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# Crises: Equilibrium Shifts and Large Shocks†

*By* Stephen Morris and Muhamet Yildiz*

*We study the informational events that trigger equilibrium shifts in coordination games with incomplete information. Assuming that the distribution of the changes in fundamentals has fat tails, we show that majority play shifts* either *if fundamentals reach a critical threshold or if there are large common shocks, even before the threshold is reached. The fat-tail assumption matters because it implies that large shocks make players more unsure about whether their payoffs are higher than others. This feature is crucial for large shocks to matter.* (*JEL* C72, C73, D83)

On July 26th, 2012, Mario Draghi gave a speech in which he promised "… to do whatever it takes to preserve the euro. And believe me, it will be enough …."[1] Many commentators have credited the "whatever it takes …" speech with shifting the Eurozone economy from a self-fulfilling "bad equilibrium"—with high sovereign debt spreads and growing fiscal deficits mutually reinforcing each other, to a self-fulfilling "good equilibrium"—with low spreads and sustainable fiscal policy.[2] There are many other economic and social contexts in which strategic complementarities are thought to give rise to the possibility of self-fulfilling equilibria in the form of currency crises, economic booms, financial panics, and revolutions.

In this paper we ask which informational events trigger a shift in self-fulfilling equilibria, such as a crisis and a recovery? We identify two distinct kinds of informational events that can trigger a crisis (or, symmetrically, a recovery). First, fundamentals fall below some critical threshold. Second, fundamentals deteriorate sharply to a level where they are somewhat weak, but still better than the critical threshold guaranteeing a crisis. The first trigger is a *level* effect: independent of

* Morris: Department of Economics, Princeton University, Princeton, NJ 08544 (email: smorris@princeton.edu); Yildiz: Department of Economics, Massachusetts Institute of Technology, The Morris and Sophie Chang Building, 50 Memorial Drive, Cambridge, MA 02142 (email: myildiz@mit.edu). Jeff Ely was the coeditor for this article. We are grateful for comments from three anonymous referees, George-Marios Angeletos, Andy Atkeson, Markus Brunnermeier, Tianjiao Dai, Ulrich Mueller, Thi Mai Anh Nguyen, Alessandro Pavan, Elia Sartori, Hyun-Song Shin, Laura Veldkamp, Ming Yang, and participants at 2015 Time, Uncertainties, and Strategies Workshop (PSE), Global Games Conference (Iowa State), 2017 Cowles Foundation Conference on Macroeconomics (Yale), 2017 Econometric Society North American Summer Meeting (St. Louis), 2018 AEA Meeting (Philadelphia) and seminars at Boston College, Cambridge University, Central European University, LSE, MIT, Oxford, Princeton, Rochester, Science Po, Stanford, University of Bristol, the University of Chicago, UCL, UCLA, UCSD, UPenn, University of Wisconsin-Madison, Warwick, and Yale. The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

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[1] See https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html for a transcript of the full speech.

[2] See Brunnermeier, James, and Landau (2016) for one discussion.

whether fundamentals are worse than expected, or by how much, sufficiently bad fundamentals trigger a crisis. The second trigger is a *change* effect and corresponds to the main large shock result of the paper. Over a wide range of levels of fundamentals, it is the size of the negative shocks that moved the economy to that level of fundamental that determines whether a crisis is triggered.

We consider a canonical coordination game with a continuum of players making a binary choice between a "good" action, namely "invest," and a "bad" action, namely "not invest." The payoff from not investing is normalized to zero. Each player's payoff from investing is increasing in the proportion of others investing and also increasing in a fundamental state, which we call "return." There is incomplete information about returns. Each player's return from investing is the sum of two components, a common component that affects every player's return, and an idiosyncratic noise term. Each player observes his own return but cannot identify the common component of fundamentals. Our key assumption is that the common component of fundamentals has fat tails (the density of the tails exceeds a power law distribution), while the idiosyncratic noise has thinner tails.

This assumption has a key statistical implication: a player who observes a large shock will be convinced that the shock is mainly due to the common component of fundamentals. In particular, the level of his shock will tell a player little about the size of the idiosyncratic component of his return. Thus a player who observes a large shock will not know whether his return is higher than other players' returns. He will have nearly "uniform rank beliefs": if asked what his rank, or percentile, in the population is with respect to his return, any percentile between 0 and 1 will be (nearly) equally likely. Thus large shocks create diffuse beliefs about what other players' information is, our key statistical observation.

Uniform rank beliefs pin down strategic behavior. For a player who has not observed a large shock and whose rank belief is not nearly uniform, invest can be rationalized for a wide range of returns by the belief that others' returns are higher than his. But consider a player who has experienced a large negative shock, and thus has nearly uniform rank beliefs and believes that other players are investing if and only if they have a higher return than him. This player would have a nearly uniform belief on the proportion of other players who are investing. He would invest only if invest is "risk dominant," in that it is a best response to a uniform belief over the proportion of his opponents choosing each action. This implies that invest can be rationalizable after the player observes a large negative shock only if it is risk dominant. Thus if invest is not risk dominant and there is a large negative shock, not invest is uniquely rationalizable. This is the large shock result.

Our large shock result uses the key argument from the "global games" literature (Carlsson and Van Damme 1993) but in a novel context, and it is useful to contrast the results. A classic benchmark result in the global game literature is the following. Suppose that players observe the payoffs of a game drawn according to a smooth prior, with a small amount of idiosyncratic noise. Look at the equilibria of the sequence of games as the amount of noise goes to zero. In the limit as the noise goes to zero, there is "global uniqueness": each player has a unique rationalizable action whatever signal she observes. In a binary action symmetric payoff game, the unique rationalizable action is the risk dominant action. The global uniqueness and the selection of the risk dominant action are consequences of the fact that, as the noise goes to zero, rank beliefs always become uniform and thus there is common certainty of uniform rank beliefs.[3]

In this paper, we do not study the case where idiosyncratic noise goes to zero, and we therefore do not have common certainty of uniform rank beliefs and we do not have global uniqueness. We instead identify some situations where there is a unique rationalizable behavior but in other cases there are multiple rationalizable actions. Thus, we identify a novel set of conditions under which uniform rank beliefs arise: after a large shock when the common component of fundamentals has fat tails. A player who observes a large shock has uniform rank beliefs and knows that all players observing larger shocks also have uniform rank beliefs. This allows us to establish a "local uniqueness" result: after observing a large shock, it is uniquely rationalizable to play the risk dominant action. We believe that this more selective use of the global game reasoning is best able to generate insights about which informational events trigger equilibrium shifts.

Our main results described above concern when we can identify the unique rationalizable behavior for a player in a static game as a function of the level of fundamentals and the size of his individual shock. This is the analytic contribution of the paper. In order to relate our results more closely to our broader motivation, we also discuss the implications of this analysis for aggregate behavior in a dynamic model where the static game is played repeatedly with evolving fundamentals. In this dynamic setup large shocks lead to equilibrium shifts. When the fundamentals exceed a critical threshold or invest is risk dominant and there was a large shock to the fundamentals, a majority of players invest. Thus, the above events trigger a shift to majority investing if they were not doing so in the previous period. Likewise, if the fundamentals go below a critical threshold or not invest is risk dominant and there was a large negative shock to the fundamentals, a majority of players stop investing, triggering a "crisis."

Our results rely on the following key feature of our model: after a large shock the players become highly uncertain about the environment, resulting in a uniform distribution on their own ranking among other players. Such increased uncertainty after large shocks has been well-documented empirically (see for example Bloom 2009). We use fat-tailed common shocks and thinner tailed idiosyncratic shocks as a practical way of modeling such beliefs. We discuss evidence that key economic variables have fat tailed distributions as well as the interpretation of idiosyncratic shocks in Section V.

Our mechanism is relevant especially when there is model uncertainty. For example, when players do not know the economic impact of a new policy (such as a new tax cut), they may attribute large shocks to their private returns to a large impact of the policy even though they know that aggregate variations under a fixed policy are very small. Fat-tailed distributions often arise when there is uncertainty about the data-generating process (aka model uncertainty). For example, if the common shock is normally distributed but its variance is unknown and distributed with an inverse

[3] Carlsson and Van Damme (1993) showed limit global uniqueness and risk dominant selection in two player two action games. Frankel, Morris, and Pauzner (2003) show limit global uniqueness in a class of supermodular global games and risk dominant selection in binary action symmetric payoffs games (see also Morris and Shin 2003). Morris, Shin, and Yildiz (2016) formalize the idea that global uniqueness and risk dominant selection follow from common certainty of uniform rank beliefs.

$\chi^2$-distribution, then the common shock has a $t$-distribution, which has fat tails. More generally, when a player has a scale-invariant prior about a multiplicative distribution parameter, his posteriors will always have fat tails regardless of how many observations he makes from that distribution (Schwarz 1999).[4]

In the next section, we introduce our model. In Section II, we define and characterize the rank belief functions that will drive our results, and give our basic characterization of equilibria and rationalizable behavior. In that section, we also illustrate our key results graphically assuming the shocks are normally distributed and the variance of common shock may not be known. Our main results are reported in Section III. We present a dynamic application of the model in Section IV. In Section V, we review what happens if we relax the assumption that common shocks have a fat-tailed distribution, motivate our assumptions further, and present the literature on fat tails and model uncertainty. We discuss our broader contribution to the global games literature in Section VI. Some proofs are relegated to the Appendix.

## I. Model

We study the following Bayesian game, parametrized by real numbers $y$ and $\sigma > 0$. There is a continuum of players $i \in N = [0,1]$. Simultaneously, each player $i$ chooses between actions invest and not invest: the chosen action is denoted by $a_i$. The payoff from not invest is normalized to zero. The payoff from invest depends on a *type* $z_i \in \mathbb{R}$ and the fraction $A$ of individuals who invest,

$$u(A, z_i) = y + \sigma z_i + A - 1. \tag{1}$$

The type $z_i$ has two components,

$$z_i = \eta + \varepsilon_i, \tag{2}$$

a *common shock* $\eta$ that affects all players' payoffs, and an *idiosyncratic shock* $\varepsilon_i$ that affects only the payoff of player $i$. Player $i$ (privately) knows the sum $z_i$, but not its components. We also refer to $z_i$ as player $i$'s (overall) *shock*.

We write

$$x_i = y + \sigma z_i \tag{3}$$

for the private *return* from investment for type $z_i$. The shock $z_i$ will have zero mean. Hence, the ex ante expectation of the return is $y$, which we call the *prior mean*. The sensitivity of the return to shock $z_i$ is $\sigma$, which we call *shock sensitivity*. Note that the coordination motives are inversely proportional to the shock sensitivity (i.e., $\frac{\partial u}{\partial A} / \frac{\partial u}{\partial z_i} = 1/\sigma$). We will pay a special attention to the case of small $\sigma$, when the coordination motives are large.

[4] Assuming that such a multiplicative parameter evolves so that the players remain uncertain, this can explain many well-known puzzles in finance (Weitzman 2007).

REMARK 1: *Throughout the paper, we will vary the prior mean y while we fix the shock sensitivity* $\sigma$ *and the distributions F and G. In particular, for a given return* $x_i$, *we will vary the associated shock* $z_i = (x_i - y)/\sigma$ *by varying the prior mean y.*

We assume that $\varepsilon_i$ and $\eta$ are independently drawn—across the players—from distributions $F$ and $G$, respectively, with positive continuous densities $f$ and $g$ everywhere on the real line. We will assume that these distributions are symmetric around zero, i.e., $f(\varepsilon) = f(-\varepsilon)$ and $g(\eta) = g(-\eta)$, and that $f$ and $g$ are weakly decreasing on $(0,\infty)$. By symmetry, the idiosyncratic shock $\varepsilon_i$ has zero mean and $F(\varepsilon) = 1 - F(-\varepsilon)$. Likewise, the common shock $\eta$ has zero mean and $G(\eta) = 1 - G(-\eta)$.

