Can Deficits Finance Themselves?*

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Abstract

We study how fiscal deficits are financed in environments with two key features: (i) nominal rigidity and (ii) a violation of Ricardian equivalence due to finite lives or liquidity constraints. In such environments, deficits contribute to their own financing via two channels: a boom in real economic activity, which expands the tax base, and a surge in inflation, which erodes the real value of nominal government debt. Our main theoretical result relates the potency of such self-financing to the timing of fiscal adjustment. Pushing the fiscal adjustment further into the future helps generate a larger and more persistent boom, leading to more self-financing. Full self-financing is possible in the limit as fiscal adjustment is delayed more and more: the government can run a deficit today, refrain from tax hikes or spending cuts in the future, and nevertheless see its debt converge back to its initial level. We conclude by arguing that a large degree of self-financing is not only theoretically possible but also quantitatively relevant.

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1 Introduction

Suppose the government runs a deficit today in order to stimulate aggregate demand. Suppose further that there is no "free lunch" of the type considered in the recent "$r < g$" literature (Blanchard, 2019; Reis, 2022); i.e., the government's net cost of borrowing is positive. What does the government have to do in order to make sure that public debt eventually returns back to its initial level?

The most conventional answer is *fiscal adjustment*: sooner or later, the government must adopt a package of tax hikes and/or spending cuts in order to pay down the accumulated debt. In this paper, we investigate a different margin—what we refer to as the *self-financing* of fiscal deficits.

The basic idea is simple. Insofar as a deficit triggers a boom, it can contribute to its own financing via two complementary channels: by expanding the tax base, which helps generate additional tax revenue without any adjustment in tax rates; and by triggering inflation, which helps reduce the real value of the government's nominal liabilities (i.e., debt erosion). Our contribution is to shed light on the theoretical properties and the quantitative relevance of this self-financing mechanism.[1]

We first verify that some self-financing obtains naturally in environments that combine two key features: a failure of Ricardian equivalence, so that deficits can stimulate aggregate demand; and nominal rigidity, so that aggregate demand can drive real economic activity and thereby also inflation. We next show that, in such environments, the degree of self-financing increases as the fiscal adjustment is delayed. Intuitively, the further the adjustment is delayed, the larger the boom induced by the initial deficit, thus raising the potency of both of our two self-financing channels. Pushing this logic to its limit, we obtain our headline result: as the fiscal adjustment is delayed more and more, the tax hike needed to bring debt back to trend vanishes. In other words, the initial deficit pays for itself. Our contribution is completed by evaluating the quantitative relevance of this result.

Environment. Our baseline model is kept purposefully close to the textbook New Keynesian model (Galí, 2008; Woodford, 2003*b*). The supply block is exactly the same and boils down to the New Keynesian Phillips curve (NKPC). The only change is in the demand block, which now consists of overlapping generations of perpetual-youth consumers (OLG à la Blanchard, 1985). Denote the survival probability by $\omega \in (0, 1]$. When $\omega = 1$, our model reduces to the standard permanent-income representative-agent (PIH-RANK) benchmark. When instead $\omega < 1$, our model shares two key properties with quantitative heterogeneous-agent (HANK) models: (i) consumers discount future disposable income more heavily than in the PIH benchmark; and (ii) they have a larger short-run propensity to consume (MPC). As will become clear, our lessons derive not from the OLG structure per se, but rather from these two more general and empirically relevant properties of consumer demand.

[1] The tax base and inflation channels bring to mind, respectively, DeLong and Summers (2012) and the Fiscal Theory of the Price Level (FTPL). We expand on these relations in due course.

Fiscal policy is represented by a rule for how tax revenue (and thereby the primary surplus) responds to changes in aggregate income and in public debt. The dependence on aggregate income captures the tax base channel—as output goes up, so automatically does tax revenue. The dependance on public debt, on the other hand, captures the speed of fiscal adjustment, or equivalently the horizon at which the government commits to hike taxes as needed to bring debt back to trend.[2] Our analysis will reveal how this policy parameter is a key determinant of the degree of self-financing.

Finally, monetary policy is represented by a rule for how nominal (and thereby real) rates adjust based on the state of the economy. For the bulk of our analysis, we concentrate on the case in which the monetary authority keeps the (expected) real rate constant. This case serves as a convenient benchmark and represents a "neutral" monetary policy in the following sense: it neither offsets the fiscal stimulus by discouraging private spending nor does it increase fiscal space by allowing the government's real cost of borrowing to fall.

The self-financing result. Starting from steady state, we shock the economy with a one-off, deficit-financed lump-sum transfer to households. We consider this experiment as a theoretical proxy for the various rounds of "stimulus checks" recently seen in the U.S. We then ask: How much of this deficit shock can be self-financed in equilibrium via the aforementioned tax base and inflation channels? Or, conversely, how large are the tax hikes needed, sooner or later, to bring public debt back to its trend?

Our first observation is that some self-financing is possible if and only if $\omega < 1$. When $\omega = 1$ (i.e., PIH-RANK), Ricardian equivalence holds, and aggregate demand is invariant to both the aforementioned shock and the timing of the subsequent fiscal adjustment. It follows that both self-financing channels are idle—the date-0 tax cut is fully financed with later tax hikes.[3] When instead $\omega < 1$, Ricardian equivalence fails, and the deficit—which now represents a net transfer from future generations to current generations or, less literally, helps some consumers overcome liquidity constraints—stimulates aggregate demand. With nominal rigidities, this increase in aggregate demand translates to an increase in real income and, via the NKPC, to an increase in the nominal price level. The two self-financing channels are thus operative: the real boom leads to higher tax revenues even without changes in tax rates; and the accompanying inflation lowers the real government debt burden.

The main question of the paper is how large such self-financing can be. We let ν denote the overall degree of self-financing; i.e., the fraction of the initial deficit that is financed via the tax base and inflation channels. The residual, $1-\nu$, then equals the discounted present value of the future tax hikes needed for each dollar of current deficit. Our headline result is that, if $\omega < 1$, then ν converges mono-

[2] In FTPL jargon, the last statement means that fiscal policy is "passive" or "Ricardian," which in turn highlights that our self-financing result is distinct from that found in the FTPL. We return to this point shortly.

[3] To be precise, these statements are true as long as we select the conventional or "fundamental" solution of the New Keynesian model. We elaborate on this below, in the paragraph entitled "Relation to FTPL."

tonically towards 1 as fiscal adjustment is pushed further into the future. In other words, delaying the fiscal adjustment reduces its magnitude, with full self-financing becoming possible in the limit.

The economics of self-financing. The economic intuition for our self-financing result can be understood through analogy with a simple two-period economy. Periods are indexed by $t \in \{0, 1\}$; prices are fully rigid, so only the tax base channel will be operative; and demand at $t = 0$ is myopic (i.e., independent of outcomes at $t = 1$) and proportional to concurrent disposable income. Echoing our full dynamic model, the government pays out transfers of size $\varepsilon > 0$ to households at $t = 0$, and then hikes taxes at $t = 1$ as much as necessary to bring total government debt down to its initial level.

By a simple static Keynesian cross logic, date-0 aggregate output is given as

$$y = \frac{\text{MPC}}{1 - \text{MPC}(1 - \tau_y)} \times \varepsilon,$$

where $\text{MPC} \in (0, 1)$ is the household's marginal propensity to consume out of disposable income and $\tau_y \in (0, 1)$ is the rate of proportional income taxation. The boom generates additional tax revenue at $t = 0$ of $\tau_y \times y$ (i.e., the tax base channel), and so the required date-1 tax hike is just $\varepsilon - \tau_y \times y$. It follows that the overall degree of self-financing in this simple economy is given as

$$\nu \equiv \frac{\tau_y \times y}{\varepsilon} = \frac{\tau_y \times \text{MPC}}{1 - \text{MPC} \times (1 - \tau_y)}.$$

We note two important properties of this expression. First, ν is strictly increasing in the MPC. This is because a larger MPC maps to both a larger partial equilibrium effect on consumer spending (numerator) and stronger general equilibrium amplification (denominator). Second, $\nu = 1$ if and only if $\text{MPC} = 1$. Intuitively, in that case, the partial equilibrium response is 1 and the Keynesian multiplier is $\frac{1}{1-(1-\tau_y)} = \frac{1}{\tau_y}$, so every dollar of initial transfer ends up generating exactly one dollar of additional tax revenue—i.e., full self-financing obtains.

Our fully-fledged dynamic model closely echoes these intuitions if and only if $\omega < 1$. To see the connection, translate "$t = 0$" in the above example to an initial time interval in our model that is both sufficiently long and sufficiently distant from the eventual tax hike (i.e., the "short run"); "$t = 1$" to the time of the delayed tax hike (i.e., the "long run"); and "MPC" as the cumulative MPC over the initial interval. Two properties of our economy then complete the connection. The first property is *discounting*: consumers discount any future disposable income, and hence also any future taxes, at a higher rate than the interest rate faced by the government. This implies that tax hikes in the far-ahead future have little effect on consumer demand in the short run, echoing the lack of feedback from date-1 taxes to date-0 demand in the two-period economy. The second property is *front-loading*: consumers spend any additional income receipt quickly. This implies that the cumulative MPC over the short run can be close to 1, and so the general equilibrium feedback plays out faster than in the

PIH benchmark. Together, these properties guarantee that the limit of our economy as the tax hike in pushed further into the future is akin to letting MPC $\to 1$ in the simple example.

Tax base vs inflation. The above intuition emphasized the tax base channel by assuming perfectly rigid prices. More generally, the fiscal stimulus partially translates into inflation rather than fully into real output, but without otherwise changing the monotonicity and limit properties of ν. The only material change is, naturally, the relative contribution of the two channels: as prices become more flexible and so the Phillips curve becomes steeper, more self-financing occurs through debt erosion.

Generality and extensions. We illustrate the versatility of the logic behind our self-financing result with a number of extensions.

Our first two extensions generalize the demand block of the economy. The first one accommodates a more flexible specification for consumer demand. This allows us to see more clearly how our self-financing result depends on measurable properties of the aggregate MPCs out of current income, future income, and wealth. Echoing the intuition provided above, the key conditions for self-financing to occur are discounting and front-loading: consumers need to respond relatively little to expectations of future taxes, so that the "stimulus checks" policy can indeed stimulate demand; and they also need to spend their transfer receipts relatively quickly, so that the resulting Keynesian boom plays out fast. The second extension accommodates investment. Under standard assumptions about firm behavior, the Keynesian cross logic behind our result continues to apply, and our self-financing result goes through almost unchanged.

The remaining extensions generalize our assumptions about fiscal and monetary policy. We first allow for the future fiscal adjustment to be distortionary, rather than lump-sum. If so, future taxes appear as additional cost-push shocks in the model's supply block, while having no effect on the demand side. Switching from lump-sum to distortionary taxes thus affects the split between tax base and inflation self-financing, but otherwise leaves the core logic of our self-financing results unchanged. By the same token, our self-financing result also readily extends from "stimulus checks" to government purchases. Lastly, we ask what happens when the monetary authority moves away from our "neutral" benchmark, either accommodating or leaning against the fiscally-led boom. If nominal rates rise less than inflation, then real rates fall, so households front-load spending further and our mechanism plays out even faster. Conversely, if real rates are increased, then the deficit-driven boom is delayed, and so is convergence to the self-financing limit. In particular, if the monetary response is too aggressive, then full self-financing ceases to be possible.

Relation to FTPL. Our self-financing result shares with the Fiscal Theory of the Price Level (FTPL) the flavor that deficits are financed not through outright fiscal adjustment, but rather through market movements in prices and/or quantities. The micro-foundations and the economic mechanisms behind our result, however, are substantially different from those behind the FTPL.[4]

The prevailing formulation of the FTPL assumes a representative, infinitely-lived, and fully rational consumer, similarly to Barro (1974)'s classic on Ricardian equivalence and thus to the $\omega = 1$ case in our model. The FTPL breaks Ricardian equivalence—i.e., it allows debt and deficits to drive aggregate demand and thereby output and inflation—by assuming that the Taylor principle is violated, opening the door to multiple self-fulfilling equilibria, and then selecting a particular such equilibrium. In our environment, this mechanism is never at play: the failure of Ricardian equivalence is grounded on fundamental reasons (finite horizons or liquidity constraints); the fiscal authority is "passive;" and our self-financing result is robust to an "active" monetary authority that satisfies the Taylor principle.[5]

Quantitative relevance. To gauge whether a meaningful degree of self-financing is plausible, we discipline the theory with appropriate evidence on its crucial elements: (i) the deviation from the PIH benchmark; (ii) the horizon or speed of fiscal adjustment; and (iii) the degree of nominal rigidity. For (iii), we draw from ample empirical evidence on the slope of the Phillips curve. For (ii), we turn to recent work that tries to measure the speed of fiscal adjustment in practice (Galí, López-Salido and Vallés, 2007; Bianchi and Melosi, 2017; Auclert and Rognlie, 2018). Finally, for (i), we review the available microeconomic evidence on the response of household consumption to income shocks (Fagereng, Holm and Natvik, 2021), including at relatively far-out horizons. We then calibrate an extended version of our baseline model—one that adds a margin of hand-to-mouth consumers—to match this empirical evidence. This extended, yet still tractable and parsimonious model, is not only capable of matching the relevant empirical evidence, but also serves as a close proxy for aggregate consumer behavior in fully-fledged HANK environments (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*).

We then use our calibrated model to quantify the effects of "stimulus checks" (defined, again, as deficit-financed lump-sum transfers to households). We find that such policies can be largely self-financed if fiscal adjustment is as slow as documented in prior empirical work. We further show that the tax base channel is the dominant force at work, and that a quantitatively large degree of self-financing is consistent with relatively small inflationary pressures, unless the Phillips curve is much steeper than prevailing estimates tend to suggest.

Literature. Our analysis relates and contributes to several strands of literature. First, we offer a different perspective on fiscal space than the "$r < g$" literature (Blanchard, 2019; Mehrotra and Sergeyev, 2021; Reis, 2022). Similar to this literature, our results suggest that deficits can be financed without

[4]Throughout we are concerned primarily with the modern, New Keynesian version of the FTPL (e.g., Cochrane, 2018, 2023), but similar points apply to its original, neoclassical version (Leeper, 1991; Sims, 1994; Woodford, 1995).

[5]By the same token, our self-financing result, unlike that of the FTPL, is also robust the kind of global-game perturbations used in Angeletos and Lian (2023) to remove the equilibrium indeterminacy of the New Keynesian model.

future tax hikes. However, unlike that literature, we do not require the real interest rate on government debt to be lower than the economy's real growth rate. Instead, we emphasize how fiscal deficits can contribute to their own financing by triggering a Keynesian boom. Mian, Straub and Sufi (2022) touch upon both these issues ("$r < g$" and deficit-driven Keynesian booms), but emphasize a self-financing mechanism different from ours: they focus on how additional debt issuance can reduce the real interest rate on government debt by triggering inflation along the ZLB.[6]

Second, we offer a different rationale for why deficits can finance themselves than that found in the FTPL literature (Leeper, 1991; Sims, 1994; Woodford, 1995; Bassetto, 2002; Cochrane, 2005)—one that relies on the failure of Ricardian equivalence due to finite horizons or liquidity constraints as opposed to equilibrium selection. In so doing, we avoid that literature's most controversial assumption—the threat to violate the government budget unless a particular equilibrium is selected (Kocherlakota and Phelan, 1999; Atkeson, Chari and Kehoe, 2010),[7] and show self-financing can instead be reconciled with the standard equilibrium selection of the New Keynesian model (i.e., with "passive" fiscal policy and "active" monetary policy). We also shift the focus from the inflation/debt erosion channel—the focal point of the FTPL literature—to the tax base channel, which in our quantitative explorations actually turns out to be dominant.

Third, we add to the large literature on the effects of fiscal policy within the New Keynesian framework. Woodford (2011) and Christiano, Eichenbaum and Rebelo (2011) are the classic references for fiscal multipliers in the PIH-RANK benchmark, while Galí, López-Salido and Vallés (2007), Kaplan, Moll and Violante (2018) and Auclert, Rognlie and Straub (2018) study environments in which Ricardian equivalence fails. It is well-understood within this literature that fiscal deficits can generate demand-led booms, thus also contributing to their own financing. From this perspective, our paper's added value is to characterize the determinants of self-financing and to highlight that self-financing could even be complete.

Finally, our take-home message echoes that of DeLong and Summers (2012). These authors provide the high-level arithmetic on fiscal multipliers necessary to generate self-financing *without* a micro-founded model. We instead characterize the economic primitives required for self-financing to be possible, characterize its determinants, and evaluate its quantitative potential.

Outline. Sections 2 and 3 begin by presenting the model and characterizing its equilibrium. Section 4 develops our self-financing result and discusses the economics behind it. Further extensions and the quantitative analysis follow in Sections 5 and 6, respectively. Section 7 concludes.

[6] This is akin to the form of monetary accommodation that obtains in our setting when we relax the assumption of neutral monetary policy and, instead, let real rates fall in response to a deficit-driven Keynesian boom.

[7] We note that Bassetto (2002) and Cochrane (2005) have disputed this interpretation. This debate, however, is outside the scope of our paper.

2 Model

For our main analysis we consider a perpetual-youth, overlapping-generations version of the textbook New Keynesian model. Similarly to Del Negro, Giannoni and Patterson (2015), Farhi and Werning (2019), and Angeletos and Huo (2021), mortality risk (finite lives) is a convenient proxy for liquidity frictions: it breaks Ricardian Equivalence and lets fiscal policy—i.e., debt and deficits—affect aggregate demand. As will become clear, this departure from the PIH benchmark is central to our results. We will show later how the insights obtained from our baseline model extend to more general aggregate demand structures, including those found in the HANK literature.[8]

Throughout we study log-linearized dynamics in response to a surprise increase in fiscal deficits. We use uppercase variables to indicate levels; unless indicated otherwise, lowercase variables denote log-deviations from the economy's deterministic steady state. Time is discrete, indexed by $t \in \{0, 1, ...\}$.

2.1 Households

We index households by $i = (i_1, i_2)$, where $i_1 \in \{0, 1, \ldots\}$ denotes their age and $i_2 \in [0, 1]$ their name. A household survives from one period to the next with probability $\omega \in (0, 1]$, so that $1 - \omega$ is the mortality rate. Whenever a household dies, it is replaced by a new household (with the same name i_2 but age reset to $i_1 = 0$). Households do not altruistically value the utility of the future households that replace them. Taking into account the mortality risk, the expected lifetime utility of any (alive) household i in period $t \in \{0, 1, \ldots\}$ is therefore given by

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\beta\omega\right)^k \left[u(C_{i,t+k}) - v(L_{i,t+k}) \right] \right], \tag{1}$$

where $C_{i,t+k}$ and $L_{i,t+k}$ denote household i's consumption and labor supply in period $t + k$ (conditional on survival), and preferences take the standard form $u(C) \equiv \frac{C^{1-1/\sigma}-1}{1-\frac{1}{\sigma}}$ and $v(L) = \chi \frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Households can save and borrow by trading an actuarially fair, risk-free nominal annuity. Conditional on survival, households therefore enjoy a nominal rate of return equal to I_t/ω, where I_t is the nominal rate on government bonds. Households furthermore receive labor income and dividend income, given respectively by $W_t L_{i,t}$ and $Q_{i,t}$ (both in real terms). Households also pay taxes. The real tax payment $T_{i,t}$ depends on both the individual's income and aggregate fiscal conditions: i.e., $T_{i,t} = \mathscr{T}(Y_{i,t}, Z_t)$, where $Y_{i,t} \equiv W_t L_{i,t} + Q_{i,t}$ is the household's total real income, Z_t captures aggregate conditions (including outstanding government debt), and $\mathscr{T}$ is a function describing tax policy (to be specified later). Finally, old households are obliged to make contributions to a "social fund" whose

[8] Also note that, consistent with Woodford (2003*b*) and Galí (2008), we will consider a "moneyless" economy. There is thus no seignorage revenue and the channel in Sargent and Wallace (1981) will not help finance deficits.

proceeds are distributed to the newborn households; the role of this fund is explained momentarily. All in all, the date-t budget constraint of household i is therefore given as

$$A_{i,t+1} = \underbrace{\frac{I_t}{\omega}}_{\text{annuity}} \left(A_{i,t} + P_t \cdot \big(\underbrace{W_t L_{i,t} + Q_{i,t}}_{Y_{i,t}} - C_{i,t} - T_{i,t} + S_{i,t}\big)\right), \tag{2}$$

where $A_{i,t}$ denotes i's nominal saving at the beginning of date t, P_t is the date-t price level, and $S_{i,t}$ is the transfer from or contribution to the fund, with $S_{i,t} = S^{\text{new}} > 0$ for newborns and $S_{i,t} = S^{\text{old}} < 0$ for old households (and where $(1-\omega)S^{\text{new}} + \omega S^{\text{old}} = 0$, ensuring that the fund is balanced).

Compared to Blanchard (1985), the only novelty in our set-up is the social fund. We will set $S^{\text{new}} = D^{ss}$ (and therefore $S^{\text{old}} = -\frac{1-\omega}{\omega} D^{ss}$), where D^{ss} is the real steady-state value of public debt (and so in equilibrium also the real steady-state value of private wealth). The fund thus ensures that all cohorts, regardless of age, enjoy the same wealth and hence the same consumption in steady state. This in turn affords two simplifications. First, it simplifies aggregation when we log-linearize the model around its steady state, with every cohort equally weighted in aggregate demand. Second, it guarantees that the steady state of our model is invariant to both ω and the steady-state level of public debt, and hence is exactly the same as its RANK counterpart. The two models thus differ only in terms of how fiscal policy influences output gaps, cleanly isolating the mechanism that we are interested in.

It remains to specify how household income is determined. First, we assume that all households receive identical shares of dividends. Second, we abstract from heterogeneity in labor supply. Specifically, we assume that labor supply is intermediated by labor unions. Those unions bargain on behalf of households, equalizing the (post-tax) real wage and the marginal rate of substitution between consumption and labor supply. Since all households work the same hours, it follows that all households receive the same labor income. Overall, the resulting household labor supply relation is exactly the same as in the textbook New Keynesian model.[9] Putting the pieces together, we conclude that $Y_{i,t} = Y_t$ and $T_{i,t} = T_t$—in any given period, all households receive the same income and face the same taxes.

2.2 Firms

The production side of the economy is the same as in the textbook New Keynesian model: there is a unit-mass continuum of monopolistically competitive retailers who set prices subject to a standard Calvo friction, hire labor on a spot market, and pay out all their profits as dividends back to households. Combined with our assumptions on household labor supply, this implies that the supply block of our economy reduces to the standard New Keynesian Phillips curve (NKPC).

[9] To be precise, this statement presumes that the tax distortion is time-invariant, which is indeed the case under our upcoming specification for taxes. Further details on the labor supply block of our economy are provided in Appendix A.1.

2.3 Policy

The government consists of two blocks: a fiscal authority issuing debt and setting taxes, and a monetary authority setting nominal interest rates.

Fiscal policy. We (for now) abstract from government spending and let B_t denote the total nominal public debt outstanding at the beginning of period t. We can then write the *nominal* flow budget constraint of the government as follows:

$$\frac{1}{I_t}B_{t+1} = B_t - P_t T_t, \tag{3}$$

where $T_t \equiv \int T_{i,t}di$ is the total real tax revenue at t. Letting $D_t \equiv B_t/P_t$ denote the *real* value of public debt, $\Pi_{t+1} \equiv P_{t+1}/P_t$ the realized inflation between t and $t+1$, and $R_t \equiv I_t/\mathbb{E}_t[\Pi_{t+1}]$ the (expected) real rate at t, we can rewrite the government budget in *real* terms as

$$D_{t+1} = R_t(D_t - T_t)\left(\frac{\mathbb{E}_t[\Pi_{t+1}]}{\Pi_{t+1}}\right).$$

This underscores how an inflation surprise between t and $t+1$ erodes the real value of the outstanding nominal debt, thus reducing the tax revenue needed to balance the government budget.

We henceforth log-linearize around a steady state in which inflation is zero, real allocations are given by their flexible-price counterparts, and the real debt burden is constant at some arbitrary level D^{ss}. Thanks to the annuities (which offset the mortality risk) and the social fund (which makes sure that all cohorts enjoy identical wealth and consumption in steady state), the steady-state real rate is the same as in the PIH-RANK counterpart: $R^{ss} = \frac{1}{\beta} > 1$.[10] Steady-state taxes then satisfy $T^{ss} = (1-\beta)D^{ss}$. While we will throughout focus on on the empirically relevant scenario with $D^{ss} > 0$, we do wish to accommodate $D^{ss} = 0$, and so we let $d_t \equiv (D_t - D^{ss})/Y^{ss}$, $b_t \equiv (B_t - B^{ss})/Y^{ss}$, and $t_t \equiv T_t/Y^{\mathrm{ss}}$—i.e., we measure fiscal variables in terms of absolute deviations (rather than log-deviations) from steady state, scaled by steady-state output. Re-writing (3) in real terms and linearizing, we obtain

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}}r_t}_{\text{expected debt burden tomorrow}} + \underbrace{\frac{D^{ss}}{Y^{ss}}(\mathbb{E}_t[\pi_{t+1}] - \pi_{t+1})}_{\text{debt erosion due to inflation surprise}} \tag{4}$$

where $r_t \equiv \log(R_t/R^{ss})$, $\pi_{t+1} \equiv \log(\Pi_{t+1}/\Pi_{ss})$, and Y^{ss} is the steady-state level of output. The government must satisfy both the above flow constraint (at each t) as well as the familiar no-Ponzi condition (in the limit as $t \to \infty$). We can thus go back and forth between the infinite sequence of flow budget constraints and the corresponding integrated intertemporal budget constraint.[11]

[10] Had we allowed for steady-state growth, this would translate to "$r > g$". Thus, and unlike the literature spurred by Blanchard (2019), we are in an environment where the real cost of government borrowing is positive.

[11] Note that this does not necessarily rule out explosive debt: debt could still explode, provided that it does so at a rate lower that the steady-state real interest. But such an explosion would be at odds, not only with the spirit of our exercise, but also with the log-linearization of the economy. Throughout the subsequent analysis, we will therefore restrict attention to

It remains to specify a rule for how taxes adjust over time to balance the intertemporal government budget. This adjustment occurs in two ways. First, there is a time-invariant distortionary tax $\tau_y \in [0,1)$ on household labor and dividend income (and thus on total income). As a result, tax revenue fluctuates automatically during booms or recessions, without the fiscal authority needing to adjust the tax code—i.e., the "tax base" effect that will play a significant role in our subsequent analysis. Second, there is an additional lump-sum component, which includes both the deficit shock as well as any subsequent fiscal adjustment. The latter captures any tax hikes used, sooner or latter, to make sure that government debt returns to steady state.[12]

The picture is completed by parameterizing the speed of fiscal adjustment. We consider two such parameterizations, each serving a distinct purpose.

1. *Baseline fiscal rule.* Our baseline rule sets total taxes as follows:

$$T_{i,t} = \tau_y Y_{i,t} + \bar{T} - \mathcal{E}_t + \tau_d \left(D_t - D^{ss} + \mathcal{E}_t \right), \tag{5}$$

   where $\tau_y \in [0,1)$ is the time-invariant tax rate on income, the "intercept" $\bar{T}$ is set to guarantee budget balance at steady state, $\mathcal{E}_t$ is a mean-zero and i.i.d. deficit shock (e.g., issuance of stimulus checks), and $\tau_d \in (0,1)$ is a scalar that parameterizes the speed of fiscal adjustment. Intuitively, fiscal adjustment is arbitrarily fast for $\tau_d \to 1^-$ and arbitrarily slow for $\tau_d \to 0^+$. Thanks to our simplifying assumption that all households receive the same income, we can drop the i index and rewrite (5), after (log-)linearization, as follows:

$$t_t = \underbrace{\tau_y y_t}_{\text{tax base effect}} + \underbrace{\tau_d \cdot (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\varepsilon_t}_{\text{deficit shock}}, \tag{6}$$

   where $\varepsilon_t \equiv \mathcal{E}_t / Y^{ss}$ is the deficit shock (conveniently rescaled), $y_t \equiv \ln(Y_t / Y^{ss})$ is the deviation of output from steady state, and d_t is the corresponding deviation of the real value of public debt.

