Online Appendix for:
SVAR (Mis-)Identification and the Real Effects of Monetary Policy Shocks

Christian K. Wolf

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This online appendix contains supplemental material for the article "SVAR (Mis-)Identification and the Real Effects of Monetary Policy Shocks". I provide (i) a sketch of other structural models used for further robustness exercises, (ii) a detailed discussion of the VAR-DSGE mapping, in particular under non-invertibility, (iii) further details on near-invertibility in the structural model of Smets & Wouters (2007), (iv) supplementary results on sign-identified sets in my monetary policy application, (v) more details on the role of relative shock persistence for the estimands of recursively identified SVARs, (vi) a discussion of the price puzzle and passive monetary policy, and (vii) further computational details for all experiments in the paper. The end of this appendix contains further proofs and auxiliary lemmas.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "B." refer to the main article.

Contents

B.1 Further Model Details

In addition to the benchmark model laboratories sketched in Section I.A, I also study SVAR estimands in (i) an extended version of the basic three-equation model and (ii) various models with passive monetary policy rules (Castelnuovo & Surico, 2010; Leeper & Leith, 2016).

B.1.1 The extended three-equation model

The extended three-equation model features persistence in the Taylor rule of the monetary authority and the three structural disturbances:

$$
\begin{aligned}
y_t &= \mathbb{E}_t\left(y_{t+1}\right) - \frac{1}{\gamma}\left(i_t - \mathbb{E}_t\left(\pi_{t+1}\right)\right) + \sigma^d \omega_t^d && \text{(IS)} \\
\pi_t &= \beta \mathbb{E}_t\left(\pi_{t+1}\right) + \kappa y_t - \sigma^s \omega_t^s && \text{(NKPC)} \\
i_t &= \phi_i i_{t-1} + (1-\phi_i) \times \left(\phi_\pi \pi_t + \phi_y y_t\right) + \sigma^m \omega_t^m && \text{(TR)}
\end{aligned}
$$

where $\omega_t^d = \rho^d \omega_{t-1}^d + \sigma^d \varepsilon_t^d$, $\omega_t^s = \rho^s \omega_{t-1}^s + \sigma^s \varepsilon_t^s$, $\omega_t^m = \rho^m \omega_{t-1}^m + \sigma^m \varepsilon_t^m$, and again $(\varepsilon_t^d, \varepsilon_t^s, \varepsilon_t^m) \sim N(0, I)$. This model is not purely static anymore; in fact, it is easy to show that the observable macro aggregates admit a VAR(1) representation. The proof of Proposition B.1 characterizes the model solution in a special case with equal shock persistence.

The model parameterization used for my numerical exercises in Section B.4.5 is summarized in Table B.1. The Phillips curve parameter κ in (NKPC) is related to the fundamental parameters shown in the table as $\kappa \equiv \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p} \times (\gamma + \varphi)$.

B.1.2 Passive monetary policy rules

For all models considered in the main text, I implicitly assume an active monetary policy rule. In this Online Appendix I consider two alternative model closures.

(i) Active-fiscal, passive-money. Similar to Leeper & Leith (2016), I append the standard three-equation New Keynesian block by a set of equations summarizing the evolution and valuation of government debt. Suppose that government debt decays at a constant rate of ρ per period. Then no-arbitrage on bonds links government debt prices p_t^m to the one-period nominal interest rate as follows:

$$
p_t^m = -i_t + \beta \rho \mathbb{E}_t p_{t+1}^m
$$

Parameterization of Three-Equation Model

Parameter	Interpretation	Value
β	discount factor	0.995
γ	CRRA coefficient	1
φ	Labor preference curvature	1
θ_p	Calvo parameter	0.75
ϕ_π	inflation coefficient	1.5
ϕ_y	output coefficient	0.1
ϕ_i	rate persistence	0.9
ρ^d	persistence demand shock	0.8
ρ^s	persistence supply shock	0.9
ρ^m	persistence monetary policy shock	0.2
σ^d	std demand shock	1.60
σ^s	std supply shock	0.95
σ^m	std monetary policy shock	0.23

Table B.1: All parameters are chosen in line with standard practice in the New Keynesian literature. The volatility ratios are selected to exactly mirror those in the mode parameterization of Smets & Wouters (2007).

Let b_t denote real government debt, and let s_t denote the fiscal surplus. The linearized government flow budget constraint then is

$$\beta(1-\rho)p_t^m + \beta b_t + (1-\beta)s_t + \pi_t = b_{t-1}$$

I assume that the surplus follows

$$s_t = \phi_b b_t + \rho_b s_{t-1} + \omega_t^b$$

where again ω_t^b follows an AR(1) process.

In the standard active-money, passive-fiscal regime, the monetary authority responds strongly to inflation (ϕ_π is large) and the fiscal authority passively adjusts the surplus to ensure debt sustainability (ϕ_b is large). In that case, the model equations describing the pricing and evolution of government debt are completely separate from the rest of the economy, so I can focus on the usual three-equation model.

For the extended active-fiscal, passive-money model I instead set $\phi_\pi = 0.8$ (passive money), $\phi_b = 0$ (active fiscal), $\rho = 0.95$ (corresponding to an average maturity of government debt of 5 years), $\rho_s = 0.8$ and $\sigma_s = 25$, all chosen to get impulse responses similar to the benchmark of Leeper & Leith (2016).

(ii) Passive-money, equilibrium selection. As a second example of a model with passive monetary policy, I replicate the benchmark environment of Castelnuovo & Surico (2010), at their preferred parameterization. They consider a three-equation model similar to mine, but with passive monetary policy and without an active fiscal block. As a result the equilibrium is indeterminate; the indeterminacy is resolved using the equilibrium selection strategy of Lubik & Schorfheide (2004).

B.2 The VAR-DSGE mapping

The analysis in this paper mostly relies on population VAR(∞) representations. This section first discusses the mapping from linear state-space model to the reduced-form VAR(∞); at the end, I briefly sketch how I use the state-space model to derive the induced VAR(p) representations necessary for the $R^2(p)$ computations in Section B.3.