Our key distributional assumptions are

(i) the distribution of idiosyncratic shocks is log-concave (i.e., $\log f$ is concave), and

(ii) the distribution of common shocks has regularly-varying tails, that is,

$$\lim_{\lambda\to\infty} \frac{g(\lambda\eta)}{g(\lambda\eta')} \in (0,\infty) \quad \text{for all } \eta, \eta' \in (0,\infty). \tag{4}$$

The log-concavity of $f$ implies that the idiosyncratic shocks have light tails (thinner than the tail of an exponential distribution, i.e., $\int e^{c|\varepsilon|} f(\varepsilon)\, d\varepsilon$ is finite for some $c > 0$). Common distributions with light tails, such as normal and exponential distributions, are log-concave. In contrast, the second part states that $g$ has regularly-varying (i.e., fat) tails, as in Pareto and $t$-distributions. In that case, $g(\eta)$ is approximately proportional to $\eta^{-\alpha}$ for some $\alpha > 1$ when $\eta$ is large, and the tails are thicker than the exponential function. Taken together, we assume that the common shock has thicker tails than the idiosyncratic shocks, reflecting our assumption that there is more tail uncertainty about the common shock. The log-concavity of $f$ also ensures that each player's belief about other players' types is increasing in his own type in the sense of the first-order stochastic dominance, making our game monotone supermodular. While such monotonicity and the tail properties are important in our analysis, log-concavity is assumed for exposition. (See the introduction and Section V for a motivation and a review of the empirical evidence for these assumptions.)

We now introduce some useful terminology. Each player $i$ has a strictly dominant strategy to invest if $x_i$ is strictly more than one, has a strictly dominant strategy to not invest if $x_i < 0$ and otherwise no action is strictly dominated. We will therefore refer to $[0,1]$ as *the undominated region* and 0 and 1 as *the dominance triggers*. When the returns are publicly observable and remain in the region $[0,1]$, there are multiple equilibria: all invest and all not invest. Game theoretic analysis suggests *refinements* to select among equilibria. An action is said to be *risk dominant* if it is a best response when each action is equally likely to be played by other players. Invest is the risk-dominant action when $x_i > 1/2$: and not invest is the risk-dominant action when $x_i < 1/2$. More generally, we say that an action is *p-dominant* if it is a best response when a player's expectation of the proportion of others taking the same action is at least $p$. An action is strictly dominant if it is zero-dominant, and it is risk dominant if it is $1/2$-dominant.

We make a host of simplifying assumptions, such as a continuum of players and independence of idiosyncratic shocks. We do not have a theoretical foundation for these assumptions. Our motivation is rather pragmatic. For example, the independence assumption ensures that the common shock is the only source of correlation among returns. Likewise, the continuum and independence assumptions together allow us to obtain a deterministic aggregate behavior as a function of the average returns.

## II. Rank Beliefs and Equilibrium Structure

In this section, we present the main ingredients of our analysis. We formally introduce the rank beliefs and identify their important properties for our analysis. Rank beliefs are key to our analysis as they determine how a player thinks his return relates to others' returns. We then describe the structure of equilibria and rationalizable strategies: rationalizable strategies are bounded by symmetric equilibria in cutoff strategies, and the return is equal to the rank belief at the equilibrium cutoffs. Finally, we illustrate these results on a canonical example in which common and idiosyncratic shocks have $t$ and normal distributions, respectively.

### A. *Rank Beliefs*

We define the *rank belief* of player $i$ as the probability he assigns to the event that another player's type $z_j$ is lower than his own,

$$R(z) = \Pr(z_j \leq z_i | z_i = z) = \frac{\int F(\varepsilon) f(\varepsilon) g(z-\varepsilon)\, d\varepsilon}{\int f(\varepsilon) g(z-\varepsilon)\, d\varepsilon}. \tag{5}$$

We refer to the function $R$ as the *rank-belief function*. While we define the rank belief to be the probability that a player assigns to one other player having a lower type, it is also equal to his expectation of the proportion of players with a lower type.

Note that the rank belief function depends only on the distributions $F$ and $G$ of idiosyncratic and common shocks, and it is independent of the prior mean $y$ and shock sensitivity $\sigma$. The following properties of rank belief functions will be important for us.

**Symmetry:** A rank belief function is said to be *symmetric* if

$$R(-z) = 1 - R(z).$$

That is, $R$ is symmetric around $1/2$ for positive and negative values. In that case, we have $R(0) = 1/2$.

**Single-Crossing Property:** A rank belief function is said to satisfy *the single-crossing property* if $R(z) > 1/2 > R(-z)$ whenever $z > 0$. That is, $R$

takes the value of $1/2$ at $z = 0$ and remains above $1/2$ for positive $z$, and symmetrically remains below $1/2$ for negative $z$.

**Uniform Limit Rank Beliefs:** A rank belief function is said to have *uniform limit rank beliefs* if

$$R(z) \rightarrow \frac{1}{2} \quad \text{as } z \rightarrow \infty.$$

That is, as $z \rightarrow \infty$, the rank belief converges back to $1/2$. Uniformity of limit rank beliefs implies immediately some further properties. The rank belief $R$ is bounded away from 0 and 1. We write $\bar{R} < 1$ for the upper bound. And the rank belief is decreasing over some interval.

Rank beliefs exhibit these properties in our model.

LEMMA 1: *The function R is differentiable*, *symmetric*, *and satisfies single-crossing and uniform limit rank belief properties.*

Here, differentiability follows from having a density that is decreasing on positive reals; rank belief function is continuous as long as the densities are continuous. (We only use continuity.) Symmetry and single crossing properties follow from the symmetry of the densities. Uniformity of limit rank beliefs is special and is the key property. We explained in the introduction how it follows from our assumption that the common shock has fat tails, and the idiosyncratic shocks have thinner tails. We plot a typical rank belief function $R$ in Figure 1 as a function of shock $z$. At $z = 0$, by symmetry, the rank belief is $1/2$. As $z$ increases, $R$ first gets larger by single-crossing property, and finally it goes back to $1/2$ by uniformity of limit rank beliefs. By symmetry, $R$ behaves symmetrically for negative shocks.

### B. *Structure of Equilibria and Rationalizable Behavior*

A (Bayesian Nash) equilibrium is defined as usual by requiring each type to play a best response. We first characterize a class of symmetric "threshold" equilibria. Suppose that each player would invest only if his type $z_i$ were greater than a critical threshold $\hat{z}$. Consider a player whose type was that critical threshold $\hat{z}$. His payoff to investing would be

$$\overbrace{y + \sigma\hat{z}}^{\text{own return}} + \overbrace{1 - R(\hat{z})}^{\text{expected proportion of others investing}} - 1.$$

The threshold type $\hat{z}$ will be indifferent only if this payoff is equal to 0, i.e.,

$$R(\hat{z}) = y + \sigma\hat{z}. \tag{6}$$

This is thus a necessary condition for there to be a $\hat{z}$-threshold equilibrium. But this is also sufficient for equilibrium. Suppose that a player anticipated that all other players were going to play a $\hat{z}$-threshold strategy, and was therefore indifferent between investing and not investing when his type was $\hat{z}$. If his type were $z_i > \hat{z}$, he

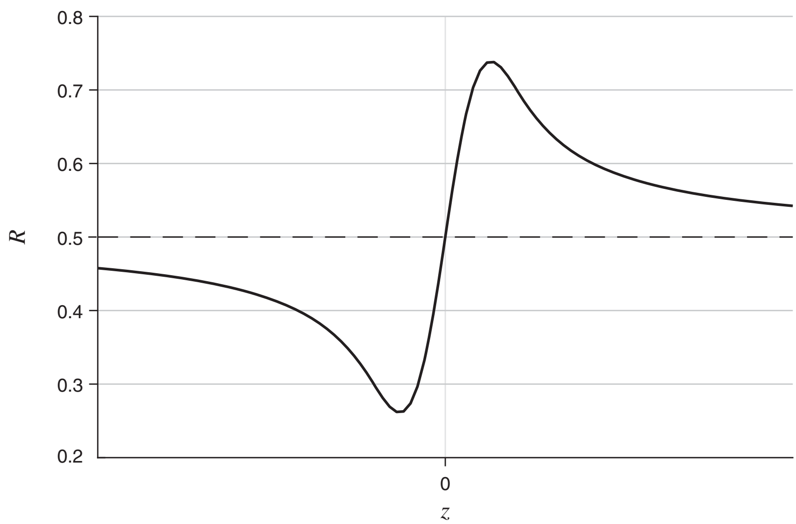


FIGURE 1. A TYPICAL RANK BELIEF FUNCTION

*Note:* Horizontal axis: shock $z$; vertical axis: rank belief $R(z)$.

would have higher incentive to invest since both his return from investment would be higher and his expectation of the proportion of the others who invest would be higher (by log-concavity of $f$).

The largest and smallest threshold strategy equilibria will play a key role in our analysis. Write $z^*$ and $z^{**}$ for the smallest and the largest solutions to (6), respectively; see Figure 2 for an illustration. We write $x^* = \sigma z^* + y$ and $x^{**} = \sigma z^{**} + y$ for the corresponding returns. Define symmetric strategies $s^*$ and $s^{**}$ associated with these cutoffs by

$$s_i^*(z_i) = \begin{cases} \text{Invest} & \text{if } z_i \geq z^* \\ \text{Not Invest} & \text{otherwise} \end{cases},$$

$$s_i^{**}(z_i) = \begin{cases} \text{Invest} & \text{if } z_i \geq z^{**} \\ \text{Not Invest} & \text{otherwise} \end{cases}.$$

Our next result establishes that $s^*$ and $s^{**}$ are Bayesian Nash equilibria and they bound all rationalizable (and hence equilibrium) strategies. In particular, $s^*$ is the equilibrium with the most investment, while $s^{**}$ is the equilibrium with the least investment.

LEMMA 2: *$s^*$ and $s^{**}$ are Bayesian Nash equilibria. Moreover, invest is uniquely rationalizable whenever $z_i > z^{**}$, and not invest is uniquely rationalizable whenever $z_i < z^*$.*

Thus the set of rationalizable actions is as follows. When $z^* \leq z_i \leq z^{**}$, both actions are rationalizable, and there is a unique rationalizable action otherwise. The unique rationalizable action is invest when $z_i > z^{**}$, and it is not invest when $z_i < z^*$. In the Appendix, we prove this result by checking that our game is monotone

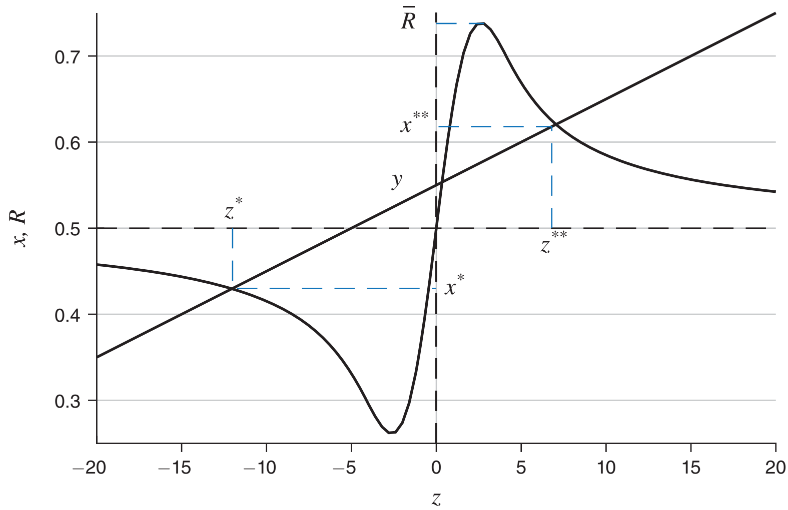


Figure 2. Extremal Cutoffs when the Variance of the Common Shock Is Unknown

*Note:* Horizontal axis: shock $z$; vertical axis: rank belief $R(z)$, non-linear function, and return $x = y + \sigma z$, linear function.

supermodular (Milgrom and Roberts 1990, Van Zandt and Vives 2007), and thus the Bayesian Nash equilibria and the rationalizable strategies are bounded by monotone Bayesian Nash equilibria. The key step in the proof is to show that, under log-concave $f$, the beliefs about the common shock are increasing in $z_i$ (i.e., $\Pr(\eta \leq \bar{\eta}|z_i)$ is decreasing in $z_i$ for any $\bar{\eta}$).

## C. *Example: t-distribution*

We now graphically illustrate the key properties of rank beliefs and the equilibrium structure with an example which also motivates our fat tail assumption with model uncertainty (model uncertainty as a foundation for fat tails is further discussed in Section V).

We assume that the idiosyncratic shocks have the standard normal distribution. We assume that the common shock is also normally distributed but its variance is not known: the reciprocal of its variance has a $\chi^2$-distribution. Such variance uncertainty leads to a $t$-distribution, and this distribution satisfies all of our distributional assumptions. We can interpret this as model uncertainty: the player does not know what is the true data generating function for a parameter that affects everybody.

