Unraveling versus Unraveling: A Memo on Competitive Equilibriums and Trade in Insurance Markets

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Abstract

Both Akerlof (1970) and Rothschild and Stiglitz (1976) show that insurance markets may "unravel". This memo clarifies the distinction between these two notions of unraveling in the context of a binary loss model of insurance. I show that the two concepts are mutually exclusive occurrences. Moreover, I provide a regularity condition under which the two concepts are exhaustive of the set of possible occurrences in the model. Akerlof unraveling characterizes when there are no gains to trade; Rothschild and Stiglitz unraveling shows that the standard notion of competition (pure strategy Nash equilibrium) is inadequate to describe the workings of insurance markets when there are gains to trade.

1 Introduction

Akerlof (1970) and Rothschild and Stiglitz (1976) have contributed greatly to the understanding of the potential problems posed by private information on the workings of insurance markets. Akerlof (1970) shows how private information can lead to an *equilibrium of market unraveling*, so that the only unique equilibrium is one in which only the worst quality good (i.e. the "lemons") are traded. Rothschild and Stiglitz (1976) show that private information can lead to an *unraveling of market equilibrium*, in which no (pure strategy) competitive equilibrium exists because insurance companies have the incentive to modify their contracts to cream skim the lower-risk agents from other firms.

While the term *unraveling* has been used to describe both of these phenomena, the distinction between these two concepts is often unclear, arguably a result of each paper's different approach to modeling the environment. Akerlof (1970) works in the context of a "supply and demand" environment with a fixed contract or asset (e.g. a used car), whereas Rothschild and Stiglitz (1976) work in the context of endogenous contracts in a stylized environment with only two types (e.g. high and low types).

This memo develops a generalized binary loss insurance model that incorporates the forces highlighted in both Akerlof (1970) and Rothschild and Stiglitz (1976). Using this unified model, I show that the *equilibrium of market unraveling* (in Akerlof) is a mutually exclusive occurrence from the *unraveling of market equilibrium* (in Rothschild and Stiglitz). Moreover, under the regularity condition that the type distribution either (a) contains a continuous interval or (b) includes $p = 1$, one of these two events must occur: either there is a Competitive (Nash) Equilibrium of no trade (Akerlof unraveling) or a Competitive (Nash) Equilibrium does not exist (Rothschild and Stiglitz unraveling). Thus, not only are these two concepts of unraveling different, but they are mutually exclusive and generically exhaustive of the potential occurrences in an insurance market with private information.

The mutual exclusivity result is more or less obvious in the canonical two-type binary loss model. The market unravels a la Rothschild and Stiglitz when the low type has an incentive to cross-subsidize the high type in order to obtain a more preferred allocation. This willingness of the good risk to subsidize the bad risk is precisely what ensures the market will not unravel a la Akerlof. Conversely, if the market unravels a la Akerlof, then the good risk is not willing to subsidize the bad risk, which implies an absence of the forces that drive non-existence in Rothschild and Stiglitz.

The intuition for the exhaustive result is also straightforward, but perhaps more difficult to see in the context of the stylized two-type model. When the support of the type distribution either (a) contains an interval or (b) contains the point $p = 1$, then trade necessarily involves cross-subsidization of types.[1] But, Rothschild and Stiglitz (1976) show that a competitive (Nash) equilibrium cannot sustain such cross-subsidization. Hence, if agents are willing to provide trade then the market unravels a la Rothschild and Stiglitz. In contrast, if no agents are willing to cross-subsidize the worse risks in the population, then there exists a unique Nash equilibrium at the endowment: no one on the margin is willing to pay the average cost of worse risks, and any potential contract (or menu of contracts) unravels a la Akerlof (1970).

The logic can be seen in the canonical two-types case. Here, the regularity condition requires one to assume that the bad risk will experience the loss with certainty. The only way for the low type (good risk) to obtain an allocation other than her endowment is to subsidize the high type (bad risk) away from her endowment. If the low type is willing to do so, the equilibrium unravels a la Rothschild and Stiglitz. If the low type is unwilling to do so, the equilibrium unravels a la Akerlof.

In the two type model, the assumption that the bad risk experiences the loss with certainty is clearly restrictive. However, for more general type distributions beyond the two-type case the regularity condition is quite weak. Any distribution can be approximated quite well by distributions that have continuously distributed regions or by distributions with an arbitrarily small amount of mass at $p = 1$. In this sense, the existence of pure strategy competitive equilibria of the type found by Rothschild and Stiglitz (1976) that yield outcomes other than the endowment is a knife-edge result. This highlights the importance of recent and future work to aid in our understanding of how best to model competition in insurance markets.