2. *Alternative fiscal rule.* Our second rule is a time-dependent variant of (5). This rule sets, after (log-)linearization,

$$t_{i,t} = t_t = \begin{cases} \tau_y y_{i,t} - \varepsilon_0 & t = 0 \\ \tau_y y_{i,t} & t \in \{1, \cdots, H-1\} \\ d_t & t \geq H \end{cases}, \tag{7}$$

   Note that this rule shuts down any deficit shocks at dates $t \geq 1$, thus allowing us to focus cleanly

policies (and equilibria) that guarantee that, following any given shock, public debt converges eventually back to steady state. Formally, we require that $\mathbb{E}_t[\lim_{k\to\infty} d_{t+k}] = 0$ for all t and all realizations of uncertainty.

[12] While our main analysis treats the required tax hikes as non-distortionary, we emphasize that this is only for the sake of simplicity in exposition. As we demonstrate in Section 5.4, allowing the tax hikes to be distortionary has minimal impact on our self-financing results.

on impulse responses to a date-0 shock.[13] The interpretation of the rule is as follows: following the date-0 shock, the government abstains from any fiscal adjustment for the first H periods, before then adjusting taxes from date H onwards to return government debt to steady state. We can thus identify H as the lag between the initial deficit and the future adjustment.

Intuitively, we can capture a longer delay in fiscal adjustment either through a low τ_d (first rule) or through a high H (second rule). This suggests that the two rules are interchangeable for our main purposes—an intuition that turns out to be correct and that we will make precise in due course.

This high-level interchangeability notwithstanding, we like to study both fiscal rules, as each serves different auxiliary purposes. On the one hand, the baseline rule (6) facilitates a tractable, recursive characterization of the equilibrium (which we use for our main theoretical results), as well as a mapping between the theory and some relevant empirical work (which we use for our quantitative exercises). Furthermore, (6) allows a sharp comparison to prior theoretical work.[14] On the other hand, the alternative rule (7) captures more transparently the timing of fiscal adjustment and thus allows us to develop a sharper intuition for our limiting self-financing result. It also makes abundantly clear that our fiscal policy is "Ricardian" or "passive" in the sense of the classical FTPL literature: (7) embeds a commitment to raise taxes as needed to make sure that debt is back in steady state at $t = H$ (and thereafter), no matter what path the economy has followed up to that point, and no matter what path it is expected to follow thereafter. We will return to this point in Section 4.5.

Monetary policy. The monetary authority sets the rate of interest on nominal bonds as

$$I_t = R^{ss}\mathbb{E}_t\left[\frac{\Pi_{t+1}}{\Pi^{ss}}\right]\left(\frac{Y_t}{Y^{ss}}\right)^{\phi}, \tag{8}$$

for some $\phi \in \mathbb{R}$. This is equivalent to saying that the monetary authority implements the following relation between the (expected) real interest rate and real output:

$$r_t = \phi y_t. \tag{9}$$

Monetary policy in our model is thus parameterized by the pro-cyclicality of the real interest rate. Since deficits will be shown to be expansionary in equilibrium (provided that $\omega < 1$, i.e., that Ricardian equivalence fails), ϕ also parameterizes the comovement between real rates and deficits. We can

[13] See Wolf (2021*a*) for a discussion of the—conceptually simple but notationally involved—mapping from policy rules in sequence space to their state-space analogues. Also, the disappearance of the term $\tau_y y_{i,t}$ in (7) for $t \geq H$ may suggest that the tax distortion on labor supply also disappears at $t \geq H$, which would complicate our upcoming NKPC by adding a non-stochastic but time-varying cost-push term. We can abstract from this nuisance by setting individual taxes as $t_{i,t} = d_t - \tau_y y_t + \tau_y y_{i,t}$ for $t \geq H$. Since $y_{i,t} = y_t$, this immediately reduces to $t_t = d_t$ for $t \geq H$, in line with specification (7) above, while also guaranteeing that the marginal tax rate faced by the individual household is time-invariant. It follows that our upcoming NKPC remains the same as we move from our baseline policy rule (6) to the variant rule (7).

[14] In particular, (6) nests the fiscal rule found in Leeper (1991). As in that paper, the term $\tau_d \cdot (d_t + \varepsilon_t)$ captures fiscal adjustment: the variation in taxes induced by the exogenous deficit shock. The key novelty here is the inclusion of the term $\tau_y y_t$, which captures the emphasized tax base channel.

thus interpret $\phi < 0$ as an "accommodative" monetary authority that, in response to a positive deficit shock, lets real interest rates fall so as to increase fiscal space. Conversely, we can interpret $\phi > 0$ as a "hawkish" or "fiscally conservative" monetary authority that leans against any boom (and any inflation) triggered by deficits through rate hikes.

For our main analysis, we let monetary policy be "neutral" in the specific sense that $\phi = 0$; that is, the real rate is kept fixed throughout. This is the same baseline policy as in Woodford (2011) and allows us to cleanly isolate how the interaction of fiscal policy and private spending shapes the scope for fiscal deficit self-financing. We will relax this restriction in Section 5.3.[15]

3 Equilibrium

This section lays the groundwork for our self-financing result by characterizing the economy's equilibrium. We start by reducing the economy to a system of three equations: one for aggregate demand, one for aggregate supply, and one for the dynamics of public debt. We then characterize the unique bounded solution to this system. Throughout this section, we employ our baseline fiscal rule (6). Derivations for the alternative rule (7) are slightly different and relegated to Appendix A.2, though the economic essence is identical.

3.1 Aggregate demand

The consumption-savings problem of a household i is to choose sequences of consumption and asset holdings to maximize (1) subject to (2). Using the simplifying property that all households receive the same income (and pay the same taxes), we can express the (log-linearized) consumption function of household i in period t as follows:

$$c_{i,t} = (1-\beta\omega)\left(\tilde{a}_{i,t} + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(y_{t+k} - t_{t+k})\right]\right) - \gamma\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{10}$$

where $\tilde{a}_{i,t}$ denotes the household's *real* financial wealth (inclusive of social fund payments) and $\gamma \equiv \sigma\beta\omega - (1-\beta\omega)\beta\frac{\tilde{A}^{ss}}{P^{ss}Y^{ss}}$ combines the intertemporal substitution and wealth effects of the real interest rate. When $\omega = 1$, (10) reduces to the consumption function of a standard permanent-income household: future disposable income is discounted at rate β, and the marginal propensity to consume (MPC) from both financial wealth and permanent income is $1-\beta$. Relative to this benchmark, $\omega < 1$ maps to both more discounting of future disposable income (and hence of future taxes) and a

[15]One may also wonder how our specification of monetary policy relates to more standard Taylor rules, and how ϕ matters for equilibrium determinacy. As we make clear in due course (Section 5.3), the answers to both of these questions are of little consequence for the lessons of our paper.

higher MPC. It is by now well understood how these qualitative properties extend to richer, more realistic, HANK-type models (e.g., see Auclert, Rognlie and Straub, 2018; Farhi and Werning, 2019; Wolf, 2021*a*); as will become clear in due course, our results are driven by these more general qualitative properties of consumer demand, and not by the specific micro-foundations behind them.

Under our baseline monetary policy, $r_t = 0$ for all t, so the last term in (10) drops out. Aggregating across households, and using the fact that aggregate private financial wealth equals total government debt, we reach the following description of aggregate consumption:

$$c_t = (1-\beta\omega)\, d_t + (1-\beta\omega)\,\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\,(y_{t+k}-t_{t+k})\right]. \tag{11}$$

Next, using (6) to express future taxes as functions of the current public debt and future output, and replacing c_t with y_t (market clearing), we arrive at the following representation of aggregate demand:

$$y_t = \mathscr{F}_1\cdot(d_t+\varepsilon_t)+\mathscr{F}_2\cdot\mathbb{E}_t\left[(1-\beta\omega)\sum_{k=0}^{\infty}(\beta\omega)^k\,y_{t+k}\right], \tag{12}$$

where $\mathscr{F}_1 \equiv \frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)}$ and $\mathscr{F}_2 = 1-\frac{(1-\omega)\tau_y}{1-\omega(1-\tau_d)}$. (12) is a key equation of this paper. The first term captures the direct or partial equilibrium effect of fiscal deficits on aggregate demand. We see that $\mathscr{F}_1 > 0$ (i.e., deficits enter positively in aggregate demand) if and only if $\omega < 1$ (no Ricardian equivalence) and $\tau_d < 1$ (no *immediate* financing). Intuitively, $\tau_d < 1$ means that deficits today are financed at least in part with taxes in the future; as long as $\omega < 1$, this means that a deficit today translates to a positive real transfer from future cohorts to current cohorts, thus increasing aggregate demand. The second term then captures the general equilibrium feedback between aggregate demand and aggregate income—the "intertemporal Keynesian cross." Note in particular that $\mathscr{F}_2$ measures the "slope" of this Keynesian cross, in the following particular sense: if we raise expectations of future spending in all periods by 1, then current spending increases by $\mathscr{F}_2$.[16]

Finally, consider how equation (12) specializes for $\omega = 1$. In this case, $\mathscr{F}_1 = 0$, $\mathscr{F}_2 = 1$, and (12) collapses to $y_t = \mathbb{E}_t\left[(1-\beta)\sum_{k=0}^{\infty}\beta^k y_{t+k}\right]$, which in turn can be rewritten in recursive form as $y_t = \mathbb{E}_t[y_{t+1}]$. This makes clear two points. First, that our aggregate demand equation is a natural extension of its RANK-PIH counterpart, namely the Euler equation of a representative, infinitely-lived, financially unconstrained consumer. And second, that debt and taxes enter this equation because and only because we have departed from PIH consumer behavior.

[16] Also note that $\mathscr{F}_2 = 1$ when $\omega = 1$ or $\tau_y = 0$, but $\mathscr{F}_2 < 1$ as soon as $\omega < 1$ and $\tau_y > 0$. The combination of finite lives (or liquidity constrains) and proportional taxes thus attenuates the Keynesian feedback. This helps explain why our economy features a unique bounded equilibrium, as we discuss below and further in Appendix A.3.

3.2 Aggregate supply

By design, the aggregate supply side of our model is *exactly* the same as its familiar RANK counterpart. In particular, labor supply is given by

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t. \tag{13}$$

Together with market clearing ($c_t = y_t$) and technology ($y_t = \ell_t$), this pins down the real wage as $w_t = \xi y_t$, where $\xi \equiv \frac{1}{\varphi} + \frac{1}{\sigma} > 0$. Firm optimality, on the other hand, pins down the optimal reset price a function of current and future real marginal costs. Following standard steps, we can then reduce the supply-side of the economy to the familiar NKPC:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right]. \tag{14}$$

where $\kappa \geq 0$ depends on ξ (the pro-cyclicality of real marginal costs) and θ (the Calvo reset probability). Since there is a one-to-one mapping between κ and θ, and since this mapping is invariant to both fiscal and monetary policy, we henceforth treat κ as an exogenous parameter and (re)parameterize the degree of price flexibility by it.

3.3 Law of motion for public debt

The remaining third equilibrium restriction comes from combining the government's flow budget constraint with the fiscal rule (6). This yields the following law of motion for public debt:

$$d_{t+1} = \beta^{-1}\big(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y y_t}_{\text{tax base}}\big) - \underbrace{\frac{D^{ss}}{Y^{ss}}\left(\pi_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right)}_{\text{debt erosion}}. \tag{15}$$

with initial condition[17]

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0. \tag{16}$$

Finally, recall that the sequence of government flow budget constraints must be complemented with the usual no-Ponzi restriction $\lim_{k\to\infty}\mathbb{E}_t\left[\beta^k d_{t+k}\right] = 0$.

3.4 Equilibrium definition and characterization

A standard equilibrium definition combines (i) individual optimality for consumers, (ii) individual optimality for firms, (iii) market clearing, and (iv) budget balance for the government (together with the no-Ponzi constraint). The preceding analysis has log-linearized the model and has reduced the first three requirements to equations (12) and (14), and the last requirement to equation (15). These

[17]Note that $d_0 = \frac{1}{P^{ss}}b_0 - \frac{D^{ss}}{Y^{ss}}\left(\pi_0 - E_{-1}[\pi_0]\right)$. Since we start in steady state (i.e., $b_0 = E_{-1}[\pi_0] = 0$), we recover (16).

equations, like the log-linearization itself, make sense only insofar the economy remains in a neighborhood of the steady state. Accordingly, our notion of equilibrium is as follows.

Definition 1. An equilibrium is a stochastic path $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$ for output, inflation, and the real value of public debt that is bounded in the sense of Blanchard and Kahn (1980) and that satisfies aggregate demand (12), aggregate supply (14), and the law of motion for public debt (15), along with the initial condition (16) and the no-Ponzi game condition $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$.

We can now state our first main result.

Proposition 1. *Suppose that $\omega < 1$ and $\tau_y > 0$. There exists a unique (bounded) equilibrium. Along this equilibrium, real output and real public debt satisfy*

$$y_t = \chi\left(d_t + \varepsilon_t\right) \qquad \textit{and} \qquad \mathbb{E}_t\left[d_{t+1}\right] = \rho_d\left(d_t + \varepsilon_t\right), \tag{17}$$

for some $\chi > 0$ and $\rho_d \in (0,1)$. These coefficients solve the following fixed point problem:

$$\chi = \frac{\mathscr{F}_1}{1 - \mathscr{F}_2 \frac{1-\beta\omega}{1-\beta\omega\rho_d}} \qquad \textit{and} \qquad \rho_d = \beta^{-1}\left(1 - \tau_d - \tau_y\chi\right). \tag{18}$$

Finally, inflation satisfies $\pi_t = \frac{\kappa}{1-\beta\rho_d} y_t$.

Condition (17) contains two relations. The first relation expresses the equilibrium level of output as a proportion χ of the private sector's real financial wealth (which itself equals d_t) and of the fiscal transfer (the deficit shock ε_t). Note that $\chi > 0$—i.e., deficits trigger booms. As emphasized previously, this is so because of the two key features of our model environment: the failure of Ricardian equivalence, which lets deficits stimulate aggregate demand; and the nominal rigidity, which lets aggregate demand drive output. The second relation gives the (expected) evolution of the real value of public debt, with ρ_d measuring the (expected) persistence of debt. But since y_t is proportional to d_t, we see that ρ_d here also measures the expected persistence of the Keynesian boom triggered by deficits.

Condition (18) summarizes the fixed-point relation between χ and ρ_d—i.e., the two-way feedback between aggregate demand and fiscal conditions. On the one hand, as long as $\tau_y > 0$, higher aggregate demand contributes to higher output, higher tax revenue and thereby to lower public debt tomorrow. This feedback is reflected in the second part of condition (18), which pins down ρ_d as a function of χ and of the two fiscal policy parameters (τ_d, τ_y). On the other hand, as long as $\omega < 1$, more delay in fiscal adjustment, or more persistence in public debt, will translate to a larger effective transfer from generations in the far future to generations in the present and the near future, thus stimulating aggregate demand both directly (the partial equilibrium effect) and indirectly (the general equilibrium Keynesian cross). This is reflected in the first part of condition (18), which pins down χ as a function of ρ_d and of the relevant aggregate-demand parameters $(\mathscr{F}_1, \mathscr{F}_2, \beta\omega)$. We emphasize that the feedback from deficits to aggregate demand is present only when $\omega < 1$, while the feedback from aggregate demand to tax revenue and thereby to public debt dynamics is present only when $\tau_y > 0$. The textbook model assumes away *both* feedbacks. We return to this point at the end of Section 4 and also in Appendix A.3, where we explain the conceptual differences between our analysis and the classical FTPL. For now, we wish to emphasize that the aforementioned two-way feedback is responsible *both* for the uniqueness of the equilibrium and for our upcoming self-financing results.[18]

We close this section with an important remark about the role of inflation. While Proposition 1 applies regardless of κ, there is a subtle difference as we move from $\kappa = 0$ to $\kappa > 0$. When prices are rigid, d_t is predetermined in the beginning of period t, and so the second part of (17) holds state-by-state, and not just in expectation. When instead prices can move, then d_t is no longer predetermined—it depends on the concurrent innovation in the price level, which itself depends on the concurrent innovation in ε_t. This is the inflation surprise term in equation (15). Proposition 1 can then be read as saying that, in general, date-0 inflation rescales the initial real debt burden d_0, but otherwise has no further effect on the dynamic shape of the output, tax, and debt impulse responses. In fact, this separation between "scale and dynamics" extends to an arbitrary Phillips curve: the latter's specification influences the equilibrium mapping of the innovations in ε_t to innovations in $d_t + \varepsilon_t$, but it does not change at all the dynamic two-way feedback between real private wealth (also, real government debt) and real economic activity characterized in Proposition 1. This observation not only explains why our upcoming self-financing result is robust to the value of κ or the specification of the NKPC, but will also be important for some of the further extensions that we consider in Section 5. All in all, Proposition 1 and the two-way feedback encapsulated in it hold the key to the rest of our analysis.

4 Self-financing of fiscal deficits

This section presents our headline result on the possibility of self-financing deficits. We first use the intertemporal government budget constraint to provide a quantitative measure of the degree of self-financing. We then show that *complete* self-financing is possible in the limit as fiscal adjustment is delayed further and further (in the sense of $\tau_d \to 0$ or $H \to \infty$), and we explain the economics behind this result. Finally, we zero in on the roles played by our environment's three other key parameters (κ, τ_y, and ω) and we contrast the economics behind our result to those behind the FTPL.

[18] Under the alternative rule (7), instead, uniqueness requires one of the following minor modifications: (i) a strengthening of the notion of boundedness to $\lim_{k\to\infty} \mathbb{E}_t \left[y_{t+k}\right] = 0$, which amounts to saying that expectations "at infinity" are anchored to the steady state; or (ii) the reinterpretation of $\phi = 0$ as the limit of $\phi \to 0$ from above, which is basically a (limit) Taylor principle. The sole role of either one of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. See Appendix A.2 for details.

4.1 Sources of fiscal financing

Iterating the debt relations (15) and (16) forward and using $\lim_{t\to\infty}\mathbb{E}_0\left[\beta^t d_t\right]=0$ (since $\rho_d\in(0,1)$), we obtain the following present-value restriction on fiscal policy:

$$\underbrace{\varepsilon_0}_{\text{deficit}} = \underbrace{\tau_d\left(\varepsilon_0+\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[d_k\right]\right)}_{\text{fiscal adjustment}} + \underbrace{\sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right]}_{\text{self-financing via tax base}} + \underbrace{\frac{D^{ss}}{Y^{ss}}\left(\pi_0-\mathbb{E}_{-1}\left[\pi_0\right]\right)}_{\text{self-financing via debt erosion}} . \tag{19}$$

The left-hand side of (19) is the exogenous shock in the initial deficit, while the right-hand side contains the three ways in which this shock will be financed over time: the first term captures the adjustment in current and future taxes triggered by the shock and any resulting accumulation of public debt; the second term collects the present value of the extra tax revenue generated by the deficit-driven boom in real economic activity; and the third term gives the erosion in the real debt burden caused by the associated innovation in inflation. Put differently, the first term captures the conventional notion of fiscal adjustment—the government actively adjusts its primary surplus to stabilize its debt—while the second and third terms reflect our two sources of self-financing. Finally, we note that a fourth source of financing—monetary accommodation—emerges if the monetary authority depresses real rates in response to deficits. In our main analysis, the assumption that $\phi=0$ means that this channel is not operative.

We can now define the (overall) degree of self-financing as follows:

Definition 2. The degree of self-financing is the fraction of the initial deficit that is financed by an expansion in the tax base and/or an erosion in the real debt burden:

$$\nu\equiv\frac{\sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right]+\frac{D^{ss}}{Y^{ss}}\pi_0}{\varepsilon_0} \tag{20}$$

This definition applies regardless of whether fiscal policy obeys the baseline rule (6) or the variant rule (7). Note next that the overall degree of self-financing can be decomposed into its two components:

$$\nu\equiv\nu_y+\nu_p$$

where

$$\nu_y\equiv\frac{1}{\varepsilon_0}\tau_y\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[y_k\right] \qquad \text{and} \qquad \nu_p\equiv\frac{1}{\varepsilon_0}\frac{D^{ss}}{Y^{ss}}\pi_0 \tag{21}$$

measure, respectively, the tax-base and debt-erosion components of self-financing. Finally, because the NKPC implies that $\pi_0=\kappa\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[y_k\right]$, it is immediate that

$$\nu_p=\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y. \tag{22}$$

This relation suggests that once we understand the two-way feedback between real economic activity and fiscal deficits in the rigid-price benchmark ($\kappa = 0$), it will be straightforward to extend the analysis to the general case ($\kappa > 0$). In the rest of this section we therefore proceed as follows: we first state our headline result for the general case; we next expand on the economics behind it under the special case of $\kappa = 0$ (rigid prices); and we finally discuss the role of letting $\kappa > 0$.

Before proceeding to our main results we close with a brief remark on our two fiscal policy specifications (6) and (7). As anticipated in Section 2.3, these two specifications are indeed equivalent ways of thinking about the effects of delays in fiscal adjustment.

Lemma 1. *Suppose that $\omega < 1$ and $\tau_y > 0$. There exists a strictly decreasing mapping $\mathcal{T} : \mathbb{N} \to (0,1]$, with $\lim_{H\to\infty} \mathcal{T}(H) = 0$, such that the degree of self-financing generated by policy (7) is the same as that generated by policy (6) if and only if $\tau_d = \mathcal{T}(H)$.*

Importantly, this result establishes that "infinite financing delay" in the sense of $H \to \infty$ under the rule (7) maps to $\tau_d \to 0$ under the rule (6), and vice versa.

4.2 The self-financing result

We can now state our main theoretical result on the possibility of self-financing.

Theorem 1. *Suppose that $\omega < 1$ and $\tau_y > 0$, and let fiscal policy follow either our baseline rule (6) or the variant rule (7). The equilibrium degree of self-financing, ν, has the following properties:*

1. *It is increasing in the delay of fiscal adjustment; i.e., ν is decreasing in τ_d for the fiscal rule (6) and increasing in H for the fiscal rule (7)).*

2. *It converges to 1 as fiscal adjustment is delayed further and further; i.e., $\nu \to 1$ (from below) as $\tau_d \to 0$ (from above) or as $H \to \infty$. Furthermore, these two limits induce the same equilibrium paths $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$, and in this common limit self-financing is sufficiently strong to return real government debt to steady state (i.e., $\lim_{k\to\infty} \mathbb{E}_t [d_{t+k}] \to 0$ for the baseline rule (6) and $\lim_{H\to\infty} \mathbb{E}_0 [d_H] \to 0$ for the variant rule (7)).*

Theorem 1 is our core result. Its main implication is that the failure of Ricardian equivalence—here encapsulated in $\omega < 1$—opens the door for fiscal deficits to finance themselves. First, as fiscal adjustment is delayed, the initial fiscal deficit induces both a larger and more persistent Keynesian boom, thus increasing the share of self-financing through higher tax revenue and a larger date-0 price level jump. Second, the limit as $\tau_d \to 0$ or $H \to \infty$ is one of *complete* self-financing: the deficit-driven boom is large and fast enough to cover the cost of the initial fiscal outlay ε_0 and to make sure that public debt automatically returns back to steady state, without any fiscal adjustment.

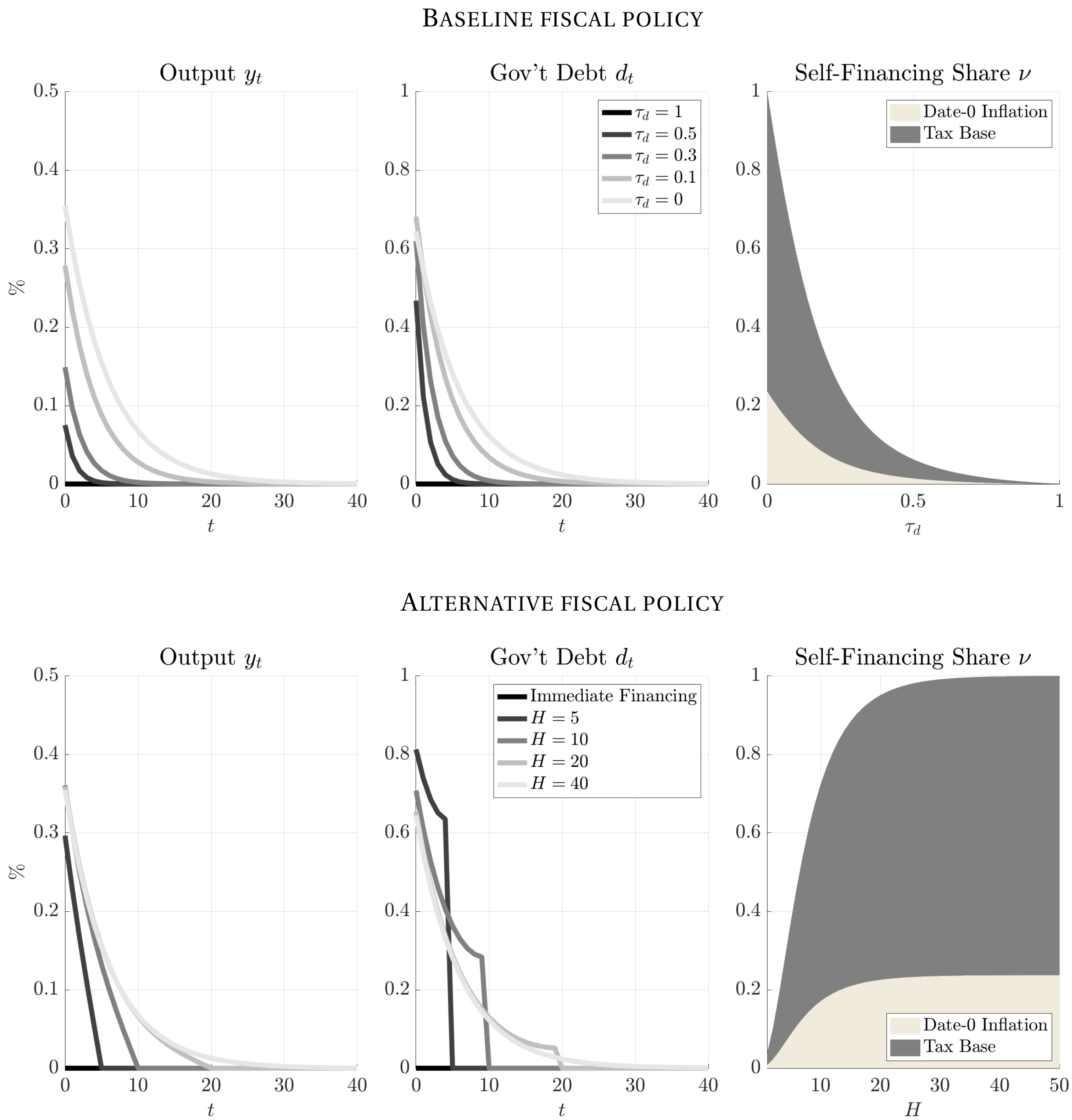

Figure 1: Top panel: impulse responses of output y_t, government debt d_t, and the self-financing share ν to the deficit shock ε_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

A visual illustration. We provide a visual illustration of Theorem 1 in Figure 1.[19] The figure shows the effects of a deficit shock ε_0 under different assumptions about fiscal adjustment. The left and middle panels in the top half of the figure begin by showing impulse responses of output y_t and government debt d_t as a function of the fiscal adjustment parameter τ_d in our baseline fiscal rule (6). Consistently with Theorem 1, we see that smaller fiscal response coefficients correspond to larger impact output booms (i.e., larger χ) and more persistent deviations of output and government debt from steady state (i.e., larger ρ_d). This boom then contributes to financing of the initial deficit ε_0 through our two self-financing channels: a tax base expansion and a jump in the date-0 price level. The top right panel of the figure then reports this degree of self-financing ν—as well as the split into ν_y and ν_p—as a function of the fiscal adjustment parameter τ_d. We see that ν is decreasing in the strength of fiscal adjustment τ_d, i.e., increasing in the delay of fiscal adjustment. In particular, as τ_d declines towards zero, the degree of self-financing converges to one.