There will be two effects of an increase in a player's shock on this rank belief. First, there will be the *reversion to mean* effect: a player will attribute some of the shock to his return to the common shock and some of it to his own idiosyncratic shock. Because of the last attribution, a player's expectation of the common shock will be further from his own shock as his own shock increases. This effect increases the rank belief as a player's shock increases. But second, there will be a *learning* effect. When the variance of the common shock is unknown, a large shock will lead a player to conclude that the variance of the common shock is higher, and he

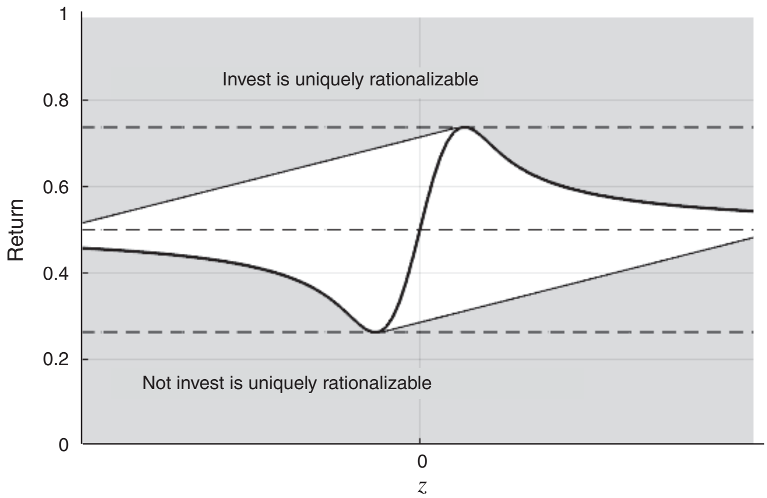


Figure 3. Rationalizable Behavior as a Function of Shock $z_i$ and Return $x_i$.

will attribute an increasing portion of his payoff shock to the common shock. This effect will tend to decrease rank beliefs. The shape of the rank belief function will then depend on which of these two effects predominates. Figure 1 plotted the rank belief function for this example: when a player's shock becomes large, he attributes it almost entirely to the common shock; the learning effect will predominate and the rank belief will approach $1/2$.

The extremal cutoffs are plotted on the same rank belief function in Figure 2. They correspond to the extremal intersections of non-monotone rank belief function $R$ and the line that represents the private return $y + \sigma z$ as a function of shock $z$. For any given $y$, invest is uniquely rationalizable when the shock is larger than $z^{**}$ or equivalently when the return is above $x^{**}$. Similarly, not invest is uniquely rationalizable when there is a large negative shock so that the return is below $x^{*}$. There will be multiplicity otherwise.

In our paper, we will study the rationalizable behavior as a function of shock $z_i$ and return $x_i$ by adjusting the prior mean $y$ accordingly. In the above example, the rationalizable behavior is as plotted in Figure 3 as a function of the shock and the return. Invest is uniquely rationalizable in the shaded region on the upper part of the figure, while not invest is uniquely rationalizable in the shaded region on the lower part of the figure. There is multiplicity in the unshaded area. First observe that invest is uniquely rationalizable for every shock when the return is more than $\bar{R}$, where $\bar{R}$ is the maximum possible rank belief; this is marked on Figure 2 and is approximately 0.738. This is the *level* effect we discussed in the introduction. This is simply because the returns $x^{*}$ and $x^{**}$ at the extremal cutoffs are always in between $1 - \bar{R}$ and $\bar{R}$. Second, for any given return $x_i > 1/2$ at which invest is risk dominant, invest is uniquely rationalizable (i.e., $x_i > x^{**}$) whenever there was a sufficiently large positive shock. This is the *change* effect we discussed in the introduction. This is simply because the rank belief approaches $1/2$ as the shock goes to $\infty$, and thus as we decrease $y$, the return $x^{**}$ at the upper cutoff approaches $1/2$. Finally, invest is uniquely rationalizable when the prior mean is very high, so that the return is above the line that is tangent to the rank belief function. The last effect is less relevant when the coordination motives are strong (i.e., when $\sigma$ is small). We will next establish these generally, as our main results.

## III. Rationalizable Behavior and the Role of Shocks

Suppose that a player has a private return $x$, having received a shock $z$ and thus having a prior mean $y = x - \sigma z$. Which actions are rationalizable? In particular, are both actions rationalizable or is invest or not invest the uniquely rationalizable action? We will show that the answers to these questions are as in Figure 3. Invest is uniquely rationalizable when the return is above the maximum rank belief, or invest is risk dominant and either there was a large shock or the prior mean was very high. Since the model is entirely symmetric between the two actions, we report formal necessary and sufficient conditions for invest to be the uniquely rationalizable action and the rest of the characterization will follow by symmetry. These characterizations will follow easily from our characterization of the rank belief function in Lemma 1 and rationalizable behavior in Lemma 2. In particular, we will be able to explain the results by appeal to the simple geometry of Figure 2.[5]

### A. *Large Shocks: Sufficient Conditions*

We first observe that when a player's private return exceeds the maximum rank belief $\bar{R}$, or equivalently when invest is $(1 - \bar{R})$-dominant, invest will be uniquely rationalizable independent of what his shock was. The critical level $\bar{R}$ depends only on the distributions $F$ and $G$ of the shocks and does not depend on the prior mean $y$ and the shock sensitivity $\sigma$.

PROPOSITION 1 (Level Trigger): *Invest is uniquely rationalizable if it is minimum rank belief dominant (i.e.,* $x_i > \bar{R}$*).*

PROOF:
Observe that, for any $\sigma$ and $y$,

$$x^{**} = R(z^{**}) \leq \bar{R},$$

where the equality is by definition of $x^{**}$ and the inequality is by definition of $\bar{R}$. Therefore, whenever $x_i > \bar{R}$, we have $x_i > x^{**}$ and invest is uniquely rationalizable. ∎

But we also see from Figure 3 that even if a player's return is less than $\bar{R}$, invest will be uniquely rationalizable when there is a large shock. In particular, invest is uniquely rationalizable whenever $x_i > 1/2$ and $z_i$ exceeds some threshold $\bar{z}$ where $\bar{z}$ is a function of $x_i$.

[5] Figure 2 has further properties that are not established in Lemma 1, such as single peakedness on the positive orthant. These properties will not be used, unless they are explicitly assumed.

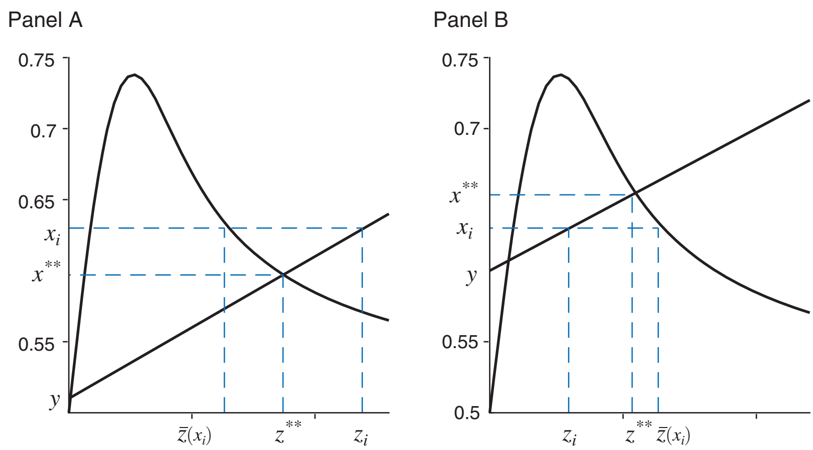


Figure 4. Effect of Shocks

*Notes:* Shock size is above the critical value on panel A and below the critical value on panel B. Horizontal axis: shock $z_i$; vertical axis: rank belief $R(z_i)$ and return $x_i = y + \sigma z_i$.

For each $x_i > 1/2$, at which invest is risk dominant, define the cutoff

$$\bar{z}(x_i) = \max R^{-1}(x_i), \tag{7}$$

where $R^{-1}(x_i) = \{z | R(z) = x_i\}$ is the pre-image of $R$ at $x_i$. The cutoff $\bar{z}(x_i)$ is illustrated in Figure 4, where we only show the part of Figure 2 where invest is risk dominant and $z_i \geq 0$. As seen in the figure, for $x_i \leq \bar{R}$, $\bar{z}(x_i)$ is the maximum level of shock under which a player's rank belief is $x_i$. (For $x_i > \bar{R}$, $\bar{z}(x_i) = -\infty$ by the convention that maximum of the empty set is $-\infty$.) Once again the critical level $\bar{z}(x_i)$ depends only on the distributions $F$ and $G$ of shocks and is independent of the prior mean $y$ and the shock sensitivity $\sigma$.

It turns out that the cutoff $\bar{z}(x_i)$ is the critical threshold for a shock to be effective in making the risk-dominant action uniquely rationalizable. This is formally established in our next result, the main result of our paper.

PROPOSITION 2 (Large Shocks): *Invest is uniquely rationalizable if it is risk dominant* (*i.e.*, $x_i > 1/2$) *and the shock is sufficiently large*, *i.e.*,

$$z_i > \bar{z}(x_i). \tag{8}$$

PROOF:

The special case of $x_i > \bar{R}$ is covered in Proposition 1. Hence, assume that $\bar{R} \geq x_i > 1/2$ and $z_i > \bar{z}(x_i)$—as in panel A of Figure 4. Then, for all $z \geq z_i$ we have

$$R(z) < x_i = y + \sigma z_i \leq y + \sigma z,$$

where the strict inequality is by definition of $\bar{z}(x_i)$ and $z > \bar{z}(x_i)$. Hence, $z^{**} < z_i$. Therefore, invest is uniquely rationalizable at $z_i$. ∎

Proposition 2 provides sufficient conditions for invest to be uniquely rationalizable: it is risk dominant (i.e., $x_i > 1/2$) and there was a large positive shock with size more than critical level $\bar{z}(x_i)$. By symmetry, this also establishes that not invest is uniquely rationalizable if it is risk dominant (i.e., $x_i < 1/2$) and there was a large negative shock, with size more than $\bar{z}(1 - x_i)$. We will refer to $\bar{z}(x_i)$ as the *critical shock size*. As in the case of critical level trigger $\bar{R}$, the critical shock size depends only on the distributions $F$ and $G$ of the shock, through the rank belief function. It is independent of the prior mean $y$ and the shock sensitivity $\sigma$. This renders the critical shock size on returns, $\sigma\bar{z}(x_i)$, proportional to $\sigma$. Hence, the latter threshold can be arbitrarily small for small $\sigma$ and arbitrarily large for large $\sigma$. For example, when coordination motives are strong, a very small positive jump in his return will lead a player to invest if investing is risk dominant. Likewise, a very small drop in his return will lead a player not to invest if not investing is risk dominant. Such behavior also arises in highly stable environments where one does not expect large shifts in returns. In the remainder of the paper, by a "large shock," we mean a shock of size that exceeds a critical shock size.

The proof of Proposition 2 is as illustrated on panel A of Figure 4. Here, $x_i = 0.63$ and hence invest is risk dominant. Moreover, the shock $z_i$ exceeds the critical shock size $\bar{z}(x_i)$. Now, for any shock level $z \geq z_i$, since $z$ is strictly greater than $\bar{z}(x_i)$, the rank belief $R(z)$ is strictly below $x_i$ (by definition of $\bar{z}(x_i)$). But clearly for any such $z$, the return $y + \sigma z$ is above $x_i$. Hence, the returns remain strictly above the rank beliefs for all $z \geq z_i$. Thus, the maximal equilibrium cutoff $z^{**}$ is strictly smaller than $z_i$. Therefore, invest is uniquely rationalizable at $z_i$. In contrast, the case on panel B illustrates that invest may not be uniquely rationalizable without a large shock. Here, the return is still as in the left panel ($x_i = 0.63$), but the shock $z_i$ is now smaller than the critical level $\bar{z}(x_i)$. In that case, the equilibrium cutoff $z^{**}$ is above $z_i$, and thus the cutoff $x^{**}$ is above $x_i$, leading to multiplicity at $x_i$.

In Figure 2, the rank belief function $R(\cdot)$ crossed the return $y + \sigma z$ in the positive orthant. But if the prior mean $y$ was high enough, the return would exceed the rank belief function for all positive shocks. This is illustrated in Figure 5. We can define cutoff $\bar{y}$ as the largest $y$ for which there exists $z > 0$ such that

$$R(z) \geq \sigma z + y. \tag{9}$$

We will refer to $\bar{y}$ as *the prior investment threshold*.[6] Define also cutoff $\underline{y} = 1 - \bar{y}$. These cutoffs play a prominent role in the remainder of the paper. When the prior mean is above the cutoff $\bar{y}$, the return remains strictly above the rank belief for non-negative shocks and hence the cutoff $z^{**}$ is negative and $x^{**} < 1/2$. Therefore, invest is uniquely rationalizable whenever it is risk dominant, regardless of the size of the shock, and it can be uniquely rationalizable even when it is not risk dominant and there is a negative shock.