VAR(∞). This section heavily draws on Fernández-Villaverde et al. (2007), Lippi & Reichlin (1994), Anderson et al. (1996) and Ljungqvist & Sargent (2012); I thus keep it purposefully brief. Consider again the linear Gaussian state-space system (2) - (3). The time-invariant innovations representation of this state-space system is[B.1]

$$\hat{s}_t = A\hat{s}_{t-1} + Ku_t \tag{B.1}$$

$$x_t = C\hat{s}_{t-1} + u_t \tag{B.2}$$

where $\hat{s}_t$ is the Kalman-filtered estimate of s_t given information on the observables up to time t, $u_t \equiv x_t - C\hat{s}_{t-1} = C(s_{t-1} - \hat{s}_{t-1}) + D\varepsilon_t$ is the Gaussian forecast error for observables and satisfies $\mathbb{E}[u_t] = 0$, $\Sigma_u \equiv \mathbb{E}[u_t u_t'] = C\Sigma_s C' + DD'$ as well as $\mathbb{E}(u_t u_{t-j}') = 0$ for $j \neq 0$, and the matrices K, Σ_s satisfy the Riccati equations

$$\Sigma_s = (A - KC)\Sigma_s(A - KC)' + BB' + KDD'K' - BD'K' - KDB' \tag{B.3}$$

$$K = (A\Sigma_s C' + BD')(C\Sigma_s C' + DD')^{-1} \tag{B.4}$$

Sufficient conditions for the existence of this time-invariant innovations representation are that (A', C') is stabilizable and that (A', B') is detectable. For further discussion I refer the interested reader to Anderson et al. (1996); I will simply assume existence from now on.

From the innovations representation we get an MA(∞) representation for x_t:

$$x_t = \left[I + C(I - AL)^{-1}KL\right] u_t \iff \left[I - C\left[I - (A - KC)L\right]^{-1} KL\right] x_t = u_t \tag{B.5}$$

[B.1] Relative to Anderson et al. (1996) and Ljungqvist & Sargent (2012), I slightly change the timing convention, using lagged aggregate states s_{t-1} in the observation equation. It is straightforward to adapt their analysis to my state-space system; see e.g. Ljungqvist & Sargent (2012, Exercise 2.22).

or, assuming that $A - KC$ is stable, the desired VAR representation

$$x_t = \sum_{j=1}^{\infty} C(A - KC)^{j-1} K x_{t-j} + u_t \tag{B.6}$$

If the state-space system is invertible for the hidden states then $\Sigma_s = 0$, so $K = BD^{-1}$, and straightforward manipulations give

$$x_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1} BD^{-1} x_{t-j} + u_t \tag{B.7}$$

VAR(p). Let $X_t(p) = [x_t', x_{t-1}', \dots, x_{t-p}']'$. To derive the model-implied VAR(p) representation, I use the Wold MA representation (B.5) to construct the variance-covariance matrix $\Sigma_x(p) \equiv \mathbb{E}(X_t(p)X_t(p)')$. From this covariance matrix, it is straightforward to derive the implied VAR(p) representation (see e.g. Kilian & Lütkepohl, 2017).[B.2] Similar covariance calculations for $[X_t(p)', \varepsilon_t']'$ immediately give the desired R^2's.

[B.2] Of course, the residuals implied by this (counterfactual) "fitted" VAR(p) model are generically not white noise, unlike the u_t's defined above.

B.3 Near-Invertibility in Smets & Wouters (2007)

The near-invertibility results for the model of Smets & Wouters (2007) discussed in Section I.C are neither sensitive to the implausible assumption of a VAR(∞), nor are they a special feature of the model's posterior mode.

Figure B.1 shows the $R^2_{0,m}$ for different finite-order VAR(p)'s in the structural model of Smets & Wouters (2007). The plot shows that the high benchmark $R^2_{0,m}$ is not sensitive to the assumption of infinitely many lags; instead, the first few lags already reveal most information about the unknown monetary policy shocks. These results agree exactly with the intuition given in Plagborg-Møller & Wolf (2019, Section 2.4).

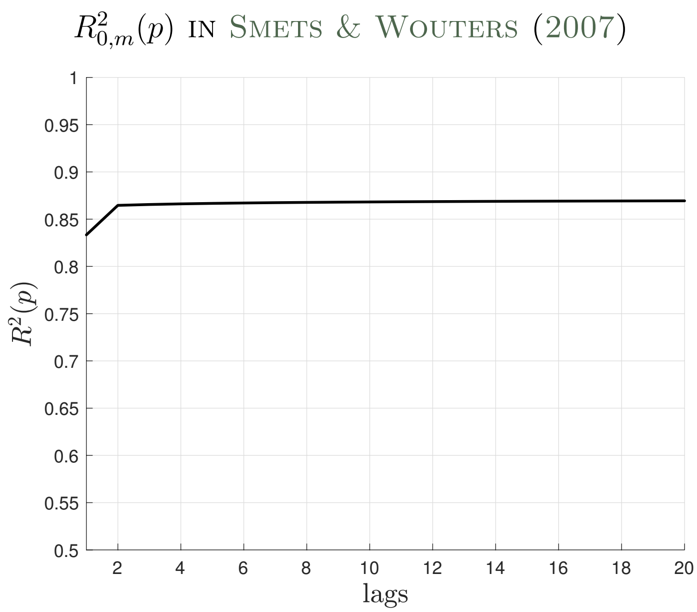

Figure B.1: $R^2_{0,m}(p)$ in the structural model of Smets & Wouters (2007), computed using the Wold-implied VAR(p) variance-covariance matrix, as outlined in Section B.2.

Next, Figure B.2 shows that the $R^2_{0,m}$ is high for most draws from the model's posterior. The orange line shows a kernel estimate of the posterior density of the maximal available shock weight $\sqrt{R^2_{0,m}}$ for the trivariate VAR, while the black line does the same for the larger five-variable VAR considered in the external-IV analysis of Section IV.B. In both cases, the maximal available weight is close to 1 for most draws from the model's posterior, so VAR inference can in principle succeed.

