level c_t—would enter the static labor wedge and thus (B.4). The model would thus be inconsistent with a wage-NKPC of my assumed form (6). I discuss this case further in Section 5.1 and Appendix C.5.

Model calibration. I first discuss the parameterization of the model's steady state, and so in particular the induced aggregate consumption function $\mathcal{C}(\bullet)$.

Parameter	Description	Value	Target	Value
Households				
ρ_e, σ_e	Income Risk	-	Kaplan & Violante (2018)	-
$\underline{e}^p$, χ_0, χ_1	Dividend Endowment	-	Illiquid Wealth Shares	-
β	Discount Rate	0.97	$\bar{b}/\bar{y}$	1.5
$\bar{r}$	Average Return	0.01	Annual Rate	0.04
ϱ	Death Rate	1/180	Average Age	45
γ	Preference Curvature	1	Standard	
φ	Labor Supply Elasticity	0.5	Standard	
ε_w	Labor Substitutability	10	Standard	
$\underline{b}$	Borrowing Limit	0	McKay et al. (2016)	
Firms				
$1-\alpha$	Returns to Scale	1	Standard	
ε_p	Goods Substitutability	16.67	Profit Share	0.06
Government				
τ_ℓ	Labor Tax	0.3	Average Labor Tax	0.30
$\bar{\tau}/\bar{y}$	Transfer Share	0.05	Transfer Share	0.05

Table B.1: HANK model, steady-state calibration.

The values of all parameters relevant for the model's deterministic steady state are displayed in Table B.1. For my quantitative analysis I slightly enrich the preferences displayed in (16) to allow for exogenous household death at rate ϱ. Preference parameters $\{\gamma, \varphi, \varrho\}$ as well as the labor substitutability ε_w are set to standard values. The average return on (liquid) assets is set in line with standard calibrations of business-cycle models, and the discount rate is then disciplined through the total amount of liquid wealth. As in McKay et al. (2016), I assume that households cannot borrow in the liquid asset. Next, for income risk, I adopt the 33-state specification of Kaplan et al. (2018), ported to discrete time. For share endowments, I assume that

$$d_{it} = \begin{cases} 0 & \text{if } e_{it}^p \leq \underline{e}^p \\ \chi_0(e_{it}^p - \underline{e}^p)^{\chi_1} \times d_t & \text{otherwise} \end{cases}$$

where e^p_{it} is the permanent component of household i's labor productivity. I set the parameters $\{\underline{e}^p, \chi_0, \chi_1\}$ as in Wolf (2020). On the firm side, I assume constant returns to scale in production, and set the substitutability between goods to a standard value. Finally, the average government tax take, transfers, and debt issuance are all set in line with direct empirical evidence. Relative to Definition 3, I slightly generalize the model to allow for non-zero government spending, giving the new market-clearing condition

$$y_t = c_t + g_t \tag{B.16}$$

Note that, for all experiments, I keep government expenditure fixed at $g_t = \bar{g}$, so its presence only matters for the steady-state fiscal tax-and-transfer system, and does not directly show up anywhere in equilibrium dynamics.

In the second step I set the remaining model parameters (which exclusively govern dynamics around the deterministic steady state). For the baseline interest rate-only policy considered in Section 4.3, I consider an interest rate rule of the form

$$\widehat{i}_{b,t} = \phi_\pi \widehat{\pi}_t + m_t$$

where m_t is the monetary shock, set to give the gradually decaying path of nominal rates displayed in Figure 4. In a slight generalization of Definition 2, I assume that the baseline interest rate-only policy is not financed through taxes and transfers adjusting period-by-period (as in (8)), but instead consider a more general fiscal financing rule of the form

$$\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \left[(1+\bar{r})\bar{b}(\widehat{i}_{b,t-1} - \widehat{\pi}_t) - \tau_\ell(\bar{w}\widehat{\ell}_t + \bar{\ell}\widehat{\ell}_t)\right] \tag{B.17}$$

and with $\rho_b \in (0,1)$. Total transfers adjust residually to balance the government budget. Since b_t evolves gradually over time, it follows that an interest rate cut at time t only feeds through to higher transfers with a delay. While the financing rule in Definition 2 was conceptually simpler, the alternative fiscal rule (B.17) has the advantage that interest rate movements are not accompanied by (counterfactual) large contemporaneous changes in transfers. This completes the specification of policy for the baseline monetary experiment. I present the rule parameterizations as well as all other model parameters in Table B.2.

Alternative calibrations. For my robustness checks in Appendix C.4 I consider two alternative model calibrations: one with less liquid wealth ($\bar{b}/\bar{y} = 0.5$, implying substan-

Parameter	Description	Value
ϕ_p	Price Calvo Parameter	0.85
ϕ_w	Wage Calvo Parameter	0.70
ϕ_π	Taylor Rule Inflation	1.5
ρ_b	Financing Rule Persistence	0.85

Table B.2: HANK model, parameters governing dynamics.

tially larger MPCs, with $\omega = 0.64$), and one with more liquid wealth ($\bar{b}/\bar{y} = 7.5$, implying substantially smaller MPCs, with $\omega = 0.12$).

B.7 Adding investment

All firms are identical, so I drop the j subscript. Analogously to the discussion in Section 2.1, we can summarize the solution to the firm problem with an investment demand function,

$$\mathbf{i} = \mathcal{I}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.18}$$

a production function,

$$\mathbf{y} = \mathcal{Y}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.19}$$

a labor demand function,

$$\boldsymbol{\ell} = \mathcal{L}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.20}$$

and a dividend function

$$\mathbf{d} = \mathcal{D}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.21}$$

where the dividend function aggregates over both intermediate goods producers and sticky-price retailers. Note that, since intermediate goods firms hire labor on a competitive spot market, and since productivities are equalized across firms j, two sequences $\boldsymbol{i_b}$ and $\boldsymbol{\tau}_f$ that induce the same paths of investment also invariably induce the same paths of output and labor hiring. However, since interest rates and investment subsidies enter the budget constraint differently, the implied dividend paths may be different.

Given investment subsidies to firms, the government budget constraint is adjusted to give

$$\frac{1 + i_{b,t-1}}{1 + \pi_t} b_{t-1} + \tau_t + \tau_{f,t}(\{i_{t-q}\}_{q=0}^{t}) = \tau_\ell w_t \ell_t + b_t \tag{B.22}$$

All other parts of the model are unchanged relative to Section 2.1. We thus arrive at the following equilibrium definition:

Definition 5. *An equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, \tau_{f,t}, b_t\}_{t=0}^{\infty}$ *and a set of aggregates* $\{c_t, \ell_t, y_t, i_t, k_t, w_t, \pi_t, d_t, p_t^I\}_{t=0}^{\infty}$ *such that:*

1. *Consumption is consistent with the aggregate consumption function* (2).
2. *Aggregate investment, output, hours worked and dividends satisfy*

$$\begin{aligned}\boldsymbol{i} &= \mathcal{I}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right)\\ \boldsymbol{y} &= \mathcal{Y}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right)\\ \boldsymbol{\ell} &= \mathcal{L}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right)\\ \boldsymbol{d} &= \mathcal{D}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right)\end{aligned}$$

3. *Wage inflation* π_t^w *together with* $\{\ell_t, c_t, w_t\}_{t=0}^{\infty}$ *are consistent with the wage-NKPC* (6).
4. *The paths* $\{\pi_t, p_t^I\}_{t=0}^{\infty}$ *are consistent with the adjusted aggregate price-NKPC* (26).
5. *The output market clears:* $y_t = c_t + i_t$ *for all* $t \geq 0$*, the government budget constraint* (B.22) *holds at all* t*, and* $\lim_{t\to\infty} \widehat{b}_t = 0$*. The bond market then clears by Walras' law.*

C Supplementary theoretical results

This section presents supplementary theoretical results. Appendices C.1 and C.2 characterize $\mathcal{C}_\tau$, $\mathcal{C}_\tau^{-1}$ as well as $\tilde{\mathcal{C}}_{i_b}$ in the analytical models of Section 2.2. Appendix C.3 presents further details on my sufficient statistics formula, and Appendix C.4 documents the accuracy of this approximation in alternative HANK model calibrations. Appendix C.5 shows that, for various alternative specifications of labor supply decisions, my policy equivalence result still holds nearly exactly. Appendix C.6 elaborates on my rule equivalence results from a recursive perspective. Finally Appendices C.7 and C.8 extend the policy equivalence result to some even richer model environments and to targeted transfers, respectively.

C.1 $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ in analytical models

This section characterizes the matrix $\mathcal{C}_\tau$ as well as its inverse in my various analytical models of non-Ricardian consumption behavior. For the overlapping-generations and bond-in-utility models, my characterization of $\mathcal{C}_\tau$ (though not $\mathcal{C}_\tau^{-1}$) will be *approximate* in the following particular sense of asymptotic equivalence.

Definition 6. *Two sequences of $n \times n$ matrices $\{A_n\}$ and $\{B_n\}$ are said to be asymptotically equivalent if*

1. *A_n and B_n are uniformly bounded in strong norm:*

$$||A_n||, ||B_n|| \leq M < \infty, \quad n = 1, 2, \ldots$$

2. *$D_n \equiv A_n - B_n$ goes to zero in weak norm as $n \to \infty$:*

$$\lim_{n\to\infty} |A_n - B_n| = \lim_{n\to\infty} |D_n| = 0$$

Here the strong (or operator) norm is defined as

$$||A|| = \max_x \left(\frac{x^*(A^*A)x}{x^*x}\right)^{\frac{1}{2}}$$

where $*$ denotes conjugate transpose, while the weak norm (or Hilbert-Schmidt norm) is defined as

$$|A| = \left(\frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{n} |a_{k,j}|^2 \right)^{\frac{1}{2}}$$

In (13) and (15) I use the symbol "≈" to indicate such asymptotic equivalence. In words, asymptotic equivalence for my purposes simply means that, as we look at farther-out rows and columns, the entries of the matrix of interest converge to the "approximate" shape that I am indicating.

Spender-saver model. A closed-form expression for $\mathcal{C}_\tau^H$ was provided in (12). Given this expression it is straightforward if tedious to verify that

$$(\mathcal{C}_\tau^H)^{-1} = \begin{pmatrix} \frac{1-(1-\omega)\bar{r}}{1-(1-\omega)(1+\bar{r})} & -\frac{(1-\omega)(1+\bar{r})}{1-(1-\omega)(1+\bar{r})}\frac{\bar{r}}{(1+\bar{r})^2} & -\frac{(1-\omega)(1+\bar{r})}{1-(1-\omega)(1+\bar{r})}\frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ -\frac{(1-\omega)(1+\bar{r})}{1-(1-\omega)(1+\bar{r})}\frac{\bar{r}}{1+\bar{r}} & \frac{1-(1-\omega)\frac{\bar{r}}{1+\bar{r}}}{1-(1-\omega)(1+\bar{r})} & -\frac{(1-\omega)(1+\bar{r})}{1-(1-\omega)(1+\bar{r})}\frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ -\frac{(1-\omega)(1+\bar{r})}{1-(1-\omega)(1+\bar{r})}\frac{\bar{r}}{1+\bar{r}} & -\frac{(1-\omega)(1+\bar{r})}{1-(1-\omega)(1+\bar{r})}\frac{\bar{r}}{(1+\bar{r})^2} & \frac{1-(1-\omega)\frac{\bar{r}}{(1+\bar{r})^2}}{1-(1-\omega)(1+\bar{r})} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.1}$$

With $\bar{r} = 0$ this expression simplifies to (23).