[6] The cutoff $\bar{y}$ lies between $1/2$ and $\bar{R}$. It is $\bar{R}$ in the limit $\sigma \to 0$, and it decreases towards $1/2$ as $\sigma$ increases. When $\sigma < \sup_z(R(z) - 1/2)/z$, the cutoff $\bar{y}$ is determined by the tangency of the line $\sigma z + y$ to $R$ and is strictly above $1/2$—as in Figure 5. In contrast, $\bar{y} = 1/2$ when $\sigma > \sup_z R'(z)$.

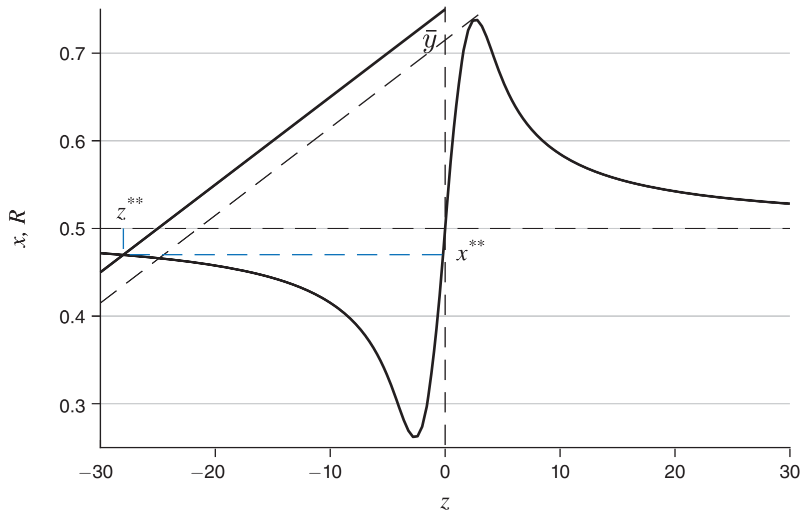


Figure 5. Equilibrium Cutoffs when Prior Mean Is Above $\bar{y}$

*Note:* Horizontal axis: shock $z$; vertical axis: rank belief $R(z)$ and return $x = y + \sigma z$.

PROPOSITION 3 (Ex Ante Level): *Invest is uniquely rationalizable if it is risk dominant (i.e., $x_i > 1/2$) and the prior mean exceeds the prior investment threshold (i.e., $y > \bar{y}$).*

PROOF:

For any $y > \bar{y}$, by definition of $\bar{y}$, the return exceeds the rank beliefs for all positive shocks: $\sigma z + y > R(z)$ for all $z > 0$. Hence, $z^{**} < 0$ as it is the largest $z$ with $R(z) = \sigma z + y$. Thus,

$$x^{**} = R(z^{**}) < 1/2,$$

where the equality is by definition of $x^{**}$ and the inequality is by the single-crossing property of rank beliefs in Lemma 1. Therefore, invest is uniquely rationalizable at each $x_i > 1/2$. ∎

We have thus shown three sufficient conditions for invest to be uniquely rationalizable: (i) it is minimum rank belief dominant; (ii) it is risk dominant and there was a large shock; and (iii) it is risk dominant and the prior mean was above $\bar{y}$. Symmetrically, not invest is uniquely rationalizable when (i) it is minimum rank belief dominant ($x_i < 1 - \bar{R}$); (ii) it is risk dominant ($x_i < 1/2$) and there was a large negative shock; and (iii) it is risk dominant and the prior mean was below $\underline{y}$.

We next consider a stark parametric example in which the cutoffs above have simple explicit form. This example does not satisfy our continuity assumptions but has an intuitive motivation.

EXAMPLE 1: *Idiosyncratic shocks are drawn from the uniform distribution on $[-1/2, 1/2]$. There is uncertainty about the distribution of the common shock: with* *probability $\pi$, there is no common shock (i.e., $\eta = 0$), but with complementary probability $1-\pi$, the common shock takes any value on the real line with an improper uniform distribution. The rank belief function is given by*[7]

$$R(z_i) = \begin{cases} \frac{1}{2} + \pi z_i & \text{if } z_i \in [-1/2, 1/2] \\ \frac{1}{2} & \text{otherwise} \end{cases}.$$

*It is non-monotone: it starts at $1/2$ for $z_i < -1/2$, drops to $(1-\pi)/2$ at $z_i = -1/2$, increases to $(1+\pi)/2$ until $z_i = 1/2$ after which it drops back to $1/2$. The return level that triggers investment as a unique rationalizable action is*

$$\bar{R} = \frac{1}{2} + \frac{\pi}{2}.$$

*The critical shock size that triggers a unique rationalizable action is*

$$\bar{z} = 1/2$$

*for each return $x_i \in (1/2, \bar{R})$. When the shock sensitivity, $\sigma$, is below $\pi$, the prior investment threshold is*

$$\bar{y} = \frac{1}{2} + \frac{\pi - \sigma}{2}.$$

*When $\sigma < \pi/2$, the sufficient conditions (i)–(iii) above characterize the rationalizable behavior. In all the remaining cases, both actions are rationalizable, and we have multiple equilibrium behavior. In particular, for any prior mean $y$ between $1-\bar{y}$ and $\bar{y}$, both actions are rationalizable whenever the shock size falls below the critical level.*

The rationalizable behavior is similar if $\sigma$ is in between $\pi/2$ and $\pi$. When $\sigma > \pi$, the rationalizable behavior is quite different: the game is dominance solvable, and the unique rationalizable action is invest when the return is above a cutoff $x^*$ and not invest when the return is below the cutoff $x^*$. Depending on $y$, the cutoff $x^*$ can take any value between $1-\bar{R}$ and $\bar{R}$. Thus, the unique rationalizable action is not determined by risk dominance or shock size.

### B. *Small Shocks: A Characterization*

The example illustrates the fact that when the shock sensitivity is small, the three sufficient conditions characterize unique rationalizability. In general, however, we

[7] Intuitively, when $z_i \notin [-1/2, 1/2]$, player $i$ learns that there is a common component and thus $\eta$ is uniformly distributed on the real line. Then, the rank belief is $1/2$ as in standard global games. When $z_i \in [-1/2, 1/2]$, a player does not learn anything about whether there is a common component. With probability $1-\pi$, there is a common component and rank beliefs are $1/2$. With probability $\pi$, there is no common component (i.e., $\eta = 0$), and $z_j$ is uniformly distributed on $[-1/2, 1/2]$ independent of $z_i$, yielding the rank belief $z_i + 1/2$. His rank belief, $1/2 + \pi z_i$, is the weighted average of $z_i + 1/2$ and $1/2$. This updating conditional on an improper common prior follows Taraldsen and Lindqvist (2016).

see that there is a gap. As in this example, in general, when shock sensitivity is high, invest can be uniquely rationalizable without being risk dominant. In particular, when $\sigma > \sup_z R'(z)$, the rationalizable action is unique, and it depends only on whether the return is above or below the rank belief, independent of the size of the shock and the risk dominant action. We next rule out such scenarios (by requiring that $\sigma \leq (\bar{R} - y)/\bar{z}(\bar{R})$), and obtain the following characterization.

PROPOSITION 4 (Characterization): *Assume $R$ is single peaked on $\mathbb{R}_+$ and $y \leq \bar{R} - \sigma\bar{z}(\bar{R})$. Then, invest is uniquely rationalizable if and only if it is risk dominant and $z_i > \bar{z}(x_i)$.*

That is, invest is uniquely rationalizable if and only if it is risk dominant and there was a large positive shock, as in Proposition 2. This also includes the case in Proposition 1 because when the return is above the maximum rank belief $\bar{R}$, the critical shock size is $-\infty$, and all shocks are considered large. In Proposition 3, invest was uniquely rationalizable whenever it was risk dominant and the prior mean was above the cutoff $\bar{y}$, even if there was a negative shock and the return was below the maximum rank belief. This case is ruled out by the condition that $y \leq \bar{R} - \sigma\bar{z}(\bar{R})$ in the hypothesis. Once that case is ruled out, for any given return level $x_i > 1/2$, the rationalizable behavior is a monotone function of shock $z_i$: both actions are rationalizable when $z_i \leq \bar{z}(x_i)$ and invest is uniquely rationalizable when $z_i > \bar{z}(x_i)$.

We must note that the condition $y \leq \bar{R} - \sigma\bar{z}(\bar{R})$ is crucial for this characterization. In general, rationalizable behavior is non-monotone in shock $z_i$ for any fixed return level $x_i > 1/2$. For example, in Figure 3, for any fixed $x_i \in (0.6, \bar{y})$, invest is uniquely rationalizable when $z_i < -(\bar{y} - x_i)/\sigma$; both actions are rationalizable when $-(\bar{y} - x_i)/\sigma \leq z_i \leq \bar{z}(x_i)$, and invest is uniquely rationalizable once again when $z_i > \bar{z}(x_i)$. As the shock sensitivity gets smaller, the lower cutoff gets smaller, making our characterization more relevant. In the limit $\sigma \rightarrow 0$, the lower cutoff approaches $-\infty$, and the rationalizable behavior is as in our characterization.

This characterization is obtained by establishing a converse to our main result under the additional conditions in the hypothesis. The proof of the converse is depicted in the right panel of Figure 4. In this figure, invest is risk dominant, but the shock $z_i$ is smaller than the critical level $\bar{z}(x_i)$. The additional conditions for the converse are also met in this example: $R$ is single-peaked, and $y \leq \bar{R} - \sigma\bar{z}(\bar{R})$, so that $R$ is decreasing at the cutoff $z^{**}$, where the line $y + \sigma z$ cuts $R$. Then, as in the figure, the equilibrium cutoff $z^{**}$ must be at least as large as $z_i$, and thus $x^{**}$ must be above $x_i$.[8] Therefore, not invest is rationalizable.

We next focus on the case of small shocks to the returns. In that case, rationalizable actions depend only on the position of the prior mean relative to the cutoffs $\bar{y}$ and $\underline{y}$.

PROPOSITION 5 (Small Shocks): *For any $\sigma < \sup_z (R(z) - 1/2)/z$, there exists $\Delta > 0$ such that whenever $|x_i - y| \leq \Delta$, invest is uniquely rationalizable if and only if $y > \bar{y}$ and not invest is uniquely rationalizable if and only if $y < \underline{y}$.*

[8] Indeed, suppose $z^{**} < z_i$. Then, since the straight line has positive slope, $x^{**}$ would have been strictly below $x_i$, and this would be a contradiction: $R$ would be decreasing from $x^{**}$ at $z^{**}$ to the larger value $x_i$ at $\bar{z}(x_i)$.

For sufficiently small shock sensitivity $\sigma$, Proposition 5 establishes that, without a large shock, invest is uniquely rationalizable when $y > \bar{y}$ and not invest is uniquely rationalizable when $y < \underline{y}$. But equilibrium play will depend on equilibrium selection when $y$ is in the intermediate range $[\underline{y}, \bar{y}]$, as the action choice depends on which equilibrium is played. When the prior mean is within this range, by definition, $z^* < 0 < z^{**}$ and hence both actions are rationalizable when the shock is sufficiently small. The proposition provides a uniform bound that guarantees multiplicity: the bound $\Delta$ is independent of $y$ although it may depend on $\sigma$ and the rank belief function.

Figure 6 illustrates the qualitative properties we have established so far as a function of prior mean $y$ and return $x_i$. In this figure, we plot equilibrium cutoffs and regions in which invest and not invest are uniquely rationalizable for the $t$-distribution example, assuming $\sigma < \sup_z \left(R(z) - 1/2\right)/z$. When $y \in [\underline{y}, \bar{y}]$, the upper equilibrium cutoff $x^{**}$ is above $\max\{y, 1/2\}$ and approaches $\max\{y, 1/2\}$ as $\sigma \to 0$ by Proposition 2. Hence, when $y > 1/2$, a large shock makes invest uniquely rationalizable. Since $x^{**} > y$, if the positive shock is sufficiently small, then both actions are rationalizable and can be played in equilibrium. Likewise, the lower cutoff $x^*$ is below $\min\{y, 1/2\}$ and approaches $\min\{y, 1/2\}$ as $\sigma \to 0$. Once again, large negative shocks make not invest uniquely rationalizable when $y < 1/2$, while both actions are rationalizable under smaller shocks. Note that when $y \in [\underline{y}, \bar{y}]$, the regions with uniquely rationalizable actions are confined to different sides of cutoff $x_i = 1/2$, and only a risk dominant action can be uniquely rationalizable. Outside of $[\underline{y}, \bar{y}]$, a non-risk-dominant action can be uniquely rationalizable. For example, when $y > \bar{y}$, the cutoff $x^{**}(y)$ is slightly below $1/2$. Whenever $x_i \in \left(x^{**}(y), 1/2\right)$, invest is uniquely rationalizable although not invest is risk dominant.

