A Smart Contract to Increase Intermediation Capacity in the Repo Market

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April 24, 2023

Abstract

We provide a mechanism that removes the current limitation on the attainable volume of transactions in the U.S. Treasury repo market when money and treasuries are appended to programmable electronic ledgers. We achieve this with a smart contract that restructures decentralized repo contracts into chains where bank intermediaries become guarantors of contracts between ultimate repo borrowers and lenders without materially altering the profile of risk or payoffs to any agent. Our smart contract increases the capacity of banks to intermediate repo transactions without requiring either a loosening of financial regulation or a fundamental restructuring of the repo market. We incorporate cryptography and hardware that preserve transaction privacy while providing regulators real-time access to relevant information. Our solution demonstrates that when financial objects are appended to programmable electronic ledgers it becomes possible to combine decentralized markets and privacy with informed regulatory oversight.

JEL Classifications: D24, D47, D82, D86, G12, G21, G23, G24, G28

Keywords: Microeconomic Engineering, Bilateral Contracting, Multilateral Contracting, Optimal Contracts, Asymmetric Information, Incentive Compatible Mechanisms, Fixed Income Se-

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1We wish to acknowledge several people who generously shared their knowledge of the repo market with us: Adam Copeland, Kevin Gaffney, Josh Galper, Bengt Holmström, Sebastain Infante, Jay Kahn, Travis Keltner, Antione Martin and Scott Skyrm. We are grateful to Nicolas Zhang for discussions pertaining to smart contracts and to Mark Serrahn for preparing the exhibits. We owe a special debt of gratitude to Madars Virza for his advice and guidance on our application of graph flow algorithms and on the architecture we propose for ensuring transaction privacy and regulator auditing. This research was supported by the George and Obie Schultz Fund. All errors are our own.
securities, Market Microstructure, Financial Intermediaries, Shadow Banking, Broker-Dealer, Bank Regulation, Capital Requirements
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1 Introduction

This paper does three things. First, we explain how the interaction of accounting rules and bank capital regulations bound volume in the repo market by limiting the volume of transactions that banks who intermediate the repo market through broker-dealer affiliates, can undertake. Second, we propose a smart contract, which can be implemented when money and collateral are appended to programmable electronic ledgers, that removes the transaction bound on banks without either requiring changes in accounting rules or capital regulations, or forcing a restructuring of the repo market. Our smart contract deploys a sophisticated algorithm to restructure repo contracts so as to remove the limitation on transactions without changing any payoff relevant variable for the contracting parties. Third, we incorporate cryptography and hardware to ensure the privacy of transactions while providing regulators with real-time access to relevant information in the decentralized market setting.

The repo market is the largest volume market for trading US Treasuries. It is an important venue for money market activity, for the pricing of US government debt and the transmission of monetary policy into the economy. A repo transaction involves two contracts entered into at the same time. There is a first-leg contract for party $i$ (the repo borrower) to immediately sell collateral to party $j$ (the repo lender), and a second-leg contract for party $i$ to (re)purchase the collateral from party $j$ at a later date. Between the first and second-leg, accounting rules require that recorded assets are increased by approximately the value of money or treasuries scheduled to be received in the second-leg transaction. Broker-dealers facilitate the movement of money and collateral between ultimate lenders and borrowers by acting as the repo lender to a hedge-fund or other entity that desires to borrow money in the repo market and as the repo borrower from a money-market fund or other entity that desires to lend money in the repo market and by transacting repo with each other in the inter-dealer market. Prompted by bottlenecks (i.e. inability to timely process all desired transactions) in the US Treasury cash and repo markets in recent years, there is concern that regulatory limits on bank leverage, defined approximately as the ratio of equity to assets, will cause more disruptions in the future, by limiting the volume of repo transactions a broker-dealer is allowed to enter into. An increase in the volume of desired transactions by ultimate repo borrowers and lenders will, by increasing the assets in the denominator, push down the leverage ratio of the bank intermediaries. When banks reach the regulatory lower bound, they cannot intermediate any additional repo transactions, which is a cause of market disruption.

To address the issue, two different approaches to regulatory reform have gained traction among academics and policymakers. One suggestion is to remove Treasuries from the assets in the calculation of the leverage ratio, which would increase the ratio at any volume of repo transaction.\footnote{See the Group of Thirty (2020) (19) position paper. The Group of Thirty’s argument for removing Treasuries from the SLR was premised on the assumption that Treasuries are a riskless asset with a negligible chance of declining in value. This argument is challenged by the failure of} Another suggestion is to mandate that all repo
transactions be centrally cleared, which would facilitate multi-lateral netting of repo transactions, thereby reducing the increase in recorded assets at any volume of repo.\textsuperscript{3} We propose a third approach to increase repo intermediation capacity which does not require any change to bank regulations. Our approach can be implemented when money and Treasuries are appended to electronic ledgers.

1.1 The Research Question and Approach

The question we address is whether it is possible to design a mechanism that fulfils the following requirements; it must increase the capacity of broker-dealers to intermediate the repo market without requiring either a loosening of financial regulation or a fundamental restructuring of the market, while preserving privacy and enabling the regulator to timely audit relevant information. Our solution involves a smart contract that re-organizes transactions after they have initially been entered into. It does so by identifying chains of repo transactions where a Treasury (or other collateral) moves from one agent to another. On one end of the chain is a repo lender; on the other end of the chain is a repo borrower. In between are broker-dealer intermediaries. The smart contract novates the initial repo contracts along the chain and replaces them with contracts between the repo lender and borrower where broker-dealer intermediaries are transformed into guarantors. We call this a replacement repo contract (“RRC”). The structure of the guarantees preserves all payoff relevant features of the initial repo contracts, other than the timing of payments. The amount of collateral and money each agent is required to send or receive is unchanged from the initial contracts. The smart contract moves the maximum feasible amount of repo volume into RRC’s. The key insight is that an RRC reduces the balance-sheet impact of the repo volume incurred by broker-dealer intermediaries on the repo chains. It does so because the increase in recorded assets for a given repo volume is lower when the agent is a guarantor of a repo transaction compared to when it is a principal.

Our solution demonstrates that when financial objects are appended to distributed ledgers it becomes possible to combine decentralized markets with informed regulatory oversight. The technology of smart contracts renders obsolete the traditional tradeoff between the efficiency of a decentralized market versus the necessity of a centralized market to generate the timely information required for competent regulation. With our smart contract, the market remains decentralized and the regulator can receive timely reports on trading activity and agent balances.\textsuperscript{4}

SVB Bank in April 2023, which was preceded by a decline in the value of its Treasury holdings.

\textsuperscript{3}See the Securities and Exchange Commission proposal for mandatory central clearing, SEC(2022) (33).

\textsuperscript{4}A key premise of the Securities and Exchange Commission proposal is that the concentration of transactions with a CCP is required to provide the regulator with timely information on trading positions of regulated entities [INSERT QUOTE].
1.2 Related Literature

This paper draws from and contributes to three areas of prior and ongoing research. One area is the literature that explores potential uses of electronic ledger technology to improve economic outcomes. Townsend (2020) (34) discusses myriad potential use-cases where outcomes can be improved when financial objects are appended to programmable participant accessible ledgers. This paper adds the example of the repo market to list of applications. Another area is the literature on the shadow banking sector where, in the aftermath of the financial crisis there has been an effort to map the system, of which the repo market is an important part. Economists at the Federal Reserve and the Department of Treasury Office of Financial Research have published a large number of studies that draw on new administrative data collected from financial firms as a result of the Dodd-Frank legislation. Finally, this paper contributes to the ongoing discussion of reforms to increase the capacity of banks to intermediate the repo market. In response to the disruptions that occured in the repo market in September 2019 and in the cash treasuries market in October 2014 and March 2020, there have been proposed regulatory reforms to reduce the likelihood and severity of future disruptions. Duffie (2018) (13) and (2020) (14) and the Group of Thirty (2020) (14) have notably contributed to this reassessment exercise. We contribute to this literature by proposing a very different approach to improving repo market performance and resiliency.

1.3 Roadmap

The rest of the paper is organized as follows. Section 2 provides a brief overview of the structure, timing and terminology of a repo market. Section 3 reviews the accounting rules for recording repo transactions on the balance-sheet and the regulation of bank leverage. Section 4 formalizes the effect of the interaction of repo accounting rules and bank leverage regulations on the volume of repo transactions a broker-dealer can undertake. Section 5 provides an introduction and preview of the smart contract. Section 6 contains the smart contract protocol for decentralized initial repo contracts implemented in a market where money and collateral are appended to programmable electronic ledgers. Section 7 contains the algorithm used to reorganize initial repo contracts into chains that connect a repo borrower to a repo lender through a sequence of matched-book repo intermediaries. Section 8 describes the repo contracts that are implemented on chains, which replace the initial repo contracts that are novated. Section 10 discusses cryptographic methods to selectively protect the privacy of transaction data from competitors while providing audit-able information on transactions and balances to regulators. Section 9 states the key proposition that the replacement repo smart contract maximizes intermediation capacity. Finally, we conclude in Section 11.

5A comprehensive set of publicly available administrative data is maintained by the Office of Financial Research (29). The studies are too numerous to list here, however we draw significant insight and data from Pozsar (2014) (30), Kahn and Olsen (2021) (26), Aguiar et.al. (2016) (2) and Infante et.al. (2018) (24).
2 The Repo Market

The repo market is characterized by an aggregation of movements of financial objects occurring on two separate dates. At the first-leg, a repo lender purchases collateral from a repo borrower, which induces a movement of money (which we label $M$) from the repo lender to the repo borrower. Collateral (which we label $T$) moves in the opposite direction, from repo borrower to repo lender. At the second-leg, the object flows reverse direction. The repo borrower purchases $T$ from the repo lender; $M$ moves from repo borrower to repo lender and $T$ moves from repo lender to repo borrower. From the repo borrower’s point of view, it has entered into a repo transaction. From the repo lender’s point of view, it has entered into a reverse-repo transaction.

Repo transactions typically form chains where, in the first-leg $T$ is moved from repo borrower repo lender in a succession of transactions and $M$ is moved in the opposite direction. In the second-leg, the direction of movement of financial objects are reversed. Transactions between agents in the interior of the chain - those who are neither ultimate lender or borrower - are denoted “inter-dealer” repo transactions. The movement of $M$ and $T$ between ultimate lender and borrower is intermediated by broker-dealers operating in the inter-dealer market. Below we provide a glossary of repo terms we use in this paper.

Repo A repo transaction between two parties is comprised of two purchase transactions that close at successive dates, with a forward purchase entered at the time of the execution of the first purchase. The first-leg of repo is a transaction in which party $j$ purchases some quantity of collateral $T$, from party $i$ for unit price $p_{j \rightarrow i}^1$ with money $M$. The second-leg of repo is the forward transaction which occurs at a later date where party $i$ purchases $T$ from party $j$ for unit price $p_{i \rightarrow j}^2$ with $M$. The repo rate is $r_{ij} = (p_{i \rightarrow j}^2 - p_{j \rightarrow i}^1)/p_{j \rightarrow i}^1$. This is the rate of return earned by the repo lender $j$ and paid by the repo borrower $i$.

Market Structure Repo transactions can be partitioned into three market segments. One is a client market where an ultimate repo borrower transacts with a broker-dealer, whom we label $BD$. We label the ultimate borrower a risk-manager (“$RM$”). Typically an $RM$ invests the $M$ it obtains at the first-leg in an investment, a margin call or a trading strategy. We label the ultimate repo lender a money manager (“$MM$”). Typically, a $MM$ trades its money for short-term liquid securities. An $RM$ or $MM$ can represent the net amount of borrowing or lending of an agent, with the remaining balanced volume in-and-out placed in the $BD$ partition.\(^6\)

Transactions We focus on “general collateral” repo, where the collateral $T$ is a member of a set of interchangeable objects, meaning that the repo borrower is required to provide a $T$ from the designated set in the second-leg transaction. For example, $T$ may be a CUSIP, such an issue of on-the-run Treasuries that mature on a particular date.

\(^6\)In Appendix B we describe the business of the ultimate lender and borrower in more detail and we derive their repo demand and supply functions.
**Rehypothecation** The T collateral is rehypothecatable, which means that a repo lender can sell the T it receives from a counterparty in the first-leg.

**Haircuts** The repo haircut is a percentage discount from the cash market price of T in first leg of repo transactions. The haircut mitigates two types of risk. One is a change in the value of the T collateral between the first and second leg. This is the risk of an increase in the interest rate. The other is a failure of the counterparty to return T on a timely basis.