Since the R^2 in Figure B.1 does not rise much after the first few lags, and since shock weights are bounded by incremental R^2 values (recall Proposition 1), we would expect weights

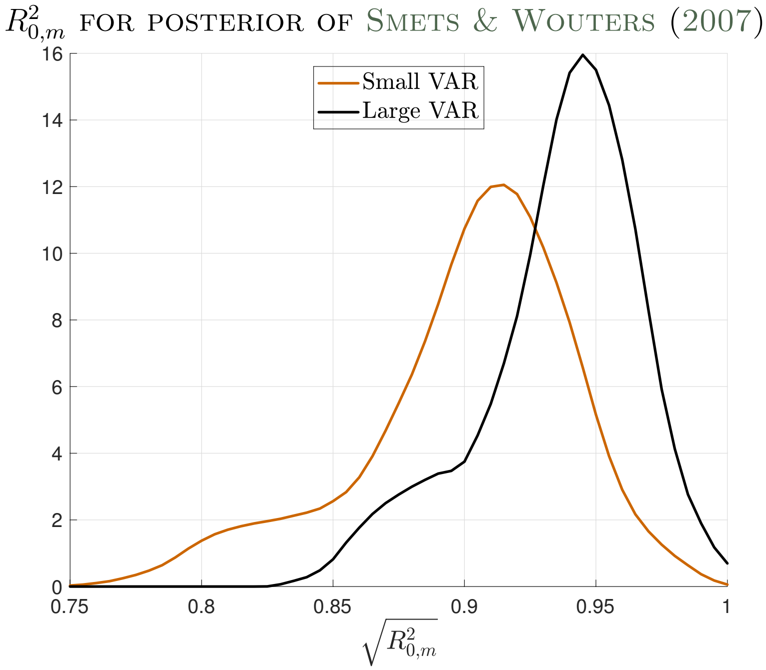

Figure B.2: I draw 1,000 times from the model's posterior, and for each draw compute the implied $R^2_{0,m}$. Kernel density estimation with a standard Epanechnikov kernel.

on lagged shocks in the polynomial (8) to be small. Figure B.3 shows that this is indeed the case for sign-identified sets in the model of Smets & Wouters.

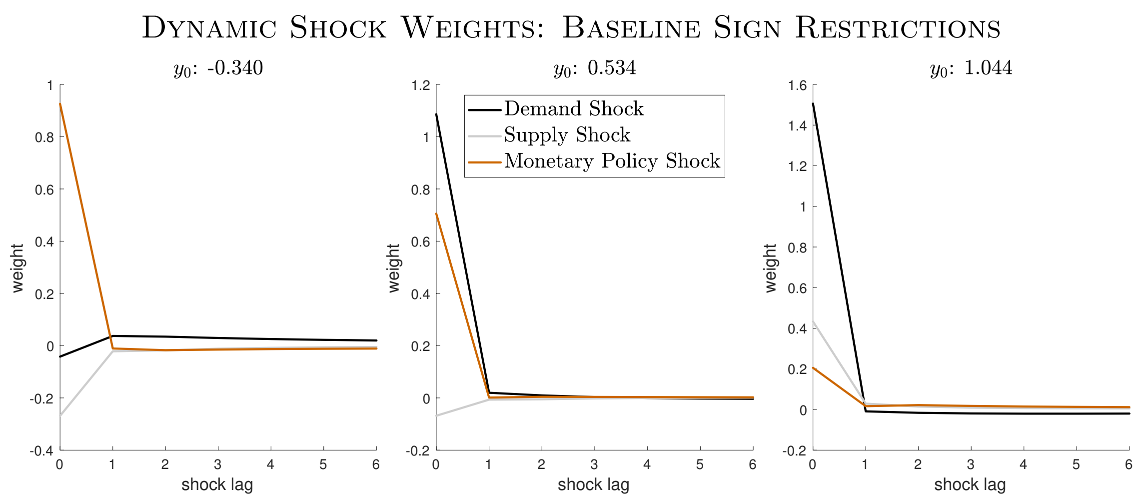

Figure B.3: Identified set of dynamic disturbance vectors (shock weights over time) as a function of the output response at horizon 0.

Given this extremely fast decay of weights, it is nearly sufficient to just look at the coefficients in the first entry P_0, as done for most of the plots displayed in Sections II to IV.

B.4 Identified sets with sign restrictions

This section complements the analysis in Sections II and III. I (i) complete the proof for the ambiguous output response in the static three-equation model, (ii) show a plot of unsmoothed shock weights for the model of Smets & Wouters (2007), (iii) show a plot for the identified set when multiple shocks are identified simultaneously, (iv) characterize identified sets for draws from the estimated posterior of the model, (v) show identified sets for the dynamic three-equation model, and (vi) show identified sets for sign restrictions combined with bounds on the impact monetary policy multiplier $\frac{dy_0}{di_0}$.

B.4.1 Ambiguous output response in static model

The response of output to a sign-identified "monetary policy shock" in the static three-equation model is ambiguous. To prove this, I first show that the structural impact matrix in Equation (1) is full rank. Its determinant is

$$-\sigma^d \sigma^m \sigma^s \times (1 + 2\kappa\phi_\pi + \kappa^2\phi_\pi^2 + 2\phi_y + 2\kappa\phi_\pi\phi_y + \phi_y^2)$$

which is guaranteed to be strictly negative in the region of the parameter space that I consider, as claimed. It is thus now immediate that there exists a vector $\tilde{p}_1$ such that

$$\frac{1}{1+\phi_y+\phi_\pi\kappa}\begin{pmatrix} \sigma^d & \phi_\pi\sigma^s & -\sigma^m \\ \kappa\sigma^d & -(1+\phi_y)\sigma^s & -\kappa\sigma^m \\ (\phi_y+\phi_\pi\kappa)\sigma^d & -\phi_\pi\sigma^s & \sigma^m \end{pmatrix} \times \tilde{p}_1 = \begin{pmatrix} \tilde{y}_{0,m} \\ \tilde{\pi}_{0,m} \\ \tilde{i}_{0,m} \end{pmatrix}$$

where $(\tilde{y}_{0,m}, \tilde{\pi}_{0,m}, \tilde{i}_{0,m})$ are some arbitrary numbers such that $\tilde{y}_{0,m} > 0$, $\tilde{\pi}_{0,m} < 0$ and $\tilde{i}_{0,m} > 0$, and similarly that there exists a vector $\tilde{p}_2$ such that $\tilde{y}_{0,m} < 0$, $\tilde{\pi}_{0,m} < 0$ and $\tilde{i}_{0,m} > 0$. Now set $p_1 \equiv \frac{\tilde{p}_1}{||\tilde{p}_1||}$ and $p_2 \equiv \frac{\tilde{p}_2}{||\tilde{p}_2||}$; clearly p_1 and p_2 lie in the identified set and give the desired positive and negative output responses, respectively.

B.4.2 Unsmoothed shock weights

The shock weights displayed in Figure 2 are smoothed. Intuitively, a given impact output response can presumably be constructed using different orthogonal rotation matrices and so different SVARs in the identified set. As a result, plots of sampled shock weights from the identified set of SVARs are likely to look somewhat erratic. Figure B.4, which plots untreated raw shock weights against their corresponding impact output response, provides an illustration. Unsurprisingly, the shock weight series are somewhat noisier than in Figure 2, but the basic message is unchanged.