The bottom half of Figure 1 provides a different perspective on the same logic, using instead the alternative fiscal rule (7), in which taxes adjust after some (finite) horizon H to perfectly balance the budget. We see again that ν is increasing in the delay of fiscal adjustment, and in particular again converges to one as fiscal adjustment is delayed further and further ($H \to +\infty$). This not only offers a complementary interpretation of what "delay" means, but also proves the following important point: our self-financing result is consistent with fiscal policy being "Ricardian" or "passive" in the strong sense that it commits on bringing debt back to steady state at $t = H + 1$ regardless of the path that the economy has taken up to that point.

4.3 The economics behind the self-financing result

To understand the economics behind Theorem 1 as transparently as possible, we in this subsection restrict attention to the special case of fully rigid prices (i.e., we set $\kappa = 0$). Based on the discussion at the end of Section 3, shifting from this rigid-price case to the general sticky-price case will be straightforward, only requiring us to re-scale all impulse responses without changing their shape. We also focus on the fiscal "H-rule" (7). As anticipated before, this fiscal rule is pedagogically useful because it makes clear both what we mean by delay in fiscal adjustment and why our results are consistent with fiscal policy being "passive" or "Ricardian" in the sense of FTPL literature.

The policy experiment studied in the remainder of this section is thus as follows: the fiscal authority pays out a lump-sum transfer to households at date 0 and promises to hike taxes at date H in order

[19] We only emphasize *qualitative* features of our results here, so we do not in detail discuss the model parameterization; it suffices to note that we set $\omega = 0.75$—a meaningful departure from Ricardian equivalence—and $\kappa = 0.1$—a rather steep NKPC, allowing a clean visual illustration of our two sources of self-financing. A more serious quantitative investigation is relegated to Section 6.

to return debt to its steady-state value at date $H+1$. The questions of interest are how this policy affects equilibrium outcomes, how large the required tax hike at H turns out to be, and what happens as we increase H. We address these questions in two steps. First, to build intuition, we analyze a simple two-period example. Second, we show how the intuition from this simple, essentially static example sheds light on the workings of our full dynamic economy.

A simple static example. We consider a two-period economy in which the government pays out a transfer ε to households at $t=0$, generates automatic tax revenue τ_y for every dollar of output, and taxes households to return debt to trend at $t=1$ (as necessary). Prices are fully rigid, so output at $t=0$ is fully demand-determined. We assume that consumer demand in period 0 is given as

$$c = \text{MPC} \cdot y_{\text{disp}}$$

where MPC $\in (0,1)$ is the marginal propensity to consume and

$$y_{\text{disp}} = (1-\tau_y)y + \varepsilon$$

is disposable income. We note that this set-up embeds a myopia assumption: date-0 consumption is invariant to date-1 outcomes, thus allowing us to characterize the date-0 equilibrium without reference to what happens later. By imposing market clearing ($y=c$), we immediately see that the date-0 equilibrium level of income is given by

$$y = \frac{\text{MPC}}{1-(1-\tau_y)\text{MPC}} \times \varepsilon \tag{23}$$

This equation is just the solution of the familiar, static Keynesian cross: MPC is the partial equilibrium effect of a unit transfer; $(1-\tau_y)$MPC is the slope of the Keynesian cross; and $\frac{1}{1-(1-\tau_y)\text{MPC}}$ is the general equilibrium multiplier.

Consider now the government's budget constraint. Since the government hands out the transfer ε and collects taxes $\tau_y y$, the net deficit at the end of date 0 is $\varepsilon - \tau_y y$. The amount of public debt inherited at date 1 is thus given by

$$\text{debt tomorrow} = R(\varepsilon - \tau_y y), \tag{24}$$

where R is the real interest rate between the two periods. Plugging (23) into (24), we conclude that

$$\text{debt tomorrow} = R(1-\nu)\varepsilon$$

where

$$\nu \equiv \frac{\tau_y y}{\varepsilon} = \frac{\tau_y \text{MPC}}{1-(1-\tau_y)\text{MPC}} \tag{25}$$

is the degree of self-financing.

Equation (25) reveals two important insights. First, we see that a higher MPC maps both to a larger partial equilibrium effect (numerator) and to a higher general equilibrium multiplier (denominator), and therefore overall to a larger degree of self-financing ν. Second, in the limit as MPC $\to 1$, the partial equilibrium effect converges to 1, the multiplier converges to $\frac{1}{\tau_y}$, and ν converges to 1—i.e., there is complete self-financing.

Back to the full model. To what extent is the simple, effectively static, example offered above informative about what is actually going on in our full dynamic economy? Note that complete self-financing ($\nu \to 1$) in the static model relies on two key properties: first, that the expected date-1 tax hike does not affect date-0 spending behavior; and second, that the date-0 transfer as well as all the additional income it generates are fully spent at date 0 (MPC $\to 1$), thus generating enough tax revenue to stabilize debt *before* the promised date-1 tax hike. The core intuition is that, as the financing delay H increases, our dynamic economy starts to emulate those two features of the static example.

To see why this is so, we begin by highlighting two important properties of our economy's aggregate consumption function. Formal details are provided in Lemma D.1 in Appendix D.

1. *Discounting.* Consider first how consumption demand at date $t \geq 0$ responds to an anticipated future change in disposable income at some future date $t+\ell$, with $\ell \geq 0$. We write this response as $\mathscr{M}_{t,t+\ell}$—the $(t, t+\ell)$ element of the matrix of intertemporal MPCs studied by Auclert, Rognlie and Straub (2018). It is straightforward to see that $\beta^{-\ell}\mathscr{M}_{t,t+\ell} = (1-\beta)$ in the PIH benchmark ($\omega = 1$). With finite horizons ($\omega < 1$), on the other hand, it can be shown that $\beta^{-\ell}\mathscr{M}_{t,t+\ell}$ is strictly decreasing in ℓ, and in particular that
$$\lim_{\ell\to\infty} \beta^{-\ell}\mathscr{M}_{t,t+\ell} = 0.$$
In words, as long as $\omega < 1$, an income change of a fixed present value that occurs further and further into the future has a diminishing and eventually vanishing effect on current consumption. We refer to this property of aggregate demand as "discounting."

2. *Front-loading.* Consider next how changes in disposable income at date $t \geq 0$ affect consumption demand at some future date-$t+\ell$, with $\ell \geq 0$. We write this response as $\mathscr{M}_{t+\ell,t}$—the $(t+\ell, t)$ element of the intertemporal MPC matrix. In the PIH benchmark, we have that $\mathscr{M}_{t+\ell,t}$ is invariant to ℓ, and in particular that $\mathscr{M}_{t+\ell,t}\beta^{-t} = (1-\beta)$, reflecting perfect consumption smoothing. With finite horizons, we instead have that $\mathscr{M}_{t+\ell,t}$ is decreasing in ℓ, with
$$\lim_{\ell\to\infty} \mathscr{M}_{t+\ell,t} = 0.$$
In words, consumers tend to spend any income receipt faster than in the PIH benchmark, and any income shock today has a vanishing effect on consumption far in enough in the future. We refer to this property as "front-loading".

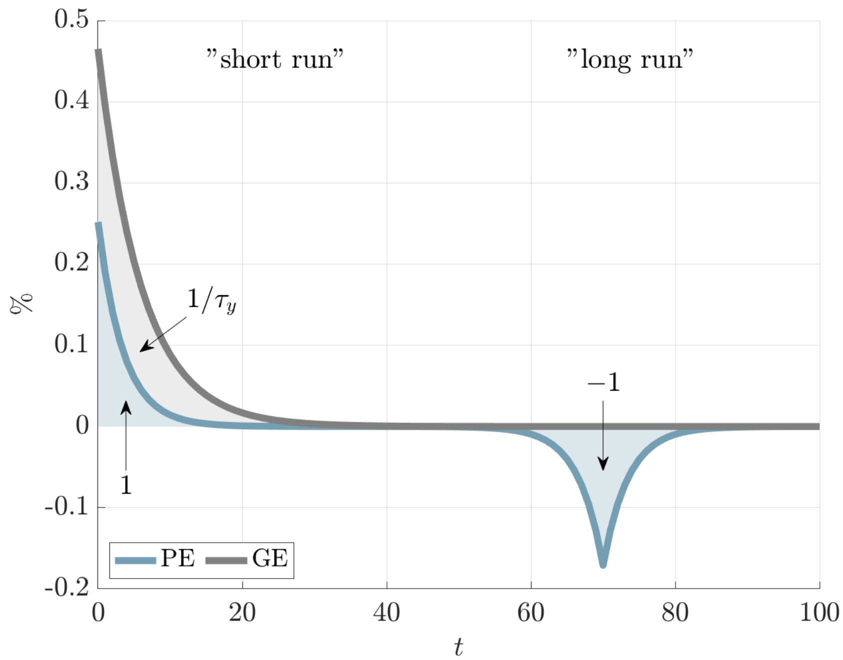

Figure 2: Direct, partial equilibrium demand effect (blue) and full general equilibrium output impulse response (grey) to a date-0 deficit shock with fiscal financing at date $H = 70$.

These two properties of the aggregate consumption function allow us to connect our full dynamic economy with the simple two-period example; a visual illustration to accompany the following discussion is furthermore provided in Figure 2.

The first object shown in the figure (in blue) is what we refer to as the "partial equilibrium" effect of the policy. It captures the following thought experiment: Suppose the government pays out a transfer of size 1 at date 0, but then counterfactually assume that aggregate output and inflation were to not move in response to that policy. Then, debt would grow at rate $R^{ss} = \beta^{-1}$ all the way up until date H, when the government hikes taxes to stabilize government debt. The blue line gives the response of consumption demand to this combination of date-0 transfer and date-H tax hike.[20] The discounting and front-loading properties allow us to understand the shape of this partial equilibrium effect. By "discounting", the date-0 cohort is essentially unaffected by the future announced tax hike; by "front-loading", it spends its lump-sum transfer receipt relatively quickly, with the cumulative MPC approaching 1 faster than in the PIH benchmark. The cohorts born shortly after date-0 are also essentially unaffected by the future tax hike, so overall spending demand is back to trend after around 20 periods. It is only around $t = 60$ that expectations of the future tax hike at $H = 70$ start to depress demand. Our dynamic economy thus echoes the static example: the future tax hike does not affect

[20]Mathematically, the blue line plots $\mathscr{M} \cdot \boldsymbol{t}^{PE}$, where $\mathscr{M}$ is the full matrix of intertemporal MPCs and $\boldsymbol{t}^{PE}$ is the "partial equilibrium" tax-and-transfer vector, which equals -1 at date 0, β^{-H} at date H, and 0 otherwise.

"short-run" spending behavior, and the "short-run" cumulative MPC approaches 1. The key is to interpret the "short run" as an interval of time that is long enough but also sufficiently distant from the tax hike; in the figure, this corresponds to, roughly, the first 20 quarters.

The second object shown in the figure (in grey) is the full general equilibrium effect of our policy experiment—the object we formally characterized in Theorem 1. In general equilibrium, since prices are rigid, the initial increase in consumer demand generates additional income. Importantly, again by the front-loading property of consumer demand, this income is spent quickly. Since the cumulative short-run MPC is 1, this delivers a front-loaded Keynesian multiplier of size $\frac{1}{\tau_y}$. Crucially, this Keynesian boom increases tax revenue (in date-0 present value terms) by $\tau_y \times \frac{1}{\tau_y} = 1$, thus returning government debt to trend far before date H (or "the long-run"). As a result, the promised subsequent tax hike at H—together with its negative effect on spending—vanishes, yet again echoing what happens as MPC $\to 1$ in the simple example above.

While the above discussion focused on the limit as $H \to \infty$, the underlying intuition also readily connects with the monotonicity of ν in H. As H is increased, the effect of the anticipated tax hike on short-run demand decreases, thereby strengthening the initial partial equilibrium demand boom. Similarly, the larger H, the longer the general equilibrium feedback—i.e., the Keynesian cross—can play out before being moderated by the future tax hike. The size of the short-run boom thus is increasing in H, and by extension so is the endogenously raised tax revenue. Finally, we note that the exact same logic also explains why the equilibrium boom becomes larger and more persistent for smaller τ_d under our baseline fiscal rule (6). Either way, as fiscal adjustment is delayed further and further, the short-run Keynesian boom on its own becomes big enough to stabilize debt.

A comment on generality. We emphasize that the intuitions offered above transcend the particular OLG economy that underlies Theorem 1 and our graphical illustration in Figure 1. We will later make this claim precise in Section 5.1, where we generalize our self-financing result to much richer aggregate demand structures. This generalization will identify sufficient conditions that—in line with the intuition offered above—ensure (i) deficits inducing in partial equilibrium a positive and front-loaded response in aggregate demand, and (ii) this translating in general equilibrium to a boom in real economic activity that is both large enough and fast enough to raise the required tax revenue prior to the promised future tax hike.

4.4 The roles of κ, τ_y, and ω

So far we have focused on how ν depends on the delay in fiscal adjustment (τ_d or H). We now study how ν changes with our model's three other key parameters: the degree of price flexibility (κ), the feedback from aggregate income to tax revenue (τ_y), and the proximity to PIH behavior (ω).

The degree of price flexibility. The intuition offered in the previous subsection considered the special case of $\kappa = 0$ (rigid prices), but extends with little change to $\kappa > 0$. In this general case, the Keynesian boom brings with it inflation, reducing the real value of public debt. Relative to the rigid-price case, this only increases the potency of self-financing. Furthermore, since the innovation in inflation is—via the NKPC—proportional to the innovation in the present discounted value of aggregate income, we can readily compute the relative importance of the two sources of self-financing, as shown in equation (22). Putting everything together, we have the following result.

Proposition 2. *Let $\omega < 1$, $\tau_y > 0$, $\kappa > 0$ and $\frac{D^{ss}}{Y^{ss}} > 0$, and consider either of our two fiscal policies.*

1. *The overall degree of self-financing, ν, increases with the degree of price flexibility κ, and the steady-state debt-to-GDP ratio, $\frac{D^{ss}}{Y^{ss}}$.*

2. *The contribution of the inflation/debt erosion channel, ν_p/ν, is increasing in κ and $\frac{D^{ss}}{Y^{ss}}$.*

The more flexible are prices, the larger the share of self-financing that comes via prices rather than quantities. In particular, as $\kappa \to \infty$, *all* self-financing comes through the date-0 jump in prices.

The strength of the tax base channel. Let us now zero in on the role of τ_y, starting with the case of rigid prices ($\kappa = 0$) and focusing on the tax base channel. In principle, τ_y has two offsetting effects on ν: first, mechanically, a larger τ_y means more tax revenue for any given boom; second, by weakening the intertemporal Keynesian cross (see (12) and (18)), a larger τ_y decreases the size and persistence of the boom. The next result verifies that the first effect dominates.

Proposition 3. *Let $\omega < 1$, $\tau_y > 0$, and $\kappa = 0$, and consider either of our two fiscal policies. The overall degree of self-financing, ν, increases with τ_y.*

When $\kappa > 0$, a third effect enters the picture: the aforementioned dampening of the real boom also dampens the inflation response, thus weakening the debt erosion channel. When κ and/or $\frac{D^{ss}}{Y^{ss}}$ are large enough, this effect may dominate. Quantitatively, however, this is never the case in the neighborhood of the model parameterization that we will consider in Section 6.

Distance from PIH. Finally, consider the importance of the deviation from permanent-income consumption behavior (i.e., $\omega < 1$). For this purpose, we find it convenient to focus on our main fiscal policy specification (6)—a specification that allows us to summarize the persistence of government debt in the single coefficient ρ_d.

Proposition 4. *Let $\omega < 1$ and $\tau_y > 0$ and consider the fiscal policy* (6).

1. *For any $\tau_d > 0$, a lower ω raises ν and decreases ρ_d; that is, a larger departure from the permanent-income benchmark yields both larger and faster self-financing.*

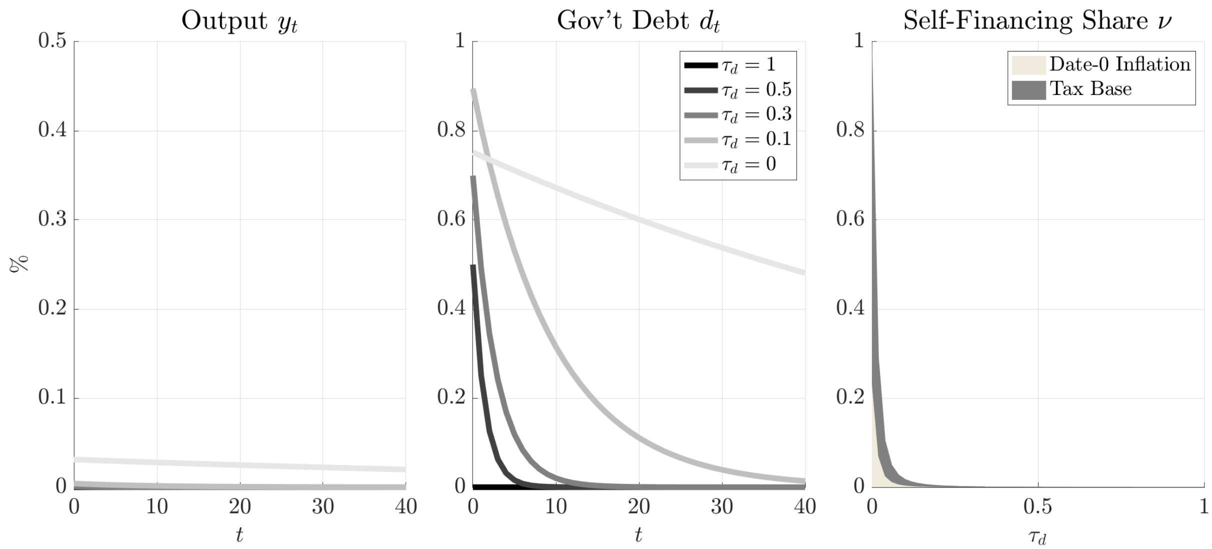

Figure 3: Impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ε_0 as a function of H in an economy with near-PIH consumers (i.e., ω close to 1).

2. *Let ρ_d^{full} be the limit of ρ_d, the persistence of government debt and output, as $\tau_d \to 0$. Then, $\rho_d^{full} < 1$ for any $\omega < 1$ but $\rho_d^{full} \to 1$ as $\omega \to 1$.*

The first part of Proposition 4 is straightforward: for any given finite delay in fiscal adjustment (i.e., for any given $\tau_d > 0$), a larger departure from the PIH benchmark (i.e., a smaller ω) implies a larger and quicker Keynesian boom, thus increasing ν and decreasing ρ_d. The second part then zeroes in on how the magnitude of this departure matters for the limit of an infinite delay in fiscal adjustment (i.e., $\tau_d \to 0$). Provided that $\omega < 1$, we can always achieve complete self-financing by pushing the fiscal adjustment sufficiently far in the future. However, the closer ω is to 1, the smaller and less front-loaded the resulting Keynesian boom is, and hence the longer it takes for public debt to return to steady state.

Figure 3 provides a visual illustration of this point. Consistent with Theorem 1, the self-financing limit still exists: $\nu \to 1$ as $\tau_d \to 0$. However, convergence to this limit is exceedingly slow: ν remains very close to 0 for all values of τ_d other than those in a small neighborhood of 0. This in turn suggests that the practical relevance of Theorem 1 critically depends on how far real-world consumer behavior is from the PIH. We tackle this question in Section 6.

4.5 Relation to the FTPL

We now zero in on the difference between our results and the Fiscal Theory of the Price Level (FTPL), which itself is built upon the PIH benchmark (i.e., $\omega = 1$). In this benchmark, and therefore also in the

FTPL, aggregate demand reduces to

$$c_t = \mathbb{E}_t \left[(1-\beta) \sum_{k=0}^{\infty} \beta^k y_{t+k} \right].$$

Ricardian equivalence thus holds in the following partial equilibrium sense: holding constant the expected path of aggregate income, optimal individual spending is invariant to fiscal policy. However, Ricardian equivalence could still fail in general equilibrium, due to a "self-fulfilling prophesy." To see what we mean by this, combine the above aggregate demand relation with output market-clearing to obtain:

$$y_t = \mathbb{E}_t \left[(1-\beta) \sum_{k=0}^{\infty} \beta^k y_{t+k} \right],$$

or equivalently $y_t = \mathbb{E}_t[y_{t+1}]$. It is immediate that this equation admits a continuum of bounded solutions. One solution of special interest is $y_t = 0$ for all t and all histories. This solution, which guarantees that Ricardian equivalence extends from partial equilibrium to general equilibrium, is known as the fundamental or minimum state variable (MSV) solution; it is the one selected by the Taylor principle; and it is also the limit of our economy's *unique* equilibrium as $\omega \to 1$ from below. But when ω is *exactly* 1, and when in addition the Taylor principle is violated, uniqueness is lost: consumers can coordinate on multiple self-fulfilling levels of aggregate spending, and fiscal policy can be used as a coordinating device. This is the crux of how the FTPL allows deficits to drive output and inflation, despite the fact that they do not enter into either aggregate demand or aggregate supply.

The plausibility of this mechanism has long been questioned. Inter alia, Kocherlakota and Phelan (1999) and Buiter (2002) have argued that the FTPL boils down to a threat by the fiscal authority to "blow up" its budget unless consumers coordinate on a very specific self-fulfilling solution. More recently, Angeletos and Lian (2023) have shown that all such solutions can be ruled out by small perturbations of social memory. These points notwithstanding, the controversies surrounding the FTPL are outside the scope of our paper. Instead, the purpose of the above discussion is to clarify the difference between our paper and the FTPL: our self-financing mechanism derives *exclusively* from a partial equilibrium failure of Ricardian equivalence, and not from any kind of self-fulfilling prophesy.[21]

5 Extensions

This section discusses several extensions of our headline self-financing result, the first two generalizing the model and the last three considering alternative specifications of fiscal and monetary policy. First, we consider a reduced-form aggregate demand relation that nests—but also goes materially beyond—our baseline OLG environment. Second, we discuss an extension featuring investment.

[21] We further elaborate on these ideas and on the characterization of the $\omega = 1$ benchmark in Appendix A.3.

Third, we relax our assumption of a "neutral" monetary policy. Fourth, in Section 5.4 we allow for fiscal adjustment to be distortionary. Finally, we show that our results extend with little change to fiscal stimulus in the form of government purchases rather than lump-sum transfers.

5.1 A more general aggregate demand structure

Extending our self-financing result to a more general aggregate demand relation allows us to: shed further light on the economics of our result; understand better the kind of environments that allow (or do not allow) for meaningful self-financing; and build a bridge to state-of-the-art HANK models.

Generalizing the demand block. Recall that, in our baseline OLG environment, aggregate consumer demand was given by the following function of current household wealth as well as current and future income net of taxes:

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{26}$$

This form of the aggregate demand relation embeds economically meaningful restrictions on consumer behavior: the MPC out of current income and wealth is the same (equal to $1-\beta\omega$), and the MPCs out of the discounted presented value of future disposable income decline at a constant rate ω. We consider a generalized aggregate demand relationship that relaxes these constraints while preserving tractability:

$$c_t = M_d\cdot d_t + M_y\cdot\left(\left(y_t-t_t\right)+\delta\cdot\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{27}$$

Equation (27) generalizes (26) in three meaningful ways. First, it allows for different MPCs out of current income and wealth, denoted respectively by $M_y \in (0,1)$ and $M_d \in (0,1)$, and where we will assume that $M_d \leq M_y$. Second, it disentangles the geometric discounting rate ω from the contemporaneous MPCs (now M_y and M_d instead of $1-\beta\omega$). And third, it additionally allows for constant discounting at rate $\delta \in [0,1]$. We will elaborate on the economics of these relaxations and give examples of several familiar models that can be nested in this structure.

A general sufficient condition for self-financing. We now revisit our self-financing result with the generalized demand relation (27) replacing our simpler baseline OLG demand block. It turns out that self-financing obtains under two key restrictions on (27), echoing the "discounting" and "front-loading" properties highlighted previously.

Assumption 1. *The aggregate consumption function features positive geometric discounting:* $\omega < 1$.

In words, MPCs out of future disposable income relative to MPCs out of current income and wealth decline strictly faster than the rate of interest β. Consistent with our discussion in Section 4.3, this is sufficient to ensure that far-ahead future tax hikes—i.e., tomorrow's tax hike in the analogy in Section 4.3—have vanishingly small effects on current aggregate demand, similar to the baseline OLG model. The fiscal deficit shock will thus lead to a demand boom around date 0.

Assumption 2. *Intertemporal MPCs are sufficiently front-loaded in the particular sense that*

$$M_d + \frac{1-\beta}{\tau_y}\left(1-\tau_y\right) M_y \left(1+\delta \sum_{k=1}^{\infty} \left(\beta\omega\right)^k\right) > \frac{1-\beta}{\tau_y}. \tag{28}$$

For (28) *to hold for all* $\tau_y \in (0,1]$, *the sufficient and necessary condition is*

$$M_d > 1-\beta \quad and \quad M_y \left(1+\delta \frac{\beta\omega}{1-\beta\omega}\right) \geq 1. \tag{29}$$

(28) is the condition required to ensure that the persistence of government debt ρ_d in the limiting full self-financing equilibrium is strictly less than 1—i.e., that government debt will return to trend even as the future tax hike becomes vanishingly small. Intuitively, this requires the general equilibrium Keynesian boom to be sufficiently front-loaded, which in turn requires households to spend any income gains sufficiently quickly. If MPCs out of income and wealth are large enough—in the precise sense of (29) or (28)—then household spending is indeed sufficiently fast.[22]

Together, Assumptions 1 and 2 suffice to generalize our self-financing result.

Theorem 2. *Suppose that aggregate demand takes the generalized form* (27) *and that Assumptions 1 and 2 hold. As* $\tau_d \to 0$ *(from above) or as* $H \to \infty$*, there exists a unique bounded equilibrium, and it is such that*

$$\nu \to 1 \text{ (from below).}$$

That is, complete self-financing obtains as fiscal adjustment is indefinitely delayed. Moreover, self-financing is sufficiently strong to return real government debt to steady state (i.e., $\lim_{k\to\infty} \mathbb{E}_t\left[d_{t+k}\right] \to 0$ *for the baseline rule* (6) *and* $\lim_{H\to\infty} \mathbb{E}_0\left[d_H\right] \to 0$ *for the variant rule* (7)*).*

Theorem 2 together with its underlying assumptions exactly echoes our intuitive discussion offered in Section 4.3. First, Assumption 1 guarantees that the future tax hike is *discounted*, so deficits will lead to a short-run boom. Second, Assumption 2 ensures that any additional income—both the initial transfer as well as all higher-order general equilibrium income gains—is spent sufficiently quickly to deliver a *front-loaded* boom that raises the required tax revenue before the promised future tax hike becomes necessary. In the remainder of this section we will discuss examples of specific models of household demand that fit into the general form (27) and either satisfy or violate our two key conditions in Assumptions 1 and 2.

[22] Specifically, the first condition in (29) (i.e., that $M_d > 1-\beta$) corresponds to the second property of the consumption function ("front-loaded MPCs") discussed in Section 4.3—that is, it ensures that $\lim_{\ell\to\infty} \mathcal{M}_{t+\ell,t} = 0$. The second condition in (29) guarantees that the general equilibrium Keynesian cross feedback is in fact front-loaded enough to stabilize debt.

What environments are consistent with self-financing? Our generalized aggregate demand block (27) is consistent with many familiar models of household consumption-savings decisions.

1. *Permanent-income consumers.* The canonical PIH model readily fits into our generalized aggregate demand structure with $M_d = M_y = 1-\beta$ and $\delta = \omega = 1$. It is immediate that Assumptions 1 and 2 are violated. First, a deficit today together with (finitely) delayed financing does not induce a demand boom, simply because future tax hikes are not discounted further. Second, even if there was a general equilibrium boom, it would never be "quick", as permanent-income households postpone part of their spending into the infinite future, violating (28).