[1] As discussed below, Riley (1979) shows this is true in the case when the support contains an interval; I show below this is also the case when the support is discrete but includes the point $p = 1$.

2 Model

Agents have wealth w and face a potential loss of size l which occurs with probability p, which is distributed in the population according to the c.d.f. $F(p)$ with support Ψ.[2] In contrast to Rothschild and Stiglitz (1976), I do not impose any restrictions on $F(p)$.[3] It may be continuous, discrete, or mixed. I let P denote the random variable with c.d.f. $F(p)$, so that realizations of P are denoted with lower-case p. Agents of type p have vNM preferences given by

$$pu(c_L) + (1-p)u(c_{NL})$$

where u is increasing and strictly concave, c_L (c_{NL}) is consumption in the event of (no) loss. I define an allocation to be a set of consumption bundles, c_L and c_{NL}, for each type $p \in \Psi$, $A = \{c_L(p), c_{NL}(p)\}_{p\in\Psi}$.

I assume there exists a large set of risk-neutral insurance companies, J, which each can offer menus of contracts $A_j = \left\{c_L^j(p), c_{NL}^j(p)\right\}_{p\in\Psi}$ to maximize expected profits[4]. Following Rothschild and Stiglitz (1976), I define a Competitive Nash Equilibrium as an equilibrium of a two stage game. In the first stage, insurance companies offer contract menus, A_j. In the second stage, agents observe the total set of consumption bundles offered in the market, $A^U = \cup_{j\in J} A_j$, and choose the bundle which maximizes their utility. The outcome of this game can be described as an allocation which satisfies the following constraints.

Definition 1. An allocation $A = \{c_L(p), c_{NL}(p)\}_{p\in\Psi}$ is a **Competitive Nash Equilibrium** if

1. A makes non-negative profits

$$\int_{p\in\Psi} \left[p(w-l-c_L(p)) + (1-p)(w-c_{NL}(p))\right] dF(p) \geq 0$$

2. A is incentive compatible

$$pu(c_L(p)) + (1-p)u(c_{NL}(p)) \geq pu(c_L(\tilde{p})) + (1-p)u(c_{NL}(\tilde{p})) \quad \forall p, \tilde{p} \in \Psi$$

3. A is individually rational

$$pu(c_L(p)) + (1-p)u(c_{NL}(p)) \geq pu(w-l) + (1-p)u(w) \quad \forall p \in \Psi$$

[2] The model is adapted from Hendren (ming), which derives the no-trade condition analogue of Akerlof in the binary loss environment but does not provide any discussion of competitive equilibriums.

[3] To my knowledge, Riley (1979) was the first paper to discuss this environment with a continuum of types.

[4] In contrast to Rothschild and Stiglitz (1976), I allow the insurance companies to offer menus of consumption bundles, consistent with the real-world observation that insurance companies offer applicants menus of premiums and deductibles.

4. A has no profitable deviations: For any $\hat{A} = \{\hat{c}_L(p), \hat{c}_{NL}(p)\}_{p \in \Psi}$, it must be that

$$\int_{p \in D(\hat{A})} \left[p\left(w - l - c_L(p)\right) + (1-p)\left(w - c_{NL}(p)\right)\right] dF(p) \leq 0$$

where

$$D\left(\hat{A}\right) = \left\{p \in \Psi | \max_{\hat{p}} \left\{pu\left(\hat{c}_L(\hat{p})\right) + (1-p)\,u\left(\hat{c}_{NL}(\hat{p})\right)\right\} > pu\left(c_L(p)\right) + (1-p)\,u\left(c_{NL}(p)\right)\right\}$$

The first three constraints require that a Competitive Nash Equilibrium must yield non-negative profits, must be incentive compatible, and must be individually rational. The last constraint rules out the existence of profitable deviations by insurance companies. For A to be a competitive equilibrium, there cannot exist another allocation that an insurance company could offer and make positive profits on the (sub)set of people who would select the new allocation (given by $D\left(\hat{A}\right)$).

2.1 Mutually Exclusive Occurrences

I first show that, in this model, the insurance market has the potential to unravel in the sense of Akerlof (1970).

