Capital and Wages*

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Abstract

Does capital accumulation increase labor demand and wages? Neoclassical production functions, where capital and labor are q-complements, ensure that the answer is yes, so long as labor markets are competitive. This result critically depends on the assumption that capital accumulation does not change the technologies being developed and used. I adapt the theory of endogenous technological change to investigate this question when technology also responds to capital accumulation. I show that there are strong parallels between the relationship between capital and wages and existing results on the conditions under which equilibrium factor demands are upward-sloping (e.g., Acemoglu, 2007). Extending this framework, I provide intuitive conditions and simple examples where a greater capital stock leads to lower wages, because it triggers more automation. I then offer an endogenous growth model with a menu of technologies where equilibrium involves choices over both the extent of automation and the rate of growth of labor-augmenting productivity. In this framework, capital accumulation and technological change in the long run are associated with wage growth, but an increase in the saving rate increases the extent of automation, and at first reduces the wage rate and subsequently depresses its long-run growth rate.

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1 Introduction

A mainstay of neoclassical capital theory is that capital accumulation increases wages, at least in the long-run. This is one of the key mechanisms via which economic growth generates "shared prosperity" (Acemoglu and Johnson, 2023). Evidence from long-run economic growth is broadly consistent with this prediction: capital-labor ratios have increased rapidly in most industrialized economies in the 20th century, and this was accompanied by growth of labor earnings (Barro and Sala-i-Martin, 2008; Acemoglu, 2009; Gordon, 2016). Nevertheless, the slowdown—or even secession—of US real wage growth over the last four decades has raised concerns about the relationship between economic growth and wage growth. Several authors have linked the decline in the labor share of national income and the slowdown of wage growth to rapid capital accumulation (Blanchard, 1995; Karabarbounis and Neiman, 2014), while others have suggested that adverse effects for labor have followed from investment in automation technologies (Autor, Levy and Murnane, 2003; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018, 2019; Acemoglu and Johnson, 2023).

Similar questions arise in the context of international capital flows. Neoclassical capital theory implies that an increase in foreign capital flows should benefit labor. This has not always been the case in reality, and ideas going back to the appropriate technology literature of the 1960s and 70s suggests that the consequences of capital flows may be more nuanced because more capital often leads to the adoption of more capital-intensive production techniques, which can have adverse effects for workers (Atkinson and Stiglitz, 1969; Schumacher, 1973; Stewart, 1977; Diwan and Rodrik, 1991; Basu and Weil, 1998; Acemoglu, 2015).

In this paper, I start by presenting the foundations of the neoclassical conclusion. When factor markets are competitive and aggregate production exhibits constant returns to scale, more capital always leads to higher wages and greater labor demand. I then explain that this prediction critically depends on the assumption that more capital does *not* change the production possibilities set of firms or the types of techniques they use. When this is the case, greater capital intensity of production can harm labor.

The heart of the paper develops a tractable model of endogenous technology choice, building on Acemoglu (2007, 2010). I show how greater capital abundance typically leads to greater *automation*—meaning a change in the organization of production such that capital becomes more important and labor becomes less important, for example, because capital is now used for tasks previously performed by labor (Acemoglu and Restrepo, 2018, 2019).

I use this framework to show how greater capital abundance can reduce wages and labor demand. I show that this can happen when the menu of technologies enables a strong automation response.

A well-known regularity is that wages have increased in most of the industrialized world for much of the 20th century even as there was rapid capital accumulation and technological change. Does this mean that configurations in which greater capital leads to lower wages are not empirically relevant? In the last part of the paper, I show that the answer to this question is no. I embed the ideas developed earlier in the paper in an extended endogenous growth model, in which the equilibrium involves both a choice over the extent of automation and over the growth rate of labor-augmenting technology. The economy admits a balanced growth path in which equilibrium wages and income per capita increase because of steady labor-augmenting technological change, but the bias of technology in favor or against labor is also determined by economic forces. Using this framework, I establish that while equilibrium wages always continue to increase in the long run, a higher saving rate can both reduce wages upon impact and also depress their long-run growth rate.

This paper is most closely related to my previous work, Acemoglu (2007, 2010) and Acemoglu and Restrepo (2018). Acemoglu (2007) provided a general framework for the analysis of the equilibrium bias of technology. The micro-founded model of technology choice presented here is based on that paper. Acemoglu (2007) develops this framework to investigate the conditions under which an increase in the supply of skilled labor shifts technology in a "skill-biased" direction (this is the *weak equilibrium bias* result), and the more stringent conditions under which greater abundance of skilled labor *increases* the long-run skill premium (this is the *strong equilibrium bias* result). Acemoglu (2010) uses a similar framework to study the conditions under which the scarcity of labor encourages innovation, as it has been observed in key historical episodes (Habakkuk, 1962; Mokyr, 1990; Allen, 2009). The focus here is on the effects of capital on wages, which has not featured in previous work. I will show, nonetheless, that there are important parallels between the strong equilibrium bias result in Acemoglu (2007) and the possibility here that greater capital intensity reduces wages via the automation response. The modeling of automation in this paper builds on Zeira (1998), Acemoglu and Zilibotti (2001), Acemoglu and Autor (2011), and especially Acemoglu and Restrepo (2018, 2019). These papers do not investigate the relationship between the direction of technology and the abundance of capital, though Acemoglu and

Restrepo (2018) consider how capital accumulation and equilibrium labor shares impact the balance between automation and new tasks.

The rest of the paper is organized as follows. Section 2 demonstrates that with exogenous technology, competitive factor markets and constant returns to scale, increasing the capital stock of the economy always raises wages. This section also highlights the importance of the exogenous technology assumption for this result. Section 3 builds on Acemoglu (1998, 2007, 2010) and provides a simple framework for endogenizing technology and its response to the abundance of capital. Section 4 reviews previous results about how endogenous and directed technology changes the relationship between factor supplies and factor prices. Specifically, it presents simplified versions of the weak and strong bias results of Acemoglu (1998, 2002, 2007). The strong bias results also have intuitions closely related to those I explore in the context of the relationship between capital and wages, as shown in Section 5. The results in this section establish that an increase in the capital stock of the economy can reduce, rather than increase, equilibrium wages when technology responds to this change in factor supplies. Section 6 provides worked-out examples of how a higher capital stock can lead to lower wages in a number of simple economies. Section 7 embeds these ideas into an endogenous growth model with multiple types of technologies—one corresponding to automation and the other to labor-augmenting technology. It establishes that, along the balanced growth path, wages increase together with capital deepening, but a higher saving rate can increase capital abundance and lead to a decline in wage levels. Section 8 concludes.

2 Capital and Wages in the Neoclassical Growth Theory

Until Section 7, I focus on static economies, where technology choices are also made statically. In this section, I demonstrate that in the neoclassical world with constant returns to scale, greater capital abundance always increases wages. This result is also exposited in the context of a static model. The terminology introduced in this section will be used throughout the rest of the paper.

Consider a neoclassical, constant returns to scale aggregate production function

$$F(L, K, \theta),$$

where L is labor used in production, K is the capital stock of the economy, and θ is an index of technology. Throughout, factor markets are assumed to be competitive, and the aggregate production function F is continuously differentiable and exhibits constant returns to scale (is linearly homogeneous in L and K). Throughout, I use subscripts to denote derivatives.

I also simplify the discussion by assuming that there is a representative household in the background (though its preferences do not play a major role until Section 7). Labor is inelastically supplied, and in this section I normalize its total supply to $L = 1$.

Given the differentiability of the production function and competitive markets, the equilibrium wage is given by $w = F_L(L, K, \theta)$. Or exploiting constant returns to scale and defining the capital-labor ratio as $k = K/L$ and per capita production function as $f(k, \theta) \equiv F(L, K, \theta)/L$ by

$$w = f(k, \theta) - kf_k(k, \theta).$$

I next prove the following result:

Proposition 1 *Holding θ constant, a higher capital stock always raises equilibrium wages. That is,*

$$\frac{dw}{dK} \geq 0.$$

Moreover, this inequality is strict whenever $f_{kk} < 0$.

The proof is straightforward and follows immediately from the fact that

$$\frac{dw}{dK} = -kf_{kk}(k, \theta) \geq 0,$$

since $f_{kk}(k, \theta) \leq 0$ by constant returns to scale (and concavity of the aggregate production function).