2. *Liquidity constraints.* Our baseline OLG model can be interpreted as a reduced-form representation of occasionally binding liquidity constraints (see Farhi and Werning, 2019). This interpretation extends to the present generalization, with the following additional flexibility: since (27) disentangles the MPC out of wealth M_d and income M_y, the present structure is also consistent with "hybrid" models that add a margin of hand-to-mouth spenders. Such models have received attention in recent work because they tend to provide a relatively accurate approximation of aggregate demand in richer HANK models (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*). They are nested herein by letting ω equal the survival rate of the OLG households and by setting $M_y = \mu + (1-\mu)(1-\beta\omega)$, $M_d = (1-\beta\omega)$, and $\delta = \frac{(1-\mu)(1-\beta\omega)}{\mu+(1-\mu)(1-\beta\omega)}$, where $\mu \in (0,1)$ is the fraction of hand-to-mouth spenders. It is then straightforward to verify that, as long as $\omega < 1$,both Assumptions 1 and 2 are satisfied, and hence our self-financing result holds. Our quantitative analysis in Section 6 will rely on a particular, calibrated version of this hybrid model.

   It is instructive to also consider an alternative environment with a mixture of hand-to-mouth and permanent-income consumers—i.e., the "spender-saver model" of Campbell and Mankiw (1989) and Galí, López-Salido and Vallés (2007). This corresponds to the knife-edge case of the previous hybrid example in which $\omega = 1$. Both Assumptions 1 and 2 then fail. The presence of a margin of permanent-income savers means that (i) the effect of the future tax hike on date-0 consumption never vanishes, and (ii) the boom is never fast enough to return public debt back to trend. As a result, the date-0 boom is always and invariably offset by a subsequent bust, with the self-financing share ν equal to zero.[23] This stark result is a feature rather than a bug of our theory: classical permanent-income savers correspond to an idealistic infinite-horizon, infinite-liquidity, and infinite-rationality scenario—a limit that can be refuted empirically on multiple grounds. These include: (i) an infinite long-run elasticity of household asset demand with respect to interest rates (e.g., Moll, Rachel and Restrepo, 2022); (ii) counterfactually low and

[23] A visual illustration is provided in Appendix B.1. Also, similar conclusions apply when a margin of PIH households is added to our baseline OLG block.

backloaded MPCs (see Section 6.1); and (iii) unrealistically strong responses to news about the future (e.g., as emphasized in the forward guidance puzzle literature). Finite horizons, wealth in the utility function of savers (Michaillat and Saez, 2021), liquidity constraints (as in HANK), and certain kinds of behavioral frictions (as discussed further below) all break this unrealistic benchmark and return us to our self-financing result.

3. *Informational or behavioral frictions.* Our general demand block (27) can also capture the essence of how aggregate demand is influenced by incomplete information (Angeletos and Lian, 2018), limited rationality (Farhi and Werning, 2019; Vimercati, Eichenbaum and Guerreiro, 2021; Angeletos and Sastry, 2021), and cognitive discounting (Gabaix, 2020). The common implication of these frictions is to dampen forward-looking behavior, which translates to a lower ω in (27). This in turn has two conflicting effects. First, the initial partial equilibrium demand boom is amplified: the future tax hike is discounted even further, and so the initial boom is larger. Second, the intertemporal Keynesian cross is weakened, as future gains in income do not feed back as strongly to today's consumer spending. We can see this in Assumption 2: a smaller ω decreases the left-hand side of (28), reflecting a Keynesian boom that is less front-loaded. We provide a quantitative illustration of these effects in Section 6.2 and Appendix C.5.

5.2 Investment

Our second extension illustrates the robustness of our insights to the presence of firm investment. We sketch the main ideas here and delegate the details to Appendix B.2.

Relative to our baseline model, this extension features a meaningfully generalized supply block, with firms employing both labor and capital, and with capital owned and accumulated by the firms themselves. Firms continue to distribute all their earnings, now net of investment costs, as dividends to the households. It follows that, similar to our baseline model, aggregate firm saving (now net of physical investment) is zero and all public debt is still owned and accumulated by the households.

For our purposes, the main insight is that equation (12) continues to hold, just now in terms of consumption c_t rather than output y_t. Put differently, the Keynesian cross applies to output *net of investment*. Furthermore, since investment is not taxed, the law of motion for public debt in equation (15) also continues to hold if we replace y_t by c_t.[24] It follows that the equilibrium continues to be characterized as in Proposition 1, with y_t re-interpreted as c_t, and so nothing of essence changes.

In particular, the *only* change has to do with how the dynamics of investment and capital influence real marginal costs and thereby date-0 inflation. This complication is absent when prices are rigid; in

[24] Note that the households continue to face a proportional tax on the sum of labor and dividend income at rate τ_y; but since dividend income is now net of investment, this tax is effectively a proportional tax on their consumption.

this case, Theorem 1 remains the same. More generally, the introduction of investment modifies the inflation/debt erosion channel. However, as emphasized at the end of Section 3, this channel has only a "scale" effect: it moderates the effective innovation in real public debt (and real household wealth), without affecting its dynamic propagation, or the nature and shape of the deficit-led Keynesian boom. Our limiting self-financing result thus again applies without change.[25]

5.3 More general monetary policy

We now return to our baseline environment, but relax the assumption of a "neutral" monetary policy. Specifically, we now consider the more general monetary rule (9), restated here for convenience:

$$r_t = \phi y_t \tag{30}$$

The $\phi > 0$ scenario can be interpreted as a hawkish monetary authority that leans against the deficit-led Keynesian boom. The $\phi < 0$ scenario, on the other hand, can help capture two kinds of monetary accommodation: the case of a monetary authority that purposefully lets the government's cost of borrowing fall when fiscal space is tight; and that of a monetary authority constrained by the ZLB.[26]

We continue to identify self-financing with the combination of the tax base and debt erosion effects,[27] and we ask how $\phi \neq 0$ affects their potency. For this we will need to restrict $\phi \in [-1/\sigma, \hat{\phi})$, where $\hat{\phi} \equiv \frac{\tau_y}{\beta \frac{D^{ss}}{Y^{ss}}}$. These bounds have a simple interpretation: when $\phi = -1/\sigma$, the monetary accommodation and the deficit-driven boom turn out to be so large that there is *immediate* self-financing; and when $\phi \geq \hat{\phi}$, the increase in the interest-rate costs of public debt is so large that the tax-base channel is fully counteracted. We now have the following generalized self-financing result.

Theorem 3. *Consider our OLG-NK environment with $\omega < 1$ and $\tau_y > 0$, and let fiscal and monetary policy obey, respectively, (6) and (30). Then there exists a $\bar{\phi} \in \left(0, \frac{\tau_y}{\beta \frac{D^{ss}}{Y^{ss}}}\right)$ such that:*

1. *If $\phi < \bar{\phi}$, complete self-financing is attained as the fiscal adjustment is infinitely delayed, i.e., $\nu \to 1$ and $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$ as $\tau_d \to 0$ (from above).*

2. *If $\phi > \bar{\phi}$, there exists a $\bar{\nu} \in (0,1)$, such that any bounded equilibrium has degree of self-financing ν with $\nu < \bar{\nu}$.*

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[25] Additional subtleties would arise if we were to relax the assumption that firms do not save or borrow. If they did, then this could influence how much of the increase in public debt translates to an increase in household wealth. This mechanism is beyond the scope of our paper. On the other hand, it is straightforward to see that nothing in our arguments hinges on the abstraction from adjustment costs or variable capacity utilization, so the result presented here is fully consistent with the production structures considered in quantitative business-cycle models (e.g., Smets and Wouters, 2007).

[26] To understand the latter case, note that, as long as $\kappa > 0$, any deficit-led boom comes together with inflation; and as long as the ZLB binds, such inflation translates one to one to negative real rates, or equivalently to $\phi < 0$.

[27] While this makes sure that our notion of self-financing remains the same as we move from $\phi = 0$ to $\phi \neq 0$, it requires an adjustment in the formal definition of ν. See Appendix B.3 for details.

The intuition underlying Theorem 3 is straightforward. If $\phi < 0$ (i.e., monetary accommodation), then, in response to the fiscally-led boom, real rates fall, which not only helps the government budget but also leads to households front-loading their spending even more. The boom is thus even quicker and debt is even less persistent—i.e., we obtain a smaller ρ_d than Theorem 1. It follows that, with monetary accommodation, even less of a delay in the promised fiscal adjustment is needed to ensure material self-financing. Conversely, if $\phi > 0$ (i.e., the monetary authority leans against the boom), then real rates rise, leading households to postpone their spending, thus delaying the boom and giving larger ρ_d. The cutoff $\bar{\phi}$ is exactly the point where the monetary policy-induced delay prevents self-financing from returning real government debt to steady state, delivering $\rho_d^{\text{full}}(\bar{\phi}) = 1$. For any strictly more aggressive monetary policy ($\phi > \bar{\phi}$), complete self-financing is not possible and the requisite fiscal adjustment is bounded away from zero: for any $\phi > \bar{\phi}$, there is a threshold $\underline{\tau}_d$ such that a (bounded) equilibrium exists, and government debt returns to steady state, if and only if $\tau_d > \underline{\tau}_d(\phi)$. In particular, in this case, self-financing is necessarily partial, with ν bounded above by $\bar{\nu} < 1$.[28]

5.4 Distortionary tax hikes

Our main analysis has treated the fiscal adjustment margin as non-distortionary. We here investigate what happens if adjustment is instead distortionary. In particular, we now replace our baseline fiscal rule (5) with the following alternative:

$$T_{i,t} = \tau_{y,t} Y_{i,t} - \mathscr{E}_t \tag{31}$$

where the time-varying distortionary tax rate $\tau_{y,t}$ is now given by

$$\tau_{y,t} = \tau_y + \tau_d \frac{D_t - D^{ss}}{Y^{ss}}. \tag{32}$$

After log-linearization, this extended model variant maps into exactly the same aggregate demand relation and law of motion for public debt as before; intuitively, in those equations, what matters is only how much tax revenue is extracted from the private sector, not whether it is done in a distortionary or lump-sum way. The only model equation that is affected is the NKPC, where the time-varying distortion in labor supply now manifests itself as a cost-push shock:

$$\pi_t = \kappa(y_t + \zeta d_t) + \beta \mathbb{E}_t[\pi_{t+1}] \tag{33}$$

[28]This discussion suggests a close connection between our results here and those for the general aggregate demand relation presented in Section 5.1. Intuitively, both an aggressive monetary authority and low MPCs can result in consumer spending being sufficiently delayed, thus preventing fiscal revenue from being raised fast enough. We confirm this intuition in Appendix B.3: as we show there, the cutoff $\bar{\phi}$ can equivalently be interpreted as being the value that delivers spending sufficiently postponed to violate our most general "front-loading" condition in Assumption 2.

where ζ is a function of model primitives such that $\zeta > 0$ if and only if $\tau_d > 0$ (see Appendix B.4). Intuitively, higher debt maps into higher distortionary taxes, and thereby to a higher labor wedge and higher real marginal cost for a given level of output.

This modification of the model again has very limited effect on our headline results. First, if $\kappa = 0$, then nothing changes, and in particular the entirety of Theorem 1 continues to hold. Second, even if $\kappa > 0$, then nothing of essence changes: as we have already emphasized, our characterization of the equilibrium dynamics of y_t and d_t is robust to the specification of the NKPC, and thus in particular to the presence of additional cost-push shocks. Furthermore, in the limit of interest (i.e., as $\tau_d \to 0$), those cost-push terms in fact vanish, and so the second part of Theorem 1 remains intact even if $\kappa > 0$. The economics of this limit case are particularly transparent: because our original self-financing limit result guarantees that fiscal adjustment is never needed, it makes no difference whether the fiscal adjustment was *hypothesized* to be distortionary or lump sum.

5.5 Government spending

While we have so far considered a policy that hands out lump-sum transfers to households, it is straightforward to see that our results extend with almost no change to deficit-financed government purchases. We sketch the main points here and relegate details to Appendix B.5.

The only change relative to our baseline economy is that the government now consumes some amount G_t of the final good. We assume that G_t is a stochastic, mean-zero spending shock, and we also shut down the lump-sum transfers featured in our baseline analysis. The linearized government budget constraint becomes

$$d_{t+1} = \frac{1}{\beta}\left[d_t - t_t + g_t + \beta\frac{D^{ss}}{Y^{ss}}(i_t - \pi_{t+1}),\right] \tag{34}$$

where $g_t = \frac{G_t}{Y^{ss}}$. We next specify taxes as follows:

$$t_t = \tau_d \cdot (d_t + (1-\tau_y)g_t) + \tau_y y_t, \tag{35}$$

where the presence of $(1-\tau_y)$ in front of g_t ensures that $\tau_d = 1$ again corresponds to a period-by-period balanced budget.[29] Finally, the aggregate output market-clearing condition is replaced by

$$y_t = c_t + g_t. \tag{36}$$

Relative to our baseline environment, switching to government purchases has two effects. First, the deficit shock immediately and one-to-one maps into demand, rather than leading to a gradual, moderately persistent increase in spending (e.g., recall Figure 2). Second, g_t translates to a cost-push

[29] To see this, one can guess and verify that, under the above specification, $\tau_d = 1$ translates in equilibrium to $y_t = g_t$, $t_t = g_t$, and $d_t = 0$ for all t (i.e., the fiscal multiplier is one, the primary surplus is zero, and debt stays in steady state).

shock in the NKPC, with the formula given in Appendix B.5. We have already discussed why the second effect will always be immaterial. The first effect, on the other hand, implies that even immediately tax-financed government purchases have real effects—the well-known unit fiscal multiplier result of Woodford (2011), Auclert, Rognlie and Straub (2018) and Hagedorn, Manovskii and Mitman (2019). Our added insight here is that the fiscal multiplier increases monotonically with the delay in tax financing, and that government spending can pay for itself if tax hikes are pushed far enough.

6 Quantitative analysis

We now ascertain the quantitative relevance of our insights. Specifically, we ask: how important is the self-financing margin for realistic departures from PIH, delays in fiscal adjustment, and nominal rigidities? We address this question in two steps. First, we present and calibrate a quantitatively relevant extension of our baseline model. Second, we use the model to compute the self-financing share ν under different, empirically plausible, assumptions on delays of fiscal financing.

6.1 Model and calibration strategy

We consider a variant of our model in Section 2 with the baseline fiscal rule (6), but with one twist: we add a margin $\mu \in (0,1)$ of hand-to-mouth spenders, along the lines discussed in Section 5.1. These agents receive the same income and pay the same taxes as the perpetual-youth consumers, but only the latter save and hold government debt. Further details are presented in Appendix C.1.

This hybrid spender-OLG model is the ideal environment for our quantitative analysis, for two reasons. First, it remains simple enough to fit into the generalized demand block of Section 5.1. As a result, we can readily invoke Theorem 2 to verify whether complete self-financing obtains in the limit as $\tau_d \to \infty$. Second, and more importantly, it is rich enough to agree with quantitative HANK models on their implied intertemporal MPCs (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*) and to match the relevant empirical evidence. Since the micro-foundations of aggregate demand affect the degree of self-financing only through those intertemporal MPCs (see Appendix A.4 for the formal discussion), it follows that our calibrated model is a suitable laboratory for evaluating whether a quantitatively meaningful degree of self-financing can obtain for realistic parameterizations of the intensity of fiscal adjustment, as herein parameterized by τ_d.[30]

Calibration. We discuss the model calibration in three steps: evidence on household consumption behavior to pin down the spender share μ and the OLG coefficient ω; evidence on fiscal adjustment to pin down τ_d; and all other parts of the model.

[30] We note that the same is not true for our baseline OLG model; see the first point of the discussion on calibration below.

Parameter	**Description**	**Value**	**Target**	**Value**
Consumer spending				
μ	Share of HtM	0.111	Average MPC	0.22
ω	OLG survival rate	0.88	Short-run MPC slope	0.88
Fiscal adjustment				
τ_d	Tax feedback	$\{0.085, 0.026, 0.004\}$	Literature, see text	
Rest of the model				
σ	EIS	1	Standard	
β	Discount factor	0.998	Annual real rate	0.01
τ_y	Tax rate	0.3	Average Labor Tax	0.3
D^{ss}/Y^{ss}	Gov't debt	1.04	Liq. wealth holdings	1.04
κ	NKPC slope	0.01	Standard	

Table 1: Hybrid OLG-spender model, calibration.

Consumer spending behavior. Empirical evidence on the household-level consumption response to income gains suggests two salient features of consumer spending behavior (Fagereng, Holm and Natvik, 2021). First, the average MPC out of income gains is elevated, with a standard quarterly value of around 0.22. Second, the income gain is spent gradually. Our baseline OLG environment of Section 2 provides a tight joint restriction on the level of the MPC and its dynamics: the level MPC (i.e., entry $\mathscr{M}_{0,0}$ of the matrix of MPCs) is $1-\beta\omega$, while the slope of the spending profile (i.e., the ratio of $\mathscr{M}_{\ell,0}$ to $\mathscr{M}_{\ell-1,0}$) is ω. This model-implied connection between level and slope is, unfortunately, inconsistent with the data; in particular, relative to the impact MPC, income in the data is subsequently spent much more quickly than predicted by the theoretical OLG-implied relationship. The spender-OLG hybrid model instead allows us to disentangle the level and the intertemporal pattern of the spending response to income gains in a way that is consistent with empirical evidence; in particular, we choose the spender share μ and the OLG coefficient ω to jointly match (i) impact and (ii) short-run (i.e., up to two years out) spending responses to lump-sum transfer receipt, as estimated by Fagereng, Holm and Natvik. A visual illustration of the implied intertemporal MPCs is provided in the top left panel of Figure 5, and a discussion of several alternative calibration strategies will follow in Section 6.2.

Current and future MPCs out of today's income gains—i.e., the estimand of Fagereng, Holm and Natvik—is of course not all that matters for our theory; for our general equilibrium Keynesian cross, it is similarly important how anticipated income changes in the (far) future affect spending today (i.e., entries $\mathscr{M}_{t,t+\ell}$ for large ℓ in the matrix of MPCs). Given the lack of empirical evidence on such responses to (far-away) income news shocks, our baseline exercise simply extrapolates these spending responses through the structure of the model. Reassuringly, as discussed in Wolf (2021*a*), our spender-OLG model extrapolates in a way very similar to quantitative HANK models. Nevertheless, we later also consider how results change with additional discounting due to limited knowledge or limited rationality, consistent with suggestive evidence from Ganong and Noel (2019).

Delay in fiscal adjustment. We henceforth focus on fiscal rule (6), in line with a large literature in the footsteps of Leeper (1991). Because prior work has estimated this type of rule, we can draw from it to discipline the calibration of τ_d. In particular, we consider a range of values taken from Galí, López-Salido and Vallés (2007), Bianchi and Melosi (2017), and Auclert and Rognlie (2018), displayed in the middle part of Table 1. A detailed literature review is provided in Auclert, Rognlie and Straub (2020).[31]

Rest of the model. The remaining model parameters are set to standard values. First, we set $\sigma = 1$, giving log preferences. Second, we set the discount factor to hit a steady-state real rate of interest of one per cent. Third, we set $\tau_y = 0.3$, in line with the discussion in DeLong and Summers (2012): for every dollar of additional output, we assume that the primary surplus automatically rises by 30 cents. Fourth, we set the slope of the NKPC to 0.01, a value consistent with the related empirical and quantitative literatures (e.g., see Galí and Gertler, 1999; Christiano, Eichenbaum and Evans, 2005; Hazell et al., 2022; Barnichon and Mesters, 2020). In one of our robustness exercises, we will also consider a slope of 0.1—an ad-hoc, materially larger value that could be relevant in the inflationary post-covid environment.

6.2 Are stimulus checks plausibly self-financed?

It is straightforward to verify that our calibrated spender-OLG model satisfies the sufficient conditions identified in Theorem 2 required for complete self-financing to be possible in the limit as $\tau_d \to \infty$. What remains is to ascertain how close we are to this limit for the range of τ_d suggested by the relevant empirical literature. Results are displayed in Figure 4.

The figure shows output (left panel) and inflation (middle panel) impulse responses to a date-0 deficit shock for our three fiscal adjustment coefficients τ_d (shades of grey); the right panel furthermore shows the degree of self-financing ν as a function of τ_d over the entire unit interval. As expected, the share of self-financing ν is decreasing in τ_d and approaches one as $\tau_d \to 0$, consistent with Theorem 2. For the purposes of the quantitative analysis here, the key takeaway is that we are already quite close to this limit for the values of τ_d estimated in the aforementioned works, with ν up to 0.95. Furthermore, given our assumed—empirically disciplined—flat NKPC, almost all of this self-financing is coming through the tax base channel rather than through inflation.

[31] Specifically, our three values for τ_d correspond to their lower bound, preferred estimate, and upper bound for the fiscal adjustment coefficient (ψ in their notation). We note that all three values correspond to "passive" fiscal rules in the terminology of Leeper (1991). See Appendix C.3 for further details.

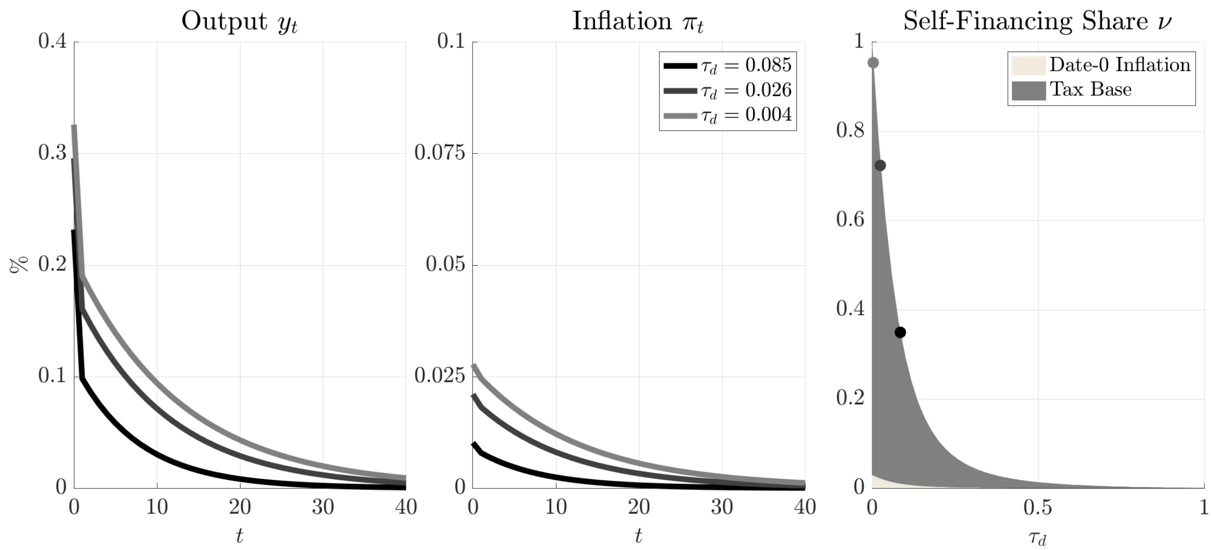

Figure 4: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ε_0 as a function of τ_d. The left and middle panels show the impulse responses for the three values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

The remainder of this section discusses several robustness exercises and extensions.

Assumptions on consumer behavior. As remarked in Section 6.1, our calibration strategy relied on two standard but potentially material assumptions: first, we disciplined the slope of the spending profile (i.e., entries $\mathscr{M}_{\ell,0}$) from evidence on relatively *short-run* spending behavior (small ℓ); and second, we relied on model structure to extrapolate from responses to contemporaneous income changes to spending behavior following income news shocks (i.e., entries $\mathscr{M}_{t,t+\ell}$). We now investigate how our conclusions change with alternative calibration strategies and modeling assumptions.

Spending responses in the tails. As we emphasized throughout Sections 4 and 5.1, sufficiently front-loaded intertemporal MPCs are important for our self-financing results—they are what ensures a front-loaded Keynesian boom. We thus now consider alternative calibration strategies that try to more directly leverage information on spending responses to lump-sum transfer receipt in the *tails* (i.e., $\mathscr{M}_{\ell,0}$ for large ℓ). These tail responses are crucial to determine how fast cumulative MPCs converge to 1 and thus how front-loaded the Keynesian boom turns out to be.[32]

Results are reported in Figure 5. Here the three panels correspond to three different models: the calibrated spender-OLG model we have worked with so far (top panel) and two alternatives (middle and bottom panel). Our empirical targets, reported as the grey areas (corresponding to 95 per cent confidence intervals) in the three "cumulative MPC" panels, are again taken from Fagereng, Holm

[32]Yet another calibration strategy is presented in Appendix C.2. There we discipline the discounting coefficient ω through empirical evidence on long-run elasticities of household asset demand. Interestingly, this very different (and much less direct) approach suggests values of ω reasonably close to our baseline calibration strategy.

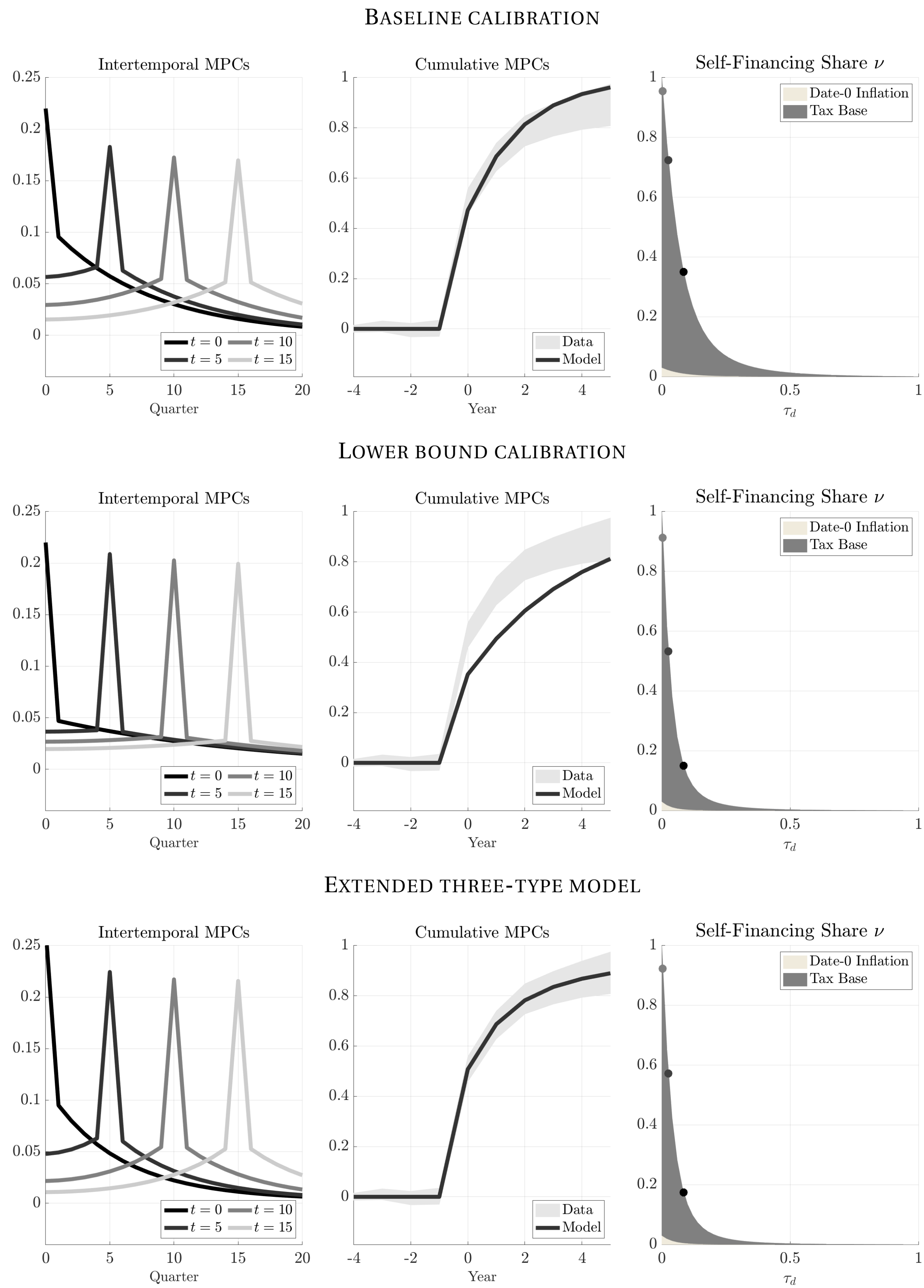

Figure 5: Top panel: iMPCs, cumulative MPCs, and self-financing share ν (as a function of τ_d) in the baseline model. Middle and bottom panels: same as above, but for models calibrated to (also) match far-ahead tail MPCs. Parameter values are reported in Appendix C.2.

and Natvik (2021) (see their Figure 2).