Overlapping generations of households. I begin with the shape of $\mathcal{C}_\tau^{OLG}$ as displayed in (13). From the discussion in Appendix B.4 it follows that the matrix $\mathcal{C}_\tau^{OLG}$—which gives dollar responses of consumption to dollar changes in income—is fully characterized by the following pair of linearized (not log-linearized) budget constraint and Euler equation:

$$\widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} = \widehat{\tau}_t \tag{C.2}$$

$$[1-\theta(1-\beta\theta)]\,\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1-\beta\theta)(1-\theta)\frac{1}{\beta}\widehat{b}_{t-1} = (1-\beta\theta)(1-\theta)\widehat{\tau}_t \tag{C.3}$$

From this system we arrive at the following result:

Proposition C.1. *Consider the consumption-savings problem of a household sector with overlapping generations, as described in Appendix B.4. Then, for any $\ell > 0$ the impulse response path $\widehat{\mathbf{c}}_H$ to an income shock at time H satisfies*

$$\lim_{H\to\infty} \widehat{c}_{H,H} = const., \quad \lim_{H\to\infty} \frac{\widehat{c}_{H+\ell,H}}{\widehat{c}_{H+\ell-1,H}} = \theta, \quad \lim_{H\to\infty} \frac{\widehat{c}_{H-\ell-1,H}}{\widehat{c}_{H-\ell,H}} = \frac{\theta}{1+\bar{r}} \tag{C.4}$$

This establishes the claimed asymptotic shape (13).

We can also use the system (C.2)–(C.3) to characterize the path of transfers $\widehat{\boldsymbol{\tau}}$ that yields any given desired consumption sequence $\widehat{\boldsymbol{c}}$. Doing so we get the following exact (not approximate) expression for the inverse $\mathcal{C}_\tau^{-1}$:

$$(\mathcal{C}_\tau^{OLG})^{-1} \;=\; \frac{1}{\omega} \;\times\; \begin{pmatrix} \frac{1-\theta(1-\frac{\theta}{1+\bar{r}})}{1-\theta} & -\frac{1}{1+\bar{r}}\frac{\theta}{1-\theta} & 0 & \cdots \\ -\frac{\theta}{1-\theta} & \frac{1+\frac{\theta^2}{1+\bar{r}}}{1-\theta} & -\frac{1}{1+\bar{r}}\frac{\theta}{1-\theta} & \cdots \\ 0 & -\frac{\theta}{1-\theta} & \frac{1+\frac{\theta^2}{1+\bar{r}}}{1-\theta} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.5}$$

and with $\omega \equiv 1 - \frac{\theta}{1+\bar{r}}$. With $\bar{r} = 0$ this expression simplifies to (24). The inverse has thus in full generality been characterized as a function of my sufficient statistics $\{\omega, \theta, \bar{r}\}$, as claimed.

Bonds in household utility. I begin with the shape of $\mathcal{C}_\tau^{BiU}$ as displayed in (15). From the discussion in Appendix B.5 it follows that the matrix $\mathcal{C}_\tau^{BiU}$ is fully characterized by the following pair of linearized (not log-linearized) budget constraint and Euler equation:

$$\widehat{c}_t + \widehat{b}_t - (1+\bar{r})\widehat{b}_{t-1} \;=\; \widehat{\tau}_t \tag{C.6}$$

$$\widehat{c}_t - \beta(1+\bar{r})\widehat{c}_{t+1} - \frac{\bar{c}}{\bar{b}}\frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]\widehat{b}_t \;=\; 0 \tag{C.7}$$

We see that the factor $\frac{\bar{c}}{\bar{b}}$ simply scales the last term in the Euler equation, so I will without loss of generality set this term equal to 1. We then arrive at the following result:

Proposition C.2. *Consider the consumption-savings problem of a household with bonds in the utility function, as described in Appendix B.5. Then, for any $\ell > 0$ the impulse response path $\widehat{\mathbf{c}}_H$ to an income shock at time H satisfies*

$$\lim_{H\to\infty} \widehat{c}_{H,H} = const., \quad \lim_{H\to\infty} \frac{\widehat{c}_{H+\ell,H}}{\widehat{c}_{H+\ell-1,H}} = \theta, \quad \lim_{H\to\infty} \frac{\widehat{c}_{H-\ell-1,H}}{\widehat{c}_{H-\ell,H}} = \beta\theta \tag{C.8}$$

where θ is now a complicated function of model primitives.

This establishes the asymptotic shape (15). The closed-form solutions in the proof of Proposition C.1 after some additional algebra yield the following exact expression for $\mathcal{C}_\tau^{-1}$:

$$(\mathcal{C}_\tau^{BiU})^{-1} \;=\; \begin{pmatrix} 1 + \frac{1}{\tilde{\omega}}\frac{1}{1+\bar{r}}\frac{\theta}{1-\beta\theta^2} & -\frac{1}{\tilde{\omega}}\frac{\beta\theta}{1-\beta\theta^2} & 0 & \cdots \\ -\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2} & 1 + \frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2}\left[\beta(1+\bar{r}) + \frac{1}{1+\bar{r}}\right] & -\frac{1}{\tilde{\omega}}\frac{\beta\theta}{1-\beta\theta^2} & \cdots \\ 0 & -\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2} & 1 + \frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2}\left[\beta(1+\bar{r}) + \frac{1}{1+\bar{r}}\right] & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.9}$$

where $\tilde{\omega}$ is given as

$$1 = \tilde{\omega}\left[\frac{(1+\bar{r})\beta\theta}{1-(1+\bar{r})\beta\theta} + \omega^{-1}\right]$$

In particular, with $\bar{r} = 0$, this expression simplifies to

$$(\mathcal{C}_\tau^{BiU})^{-1} \quad = \quad \frac{1}{\omega} \times \begin{pmatrix} \frac{1-\beta\theta(1-\theta)}{1-\beta\theta} & -\frac{\beta\theta}{1-\beta\theta} & 0 & \dots \\ -\frac{\theta}{1-\beta\theta} & \frac{1+\beta\theta^2}{1-\beta\theta} & -\frac{\beta\theta}{1-\beta\theta} & \dots \\ 0 & -\frac{\theta}{1-\beta\theta} & \frac{1+\beta\theta^2}{1-\beta\theta} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.10}$$

We thus as claimed find an inverse with the same shape as that for the OLG model.

C.2 $\tilde{\mathcal{C}}_{i_b}$ in analytical models

For all of the analytical models presented in Section 2.2, the two consumption derivative matrices $\mathcal{C}_\tau$ and $\tilde{\mathcal{C}}_{i_b}$ are extremely tightly related. This section establishes this claim and reveals the precise shape of the relationship.

I will first show that, for the permanent-income, spender-saver, and OLG models, the relationship is simply

$$\tilde{\mathcal{C}}_{i_b} = -\frac{1}{\gamma}(I - \mathcal{C}_\tau)\begin{pmatrix} 1 & 1 & 1 & \dots \\ 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.11}$$

I begin with the permanent-income model. By the discussion in Appendix B.2 it follows that $\mathcal{C}_\tau$ and $\tilde{\mathcal{C}}_{i_b}$ here are characterized by the linearized system

$$\widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} \quad = \quad \widehat{\tau}_t \tag{C.12}$$

$$\widehat{c}_t - \widehat{c}_{t+1} \quad = \quad -\gamma\widehat{i}_{b,t} \tag{C.13}$$

For all of the following I will define $\widehat{c}_t^* \equiv \widehat{c}_t - \widehat{\tau}_t$. Now consider first an income shock at date 0. Plugging this into the optimality conditions and re-arranging, we see that the impulse response of c^* is identical to the impulse response of c to a date-0 interest rate change scaled by γ. That is, we have

$$\mathcal{C}_\tau^*(\bullet, 1) = \mathcal{C}_\tau(\bullet, 1) - e_1 = \gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 1)$$

where $e_1 = (1, 0, 0, \dots)'$. This gives the first column of (C.11):

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 1) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1))$$

Similarly, for date-1 shocks, we have

$$\mathcal{C}_\tau^*(\bullet, 2) = \mathcal{C}_\tau(\bullet, 2) - e_2 = -\gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 1) + \gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 2)$$

where $e_2 = (0, 1, 0, \dots)'$. Thus we get

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 2) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1) + e_2 - \mathcal{C}_\tau(\bullet, 2))$$

giving the second column of (C.11). All other columns follow analogously.

It is immediate that this argument applies without change to the spender-saver model. For the OLG model, the two consumption derivative matrices are characterized by the system

$$\widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} = \widehat{\tau}_t \tag{C.14}$$

$$[1 - \theta(1 - \beta\theta)]\,\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1 - \beta\theta)(1 - \theta)\frac{1}{\beta}\widehat{b}_{t-1} = (1 - \beta\theta)(1 - \theta)\widehat{\tau}_t - \gamma\beta\theta\widehat{i}_{b,t} \tag{C.15}$$

Again defining $\widehat{c}_t^* \equiv \widehat{c}_t - \widehat{\tau}_t$, we see that the impulse response of c^* to a date-0 income shock is again identical to the impulse response of c to a date-0 interest rate change scaled by γ. The previous argument thus continues to apply unchanged, and in particular also applies for all further columns of $\tilde{\mathcal{C}}_{i_b}$.

Finally, the argument is only slightly more involved for the bond-in-utility model. Here the two consumption derivative matrices are characterized by the system

$$\widehat{c}_t + \widehat{b}_t - (1 + \bar{r})\widehat{b}_{t-1} = \widehat{\tau}_t \tag{C.16}$$

$$\widehat{c}_t - \beta(1 + \bar{r})\widehat{c}_{t+1} - \frac{\bar{c}}{\bar{b}}\frac{\eta}{\gamma}\,[1 - \beta(1 + \bar{r})]\,\widehat{b}_t = -\beta(1 + \bar{r})\frac{1}{\gamma}\widehat{i}_{b,t} \tag{C.17}$$

In this case the impulse response of c^* to a date-0 income shock is identical to the impulse response of c to a date-0 interest rate change scaled by $\frac{\gamma}{\beta(1+\bar{r})}$. That is, we have

$$\mathcal{C}_\tau^*(\bullet, 1) = \mathcal{C}_\tau(\bullet, 1) - e_1 = \frac{\gamma}{\beta(1 + \bar{r})}\tilde{\mathcal{C}}_{i_b}(\bullet, 1)$$

and so

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 1) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1))\left[\beta(1+\bar{r})\right]$$

Proceeding similarly for a date-1 shock we find that

$$\mathcal{C}_\tau^*(\bullet, 2) = \mathcal{C}_\tau(\bullet, 2) - e_2 = -\frac{\gamma}{[\beta(1+\bar{r})]^2}\tilde{\mathcal{C}}_{i_b}(\bullet, 1) + \frac{\gamma}{\beta(1+\bar{r})}\tilde{\mathcal{C}}_{i_b}(\bullet, 2)$$

and so

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 2) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1))\left[\beta(1+\bar{r})\right] - \frac{1}{\gamma}(e_2 - \mathcal{C}_\tau(\bullet, 2))\left[\beta(1+\bar{r})\right]^2$$

Continuing in this fashion we find that overall we now have

$$\mathcal{C}_{i_b}^{BiU} = -\frac{1}{\gamma}(I - \mathcal{C}_\tau^{BiU})\begin{pmatrix} [\beta(1+\bar{r})] & [\beta(1+\bar{r})]^2 & [\beta(1+\bar{r})]^3 & \dots \\ 0 & [\beta(1+\bar{r})] & [\beta(1+\bar{r})]^2 & \dots \\ 0 & 0 & [\beta(1+\bar{r})] & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

The analysis in this section confirms my claim in Footnote 10: in my analytical models, knowledge of $\mathcal{C}_\tau$ (and thus my three sufficient statistics $\{\omega, \theta, \bar{r}\}$) together with the intertemporal elasticity of substitution γ^{-1} already suffices to also fully characterize $\tilde{\mathcal{C}}_{i_b}$.

C.3 The sufficient statistics formula

My sufficient statistics formula maps the three observables $\{\omega, \theta, \bar{r}\}$ into the matrix $\mathcal{C}_\tau$ (and thus its inverse $\mathcal{C}_\tau^{-1}$). The formula proceeds in two steps.

