Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

Christian K. Wolf[†]

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Abstract: I derive a general condition on consumer behavior ensuring that, in a standard model of demand-determined output, any path of inflation and output that is implementable via interest rate policy is also implementable through time-varying uniform transfers. In an analytical model with occasionally-binding borrowing constraints, my condition holds generically. In a quantitative HANK model, the transfer policy that closes any given demand shortfall is furthermore well-characterized by a small number of measurable sufficient statistics. My results extend to environments with investment if transfers are supplemented by another standard fiscal tool—bonus depreciation.

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1 Introduction

The prescription of standard New Keynesian theory is to conduct stabilization policy through changes in short-term interest rates. Over the past decade, much policy and academic interest has centered on the question of whether—and if so, how—alternative policy tools could be used to replicate monetary stimulus when nominal interest rates are constrained by a zero or effective lower bound (ELB).[1] Prior work has in particular identified tax policy, often labeled *unconventional* fiscal policy, as an attractive option (Correia et al., 2008, 2013): time-varying tax rates manipulate intertemporal prices just like monetary policy and thus can replicate any desired monetary allocation.

In this paper I ask whether *conventional* fiscal policy—that is, fiscal instruments that are already part of the standard stabilization policy toolkit—is similarly sufficient to replicate any given monetary policy. The setting for much of my analysis is a textbook business-cycle model with nominal rigidities and without capital, extended to allow for more general, non-Ricardian household consumption behavior. The fiscal stabilization tool that I consider are uniform, deficit-financed transfers ("stimulus checks"), an instrument used in all recent U.S. recessions. I make three contributions. First, I identify a general sufficient condition under which any time paths of aggregate output and inflation that are implementable via interest rate policy are also implementable solely by adjusting the time path of such uniform lump-sum taxes and transfers. Second, I check this condition in an analytical model of occasionally-binding household borrowing constraints (e.g., as in Woodford, 1990; Farhi & Werning, 2019; Angeletos et al., 2023), and prove that it holds *generically*—borrowing constraints just need to bind some of the time. Third, I then numerically verify the sufficient condition in a rich quantitative heterogeneous-agent (HANK) model. In this environment, the stimulus check policy that closes any given demand shortfall is furthermore well-characterized by a small number of measurable "sufficient statistics." Finally, I establish that all of these conclusions extend to richer models with investment as long as stimulus checks are supplemented by a second, similarly conventional fiscal tool: bonus depreciation.

Environment & general equivalence condition. My model economy features a policymaker with access to two instruments: nominal interest rates and uniform, lump-sum taxes and transfers. My objective is to characterize the space of allocations implementable

[1] A notable early example is Bernanke (2002). Important recent contributions include Correia et al. (2008), Correia et al. (2013), Galí (2020), and Reis & Tenreyro (2022).

through manipulation of these two instruments. All results apply to (linearized) perfect foresight transition paths, or equivalently to the model's first-order perturbation solution with aggregate risk. Key for me are properties of the two matrices $\mathcal{C}_{i_b}$ and $\mathcal{C}_\tau$, whose (t, s)th entries are, respectively, the derivatives of partial equilibrium consumption demand at time t with respect to a) a change in the time-s rate of interest on bonds and b) a uniform lump-sum transfer paid out at time s. Note that $\mathcal{C}_\tau$ is a matrix of intertemporal marginal propensities to consume (iMPCs), as studied first in Auclert et al. (2018).

I establish that, if $\mathcal{C}_\tau$ is invertible—a condition that I refer to as *strong Ricardian non-equivalence*—, then any sequence of aggregate output and inflation that can be attained via interest rate policy is similarly implementable by only adjusting the time profile of uniform lump-sum taxes and transfers. The proof begins with the household consumption-savings problem. A feasible monetary policy is a path of interest rates together with a path of lump-sum taxes or transfers that ensures a balanced government budget. Through the household problem, this policy induces some path of net excess consumption demand. Can a transfer-only policy—that is, a policy that only changes the time profile of taxes or transfers, again subject to budget balance—engineer the same path of demand? For Ricardian households, the answer is clearly no: for them, only the net present value of transfers matters, so any budget-feasible transfer policy leaves spending unchanged. Mathematically, this is reflected in $\mathcal{C}_\tau$ being rank-1. If instead the *timing* of transfers matters (in the strong sense that $\mathcal{C}_\tau$ is invertible), then there does exist some path of transfers alone that perturbs demand in exactly the same way as the baseline monetary policy. Since this monetary policy was by assumption budget-balanced, the equivalent transfer policy is feasible as well. The argument is completed by showing that, in my environment, two policies that generate the same net excess consumption demand paths must be accommodated in general equilibrium through the same market-clearing adjustments in prices and quantities.[2]

Under the conditions of my equivalence result, transfer payments can serve as a perfect substitute for interest rate policy in the eyes of a conventional "dual mandate" policymaker. Formally, my results imply that systematic policy *rules* (e.g., flexible inflation forecast target criteria, as discussed in Woodford, 2011) continue to be implementable even if nominal rates are constrained by a binding lower bound. In particular, this conclusion holds completely independently of the menu of non-policy disturbances hitting the economy.

[2]Though revenue-equivalent in net present value terms, the two policies do invariably induce different short-run government debt dynamics: while interest rate policy can in principle have aggregate effects even if outstanding debt is fixed, uniform stimulus checks work *only* because they change the time path of government bonds held by the private sector.

Analytical equivalence result. I next verify the condition underlying my equivalence result in an analytically tractable model of occasionally-binding household borrowing constraints. Building on Farhi & Werning (2019), I consider a perpetual-youth model of household consumption-savings decisions (à la Blanchard, 1985), where the survival probability $\theta \in [0, 1]$ can be re-interpreted as one minus the probability of borrowing constraints binding at any given date (see also Woodford, 1990; Angeletos et al., 2023). I further generalize this environment by allowing for an arbitrary number of household types i that differ in θ_i—i.e., the extent to which they are subject to binding borrowing constraints.

My main result is that, for policy equivalence to hold, it suffices for borrowing constraints to bind *some of the time.* The argument proceeds in two steps. First, I begin by considering a model with only one household type and a borrowing constraint coefficient of θ. I here can prove that, as long as $\theta < 1$—i.e., borrowing constraints that just bind some of the time—$\mathcal{C}_\tau$ is invertible. The intuition is straightforward: occasionally-binding borrowing constraints shorten spending horizons, and so transfers today financed by taxes in the future lead to a front-loading of spending, mimicking the effects of interest rate cuts. Second, I go back to the general environment featuring a mixture of household types, with potentially heterogeneous θ_i's. I here establish that, if θ_i for all types i is below—but potentially arbitrarily close to—1, then the equivalence result yet again holds.[3]

Finally, I establish that the policy equivalence result is not just theoretically general, but also empirically relevant. In my environment, transfer policy can induce any desired path of net excess demand if and only if the economywide average MPC strictly exceeds $\frac{\bar{r}}{1+\bar{r}}$, where $\bar{r}$ is the real rate of interest—i.e., an MPC above the classical permanent-income level. This requirement is well-known to receive ample support in the data (e.g., see Parker et al., 2013; Fagereng et al., 2018; Borusyak et al., 2021).[4]

Quantitative explorations. I complement my analytical discussion with a quantitative exploration in a rich HANK model (Kaplan et al., 2018). This analysis serves a dual purpose. First, it allows me to verify that—as expected given my analytical results—the sufficient condition for policy equivalence also holds in state-of-the-art quantitative models

[3]The formal proposition does not cover the limit case of *some* households with θ_i exactly equal to 1 due to economically inessential technicalities (see the discussion after Proposition 2).

[4]My derivations throughout assume that $\beta(1+\bar{r}) = 1$, as in Farhi & Werning (2019). If instead $\beta(1+\bar{r}) > 1$, then consumers have a natural tendency to postpone spending, so my results instead require that borrowing constraints bind *often enough*. Importantly, however, this generalized condition is still equivalent to MPCs above the annuity value of income gains, and thus testable in the same way.

of the consumption-savings decision. Second, I document that the policy equivalence result is also practically relevant, in the following sense. Building on Auclert et al. (2018), I find that, in my quantitative HANK model, the inverse $\mathcal{C}_\tau^{-1}$ not only exists, but also can be characterized to a very high degree of accuracy with just three empirically measurable numbers: the economy's average MPC ω; the slope θ of how consumers spend a lump-sum income gain over time; and the steady-state rate of interest $\bar{r}$. Crucially, for empirically relevant levels of these three statistics, my formula for the inverse matrix $\mathcal{C}_\tau^{-1}$ reveals that even moderate increases in transfers suffice for meaningful aggregate cyclical stabilization.

Extensions & limitations. My results extend to environments that also feature capital. Monetary policy here now operates through two levers: by directly affecting (i) the household consumption-savings decision and (ii) the firm investment decision. Transfer policy is still enough to replicate arbitrary stimulus to consumer excess demand. For firm investment, it is straightforward to show that a second, similarly conventional fiscal instrument will suffice: bonus depreciation stimulus. I thus conclude that, even in this extended environment, two entirely conventional fiscal tools suffice to replicate any desired monetary stimulus.

While most of my analysis is concerned with aggregates, I also briefly discuss important limitations to my equivalence result in the presence of microeconomic heterogeneity. First, I emphasize that—for my equivalence result to also apply to HANK-type models—I require particular (though standard) assumptions on wage-setting. These assumptions are consistent with arbitrary *levels* of wealth effects in labor supply, but not with cross-sectional *dispersion* in those wealth effects. I review the empirical evidence on such dispersion and conclude that the scope for this channel to materially affect the equivalence result is limited as long as stimulus checks are moderate in size, with magnitudes on the order of what has been observed in practice. Second, my macro-equivalent interest rate and lump-sum transfer policies are generally *not* equivalent household-by-household: nominal interest rate cuts mostly act by directly stimulating consumption at the top of the liquid wealth distribution, whereas the macro-equivalent transfer stimulus almost exclusively acts at the bottom. Thus, compared to a given interest rate policy, the macro-equivalent transfer delivers the same macro stimulus at smaller cross-sectional consumption dispersion. The normative implications of this positive observation are studied in McKay & Wolf (2022a).

Related literature. The paper relates and contributes to several strands of literature.

First, the analysis is motivated by the recent experience of limits to conventional monetary policy space. Prior work has argued that *unconventional* fiscal instruments can be used to substitute for monetary stimulus if needed (Correia et al., 2008, 2013); I instead clarify the conditions under which more *conventional* fiscal stimulus tools can do the same.[5]

Second, I relate to prior work on the stimulative effects of transfer payments in the absence of Ricardian equivalence. Blanchard (1985), Woodford (1990) and Bilbiie et al. (2013) all emphasize that, if private planning horizons are finite (due to death) or borrowing constraints bind, then public debt can stimulate spending through its role as private liquidity. Bilbiie et al. (2021) furthermore show that redistribution from savers to spenders in a two-type model can perfectly mimic monetary stimulus. Relative to this line of work, my contribution is to: (i) identify a general condition under which uniform, time-varying taxes and transfers are stimulative in the precise sense that they can replicate monetary policy; (ii) verify this condition in a quite general analytical model of occasionally-binding borrowing constraints; and (iii) show that the required paths of taxes and transfers can, in quantitatively relevant models, be characterized as a function of a small number of empirically measurable statistics.

Third, my proof of policy equivalence relies heavily on equilibrium characterizations in sequence space (see Boppart et al., 2018; Auclert et al., 2019). So far, the sequence-space set-up has been used to analytically characterize general equilibrium effects (Auclert & Rognlie, 2018; Auclert et al., 2018) or to construct general equilibrium counterfactuals for unobserved shocks (Wolf, 2020). I instead use the same observations to sidestep constraints on policy space. Echoing classical general equilibrium theory (Arrow & Debreu, 1954), the sequence-space perspective reveals that two policies are equivalent if they induce the same net excess demand paths. As such, my equivalence results are conceptually distinct from Correia et al. (2013) and Farhi et al. (2014)—there, equivalence is shown via identical wedges in optimality conditions. The advantage of my approach is that it can be applied readily to conventional fiscal instruments (like stimulus checks); the obvious challenge is that characterization of the equivalent policy requires additional arguments (my formulas for $\mathcal{C}_\tau^{-1}$).

Outlook. The rest of the paper proceeds as follows. Section 2 sets up the baseline model, and Section 3 presents the main theoretical results on policy equivalence. The quantitative analysis, including in particular my sufficient statistics formula, follows in Section 4. I discuss the role of micro heterogeneity in Section 5, and finally extend my results to a model with investment in Section 6. Section 7 concludes.

[5] Another related policy tool that has received both academic as well as substantial policy interest are helicopter drops of money—that is, *money-financed* transfer payments (e.g., see Bernanke, 2002; Galí, 2020; Reis & Tenreyro, 2022). My results reveal that, under natural assumptions on consumer spending behavior, even *deficit-financed*, *uniform* transfer payments are sufficient to replicate monetary stimulus.

2 Environment

I begin with a description of the model environment in Section 2.1. My assumptions on consumption behavior will be purposefully general, requiring only the existence of an aggregate consumption function. Section 2.2 then discusses the particular analytical and quantitative consumption-savings problems that I will consider in Sections 3 and 4.

2.1 Model outline

Time is discrete and runs forever, $t = 0, 1, \ldots$. The model economy is populated by households, unions, firms, and a government, and is initially at its deterministic steady state. I study linearized perfect foresight transition paths.[6] The set-up is kept deliberately close to the textbook New Keynesian business-cycle framework (Woodford, 2011; Galí, 2015).

At time $t = 0$, the policymaker announces paths for her policy instruments. My objective is to characterize the set of allocations that she can implement. The realization of a variable x at time t along the equilibrium perfect foresight transition path will be denoted x_t, while the entire time path will be denoted $\boldsymbol{x} = \{x_t\}_{t=0}^{\infty}$. Hats denote (log-)deviations from the deterministic steady state and bars denote steady-state values.

The aggregate consumption function. Households consume and supply labor, with total consumption and hours worked denoted by c_t and ℓ_t, respectively. Due to frictions in the labor market, hours worked are taken as given by households and set by optimizing labor unions, to be discussed in detail later.[7] Given paths of income, households then decide on their consumption and savings. Rather than specifying the details of this consumption-savings problem, I here simply summarize its solution in the form of an aggregate consumption function. Section 2.2 will consider several particular models of household consumption behavior that fit into this general framework.

Before stating and discussing the aggregate consumption function I begin with the household budget constraint. Total income e_t of the household sector consists of: labor earnings $(1 - \tau_\ell)w_t\ell_t$, where w_t is the real wage and τ_ℓ is the (assumed fixed) labor tax rate; uniform

[6] My results can thus equivalently be interpreted as applying to the first-order perturbation solution of an analogous model with aggregate risk (e.g. Boppart et al., 2018; Auclert et al., 2019).

[7] I allow for unions in the interest of generality and empirical relevance. For all models except for the quantitative heterogeneous-household model of Section 4, the alternative standard case of frictionless labor supply will correspond to the flexible-wage limit of my union model. I further discuss the role of labor supply in HANK models in Section 5.1.

lump-sum transfer receipts τ_t; and dividends d_t. Households can invest in nominally risk-free liquid bonds with nominal returns $i_{b,t}$. The real return to saving is affected by the inflation rate π_t. The period-t budget constraint of the aggregated household sector is thus

$$c_t + b_t = \underbrace{\overbrace{(1-\tau_\ell)w_t\ell_t + \tau_t + d_t}^{\text{total income } e_t} + \frac{1+i_{b,t-1}}{1+\pi_t}b_{t-1}}_{\text{total cash on hand } x_t} \tag{1}$$

where b_t denotes real bond holdings. Given any sequence of total household income and asset returns, optimal household behavior yields time paths of aggregate consumption demand $\boldsymbol{c}$ and asset supply $\boldsymbol{b}$. I summarize optimal household consumption behavior in the form of an aggregate consumption function $\mathcal{C}(\bullet)$ (Farhi & Werning, 2019; Auclert et al., 2018):[8]

$$\boldsymbol{c} \quad = \quad \mathcal{C}(\underbrace{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}}_{\text{eq'm aggregates}}; \underbrace{\boldsymbol{\tau}, \boldsymbol{i}_b}_{\text{policy}}) \tag{2}$$

By definition, the aggregate consumption function evaluated at steady state satisfies

$$\bar{\boldsymbol{c}} \quad = \quad \mathcal{C}\left(\bar{\boldsymbol{w}}, \bar{\boldsymbol{\ell}}, \bar{\boldsymbol{\pi}}, \bar{\boldsymbol{d}}; \bar{\boldsymbol{\tau}}, \bar{\boldsymbol{i}}_b\right)$$

In my linearized environment, policy equivalence will be fully governed by the properties of $\mathcal{C}(\bullet)$ around this deterministic steady state. Linearizing (2), we can write

$$\widehat{\boldsymbol{c}} \quad = \quad \mathcal{C}_w\widehat{\boldsymbol{w}} + \mathcal{C}_\ell\widehat{\boldsymbol{\ell}} + \mathcal{C}_\pi\widehat{\boldsymbol{\pi}} + \mathcal{C}_d\widehat{\boldsymbol{d}} + \mathcal{C}_\tau\widehat{\boldsymbol{\tau}} + \mathcal{C}_{i_b}\widehat{\boldsymbol{i}}_b \tag{3}$$

where, for each $q \in \{w, \ell, \pi, d, \tau, i_b\}$, I have defined

$$\mathcal{C}_q \quad \equiv \quad \frac{\partial \mathcal{C}(\bullet)}{\partial \boldsymbol{q}}, \tag{4}$$

with the derivative evaluated at the deterministic steady state. The (t, s)th entry of each of those infinite-dimensional linear maps is the response of aggregate consumption demand at time t to a marginal change in input q at time s. The linear map $\mathcal{C}_\tau$—which indicates how aggregate consumption demand will respond to changes in lump-sum transfers—will play a central role in characterizing the allocations implementable by the policymaker through stimulus check policy. To build further intuition I will in Section 2.2 provide closed-form characterizations of this map in various familiar models of household consumption decisions.

[8] To be precise, I, as in Auclert et al. (2018), assume that this consumption function is Fréchet-differentiable around the steady state, and its derivatives in (4) are bounded linear operators from ℓ^∞ to ℓ^∞.

Unions & firms. I summarize the production and wage bargaining block through three key relations. First, a unit continuum of firms produces the final output good:

$$y_t = y(\ell_t)$$

Second, price-setting is subject to the usual Rotemberg adjustment costs, giving a textbook New Keynesian Phillips Curve (NKPC) in prices (Galí, 2015):

$$\widehat{\pi}_t = \kappa_p \times \underbrace{\chi_p(\widehat{w}_t, \widehat{\ell}_t)}_{\text{deviation from price target}} + \beta \widehat{\pi}_{t+1} \tag{5}$$

where $\chi_p(\bullet)$ gives the deviation from the price target and κ_p is the slope of the price-NKPC. Third, wage bargaining is similarly subject to adjustment costs and so induces a general wage-NKPC, linking wage inflation to the static labor optimality wedge (Erceg et al., 2000):

$$\widehat{\pi}^w_t = \kappa_w \times \underbrace{\chi_w(\widehat{w}_t, \widehat{\ell}_t, \widehat{c}_t)}_{\text{deviation from wage target}} + \beta \widehat{\pi}^w_{t+1} \tag{6}$$

where $1 + \pi^w_t = \frac{w_t}{w_{t-1}}(1 + \pi_t)$ denotes wage inflation, $\chi_w(\bullet)$ gives the deviation from the wage target, and κ_w is the slope of the wage-NKPC. I discuss the derivation of (6) for particular assumptions on union behavior and household preferences in Appendices B.2 and B.3.

Policy. The government flow budget constraint is

$$\frac{1 + i_{b,t-1}}{1 + \pi_t} b_{t-1} + \tau_t = \tau_\ell w_t \ell_t + b_t \tag{7}$$

The policymaker sets nominal interest rates $i_{b,t}$ and uniform lump-sum taxes and transfers τ_t subject to the flow budget constraint (7) and the requirement that $\lim_{t\to\infty} b_t = \bar{b}$.

The remainder of this paper studies the implications of constraints on this policy toolkit. I focus on two kinds of restrictions: *transfer-only* policies and *interest rate-only* policies.

Definition 1. *A* transfer-only *policy is a policy that sets $i_{b,t} = \bar{i}_b$ for all t.*

In a transfer-only policy, the policymaker is forced to keep nominal interest rates fixed, perhaps due to a binding effective lower bound. In light of the ELB's empirical relevance, most discussion in this paper will center on the extent to which a restriction to transfer-only policies meaningfully constrains the policymaker. I note that, for such a policy, the direct mapping from policy instrument to consumer spending is fully governed by the matrix $\mathcal{C}_\tau$.

Definition 2. *An* interest rate-only *policy is a policy that sets, for all* $t = 0, 1, \ldots,$

$$\tau_t = \tau_\ell w_t \ell_t + (1 - \frac{1 + i_{b,t-1}}{1 + \pi_t})\bar{b}. \tag{8}$$

Definition 2 gives the natural opposite to a transfer-only policy: in an interest rate-only policy, the policymaker is free to adjust the path of nominal rates, but forced to passively adjust lump-sum transfers/taxes to balance the budget period-by-period. This policy thus operates by manipulating intertemporal prices, without any time variation in the total amount of government debt in the hands of households.[9] For such a policy, the direct mapping from policy instrument to consumer spending is governed by the following map:

$$\tilde{\mathcal{C}}_{i_b} \quad \equiv \quad \mathcal{C}_{i_b} \quad - \quad \bar{b} \times \mathcal{C}_{\tau,\bullet,-1}, \tag{9}$$

Intuitively, $\tilde{\mathcal{C}}_{i_b}$ combines the direct effect of the nominal rate change itself ($\mathcal{C}_{i_b}$) together with the implied movement in taxes required to balance the budget (second term).

Equilibrium. I am now in a position to define a perfect foresight transition equilibrium in this economy. As usual, I throughout restrict attention to equilibria in which all sequences of policies and macroeconomic aggregates are bounded (Woodford, 2011).

Definition 3. *An equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$ *and a set of macroeconomic aggregates* $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ *such that:*

1. *Consumption is consistent with the aggregate consumption function* (2)*.*
2. *Wage inflation* $\{\pi_t^w\}_{t=0}^{\infty}$ *and* $\{\ell_t, c_t, w_t\}_{t=0}^{\infty}$ *are consistent with the wage-NKPC* (6)*.*
3. *The paths* $\{\pi_t, w_t, \ell_t\}_{t=0}^{\infty}$ *are consistent with the price-NKPC* (5)*, and dividends are given as* $d_t = y_t - w_t \ell_t$*.*
4. *The output market clears:* $y_t = c_t$ *for all* $t \geq 0$*, the government budget constraint* (7) *holds at all* t*, and* $\lim_{t\to\infty} b_t = \bar{b}$*. The bond market then clears by Walras' law.*

An allocation of macroeconomic aggregates $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ *is said to be implementable if it can be supported as an equilibrium sequence.*

[9] More generally, an interest rate policy is a policy that freely sets nominal rates $i_{b,t}$ and then ensures budget balance in net present value terms (i.e., $\lim_{t\to\infty} \widehat{b}_t = 0$) through some fixed tax-and-transfer adjustment rule. (8) is simply a particularly transparent example of such a financing rule.

Note that Definition 3 specifies policy simply as a path of policy instruments $\{i_{b,t}, \tau_t, b_t\}_{t=0}^{\infty}$. As is well-known, policies of this sort generically do not induce unique equilibria (Sargent & Wallace, 1975). To address this challenge, I will later also discuss equilibria induced by policy *rules* for interest rates and transfers. I call an allocation $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$ uniquely implementable if it is the only equilibrium sequence consistent with those rules.

2.2 Detailed models of household consumption behavior

This section gives examples of several canonical models of household behavior that are consistent with the general aggregate consumption function (2). The purpose of this is threefold: first, to illustrate the generality of the model set-up in Section 2.1; second, to introduce the analytical and quantitative models that I will later study in Sections 3 and 4; and third, to already give some intuition for the shape and properties of the key matrix $\mathcal{C}_\tau$.

Analytical model. All analytical results in the remainder of the paper rely on a general yet tractable model of household consumption decisions, building very closely on Woodford (1990), Farhi & Werning (2019), and Angeletos et al. (2023). My presentation of the model will be brief, focusing mostly on what the model implies for the key derivative matrix $\mathcal{C}_\tau$. Supplementary details are provided in Appendix B.2.

There is a unit continuum of households, split into N distinct types with mass μ_i, where $\sum_{i=1}^{N} \mu_i = 1$. Households of each type i form a perpetual-youth block (e.g., as in Blanchard, 1985) with survival probability $\theta_i \in [0, 1]$. Preferences of a household j of type i are

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_i)^k \left[u(c_{ijt+k}) - v(\ell_{ijt+k}) \right] \right] \tag{10}$$

where the expectation operator captures idiosyncratic mortality risk, and I will assume that preferences take the standard form $u(c) = (c^{1-\gamma} - 1)/(1-\gamma)$ and $\nu(\ell) = \chi \ell^{1+\frac{1}{\varphi}}/(1+\frac{1}{\varphi})$. For each type i of household, the aggregate type budget constraint is exactly like (1), i.e.,

$$c_{it} + b_{it} = \underbrace{\overbrace{(1-\tau_\ell)w_t\ell_t + \tau_t + d_t}^{\text{total income } e_{it}} + \frac{1 + i_{b,t-1}}{1 + \pi_t} b_{it-1}}_{\text{cash-on-hand } x_{it}} \tag{11}$$

Note that all types receive the same income, but may differ in their consumption and wealth holdings. Within types, households trade in actuarially fair, risk-free nominal annuities, with

$\beta(1+\bar{r}) = 1$. Newborns furthermore receive a transfer that facilitates aggregation, exactly as in Angeletos et al. (2023). As argued in Woodford (1990) and Farhi & Werning (2019), this perpetual-youth set-up can be re-interpreted as a model of occasionally-binding borrowing constraints, with $1-\theta_i \in [0,1]$ giving the probability that the borrowing constraint will bind in any given time period. For $N = 1$, my model is identical to theirs, with all households subject to binding borrowing constraints with some common probability $1-\theta$. For $N > 1$, borrowing constraints can in principle bind differentially often for different types.