### C. *Aggregate Implications*

We now focus on the implications of our result on aggregate behavior, showing that there will be a shift in aggregate investment when the size of the common shock exceeds the critical level. We define the fundamental state (or fundamentals) as

$$\theta = y + \sigma\eta,$$

which is the average return, as a function of the common shock. Since we assume a continuum of players with independently distributed idiosyncratic shocks, there is no aggregate uncertainty conditional on the fundamental state. In particular, conditional on common shock $\eta$, the fraction of players with shocks below a given threshold $z$ is $F(z - \eta)$. Thus, the effect of a shock on individuals' behavior directly translates as an effect of a common shock on majority behavior:[9]

COROLLARY 1: *Invest is uniquely rationalizable for a majority if it is risk dominant for the median player* (*i.e.*, $\theta > 1/2$) *and, in addition, one of the following is true:* ($i$) *invest is minimum rank belief dominant for the median player*

[9] By a majority we mean a set of players whose Lebesgue measure is more than $1/2$.

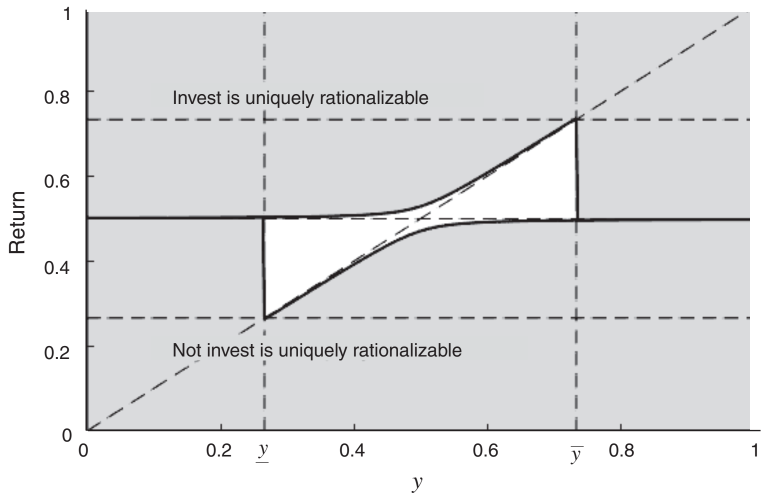


Figure 6. Equilibrium Cutoffs and Rationalizability as a Function of Prior Mean $y$ in the $t$-distribution Example ($\sigma = 0.01$)

*Note:* Horizontal axis: prior mean $y$; vertical axis: return $x = y + \sigma z$.

*(i.e.,* $\theta > \bar{R}$); $(ii)$ *there is a large common shock (i.e.,* $\eta > \bar{z}(\theta)$); *or* (3) *the prior mean exceeds the prior investment threshold* ($y > \bar{y}$).

PROOF:

Clearly, invest is uniquely rationalizable for a majority if and only if it is uniquely rationalizable for the player with the median return, for which $x_i = \theta$ and $z_i = \eta$. We obtain our corollary by substituting these equalities in Propositions 1, 2, and 3. ∎

That is, under rationalizability a majority must invest if doing so is risk dominant for that fundamental state and there was a large positive common shock. When the shock sensitivity $\sigma$ is small and the common shock is away from the critical level, nearly all players take the same action. In that case, the aggregate investment is near 1 when investing is risk dominant and there was a large common shock.

It is straightforward to extend this result to an arbitrary percentile of players. For any $p \in (0,1)$ and for any $\theta \in \left(1/2 + \sigma F^{-1}(p), \bar{R}\right]$, invest is uniquely rationalizable for a fraction $p$ of the players if the common shock $\eta$ exceeds a critical shock size

$$\bar{z}_{p,\sigma}(\theta) = \bar{z}\left(\theta - \sigma F^{-1}(p)\right) + F^{-1}(p).$$

As $\sigma \rightarrow 0$, the critical shock size, $\bar{z}_{p,\sigma}(\theta)$, decreases to

$$\bar{z}_{p,0}(\theta) = \bar{z}(\theta) + F^{-1}(p),$$

a translation of the critical shock size for the majority.

## IV. Dynamic Application

Our motivation for studying this problem comes from thinking about a dynamic model. In this section, we describe a dynamic model that is simply a sequence of plays of the static model. This analysis provides an interpretation of and a motivation for our earlier results.

For a fixed $\sigma$, the static game that we have analyzed can be parameterized by the prior mean $y$, and we will denote that game by $\Gamma(y)$. We will now consider the following dynamic game. At the beginning of each period $t \geq 0$, there is an expected return $y_t$. In each period $t$, the static game $\Gamma(y_t)$ is played by a continuum of players. That is, a common shock $\eta_t$ and idiosyncratic shocks $\varepsilon_{it}$ are independently drawn across players, and players with types $z_{it} = \eta_t + \varepsilon_{it}$ make investment choices as in the game $\Gamma(y_t)$. The expected return $y_{t+1}$ at period $t+1$ is a function of the fundamental state $\theta_t = y_t + \sigma\eta_t$ at $t$,

$$y_{t+1} = Y(\theta_t),$$

for some known function $Y: \mathbb{R} \to \mathbb{R}$. At the beginning of $t$, the current expected return $y_t$ and previous aggregate investment $A_{t-1}$ (the fraction of players who invested in the previous period) are publicly observable. Our interpretation is that $y_t$ is the expected productivity in the economy. In each period, there is a common shock to productivity. The shock is persistent, but there may be a reversion to a mean productivity, as in the example below.

We now identify equilibrium shifts (when does equilibrium play switch from investment to non-investment and back?) and the role the shocks play in such shifts. We will focus on the *hysteresis equilibrium*: each player $i$ invests at any period $t$ if and only if $z_{it} > \hat{z}_t$ where $\hat{z}_t = z^*(y_t)$ if $t = 0$ or $A_{t-1} \geq 1/2$ and $\hat{z}_t = z^{**}(y_t)$ otherwise. The cutoff $\hat{z}_t$ is a function of the current expected return and the previous aggregate investment. If a majority invested in the previous period, each player invests as long as investing is rationalizable for him, using the lowest equilibrium cutoff $z^*(y_t)$ in the static game. Likewise, if majority did not invest in the previous period, he does not invest unless investing is the only rationalizable option for him. This leads to inertia in majority behavior: majority behavior changes if and only if the action taken by a majority in the previous period is no longer rationalizable for a majority. Combined with this simple characterization, our previous results lead to the following description of equilibrium shifts under hysteresis. (We say that there is *majority investment* at $t-1$ if $A_{t-1} > 1/2$ and there is *minority investment* if $A_{t-1} < 1/2$.)

COROLLARY 2: *Under the hysteresis equilibrium, at any $t > 0$, if there was minority investment in the previous period, equilibrium shifts to majority investment whenever invest is risk dominant for the median player and, in addition, one of the following conditions hold: (i) invest is minimum rank belief dominant (i.e., $\theta_t > \bar{R}$); (ii) there is a large common shock (i.e., $\eta_t > \bar{z}(\theta_t)$); or (iii) the prior mean exceeds the prior investment threshold ($y_t > \bar{y}$). If $R$ is single peaked on $\mathbb{R}_+$, then equilibrium shifts to majority investment can occur only if (i) invest is minimum rank belief dominant (i.e., $\theta_t > \bar{R}$); (ii) invest is risk dominant for the median player and*

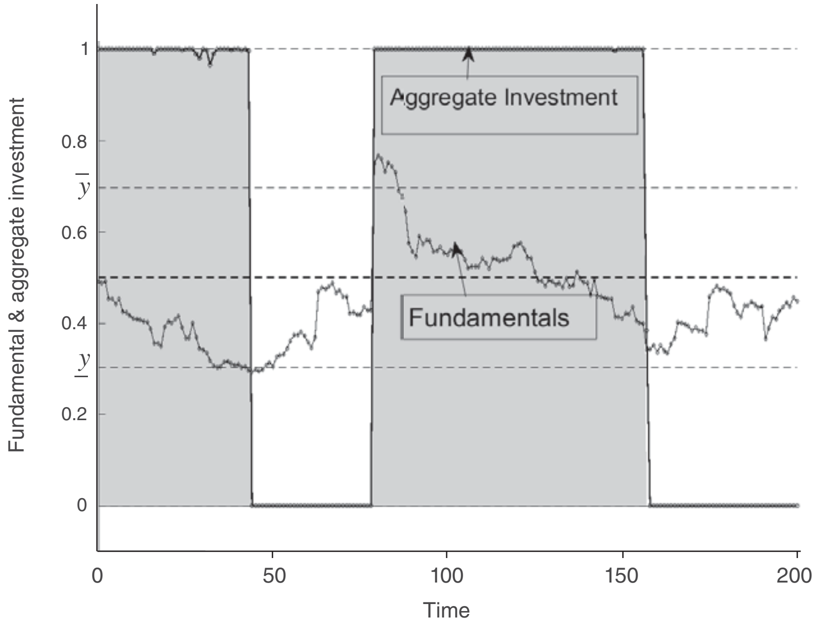


Figure 7. Equilibrium Shifts on a Typical Sample Path

*there is a large common shock (i.e.,* $\eta_t > \bar{z}(\theta_t)$*); or (iii) the prior mean exceeds* $\bar{R} - \sigma\bar{z}(\bar{R})$.

PROOF:

Equilibrium shifts to majority investment if and only if invest is uniquely rationalizable for the median type $z_{it} = \eta_t$, i.e., $\eta_t > z^{**}(y_t)$. Then, the corollary immediately follows from Propositions 1–5. ■

Under hysteresis, Corollary 2 provides nearly a characterization of when equilibrium shifts to majority investment occur: invest is risk dominant under the median return and either the expected return is above $\bar{y}$ or there was a large positive shock at $t$. The converse rules out an equilibrium shift for all but a few remaining cases discussed in Section III.

The dynamics under the hysteresis equilibrium is illustrated in Figure 7.[10] Time is on the horizontal axis. A sample path of fundamentals and aggregate investment are plotted on the vertical axis. Because shock sensitivity is small, aggregate investment is always close to one or close to zero, so majority investment and minority investment correspond to almost all investing and almost all not investing, respectively. There are two periods of majority investment (the shaded areas) interspersed with minority investment. At the beginning, there is majority investment with aggregate investment nearly 1. The majority keeps investing due to hysteresis, although

[10]In this example, we take $y_{t+1} = 1/2 + \kappa(\theta_t - 1/2)$ with parameter $\kappa = 0.99$, so that fundamentals follow an AR(1) process around 1/2: $\theta_{t+1} - 1/2 = \kappa(\theta_t - 1/2) + \sigma\eta_{t+1}$. The distributions of shocks are as in the $t$-distribution example. We also take $\sigma = 0.01$.

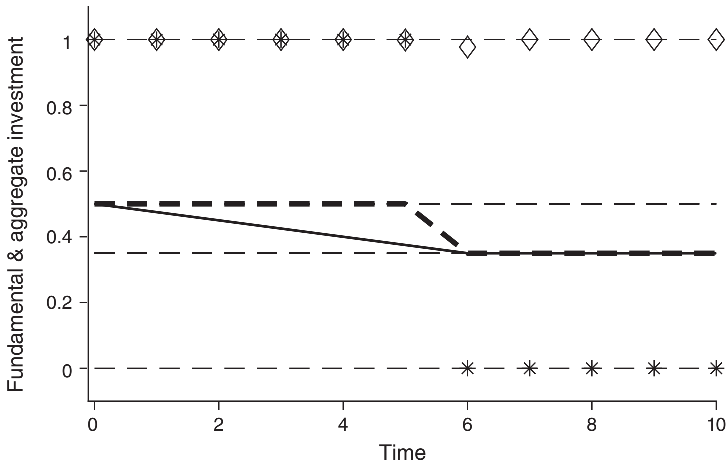


FIGURE 8. AGGREGATE INVESTMENT UNDER SOFT-LANDING (SOLID, ◇) AND SUDDEN DROP (DASHED, *)

the fundamental drifts downward and investing is not risk dominant. The equilibrium shifts when the prior mean of the fundamental drifts below $\underline{y}$:[11] the majority stop investing and aggregate investment drops near zero. This shift illustrates level condition 3 in Corollary 2. This is the end of the first period of majority investment. After that the fundamental fluctuates, but aggregate investment remains near zero. In particular, in this no investment period, a large positive shock has no discernible impact on aggregate investment as not invest remains risk dominant under the median return. Later, the arrival of a major large positive shock makes investing risk dominant for the median player and shifts equilibrium back to majority investment. Since investment also becomes minimum rank belief dominant, both level condition 1 and shock condition 2 in Corollary 2 make the shift necessary. Thereafter, fundamentals drift down with occasional negative shocks, and a large negative shock ends the second investment period as it arrives when not invest is risk dominant for the median player. This shift illustrates shock condition 2 in Corollary 2.