First, given $\{\theta, \bar{r}\}$, I construct a matrix $\mathcal{C}_\tau^{(1)}$ that has the same asymptotic shape as that of the overlapping-generations and bond-in-utility models. Specifically, I first set

$$\mathcal{C}_\tau^{(1)}(\bullet, 1) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \theta, \theta^2, \dots\right\}'}_{\text{spending decay}}$$

and

$$\mathcal{C}_\tau^{(1)}(1, \bullet) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \frac{\theta}{1+\bar{r}}, \left(\frac{\theta}{1+\bar{r}}\right)^2, \dots\right\}}_{\text{anticipation effects}}$$

In words, an income gain at date 0 is spent at rate θ (the first column of $\mathcal{C}_\tau^{(1)}$), and date-0 anticipation effects of future income gains decline at rate $\frac{\theta}{1+\bar{r}}$ (the first row of $\mathcal{C}_\tau^{(1)}$), as in (13). I then construct the remaining columns of $\mathcal{C}_\tau^{(1)}$ as a function of the first row and column, and in a way that respects budget constraints. For the second column, I set

$$\mathcal{C}_\tau^{(1)}(\bullet, 2) = \mathcal{C}_\tau^{(1)}(1,2) \times \begin{pmatrix} 1 \\ -\mathcal{C}_\tau^{(1)}(\bullet, 1)(1+\bar{r}) \end{pmatrix} + \begin{pmatrix} 0 \\ \mathcal{C}_\tau^{(1)}(\bullet, 1) \end{pmatrix}$$

This expression builds on the intuition of the "fake-news" algorithm of Auclert et al. (2019): the first term is the response of households to a date-1 income shock announced at date-0 but then reversed at date 1; the second term then undoes that date-1 reversal, ensuring that the sum gives us the actual response to a date-1 income shock—that is, $\mathcal{C}_\tau^{(1)}(\bullet, 2)$. Relative to the more naive strategy of simply imposing the asymptotic shape (13) throughout, this alternative approach has the appeal that household budget constraints are always satisfied. I thank an anonymous referee of alerting me of this important point.

Continuing recursively, I then set

$$\mathcal{C}_\tau^{(1)}(\bullet, h) = \mathcal{C}_\tau^{(1)}(1,h) \times \begin{pmatrix} 1 \\ -\mathcal{C}_\tau^{(1)}(\bullet, 1)(1+\bar{r}) \end{pmatrix} + \begin{pmatrix} 0 \\ \mathcal{C}_\tau^{(1)}(\bullet, h-1) \end{pmatrix}, \quad h = 3, 4, \ldots$$

This specifies the entire matrix $\mathcal{C}_\tau^{(1)}$ as a function only of $\{\theta, \bar{r}\}$, and does so in a way that imposes the asymptotic shape (13) yet does not violate any budget constraints.

Second, I add a margin of spenders to disentangle the MPC ω and the spending slope θ. Note that, in my construction of $\mathcal{C}_\tau^{(1)}$, the MPC is mechanically given as $\left(1 - \frac{\theta}{1+\bar{r}}\right)$. To match any desired arbitrary MPC ω I then simply set

$$\mathcal{C}_\tau \;=\; \frac{\theta - (1-\omega)(1+\bar{r})}{\theta} \times I \;+\; \frac{(1-\omega)(1+\bar{r})}{\theta} \times \mathcal{C}_\tau^{(1)}$$

It is straightforward to verify that the resulting $\mathcal{C}_\tau$ matches the desired MPC ω. I have thus mapped my three sufficient statistics $\{\omega, \theta, \bar{r}\}$ into a matrix $\mathcal{C}_\tau(\omega, \theta, \bar{r})$ that (i) matches the average MPC ω and spending slope θ, and (ii) is consistent with lifetime household budget constraints. From here I can then also construct $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r})$. Note that existence of this inverse is ensured as long as $\theta < 1$ and $\omega \geq 1 - \frac{\theta}{1+\bar{r}}$, by the proof of Proposition 2.

Characterizing $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r})$. Unlike the analytical models considered in Section 2.2, the mixture model underlying the sufficient statistics formula does not admit a straightforward characterization of the $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r})$. However, what *can* be characterized in closed form are

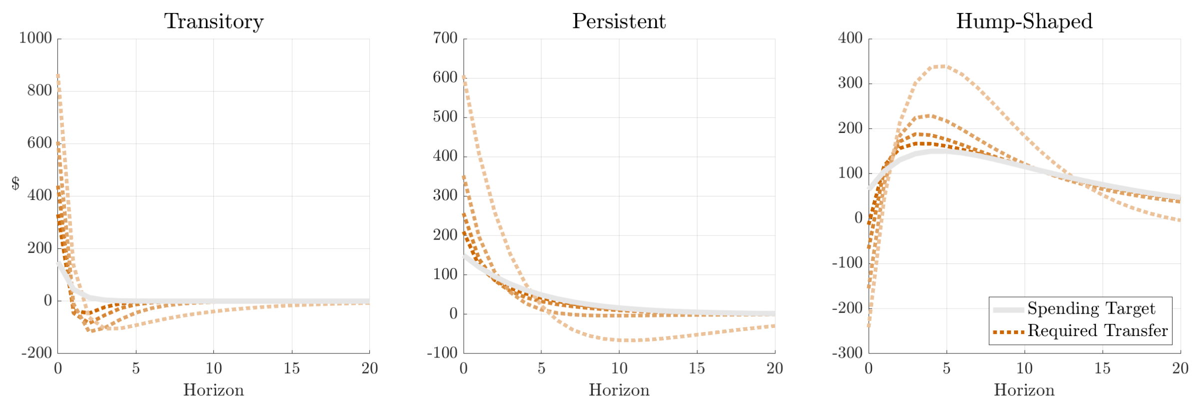

Figure C.1: Three desired net excess demand paths $\widehat{\boldsymbol{c}}^{PE}$ (grey) and the required sequences of uniform lump-sum taxes and transfers, $\mathcal{C}_\tau^{-1}(\omega,\theta,\bar{r}) \times \widehat{\boldsymbol{c}}^{PE}$ for a range of values of my sufficient statistics (orange dashed, see text), with lighter shades indicating lower MPCs ω.

the limits $\mathcal{C}_\tau^{-1}(\omega,\theta,\bar{r})(H+\ell,H)$ as $H \to \infty$—that is, we can characterize the limiting shape of $\mathcal{C}_\tau^{-1}(\omega,\theta,\bar{r})$.[25] Assuming for simplicity that $\bar{r}=0$, these limiting entries are given as[26]

$$\lim_{H\to\infty} \mathcal{C}_\tau^{-1}(\omega,\theta,0)(H+\ell,H) = \frac{1}{2\pi}\int_0^{2\pi} e^{-i\ell\lambda}\frac{1+\theta^2-2\theta\cos(\lambda)}{(2\omega+\theta^2-1)-2(\theta+\omega-1)\cos(\lambda)}d\lambda \quad \text{(C.18)}$$

Given (C.18), it is straightforward to plug in any desired value for my sufficient statistics and (approximately) characterize $\mathcal{C}_\tau^{-1}$.

Perturbing the sufficient statistics. I here further substantiate my claim that, for empirically relevant values of the sufficient statistics, even moderately sized stimulus check policies suffice to close meaningful shortfalls in aggregate spending. To do so I repeat the exercise of Figure 3 for a range of values of my sufficient statistics $\{\omega,\theta,\bar{r}\}$. Specifically, I continue to fix $\bar{r}=0.01$, consider $\omega \in \{0.2, 0.3, 0.4, 0.5\}$ and then pin down the slope θ by requiring the same ratio $\frac{\theta}{1-\omega}$ as in my quantitative heterogeneous-household model. The range of MPCs ω that I consider is chosen to contain and in fact go beyond the range of estimates available from the literature (e.g. Parker et al., 2013; Fagereng et al., 2018).

Results are displayed in Figure C.1. The figure illustrates my claim: across the range of empirically relevant values for my sufficient statistics, the required stimulus check policies

[25]Inspecting Figure 2, we see that convergence to these limits tends to be very quick.

[26]To derive (C.18), I leverage the fact that $\mathcal{C}_\tau(\omega,\theta,\bar{r})$ is asymptotically Topelitz, implying that its inverse is also asymptotically Toeplitz. I then leverage known expressions for that inverse (see Gray, 2006). The detailed argument is available upon request.

(a) Low liquid wealth calibration (b) High liquid wealth calibration

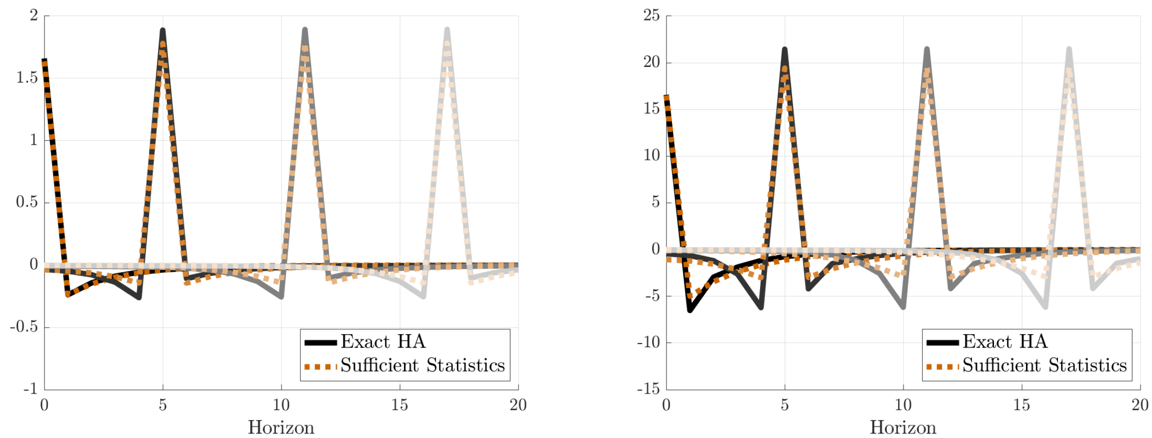

Figure C.2: See the caption of Figure 2.

are moderate in size.

C.4 $\mathcal{C}_\tau^{-1}$ in alternative HANK calibrations

The analysis in Section 4.3 confirmed the accuracy of my three-parameter sufficient statistics formula for $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ in the baseline calibration of my heterogeneous-household model. I here repeat the same exercise for two materially different model calibrations: one with very low liquid wealth (implying a counterfactually large average MPC of $\omega = 0.64$) and one with a lot of liquid wealth (implying a counterfactually small average MPC of $\omega = 0.12$). Results are reported in Figures C.2 and C.3.

The takeaways from these figures are twofold. First, changing the model calibration materially affects the model-implied consumption map $\mathcal{C}_\tau$ and its inverse $\mathcal{C}_\tau^{-1}$. As expected, for low liquid wealth, the inverse $\mathcal{C}_\tau^{-1}$ looks closer to a simple spender-saver model (recall Figure 1 and notice the y-axis scale) and the required transfer stimulus policies are even smaller than in my baseline analysis. For high liquid wealth, the inverse $\mathcal{C}_\tau^{-1}$ looks closer to an overlapping-generations or bond-in-utility model (again recall Figure 1 and notice the y-axis scale), and the transfer stimulus policies required to close a given shortfall in demand are now much larger. Second, even though $\mathcal{C}_\tau$ looks very different across calibrations, my sufficient statistics formula throughout approximates $\mathcal{C}_\tau^{-1}$ and thus the implied equivalent transfer stimulus policies very well.