Following the derivations of Angeletos et al., it can be shown that total consumption of households of type i satisfies the following linearized demand function:

$$\widehat{c}_{it} = \underbrace{\left(1 - \frac{\theta_i}{1+\bar{r}}\right)}_{\text{MPC}} \cdot \underbrace{\left\{\widehat{x}_{it} + \sum_{k=1}^{\infty} \left(\frac{\theta_i}{1+\bar{r}}\right)^k \widehat{e}_{it+k}\right\}}_{\text{current cash-on-hand + future income}} - \underbrace{\sigma_i \sum_{k=0}^{\infty} \left(\frac{\theta_i}{1+\bar{r}}\right)^k \left(\widehat{i}_{b,t+k} - \widehat{\pi}_{t+k+1}\right)}_{\text{interest rate effects}} \quad (12)$$

where σ_i is a function of primitives. For the purposes of this paper, what matters most is the first part of (12)—the mapping from current and expected future income, including in particular transfer income, to consumer demand. Consistent with the borrowing-constraint interpretation of Farhi & Werning, we see that $\theta_i < 1$—i.e., occasionally-binding constraints—has two important effects: first, the contemporaneous MPC $1-\theta_i/(1+\bar{r})$ is elevated; and second, future income is subject to additional discounting relative to the steady-state rate of interest. These two features of (12) will loom large throughout the paper.

Combining the demand relation (12) with the budget constraint (11) yields a type-specific consumption function $\mathcal{C}^i(\bullet)$—i.e., a mapping from perfect-foresight sequences of equilibrium aggregates $\{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}\}$ and policy variables $\{\boldsymbol{\tau}, \boldsymbol{i_b}\}$ into sequences of type-specific consumption $\boldsymbol{c}_i$. Aggregating across types, we obtain the aggregate consumption function $\mathcal{C}(\bullet)$ in (2), and in particular

$$\mathcal{C}_q \equiv \sum_{i=1}^{N} \mu_i \cdot \mathcal{C}_q^i, \quad q \in \{w, \ell, \pi, d, \tau, i_b\}. \quad (13)$$

The main results in Sections 3 and 4 of this paper concern $\mathcal{C}_\tau$. The remainder of this section thus characterizes the type-specific matrix $\mathcal{C}_\tau^i$ under different assumptions on θ_i.

1. *Permanent-income consumer:* $\theta_i = 1$. The familiar case of $\theta_i = 1$ corresponds to standard permanent-income consumers, with borrowing constraints never binding. We see from (12) that, in this case, the MPC is simply equal to $\frac{\bar{r}}{1+\bar{r}}$—in response to any lump-sum income gain, households increase their consumption by the annuity value of that gain at each date. This is true both for contemporaneous income gains as well as for future income

gains, with those future income gains discounted at the steady-state rate of interest, but not any further. Combining (12) with (11) we arrive at the following expression for $\mathcal{C}_\tau$ in the permanent-income case, denoted $\mathcal{C}_\tau^R$ for future reference:

$$\mathcal{C}_\tau^R = \begin{pmatrix} \frac{\bar{r}}{1+\bar{r}} & \frac{\bar{r}}{(1+\bar{r})^2} & \frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ \frac{\bar{r}}{1+\bar{r}} & \frac{\bar{r}}{(1+\bar{r})^2} & \frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ \frac{\bar{r}}{1+\bar{r}} & \frac{\bar{r}}{(1+\bar{r})^2} & \frac{\bar{r}}{(1+\bar{r})^3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{14}$$

We see that $\mathcal{C}_\tau^R$ is rank-1, and in particular that any sequence of transfers with zero net present value does not affect household consumption at all—i.e., the familiar Ricardian equivalence result (Barro, 1974).

2. *Hand-to-mouth consumer: $\theta_i = 0$.* The case of $\theta_i = 0$ corresponds to perpetually-binding borrowing constraints—households immediately and fully consume any lump-sum income gain, i.e., they are hand-to-mouth. It is straightforward to see that $\mathcal{C}_\tau$, for future reference here denoted as $\mathcal{C}_\tau^H$, is given as

$$\mathcal{C}_\tau^H = I \tag{15}$$

where I is the identity matrix. Any sequence of taxes and transfers is thus passed through one-to-one into consumption; in particular, this means that even transfer sequences with zero net present value will affect consumer demand.

3. *Occasionally-binding constraints: $\theta_i \in (0,1)$.* The intermediate case $\theta_i \in (0,1)$—which is the one studied, for example, in Farhi & Werning (2019) or Angeletos et al. (2023)—is one of occasionally-binding constraints, leading to elevated MPCs and partial discounting of future income. Appendix C.1 provides the full expression for $\mathcal{C}_\tau$, denoted $\mathcal{C}_\tau^{OLG}$. For the purposes of the discussion here it suffices to consider the following approximate expression, with the sense of the approximation made precise in Appendix C.1 (see Lemma C.1):

$$\mathcal{C}_\tau^{OLG} \approx \underbrace{\left(1 - \frac{\theta_i}{1+\bar{r}}\right)}_{\text{baseline MPC}} \cdot \underbrace{\begin{pmatrix} 1 & \frac{\theta_i}{1+\bar{r}} & (\frac{\theta_i}{1+\bar{r}})^2 & \cdots \\ \theta_i & 1 & \frac{\theta_i}{1+\bar{r}} & \cdots \\ \theta_i^2 & \theta_i & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}}_{\text{intertemporal spending profile}}. \tag{16}$$

As discussed above, if $\theta_i < 1$, then MPCs are elevated. The second term in (16) tells us about the implied *intertemporal* consumer spending profile. First, because of current and anticipated future binding borrowing constraints, households frontload spending, decaying at rate θ_i (i.e., entries below the main diagonal of $\mathcal{C}_\tau^{OLG}$). Second, borrowing constraints binding between now and the future also means that future income receipt only partially affects spending today, leading to anticipation effects (i.e., entries above the main diagonal of $\mathcal{C}_\tau^{OLG}$) that similarly decay, now at rate $\theta_i/(1+\bar{r})$. It follows that even transfer sequences with zero present value affect consumer spending, unlike the permanent-income baseline; compared to the simple hand-to-mouth limit, however, the passthrough from transfers to demand is now not immediate (i.e., $\mathcal{C}_\tau^{OLG} \neq I$), instead exhibiting the more complicated, geometrically decaying dynamics displayed in the far-right part of (16).

My general multi-type analytical model nests each of these canonical models as special cases, as well as arbitrary mixtures between them. The theoretical results in Section 3 will all apply to such general mixtures. Notable special examples include the spender-saver model of Campbell & Mankiw (1989) and the OLG-spender hybrid of Angeletos et al. (2023). The latter will in particular be important in my quantitative analysis of $\mathcal{C}_\tau$ in Section 4.[10]

A quantitative "HANK"-type model. The set-up of Section 2.1 is similarly consistent with quantitative heterogeneous-agent models of the consumption-savings problem. In a canonical "HANK" model, a unit continuum of households $i \in [0,1]$ has preferences

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k \left[u(c_{it+k}) - v(\ell_{it+k}) \right] \right] \tag{17}$$

where expectations are now taken over idiosyncratic household productivity ε_{it}, with $\int_0^1 \varepsilon_{it} di = 1$ for all t. The individual household budget constraint is

$$c_{it} + b_{it} = (1-\tau_\ell) w_t \varepsilon_{it} \ell_t + \tau_t + d_t + \frac{1+i_{b,t-1}}{1+\pi_t} b_{it-1}, \quad b_{it} \geq \underline{b} \tag{18}$$

where household borrowing is now subject to an-hoc (tight) borrowing constraint $\underline{b}$.

The household consumption-savings decision is to choose sequences of consumption and savings, $\{\boldsymbol{c}_i, \boldsymbol{b}_i\}$ to maximize (17) subject to (18). Aggregating across all households, we yet

[10] My analytical results extend with rather little change to the most popular alternative model of tractable, non-Ricardian consumer behavior: bond-in-utility models, as considered for example in Michaillat & Saez (2018). I provide a discussion of such models in Appendices B.4 and C.1.

again obtain an aggregate consumption function (2). This consumption function—and in particular the derivative matrix $\mathcal{C}_\tau$—now does not admit any closed-form characterization. However, these objects can be recovered straightforwardly using the computational methods developed by Boppart et al. (2018) and Auclert et al. (2019). The numerical explorations in Section 4 will study the properties and shape of $\mathcal{C}_\tau$ in such models.

3 Interest rate cuts vs. stimulus checks

This section presents my core theoretical results on how stimulus checks can replicate nominal interest rate policy. I proceed in three steps. First, in Section 3.1, I present a general sufficient condition for policy equivalence. This general condition only uses the aggregate consumption function of Section 2.1. Second, in Section 3.2, I show that, in my analytical model, stimulus checks can *generically* replicate arbitrary interest rate policy, in the sense that borrowing constraints only need to bind some of the time.

My analysis in Sections 3.1 and 3.2 is couched in terms of linearized perfect-foresight transition sequences. Section 3.3, building on discussions in Auclert et al. (2019) and McKay & Wolf (2022b), translates these conclusions to equivalence in terms of policy *rules* in models with aggregate risk, solved using standard first-order perturbation techniques.

3.1 A sufficient condition for aggregate policy equivalence

I begin with a preliminary definition: a high-level property of the consumption derivative map $\mathcal{C}_\tau$ that I refer to as *strong Ricardian non-equivalence.* This property will turn out to be a general sufficient condition for my core equivalence result.

Definition 4. *A consumption function $\mathcal{C}(\bullet)$ exhibits strong Ricardian non-equivalence if the linear map $\mathcal{C}_\tau$ is invertible. I denote its inverse by $\mathcal{C}_\tau^{-1}$.*

Under the Barro (1974) definition of Ricardian equivalence, the time path of (lump-sum) taxes used to finance any given fiscal expenditure is completely irrelevant for consumption—only the present value matters. We already saw this in the expression for $\mathcal{C}_\tau$ in the permanent-income model (see (14)): there $\mathcal{C}_\tau$ is rank-1, and so in particular it is not invertible. With non-Ricardian households, on the other hand, the timing of transfers also begins to matter, thus increasing the rank of $\mathcal{C}_\tau$; strong Ricardian non-equivalence corresponds to the limit case of invertibility. Section 3.2 will establish that perpetual-youth models satisfy this property as soon as $\theta < 1$—i.e., as soon as borrowing constraints bind at least occasionally.

The equivalence result. I am now in a position to state the policy equivalence result.

Proposition 1. *Consider the model of Section 2.1, and let* $\widehat{\boldsymbol{c}}$ *be a path of household consumption with zero net present value, i.e.,* $\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_t = 0$. *Suppose that*

$$\widehat{\boldsymbol{c}} \;\in\; image(\mathcal{C}_\tau) \quad \Leftrightarrow \quad \widehat{\boldsymbol{c}} \;\in\; image(\tilde{\mathcal{C}}_{i_b}) \tag{19}$$

Then τ *and* i_b *are macro-equivalent: any aggregate allocation that is implementable with interest rate-only policy is also implementable with transfer-only policy, and vice-versa.*

An easy-to-interpret condition ensuring the direction "$\Leftarrow$" in (19) is strong Ricardian non-equivalence: if $\mathcal{C}_\tau$ is invertible, then the space of aggregate allocations implementable using transfer-only policies is at least as large that implementable using rate-only policies.

Proof sketch. Key to the proof of Proposition 1 is the insight that both interest rate as well as transfer policies only directly perturb the model's equilibrium conditions in two places: first, the left-hand side of the output market-clearing condition $\mathcal{C}(\bullet) = \boldsymbol{y}(\boldsymbol{\ell})$; and second, the sequence of government budget constraints (7).

In partial equilibrium—i.e., prior to general equilibrium price and quantity adjustments—a feasible monetary policy is simply a budget-neutral perturbation of relative intertemporal prices, inducing a path of net excess consumption demand

$$\widehat{\boldsymbol{c}}_{i_b}^{PE} \quad \equiv \quad \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b$$

Note that, since my definition of an interest rate policy includes its financing, this demand path necessarily has zero net present value. By (19), we can find *some* transfer sequence $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ that induces the exact same perturbation of net excess demand. Since the initial monetary policy was consistent with fiscal budget balance, and so since $\widehat{\boldsymbol{c}}_{i_b}^{PE}$ has zero net present value, it then follows from the household budget constraint (1) that the equivalent transfer also necessarily has zero net present value,

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_{t,i_b}^{PE} \;=\; \sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t(\widehat{\boldsymbol{i}}_b) \;=\; 0$$

Thus, prior to any general equilibrium feedback, $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ is also consistent with budget balance, in the sense that the induced debt path $\widehat{\boldsymbol{b}}$ satisfies $\lim_{t\to\infty} \widehat{b}_t = 0$. Finally note that the argument also works in reverse: any transfer-only policy that is consistent with the government budget constraint (7) (i.e., $\lim_{t\to\infty} \widehat{b}_t = 0$) necessarily induces a perturbation of consumption demand with zero net present value, and so by (19) we can always find an equivalent interest rate-only policy. To summarize, this first step leverages the fact that both policies equally flexibly manipulate the same lever: the consumption-savings decision of households.

Given that the transfer and nominal interest rate policies are both budget-feasible and both perturb the output market-clearing condition by exactly the same amounts time period by time period, it then follows via the implicit function theorem that they must also induce the same general equilibrium paths of inflation, hours worked, wages, and dividends. The intuition is simple: if, for example, the excess demand path induced by some monetary policy is accommodated in general equilibrium through increases in inflation and hours worked, then the same inflation and hours worked paths are also consistent with agent optimality and market-clearing after the equivalent transfer policy. This proof strategy is a simple application of the Arrow & Debreu (1954) approach to general equilibrium characterization using state-by-state (here t-by-t) net excess demand functions. Finally, the proof also reveals that the two policies are *revenue-equivalent* in general equilibrium: e.g., if increases in economic activity and inflation lead to an additional budget surplus after a rate cut, then the exact same budget surplus also opens up after the equivalent transfer stimulus.

Discussion of assumptions. The first step of my argument relied on two assumptions: (i) the existence of an aggregate consumption function $\mathcal{C}(\bullet)$, and (ii) the restriction that both policies operate *only* by manipulating that function. Many macroeconomic models induce such consumption functions—e.g., ranging from the models reviewed in Section 2.2 to those with behavioral biases (like Laibson et al., 2020; Lian, 2021)—, and so strong Ricardian non-equivalence is a useful, quite widely applicable sufficient condition. By (ii), however, my conclusions do not immediately extend to models in which monetary policy acts through multiple levers, e.g., firm investment. I discuss such extensions in Section 6.

The second (general equilibrium) step of my proof requires that two policies with identical effects on partial equilibrium net excess consumer demand are accommodated in the same way in general equilibrium. The shape of the wage-NKPC (6) plays an important role in ensuring that this is indeed the case. While this shape is entirely standard for representative-agent models and for those in particular nests the limiting special case of flexible labor supply, it requires additional assumptions in the case of cross-sectional household heterogeneity (e.g., as in quantitative HANK models). I will return to this point in Section 5.1.

3.2 Strong Ricardian non-equivalence in analytical models

I now investigate whether the high-level condition stated in Proposition 1 holds in particular models of household consumption behavior. The key takeaway will be that, in the analytical model of occasionally-binding borrowing constraints of Section 2.2, uniform lump-sum taxes and transfers can *generically* be used to replicate arbitrary interest rate policy, in the precise sense that borrowing constraints only need to bind some of the time.

$\mathcal{C}_\tau$ WHEN BORROWING CONSTRAINTS BIND. I begin by studying $\mathcal{C}_\tau^i$ for an individual type i. My first result is that strong Ricardian non-equivalence holds—that is, $\mathcal{C}_\tau^i$ is invertible—if and only if $\theta_i < 1$, i.e., type i is subject to occasionally-binding borrowing constraints. The proof is constructive, involving a direct characterization $(\mathcal{C}_\tau^i)^{-1}$.

Lemma 1. *Consider the analytical model of occasionally-binding borrowing constraints of Section 2.2. If $\theta_i < 1$, then the transfer derivative matrix $\mathcal{C}_\tau^i$ satisfies strong Ricardian non-equivalence, with its inverse given as*

$$(\mathcal{C}_\tau^i)^{-1} = \frac{1}{1-\frac{\theta_i}{1+\bar{r}}} \cdot \begin{pmatrix} \frac{1-\theta_i(1-\frac{\theta_i}{1+\bar{r}})}{1-\theta_i} & -\frac{1}{1+\bar{r}}\frac{\theta_i}{1-\theta_i} & 0 & \cdots \\ -\frac{\theta_i}{1-\theta_i} & \frac{1+\frac{\theta_i^2}{1+\bar{r}}}{1-\theta_i} & -\frac{1}{1+\bar{r}}\frac{\theta_i}{1-\theta_i} & \cdots \\ 0 & -\frac{\theta_i}{1-\theta_i} & \frac{1+\frac{\theta_i^2}{1+\bar{r}}}{1-\theta_i} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{20}$$

Note that the expression in (20) is exact, and not approximate as in (16)—characterizing $(\mathcal{C}_\tau^i)^{-1}$ is actually easier than characterizing $\mathcal{C}_\tau^i$ itself. The particular shape of $(\mathcal{C}_\tau^i)^{-1}$ is also intuitive. First, as $\theta_i \to 1$, we see that its entries diverge, reflecting the lack of invertibility in the permanent-income limit. Second, the constant probability of binding borrowing constraints—which led to the simple geometric shape of intertemporal spending displayed in (16)—means that the inverse also has a simple shape. Intuitively, to engineer one dollar of additional spending at some date t, a policymaker would need to give money to households at date t and then take money away at dates $t-1$ and $t+1$, exactly offsetting the leakage of date-t income to demand in adjacent periods.[11] Finally, if $\theta_i = 0$ (i.e., hand-to-mouth spending), then there is no cross-period leakage, so we just have $(\mathcal{C}_\tau^i)^{-1} = I$, as expected.

Lemma 1 reveals that occasionally-binding borrowing constraints suffice to ensure that

[11] As I show in Appendix C.1, $\mathcal{C}_\tau^{-1}$ has (almost) the exact same shape in bond-in-utility models.

(bounded) sequences of taxes and transfers can be used to induce arbitrarily complicated (bounded) paths of consumer demand. It thus follows that the popular models of occasionally-binding constraints considered in previous work—e.g., Farhi & Werning (2019) and Angeletos et al. (2023), which correspond to a single-type, $N = 1$ case of my general model—indeed satisfy policy equivalence. I now go one step further and return to the general model.

Proposition 2. *Consider the analytical model of occasionally-binding borrowing constraints of Section 2.2, and let* $\widehat{\boldsymbol{c}}$ *be a square-summable path of consumption with zero net present value. Then, if* $\theta_i < 1$ *for all* i*, and if* $\bar{r}$ *is sufficiently close to* 0*,*

$$\widehat{\boldsymbol{c}} \;\in\; image(\mathcal{C}_\tau), \tag{21}$$

and any aggregate allocation that is implementable with interest rate-only policy is also implementable with transfer-only policy.

The proof of Proposition 2 reveals that, for uniform transfer policy to be able to engineer any possible (square-summable, zero net present value) path of aggregate demand, it suffices if borrowing constraints bind some of the time *for some households*—i.e., that $\theta_i < 1$ *for some* i with $\mu_i > 0$. The restriction that θ_i is (at least marginally) below 1 for *all* types i is then just added for technical reasons, to ensure that the monetary policy-induced net excess demand path $\widehat{\boldsymbol{c}}_{i_b}^{PE}$ is in fact also square-summable.[12] Intuitively, as soon as there is at least one non-Ricardian type, consumer spending at date t becomes anchored to consumer income at date t, moving $\mathcal{C}_\tau$ away from the flat permanent-income shape and towards an identity matrix. As a result, the policymaker has all the leeway needed to flexibly manipulate excess demand through transfers, as required for my equivalence results.

Connection to empirical evidence. Proposition 2 has established that the high-level condition underlying Proposition 1 is a generic feature of a particular class of popular models. It turns out to also be empirically relevant. To show this I now map the conditions underlying Proposition 2 into empirically testable properties of the matrix $\mathcal{C}_\tau$.

Proposition 3. *Consider the analytical model of occasionally-binding borrowing constraints of Section 2.2. Then the condition in (21) is equivalent to either of the following two (interchangeable) properties of* $\mathcal{C}_\tau$*:*

[12] In the presence of permanent-income consumers, the net excess demand path induced by a given target monetary policy is generally not square-summable, by the usual unit-root property. This limitation, however, is of only limited practical relevance, as even a marginal deviation from permanent-income behavior (i.e., $\theta_i < 1$) breaks the unit-root property and restores square-summability.

1. *The average contemporaneous MPC is strictly above the permanent-income level, i.e.,*

$$\mathcal{C}_\tau(1,1) \ > \ \frac{\bar{r}}{1+\bar{r}} \tag{22}$$

2. *The average consumer spending response to a lump-sum income gain is front-loaded, i.e.,*

$$\mathcal{C}_\tau(1,1) \ > \ \mathcal{C}_\tau(2,1) \tag{23}$$

The two conditions in Proposition 3 capture the same intuition. In the permanent-income limit, the impact MPC $\mathcal{C}_\tau(1,1)$ is equal to the annuity value of the income gain, and the dynamic MPC profile is flat. Binding borrowing constraints elevate the impact MPC beyond the annuity value—condition (22)—and thus at the same time front-load overall consumption demand—condition (23). Importantly, both of these conditions are testable using evidence on how consumers spend lump-sum income gains (e.g., see Parker et al., 2013; Fagereng et al., 2018; Borusyak et al., 2021). A consistent finding here is that impact MPCs are elevated and that dynamic spending profiles are front-loaded and peak at income receipt. The policy equivalence result is thus not just theoretically general but also empirically relevant.

Extensions and limitations. In Appendix C.1 I extend the analysis to an even richer consumption function. Relative to (12), this alternative specification of consumer demand is more general in that it disentangles the impact MPC from the dynamic discounting factor θ. This extension is economically interesting for at least two reasons.

First, it nests the case $\beta(1+\bar{r}) \neq 1$.[13] If $\beta(1+\bar{r}) < 1$ (e.g., as is the case in models with precautionary savings), then relative household impatience increases MPCs and front-loads consumer spending even more, and so my conclusions about invertibility of $\mathcal{C}_\tau$ are entirely unaffected. If instead $\beta(1+\bar{r}) > 1$, then invertibility of $\mathcal{C}_\tau^i$ requires that borrowing constraints are binding *enough*; e.g., with log preferences, the condition becomes

$$\theta_i \leq \frac{1}{\beta(1+\bar{r})} < 1 \tag{24}$$

Intuitively, $\beta(1+\bar{r}) > 1$ leads households to *back-load* their spending. This is problematic for my invertibility requirement, for the following reason. Suppose a policymaker wishes to

[13] With $\beta(1+\bar{r}) \neq 1$ the analytical model need not have a well-defined steady-state level of consumption. I thus simply consider a first-order approximation to the partial equilibrium consumption-savings problem, characterize the implied $\mathcal{C}_\tau$, and ask when it is invertible (see Farhi & Werning, 2019).

increase spending by $1 today. If consumers postpone their spending, then the transfer that suitably increases demand today at the same time increases demand by *even more* tomorrow, thus necessitating further—and larger—transfer movements tomorrow. In Appendix C.1 I discuss a specific example in which this iteration diverges, breaking invertibility even though variations in the timing of transfers do invariably impact spending.[14] Borrowing constraints binding *often enough*—as ensured by the bound on θ_i in (24)—counteracts this effect, front-loads spending, and thus returns us to invertibility. Importantly, (24) is implied by either of the two testable conditions in Proposition 3, thus further reinforcing the practical relevance of the equivalence result.

3.3 From perfect foresight transitions to policy rules

The equivalence result in Proposition 1 was phrased in terms of perfect foresight transition paths, or equivalently in terms of impulse responses to policy shocks in a linearized model with aggregate risk. By the results in McKay & Wolf (2022b), such equivalence in terms of responses to policy *shocks* also implies equivalence in terms of policy *rules*: in response to any set of non-policy shocks, the aggregate outcomes implied by any given nominal interest rate rule can equivalently be implemented using a transfer-only rule. This subsection briefly elaborates on this observation, with further details provided in Appendix D.

A SKETCH OF POLICY RULE EQUIVALENCE. Consider an extended version of the baseline model that features a rich menu of non-policy shocks—wedges to the aggregate consumption function (2) as well as the two Phillips curves (5) - (6), corresponding to simple reduced-form representations of canonical demand and supply shocks. Given such a menu of shocks, I now ask whether the space of aggregate allocations that the policymaker can implement through commitment to policy rules is affected by constraints on nominal rates $i_{b,t}$ (e.g., a binding ELB). Under the conditions of Proposition 1, the answer turns out to be "no".