Our result implies that it is preferable to avoid large negative shocks in good times in order to avoid crises, and preferable to have large positive shocks in the aftermath of a crisis especially after a substantial improvement of fundamentals in order to hasten the economic recovery. This is illustrated in Figure 8, where we compare two alternative hypothetical paths in our $t$-distribution example for $\sigma = 0.01$ and $y_t = \theta_{t-1}$. On both paths, the fundamental state starts at 0.5 and drops to 0.35. The paths differ in terms of how that change happens. On one path (in solid lines), fundamentals drop smoothly, as in the soft landing of a bubble. In that case, the aggregate investment remains nearly one throughout (marked with ◇). On the other path (dashed lines), fundamentals drop suddenly after remaining high for a long while. In that case, the negative shock triggers a long-lasting crisis, dropping aggregate investment near zero (marked with *).

[11] This happens when $Y(\theta_{t-1}) < \underline{y}$—when $\theta_{t-1} < 1/2 + (\underline{y} - 1/2)/\kappa \cong \underline{y}$ in this particular example.

It is useful to compare the dynamics here to two usual solution concepts. First, consider the hysteresis equilibrium where the returns are publicly observable and identical to each other (as in Cooper 1994) where (all) players switch their action only when the previous action becomes inconsistent with equilibrium, switching to all investing when $\theta$ goes above 1 and switching to nobody investing when $\theta$ goes below 0. Under this equilibrium, in Figure 7, the players keep investing throughout because the fundamental never drops below 0. There are more equilibrium shifts in our model in general because equilibrium shifts even before the fundamental reaches the cutoffs 0 and 1. Second, suppose players always play the risk dominant action, as they do in the noise free limit in the classic global games analysis. The equilibrium shifts as the fundamental crosses $1/2$, resulting in frequent equilibrium shifts when the fundamental is near $1/2$ and no shift away from the cutoff. Our equilibrium is not sensitive to the cutoff $1/2$ per se, but the outcomes correlate because shocks revert to the solution under risk dominance if they happen to be in the right direction.[12]

Hysteresis as a selection device is often assumed as a modeling device, see Krugman (1991) and Cooper (1994) among others. Romero (2015) has tested hysteresis in the laboratory, confirming its existence in a setting with evolving complete information payoffs. The switches occur before dominance regions are reached, consistent with our results. Chamley (1999) develops a dynamic model of global games in which hysteresis arises as a unique equilibrium. In his model, players can learn about the previous fundamentals when the fundamentals reach near dominance regions, when the equilibrium shifts occur. In another dynamic model with small amount of hysteresis in players' actions, Burdzy, Frankel, and Pauzner (2001) obtain risk dominant selection as the unique equilibrium. Finally, Angeletos, Hellwig, and Pavan (2007) study a dynamic model of global games with regime change. In their model, fundamentals do not change over time, but players learn about them as they observe the outcomes of the past play; learning leads to multiple equilibria and interesting dynamics.

## V. Rank Beliefs, Fat Tails, and Model Uncertainty

We made primitive assumptions on $f$ and $g$ that implied properties of rank beliefs (in Lemma 1) which then had implications for rationalizable behavior. In this section, we want to assess the role that our assumptions play in our argument, present the empirical evidence for them, and discuss informally what results hold under alternative assumptions and how they relate to the literature.

The structure of rationalizable strategies is determined by the shape of rank beliefs and does not depend on the specific assumptions one makes to derive them. As long as the properties in Lemma 1 hold, the results in this paper are valid. More generally, in a follow-up paper (Morris and Yildiz 2019), we show that if a limit $R_\infty \equiv \lim_{z \to \infty} R(z)$ exists, then invest is uniquely rationalizable if the return is above

[12] In our model, by Proposition 2, when $y > 1/2$, equilibrium shifts to majority investing whenever $\eta > \bar{z}(y)$. Hence, in the limit $\sigma \to 0$, equilibrium shifts occur near the cutoff $1/2$ almost surely. However, as in Figure 7, even for $\sigma = 0.01$, equilibrium shifts typically occur away from the cutoff $1/2$ (because the odds of getting large shocks $\eta > \bar{z}(1/2 + \varepsilon)$ are also very small for small $\varepsilon$).

the limit rank belief $R_\infty$ and there was a large positive shock. Under the limit uniformity of rank beliefs, $R_\infty$ coincides with the risk-dominance threshold $1/2$, yielding our large shock result. Our large shock result does rely on non-monotonicity of rank beliefs however. Under a monotone rank belief function, shocks per se do not lead to equilibrium shifts: if invest is uniquely rationalizable for some return $x_i$ and shock $z_i$, it would have been uniquely rationalizable for return $x_i$ and a smaller shock $z_i' < z_i$.[13]

It is instructive to compare our results to the case in which both common and idiosyncratic shocks are normally distributed. This case has been studied extensively in the global games literature but does not satisfy our fat-tail assumption. The literature focuses on the case of large shock sensitivity $(\sigma > R'(0))$, where coordination motives are weak. In that case, the game is dominance solvable. Here, we focus on the case of small shock sensitivity $(\sigma < R'(0))$, where the coordination motives are strong. For this case, we plot the rank belief function and the rationalizable solution in Figure 9. The rank-belief function is monotone and traces the entire undominated region $(0,1)$ as the shock level varies. As a result, our large-shock result disappears, and the return levels that trigger a unique rationalizable action coincide with the dominance triggers. Our result about the ex ante level (Proposition 3) remains to hold, so that invest is uniquely rationalizable when the prior mean exceeds the ex ante investment threshold. In this case, risk dominance does not play a role, and the rationalizable behavior is monotone with respect to shocks. In particular, for any return level $x_i \in (0,1)$, invest is uniquely rationalizable if there was a very large *negative* shock; not invest is uniquely rationalizable if there was a very large *positive* shock, and both actions are rationalizable in between. As $\sigma$ gets small, multiplicity becomes prevalent. In contrast, as shown in Figure 3, when the common shock has *t*-distribution, risk dominance plays a central role: the risk-dominant action is uniquely rationalizable when there is a large shock, and there is multiplicity otherwise.

This paper illustrates a mechanism in which large shocks lead to increased uncertainty about the relative ranking of players, leading them to play according to risk dominance. As we have mentioned above, it is sufficient that the rank beliefs are uniform at the limit $z \to \infty$. In our model, we used fat-tailed common shocks and thinner tailed idiosyncratic shocks to model such beliefs—and we motivate fat tails by model uncertainty. We next briefly present empirical evidence for fat tails and other studies that address model uncertainty in related contexts.

There is a long-standing empirical literature that establishes that changes in key economic variables have fat tailed distributions, going back to Pareto's observation about income distribution (see surveys by Benhabib and Bisin 2018, Gabaix 2009 and Ibragimov and Prokhorov 2017). For example, changes in GDP, prices, asset returns, and foreign exchange rates all have fat-tailed distributions (see pioneering works of Mandelbrot 1963 and Fama 1963; as well as contemporary studies such as Cont 2001; Gabaix et al. 2006; and Acemoglu, Ozdaglar, and Tahbaz-Salehi 2017). Moreover, many commonly used theoretical models, such as GARCH models and

[13] Monotonicity of beliefs play a central role in a number of economic models. For example, in observational learning, non-monotonicity of posterior beliefs (as a function of prior beliefs) is what gives informational cascades. In a model with two states, when the density of the log-ratio of the beliefs is log-concave, the posterior beliefs are monotone, and informational cascades do not arise (see Smith and Sørensen 2000 and Smith and Sørensen 2011).

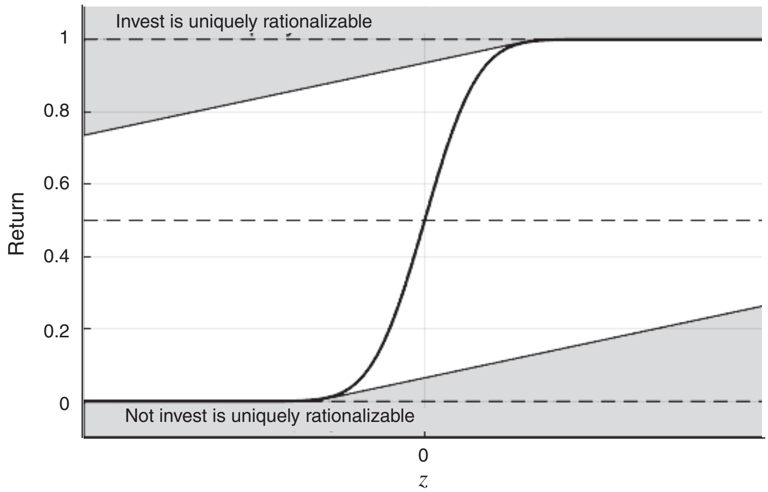


Figure 9. Rationalizable Actions when Both Common and Idiosyncratic Shocks Are Normally Distributed

*Note:* Both actions are rationalizable in the unshaded area.

models with stochastic volatility, naturally lead to fat tailed changes in the fundamental, as in the example of *t*-distribution above.

We also assume that the idiosyncratic components of the shocks have thinner tails than the common component, so that the tails of the changes in returns are as thick as the tails of common shocks. This is similar to the fact that the empirical tail indices of stock and market returns are both approximately 3 (Gabaix 2009). This assumption is plausible especially when the players learn the distribution of the shocks from the past realizations; the individual shocks generate a rich cross-sectional data while there is only a single time series about the common shocks. However, one must be cautious about mapping our highly stylized model to macroeconomic data. Macroeconomic aggregate productivity shocks tend to be much smaller than idiosyncratic shocks.[14] If one were to take our model literally (by identifying aggregate and idiosyncratic shocks with common and idiosyncratic shocks in our model, respectively), then our mechanism would be relevant only for extremely large aggregate shocks. But idiosyncratic variation can arise from many sources. It is enough for players to observe noisy signals of the fundamental; under this interpretation, thinner idiosyncratic tails correspond to a well understood (if noisy) observation technology.

Our focus on fat tails is motivated by model uncertainty. Model uncertainty also plays an important role in some other models. Chen and Suen (2016) study a coordinated attack problem in which players are uncertain about how easy it is to change a regime. An unexpectedly successful attack by the previous cohort dramatically increases the probability that changing the regime is easy, enticing players to attack. Hence, successful attacks lead to further attacks by other players. Acemoglu, Chernozhukov, and Yildiz (2016) study learning and asymptotic

[14] For example, the standard deviation of the changes in GDP is only about 0.02 while the standard deviation of firm-level productivity shocks is estimated to be 0.45 (Cooper and Haltiwanger 2006; see also Pischke 1995 and Bloom, Sadun, and Van Reenen 2017).

agreement when players do not know the conditional distribution of signals. Such model uncertainty leads to asymptotic belief disagreement and possibly non-monotone beliefs (as in our paper). Such model uncertainty is also central to Liang (2016), who studies robustness of solution concepts to uncertainty about the statistical rules players use to learn the fundamentals. Kozlowski, Veldkamp, and Venkateswaran (2017) study a macroeconomic model in which the players do not know the distribution of shocks and update their beliefs by using a normal kernel estimation method. When they observe large unexpected shocks, they update their beliefs about tail probability drastically. Large shocks have large and long-lasting impact on the economy as a result.

## VI. Discussion

In an economic environment with multiple equilibria, what explains which equilibrium is played? There are two versions of this question. In a static setting, how can we explain which equilibrium is played? In a dynamic setting, how can we explain switches among equilibria?