Figures 1 and 2 display the movement of objects M and T along a chain between clients RM and MM, intermediated by BD_i and BD_j at the first and second legs of a repo transaction. Note that the movement of T across the BD’s in the first-leg involves rehypothecation.

\[
\begin{align*}
RM & \xrightarrow{T} BD_i & \xrightarrow{T} BD_j & \xrightarrow{T} MM \\
RM & \xleftarrow{M} & \xleftarrow{M} & \xleftarrow{M} \\
RM \text{ sale of } T \text{ to } BD_i & & \text{inter-dealer sale of } T & \text{BD}_j \text{ sale of } T \text{ to } MM
\end{align*}
\]

**Figure 1: First Leg of Repo Contracts and Object Flows**

\[
\begin{align*}
RM & \xleftarrow{T} BD_i & \xleftarrow{T} BD_j & \xleftarrow{T} MM \\
BD_i & \xrightarrow{M} & BD_j & \xrightarrow{M} MM \\
BD_i \text{ sale of } T \text{ to RM} & & \text{inter-dealer sale of } T & \text{MM sale of } T \text{ to } BD_j
\end{align*}
\]

**Figure 2: Second Leg of Repo Contracts and Object Flows**

3 **Repo Accounting Rules and Leverage Regulations**

In this section we briefly review repo accounting standards and bank leverage regulations. We argue that two reforms which were introduced after the 2008 financial crisis have, in combination, had the unintended consequence of limiting the transaction capacity of the repo market. Repo accounting rules were reformed to close loopholes that some regulated broker-dealers had used to conceal assets. This reform requires approximately all second leg inflows and outflows of M and T to be recorded on the BD’s balance sheet when the repo contract is executed at the first leg.\(^7\) Bank leverage regulations were reformed to increase the minimum capital and liquid assets held by banks. These regulations were intended to ensure banks are able meet depositor withdrawals and debt payments and remain solvent in a state where asset prices drop significantly and the ability to liquidate securities without incurring large

\(^7\)Salerno et.al. (2016) (32) explains the effect of the changes to repo accounting rules.
losses is limited. One regulation, called the supplementary leverage ratio (“SLR”),
sets a lower bound on the ratio of unweighted on and off balance-sheet bank assets to
equity (BIS (2014) (6)).\textsuperscript{8} Repo accounting reforms and the SLR address independent
concerns related to the resilience of banks to withstand a future financial crisis. We
shall argue that in combination they limit the capacity of BD’s to intermediate repo
transactions.

3.1 Repo Accounting Rules

Financial accounting standards in the US are set by the Financial Accounting Stan-
dards Board (“FASB”).\textsuperscript{9} Currently, repo transactions are accounted for as secured
loans, with the first-leg purchaser treated as the lender and the first-leg seller treated
as the borrower. The balance sheet impact after the first-leg is roughly the same for
both counterparties. Assets and liabilities are increased by approximately the amount
the second-leg purchase price of the collateral.\textsuperscript{10} We will refer to the approximate
recording obligation. For example, suppose the second-leg of a repo transaction be-
tween BD\textsubscript{i} and BD\textsubscript{j} requires BD\textsubscript{i} to repurchase T from BD\textsubscript{j} for a price of
M = \(p_{i\rightarrow j}^2 T\) and that the transaction is treated as a secured loan under the accounting rules. At
the time of the first-leg transaction the counterparties are required to record approx-
imately the value of M or T scheduled to be received in the second-leg transaction.
BD\textsubscript{i} would record an asset of \(p_{i\rightarrow j}^2 T\) and a liability of \(p_T T\) on its balance sheet relat-
ing to the second-leg, where \(p_T\) is the market value of collateral. BD\textsubscript{j} would record
those same amounts, with the asset and liability designations reversed. By contrast,
if the first-leg of the repo transaction was treated as a final sale, nothing relating to
the second-leg would be recorded on the balance-sheet. Prior to the recent change
in FASB repo accounting rules, it was possible to structure the first-leg of a repo
transaction as a final sale (though it was not commonly done). In that case, the
second-leg obligation is not recorded on the balance sheet. Two notable instances of
perceived abuse led to changes in FASB accounting rules to prevent this practice.

In one instance of abuse, the investment bank Lehman Brothers devised a transaction
structure, called repo 105, which it employed around financial disclosure dates for
several years prior to its 2008 bankruptcy. The manoeuvre enabled Lehman to conceal
billions of dollars of subprime mortgage exposure prior to the 2008 financial crisis.
In another instance of abuse, MF Global took advantage of a rule called ’repo-to-
maturity’ which allowed the first leg repo transaction to be treated as a final sale
when the second-leg was scheduled near the maturity date of the collateral security.
MF Global exploited the loophole to conceal billions of dollars of exposure to low
rated sovereign debt in the aftermath of the 2008 financial crisis. In Appendix A we

\textsuperscript{8}Walter (2019) (37) reviews, inter-alia, the changes to bank capital regulations since the 2008
financial crisis.

\textsuperscript{9}Changes to repo accounting standards can be accessed at Financial Accounting Standard Board
(2022) (15).

\textsuperscript{10}The actual balance sheet impacts are affected by the haircut and the book versus fair market
value of the collateral. We ignore these features because they typically involve minor adjustments. See Salerno et.al. (2016) (32).
discuss the strategies employed by Lehman and MF Global.

3.1.1 FASB Reforms to Repo Accounting Rules

The rules that allowed repo 105 and RTM accounting were closed by subsequent FASB rule changes. In 2011, FASB issued ASU 2011-03, which treats repo 105 type transactions as secured loans. In 2014, FASB issued Accounting Standards Update (ASU) 2014-11, which treats repo-to-maturity as a secured loan\textsuperscript{11}. At the present time, banks and BD’s are required to record the approximate second leg obligation on their balance sheet.

3.2 Bank Leverage Regulations

The steep decline in asset prices and the impairment of inter-bank lending that marked the 2008 financial crisis made almost all large US banks insolvent and unable to meet the short term borrowing needs of their customers. Shortly thereafter, the US Congress passed a set of financial market reform measures known as Dodd-Frank and the Bank for International Settlements finalized the Basel III version of its bank regulatory guidelines\textsuperscript{12}. A key aim of the reforms was to reduce the risk of bank insolvency (i.e. liabilities exceeding assets) by limiting the amount of leverage a bank is allowed assume.

There are several regulatory rules that place limitations on bank leverage. The SLR represents a departure from previous leverage limits in that it does not weight assets in terms of risk. For example, a US Treasury bond and a Greek sovereign bond with the same coupon, payoff amount and maturity date each increase the denominator of the SLR leverage ratio by the same amount. The SLR places a lower bound on the ratio of bank capital to unweighted balance sheet assets and certain off balance-sheet exposures (BIS (2014 (1) (6))). The SLR lower bound is 3% with an additional 2% for large globally systemically important banks ("GSIB”’s). We denote the SLR lower bound $L$ as follows.\textsuperscript{13}

\[
\text{SLR: equity/assets} \geq 3\% + 2\% \text{ for GSIB’s} = L
\]

An opinion expressed by many financial industry participants, academics and regulators is that the SLR is limiting the intermediation capacity of BD’s in the cash and treasury repo markets. This is reflected in the approximate 1.5% increase in the capital to asset ratio for US banks from the period that preceded the 2008 financial crisis to 2022\textsuperscript{14}. The Board of Governors of the Federal Reserve System made the

\textsuperscript{11}For a brief history of changes to repo accounting rules over time see \cite{Hartwell2016}. For an example of the difference between a repo transaction accounted for as a final sale versus a secured financing, see \cite{SalernoRuddyRajan2016}.

\textsuperscript{12}See Walter (2019) (37).

\textsuperscript{13}This is a simplification. The SLR denominator is actually “exposures”, which include balance sheet assets plus other off-balance sheet items that represent contingent obligations of the bank. The SLR numerator actually includes convertible debt.

\textsuperscript{14}World Bank, Bank Capital to Total Assets for United States [DDS103USA156NWDB], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/
following statement in the Congressional Record in 2020.

Large holding companies have cited balance sheet constraints for their broker-dealer subsidiaries as an obstacle to supporting the Treasury market. Specifically, the supplementary leverage ratio can limit holding companies’ ability to own Treasuries outright...

The channel through the SLR affects a BD’s capacity to intermediate in the repo market is the requirement to record second-leg obligations on the balance sheet. The second leg obligations involve an asset for both parties to a repo transaction. The lender records the $M$ it has contracted to receive and the borrower records the $T$ it has contracted to receive.

3.3 The Interaction of the FASB Accounting Rule and the SLR on Intermediation Capacity

The FASB reforms to repo accounting were designed to close loopholes that can enable a bank or broker-dealer to conceal the true state of its balance sheet to regulators and investors. The SLR was designed to provide a hard backstop to bank insolvency risk. The different aims of the two sets of regulations address distinct risks associated with banking. However, in combination, they have the unintended effect of reducing the capacity of BD’s to intermediate repo transactions. The FASB rules that require the second leg obligations to be recorded on the balance sheet increase assets, which pushes down the leverage ratio. The SLR pushes up the regulatory lower bound to a leverage ratio that counts a Treasury the same as any other security. This results, inter alia, in a reduction in the maximum volume of repo transactions a BD is allowed to undertake.

4 A Model of Broker-Dealer Leverage Constraints

In this section we model the effects of repo accounting regulations and bank leverage rules on broker-dealer repo transaction capacity.

4.1 Balance Sheet and Leverage Impact of a Repo Transaction

Intermediation requires that a BD carries out two repo transactions to move objects from one counterparty to another along a repo chain. At the first-leg, the assets and liabilities of a BD increases by approximately the volume of the two repo transactions it enters into. For example, suppose that $BD_j$ enters into a reverse-repo transaction to lend $M = p^1_{j \rightarrow i}$ to $BD_i$ and a repo transaction to borrow $M = p^2_{k \rightarrow j}$ from $BD_k$. The second-leg contract with $BD_i$ requires $BD_i$ to send $M = p^2_{i \rightarrow j} T$ to $BD_j$ in exchange for $T$. FASB requires $BD_j$ to record $p^T_{i \rightarrow j} T$ as an asset at the first-leg. Similarly, the second-leg contract with $BD_k$ requires $BD_k$ to convey $T$ to $BD_j$ in exchange for $T$. 

\footnote{Federal Reserve (2020) (16).}
\[ M = p_{j \rightarrow k}^2 T. \] FASB requires \( BD_j \) to record the value of \( T = p_T T \) as an asset at the first-leg. For the two transactions, \( BD_j \) will record an increase of \( p_{i \rightarrow j}^2 T + p_T T \) assets on its balance-sheet at the first-leg. This amount is added to the denominator of the SLR, which pushes the SLR closer to the regulatory lower bound.

4.2 Effect of SLR on Repo Volume when Capital is Fixed

Equation 1 shows that an increase in repo transactions pushes down the leverage ratio by increasing assets in the denominator; \( \Delta A = p_{i \rightarrow j}^2 T + p_T T \). The ratio cannot drop below the SLR bound \( L \).

\[
L \leq \frac{\text{equity}}{\text{assets} + \Delta A} \tag{1}
\]

Rearranging the terms in Equation 1 and denoting the \( \Delta \) repo volume \( \approx \frac{1}{2}(p_{i \rightarrow j}^2 T + p_T T) \) expresses the effect of the SLR on a \( BD \)'s repo intermediation capacity.

\[
\Delta \text{repo volume} \leq \frac{1}{2} \frac{\text{equity}}{L} - \text{assets} \tag{2}
\]

Equation 2 shows that a leverage lower bound \( L \) places a limit on repo volume and that attainable repo volume declines as the lower bound increases. When the SLR is binding, an increase in the leverage lower bound will reduce the volume of repo transactions a bank will transact, holding fixed the rest of its portfolio.