The core intuition for this policy rule equivalence result is straightforward and follows from McKay & Wolf (2022b). In a general linearized system, a policy rule is nothing but a mapping from lagged, current, and future expected macroeconomic outcomes to current and expected future values of the policy instrument. Under the conditions of Proposition 1, for every path of current and expected nominal interest rates, we can find a path of current and

[14]Formally, in this example, the linear map $\mathcal{C}_\tau$ is *injective*—variations in the timing of transfers affect consumption—but not *surjective*—there exist bounded paths of consumption that cannot be induced through bounded paths of transfers. I thank an anonymous referee for suggesting the insightful counterexample.

expected future transfers that perturbs all equilibrium conditions by the same amount, thus ensuring that current and expected future values of all macroeconomic outcomes—including in particular output and inflation—are identical. For a concrete, canonical example, consider the "forecast targeting rule" of a textbook dual-mandate policymaker (Woodford, 2011),

$$\widehat{\pi}_t + \lambda(\widehat{y}_t - \widehat{y}_{t-1}) \ = \ 0, \quad t = 0, 1, 2, \ldots \tag{25}$$

where λ is a function of policymaker preferences and model primitives. Under the conditions of Proposition 1, if the rule (25) induces a unique equilibrium when implemented through an interest rate-only policy, then (25) implemented through a transfer-only policy also induces a unique equilibrium, featuring the exact same aggregate allocation.[15] Appendix D substantiates the claims made here; in particular, I there show how transfers can replicate a general set of interest rate-only rules, including both *implicit* rules—like the forecast targeting criterion (25)—and *explicit* rules—like Taylor-type rules.

I note that my arguments on mapping one policy rule into an equivalent alternative policy rule sensitively rely on the invariance of the aggregate consumption function—including in particular $\mathcal{C}_\tau$—with respect to changes in policy rules. This model property is an immediate implication of linearity: I study the response of the economy to "small" incremental business-cycle shocks, linearizing around a deterministic steady state that is not affected by the change from one policy rule to the other. Naturally, focusing on such linearized dynamics could be misleading if my policy equivalence construction maps a given (moderately sized) nominal interest rate path into very large tax-and-transfer movements—i.e., policy changes that will plausibly move households far away from or meaningfully towards binding borrowing constraints, and are thus likely to materially change $\mathcal{C}_\tau$. This could happen if households are *almost* Ricardian: for example, if the economy is populated by just one consumer type, and if that consumer type has θ below but close to 1, then $\mathcal{C}_\tau^{-1}$ takes the form displayed in (20), with its entries diverging as $\theta \to 1$. As a result, to move consumer demand by a given target

[15] I picked (25) as my example because of its clear practical relevance. Notably, Bernanke (2015) summarizes the salience of the implicit targeting perspective embedded in (25) for Federal Reserve policymaking:

> "The Fed has a rule. The Fed's rule is that we will go for a 2 percent inflation rate. We will go for the natural rate of unemployment. We put equal weight on those two things. We will give you information about our projections about our interest rates. That is a rule and that is a framework that should clarify exactly what the Fed is doing."

My main result simply states that, under my assumptions, this rule can equivalently be implemented using a different policy instrument, giving exactly the same equilibrium outcomes of inflation and output.

amount, impractically large changes in lump-sum transfers would be required, invalidating my linear approximations. I will return to this important point in Section 4.

Taking stock. The analysis in this section has argued that my policy equivalence results are quite general: they hold under economically weak and empirically relevant assumptions, and they suffice for equivalence in the strong sense of delivering identical aggregate outcomes for an arbitrary menu of shocks hitting the economy.

Where the analysis in this section has fallen short, however, is on *characterization* of the equivalent stimulus check policies. In particular, while *existence* of the inverse $\mathcal{C}_\tau^{-1}$ may be general, a similarly important piece of information for a policymaker is the *shape* of that inverse—only with this knowledge can she figure out the particular policy that provides the desired macroeconomic stabilization. Section 4 will provide that characterization.

4 Quantitative analysis and policy characterization

The objective of this section is to provide a simple yet empirically relevant characterization of $\mathcal{C}_\tau^{-1}$—the potentially complicated infinite-dimensional object that governs the mapping from any possible shortfall in consumer spending to the transfer policy that would offset it. To this end I study the properties of $\mathcal{C}_\tau^{-1}$ in a quantitative HANK model.

Overall, this section establishes three main results. First, I verify numerically that $\mathcal{C}_\tau$ in my HANK model is indeed invertible, consistent with the theoretical discussion surrounding Proposition 2. Second, I show that $\mathcal{C}_\tau^{-1}$ takes a simple form, being robustly well-characterized by a small number of measurable sufficient statistics. Both results hold across a range of model parameterizations. Third, for empirically relevant values of the sufficient statistics, I find that moderately sized transfers already suffice for meaningful aggregate stabilization.

4.1 Heterogeneous-household model

My analysis relies on a relatively standard one-asset HANK model. This section presents the parameterization and discusses the model-implied consumption behavior.

Parameterization. I consider a calibrated version of the model of Section 2.1, with the aggregate consumption function coming from the heterogeneous-household consumption-savings problem described in Section 2.2. I only present a very brief overview of the (standard) parameterization here, and relegate further details to Appendix B.3.

Households face the same income process as in Kaplan et al. (2018), and can self-insure by saving, but not borrowing. I calibrate total liquid bond holdings to the amount of liquid wealth in the U.S. economy; corporate wealth, instead, is perfectly illiquid, with households receiving dividend payments as a function of their labor productivity (so that d_{it} is actually i-specific). For my robustness exercises, I consider alternative—less empirically relevant—parameterizations with materially higher or lower wealth holdings, thus delivering materially lower or higher average marginal propensities to consume, respectively.

The economy is closed with a constant-returns-to-scale production function as well as conventional degrees of nominal wage and price stickiness. This general equilibrium closure is irrelevant for all of the main results of this section (which only concern partial equilibrium consumer behavior, i.e. $\mathcal{C}_\tau$); rather, the model closure will only start to matter once I report general equilibrium experiments (as I do at the end of Section 4.2 as well as in Section 5).

Properties of $\mathcal{C}_\tau$. In an important contribution, Auclert et al. (2018) establish that the first column of $\mathcal{C}_\tau$ in HANK models—i.e., the response of consumption demand over time to a lump-sum income gain today—is consistent with empirical evidence on consumer spending behavior (e.g., see Parker et al., 2013; Fagereng et al., 2018). They furthermore show that, in both model and data, this first column actually has a rather simple shape: the impact MPC—say, within quarter of income receipt—is elevated; the MPC in the following period is then meaningfully smaller; and finally, from thereon out, intertemporal MPCs approximately decline at a constant, geometric rate.[16] This second observation suggests that the first column of $\mathcal{C}_\tau$ in quantitatively relevant HANK-type models may be parameterized with three "sufficient statistics": the impact MPC ω; the dynamic rate of decay θ from horizon $t = 2$ onwards; and finally the interest rate $\bar{r}$ which—via the fact that discounted lifetime MPCs necessarily add up to 1—delivers the delayed date-1 MPC. The sufficient statistics formula developed in the next section shows that these three numbers in fact suffice to provide an accurate characterization of the *entirety* of $\mathcal{C}_\tau$ in HANK, and thus also of $\mathcal{C}_\tau^{-1}$.

4.2 A three-coefficient sufficient statistics approximation

The analysis in this section proceeds in three steps. First, I propose a simple three-parameter sufficient statistics formula for $\mathcal{C}_\tau^{-1}$, leveraging the analytical models of Section 3.2. Second,

[16] Empirical evidence and some quantitative HANK modeling suggest that this rate of decay of intertemporal MPCs may slow down at longer horizons, in the tail. I investigate this point in Appendix C.3 and find that those tail MPCs are essentially irrelevant for my characterization of $\mathcal{C}_\tau^{-1}$.

I document the accuracy of that formula in my quantitative HANK model. Third, I discuss some practical policy implications.

The sufficient statistics formula. The proposed sufficient statistics formula leverages a special case of the general analytical model of Section 2.2—a simple two-type version that is rich enough to agree with both empirical evidence and quantitative HANK models on the shape of the first column of $\mathcal{C}_\tau$. Specifically, I consider a model populated by perpetual-youth consumers (i.e., with $\theta \in (0,1)$) together with a residual margin μ of spenders. By the discussion in Section 2.2 and Appendix C.1, the first column of $\mathcal{C}_\tau$ in such a simple mixture model satisfies

$$\mathcal{C}_\tau(\bullet, 1) \quad = \quad \underbrace{\mu \cdot \left(1 - \frac{\theta}{1+\bar{r}}\right) \cdot \begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \vdots \end{pmatrix}}_{\text{perpetual-youth part}} + \underbrace{(1-\mu) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}}_{\text{hand-to-mouth part}} \tag{26}$$

We see from (26) that the two-type hybrid model is rich enough to match arbitrary values of the three statistics $\{\omega, \theta, \bar{r}\}$—$\theta$ and $\bar{r}$ are given directly, and μ is then recovered residually to match the impact MPC ω. Importantly, however, in my hybrid model, these three statistics then characterize the entirety of $\mathcal{C}_\tau \equiv (1-\mu) \cdot \mathcal{C}_\tau^{OLG}(\theta, \bar{r}) + \mu \cdot I$ and thus also $\mathcal{C}_\tau^{-1}$. This is my proposed three-parameter sufficient statistics approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$.

The inverse $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ has a simple and intuitive shape, displayed as the orange dashed lines in Figure 1. Recall from the discussion in Section 3.2 that, for a single perpetual-youth consumer type, $\mathcal{C}_\tau^{-1}$ is tridiagonal—to engineer a dollar of excess demand at some arbitrary date t, it suffices to hand out a transfer at date t and then reduce transfers at the adjacent dates $t-1$ and $t+1$, to prevent any cross-period leakage. In the two-type model underlying my sufficient statistics formula the shape of $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ is similar: it has peaks along the main diagonal, just now the off-diagonal entries decay to zero more gradually, simply because the cross-period spending leakage takes a more complicated (i.e., non-geometric) form.

Approximation accuracy. The main result of this section is that the proposed three-dimensional approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ actually provides a quite accurate description of $\mathcal{C}_\tau^{-1}$ in quantitative HANK-type models. This conclusion closely echoes the findings of Auclert et al. (2018), who show that a mixture of spender-saver and bond-in-utility models can match

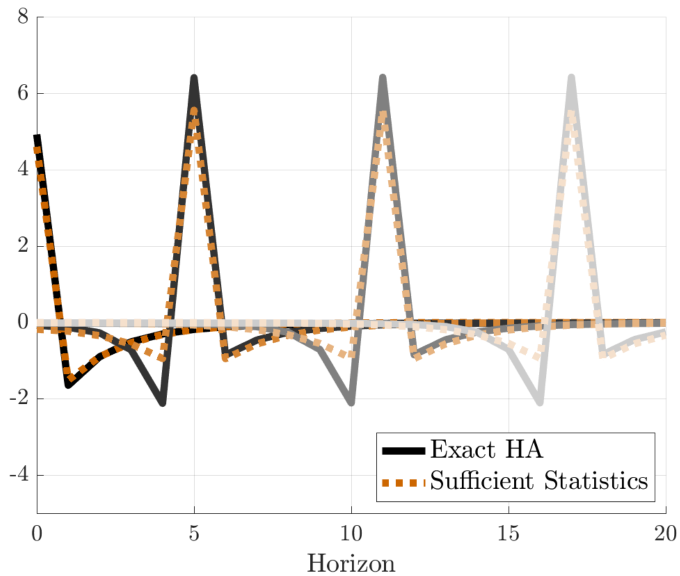

Figure 1: Entries of $\mathcal{C}_\tau^{-1}$ in the quantitative heterogeneous-agent model (shades of grey) and in the sufficient statistics approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ (shades of orange, dashed). Here $\bar{r}$ is set as in the heterogeneous-agent model, and $\{\omega, \theta\}$ are set to match $\mathcal{C}_\tau(1,1)$ and $\mathcal{C}_\tau(2,1)$. The lines correspond to columns $\{1, 6, 11, 16\}$, with lighter shades indicating farther-out columns.

HANK reasonably well on the entirety of $\mathcal{C}_\tau$.[17] While the analysis in this section will only look at the preferred calibration of my HANK model, I in Appendix C.3 document the robustness of my conclusions by also looking at alternative, materially different parameterizations.

Figure 1 begins by displaying several individual columns of $\mathcal{C}_\tau^{-1}$ taken from both (i) the full heterogeneous-agent model (shades of grey) and (ii) the sufficient statistics approximation (orange dashed), with $\{\omega, \theta, \bar{r}\}$ set to agree with the heterogeneous-agent model: $\omega = 0.30$, $\theta = 0.82$, and $\bar{r} = 0.01$—all values that are broadly consistent with the empirical evidence.[18] The key takeaway from the figure is that the orange lines are throughout quite close to the grey ones.[19] The intuition is that, in the HANK model, some households are up against the

[17]Relative to Auclert et al., the further value-added of my analysis here is to provide an explicit mapping from $\{\omega, \theta, \bar{r}\}$ into $\mathcal{C}_\tau^{-1}$, and to discuss how these three statistics affect the shape of $\mathcal{C}_\tau^{-1}$.

[18]To be precise, I set $\bar{r}$ to its value in the heterogeneous-agent model, $\omega = \mathcal{C}_\tau(1,1)$, and finally θ is set to ensure that $\mathcal{C}_\tau(\omega, \theta, \bar{r})(2,1) = \mathcal{C}_\tau(2,1)$.

[19]The figure reveals that there is some inaccuracy above the main diagonal of $\mathcal{C}_\tau^{-1}$, reflecting the fact that anticipation effects in the hybrid model somewhat differ from those in HANK. In principle, a generalized four-parameter sufficient statistics formula could better capture these effects, as discussed in Appendix C.3. In practice, however, the simpler three-parameter formula already provides an excellent approximation, as in particular is visible in my policy applications in Figure 2.

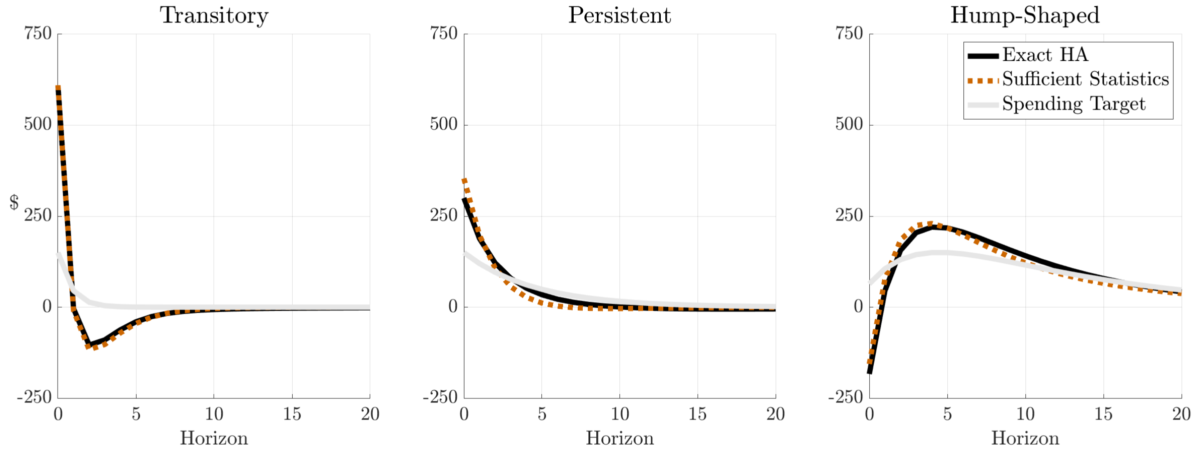

Figure 2: Three desired net excess demand paths $\widehat{\boldsymbol{c}}^{PE}$ (grey) and the required sequences of uniform lump-sum taxes and transfers, $\mathcal{C}_\tau^{-1} \times \widehat{\boldsymbol{c}}^{PE}$, as implied by the full heterogeneous-agent model (black) and my sufficient statistics formula (orange dashed).

borrowing constraint (so they act like spenders), while others will be up against it in the future with positive probability, so they act like perpetual-youth consumers with intermediate θ. Importantly, three simple (and measurable!) parameters are already enough to capture these spending dynamics—and their implications for $\mathcal{C}_\tau^{-1}$—quite well.

Next, Figure 2 shows several different "typical" target paths of net excess demand that a policymaker may wish to implement (grey) together with the sequences of transfers and taxes that do so (black and orange dashed). More precisely, given the three distinct paths of desired net excess demand $\widehat{\boldsymbol{c}}_\tau^{PE}$ (grey), the figure plots

$$\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}_\tau^{PE}) \;\equiv\; \mathcal{C}_\tau^{-1} \;\times\; \widehat{\boldsymbol{c}}_\tau^{PE},$$

where $\mathcal{C}_\tau^{-1}$ is either taken from the full quantitative HANK model (black) or from my simple sufficient statistics approximation (orange dashed). The three time paths of desired spending that I consider all have a peak of one per cent of steady-state consumption, but they are quite distinct in shape: short-lived in the left panel, more persistent in the middle panel, and hump-shaped in the right panel, capturing a range of policy-relevant scenarios. As expected in light of Figure 1, I find that, for all three paths, the actual required transfer sequence and the sufficient statistics prediction are close. I will return to the *magnitudes* of the required stimulus check policies when discussing practical policy implications.

Appendix C.3 repeats the exercises of Figures 1 and 2 for two possible alternative calibrations of the HANK model: one with much lower household wealth and thus higher MPCs,

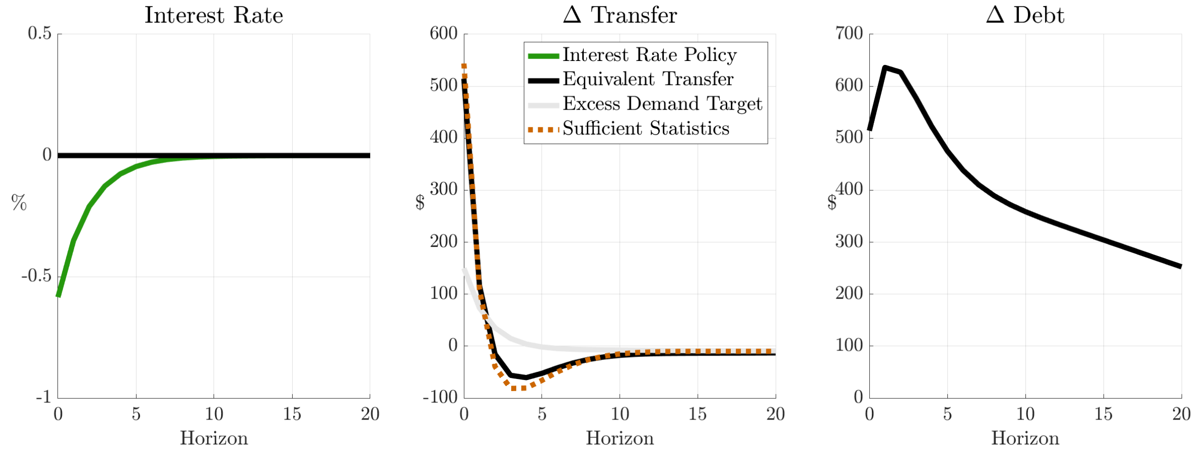

Figure 3: Impulse responses of nominal interest rates, transfers, and debt to interest rate (green) and transfer (black) policies in the calibrated HANK model, chosen to generate the net excess demand path displayed in the middle panel (grey). For transfers and debt I show the *differences* between their paths under the two policies as the black line.

and the other one with very high wealth and so low MPCs. The matrix $\mathcal{C}_\tau^{-1}$ and so the time paths $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}_\tau^{PE})$ materially differ across all of these models, but crucially my three-parameter sufficient statistics approximation remains accurate for all of them.

Practical policy implications. So far I have emphasized that my sufficient statistics formula yields an approximation $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ that accurately reflects consumer behavior in HANK models, across a range of parameterizations. A second important point is that, for *empirically relevant* values of the "sufficient statistics" $\{\omega, \theta, \bar{r}\}$, the entries of this inverse are moderate in size. We can see this in Figure 1, with the diagonal entries of $\mathcal{C}_\tau(\omega, \theta, \bar{r})^{-1}$ somewhat larger than ω^{-1}, and the off-diagonal entries relatively small and quickly converging to zero away from the main diagonal. Given that empirical evidence suggests elevated average MPCs ω, it follows that even moderately sized transfer stimulus suffices to close meaningful aggregate spending shortfalls. We also see this in Figure 2, which reveals that stimulus checks of the magnitudes observed in practice (e.g., around \$600, as in 2008), are predicted to close aggregate demand shortfalls of around one per cent.[20]

Figure 3 provides a final illustration by studying stabilization policy in response to a contractionary demand shock—that is, a shock that temporarily depresses partial equilibrium consumer spending, here with a peak effect of -1 per cent. By the classical divine coincidence

[20] Appendix C.3 elaborates further: there, I repeat the exercises in Figure 2 for a *range* of credible values of my sufficient statistics, and find that moderately sized checks throughout deliver meaningful stabilization.

logic, it is possible to perfectly stabilize inflation and output in the face of such a shock. To do so, policy needs to increase consumer spending to offset the 1 per cent contraction in demand (= \$150 per household), shown in the middle (grey). The usual policy prescription would be to do so through a cut in rates (left panel, green); in my heterogeneous-agent model, this requires a relatively short-lived rate cut with a cumulative total of around 150 basis points.[21] The black lines indicate how to achieve the same stabilization instead using lump-sum tax-and-transfer policy alone: transfers initially go up (here by around \$600), before then being financed through higher taxes down the line. Unsurprisingly (in light of Figures 1 and 2), this time path is again predicted almost perfectly by my sufficient statistics formula (orange, dashed). Finally, the right panel shows that the moderate stimulus check policy brings with it a moderate, transitory increase in government debt. Cyclical stabilization through stimulus check policy thus here requires no implausibly large or erratic fluctuations in taxes, deficits, or aggregate government debt. This is important: had meaningfully larger stimulus check policies been required, then the linearity assumption underlying my theoretical derivations in Section 3 would have been much less plausible.

Finally, while my experiments here throughout assumed uniform transfers, I emphasize that all results extend without any change if transfers are instead targeted at particular sub-populations of households, more similar to the more recent rounds of stimulus check policies observed in U.S. policy practice. A detailed discussion is provided in Appendix C.7.

5 The role of microeconomic heterogeneity

My analysis so far has been concerned exclusively with policy instrument equivalence at the aggregate level. Microeconomic heterogeneity played a role only to the extent that popular heterogeneous-agent consumption models are a natural (and empirically relevant) candidate to satisfy my sufficient condition of strong Ricardian non-equivalence.

This section sheds further light on the scope and the limitations of the policy equivalence result in the face of microeconomic heterogeneity. First, in Section 5.1, I gauge the extent to which microeconomic heterogeneity in labor supply decisions can break the equivalence result. I argue that, while this is possible in theory, it is unlikely to matter too much in practice. Second, in Section 5.2, I then note that equivalence at the aggregate level does not necessarily imply equivalence household-by-household.

[21] This implied direct mapping from interest rates to consumer net excess demand is broadly consistent with recent empirical evidence (e.g., see Table 4 in Crawley & Kuchler, 2021).

5.1 Wealth effects and household labor supply

To understand how household heterogeneity and wealth effects in labor supply can in principle challenge my equivalence result, it will be useful to recall the proof sketch of Proposition 1. As the first step of the argument, I consider nominal interest rate and transfer policies that induce identical paths of spending. This however is in general not enough to ensure equivalence in general equilibrium—the two policies also need to induce identical responses of total household labor supply. In my environment, the specific wage-NKPC (6)—which importantly depends only on aggregate consumption c_t—ensures that interest rate and stimulus check policies with identical direct effects on consumer spending indeed also induce identical labor supply responses, as required.

I conclude from this discussion that it is not wealth effects in labor supply *per se* that threaten the equivalence result; rather, a potential challenge are *heterogeneous* wealth effects, which would allow two policies with identical effects on total spending to potentially lead to different responses of total labor supply. I here use two experiments to argue that the scope for such heterogeneity to materially threaten the equivalence result is likely to be limited, at least for stimulus check policies of the size considered in my applications.

An alternative union bargaining protocol. Some form of nominal wage rigidity is widely argued to be necessary to match business-cycle dynamics in general (Christiano et al., 2005; Smets & Wouters, 2007) and consumption responses to macro shocks in particular (Auclert et al., 2020; Broer et al., 2020); importantly, it is also consistent with microeconomic evidence (Grigsby et al., 2019). The derivation of my wage-NKPC (6) relies on one particular union bargaining protocol that only responds to changes in aggregate consumption, thus ensuring that my two candidate interest rate and stimulus check policies also lead to identical responses of labor supply. The exact same bargaining protocol has also been used in other recent contributions to the HANK literature (Auclert et al., 2021; Aggarwal et al., 2022; McKay & Wolf, 2022a). An alternative union bargaining protocol (used e.g. in Auclert et al., 2018) instead has the union respond to a weighted average of individual household marginal utilities (rather than marginal utility at the average, as in my protocol). Appendix C.4 extends my HANK model environment to such an alternative protocol.

The main takeaway from my analysis in Appendix C.4 is that the equivalence result continues to hold *almost exactly* even under the alternative protocol. Intuitively, equivalence is now not exact because the interest rate and transfer policies that induce identical responses of consumer spending will generically *not* induce identical changes in the weighted average of consumer marginal utilities that enters the union problem. As a result, the adjustments in labor supply—$\widehat{\boldsymbol{\ell}}_{i_b}^{PE}$ and $\widehat{\boldsymbol{\ell}}_{\tau}^{PE}$—are not the same, and equivalence fails. However, marginal utility at the average and average marginal utility still co-move closely, so $\widehat{\boldsymbol{\ell}}_{i_b}^{PE}$ and $\widehat{\boldsymbol{\ell}}_{\tau}^{PE}$ remain quite similar, and aggregate outcomes are still nearly identical under the two policies.[22]

Matching empirical evidence on labor supply responses. Empirical evidence on household labor supply suggests that *marginal propensities to earn* (MPE)—that is, the response of earned income to a one-time, unexpected lump-sum transfer—are moderate in size, ranging from around 1% - 3% (Cesarini et al., 2017; Golosov et al., 2021), and somewhat increasing in household income, roughly doubling from the lowest to the highest income quartile (Golosov et al., 2021). Standard heterogeneous-household models with flexible labor supply struggle with both observations (Auclert et al., 2020): MPEs are predicted to be of the same order of magnitude as MPCs, and MPEs tend to be highest for high-MPC households. My solution is to adjust household preferences: I consider a hybrid of standard separable and Greenwood et al. (1988) GHH preferences (as originally proposed by Auclert et al., 2020) to match average MPEs, and then allow for preference heterogeneity across households to also match the cross-sectional MPE gradient. To make the heterogeneity particularly stark I consider a simple two-type model with low-MPC savers and high-MPC spenders, with their MPEs matched to the first and fourth quartiles of income reported in Golosov et al.. Full results are again reported in Appendix C.4.