One response to the static question is to observe that the multiplicity may be an artifact of the assumption of complete information, or common certainty of the game's payoffs. A first generation of *global game* models (Carlsson and Van Damme 1993, Morris and Shin 1998 and Morris and Shin 2003) argued that if the common certainty assumption is relaxed in a natural way, there is a unique equilibrium selection—the risk dominant one in two player two action games. The natural relaxation is to allow players to observe very accurate noisy signals of the state of the world.[15] Morris, Shin, and Yildiz (2016) formalize the idea that this information structure gives rise to (common certainty of) uniform rank beliefs, and this is what drives the results. Note that in this literature, the focus is on *global* uniqueness: there is a unique prediction of play for any signal that a player might observe.

Two basic criticisms of this first generation of global models are the following. First, with respect to assumptions, common knowledge of uniform rank beliefs will not hold even approximately in many environments (for example, when there are very accurate public signals).[16] Second, with respect to predictions, as long as rank beliefs are approximately uniform throughout a model, outcomes will be largely determined by fundamentals. Thus in a dynamic model, the prediction would be that equilibrium play would always be switching when fundamentals crossed a threshold (which we call the risk-dominance threshold). Both predictions seem counter-factual.

In this paper, we made an intermediate set of assumptions, relative to complete information and first generation global games. Like the first generation global games literature, we relax complete information and use the vital insight that properties of rank beliefs sometimes lead to unique predictions.[17] Like the complete information

[15] Weinstein and Yildiz (2007) pointed out that it mattered exactly how common certainty assumptions were relaxed: any rationalizable action in the underlying complete information game is a uniquely rationalizable action for a type of a player that is "close" to the complete information type, where closeness is in the product topology in the universal belief space of Mertens and Zamir (1985).

[16] Angeletos and Werning (2006) give a price revelation foundation for the assumption that idiosyncratic shocks should have no less variation than common shocks.

[17] We allow for a qualitatively richer class of rank beliefs than the first generation literature. The existing literature exclusively focuses on two cases (see Morris and Shin 2003 for a discussion of both cases). First, the case

literature, we allowed for the possibility that information alone does not determine behavior and that some other factor or factors determine equilibrium choice; our focus was on hysteresis as that factor.

This approach generated three novel and intuitive predictions. First, if we look at the relationship between fundamentals and outcomes, play must shift once fundamentals cross a fundamental threshold that arises before an action becomes dominant. Second, large shocks can trigger a shift before that threshold is reached. And third, those shifts can only occur once an action is risk dominant; i.e., the best response to uniform rank beliefs and thus the first generation global game prediction.

We conclude by contrasting our explanation and modeling with the conventional account that equilibrium shifts are triggered by the arrival of public signals: even though the financial system has been coming under continuing pressure, a public event (such as the collapse of Lehman) triggers the shift to a bad equilibrium (a financial crisis); even though European fiscal and sovereign debt positions had been improving for some years, it was a public event (Draghi's speech) that triggered the shift to the good equilibrium. Such explanations are common in a wide variety of settings; see Chwe (2001) for many examples across the social sciences. Such explanations are based on the idea that public signals restore sufficient approximate common knowledge in the sense of high common $p$-belief, leading to multiple equilibria.[18] But is it really the case that there is more common knowledge after such large shocks? Surely people are more uncertain what other people are thinking after a large shock. We offer the alternative explanation that a large shock gives rise to *less* common knowledge in the sense of uncertainty about others' relative optimism, i.e., *more* uniform rank beliefs, and it is this that triggers a shift to a new equilibrium. While both explanations appeal to large shocks, the mechanisms are opposite in terms of the properties of rank beliefs generating the results. Moreover, while the conventional account shows public signals *may* lead to an equilibrium shift as one of many equilibrium outcomes, we show that the large shocks *will* lead to equilibrium shift as the only rationalizable outcome.

## Appendix A: Omitted Proofs

### A. *Properties of Beliefs and Equilibria*

In this section, we present a couple of basic properties of beliefs and prove Lemma 2, showing that the extremal equilibria $s^*$ and $s^{**}$ bound all rationalizable strategies. We write $F(\eta, z_{-i}|z_i)$ for the cumulative distribution function of $(\eta, z_{-i})$ conditional on $z_i$, which represents the interim beliefs of type $z_i$ about the common shock and the other players' types.

where we fix the distribution of common shocks/public signals and let the noise in idiosyncratic shocks/private signals go to zero; in this case there is common certainty of uniform rank beliefs in the limit. Second, the case where both common shocks/public signals and idiosyncratic shocks/private signals are normally distributed. As we noted in Section V, rank beliefs are monotonic in this case and our large shock results cannot arise.

[18] There can be a unique equilibrium when public signals are sufficiently accurate to break uniform rank beliefs—and thus global selection of the risk dominant equilibrium—but not sufficiently accurate to break globally unique equilibrium. In such models, large shocks/public signals will play a disproportionate role in selecting the unique equilibrium (see Morris and Shin 2003, Morris and Shin 2004). Such existing models have monotone rank beliefs, under which large shocks do not lead to equilibrium shifts per se (see Section V).

LEMMA 3: *The interim beliefs are increasing in types in the sense of first-order stochastic dominance. Moreover, f has thinner tails than g:*

$$\lim_{\lambda\to\infty}\frac{f(\lambda z)}{g(\lambda z')} = 0 \quad \left(\forall\, z, z' \in \mathbb{R}\backslash\{0\}\right). \tag{A1}$$

PROOF:

(Part 1) Since $z_j = \eta + \sigma\varepsilon_j$ where $\varepsilon_j$ is independent of $\varepsilon_i$ and $\eta$ for each $j \neq i$, it suffices to show that $F(\eta|z_i)$ is decreasing in $z_i$, where $F(\eta|z_i)$ is the conditional distribution of the common shock. To do this, it suffices to show that $\eta$ and $z_i$ are affiliated, i.e., the joint density $h$ of $(\eta, z_i)$ is log-supermodular. But since $h(\eta, z_i) = g(\eta)f(z_i - \eta)$, $\log h$ is supermodular,

$$\log h(\eta, z_i) = \log g(\eta) + \log f(z_i - \eta).$$

Here, $\log g(\eta)$ is trivially supermodular, and $\log f(z_i - \eta)$ is supermodular because $\log f$ is concave.

(Part 2) Since $f$ is log-concave, it is well known that $f$ has light tails, i.e.,

$$\lim_{z\to\infty}\frac{f(z)}{\exp(-cz)} = 0$$

for some $c > 0$. Thus, for any non-zero $z$ and $z'$,

$$\lim_{\lambda\to\infty}\frac{f(\lambda z)}{g(\lambda z')} = \lim_{\lambda\to\infty}\frac{f(\lambda z)}{\exp(-c\lambda z)}\frac{\exp(-c\lambda z)}{g(\lambda z)}\frac{g(\lambda z)}{g(\lambda z')} = 0.$$

(The limits of $\exp(-cz)/g(\lambda z)$ and $g(\lambda z)/g(\lambda z')$ are finite because $g$ has regularly varying tails.) ∎

The first part of the lemma is the main step in the proof of Lemma 2.

PROOF OF LEMMA 2:

It suffices to verify that our game is monotone supermodular, as in Van Zandt and Vives (2007) (who also assume that the set of players is finite but their proof also applies to our game). It is straightforward to verify the continuity and compactness assumptions as well as supermodularity of the payoff functions. Lemma 3 further establishes that the beliefs are monotone, and this fact immediately implies that $s^*$ and $s^{**}$ are Bayesian Nash equilibria. Since the game is monotone supermodular, all rationalizable strategies are bounded by $s_i^*$ and $s_i^{**}$. In particular, all rationalizable strategies coincide whenever $s_i^*(z_i) = s_i^{**}(z_i)$. ∎

## B. *Properties of Rank Beliefs*

In this section, we prove Lemma 1. We start with some useful notation. For any two functions $h_1$ and $h_2$ from reals to reals, we define *convolution* $h_1 * h_2$ of $h_1$ and $h_2$ by

$$h_1 * h_2(z) = \int h_1(\varepsilon)\, h_2(z - \varepsilon)\, d\varepsilon. \tag{A2}$$

Observe that

$$(A3) \qquad R(z) = \frac{Ff * g(z)}{f * g(z)}.$$

Since $F(-\varepsilon) = 1 - F(\varepsilon)$ and $f$ and $g$ are even functions, we have the following useful properties:

$$(A4) \qquad f * g(z) = f * g(-z);$$

$$(A5) \qquad R(-z) = \frac{(1 - F)f * g(z)}{f * g(z)},$$

where $1 - F$ is the complementary cdf. The first property states that $f * g$ is even, and the second property states that $R(-z)$ is simply computed by using the complementary cdf. Hence,

$$(A6) \qquad R(z) - R(-z) = \frac{(2F - 1)f * g(z)}{f * g(z)}$$

$$= \frac{\int_0^\infty (2F(\varepsilon) - 1) f(\varepsilon)(g(z - \varepsilon) - g(z + \varepsilon))\, d\varepsilon}{f * g(z)},$$

where the first equality is by (A3), (A4), and (A5), and the last equality is by the fact that $2F - 1$ is an odd function while $f$ is even.

PROOF OF LEMMA 1:

**Differentiability:** By monotonicity property of $g$, $g$ is differentiable almost everywhere. In the computation of convolutions, one can exclude the zero probability event on which $g'$ is not defined. With that exclusion, the function $g'$ is integrable: $\int |g'(z)|\, dz = 2g(0)$. Thus, both $Ff * g$ and $f * g$ are differentiable, showing that $R$ is differentiable.

**Symmetry:** By (A5),

$$R(-z) = \frac{(1 - F)f * g(z)}{f * g(z)} = \frac{f * g(z) - Ff * g(z)}{f * g(z)} = 1 - R(z).$$

**Single Crossing:** For any $z > 0$, observe that $g(z - \varepsilon) - g(z + \varepsilon) \geq 0$ and the inequality is strict with positive probability; equality holds only if $g$ is constant over the relevant range. Hence, by (A6), $R(z) - R(-z) > 0$. Since $R(-z) = 1 - R(z)$, this also implies that $R(z) > 1/2 > R(-z)$.

**Uniform Limit Rank Beliefs:** Fix any $\epsilon \in (0,1)$. Since $g$ has regularly varying tails (4), there exist $\beta > 0$ and $\eta_0$ such that for all $\eta' > \eta \geq \eta_0$,

$$(A7) \qquad \frac{g(\eta)}{g(\eta')} \leq (1 + \epsilon/2)(\eta/\eta')^{-\beta}.$$

Fix also $\gamma \in (0,1)$ such that

$$(1+\epsilon/2)\left(\frac{1-\gamma}{1+\gamma}\right)^{-\beta} < 1+\epsilon. \tag{A8}$$

Now, by definition, for any $z > 0$,

$$R(z) \leq (I_1 + I_2)/I_3$$

where

$$I_1 = \int_{-\gamma z}^{\gamma z} f(\varepsilon)F(\varepsilon)g(z-\varepsilon)\,d\varepsilon,$$

$$I_2 = \int_{\varepsilon \notin (-\gamma z, \gamma z)} f(\varepsilon)F(\varepsilon)g(z-\varepsilon)\,d\varepsilon,$$

$$I_3 = \int_{-\gamma z}^{\gamma z} f(\varepsilon)g(z-\varepsilon)\,d\varepsilon.$$

To find an upper bound on $R(z)$, we next find bounds for these integrals. First, observe that

$$g(z-\gamma z) \geq g(z-\varepsilon) \geq g(z+\gamma z) \quad \big(\forall \varepsilon \in [-\gamma z, \gamma z]\big),$$

as $z+\gamma z \geq z-\varepsilon \geq z-\gamma z > 0$ and $g$ is decreasing on positive reals. The inequality $g(z-\gamma z) \geq g(z-\varepsilon)$ yields an upper bound for $I_1$:

$$\begin{aligned} I_1 \leq g(z-\gamma z)\int_{-\gamma z}^{\gamma z} f(\varepsilon)F(\varepsilon)\,d\varepsilon &= \frac{1}{2}g(z-\gamma z)\big(F(\gamma z)^2 - F(-\gamma z)^2\big) \\ &= \frac{1}{2}g(z-\gamma z)\big(F(\gamma z) - F(-\gamma z)\big), \end{aligned}$$

where the integral is computed by change of variable $u = F(\varepsilon)$, and the last equality is by symmetry of $F$, $F(\gamma z) + F(-\gamma z) = 1$. Likewise, the inequality $g(z-\varepsilon) \geq g(z+\gamma z)$ yields a lower bound for $I_3$:

$$I_3 \geq \int_{-\gamma z}^{\gamma z} f(\varepsilon)g(z+\gamma z)\,d\varepsilon = \big(F(\gamma z) - F(-\gamma z)\big)g(z+\gamma z).$$