Theorem 1. *The endowment, $\{(w, w-l)\}_{p \in \Psi}$, is the unique Competitive Nash Equilibrium if and only if*

$$\frac{p}{1-p}\frac{u'(w-l)}{u'(w)} \leq \frac{E\left[P|P \geq p\right]}{1 - E\left[P|P \geq p\right]} \quad \forall p \in \Psi \backslash \{1\} \tag{1}$$

Proof. The no-trade theorem of Hendren (ming) shows that Condition (1) characterizes when the endowment is the only allocation satisfying incentive compatibility, individual rationality, and non-negative profits. Now, suppose $A = \{(w-l, l)\}$ and consider any allocation, $\hat{A} \neq \{(w-l, l)\}_{p \in \Psi}$. Suppose $\hat{A}$ delivers positive profits. Because A is the endowment, I can WLOG assume all agents choose $\hat{A}$ (since $\hat{A}$ can provide the endowment to types p at no cost). But then $\hat{A}$ would be an allocation other than A satisfying incentive compatibility, individual rationality, and non-negative profits, contradicting the no-trade theorem of Hendren (ming). □

The market unravels a la Akerlof (1970) if and only if no one is willing to pay the pooled cost of worse risks in order to obtain some insurance. This is precisely the logic of Akerlof (1970) but provided in an environment with an endogenous contract space. When Condition (1) holds, no contract or menu of contracts can be traded because they would not deliver positive profits given the set of risks that would be attracted to the contract. This is precisely the unraveling intuition provided in Akerlof (1970) in which the demand curve lies everywhere below the average cost curve. Notice that when this no-trade condition holds, the endowment is indeed a Nash equilibrium. Since no one is willing to pay the pooled cost of worse risks to obtain insurance, there exist no profitable deviations for insurance companies to break the endowment as an equilibrium.

Theorem 1 also shows that whenever the no-trade condition holds, there must exist a Competitive Nash Equilibrium. Thus, whenever the market unravels a la Akerlof (1970), the competitive equilibrium cannot unravel a la Rothschild and Stiglitz (1976). Unraveling in the sense of Akerlof (1970) is a mutually exclusive occurrence from unraveling in the sense of Rothschild and Stiglitz (1976).

Two-type case To relate to previous literature, it is helpful to illustrate how Theorem 1 works in the canonical two-type model of Rothschild and Stiglitz (1976). So, let $\Psi = \{p^L, p^H\}$ with $p^H > p^L$ denote the type space and let λ denote the fraction of types p^H. When $p^H < 1$, Corollary 1 of Hendren (ming) shows that the market cannot unravel a la Akerlof.[5] Hence, the mutual exclusivity of Akerlof and Rothschild and Stiglitz holds trivially. But, when $p^H = 1$, the situation is perhaps more interesting. To see this, Figure 1 replicates the canonical Rothschild and Stiglitz (1976) graphs in the case when $p^H = 1$.

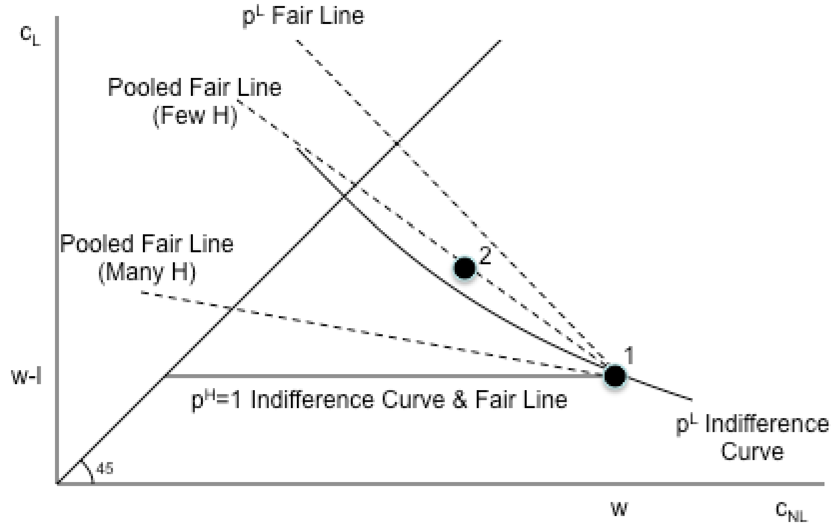
The vertical axis is consumption in the event of a loss, c_L; the horizontal axis is consumption in the event of no loss, c_{NL}. Point 1 is the endowment $\{w - l, w\}$. Because $p^H = 1$, the horizontal line running through the endowment represents both the indifference curve of type p^H and the actuarially fair line for type p^H. Notice that type p^H prefers any allocation bundle that lies above this line (intuitively, she cares only about consumption in the event of a loss).

The low type indifference curve runs through the endowment (point 1) and intersects the 45-degree line parallel to her actuarially fair line. As noted by Rothschild and Stiglitz (1976), the outcomes in this environment depend crucially on the fraction of low versus high types. Figure 1 illustrates the two cases. If there are few p^H types (λ is small), then point 2 is a feasible pooling deviation from the endowment. When such a deviation is feasible, unraveling a la Akerlof does not occur: the low type is willing to pay the pooled cost of the worse risks. But, the existence of such a deviation is precisely what breaks the existence of a competitive equilibrium in Rothschild and Stiglitz (1976). Point 2 involves pooling across types and cannot be a competitive equilibrium. Hence, if the market unravels a la Akerlof, the endowment is the unique competitive equilibrium. If the market unravels a la Rothschild and Stiglitz, there exists implementable allocations other than the endowment and Akerlof's notion of unraveling does not occur.