4.3 Effect of SLR on Repo Volume when Capital is Adjustable

A bank can set its capacity to intermediate repo by allocating more or less capital to its \( BD \). This can be accomplished by raising or repaying capital or by re-allocating internal capital to the \( BD \). We analyze the bank’s choice of capital during a period of sufficient length for the bank to re-set its entire capital allocation. Equation 2 implies that the incremental amount of capital required per unit of repo volume is \( \frac{\Delta \text{equity}}{\Delta \text{repo volume}} = 2L \). Let \( c \) denote the unit cost of capital. We evaluate the decision problem of a \( BD \) who enters into a repo transaction with \( MM \) while it is at the SLR lower bound by including a marginal cost of repo \( 2cL \) when repo volume exceeds the initial SLR threshold amount \( D = \frac{1}{2} \frac{\text{equity}}{L} - \text{assets} \). The \( BD \)'s problem is to set the repo rate \( r_{MM} \) it offers to \( MM \) to maximize its profit, given the \( MM \)'s demand function for repo volume in units of \( T \), \( D_{r_{MM}} \), and the inter-dealer repo rate \( r_{\text{inter-dealer}} \) (which we take to be market determined and exogenous) at which it is able to offset its transaction with \( MM \) by entering into a reverse-repo transactions with another \( BD \). Equation 3 describes the \( BD \)'s decision problem.
argmax \( r_{MM} \) \( \frac{1}{2} (r_{\text{inter-dealer}} - r_{MM}) D'_{r_{MM}} - 1 \{ D_{r_{MM}} > \overline{D} \} 2cL D_{r_{MM}} \) \( (3) \)

In Appendix B we derive an increasing concave \( MM \) repo demand function \( D_{r_{MM}} \). The derivative of Equation 3 when \( D_{r_{MM}} > \overline{D} \) is

\[
f = \frac{1}{2} \left[ (r_{\text{inter-dealer}} - r_{MM}) D'_{r_{MM}} - D_{r_{MM}} \right] - \frac{D'_{r_{MM}} 2cL}{2} \]

Concavity is proven as follows. Noting that \( D' > 0 \) and \( D'' < 0 \), the second derivative of the profit term is

\[
\frac{1}{2} \left[ r_{\text{inter-dealer}} D''_{r_{MM}} - 2D'_{r_{MM}} \right] < 0
\]

Since \( D''_{r_{MM}} < 0 \), it follows that the second derivative of the cost term is positive. Therefore, the second derivative of Equation 3 is negative, thus proving that it is concave. There is a unique repo rate \( r_{MM} \) that solves the \( BD \)'s problem. Finally, we can derive the effect of an increase in the SLR lower bound \( L \) on repo volume by applying the Implicit Function Theorem to Equation 4.\(^{16}\)

\[
\frac{dr_{MM}}{dL} = -\frac{df/dL}{df/dr_{MM}} < 0
\]

Since \( D_{r_{MM}} \) is an increasing function of \( r_{MM} \), it follows that an increase in the SLR lower bound causes a reduction in the volume of repo transacted between broker-dealers and ultimate repo lenders.

The problem faced by a \( BD \) transacting with an ultimate repo borrower, such as a hedge fund, whom we label \( RM \), is a reflection of the problem faced by the \( BD \) who transacts with an \( MM \). In this case, the \( BD \) sets the repo rate \( r_{RM} \) it offers to the \( RM \) to maximize its profit, given the \( RM \)'s supply function for repo volume in units of \( T \), \( S_{r_{RM}} \) and the inter-dealer repo rate \( r_{\text{inter-dealer}} \) at which it is able to offset its transaction with \( RM \). The \( BD \)'s problem is expressed by re-arranging the terms of Equation 3 to

\[
argmax \( r_{MM} \) \( \frac{1}{2} (r_{RM} - r_{\text{inter-dealer}}) S_{r_{RM}} - 1 \{ S_{r_{RM}} > \overline{S} \} 2cL S_{r_{RM}} \) \( (5) \)

\(^{16}\)The inequality follows from \( df/dL = -D'_{r_{MM}} 2c < 0 \) and \( df/dr_{MM} < 0 \) by the concavity of the \( BD \)'s problem. Therefore, the numerator of the expression is positive and the denominator is negative.
If the $RM$ repo supply function, $S_{rRM}$, is decreasing concave function of $r_{RM}$, the reader can work out that an increase in the SLR lower bound $L$ will induce the $BD$ to increase $r_{RM}$ thereby reducing the repo transaction volume of the ultimate repo borrowers. In Appendix B we derive a decreasing concave repo supply function $S_{rRM}$.

We have demonstrated that, subject to the concavity of the demand and supply functions of the ultimate repo lenders and borrowers, the maximum volume of repo transactions is a declining function of the SLR lower bound, whether or not broker-dealers are able to adjust their capital.

5 A Smart Contract to Reduce $BD$ Leverage

To motivate our smart contract (SC) we introduce the concept of a replacement repo contract (“RRC”) and demonstrate how it can reduce the impact of a repo transaction on a $BD$’s balance sheet. The RRC replaces the repo contracts that agents have entered into before $M$ or $T$ have changed hands. The parties to an RRC are agents along a chain. A chain connects an $RM_j$ to an $MM_i$ through one or more $BD$’s where each agent has an initial repo contract with its neighbors. Along the chain, each initial repo contract is scaled to a volume of first-leg $T$ that is less than the smallest volume in any initial repo contract between agents in the chain. The SC partitions the network formed by the initial repo contracts into chains and novates and replaces the initial repo contracts with RRC’s, one for each chain. The $MM_i$ and $RM_j$ located at the ends of the chain are the contract principals. The $BD$’s, which lie in the middle section of the chain, are contingent guarantors.

We proceed by explaining how the RRC reduces the impact of a repo transaction on the $BD$’s SLR by transforming the $BD$ from a contract principal into a guarantor. We then informally demonstrate that the novation of the initial repo contract and its replacement by a RRC does not alter any payoff relevant variable in any state. These facts imply that the resulting pattern of repo trades will replicate the pattern that would appear in the counterfactual world where the SLR lower bound was reduced and there was no replacement repo contract. Intuitively, we expect that the total volume of repo transactions would weakly increase in the counterfactual world.

5.1 The Impact of a Repo Guaranty on the SLR Leverage Ratio

In Section 3 we cited the FASB repo accounting rules which require the second-leg inflows and outflows to be recorded on the balance-sheet of the transacting parties at the first-leg. This rule does not apply to a contingent guarantor of a repo contract. The applicable rule for a guarantor is to add a measure of counterparty credit risk (CCR) to its liabilities,
which is a significantly smaller number than the obligation that is being guaranteed.\textsuperscript{17} The increase in liabilities automatically reduces equity (since the sum of liabilities and equity must equal assets), which pushes down the SLR. The formula for determining CCR is the greater of zero and the difference in fair market value (FMV) between the financial object the counterparty is required to deliver and the financial object the indemnified party is required to deliver. An analysis of the SLR rule by the law firm Davis Polk states,\textsuperscript{18}

The measure for counterparty credit risk includes where a banking organization acts as an agent for a repo-style transaction and indemnifies the customer with respect to the performance of the customer’s counterparty in an amount limited to the difference between the fair value of the security or cash its customer has lent and the fair value of the collateral the borrower has provided.

One reason the CCR is a fraction of the size of the insured obligation, additional to the low probability of its occurrence, arises from the similarity between repo and a secured loan. A second-leg default by one party to a repo contract only becomes eligible for legal redress when the counterparty has delivered its contracted financial object. In other words, if $BD_i$ fails to deliver its payment at the second-leg, $BD_j$ has no legal case against $BD_i$ unless it has presented the $T$ it is required to deliver. Consequently, the exposure taken on by the guarantor of performance of a party to a repo contract is always less than the value of the object the party is required to deliver. Drawing from the example above, suppose that $BD_j$ becomes the guarantor of a contract between $BD_i$ and $BD_k$ where the payment and delivery obligations of the contracting parties remain unchanged. $BD_j$ retains the margin of profit \[ \pi = (p^2_{i\rightarrow j} - p^1_{j\rightarrow i})T \] and guarantees second-leg delivery of the required financial objects to each contracting party. $BD_i$’s CCR is approximately, \[ |p^2_{i\rightarrow j} - p_T|T. \] This is substantially below the increase in assets of $(p^2_{i\rightarrow j} + p_T)T$ that $BD_j$ would have been required to record when it transacts directly with $BD_i$ and $BD_k$.\textsuperscript{19}

5.2 The Replacement Repo Contract

The smart contract simulates a reorganization of the initial repo contracts into chains. We explain the method by which this is accomplished in Section 7 below. The smart contract cannot add or subtract to the volume of $T$ contained in the initial repo contract between two agents. All it can do is to partition the contract. The purpose of doing this is to reorganize transactions into chains. Drawing from the example above, suppose that $BD_j$ was additionally a repo borrower from an agent $BD_l$ for a volume $T$. The smart contract might split up $BD_i$’s contracts such that $BD_i$ was a borrower of $T$ from $BD_l$ (in effect a $RM_i$) at the end of chain 1, and a borrower and lender of $T$ on chain 2 where its neighbors are $BD_i$ and $BD_k$. Having organized the transactions into chains, the replacement repo contracts are implemented. On chain 1 $BD_j$ is a contracting party (with the MM node at

\begin{itemize}
  \item \textsuperscript{17}The guarantor is also required to add to its assets any fee it is scheduled to receive at the second-leg. We are focused on the SLR, which is not impacted by assets.
  \item \textsuperscript{18}See pp.33-36 of Davis Polk (2014) (12)
  \item \textsuperscript{19}It should be noted that a given increase in liabilities will reduce the SLR by more than an increase in assets, so long as assets are greater than equity, since \( \frac{dSLR}{dL}/\frac{dSLR}{dA} = A/e > 1. \) Therefore, the effect described in the text holds because the absolute value of the decrease in assets exceeds the increase in liabilities - equivalently the decrease in equity - by an order of magnitude.
\end{itemize}
the other end of the chain). On chain 2, \( BD_j \) is a guarantor. We explain the details of the guarantee in Section 8.

5.2.1 Intuition

Intuition for the RRC can be gained by focusing the movement of \( T \) under the initial repo contracts as displayed in Figures 1 and 2. In the first-leg, \( T \) flows from \( RM \) to \( MM \) and in the opposite direction in the second leg. Under the sequence of initial repo contracts between neighbors on the chain, the \( BD \)'s do not contribute any \( T \), they just move it along the chain. Focusing on the second leg, in substance \( BD_j \)'s role is to guarantee to \( RM \) that it will receive \( T \) from \( MM \). In turn, \( BD_i \) provides a back-up guarantee that \( MM \) will send \( T \) and finally \( MM \) makes a direct guarantee to \( BD_i \) that it will send \( T \) at the second leg. A RRC matches the substance of the \( BD \)'s role as guarantor with a contract form in which the ultimate providers of traded objects, \( MM \) and \( RM \), are principals and the intermediary \( BD \)'s are guarantors.

5.2.2 The Pattern of Guarantees

The initial repo contracts authorize the smart contract to novate and replace provided the replacement contracts do not add risk or alter the profit of either counterparty. Consider a chain of initial repo contracts connecting \( MM_i \) to \( RM_j \) through two intermediaries, \( BD_1 \) and \( BD_2 \). The RRC for the first and second-leg are denoted \( C_{RM_j \rightarrow MM_i}^1 \) and \( C_{MM_i \rightarrow RM_j}^2 \) respectively. Suppose that, under the RRC \( MM_i \) and \( RM_j \) purchase and sell \( T \) at the same prices (first and second-leg) that they transacted with their \( BD \) counterparties in their initial repo contracts and that the obligations and payoffs to each \( BD \) is unchanged by the RRC. The pattern of guarantees under the RRC is displayed in Figure 3 and Table 1.

![Figure 3: Replacement Repo Contract: Pattern of Client and BD Guarantees](image-url)
<table>
<thead>
<tr>
<th>Guarantee #</th>
<th>Guarantor</th>
<th>Indemnified Party</th>
<th>Indemnification</th>
<th>Accrued Claim</th>
</tr>
</thead>
<tbody>
<tr>
<td># 1</td>
<td>BD_2</td>
<td>RM</td>
<td>Default by MM</td>
<td>v.s. BD_1</td>
</tr>
<tr>
<td># 2</td>
<td>BD_1</td>
<td>MM</td>
<td>Default by RM</td>
<td>v.s. BD_2</td>
</tr>
<tr>
<td># 3</td>
<td>BD_1</td>
<td>BD_2</td>
<td>Reimb BD_2 re Guar # 1</td>
<td>v.s. MM</td>
</tr>
<tr>
<td># 4</td>
<td>BD_2</td>
<td>BD_1</td>
<td>Reimb BD_1 re Guar # 2</td>
<td>v.s. RM</td>
</tr>
</tbody>
</table>

Table 1: Pattern of Client and BD Guarantees

The rightmost “Accrued Claim” column of Table 1 reflects that when a guarantor delivers an object to an indemnified party, it inherits the claim the indemnified party has against the defaulting party. An important feature of the replacement contract is that it does not cause either MM_i or RM_j to incur an increase in contractual risk. Each is responsible to send its contracted object. Each is guaranteed by its initial counterparty BD the receipt of the object it has contracted to receive in exchange. The patterns displayed in Figure 3 can be generalized to intermediation chains of any length. Any BD located between BD_1 and BD_2 provides a reimbursement guarantee to each BD’s with whom it initially contracts. For example, if BD_i stands between BD_1 and BD_2, it would indemnify BD_1 for delivery of M to MM_i (and accrue a claim against BD_2 if it sends M to BD_1) and it would indemnify BD_2 for delivery of T to RM_j (and accrue a claim against BD_1 if it sends T to BD_2).