Consider first the top panel. By construction, this model matches MPCs—and thus also cumulative MPCs—over the first couple of years after income receipt. However, as revealed by the middle panel, the implied cumulative MPC appears to converge to 1 somewhat too quickly, reflecting intertemporal MPCs that converge to 0 relatively fast, at rate ω. The middle panel—labeled "lower bound calibration"—instead disciplines the model parameters (μ, ω) by matching (i) the same impact MPC as before and then (ii) targeting the *lower bound* of the estimated confidence interval five years after the income receipt. This calibration strategy unsurprisingly delivers a larger ω (equal to 0.94), mapping into materially flatter intertemporal MPC profiles (left part), inconsistent with evidence on short-run spending responses (middle part). As expected, convergence to our self-financing limit is slowed, though even in this calibration the degree of self-financing remains rather substantial for our various fiscal adjustment rules. Finally, in the bottom panel, we consider an extended model—one featuring spenders together with *two* types of OLG blocks, with heterogeneous ω_1 and ω_2—that is rich enough to provide a tight fit to the entire dynamic profile of cumulative MPCs, up to five years out. The left part of the bottom panel reveals that this model looks rather similar to our baseline calibration in the periods around income receipt, but then has somewhat flatter tail MPCs. Thus, as in the middle panel, the speed of convergence to the self-financing limit is slowed down, though the predicted degree of self-financing again remains meaningful.

Spending response to news about future income. All models discussed above are calibrated to be consistent with evidence on consumer spending behavior in response to income shocks *today*; the similarly important consumption response to "news shocks" (i.e., $\mathscr{M}_{t,t+\ell}$), on the other hand, is extrapolated through the model structure. We here briefly discuss results from two additional models that extrapolate somewhat differently, with details in Appendices C.4 and C.5. First, in Appendix C.4, we consider a fully-fledged quantitative HANK model. Consistent with the results in Auclert, Rognlie and Straub (2018) and Wolf (2021*a*), we find that this model extrapolates MPCs across horizons in almost the same way as our reduced-form spender-OLG hybrid model, so our conclusions on self-financing are largely unchanged. Second, we consider a variant of our spender-OLG model with cognitive discounting. As discussed in Section 5.1, we expect this behavioral friction to introduce two offsetting effects on ν—the future tax hike is discounted by more, but higher future income feeds back less strongly to the present. Our model simulations confirm this intuition: we find that ν is higher than in our baseline model for intermediate values of τ_d, but converges to the full self-financing limit somewhat more gradually.[33]

[33] We expect similar results in behavioral model variants that replace cognitive discounting à la Gabaix, 2020 with incomplete information à la Angeletos and Lian (2018) and/or bounded rationality à la Farhi and Werning (2019), Vimercati, Eichenbaum and Guerreiro (2021), and Angeletos and Sastry, 2021.

Steeper Phillips curve. In all quantitative experiments displayed above, self-financing occurred almost exclusively through the tax base channel, reflecting the fact that our assumed NKPC was—consistent with empirical evidence and much quantitative modeling—very flat. Larger values of the slope coefficient κ have two effects, as discussed in Section 4.4: first, for any given τ_d, the total amount of self-financing ν increases; and second, the split of self-financing moves from the tax base effect to inflation. In Appendix C.6, we repeat our analysis with somewhat more flexible prices, setting $\kappa = 0.1$. This value lies at the upper end of recent empirical estimates pertaining to the inflationary post-covid episode. We find that, with this alternative calibration, around 20 per cent of self-financing comes through date-0 inflation.

Monetary policy reaction. Our baseline exercises assumed a monetary authority that neither accommodates the fiscal deficit nor leans against the resulting Keynesian boom. Conceptually, it allows us to cleanly isolate the quantitative potential of the interaction between consumer spending behavior and fiscal financing delays at the heart of our paper. Practically, it at least broadly consistent with the recent post-covid U.S. policy experience. In Appendix C.7, we nevertheless consider what happens when the monetary authority leans more aggressively against the fiscal boom, consistent with a standard Taylor-type rule. The results from this exercise are consistent with the insights in Theorem 3. First, for moderately active monetary reaction, full self-financing is still possible, though convergence as $\tau_d \to 0^+$ is now slowed down. Second, if the monetary response is very aggressive, then for a bounded equilibrium to exist fiscal adjustment needs to be sufficiently strong, and so complete self-financing is not possible anymore. That being said, for empirically relevant calibrations of the Taylor rule and of the slope of the NKPC, the degree of self-financing can still be substantial.

7 Conclusion

The central contribution of this paper is to clarify the conditions under which fiscal deficits can finance themselves. Our analysis applies to environments with two empirically relevant features: (i) nominal rigidities and (ii) a violation of Ricardian equivalence due to finite lives or liquidity constraints. The headline result is that, as the fiscal authority delays the tax adjustment, the demand-led boom gets larger and more persistent, and this contributes to a larger share of self-financing. In the limit, deficits can even fully pay for themselves.

Our results have important conceptual as well as practical implications. *Conceptually,* they suggest that one specific idea from the classical Fiscal Theory of the Price Level—that future tax hikes are not necessary to finance today's deficits—is more robust than commonly believed. Our accommodation of this idea does not rely on the threat of a government violating its budget constraint, nor is it vulnerable to debates regarding equilibrium selection. Rather, it is grounded in realistic departures from permanent-income consumer behavior, together with sufficient delay in fiscal adjustment. *Practically*, our results suggest that a meaningful degree of self-financing is empirically plausible, especially in the absence of aggressive monetary policy responses. These insights seem relevant for recent U.S. fiscal stimulus experiments and for the related debate on "excess savings" (Auclert, Rognlie and Straub, 2023): fiscal deficits without a quick tax offset are predicted to lead to a long-lived boom and to largely pay for themselves.

We leave several important avenues for future work. First, to assess the practical relevance of our results, we disciplined the theory with the best available evidence on consumer behavior and the degree of nominal rigidity. A more direct empirical test would compare the dynamic causal effects of equally large but differentially financed changes in government spending, using the standard macroeconometric toolkit (see Ramey, 2016). Second, a meaningful limitation of our theory was the assumption of a closed economy. In an open economy, some of the deficit-led demand boom will leak abroad, dampening the general equilibrium Keynesian amplification. We therefore expect our mechanism to be more relevant for large closed economies (like the U.S.) than for small open economies. Third, note that our analysis here has been entirely positive, not normative. The implications of our results for optimal fiscal policy seem like another important direction for future work.

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Online Appendix for:
Can Deficits Finance Themselves?

This online appendix contains supplemental material for the article "Can Deficits Finance Themselves?". We provide: (i) supplementary discussion of our baseline model and results in Sections 2-4; (ii) details for the extensions considered in Section 5; (iii) additional analysis and alternative results for our quantitative investigation in Section 6. The end of this appendix contains all proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"D." refer to the main article.

A Supplementary details

We here provide some additional discussion of our baseline model. Subsection A.1 characterizes labor supply and explains how our model's supply block reduces to the standard NKPC. Subsection A.2 explains how we characterize equilibria under the alternative fiscal policy rule (7). Subsection A.3 provides a detailed discussion of the PIH benchmark ($\omega = 1$). Finally, Subsection A.4 interprets our self-financing result from a sequence-space perspective.

A.1 The supply block

Unions equalize the post-tax real wage and the average marginal rate of substitution between labor supply and consumption. Since we abstract from heterogeneity in labor supply $L_{i,t} = L_t$, the optimal labor supply relation is

$$(1-\tau_y)W_t = \frac{\chi L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-1/\sigma} di}, \tag{A.1}$$

With Log-linearizing, we obtain (13).

Optimal firm pricing decisions as usual give inflation as a function of real marginal costs. With a standard constant-returns-to-scale, labor-only production function this gives (e.g., see the textbook derivations in Woodford, 2003*a*; Galí, 2008)

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.2}$$

where $\tilde{\kappa}$ is a function of firm-side primitives, including in particular the stickiness of prices. Combining (A.2) with (13) and imposing that $c_t = \ell_t = y_t$ we obtain

$$\pi_t = \underbrace{\tilde{\kappa}\left(\frac{1}{\varphi} + \frac{1}{\sigma}\right)}_{\equiv\kappa} y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right], \tag{A.3}$$

as required.

A.2 Equilibrium characterization with the alternative fiscal rule

We characterize the equilibrium in our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, and the alternative fiscal rule (7) here. The aggregate demand relation (11) together with monetary policy (8) and market clearing $y_t = c_t$ lead to the following recursive aggregate demand curve[34]

$$y_t = \frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(d_t - t_t\right) + \mathbb{E}_t\left[y_{t+1}\right], \tag{A.4}$$

where we use that $\mathbb{E}_t\left[d_{t+1}\right] = \frac{1}{\beta}\left(d_t - t_t\right)$ from (4).

We characterize the bounded equilibrium path through backward induction. Given the alternative fiscal rule (7), we know that, for $t \geq H$,

$$d_t - t_t = 0 \Longrightarrow y_t = \mathbb{E}_t\left[y_{t+1}\right].$$

We focus on the equilibrium with $y_t = 0$ for $t \geq H$. As discussed in Footnote 18, this equilibrium selection can be justified in two ways: strengthening the boundedness requirement in the equilibrium definition or considering limits to $\phi = 0$ from above. The sole role of any of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. Given this selection we use (A.4) to find the equilibrium path of $\left\{y_t, d_t\right\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0 d_H \quad \text{with} \quad \chi_0 = 0. \tag{A.5}$$

For $t \leq H-1$, substitute the alternative fiscal rule (7) into (A.4), giving

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\left(d_t + \varepsilon_t\right) + \frac{1}{1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

[34] Note that (12) does not apply here, since its derivation uses the baseline fiscal policy (6).

As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}(d_t + \varepsilon_t) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y} + \frac{\frac{1}{\beta}\left(1-\tau_y\chi_{H-t}\right)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\chi_{H-t-1}, \tag{A.6}$$

where, according to (7), $\varepsilon_t = 0$ for all $t \neq 0$. Rearranging terms, we find the following recursive formula for the χs:

$$\chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}\right)\tau_y} = g\left(\chi_{H-t-1}\right), \tag{A.7}$$

where

$$g(\chi) = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\tau_y} \quad \text{and} \quad g'(\chi) = \frac{1}{\beta}\frac{1}{\left(1+\tau_y\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\right)^2} \geq 0 \quad \forall \chi \geq 0. \tag{A.8}$$

We thus know that

$$\chi_k \in (0, \frac{1}{\tau_y}) \quad \forall k \geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{A.9}$$

From (4), $r_t = 0$, (7), and (A.6), we also know that

$$\mathbb{E}_0\left[d_t\right] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)(d_0+\varepsilon_0) \tag{A.10}$$

To further characterize the equilibrium we begin by considering an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Let ν' denote the self financing share in this alternative economy, i.e.,

$$\nu' \cdot \varepsilon_0 = \nu'_y \cdot \varepsilon_0 \equiv \sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0\left[y_k\right].$$

In this economy, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (A.6) and (A.10), we have that

$$\nu' = \sum_{t=0}^{H-1} \Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)\tau_y\chi_{H-t} = 1 - \Pi_{j=0}^{H-1}\left(1-\tau_y\chi_{H-j}\right). \tag{A.11}$$

We can now return to the general case with $\kappa \geq 0$. From the NKPC (14) as well as the definitions in (20) – (21), we have that

$$\nu_p = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa\frac{D^{ss}}{Y^{ss}}}\nu \tag{A.12}$$

Finally, from the formula of d_0 (16), we know

$$d_0 = -\nu_p \varepsilon_0 \quad \text{and} \quad \nu_y = \left(1-\nu_p\right)\nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}. \tag{A.13}$$

This completes our characterization of the equilibrium.

A.3 The PIH/RANK benchmark ($\omega = 1$)

This section provides a detailed comparison between our economy ($\omega < 1$) and its textbook, representative-agent counterpart ($\omega = 1$).

The aggregate demand block. Recall that, with $\omega = 1$, the aggregate demand block becomes

$$y_t = \mathbb{E}_t\left[(1-\beta)\sum_{k=0}^{\infty}\beta^k y_{t+k}\right] \tag{A.14}$$

or equivalently

$$y_t = \mathbb{E}_t[y_{t+1}]. \tag{A.15}$$

Although the Euler equation (A.15) is more familiar, the aggregate demand relation (A.14) is more insightful, for several reasons. First, it reveals the general equilibrium interactions hidden behind the representative consumer's Euler condition—i.e., the intertemporal Keynesian cross (IKC). Second, it translates this IKC to a dynamic game of strategic complementarity among consumers: when a consumer expects others to spend more, she finds it optimal to spend more herself, because high spending by others means higher income for her. Third, it makes clear why this game admits multiple equilibria—because the cumulative marginal propensity to consume is one. And fourth, it reveals that fiscal policy does not enter the payoffs and the best responses of this game: holding constant aggregate spending, a consumer's optimal spending is invariant to fiscal policy. It follows that public debt and deficits can drive spending if and only if they serve as coordination devices (sunspots).

What are solutions of the aggregate demand block (A.14), studied in isolation? An obvious one is $y_t = 0$ for all t and states of nature, which means that the economy stays at steady state for ever. But $y_t = \bar{y}$ is also a solution, for arbitrary $\bar{y}$; this corresponds to consumers at every t spending $\bar{y}$ because, and only because, they expect all future consumers to do the same. Finally, there exist equilibria in which $y_t = \xi \sum_{k=0}^{t} \varepsilon_{t-k}$, for arbitrary $\xi \in \mathbb{R}$. In any such equilibrium, consumers vary their spending with the current deficit by an amount equal to ξ because and only because they expect future consumers to keep responding to the current deficit in the same away, in perpetuity.

Feedback between consumers & fiscal policy in RANK. The above statements are valid characterizations of solutions to (A.14), without reference to fiscal policy. Conventional notions of equilibrium—including in particular our Definition 1—however also require that debt does not explode, thus putting the government budget constraint back into the picture. This in turn puts joint restrictions on fiscal policy and consumer behavior, despite the fact that consumers do not care for fiscal policy in the precise sense of the game described above.

What are these restrictions? For this it will prove instructive to consider first the case in which $\tau_y = 0$ and $\kappa = 0$. In this case, the law of motion for public debt reduces to

$$d_{t+1} = \beta^{-1}(1-\tau_d)(d_t + \varepsilon_t).$$

This relation has two key properties. First, it is invariant to consumer behavior. And second, it has debt converge back to steady state if and only if $\tau_d > 1-\beta$. We thus arrive at the following conclusion.

Proposition A.1. *Suppose* $\omega = 1$, $\tau_y = 0$, *and* $\kappa = 0$. *Then:*

1. *If* $\tau_d > 1-\beta$, *there exist multiple self-fulfilling (and bounded) equilibria. In particular,* $y_t = \bar{y} + \xi \sum_{k=0}^{t} \varepsilon_{t-k}$ *is an equilibrium for any* $\bar{y} \in \mathbb{R}$. *and any* $\xi \in \mathbb{R}$.

2. *If instead* $\tau_d < 1-\beta$, *there does not exist a (bounded) equilibrium.*

It is important to recognize that the non-existence of equilibrium in case (ii) is the byproduct of "bad" fiscal policy: when $\tau_d < 1-\beta$, there is no problem in finding a fixed point in the GE interaction among the consumers, but there is also no way for public debt to be sustainable (i.e., not to explode).

Now let us relax the restrictions $\tau_y = 0$ and/or $\kappa = 0$ (while maintaining $\omega = 1$). This does not change either condition (A.14) or our aforementioned logic about consumer behavior: fiscal policy continues not to enter the game among the consumers. But the opposite is no more true: consumer behavior now enters the government's budget via the tax base (when $\tau_y > 0$) and/or the real value of public debt (when $\kappa > 0$). This in turn modifies the second (but not the first) part of Proposition A.1.

Proposition A.2. *Suppose* $\omega = 1$ *and* $\max\{\tau_y, \kappa\} > 0$. *Then:*

1. *If* $\tau_d > 1-\beta$, *there exist multiple self-fulfilling (and bounded) equilibria.*

2. *If instead* $\tau_d < 1-\beta$, *there exists a unique bounded equilibrium.*

To understand the content of Proposition A.2, consider first the case with rigid prices ($\kappa = 0$). Had τ_y been zero, then $\tau_d < 1-\beta$ would have caused debt to explode regardless of how consumers behave. But now that $\tau_y > 0$, the following possibility is logically coherent (although, at least in our view, rather implausible): consumers coordinate on an equilibrium among them that generates just enough tax revenue to offset the adverse effects of deficit shocks and to make sure that debt does not explode. By direct analogy to the classical FTPL, we call this the "Fiscal Theory of Y" (FTY): in a world with rigid prices ($\kappa = 0$) and passive monetary policy ($\phi = 0$), consumers may coordinate on multiple self-fulling spending behaviors and hence to multiple self-fulfilling levels of aggregate income, but only one of them makes sure that debt does not explode when $\tau_d < 1-\beta$. Next, if we let $\tau_y = 0$ but $\kappa > 0$, then we recover the original Fiscal Theory of the Price Level (FTPL): the set of multiple self-fulling spending behaviors remains the same, but now there is a different unique selection among them that makes sure that debt does not explode when $\tau_d < 1-\beta$. And finally, a hybrid of these two extreme cases obtains (and a different equilibrium is selected) when both $\tau_y > 0$ and $\kappa > 0$.

We note that there is a long-standing theoretical debate about the plausibility of the FTPL. While this debate is *not* our paper's main theme or contribution, we hope that the above analysis helps make clear the following points: (i) the precise sense in which fiscal policy does not enter the Intertemporal Keynesian Cross (or the game among the consumers, when $\omega = 1$); (ii) the reason why this game admits multiple self-fulfilling equilibria; (iii) that the entirety of this multiplicity is consistent with non-explosive debt when $\tau_d > 1-\beta$; (iv) that no equilibrium is consistent with non-explosive debt when $\tau_d < 1-\beta$ and $\tau_y = \kappa = 0$; (v) that $\tau_y > 0$ or $\kappa > 0$ allows debt to be non-explosive when $\tau_d < 1-\beta$, but only insofar as consumers coordinate on a particular self-fulfilling equilibrium among the many of the aforementioned game; and (vi) that both the FTPL and the FTY translate to specific selections among the multiple self-fulfilling solutions of the IKC. Last but not least, the unique "fundamental" (or Markov Perfect) equilibrium of the aforementioned game is one where debt and deficits do not influence aggregate demand, and debt is sustainable along this equilibrium if and only if $\tau_d > 1-\beta$.

Returning to our environment. How does our environment fit into this picture? Once $\omega < 1$, fiscal policy becomes payoff-relevant in the game among the consumers. This then introduces a feedback from fiscal policy to aggregate demand, differently from the permanent-income baseline.

To see this most clearly, again suppose first that $\tau_y = \kappa = 0$, which means that consumer behavior does not enter the government budget. Similarly to the $\omega = 1$ case, this implies that public debt is non-explosive if and only if $\tau_d > 1-\beta$. Furthermore, it remains true that the game among consumers admits multiple equilibria. For instance, if there are no deficit shocks, then $y_t = \bar{y}$ is an equilibrium for any constant $\bar{y}$. Differently from the case above, however, the response of output to deficits is now uniquely pinned down.

Proposition A.3. *Suppose $\omega < 1$ but $\tau_y = \kappa = 0$. When $\tau_d < 1-\beta$, there does not exist an equilibrium.*

And when $\tau_d > 1-\beta$, *there exist multiple bounded equilibria. The set of equilibria is then given by*

$$y_t = \bar{y} + \chi(d_t + \varepsilon_t)$$

for arbitrary $\bar{y} \in \mathbb{R}$ *and for a unique* $\chi > 0$, *given by*

$$\chi = \frac{\mathscr{F}_1}{1 - \frac{(1-\beta\omega)}{1-\beta\omega(1-\tau_d)}}.$$

In short, the "intercept" or "level" remains indeterminate (because the dynamic strategic complementarity is still 1), but the response to debt and deficits is uniquely pinned down (because debt and deficits cease to be sunspots).

Finally return to the case where $\tau_y > 0$—i.e., our baseline case. Then and only then there is a *two-way* feedback between aggregate demand and the government budget. Furthermore, this two-way feedback translates to a reduction in the effective degree of strategic complementary. From

$$\mathscr{F}_2 = 1 - \frac{(1-\omega)\,\tau_y}{1-\omega\,(1-\tau_d)},$$

it is immediate that $\mathscr{F}_2 < 1$ if and only if *both* $\omega < 1$ and $\tau_y > 0$. This in turn helps explain why the multiplicity disappears when and only when *both* $\omega < 1$ and $\tau_y > 0$. On other hand, the presence of a feedback from output to the government budget constraint helps explain why a bounded equilibrium exists not only for $\tau_d > 1-\beta$ but also for $\tau_d < 1-\beta$.

Summary. From the preceding analysis we conclude that Proposition 1 combines the following subtle lessons: $\omega < 1$ alone explains why debt and deficits become fundamentals and feed into aggregate demand; $\tau_y > 0$ in turn explains how aggregate demand feeds into the government budget and can aid debt sustainability even when $\tau_d < 1-\beta$; and the combination of the two feedbacks finally helps explain why our economy features a unique bounded equilibrium, despite the apparent failure of the Taylor principle ($\phi = 0$).

A.4 A sequence-space perspective

All results in the main part of this paper are stated and proved using a standard state-space approach to equilibrium characterization. We can, however, develop some additional insights by instead adopting a sequence-space perspective. In the context of the paper as a whole the purpose of the sequence-space analysis in this section is twofold. First, by adopting this sequence-space perspective, we will be able to very easily substantiate a claim made in Section 6—that intertemporal MPCs fully characterize limiting self-financing equilibria. Second, we provide a different perspective on Assumption 2, re-phrasing it as a sufficient condition ensuring that intertemporal MPCs decay "sufficiently quickly."

Equilibrium. For the analysis in this section, we substantially generalize the aggregate demand relation (11) to the following sequence-space relation:

$$\boldsymbol{c} = \mathscr{M} \times \left(\boldsymbol{y} - \boldsymbol{t}\right) + \mathscr{M}_i \times \boldsymbol{i} + \mathscr{M}_\pi \times \boldsymbol{\pi} \tag{A.16}$$

where boldface denotes sequences. Our objective in this section is to shed further light on the possibility of complete self-financing in the limit with infinite delay. We maintain the assumption of a neutral monetary policy and consider the limiting case of $\tau_d = 0$. For simplicity, we also assume that prices are rigid ($\kappa = 0$), thus focusing on the tax base channel.[35]

Imposing market-clearing and constant real rates, equation (A.16) becomes

$$\boldsymbol{y} = \mathscr{M} \times \left(\boldsymbol{y} - \boldsymbol{t}\right) \tag{A.17}$$

Now note that, under our assumptions on fiscal policy, taxes are given

$$\boldsymbol{t} = \tau_y \times \boldsymbol{y} - \boldsymbol{\varepsilon} \tag{A.18}$$

where $\boldsymbol{\varepsilon}$ captures the exogenous policy intervention. For our "stimulus check" experiment, this a vector that has zeros everywhere but in its first entry. Combining (A.17) and (A.18), we find that output in the limiting, complete self-financing equilibrium is characterized through the following system of dynamic equations:

$$\boldsymbol{y} = (1 - \tau_y)\mathscr{M} \times \boldsymbol{y} + \mathscr{M} \times \boldsymbol{\varepsilon} \tag{A.19}$$

(A.19) is a variant of the intertemporal Keynesian cross studied previously in Auclert, Rognlie and Straub (2018), but with a crucial difference: automatic tax financing is embedded in the tax revenue term $\tau_y \times \boldsymbol{y}$, rather than being specified directly as part of the policy intervention (here $\boldsymbol{\varepsilon}$). This seemingly subtle distinction has important implications and in particular connects tightly with our self-financing results in Sections 4 and 5.1.

Discussion. The above analysis substantiates the claim made in Section 6: for a large family of models (including in particular our spender-OLG hybrid), the matrix of the intertemporal MPCs together with the value of τ_y pin down the dynamics of output in the limiting self-financing equilibrium. It

[35] By an argument analogous to that surrounding Theorem 1, the extension to the partially sticky price case is conceptually straightforward. The only delicate part of the ensuring discussion is that we directly set $\tau_d = 0$, instead of taking the limit as $\tau_d \to \infty$ from above, or proving the equivalence to $H \to \infty$. These details are fully taken care in our main analysis.

remains to further characterize the solution of (A.19), allowing us to connect with the economic intuitions offered in Sections 4 and 5.1.

The remainder of the discussion here will leverage a crucial property of the intertemporal MPC matrix $\mathscr{M}$. Letting $\boldsymbol{r} \equiv (1, \frac{1}{1+\bar{r}}, \frac{1}{(1+\bar{r})^2}, \dots)$, we have that $\boldsymbol{r}' \cdot \mathscr{M}(\bullet, h) = \frac{1}{(1+\bar{r})^h}$—i.e., every dollar of income is spent at some point. It follows from this property that any solution $\boldsymbol{y}$ of (A.19) necessarily has net present value equal to $\frac{1}{\tau_y}$ times the net present value of the fiscal stimulus:

$$\boldsymbol{r}'\boldsymbol{y} = (1-\tau_y)\boldsymbol{r}'\mathscr{M} \times \boldsymbol{y} + \boldsymbol{r}'\mathscr{M} \times \boldsymbol{\varepsilon}$$

and so from the properties of $\mathscr{M}$ we obtain that indeed

$$\tau_y \times \boldsymbol{r}'\boldsymbol{y} = \boldsymbol{r}'\boldsymbol{\varepsilon}, \tag{A.20}$$

as claimed. Next we note that the solution of (A.19) takes the simple form

$$\boldsymbol{y} = \left[I - (1-\tau_y)\mathscr{M}\right]^{-1} \times \mathscr{M} \times \boldsymbol{\varepsilon} \tag{A.21}$$

where for the purpose of the discussion here we simply assume that the stated inverse exists.[36] Our self-financing results in Theorems 1 and 2 concern the question of whether, as fiscal financing is gradually delayed further and further, we indeed converge to the general self-financing equilibrium characterized by (A.21). As discussed following Theorem 1, the condition required for such convergence to occur is that the Keynesian boom happens sufficiently quickly, raising all required revenue before fiscal adjustment is ever actually necessary. In (A.21), the "quickness" of the Keynesian boom is entirely governed by the properties of $\left[I-(1-\tau_y)\mathscr{M}\right]^{-1}$: if the off-diagonal entries of $\mathscr{M}$ decay to zero sufficiently quickly, then the same is true for the off-diagonal entries of $\left[I-(1-\tau_y)\mathscr{M}\right]^{-1}$ (e.g., see Bickel and Lindner, 2012). This then ensures that the solution $\boldsymbol{y}$ and thus the debt path $\boldsymbol{d}$ converge to zero, which in turn is precisely what is needed for self-financing to obtain as fiscal adjustment is delayed further and further. For our general aggregate demand relation (27), the condition stated in Assumption 2 is simply what is needed to ensure that indeed the off-diagonal entries of $\mathscr{M}$ and thus $\left[I-(1-\tau_y)\mathscr{M}\right]^{-1}$ decay to zero sufficiently quickly.

[36] Our analysis in the main text implies that, for standard models of the consumption-savings problem and if $\tau_y > 0$, then this inverse indeed exists.

B Details on model extensions

We elaborate on the various extensions considered in Section 5: the richer aggregate demand block in Appendix B.1; a model with investment in Appendix B.2; equilibria under more general monetary rules in Appendix B.3; fiscal financing through distortionary taxes in Appendix B.4; and fiscal stimulus in the form of government purchases in Appendix B.5.

B.1 A more general aggregate demand relation

In Section 5.1 we showed explicitly how several popular models of the household consumption-savings problem can be written in our general form (27). We here elaborate on: (i) our discussion of the well-known spender-saver model; and (ii) the model of cognitive discounting of Gabaix (2020).

Self-financing in the spender-saver model. We provide a visual illustration of self-financing—or the lack thereof—in Figure B.1. The top panel shows impulse responses and the self-financing share in the spender-saver model, while the bottom panel does the same for a spender-OLG hybrid model.