Intuitively, with a constant returns to scale production function, capital and labor are q-complements—meaning that an increase in the use of one of these factors raises the marginal product of the other, or $F_{LK} \geq 0$. This feature is implied because, under constant returns to scale, F_{LK} is proportional to $-kf_{kk}$, and $f_{kk} \leq 0$. Hence, more capital always increases the marginal product of labor. So long as the wage is proportional to labor's marginal product (as it is in a perfectly competitive labor market), the conclusion follows.

It is also straightforward to embed this result in a standard (exogenous or endogenous) growth model. For example, a higher saving rate starts raising the capital stock immediately and takes the economy towards a new steady state (balanced growth path) with higher capital-labor ratio. After the rise in the saving rate, the wage rate starts increasing (relative to the baseline) and the new steady state will have permanently higher wages. I will contrast this result to the pattern that obtains in a dynamic economy with a menu of technologies and Section 5.

The fact that we are holding technology fixed is important for these results. Imagine we also had $\frac{d\theta}{dK} \neq 0$. In that case, greater capital abundance would change the aggregate production function (e.g., alter how important capital and labor are in the production process), and this would also impact wages. For example, if θ corresponds to a measure of automation and we have $\frac{d\theta}{dK} > 0$ (more capital and encourages more automation) and $\frac{\partial w}{\partial \theta} < 0$ (automation reduces wages; see Acemoglu and Restrepo, 2019), then the overall impact of greater capital abundance could be to reduce wages.

To explore these issues in detail, we need a way of endogenizing technology choices, which is what I start with in the next section.

3 Endogenizing Technology

This section is based on Acemoglu (2007). I provide a brief exposition to avoid repetition and economize on space.

Each firm $i \in \mathcal{F}$ has access to a production function

$$y^i = \alpha^{-\alpha} (1-\alpha)^{-1} F(L^i, K^i, \Theta)^{\alpha} q^i (\Theta)^{1-\alpha}, \tag{1}$$

where L^i is the firm's employment, K^i is its capital stock, $q^i(\Theta)$ is the quantity of intermediate good embedding technology Θ, $\alpha \in (0,1)$, and $\Theta = (\theta_1, \ldots, \theta_N) \in \mathcal{O} \subset \mathbb{R}^N$ is a measure of technology that applies to all firms in the economy. I assume throughout that $\mathcal{O}$ is a lattice (see Topkis, 1998), and F is again taken to be a neoclassical production function with constant returns to scale. I use lower case y^i to denote the output of firm i, and use Y to denote net aggregate output below. The term $\alpha^{-\alpha} (1-\alpha)^{-1}$ is included as a convenient normalization.

This production structure is similar to models of endogenous technology (e.g., Romer, 1990, Grossman and Helpman, 1991, Aghion and Howitt, 1992), but is somewhat more general since it does not impose that technology necessarily takes a factor-augmenting form.

The monopolist can create (a single) technology $\Theta \in \mathcal{O}$. The cost of inventing this technology is $\Gamma(\Theta)$. I often set this cost to zero, which is without loss of any generality, since Θ is already part of the production function, so any additional costs can be incorporated into

F. In line with Romer's (1990) emphasis that technology has a "non-rivalrous" character and can thus be produced at relatively low cost once invented, I assume that once Θ is created, the intermediate good embodying technology Θ can be produced at constant per unit cost normalized to $1-\alpha$ units of the final good (this is also a convenient normalization). The monopolist can then set a (linear) price per unit of the intermediate good of type Θ, denoted by χ.[1]

I continue to assume that labor is inelastically supplied, with total supply denoted by $\bar{L}$, and I take the supply of capital, $\bar{K}$, as given as well. All factor markets are, once again, competitive, and each firm takes the available technology, Θ, and the price of the intermediate good embodying this technology, χ, as given and maximizes

$$\max_{L^i, K^i, q^i(\Theta)} \pi(L^i, K^i, q^i(\Theta) \mid \Theta, \chi) = \alpha^{-\alpha} (1-\alpha)^{-1} F(L^i, K^i, \Theta)^{\alpha} q^i(\Theta)^{1-\alpha} - wL^i - RK^i - \chi q^i(\Theta), \tag{2}$$

which gives the following simple inverse demand for intermediates of type Θ as a function of its price, χ, and the factor employment levels of the firm as

$$q^i\left(\chi, L^i, K^i \mid \Theta\right) = \alpha^{-1} F(L^i, K^i, \Theta) \chi^{-1/\alpha}. \tag{3}$$

The problem of the monopolist is to maximize its profits:

$$\max_{\Theta, \chi, [q^i(\chi, L^i, K^i \mid \Theta)]_{i \in \mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i \in \mathcal{F}} q^i\left(\chi, L^i, K^i \mid \Theta\right) di - \Gamma(\Theta) \tag{4}$$

subject to (3). Given the supplies of labor and capital, market clearing requires:

$$\int_{i \in \mathcal{F}} L^i di \leq \bar{L} \text{ and } \int_{i \in \mathcal{F}} K^i di \leq \bar{K}. \tag{5}$$

An *equilibrium* is a set of firm decisions $\{L^i, K^i, q^i(\chi, L^i, K^i \mid \Theta)\}_{i \in \mathcal{F}}$, technology choice and pricing decisions by the technology monopolist (Θ, χ), and factor prices (w, R) such that $\{L^i, K^i, q^i(\chi, L^i, K^i \mid \Theta)\}_{i \in \mathcal{F}}$ solve (2) given (w, R) and (Θ, χ); (5) holds; and (Θ, χ) maximize (4) subject to (3).

This definition emphasizes that factor demands and technology are decided by different agents (the former by the final good producers, the latter by the technology monopolist). This is an important feature both theoretically and as a representation of how technology is determined in practice (see Acemoglu, 2007, for more discussion). Since factor demands

[1] See Acemoglu (2007) for the oligopolistic competition case and also for environments exhibiting technological externalities, which give identical results so far as the comparative statics of technology are concerned.

and technology are decided by different agents, $F(L^i, K^i, \Theta)$ need not be jointly concave in (L^i, K^i, Θ). Instead, it is sufficient that F is concave in (L^i, K^i), which is already assume given our constant returns to scale restriction.

To characterize the equilibrium, note that (3) defines a constant elasticity demand curve, so the profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and, given our normalizations, is equal to $\chi = 1$. Consequently, $q^i(\Theta) = q^i(\chi = 1, L^i, K^i \mid \Theta) = \alpha^{-1} F(L^i, K^i, \Theta)$ for all $i \in \mathcal{F}$. Substituting this into (4), integrating over the set of firms $\mathcal{F}$ and using the fact that F exhibits constant returns to scale and $\chi = 1$, we have that $\Pi = (\chi - (1-\alpha))\, \alpha^{-1} F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta) = F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta)$. Therefore, the maximization problem of the monopolist can be expressed as

$$\max_{\Theta \in \mathcal{O}} H(\bar{L}, \bar{K}, \Theta) \equiv F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta).$$

Thus we have established:

Proposition 2 Any equilibrium technology Θ^* is a solution to

$$\max_{\Theta \in \mathcal{O}} H(\bar{L}, \bar{K}, \Theta), \tag{6}$$

and any solution to this problem is an equilibrium technology.

This proposition shows that the equivalence between equilibrium technology and the maximizers of $H(\bar{L}, \bar{K}, \Theta) \equiv F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta)$. Notice also that the function H inherits all the properties of the production function F with respect to L and K. However, because of the monopoly markup, there are distortions, and equilibrium technology is not at the level that maximizes net output. I now use the fact that the profit-maximizing monopoly price is $\chi = 1$ and substitute (3) into the production function (1), and then subtract the cost of technology choice, $\Gamma(\Theta)$, and the cost of production of the machines, $(1-\alpha)\, \alpha^{-1} F(L, K, \Theta)$, from gross output. This gives net output in this economy as

$$Y(\bar{L}, \bar{K}, \Theta) \equiv \frac{2-\alpha}{1-\alpha} F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta). \tag{7}$$

Observe that the coefficient in front of $F(\bar{L}, \bar{K}, \Theta)$ is always greater than one, so equilibrium technology will generally fail to maximize net output. A more systematic discussion of distortions in the direction of innovation is provided in Acemoglu (2023).