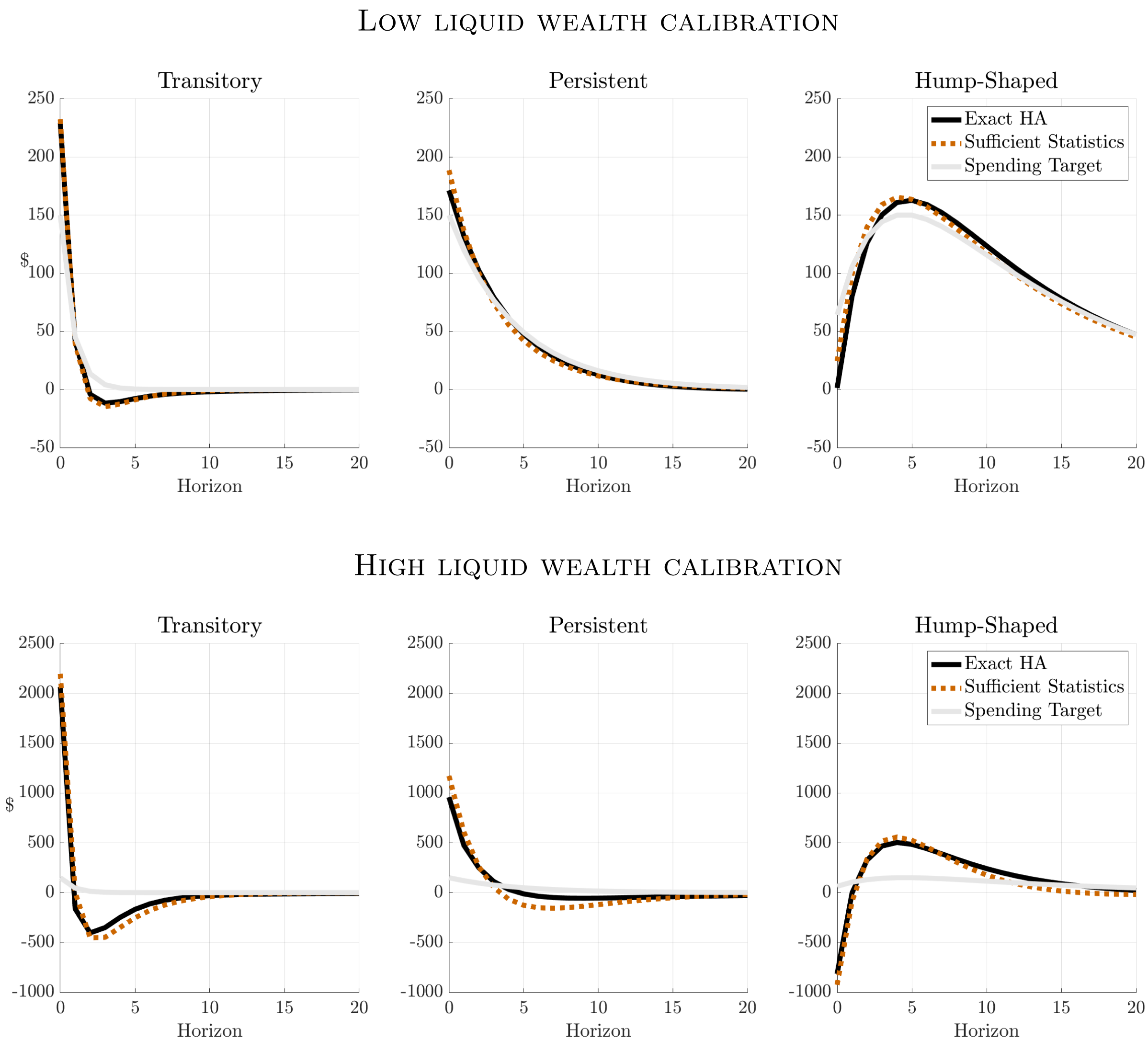

Figure C.3: See the caption of Figure 3.

C.5 Heterogeneous wealth effects in labor supply

This section elaborates on my discussion of the role of heterogeneity in wealth effects in labor supply across households (see Section 5.1). I first present results for an alternative union bargaining protocol and then consider an alternative model with preference heterogeneity, designed to match empirical evidence on heterogeneity in marginal propensities to earn (Golosov et al., 2021). Results for both are reported in Figure C.4.

Alternative bargaining results. I return to my main quantitative HANK model, but with one twist: the wage-NKPC (B.4) is replaced by the alternative formulation

$$\widehat{\pi}_t^w = \kappa_w \times \left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma\widehat{c}_t^*)\right] + \beta\widehat{\pi}_{t+1}^w \tag{C.19}$$

where

$$c_t^* \equiv \left[\int_0^1 e_{it}c_{it}^{-\gamma}di\right]^{-\frac{1}{\gamma}} \tag{C.20}$$

This is the specification of the wage-NKPC originally derived in Auclert et al. (2018) and implied by the union objective (B.15). I note that, in this case, my policy equivalence result will not hold *exactly*: two interest rate and stimulus check policies with identical direct effects on net excess demand (and so c_t) will not necessarily have identical direct effects on c_t^*, thus inducing different wedges in the economy's aggregate supply relation (C.19).

Are these differential labor supply effects likely to materially undermine the policy equivalence result? The left panel Figure C.4 suggests that the answer is "no". To construct the panel, I first compute impulse responses to a gradual monetary policy shock (with persistence 0.6), normalized to in general equilibrium increase consumption on impact by one per cent (grey). I then follow the steps in the proof of Proposition 1 to construct a stimulus check policy with identical effects on partial equilibrium consumer spending. The general equilibrium impulse response of consumption to this policy is displayed as the blue dashed line. The main takeaway is that the two lines are very close, with the stimulus check policy overall slightly more stimulative than the (not-quite-)equivalent interest rate cut.

The intuition for the results displayed in Figure C.4 is somewhat subtle. Both policies by design lead to a response of partial equilibrium consumption demand with zero present value—initially positive and then later on negative. Under my baseline wage-NKPC (6), this initial decrease and later increase in the average marginal utility of consumption leads to an initial decrease and later increase of union labor supply. With the alternative formulation (C.19), on the other hand, the labor supply response is governed by average marginal utility. Since most of the consumption adjustment following the transfer policy comes from high-MPC households (who tend to consume less), average marginal utility of consumption changes by more than after the equivalent rate cut—it drops by more initially and expands by more later on. It then follows that labor supply also contracts by more initially and expands by more later on, so the overall partial equilibrium net excess demand path (which equals consumption demand less labor supply) is more front-loaded after the stimulus check

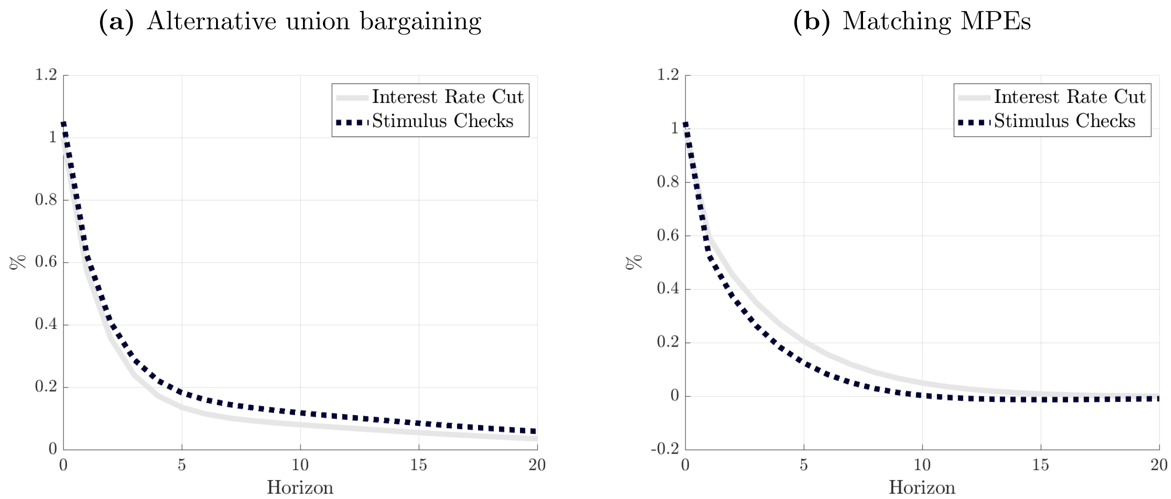

Figure C.4: Left panel: impulse response of consumption to a monetary policy shock with persistence $\rho_m = 0.6$ and peak effect of 1% (grey) and the "equivalent" stimulus check policy (blue dashed) in a HANK model with labor supply relation (C.19). Right panel: analogous figure for my hybrid spender-saver model described below.

policy. But we know from Auclert et al. (2018) that more front-loaded excess demand paths lead to a bigger boom in general equilibrium, so stimulus check policy will stimulate output by more—exactly as we see in Figure C.4. This effect however is moderate in size, for two reasons. First, average marginal utility and marginal utility at the average are still not too different, so the desired labor supply response is also not too different. Second, any such transitory differences in desired labor supply matter even less in general equilibrium as long as wages are at least moderately sticky (Christiano, 2011), as is the case in my model.

Mixture model results. My second exercise is designed to speak as closely as possible to the empirical evidence reported in Golosov et al. (2021). Those authors report marginal propensities to earn (MPEs)—defined as the response of labor income to an unearned lump-sum wealth gain—of up to \$3 per additional \$100 in wealth, with the response roughly two times larger for the highest-income households compared to the lowest-income ones (see their Table 3.2). These estimates are roughly twice as large as those reported in prior work, notably Cesarini et al. (2017) (see Table J.1 of Golosov et al.). While it is straightforward to match such *average* MPEs in heterogeneous-agent models (see Auclert et al., 2020), it is much harder to match the *cross-sectional dispersion* in MPEs (which is what matters for my policy equivalence result). Intuitively, the challenge is that, in standard models of household consumption and labor supply, MPEs are increasing (in absolute value) with MPCs.[27] Since poorer households tend to have higher MPCs, this would also imply that they have higher MPEs (in absolute value), inconsistent with the empirical evidence reviewed above.

My solution is to consider a two-type spender-saver model with preference heterogeneity chosen to ensure higher MPEs for low-MPC households. Specifically, I consider a model with so-called "GHH-plus preferences" (Auclert et al., 2020); savers have preferences

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(c_t - \psi_R \delta_R \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} - \psi_R (1-\delta_R) \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{C.21}$$

while spenders have static per-period preferences

$$\frac{\left(c_t - \psi_H \delta_H \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{1-\gamma} - 1}{1-\gamma} - \psi_H (1-\delta_H) \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{C.22}$$

Here the two coefficients $\{\delta_R, \delta_H\}$ control the strength of wealth effects in labor, with $\delta = 0$ corresponding to standard separable preferences and $\delta = 1$ corresponding to GHH preferences. I calibrate the consumer part of the model to induce consumption behavior similar to my baseline HANK model—matching in particular $\omega = 0.3$—and MPEs consistent with Golosov et al. (2021)—an MPE of \$3.5 for savers and an MPE of \$1.8 for spenders. The model parameterization is reported in Table C.1. All other parameters are set exactly as in my baseline HANK model, as reported in Tables B.1 and B.2.[28]

The model features cross-sectional heterogeneity in wealth effects in labor supply, so the policy equivalence result will not hold exactly. The right panel of Figure C.4 however reveals that it continues to approximately hold. I already in the main text gave the intuition for why the *magnitude* of the inaccuracy is so small (recall Section 5.1). I here instead focus on the *direction* of the error. The intuition is exactly opposite to that of the adjusted HANK model studied above. Savers have a larger MPE, so labor supply initially contracts by relatively more after an interest rate cut, and later on increases by relatively more. The total implied

[27] This follows straightforwardly from the standard labor supply optimality condition with separable preferences over consumption and labor supply. See Auclert et al. (2020) for details.

