Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

Christian K. Wolf†
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Abstract: I derive a general condition on consumer behavior ensuring that, in a standard model of demand-determined output, any path of inflation and output that is implementable via interest rate policy is also implementable through time-varying uniform transfers. In an analytical model with occasionally-binding borrowing constraints, my condition holds generically. In a quantitative HANK model, the transfer policy that closes any given demand shortfall is furthermore well-characterized by a small number of measurable sufficient statistics. My results extend to environments with investment if transfers are supplemented by another standard fiscal tool—bonus depreciation.

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1 Introduction

The prescription of standard New Keynesian theory is to conduct stabilization policy through changes in short-term interest rates. Over the past decade, much policy and academic interest has centered on the question of whether—and if so, how—alternative policy tools could be used to replicate monetary stimulus when nominal interest rates are constrained by a zero or effective lower bound (ELB).\(^1\) Prior work has in particular identified tax policy, often labeled unconventional fiscal policy, as an attractive option (Correia et al., 2008, 2013): time-varying tax rates manipulate intertemporal prices just like monetary policy and thus can replicate any desired monetary allocation.

In this paper I ask whether conventional fiscal policy—that is, fiscal instruments that are already part of the standard stabilization policy toolkit—is similarly sufficient to replicate any given monetary policy. The setting for much of my analysis is a textbook business-cycle model with nominal rigidities and without capital, extended to allow for more general, non-Ricardian household consumption behavior. The fiscal stabilization tool that I consider are uniform, deficit-financed transfers (“stimulus checks”), an instrument used in all recent U.S. recessions. I make three contributions. First, I identify a general sufficient condition under which any time paths of aggregate output and inflation that are implementable via interest rate policy are also implementable solely by adjusting the time path of such uniform lump-sum taxes and transfers. Second, I check this condition in an analytical model of occasionally-binding household borrowing constraints (e.g., as in Woodford, 1990; Farhi & Werning, 2019; Angeletos et al., 2023), and prove that it holds generically—borrowing constraints just need to bind some of the time. Third, I then numerically verify the sufficient condition in a rich quantitative heterogeneous-agent (HANK) model. In this environment, the stimulus check policy that closes any given demand shortfall is furthermore well-characterized by a small number of measurable “sufficient statistics.” Finally, I establish that all of these conclusions extend to richer models with investment as long as stimulus checks are supplemented by a second, similarly conventional fiscal tool: bonus depreciation.

Environment & general equivalence condition. My model economy features a policymaker with access to two instruments: nominal interest rates and uniform, lump-sum taxes and transfers. My objective is to characterize the space of allocations implementable

\(^1\)A notable early example is Bernanke (2002). Important recent contributions include Correia et al. (2008), Correia et al. (2013), Galí (2020), and Reis & Tenreyro (2022).
through manipulation of these two instruments. All results apply to (linearized) perfect foresight transition paths, or equivalently to the model’s first-order perturbation solution with aggregate risk. Key for me are properties of the two matrices $C_{ib}$ and $C_{\tau}$, whose $(t, s)$th entries are, respectively, the derivatives of partial equilibrium consumption demand at time $t$ with respect to a) a change in the time-$s$ rate of interest on bonds and b) a uniform lump-sum transfer paid out at time $s$. Note that $C_{\tau}$ is a matrix of intertemporal marginal propensities to consume (iMPCs), as studied first in Auclert et al. (2018).