In the model with empirically relevant and heterogeneous MPEs, I find that the policy equivalence result again holds almost exactly. The intuition is simply that heterogeneity in MPEs is small relative to the *level* of the average MPC. For example, with an MPC of 30% (an empirically relevant number for checks of the size studied in Section 4) and MPEs of 2% for spenders and 4% for savers (in line with Golosov et al.), the direct demand stimulus associated with either transfers or the equivalent rate cut is an order of magnitude larger than the difference in the labor supply response across households. Aggregate equilibrium dynamics are thus still dominated by the demand effects that are at the heart of Proposition 1.

Takeaways & qualifiers. The analysis in this section suggests that dispersion in wealth effects in labor supply is unlikely to materially affect the aggregate policy equivalence result.

[22] Furthermore, *even if* the two policies were to induce quite heterogeneous wealth effects in labor supply, nominal wage rigidity of the degree usually assumed in quantitative work would mean that this heterogeneity would matter little in general equilibrium, at least for transitory policy shocks (Christiano, 2011).

However, it is important to acknowledge that this conclusion is sensitively tied to the fact that throughout I am looking at stimulus check policies that are relatively moderate in size and transitory. For larger stimulus checks the cross-sectional heterogeneity in labor supply responses would likely become larger relative to the size of the demand stimulus (i.e., bigger MPE relative to MPC), thus moving the economy further away from policy equivalence.

5.2 Non-equivalence at the household level

It is important to note that the equivalence result of this paper applies to macroeconomic aggregates, but does not necessarily hold household-by-household.

Household-level consumption. Consider the general heterogeneous-household environment described in Section 2.2. Consumption of an individual household i is given as

$$\boldsymbol{c}_i \quad = \quad \mathcal{C}_i(\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}; \boldsymbol{\tau}, \boldsymbol{i}_b)$$

where the individual consumption function $\mathcal{C}_i(\bullet)$ is the solution to individual i's consumption-savings problem, indexed by that individual's initial asset holdings and productivity. Now consider two macro-equivalent interest rate and stimulus check policies $\widehat{\boldsymbol{i}}_b$ and $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$, constructed as in the proof of Proposition 1, and let $\Delta_i(\widehat{\boldsymbol{i}}_b)$ denote the difference in household i's consumption under the two policies; that is, let

$$\Delta_i(\widehat{\boldsymbol{i}}_b) \;\; \equiv \;\; \widehat{\boldsymbol{c}}_{i,i_b} - \widehat{\boldsymbol{c}}_{i,\tau(i_b)}$$

where i_b and τ subscripts indicate transition paths corresponding to interest rate and transfer policy, respectively. Since by construction both policies induce the same general equilibrium price and quantity responses (notably $\{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}\}$), we find that this difference satisfies

$$\Delta_i(\widehat{\boldsymbol{i}}_b) \;\; = \;\; \tilde{\mathcal{C}}_{i,i_b}\widehat{\boldsymbol{i}}_b \; - \; \mathcal{C}_{i,\tau}\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$$

where $\tilde{\mathcal{C}}_{i,i_b}$ and $\mathcal{C}_{i,\tau}$ are defined like $\tilde{\mathcal{C}}_{i_b}$ and $\mathcal{C}_\tau$, just now for each i. In words, the two policies may result in differences in consumption household-by-household because of potentially differential *direct* effects on consumer spending. Intuitively, while the two policies by design induce the same total direct stimulus (i.e., $\tilde{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b = \mathcal{C}_\tau\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ and so $\int_0^1 \Delta_i(\widehat{\boldsymbol{i}}_b)di = \boldsymbol{0}$), they may do so by affecting consumption at different points in the cross-section of households (i.e., we can have $\tilde{\mathcal{C}}_{i,i_b}\widehat{\boldsymbol{i}}_b \neq \mathcal{C}_{i,\tau}\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{i}}_b)$ and so $\Delta_i(\widehat{\boldsymbol{i}}_b) \neq \boldsymbol{0}$ for individual i).

Distributional outcomes in quantitative HANK models. I provide a numerical illustration of this non-equivalence at the household level by returning to the quantitative HANK model of Section 4. In this environment I compute the evolution of consumption along the household wealth distribution in response to the macro-equivalent nominal interest rate and stimulus check policies displayed in Figure 3. I only briefly discuss the main results here, with all further details relegated to Appendix C.5.

The headline result is that, while macro-equivalent, interest rate and transfer stimulus policies can have quite materially different effects in the cross-section of households. As discussed above, both policies induce the same general equilibrium feedback effects, household-by-household. The direct effects, however, are heterogeneous. On the one hand, rate cuts work mostly by directly stimulating the consumption of the rich. This is not surprising: wealthy households substitute intertemporally, while poor households are close to their borrowing constraint and so do not. On the other hand, the equivalent lump-sum transfer policy mostly acts at the bottom: it relaxes borrowing constraints and so stimulates consumption of the poor, while rich households barely respond (since the policy has zero present value). Relative to a given rate cut, stimulus checks that deliver the same aggregate stabilization thus do so at strictly smaller cross-sectional consumption dispersion. The normative implications of this positive observation are explored in McKay & Wolf (2022a).

6 Extension to investment

As the final step in my argument, I show that the policy equivalence result extends straightforwardly to a richer environment with investment if stimulus checks are complemented by a second, similarly standard fiscal tool: bonus depreciation stimulus.

6.1 A brief sketch of the environment

I augment the model of Section 2.1 to allow for productive capital. The firm block in this extended environment closely follows the tradition of standard business-cycle modeling (e.g., Smets & Wouters, 2007; Justiniano et al., 2010). I provide a brief sketch of this familiar model here, and relegate further details to Appendix B.5.

Production. A unit continuum of identical, perfectly competitive firms $j \in [0, 1]$ produces a homogeneous intermediate good, sold at real relative price p_t^I. The problem of firm j along the perfect foresight transition path is to

$$\max_{\{d_{jt},\ell_{jt},k_{jt},b^f_{jt}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty}\left(\prod_{q=0}^{t-1}\frac{1+\pi_{q-1}}{1+i_{b,q}}\right)d_{jt}$$

subject to the flow budget constraint

$$d_{jt} = p_t^I y(\ell_{jt},k_{jt-1}) - w_t\ell_{jt} - \underbrace{[k_{jt}-(1-\delta)k_{jt-1}]}_{\equiv i_{jt}} + \tau_{f,t}(\{i_{jt-q}\}_{q=0}^{t}) - \phi(k_{jt},k_{jt-1},i_{jt},i_{jt-1}) \quad (27)$$

where $\phi(\bullet)$ captures costs on either stock (capital) or flow (investment) adjustment. Intermediate goods producers thus hire labor on spot markets, invest, and pay out dividends. My only twist to this familiar production block is that I allow for a general fiscal investment stimulus policy $\boldsymbol{\tau}_f(\bullet)$, mapping investment today into future payments to the firm. Importantly, this set-up nests the popular bonus depreciation stimulus policy, in which investment today reduces tax liabilities in the future (Zwick & Mahon, 2017; Koby & Wolf, 2020). I will index time-t investment stimulus policies by a single parameter $\tau_{f,t}$.

Proceeding exactly as in Section 2.1, we can define an aggregate investment function $\mathcal{I}(\bullet)$; I relegate a discussion of the arguments of this function to Appendix B.5, as it is not essential here. For my purposes, the only important consideration is that fiscal stimulus is one of those arguments, with the direct effects of such stimulus summarized by the following derivative matrix:

$$\mathcal{I}_{\tau_f} \quad \equiv \quad \frac{\partial \mathcal{I}(\bullet)}{\partial \boldsymbol{\tau}_f}$$

Statements about the degree to which conventional fiscal instruments can be used to replicate monetary stimulus will be statements about the properties of $\mathcal{I}_{\tau_f}$ (and of course $\mathcal{C}_\tau$, as before).

Rest of the economy. The intermediate good is sold to monopolistically competitive retailers subject to nominal rigidities, summarized again with a general price-NKPC:

$$\widehat{\pi}_t \quad = \quad \kappa_p \ \times \ \widehat{p}_t^I + \beta\widehat{\pi}_{t+1} \quad (28)$$

The remainder of the model is unchanged. The extension of the equilibrium definition in Definition 3 is then straightforward, and provided in Appendix B.5.

6.2 Policy equivalence with conventional fiscal instruments

In this extended model, monetary policy operates through two channels: first, as before, it affects household consumption demand, and second, it changes firm investment and so labor hiring as well as intermediate goods production. Thus, transfer stimulus policy alone is now insufficient to replicate the effects of (infeasible) conventional monetary policy—exactly as in Correia et al. (2013), an additional instrument is needed. In a straightforward generalization of Proposition 1, Proposition 4 shows that invertibility of $\mathcal{C}_\tau$ and $\mathcal{I}_{\tau_f}$ is sufficient to leave the space of implementable output-inflation allocations unchanged.

Proposition 4. *Consider the extended model of Section 6.1. Suppose that $\mathcal{C}_\tau$ and $\mathcal{I}_{\tau_f}$ are both invertible, and consider an allocation $\{\pi_t^*, y_t^*\}_{t=0}^{\infty}$ that is implementable using an interest rate-only policy. Then it is similarly implementable through time-varying uniform transfer and bonus depreciation policies alone.*

Transfer stimulus now only replicates the consumption channel of monetary policy transmission. If additionally $\mathcal{I}_{\tau_f}$ is invertible, then the general form of investment stimulus considered in (27) suffices to replicate the investment channel, and so leave the set of implementable aggregate allocations unchanged. The final step of my argument is to ascertain that bonus depreciation stimulus can indeed perturb firm investment demand over time as required in Proposition 4. Unlike the stimulus check case, however, this logic is now entirely straightforward and in particular can closely follow the work of Correia et al. (2013). Since my model features no firm-level financial frictions, it is straightforward to see that bonus depreciation is equivalent to a standard investment subsidy (e.g., see the discussion in Winberry, 2021; Koby & Wolf, 2020). The analysis of Correia et al. (2013) now applies unchanged: interest rates $i_{b,t}$ and the subsidy $\tau_{f,t}$ both enter investment optimality conditions as wedges, and so the investment channel of monetary policy can be replicated by matching those wedges.

Taking stock. Taken together, the results in this section as well as in Section 4 give the headline practical takeaway of my paper: even in quite large-scale quantitative business-cycle models, a conventional mix of fiscal instruments—lump-sum transfers together with bonus depreciation tax stimulus—suffice to replicate the effects of any desired monetary policy on macroeconomic aggregates. The investment side of the argument is straightforward: bonus depreciation as the standard fiscal tool is sufficiently close to an investment subsidy that I was able to adapt the original results of Correia et al. with little change. On the consumption side, on the other hand, an entirely different argument was needed—one that I provided in Sections 3 and 4, constituting the main contribution of the paper.

7 Conclusion

Over the past decade, much academic and applied policy interest has centered on the question of how to replicate monetary stimulus when nominal interest rates are constrained.

The central contribution of this paper is to show that, in business-cycle models that are entirely standard except for the presence of non-Ricardian consumers, a conventional mix of fiscal instruments—in particular including uniform lump-sum stimulus checks—suffices to replicate the aggregate effects of an arbitrary interest rate policy. The core insight is a formalization of the notion that interest rate and stimulus check policies can manipulate consumer demand "equally flexibly." My main theoretical result is to establish that, in a rich but analytically tractable model of occasionally-binding borrowing constraints, this condition holds *generically.* As a secondary contribution, I provide an explicit characterization of the transfer policy that is needed to close any given shortfall in demand; in particular I show that, even in state-of-the-art quantitative heterogeneous-household models, this transfer policy is well-characterized by a very small number of measurable sufficient statistics.

I leave several important extensions for future work. First, it would be interesting to compare interest rate and stimulus check policies in a full Ramsey problem. Steps in this direction are taken in McKay & Wolf (2022a). Second, to the extent that the linear map $\mathcal{C}_\tau$ changes over the cycle, the required transfer stimulus will also depend on the aggregate state of the economy. Future empirical work should try to better measure that state dependence.

A Appendix

A.1 Proof of Proposition 1

Linearizing the government budget constraint (7), we find

$$\bar{b}\widehat{i}_{b,t-1} - (1+\bar{r})\bar{b}\widehat{\pi}_t + (1+\bar{r})\widehat{b}_{t-1} + \widehat{\tau}_t = \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + \widehat{b}_t \tag{A.1}$$

Using (A.1), I will decompose total transfers into two parts: an endogenous "general equilibrium" component related to labor tax revenue and inflation debt servicing costs,

$$\widehat{\tau}^e_t \equiv \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}\widehat{\pi}_t \tag{A.2}$$

and an exogenous "policy" component

$$\widehat{\tau}^x_t \equiv \widehat{\tau}_t - \widehat{\tau}^e_t \tag{A.3}$$

I now present a constructive proof of Proposition 1: leveraging (19) I will show how to construct a transfer-only policy replicating any interest rate-only policy, and vice-versa. The decomposition in (A.2) and (A.3) will prove useful in this constructive proof.

1. An interest rate-only policy is a tuple $\{i_{b,t}, \tau_t\}_{t=0}^\infty$ with

   $$\widehat{\tau}_t = \widehat{\tau}^e_t - \bar{b}\widehat{i}_{b,t-1}$$

   so that $\widehat{\tau}^x_t = -\bar{b}\widehat{i}_{b,t-1}$. By (19), there exists a path of transfers $\{\tau^*_t\}_{t=0}^\infty$ such that

   $$\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^* = \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b + \mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x \tag{A.4}$$

   Since the interest rate-only policy by construction has zero net present value, it follows that the transfer-only policy $\widehat{\boldsymbol{\tau}}^*$—which induces the exact same (zero-NPV) consumption sequence—also has zero NPV:

   $$\sum_{t=0}^\infty \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}^*_t = 0 \tag{A.5}$$

   Now consider the transfer-only policy tuple $\{\bar{i}_b, \bar{\tau} + \widehat{\tau}^e_t + \widehat{\tau}^*_t\}_{t=0}^\infty$. I will verify that, at the initial $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty$, all markets still clear and all agents still behave optimally. First, by (A.5) and the definition of $\widehat{\tau}^e_t$, we still have that $\lim_{t\to\infty} \widehat{b}_t = 0$. Second, by construction of $\widehat{\tau}^*_t$ in (A.4), the path $\widehat{\mathbf{c}}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; \bar{i}_b, \bar{\tau} + \widehat{\tau}^e_t + \widehat{\tau}^*_t\}_{t=0}^\infty$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

2. A transfer-only policy is a tuple $\{\bar{i}_b, \tau_t\}_{t=0}^{\infty}$ with

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t^x = 0$$

By (19), there exists a path of interest rates $\{i_{b,t}^*\}_{t=0}^{\infty}$ with $\widehat{\tau}_t^* = -\widehat{\bar{b} i}_{b,t-1}^*$ such that

$$\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x = \tilde{\mathcal{C}}_{i_b} \times \widehat{\boldsymbol{i}}_b^* = \mathcal{C}_{i_b} \times \widehat{\boldsymbol{i}}_b^* + \mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^* \tag{A.6}$$

and where by construction $\{\widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$ has zero NPV:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\tau}_t^* + \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \widehat{\bar{b} i}_{b,t-1}^* = 0 \tag{A.7}$$

Now consider the interest rate-only policy tuple $\{i_{b,t}^*, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. As before I will verify that, at the initial $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^{\infty}$, all markets still clear and all agents still behave optimally. First, by (A.7) and the definition of $\widehat{\tau}_t^e$, we have that $\widehat{b}_t = 0$ for all t, so indeed the policy is a valid interest rate-only policy. Second, by construction of $\widehat{i}_{b,t}^*$ in (A.6), the path $\widehat{\mathbf{c}}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; i_{b,t}^*, \bar{\tau} + \widehat{\tau}_t^e + \widehat{\tau}_t^*\}_{t=0}^{\infty}$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

□

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Online Appendix for: Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

This online appendix contains supplemental material for the article "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result". I provide (i) details for the various structural models used in the paper, (ii) supplementary theoretical results, and (iii) a detailed discussion of equivalence in terms of policy rules. The end of this appendix contains further proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded "B."—"E." refer to the main article.

Contents

B Model details

This appendix contains supplementary model details. I begin in Appendix B.1 by discussing in more detail the price-NKPC (5). Appendices B.2 and B.3 provide further details regarding the consumption-savings models of Section 2.2, including in particular a discussion of labor supply. In Appendix B.4 I sketch an alternative model of non-Ricardian consumer behavior—a model with bonds in the utility function—, and in particular discuss a variant of that model with generalized Greenwood et al. preferences. Finally, Appendix B.5 presents the extended model with investment.

B.1 Sticky-price retailers

To derive the price-NKPC (5), I let $1-\alpha$ denote the elasticity of output with respect to total labor input at the steady state, $1-\theta_p \in (0,1)$ denote the probability of a price re-set, and ϵ_p the substitutability between different retail varieties in aggregation to the final good. We can then follow the standard derivations in Galí (2015) to arrive at the following log-linearized aggregate price-NKPC:

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}})}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}\left(\widehat{w}_t + \alpha\widehat{\ell}_t\right) + \beta\widehat{\pi}_{t+1} \tag{B.1}$$

(B.1) is a special case of (5).

B.2 Further details on the analytical model

I first provide some additional details on the household consumption-savings problem—i.e., the mapping from sequences of income and interest rates to consumer demand. I then discuss household labor supply decisions.

Consumption-savings decisions. Consider an individual household type i. I write their steady-state consumption level as $\bar{c}_i$ and their steady-state wealth holdings as $\bar{b}_i$. Under my stated assumptions on transfers to newborns, the consumption-savings problem of type i is identical to the consumer demand block studied in Angeletos et al. (2023). In particular, the aggregate demand relation in their equation (11) is my linearized optimality relation (12), reproduced below for convenience:

$$\widehat{c}_{it} = \left(1 - \frac{\theta_i}{1+\bar{r}}\right) \cdot \left\{ \widehat{x}_{it} + \sum_{k=1}^{\infty} \left(\frac{\theta_i}{1+\bar{r}}\right)^k \widehat{e}_{it+k} \right\} - \sigma_i \sum_{k=0}^{\infty} \left(\frac{\theta_i}{1+\bar{r}}\right)^k \left(\widehat{i}_{b,t+k} - \widehat{\pi}_{t+k+1}\right) \quad \text{(B.2)}$$

where $\sigma_i \equiv \beta\theta_i\gamma^{-1} - (1-\beta\theta_i)\beta\bar{b}_i/\bar{c}_i$. The second equation that is needed to characterize the consumption function $\mathcal{C}^i(\bullet)$ of a consumer type i is the budget constraint. Here we have

$$c_{it} + b_{it} = (1-\tau_\ell)w_t\ell_t + \tau_t + d_t + \frac{1+i_{b,t-1}}{1+\pi_t} b_{it-1} \quad \text{(B.3)}$$

Linearizing (B.3) and combining with (B.2), we obtain a mapping from $\{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}, \boldsymbol{\tau}, \boldsymbol{i}_b\}$ to the type-i consumer demand sequence $\boldsymbol{c}_i$—i.e., the matrices $\{\mathcal{C}^i_w, \mathcal{C}^i_\ell, \mathcal{C}^i_\pi, \mathcal{C}^i_d, \mathcal{C}^i_\tau, \mathcal{C}^i_{i_b}\}$.

Labor supply. I begin by considering a model with a single household type i. I assume that labor is assigned so that all households work the same amount of hours, and furthermore that unions bargain as described in Auclert et al. (2018). Then, since all households have the same steady-state consumption level, we get the following standard aggregate log-linearized wage-NKPC, exactly as in Erceg et al. (2000):

$$\widehat{\pi}^w_t = \kappa_w \times \underbrace{\left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma\widehat{c}_t)\right]}_{\text{wedge in labor supply}} + \beta\widehat{\pi}^w_{t+1} \quad \text{(B.4)}$$

where κ_w is a function of model primitives, satisfying

$$\kappa_w = \frac{(1 - \frac{1}{1+\bar{r}}\phi_w)(1-\phi_w)}{\phi_w(\varepsilon_w\frac{1}{\varphi} + 1)},$$

with ϕ_w indicating the degree of wage stickiness, and ε_w indicating the elasticity of substitution between different types of labor. As in the price-NKPC case, (B.4) is a special case of the general relation (6).

Matters are slightly more subtle in my most general multi-type model. There, if different types have different steady-state wealth holdings, then their steady-state consumption also invariably differs, implying that standard union wage bargaining does not exactly map into a representation like (B.4).[23] Type-specific transfers that equalize steady-state consumption

[23]For example, in a two-type spender-saver model with types R and H, the static wedge in labor supply

across types i would thus be needed to return the model to a standard wage-NKPC. I investigate the importance of labor supply wedges in empirically relevant models in Section 5.1.

B.3 Further details on the heterogeneous-household model

This section completes the description of the quantitative heterogeneous-agent model introduced in Section 2.2 and studied in Section 4. I first discuss my assumptions on union bargaining (which are revisited in Section 5.1) and then describe the model calibration.

Union bargaining. I assume that unions order aggregate consumption and employment streams according to "as-if" representative-agent preferences:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}-1}{1-\gamma} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{B.5}$$

Given this particular choice of union preferences, we can yet again follow the same steps as in Erceg et al. (2000) or Auclert et al. (2018) to arrive at (B.4).

Note that, if unions instead maximized an equal-weighted average of household utility (i.e., the baseline specification of Auclert et al., 2018),

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 \left\{ \frac{c_{it}^{1-\gamma}-1}{1-\gamma} - \psi \frac{\ell_{it}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} di = \sum_{t=0}^{\infty} \beta^t \int_0^1 \left\{ \frac{c_{it}^{1-\gamma}-1}{1-\gamma} - \psi \frac{\ell_{t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} di \tag{B.6}$$

then a weighted average of household marginal consumption utilities—rather than marginal consumption utility evaluated at the aggregate consumption level c_t—would enter the static labor wedge and thus (B.4). The model would thus be inconsistent with a wage-NKPC of my assumed form (6). I discuss this case further in Section 5.1 and Appendix C.4.

Model calibration. I first discuss the parameterization of the steady state. Recall that this is all that matters for the consumption function $\mathcal{C}(\bullet)$, and so in particular for $\mathcal{C}_\tau$.

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would be

$$\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma \widehat{c}_t) + \gamma \left[\mu_R \widehat{c}_{R,t} + \mu_H \widehat{c}_{H,t} - \widehat{c}_t\right],$$

where the last term in brackets is evidently equal to zero if $\bar{c}_R = \bar{c}_H = \bar{c}$, but not in general. Prior work in such models thus often assumes identical steady-state consumption shares, allowing straightforward aggregation to (B.4) (e.g., Bilbiie et al., 2021).

Parameter	Description	Value	Target	Value
Households				
ρ_ε, σ_ε	Income Risk	-	Kaplan & Violante (2018)	-
$\underline{\varepsilon}^p$, χ_0, χ_1	Dividend Endowment	-	Illiquid Wealth Shares	-
β	Discount Rate	0.97	$\bar{b}/\bar{y}$	1.5
$\bar{r}$	Average Return	0.01	Annual Rate	0.04
ϱ	Death Rate	1/180	Average Age	45
γ	Preference Curvature	1	Standard	
φ	Labor Supply Elasticity	0.5	Standard	
ε_w	Labor Substitutability	10	Standard	
$\underline{b}$	Borrowing Limit	0	McKay et al. (2016)	
Firms				
$1-\alpha$	Returns to Scale	1	Standard	
ε_p	Goods Substitutability	16.67	Profit Share	0.06
Government				
τ_ℓ	Labor Tax	0.3	Average Labor Tax	0.30
$\bar{\tau}/\bar{y}$	Transfer Share	0.05	Transfer Share	0.05

Table B.1: HANK model, steady-state calibration.

The values of all parameters relevant for the model's deterministic steady state are displayed in Table B.1. For my quantitative analysis I slightly enrich the preferences displayed in (17) to allow for exogenous household death at rate ϱ. Preference parameters $\{\gamma, \varphi, \varrho\}$ as well as the labor substitutability ε_w are set to standard values. The average return on (liquid) assets is set in line with standard calibrations of business-cycle models, and the discount rate is then disciplined through the total amount of liquid wealth. As in McKay et al. (2016), I assume that households cannot borrow in the liquid asset. Next, for income risk, I adopt the 33-state specification of Kaplan et al. (2018), ported to discrete time. For share endowments, I assume that

$$d_{it} = \begin{cases} 0 & \text{if } \varepsilon_{it}^p \leq \underline{\varepsilon}^p \\ \chi_0(\varepsilon_{it}^p - \underline{\varepsilon}^p)^{\chi_1} \times d_t & \text{otherwise} \end{cases}$$

where ε_{it}^p is the permanent component of household i's labor productivity. I set the parameters $\{\underline{\varepsilon}^p, \chi_0, \chi_1\}$ as in Wolf (2020). On the firm side, I assume constant returns to scale in production, and set the substitutability between goods to a standard value. Finally, the average government tax take, transfers, and debt issuance are all set in line with direct empirical evidence. Relative to Definition 3, I slightly generalize the model to allow for non-zero government spending, giving the new market-clearing condition

$$y_t = c_t + g_t \tag{B.7}$$

Note that, for all experiments, I keep government expenditure fixed at $g_t = \bar{g}$, so its presence only matters for the steady-state fiscal tax-and-transfer system, and does not directly show up anywhere in equilibrium dynamics.