[28] Except for δ_H and δ_R and thus their wealth effects in labor supply, I treat spenders and savers entirely symmetrically.

Parameter	Description	Value
α	Wealth preference level	0.10
η	Wealth preference curvature	0.50
δ_R	Savers GHH+ coefficient	0.03
δ_H	Spenders GHH+ coefficient	0.95
μ	Share of spenders	0.25

Table C.1: Mixture model with heterogeneous wealth effects, preference parameterization.

net excess demand path is thus more frontloaded after the interest rate policy, and so now the interest rate cut is slightly more expansionary, as seen in the right panel of Figure C.4.

Summary. My conclusion from the previous two experiments is that cross-sectional heterogeneity in wealth effects in labor supply is unlikely to materially threaten my headline policy equivalence result. However, it is important to note that this takeaway hinges on the equivalent stimulus check policy being moderate in size: by the evidence in Golosov et al. (2021), for very large transfers, we would expect the cross-sectional heterogeneity in labor supply responses to become larger relative to the demand stimulus of the policy (i.e., MPEs are larger relative to MPCs). The fact that equivalent stimulus check policies are moderate in size—the key takeaway of Section 4—is thus an integral part of my argument.

C.6 A recursive perspective on equivalent fiscal rules

The equivalence results for policy rules in Section 3.2 were presented using perfect-foresight notation. I here elaborate on the interpretation of my equivalent rules from a recursive aggregate risk perspective. To build further intuition I also provide a closed-form example of an equivalent explicit transfer rule in a simple overlapping-generations environment.

From perfect-foresight transitions to recursive rules. Recall that my discussion in Section 3.2 was split into two parts: implicit targeting rules and explicit instrument rules. For implicit targeting rules, we saw that the policy rule itself remains the same—only the instrument that is used to *implement* the rule changes. By the proof of Corollary 1, we can write the path of transfers that implements the desired implicit targeting rule as

$$\widehat{\boldsymbol{\tau}} \;=\; \mathcal{C}_\tau^{-1} \times \widehat{\boldsymbol{c}}^{PE} \tag{C.23}$$

where $\widehat{\boldsymbol{c}}^{PE}$ is the net excess demand sequence that ensures aggregate output market-clearing, characterized explicitly in the proof of Corollary 1. What happens in the analogous linearized economy with aggregate risk? Prior work has established that linearized perfect-foresight transition paths are identical to *shock impulse responses*—i.e., conditional expectations—in linearized economies with aggregate risk (Boppart et al., 2018; Auclert et al., 2019). So consider the analogous linearized economy with aggregate risk at its initial date 0, subject to some initial date-0 shocks. It then follows from (C.23) that, to implement her desired implicit targeting rule, the policymaker at date 0 needs to sets current and expected future values of transfers as

$$\mathbb{E}_0\left(\widehat{\boldsymbol{\tau}}^0\right) \;=\; \mathcal{C}_\tau^{-1} \times \mathbb{E}_0\left(\widehat{\boldsymbol{c}}^{0,PE}\right) \tag{C.24}$$

where the notation $\boldsymbol{x}^t = (x_t, x_{t+1}, \dots)'$ indicates time paths from date t onwards. At date 1 additional shocks hit the economy; adding up impulse responses to the initial date-0 shocks and the new date-1 shocks, we see that transfers at date 1 satisfy

$$\mathbb{E}_1\left(\widehat{\boldsymbol{\tau}}^1\right) \;=\; \underbrace{\mathbb{E}_0\left(\widehat{\boldsymbol{\tau}}^1\right)}_{\text{impulse response to date-0 shocks}} + \underbrace{\mathcal{C}_\tau^{-1} \times \left[\mathbb{E}_1\left(\widehat{\boldsymbol{c}}^{1,PE}\right) - \mathbb{E}_0\left(\widehat{\boldsymbol{c}}^{1,PE}\right)\right]}_{\text{impulse response to date-1 shocks}} \tag{C.25}$$

or more compactly

$$\hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{\tau}}^1\right) \;=\; \mathcal{C}_\tau^{-1} \times \hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{c}}^{1,PE}\right) \tag{C.26}$$

where $\hat{\mathbb{E}}_{t,t-1}$ denotes the *change* in expectations between dates t and $t-1$. Continuing recursively, we in general find that transfers in the linearized economy with aggregate risk satisfy the recursion

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) \;=\; \mathcal{C}_\tau^{-1} \times \hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{c}}^{t,PE}\right) \tag{C.27}$$

In words, at each date t, the policymaker revises current and expected future transfers to households to make sure that she generates the time path of current and expected future consumer spending required to implement her desired targeting rule. Starting from date 0, the recursion (C.27) affords a full characterization of the equilibrium sequence of transfers in the linearized economy with aggregate risk.

A similar argument applies to explicit instrument rules. Recall that, for the perfect-foresight transition equilibrium, the equivalent explicit transfer rule takes the form

$$\widehat{\boldsymbol{\tau}} \;=\; \tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}} \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}\right) \tag{C.28}$$

By exactly the same "adding up impulse responses" logic as above, in the analogous linearized economy with aggregate risk, the mapping from macro aggregates to current and expected future policy instruments embedded in (C.28) now needs to be imposed in expectation in response to any macro shock. At date 0 the policymaker thus sets current and expected future transfers as

$$\mathbb{E}_0\left(\widehat{\boldsymbol{\tau}}^0\right) \;=\; \mathbb{E}_0\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^0 + \widehat{\boldsymbol{\ell}}^0) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^0 \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi\widehat{\boldsymbol{\pi}}^0 + \mathcal{B}_y\widehat{\boldsymbol{y}}^0\right)\right] \tag{C.29}$$

Building up the recursion just like was done above, we in general have

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) \;=\; \hat{\mathbb{E}}_{t,t-1}\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^t + \widehat{\boldsymbol{\ell}}^t) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^t \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi\widehat{\boldsymbol{\pi}}^t + \mathcal{B}_y\widehat{\boldsymbol{y}}^t\right)\right] \tag{C.30}$$

(C.30) is an explicit instrument rule in the sense of Giannoni & Woodford (2002): it specifies, at each date t, the current and expected future values of the policy instrument as a function of lagged, current, and expected future values of macro aggregates. Note that the dependence on lagged aggregates is here encoded in the lagged instrument term $\mathbb{E}_{t-1}\left(\widehat{\boldsymbol{\tau}}^t\right)$.

It is important to note that the shape of the mapping from macro aggregates to policy instruments in (C.30) may in general be quite complicated, and in particular can be much more complicated than the policy rules typically considered in recursively specified models with aggregate risk. To see this, consider instead a familiar simple Taylor rule:

$$\widehat{i}_{b,t} \;=\; \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t \tag{C.31}$$

Mapped into my general recursion notation, such a conventional Taylor rule is equivalent to the rule

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{i}}_b^t\right) \;=\; \hat{\mathbb{E}}_{t,t-1}\left[\mathcal{B}_\pi\widehat{\boldsymbol{\pi}}^t + \mathcal{B}_y\widehat{\boldsymbol{y}}^t\right] \tag{C.32}$$

with the two matrices $\mathcal{B}_\pi$ and $\mathcal{B}_y$ given as $\mathcal{B}_\pi = \phi_\pi \times I$ and $\mathcal{B}_y = \phi_y \times I$. In the case of the Taylor rule, the very general recursion (C.32) is equivalent to the single equation (C.31) precisely because the implied matrices $\mathcal{B}_\pi$ and $\mathcal{B}_y$ are very simple, allowing one date-t relation like (C.31) to characterize the potentially complicated time path of expected future interest rates embedded in (C.32). Things are generally not as simple for the equivalent transfer rule (C.30)—the matrix product $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$ need not be proportional to an identity matrix, so current and future transfers may depend in a much more complicated way on current and expected future output and inflation. The next paragraph provides an explicit worked-out example illustrating this discussion using my analytical OLG model.

A simple explicit transfer rule. By (C.30), the recursively written explicit transfer rule that replicates the simple Taylor rule (C.31) sets transfers as

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) = \hat{\mathbb{E}}_{t,t-1}\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^t + \widehat{\boldsymbol{\ell}}^t) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^t + \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\phi_\pi \widehat{\boldsymbol{\pi}}^t + \phi_y \widehat{\boldsymbol{y}}^t\right)\right] \tag{C.33}$$

To further characterize this general rule expression I consider the special case of a simple OLG model, leveraging my closed-form expressions for $\tilde{\mathcal{C}}_{i_b}$ from Appendix C.2. Straightforward algebra reveals that for this model we have that

$$(\mathcal{C}_\tau^{OLG})^{-1} \times (\tilde{\mathcal{C}}_{i_b}^{OLG}) = \frac{1}{\gamma}\frac{1}{\omega}\begin{pmatrix} -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & 0 & 0 & \dots \\ \frac{\theta}{1-\theta} & -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & 0 & \dots \\ 0 & \frac{\theta}{1-\theta} & -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.34}$$

Plugging (C.34) into (C.33), we see that mapping the simple Taylor rule (C.31) into transfer space does indeed result in an almost equally simple explicit transfer-only rule:

$$\widehat{\tau}_t = \underbrace{\tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{\pi}\widehat{\pi}_t}_{\text{deficit response}} + \underbrace{\frac{1}{\gamma\omega}\left[-\frac{\frac{\theta}{1+\bar{r}}}{1-\theta}\left(\phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t\right) + \frac{\theta}{1-\theta}\left(\phi_\pi\widehat{\pi}_{t-1} + \phi_y\widehat{y}_{t-1}\right)\right]}_{\text{active demand management}} \tag{C.35}$$

If transfers are set according to this simple policy rule, then they in the linearized equilibrium with aggregate risk indeed satisfy the general recursion (C.33). Thus, in this particular environment, an interest rate policy that responds to contemporaneous macro aggregates is equivalent to a still quite simple transfer-only policy that responds to current and one-period-lagged aggregates, with the response coefficients given from (C.34). The rule representation in this special case is so simple because the map $(\mathcal{C}_\tau^{OLG})^{-1} \times (\tilde{\mathcal{C}}_{i_b}^{OLG})$—while not proportional to an identity matrix—is nevertheless quite special: it is tridiagonal with repeating rows, so we can summarize the potentially very complicated expectation revisions in (C.33) with just one simple equation, (C.35). This is exactly analogous to the Taylor rule (C.31) being equivalent to the more complicated expression (C.32).

I emphasize, however, that the precise shape of the equivalent explicit transfer-only rule invariably depends on $\tilde{\mathcal{C}}_{i_b}$—an object that is likely to be shaped by forces outside of my particular model, most notably household mortgage refinancing. Those forces may result in a more complicated $\tilde{\mathcal{C}}_{i_b}$, implying that we may in general not be able to collapse the general recursive representation (C.30) to a very simple explicit rule like (C.35).[29] What will not depend on this precise shape, however, is the fact that—given my characterization of $\mathcal{C}_\tau$—practical implementation of the potentially complicated equivalent transfer rule (C.30) will robustly only require moderately sized fluctuations in transfers.

C.7 Other model extensions

I consider three further model extensions: behavioral models of the consumption-savings decision, durable consumption, and a richer network production structure.