I establish that, if $C_{\tau}$ is invertible—a condition that I refer to as strong Ricardian non-equivalence—, then any sequence of aggregate output and inflation that can be attained via interest rate policy is similarly implementable by only adjusting the time profile of uniform lump-sum taxes and transfers. The proof begins with the household consumption-savings problem. A feasible monetary policy is a path of interest rates together with a path of lump-sum taxes or transfers that ensures a balanced government budget. Through the household problem, this policy induces some path of net excess consumption demand. Can a transfer-only policy—that is, a policy that only changes the time profile of taxes or transfers, again subject to budget balance—engineer the same path of demand? For Ricardian households, the answer is clearly no: for them, only the net present value of transfers matters, so any budget-feasible transfer policy leaves spending unchanged. Mathematically, this is reflected in $C_{\tau}$ being rank-1. If instead the timing of transfers matters (in the strong sense that $C_{\tau}$ is invertible), then there does exist some path of transfers alone that perturbs demand in exactly the same way as the baseline monetary policy. Since this monetary policy was by assumption budget-balanced, the equivalent transfer policy is feasible as well. The argument is completed by showing that, in my environment, two policies that generate the same net excess consumption demand paths must be accommodated in general equilibrium through the same market-clearing adjustments in prices and quantities.\footnote{Though revenue-equivalent in net present value terms, the two policies do invariably induce different short-run government debt dynamics: while interest rate policy can in principle have aggregate effects even if outstanding debt is fixed, uniform stimulus checks work only because they change the time path of government bonds held by the private sector.}

Under the conditions of my equivalence result, transfer payments can serve as a perfect substitute for interest rate policy in the eyes of a conventional “dual mandate” policymaker. Formally, my results imply that systematic policy rules (e.g., flexible inflation forecast target criteria, as discussed in Woodford, 2011) continue to be implementable even if nominal rates are constrained by a binding lower bound. In particular, this conclusion holds completely independently of the menu of non-policy disturbances hitting the economy.
Analytical equivalence result. I next verify the condition underlying my equivalence result in an analytically tractable model of occasionally-binding household borrowing constraints. Building on Farhi & Werning (2019), I consider a perpetual-youth model of household consumption-savings decisions (à la Blanchard, 1985), where the survival probability $\theta \in [0,1]$ can be re-interpreted as one minus the probability of borrowing constraints binding at any given date (see also Woodford, 1990; Angeletos et al., 2023). I further generalize this environment by allowing for an arbitrary number of household types $i$ that differ in $\theta_i$—i.e., the extent to which they are subject to binding borrowing constraints.

My main result is that, for policy equivalence to hold, it suffices for borrowing constraints to bind some of the time. The argument proceeds in two steps. First, I begin by considering a model with only one household type and a borrowing constraint coefficient of $\theta$. I here can prove that, as long as $\theta < 1$—i.e., borrowing constraints that just bind some of the time—$C_T$ is invertible. The intuition is straightforward: occasionally-binding borrowing constraints shorten spending horizons, and so transfers today financed by taxes in the future lead to a front-loading of spending, mimicking the effects of interest rate cuts. Second, I go back to the general environment featuring a mixture of household types, with potentially heterogeneous $\theta_i$’s. I here establish that, if $\theta_i$ for all types $i$ is below—but potentially arbitrarily close to—1, then the equivalence result yet again holds.

Finally, I establish that the policy equivalence result is not just theoretically general, but also empirically relevant. In my environment, transfer policy can induce any desired path of net excess demand if and only if the economywide average MPC strictly exceeds $\beta(1+\bar{r})$, where $\bar{r}$ is the real rate of interest—i.e., an MPC above the classical permanent-income level. This requirement is well-known to receive ample support in the data (e.g., see Parker et al., 2013; Fagereng et al., 2018; Borusyak et al., 2021).

Quantitative explorations. I complement my analytical discussion with a quantitative exploration in a rich HANK model (Kaplan et al., 2018). This analysis serves a dual purpose. First, it allows me to verify that—as expected given my analytical results—the sufficient condition for policy equivalence also holds in state-of-the-art quantitative models.

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3The formal proposition does not cover the limit case of some households with $\theta_i$ exactly equal to 1 only because of economically inessential technicalities; see the discussion after Proposition 2.

4My derivations throughout assume that $\beta(1+\bar{r}) = 1$, as in Farhi & Werning (2019). If instead $\beta(1+\bar{r}) > 1$, then consumers have a natural tendency to postpone spending, so my results instead require that borrowing constraints bind often enough. Importantly, however, this generalized condition is still equivalent to MPCs above the annuity value of income gains, and thus testable in the same way.
of the consumption-savings decision. Second, I document that the policy equivalence result is also practically relevant, in the following sense. Building on Auclert et al. (2018), I find that, in my quantitative HANK model, the inverse $C_{\tau}^{-1}$ not only exists, but also can be characterized to a very high degree of accuracy with just three empirically measurable numbers: the economy’s average MPC $\omega$; the slope $\theta$ of how consumers spend a lump-sum income gain over time; and the steady-state rate of interest $\bar{r}$. Crucially, for empirically relevant levels of these three statistics, my formula for the inverse matrix $C_{\tau}^{-1}$ reveals that even moderate increases in transfers suffice for meaningful aggregate cyclical stabilization.