Finally, it can be verified that the equilibrium wage is now

$$w = \frac{1}{1-\alpha} F_L(\bar{L}, \bar{K}, \Theta),$$

and thus proportional to the standard wage equation. This wage equation can be rewritten as

$$\begin{aligned} w &= \frac{1}{1-\alpha}\left[f(k,\Theta) - f_k(k,\Theta)k\right] \\ &= \frac{1}{1-\alpha}\left[h(k,\Theta) + \Gamma(\Theta) - h_k(k,\Theta)k\right], \end{aligned}$$

where $h(k,\Theta) \equiv f(k,\Theta) - \Gamma(\Theta)$.

4 Review of Weak and Strong Bias Results

Here I review the results presented in Acemoglu (2007), which in turn draw on and significantly generalize those in Acemoglu (1998, 2002). The focus will be on how the change in the supply of labor changes the demand for labor and the equilibrium wage. For this purpose, I consider a setting, as in Acemoglu (2007), where there are potentially several factors, which would include capital, summarized by the vector Z, and additionally, labor L. I assume for simplicity that all of these factors are inelastically supplied, with supplies denoted by $(\bar{L}, \bar{Z})$. Hence, with analogy to the previous section, equilibrium technology is now given by the solution to

$$\max_{\Theta \in \mathcal{O}} H(L, Z, \Theta).$$

Definition 1 *An increase in technology θ_j for $j = 1, ..., N$ is (absolutely) biased towards L at supplies $(\bar{L}, \bar{Z})$ if $\partial w_L / \partial \theta_j \geq 0$.*

This definition only considers small changes in technology at the *current* factor proportions $(\bar{L}, \bar{Z})$, which simplifies the results. The more general case is presented in Acemoglu (2007). I also assume throughout that all the relevant functions are continuously differentiable and the relevant first-order conditions hold (and I discuss this issue further in footnote).

Definition 2 Denote the equilibrium technology at factor supplies $\left(\bar{L},\bar{Z}\right)$ by $\Theta^*\left(\bar{L},\bar{Z}\right)$ and assume that $\partial\theta_j^*/\partial L$ exists at $\left(\bar{L},\bar{Z}\right)$ for all $j = 1,...,N$.[2] Then there is *weak (absolute) equilibrium bias* at $\left(\bar{L},\bar{Z},\Theta^*\left(\bar{L},\bar{Z}\right)\right)$ if

$$\sum_{j=1}^{N}\frac{\partial w_L}{\partial\theta_j}\frac{\partial\theta_j^*}{\partial L}\geq 0. \tag{8}$$

This definition requires the (total) induced change in technology resulting from an increase in L to raise the marginal product of labor. The summation ensures that the distinct effects of different components of technology are taken into account on the demand for labor and thus the bias of technology.

Proposition 3 Let the equilibrium technology at factor supplies $\left(\bar{L},\bar{Z}\right)$ be $\Theta^*\left(\bar{L},\bar{Z}\right)$ and assume that $\Theta^*\left(\bar{L},\bar{Z}\right)$ is in the interior of $\mathcal{O}$ and that $\partial\theta_j^*/\partial Z$ exists at $\left(\bar{L},\bar{Z}\right)$ for all $j = 1,...,N$. Then, there is *weak (absolute) equilibrium bias* at all $\left(\bar{L},\bar{Z}\right)$, meaning that

$$\sum_{j=1}^{N}\frac{\partial w_L}{\partial\theta_j}\frac{\partial\theta_j^*}{\partial L}\geq 0 \text{ for all feasible } \left(\bar{L},\bar{Z}\right), \tag{9}$$

with strict inequality if $\partial\theta_j^*/\partial L \neq 0$ for some $j = 1,...,N$.

This proposition establishes an unambiguous and at first surprising result: without any further assumptions on the production technology, an increase in the supply of a factor, say labor, always induces technology to become more complementary to that factor. Intuitively, an increase in the supply of factor raises the benefit of technologies that make better use of that factor, and this leads to change in the bias of technology in favor of that factor.

There is a parallel between Proposition 3 and Samuelson's LeChatelier principle, which states that "long-run" factor demand curves are more elastic than "short-run" factor demand curves that hold some factors constant. Proposition 3, on the other hand, states that long-run changes in marginal products (and factor prices) will be less than those in the short run because of induced technological change. However, there are also some major differences.

[2] The assumption that $\partial\theta_j^*/\partial L$ exists at $\left(\bar{L},\bar{Z}\right)$ entails two restrictions. The first is the usual non-singularity requirement to enable an application of the Implicit Function Theorem, i.e., that the Hessian of H with respect to Θ, is non-singular at the point Θ^*. Second, a small change may shift the technology choice from one local optimum to another, in which case $\partial\theta_j^*/\partial L$ is undefined. This possibility is also ruled out by this assumption. The assumption that $\partial\theta_j/\partial L$ exists at $\left(\bar{L},\bar{Z}\right)$ can be replaced by an assumption on primitives as shown in Acemoglu (2007). Here I omit the details.

First, this proposition concerns how marginal products change as a result of technological responses to factor supplies—rather than the elasticity of short-run and long-run demand curves. Second, the result here applies to the equilibrium of an economy, not to the maximization problem of a single firm. This last distinction is central for the next result I present, which shows that labor demand curves can be upward-sloping, a possibility that is ruled out for price-taking firms.

While weak (absolute) bias is about how the technology changes, *strong bias* is about how a change in supplies affects factor prices. In competitive factor markets with exogenous technology, the increase in the supply of a factor, say labor, always reduces its price. Hence, labor demand curves are downward-sloping. Strong bias applies when this result no longer holds—that is, the increase in the supply of labor increases the equilibrium wage. This is now defined more formally.

Definition 3 Denote the equilibrium technology at factor supplies $\left(\bar{L},\bar{Z}\right)$ by $\Theta^*\left(\bar{L},\bar{Z}\right)$ and suppose that $\partial\theta_j^*/\partial L$ exists at $\left(\bar{L},\bar{Z}\right)$ for all $j=1,...,N$. Then there is *strong (absolute) equilibrium bias* at $\left(\bar{L},\bar{Z}\right)$ if

$$\frac{dw_L}{dL}=\frac{\partial w_L}{\partial L}+\sum_{j=1}^{N}\frac{\partial w_L}{\partial \theta_j}\frac{\partial \theta_j^*}{\partial L}>0.$$

In this definition, dw_L/dL denotes the total derivative, while $\partial w_L/\partial L$ denotes the partial derivative holding $\Theta=\Theta^*\left(\bar{L},\bar{Z}\right)$. Recall also that if H is jointly concave in (L,Θ) at $\left(\bar{L},\Theta^*\left(\bar{L},\bar{Z}\right)\right)$, its Hessian with respect to (L,Θ), $\nabla^2 H_{(L,\Theta)(L,\Theta)}$, is negative semi-definite at this point (though negative semi-definiteness is not sufficient for local joint concavity).

Proposition 4 *Assume that Θ^* is in the interior of $\mathcal{O}$ and that $\partial\theta_j^*\left(\bar{L},\bar{Z}\right)/\partial L$ exists at $\left(\bar{L},\bar{Z}\right)$ for all $j=1,...,N$. Then there is strong (absolute) equilibrium bias at $\left(\bar{L},\bar{Z}\right)$ if and only if $H\left(L,Z,\Theta\right)$'s Hessian in (L,Θ), $\nabla^2 H_{(L,\Theta)(L,\Theta)}$, is **not negative semi-definite** at $\left(\bar{L},\bar{Z},\Theta^*\left(\bar{L},\bar{Z}\right)\right)$.*

Three remarks are useful. First, Proposition 4 shows that with endogenous technology, labor (more generally, factor) demand curves can be upward-sloping. With exogenous technology and price-taking firms, downward-sloping factor demands follow from cost minimization when firms take factor prices as given. But the response of technology to factor supplies implies that demand for a factor increases when its supply does (Proposition 3), and this induced response can be larger than the direct impact of the supply increase. Acemoglu (1998, 2002, 2023) discuss a number of applications of this result and the empirical evidence from healthcare, agriculture, the energy sector, robotics, and economic history, documenting the response of technology to factor supplies and the possibility of upward-sloping factor demand curves.