Behavioral consumption models. The equivalence result extends without change to standard behavioral models of the household consumption-savings decision. For example, the classical cognitive discounting (Gabaix, 2020) and sticky information (Mankiw & Reis, 2002) models can be fit into my general aggregate consumption function set-up by proceeding as follows. Define $\mathcal{E}$ as a linear map whose (i, j)th entry indicates the fraction of households at time i that are aware of a future shock that will materialize at date j. In the cognitive discounting and sticky information cases, this map is respectively given as

$$\mathcal{E}^{CD} = \begin{pmatrix} 1 & 1-\psi & (1-\psi)^2 & \dots \\ 1 & 1 & 1-\psi & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathcal{E}^{SI} = \begin{pmatrix} 1 & 1-\psi & 1-\psi & \dots \\ 1 & 1 & 1-\psi^2 & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where the coefficient ψ in both cases indicates the severity of the behavioral frictions—cognitive discounting in the first case, and the arrival of information about future shocks in the sticky information case. For any input $q \in \{w, \ell, \pi, d, \tau, i_b\}$ to the aggregate consumption function, we then obtain the behavioral analogue of the consumption derivative matrix, denoted $\mathcal{C}_q^\dagger$, from its full-information analogue $\mathcal{C}_q$ by computing (see Auclert et al., 2019)

$$\mathcal{C}_q^\dagger(t, s) = \sum_{p=1}^{\min(t,s)} (\mathcal{E}_{p,s} - \mathcal{E}_{p-1,s})\mathcal{C}_q(t-p+1, s-p+1) \tag{C.36}$$

I emphasize two main takeaways from this analysis. First, it illustrates my claims in Sec-

[29] Of course that is not surprising. If interest rates and transfers induce spending dynamics with very different time profiles, then simple rules for one instrument (like a Taylor rule) will be equivalent to quite complicated rules for the other.

tion 3.1: the sufficient condition of strong Ricardian non-equivalence is general, and applies in particular also to standard behavioral models of consumption. Second, in quantitatively relevant but behavioral models of consumption (as e.g. in Auclert et al., 2019), an extended version of my sufficient statistics formula applies: by (C.36), $\mathcal{C}_\tau$ is now characterized by my three sufficient statistics $(\omega, \theta, \bar{r})$ (which give the full-information benchmark) together with a fourth statistic (ψ) reflecting the severity of behavioral frictions. Empirical evidence along the lines of Ganong & Noel (2019) could be used to discipline this additional parameter and implement an extended version of my sufficient statistics formula.

Durable goods. The results in this paper extend without change to a model with durables as long as durables and non-durables can be produced costlessly out of some common final good; that is, if real relative prices of the two goods are always 1 and we can write the aggregate resource constraint as

$$y_t = \underbrace{c_t + d_t^h - (1-\delta)d_t^h}_{e_t} \tag{C.37}$$

where e_t is total household expenditure, c_t is non-durables consumption, d_t^h is the stock of durables, and δ is the depreciation rate. Letting $\mathcal{E}_\tau$ denote the analogous derivative map for the response of total spending to lump-sum income, the key condition for my results to extend to this model is that $\mathcal{E}_\tau$ is invertible—strong Ricardian non-equivalence now applied to total spending. The details of the argument are straightforward and thus omitted: interest rate and transfer policies can perturb net excess demand for the common final good equally flexibly and are thus also equivalent in general equilibrium, by exactly the same argument as in the proof of Proposition 1.[30]

I emphasize that the assumptions underlying this extended equivalence result are empirically relevant: relative durable goods prices tend to not respond much to standard business-cycle fluctuations (House & Shapiro, 2008; McKay & Wieland, 2019; Beraja & Wolf, 2020), suggesting that the aggregation to a common aggregate resource constraint (C.37) is sensible. It is furthermore also an assumption made in recent quantitative structural explorations of durable goods spending (e.g., Berger & Vavra, 2015).

[30] If non-durables and durables were not produced out of a common final good (and so their relative prices could fluctuate), then it would of course still be possible to engineer a sequence of transfers that mimics a given interest rate policy's effect on *total* spending. Nothing guarantees, however, that the *composition* of that spending would be the same. If relative prices can move then the composition will matter in general equilibrium, thus breaking equivalence.

Network production. The policy equivalence result leverages properties of consumer spending behavior and as such is robust to many different possible model extensions on the production side of the economy. I here provide one illustration using a simple model of roundabout production (e.g., see Phaneuf et al., 2018).

Differently from my baseline model, intermediate goods firms now produce using both labor as well as the intermediate good itself, with production function

$$y_t = q_t^{\phi} \ell_t^{(1-\alpha)(1-\phi)}$$

where $\phi \in [0,1)$ denotes the share of intermediates in production. A standard cost minimization problem gives marginal costs as

$$mc_t = \left(\frac{1}{(1-\alpha)(1-\phi)\phi^{\frac{\phi}{1-\phi}}} \right)^{1-\phi} w_t^{1-\phi} \ell_t^{\alpha(1-\phi)}$$

and so, in log deviations,

$$\widehat{mc}_t = (1-\phi)\left(\widehat{w}_t + \alpha \widehat{\ell}_t\right)$$

Following the same steps as in the derivation of (B.1) we thus find that

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}})}{\theta_p} \frac{(1-\alpha)(1-\phi)}{1-\alpha+\alpha\varepsilon_p} \left(\widehat{w}_t + \alpha\widehat{\ell}_t\right) + \beta \widehat{\pi}_{t+1} \tag{C.38}$$

The only effect of roundabout production is thus to flatten the price-NKPC, leaving the headline policy equivalence result entirely unchanged.

C.8 Targeted transfers

My analysis throughout was focussed on *uniform* lump-sum taxes and transfers. This was by design: my objective was to establish that, in standard models of non-Ricardian consumption behavior, manipulating taxes and transfers *over time* can manipulate spending just like changes in intertemporal prices—that is, stimulus checks are stimulative even without any redistribution. In models with microeconomic heterogeneity (like HANK), it is of course also possible to consider transfer policies aimed at sub-populations of households and thus (in part) operational through redistribution. My results extend with little change to such alternative policy experiments.

Recall from the proof of Proposition 1 that the key requirement for policy equivalence is that, for ay excess demand sequence $\widehat{\boldsymbol{c}}$ with zero net present value, we can find a transfer policy that induces a net excess demand path of $\widehat{\boldsymbol{c}}$. To see how this can be done using targeted transfers, consider a transfer targeted at some subgroup of households (group a) and financed using taxes on another subgroup (group b). I denote the transfer to group a by $\widehat{\boldsymbol{\tau}}^x$ and write the corresponding tax financing as $\widehat{\boldsymbol{\tau}}^e \equiv \mathcal{T}_\tau \widehat{\boldsymbol{\tau}}^x$, where $\mathcal{T}_\tau$ is such that

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t (\widehat{\tau}_t^x + \widehat{\tau}_t^e(\widehat{\boldsymbol{\tau}}^x)) = 0$$

Letting $\mathcal{C}_\tau^{(a)}$ and $\mathcal{C}_\tau^{(b)}$ denote the consumption derivative matrices for subgroups a and b, respectively, the effect of any given transfer policy $\widehat{\boldsymbol{\tau}}^x$ on net excess demand is given as

$$\underbrace{\left(\mathcal{C}_\tau^{(a)} + \mathcal{C}_\tau^{(b)} \mathcal{T}_\tau\right)}_{\equiv \mathcal{C}_\tau^x} \widehat{\boldsymbol{\tau}}^x$$

Analogously to the proof of Proposition 1, a sufficient condition for policy equivalence is now simply that every net excess demand path with zero net present value lies in the image of $\mathcal{C}_\tau^x$. Differently from my main analysis, characterizing $\mathcal{C}_\tau^x$ does not require MPCs averaged across the entire household cross-section, but MPCs averaged across the subgroups a and b.

For practical policy purposes, there are two key differences between my uniform policies and such targeted policies. First, the latter also work explicitly through redistribution across households, and thus in particular can affect net excess demand even with period-by-period budget balance and without any fluctuations in aggregate government debt. Second, it is unclear ex ante whether targeted transfers need to be larger or smaller in per capita terms. On the one hand, to engineer a given spending response by targeting a smaller group of households, the required transfer size per capita increases mechanically. On the other hand, if targeted households have larger MPCs, the required transfer decreases in size. I leave a detailed characterization of such macro-equivalent targeted transfers to future work.

D Proofs and auxiliary lemmas

D.1 Proof of Corollary 1

I begin with some preliminary simplifications. The non-policy block of the economy can be summarized by the following system of equations, now written in compact sequence-space notation (as in Auclert et al., 2019). First, the price-NKPC,

$$\widehat{\boldsymbol{\pi}} = \Pi_w \widehat{\boldsymbol{w}} + \Pi_\ell \widehat{\boldsymbol{\ell}} + \beta \widehat{\boldsymbol{\pi}}_{+1} + \boldsymbol{\varepsilon}^p$$

Second, the production function

$$\widehat{\boldsymbol{y}} = \mathcal{Y}_\ell \widehat{\boldsymbol{\ell}}$$

Third, firm dividends,

$$\widehat{\boldsymbol{d}} = \mathcal{D}_y \widehat{\boldsymbol{y}} + \mathcal{D}_w \widehat{\boldsymbol{w}} + \mathcal{D}_\ell \widehat{\boldsymbol{\ell}}$$

Fourth, consumer demand

$$\widehat{\boldsymbol{c}} = \mathcal{C}_w \widehat{\boldsymbol{w}} + \mathcal{C}_\ell \widehat{\boldsymbol{\ell}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{C}_d \widehat{\boldsymbol{d}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}} + \mathcal{C}_{i_b} \widehat{\boldsymbol{i}}_b + \mathcal{C}_c \boldsymbol{\varepsilon}^c$$

Fifth, the wage-NKPC,

$$\widehat{\boldsymbol{\ell}} = \mathcal{L}_w \widehat{\boldsymbol{w}} + \mathcal{L}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{L}_c \widehat{\boldsymbol{c}} + \boldsymbol{\varepsilon}^w$$

And sixth, the output market-clearing condition

$$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{c}}$$

Using the price-NKPC, the production function, the equation for firm dividends, and the output market-clearing condition, we can substitute out $\{\boldsymbol{c}, \boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{d}\}$ in the consumer demand relation. This gives

$$\bar{\mathcal{C}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b + \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}} + \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \tag{D.1}$$

where $\bar{\mathcal{C}}_{i_b} = \mathcal{C}_{i_b}$, $\bar{\mathcal{C}}_\tau = \mathcal{C}_\tau$, $\bar{\mathcal{C}}_c = \mathcal{C}_c$ and $\{\bar{\mathcal{C}}_y, \bar{\mathcal{C}}_\pi\}$ are functions of model primitives. Similarly, using the price-NKPC as well as the production function and the output market-clearing condition, we can substitute out $\{\boldsymbol{\ell}, \boldsymbol{w}, \boldsymbol{c}\}$ in the wage-NKPC to write it as

$$\bar{\mathcal{L}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{L}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \tag{D.2}$$

where $\{\bar{\mathcal{L}}_y, \bar{\mathcal{L}}_\pi, \bar{\mathcal{L}}_p, \bar{\mathcal{L}}_w\}$ are functions of model primitives.

Now consider first the case where the desired implicit targeting rule (19) is implemented using an interest rate-only policy. Plugging the price-NKPC and the production function into the financing rule (8), we can write the financing rule compactly as

$$\widehat{\boldsymbol{\tau}} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}} + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b \tag{D.3}$$

where $\{\bar{\mathcal{T}}_y, \bar{\mathcal{T}}_\pi, \bar{\mathcal{T}}_{i_b}\}$ are functions of the financing rule (8) and model primitives. Using the simplifications in (D.1), (D.2) and (D.3), we can write the equilibrium system as the following stacked linear system:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_y & \mathcal{B}_\pi & \mathbf{0} & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{D.4}$$

By assumption, the system (D.4) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the question of whether the same implicit targeting rule can be implemented using a transfer-only policy. Using the simplifications from above, we can write the equilibrium system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_y & \mathcal{B}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{D.5}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (D.5). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where $\widehat{\boldsymbol{\tau}}^{**}$ solves

$$\left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* + \bar{\mathcal{C}}_\tau \left(\bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^*\right) = \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**}$$

We know by the conditions of Proposition 1 (which recall are assumed for Corollary 1) that such a $\widehat{\boldsymbol{\tau}}^{**}$ exists. Plugging into (D.1), we get

$$\bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c$$

$$\Leftrightarrow \quad \left(\bar{\mathcal{C}}_y - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y\right) \widehat{\boldsymbol{y}}^* + \left(\bar{\mathcal{C}}_\pi - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi\right) \widehat{\boldsymbol{\pi}}^* - \left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c$$
$$\Leftrightarrow \quad \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c$$

Thus (D.1) still holds. It is immediate that all other relations in (D.5) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (D.5).