6 Implementation of the Repo Smart Contract

In this section we describe a baseline protocol for a smart contract at the API layer of abstraction where the financial objects M and T are appended to programmable electronic ledgers. Each agent is a node in a network and can send and receive financial objects, messages and contracts. We assume the repo contract terms between agents are negotiated offline and transmitted to the SC. A more comprehensive analysis could extend the SC to include a mechanism to negotiate contract terms, such as an auction or structured exchange. The protocol replicates the outcome of a decentralized repo market without a ledger or a CCP, but with bi-lateral netting of contracts. We do not include an analysis of the legal requirements of the SC, which remain unsettled under US law [PROVIDE CITATION].

6.1 Notation

The set of agents (interchangeably referred to as nodes) is A. The notation we use is consistent throughout the protocol, but the indexing of agents is adjusted for different purposes. When discussing the initial repo contracts between agents, we index the contracting pair of counterparties by i, j and other lower case letters. When discussing the aggregation of inflows of financial objects to agent i sent from multiple counterparties and the aggregation of outflows of financial objects from agent i sent to multiple counterparts, we index the latter by T/M_{j-1→i} and the former by T/M_{i→i+1}. Finally, when discussing the contracts

---

20T and M can be appended to separate ledgers.
21The extension of the SC to contract negotiation is the subject of planned future research.
and flows of objects across a linear chain of agents, we index successive agents in the chain by \{1, 2, ..., n\}.

I. Initial Repo Contracts

The initial repo contracts are entered into by pairs of agents who we denote \(i\) and \(j\).

Quantities:

First-leg \(T\) contracted to be sent from \(i\) to \(j\) is denoted by \(T_{i\rightarrow j}\).

Second-leg \(T\) contracted to be sent from \(j\) to \(i\) is denoted by \(T_{j\rightarrow i}\).

\(T_{i\rightarrow j} = T_{j\rightarrow i}\) i.e. first-leg and second leg collateral volume is equal.

Timing:

Both contracts entered into at \(t_1\). The first-leg contract executes at \(t_1\) and the second-leg contract executes after \(t_1\) at \(t_2\).

Prices

First-leg: \(p_{j\rightarrow i}^1\) is the unit price of \(T\) paid by \(i\) to \(A_j\) in first-leg contract. The movement of money is \(M_{j\rightarrow i} = T_{i\rightarrow j}p_{j\rightarrow i}^1\).

Second-leg: \(p_{i\rightarrow j}^2\) is the unit price of \(T\) paid by \(j\) to \(i\) in second-leg contract. The movement of money is \(M_{i\rightarrow j} = T_{j\rightarrow i}p_{i\rightarrow j}^2\).

Contracts

First-leg contract - following the movement of \(T\) is denoted by \(C_{i\rightarrow j}^1 = \{T_{i\rightarrow j}, p_{j\rightarrow i}^1\}\).

Second-leg contract - following the movement of \(T\) is denoted by \(C_{j\rightarrow i}^2 = \{T_{j\rightarrow i}, p_{i\rightarrow j}^2\}\).

II. Network Structure

The set of initial repo contracts induces a network between agents where \(T\) and \(M\) move along edges that connect pairs of nodes. Figure 4 depicts such a network where the nodes represent agents and each edge is created by repo contracts along which \(M\) and \(T\) are contracted to be sent and received, in the first and second-leg, by the nodes at either end.
III. Aggregation

At each leg an agent $i$ sends objects - $T$ and $M$ - and receives objects along the edges that connect it to other agents. If $i$ enters into contracts with several counterparties, it will send and receive objects along multiple edges. The notation for an agent $i$’s aggregate flows of $T$ and $M$ is as follows.

First-leg contract sales of $T$ are $\sum_{j/i} T_{i\to j} = T_{i\Rightarrow}$.  
First-leg contract payments received for $T$ are $\sum_{j/i} p_{j\to i}^1 T_{i\to j} = p_{i\Leftarrow}^1 T_{i\Rightarrow}$.  
The set of first-leg counterparty agents to whom $i$ sells $T$ is $A_{i\Rightarrow}$.  
The set of first-leg contracts where $i$ sells $T$ is $\{C_{j\to i}^1 \mid T_{i\to j} \neq 0 \} = C_{i\Leftarrow}^1$.  
First-leg contract purchases of $T$ are $\sum_j T_{j\to i} = T_{\Rightarrow i}$.  
First-leg contract payments made for $T$ are $\sum_j p_{j\to i}^1 T_{j\to i} = p_{i\Leftarrow i-1}^1 T_{\Rightarrow i}$.  
The set of first-leg counterparty agents from whom $A_i$ purchases $T$ is $A_{\leftarrow i}$.  
The set of first-leg contracts where $i$ purchases $T$ is $\{C_{i\to j}^1 \mid T_{j\to i} \neq 0 \} = C_{i\Leftarrow}^1$.  

A similar notation applies to the second-leg, which price indexed by $P^2$. Figure 5 depicts the aggregation of agent $i$’s flows of $T$. On the left side of $i$ are the $T$ sent to $i$ from other agents and on the right side are the $T$ sent from $i$ to other agents.
6.2 Protocol for Initial Repo Contracts

Steps 1 - 6 of the protocol, which describe the formation of the initial repo contracts, apply to a repo transaction between two agents, $A_i$ and $A_j$. The remaining steps apply to the resulting flow of objects into and out of a single agent $A_i$. We suppress from the notation that each agent has its own escrow in the SC into which it sends and from which it receives $T$ and $M$. The SC moves financial objects between the agent escrows and sends financial objects to agents from their escrows.
Contract Formation between $i$ and $j$

1. Each pair $i$ and $j$ negotiates the terms of their - possibly multiple - first and second-leg repo contracts offline.

2. For each transaction, $i$ and $j$ each send details on their first and second-leg prices and volume of $T$, $\{T_{i \rightarrow j}, p_{i \rightarrow j}^1\}$ and $\{T_{j \rightarrow i}, p_{j \rightarrow i}^2\}$, to the SC.

3. For each transaction, the SC verifies that the terms sent by $i$ and $j$.

4. For each transaction, the SC sends two contracts, $\{C_{j \rightarrow i}^1, C_{i \rightarrow j}^2\}$, to $A_i$ and $A_j$ reflecting the terms. Each contract authorizes the SC to replace it with a netted contract followed by a replacement repo contract.

5. $i$ and $j$ send signed contracts to the SC.

6. The SC aggregates all repo transactions between $i$ and $j$ and replaces the separate repo contracts with a single contract for each leg $\{C_{j \rightarrow i}^1, C_{i \rightarrow j}^2\}$ which net the transactions between $i$ and $j$. The SC sends the netting contract to $i$ and $j$. No signatures are required since the RRC method is approved in the initial repo contracts.

Flow of objects into and out of $i$

7. $i$ sends its first-leg contracted objects, $T_{i \rightarrow i}$ and $M = p_{i \rightarrow i}^1 T_{i \rightarrow i}$, into its SC escrow, subject to the following;

   (i) Treasuries if $A_{i-1}$ send $T_{i \rightarrow i}$ into the SC escrow in accordance with contacts $C_{j \rightarrow i}^1$, then $A_i$ is required to send the gap $(T_{i \rightarrow i} - T_{i \rightarrow i})^+$ into the SC escrow.

   (ii) Money if $A_{i+1}$ send $p_{i \rightarrow i}^1 T_{i \rightarrow i}$ into the SC escrow in accordance with contacts $C_{i \rightarrow i}^1$ then $A_i$ is required to send the gap $(p_{i \rightarrow i}^1 T_{i \rightarrow i} - p_{i \rightarrow i}^1 T_{i \rightarrow i})^+$ into the SC escrow.

   For any counterparties $i$ and $j$, if either one fails to send in its contracted object, the SC sends a message to the agents that the first and second-leg contracts between $i$ and $j$ are cancelled.

8. For the contracts that remain in force, the SC sends to $A_i$ the objects as follows;

   \[\text{collateral} \ (T_{i \rightarrow i} - T_{i \rightarrow i})^+, \text{ money} \ (p_{i \rightarrow i}^1 T_{i \rightarrow i} - p_{i \rightarrow i}^1 T_{i \rightarrow i})^+ = 0\]

Margining Between First and Second-Leg

9. At scheduled intervals after step 8 the SC takes input from an oracle for Treasuries price to compute the amount required to offset any decline in the value of second-leg $T_{j \rightarrow i}$ under each contract $C_{j \rightarrow i}^2$. If the price of of $T$ has declined, the SC sends a message to $A_i$ to send $M = \Delta P T_{i \rightarrow i}$, into the SC escrow, where $P$ is the market price of a unit of $T$.

10. If $i$ fails to send the additional $M$ into escrow, the SC sends a message to each counterparty $j$ stating that $i$ is in default under $C_{j \rightarrow i}^2$. 

18
Second Leg Protocol

7'. $i$ sends its second-leg contracted objects into its $SC$ escrow, subject to the following

(i) *Treasuries* if $A \xrightarrow{\bullet} T_{i\leftarrow}$ into the SC escrow in accordance with contacts $C^2_{i\leftarrow}$, then $A_i$ is required to send the gap $(T_{i\leftarrow} - T_{i\leftarrow})^+$ into the SC escrow

(ii) *Money* if $A \xrightarrow{\bullet} T_{i\leftarrow}$ into the SC escrow in accordance with contacts $C^2_{i\leftarrow}$ then $A_i$ is required to send the gap $(p_{i\leftarrow}^2 - p_{i\leftarrow}^2 T_{i\leftarrow})^+$ into the SC escrow

For any counterparties $i$ and $j$, if $i$ fails to send in its contracted object, the SC sends a message to $i$ and the affected counterparties stating that $i$ is in default under the contract with its counterparty.

If $i$ and $j$ both fail to send in its contracted object, the SC sends a message to both parties that their second-leg contract is cancelled.

8'. If neither counterparty is in default, the SC sends to $i$ the objects as follows

*Treasuries* $(T_{i\leftarrow} - T_{i\leftarrow})^+$

*Money* $(p_{i\leftarrow}^2 T_{i\leftarrow} - p_{i\leftarrow}^2 T_{i\leftarrow})^+ + (M_{i\leftarrow} - M_{i\leftarrow})^+$.

The SC operates in the same legal environment as all other financial transactions. When a counterparty defaults, the non-defaulting a party has recourse to arbitration or court in the jurisdiction in which the transaction takes place. The following protocol enables agents to interact with a court of law in the event a counterparty defaults. A similar protocol would govern the interaction with an arbitrator, if the repo contract called for arbitration.

Second-Leg Protocol (continued)

Protocol for defaults

If, after a cure period of time $\Delta$, $i$ is in default to $j$ and $j$ is not in default to $i$

9'. Then the SC sends a message to both agents that $i$ is in default to $j$

(i) $j$ can present the default notice as conclusive evidence of the default in a Court of Law. In that case the SC sends the financial object in $j$’s escrow to the Court where the default claim is adjudicated, or

(ii) $j$ sends the SC a message that it desires to cancel the transaction. The SC sends to $j$ the financial object in its escrow and sends messages to $i$ and $j$ that the second leg contract is canceled. In addition, the SC sends the margin escrow $M_{i\rightarrow j}$ to $j$, or

(iii) $i$ and $j$ each send contract details containing a revised price and volume of $T$ to the $SC$. The SC moves to step 3, but only for the new (single) contract. In step 4 the new contract replaces the pre-existing second-leg contract. If the contract does not proceed to contract execution at step 6 or the contracted objects are not sent into the SC escrows in step 7, the SC sends a message to $A_i$ and $A_j$ that the pre-existing contract remains in force and the SC moves to step 9'.