IDENTIFIED SET OF SHOCK WEIGHTS: UHLIG (2005) SIGN RESTRICTIONS

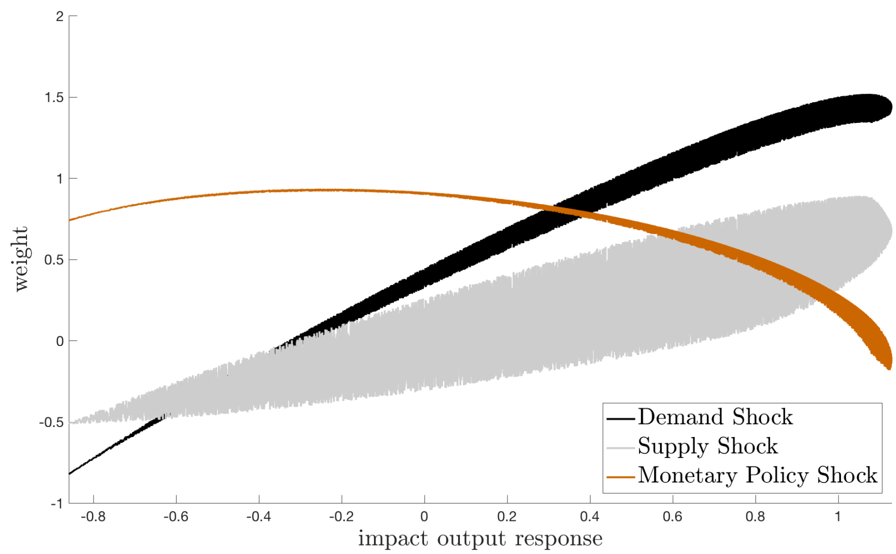

Figure B.4: Identified set of unsmoothed static shock weights as a function of the output response at horizon 0. I display results for 20,000 draws from the model's posterior.

B.4.3 Simultaneous shock identification

A potential remedy to the masquerading problem is the simultaneous identification of multiple structural shocks, as in fact first advocated in Uhlig (2005). Intuitively, by requiring the identified shock to be orthogonal to other identified shocks that have the properties of demand and supply shocks, the masquerading problem may become less severe. As Figure B.5 shows, however, this is not the case in the monetary policy shock application.[B.3]

The identified set is almost as wide as in my benchmark analysis; plots of shock weights reveal that the underlying masquerading story also survives unchanged.

[B.3] I am not able to prove convexity of the identified set in this application. As a result, the plots should strictly speaking be interpreted as the convex hull of the identified set. My draws from the identified set however suggest that it is indeed convex everywhere.

Identified Set of Impulse Responses: Multiple Shocks

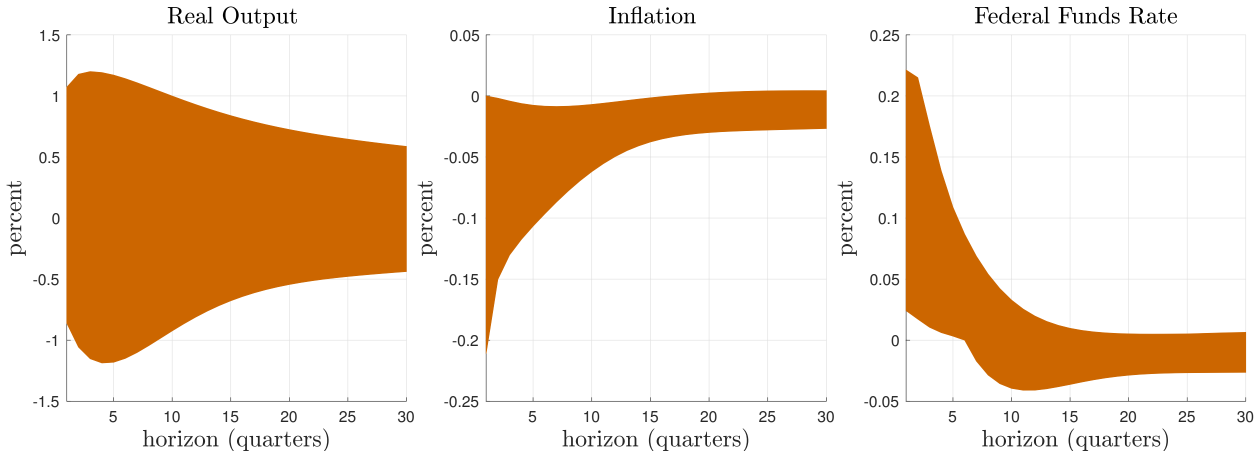

Figure B.5: See Figure 1. Additionally monetary policy shock vectors now need to be orthogonal to shock vectors that move output, inflation and interest rates up (demand shocks) as well as shocks that move output up and inflation and interest rates down (supply shocks), again for six quarters.

B.4.4 Posterior draws from Smets & Wouters (2007)

Figure B.6 shows the model-implied posterior distribution over the sign-identified output response to a monetary policy shock.

Identified Set of Impulse Responses: Uhlig (2005) Sign Restrictions

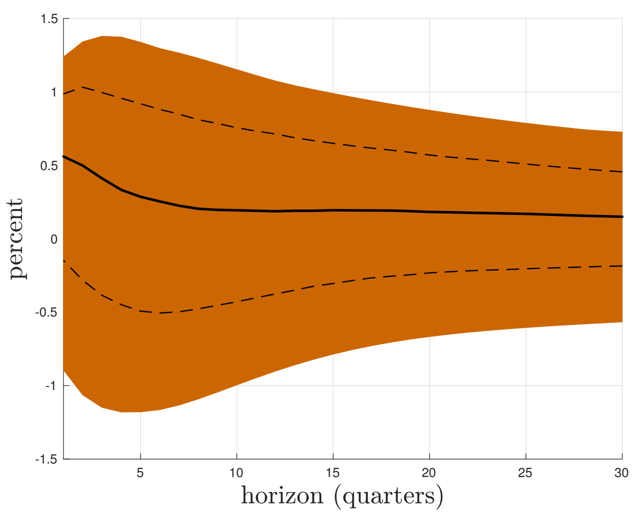

Figure B.6: Identified set for output, identifying restrictions as in Figure 1. Posterior uncertainty via uniform Haar prior and draws from the model posterior (1,000 draws).

I construct Figure B.6 as follows. First, I draw 1,000 times from the model's posterior.