To show uniqueness, suppose for a contraction that (D.5) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. By the assumptions of Proposition 1 we can thus find a bounded tuple $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^\dagger + \bar{\mathcal{C}}_\tau \left(\bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger\right) = \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^\dagger$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^\dagger$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (D.4). Contradiction. □

D.2 Proof of Corollary 2

Proceeding as in the proof of Corollary 1, we arrive at the following equilibrium system for the explicit interest rate rule:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ -\mathcal{B}_y & -\mathcal{B}_\pi & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{D.6}$$

By assumption, the system (D.6) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the equilibrium system corresponding to the proposed transfer-only rule (22). Using the simplifications from above, we can write that system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y - \mathcal{C}_\tau^{-1} \mathcal{C}_{i_b} \mathcal{B}_y & -\bar{\mathcal{T}}_\pi - \mathcal{C}_\tau^{-1} \mathcal{C}_{i_b} \mathcal{B}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{D.7}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (D.7). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where

$$\widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^* + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^* + \mathcal{C}_\tau^{-1} \mathcal{C}_{i_b} \widehat{\boldsymbol{i}}_b^*$$

Plugging the candidate solution into the consumer demand function (D.1), we get

$$\begin{aligned}
& \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\
\Leftrightarrow \quad & \left(\bar{\mathcal{C}}_y - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y\right) \widehat{\boldsymbol{y}}^* + \left(\bar{\mathcal{C}}_\pi - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi\right) \widehat{\boldsymbol{\pi}}^* - \left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\
\Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c
\end{aligned}$$

Thus (D.1) still holds. It is immediate that all other relations in (D.7) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (D.7).

To show uniqueness, suppose for a contraction that (D.7) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. Now consider the tuple $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\widehat{\boldsymbol{i}}_b^\dagger = \mathcal{B}_y \widehat{\boldsymbol{y}}^\dagger + \mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^\dagger$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^\dagger$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (D.6). Contradiction. □

D.3 Proof of Proposition 2

I consider each of the three analytical models in turn.

1. *Spender-saver model.* It is straightforward to verify that, expressed in terms of μ and $\bar{r}$, the inverse $(\mathcal{C}_\tau^H)^{-1}$ satisfies

$$(\mathcal{C}_\tau^H)^{-1} = \begin{pmatrix} \frac{1-(1-\mu)\frac{\bar{r}}{1+\bar{r}}}{\mu} & -\frac{1-\mu}{\mu}\frac{\bar{r}}{(1+\bar{r})^2} & -\frac{1-\mu}{\mu}\frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ -\frac{1-\mu}{\mu}\frac{\bar{r}}{1+\bar{r}} & \frac{1-(1-\mu)\frac{\bar{r}}{(1+\bar{r})^2}}{\mu} & -\frac{1-\mu}{\mu}\frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ -\frac{1-\mu}{\mu}\frac{\bar{r}}{1+\bar{r}} & -\frac{1-\mu}{\mu}\frac{\bar{r}}{(1+\bar{r})^2} & \frac{1-(1-\mu)\frac{\bar{r}}{(1+\bar{r})^3}}{\mu} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{D.8}$$

   This completes the constructive proof: as long as $\mu \in (0, 1]$, this linear map is well-defined and maps bounded sequences of $\widehat{\boldsymbol{c}}$ into bounded sequences of $\widehat{\boldsymbol{\tau}}$.

2. *Overlapping-generations model.* I will show that, for every bounded sequence of consumption $\widehat{\boldsymbol{c}}^*$, there exists a bounded sequence of transfers $\widehat{\boldsymbol{\tau}}^*$ that induces this sequence of consumption, thus establishing invertibility. From the Euler equation (C.3) it follows that we must have

$$\widehat{\tau}_t^* + \frac{1}{\beta}\widehat{b}_{t-1}^* = \frac{[1-\theta(1-\beta\theta)]\widehat{c}_t^* - \beta\theta\widehat{c}_{t+1}^*}{(1-\beta\theta)(1-\theta)}$$

Since $\widehat{b}_{-1} = 0$ this expression already gives $\widehat{\tau}_0^*$ (as long as $\theta < 1$, as assumed). Next we get from the budget constraint (C.2) that

$$\widehat{b}_t^* = \frac{\beta\theta}{(1-\beta\theta)(1-\theta)}(\widehat{c}_t^* - \widehat{c}_{t+1}^*)$$

Combining this with the expression for $\widehat{\tau}_t^*$ above we find that, for $t > 0$,

$$\widehat{\tau}_t^* = \frac{-\theta\widehat{c}_{t-1}^* + (1+\beta\theta^2)\widehat{c}_t^* - \beta\theta\widehat{c}_{t+1}^*}{(1-\beta\theta)(1-\theta)}$$

establishing the claim. Collecting coefficients, we obtain the closed-form expression (C.5).

3. *Bond-in-utility model.* I will proceed as I did for the overlapping-generations model. From the Euler equation (C.7) it now follows that we must have

$$\widehat{b}_t^* = \frac{1}{\frac{\eta}{\gamma}[1-\beta(1+\bar{r})]}\left[\widehat{c}_t^* - \beta(1+\bar{r})\widehat{c}_{t+1}^*\right]$$

As long as $\alpha, \eta > 0$ we have that $\beta(1+\bar{r}) < 1$, so the denominator is positive and the expression is well-defined. Next we get from the budget constraint (C.6) that

$$\widehat{y}_t^* = \widehat{c}_t^* + \widehat{b}_t^* - (1+\bar{r})\widehat{b}_{t-1}^*$$

Thus we have that

$$\begin{aligned}\widehat{\tau}_t^* = \widehat{c}_t^* &+ \frac{1}{\frac{\eta}{\gamma}[1-\beta(1+\bar{r})]}\left[\widehat{c}_t^* - \beta(1+\bar{r})\widehat{c}_{t+1}^*\right] \\ &- (1+\bar{r})\frac{1}{\frac{\eta}{\gamma}[1-\beta(1+\bar{r})]}\left[\widehat{c}_{t-1}^* - \beta(1+\bar{r})\widehat{c}_t^*\right], \quad t = 1, 2, \ldots\end{aligned}$$

as well as

$$\widehat{\tau}_0^* = \widehat{c}_0^* + \frac{1}{\frac{\eta}{\gamma}[1-\beta(1+\bar{r})]}\left[\widehat{c}_0^* - \beta(1+\bar{r})\widehat{c}_1^*\right]$$

establishing the claim.

□

Finally I note that the above arguments also immediately imply invertibility whenever an overlapping-generations or bond-in-utility matrix is combined with a margin of spenders, as done in my sufficient statistics formula:

$$\mathcal{C}_\tau = (1-\mu)\times\mathcal{C}_\tau^{OLG} + \mu\times I, \quad \text{or} \quad \mathcal{C}_\tau = (1-\mu)\times\mathcal{C}_\tau^{BiU} + \mu\times I, \quad \mu\in(0,1)$$

The key step in the argument is to note that the eigenvalues of $\mathcal{C}_\tau^{OLG}$ and $\mathcal{C}_\tau^{BiU}$ are all strictly positive.[31] Since the eigenvalues of $\mathcal{C}_\tau$ are equal to $\mu>0$ plus $(1-\mu)$ times the eigenvalues of $\mathcal{C}_\tau^{OLG}$ or $\mathcal{C}_\tau^{BiU}$, it follows that $\mathcal{C}_\tau$ is also invertible.

D.4 Proof of Proposition 3

By assumption, the allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$ is implementable using an interest rate-only policy tuple $\{i_{b,t}^*, \tau_t^*, 0\}_{t=0}^\infty$. Now consider the alternative policy tuple $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \widehat{\tau}_{f,t}^\dagger\}_{t=0}^\infty$ where

$$\widehat{\boldsymbol{\tau}}_f^\dagger = \mathcal{I}_{\tau_f}^{-1}\mathcal{I}_{i_b}\widehat{\mathbf{i}}_b^* \tag{D.9}$$

and

$$\widehat{\boldsymbol{\tau}}^\dagger = \mathcal{C}_\tau^{-1}\left[\mathcal{C}_{i_b}\widehat{\mathbf{i}}_b^* + \mathcal{C}_d\left(\mathcal{D}_{i_b}\widehat{\mathbf{i}}_b^* - \mathcal{D}_{\tau_f}\widehat{\boldsymbol{\tau}}_f^\dagger\right)\right] \tag{D.10}$$

I now claim that this policy tuple similarly engineers the allocation $\{\pi_t^*, y_t^*\}_{t=0}^\infty$. First, with $\widehat{\boldsymbol{\tau}}_f^\dagger$ set as in (D.9), the investment, output and labor demand paths are unchanged; however, as remarked in Appendix B.7, the dividend paths may be different. The transfer path $\widehat{\boldsymbol{\tau}}^\dagger$ is constructed to offset both the missing monetary stimulus as well as neutralize any potential dividend-related effects: to see this, note that we have

$$\begin{aligned}\widehat{\mathbf{c}}^\dagger &= \mathcal{C}_\tau(\widehat{\boldsymbol{\tau}}^* + \widehat{\boldsymbol{\tau}}^\dagger) + \mathcal{C}_d\mathcal{D}_{\tau_f}\widehat{\boldsymbol{\tau}}_f^\dagger + \text{non-policy terms} \\ &= \mathcal{C}_\tau\widehat{\boldsymbol{\tau}}^* + \mathcal{C}_{i_b}\widehat{\mathbf{i}}_b^* + \mathcal{C}_d\mathcal{D}_{i_b}\widehat{\mathbf{i}}_b^* + \text{non-policy terms} = \widehat{\mathbf{c}}^*\end{aligned}$$

Next note that, since they induce the same paths of consumption, investment, hours worked and production, and since by assumption wages are unchanged, the initial policy $\{i_{b,t}^*, \tau_t^*, 0\}_{t=0}^\infty$

[31] This follows from my closed-form expressions for $(\mathcal{C}_\tau^{OLG})^{-1}$ and $(\mathcal{C}_\tau^{BiU})^{-1}$ in Appendix C.1: both matrices are positive definite (they are both diagonally dominant with positive diagonals) so all their eigenvalues are strictly positive, and so the same is true for $\mathcal{C}_\tau^{OLG}$ and $\mathcal{C}_\tau^{BiU}$.

and the new policy $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \widehat{\tau}_{f,t}^\dagger\}_{t=0}^\infty$ have the same present value in the augmented government budget constraint (B.22), exactly as in the proof of Proposition 1. With $\lim_{t\to\infty} \widehat{b}_t = 0$ in the initial equilibrium, it then follows that we must also have $\lim_{t\to\infty} \widehat{b}_t = 0$ in the new one, as required. All other model equations are unaffected, so the guess is verified. □

D.5 Proof of Proposition C.1

I will guess and verify that lagged wealth is the only endogenous state, so the decision rules take the general form

$$
\begin{aligned}
\widehat{c}_t &= \vartheta_{cb}\widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{cyh}\widehat{\tau}_{t-h} \\
\widehat{b}_t &= \vartheta_{bb}\widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{byh}\widehat{\tau}_{t-h}
\end{aligned}
$$