In the second step I set the remaining model parameters (which exclusively govern dynamics around the deterministic steady state). For the baseline interest rate-only policy studied in Section 4.2, I consider an interest rate rule of the form

$$\widehat{i}_{b,t} = \phi_\pi \widehat{\pi}_t + m_t$$

where m_t is the monetary shock, set to give the gradually decaying path of nominal rates displayed in Figure 3. In a slight generalization of Definition 2, I assume that the baseline interest rate-only policy is not financed through taxes and transfers adjusting period-by-period (as in (8)), but instead consider a more general fiscal financing rule of the form

$$\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \left[(1+\bar{r})\bar{b}(\widehat{i}_{b,t-1} - \widehat{\pi}_t) - \tau_\ell(\bar{w}\widehat{\ell}_t + \bar{\ell}\widehat{\ell}_t)\right], \tag{B.8}$$

and with $\rho_b \in (0,1)$. Total transfers adjust residually to balance the government budget. Since b_t evolves gradually over time, it follows that a nominal interest rate cut at time t only feeds through to higher transfers with a delay. While the financing rule in Definition 2 was conceptually simpler, the alternative fiscal rule (B.8) has the advantage that nominal interest rate movements are not accompanied by (counterfactual) large contemporaneous changes in transfers. This completes the specification of policy for the baseline monetary experiment. I present the rule parameterizations as well as all other model parameters in Table B.2.

Alternative calibrations. For my robustness checks in Appendix C.3 I consider two alternative model calibrations: one with less liquid wealth ($\bar{b}/\bar{y} = 0.5$, implying substantially larger MPCs, with $\omega = 0.64$), and one with more liquid wealth ($\bar{b}/\bar{y} = 7.5$, implying

Parameter	Description	Value
ϕ_p	Price Calvo Parameter	0.85
ϕ_w	Wage Calvo Parameter	0.70
ϕ_π	Taylor Rule Inflation	1.5
ρ_b	Financing Rule Persistence	0.85

Table B.2: HANK model, parameters governing dynamics.

substantially smaller MPCs, with $\omega = 0.12$).

B.4 Bond-in-utility models

While my main analysis considers (mixtures of) perpetual-youth overlapping-generations models as particularly convenient models of non-Ricardian consumer behavior, I emphasize that my results extend with very little change to an alternative popular model variant: bond-in-utility models, as considered in Michaillat & Saez (2018).

A baseline bond-in-utility model. Household preferences are now

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{B.9}$$

The budget constraint is still exactly as in Section 2.2. The log-linearized optimality conditions are then

$$\bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t = (1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}(\widehat{b}_{t-1} + \widehat{i}_{b,t-1} - \widehat{\pi}_t) + \bar{\tau}\widehat{\tau}_t + \bar{d}\widehat{d}_t \tag{B.10}$$

$$\widehat{c}_t = \beta(1+\bar{r})\widehat{c}_{t+1} + \frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]\widehat{b}_t - \frac{1}{\gamma}\beta(1+\bar{r})\left(\widehat{i}_{b,t} - \widehat{\pi}_{t+1}\right) \tag{B.11}$$

where $\beta(1+\bar{r}) < 1$ as long as $\alpha, \eta > 0$. Together, the two relations (B.10) - (B.11) fully characterize the model-implied consumption derivative matrix $\mathcal{C}_\tau^{BiU}$. I provide a closed-form expression for its inverse $(\mathcal{C}_\tau^{BiU})^{-1}$ in Appendix C.1.

Finally I also note that, since there is a single representative household with separable preferences over consumption, wealth, and hours worked, union bargaining again gives (B.4). The analytical bond-in-utility model is thus also consistent with all of the high-level assumptions on labor supply made in Section 2.1.

Parameter	Description	Value
α	Wealth preference level	0.10
η	Wealth preference curvature	0.50
δ_R	Savers GHH+ coefficient	0.03
δ_H	Spenders GHH+ coefficient	0.95
μ	Share of spenders	0.25

Table B.3: Mixture model with heterogeneous wealth effects, parameterization of preferences.

GHH+ PREFERENCES. For my second investigation of the role of wealth effects in labor supply in Section 5.1, I consider a two-type model with generalized Greenwood et al. preferences, as originally proposed in Auclert et al. (2020). I assume that savers have bonds in their utility function; that is, their preferences are[24]

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(c_t - \psi_R \delta_R \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} - \psi_R (1-\delta_R) \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{B.12}$$

while spenders have static per-period preferences

$$\frac{\left(c_t - \psi_H \delta_H \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{1-\gamma} - 1}{1-\gamma} - \psi_H (1-\delta_H) \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{B.13}$$

Here the two coefficients $\{\delta_R, \delta_H\}$ control the strength of wealth effects in labor, with $\delta = 0$ corresponding to standard separable preferences and $\delta = 1$ corresponding to GHH preferences. I calibrate the consumer part of the model to induce consumption behavior similar to my baseline HANK model—matching in particular $\omega = 0.3$—and MPEs consistent with Golosov et al. (2021)—an MPE of \$3.5 for savers and an MPE of \$1.8 for spenders. The model parameterization is reported in Table B.3. All other parameters are set exactly as in my baseline HANK model, as reported in Tables B.1 and B.2.[25] Detailed results on the

[24]The perpetual-youth model with generalized Greenwood et al. preferences is substantially less tractable, so I consider the bond-in-utility variant instead.

[25]Except for δ_H and δ_R and thus their wealth effects in labor supply, I treat spenders and savers entirely symmetrically, ensuring in particular identical steady-state consumption.

accuracy of my policy equivalence result in this alternative model environment are discussed in Appendix C.4.

B.5 Adding investment

All firms are identical, so I drop the j subscript. Analogously to the discussion in Section 2.1, we can summarize the solution to the firm problem with an investment demand function,

$$\mathbf{i} = \mathcal{I}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.14}$$

a production function,

$$\mathbf{y} = \mathcal{Y}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.15}$$

a labor demand function,

$$\boldsymbol{\ell} = \mathcal{L}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.16}$$

and a dividend function

$$\mathbf{d} = \mathcal{D}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.17}$$

where the dividend function aggregates over both intermediate goods producers and sticky-price retailers. Note that, since intermediate goods firms hire labor on a competitive spot market, and since all firms j are identical, two sequences $\boldsymbol{i_b}$ and $\boldsymbol{\tau}_f$ that induce the same paths of investment also invariably induce the same paths of output and labor hiring. However, since interest rates and investment subsidies enter the firm budget constraint differently, the implied dividend paths may be different.

Given investment subsidies to firms, the government budget constraint is adjusted to give

$$\frac{1 + i_{b,t-1}}{1 + \pi_t} b_{t-1} + \tau_t + \tau_{f,t}(\{i_{t-q}\}_{q=0}^t) = \tau_\ell w_t \ell_t + b_t \tag{B.18}$$

All other parts of the model are unchanged relative to Section 2.1. We thus arrive at the following equilibrium definition:

Definition 5. *An equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, \tau_{f,t}, b_t\}_{t=0}^\infty$ *and a set of aggregates* $\{c_t, \ell_t, y_t, i_t, k_t, w_t, \pi_t, d_t, p_t^I\}_{t=0}^\infty$ *such that:*

1. *Consumption is consistent with the aggregate consumption function* (2).

2. *Aggregate investment, output, hours worked and dividends satisfy*

$$\begin{aligned}
\boldsymbol{i} &= \mathcal{I}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right) \\
\boldsymbol{y} &= \mathcal{Y}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right) \\
\boldsymbol{\ell} &= \mathcal{L}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right) \\
\boldsymbol{d} &= \mathcal{D}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right)
\end{aligned}$$

3. *Wage inflation* $\{\pi_t^w\}_{t=0}^\infty$ *and* $\{\ell_t, c_t, w_t\}_{t=0}^\infty$ *are consistent with the wage-NKPC* (6)*.*

4. *The paths* $\{\pi_t, p_t^I\}_{t=0}^\infty$ *are consistent with the adjusted aggregate price-NKPC* (28)*.*

5. *The output market clears:* $y_t = c_t + i_t$ *for all* $t \geq 0$*, the government budget constraint* (B.18) *holds at all* t*, and* $\lim_{t\to\infty} b_t = \bar{b}$*. The bond market then clears by Walras' law.*

C Supplementary results

This section presents supplementary theoretical results. Appendices C.1 and C.2 characterize $\mathcal{C}_\tau$, $\mathcal{C}_\tau^{-1}$ as well as $\tilde{\mathcal{C}}_{i_b}$ in the analytical model of Section 2.2. Appendix C.3 elaborates on my sufficient statistics formula and discusses its accuracy in other models. Results supplementing my discussion of wealth effects in labor supply and non-equivalence at the household level are provided in Appendices C.4 and C.5. Finally Appendices C.6 and C.7 extend the policy equivalence result to some richer model environments and to targeted transfers.

C.1 $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ in analytical models

I here characterize the matrix $\mathcal{C}_\tau$ as well as its inverse $\mathcal{C}_\tau^{-1}$ in my analytical models of non-Ricardian consumption behavior. I first present results for a perpetual-youth consumer block (as in Section 2.2, with some general θ) and then consider a further extended model with an arbitrary contemporaneous MPC, thus allowing me to nest environments with $\beta(1+\bar{r}) \neq 1$ and/or behavioral frictions. Finally I sketch results for bond-in-utility models.

Characterizing $\mathcal{C}_\tau$. I begin with the shape of $\mathcal{C}_\tau^{OLG}$ as displayed in (16). From the discussion in Appendix B.2 it follows that the matrix $\mathcal{C}_\tau^{OLG}$ is fully characterized by the following pair of equations:

$$\widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} = \widehat{\tau}_t, \tag{C.1}$$

$$[1-\theta(1-\beta\theta)]\,\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1-\beta\theta)(1-\theta)\frac{1}{\beta}\widehat{b}_{t-1} = (1-\beta\theta)(1-\theta)\widehat{\tau}_t, \tag{C.2}$$

where (C.2) is the Euler equation representation of the aggregate demand relation (B.2).

From this system we arrive at the following characterization of $\mathcal{C}_\tau^{OLG}$. First, it is straightforward to see that the first column and the first row of $\mathcal{C}_\tau^{OLG}$ are respectively given as

$$\mathcal{C}_\tau^{OLG}(\bullet,1) = \left(1 - \frac{\theta}{1+\bar{r}}\right) \times \left\{1, \theta, \theta^2, \dots\right\}'$$

and

$$\mathcal{C}_\tau^{OLG}(1,\bullet) = \left(1 - \frac{\theta}{1+\bar{r}}\right) \times \left\{1, \frac{\theta}{1+\bar{r}}, \left(\frac{\theta}{1+\bar{r}}\right)^2, \dots\right\}$$

Second, all higher-order columns are given recursively as

$$\mathcal{C}_\tau^{OLG}(\bullet, h) = \mathcal{C}_\tau^{OLG}(1, h) \times \begin{pmatrix} 1 \\ -\mathcal{C}_\tau^{OLG}(\bullet, 1)(1+\bar{r}) \end{pmatrix} + \begin{pmatrix} 0 \\ \mathcal{C}_\tau^{OLG}(\bullet, h-1) \end{pmatrix}, \quad h = 2, 3, 4, \ldots$$

This expression—which is straightforward to verify from (C.1) - (C.2)—reflects the intuition of the "fake-news" algorithm of Auclert et al. (2019): the first term is the response of households to a date-h income shock announced at date-0 but then reversed at date 1; the second term then undoes that date-1 reversal, ensuring that the sum gives us the actual response to a date-h income shock—that is, $\mathcal{C}_\tau^{OLG}(\bullet, h)$. This somewhat complicated exact shape of $\mathcal{C}_\tau^{OLG}$ corresponds to the approximate shape displayed in (16), as established in the following result.

Lemma C.1. *Consider the consumption-savings problem of a perpetual-youth household block, as described in Section 2.2. Then, for any $\ell > 0$ the impulse response path $\widehat{\mathbf{c}}_H$ to an income shock at time H satisfies*

$$\lim_{H\to\infty} \widehat{c}_{H,H} = const., \quad \lim_{H\to\infty} \frac{\widehat{c}_{H+\ell,H}}{\widehat{c}_{H+\ell-1,H}} = \theta, \quad \lim_{H\to\infty} \frac{\widehat{c}_{H-\ell-1,H}}{\widehat{c}_{H-\ell,H}} = \frac{\theta}{1+\bar{r}} \tag{C.3}$$

In words, for large H, the intertemporal spending profile in $\mathcal{C}_\tau$ looks as indicated in (16). This is the sense of the approximation $\approx$ in that relation.

Characterizing $\mathcal{C}_\tau^{-1}$. Given an arbitrary target consumption sequence $\widehat{\boldsymbol{c}}$, we can solve the system (C.1) - (C.2) for $\{\widehat{\boldsymbol{\tau}}, \widehat{\boldsymbol{b}}\}$, with $\widehat{b}_{-1} = 0$. This gives the solution displayed in (20). The detailed steps are provided in the proof of Lemma 1.

Extension to arbitrary MPCs. I next consider an even further-generalized aggregate demand relation of the following form:

$$\widehat{c}_t = M \cdot \left\{ \widehat{x}_t + \sum_{k=1}^{\infty} \left(\frac{\theta}{1+\bar{r}}\right)^k \widehat{e}_{t+k} \right\} - \sigma \sum_{k=0}^{\infty} \left(\frac{\theta}{1+\bar{r}}\right)^k \left(\widehat{i}_{b,t+k} - \widehat{\pi}_{t+k+1}\right). \tag{C.4}$$

Relative to (B.2), (C.4) additionally disentangles the impact MPC M from the discounting factor θ applied to future income. This allows me to study two meaningful extensions of the baseline model. First, in an environment with $\beta(1+\bar{r}) \neq 1$, and under the simplifying assumption of log preferences, (C.4) applies with $M = 1 - \beta\theta$ (see Farhi & Werning, 2019, for the continuous-time analogue).[26] Second, in a model with cognitive discounting and occasionally-binding borrowing constraints, we have $M = 1 - \frac{\theta_1}{1+\bar{r}}$ and $\theta = \theta_1 \cdot \theta_2$, where θ_1 and θ_2 are the borrowing constraint and behavioral discounting coefficients, respectively.

The following result characterizes existence and shape of $\mathcal{C}_\tau^{-1}$ in this environment.

Lemma C.2. *Consider a variant of the analytical consumption-savings problem of Section 2.2 with generalized aggregate demand relation (C.4). Suppose that $M \in \left[\frac{\bar{r}}{1+\bar{r}}, 1 - \frac{\theta}{1+\bar{r}}\right]$. Then, if $\theta < 1$, $\mathcal{C}_\tau$ is invertible, with*

$$\mathcal{C}_\tau^{-1} = \frac{1}{M} \cdot \begin{pmatrix} \frac{1-M\theta}{1-\theta} & -\frac{1}{1+\bar{r}}\frac{\theta}{1-\theta} & 0 & \cdots \\ -\frac{(1-M)(1+\bar{r})}{1-\theta} & \frac{1+\theta(1-M)}{1-\theta} & -\frac{1}{1+\bar{r}}\frac{\theta}{1-\theta} & \cdots \\ 0 & -\frac{(1-M)(1+\bar{r})}{1-\theta} & \frac{1+\theta(1-M)}{1-\theta} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.5}$$

Note that Lemma C.2 imposes bounds on M. Here, the upper bound corresponds to the baseline perpetual-youth model—so all discounting of future income θ also increases contemporaneous MPCs—while the lower bound ensures that $\mathcal{C}_\tau$ is actually a bounded operator.[27] Importantly, Lemma C.2 allows me to substantiate two claims made in Section 3.2. First, for the special case of a perpetual-youth model with $\beta(1+\bar{r}) > 1$ and with log preferences, the requirement on M that $M \geq \frac{\bar{r}}{1+\bar{r}}$ becomes

$$\theta \leq \frac{1}{\beta(1+\bar{r})} < 1,$$

—a condition that is strictly tighter than my baseline perpetual-youth requirement of $\theta < 1$. In words, borrowing constraints now need to bind *often enough*. This is precisely what is needed to counteract the backloading implied by $\beta(1+\bar{r}) > 1$, implying that $\mathcal{C}_\tau$ is a bounded operator whose inverse takes the shape (C.5). Second, a standard model with behavioral discounting corresponds to the special case where $M = \frac{\bar{r}}{1+\bar{r}}$ as well as $\theta < 1$. By Lemma C.2, this is again sufficient to ensure invertibility of $\mathcal{C}_\tau$, further underscoring my claims about the generality of this property of aggregate consumption functions.

[26] Without log preferences, a consumption function like (C.4) still obtains, but the mapping into M is more complicated—we have $M = \gamma^{-1}[1-\beta\theta] + (1-\gamma^{-1})[1 - \frac{\theta}{1+\bar{r}}]$ (see Appendix 1.2 of Farhi & Werning, 2019). The characterization of $\mathcal{C}_\tau^{-1}$ in Lemma C.2 of course continues to apply, however, and so my conclusions are unchanged—invertibility obtains if MPCs are large enough and spending is front-loaded.

[27] To see this, consider for example the first column of $\mathcal{C}_\tau$. It is straightforward to see that its entries grow at rate $(1-M)(1+\bar{r})$. Thus, if $M < \frac{\bar{r}}{1+\bar{r}}$, the implied consumption path diverges.

Front-loaded spending and a counterexample. To even more clearly see the importance of front-loading in consumer spending it will prove useful to consider a particularly transparent case: a simple two-period OLG model in which households have log preferences and receive income (including transfers) only when young.[28] In that case we have

$$\mathcal{C}_\tau = \begin{pmatrix} \frac{1}{1+\beta} & 0 & 0 & \dots \\ \frac{\beta(1+\bar{r})}{1+\beta} & \frac{1}{1+\beta} & 0 & \dots \\ 0 & \frac{\beta(1+\bar{r})}{1+\beta} & \frac{1}{1+\beta} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Note that spending here is back-loaded if and only if $\beta(1+\bar{r}) > 1$. In that case, the majority of a dollar of income received at date t is spent at date $t+1$, not t.

Straightforward algebra reveals that, if $\mathcal{C}_\tau^{-1}$ exists, it is lower-triangular, with off-diagonal elements that (in absolute value) decay at rate $\beta(1+\bar{r})$. $\beta(1+\bar{r}) \leq 1$ is thus necessary (and here also sufficient) for invertibility. Intuitively, if households have a natural tendency to postpone their spending, then a transfer today designed to induce spending today will require ever-larger transfers in the future to offset it. As a result, for a given target sequence of consumption, it becomes impossible to find a bounded sequence of transfers that induces it. This instructive example reveals that $\mathcal{C}_\tau$ may well be *injective*—changing the timing of transfers invariably affects consumption—yet fail to be *surjective*—certain bounded sequences of demand cannot be induced via bounded sequences of transfers. Households front-loading their spending—which is ensured in my headline environment by occasionally-binding borrowing constraints, and which is a robust feature of actual consumer behavior—prevents the divergence that here is causing non-invertibility.[29]

Bond-in-utility model. Recall from the discussion in Appendix B.4 that the consumption derivative matrix $\mathcal{C}_\tau^{BiU}$ is fully characterized by the following pair of equations:

$$\widehat{c}_t + \widehat{b}_t - (1+\bar{r})\widehat{b}_{t-1} = \widehat{\tau}_t \tag{C.6}$$

$$\widehat{c}_t - \beta(1+\bar{r})\widehat{c}_{t+1} - \frac{\bar{c}}{\bar{\bar{b}}}\frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]\widehat{b}_t = 0 \tag{C.7}$$

[28] I thank an anonymous referee for bringing this illuminating example to my attention.

[29] An elevated impact MPC, on the other hand, is not sufficient to rule out a linear map like $\mathcal{C}_\tau$. This is why, in Proposition 3, I highlight *both* the elevated MPC as well as the front-loaded spending profile, even though in my particular model of occasionally-binding borrowing constraints the two are interchangeable.

We see that the factor $\frac{\bar{c}}{\bar{b}}$ simply scales the last term in the Euler equation, so I will without loss of generality set this term equal to 1. Algebra similar to that in the proof of Lemma 1 then yields the following exact expression for $\mathcal{C}_\tau^{-1}$:

$$(\mathcal{C}_\tau^{BiU})^{-1} = \begin{pmatrix} 1+\frac{1}{\tilde{\omega}}\frac{1}{1+\bar{r}}\frac{\theta}{1-\beta\theta^2} & -\frac{1}{\tilde{\omega}}\frac{\beta\theta}{1-\beta\theta^2} & 0 & \dots \\ -\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2} & 1+\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2}\left[\beta(1+\bar{r})+\frac{1}{1+\bar{r}}\right] & -\frac{1}{\tilde{\omega}}\frac{\beta\theta}{1-\beta\theta^2} & \dots \\ 0 & -\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2} & 1+\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2}\left[\beta(1+\bar{r})+\frac{1}{1+\bar{r}}\right] & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.8}$$

where $\omega = \mathcal{C}_\tau^{BiU}(1,1)$, $\theta = \mathcal{C}_\tau^{BiU}(2,1)/\mathcal{C}_\tau^{BiU}(1,1)$, and $\tilde{\omega}$ is given via

$$1 = \tilde{\omega}\left[\frac{(1+\bar{r})\beta\theta}{1-(1+\bar{r})\beta\theta} + \omega^{-1}\right]$$

We see that $(\mathcal{C}_\tau^{BiU})^{-1}$ has the same tridiagonal shape as $(\mathcal{C}_\tau^{OLG})^{-1}$, as claimed.

C.2 $\tilde{\mathcal{C}}_{i_b}$ in analytical models

This section offers additional results on the shape and properties of $\tilde{\mathcal{C}}_{i_b}$ in the one-type perpetual-youth OLG model. I proceed in two steps. First, I provide a closed-form expression for $\tilde{\mathcal{C}}_{i_b}$. Second, I establish that, if $\theta > 0$ (i.e., not pure spender behavior), then interest rate policy can similarly be used to induce any sequence of net excess consumption demand with zero net present value.

A CLOSED-FORM EXPRESSION FOR $\tilde{\mathcal{C}}_{i_b}$. Recall that the matrices $\mathcal{C}_\tau$ and $\tilde{\mathcal{C}}_{i_b}$ in the baseline (one-type) perpetual-youth model are characterized by the following pair of equations:

$$\widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} = \widehat{\tau}_t, \tag{C.9}$$

$$[1-\theta(1-\beta\theta)]\,\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1-\beta\theta)(1-\theta)\frac{1}{\beta}\widehat{b}_{t-1} = (1-\beta\theta)(1-\theta)\widehat{\tau}_t - \gamma\beta\theta\widehat{i}_{b,t}. \tag{C.10}$$

From here it is straightforward to see that

$$\tilde{\mathcal{C}}_{i_b} = -\frac{1}{\gamma}(I-\mathcal{C}_\tau)\begin{pmatrix} 1 & 1 & 1 & \dots \\ 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{C.11}$$

To formally establish this relation, I will define $\widehat{c}_t^* \equiv \widehat{c}_t - \widehat{\tau}_t$. Now consider first an income shock at date 0. Plugging this into the optimality conditions and re-arranging, we see that the impulse response of c^* is identical to the impulse response of c to a date-0 interest rate change scaled by γ. That is, we have

$$\mathcal{C}_\tau^*(\bullet, 1) = \mathcal{C}_\tau(\bullet, 1) - e_1 = \gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 1)$$

where $e_1 = (1, 0, 0, \dots)'$. This gives the first column of (C.11):

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 1) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1))$$

Similarly, for date-1 shocks, we have

$$\mathcal{C}_\tau^*(\bullet, 2) = \mathcal{C}_\tau(\bullet, 2) - e_2 = -\gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 1) + \gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 2)$$

where $e_2 = (0, 1, 0, \dots)'$. Thus we get

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 2) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1) + e_2 - \mathcal{C}_\tau(\bullet, 2))$$

giving the second column of (C.11). All other columns follow analogously. We thus see that the perpetual-youth model admits a straightforward relation between $\mathcal{C}_\tau$ and $\tilde{\mathcal{C}}_{i_b}$, with the mapping between the two fully governed by γ.

Establishing equivalence. Consider an arbitrary consumption sequence $\widehat{\boldsymbol{c}}$ such that $\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_t = 0$—i.e., it has zero net present value. I will now provide a constructive argument showing that we can find a bounded sequence of interest rates $\widehat{\boldsymbol{i}}_b^*$ such that $\tilde{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b^* = \widehat{\boldsymbol{c}}$. For this I first of all note that, from (C.9), we must have that

$$\widehat{b}_t = \frac{1}{\beta}\widehat{b}_{t-1} - \widehat{c}_t$$

Since $\widehat{\boldsymbol{c}}$ has zero net present value, it follows that $\widehat{b}_t \to 0$. Next, from (C.10), it follows that it suffices to set

$$\widehat{i}_{b,t} = -\frac{1}{\gamma\beta\theta} \cdot \left([1 - \theta(1 - \beta\theta)]\,\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1 - \beta\theta)(1 - \theta)\frac{1}{\beta}\widehat{b}_{t-1}\right)$$

Since $\widehat{\boldsymbol{c}}$ is bounded by construction and $\widehat{b}_t \to 0$ by the argument above, it follows that—if $\theta > 0$—we can find a bounded sequence $\widehat{\boldsymbol{i}}_b^*$ that induces net excess demand $\widehat{\boldsymbol{c}}$, as claimed.

C.3 The sufficient statistics formula and its accuracy

I begin with some additional details on the sufficient statistics formula. The formula maps the three observables $\{\omega, \theta, \bar{r}\}$ into the matrix $\mathcal{C}_\tau$ (and thus its inverse $\mathcal{C}_\tau^{-1}$). The formula proceeds in two steps.