As one might gather from Figure 1, when types are arbitrarily close to 1, the only feasible competitive equilibrium is the endowment – i.e. there is no possibility of a pair of separating contracts with the p^L-type receiving partial coverage in equilibrium. I now make this point in the more general setting that does not require any mass of types at $p^H = 1$.

[5] If $p^H < 1$, then equation (1) would be violated at $p^H = 1$ by the assumption of strict concavity of u.

Figure 1



2.2 Exhaustive Occurrences

I now show that not only are these two notions of unraveling mutually exclusive, but they are also exhaustive of the possibilities that can occur in model environments when the type distribution satisfies the following regularity condition.

Assumption 1. *Either (a) there exists* $a < b$ *such that* $[a, b] \subset \Psi$ *or (b)* $1 \in \Psi$ *(i.e.* $F(p) < 1$ *for all* $p < 1$*).*

Assumption 1 assumes that the support of the type distribution includes either (a) a continuous interval or (b) the point $p = 1$. Note any distribution can be approximated arbitrarily closely by distributions satisfying this regularity condition.

I now show that competitive equilibriums cannot sustain cross-subsidization, an insight initially provided in Rothschild and Stiglitz (1976).

Lemma 1. *(Rothschild and Stiglitz (1976)) Suppose A is a Competitive Nash Equilibrium. Then*

$$pc_L(p) + (1-p)\,c_{NL}(p) = w - pl \quad \forall p \in \Psi$$

Proof. See Rothschild and Stiglitz (1976) for a full discussion. Clearly, competition requires zero profits on any consumption bundle. Hence, it suffices to show that no allocation can pool types into the same consumption bundle other than the endowment. Suppose multiple types are allocated to the same consumption bundle (distinct from the endowment). Then, an insurance company could offer a new allocation, $\hat{A}$, arbitrarily close to the current allocation but that is only preferred by the lowest p in the pool. Hence, this allocation will provide strictly positive profits and will render the original consumption bundle unprofitable, thereby breaking the Nash equilibrium with pooling. Therefore, $pc_L(p) + (1-p)\,c_{NL}(p) = w - pl$ for all p. □

Now, consider the two cases in Assumption 1. If $p = 1$ is in the support of the type distribution, it is straightforward to see that there cannot exist any Competitive Nash Equilibrium other than the endowment, since trade requires cross-subsidization toward types near $p = 1$. Now, suppose that $p = 1$ is not in the support of the type distribution but that the type distribution contains an interval. Here, the non-existence of a Nash equilibrium is perhaps less straightforward, but a proof is actually contained in Riley (1979)[6]. Given the interval $[a, b] \subset \Psi$, Riley's derivations show that there exists a profitable deviation which pools types near $p = b$; hence there can be no Nash equilibrium other than the endowment. Theorem 2 follows.

Theorem 2. *Suppose Assumption 1 holds. Then, there exists a Competitive Nash Equilibrium if and only if Condition* (1) *holds.*

When Assumption1 holds, trade requires risk types to be willing to enter risk pools which pool ex-ante heterogeneous types. Such ex-ante pooling is not possible in a Competitive Nash Equilibrium. So, when the no-trade condition (1) does not hold, there does not exist any Competitive Nash Equilibrium: the equilibrium unravels a la Rothschild and Stiglitz (1976).

3 Conclusion

This memo uses a generalized binary model of insurance to highlight the distinction between Akerlof's notion of unraveling, in which an equilibrium exists in which no trade can occur, and Rothschild and Stiglitz' notion of unraveling, in which a standard notion of competitive equilibrium (pure strategy Nash) cannot exist. In the latter case, there are (Pareto) gains to trade; but in a generic sense described in Assumption 1, the realization of these gains to trade require cross-subsidization of types. Such cross-subsidization cannot be sustained under the canonical notion of competition.[7] Hence, Akerlof unraveling shows when private information can lead to the absence of trade in insurance markets. Rothschild and Stiglitz unraveling shows that the canonical model of competition (Nash equilibrium) is inadequate to describe the behavior of insurance companies in settings where there are potential gains to trade.

References

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[6]See Theorem 3 in Section 4, pages 341-3

[7]In the modified models of competition, proposed by Miyazaki (1977), Wilson (1977), or Spence (1978), such gains to trade will be realized in equilibrium.

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