Plugging into the optimality conditions (C.2) - (C.3) and matching coefficients, we get the following system of equations characterizing behavior in response to an anticipated income shock H periods into the future:

$$
\begin{aligned}
\vartheta_{cb} + \vartheta_{bb} - \frac{1}{\beta} &= 0 && \text{(D.11)} \\
\vartheta_{c\tau h} + \vartheta_{b\tau h} &= 0, \quad h = 0, 1, \ldots, H-1 && \text{(D.12)} \\
\vartheta_{c\tau H} + \vartheta_{b\tau H} &= 1 && \text{(D.13)} \\
[1 - \theta(1-\beta\theta)]\, \vartheta_{cb} - \beta\theta\vartheta_{cb}\vartheta_{bb} - (1-\beta\theta)(1-\theta)\frac{1}{\beta} &= 0 && \text{(D.14)} \\
[1 - \theta(1-\beta\theta)]\, \vartheta_{c\tau h} - \beta\theta\left[\vartheta_{cb}\vartheta_{b\tau h} + \vartheta_{c\tau h+1}\right] &= 0, \quad h = 0, 1, \ldots, H-1 && \text{(D.15)} \\
[1 - \theta(1-\beta\theta)]\, \vartheta_{c\tau H} - \beta\theta\vartheta_{cb}\vartheta_{b\tau H} &= (1-\beta\theta)(1-\theta) && \text{(D.16)}
\end{aligned}
$$

I will begin by characterizing the solution of this system. From (D.11) and (D.14) we have

$$
\begin{aligned}
\vartheta_{bb} &= \theta \\
\vartheta_{cb} &= \frac{1}{\beta} - \theta
\end{aligned}
$$

Next, from (D.13) and (D.16), we have that

$$
\vartheta_{c\tau H} = 1 - \beta\theta
$$

$$\vartheta_{b\tau H} \quad = \quad \beta\theta$$

Finally, from (D.12) and (D.15),

$$\begin{aligned}
\vartheta_{c\tau h} &= \beta\theta\vartheta_{c\tau h+1} = (\beta\theta)^{H-h}(1-\beta\theta) \\
\vartheta_{b\tau h} &= -\beta\theta\vartheta_{c\tau h+1} = -(\beta\theta)^{H-h}(1-\beta\theta)
\end{aligned}$$

This characterizes the full solution.

I can now prove the various asymptotic statements of Proposition C.1. First we have that

$$\widehat{c}_{H,H} = \vartheta_{cb} \sum_{\ell=0}^{H-1} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H}$$

Plugging in from the closed-form expressions above and simplifying:

$$\lim_{H\to\infty} c_{H,H} = \frac{(1-\theta)(1-\beta\theta)}{1-\beta\theta^2} = \text{const.}$$

Next looking below the main diagonal:

$$\widehat{c}_{H+1,H} = \vartheta_{cb} \sum_{\ell=0}^{H} \vartheta_{bb}^{\ell} \vartheta_{d\tau H-\ell}$$

and so

$$\frac{1}{\theta}\widehat{c}_{H+1,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}(\vartheta_{bb}\widehat{b}_{H-1,H} + \vartheta_{b\tau H}) = \vartheta_{cb}\widehat{b}_{H-1,H} + (1-\theta\beta) = \widehat{c}_{H,H}$$

Similarly

$$\frac{1}{\theta}\widehat{c}_{H+\ell,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}\vartheta_{bb}\widehat{b}_{H+\ell-2,H} = \widehat{c}_{H+\ell-1,H}$$

The proof reveals that the result holds for any H, not just $H \to \infty$.

Finally I look above the main diagonal. Here we have

$$\widehat{c}_{H-1,H} = \vartheta_{cb} \sum_{\ell=0}^{H-2} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-2} + \vartheta_{cyH-1}$$

We thus have that

$$\frac{1}{\beta\theta}\widehat{c}_{H-1,H} = \vartheta_{cb} \sum_{\ell=0}^{H-2} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H} = \widehat{c}_{H,H} - \vartheta_{cb}\vartheta_{bb}^{H-1}\vartheta_{b\tau 0}$$

The last term goes to zero as $H \to \infty$. Similarly we have

$$\widehat{c}_{H-\ell,H} = \vartheta_{cb} \sum_{\ell=0}^{H-\ell-1} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-2} + \vartheta_{c\tau H-\ell}$$

and so

$$\frac{1}{\beta\theta} \widehat{c}_{H-\ell,H} = \vartheta_{cb} \sum_{\ell=0}^{H-\ell-1} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H-\ell+1} = \widehat{c}_{H-\ell+1,H} - \vartheta_{cb} \vartheta_{bb}^{H-\ell} \vartheta_{b\tau 0}$$

where the last term again goes to zero as $H \to \infty$, completing the argument. □

D.6 Auxiliary lemmas for the proof of Proposition C.2

I will guess and verify that lagged wealth is the only endogenous state, so the decision rules take the general form

$$\begin{aligned}
\widehat{c}_t &= \vartheta_{cb} \widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{cyh} \widehat{\tau}_{t-h} \\
\widehat{b}_t &= \vartheta_{bb} \widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{byh} \widehat{\tau}_{t-h}
\end{aligned}$$

Plugging into the optimality conditions (C.6) - (C.7) and matching coefficients, we get the following system of equations characterizing behavior in response to an anticipated income shock H periods into the future:

$$\begin{aligned}
\vartheta_{cb} + \vartheta_{bb} - (1+\bar{r}) &= 0 && \text{(D.17)} \\
\vartheta_{c\tau h} + \vartheta_{b\tau h} &= 0, \quad h = 0, 1, \ldots, H-1 && \text{(D.18)} \\
\vartheta_{c\tau H} + \vartheta_{b\tau H} &= 1 && \text{(D.19)} \\
\vartheta_{cb} - \beta(1+\bar{r}) \vartheta_{cb} \vartheta_{bb} - \frac{\eta}{\gamma} \left[1 - \beta(1+\bar{r})\right] \vartheta_{bb} &= 0 && \text{(D.20)} \\
\vartheta_{c\tau h} - \beta(1+\bar{r}) \left[\vartheta_{cb} \vartheta_{b\tau h} + \vartheta_{c\tau h+1}\right] - \frac{\eta}{\gamma} \left[1 - \beta(1+\bar{r})\right] \vartheta_{b\tau h} &= 0, \quad h = 0, 1, \ldots, H-1 && \text{(D.21)} \\
\vartheta_{c\tau H} - \beta(1+\bar{r}) \vartheta_{cb} \vartheta_{b\tau H} - \frac{\eta}{\gamma} \left[1 - \beta(1+\bar{r})\right] \vartheta_{b\tau H} &= 0 && \text{(D.22)}
\end{aligned}$$

I now state and prove two useful properties of this system.

Lemma D.1. *Consider the system* (D.17) - (D.22)*. The solution satisfies*

$$\frac{\vartheta_{cb}}{\vartheta_{bb}}\vartheta_{b\tau H} = \vartheta_{c\tau H} \tag{D.23}$$

Proof. By (D.20) we have that

$$\frac{\vartheta_{cb}}{\vartheta_{bb}} = \beta(1+\bar{r})\vartheta_{cb} + \frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]$$

(D.22) furthermore gives

$$\frac{\vartheta_{c\tau H}}{\vartheta_{b\tau H}} = \beta(1+\bar{r})\vartheta_{cb} + \frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]$$

Combining the two, the claim follows. □

Lemma D.2. *Consider the system* (D.17) - (D.22)*. The solution satisfies*

$$\frac{\vartheta_{c\tau h}}{\vartheta_{c\tau h+1}} = \beta\vartheta_{bb}, \quad h = 0, 1, \ldots, H-1 \tag{D.24}$$

and

$$\frac{\vartheta_{b\tau h}}{\vartheta_{b\tau h+1}} = \beta\vartheta_{bb}, \quad h = 0, 1, \ldots, H-2 \tag{D.25}$$

Proof. (D.18) and (D.21) together imply that

$$\vartheta_{c\tau h} = \frac{\beta(1+\bar{r})}{1+\beta(1+\bar{r})\vartheta_{cb} + \frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]}\vartheta_{c\tau h+1}$$

(D.17) and (D.20) furthermore give

$$\vartheta_{bb} = \frac{(1+\bar{r})}{1+\beta(1+\bar{r})\vartheta_{cb} + \frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]}$$

The first part of the statement thus follows. Finally, since we for all $h = 0, 1, \ldots, H-2$ have that

$$\frac{\vartheta_{c\tau h}}{\vartheta_{c\tau h+1}} = \frac{\vartheta_{b\tau h}}{\vartheta_{b\tau h+1}}$$

the second part follows as well. □

D.7 Proof of Proposition C.2

The proof of Proposition C.2 relies on Lemmas D.1 and D.2. My derivations below furthermore leverage the fact that the impulse response of wealth holdings is given as

$$\widehat{b}_{t,H} = \sum_{\ell=0}^{t} \vartheta_{bb}^{\ell} \vartheta_{b\tau t-\ell}$$

I begin with the MPC at time H. Here we have that

$$\widehat{c}_{H,H} = \vartheta_{cb} \sum_{\ell=0}^{H-1} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H}$$

Plugging in from the closed-form expressions above and simplifying:

$$\lim_{H\to\infty} \widehat{c}_{H,H} = \frac{\vartheta_{cb}}{1-\beta\vartheta_{bb}^2} \vartheta_{byH-1} + \vartheta_{c\tau H}$$

But by (D.18) - (D.19) and (D.21) - (D.22) $\vartheta_{b\tau H-1}$ and $\vartheta_{c\tau H}$ are actually independent of H, so $\widehat{c}_{H,H}$ converges to some constant.

Next I look below the main diagonal. Here we have that

$$\widehat{c}_{H+1,H} = \vartheta_{cb} \sum_{\ell=0}^{H} \vartheta_{bb}^{\ell} \vartheta_{d\tau H-\ell}$$

Now let $\theta \equiv \vartheta_{bb}$. Then we have

$$\frac{1}{\theta}\widehat{c}_{H+1,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}(\vartheta_{bb}\widehat{d}_{H-1,H} + \vartheta_{b\tau H}) = \vartheta_{cb}\widehat{d}_{H-1,H} + \vartheta_{c\tau H} = \widehat{c}_{H,H}$$

Similarly

$$\frac{1}{\theta}\widehat{c}_{H+\ell,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}\vartheta_{bb}\widehat{d}_{H+\ell-2,H} = \widehat{c}_{H+\ell-1,H}$$

I have thus proven the half of (C.8) below the main diagonal. The proof reveals that the result holds for any H, not just $H \to \infty$.

Finally I look above the main diagonal. Here we have

$$\widehat{c}_{H-1,H} = \vartheta_{cb} \sum_{\ell=0}^{H-2} \vartheta_{bb}^{\ell} \vartheta_{b\tau H-\ell-2} + \vartheta_{c\tau H-1}$$

We thus have that

$$\frac{1}{\beta\theta}\widehat{c}_{H-1,H} = \vartheta_{cb}\sum_{\ell=0}^{H-2}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H} = \widehat{c}_{H,H} - \vartheta_{cb}\vartheta_{bb}^{H-1}\vartheta_{b\tau 0}$$

But with ϑ_{bb} inside the unit circle—the stable solution of the system (D.17) and (D.20)—it follows that second term converges to zero for $H \to \infty$, establishing the claim.[32] Proceeding analogously for any $\ell > 0$, we have

$$\widehat{c}_{H-\ell,H} = \vartheta_{cb}\sum_{\ell=0}^{H-\ell-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-2} + \vartheta_{c\tau H-\ell}$$

and so

$$\frac{1}{\beta\theta}\widehat{c}_{H-\ell,H} = \vartheta_{cb}\sum_{\ell=0}^{H-\ell-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H-\ell+1} = \widehat{c}_{H-\ell+1,H} - \vartheta_{cb}\vartheta_{bb}^{H-\ell}\vartheta_{b\tau 0}$$

thus completing the proof of the half of (C.8) above the main diagonal. □

[32] This argument leverages the fact that $\theta_{b\tau 0} \to 0$, which follows from Lemma D.2 together with the fact θ_{bb} is inside the unit circle.