**Extensions & Limitations.** My results extend to environments that also feature capital. Monetary policy here now operates through two levers: by directly affecting (i) the household consumption-savings decision and (ii) the firm investment decision. Transfer policy is still enough to replicate arbitrary stimulus to consumer excess demand. For firm investment, it is straightforward to show that a second, similarly conventional fiscal instrument will suffice: bonus depreciation stimulus. I thus conclude that, even in this extended environment, two entirely conventional fiscal tools suffice to replicate any desired monetary stimulus.

While most of my analysis is concerned with aggregates, I also briefly discuss important limitations to my equivalence result in the presence of microeconomic heterogeneity. First, I emphasize that—for my equivalence result to also apply to HANK-type models—I require particular (though standard) assumptions on wage-setting. These assumptions are consistent with arbitrary levels of wealth effects in labor supply, but not with cross-sectional dispersion in those wealth effects. I review the empirical evidence on such dispersion and conclude that the scope for this channel to materially affect the equivalence result is limited as long as stimulus checks are moderate in size, with magnitudes on the order of what has been observed in practice. Second, my macro-equivalent interest rate and lump-sum transfer policies are generally not equivalent household-by-household: nominal interest rate cuts mostly act by directly stimulating consumption at the top of the liquid wealth distribution, whereas the macro-equivalent transfer stimulus almost exclusively acts at the bottom. Thus, compared to a given interest rate policy, the macro-equivalent transfer delivers the same macro stimulus at smaller cross-sectional consumption dispersion. The normative implications of this positive observation are studied in McKay & Wolf (2022a).

**Related Literature.** The paper relates and contributes to several strands of literature. First, the analysis is motivated by the recent experience of limits to conventional monetary policy space. Prior work has argued that unconventional fiscal instruments can be used
to substitute for monetary stimulus if needed (Correia et al., 2008, 2013); I instead clarify the conditions under which more conventional fiscal stimulus tools can do the same.\textsuperscript{5}

Second, I relate to prior work on the stimulative effects of transfer payments in the absence of Ricardian equivalence. Blanchard (1985), Woodford (1990) and Bilbiie et al. (2013) all emphasize that, if private planning horizons are finite (due to death) or borrowing constraints bind, then public debt can stimulate spending through its role as private liquidity. Bilbiie et al. (2021) furthermore show that redistribution from savers to spenders in a two-type model can perfectly mimic monetary stimulus. Relative to this line of work, my contribution is to: (i) identify a general condition under which uniform, time-varying taxes and transfers are stimulative in the precise sense that they can replicate monetary policy; (ii) verify this condition in a quite general analytical model of occasionally-binding borrowing constraints; and (iii) show that the required paths of taxes and transfers can, in quantitatively relevant models, be characterized as a function of a small number of empirically measurable statistics.

Third, my proof of policy equivalence relies heavily on equilibrium characterizations in sequence space (see Boppart et al., 2018; Auclert et al., 2019). So far, the sequence-space setup has been used to analytically characterize general equilibrium effects (Auclert & Rognlie, 2018; Auclert et al., 2018) or to construct general equilibrium counterfactuals for unobserved shocks (Wolf, 2020). I instead use the same observations to sidestep constraints on policy space. Echoing classical general equilibrium theory (Arrow & Debreu, 1954), the sequence-space perspective reveals that two policies are equivalent if they induce the same net excess demand paths. As such, my equivalence results are conceptually distinct from Correia et al. (2013) and Farhi et al. (2014)—there, equivalence is shown via identical wedges in optimality conditions. The advantage of my approach is that it can be applied readily to conventional fiscal instruments (like stimulus checks); the obvious challenge is that characterization of the equivalent policy requires additional arguments (my formulas for $C^{-1}$).