Second, the proposition provides necessary and sufficient conditions for this result that turn on a form of "non-convexity": factor demand curves are downward-sloping when the production possibilities set is convex, or the maximization problem is concave, in the relevant factor and technology (e.g., in labor and Θ); conversely, they are upward-sloping when there is a non-convexity at $\left(\bar{L}, \bar{Z}\right)$. Third, this result also highlights the importance of technology and factor demands being decided by different agents. If the same firms chose technology and factor demands at the same time, then the second-order conditions of their maximization problem would imply that all factor demands are downward-sloping. However, when labor is chosen by final good producers, while technology is decided by different firms, as in the setting here, then this type of non-convexity is not ruled out. Specifically, each agent may still have a well-defined concave objective function, while their joint problem features a non-convexity (see Acemoglu, 2007, for further discussion). This is essential for upward-sloping factor demand curves, and we will see that it is also important for the relationship between capital and wages.

5 Capital and Wages with Endogenous Technology

In this section, I specialize the economy with endogenous technology to one with a single dimension of technology, so that $\Theta = \theta$, and assume that the equilibrium technology θ^* is interior. I normalize labor supply to $\bar{L} = 1$. I also set $\Gamma \equiv 0$, so that there are no external costs of choosing different values of θ. This implies that there is no difference between F and H in this section, and the first-order condition for technology choice is

$$F_\theta(\bar{L}, \bar{K}, \theta) = H_\theta(\bar{L}, \bar{K}, \theta) = 0. \tag{10}$$

Acemoglu (2007) provides sufficient conditions for this to be the case and for the second-order conditions to hold with strict inequality (i.e., $h_{\theta\theta}(k, \theta) < 0$). I omit these details here.

We can write the total impact of a greater capital stock on equilibrium wages as

$$\frac{dw}{dK} = \frac{\partial w}{\partial K} + \frac{\partial w}{\partial \theta}\frac{d\theta^*}{dK}, \tag{11}$$

where $\partial w/\partial K$ is the partial derivative, holding technology constant at $\theta = \theta^*$, and thus is identical to the expression derived above:

$$\frac{\partial w}{\partial K} = -\frac{1}{1-\alpha} f_{kk}\left(k, \theta^*\right) k > 0.$$

Following the same steps as in Acemoglu (2007), we have:

$$\begin{aligned} \frac{\partial w}{\partial \theta} &= \frac{1}{1-\alpha}\left[f_\theta(k,\theta^*) - f_{\theta k}(k,\theta^*)k\right] \\ &= -\frac{1}{1-\alpha} f_{\theta k}(k,\theta^*)k, \end{aligned}$$

since in the technology equilibrium, $f_\theta(k,\theta^*) = 0$.

Now, $d\theta^*/dK$ can be obtained from the Implicit Function Theorem from the first-order condition (10):

$$\frac{d\theta^*}{dK} = -\frac{f_{\theta k}(k,\theta^*)}{f_{\theta\theta}(k,\theta^*)}.$$

Substituting this into (11), we obtain

$$\begin{aligned} \frac{dw}{dK} &= \frac{1}{1-\alpha}\left[-f_{kk}(k,\theta^*)k + f_{\theta k}\left(k,\theta^*\right)\frac{f_{k\theta}(k,\theta^*)k}{f_{\theta\theta}(k,\theta^*)}\right] \\ &= -\frac{k}{(1-\alpha)f_{\theta\theta}(k,\theta^*)}\left[f_{kk}(k,\theta^*)f_{\theta\theta}(k,\theta^*) - \left(f_{\theta k}\left(k,\theta^*\right)\right)^2\right]. \end{aligned}$$

Since $f_{\theta\theta}(k,\theta^*) < 0$, $\frac{dw}{dK} > 0$ if and only if the square bracketed term is positive. This is of course nothing but the condition for the joint concavity of the function f in (k,θ). If technology were chosen by the same agent as labor demand, this would have to be satisfied by the second-order conditions. But since technology chosen by a monopolist, while labor demand is decided by final good producers, there is no guarantee that it is satisfied, as explained in the context of strong bias result above. This discussion establishes:

Proposition 5 With endogenous technology (θ responding to capital), a greater capital stock can reduce equilibrium wages. That is, $\frac{dw}{dK} < 0$ is possible.

In particular, we have $\frac{dw}{dK} \geq 0$ whenever the per capita production function f is jointly concave in (k,θ), and $\frac{dw}{dK} < 0$ whenever f is not locally jointly concave (its Hessian is not negative semi-definite) in (k,θ).

The parallel between Propositions 4 and 5 is clear. They both require a local failure of convexity (or the relevant maximization problem not to be locally concave). This reflects the fact that in both cases the desired result obtains when the indirect effects working through the response of technology are more powerful than the direct impact.

It is also interesting to consider the conditions under which this type of indirect technology effect can be sufficiently powerful. Clearly, when $f_{\theta k}(k, \theta^*)$ is small, the indirect effect will be weak and the result will be similar to the exogenous technology case. Hence, we need a strong "complementarity" between technology and capital, which is closely linked to the presence of "automation-type" technologies. As emphasized in Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018, 2019), automation corresponds to technologies enabling capital to take over tasks previously performed by labor. This increases the importance of capital, raising its marginal product all else equal, and tends to have a negative impact on the marginal product of labor.

In this context, one might conjecture that the direction of change of θ is important. This is not necessarily the case, however: the case in which θ corresponds to automation that complements capital and substitutes for labor, and the one in which it complements labor and substitutes for capital will have similar properties. This is because in one case a higher capital stock will increase θ, while in the other it will decrease it.[3] Hence, what matters is that the available technologies feature strong complementarities to either capital or labor, which will be the case when $|f_{\theta k}(k, \theta^*)|$ is high. In the examples discussed in the next section, I will always include automation-type technologies and for specificity, will adopt the convention that higher θ corresponds to more automation (though this is not important as explained in this paragraph).

6 When Capital Reduces Wages

In this section, I provide several example economies in which a higher capital stock reduces equilibrium wages.

6.1 An Example with Linear Technology

The simplest economy is one in which the marginal product of labor is independent of capital and there is an automation-type technology, θ, that affects the importance of capital and

[3] This is in fact related to the reason why the weak bias result of Acemoglu (2002, 2007) holds regardless of the elasticity of substitution and the exact details of technology—the direction of change is always dictated by the (relative) abundance of factors.

labor. Consider the production function

$$F(L, K, \theta) = (1 - \theta)L + \theta K,$$

and assume that $K > L$.

Suppose that the cost of producing this technology is given by $\Gamma(\theta)$, which is assumed to be nondecreasing and convex. The technology equilibrium will then maximize

$$H(L, K, \theta) = (1 - \theta)L + \theta K - \Gamma(\theta).$$

Suppose we have an interior solution (which can be guaranteed if we assume that the derivative of $\Gamma(\theta)$ satisfies Inada-type boundary conditions; in particular, $\Gamma'(0) = 0$ and $\lim_{\theta \to 1} \Gamma'(\theta) = \infty$). Then we have

$$H_\theta(L, K, \theta) = 0 \Longleftrightarrow K - L = \Gamma'(\theta),$$

with the second-order condition

$$H_{\theta\theta} < 0 \Longleftrightarrow \Gamma''(\theta) > 0$$

always satisfied by assumption.

We also have

$$\frac{d\theta}{dK} = \frac{1}{\Gamma''(\theta)} > 0,$$

meaning that a greater stock of capital induces more automation. The equilibrium wage rate is simply $w = 1 - \theta$, because, with the linear production technology, factor supplies do not affect the marginal product of labor. The impact of θ on the wage can then be computed as $\frac{\partial w}{\partial \theta} = -1$, meaning that more automation always reduces the equilibrium wage.

Moreover, we have $\frac{\partial w}{\partial K} = 0$, given the linear technology, and thus

$$\begin{aligned} \frac{dw}{dK} &= \frac{\partial w}{\partial \theta}\frac{d\theta}{dK} \\ &= -\frac{1}{\Gamma''(\theta)} < 0. \end{aligned}$$

This establishes:

Proposition 6 With a linear aggregate production function and endogenous technology, a higher capital stock always (strictly) reduces the equilibrium wage.

Intuitively,with a linear production function, the positive effect of the capital stock on the wage is removed, and, given the response of automation to capital, a higher capital stock always reduces the equilibrium wage, establishing Proposition 6.