Second, for each posterior draw, I draw *once* from the Haar-implied Bayesian posterior over the identified set. The solid and dotted black lines show posterior 16th, 50th and 84th percentile bands for the response of output to an identified monetary policy shock, while the boundaries of the orange region give the largest and smallest responses across all posterior draws. The experiment is designed to emulate as closely as possible standard empirical practice. In particular, the displayed confidence bands are constructed to reflect posterior estimation and identification uncertainty exactly as done in popular Bayesian implementations of sign-identifying schemes. A frequentist approach (which would be more in keeping with my plots of full identified sets, as in Figure 1), would instead present confidence sets for full identified sets, constructed separately for each draw from the underlying structural model's posterior.

B.4.5 Dynamic three-equation model

Figure B.7 shows identified sets (and Haar-induced posterior distributions) in the dynamic three-equation model of Section B.1.1.

Identified Set of Impulse Responses: Uhlig (2005) Sign Restrictions

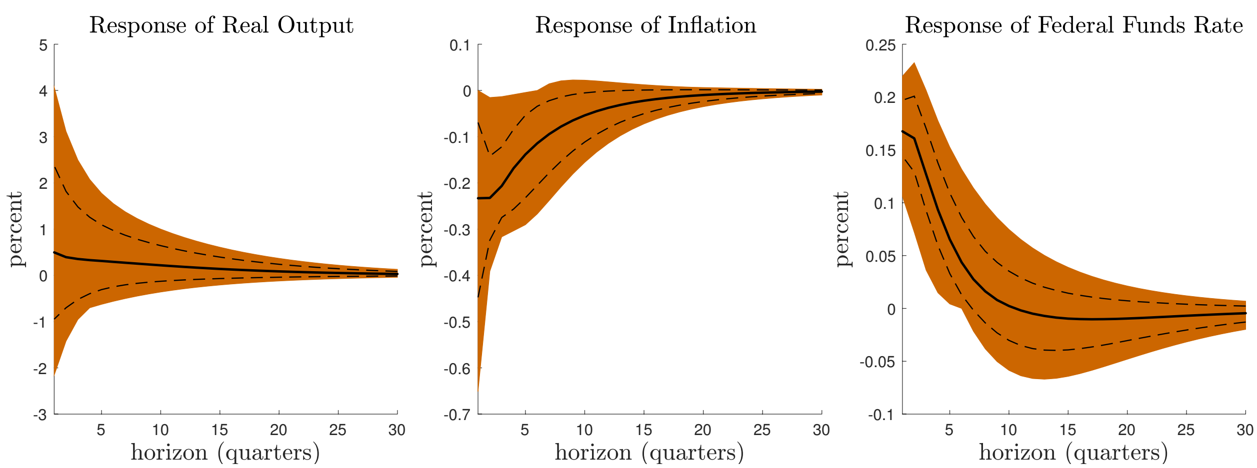

Figure B.7: Identified sets of the responses of output, inflation and policy rate to a one standard deviation shock to the monetary policy rule, identified through sign restrictions on inflation and policy rate (imposed for six quarters). Posterior uncertainty via Haar prior.

The conclusions echo those of Figure 1 for the Smets-Wouters model. First, identified sets are very wide, with both positive and negative output responses consistent with the imposed sign restrictions. Second, the Haar-induced posterior distribution over the identified set for output again assigns most mass to counterfactual positive output responses.

Figure B.8 shows that the economic interpretation of the displayed identified sets is also exactly as in the larger Smets-Wouters model. By invertibility, the true monetary policy shock lies in the identified set. As before, the long tail of mis-identified positive output responses corresponds to positive demand and supply shocks masquerading as contractionary monetary policy shocks.

Identified Set of Shock Weights: Uhlig (2005) Sign Restrictions

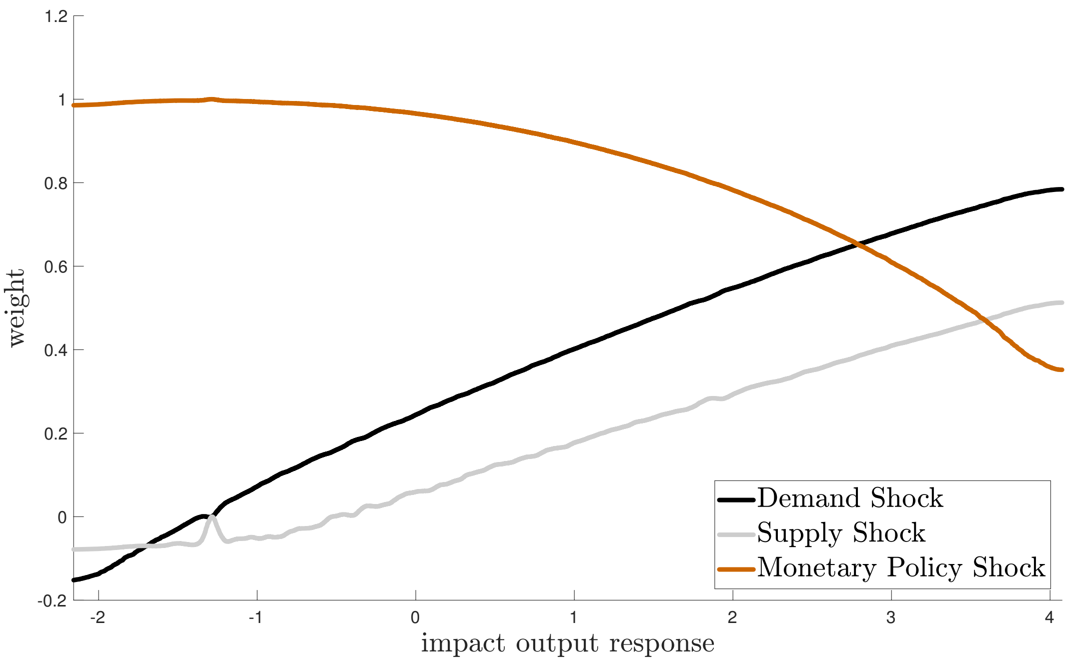

Figure B.8: Identified set of static shock weights as a function of the output response at horizon 0. The true impact response of output is -1.274. Note that I enforce the true shock vector $(0, 0, 1)$ to lie in the displayed (smoothed) identified set.

B.4.6 Bounds on impact multipliers

Figure 5 shows that an additional hard zero restriction on output is enough to tighten sign-identified sets around conventional negative output responses. Figure B.9 shows that, in fact, even weaker *bounds* on the impact multiplier $|\frac{dy_0}{di_0}|$ are enough to afford a substantial tightening. In particular, even for relatively weak bounds that are consistent with the large impact output response of Smets & Wouters (2007), the identified set for the output response after around one year only contains negative entries. The intuition is exactly analogous to that discussed in Section III.