Finally, note the SC can incorporate an option to delay delivery of $T_i$ in any second-leg contract by payment of a delay fee.
6.2.1 Netting of repo contracts

In Step 6 the SC nets contracts between pairs of agents. This reduces the balance-sheet impact of transactions between two agents up to the net borrowing or lending between them. Borrowing and lending transactions cancel each other out and thus reduce the repo transaction volume. This reduces the second-leg \( T \) and \( M \) recorded on the balance-sheets of the counterparties. The protocol implements a multilateral netting of the obligation to send objects in the second-leg. This is shown in steps 7 and 7' of the protocol, where each agent is required to send quantities of \( M \) and \( T \) equal to its aggregate net obligation. However, this does not result in a reduction in balance-sheet impact since the contractual obligation to send and receive objects in the second-leg between counterparties remains in place.

In the sequel, the initial repo contract refers to the netted contracts from Step 6.

7 Constructing Replacement Repo Chains

In order to achieve the increase in intermediation capacity outlined in Section 5 the SC reorganizes transactions. In this section we explain how the SC partitions the flow of first-leg \( T \) into chains of nodes where a unit of \( T \) moves from one node to the next. Section 7.1 describes the method to divide the initial repo contract between counterparties \( i \) and \( j \) into two parts which enables the transformation of the initial repo graph into a directed graph where first-leg \( T \) enters on the left side and flows to the right side. We call this a repo flow network. The resultant repo flow graph meets the requirement of a flow network, the properties of which are described in Section 7.2. Section 7.3 states concepts and results from the literature on algorithms that are used in developing the method for constructing RRC’s and estimates of the computational steps required to do so. The section culminates with the RRC Algorithm, which is a computationally feasible procedure to construct RRC’s. Section 7.4 reviews the partitioning of repo contracts implied by the transformation of the initial repo graph into a set of RRC’s and cycles.

7.1 Transforming the Initial Repo Graph into a Flow Network

Transforming the initial repo graph into a repo flow network involves four steps. The first step is to aggregate the inflows and outflows of first-leg \( T \) for each node \( i \) as depicted in figure 5. These aggregates denote the flows of first-leg \( T \) that are in the initial repo contracts between the subject node \( i \) and its counterparties. The second step is to divide each node \( i \) into possibly two nodes; one node has equal inflow and outflow of \( T \) (the “balanced node”) and the other node has the excess of inflow or outflow (the “excess flow node”), if any. This split is effectuated by conceptually dividing one or more of node \( i \)’s initial repo contracts into two separate contracts with one part of the split contract assigned to the balanced node and the other part assigned to the excess flow node. The division of a repo contract involves allocating \( \lambda \% \) of the first-period \( T \) (and other flows of first and second-leg \( T \) and \( M \)) to one contract and \( (1 - \lambda) \% \) to the other contract. The split of a netted initial repo contract into two separate contracts does not alter any contractual terms between node \( i \) and its counterparty. It is a device to re-arrange the repo flow graph. Figure 6 depicts the partitioning of the initial repo contracts of agent \( i \) into a balanced node, which corresponds to a matched-book intermediary \( BD \), and an excess (in)flow node, which corresponds to an
MM repo lender.\textsuperscript{22}

Figure 6: Assignment of Netted Initial Repo Contracts

The third step is to arrange the graph to depict a flow of $T$ from left to right. For each node $i$ in the initial repo flow graph, the balanced node is placed in the middle of the graph and is labeled $BD_i$. If node $i$ has a net outflow of first-leg $T$, the excess flow node is moved to the left side of the graph and labeled $RM_i$. If node $i$ has a net inflow of first-leg $T$, the excess flow node is moved to the right side of the graph and labeled $MM_i$. Figure 7 displays the transformation when node $i$ has a net outflow of first-leg $T$ under its initial repo contracts. Recall from figure 5 that $E$ represents the net outflow of node $i$. In this case, where $E$ is positive, $E$ is moved to $RM_i$. Counterparty node $k$ still receives the initial repo contract amount $T_{i\rightarrow k}$ from $i$, but the transformed graph splits the volume with $T_{i\rightarrow k} - E$ sent from node $BD_i$ and $E$ sent from node $RM_i$. Figure 8 displays the transformation when node $i$ has a net inflow of first-leg $T$ under its initial repo contracts. In this case, since $E$ is negative, $E$ is moved to $MM_i$. Counterparty node $h$ still sends the initial repo contract amount $T_{h\rightarrow i}$ to $i$, but the transformed graph splits the volume with $T_{h\rightarrow i} - E$ sent to node $BD_i$ and $E$ sent to node $MM_i$.

\textsuperscript{22}When node $i$ has initial repo contracts with multiple counterparties, the contracts with one counterparty can be allocated to one of the two new nodes or split between them.
The final step is to add nodes $s$ and $t$ to the left and right ends of the graph. Node $s$ is placed on the left end of the graph and has a fictitious volume of $T$ with each $RM_i$ whereby $RM_i$ can receive an unlimited inflow of $T$ from $s$. Node $t$ is placed on the right end of the graph and has a fictitious volume of $T$ with each $MM_i$ whereby $MM_i$ can send an unlimited outflow of $T$ to $t$. Figure 9 is an example flow network. The edges connecting to $s$ and $t$ enable computation of the RRC path, but are not otherwise relevant. The result is an directed graph where the flow of first-leg $T$ moves from left to right. The $RM$ nodes send first-leg $T$ (and receive no $T$); the $BD$ nodes receive and send an equal amount of first-leg $T$ and the $MM$ nodes receive and do not send first-leg $T$. In this case, The initial repo contracts for nodes $k$, $o$ and $p$ are balanced, so they are re-labeled as $BD$’s without being
split. The initial repo contracts for nodes $i, j, l$ and $m$ are not balanced and are split into a balanced $BD$ node and either an $RM$ outflow node or a $MM$ inflow node. The outlined path $RM_l \rightarrow BD_o \rightarrow BD_p \rightarrow BD_j \rightarrow BD_k \rightarrow BD_i \rightarrow MM_j$, which we shall call $P_{example}$, is a potential RRC. We call the resultant graph the repo flow network (“RFN”).

![Figure 9: The Repo Flow Network](image)

### 7.2 Properties of the Repo Flow Network

The first-leg flows of $T$ in the RFN form a network $R = (A, E)$ where nodes $\rho \in \{1, ..., A\}$ denote agents; directed edges $(\rho, \gamma) \in E$ which have capacity equal to the volume $T_{\rho \rightarrow \gamma}$ of first-leg $T$ in the portion of the netted initial repo contract between nodes $\rho$ and $\gamma$ that is allocated to the edge.\(^{23}\) A flow in $R$ is a real valued function $f : A \times A \rightarrow \mathbb{R}$, which represents the flow of first-leg $T$ between a pair of agents that are connected by an edge, i.e. that have entered into an initial repo contract. A flow network satisfies the following two properties:\(^{24}\)

**Flow Conditions**

**Capacity constraint:** For all $\rho, \gamma \in I$, $0 \leq f(\rho, \gamma) \leq$ first-leg $T_{\rho \rightarrow \gamma}$

**Flow conservation:** For all $\rho \in I - \{s, t\}$,

$$\sum_{\rho \in A} f(\rho, \gamma) = \sum_{\rho \in A} f(\gamma, \rho)$$

Where $(\rho, \gamma) \notin E$, $f(\rho, \gamma) = 0$

It is immediate that the RFN is a flow network. Noting that in the RFN node $s$ sends flow to nodes labeled $RM_i$, which in turn send flow to nodes labeled $BD_i$, the measurement of flow is the volume of first-leg $T$ sent by $RM_i$ nodes.

$$|f| = \sum_{\rho \in A} f(s, \rho) = \sum_{RM_i \in A} \sum_{BD_j \in A} f(RM_i, BD_j)$$

\(^{23}\)We denote nodes as $\{\rho, \gamma\}$ rather than $\{i, j\}$ to underscore that nodes in the MFRN are not the same as nodes in the initial repo graph.

\(^{24}\)See Cormen et.al. (11) Chapter 26.1.
A cut \((S,T)\) of the RFN is a partition of \(A\) into \(S\) and \(T = A - S\) such that \(s \in S\) and \(t \in T\). The flow \(f(S,T)\) across the cut \((S,T)\) is defined to be

\[
f(S,T) = \sum_{\rho \in S} \sum_{\gamma \in T} f(\rho, \gamma).
\]

Since the outflow of the \(RM_i\) nodes to \(BD_i\) nodes equals the inflow received from \(s\), flow in the RFN is the volume across the cut that has \(s\) and the \(RM_j\) nodes in \(S\) and all other nodes in \(T\). Corman et.al. (11) show that the volume of flow across any cut in a flow network is equal to the network flow.

**Lemma: Equal Flow Across Cuts.** Let \(f\) be a flow in a flow network \(R\) with source \(s\) and sink \(t\), and let \((S,T)\) be any cut of \(R\). The flow across \((S,T)\) is \(f(S,T) = |f|\).

**Proof.** See Corman et.al. (11) Lemma 26.4, p.722.

An implication of Lemma 26.4 is that the flow of first-leg \(T\) into the \(MM_i\) nodes is equal to first-leg \(T\) outflow from the \(RM_i\) nodes. This can be seen by evaluating the flow across \((S,T)\) when \(t\) and the nodes labeled \(MM_i\) are assigned to \(T\) and all other nodes are assigned to \(S\).

### 7.3 Constructing the RRC’s

RRC’s can be constructed by any algorithm that iteratively identifies paths with positive flow along each edge connecting \(s\) to \(t\) in the RFN (equivalent to connecting an \(RM\) node to an \(MM\) node), called simple paths, and a volume of \(T\) that does not exceed the capacity of any edge along the path. The algorithm follows from the flow decomposition lemma in Williamson (38).

**Lemma: Flow Decomposition.** Given an \(s-t\) flow \(f\), there exist flows \(f_1, \ldots, f_\ell\), for some \(\ell \leq m\) such that \(f = \sum_{i=1}^{\ell} |f_i|\). and for each \(i\), the arcs of \(f_i\) which positive flow form either a simple \(s-t\) path or a cycle.

**Proof.** See Williamson (38) Lemma 2.20, p.61.

Where \(m\) is the number of edges in the network, an arc is the sequence of edges on a path and a cycle is a flow that starts from a node and comes back into that node.\(^{25}\) A key observation is that no \(RM_i\) or \(MM_i\) can be part of a cycle. Neither \(s\) nor \(t\) can be part of a cycle, since the former does not receive inflow and the latter does not send outflow. Therefore, no \(RM_i\) can be part of a cycle, since it only receives flow from \(s\) and no \(MM_i\) can be part of a cycle, since it only sends flow to \(t\).

Combining the observations that the outflow of first-leg \(T\) from \(RM_i\) nodes is equal to the inflow of first-leg \(T\) to \(MM_i\) nodes and that none of those nodes can be part of a cycle, the flow decomposition lemma implies that there is a (possibly non-unique) set of chains connecting \(RM_i\)’s to \(MM_i\) that contain the entire outflow of the former and inflow of the latter. Each chain thus constructed is an RRC. The flow decomposition lemma also implies that flows of first-leg \(T\) that form cycles connecting \(BD_i\) nodes cannot be placed in RRC’s.

\(^{25}\)In the RFN Figure 9 the path \(BD_j \rightarrow BD_k \rightarrow BD_i \rightarrow BD_j\) is a cycle.
7.3.1 An Algorithm to Identify RRC’s

An augmenting path in the network $R$ is a simple path $P$ from $s$ to $t$ along which there is positive capacity on each edge in the arc. Denote the minimum capacity edge in $P$ as $q(P) = \min_{(\rho, \gamma) \in P} f(\rho, \gamma)$. The RFN has capacity $f(\rho, \gamma)$ on the edge that connects node $\rho$ to node $\gamma$. The residual graph with respect to flow $f$ is the graph $R_f$ that remains after removal of an augmenting path. The residual capacity of an edge in $R_f$ is defined as $r(\rho, \gamma) = f(\rho, \gamma) - \delta q(P)$, where $\delta = 1$ if $(\rho, \gamma)$ is on the augmenting path and 0 otherwise. The residual graph of $R_f$ is defined iteratively. A set of RRC’s containing the entire RFN flow between $RM_i$’s and $MM_i$’s can be constructed using the following Algorithm.