6.2 Constant Elasticity of Substitution

This example generalizes the previous one to a constant elasticity setting:[4]

$$F(L,K,\theta)=\left[(1-\theta)L^{\frac{\sigma-1}{\sigma}}+\theta K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The cost of choosing the technologies again $\Gamma(\theta)>0$, where Γ is increasing, differentiable and strictly convex, and its derivative $\Gamma'(\theta)$ satisfies the same boundary conditions as in the previous subsection: $\Gamma'(0)=0$ and $\lim_{\theta\to 1}\Gamma'(\theta)=\infty$. I also assume that $K>L$. The maximization problem of the technology monopolist is

$$\max_{\theta\in[0,1]} H(L,K,\theta)=\left[(1-\theta)L^{\frac{\sigma-1}{\sigma}}+\theta K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}-\Gamma(\theta).$$

Technology optimality condition is now:

$$\begin{aligned} H_\theta &= 0 \Longleftrightarrow \\ \frac{\sigma}{\sigma-1}\left(K^{\frac{\sigma-1}{\sigma}}-L^{\frac{\sigma-1}{\sigma}}\right)F(L,K,\theta)^{\frac{1}{\sigma}} &= \Gamma'(\theta) \end{aligned}$$

(which is always satisfied as equality given $K>L$ and the Inada-type conditions on $\Gamma'(\theta)$ imposed above).

The second-order condition is $H_{\theta\theta}<0$, and can be written as

$$\frac{\sigma}{(\sigma-1)^2}\left(K^{\frac{\sigma-1}{\sigma}}-L^{\frac{\sigma-1}{\sigma}}\right)^2 F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}-\Gamma''(\theta)<0.$$

In what follows, I assume that this condition is satisfied, and note that for large values of σ (on which I will focus below), the first term disappears and this second-order condition is equivalent to $-\Gamma''(\theta)<0$, which is satisfied by assumption.

The response of technology to capital is given by

$$\frac{d\theta}{dK}=\frac{K^{-1/\sigma}F(L,K,\theta)^{\frac{1}{\sigma}}+\frac{\theta}{\sigma-1}K^{-1/\sigma}\left(K^{\frac{\sigma-1}{\sigma}}-L^{\frac{\sigma-1}{\sigma}}\right)F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}}{\Gamma''(\theta)-\frac{\sigma}{(\sigma-1)^2}\left(K^{\frac{\sigma-1}{\sigma}}-L^{\frac{\sigma-1}{\sigma}}\right)^2 F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}}.$$

Although the sign of this expression is in general ambiguous, when σ is large, the second term in the numerator and the second term in the denominator become small, and consequently,

[4] This is related to, but simplified from, the constant elasticity of substitution representation that Acemoglu and Restrepo (2019) derived from a task-based model. The simplification is adopted for expositional clarity and does not have any substantive implications.

we have $\frac{d\theta}{dK} > 0$. In other words, for a sufficiently high elasticity of substitution between capital and labor, a greater capital stock always induces more automation.

The equilibrium wage rate is

$$w = (1-\theta)L^{-\frac{1}{\sigma}}F(L,K,\theta)^{\frac{1}{\sigma}},$$

and the impact of θ on the wage can be written as

$$\frac{\partial w}{\partial \theta} = -L^{-\frac{1}{\sigma}}F(L,K,\theta)^{\frac{1}{\sigma}} + \frac{1-\theta}{\sigma-1}L^{-\frac{1}{\sigma}}\left(K^{\frac{\sigma-1}{\sigma}} - L^{\frac{\sigma-1}{\sigma}}\right)F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}.$$

Moreover, we have

$$\frac{\partial w}{\partial K} = \frac{(1-\theta)\theta}{\sigma}L^{-\frac{1}{\sigma}}K^{-\frac{1}{\sigma}}F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}.$$

Then

$$\frac{dw}{dK} = \frac{\partial w}{\partial K} + \frac{\partial w}{\partial \theta}\frac{d\theta}{dK}.$$

The sign of this expression is in general ambiguous. But notice that as σ gets large, $\frac{\partial w}{\partial K} \to 0$, and $\frac{\partial w}{\partial \theta} \to -1$ (as in the previous example). Moreover, in this case we also have $\frac{d\theta}{dK} > 0$, as noted above. Therefore, there exists σ^* such that for all $\sigma > \sigma^*$, a greater capital stock reduces wages. This discussion establishes:

Proposition 7 With a constant elasticity of substitution aggregate production function, there exists $\sigma^* < \infty$, such that whenever the elasticity of substitution between capital and labor σ is greater than σ^*, a higher capital stock reduces the equilibrium wage.

6.3 A Cobb-Douglas Economy

I now discuss a simple Cobb-Douglas production function, which will be a key ingredient of the full growth model in the next section. Suppose now that

$$F(L,K,\theta) = (1-\theta)L^{1-\theta}K^{\theta}, \tag{12}$$

and also set $\Gamma = 0$ and assume that $K >> L$, so that $\ln k > 1$. This functional form is also related to automation. Acemoglu and Restrepo (2018, 2019) show that when there is automation and tasks are combined with the unit elasticity of substitution, the equilibrium representation of aggregate output takes a Cobb-Douglas form, with exponents corresponding to the extent of automation, as in (12).

The equilibrium wage in this case is

$$w = (1-\theta)^2 k^\theta.$$

Moreover, the first-order condition for technology choice is

$$-k^\theta + (1-\theta)k^\theta \ln k = 0,$$

which is always satisfied at an interior solution provided that $\ln k > 1$. The second-order condition is

$$-\ln k < 0,$$

which is also satisfied since we have already imposed that $\ln k > 0$. Hence, we have that equilibrium technology satisfies

$$1-\theta^* = \frac{1}{\ln k}.$$

Next, observe that

$$\frac{\partial w}{\partial k} = \theta(1-\theta)^2 k^{\theta-1} > 0,$$

while

$$\frac{\partial w}{\partial \theta} = -2(1-\theta)k^\theta + (1-\theta)^2 k^\theta \ln k,$$

which can be negative or positive. Nevertheless, using the optimal technology relationship to substitute out $\ln k$, we obtain

$$\frac{\partial w}{\partial \theta} \propto -2(1-\theta^*) + (1-\theta^*) = -(1-\theta^*),$$

and thus higher θ always reduces wages.

Finally, we have

$$\frac{d\theta^*}{dK} = \frac{1}{k(\ln k)^2} > 0.$$

Therefore, a higher capital stock always induces greater automation, increasing θ. Moreover, this indirect effect of capital on wages becomes negative and dominates the direct effect provided that the capital-labor ratio is not too high. Specifically,

$$\begin{aligned}
\frac{dw}{dK} &= \frac{\partial w}{\partial k} + \frac{\partial w}{\partial \theta^*}\frac{d\theta^*}{dK} \\
&= k^{\theta^*-1}\left[\theta^*(1-\theta^*)^2 - 2(1-\theta^*)\frac{1}{(\ln k)^2} + (1-\theta^*)^2\frac{1}{\ln k}\right] \\
&= k^{\theta^*-1}(1-\theta^*)^2\left[\theta^* - (1-\theta^*)\right] \\
&= -k^{\theta^*-1}(1-\theta^*)^2(1-2\theta^*).
\end{aligned}$$

This expression is negative if and only if

$$\theta^* < \frac{1}{2} \Longleftrightarrow \ln k < 2.$$

Hence we have:

Proposition 8 With the Cobb-Douglas production function given in (12), a higher capital stock reduces the equilibrium wage provided that $\ln k < 2$.

Intuitively, when there is more capital, technology adjusts (the direction of greater automation), and this can create a negative indirect effect on the equilibrium wage. As emphasized in Acemoglu and Restrepo (2018, 2019), in general the impact of automation on the equilibrium wage can be negative or positive. When the negative displacement effect is stronger than the positive productivity effect (due to the fact that automation increases productivity by substituting cheaper capital for labor), the overall impact is negative. When the displacement effect is weaker than the productivity effect, the overall impact is positive. So we obtain a negative relationship between capital and wages, when the direct positive effect of the capital stock on the wage is not too large, the automation response is strong, and the productivity effect from automation is not too large. This intuition also sheds light on why there is no negative relationship between capital and wages when $\ln k$ is greater than 2. In this case, θ is already high, so a further increase does not create much displacement but raises productivity, and via this channel, it tends to boost the equilibrium wage.