Consistent with the empirical results in Uhlig (2005, Figure 12), similarly moderate

Identified Set of Impulse Responses: Uhlig (2005) + Quantity Bounds

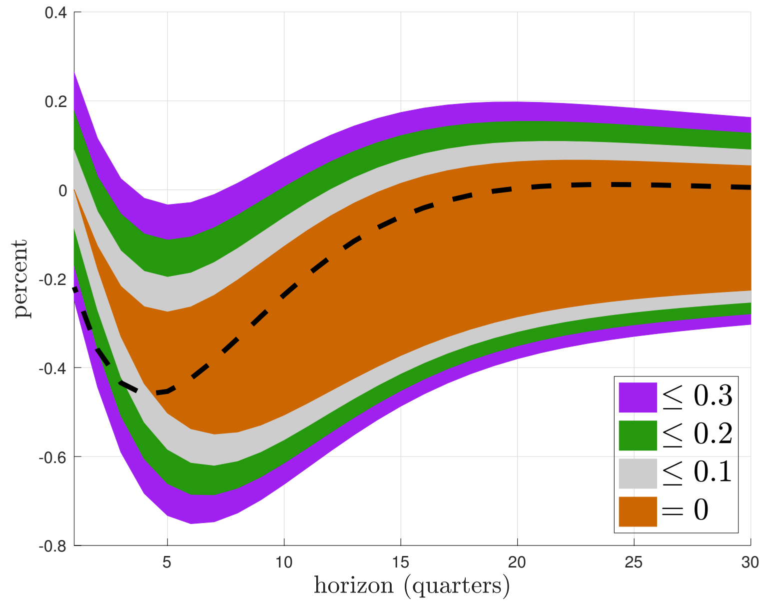

Figure B.9: Identified sets of output impulse responses; sign restrictions of Uhlig (2005) plus bounds on the impact output multiplier $|\frac{dy_0}{di_0}|$ (for the annualized nominal rate). The dotted line indicates the true output impulse response.

impact multiplier bounds are also sufficient to tighten *empirical* identified sets around significant negative output responses to identified monetary policy shocks. Theses results are discussed in the earlier working paper Wolf (2017).

B.5 Recursive identification and shock persistence

The long-horizon impulse response functions displayed in panel (a) of Figure 5 are largely dominated by the extremely persistent effects of underlying contractionary technology shocks. In this section I use a dynamic version of the three-equation model (sketched in Section B.1) to provide a formal explanation. In particular, I establish two results. First, shock weights are exclusively governed by *impact* impulse responses. Second, conditional on shock weights, dynamic impulse responses are governed exclusively by relative shock *persistence*.

Proposition B.1. *In the extended three-equation model, a recursive ordering, with the monetary policy equation ordered last, yields an identified monetary policy shock e_t^m and shock weights p_m with the following properties:*

(i) The shock weights depend only on the 3×3 *matrix of* impact *impulse response functions.*

(ii) If demand and supply shocks are as persistent as monetary policy shocks, then the recursively identified impulse response of inflation and output is 0 *at all horizons.*

Proof. Please see Section B.8. □

The stark dichotomy underlying Proposition B.1 is tied sensitively to the model's invertibility. However, since shock weights decay extremely fast in the structural model of Smets & Wouters (2007) (cf. the discussion in Appendix A.A2), the same intuition applies.

B.6 Price puzzle and passive monetary policy

This paper is largely concerned with the response of *output* to (mis-)identified monetary policy shocks. For the recursive identification scheme considered in Section III, another notable feature of empirical estimates is the so-called price puzzle, in particular for recursive VARs estimated on early samples (Christiano et al., 1996).

This section adds two further results related to the price puzzle and recursive identification in my controlled model environments. First, I show that, while the benchmark parameterization of the Smets-Wouters model does not generate a price puzzle, the estimated inflation response is only mildly negative, and in particular much less pronounced than the actual true inflation impulse response. Second, building on earlier results in Leeper & Leith (2016), I show that passive monetary policy can naturally generate the price puzzle in VAR estimands, and in particular does so while still generating a substantial under-estimate of the *real* effects of monetary policy, consistent with the main results in this paper.

Price Puzzle in Smets & Wouters (2007). Figure B.10 shows full identified sets for the zero and recursive identification schemes studied in Section III, applied to the structural model of Smets & Wouters (2007).

Identified Set of Impulse Responses: Zero Restrictions

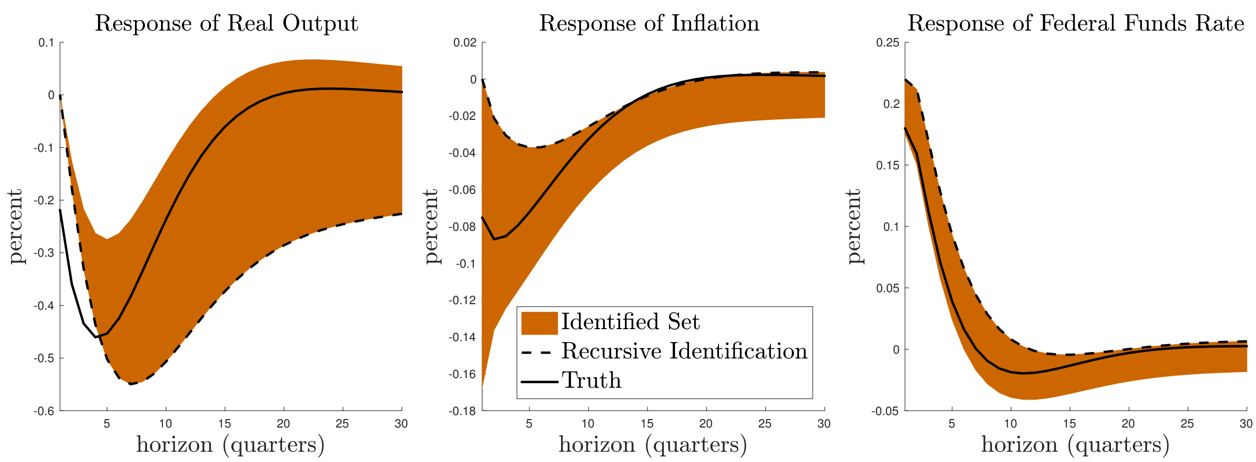

Figure B.10: Identified sets of output, inflation, and interest rates. Inflation and interest rates are restricted to move in opposite directions for six quarters; additionally, the impact output response is restricted to be 0. Dotted lines show the point-identified recursive impulse responses (with the policy shock ordered last) as well as the true impulse response.