First, given $\{\theta, \bar{r}\}$, I construct a matrix $\mathcal{C}_\tau^{(1)}$ that has the same shape as in a one-type perpetual-youth model. By the discussion in Appendix C.1, this means that

$$\mathcal{C}_\tau^{(1)}(\bullet, 1) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \theta, \theta^2, \dots\right\}'}_{\text{spending decay}}$$

and

$$\mathcal{C}_\tau^{(1)}(1, \bullet) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \frac{\theta}{1+\bar{r}}, \left(\frac{\theta}{1+\bar{r}}\right)^2, \dots\right\}}_{\text{anticipation effects}}$$

together with

$$\mathcal{C}_\tau^{(1)}(\bullet, h) = \mathcal{C}_\tau^{(1)}(1, h) \times \begin{pmatrix} 1 \\ -\mathcal{C}_\tau^{(1)}(\bullet, 1)(1+\bar{r}) \end{pmatrix} + \begin{pmatrix} 0 \\ \mathcal{C}_\tau^{(1)}(\bullet, h-1) \end{pmatrix}, \quad h = 2, 3, 4, \dots$$

This specifies the entire matrix $\mathcal{C}_\tau^{(1)}$ as a function only of $\{\theta, \bar{r}\}$.

Second, I add a margin of spenders to disentangle the MPC ω and the spending slope θ. Note that, in my construction of $\mathcal{C}_\tau^{(1)}$, the MPC is mechanically given as $\left(1 - \frac{\theta}{1+\bar{r}}\right)$. To match any desired arbitrary MPC ω I then simply set

$$\mathcal{C}_\tau \quad = \quad \frac{\theta - (1-\omega)(1+\bar{r})}{\theta} \times I \quad + \quad \frac{(1-\omega)(1+\bar{r})}{\theta} \times \mathcal{C}_\tau^{(1)}$$

It is straightforward to verify that the resulting $\mathcal{C}_\tau$ matches the desired MPC ω. I have thus mapped my three sufficient statistics $\{\omega, \theta, \bar{r}\}$ into a matrix $\mathcal{C}_\tau(\omega, \theta, \bar{r})$ that (i) matches the average MPC ω and spending slope θ, and (ii) is by construction consistent with lifetime household budget constraints. From here I can then also construct $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r})$. Note that this inverse exists as long as $\theta < 1$ and $\omega \geq 1 - \frac{\theta}{1+\bar{r}}$, by the proof Proposition 2.

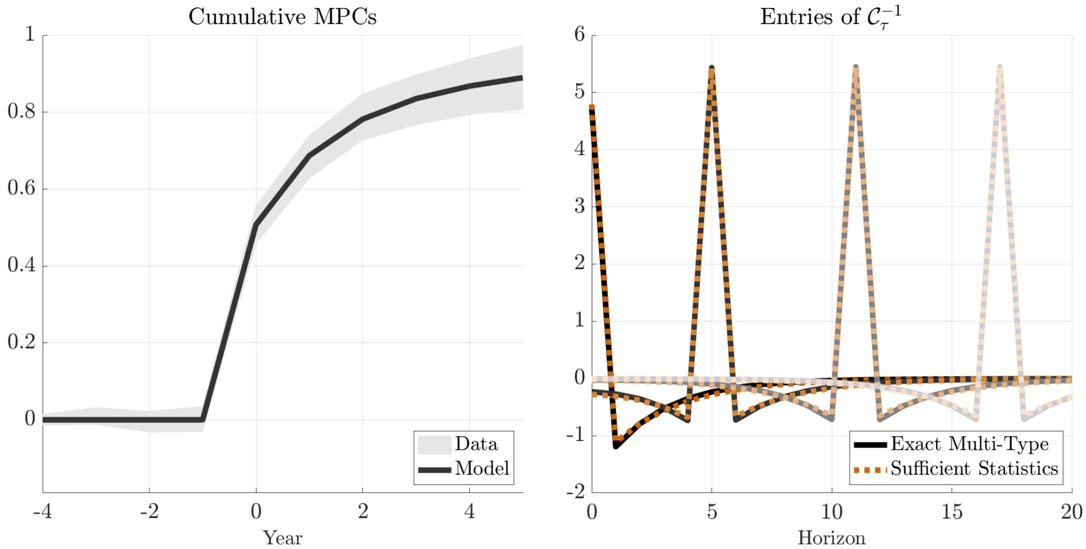

Figure C.1: Left panel: cumulative MPCs in the data (taken from Fagereng et al., 2018) and in the three-type model of Angeletos et al. (2023). Right panel: model-implied entries of $\mathcal{C}_\tau^{-1}$ (blue) vs. predicted values from my sufficient statistics formula (orange dashed).

The role of tail MPCs. The sufficient statistics formula imposes a constant rate of decay θ of intertemporal marginal propensities to consume. As discussed in Angeletos et al. (2023), empirical evidence on the other hand suggests that this rate of iMPC decay slows down in the far tails, at long horizons. To gauge whether this mismatch in the tails actually matters for the purposes of my results here, I consider the model used in Angeletos et al. to match empirical evidence on the entire intertemporal MPC profile—a hybrid model with *two* types of perpetual-youth consumers, together with a margin of spenders. The left panel of Figure C.1, taken from Angeletos et al., shows that this model indeed matches empirical evidence on far-ahead MPCs very well. The right panel then compares the true model-implied $\mathcal{C}_\tau^{-1}$ with the prediction from my sufficient statistics formula.[30] We see that the two are almost indistinguishable—i.e., for the purposes of the analysis here, a mild mismatch in the far-ahead tails is essentially irrelevant.

Perturbing the sufficient statistics. I here further substantiate my claim that, for empirically relevant values of the sufficient statistics, even moderately sized stimulus check

[30] I recover the coefficients for the sufficient statistics formula exactly as done for my quantitative HANK model: I set $\bar{r}$ to its true model-implied value, and then set ω and θ so that $\mathcal{C}_\tau(\omega, \theta, \bar{r})(1,1)$ and $\mathcal{C}_\tau(\omega, \theta, \bar{r})(2,1)$ match the extended hybrid model.

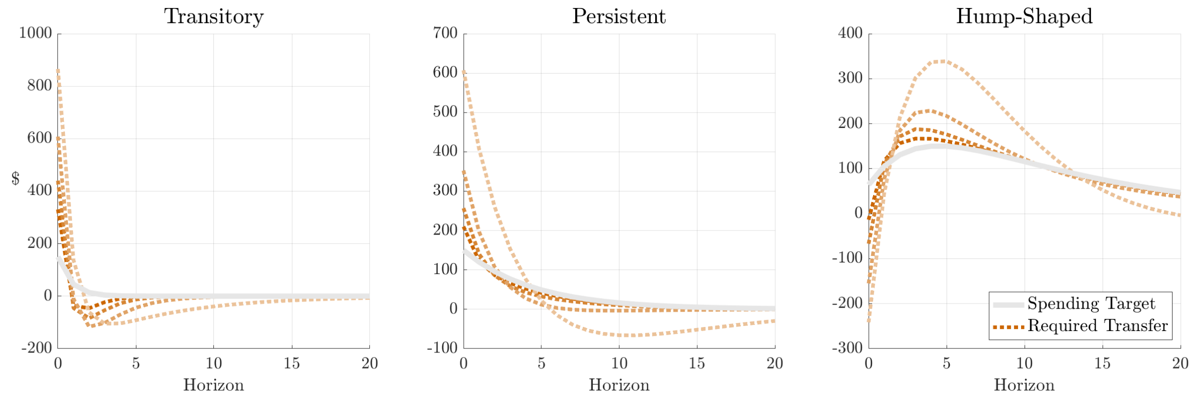

Figure C.2: Three desired net excess demand paths $\widehat{\boldsymbol{c}}^{PE}$ (grey) and the required sequences of uniform lump-sum taxes and transfers, $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r}) \times \widehat{\boldsymbol{c}}^{PE}$ for a range of values of my sufficient statistics (orange dashed, see text), with lighter shades indicating lower MPCs ω.

policies suffice to close meaningful shortfalls in aggregate spending. To do so I repeat the exercise of Figure 2 for a range of values of my sufficient statistics $\{\omega, \theta, \bar{r}\}$. Specifically, I continue to fix $\bar{r} = 0.01$, consider $\omega \in \{0.2, 0.3, 0.4, 0.5\}$ and then pin down the slope θ by requiring the same ratio $\frac{\theta}{1-\omega}$ as in my quantitative heterogeneous-household model. The range of MPCs ω that I consider is chosen to contain and in fact go beyond the range of estimates available from the literature (e.g. Parker et al., 2013; Fagereng et al., 2018).

Results are displayed in Figure C.2. The figure illustrates my claim: across the range of empirically relevant values for my sufficient statistics, the stimulus check policies that close the indicated shortfalls in aggregate consumer spending are moderate in size. As discussed in Section 4.2, this finding is important to ensure that my theoretical results—which rely on linearization at the aggregate level—are actually practically relevant.

$\mathcal{C}_\tau^{-1}$ IN ALTERNATIVE HANK CALIBRATIONS. The analysis in Section 4.2 confirmed the accuracy of the sufficient statistics formula for $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ in the baseline calibration of my heterogeneous-household model. I here repeat the same exercise for two materially different model calibrations: one with very low liquid wealth (implying a counterfactually large average MPC of $\omega = 0.64$) and one with a lot of liquid wealth (implying a counterfactually small average MPC of $\omega = 0.12$). Results are reported in Figures C.3 and C.4.

The takeaways from these figures are twofold. First, changing the model calibration materially affects the model-implied consumption map $\mathcal{C}_\tau$ and its inverse $\mathcal{C}_\tau^{-1}$. As expected, for low liquid wealth, the inverse $\mathcal{C}_\tau^{-1}$ looks closer to a simple spender-saver model and the required transfer stimulus policies are even smaller than in my baseline analysis. For

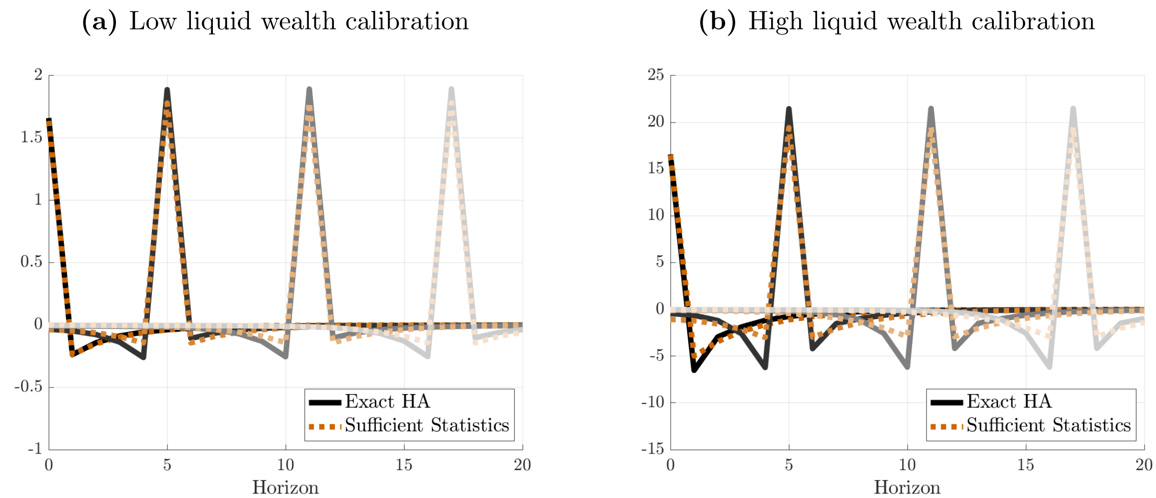

Figure C.3: See the caption of Figure 1.

high liquid wealth, the inverse $\mathcal{C}_\tau^{-1}$ looks closer to a perpetual-youth overlapping-generations model, and the transfer stimulus policies required to close a given shortfall in demand are now much larger. Second, even though $\mathcal{C}_\tau$ looks very different across calibrations, my sufficient statistics formula throughout approximates $\mathcal{C}_\tau^{-1}$ and thus the implied equivalent transfer stimulus policies very well.

A GENERALIZED SUFFICIENT STATISTICS FORMULA. My three-parameter sufficient statistics formula imposes that the same coefficient θ governs both the decay of intertemporal MPCs after spending receipt as well as the strength of anticipation effects. It is in principle straightforward to disentangle the two by allowing for an additional degree of freedom in the first row of $\mathcal{C}_\tau$. For example, one simple and natural choice would be to set[31]

$$\mathcal{C}_\tau^{(1)}(1,\bullet) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \psi \cdot \frac{\theta}{1+\bar{r}}, \psi \cdot \left(\frac{\theta}{1+\bar{r}}\right)^2, \dots\right\}}_{\text{anticipation effects}}$$

where the coefficient ψ could be recovered from empirical evidence on the strength of anticipation effects in MPCs (e.g., Ganong & Noel, 2019). The rest of $\mathcal{C}_\tau$ would then be completed exactly as in the baseline sufficient statistics formula. Unsurprisingly, with this additional de-

[31]Here anticipation effects are additionally discounted by a constant factor ψ. An alternative—which I have found to be less accurate in my quantitative HANK models—is to discount future income receipts at some constant rate ψ (that is allowed to be different from θ).

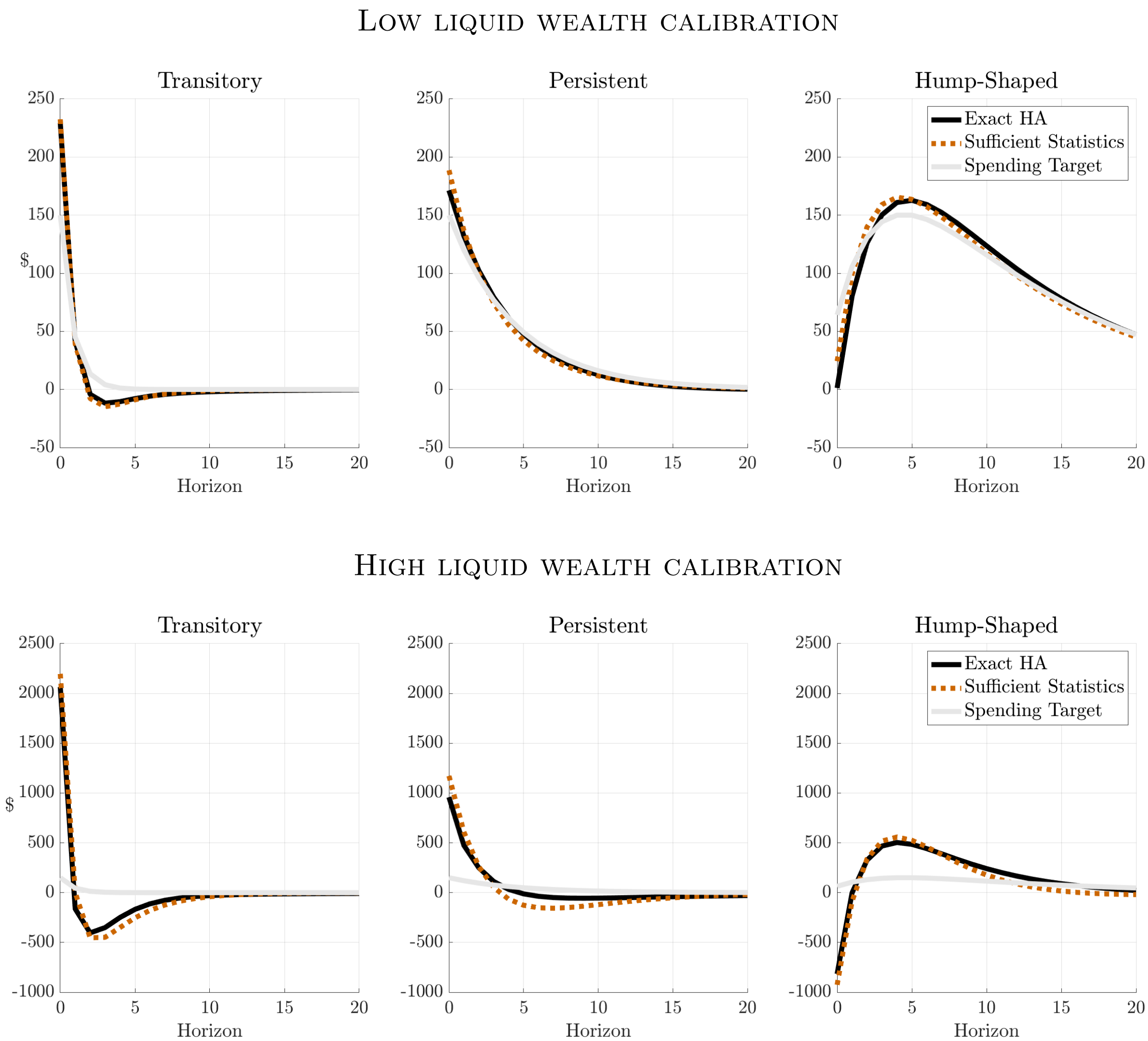

Figure C.4: See the caption of Figure 2.

gree of freedom, the approximation becomes even more accurate, with the difference between actual and approximate $\mathcal{C}_\tau$ now barely visible (figure available upon request). However, as argued in Section 4.2, my simpler three-coefficient formula already provides a very accurate approximation, so I focus on results from that simpler specification instead. Intuitively, at least in my HANK model, anticipation effects are not particularly far from being governed by the iMPC decay rate θ, and so the simpler three-parameter formula suffices.

C.4 Heterogeneous wealth effects in labor supply

This section elaborates on my discussion of the role of heterogeneity in wealth effects in labor supply across households (see Section 5.1). I first present results for an alternative union bargaining protocol and then consider an alternative model with preference heterogeneity, designed to match empirical evidence on heterogeneity in marginal propensities to earn (from Golosov et al., 2021). Results for both are reported in Figure C.5.

Alternative bargaining results. I return to my main quantitative HANK model, but with one twist: the wage-NKPC (B.4) is replaced by the alternative formulation

$$\widehat{\pi}_t^w = \kappa_w \times \left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma\widehat{c}_t^*)\right] + \beta\widehat{\pi}_{t+1}^w \tag{C.12}$$

where

$$c_t^* \equiv \left[\int_0^1 e_{it} c_{it}^{-\gamma} di\right]^{-\frac{1}{\gamma}} \tag{C.13}$$

This is the specification of the wage-NKPC originally derived in Auclert et al. (2018) and implied by the union objective (B.6). I note that, in this case, my policy equivalence result will not hold *exactly*: two nominal interest rate and stimulus check policies with identical direct effects on net excess demand (and so c_t) will not necessarily have identical direct effects on c_t^*, thus inducing different wedges in the economy's aggregate supply relation (C.12).

Are these differential labor supply effects likely to materially undermine the policy equivalence result? The left panel Figure C.5 suggests that the answer is "no". To construct the panel, I first compute impulse responses to a gradual monetary policy shock (with persistence 0.6), normalized to in general equilibrium increase consumption on impact by one per cent (grey). I then follow the steps in the proof of Proposition 1 to construct a stimulus check policy with identical effects on partial equilibrium consumer spending. The general equilibrium impulse response of consumption to this policy is displayed as the blue dashed line. The main takeaway is that the two lines are very close, with the stimulus check policy overall slightly more stimulative than the (not-quite-)equivalent interest rate cut.

The intuition for the results displayed in Figure C.5 is somewhat subtle. Both policies by design lead to a response of partial equilibrium consumption demand with zero present value—initially positive and then later on negative. Under my baseline wage-NKPC (6), this initial decrease and later increase in the average marginal utility of consumption leads to an initial decrease and later increase of union labor supply. With the alternative formu-

(a) Alternative union bargaining **(b)** Matching MPEs

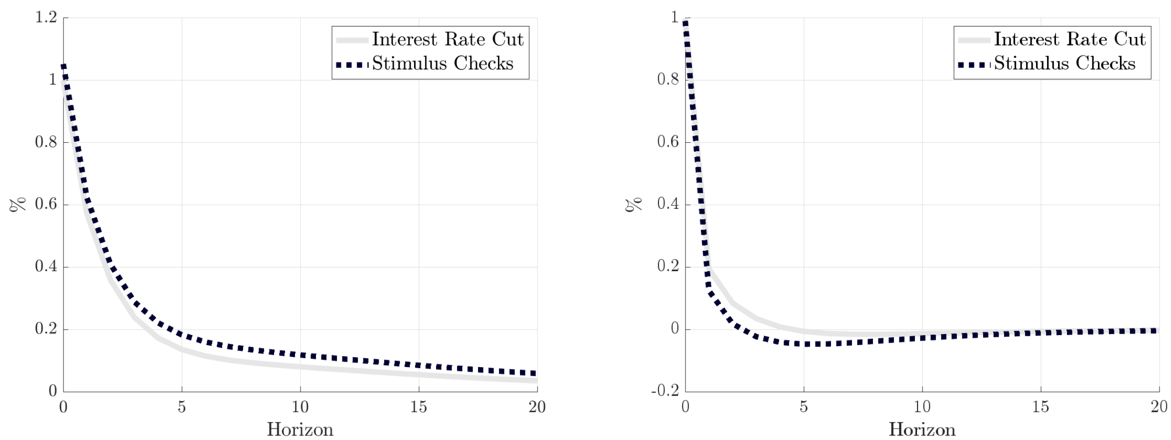

Figure C.5: Left panel: impulse response of consumption to a monetary policy shock with persistence $\rho_m = 0.6$ and peak effect of 1% (grey) and the "equivalent" stimulus check policy (blue dashed) in a HANK model with labor supply relation (C.12). Right panel: analogous figure for my hybrid spender-saver model described below.

lation (C.12), on the other hand, the labor supply response is governed by average marginal utility. Since most of the consumption adjustment following the transfer policy comes from high-MPC households (who tend to consume less), average marginal utility of consumption changes by more than after the equivalent rate cut—it drops by more initially and expands by more later on. It then follows that labor supply also contracts by more initially and expands by more later on, so the overall partial equilibrium net excess demand path (which equals consumption demand less labor supply) is more front-loaded after the stimulus check policy. But we know from Auclert et al. (2018) that more front-loaded excess demand paths lead to a bigger boom in general equilibrium, so stimulus check policy will stimulate output by more—exactly as we see in Figure C.5. This effect however is moderate in size, for two reasons. First, average marginal utility and marginal utility at the average are still not too different, so the desired labor supply response is also not too different. Second, any such transitory differences in desired labor supply matter even less in general equilibrium as long as wages are at least moderately sticky (Christiano, 2011), as is the case in my model.

Mixture model results. My second exercise is designed to speak as closely as possible to the empirical evidence reported in Golosov et al. (2021). Those authors report marginal propensities to earn (MPEs)—defined as the response of labor income to an unearned lump-sum wealth gain—of up to \$3 per additional \$100 in wealth, with the response roughly two times larger for the highest-income households compared to the lowest-income ones (see their Table 3.2). These estimates are roughly twice as large as those reported in prior work, notably Cesarini et al. (2017) (see Table J.1 of Golosov et al.). While it is straightforward to match such *average* MPEs in heterogeneous-agent models (see Auclert et al., 2020), it is much harder to match the *cross-sectional dispersion* in MPEs (which is what matters for my policy equivalence result). Intuitively, the challenge is that, in standard models of household consumption and labor supply, MPEs are increasing (in absolute value) with MPCs.[32] Since poorer households tend to have higher MPCs, this would also imply that they have higher MPEs (in absolute value), inconsistent with the empirical evidence reviewed above.

My solution is to consider a two-type spender-saver model with preference heterogeneity chosen to ensure higher MPEs for low-MPC households—i.e., the model sketched in Appendix B.4. As discussed there, that model is calibrated to be consistent with empirical evidence on household MPEs. Importantly, since the model features cross-sectional heterogeneity in wealth effects in labor supply, the policy equivalence result will not hold exactly. The right panel of Figure C.5 however reveals that it continues to approximately hold. I already in the main text gave the intuition for why the *magnitude* of the inaccuracy is so small (recall Section 5.1). I here instead focus on the *direction* of the error. The intuition is exactly opposite to that of the adjusted HANK model studied above. Savers have a larger MPE, so labor supply initially contracts by relatively more after an interest rate cut, and later on increases by relatively more. The total implied net excess demand path is thus more frontloaded after the interest rate policy, and so now the interest rate cut is slightly more expansionary, as seen in the right panel of Figure C.5.

Summary. My conclusion from the previous two experiments is that cross-sectional heterogeneity in wealth effects in labor supply is unlikely to materially threaten my headline policy equivalence result. However, it is important to note that this takeaway hinges on the equivalent stimulus check policy being moderate in size: by the evidence in Golosov et al. (2021), for very large transfers, we would expect the cross-sectional heterogeneity in labor supply responses to become larger relative to the demand stimulus of the policy (i.e., MPEs are larger relative to MPCs). The fact that equivalent stimulus check policies are moderate in size—the key takeaway of Section 4—is thus an integral part of my argument.

[32] This follows straightforwardly from the standard labor supply optimality condition with separable preferences over consumption and labor supply. See Auclert et al. (2020) for details.