OUTLOOK. The rest of the paper proceeds as follows. Section 2 sets up the model, and Section 3 presents the main theoretical results. The quantitative analysis follows in Section 4. I discuss the role of micro heterogeneity in Section 5, and finally extend my results to a model with investment in Section 6. Section 7 concludes.\textsuperscript{6}

\textsuperscript{5}Another related policy tool that has received both academic as well as substantial policy interest are helicopter drops of money—that is, \textit{money-financed} transfer payments (e.g., see Bernanke, 2002; Galí, 2020; Reis & Tenreyro, 2022). My results reveal that, under natural assumptions on consumer spending behavior, even \textit{deficit-financed}, \textit{uniform} transfer payments are sufficient to replicate monetary stimulus.

\textsuperscript{6}Replication files are available online; see https://github.com/ckwolf92/mpfp_equiv.
2 Environment

I begin with a description of the model environment in Section 2.1. My assumptions on consumption behavior will be purposefully general, requiring only the existence of an aggregate consumption function. Section 2.2 then discusses the particular analytical and quantitative consumption-savings problems that I will consider in Sections 3 and 4.

2.1 Model outline

Time is discrete and runs forever, $t = 0, 1, \ldots$. The model economy is populated by households, unions, firms, and a government, and is initially at its deterministic steady state. I study linearized perfect foresight transition paths. The set-up is kept deliberately close to the textbook New Keynesian business-cycle framework (Woodford, 2011; Gali, 2015).

At time $t = 0$, the policymaker announces paths for her policy instruments. My objective is to characterize the set of allocations that she can implement. The realization of a variable $x$ at time $t$ along the equilibrium perfect foresight transition path will be denoted $x_t$, while the entire time path will be denoted $x = \{x_t\}_{t=0}^\infty$. Hats denote (log-)deviations from the deterministic steady state and bars denote steady-state values.

The aggregate consumption function. Households consume and supply labor, with total consumption and hours worked denoted by $c_t$ and $\ell_t$, respectively. Due to frictions in the labor market, hours worked are taken as given by households and set by optimizing labor unions, to be discussed in detail later. Given paths of income, households then decide on their consumption and savings. Rather than specifying the details of this consumption-savings problem, I here simply summarize its solution in the form of an aggregate consumption function. Section 2.2 will consider several particular models of household consumption behavior that fit into this general framework.

Before stating and discussing the aggregate consumption function I begin with the household budget constraint. Total income $e_t$ of the household sector consists of: labor earnings $(1 - \tau_t)w_t\ell_t$, where $w_t$ is the real wage and $\tau_t$ is the (assumed fixed) labor tax rate; uniform

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7My results can thus equivalently be interpreted as applying to the first-order perturbation solution of an analogous model with aggregate risk (e.g. Boppart et al., 2018; Auclert et al., 2019).

8I allow for unions in the interest of generality and empirical relevance. For all models except for the quantitative heterogeneous-household model of Section 4, the alternative standard case of frictionless labor supply will correspond to the flexible-wage limit of my union model. I further discuss the role of labor supply in HANK models in Section 5.1.
lump-sum transfer receipts $\tau_t$; and dividends $d_t$. Households can invest in nominally risk-free liquid bonds with nominal returns $i_{b,t}$. The real return to saving is affected by the inflation rate $\pi_t$. The period-$t$ budget constraint of the aggregated household sector is thus

$$c_t + b_t = \left(1 - \tau_t\right)w_t\ell_t + \tau_t + d_t + \frac{1 + i_{b,t-1}}{1 + \pi_t}b_{t-1}$$

where $b_t$ denotes real bond holdings. Given any sequence of total household income and asset returns, optimal household behavior yields time paths of aggregate consumption demand $c$ and asset supply $b$. I summarize optimal household consumption behavior in the form of an aggregate consumption function $C(\bullet)$ (Farhi & Werning, 2019; Auclert et al., 2018):

$$c = C(\underbrace{w, \ell, \pi, d, \tau, i_b}_{\text{eq’m aggregates}}, \underbrace{\tau, i_b}_{\text{policy}})$$