The middle panel reveals that, while the model does not generate a price puzzle, the estimated inflation response is much less pronounced than the true model-implied IRF. Intuitively, to ensure a zero impact inflation response, the identified "monetary policy shock" is contaminated by an inflationary supply shock, thus pushing up the inflation response throughout. In particular, the recursive inflation response is the upper bound of the identified set under weaker zero-plus-sign restrictions, and thus unsurprisingly close to 0 throughout.

PASSIVE MONETARY POLICY. Leeper & Leith (2016) argue that, with passive monetary policy, the "price puzzle" need not be a puzzle after all – contractionary monetary policy shocks may actually push inflation up, and recursively identified SVARs may simply be picking up these true impulse responses. Since U.S. monetary policy is often argued to have been passive prior to Volcker disinflation – and since the price puzzle is particularly pronounced in such early sample periods –, this explanation is intuitively appealing. An important limitation, however, is that the analysis of Leeper & Leith (2016) is throughout concerned only with *true* impulse responses, and never asks whether recursive SVAR *estimands* also display the price puzzle.

To address this shortcoming, I solve the passive-money, active-fiscal model of Section B.1.2, and study the implied recursive SVAR estimand. Figure B.11 shows the results.

RECURSIVE IDENTIFICATION, ACTIVE-FISCAL PASSIVE-MONEY

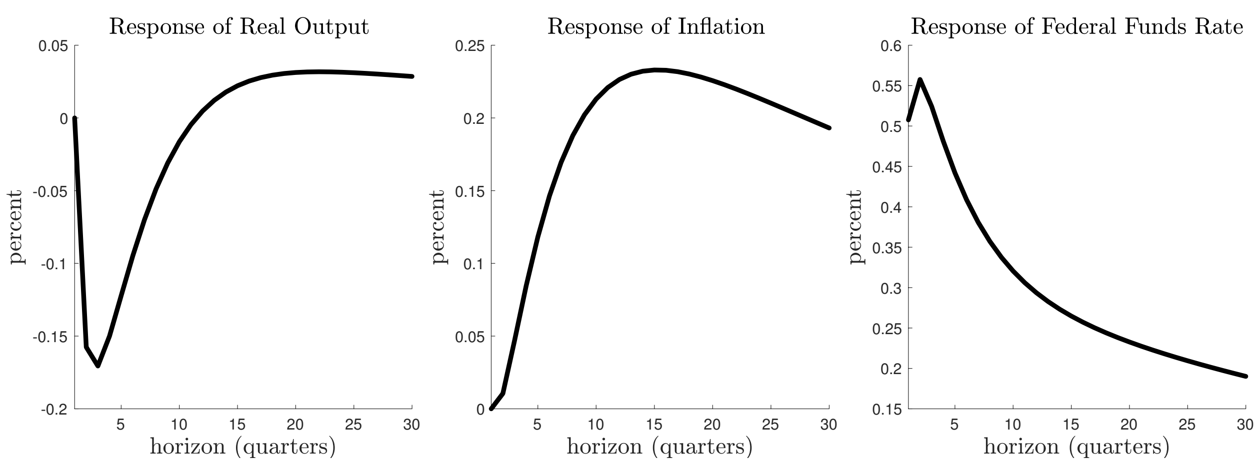

Figure B.11: Recursively identified impulse responses for output, inflation and interest rates in the passive-money, active-fiscal model.

Encouragingly, the estimated VAR displays a strong price puzzle – inflation is *estimated* to increase, and of course, as remarked by Leeper & Leith, it truly does increase even after actual monetary policy shocks. At the same time, output is still estimated to drop, as is also the case in the actual underlying model. The estimated output response is, however, again much smaller in magnitude than the actual output response (now by a factor of almost 10), consistent with the results displayed and the intuition given in Section III.

Castelnuovo & Surico (2010), in a somewhat different model environment and with a different solution strategy, also argue that passive monetary policy – in their case not complemented by an active-fiscal block – can generate a price puzzle in estimated recursive VARs. Figure B.12 replicates their simulation analysis in population, and indeed confirms that the population estimand also displays a price puzzle. More importantly for the purposes of this paper, the estimated negative output response is again much less pronounced than the true response.[B.4]

Recursive Identification, Castelnuovo & Surico (2010)

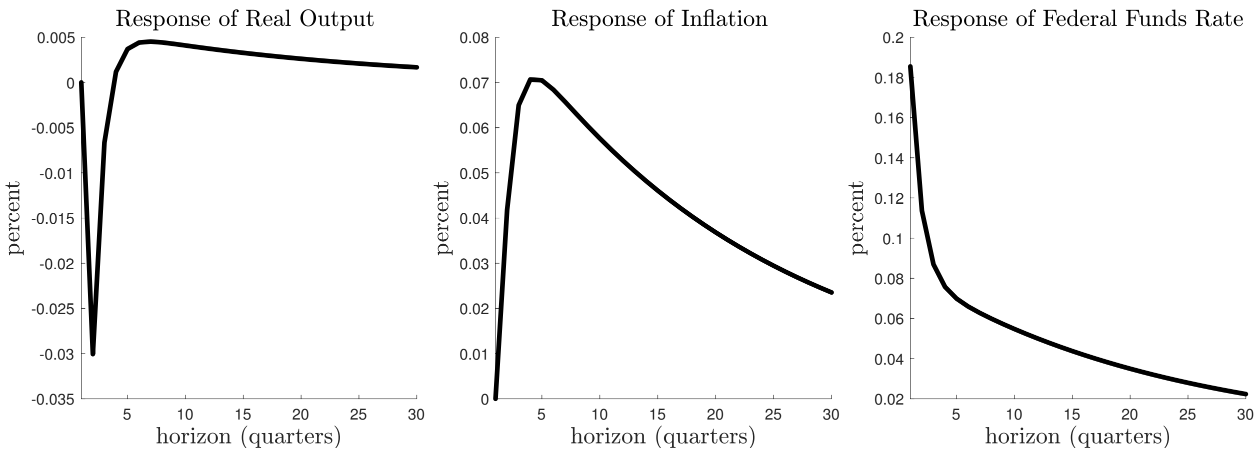

Figure B.12: Recursively identified impulse responses for output, inflation and interest rates in the model of Castelnuovo & Surico (2010).

[B.4] This holds not just (trivially) on impact, but along the entire path: The true peak output response is around five times larger than the estimated peak response.

B.7 Computational details

This section provides further details on the implementation of the computational experiments considered throughout the paper.