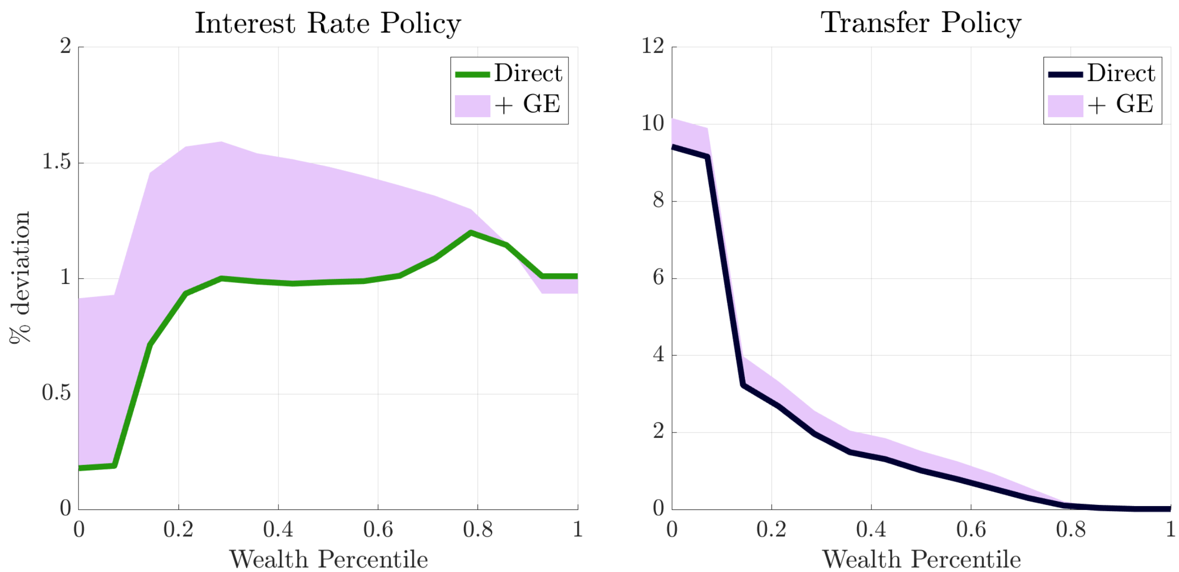

Figure C.6: Left panel: The green line shows the time-0 *direct* household spending response by liquid wealth percentile to the nominal interest rate policy of Figure 3. The shaded purple area gives the corresponding *indirect* general equilibrium effects. Right panel: Analogous figure for the equivalent uniform transfer stimulus (dark blue and shaded purple).

C.5 Non-equivalence at the household level

As discussed in Section 5.2, macro-equivalent interest rate and transfer stimulus policies need not and generally will not be equivalent in the cross-section of households. Figure C.6 provides an illustration.

The figure shows the evolution of consumption along the household wealth distribution in response to the macro-equivalent nominal interest rate and stimulus check policies displayed in Figure 3. The impact consumption response is split by household liquid wealth percentile (x-axis) into (a) the *direct* effects of the policy instrument (green and blue)—defined as the response of consumption demand to the policy instruments $\{\boldsymbol{i}_b, \boldsymbol{\tau}\}$ alone, fixing all non-policy variables at their steady state values forever—and (b) the residual *indirect* effects (shaded purple) coming from general equilibrium feedback. We see that the direct and thus overall effects are very heterogeneous in the cross-section of households.

C.6 Other model extensions

I consider two further model extensions: durable consumption, and a richer network production structure.

Durable goods. My results extend without change to a model with durables as long as durables and non-durables can be produced costlessly out of some common final good; that is, if real relative prices of the two goods are always one and we can write the aggregate resource constraint as

$$y_t = \underbrace{c_t + d_t^h - (1-\delta)d_t^h}_{e_t} \tag{C.14}$$

where e_t is total household expenditure, c_t is non-durables consumption, d_t^h is the stock of durables, and δ is the depreciation rate. Letting $\mathcal{E}_\tau$ denote the analogous derivative map for the response of total spending to lump-sum income, the key condition for my results to extend to this model is that $\mathcal{E}_\tau$ is invertible—strong Ricardian non-equivalence now applied to total spending. The details of the argument are straightforward and thus omitted: interest rate and transfer policies can perturb net excess demand for the common final good equally flexibly and are thus also equivalent in general equilibrium, by exactly the same argument as in the proof of Proposition 1.[33]

I emphasize that the assumptions underlying this extended equivalence result are empirically relevant: relative durable goods prices tend to not respond much to standard business-cycle fluctuations (House & Shapiro, 2008; McKay & Wieland, 2019; Beraja & Wolf, 2020), suggesting that the aggregation to a common aggregate resource constraint (C.14) is sensible. It is furthermore also an assumption made in recent quantitative structural explorations of durable goods spending (e.g., Berger & Vavra, 2015).

Network production. The policy equivalence result leverages properties of consumer spending behavior and as such is robust to many different possible model extensions on the production side of the economy. I here provide one illustration using a simple model of roundabout production (e.g., see Phaneuf et al., 2018).

Differently from my baseline model, intermediate goods firms now produce using both labor as well as the intermediate good itself, with production function

$$y_t = q_t^{\phi} \ell_t^{(1-\alpha)(1-\phi)}$$

[33]If non-durables and durables were not produced out of a common final good (and so their relative prices could fluctuate), then it would of course still be possible to engineer a sequence of transfers that mimics a given interest rate policy's effect on *total* spending. Nothing guarantees, however, that the *composition* of that spending would be the same. If relative prices can move then the composition will matter in general equilibrium, thus breaking equivalence.

where $\phi \in [0, 1)$ denotes the share of intermediates in production. A standard cost minimization problem gives marginal costs as

$$mc_t = \left(\frac{1}{(1-\alpha)(1-\phi)\phi^{\frac{\phi}{1-\phi}}} \right)^{1-\phi} w_t^{1-\phi} \ell_t^{\alpha(1-\phi)}$$

and so, in log deviations,

$$\widehat{mc}_t = (1-\phi)\left(\widehat{w}_t + \alpha \widehat{\ell}_t\right)$$

Following the same steps as in the derivation of (B.1) we thus find that

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}})}{\theta_p} \frac{(1-\alpha)(1-\phi)}{1-\alpha+\alpha\varepsilon_p} \left(\widehat{w}_t + \alpha\widehat{\ell}_t\right) + \beta\widehat{\pi}_{t+1} \tag{C.15}$$

The only effect of roundabout production is thus to flatten the price-NKPC, leaving the headline policy equivalence result entirely unchanged.

C.7 Targeted transfers

My analysis throughout was focussed on *uniform* lump-sum taxes and transfers. This was by design: my objective was to establish that, in standard models of non-Ricardian consumption behavior, manipulating taxes and transfers *over time* can manipulate spending just like changes in intertemporal prices—that is, stimulus checks are stimulative even without any redistribution. In models with microeconomic heterogeneity (like HANK), it is of course also possible to consider transfer policies aimed at sub-populations of households and thus (in part) operational through redistribution. My results extend with little change to such alternative policy experiments.

Recall from the proof of Proposition 1 that the key requirement for policy equivalence is that, for any excess demand sequence $\widehat{\boldsymbol{c}}$ with zero net present value, we can find a transfer policy that induces a net excess demand path of $\widehat{\boldsymbol{c}}$. To see how this can be done using targeted transfers, consider a transfer targeted at some subgroup of households (group a) and financed using taxes on another subgroup (group b). I denote the transfer to group a by $\widehat{\boldsymbol{\tau}}^x$ and write the corresponding tax financing as $\widehat{\boldsymbol{\tau}}^e \equiv \mathcal{T}_\tau \widehat{\boldsymbol{\tau}}^x$, where $\mathcal{T}_\tau$ is such that

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t (\widehat{\tau}_t^x + \widehat{\tau}_t^e(\widehat{\boldsymbol{\tau}}^x)) = 0$$

Letting $\mathcal{C}_\tau^{(a)}$ and $\mathcal{C}_\tau^{(b)}$ denote the consumption derivative matrices for subgroups a and b, respectively, the effect of any given transfer policy $\hat{\boldsymbol{\tau}}^x$ on net excess demand is given as

$$\underbrace{\left(\mathcal{C}_\tau^{(a)} + \mathcal{C}_\tau^{(b)}\mathcal{T}_\tau\right)}_{\equiv \mathcal{C}_\tau^x} \hat{\boldsymbol{\tau}}^x$$

Analogously to the proof of Proposition 1, a sufficient condition for policy equivalence is now simply that every net excess demand path with zero net present value lies in the image of $\mathcal{C}_\tau^x$. Differently from my main analysis, characterizing $\mathcal{C}_\tau^x$ does not require MPCs averaged across the entire household cross-section, but MPCs averaged across the subgroups a and b.

For practical policy purposes, there are two key differences between my uniform policies and such targeted policies. First, the latter also work explicitly through redistribution across households, and thus in particular can affect net excess demand even with period-by-period budget balance and without any fluctuations in aggregate government debt. Second, it is unclear ex ante whether targeted transfers need to be larger or smaller in per capita terms. On the one hand, to engineer a given spending response by targeting a smaller group of households, the required transfer size per capita increases mechanically. On the other hand, if targeted households have larger MPCs, the required transfer decreases in size. I leave a detailed characterization of such macro-equivalent targeted transfers to future work.

D Equivalence in terms of policy rules

This appendix elaborates on the implications of my equivalence results for systematic policy *rules*. Appendix D.1 begins by formalizing the claims made in Section 3.3 from a sequence-space perspective. Appendix D.2 then translates all arguments to recursive notation. Finally, in Appendix D.3, I provide a worked-out example, deriving the transfer rule that replicates a standard Taylor rule in the context of the perpetual-youth consumption-savings model.

D.1 From policy paths to rules

I augment the baseline model of Section 2.1 to additionally feature wedges $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$ to the aggregate consumption function (2) as well as the Phillips curves (5) - (6), corresponding to reduced-form representations of canonical demand and supply shocks. Given a specification of policy in the form of policy rules, a bounded perfect-foresight transition path in response to any of these wedges corresponds to impulse response functions in the analogous linearized economy with aggregate risk (Boppart et al., 2018; Auclert et al., 2019). I will argue that, for any interest rate-only policy rule, there exists a transfer-only policy rule that implies the exact same impulse response of macroeconomic aggregates, including in particular aggregate output and inflation.

I will present my equivalence results for two particular kinds of interest rate policy rules: implicit targeting rules and explicit instrument rules (Giannoni & Woodford, 2002).

Implicit rules. A classical *implicit* targeting rule specifies a relationship between policy targets. For a standard dual-mandate policymaker, and written in perfect-foresight notation (e.g., see McKay & Wolf, 2022b), such a rule takes the general form

$$\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} = \mathbf{0} \tag{D.1}$$

(D.1) specifies a relationship between inflation and output along the perfect-foresight transition path. It nests as special cases strict inflation targeting ($\widehat{\pi}_t = 0$ and so $\mathcal{B}_\pi = I$, $\mathcal{B}_y = \mathbf{0}$), strict output targeting ($\widehat{y}_t = 0$ and so $\mathcal{B}_\pi = \mathbf{0}$, $\mathcal{B}_y = I$), as well as the canonical optimal implicit targeting rule of a dual-mandate policymaker, (25), mentioned in the main text. Strong Ricardian non-equivalence is sufficient to ensure that, if a rule of the general form (D.1) can be (uniquely) implemented using an interest rate-only policy (i.e., with a policy as in Definition 2), then the same is true for a transfer-only policy.

Corollary D.1. *Suppose that, given a sequence of shocks* $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$*, the implicit targeting rule* (D.1) *implemented through an interest rate-only policy induces a unique equilibrium. Then, under the conditions of Proposition 1 and strong Ricardian non-equivalence, the rule* (D.1) *implemented through a transfer-only policy also induces a unique equilibrium featuring the same aggregate allocation.*

When implementing the targeting rule (D.1) through interest rate policy, the policymaker in the background sets nominal interest rates so that aggregate demand is consistent with output and inflation sequences satisfying (D.1). By my high-level assumption of strong Ricardian non-equivalence, she can engineer that exact same required time path of aggregate excess demand through transfers—she simply needs to set transfers equal to

$$\widehat{\boldsymbol{\tau}} \;=\; \mathcal{C}_\tau^{-1} \;\times\; \text{demand target}$$

The proof of Corollary D.1 formalizes this argument.

Explicit rules. The same logic as above extends to *explicit* instrument rules—that is, rules that explicitly specify the value of the policy instrument as a function of observables. Again written in linearized perfect-foresight notation, a typical explicit interest rate rule takes the general form

$$\widehat{\boldsymbol{i}}_b \;=\; \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} \tag{D.2}$$

(D.2) here specifies a mapping from inflation and output into nominal interest rates along the perfect-foresight transition path. For example, a simple Taylor rule would take the form

$$\widehat{i}_{b,t} \;=\; \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t, \quad t = 0, 1, 2, \ldots$$

and so $\mathcal{B}_\pi = \phi_\pi \times I$, $\mathcal{B}_y = \phi_y \times I$. With interest rates set according to (D.2), taxes under my definition of an interest rate-only policy rule adjust in the background to ensure a balanced government budget (recall Definition 2). In particular, for my environment in Section 2.1, taxes by (8) follow

$$\widehat{\boldsymbol{\tau}} \;=\; \tau_\ell \bar{w} \bar{\ell} (\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) - \bar{b} \widehat{\boldsymbol{i}}_{b,-1} + (1+\bar{r}) \bar{b} \widehat{\boldsymbol{\pi}} \tag{D.3}$$

As before, strong Ricardian non-equivalence is sufficient to ensure that the equilibrium dynamics induced by a rule of the form (D.2)-(D.3) can equivalently be implemented uniquely through an explicit transfer-only policy rule.

Corollary D.2. *Suppose that, given a sequence of shocks $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$, the explicit interest rate rule* (D.2)-(D.3) *induces a unique equilibrium. Then, under the conditions of Proposition 1 and strong Ricardian non-equivalence, the transfer-only policy rule*

$$\widehat{\boldsymbol{\tau}} = \underbrace{\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}}_{\text{deficit response}} + \underbrace{\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}\right)}_{\text{active demand management}} \tag{D.4}$$

together with $\widehat{\boldsymbol{i}}_b = \mathbf{0}$ uniquely implements the exact same aggregate allocation.

(D.4) is an explicit instrument rule for taxes and transfers. Just like (D.2) did for interest rates, (D.4) is a rule that gives the time path of taxes and transfers as a function of time paths of inflation and output (as well as the deficit). Intuitively, the rules (D.2) and (D.4) are equivalent because they both imply the same mapping from macroeconomic aggregates—output and inflation—into aggregate demand. The only difference is the instrument that is used to achieve that mapping.

D.2 A recursive aggregate-risk perspective

The equivalent policy rules characterized in Appendix D.1 were written in sequence-space perfect-foresight notation. Here I discuss how to interpret such rules in the analogous linearized economy with aggregate risk. For implicit rules no further arguments are needed, simply because the policy rule does not directly involve the instrument. For example, written in recursive aggregate-risk notation, suppose the economy was closed with either the interest rate-only rule

$$\begin{aligned} \widehat{\pi}_t + \lambda(\widehat{y}_t - \widehat{y}_{t-1}) &= 0 \\ \widehat{\tau}_t &= (1+\bar{r})\bar{b}\widehat{\pi}_t + \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + \widehat{b}_t \end{aligned}$$

or the transfer-only rule

$$\begin{aligned} \widehat{\pi}_t + \lambda(\widehat{y}_t - \widehat{y}_{t-1}) &= 0 \\ \widehat{i}_{b,t} &= 0. \end{aligned}$$

Combining Corollary D.1 with the equivalence of linearized perfect-foresight solutions and linearized shock impulse responses (e.g., Boppart et al., 2018; Auclert et al., 2019), we can conclude that aggregate outcomes in the stochastic linearized economy with aggregate risk would be exactly the same under the two specifications of policy given above. I will thus from now on focus on explicit rules, where additional arguments are needed.

THE INTEREST RATE RULE. Consider first the general perfect-foresight explicit interest rate rule (D.2), re-stated here for convenience:

$$\widehat{\boldsymbol{i}}_b = \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}$$

This rule specifies a relationship between sequences of interest rates—the policy instrument—and sequences of output and inflation—the arguments of the policy rule—along the perfect-foresight transition path. My objective now is to provide an interpretation of that rule in the analogous linearized economy with aggregate risk.

The key building block result is yet again that linearized perfect-foresight transition paths are identical to *shock impulse responses*—i.e., to conditional expectations—in analogous linearized economies with aggregate risk. Specifically, begin by considering the analogous linearized economy with aggregate risk at its initial date 0, subject to some initial date-0 shocks. The explicit nominal interest rate rule (D.2) then simply says that current and expected future rates at date 0 satisfy

$$\mathbb{E}_0\left(\widehat{\boldsymbol{i}}_b^0\right) = \mathbb{E}_0\left[\mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^0 + \mathcal{B}_y \widehat{\boldsymbol{y}}^0\right] \tag{D.5}$$

where the notation $\boldsymbol{x}^t = (x_t, x_{t+1}, \dots)'$ indicates time paths from date t onwards. That is, current and future expected interest rates are given as a simple function of date-0 expectations of current and future output. At date 1 additional shocks hit the economy; adding up impulse responses to the initial date-0 shocks and the new date-1 shocks, we see that interest rates at date 1 satisfy

$$\mathbb{E}_1\left(\widehat{\boldsymbol{i}}_b^1\right) = \underbrace{\mathbb{E}_0\left(\widehat{\boldsymbol{i}}_b^1\right)}_{\text{response to date-0 shocks}} + \underbrace{\mathcal{B}_\pi \times \left[\mathbb{E}_1\left(\widehat{\boldsymbol{\pi}}^1\right) - \mathbb{E}_0\left(\widehat{\boldsymbol{\pi}}^1\right)\right] + \mathcal{B}_y \times \left[\mathbb{E}_1\left(\widehat{\boldsymbol{y}}^1\right) - \mathbb{E}_0\left(\widehat{\boldsymbol{y}}^1\right)\right]}_{\text{response to date-1 shocks}} \tag{D.6}$$

or more compactly

$$\hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{i}}_b^1\right) = \mathcal{B}_\pi \times \hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{\pi}}^1\right) + \mathcal{B}_y \times \hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{y}}^1\right) \tag{D.7}$$

where $\hat{\mathbb{E}}_{t,t-1}$ denotes the *change* in expectations between t and $t-1$. Continuing recursively, we in general find that interest rates in the linearized economy with aggregate risk satisfy the recursion

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{i}}_b^t\right) \;=\; \mathcal{B}_\pi \;\times\; \hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\pi}}^t\right) \;+\; \mathcal{B}_y \;\times\; \hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{y}}^t\right) \tag{D.8}$$

In words, at each t, the policymaker revises current and expected future paths of nominal interest rates in line with revisions about expectations of future inflation and output. This is an explicit instrument rule in the sense of Giannoni & Woodford (2002): it specifies, at each date t, the current and expected future values of the policy instrument as a function of lagged, current, and expected future values of macro aggregates. Note that the dependence on lagged aggregates is here encoded in the lagged instrument term $\mathbb{E}_{t-1}\left(\widehat{\boldsymbol{i}}_b^t\right)$.

The preceding discussion applies for arbitrarily complicated matrices $\{\mathcal{B}_\pi, \mathcal{B}_y\}$ specifying the mapping from expectations of macro aggregates to expectations of interest rates. Canonical recursive policy rules—like textbook Taylor rules—on the other hand restrict this mapping to have a particularly simple form. A standard Taylor rule maps into the general explicit rule form (D.2) with $\mathcal{B}_\pi = \phi_\pi \times I$ and $\mathcal{B}_y = \phi_y \times I$, where I denotes the identity map. If $\{\mathcal{B}_\pi, \mathcal{B}_y\}$ take such a simple diagonal form, then the in principle very complicated expectational revisions embedded in (D.8) are equivalent to one simple static equation—the familiar relation

$$\widehat{i}_{b,t} \;=\; \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t \tag{D.9}$$

Both (D.8) and (D.9) are valid explicit rules: at each date t, they specify a mapping from lagged, current, and expected future inflation and output into current and expected future policy instruments. The only difference is that in one case this mapping is restricted to have a very simple form, while in the other it is allowed to be much more general.

THE EQUIVALENT TRANSFER RULE. Now consider the macro-equivalent transfer rule, written in perfect-foresight sequence-space notation as (D.4). By exactly the same arguments as above, it corresponds to the following recursive formulation in the analogous linearized economy with aggregate risk:

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) \;=\; \hat{\mathbb{E}}_{t,t-1}\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^t + \widehat{\boldsymbol{\ell}}^t) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^t \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^t + \mathcal{B}_y \widehat{\boldsymbol{y}}^t\right)\right] \tag{D.10}$$

Equation (D.10) is an explicit instrument rule in exactly the same way as (D.8): it specifies, at each date t, the current and expected future values of the policy instrument—here transfers—as a function of lagged, current, and expected future values of macro aggregates. Of course, since at this point I am imposing no further restrictions on the product $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$, a rule that may be "simple" in interest rate space—like a conventional Taylor rule, as discussed above—

may be complicated in transfer space, in the sense that the general set of restrictions (D.10) cannot be reduced to a single static relation like (D.9). The next subsection provides an explicit worked-out example in the special case of a one-type perpetual-youth economy. In that particular setting, policy rules that are simple in interest rate space also turn out to be simple in transfer space, and vice-versa, simply because the matrix product $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$ takes a very simple form.

D.3 A worked-out example

I consider a special case of my economy in Section 2.1 where the aggregate consumption function $\mathcal{C}(\bullet)$ is that implied by a one-type perpetual-youth consumer demand structure, as discussed in Section 2.2. I suppose that the monetary policymaker wishes to replicate the outcomes implied by the Taylor-type rule (D.9) by relying on transfers instead. By (D.10), the recursively written explicit transfer rule that does so is

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) \;=\; \hat{\mathbb{E}}_{t,t-1}\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^t + \widehat{\boldsymbol{\ell}}^t) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^t \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\phi_\pi \widehat{\boldsymbol{\pi}}^t + \phi_y \widehat{\boldsymbol{y}}^t\right)\right] \tag{D.11}$$

The particular one-type perpetual-youth structure allows us to now further simplify the term $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$. Putting together the results from Appendices C.1 and C.2, we obtain

$$\mathcal{C}_\tau^{-1} \times \tilde{\mathcal{C}}_{i_b} \;=\; \frac{1}{\gamma}\frac{1}{1-\frac{\theta}{1+\bar{r}}}\begin{pmatrix} -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & 0 & 0 & \cdots \\ \frac{\theta}{1-\theta} & -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & 0 & \cdots \\ 0 & \frac{\theta}{1-\theta} & -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{D.12}$$

Plugging (D.12) into (D.11), we see that mapping the simple Taylor rule (D.9) into transfer space does indeed result in an almost equally simple explicit transfer-only rule:

$$\widehat{\tau}_t = \underbrace{\tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{\pi}\widehat{\pi}_t}_{\text{deficit response}} + \underbrace{\frac{1}{\gamma\left(1-\frac{\theta}{1+\bar{r}}\right)}\left[-\frac{\frac{\theta}{1+\bar{r}}}{1-\theta}\left(\phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t\right) + \frac{\theta}{1-\theta}\left(\phi_\pi\widehat{\pi}_{t-1} + \phi_y\widehat{y}_{t-1}\right)\right]}_{\text{active demand management}} \tag{D.13}$$

If transfers are set according to this simple policy rule, then they in the linearized equilibrium with aggregate risk indeed satisfy the general recursion (D.11). Thus, in this particular environment, an interest rate policy that responds to contemporaneous macro aggregates is equivalent to a still quite simple transfer-only policy that responds to current and one-period-lagged aggregates, with the response coefficients given from (D.12). The rule representation here is so simple because the map $\mathcal{C}_{\tau}^{-1}\tilde{\mathcal{C}}_{i_b}$—while not proportional to an identity matrix, like $\{\mathcal{B}_{\pi}, \mathcal{B}_{y}\}$—is nevertheless quite special: it is tridiagonal with repeating rows, and so we can summarize the potentially very complicated expectation revisions in (D.11) with just one simple equation, (D.13). This is exactly analogous to the Taylor rule (D.9) being equivalent to the more complicated general expression (D.8) when $\mathcal{B}_{\pi} = \phi_{\pi} \times I$ and $\mathcal{B}_{y} = \phi_{y} \times I$.

E Proofs and auxiliary lemmas

E.1 Proof of Lemma 1

Re-arranging (C.2) we obtain

$$\widehat{\tau}_t + \frac{1}{\beta}\widehat{b}_{t-1} = \frac{[1-\theta(1-\beta\theta)]\widehat{c}_t - \beta\theta\widehat{c}_{t+1}}{(1-\beta\theta)(1-\theta)}.$$

From the budget constraint (C.1) it follows that the left-hand side equals $\widehat{c}_t + \widehat{b}_t$. Re-arranging, we thus obtain

$$\widehat{b}_t = \frac{\beta\theta}{(1-\beta\theta)(1-\theta)}(\widehat{c}_t - \widehat{c}_{t+1})$$

and so, from the Euler equation (C.2),

$$\widehat{\tau}_t = \frac{[1-\theta(1-\beta\theta)]\widehat{c}_t - \beta\theta\widehat{c}_{t+1}}{(1-\beta\theta)(1-\theta)} - \frac{\theta}{(1-\beta\theta)(1-\theta)}(\widehat{c}_{t-1} - \widehat{c}_t).$$

Stacking these coefficients as the matrix $\mathcal{C}_\tau^{-1}$ (with $\beta(1+\bar{r}) = 1$), we obtain (20). □

E.2 Proof of Proposition 2

Key to the proof is the following auxiliary lemma.