By definition, the aggregate consumption function evaluated at steady state satisfies

$$\bar{c} = C(\bar{w}, \bar{\ell}, \bar{\pi}, \bar{d}, \bar{\tau}, \bar{i}_b)$$

In my linearized environment, policy equivalence will be fully governed by the properties of $C(\bullet)$ around this deterministic steady state. Linearizing (2), we can write

$$\hat{c} = C_w\hat{w} + C_\ell\hat{\ell} + C_\pi\hat{\pi} + C_d\hat{d} + C_{\tau}\hat{\tau} + C_{i_b}\hat{i}_b$$

where, for each $q \in \{w, \ell, \pi, d, \tau, i_b\}$, I have defined

$$C_q \equiv \frac{\partial C(\bullet)}{\partial q}.$$ 

with the derivative evaluated at the deterministic steady state. The $(t, s)$th entry of each of those infinite-dimensional linear maps is the response of aggregate consumption demand at time $t$ to a marginal change in input $q$ at time $s$. The linear map $C_{\tau}$—which indicates how aggregate consumption demand will respond to changes in lump-sum transfers—will play a central role in characterizing the allocations implementable by the policymaker through stimulus check policy. To build further intuition I will in Section 2.2 provide closed-form characterizations of this map in various familiar models of household consumption decisions.

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9To be precise, I, as in Auclert et al. (2018), assume that this consumption function is Fréchet-differentiable around the steady state, and its derivatives in (4) are bounded linear operators from $\ell_\infty$ to $\ell_\infty$. 

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Unions & Firms. I summarize the production and wage bargaining block through three key relations. First, a unit continuum of firms produces the final output good:

\[ y_t = y(\ell_t) \]

Second, price-setting is subject to the usual Rotemberg adjustment costs, giving a textbook New Keynesian Phillips Curve (NKPC) in prices (Galí, 2015):

\[ \hat{\pi}_t = \kappa_p \times \chi_p(\hat{w}_t, \hat{\ell}_t) + \beta \hat{\pi}_{t+1} \]

where \( \chi_p(\bullet) \) gives the deviation from the price target and \( \kappa_p \) is the slope of the price-NKPC. Third, wage bargaining is similarly subject to adjustment costs and so induces a general wage-NKPC, linking wage inflation to the static labor optimality wedge (Erceg et al., 2000):

\[ \hat{\pi}_w = \kappa_w \times \chi_w(\hat{w}_t, \hat{\ell}_t, \hat{c}_t) + \beta \hat{\pi}_{w,t+1} \]

where \( 1 + \pi_t = \frac{w_t}{w_{t-1}}(1 + \pi_t) \) denotes wage inflation, \( \chi_w(\bullet) \) gives the deviation from the wage target, and \( \kappa_w \) is the slope of the wage-NKPC. I discuss the derivation of (6) for particular assumptions on union behavior and household preferences in Appendices B.2 and B.3.

Policy. The government flow budget constraint is

\[ \frac{1 + i_{b,t-1}}{1 + \pi_t} b_{t-1} + \tau_t = \tau w_t \ell_t + b_t \]

The policymaker sets nominal interest rates \( i_{b,t} \) and uniform lump-sum taxes and transfers \( \tau_t \) subject to the flow budget constraint (7) and the requirement that \( \lim_{t \to \infty} b_t = \bar{b} \).

The remainder of this paper studies the implications of constraints on this policy toolkit. I focus on two kinds of restrictions: transfer-only policies and interest rate-only policies.

Definition 1. A transfer-only policy is a policy that sets \( i_{b,t} = \bar{i}_b \) for all \( t \).