**RRC Algorithm [Adaptation of Williamson (38) Algorithm 2.3]**

1. initialize RRCSets = [ ]
2. $R = (A, E)$
3. initialize flow $f$ at 0
4. $f(\rho, \gamma) \leftarrow 0$ for all $(\rho, \gamma) \in R$
5. while there is an augmenting path in $R$
   1. Let $P$ be the augmenting path that maximizes $\min_{(\rho, \gamma) \in P} f(\rho, \gamma)$
   2. RRCSets $\leftarrow$ RRCSets $\cup$ $P$
   3. $R \leftarrow R - \{P\}$
6. return RRCSets

A notable property of the RRC Algorithm is that, for each augmenting path $P$ that is removed from the graph, the edge with the lowest capacity is set to zero and is removed from the graph. This suggests a limit on the number of computations that must be made to construct all RRC’s. Let $n$ denote the number of nodes in the RFN and $U = \max_{(\rho, \gamma) \in R_f} f(\rho, \gamma)$ i.e. the largest capacity on any edge in the RFN. The next theorem from Williamson (38) states the bound.

**Theorem: A Computational Bound.** Algorithm 2.3 [the RRC Algorithm] can be implemented to run in $O(m \log(mU)(m + n \log n))$ time.

**Proof.** See Williamson (38) Theorem 2.24, pp.47-50.

[Obtain upper bound estimates of $m$, $n$ and $U$ from OFR-Treasury studies of the UST repo market]

The RRC Algorithm is not necessarily the fastest. The takeaway from this section is that there is a cost-feasible method to construct RRC chains.

7.4 The Partitioning of Repo Contracts

The transformation of the RFN into a set of chains, RRCSets, and cycles implies a further partitioning of the initial netted repo contracts into separate nodes on the graph. Figure ?? depicts the partitioning of the first-leg $T$ flows of agent $i$ into separate inclusions in RRC’s and cycles. For convenience the various RRC’s into which portions of agent $i$’s flow
is assigned are labeled $A$, $B$ and $C$. The cycle to which a portion of agent $i$’s flow is assigned is labeled $A$. The figure uses node labels from the RFN. Overall, the inflows and outflows of first-leg $T$ between agent $i$ and its netted initial repo contract counterparties is unchanged. The partitioning of flows enables a redesign of the Initial Repo Graph (and the RFN) to reflect particular subgraphs that are embedded within it. The partitioning of the flows enables the creation of a new set of contracts to replace the netted initial repo contracts.

![Figure 10: Assignment of Repo Flows to RRC’s and Cycles](image)

8 The Replacement Repo Contract

In this section we characterize the replacement repo contract that is implemented on an RRC and show that it does not alter any payoff relevant variable as compared to the netted initial repo contract. The timing works as follows. First, agents enter into initial repo contracts. Next, after the initial contracts have been netted but before any objects - $T$ or $M$ - have been sent, the smart contract partitions the flow of first-leg $T$ in the network formed by the netted initial repo contracts into RRC’s. An RRC is a directed graph with a finite set of nodes connected by edges where each node - representing an agent - has at most one parent and one child and a fixed amount of $T$ moves along the edges in the first-leg from parent to child. Then, the smart contract replaces the initial repo contracts with new contracts, one for each chain. The volume of $T$ transacted on the chain cannot exceed the capacity of any edge. Finally, the protocol for RRC’s is implemented.

Section 8.1 provides defines framework we use to analyze RRC’s. Section 8.2 describes the replacement repo contract entered into be the $RM$ and $MM$ nodes that occupy the end positions on an RRC. Section 8.3 describes the intermediation fees paid and received by the intermediate $BD$ nodes in an RRC. Section 8.4 describes the structure of guarantees provided by the intermediate nodes. Section 8.5 states the protocol followed by the SC to implement the replacement repo contract on an RRC. Section 8.6 describes the scaling of the netted initial repo contracts between nodes on a cycle path.

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26 We refer to the chain and the contract as an RRC.

27 As with the initial repo contracts, $M$ moves along the chain in the opposite direction from $T$ and the direction of movement reverses in the second-leg.
8.1 RRC Nodes and Flows

To simplify notation, and in order to distinguish the nodes on an RRC from the nodes in the Initial Repo Graph, we index agents in an RRC chain by numbers from 1 to \( n \), where 1 denotes the \( RM \) node and \( n \) denotes the \( MM \) node and the nodes in between, indexed by \( \rho \), denote the \( BD \)'s involved in transactions along the chain. An RRC is denoted by \( \{1, 2, ..., \rho, ..., n\} \) with first-leg \( T \) moving rightward starting at 1. In the RRC the nodes at either end of the chain, \( RM_i = 1 \) and \( MM_j = n \), become the principal contracting parties. The intermediate \( BD \) nodes \( \{2, ..., \rho, ..., n-1\} \) become guarantors of performance of the contract.

At step 6 of the initial smart contract protocol the SC implements the RRC. We now describe the key properties.

The flow of first-leg \( T \) between neighboring nodes on an RRC reflects the netted initial repo contracts between them (or portions thereof). Those contracts are initially unchanged except to the extent that the volume of \( M \) and \( T \) transacted is scaled to the portion of the first-leg \( T \) that is assigned to the chain. Consequently, the basis of comparison of the RRC contracts on a chain is the scaled netted initial repo contracts between nodes on the chain.

8.2 Contract between 1 and \( n \)

1 and \( n \) are the contracting parties. Their first and second leg repo prices remain the same as in the initial repo contracts. We denote the traded collateral in the RRC by \( T_R \), which is equal in both legs.

**First-leg:**

The contract is denoted \( C^1(R)_{1\to n} = \{T_R, \bar{p}(R)_{1\to n}^1\} \) where \( \bar{p}(R)_{n\to 1}^1 = \{p_{2\to 1}^1, p_{n\to n-1}^1\} \) and \( T_R \) is the first-leg volume of \( T \) that moves across the chain. Notably, the unit price \( p_{2\to 1}^1 \) received by 1 equals the unit price paid by 2 to 1 in their initial repo contract. Similarly, the unit price \( p_{n\to n-1}^1 \) paid by \( n \) equals the unit price paid by \( n \) to \( n-1 \) in their initial repo contract. Any excess of \( p_{2\to 1}^1 \) over \( p_{n\to n-1}^1 \) is covered by \( M \) sent to the SC by intermediate agents.

**Second-leg:**

The contract is denoted \( C^2(R)_{n\to 1} = \{T_R, \bar{p}(R)_{2\to n}^2\} \) where \( \bar{p}(R)_{n\to 1}^2 = \{p_{1\to 2}^2, p_{n-1\to n}^2\} \).

The interpretation of the prices is analogous to that of the first-leg.

8.3 Intermediation Fees

Intermediation fees refer to the \( M \) sent by intermediate agents into the SC and received from the SC. The fee structure in the RRC is designed to accomplish two objectives; (i) to
ensure performance of the second-leg contracts between 1 and n and (ii) to ensure that each intermediate agent earns the same fee as it would have earned in its initial repo contract.

At the second leg agent 1 sends $T_R$ to the SC and the SC sends $T_R$ to agent 1. The matching volume implies there is no need for intermediate agents to send any $T$ into the SC to enable all obligations to be met. On the other hand, when $p_{1\to2} < p_{n-1\to n}$, the SC must receive $M$ from intermediate agents to enable it to send $n$ its contracted payment. In order to ensure that objectives (i) and (ii) are met, the RRC implements the following fee structure.

At step 7 (at the first-leg) the intermediaries, $\{2, \ldots, \rho, \ldots, n-1\}$ are required to send into the SC escrow the first and second leg net payments of $M$ that would have been required at steps 7 and 7’ in the first and second-legs, respectively, if the RRC operated as the initial contract scaled to $T_R$. The amount $\rho$ is required to send into the SC st the first-leg is the sum:

$$\{ (p^1_{\rho+1\to\rho} - p^1_{\rho\to\rho-1})^+ + (p^2_{\rho-1\to\rho} - p^2_{\rho\to\rho+1})^+ \} T_R = \rho RRC participation fee$$

The SC retains the $t_1$ RRC escrow until the second leg, so as to ensure that there is sufficient $M$ to send $n$ the payment due in $C(R)_{n\to1}$ (contingent on 1 sending its required $p_{1\to2}T_R$ into escrow). The $t_1$RRC participation fee paid by intermediary agents $\{\rho\}_{i=2,\ldots,\rho,\ldots,n-1}$ ensures that there are sufficient funds in the SC escrow to cover the difference between the price owed to $n$ and the price paid by 1 in the second-leg. Therefore, if 1 and n each meet their obligation under $C(R)_{n\to1}$, the contract will be fulfilled. At Step 8’ of the second-leg, assuming n sends $T_R$ into the SC escrow, the intermediary $i$ receives the sum:

$$\{ (p^1_{\rho\to\rho-1} - p^1_{\rho+1\to\rho})^+ + (p^2_{\rho-1\to\rho} - p^2_{\rho-1\to\rho})^+ \} T_R = \rho RRC guarantee fee$$

An intermediate agent sends into the SC escrow the RRC participation fee at the first-leg and the SC sends the RRC guarantee fee at the second-leg. The net payment received by the intermediary is identical to what the intermediary would have received from its initial contracts that are transferred to the chain, scaled to $T_R$. This can be seen by adding together Equations 6 and 7.

8.4 Guarantees

In the RRC, the intermediary agents guarantee the performance of the second-leg repo contract $C(R)_{1\to n}$ between 1 and n. Each intermediate agent on the RRC chain guarantees to its right-side neighbor the performance of 1’s obligation and guarantees its left-side neighbor the performance of n’s obligation.

Definition 8.1 (The RRC Guarantee Structure). At the second-leg,

if 1 does not send $p_{1\to2}T_R$ into the SC, then $\rho$ s.t. $i \in \{2, \ldots, \rho, \ldots, n-1\}$ guarantees to $\rho + 1$ that it will send the deficiency amount $p^2_{n-1\to n}T_R$ in the SC. If any agent fulfills its guaranty, n transfers its default claim against 1 to 2.

if n does not send $T_R$ into the SC, then $\rho$ s.t. $\rho \in \{2, \ldots, \rho, \ldots, n-1\}$ guarantees to $\rho - 1$ that it will send the object in the SC. If any agent fulfills its guaranty, $A_1$ transfers its default claim against n to $n-1$. 

28
There are two salient features of the RRC guaranty structure in relation to the pattern of guarantees in the initial repo contracts between neighbors on the chain. One is that the pattern of claims are the same in both instances. When \( \rho - 1 \) fails to send to the SC a required payment - in the RRC case it is the deficiency owed to \( n - \rho \) accrues a claim. Similarly, when \( \rho + 1 \) fails to send to the SC the required collateral - in the RRC case it is \( T_R - \rho \) accrues a claim. The other is that payment guarantee of \( p_{1\to2}T_R \) in the RRC may differ from the payment guarantee of \( p_{\rho-1\to\rho}T_R \) in the initial repo contract, scaled to \( T_R \).

We look at the operation of the guarantees when either 1 or \( n \) defaults on its second-leg obligation.

8.4.1 Agent 1 fails to send contracted payment

The second-leg payment that must be sent to the SC to enable it to send to \( n \) the contract payment amount of \( p_{n-1\to n}^2 \) is the sum of iRRC participation fee’s sent by agents in the first-leg and \( p_{1\to2}^2 \).

\[
\sum_{\rho=2}^{n-1} \rho \text{ RRC participation fee} + p_{1\to2}^2 T_R = p_{n-1\to n}^2
\]

If 1 fails to send its second-leg payment into the SC escrow, the SC sends to \( n \) the aggregate participation fee. \( n \) accrues a default claim in the amount of the deficiency \( p_{n-1\to n}^2 T_R \) against \( n - 1 \), which is the counterparty against whom it would have held a claim under the initial repo contract if the counterparty failed to send the contract payment. In addition, the deficiency in the payment to \( n \) triggers guarantees of intermediate agents to send in \( p_{1\to2}^2 T_R \). There are two scenarios to consider.

One scenario is where no agent sends \( p_{1\to2}^2 T_R \) into the SC. Each intermediate agent \( \rho, \rho \in \{2, \ldots, \rho, \ldots, n - 1\} \) holds a claim for default against \( \rho - 1 \), which is the counterparty against whom it would have held a claim under the initial repo contract if the counterparty failed to send the contract payment. Similarly, \( \rho \) is subject to a claim for default from \( \rho + 1 \), which is the counterparty who would have held a claim under the initial repo contract if \( \rho \) failed to send the contract payment.