7 Endogenous Growth with a Menu of Technologies

For much of the 20th century, capital accumulation in the industrialized world went hand-in-hand with rising wages (e.g., Barro and Sala-i- Martin, 2008; Acemoglu, 2009; Gordon, 2016). Hence, the simple negative relationship highlighted in the previous two sections cannot account for the long-run relationship between capital and wages. In this section, I develop an endogenous growth model with a menu of technologies—automation and labor-augmenting technological change—and show that secular increases in the equilibrium wage can coexist together with a negative relationship between capital intensity and labor demand.

Specifically, I consider a dynamic economy in discrete time with two technologies, $\Theta_t = (\theta_t, A_t)$ and assume that the aggregate production function is

$$Y_t = (1 - \theta_t)\, K_t^{\theta_t} (A_t L)^{1-\theta_t}. \tag{13}$$

I assume, to simplify the analysis of dynamics, that the economy is inhabited by a representative household with a constant saving rate $s \in (0, 1)$ and normalize labor supply to $L = 1$. Hence aggregate and per capita consumption is

$$C_t = (1 - s)(1 - \theta_t) K_t^{\theta_t} A_t^{1-\theta_t}.$$

I continue to assume that labor is inelastically supplied and normalize it to $L = 1$. I also simplify the notation by assuming that capital does not depreciate. The evolution of the capital stock is then given by

$$K_{t+1} = K_t + s(1 - \theta_t) K_t^{\theta_t} A_t^{1-\theta_t}. \tag{14}$$

The cost of choosing technology combination (θ_t, A_t) at time t is assumed to be

$$\gamma\left(\frac{A_t}{A_{t-1}}\right) Y_t,$$

where γ is differentiable, increasing and strictly convex, with derivative denoted by γ'. I also impose the following Inada-type condition: $\gamma'(1) = 0$. This specification implies that current increases in the labor-augmenting technology A_t build on past advances. Additionally, Y_t is included in the cost so that the cost of increasing A_t over time is proportional to current output.

Note finally that varying θ_t has no external technology costs beyond its impact via (13), as in the Cobb-Douglas example in the previous section.

Let us define

$$k_t \equiv \frac{K_t}{A_t}.$$

I take the initial endowment of capital K_0 and the initial labor-augmenting technology A_0 to be such that $\ln k_0 > 1$, as imposed in the previous section in the context of the analysis of the Cobb-Douglas economy.

With this notation, the equilibrium wage is

$$w_t = (1 - \theta_t)^2 A_t k_t^{\theta_t}. \tag{15}$$

Moreover, the first-order conditions for the two components of technology are given as

$$-k_t^{\theta_t} + (1 - \theta_t) k_t^{\theta_t} \ln k_t = 0,$$

and

$$\frac{(1-\theta_t)\,Y_t}{A_t} = \frac{1}{A_{t-1}}\gamma'\left(\frac{A_t}{A_{t-1}}\right)Y_t.$$

Rearranging these equations, we obtain

$$1-\theta_t = \frac{1}{\ln k_t}, \tag{16}$$

as in the previous section, and also

$$1-\theta_t = \frac{A_t}{A_{t-1}}\gamma'\left(\frac{A_t}{A_{t-1}}\right).$$

Therefore, $1-\theta_t = (1+g_t)\gamma'(1+g_t)$, where $1+g_t = \frac{A_t}{A_{t-1}}$. This equation always holds in equilibrium given the assumption that $\gamma'(1) = 0$. For notational convenience I define:

$$\begin{aligned} 1-\theta &= Z(1+g) \equiv (1+g)\gamma'(1+g), \text{ and} \\ 1+g &= G(1-\theta) \equiv Z^{-1}(1-\theta). \end{aligned}$$

Notice that because Z is increasing, so is G. Moreover, in what follows, I assume that G is strictly concave.[5] With this notation, we can write

$$1+g_t = G(1-\theta_t). \tag{17}$$

Finally, rewriting (14) in terms of the effective capital-labor ratio k_t, equilibrium dynamics satisfy

$$k_{t+1} = \frac{1}{1+g_{t+1}}\left[k_t + s(1-\theta_t)k_t^{\theta_t}\right], \tag{18}$$

where I used the fact that $A_{t+1}/A_t = 1+g_{t+1}$. Given the initial conditions k_0 and A_0, a dynamic equilibrium path $\{k_t, A_t, \theta_t\}_{t=0}^{\infty}$ is characterized by equations (16), (17) and (18).[6] Equilibrium aggregate output and consumption can be obtained from these variables. Finally, the equilibrium wage rate is given by (15).

Let us define a balanced growth path (BGP) as an equilibrium in which θ_t^* and k_t are constant, and A_t^* grows at a constant rate $g_t = g^*$. Then a BGP, represented by (k^*, θ^*, g^*), satisfies:[7]

[5] The elasticity of the γ'' function being less than 2, i.e.,$-(1+g)\gamma'''(1+g)/\gamma''(1+g) < 2$ for all $g > 0$, is sufficient for the concavity of G.

[6] There is no initial condition for θ, which is not a state variable and adjusts immediately. Hence, θ_0 is an equilibrium object.

[7] To obtain this, substitute from (16) and (17) into (18), use the fact that, for any $x > 0$, $x^{-\frac{1}{\ln x}} = e^{-1}$, and then impose $k_t = k_{t+1} = k^*$.

$$G\left(\frac{1}{\ln k^*}\right) - 1 = +\frac{s}{e}\frac{1}{\ln k^*}, \tag{19}$$

with

$$1 - \theta^* = \frac{1}{\ln k^*}, \text{ and} \tag{20}$$

$$g^* = G\left(\frac{1}{\ln k^*}\right) - 1, \tag{21}$$

Notice also that the BGP growth rate of wages is given by g^* in view of (15) and the fact that k_t and θ_t are constant.

I will also assume that

$$\begin{aligned} G(1) &< 1 + \frac{s}{e}, \text{ and} \\ G\left(\frac{1}{2}\right) &> 1 + \frac{1}{2}\frac{s}{e}. \end{aligned} \tag{22}$$

The conditions in (22) ensure that the (unique) solution to (19) satisfies $\ln k^* \in (1, 2)$, as I explain below.[8]

Proposition 9 Suppose G is strictly concave and satisfies (22). Then there exists a unique BGP where labor-augmenting technology A_t, the capital stock K_t, GDP Y_t, and aggregate consumption C_t all grow at the rate g^* and the effective capital-labor ratio k^* is constant and satisfies (19) with $\ln k^* \in (1, 2)$. Given k^*, θ^* is constant and satisfies (20), with $\theta^* < 1/2$, while g^* is given by (21).

This unique BGP is globally stable, meaning that starting with any initial conditions k_0 and A_0, the dynamic equilibrium converges to the unique BGP (k^*, θ^*, g^*). Moreover, this convergence is monotone.

Proof. First I prove that a BGP (k^*, θ^*, g^*) satisfies (19), (20) and (21) and exists. A preliminary step is to establish that (16), (17) and (18) define a well-defined dynamical

[8]In terms of γ, (22), can be written as

$$\begin{aligned} 1 &< \left(1 + \frac{sB}{e}\right)\gamma'\left(1 + \frac{sB}{e}\right) \text{ and} \\ \frac{1}{2} &> \left(1 + \frac{1}{2}\frac{sB}{e}\right)\gamma'\left(1 + \frac{1}{2}\frac{sB}{e}\right). \end{aligned}$$