The Computational Experiment. Having specified a set of macro observables x_t, and given my assumption that the econometrician observes infinitely many data, I treat the model-implied reduced-form VAR representation (4) as known. In particular, I use the model-implied matrices (A, B, C, D) to derive the reduced-form VAR representation in line with (B.3) - (B.7). Given the reduced-form VAR representation, identified sets are defined as in Definition 1 and Definition 2, and shock weights follow using (6) or (8).

For all experiments involving external instruments, I add to the Smets-Wouters model an additional equation characterizing a new variable (the instrument) z_t, where

$$z_t = \alpha \varepsilon_t^m + \sigma_v v_t$$

with $\alpha > 0$ and $v_t \sim N(0, 1)$, independent of any other disturbances of the model. This artificial instrument evidently satisfies the required relevance and exclusion restrictions (Stock & Watson, 2017). In particular, because I only consider population limits, weak instrument issues are irrelevant, and so my analysis is independent of the chosen values for α and σ_v.

Numerical Details. To approximate the required VAR(∞) and VMA(∞) representations, I need to choose truncation lag lengths. I truncate the population VAR at horizon 250, and the VMA representation at horizon 350. Further increasing these horizons does not materially affect any results.

I numerically characterize sign-identified sets in line with established numerical practice. Specifically, I sample 20,000 times from the identified set using the techniques of Rubio-Ramírez et al. (2010). Such comprehensive sampling from the identified set is useful for my purposes since (i) I am intrinsically interested in the Haar-induced posterior distribution over the identified set and (ii) I need draws of shock weights over the identified set to construct my plots of the identified set of shock weights. I have verified in all cases that further increases in the number of sampled matrices leave the overall identified set unchanged.

B.8 Additional proofs and auxiliary lemmas

B.8.1 Proof of Proposition B.1

(i) We need to solve the system

$$\begin{pmatrix} 0 \\ 0 \\ \star \end{pmatrix} = IRF_0 \times \begin{pmatrix} p_{md} \\ p_{ms} \\ p_{mm} \end{pmatrix}$$

where IRF_0 is a 3×3 matrix of structural impact impulse responses and $\star$ is some arbitrary strictly positive number, ensuring that p has unit length. Solving this system, we see that the solution has the form

$$\begin{aligned} p_{md} &= \frac{1}{C} \times [\mathrm{IRF}_{\pi,s,0} \cdot \mathrm{IRF}_{y,m,0} - \mathrm{IRF}_{y,s,0} \cdot \mathrm{IRF}_{\pi,m,0}] \\ p_{ms} &= \frac{1}{C} \times [\mathrm{IRF}_{\pi,m,0} \cdot \mathrm{IRF}_{y,d,0} - \mathrm{IRF}_{y,m,0} \cdot \mathrm{IRF}_{\pi,d,0}] \\ p_{mm} &= \frac{1}{C} \times [\mathrm{IRF}_{\pi,d,0} \cdot \mathrm{IRF}_{y,s,0} - \mathrm{IRF}_{y,d,0} \cdot \mathrm{IRF}_{\pi,s,0}] \end{aligned} \tag{B.8}$$

for some $C = C(\star)$ In particular, the statement (i) of the proposition follows.

(ii) Consider first the case where $\rho^d = \rho^s = \rho^m = \rho$ and $\phi_i = 0$. Then the state variables of the system are $s_t = (\omega_t^d, \omega_t^s, \omega_t^m)'$ and the state-space representation has $A = \rho I$, $B = \mathrm{diag}(\sigma^d, \sigma^s, \sigma^m)$. For C and D, coefficient matching immediately gives $C = \rho D$. Thus, for a given orthogonal rotation matrix Q, the implied impulse response functions satisfy

$$IRF_h \times Q = \rho^h \times IRF_0 \times Q$$

But the impact impulse responses of output and inflation are 0 by construction, so the same holds at all future horizons. With $\rho^d = \rho^s = \phi_i = \rho$ and $\rho^m = 0$ the argument is completely analogous – only now i_{t-1} appears as an additional state variable, not ω_{t-1}^m.

$\square$

B.8.2 Auxiliary lemmas

Lemma B.1. *Fernández-Villaverde et al. (2007) The reduced-form residuals u_t satisfy*

$$u_t = \sum_{\ell=0}^{\infty} M_\ell \varepsilon_{t-\ell}$$

for some $n_x \times n_\varepsilon$ matrices $\{M_\ell\}$.

Proof. Recall that $u_t = C(s_{t-1} - \hat{s}_{t-1}) + D\varepsilon_t$. Using this, we can combine state-space representation and innovations representation to get

$$\begin{aligned} \begin{pmatrix} s_t \\ \hat{s}_t \end{pmatrix} &= \begin{pmatrix} A & 0 \\ KC & A - KC \end{pmatrix} \begin{pmatrix} s_{t-1} \\ \hat{s}_{t-1} \end{pmatrix} + \begin{pmatrix} B \\ KD \end{pmatrix} \varepsilon_t \\ u_t &= \begin{pmatrix} C & -C \end{pmatrix} \begin{pmatrix} s_{t-1} \\ \hat{s}_{t-1} \end{pmatrix} + D\varepsilon_t \end{aligned}$$

Solving the first equation for $(s_t, \hat{s}_t)'$ and plugging into the second we get the result:

$$u_t = \left\{ D + \begin{pmatrix} C & -C \end{pmatrix} \left(I - \begin{pmatrix} A & 0 \\ KC & A - KC \end{pmatrix} L \right)^{-1} \begin{pmatrix} B \\ KD \end{pmatrix} L \right\} \varepsilon_t$$

□

Lemma B.2. *All relevant information for inference on shock $\varepsilon_{j,t}$ is summarized in the contemporaneous reduced-form forecasting errors u_t.*

Proof. By invertibility of the reduced-form VAR for the Wold errors u_t, we know that

$$\text{Var}\left(\varepsilon_{j,t} \mid \{x_\tau\}_{-\infty<\tau\leq t}\right) = \text{Var}\left(\varepsilon_{j,t} \mid \{u_\tau\}_{-\infty<\tau\leq t}\right)$$

But by construction $\text{Cov}\left(\varepsilon_{j,t}, u_{t-\ell}\right) = 0$ and $\text{Cov}\left(u_t, u_{t-\ell}\right) = 0$ for all $\ell \geq 1$. Thus

$$\text{Var}\left(\varepsilon_{j,t} \mid \{x_\tau\}_{-\infty<\tau\leq t}\right) = \text{Var}\left(\varepsilon_{j,t} \mid u_t\right)$$

□

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