The other scenario is where an intermediate agent \( \rho \) sends \( p_{1\to2}^2 T_R \) to the SC. The claims are the same as the first case, except that no agent with index \( \rho \) or higher is in default. This outcome is similar to the outcome that would occur under the initial repo SC since \( \rho \)’s payment fulfills the obligations of the higher numbered agents, i.e. \( \rho \) has paid the amount necessary for \( \rho + 1 \) to meet its commitment to \( \rho + 2 \) and so forth. Figure 11 displays the outcome in a six agent RRC chain where agent 4 sends \( p_{1\to2}^2 T_R \) to the SC. The SC sends to 6 the aggregate participation fee plus the payment sent by 4 so that 6 receives its payment in full. 4 has a claim against 3. 3 has a claim against 2 and, since 6 transfers to 2 its claim against 1, 2 has a claim against 1.

This is the same pattern of payments and claims that would occur under the initial repo contracts along the chain if 1, 2 and 3 each failed to send their contract payments into the SC. In that case each agent has a claim against its left-side neighbor until we reach 5.
has no claim against 4 since 4 sent its object into the SC. 6 has no claim against 5 since the SC passes to 6 the payment sent by 4 to 5’s escrow account.

![Diagram of SC Escrow]

Figure 11: Node 1 RRC Default

8.4.2 Agent \( n \) fails to send contracted collateral

A failure of \( n \) to send \( T_R \) into the SC escrow triggers the same pattern of claims as above with the numbering reversed e.g. 1 transfers its default claim against \( n \) to \( n - 1 \) and it accrues a claim against 2.

8.5 Protocol for Replacement Repo Contracts

The protocol for an RRC is a modification of the initial repo contract protocol in Section 6.2. There are changes to each leg.

8.5.1 RRC Modifications to the First-Leg Steps

Step 7 is modified to reflect that a failure by an agent \( \rho \) to send its object will cause the SC to replace the RRC with a sequence of bi-lateral repo contracts which we call “Alternative Repo Contracts” between neighbors \( \rho \) and \( \rho - 1 \), where \( C^1_{\rho-1\to\rho} = T_R; p^1_{\rho\to\rho-1} \) and \( C^2_{\rho\to\rho-1} = T_R; p^2_{\rho-1\to\rho} \). The protocol for these contracts are the same as the initial decentralized repo contracts in Section 6.2.
RRC First-Leg Protocol Modifications

6. For each pair of neighbors on the RRC chain, the SC deducts $T_R$ from their netted initial repo contracts. The SC sends the RRC to each agent on the chain. No signatures are required since the RRC method is approved in the initial repo contracts.

7. $\rho$ sends its first-leg contracted objects as follows
   
   (i) Treasuries 1 sends $T_R$ into the SC escrow.
   
   (ii) Money $n$ sends $p_{n\rightarrow n-1}T_R$ and each $\rho, \rho \in \{2, ..., \rho, ..., n-1\}$ sends $i$RRC participation fee into the SC escrow.

   If any $\rho$ or $\gamma$ fails to send in its contracted object, the SC sends a message to the agents that the RRC is replaced by separate repo contracts - the “Alternate Repo Contracts” between each pair of agents $\{\rho, \gamma\} \rho \in \{1, ..., \rho, ..., n\}$ with terms $C^1_{\rho-1\rightarrow \rho} = T_R, p^1_{\rho\rightarrow \rho-1}$ and $C^2_{\rho\rightarrow \rho-1} = T_R, p^2_{\rho-1\rightarrow \rho}$. Each Alternate Repo Contract moves to Step 7. of the initial repo contract protocol.

8. If the RRC remains in force, the SC sends to 1 and $n$ objects as follows:
   
   1 receives $p^1_{1\rightarrow 2}T_R$. $n$ receives $T_R$.

8.5.2 RRC Modifications to the Second-Leg Steps

Step 8” is modified to reflect the guarantees described in Section 8.4 that are triggered by a failure of 1 to send $p_{1\rightarrow 2}T_R$ or $n$ to send $T_R$ to the SC escrow.

RRC Second Leg Protocol Modifications

7”. Either 1 and $n$ respectively send their contracted objects, $T_R$ and $M = p^1_{1\rightarrow 2}T_R$, into the SC escrow, or

(i) If 1 fails to send $p^2_{1\rightarrow 2}T_R$ into the SC escrow, the SC sends the aggregate participation fee to $n$ and sequentially notifies the agents as follows

First, the SC sends a message to 1 and 2 stating that 1 failed to send its object into escrow and is in default to 2 and that, pursuant to its guarantee, 2 is required to send $p^1_{1\rightarrow 2}T_R$ into the SC escrow.

If the SC has not received the payment from 1 in $\Delta$ time, it sends default notifications to agents in ascending order starting from $\rho = 2$; if $\rho - 1$ fails to send $p^1_{\rho-1\rightarrow 2}T_R$ into the SC escrow in $\Delta$ time, then the SC sends a message to $\rho - 1$ and $\rho$ stating that $\rho - 1$ failed to send its object into escrow and is in default to $\rho$ and that, pursuant to its guarantee, $\rho$ is required to send $p^2_{1\rightarrow 2}T_R$ into the SC escrow. If an agent sends the payment to the SC, the SC sends $p^1_{\rho\rightarrow n}$ to $A_n$ and the SC sends no more default notifications to agents.

(ii) If $n$ fails to send in its contracted $T_R$ into the SC escrow, the SC follows the same protocol as in (ii) except that it proceeds in descending order starting from $\rho = n - 1$.

(iii) If 1 and $n$ each fail to send their respective contracted objects into the SC escrow, the SC sends a message to all agents that the second-leg contract is cancelled and returns to each $\rho$ its $\rho$RRC participation fee.

8”. If 1 and $n$ send their contracted financial objects into the SC escrow, the SC sends out the objects as follows.

1 receives $T_R$. $n$ receives $p^2_{n\rightarrow n-1}$. Each $ii \in \{2, ..., \rho, ..., n-1\}$ receives $\{(p^1_{i\rightarrow \rho-1} - p^1_{\rho+1\rightarrow \rho})^+ + (p^2_{\rho\rightarrow \rho+1} - p^2_{\rho-1\rightarrow \rho})^+\}T_R = t_2$RRC guarantee fee.
8.6 Cycles

The flow of first-leg $T$ between neighboring nodes $\rho$ and $\gamma$ in a cycle reflects the netted initial repo contracts between them (or portions thereof). Those contracts are unchanged except to the extent that the volume of $M$ and $T$ transacted is scaled to the portion of the contract that is placed in the cycle. Flows of first-leg $T$ in a cycle can be understood as a chain of inter-dealer repo transactions. A curious feature of a cycle is that at least one node must incur a negative net repo rate. To see this, consider a node $\rho$ that earns repo rate $r$ in a transaction where it sends first-leg $T$ to node $\gamma$. For $\gamma$ to earn a net profit on the cycle, it must earn a rate above $r$ from the node to which it sends first-leg $T$. The progression if increasing rates around the cycle means that $\rho$ will pay a rate higher than $r$ in the transaction where it receives first-leg $T$. Therefore, the existence of a cycle reflects an inefficiency in the trading activity of at least one node. In practice, a cycle may arise from the decentralized trading of individuals and units within an organization.

8.7 Backdoor

An important observation is that, if an agent is providing its own net collateral it will stand at the end of a repo chain as a principal to the RRC contract and the second-leg transaction will appear on its balance-sheet. This prevents a backdoor whereby an agent can sell a security in the first leg and remove the security from its balance sheet, as Lehman did with Repo 105 and MF Global did with its RTM trade. When an agent provides its own collateral it will, to the extent of its contribution of collateral, stand at the end of a repo chain as a principal to the RRC contract and the second-leg transaction will appear on its balance-sheet.

9 A Comparison of the Decentralized Repo Market to a Centralized Repo Market under Mandatory Central Clearing

In this section we compare the repo smart contract to decentralization without the smart contract and mandatory central clearing in terms of the distribution of risk, regulator auditing and the attainable repo transaction volume.

The smart contract retains the pattern of risk that is incorporated in the initial repo contracts. In both cases, the risk of second-leg default is borne by the counterparties to each initial repo contract, which creates a dispersion of risk. Under mandatory central clearing all second-leg default risk is concentrated on the CCP, which represents a mutualization of risk among the agents in the market (so that risk is actually dispersed). This difference in risk bearing induces different incentives to monitor risk. Under decentralization each agent has an incentive to invest in evaluating and pricing the risk of counterparty default. Under mandatory central clearing, only the CCP has an incentive to do those things. In the event of a major default, in both cases, the regulator could obtain timely audit information that identified the default, in response to which the regulator could inject money into the CCP or the counterparties of the defaulting agent, respectively, to avoid a contagion of default across repo market participants. This would not be possible to do in a decentralized market without a smart contract.

The decentralized repo market without a smart contract sets a baseline for attainable trans-
action volume. There is no netting of transactions in this context. Both the smart contract and mandatory central clearing achieve multilateral netting of initial repo contracts. In both cases the un-netted transactions cause assets to increase by the value of the second-leg inflows of $M$ or $T$. Both regimes require agents to record liabilities associated with CCR charges, which reduce the SLR. Under the smart contract the charges arise from the guarantees. Under mandatory central clearing, the charges arise from the concentration of counterparty and mutualization risk to the CCP. It is not possible to determine in advance the comparative amount of reduction in the SLR.

The most salient difference between the smart contract and mandatory central clearing may be that the latter does not impose any fundamental change in the organization of trade in the market whereas the latter requires an initially decentralized market to be reshaped into a centralized market. An evaluation of the welfare effects of the centralization of the repo market lies outside the scope of this paper.

10 Privacy and Auditing

In this section we outline a framework for preserving the privacy of transactions while enabling a regulator or a designated counterparty to audit selected transaction data. An agent has preferences over the disclosure of its balances of financial objects, $M$ and $T$, in its account and its activity on the ledgers, namely sending and receiving transaction terms, contracts and financial objects with the SC (collectively, the “messages”). Typically, the agent will want to limit disclosure to selected information to named agents (e.g. a lender or a prime broker). At the same time, a regulator will require disclosure of certain information. The goal is to design a network architecture that achieves the following:

(i) The balances, contracts and trading activity of each agent are private except for selected disclosures to approved agents and regulators.

(ii) A regulator can audit the ledger items and messages it has a legal right to observe.

(iii) The audited information is accurate.

There are three mechanisms that can be employed to ensure privacy and the accuracy of audits. One mechanism is legal enforcement. For example, under U.S. securities law a clearing agent, who observes transactions between agents, cannot disclose trades to third parties and cannot use the information gained to front-run trades. A second mechanism is encryption whereby only select agents possess the key to decrypt and observe the underlying data. The third mechanism is use of trusted hardware to perform computations that cannot be observed by any agent, but which can generate an encrypted transcript that can be observed by select agents that have a decryption key. Up to the present time, only the first mechanism, legal enforcement, has been used. When the traded financial objects and contracts are appended to digital ledgers, it is possible to achieve the privacy and audit goals with the latter two mechanisms. We outline an architecture with the following elements.

(1) All messages between agents and the SC are encrypted.

(2) The messages contain noise which prevents an observer from inferring trading relationships between agents.
The smart contract is a trusted execution environment whereby no agent can observe the computations and movement of objects between agent escrows.

An auditor is able to decrypt into plaintext (i.e. the underlying data) exactly the information that it is authorized to observe.

We briefly review each element.

**Encryption**

To hide plaintext contents and values, messages sent between an agent and the smart contract are encrypted.\(^{28}\) We denote a message sent from agent \(i\) to the smart contract as \(\text{Enc}(\uparrow m_i, pk_{TEE})\) where \(\uparrow m_i\) is the message; \(pk_{TEE}\) is a public key that authenticates the message; \(\text{Enc}(\ )\) is the encryption of the message and \(sk_{TEE}\) is a secret key held by the smart contract (i.e. the “trusted execution environment”) which enables it to decrypt the message it receives. The notation for a message sent from the smart contract to agent \(i\) is \(\text{Enc}(\downarrow m_i, pk_i)\) with secret decryption key, held by agent \(i\), denoted \(sk_i\).