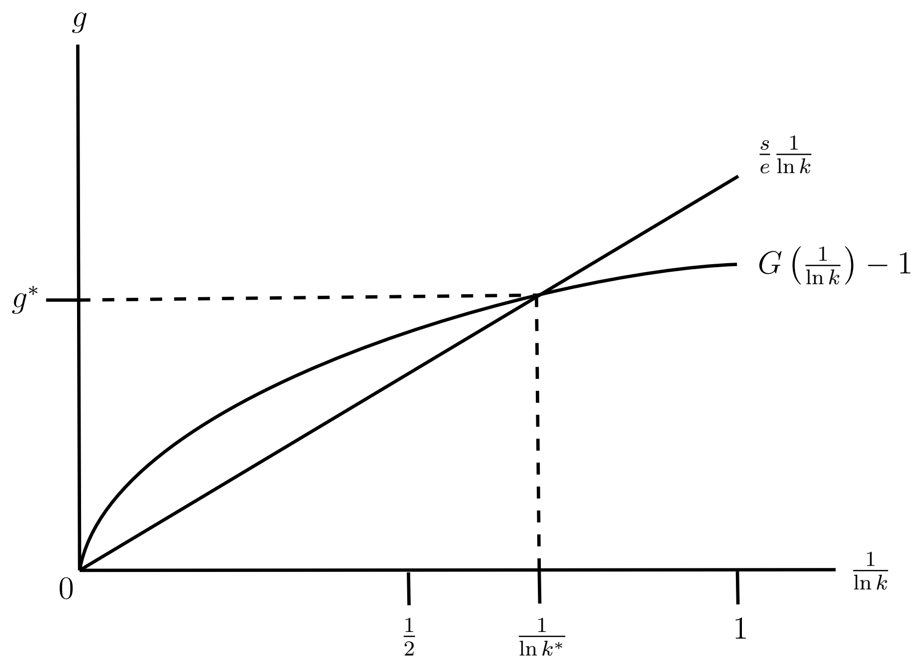

Figure 1: *Existence and uniqueness of the BGP.* $G\left(\frac{1}{\ln k}\right) - 1$ is the left-hand side of (19), while $\frac{s}{e}\frac{1}{\ln k}$ is the right-hand side. The two curves both start at 0, and given the concavity of G, there can be at most one intersection where the left-hand side intersects the right-hand side from above. The first condition in (22) ensures that there is always such an intersection and hence a unique BGP.

system. With the same steps as outlined before the proposition, equation (18), combined with (16), can be written as

$$G\left(\frac{1}{\ln k_{t+1}}\right) k_{t+1} = \left(1 + \frac{s}{e}\frac{1}{\ln k_t}\right) k_t \text{ for all } k_t > 0. \tag{23}$$

The left-hand side of (23) satisfies $G(1)e < e + sB$ by (22), and $\lim_{k\to\infty} G(1/\ln k)k = \infty$, and thus for any k_t, there exists a solution $k_{t+1} \in (e, \infty)$ to this equation by the Intermediate Value Theorem. Moreover, because $G(1/\ln k)k$ is strictly increasing, this solution is unique, establishing that this relationship defines a first-order difference equation. The effective capital-labor ratio k must be constant in BGP, and hence, its BGP value k^* can be written as a fixed point of this difference equation, which gives (19).

Given (22), the Intermediate Value Theorem ensures the existence of a solution $\ln k^* \in (1, 2)$. Since G is concave and given (22), (19) can have at most one solution on $k \in (0, \infty)$, as illustrated in Figure 1, and (22) guarantees that this solution is between $\ln k = 1$ and $\ln k = 2$.

Next, I verify that this BGP is globally stable under (22). The preceding argument establishes that (23) is a well-defined difference equation and has a single fixed point, and moreover, as implied by Figure 1, the left-hand side of (23) intersects the right-hand side from above. More formally, first observe that the left-hand side of (23), $G(1/\ln k)\, k$, is strictly increasing in k whenever γ is convex, as assumed. This follows since:

$$\begin{aligned}\frac{d\ln\left(G\left(\frac{1}{\ln k}\right)k\right)}{d\ln k} &= 1-\frac{G'\left(\frac{1}{\ln k}\right)}{G\left(\frac{1}{\ln k}\right)}\left(\frac{1}{\ln k}\right)^{2} \\ &= 1-\frac{1}{\ln k}\frac{1}{G\left(\frac{1}{\ln k}\right)\frac{\gamma''\left(G\left(\frac{1}{\ln k}\right)\right)}{\gamma'\left(G\left(\frac{1}{\ln k}\right)\right)}+1}>0.\end{aligned}$$

Moreover, the right-hand side of (23), $\left(1+\frac{s}{e}\frac{1}{\ln k}\right)k$, is also increasing in k, which follows by direct differentiation as well.

Now consider any $k_t \in (e, k^*)$. Then (23) implies that $k_{t+1} < k^*$. To see this, observe that:

$$\begin{aligned}G\left(\frac{1}{\ln k_{t+1}}\right)k_{t+1} &= \left(1+\frac{s}{e}\frac{1}{\ln k_t}\right)k_t \\ &< \left(1+\frac{s}{e}\frac{1}{\ln k^*}\right)k^* \\ &= G\left(\frac{1}{\ln k^*}\right)k^*,\end{aligned}$$

where the first line repeats (23), while the second line uses the fact that the right-hand side of (23) is strictly increasing, as stated above, and $k_t < k^*$.

Next, also observe that $k_{t+1} > k_t$. This follows because

$$G\left(\frac{1}{\ln k_{t+1}}\right)k_{t+1} = \left(1+\frac{s}{e}\frac{1}{\ln k_t}\right)k_t > G\left(\frac{1}{\ln k_t}\right)k_t,$$

in view of the fact that $k_t < k^*$. Since the left-hand side of (23) is strictly increasing, as established above, the outer inequality implies that $k_{t+1} > k_t$.

These two steps together imply $k_{t+1} \in (k_t, k^*)$, and thus the effective capital-labor ratio monotonically converges to the BGP k^* from below. With the same argument, starting with any $k_t > k^*$, we have $k_{t+1} \in (k^*, k_t)$, guaranteeing that this time there will be monotonic convergence from above to the unique BGP k^*. Given this, θ_t and g_t also monotonically converge to their unique BGP values (θ^*, g^*), and this completes the proof. ■

Proposition 9 establishes the existence of a unique and globally stable BGP, in which there is constant labor-augmenting technological change, but in addition, the extent to

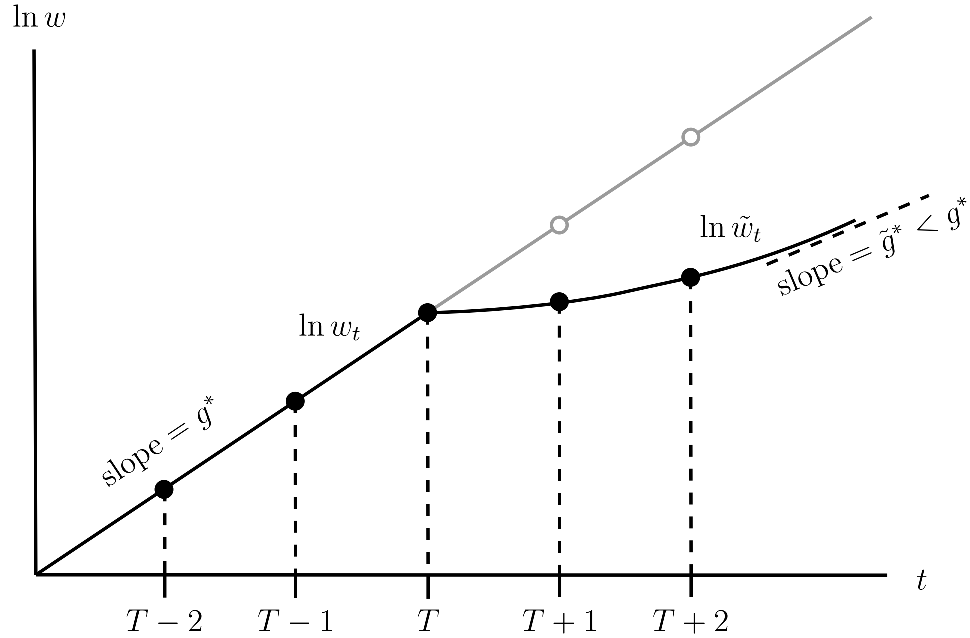

Figure 2: *Wage dynamics after a permanent increase in the saving rate s at time T.* This reduces the capital stock at $T+1$, and reduces the new equilibrium wage $\tilde{w}_{T+1}$ below what it would have been without the change in saving rate. The growth rate of the new equilibrium wage converges to a lower value than its growth rate $\tilde{g}^*$ than the growth rate before the change in the saving rate g^*.

which tasks are automated is also endogenously determined. In addition, the assumption that $G\left(\frac{1}{2}\right) > 1 + \frac{1}{2}\frac{s}{e}$ (imposed in (22)) ensures that this unique BGP involves $\ln k^* < 2$, placing us in the range where a greater capital stock reduces the equilibrium wage.