Finally, since $f$ is decreasing in the absolute value of $\varepsilon$, for all $\varepsilon \notin (-\gamma z, \gamma z)$, we have $f(\varepsilon) \leq f(\gamma z)$, yielding $f(\varepsilon)F(\varepsilon)g(z-\varepsilon) \leq f(\gamma z)g(z-\varepsilon)$. Hence,

$$I_2 \leq f(\gamma z)\int_{\varepsilon \notin (-\gamma z, \gamma z)} g(z-\varepsilon)\,d\varepsilon \leq f(\gamma z)\int_{-\infty}^{\infty} g(z-\varepsilon)\,d\varepsilon = f(\gamma z).$$

Combining the bounds for the integrals, we conclude that

$$R(z) \leq \frac{1}{2}\frac{g(z-\gamma z)}{g(z+\gamma z)} + \frac{f(\gamma z)}{\big(F(\gamma z) - F(-\gamma z)\big)g(z+\gamma z)}. \tag{A9}$$

Now, by (A7) and (A8),

$$\frac{1}{2}\frac{g(z-\gamma z)}{g(z+\gamma z)} \leq \frac{1}{2}(1+\epsilon/2)\left(\frac{1-\gamma}{1+\gamma}\right)^{-\beta} < 1/2+\epsilon/2$$

for any $z > \eta_0/(1-\gamma)$. Moreover, by (A1), there exists $\hat{z} > \eta_0/(1-\gamma)$ such that for all $z > \hat{z}$,

$$\frac{f(\gamma z)}{\big(F(\gamma z)-F(-\gamma z)\big)g(z+\gamma z)} < \epsilon/2.$$

Substituting the two displayed inequalities in (A9), we obtain $R(z) < 1/2+\epsilon$ for all $z > \hat{z}$, as desired. ∎

## C. *Omitted Proofs of Main Results*

We next prove Propositions 4 and 5.

PROOF OF PROPOSITION 4:

By Proposition 2, it suffices to prove the necessity. Take any $y \leq \bar{R}-\sigma\bar{z}(\bar{R})$. Since $y \leq \bar{R}-\sigma\bar{z}(\bar{R})$,

$$R\big(\bar{z}(\bar{R})\big) = \bar{R} \geq y+\sigma\bar{z}(\bar{R}).$$

Since $R(z) < y+\sigma z$ for large values of $z$, by the intermediate-value theorem, this implies that $z^{**} \geq \bar{z}(\bar{R}) > 0$. Thus,

$$x^{**} > \max\{y, 1/2\}.$$

(Clearly, $x^{**} = y+\sigma z^{**} > y$ and $x^{**} = R(z^{**}) > 1/2$.) Hence, if invest is not risk dominant (i.e., $x_i \leq 1/2$), then $x^{**} > x_i$, and therefore invest is not uniquely rationalizable. Now, assume that invest is risk dominant (i.e., $x_i > 1/2$) but inequality (8) does not hold, as in panel B of Figure 4,

$$z_i \leq \bar{z}(x_i). \tag{A10}$$

We claim that, if in addition $R$ is single peaked, then (A10) implies that $x^{**} \geq x_i$, and therefore invest is not uniquely rationalizable. To prove the claim that $x^{**} \geq x_i$, suppose $x^{**} < x_i$ and equivalently

$$z^{**} < z_i. \tag{A11}$$

Now, since $z^{**} \geq \bar{z}(\bar{R})$, by (A10) and (A11), we have

$$\bar{z}(\bar{R}) \leq z^{**} < z_i \leq \bar{z}(x_i).$$

However, since $R$ is single-peaked with a peak at $\bar{z}(\bar{R})$, this implies that

$$x^{**} = R(z^{**}) \geq R(\bar{z}(x_i)) = x_i,$$

contradicting that $x^{**} < x_i$. ■

We will below vary $y$ and write the cutoffs $x^*$ and $x^{**}$ as functions of $y$.

PROOF OF PROPOSITION 5:

Set $\Delta = \min\{x^{**}(\bar{y}) - \bar{y}, \bar{y} - 1/2\}$. Observe that

$$\text{(A12)} \quad \min_{y \geq \underline{y}}(y - x^*(y)) = \underline{y} - x^*(\underline{y}) = x^{**}(\bar{y}) - \bar{y} = \min_{y \leq \bar{y}}(x^{**}(y) - y) > 0,$$

where the first and the last equalities are by the fact that $y - x^*(y) = -\sigma z^*(y)$ is increasing while $x^{**}(y) - y = \sigma z^{**}(y)$ is decreasing, and the middle equality is by symmetry. To see the last inequality, observe that, since $\sigma < \sup_z(R(z) - 1/2)/z$, we have $\bar{y} > 0$, and by definition of $\bar{y}$, there exists $z > 0$ such that $R(z) \geq \bar{y} + \sigma z$, showing that $z^{**}(\bar{y}) > 0$. Therefore, $x^{**}(\bar{y}) - \bar{y} = \sigma z^{**}(\bar{y}) > 0$.

Consider any $y > \bar{y}$. Since $\Delta \leq \bar{y} - 1/2$, for any $x_i$ with $|x_i - y| \leq \Delta$, we have $x_i \geq y - \Delta > 1/2$. Then, by Proposition 3, invest is uniquely rationalizable at $x_i$ under $y$. Now consider any $y \leq \bar{y}$. By (A12),

$$x^{**}(y) - y \geq x^{**}(\bar{y}) - \bar{y} \geq \Delta.$$

Hence, for any $x_i$ with $|x_i - y| \leq \Delta$, we have

$$x_i \leq y + \Delta \leq x^{**}(y),$$

showing that invest is not uniquely rationalizable at $x_i$ under $y$. ■

## REFERENCES

**Acemoglu, Daron, Victor Chernozhukov, and Muhamet Yildiz.** 2016. "Fragility of Asymptotic Agreement under Bayesian Learning." *Theoretical Economics* 11 (1): 187–225.

**Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi.** 2017. "Microeconomic Origins of Macroeconomic Tail Risks." *American Economic Review* 107 (1): 54–108.

**Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan.** 2007. "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks." *Econometrica* 75 (3): 711–56.

**Angeletos, George-Marios, and Iván Werning.** 2006. "Crises and Prices: Information Aggregation, Multiplicity and Volatility." *American Economic Review* 96 (5): 1720–36.

**Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan.** 2007. "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks." *Econometrica* 75 (3): 711–56.

**Benhabib, Jess, and Alberto Bisin.** 2018. "Skewed Wealth Distributions: Theory and Empirics." *Journal of Economic Literature* 56 (4): 1261–91.

**Bloom, Nicholas.** 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77 (3): 623–85.

**Bloom, Nicholas, Raffaella Sadun, and John Van Reenen.** 2017. "Management as a Technology?" NBER Working Paper 22327.

**Brunnermeier, Markus K., Harold James, and Jean-Pierre Landau.** 2016. *The Euro and the Battle of Ideas.* Princeton, NJ: Princeton University Press.

**Burdzy, Krzysztof, David M. Frankel, and Ady Pauzner.** 2001. "Fast Equilibrium Selection by Rational Players Living in a Changing World." *Econometrica* 69 (1): 163–89.

**Carlsson, Hans, and Eric Van Damme.** 1993. "Global Games and Equilibrium Selection." *Econometrica* 61 (5): 989–1018.

**Chamley, Christophe.** 1999. "Coordinating Regime Switches." *Quarterly Journal of Economics* 114 (3): 869–905.

**Chen, Heng, and Wing Suen.** 2016. "Falling Dominoes: A Theory of Rare Events and Crisis Contagion." *American Economic Journal: Microeconomics* 8 (1): 228–55.

**Chwe, Michael Suk-Young.** 2001. *Rational Ritual: Culture, Coordination, and Common Knowledge.* Princeton, NJ: Princeton University Press.

**Cont, Rama.** 2001. "Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues." *Quantitative Finance* 1 (2): 223–236.

**Cooper, Russell W.** 1994. "Equilibrium Selection in Imperfectly Competitive Economies with Multiple Equilibria." *Economic Journal* 104 (426): 1106–22.

**Cooper, Russell W., and John C. Haltiwanger.** 2006. "On the Nature of Capital Adjustment Costs." *Review of Economic Studies*, 73 (3): 611–633.

**Fama, Eugene F.** 1963. "Mandelbrot and the Stable Paretian Hypothesis." *Journal of Business* 36 (4): 420–29.

**Frankel, David M., Stephen Morris, and Ady Pauzner.** 2003. "Equilibrium Selection in Global Games with Strategic Complementarities." *Journal of Economic Theory* 108 (1): 1–44.

**Gabaix, Xavier.** 2009. "Power Laws in Economics and Finance." *Annual Review of Economics* 1 (1): 255–94.

**Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou, and H. Eugene Stanley.** 2006. "Institutional Investors and Stock Market Volatility." *Quarterly Journal of Economics* 121 (2): 461–504.

**Ibragimov, Rustam, and Artem Prokhorov.** 2017. *Heavy Tails and Copulas: Topics in Dependence Modelling in Economics and Finance.* Singapore: World Scientific.

**Kozlowski, Julian, Laura Veldkamp, and Venky Venkateswaran.** 2017. "The Tail that Wags the Economy: Beliefs and Persistent Stagnation." Unpublished.

**Krugman, Paul.** 1991. "History versus Expectations." *Quarterly Journal of Economics* 106 (2): 651–67.

**Liang, Annie.** 2016. "Games of Incomplete Information Played by Statisticians." Unpublished.

**Mandelbrot, Benoit.** 1963. "The Variation of Certain Speculative Prices." *Journal of Business* 36: 394–419.

**Mertens, Jean-François, and Shmuel Zamir.** 1985. "Formulation of Bayesian Analysis for Games with Incomplete Information." *International Journal of Game Theory* 14 (1): 1–29.

**Milgrom, Paul, and John Roberts.** 1990. "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities." *Econometrica* 58 (6): 1255–77.

**Morris, Stephen, and Hyun Song Shin.** 1998. "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks." *American Economic Review* 88 (3): 587–97.

**Morris, Stephen, and Hyun Song Shin.** 2003. "Global Games: Theory and Applications." In *Advances in Economics and Econometrics,* edited by M. Dewatripont, L. Hansen, and S. Turnovsky 56–114. Cambridge: Cambridge University Press.

**Morris, Stephen, and Hyun Song Shin.** 2004. "Coordination Risk and the Price of Debt." *European Economic Review* 48 (1): 133–53.

**Morris, Stephen, Hyun Song Shin, and Muhamet Yildiz.** 2016. "Common Belief Foundations of Global Games." *Journal of Economic Theory* 163: 826–48.

**Morris, Stephen, and Muhamet Yildiz.** 2019. "Rank Beliefs." Unpublished.

**Pischke, Jörn-Steffen.** 1995. "Individual Income, Incomplete Information and Aggregate Consumption." *Econometrica* 63 (4): 805–40.

**Romero, Julian.** 2015. "The Effect of Hysteresis on Equilibrium Selection in Coordination Games." *Journal of Economic Behavior and Organization* 111: 88–105.

**Schwarz, Michael.** 1999. *Decision Making Under Extreme Uncertainty.* Stanford University Ph.D. Diss.

**Smith, Lones, and Peter Norman Sørensen.** 2011. "Observational learning." *The New Palgrave Dictionary of Economics Online Edition*, 29–52.

**Smith, Lones, and Peter Sørensen.** 2000. "Pathological Outcomes of Observational Learning." *Econometrica* 68 (2): 371–98.

**Taraldsen, Gunnar, and Bo Henry Lindqvist.** 2016. "Conditional Probability and Improper Priors." *Communications in Statistics - Theory and Methods* 45 (17): 5007–16.

**Van Zandt, Timothy, and Xavier Vives.** 2007. "Monotone Equilibria in Bayesian Games of Strategic Complementarities." *Journal of Economic Theory* 134 (1): 339–60.

**Weinstein, Jonathan, and Muhamet Yildiz.** 2007. "A Structure Theorem for Rationalizability with Application to Robust Predictions of Refinements." *Econometrica* 75 (2): 365–400.

**Weitzman, Martin L.** 2007. "Subjective Expectations and Asset-Return Puzzles." *American Economic Review* 97 (4): 1102–30.

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