Lemma E.1. *If $\theta_i < 1$, then, for $\bar{r}$ is sufficiently close to (but weakly above) zero, $\mathcal{C}_\tau^i$ is a positive operator (i.e., $\boldsymbol{\tau}'\mathcal{C}_\tau^i\boldsymbol{\tau} > 0$ for any $\boldsymbol{\tau} \neq \mathbf{0}$).*

Proof. $\mathcal{C}_\tau^i$ is positive if and only if its inverse is positive, so I will instead establish that $(\mathcal{C}_\tau^i)^{-1}$ is positive. Recall that

$$(\mathcal{C}_\tau^i)^{-1} = \begin{pmatrix} a_i & c_i & 0 & 0 & \dots \\ b_i & d_i & c_i & 0 & \dots \\ 0 & b_i & d_i & c_i & \dots \\ 0 & 0 & b_i & d_i & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where

$$a_i = \frac{1-\theta_i\left(1-\frac{\theta_i}{1+\bar{r}}\right)}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)}$$

$$
\begin{aligned}
c_i &= -\frac{\frac{\theta_i}{1+\bar{r}}}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)} \\
b_i &= -\frac{\theta_i}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)} \\
d_i &= \frac{1+\frac{\theta_i^2}{1+\bar{r}}}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)}
\end{aligned}
$$

From now on, to simplify notation, I will suppress all i subscripts. To prove that $\mathcal{C}_\tau^{-1}$ is positive I will decompose $\tilde{\mathcal{C}}_\tau^{-1} \equiv \frac{1}{2}\left(\mathcal{C}_\tau^{-1} + \mathcal{C}_\tau^{-1\prime}\right)$ or

$$
\tilde{\mathcal{C}}_\tau^{-1} = \begin{pmatrix}
a & \frac{b+c}{2} & 0 & 0 & \dots \\
\frac{b+c}{2} & d & \frac{b+c}{2} & 0 & \dots \\
0 & \frac{b+c}{2} & d & \frac{b+c}{2} & \dots \\
0 & 0 & \frac{b+c}{2} & d & \dots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

as $\tilde{\mathcal{C}}_\tau^{-1} = \mathcal{L}'\mathcal{L}$, where $\mathcal{L}$ is upper-triangular with real and positive diagonal entries. If I can find such an operator then $\tilde{\mathcal{C}}_\tau^{-1}$ (and so $\mathcal{C}_\tau^{-1}$) is positive. For this consider the candidate

$$
\mathcal{L} = \begin{pmatrix}
\sqrt{\delta_1} & \frac{\alpha}{\sqrt{\delta_1}} & 0 & 0 & \dots \\
0 & \sqrt{\delta_2} & \frac{\alpha}{\sqrt{\delta_2}} & 0 & \dots \\
0 & 0 & \sqrt{\delta_3} & \frac{\alpha}{\sqrt{\delta_3}} & \dots \\
\vdots & \vdots & \vdots & \ddots &
\end{pmatrix}
$$

where $\alpha \equiv \frac{b+c}{2}$ and δ_j follows the recursion

$$
\delta_j = d - \frac{\alpha^2}{\delta_{j-1}}
$$

with initial condition $\delta_1 = a > 0$. If $\delta_i > 0$ for all i and if the δ_i's are bounded, then it is straightforward to verify that $\tilde{\mathcal{C}}_\tau^{-1} = \mathcal{L}'\mathcal{L}$, and so that $\tilde{\mathcal{C}}_\tau^{-1}$ as well as $\mathcal{C}_\tau^{-1}$ (and so $\mathcal{C}_\tau$) are positive, bounded operators. To establish these properties of the δ_i's, note first of all that

$$
\alpha = \frac{b+c}{2} = -\frac{1}{2}\frac{\theta + \frac{\theta}{1+\bar{r}}}{\left(1-\frac{\theta}{1+\bar{r}}\right)(1-\theta)} < 0
$$

is a well-defined, finite number. Now consider the recursion for δ_j. Write $\delta_j = f(\delta_{j-1})$ and note that, for $\delta_{j-1} > 0$, $f(\bullet)$ is a strictly increasing, strictly concave function. The fixed points $\bar{\delta}$ and $\underline{\delta}$ satisfy

$$\bar{\delta} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - \alpha^2}, \quad \underline{\delta} = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \alpha^2}$$

Let me first establish that the argument under the square root is indeed strictly positive. For this it suffices to show that $d > |b + c|$, or that

$$1 + \frac{\theta^2}{1+\bar{r}} > \theta + \frac{\theta}{1+\bar{r}}$$

which holds for $\bar{r} \geq 0$ and $\theta < 1$. Next I will argue that $a > \underline{\delta}$. This requires

$$1 - \theta\left(1 - \frac{\theta}{1+\bar{r}}\right) > \frac{1}{2}\left(1 + \frac{\theta^2}{1+\bar{r}}\right) - \frac{1}{2}\sqrt{\left(1 + \frac{\theta^2}{1+\bar{r}}\right)^2 - \left(\theta + \frac{\theta}{1+\bar{r}}\right)^2}$$

Note that, for $\bar{r} = 0$, this becomes

$$1 - \theta(1-\theta) > \frac{1}{2}(1+\theta^2) - \frac{1}{2}\sqrt{(1+\theta^2)^2 - 4\theta^2}$$

which holds for $\theta < 1$. Thus $a > \underline{\delta}$, and so this also holds for $\bar{r} \in (0, r^*)$ for some upper bound r^*. Proceeding identically we can show that $a < \bar{\delta}$ for $\bar{r}$ sufficiently close to zero. But then it follows from the properties of $f(\bullet)$ that the sequence $\{\delta_j\}$ will converge monotonically from $\delta_1 = a$ to $\bar{\delta} > a > 0$. Thus the entire sequence consists of well-defined, finite, strictly positive numbers. We conclude that the candidate operator $\mathcal{L}$ exists and is well-defined. It follows that $\mathcal{C}_\tau^{-1}$ and so $\mathcal{C}_\tau$ are positive, as claimed. $\square$

The proof of Proposition 2 leverages Lemma E.1.[34] I will first of all show that the presence of *some* constrained households (i.e., $\theta_i < 1$ for at least some i with $\mu_i > 0$) suffices to ensure that transfer policy can induce any square-summable $\widehat{\boldsymbol{c}}$. I decompose

$$\mathcal{C}_\tau = \underbrace{\sum_{i=1}^{N} \mu_i \mathcal{C}_\tau^i}_{\equiv |\mathcal{I}| \times \mathcal{C}_\tau^{\mathcal{I}}} + (1 - |\mathcal{I}|)\mathcal{C}_\tau^R$$

where $\mathcal{I}$ denotes the set of groups with $\theta_i < 1$, and where $|\mathcal{I}| \equiv \sum_{i \in \mathcal{I}} \mu_i > 0$ by assumption. I

[34] I thank an anonymous referee for several suggestions that helped fix technical issues with the proof.

begin by studying $\mathcal{C}_\tau^{\mathcal{I}}$. This operator maps square-summable sequences into square-summable sequences, so from now on I will operate in ℓ^2, not ℓ^∞.[35] First suppose that $\bar{r}=0$. $\mathcal{C}_\tau^{\mathcal{I}}$ is then the sum of linear operators that are, by Lemmas 1 and E.1, both invertible and positive. Since $\bar{r}=0$ they are also symmetric, and so we can conclude from Proposition 1.5 in Mortad (2020) that the sum $\mathcal{C}_\tau^{\mathcal{I}}$ is also invertible. Next, by the representation of $\mathcal{C}_\tau^i$ in Appendix C.1, $\mathcal{C}_\tau^{\mathcal{I}}$ is continuous in $\bar{r}$. Since in any Banach space the set of invertible operators is open, it follows that $\mathcal{C}_\tau^{\mathcal{I}}$ is also invertible for $\bar{r}$ sufficiently close to (but above) 0.

Now return to $\mathcal{C}_\tau$. It remains to establish that any square-summable sequence $\widehat{\boldsymbol{c}}$ with zero net present value lies in the image of $\mathcal{C}_\tau$. By the previous results it follows that we can find a square-summable $\widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})$ such that

$$\mathcal{C}_\tau^{\mathcal{I}} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}}) = \widehat{\boldsymbol{c}}$$

Note that $\widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})$ necessarily has zero net present value, since $\mathcal{C}_\tau^{\mathcal{I}}$ embeds all individual agents' budget constraints (C.1). Now consider setting

$$\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}) = \frac{1}{|\mathcal{I}|} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})$$

Then, since $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}})$ also has zero net present value, it follows from the properties of $\mathcal{C}_\tau^R$ that

$$\mathcal{C}_\tau \cdot \widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}) = \underbrace{|\mathcal{I}|\mathcal{C}_\tau^{\mathcal{I}} \cdot \frac{1}{|\mathcal{I}|} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})}_{=\widehat{\boldsymbol{c}}} + \underbrace{(1-|\mathcal{I}|)\mathcal{C}_\tau^R \cdot \frac{1}{|\mathcal{I}|} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})}_{=\boldsymbol{0}} = \widehat{\boldsymbol{c}}.$$

It finally remains to note that, if $\theta_i < 1$, then $\tilde{\mathcal{C}}_{i_b}^i \cdot \widehat{\boldsymbol{i}}_b$ is square-summable (for square-summable $\widehat{\boldsymbol{i}}_b$), and thus so is $\widehat{\boldsymbol{c}} = \widehat{\boldsymbol{c}}_{i_b}^{PE} = \tilde{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b$ (if $\theta_i < 1 \; \forall i$).[36] This completes the argument. $\square$

E.3 Proof of Proposition 3

It follows from the discussion in Appendix C.1 that

$$\mathcal{C}_\tau(1,1) \;=\; \sum_{i=1}^N \mu_i \left(1 - \frac{\theta_i}{1+\bar{r}}\right)$$

[35] With the entries of $\mathcal{C}_\tau^i$ decaying exponentially in rows and columns away from the main diagonal (recall Lemma C.1 with $\theta_i < 1$), $\mathcal{C}_\tau^i$ is a bounded linear operator on ℓ^2; see also Auclert et al. (2023, Section 3.3).

[36] By (C.11), the off-diagonal entries of $\tilde{\mathcal{C}}_{i_b}^i$ also decay exponentially in rows and columns away from the main diagonal, like $\mathcal{C}_\tau^i$.

$$\mathcal{C}_\tau(2,1) \;=\; \sum_{i=1}^{N} \mu_i \theta_i \left(1 - \frac{\theta_i}{1+\bar{r}}\right)$$

We thus see that $\theta_i < 1$ for at least one i with $\mu_i > 0$ suffices to ensure that both $\mathcal{C}_\tau(1,1) > \frac{\bar{r}}{1+\bar{r}}$ and $\mathcal{C}_\tau(1,1) > \mathcal{C}_\tau(2,1)$, as claimed. □

E.4 Proof of Proposition 4

By assumption, the allocation $\{\pi_t^*, y_t^*\}_{t=0}^{\infty}$ is implementable using an interest rate-only policy tuple $\{i_{b,t}^*, \tau_t^*, 0\}_{t=0}^{\infty}$. Now consider the alternative policy tuple $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \widehat{\tau}_{f,t}^\dagger\}_{t=0}^{\infty}$ where

$$\widehat{\boldsymbol{\tau}}_f^\dagger = \mathcal{I}_{\tau_f}^{-1} \mathcal{I}_{i_b} \widehat{\mathbf{i}}_b^* \tag{E.1}$$

and

$$\widehat{\boldsymbol{\tau}}^\dagger = \mathcal{C}_\tau^{-1} \left[\mathcal{C}_{i_b} \widehat{\mathbf{i}}_b^* + \mathcal{C}_d \left(\mathcal{D}_{i_b} \widehat{\mathbf{i}}_b^* - \mathcal{D}_{\tau_f} \widehat{\boldsymbol{\tau}}_f^\dagger\right)\right] \tag{E.2}$$

I now claim that this policy tuple similarly engineers the allocation $\{\pi_t^*, y_t^*\}_{t=0}^{\infty}$. First, with $\widehat{\boldsymbol{\tau}}_f^\dagger$ set as in (E.1), the investment, output and labor demand paths are unchanged; however, as remarked in Appendix B.5, the dividend paths may be different. The transfer path $\widehat{\boldsymbol{\tau}}^\dagger$ is constructed to offset both the missing monetary stimulus as well as neutralize any potential dividend-related effects: to see this, note that we have

$$\begin{aligned}
\widehat{\mathbf{c}}^\dagger &= \mathcal{C}_\tau(\widehat{\boldsymbol{\tau}}^* + \widehat{\boldsymbol{\tau}}^\dagger) + \mathcal{C}_d \mathcal{D}_{\tau_f} \widehat{\boldsymbol{\tau}}_f^\dagger + \text{non-policy terms} \\
&= \mathcal{C}_\tau \widehat{\boldsymbol{\tau}}^* + \mathcal{C}_{i_b} \widehat{\mathbf{i}}_b^* + \mathcal{C}_d \mathcal{D}_{i_b} \widehat{\mathbf{i}}_b^* + \text{non-policy terms} = \widehat{\mathbf{c}}^*
\end{aligned}$$

Next note that, since they induce the same paths of consumption, investment, hours worked and production, and since by assumption wages are unchanged, the initial policy $\{i_{b,t}^*, \tau_t^*, 0\}_{t=0}^{\infty}$ and the new policy $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \widehat{\tau}_{f,t}^\dagger\}_{t=0}^{\infty}$ have the same present value in the augmented government budget constraint (B.18), exactly as in the proof of Proposition 1. With $\lim_{t\to\infty} \widehat{b}_t = 0$ in the initial equilibrium, it then follows that we must also have $\lim_{t\to\infty} \widehat{b}_t = 0$ in the new one, as required. All other model equations are unaffected, so the guess is verified. □

E.5 Proof of Lemma C.1

I will guess and verify that lagged wealth is the only endogenous state, so the decision rules take the general form

$$\begin{aligned}\widehat{c}_t &= \vartheta_{cb}\widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{cyh}\widehat{\tau}_{t-h} \\ \widehat{b}_t &= \vartheta_{bb}\widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{byh}\widehat{\tau}_{t-h}\end{aligned}$$

Plugging into the optimality conditions (C.1) - (C.2) and matching coefficients, we get the following system of equations characterizing behavior in response to an anticipated income shock H periods into the future:

$$\vartheta_{cb} + \vartheta_{bb} - \frac{1}{\beta} = 0 \tag{E.3}$$

$$\vartheta_{c\tau h} + \vartheta_{b\tau h} = 0, \quad h = 0, 1, \ldots, H-1 \tag{E.4}$$

$$\vartheta_{c\tau H} + \vartheta_{b\tau H} = 1 \tag{E.5}$$

$$[1 - \theta(1-\beta\theta)]\,\vartheta_{cb} - \beta\theta\vartheta_{cb}\vartheta_{bb} - (1-\beta\theta)(1-\theta)\frac{1}{\beta} = 0 \tag{E.6}$$

$$[1 - \theta(1-\beta\theta)]\,\vartheta_{c\tau h} - \beta\theta\left[\vartheta_{cb}\vartheta_{b\tau h} + \vartheta_{c\tau h+1}\right] = 0, \quad h = 0, 1, \ldots, H-1 \tag{E.7}$$

$$[1 - \theta(1-\beta\theta)]\,\vartheta_{c\tau H} - \beta\theta\vartheta_{cb}\vartheta_{b\tau H} = (1-\beta\theta)(1-\theta) \tag{E.8}$$

I will begin by characterizing the solution of this system. From (E.3) and (E.6) we have

$$\begin{aligned}\vartheta_{bb} &= \theta \\ \vartheta_{cb} &= \frac{1}{\beta} - \theta\end{aligned}$$

Next, from (E.5) and (E.8), we have that

$$\begin{aligned}\vartheta_{c\tau H} &= 1 - \beta\theta \\ \vartheta_{b\tau H} &= \beta\theta\end{aligned}$$

Finally, from (E.4) and (E.7),

$$\vartheta_{c\tau h} = \beta\theta\vartheta_{c\tau h+1} = (\beta\theta)^{H-h}(1-\beta\theta)$$

$$\vartheta_{b\tau h} \;\; = \;\; -\beta\theta\vartheta_{c\tau h+1} = -(\beta\theta)^{H-h}(1-\beta\theta)$$

This characterizes the full solution.

I can now prove the various asymptotic statements of Lemma C.1. First we have that

$$\widehat{c}_{H,H} = \vartheta_{cb}\sum_{\ell=0}^{H-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H}$$

Plugging in from the closed-form expressions above and simplifying:

$$\lim_{H\to\infty} c_{H,H} = \frac{(1-\theta)(1-\beta\theta)}{1-\beta\theta^2} = \text{const.}$$

Next looking below the main diagonal:

$$\widehat{c}_{H+1,H} = \vartheta_{cb}\sum_{\ell=0}^{H}\vartheta_{bb}^{\ell}\vartheta_{d\tau H-\ell}$$

and so

$$\frac{1}{\theta}\widehat{c}_{H+1,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}(\vartheta_{bb}\widehat{b}_{H-1,H} + \vartheta_{b\tau H}) = \vartheta_{cb}\widehat{b}_{H-1,H} + (1-\theta\beta) = \widehat{c}_{H,H}$$

Similarly

$$\frac{1}{\theta}\widehat{c}_{H+\ell,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}\vartheta_{bb}\widehat{b}_{H+\ell-2,H} = \widehat{c}_{H+\ell-1,H}$$

The proof reveals that the result holds for any H, not just $H \to \infty$.

Finally I look above the main diagonal. Here we have

$$\widehat{c}_{H-1,H} = \vartheta_{cb}\sum_{\ell=0}^{H-2}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-2} + \vartheta_{cyH-1}$$

We thus have that

$$\frac{1}{\beta\theta}\widehat{c}_{H-1,H} = \vartheta_{cb}\sum_{\ell=0}^{H-2}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H} = \widehat{c}_{H,H} - \vartheta_{cb}\vartheta_{bb}^{H-1}\vartheta_{b\tau 0}$$

The last term goes to zero as $H \to \infty$. Similarly we have

$$\widehat{c}_{H-\ell,H} = \vartheta_{cb}\sum_{\ell=0}^{H-\ell-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-2} + \vartheta_{c\tau H-\ell}$$

and so

$$\frac{1}{\beta\theta}\widehat{c}_{H-\ell,H} = \vartheta_{cb}\sum_{\ell=0}^{H-\ell-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H-\ell+1} = \widehat{c}_{H-\ell+1,H} - \vartheta_{cb}\vartheta_{bb}^{H-\ell}\vartheta_{b\tau 0}$$

where the last term again goes to zero as $H \to \infty$, completing the argument. $\square$

E.6 Proof of Lemma C.2

I begin with the budget constraint and the Euler equation. For notational convenience I suppress all i subscripts. The budget constraint is

$$\widehat{c}_t + \widehat{b}_t - (1+\bar{r})\widehat{b}_{t-1} = \widehat{\tau}_t. \tag{E.9}$$

Combining budget constraint and the sequential formulation of aggregate demand in (C.4), we obtain the following Euler equation:

$$(1-M\theta)\widehat{c}_t - \frac{\theta}{1+\bar{r}}\widehat{c}_{t+1} - M(1-\theta)(1+\bar{r})\widehat{b}_{t-1} = M(1-\theta)\widehat{\tau}_t. \tag{E.10}$$

I now proceed by combining (C.1) and (E.10) as in the proof of Lemma 1. First, we have

$$\widehat{\tau}_t + (1+\bar{r})\widehat{b}_{t-1} = \frac{(1-M\theta)\widehat{c}_t - \frac{\theta}{1+\bar{r}}\widehat{c}_{t+1}}{M(1-\theta)}$$

which again also equals $\widehat{c}_t + \widehat{b}_t$, from the budget constraint. Thus

$$\widehat{b}_t = \frac{1-M}{M(1-\theta)}\widehat{c}_t - \frac{\frac{\theta}{1+\bar{r}}}{M(1-\theta)}\widehat{c}_{t+1}$$

and so

$$\widehat{\tau}_t = \frac{(1-M\theta)\widehat{c}_t - \frac{\theta}{1+\bar{r}}\widehat{c}_{t+1}}{M(1-\theta)} - \frac{(1+\bar{r})(1-M)}{M(1-\theta)}\widehat{c}_{t-1} - \frac{\theta}{M(1-\theta)}\widehat{c}_t.$$

Stacking, we obtain (C.5).

E.7 Proof of Corollary D.1

I begin with some preliminary simplifications. The non-policy block of the economy can be summarized by the following system of equations, now written in compact sequence-space notation (as in Auclert et al., 2019). First, the price-NKPC,

$$\widehat{\boldsymbol{\pi}} = \Pi_w \widehat{\boldsymbol{w}} + \Pi_\ell \widehat{\boldsymbol{\ell}} + \beta \widehat{\boldsymbol{\pi}}_{+1} + \boldsymbol{\varepsilon}^p$$

Second, the production function

$$\widehat{\boldsymbol{y}} = \mathcal{Y}_\ell \widehat{\boldsymbol{\ell}}$$

Third, firm dividends,

$$\widehat{\boldsymbol{d}} = \mathcal{D}_y \widehat{\boldsymbol{y}} + \mathcal{D}_w \widehat{\boldsymbol{w}} + \mathcal{D}_\ell \widehat{\boldsymbol{\ell}}$$

Fourth, consumer demand

$$\widehat{\boldsymbol{c}} = \mathcal{C}_w \widehat{\boldsymbol{w}} + \mathcal{C}_\ell \widehat{\boldsymbol{\ell}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{C}_d \widehat{\boldsymbol{d}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}} + \mathcal{C}_{i_b} \widehat{\boldsymbol{i}}_b + \mathcal{C}_c \boldsymbol{\varepsilon}^c$$

Fifth, the wage-NKPC,

$$\widehat{\boldsymbol{\ell}} = \mathcal{L}_w \widehat{\boldsymbol{w}} + \mathcal{L}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{L}_c \widehat{\boldsymbol{c}} + \boldsymbol{\varepsilon}^w$$

And sixth, the output market-clearing condition

$$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{c}}$$

Using the price-NKPC, the production function, the equation for firm dividends, and the output market-clearing condition, we can substitute out $\{\boldsymbol{c}, \boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{d}\}$ in the consumer demand relation. This gives

$$\bar{\mathcal{C}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b + \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}} + \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \tag{E.11}$$

where $\bar{\mathcal{C}}_{i_b} = \mathcal{C}_{i_b}$, $\bar{\mathcal{C}}_\tau = \mathcal{C}_\tau$, $\bar{\mathcal{C}}_c = \mathcal{C}_c$ and $\{\bar{\mathcal{C}}_y, \bar{\mathcal{C}}_\pi\}$ are functions of model primitives. Similarly, using the price-NKPC as well as the production function and the output market-clearing condition, we can substitute out $\{\boldsymbol{\ell}, \boldsymbol{w}, \boldsymbol{c}\}$ in the wage-NKPC to write it as

$$\bar{\mathcal{L}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{L}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \tag{E.12}$$

where $\{\bar{\mathcal{L}}_y, \bar{\mathcal{L}}_\pi, \bar{\mathcal{L}}_p, \bar{\mathcal{L}}_w\}$ are functions of model primitives.

Now consider first the case where the desired implicit targeting rule (D.1) is implemented using an interest rate-only policy. Plugging the price-NKPC and the production function into the financing rule (8), we can write the financing rule compactly as

$$\widehat{\boldsymbol{\tau}} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}} + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b \tag{E.13}$$

where $\{\bar{\mathcal{T}}_y, \bar{\mathcal{T}}_\pi, \bar{\mathcal{T}}_{i_b}\}$ are functions of the financing rule (8) and model primitives. Using the simplifications in (E.11), (E.12) and (E.13), we can write the equilibrium system as the following stacked linear system:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_y & \mathcal{B}_\pi & \mathbf{0} & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.14}$$

By assumption, the system (E.14) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the question of whether the same implicit targeting rule can be implemented using a transfer-only policy. Using the simplifications from above, we can write the equilibrium system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_y & \mathcal{B}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.15}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (E.15). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where $\widehat{\boldsymbol{\tau}}^{**}$ solves

$$\left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* + \bar{\mathcal{C}}_\tau \left(\bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^*\right) = \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**}$$

We know by the conditions of Proposition 1 (which recall are assumed for Corollary D.1) that such a $\widehat{\boldsymbol{\tau}}^{**}$ exists. Plugging into (E.11), we get

$$\begin{aligned} & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \Leftrightarrow \quad & \left(\bar{\mathcal{C}}_y - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y\right) \widehat{\boldsymbol{y}}^* + \left(\bar{\mathcal{C}}_\pi - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi\right) \widehat{\boldsymbol{\pi}}^* - \left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \end{aligned}$$

Thus (E.11) still holds. It is immediate that all other relations in (E.15) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (E.15).

To show uniqueness, suppose for a contraction that (E.15) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. By the assumptions of Proposition 1 we can thus find a bounded tuple $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right)\widehat{\boldsymbol{i}}_b^\dagger + \bar{\mathcal{C}}_\tau\left(\bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger\right) = \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^\dagger$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^\dagger$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (E.14). Contradiction. □

E.8 Proof of Corollary D.2

Proceeding as in the proof of Corollary D.1, we arrive at the following equilibrium system for the explicit interest rate rule:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ -\mathcal{B}_y & -\mathcal{B}_\pi & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.16}$$

By assumption, the system (E.16) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the equilibrium system corresponding to the proposed transfer-only rule (D.4). Using the simplifications from above, we can write that system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y - \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_y & -\bar{\mathcal{T}}_\pi - \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.17}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (E.17). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where

$$\widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^* + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^* + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b} \widehat{\boldsymbol{i}}_b^*$$

Plugging the candidate solution into the consumer demand function (E.11), we get

$$\begin{aligned} & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \Leftrightarrow \quad & \left(\bar{\mathcal{C}}_y - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y\right) \widehat{\boldsymbol{y}}^* + \left(\bar{\mathcal{C}}_\pi - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi\right) \widehat{\boldsymbol{\pi}}^* - \left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \end{aligned}$$

Thus (E.11) still holds. It is immediate that all other relations in (E.17) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (E.17).

To show uniqueness, suppose for a contraction that (E.17) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. Now consider the tuple $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\widehat{\boldsymbol{i}}_b^\dagger = \mathcal{B}_y \widehat{\boldsymbol{y}}^\dagger + \mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^\dagger$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^\dagger$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (E.16). Contradiction. $\square$