The top panel reveals that, in the spender-saver model, the self-financing share ν is always 0. In particular, we see that the date-0 boom is always exactly offset (in present-value terms) by a bust at date H—the date of the delayed fiscal adjustment. The intuition is that the presence of PIH households breaks our discounting and front-loading properties: PIH households mean that the date-H tax hike affects date-0 demand, and similarly delay any date-0 boom to the infinite future. The empirically testable flipside of this "connection at infinity" is an infinitely large elasticity of household asset demand (e.g., see the discussion in Kaplan and Violante, 2018). With $\omega < 1$ we break this unrealistic feature of the model, return to our discounting and front-loading properties, and thus see that the date-H bust endogenously gets smaller and smaller as we again converge to full self-financing.

Cognitive discounting. Under cognitive discounting, a shock h periods in the future is additionally discounted by a factor of θ, with $\theta = 1$ corresponding to the standard full-information, rational-expectations model and $\theta = 0$ corresponding to myopic households. It is immediate that cognitive discounting added to our baseline OLG model gives the adjusted aggregate demand relation

$$c_t = (1-\beta\tilde{\omega})\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\tilde{\omega}\theta)^k (y_{t+k} - t_{t+k})\right]\right). \tag{B.1}$$

This fits into our demand structure (27) with $M_y = M_d = 1-\beta\tilde{\omega}$, $\delta = 1$ and $\omega = \tilde{\omega}\theta$. It is immediate that, for $\tilde{\omega} < 1$ and $\theta < 1$, Assumption 1 holds. Differently from the baseline OLG case, however, Assumption

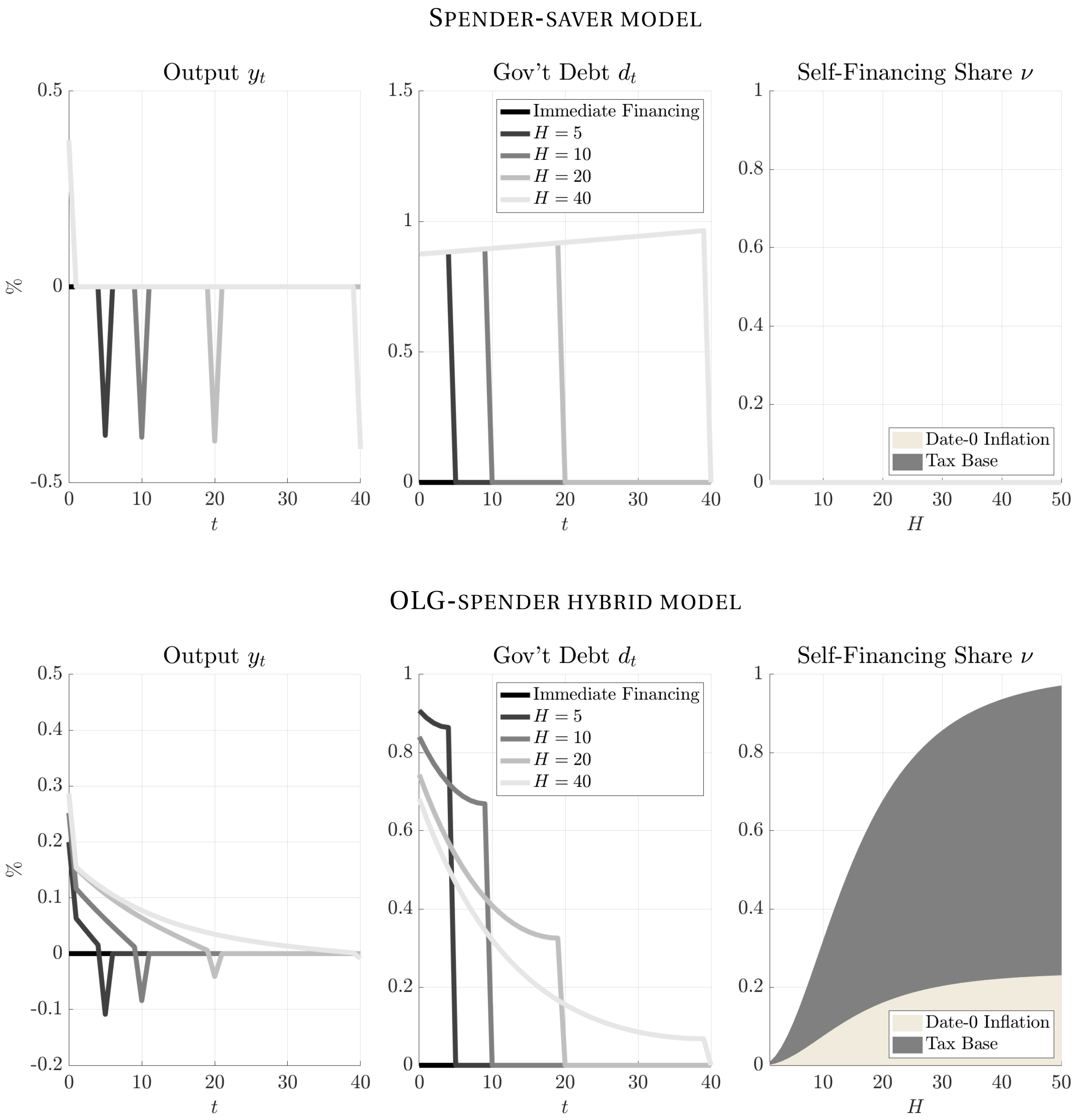

Figure B.1: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ε_0 as a function of H in a spender-saver model. Bottom panel: same as above, but in an OLG-spender hybrid economy.

2 does not hold automatically; plugging in to (28) and re-arranging we find that we need

$$\tau_y > \frac{\tilde{\omega}(1-\theta)}{1-\tilde{\omega}\theta}\frac{1-\beta}{1-\beta\tilde{\omega}} \tag{B.2}$$

This relation holds automatically for $\theta = 1$, but need not hold for $\theta < 1$; intuitively, as already discussed in the main text, $\theta < 1$ dampens demand spillovers from the future to the present and thus slows down the Keynesian boom. (B.2) is, however, a very mild condition: even for $\theta = 0$, as long as β is close to one and for values of $\tilde{\omega}$ as considered in Section 6, Assumption 2 holds even for small τ_y.

B.2 Investment

This section provides the missing details on the extension to models with investment discussed in Section 5.2. We begin by stating the (linearized) equations of the extended model before then characterizing its equilibrium.

Model equations. The household block changes very little. Households still receive labor income and dividends; we now denote this total household income by e_t (which in equilibrium will be equal to total household consumption rather than total aggregate income). The linearized household demand relation is now

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(e_{t+k} - t_{t+k})\right]\right) - \gamma\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{B.3}$$

while labor supply still satisfies

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t \tag{B.4}$$

The firm block on the other hand changes materially relative to our baseline model. Since this production side is entirely standard our discussion here will be brief and only present linearized optimality conditions; a detailed discussion of an almost identical model is offered in Wolf (2021*b*). The production sector consists of three parts: perfectly competitive intermediate goods producers who accumulate capital and hire labor on spot markets; monopolistically competitive retailers who purchase the intermediate good and costlessly differentiate it, subject to nominal rigidities; and a competitive final goods aggregator. Profits of the corporate sector as a whole are returned to households, subject to the time-invariant tax τ_y. The relevant equilibrium relations follow from the behavior of the intermediate goods producers and the retailers.

1. *Intermediate goods producers.* The production function takes a standard Cobb-Douglas form with capital share α, and capital depreciates at rate δ. We let p_t^I denote the real relative price of

the intermediate good. Optimal labor demand gives the static relation

$$w_t = p_t^I + \alpha k_{t-1} - \alpha \ell_t \tag{B.5}$$

while optimal capital accumulation gives[37]

$$\frac{1}{\beta}(i_t - \mathbb{E}_t[\pi_{t+1}]) = \left(\frac{1}{\beta} - 1 + \delta\right) \times \mathbb{E}_t\left[p_{t+1}^I + (\alpha - 1)k_t + (1-\alpha)\ell_{t+1}\right] \tag{B.6}$$

By our assumptions on the production function total output is given as

$$y_t = \alpha k_{t-1} + (1-\alpha)\ell_t \tag{B.7}$$

and finally investment x_t satisfies

$$x_t = \frac{1}{\delta}(k_t - (1-\delta)k_{t-1}) \tag{B.8}$$

2. *Retailers.* Optimal price-setting as usual relates real marginal costs—here the relative price of the intermediate good, p_t^I—to aggregate inflation:

$$\pi_t = \kappa p_t^I + \beta \mathbb{E}_t[\pi_{t+1}] \tag{B.9}$$

Aggregating dividend payments from intermediate goods producers and retailers, we obtain

$$Q_t = Y_t - W_t L_t - X_t \tag{B.10}$$

which implies that total household income E_t (in levels) is given as

$$E_t = W_t L_t + Q_t = Y_t - X_t \tag{B.11}$$

Aggregate output market-clearing dictates that

$$y_t = \frac{C^{ss}}{Y^{ss}} c_t + \frac{X^{ss}}{Y^{ss}} x_t \tag{B.12}$$

Finally we return to the government. The monetary rule (8) and the government budget constraint (4) are unchanged. The fiscal policy rules (6) or (7) are also unchanged up to the tax base revenue term:

[37] Adjustment costs on the capital stock or investment flows would complicate this relation but not affect any of our subsequent arguments.

since the government taxes labor and dividend income, this term now equals $\tau_y \times E_t$.

Equilibrium characterization. Our key building block result is that we can reduce the equilibrium of this extended model to a system of equations almost as simple as that of our baseline model in Section 2. First, combining market-clearing and the policy rules with private-sector demand we obtain

$$c_t = \mathscr{F}_1 \cdot (d_t + \varepsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[(1-\beta\omega) \sum_{k=0}^{\infty} (\beta\omega)^k c_{t+k} \right]. \tag{B.13}$$

Relative to our baseline model, the only change is that this equilibrium demand relationship is in aggregate consumption c_t rather than aggregate output y_t. We emphasize that this is possible precisely because the government taxes dividend and labor income, which as discussed above in equilibrium is equal to total consumption. Second, the law of motion for aggregate debt is now

$$d_{t+1} = \beta^{-1} \left(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y c_t}_{\text{tax base}} \right) - \underbrace{\frac{D^{ss}}{Y^{ss}} (\pi_{t+1} - \mathbb{E}_t [\pi_{t+1}])}_{\text{price level}}, \tag{B.14}$$

with real debt at date 0 given as

$$d_0 = b_0 - \frac{D^{ss}}{Y^{ss}} (\pi_0 - E_{-1} [\pi_0]) = -\frac{D^{ss}}{Y^{ss}} \pi_0. \tag{B.15}$$

Again, relative to the baseline model, the only change is that now it is aggregate consumption rather than aggregate output appearing in (B.14).

We note that (B.13) - (B.14) is a system in $\{c_t, d_t\}_{t=0}^{\infty}$ that depends on the rest of the economy—and so in particular the investment block—*only* through the presence of π_0. π_0 on the other hand can be obtained as a function of the consumption path $\{c_t\}_{t=0}^{\infty}$ by solving the system (B.4), (B.5), (B.6), (B.7), (B.8), (B.9) and (B.12) given consumption, and with the monetary policy rule (8) imposed. We write this function as

$$\pi_0 = \Pi_0 \left(\{c_t\}_{t=0}^{\infty} \right) \tag{B.16}$$

The equilibrium described by equations (B.13) - (B.16) is straightforward to characterize given our earlier analysis of the model without investment in Sections 2 and 4. We begin with the case of perfectly rigid prices ($\kappa = 0$), and for simplicity restrict attention to the limiting self-financing case ($\tau_d \to 0$ or $H \to \infty$). In that case $\pi_0 = 0$, so we can focus on the bivariate system (B.13) - (B.14) in $\{c_t, d_t\}_{t=0}^{\infty}$. Crucially, this system is *exactly* the same as that covered in Theorem 1, so the equilibrium characterization underlying that result applies unchanged, with c_t replacing y_t.

We now turn to the case of general κ. To this end let $c_{t,0}$ denote the solution of the rigid-price system, and furthermore let $p^I_{t,0}$ denote the corresponding equilibrium intermediate goods price obtained by solving the system (B.4), (B.5), (B.6), (B.7), (B.8), and (B.12) for p^I given $\{c_{t,0}\}_{t=0}^{\infty}$. Proceeding analogously to the proof of Theorem 1, we will now construct the equilibrium for general κ by simply scaling the $\kappa = 0$ equilibrium. To this end conjecture that equilibrium consumption satisfies $a \times c_{t,0}$, for some scalar a. It is then immediate that then we would also have $p^I_t = a \times p^I_{t,0}$. But then, from (B.9), we have that

$$\pi_0 = a \times \kappa \times \sum_{t=0}^{\infty} \beta^t p^I_{t,0} \tag{B.17}$$

Finally it follows from the government budget constraint that—again in our limiting self-financing equilibrium—we must have

$$\varepsilon_0 = a \times \tau_y \times \sum_{t=0}^{\infty} \beta^t c_{t,0} + a \times \frac{D^{ss}}{Y^{ss}} \times \kappa \times \sum_{t=0}^{\infty} \beta^t p^I_{t,0}$$

Solving this equation for a we obtain consumption and thus inflation as well as government debt in the general sticky-price equilibrium. In particular we see that self-financing yet again obtains exactly as in our baseline economy. We summarize these observations in the following corollary.

Corollary B.1. *Consider the extended OLG-NK environment with investment. Fiscal adjustment is never necessary in equilibrium—that is, $\nu \to 1$—if the tax response is infinitely delayed, i.e., $\tau_d \to 0$ or $H \to \infty$. These two limits induce the same equilibrium paths $\{c_t, \pi_t, d_t\}_{t=0}^{\infty}$, and in this common limit self-financing is sufficient to return government debt to steady state (i.e., $\rho_d \in (0,1)$).*

B.3 More general monetary policy

Here we supplement the discussion of monetary policy in Subsection 5.3. First, we explain how we measure the degree of self-financing when real rates are variable. Second, we investigate the model's determinacy regions, extending Leeper (1991). Finally, we elaborate on the connection between monetary policy feedback and our analysis of more general aggregate demand relations.

Measuring the degree of self-financing when $\phi \neq 0$. With variable real rates, the government's intertemporal budget constraint in (19) has to be re-written as follows:

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} E_0[r_k] = \tau_d \left(\varepsilon_0 + \sum_{k=0}^{+\infty} \beta^k E_0[d_k] \right) + \sum_{k=0}^{+\infty} \tau_y \beta^k E_0[y_k] + \frac{D^{ss}}{Y^{ss}} (\pi_0 - \mathbb{E}_{-1}[\pi_0]).$$

The right hand side is the same as before, while the new term on the left-hand side, $\frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} E_0[r_k]$, captures how the time-varying real interest rate in (9) changes the costs of servicing the outstanding public debt. We interpret this new term as analogous to the deficit shock itself and accordingly define

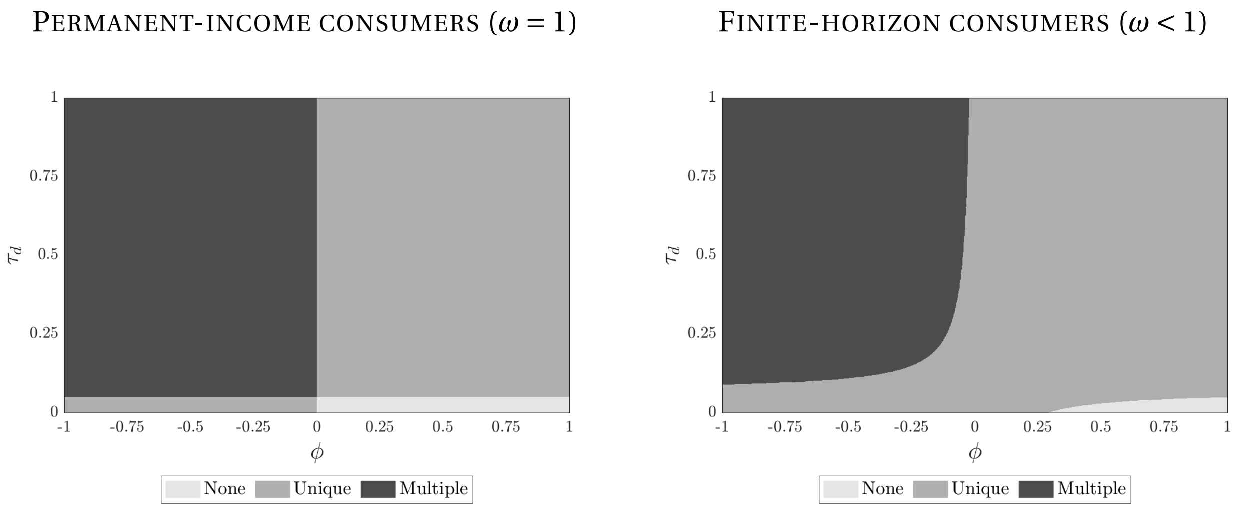

Figure B.2: Determinacy regions for $\omega = 1$ (left panel) and $\omega < 1$ (right panel) as a function of the fiscal and monetary policy rule coefficients τ_d (y-axis) and ϕ (x-axis).

the share of self-financing as

$$\nu \equiv \frac{\sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right] + \frac{D^{ss}}{Y^{ss}} \pi_0}{\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} E_0 \left[r_k \right]} \tag{B.18}$$

This reduces to the original definition when $\phi = 0$.

Determinacy regions. We begin by providing a visual illustration of equilibrium determinacy—i.e., the famous Leeper (1991) regions—in our OLG model. The two panels in Figure B.2 show whether a bounded equilibrium (i.e., the standard solution concept of Blanchard and Kahn (1980)) exists and is unique under different assumptions on monetary policy (ϕ) and fiscal policy (τ_d in (6)), for a standard permanent-income model (left panel) and our OLG economy (right panel).

The figure reveals that the determinacy properties of the two economies are materially different. Results for the permanent-income model are well-known and require little explanation: equilibrium uniqueness requires that fiscal policy is passive ($\tau_d > 1 - \beta$) and monetary policy is active ($\phi > 0$), or vice-versa; if both rules are active then no bounded equilibrium exists, and if both are passive then there are multiple bounded equilibria. With discounting on the household side (i.e., $\omega < 1$), the regions of equilibrium determinacy look rather different. Perhaps most importantly, the benchmark monetary rule of $\phi = 0$ now induces unique bounded equilibrium for any τ_d, consistent with Proposition 1. Intuitively, with $\omega < 1$, determinacy comes from the fact that public debt directly enters the aggregate demand relation. Moreover, existence comes from the fact, with $\tau_y > 0$, output also directly affects the government budget. Self-financing is now strong enough to pull debt as well as spending towards zero, even if interest rates do not provide any further Euler equation tilting. This automatic stabilization of government debt also shrinks the equilibrium non-existence region in the bottom right corner of the figure. See the discussion in Appendix A.3 for further details.

Connection between general monetary policy (9) and general aggregate demand (27). As discussed in the main text, the cutoff $\bar{\phi}$ plays the same role as Assumption 2—they both make sure that the deficit-driven Keynesian boom is large enough to play out in finite time. In fact, when $\phi = \bar{\phi}$ or when (28) holds with equality, the expected persistence of real value of debt (ρ_d in (17)) becomes 1 when fiscal adjustment is infinitely delayed ($\tau_d \to 0$). This then prevents the existence of a bounded complete self-financing equilibrium.

To further elaborate on this connection we note that, when $\frac{D^{ss}}{Y^{ss}} = 0$, it is possible to re-write the OLG model's aggregate demand relation under a general monetary reaction in a way similar to the generalized aggregate demand relation (27) in Section 5.1. Specifically, aggregating (10) and using asset market clearing, monetary policy (30) and $\frac{D^{ss}}{Y^{ss}} = 0$, we reach the following description of aggregate consumption

$$c_t = (1-\beta\omega)\, d_t + (1-\beta\omega-\phi\sigma\beta\omega)\, E_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k\, y_{t+k}\right] - (1-\beta\omega)\, E_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k\, t_{t+k}\right], \tag{B.19}$$

which can be nested by a general aggregate demand equation:

$$c_t = M_y\left(y_t + E_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k y_{t+k}\right]\right) - M_t\left(t_t + E_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k t_{t+k}\right]\right) + M_d d_t, \tag{B.20}$$

with the only slight difference from (27) being that the MPC out of income M_y is now allowed to differ from the MPC out of taxes M_t. This difference however is immaterial: we can generalize the proof of Theorem 2 to get a more general version of Assumption 2, requiring that

$$M_d + \frac{1-\beta}{\tau_y}\left(M_y - \tau_y M_t\right)\left(1 + \sum_{k=1}^{+\infty}(\beta\omega)^k\right) > \frac{1-\beta}{\tau_y}. \tag{B.21}$$

(B.20) nests (B.19) with $M_y = 1-\beta\omega-\phi\sigma\beta\omega$ and $M_d = M_t = 1-\beta\omega$. We see that, when $\phi = \bar{\phi}$ in (D.31), (B.21) holds with equality. This formalizes our claim that the cutoff $\bar{\phi}$ indeed plays the same role as Assumption 2 in Section 5.1.

B.4 Distortionary tax hikes

We begin by showing how the equilibrium relations of the model change with time-varying distortionary taxes. We then discuss implications for our limiting self-financing equilibria.

Environment. With time-varying distortionary taxes the optimal labor supply relation becomes

$$(1-\tau_{y,t})W_t = \frac{\chi \int_0^1 L_{i,t}^{\frac{1}{\varphi}} di}{\int_0^1 C_{i,t}^{-1/\sigma} di} \tag{B.22}$$

Log-linearizing, we find that

$$w_t - \frac{1}{\sigma}c_t - \frac{1}{1-\tau_y}\widehat{\tau}_{y,t} = \frac{1}{\varphi}\ell_t \tag{B.23}$$

where $\widehat{\tau}_t \equiv \tau_{y,t} - \tau_y$. Next note that the firm optimal pricing relationship is still

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t[\pi_{t+1}] \tag{B.24}$$

Combining (B.23), (B.24), and the modified baseline fiscal (32), we obtain

$$\pi_t = \kappa \left(y_t + \underbrace{\frac{1}{\left(\frac{1}{\varphi}+\frac{1}{\sigma}\right)(1-\tau_y)}\tau_d}_{\equiv \zeta} d_t \right) + \beta \mathbb{E}_t[\pi_{t+1}], \tag{B.25}$$

where $\kappa = \tilde{\kappa}\left(\frac{1}{\varphi}+\frac{1}{\sigma}\right)$. All other equilibrium relations of the model are unaffected.

Self-financing result. Note that, as $\tau_d \to 0$, we obtain that $\zeta \to 0$ for all t. It thus follows that the equilibrium characterization of Theorem 1 for the self-financing limit applies without any change to the alternative economy in which fiscal adjustment is distortionary. Intuitively, since the adjustment is not necessary in equilibrium, it is immaterial whether the adjustment *would have been* distortionary or lump-sum.

B.5 Government spending

The government spending policy experiment was already described in Section 5.5. The only missing ingredient for equilibrium characterization is the adjusted NKPC. By standard arguments (e.g., see Galí, 2008), it is now given as

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}] - \kappa \frac{\frac{1}{\sigma}}{\frac{1}{\varphi}+\frac{1}{\sigma}} g_t$$

Intuitively, the last term reflects the fact that, if higher output comes from higher government purchases (rather than higher consumption), then household labor supply is larger for standard wealth effects reasons—i.e., a negative cost-push shock. Since the overall analytics of the self-financing result

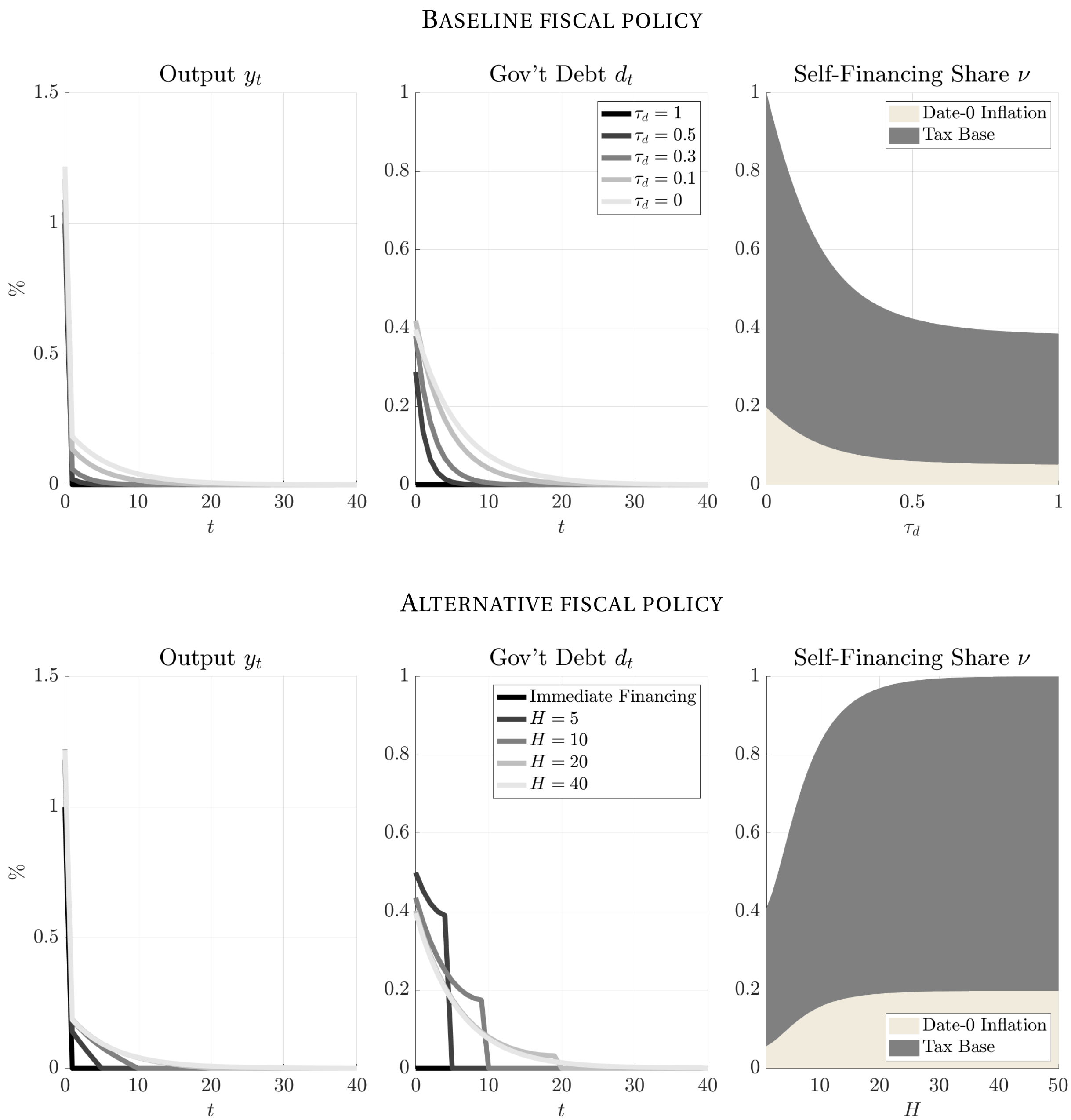

Figure B.3: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the government spending shock ε_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

with government purchases are exactly analogous to our baseline "stimulus checks" case, we do not repeat those derivations here and instead just provide a visual illustration of the self-financing result.

We summarize our results in Figure B.3—the government spending analogue of Figure 1. We emphasize two main takeaways. First, as $\tau_d \to 0$ or $H \to \infty$, we indeed again converge to a complete self-financing limit. Second, even immediately tax-financed fiscal purchases actually have a positive spending multiplier, and thus the share of self-financing ν for $\tau_d = 1$ (top panel) and $H = 0$ (bottom panel) is already strictly positive.[38]

C Quantitative analysis

This section supplements our quantitative analysis in Section 6. We: provide some missing details on the specification of our spender-OLG hybrid model in Appendix C.1; discuss alternative calibration strategies in Appendix C.2; review empirical evidence on fiscal adjustment in Appendix C.3; present results from a HANK model and a model with cognitive discounting in Appendices C.4 and C.5; and consider model variants with more flexible prices and more aggressive monetary reactions in Appendices C.6 and C.7.