Let us next consider the comparative dynamics of this BGP in response to an increase in the saving rate s. It is straightforward to verify that this raises the BGP effective capital-labor ratio k^* given by (19). Consequently, the BGP value of θ^* also increases from (20), but the growth rate of labor-augmenting technology A_t decreases from (21). Hence, even though wages continue to grow in the BGP, their growth rate is reduced. In addition, we can trace the full equilibrium response of wages to this increase in the saving rate of the economy. As soon as the saving rate increases, θ_t also increases with k_t as dictated by (16). What about A_t? From (17), greater θ_t leads to a lower rate of increase of A_t. Hence, the immediate impact of the higher saving rate is to reduce the equilibrium wage relative to the counterfactual of a constant saving rate.[9]

[9]Whether the equilibrium wage actually declines depends on how strong the response of A_t is. Holding A_t constant, the increase in θ between T and $T+1$ would lead to a lower equilibrium wage, since we are in the range where $\ln k^* \in (1, 2)$. However, between these two dates, A_t increases as well. If γ' is high

We summarize this discussion in the next proposition and Figure 2 (proof in the text).

Proposition 10 *Consider a permanent increase in the saving rate s. This immediately reduces the equilibrium wage relative to the baseline of constant saving rate and also depresses the rate of labor-augmenting productivity growth. In the long run, the economy converges to a new BGP in which technology involves greater automation and the growth rate of the equilibrium wage is lower.*

The consequences of Proposition 10 are illustrated in Figure 2. Until time T, the saving rate is constant at s and the economy is assumed to be in BGP, so the equilibrium wage grows at the rate g^*. At T, the saving rate increases to $s' > s$. This leads to a larger increase in the capital stock at time $T+1$, K_{T+1}, and the new equilibrium wage $\tilde{w}_{T+1}$ drops below w_{T+1}. Thereafter, the rate of labor-augmenting technological change slows down, so the growth rate of the equilibrium wage $\tilde{w}_t$ converges to $\tilde{g}^* < g^*$. Hence, this proposition shows that the economic forces highlighted in our static model are present in this dynamic setup. In particular, greater capital abundance induces further automation, potentially harming workers. This possibility does not contradict the growth of equilibrium wages together with technological change and capital accumulation along the BGP.

8 Conclusion

A celebrated result in neoclassical growth theory maintains that a greater capital stock—and hence capital accumulation—raises labor demand and equilibrium wages. This is a critical channel for "shared prosperity": any process of capital accumulation, e.g., because investment has become more profitable, firms have greater retained earnings, or capitalists have greater wealth, will create a powerful force towards some of these gains being shared with workers, whose incomes depend on the labor market wage.

The last several decades during which wages have stagnated in the United States and have increased only slowly in many other industrialized nations create a challenge for this perspective, however.

In this paper, I argued that the impact of a greater stock of capital on wages is more complex, because technology responds to the availability of more abundant capital. Under

and the equilibrium g_T is low, the effect through θ can dominate and we may first get a decline in the equilibrium wage. But in any case, the equilibrium wage always falls below its counterfactual trajectory under the constant saving rate.

reasonable conditions, a greater capital stock induces further automation, and automation could reduce (real) wages.

I showed that the conditions under which more capital reduces equilibrium wages are strongly tied to the conditions under which the demand for labor (or other factors) are upward-sloping, because of technology responses (e.g., Acemoglu, 2002, 2007). I then illustrated how simple economies with broadly neoclassical features can reverse this result as soon as the direction of technology is endogenize.

The last part of the paper shows that the economic forces I have emphasized do not imply that wages should fall steadily along the process of economic growth with capital accumulation. I constructed a model of endogenous growth with a menu of technologies, whereby firms decide both the extent of automation and the pace of labor-augmenting technological change. The long-run equilibrium of the economy involves constant wage growth. Nevertheless, an increase in the saving rate has both a negative impact effect and leads to lower long-run growth rate of wages, because it induces greater automation.

Future interesting directions of research include more detailed analysis of the interplay between capital accumulation, technology choices and wages in models in which there are non-competitive elements in product or labor markets, as well as even richer menus of technologies available to firms. An even more important area for future research is the empirical exploration of the channels highlighted here. These include both careful estimation of the impact of greater capital stock on labor demand, and the modeling and estimation of the joint dynamics of capital, technology (automation) and equilibrium wages.

References

Acemoglu, Daron (1998) "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics,* 113(4): 1055-1089.

Acemoglu, Daron (2002) "Directed Technical Change," *Review of Economic Studies,* 69(4): 781–810.

Acemoglu, Daron (2003) "Labor- and Capital-Augmenting Technical Change," *Journal of European Economic Association,* 1(1): 1–37.

Acemoglu, Daron (2007) "Equilibrium Bias of Technology," *Econometrica,* 75(5):

1371–1410.

Acemoglu, Daron (2009) *Interaction to Modern Economic Growth*, Princeton University Press, New York.

Acemoglu, Daron (2010) "When Does Labor Scarcity Encourage Innovation?" *Journal of Political Economy,* 118(6): 1037–1078.

Acemoglu, Daron (2015) "Localised and Biased Technologies: Atkinson and Stiglitz's New View, Induced Innovations, and Directed Technological Change." *Economic Journal*, 125(583): 443-463.

Acemoglu, Daron (2023) "Distorted Innovation: Does the Market Get the Direction of Technology Right?" AEA Distinguished Lecture, *American Economic Review Papers and Proceedings*, 113(4), 1–28.

Acemoglu, Daron and David Autor (2011) "Skills, Tasks and Technologies: Implications for Employment and Earnings," *Handbook of Labor Economics*, 4: 1043–1171.

Acemoglu, Daron and Simon Johnson (2023)

Acemoglu, Daron and Pascual Restrepo (2018) "The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment" *American Economic Review*, 108(6): 1488–1542.

Acemoglu, Daron and Fabrizio Zilibotti (2001) "Productivity Differences," *Quarterly Journal of Economics,* 116(2): 563–606.

Aghion, Philippe and Peter Howitt (1992) "A Model of Growth Through Creative Destruction," *Econometrica,* 60(2): 323–351.

Allen, Robert C. (2009) *British Industrial Revolution in Global Perspective*, Cambridge University Press, New York.

Atkinson, Anthony B. and Joseph E. Stiglitz (1969) "A New View of Technological Change," *Economic Journal*, 79(2): 573-78.

Autor, David H., Frank Levy and Richard J. Murnane (2003) "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics,* 118(4): 1279–1333.

Barro, Robert and Xavier Sala-i-Martin (1995) *Economic Growth*, McGraw Hill, New York.

Basu, Susanto and David N. Weil (1998) "Appropriate Technology and Growth," *Quarterly Journal of Economics*, 113(3): 1025-10.

Blanchard, Olivier (1998) "The Medium Run" *Brookings Papers on Economic Activity.*

Diwan, I. and Dani Rodrik (1991) "Patents, Appropriate Technology and North-South Trade" *Journal of International Economics*, 30(1): 27-47.

Gordon, Robert (2016) *The Rise and Fall of American Growth*, Princeton: Princeton University Press,

Grossman, Gene M. and Elhanan Helpman (1991) "Quality Ladders in the Theory of Growth," *The Review of Economic Studies,* 58(1): 43–61.

Habakkuk, H.J. (1962) *American and British Technology in the Nineteenth Century.* London: Cambridge University Press.

Jones, Charles I. (2005) "The Shape of Production Functions and the Direction of Technical Change," *The Quarterly Journal of Economics,* 120(2): 517–549.

Karabarbounis, Loukas and Brent Neiman (2014) "The Global Decline of the Labor Share," *The Quarterly Journal of Economics*, 129(1): 61–103.

Mokyr, Joel (1990) *The Lever of Riches: Technological Creativity and Economic Progress,* Oxford University Press: New York.

Romer, Paul M. (1990) "Endogenous Technological Change," *Journal of Political Economy*, S30, S71-S102.

Schumacher, Ernst F. (1973) *Small is Beautiful: Economics as if People Mattered*, Blond & Briggs Ltd., London, England..

Stewart, Frances (1977) *Technology and Underdevelopment*, The Macmillan Press, Ltd., London, England.

Zeira, Joseph (1998) "Workers, Machines, and Economic Growth," *Quarterly Journal of Economics,* 113(4): 1091–1117.