**Hiding Transaction Flows**

The encrypted messages sent between agents and the smart contract do not have to be public. For example, visibility can be restricted to an entity that runs the platform. Nevertheless, some agent will observe the flow of traffic between agents and the smart contract. The pattern of messages might reveal trading volumes or trading relationships. For example, different volumes of messaging between agents and the smart contract might imply differences in trading volumes, or a pattern of synchronous messaging among two agents might imply they have a trading relationship. Traditionally, the privacy of agent trading behavior is enforced by legal rules. Encryption provides an alternative method. Since the plaintext underlying messages cannot be observed by unauthorized parties, the signal provided by message traffic can be scrambled by sending encrypted messages with null content. For example, if the volume of encrypted messages sent between each agent and the smart contract was set at a level that matched the highest volume of messaging involving any agent, the encrypted message traffic would not reveal any information about trading patterns.

**Trusted Execution Environment**

The smart contract is a trusted execution environment where the encrypted messages received from agents are decrypted and mathematical operations are performed by hardware that cannot be directly observed by any agent.\(^{29}\) It is necessary to decrypt the message in order to perform the operations required to net transactions and reform the repo graph into chains on the plaintext data.\(^{30}\) Agents and auditors may require a way to verify the correctness of the operations inside the smart contract, particularly when a dispute arises. This can be handled by encoding into the smart contract a transcript of the operations and

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\(^{28}\)See Townsend et.al. (35) for an introduction to cryptographic methods that are employed on distributed ledgers.

\(^{29}\)For an introduction to trusted execution environments see the Wikipedia entry (39).

\(^{30}\)Homomorphic encryption, which enables addition and multiplication of the plaintext to be performed on the encrypted message, does not suffice in our setting for two reasons. One reason is that separate encryptions from each agent are involved and homomorphic encryption does not apply in that case. The second reason is that performing on encrypted data some of the mathematical operations involved in reforming the repo graph and into chains cannot be reflected in the plaintext under any known encryption scheme [Cite Source].
a proof of consistency. In order to reveal this information requires that someone is able to obtain it. We do not offer a recommendation of whom this party should be, but we mention a few options. One option is to have a secret key held in cold storage at a custodian or a court of law. In either of these cases, the enforcement mechanism would ultimately be legal enforcement. Another option would be to have multi-party signatures required to open the proof and transcript. The choice of parties would be tailored to the situation and objectives.

Auditing and Regulation

Regulators and named counterparties (such as e.g. prime brokers) are allowed to audit certain elements of an agent’s account balances, messages, contracts and transactions. The auditor needs assurance that the information provided is accurate. The agent needs assurance that the auditor observes only the data it is legally entitled to obtain. These two requirements can be achieved with zero-knowledge proofs. The protocol for verification of plaintext data is for the auditor to issue a query to the agent, such as “What volume of $M$ did you receive today” or “what volume to $M$ are you required to pay tomorrow?” or “what volume of $T$ are you holding at time $t$? The details of the design will be tailored to the market environment. An auditor will hold a token or other object that verifies its identity and the information it is entitled to see. Upon presentation of the query, the information will be disclosed along with a proof of its validity. The zkLedger paper by Narula et.al (28) provides an audit mechanism using zero-knowledge proofs in a market where cash is moved between bank accounts. The repo market is a somewhat different environment, but the query and proof mechanism is similar. By making the requisite enquiries to the agents, a regulator can obtain complete, timely and accurate information on the data it has a legal right to collect.31

Figure 12 displays the framework for privacy and auditing outlined above. Agents send and receive encrypted messages with the smart contract. The operations of netting initial repo contracts and reforming transactions into chains takes place in a trusted execution environment and auditors are able to receive timely and accurate reports of selected information.  

31A refusal by an agent to provide the requisite proof is irrefutable evidence of a violation by the agent, which the regulator can present in a court of law. Moreover, a secret key to decrypt could be deposited with the court, with cryptographic safeguards such as a requirement for multiple signatures from different parties to open the plaintext.
Conclusion

We provide a new approach to increasing intermediation capacity in the repo market. The RRC SC reduces the balance-sheet impact of repo volume on the balance-sheets of broker-dealer subsidiaries of regulated banks, which increases the volume of repo they can transact. Our approach does not require a relaxation of bank capital regulations nor does it compel agents to engage in central clearing and incur a mutualization of risk. More generally, the RRC SC demonstrates that an algorithmic smart contract operating on a distributed ledger can support a market where agents are free to choose their counterparties while providing the regulator with timely information on market conditions. This feature underscores that when financial objects are appended to distributed ledgers it becomes possible to reconcile decentralization with efficient regulatory oversight.

References


[29] Office of Financial Research US Department of Treasury
OfficeofFinancialResearchUSDepartmentofTreasury


A Lehman and MF Global Repo Strategies

In this appendix we discuss the strategies employed by Lehman and MF Global to use repo transactions to conceal risk positions.

A.0.1 Lehman’s Repo 105

The investment bank Lehman Brothers devised a transaction structure, called repo 105, which it employed around financial disclosure dates for several years prior to its 2008 bankruptcy. The maneuver involved an arbitrage between US and UK accounting rules. Lehman executed the repo transaction in the UK through an offshore subsidiary. The first leg was treated as a final sale under UK law. The reporting of the final sale was consolidated up to the US holding company balance sheet. With repo 105 Lehman used its holdings of subprime mortgage securities as collateral and applied the proceeds of the first leg sale to pay down debt. This enabled Lehman to simultaneously conceal its subprime exposure and to understate its indebtedness. Between the first-leg and the second-leg, Lehman’s subprime securities were owned by its counterparty, so they were no longer recorded on Lehman’s balance sheet during that time interval. Meanwhile, Lehman received a payment of $M\xi$ at the first leg, which it used to pay down lines of credit. As a consequence, by timing repo 105 so that its quarterly release of financial information occurred between the first and second leg, Lehman was able to report a balance sheet with a lower volume of subprime securities holdings and less debt than would be the case after the second leg (which it had a legal obligation to complete). It is estimated that, for several years prior to its bankruptcy, Lehman’s use of repo 105 enabled it to under-report its holdings of subprime securities and its debt by approximately $50 billion.\footnote{For a description of how repo 105 worked see Chang et.al (2011) (8) and Hartwell (2016) (22).}

A.0.2 MF Global’s Repo-to-Maturity Program

In a repo-to-maturity transaction, the lender collects the payoff from the security issuer on the maturity date. The borrower is only obligated to make a payment in the event the security issuer defaults on its obligation to repay. For two years prior to its 2011 bankruptcy, MF Global entered into repo-to-maturity transactions to conceal from investors and regulators its exposure to low-rated sovereign debt. An example of the strategy worked as follows. MF Global draws on a line of credit to purchase a risky high yielding Greek sovereign bond for a price of $M\xi$. The purchase price of the bond is below its payoff at maturity, which is $M+\xi$ (we normalize the interest rate to zero for simplicity). The discount on the purchase price is $\xi$. Shortly thereafter, MF Global enters into a repo transaction in which it sells the Greek bond in the first leg for a price of $M+\frac{1}{2}\xi$ and sets the second leg date to match the maturity date of the Greek bond. MF Global is able to sell the bond for a higher price than it paid because it protects its counterparty against default risk by agreeing to periodically pay into a margin account sufficient $M$ to cover the implied risk of loss reflected in the cost of credit default swaps linked to the Greek bonds. MF Global uses the first leg sale proceeds to pay down the line of credit it used to acquire the Greek bonds and retains a profit equal to a portion of the discount on its purchase, $\frac{1}{2}\xi$. The repo-to-maturity accounting exempts MF Global from recording the second leg transaction on its balance sheet. In addition, since the margining obligation is off-balance sheet, it
does not get reported. The repo-to-maturity strategy unraveled in 2011 when the implied risk on Greek bonds increased, which required MF Global to make large margin payments. The need for cash ultimately resulted in the misappropriation of millions of dollars from customer accounts, and the company filed for bankruptcy.

B Ultimate Repo Lender and Borrower Demand and Supply Functions

In this appendix we derive the concave demand and supply functions for the ultimate repo lenders (money managers) and borrowers (risk managers) that appeared in the analysis of capital regulations in Section 4.3. After labeling agent net repo lending as \( MM_i \) and agent net repo borrowing as \( RM_j \), all repo transaction chains run between these two types of agents.

B.1 The Money Manager \( MM \) Demand for \( T \)

\( MM_i \) invests \( M \) as an agent for investors whom we call cash pools ("CP"’s). A CP is an individual or entity acting in the capacity of allocating \( M \) that has been designated to be invested in safe, liquid assets such as insured bank accounts, commercial paper, short-term government debt and repo. An insured bank account is a safe asset and it is the most liquid because all purchases are made by transferring a commercial bank deposit. A \( T \) overnight repo ranks second in terms of safety and liquidity; the overnight repayment minimizes interest rate risk and the repo lender can sell the \( T \) immediately upon counterparty default in the secondary market for \( T \), which is highly liquid. We model the CP’s investment problem assuming that it already holds the maximum amount of insured bank deposits. This simplification is justified by the observation that, in aggregate, CP’s hold bank deposits above the maximum insured limit, which is the product of the per bank insured deposit limit and the number of bank. We aggregate securities other than \( T \) repo into a single alternative denoted by \( D \), which offers a (risky) return of \( r_D \). Normalizing the \( T \) in a repo transaction to be risk-free enables us to model the objective of the CP as a quasi-linear function. The \( T \) repo enters linearly and the risky alternative security enters as a concave function.\(^{35}\)

\(^{33}\)For a detailed examination of the motives and investment options faced by CP’s, see Pozzar (2014) (30).

\(^{34}\)See Poszar (2011) (31).

\(^{35}\)The expression \( r_{MM}^T \) in the utility function represents rate of return \( \times \) investment, where rate of return \( = \frac{p_f^T - p_t^T}{p_t^T} \) and investment \( = p_t^T \).
B.1.1  \( MMi \)'s Demand Function for Treasury Repo

\( MMi \)'s CP chooses an allocation of investment between Treasury repo (represented by \( T^* \)) and other securities (represented by \( D^* \)).

\[
\argmax_{T^*, D^*} U(r_{MMi}p_T T^*, r_D D^*) = r_{MMi}p_T T^* + \log(1 + r_D D^*)
\]

subject to \( p_T T^* + D^* = M_i \theta_{MMi} \)  \( (8) \)

Concavity implies that an incremental increase in holdings of \( D \) above some level increases utility by less than an incremental increase in \( T \) repo, even when the return offered on \( D \), \( r_D \), exceeds the return on \( T \) repo, \( r_{MMi} \). This reflects the risk aversion of the \( CP \). The \( CP \)'s problem is bounded by the restriction \( p_T T^* + D^* = \bar{M}_i \theta_{MMi} \), which is the total \( M \) it allocates between securities, multiplied by the shock \( \theta_{MMi} \sim f_{MMi} \). Substituting the constraint into the utility function and expressing \( D \) in terms of \( T \), the \( CP \)'s problem becomes

\[
\argmax_{T^*} r_{MMi}p_T T^* + \log(1 + r_D [\bar{M}_i \theta_{MM} - p_T T^*])
\]

This yields \( MMi \)'s demand function for Treasury repo,

\[
D_{r_{MMi}} = T^* = \frac{1}{p_T} \left[ \frac{1}{r_D} + \frac{\bar{M}_i \theta_{MMi}}{p_T \bar{M}_{MMi}} \right] - \frac{1}{p_T \bar{M}_{MMi}} \quad (9)
\]

\( D_{MMi} \) is a concave increasing function of \( r_{MMi} \). \( ^{37} \)

\( ^{36} \)The expression \( r_{MMi}p_T T \) in the utility function represents rate of return \( \times \) investment, where rate of return = \( \frac{p_T - p_T}{p_T} \) and investment = \( p_T T \).

\( ^{37} \)One observation from industry participants is that an \( MM \) will at times make an allocation of \( M \) to repo which they expect their \( BD \) counterparties to intermediate. They also stated that they are not price sensitive (within a certain range). We can capture this in the \( MM \) demand function by multiplying the repo rate term by a scaler \( \Omega \in (0,1] \), which reduces the sensitivity of demand to the repo rate. We re-scale the \( MMi \) demand function, equation 9 as

\[
D_{r_{MMi}} = T^* = \frac{1}{p_T} \left[ \frac{1}{r_D} + \frac{\bar{M}_i \theta_{MMi}}{p_T \bar{M}_{MMi}} \right] - \Omega \times \frac{1}{p_T \bar{M}_{MMi}} \quad (2')
\]

The curvature properties of the demand function remain unchanged.