C.1 Further details on the hybrid model

We first elaborate on the model environment and discuss in greater detail the model's implications for household consumption behavior, contrasting it in particular with the predictions of quantitative HANK-type models.

Model. The only change relative to our baseline model of Section 2 is that we generalize the household block to also feature a margin μ of spenders—that is, households who do not hold any assets and immediately spend any income they receive. The remaining fraction $1-\mu$ of households are exactly as described in Section 2.1. Both groups of households receive labor income as well as dividends and pay taxes, but only the OLG block holds government bonds.

We will make assumptions ensuring that both groups of households receive the same labor and dividend income, pay the same taxes (up to a between-group steady-state transfer), and have identical steady-state consumption. First, we assume that unions assign identical hours worked to both groups, and that dividends also accrue equally to both. Second, we assume that the government in lump-sum fashion redistributes between the two groups to ensure identical steady-state consumption; given that government bonds are held by the OLG block, this will generally require lump-sum transfers to spenders. Under those assumptions, it is first of all immediate that the supply block of the economy—notably (14)—is unchanged. Next, the demand block of the economy generalizes (26) as follows:

$$c_t = (1-\beta\omega)\cdot d_t + \left[\mu + (1-\mu)(1-\beta\omega)\right]\cdot\left((y_t - t_t) + (1-\mu)\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\omega\beta)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{C.1}$$

[38] As discussed in Section 5.5, if prices were rigid, then in this immediately tax-financed case $\nu = \nu_y = \tau_y$. With partially sticky prices the initial inflation further increases the degree of self-financing.

Replacing (26) by (C.1) is the only difference between our baseline OLG economy and the generalized hybrid model. Relative to (26), the most important change in (C.1) is that we allow the MPC out of income to be larger than that out of wealth. As we discuss next, this minimal departure from our baseline OLG model is all that is needed to ensure (approximate) consistency with consumption-savings behavior even in quantitative HANK-type models.

Household consumption-savings behavior. By our discussion in Appendix A.4 we know that the role of the household consumption-savings decision in driving our self-financing result is fully governed by the matrix of intertemporal marginal propensities to consume. The left top panel of Figure 5 provides a visual illustration of this matrix in our spender-OLG hybrid model, as implied by the generalized demand block (C.1).

The figure plots the spending response over time to (anticipated) income gains at different dates. We emphasize two key takeaways. First, the response to a date-0 income gain—that is, the first column of $\mathscr{M}$—agrees closely with prior empirical evidence (Fagereng, Holm and Natvik, 2021), as already discussed in Section 6.1. Second, the higher-order columns are qualitatively and quantitatively similar to those implied by quantitative HANK-type models. This observation has been made previously in Auclert, Rognlie and Straub (2018) and Wolf (2021*a*). For our purposes, the important takeaway is that our analysis is indeed quantitatively relevant—as far as our question of self-financing is concerned, our model will have very similar predictions as richer quantitative HANK-type models. We further illustrate this observation in Appendix C.4.

C.2 Alternative calibration approaches for the household block

For our baseline analysis in Section 6 we discipline our model's departure from permanent-income behavior by requiring consistency with empirical evidence on the *level* and *slope* of (short-run) household consumption behavior lump-sum income receipt, as in Auclert, Rognlie and Straub (2018) and Wolf (2021*a*). We here instead discuss two different approaches: one based on farther-out spending responses, and one based on long-run price elasticities of household asset demand.

Calibration via tail MPCs. The calibration strategies were already discussed in Section 6.2: the "lower bound" hybrid model matches the impact MPC and the estimated lower bound of the five-year cumulative MPC, while the generalized three-type model is parameterized to match the five-year cumulative MPC path as well as possible, in a standard least-squares sense. For the "lower bound" calibration of the hybrid model we set $\mu = 0.17$ and $\omega = 0.94$; all other parameters are as in Table 1. For the three-type model, we set $\omega_1 = 0.97$ and $\omega_2 = 0.83$, with the fractions of the two groups equal to $\chi_1 = 0.22$ and $\chi_2 = 0.63$. The residual fraction $1 - \chi_1 - \chi_2$ are hand-to-mouth.

Calibration via asset supply elasticities. For this approach we combine evidence on level MPCs with long-run price elasticities of household asset demand. This calibration strategy is promising because models with permanent-income savers invariably imply a (counterfactual) infinite elasticity of household asset demand (e.g., Kaplan and Violante, 2018).

Our main building block result for this calibration approach is Proposition C.1. We there give the long-run elasticity of household asset demand as a function of model primitives.

Proposition C.1. *Consider the spender-OLG hybrid model. Let η denote the long-run interest rate elasticity of household asset demand—that is, the long-run response of asset demand to a permanent change in real interest rates. It is given as*

$$\eta \;=\; (1-\mu)\times\frac{\sigma}{1-\beta}\times\left(\frac{1}{1-\omega}-\frac{1}{1-\beta\omega}\right) \tag{C.2}$$

Empirical work suggests a range for η of around 1.25 to 35 (see Moll, Rachel and Restrepo, 2022). Setting $\beta = 0.99$, $\sigma = 1$, and requiring the model to generate an impact MPC of 22 per cent (all as in our baseline calibration), we find $\omega \in [0.21, 0.85]$. Our baseline calibration lies somewhat beyond the upper end of this range and is thus conservative.

C.3 Empirical evidence on fiscal adjustment

Notable prior work that has estimated fiscal financing rules and thus in particular the speed of fiscal adjustment in response to deficits includes Galí, López-Salido and Vallés (2007), Bianchi and Melosi (2017), and Auclert and Rognlie (2018). Auclert, Rognlie and Straub (2020) (Appendix D.1) survey this literature and conclude that the *annual* tax adjustment parameter—ϕ in their notation—lies between 0.015 and 0.3, with their preferred estimate equal to 0.1. Our displayed values for τ_d correspond to the quarterly analogues of these values. We note that all of our values strictly exceed $\bar{r}$ and thus correspond to "passive" fiscal rules in the terminology of Leeper (1991).

C.4 A full HANK model

This section provides a sketch of the quantitative HANK model that we use to numerically illustrate the generality of our self-financing result. The discussion is brief because the household block of the model is essentially borrowed from Wolf (2021*a*).

Model sketch & calibration. The model economy is exactly as in Section 2, but with one twist: the OLG household block is replaced by a unit continuum of households $i \in [0, 1]$ that face uninsurable income risk. Households have preferences

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^t \left[u(C_{i,t+k}) - \nu(L_{i,t+k}) \right] \right]$$

Households save and borrow (subject to a constraint) in a nominally risk-free bond, as in our baseline model. They receive labor and dividend income in proportion to their (stochastic) productivity, pay a proportional tax τ_y on that income, and finally pay additional lump-sum uniform taxes $\tilde{T}_t$. We can thus write the household budget constraint in real terms as

$$C_{i,t} + D_{i,t+1} = (1-\tau_y) e_{i,t} Y_t - \tilde{T}_t + \frac{I_{t-1}}{\Pi_t} D_{i,t}, \quad D_{i,t+1} \geq \underline{D}$$

Whenever possible we calibrate the model exactly as our baseline model. The remaining HANK-specific parameters are: the income risk process; the borrowing constraint; and the discount factor. The income risk process is taken from Kaplan, Moll and Violante (2018), just ported to discrete time as in Wolf (2021*b*). The borrowing constraint $\underline{D}$ is set to zero, and the discount factor β is backed out residually to clear the asset market. Finally, we need to make one more change relative to our baseline model: in the model set-up as described so far, tax revenue $\tau_y \times Y^{ss}$ would far exceed debt servicing costs, so the government would make a substantial uniform transfer, thus materially dampening household MPCs. We instead set the steady-state transfer share as in the data (following Kaplan, Moll and Violante, 2018, which gives $\tilde{T}^{ss}/Y^{ss} = 0.06$), and then clear the government budget by additionally allowing for positive (and time-invariant) government purchases.

Results. We use the quantitative HANK model to revisit our numerical exercises in Section 6.2. Exactly as done there, we here compute the aggregate effects of one-off fiscal stimulus for different assumptions on the delay in fiscal financing. Results are reported in Figure C.1.

Our results closely echo those of Section 6.2. We emphasize two main takeaways. First, Figure C.1 is *qualitatively* very similar to Figure 4: output and inflation responses as well as the share of self-financing ν are all increasing in the delay in fiscal adjustment (i.e., decreasing in τ_d). Furthermore, as $\tau_d \to 0$, we again converge to a full self-financing limit. We have also verified numerically that, for each $\tau_d \in [0,1]$ considered in construction of Figure C.1, the constructed equilibrium is (locally) unique, again echoing our baseline analysis. Second, the two figures are also *quantitatively* similar: for our three values of τ_d taken from prior work, the impulse responses of output and inflation as well as the share of self-financing ν are very similar to the spender-OLG hybrid model. This conclusion confirms prior work arguing that, as far the dynamics of macroeconomic aggregates are concerned, spender-OLG hybrid models and fully specified HANK models look extremely similar (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*)

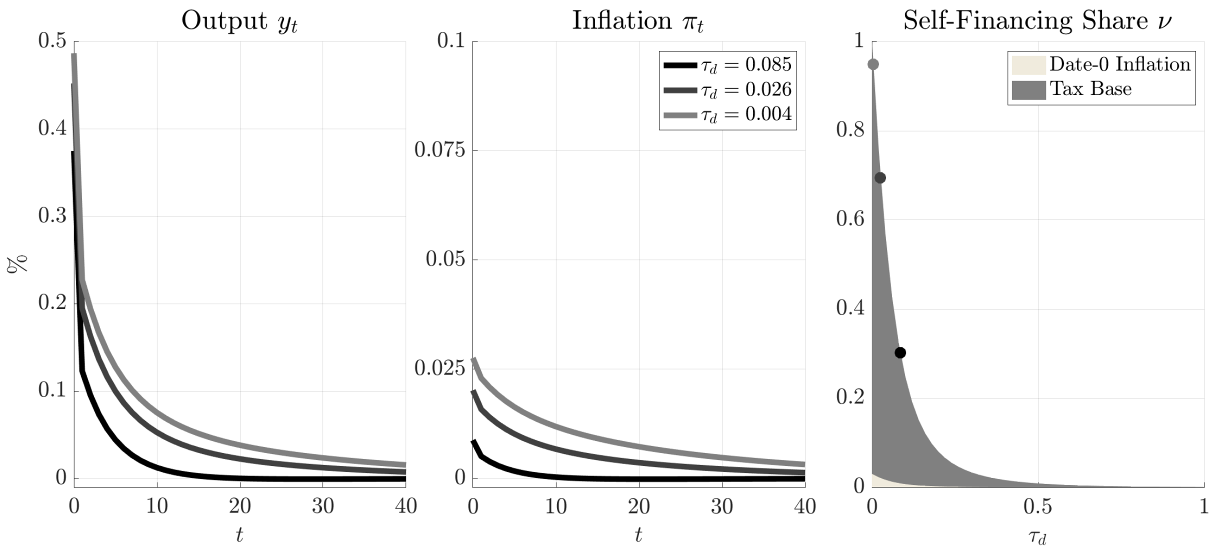

Figure C.1: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ε_0 as a function of τ_d, for the quantitative HANK model. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

C.5 The effects of cognitive discounting

Figure C.2 repeats our analysis of Section 6.2 in a variant of our spender-OLG hybrid model with cognitive discounting. To illustrate the effects of discounting as clearly as possible we consider a rather significant degree of discounting ($\theta = 0.25$).

The figure illustrates the two effects described in Section 5.1. First, for τ_d close to one, the Keynesian boom and thus the share of self-financing ν are larger than in our baseline model. Intuitively, in this case, the strong discounting of the not-so-distant tax hike meaningfully amplifies the initial boom. Second, for τ_d close to one, the self-financing limit is approached somewhat more slowly, reflecting a weakening of the intertemporal Keynesian cross.

C.6 Self-financing with more flexible prices

Figure C.3 repeats our analysis of Section 6.2 in a variant of the baseline model with more flexible prices, setting $\kappa = 0.1$—a value at the large end of recent empirical evidence and arguably more relevant for the inflationary post-covid environment. With more flexible prices, a non-trivial share—here around 20 per cent—of self-financing comes from adjustments in prices rather than quantities.

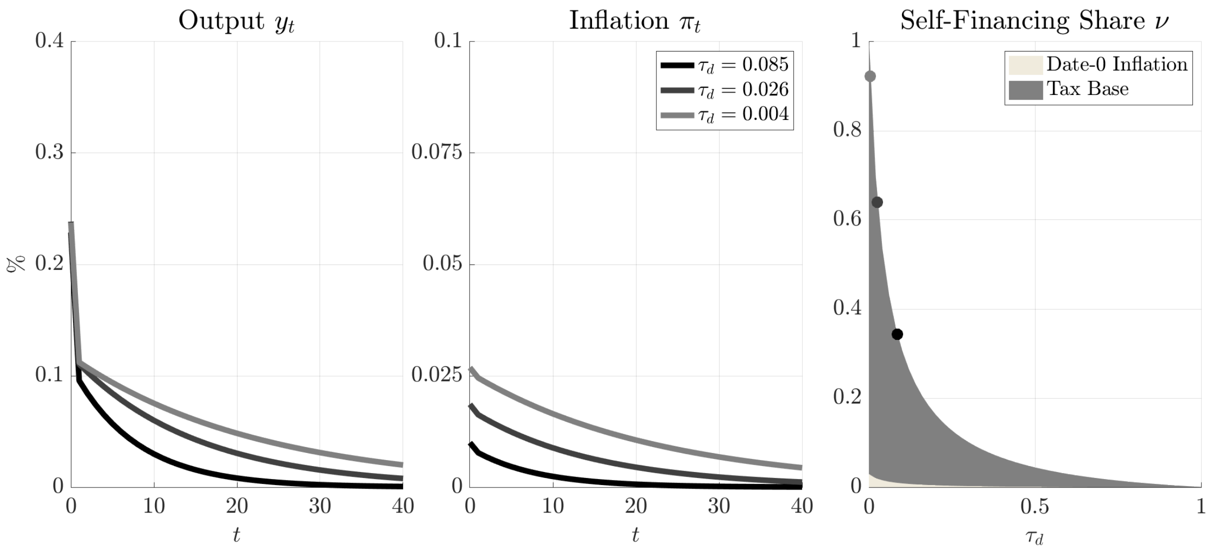

Figure C.2: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ε_0 as a function of τ_d, with cognitive discounting. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

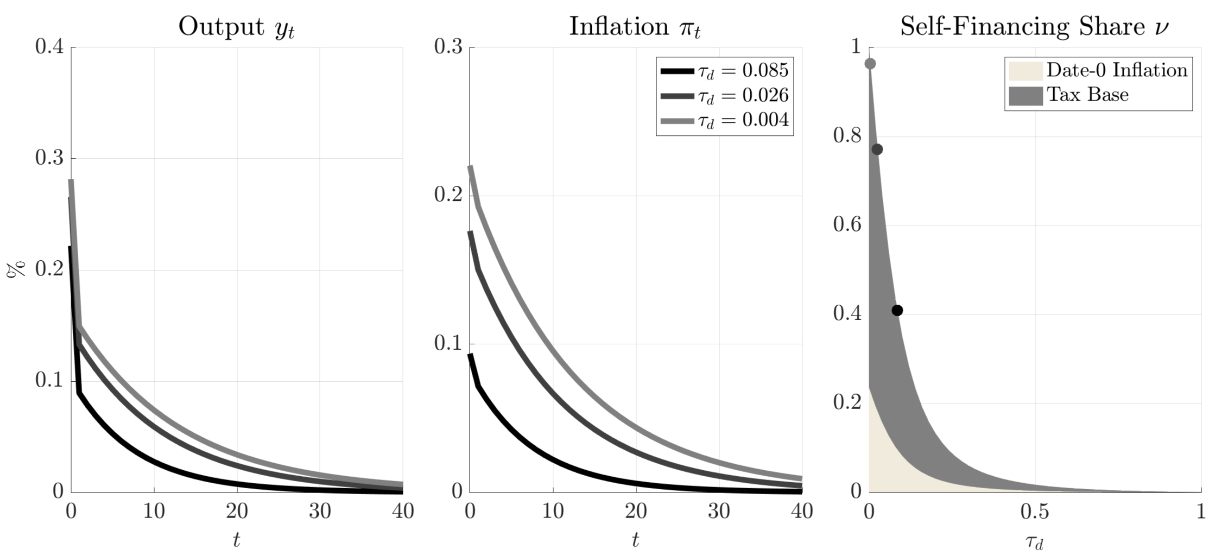

Figure C.3: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ε_0 as a function of τ_d, with more flexible prices. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

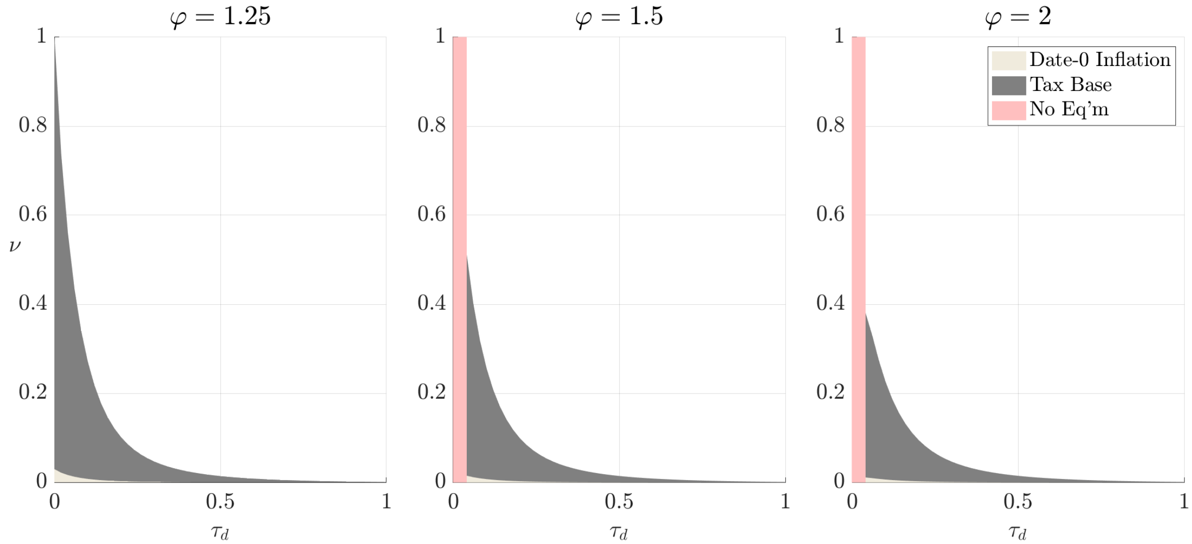

Figure C.4: Total self-financing share ν in response to the deficit shock ε_0 as a function of τ_d, for the standard Taylor-type monetary policy rule $i_t = \varphi\pi_t$, with $\varphi \in \{1.25, 1.5, 2\}$. The red areas indicate (bounded) equilibrium non-existence.

C.7 Active monetary reaction

For our final extension we consider a variant of the baseline quantitative model in which the monetary authority instead follows a standard Taylor-type rule, setting

$$i_t = \varphi\pi_t$$

We consider $\varphi \in \{1.25, 1.5, 2\}$—a range of values, all corresponding to active monetary policy responses in the classical terminology of Leeper (1991). The three panels in Figure C.4 show the self-financing share ν as a function for τ_d for the three displayed values of φ. In the middle and right panels the red regions indicate equilibrium non-existence for the underlying policy mix.

The figure illustrates the robustness of our conclusions to active monetary policy reactions. First, for a moderate monetary response ($\varphi = 1.25$), complete self-financing is still possible, consistent with Theorem 3. Compared to our baseline exercises in Section 6.2, the more aggressive monetary reaction delays the fiscal boom, so ν is strictly smaller for any $\tau_d > 0$, though still converging to 1 as $\tau_d \to 0^+$. Second, for even stronger monetary reactions ($\varphi = 1.5$ or $\varphi = 2$), existence of a bounded equilibrium requires that τ_d is sufficiently far above 0, as indicated by the shaded red areas. We nevertheless see that, for small values of τ_d outside of these non-existence regions, there can still be meaningful degrees of self-financing, even for a central bank that quite aggressively leans against the fiscal boom.

D Proofs and auxiliary lemmas

D.1 Proof of Proposition 1

Note that we restrict that $\omega \in (0,1)$, $\tau_y \in (0,1)$, and $\tau_d \in [0,1)$. We first write (12) recursively:

$$\begin{aligned} y_t - \mathscr{F}_1 \cdot (d_t + \varepsilon_t) &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega \mathbb{E}_t \left[y_{t+1} - \mathscr{F}_1 \cdot (d_{t+1} + \varepsilon_{t+1}) \right] \\ &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega \mathbb{E}_t \left[y_{t+1} - \mathscr{F}_1 \cdot \frac{1}{\beta} \left[(1-\tau_d)(d_t + \varepsilon_t) - \tau_y y_t \right] \right]. \end{aligned}$$

After rearranging terms and using the formula of $\mathscr{F}_1$ and $\mathscr{F}_2$ (as stated after (12)), we have

$$\begin{aligned} y_t &= \frac{(1-\omega(1-\tau_d))\mathscr{F}_1}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}(d_t+\varepsilon_t) + \frac{\beta\omega}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}\mathbb{E}_t\left[y_{t+1}\right] \\ &= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t+\varepsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right]. \end{aligned}$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (15), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y \end{pmatrix} \begin{pmatrix} d_t+\varepsilon_t \\ y_t \end{pmatrix} \tag{D.1}$$

The two eigenvalues of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \frac{1}{\beta}(1-\tau_d) = 0,$$

with

$$\begin{aligned} \lambda_1 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &> \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1 \end{aligned} \tag{D.2}$$

and

$$\begin{aligned} \lambda_2 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &< \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \left|\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2} \leq 1, \end{aligned} \tag{D.3}$$

with $\lambda_2 > 0$ too since $\lambda_1 \lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $(1, \chi_2)'$ denote the eigenvector associated with λ_2, where

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_2} > 0. \tag{D.4}$$

This means that any bounded equilibrium path $\{d_t, y_t\}_{t=0}^{+\infty}$ of (D.1)[39] takes the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\varepsilon_t),$$

where χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1). \tag{D.5}$$

In other words, the equilibrium takes the form of (17) and satisfies (18).[40]

To prove equilibrium uniqueness, note that the total amount of nominal public debt outstanding at the start of $t=0$, $B_0 = B^{ss}$, is given. From (14) and (16), we know d_0 is uniquely pinned down by

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0\left[y_k\right] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0. \tag{D.6}$$

Similarly, for $t \geq 1$,

$$\begin{aligned} d_t - \mathbb{E}_{t-1}[d_t] &= -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t\left[y_{t+k}\right]-\mathbb{E}_{t-1}\left[y_{t+k-1}\right]\right) \\ &= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_t-\mathbb{E}_{t-1}[d_t]+\varepsilon_t) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_t. \end{aligned}$$

This pins down the unique bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ with (14) and (17).

D.2 Proof of Lemma 1

The result follows directly from two facts in the proof of Proposition 1 and Theorem 1. First, under fiscal rule (6), ν decreases in τ_d, $\lim_{\tau_d\to 0^+}\nu \to 1$, and $\lim_{\tau_d\to 1^-}\nu \to 0$. Second, under fiscal rule (7), ν increases in H, $\nu = 0$ with $H = 0$, and $\lim_{H\to\infty}\nu \to 1$.

[39] Boundedness means that $\lim_{k\to+\infty}\mathbb{E}_t[d_{t+k}]$ and $\lim_{k\to+\infty}\mathbb{E}_t\left[y_{t+k}\right]$ are bounded for any t, $d_t+\varepsilon_t$, and y_t, similar to Blanchard and Kahn (1980).

[40] To see the first part of (18), combine (12) with (17).

D.3 Proof of Theorem 1

We start with the case based on the baseline fiscal policy (6). We begin by considering an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Let ν' denote the self financing share in this alternative economy, which similarly to (21) is given as

$$\nu' \cdot \varepsilon_0 = \nu'_y \cdot \varepsilon_0 \equiv \sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right].$$

Note that in this alternative economy all self-financing comes from tax base changes. In particular, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (17) and (21) we know that

$$\nu' = \tau_y \frac{\chi}{1 - \beta \rho_d}. \tag{D.7}$$

Now, consider the general case with $\kappa \geq 0$. From NKPC (14) and the definitions in (20) – (21), we have

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu \tag{D.8}$$

From the formula of d_0 (16), we know

$$d_0 = -\nu_p \varepsilon_0 \quad \text{and} \quad \nu_y = \left(1 - \nu_p\right) \nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}. \tag{D.9}$$

From (18), we know that

$$\frac{\chi}{1 - \beta \rho_d} = \frac{\chi}{\tau_d + \tau_y \chi}. \tag{D.10}$$

From (D.3) and (D.5), we know

$$\rho_d = \lambda_2 = f(a, b) \equiv \frac{a + b + 1 - \sqrt{(a + b - 1)^2 + 4b}}{2} \tag{D.11}$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}} > 0$,we know that ρ_d decreases with τ_d. From (D.4) and (D.5), we then know $\chi = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y - \rho_d}$ also decreases in τ_d. From (D.10), we know $\frac{\chi}{1-\beta\rho_d}$ decreases in τ_d. Finally, from (D.7) and (D.9), we know ν decreases in τ_d. This finishes the proof of Part 1.

For Part 2, from (D.3) and (18), we know that ρ_d and χ are continuous in $\tau_d \in [0,1)$, and

$$\rho_d^{\text{full}} \equiv \lim_{\tau_d \to 0^+} \rho_d = \frac{\left(\frac{1}{\beta} + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1 - \frac{1}{\beta} - \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \tag{D.12}$$

$$< \frac{\frac{1}{\beta} + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) - \left|\frac{1}{\beta} + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) - 1\right|}{2} \le 1$$

$$\chi^{\text{full}} \equiv \lim_{\tau_d \to 0^+} \chi = \frac{1-\beta\rho_d^{\text{full}}}{\tau_y} > 0 \tag{D.13}$$

From (D.10), we know $\lim_{\tau_d \to 0^+} \frac{\chi}{1-\beta\rho_d} = \frac{1}{\tau_y}$. From (D.7) and (D.9), we know $\lim_{\tau_d \to 0^+} \nu = 1$. Finally, $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$ follows directly from $\rho_d^{\text{full}} < 1$.

Now we turn to the alternative fiscal policy rule in (7), for which we use the equilibrium characterization in Appendix A.2. For the case with rigid prices ($\kappa = 0$), one can see from (A.11) that ν' increases in H, which proves Part I. For Part II and to find $\lim_{H\to\infty} \nu'$, first note that, from (A.9), $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, increasing sequence. As a result, there exists $\chi^{\text{full,NM}}$ such that $\lim_{k\to\infty} \chi_k = \chi^{\text{full,NM}}$ and $\chi^{\text{full,NM}} = g\left(\chi^{\text{full,NM}}\right) \in \left(0, \frac{1}{\tau_y}\right)$. From (A.11), we know that $\lim_{H\to\infty} \nu' = 1$. From (A.10), we know that $\lim_{H\to\infty} \mathbb{E}_0[d_H] = 0$. From (D.4) and (D.5), we also know that $g\left(\chi^{\text{full}}\right) = \chi^{\text{full}}$ where χ^{full} defined in (D.13) parametrizes the output response in the complete self-financing limit ($\tau_d \to 0$) with the baseline fiscal rule (6). From the definition of $g(\cdot)$ in (A.8), we know that there is a unique $\chi > 0$ such that $g(\chi) = \chi$ when $\omega < 1$ and $\tau_d \in (0,1)$. As a result, $\chi^{\text{full,NM}} = \chi^{\text{full}} < \frac{1}{\tau_y}$ and $\lim_{k\to+\infty} \chi_k = \chi^{\text{full}}$. That is, these two limits ($\tau_d \to 0$ and $H \to \infty$) share the same equilibrium path. Finally, for the general case with $\kappa \ge 0$, the desired result follows directly from the rigid price case together with (A.13) and (D.9).

D.4 Properties of the consumption function

Lemma D.1. *Let $\mathcal{M}$ denote the the matrix of intertemporal MPCs corresponding to our consumption function* (11)*. Then, if and only if $\omega < 1$:*

1. *As ℓ increases, one unit anticipated income changes at date $t+\ell$ (in terms of present value at t) have a vanishing effect on consumer demand at date t:*

$$\lim_{\ell\to\infty} \beta^{-\ell} \mathcal{M}_{t,t+\ell} = 0$$

2. *As ℓ increases, one unit income changes at date t have a vanishing effect on consumer demand at date $t+\ell$:*

$$\lim_{\ell\to\infty} \mathscr{M}_{t+\ell,t} = 0$$

We prove the two parts of the lemma in turn. The proof leverages results on the properties of the intertemporal MPC matrix $\mathscr{M}$ in OLG models from Wolf (2021*a*).

1. The proof is by induction. First of all we have

$$\mathscr{M}_{0,\ell}\beta^{-\ell} = (1-\beta\omega)\omega^{\ell}$$

Thus the claim holds for $t=0$. Now suppose the claim holds for some $t-1$ (where $t \geq 1$), and consider horizon t. Here we have, for $\ell \geq 0$,

$$\mathscr{M}_{t,t+\ell}\beta^{-\ell} = -(1-\beta\omega)^2\beta^{t-1}\omega^{t+\ell+1} + \mathscr{M}_{t-1,t-1+\ell}\beta^{-(\ell-1)}\beta^{-1}$$

As $\ell \to \infty$ the first term converges to zero since $\omega < 1$ while the second term converges to zero by the inductive assumption, completing the argument.

2. The proof is again by induction. Begin again with $t=0$. Here we have

$$\mathscr{M}_{\ell,0} = (1-\beta\omega)\omega^{\ell}$$

and so the statement holds. Now suppose it holds for some $t-1$ (where $t \geq 1$), and consider horizon t. Here we have, for $\ell \geq 0$,

$$\mathscr{M}_{t+\ell,t} = -(1-\beta\omega)^2\beta^{t-1}\omega^{2t+\ell-1} + \mathscr{M}_{t-1+\ell,t-1}$$

The first term converges to zero as $\ell \to \infty$, for any t. The second term furthermore also converges to zero (by the inductive hypothesis), completing the argument.

D.5 Proof of Proposition 2

For part 1 of the Proposition, we start with the case of the baseline fiscal policy (6). From (D.11), we know that ρ_d is independent of the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$. From (18) and (D.7), we know that ν' is independent of the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$. From (D.9), we know that ν is increasing in the degree of price flexibility κ and steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$.

Now we turn to part 1 with the alternative fiscal policy in (7), for which we use the equilibrium characterization in Appendix A.2. From (A.7) and (A.11), we know that ν' is independent of the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$. From (A.13), we know that ν increases in the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$.

Part 2 of the Proposition follows directly from (22), which holds for both fiscal policies and gives

$$\nu_y = \frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \times \nu \quad \text{and} \quad \nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \times \nu.$$

D.6 Proof of Proposition 3

We start with the case of the baseline fiscal policy (6). From (D.11), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2-4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

which decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$. As a result, ρ_d decreases in τ_y. From the second part of 18, we know that $\tau_y\chi$ increases with τ_y. From (D.7) and D.10, we know that, when $\kappa = 0$, $\nu = \frac{\tau_y\chi}{\tau_d+\tau_y\chi}$ increases with τ_y.

Now we turn to part 1 with the alternative fiscal policy in (7), for which we use the equilibrium characterization in Appendix A.2. From (A.7), we know that

$$\tau_y\chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y + \frac{\chi_{H-t-1}}{\beta}\tau_y}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y + \frac{\chi_{H-t-1}}{\beta}\tau_y} = \mathcal{G}\left(\tau_y, \chi_{H-t-1}\tau_y\right),$$

where

$$\mathcal{G}(x,y) = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}x + \frac{1}{\beta}y}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}x + \frac{1}{\beta}y}$$

increases in $x \in (0,1)$ and $y > 0$. Together with (A.5), we know that $\tau_y\chi_k$ increases with τ_y for each $k \geq 1$. Together with (A.11), when $\kappa = 0$, ν increases with τ_y.

D.7 Proof of Proposition 4

Consider the baseline fiscal policy (6). From (D.11), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2-4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

where $a = \frac{1}{\beta}(1-\tau_d) \in (0,1)$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. For the second part of the equation, we know that ρ_d decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$.

From the second half of (18), we know χ decreases in ω. From, (D.7), (D.9) and (D.10), we know that ν decreases in ω. This proves Part 1.

For Part 2, from Theorem 1, we know that $\rho_d^{\text{full}} < 1$ for any $\omega < 1$. From (D.12), we know that $\lim_{\omega\to 1}\rho_d^{\text{full}} = \frac{\left(\frac{1}{\beta}+1\right)-\sqrt{\left(1-\frac{1}{\beta}\right)^2}}{2} = 1$.

D.8 Proof of Theorem 2

We start with the baseline fiscal policy (6). We focus on a bounded equilibrium similar to (17), taking the form of

$$y_t = \chi_d d_t + \chi_\varepsilon \varepsilon_t, \quad E_t[d_{t+1}] = \rho_d d_t + \rho_\varepsilon \varepsilon_t \quad \text{with} \quad \chi_d, \chi_\varepsilon > 0, \ \rho_d \in (0,1). \tag{D.14}$$

For (D.14) to be an equilibrium, it needs to satisfy (15) and (27). For (D.14) to satisfy to the government budget (15), we need

$$\rho_d = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_d\right) \quad \text{and} \quad \rho_\varepsilon = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_\varepsilon\right). \tag{D.15}$$

For (D.14) to satisfy aggregate demand (27) (together with market clearing $c_t = y_t$), we need

$$\chi_d = M_d + M_y\left[\left(1-\tau_y\right)\chi_d - \tau_d\right]\left(1+\delta\sum_{k=1}^{+\infty}\left(\beta\omega\rho_d\right)^k\right) = \frac{M_d - \tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}. \tag{D.16}$$

and

$$\chi_\varepsilon = M_y\left(1+\left(\left(1-\tau_y\right)\chi_\varepsilon - \tau_d + \delta\left(\left(1-\tau_y\right)\chi_d-\tau_d\right)\sum_{k=1}^{+\infty}(\beta\omega)^k\rho_d^{k-1}\rho_\varepsilon\right)\right). \tag{D.17}$$

(D.15) and (D.16) together mean that ρ_d needs to be the root of the following equation:

$$h\left(\rho_d;\tau_d\right) = \frac{1-\tau_d-\beta\rho_d}{\tau_y} - \frac{M_d - \tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)} = 0,$$

with $\lim_{\tau_d\to 0^+} h(0;\tau_d) = \frac{1}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)} \geq 0$ because $\tau_y > 0$, $M_y \in (0,1)$ and $M_y \geq M_d$.

When Assumption 2 holds, we first show that there exists $\rho_d^{\text{full}} \in [0,1)$ such that $\lim_{\tau_d\to 0^+} h\left(\rho_d^{\text{full}};\tau_d\right) =$

0. There are two cases. First, $M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right)>1$. In this case, there exists $\bar{\rho}\in(0,1)$ such that

$$M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\bar{\rho}}{1-\beta\omega\bar{\rho}}\right)=1,$$

and $\lim_{\tau_d\to0^+,\rho_d\to(\bar{\rho})^-}h\left(\rho_d;\tau_d\right)=-\infty$. As a result, there exists a unique $\rho_d^{\text{full}}\in[0,1)$ such that we have $\lim_{\tau_d\to0^+}h\left(\rho_d^{\text{full}};\tau_d\right)=0$.[41]

Second, $M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right)<1$. In this case,

$$\lim_{\tau_d\to0^+}h\left(1;\tau_d\right)=\frac{1-\beta}{\tau_y}-\frac{M_d}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right)}<0$$

from Assumption 2. As a result, there exists a unique $\rho_d^{\text{full}}\in[0,1)$ such that $\lim_{\tau_d\to0^+}h\left(\rho_d^{\text{full}};\tau_d\right)=0$.

Since $h\left(\rho_d;\tau_d\right)$ is continuous, we know that, for each τ_d in a right neighborhood of 0, there exists a unique $\rho_d\left(\tau_d\right)\in[0,1)$ such that $h\left(\rho_d\left(\tau_d\right);\tau_d\right)=0$ and $\lim_{\tau_d\to0^+}\rho_d\left(\tau_d\right)=\rho_d^{\text{full}}\in[0,1)$. For each τ_d in this neighborhood, given $\rho_d\left(\tau_d\right)$, one can find $\rho_\varepsilon\left(\tau_d\right)$ and $\chi_d\left(\tau_d\right),\chi_\varepsilon\left(\tau_d\right)>0$ from (D.15) – (D.17) to constitute the unique bounded equilibrium in the form of (D.14). Next note that from boundedness we have $\lim_{\tau_d\to0^+}\frac{\tau_d\left(\varepsilon_0+\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[d_k\right]\right)}{\varepsilon_0}=0$ and $\lim_{\tau_d\to0^+}\nu=1$, using also (19) and (20). Finally, $\lim_{k\to\infty}\mathbb{E}_t\left[d_{t+k}\right]\to0$ follows directly from $\rho_d^{\text{full}}\in[0,1)$. This finishes the proof with the baseline fiscal policy (6).

We now turn to the case with (7). Together with market clearing $c_t=y_t$, we first write the aggregate demand in (27) recursively

$$\begin{aligned}
y_t&=\frac{M_d}{1-M_y}d_t-\frac{M_y}{1-M_y}t_t+\delta\beta\omega\frac{M_y}{1-M_y}\mathbb{E}_t\left[y_{t+1}-t_{t+1}\right]+\beta\omega\mathbb{E}_t\left[y_{t+1}-\frac{M_d}{1-M_y}d_{t+1}+\frac{M_y}{1-M_y}t_{t+1}\right]\\
&=\frac{M_d}{1-M_y}d_t-\frac{M_y}{1-M_y}t_t+\delta\beta\omega\frac{M_y}{1-M_y}\mathbb{E}_t\left[y_{t+1}-t_{t+1}\right]+\beta\omega\mathbb{E}_t\left[y_{t+1}+\frac{M_y}{1-M_y}t_{t+1}\right]-\omega\frac{M_d}{1-M_y}\left(d_t-t_t\right)\\
&=\frac{M_d\left(1-\omega\right)}{1-M_y}d_t-\frac{M_y-\omega M_d}{1-M_y}t_t+\beta\omega\left(\frac{1-\left(1-\delta\right)M_y}{1-M_y}\right)\mathbb{E}_t\left[y_{t+1}\right]+\beta\omega\frac{M_y}{1-M_y}\left(1-\delta\right)\mathbb{E}_t\left[t_{t+1}\right].
\end{aligned}\tag{D.18}$$

From (7), we know that $t_t=d_t$ for all $t\geq H$. As a result, $d_{t+1}=0$ for all $t\geq H$. Similar to the argument in Appendix A.2, we can then focus on the case that $y_t=d_t=0$ for $t\geq H+1$. At $t=H$, from (D.18), we have

$$y_H=\frac{-\left(M_y-M_d\right)}{1-M_y}d_H=\chi_0 d_H\ \text{ with }\ \chi_0=\frac{-\left(M_y-M_d\right)}{1-M_y}.\tag{D.19}$$

Similar to the main analysis in Appendix A.2, we will now use (D.18) to find the equilibrium path of

[41] Note that, for $\rho_d\in\left(\bar{\rho},1\right)$, $\lim_{\tau_d\to0^+}\frac{M_d-\tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}<0$ and so $\lim_{\tau_d\to0^+}h\left(\rho_d;\tau_d\right)>0$.

$\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction. At $t = H-1$, from (7) and (D.18)

$$
\begin{aligned}
y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \beta\omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}} d_H \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}} \left(d_{H-1} - \tau_y y_{H-1}\right) \\
y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y} + \omega\tau_y\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]} d_{H-1} \\
&= \chi_1 d_{H-1},
\end{aligned} \tag{D.20}
$$

with

$$
\begin{aligned}
\chi_1 &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\tau_y\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)\tau_y}.
\end{aligned} \tag{D.21}
$$

For $1 \le t \le H-2$, from (7) and (D.18),

$$
\begin{aligned}
y_t &= \frac{\frac{M_d(1-\omega)}{1-M_y} d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega\frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\mathbb{E}_t\left[y_{t+1}\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega\frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\left(d_t - \tau_y y_t\right)\chi_{H-t-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}} d_t \\
&= \chi_{H-t} d_t \ \text{ with } \ \chi_{H-t} = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}}
\end{aligned} \tag{D.22}
$$

Finally, for $t = 0$, from (7) and (D.18), we know

$$\begin{aligned}
y_0 &= \frac{\frac{M_d(1-\omega)}{1-M_y} d_0 + \frac{M_y-\omega M_d}{1-M_y}\varepsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\mathbb{E}_0\left[y_1\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} d_0 + \frac{M_y-\omega M_d}{1-M_y}\varepsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\left(d_0+\varepsilon_0-\tau_y y_0\right)\chi_{H-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}} d_0 + \frac{\frac{M_y-\omega M_d}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}\varepsilon_0 \\
&= \chi_H d_t + \chi_H^{\varepsilon}\varepsilon_0 \;\text{ with }\; \chi_H = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}, \qquad \text{(D.23)}
\end{aligned}$$

and $\chi_H^{\varepsilon} = \frac{\frac{M_y-\omega M_d}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}$. Define

$$g\left(\chi\right) \equiv \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi} = \frac{1}{\tau_y} - \frac{\frac{M_y-M_d}{1-M_y} + \frac{1}{\tau_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi}. \qquad \text{(D.24)}$$

From (D.21), we know that $\chi_1 = g\left(\chi_0^{'}\right)$ with

$$\chi_0^{'} = \frac{M_y}{1-\left(1-\tau_y\right)(1-\delta)M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right). \qquad \text{(D.25)}$$

From (D.22) and (D.23) we have $\chi_k = g\left(\chi_{k-1}\right)$ for all $k \in \{2, \cdots, H\}$. We first find the fixed point of $f\left(\chi\right)$:

$$\chi_{MSV} = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}}, \qquad \text{(D.26)}$$

which is equivalent to

$$\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}^2 + \chi_{MSV}\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right) - \frac{M_d(1-\omega)}{1-M_y} = 0. \qquad \text{(D.27)}$$

Let $\chi_{MSV,1}$ denote the smaller root and $\chi_{MSV,2}$ denote the larger root:

$$\chi_{MSV,1} = \frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)-\sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2+4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}$$

$$\chi_{MSV,2} = \frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)+\sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2+4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}. \tag{D.28}$$

If Assumption 1 holds ($\omega < 1$), we know that $\chi_{MSV,1}\chi_{MSV,2} < 0$ so $\chi_{MSV,1} < 0$ and $\chi_{MSV,2} > 0$. Note that $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1} > 0$,[42] we have $g(\chi) > \chi$ if $\chi \in (\chi_{MSV,1}, \chi_{MSV,2})$ and $g(\chi) < \chi$ if $\chi \in (\chi_{MSV,2}, +\infty)$. From (D.24), we also know that $g(\chi)$ increases if $\chi \in [\chi_{MSV,1}, +\infty]$.

Moreover, from (D.25), we know that $\chi_0' \geq \chi_{MSV,1}$. To see this, define the left-hand side of (D.27) as

$$h(\chi) = \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi^2 + \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)\chi - \frac{M_d(1-\omega)}{1-M_y}.$$

We have

$$\begin{aligned}
h\left(\chi_0'\right) &= \left(\omega\frac{\tau_y M_y}{1-M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right)+1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)\chi_0' - \frac{M_d(1-\omega)}{1-M_y}\\
&= \left(1+\frac{\tau_y M_y}{1-M_y}\left(1-\omega\delta\frac{1-M_d}{1-M_y}\right)\right)\chi_0' - \omega\frac{M_y}{1-M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right) - \frac{M_d(1-\omega)}{1-M_y}\\
&= \frac{M_y\left(1+\frac{\tau_y M_y}{1-M_y}\left(1-\omega\delta\frac{1-M_d}{1-M_y}\right)\right)}{1-(1-\tau_y)(1-\delta)M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right) - \frac{M_d}{1-M_y}\left[1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right]\\
&= \frac{(1-M_y)^2+\tau_y M_y(1-M_y-\omega\delta(1-M_d))}{1-(1-\tau_y)(1-\delta)M_y}\frac{M_d}{1-M_y}\left(1-\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right) - \frac{M_d}{1-M_y}\left[1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right]
\end{aligned}$$

Since $\frac{(1-M_y)^2+\tau_y M_y-\tau_y M_y(M_y+\omega\delta(1-M_d))}{1-(1-\tau_y)(1-\delta)M_y} < 1$ and $1-\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y} < 1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}$, we know that $h\left(\chi_0'\right) < 0$ so $\chi_0' \geq \chi_{MSV,1}$. The fact that $\chi_0' \geq \chi_{MSV,1}$ together with the aforementioned property of $g(\chi)$ means that $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, monotonic sequence converging to $\lim_{k\to+\infty}\chi_k = \chi_{MSV,2} > 0$.

[42] If $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1} < 0$, $\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV,1} < 0$, and $\chi_{MSV,1} > 0$ from (D.26), a contradiction.

If Assumption 2 holds ($\omega < 1$), $\chi_{MSV,2} > \frac{1-\beta}{\tau_y}$. To see this, we have

$$\begin{aligned}
h\left(\frac{1-\beta}{\tau_y}\right) &= \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)^2}{\tau_y} + \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)\frac{1-\beta}{\tau_y} - \frac{M_d(1-\omega)}{1-M_y} \\
&= -\beta\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y} + \left(1+\frac{M_y}{1-M_y}\tau_y\right)\frac{1-\beta}{\tau_y} - \frac{(1-\beta\omega)M_d}{1-M_y} \\
&< -\beta\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y} + \left(1+\frac{M_y}{1-M_y}\tau_y - \frac{(1-\beta\omega)\left[1-(1-\tau_y)M_y\left(1+\delta\frac{\beta\omega}{1-\beta\omega}\right)\right]}{1-M_y}\right)\frac{1-\beta}{\tau_y} \\
&= -\beta\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y} + \beta\omega\left(\frac{1-(1-\tau_y)M_y(1-\delta)}{1-M_y}\right)\frac{1-\beta}{\tau_y} = 0.
\end{aligned}$$

Similar to (A.10),

$$\mathbb{E}_0[d_t] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)(d_0+\varepsilon_0).$$

Since $\lim_{k\to+\infty}\chi_k = \chi_{MSV,2} > \frac{1-\beta}{\tau_y}$, we know that $\lim_{H\to\infty}\mathbb{E}_0[d_H] \to 0$. This finishes the proof with the alternative fiscal policy (7).

D.9 Proof of Theorem 3

As mentioned in the main text, we restrict $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}})$. Aggregating individual demand relation (10), together with monetary policy (30), goods and asset market clearing, and the government budget (4) lead to the following aggregate demand relation:

$$\begin{aligned}
y_t &= (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right) - \left(\sigma\omega - (1-\beta\omega)\frac{D^{ss}}{Y^{ss}}\right)\beta\phi\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k y_{t+k}\right] \\
&= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}(d_t-t_t) + \frac{1}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}\mathbb{E}_t\left[y_{t+1}\right]
\end{aligned}$$

Together with the baseline fiscal policy (6) we arrive at the following equation:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t+\varepsilon_t) + \frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying the period-t expectation operator $\mathbb{E}_t[\cdot]$ to (4), we have

$$\begin{pmatrix}\mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t\left[y_{t+1}\right]\end{pmatrix} = \begin{pmatrix}\frac{1-\tau_d}{\beta} & -\frac{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega}\end{pmatrix}\begin{pmatrix}d_t+\varepsilon_t \\ y_t\end{pmatrix} \qquad \text{(D.29)}$$

The two eigenvalues are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + (1+\sigma\phi)\frac{1-\tau_d}{\beta} = 0. \tag{D.30}$$

From $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}]$ and $\tau_d \in [0,1]$, we know that $\lambda_1 + \lambda_2 \geq 0$ and $\lambda_1\lambda_2 \geq 0$, so $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

We first prove Part 1 of Theorem 3. That is, if

$$\phi < \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma(1-\beta) + \frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}} < \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}, \tag{D.31}$$

complete self-financing is achieved as the fiscal adjustment is infinitely delayed. That is, there exists a bounded equilibrium of the form (17) with $\lim_{\tau_d \to 0^+} \rho_d \in (0,1)$ and $\lim_{\tau_d \to 0^+} \nu = 1$.

Since the eigenvalue of (D.30) is continuous in τ_d at 0, we have

$$\begin{aligned}\lim_{\tau_d \to 0^+} \lambda_1 &= \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \sqrt{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 - 4\frac{1+\sigma\phi}{\beta}}}{2} \\ &= \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \sqrt{\left(1 + \sigma\phi - \frac{1}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 + 4(1+\sigma\phi)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}\end{aligned}$$

and

$$\begin{aligned}\lim_{\tau_d \to 0^+} \lambda_2 &= \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) - \sqrt{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 - 4\frac{1+\sigma\phi}{\beta}}}{2} \\ &= \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) - \sqrt{\left(1 + \sigma\phi - \frac{1}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 + 4(1+\sigma\phi)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}.\end{aligned} \tag{D.32}$$

When $\phi \in [-1/\sigma, 0)$, from (D.32),

$$\lim_{\tau_d \to 0^+} \lambda_2 \leq \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) - \left|1 + \sigma\phi - \frac{1}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \leq 1 + \sigma\phi < 1$$

When $\phi \in [0, \bar{\phi})$, from (D.31), we have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1}{\beta} - 1\right).$$

Hence

$$\lim_{\tau_d \to 0^+} \lambda_2 = \frac{2\frac{1+\sigma\phi}{\beta}}{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}} \tag{D.33}$$

$$< \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = \frac{2\frac{1+\sigma\phi}{\beta}}{1+\frac{1+\sigma\phi}{\beta}+\left|\frac{1+\sigma\phi}{\beta}-1\right|} = 1.$$

Thus, as long as (D.31) holds, a bounded equilibrium in the form of (17) with $\rho_d = \lambda_2$ and $\chi = \frac{1-\tau_d-\beta\rho_d}{\tau_y} > 0$ will be a solution of (D.29), with $\lim_{\tau_d \to 0^+} \rho_d < 1$. Given (17), one can then back out $\{\pi_t, d_t - \mathbb{E}_{t-1}[d_t]\}$ from (14) – (16). The fact that $\lim_{\tau_d \to 0^+} \nu \to 1$ follows directly from the definition of ν in (B.18) and the fact that $\lim_{\tau_d \to 0^+}\left(\tau_d \frac{\varepsilon_0+\sum_{k=0}^{+\infty}\beta^k E_0[d_k]}{\varepsilon_0}\right) = 0$ given $\lim_{\tau_d \to 0^+} \rho_d \in (0,1)$.

For Part 2 of Theorem 3, note that, when $\phi > \bar{\phi}$, we from (D.31) have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) < \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence, from (D.33),

$$\lim_{\tau_d \to 0^+} \lambda_2 > \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = 1.$$

As a result, there exists no bounded equilibrium if the fiscal adjustment is infinitely delayed (i.e., if $\tau_d \to 0$ from above). For $\tau_d > 0$, we have[43]

$$\lambda_2 = f(a,b) \equiv \frac{a+b+1+\sigma\phi-\sqrt{(a+b-(1+\sigma\phi))^2+4(1+\sigma\phi)b}}{2}$$

where $f(a,b)$, $a = \frac{1}{\beta}(1-\tau_d) > 0$, and $b = \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{a+b-(1+\sigma\phi)}{2\sqrt{(a+b-(1+\sigma\phi))^2+4(1+\sigma\phi)b}} > 0$, we know that ρ_d decreases with τ_d and $\lim_{\tau_d \to 1^-} \lambda_2 = 0$. As a result, for $\phi > \bar{\phi}$, there exists an $\underline{\tau}_d(\phi) \in (0,1)$ such that $\lambda_2 < 1$ if and only if $\tau_d > \underline{\tau}_d(\phi)$. As a result, for $\phi > \bar{\phi}$, any bounded equilibrium exists if and only if $\tau_d > \underline{\tau}_d(\phi)$.

From (14) and (16),

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0[y_k] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0.$$

[43] The formula for the function f is slightly adjusted compared to (D.11) in the baseline analysis, to accommodate $\phi \neq 0$.

From the definition (B.18), we know

$$\nu = \frac{\frac{\left(\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}\right)\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}}{1+\frac{\beta\frac{D^{ss}}{Y^{ss}}\phi\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}} = \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{\chi}{1-\beta\rho_d}}{1+\left(\kappa+\beta\phi\right)\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}} = \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\chi}{\tau_d+\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\chi},$$

where I use $\rho_d = \frac{1-\tau_d}{\beta} - \frac{\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta}\chi$ from (D.29). For any $\phi \in \left(\bar{\phi}, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}\right)$, $\chi = \frac{1-\tau_d-\beta\rho_d}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}} \leq \frac{1-\bar{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}$ and $\nu \leq \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{1-\bar{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}}{\bar{\tau}_d(\phi)+\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{1-\bar{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}} \equiv \bar{\nu}(\phi) < 1$. This proves Part 2 of Theorem 3.

D.10 Proof of Proposition C.1

We note that the proof heavily leverages results from Wolf (2021*a*). Following that paper, all arguments are established using sequence-space notation, with boldface denoting time paths.

The sequence of wealth holdings associated with an interest rate sequence $\boldsymbol{r}$ (both in deviation from steady state) is given as

$$\boldsymbol{d}(\boldsymbol{r}) = \mathscr{D}_r \times \boldsymbol{r}$$

where $\mathscr{D}_r$ is the sequence-space Jacobian of wealth holdings with respect to interest rates. The desired long-run elasticity η is the long-run response of asset holdings to a permanent change in interest rates; that is, it is given as the limit (if it exists) of the sequence $\boldsymbol{d}(\mathbf{1})$.

It follows from the aggregate household budget constraint that the savings matrix $\mathscr{D}_r$ and the analogous consumption matrix $\mathscr{M}_r$ are related as[44]

$$\mathscr{M}_r + \frac{1}{1+\bar{r}}\mathscr{D}_r = \begin{pmatrix} \mathbf{0}' \\ \mathscr{D}_r \end{pmatrix} \tag{D.34}$$

where $1+\bar{r} = \beta^{-1}$. Since by definition

$$\eta = \lim_{H\to\infty} \mathscr{D}_r(H, \bullet) \times \mathbf{1}$$

it follows from (D.34) that we have

$$\eta = \frac{1+\bar{r}}{\bar{r}} \lim_{H\to\infty} \mathscr{M}_r(H, \bullet) \times \mathbf{1} \tag{D.35}$$

It thus remains to characterize $\mathscr{M}_r$. For this we momentarily assume that there are no spenders ($\mu =$

[44] Note that this construction removes income effects related to steady-state wealth holdings.

0); the extension to the full spender-OLG model is straightforward and will come at the end. It follows from the results in Wolf (2021*a*) that $\mathcal{M}_r$ has the following limiting properties:

$$\begin{aligned}\lim_{H\to\infty} \mathcal{M}_r(H,H) &= -\sigma\beta\omega\frac{1-\omega}{1-\beta\omega^2} \\ \lim_{H\to\infty} \mathcal{M}_r(H,H-1) &= \sigma\frac{\omega(1-\beta\omega)}{1-\beta\omega^2}\end{aligned}$$

as well as

$$\begin{aligned}\lim_{H\to\infty} \frac{\mathcal{M}_r(H,H-s)}{\mathcal{M}_r(H,H-s+1)} &= \omega\beta, \quad s\geq 2 \\ \lim_{H\to\infty} \frac{\mathcal{M}_r(H,H+s)}{\mathcal{M}_r(H,H+s-1)} &= \omega, \quad s\geq 1\end{aligned}$$

Plugging those relations into (D.35) and simplifying, we find

$$\eta = \frac{1}{1-\beta}\sigma\left[\frac{1}{1-\theta} - \frac{1}{1-\beta\theta}\right] \tag{D.36}$$

Finally, if there is a margin of spenders, then the elasticity is simply scaled down to correspond to the margin of OLG households $(1-\mu)$, thus giving (C.2).