In a transfer-only policy, the policymaker is forced to keep nominal interest rates fixed, perhaps due to a binding effective lower bound. In light of the ELB’s empirical relevance, most discussion in this paper will center on the extent to which a restriction to transfer-only policies meaningfully constrains the policymaker. I note that, for such a policy, the direct mapping from policy instrument to consumer spending is fully governed by the matrix \( C_\tau \).
Definition 2. An interest rate-only policy is a policy that sets, for all \( t = 0, 1, \ldots \),

\[
\tau_t = \tau \ell w_t \ell_t + (1 - \frac{1 + i_{b,t-1}}{1 + \pi_t}) \bar{b}.
\] (8)

Definition 2 gives the natural opposite to a transfer-only policy: in an interest rate-only policy, the policymaker is free to adjust the path of nominal rates, but forced to passively adjust lump-sum transfers/taxes to balance the budget period-by-period. This policy thus operates by manipulating intertemporal prices, without any time variation in the total amount of government debt in the hands of households.\(^\text{10}\) For such a policy, the direct mapping from policy instrument to consumer spending is governed by the following map:

\[
\tilde{C}_{ib} \equiv C_{ib} - \bar{b} \times C_{\tau,\pi,-1},
\] (9)

Intuitively, \( \tilde{C}_{ib} \) combines the direct effect of the nominal rate change itself (\( C_{ib} \)) together with the implied movement in taxes required to balance the budget (second term).

**Equilibrium.** I am now in a position to define a perfect foresight transition equilibrium in this economy. As usual, I throughout restrict attention to equilibria in which all sequences of policies and macroeconomic aggregates are bounded (Woodford, 2011).

Definition 3. An equilibrium is a set of government policies \( \{i_{b,t}, \tau_t, b_t\}_{t=0}^\infty \) and a set of macroeconomic aggregates \( \{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty \) such that:

1. Consumption is consistent with the aggregate consumption function (2).
2. Wage inflation \( \{\pi^w_t\}_{t=0}^\infty \) and \( \{\ell_t, c_t, w_t\}_{t=0}^\infty \) are consistent with the wage-NKPC (6).
3. The paths \( \{\pi_t, w_t, \ell_t\}_{t=0}^\infty \) are consistent with the price-NKPC (5), and dividends are given as \( d_t = y_t - w_t \ell_t \).
4. The output market clears: \( y_t = c_t \) for all \( t \geq 0 \), the government budget constraint (7) holds at all \( t \), and \( \lim_{t \to \infty} b_t = \bar{b} \). The bond market then clears by Walras’ law.

An allocation of macroeconomic aggregates \( \{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty \) is said to be implementable if it can be supported as an equilibrium sequence.

\(^{10}\)More generally, an interest rate policy is a policy that freely sets nominal rates \( i_{b,t} \) and then ensures budget balance in net present value terms (i.e., \( \lim_{t \to \infty} \hat{b}_t = 0 \)) through some fixed tax-and-transfer adjustment rule. (8) is simply a particularly transparent example of such a financing rule.
Note that Definition 3 specifies policy simply as a path of policy instruments \( \{i_{b,t}, \tau_t, b_t\}_{t=0}^\infty \). As is well-known, policies of this sort generically do not induce unique equilibria (Sargent & Wallace, 1975). To address this challenge, I will later also discuss equilibria induced by policy rules for interest rates and transfers. I call an allocation \( \{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty \) uniquely implementable if it is the only equilibrium sequence consistent with those rules.

### 2.2 Detailed models of household consumption behavior

This section gives examples of several canonical models of household behavior that are consistent with the general aggregate consumption function (2). The purpose of this is threefold: first, to illustrate the generality of the model set-up in Section 2.1; second, to introduce the analytical and quantitative models that I will later study in Sections 3 and 4; and third, to already give some intuition for the shape and properties of the key matrix \( C_\tau \).

**Analytical model.** All analytical results in the remainder of the paper rely on a general yet tractable model of household consumption decisions, building very closely on Woodford (1990), Farhi & Werning (2019), and Angeletos et al. (2023). My presentation of the model will be brief, focusing mostly on what the model implies for the key derivative matrix \( C_\tau \). Supplementary details are provided in Appendix B.2.

There is a unit continuum of households, split into \( N \) distinct types with mass \( \mu_i \), where \( \sum_{i=1}^N \mu_i = 1 \). Households of each type \( i \) form a perpetual-youth block (e.g., as in Blanchard, 1985) with survival probability \( \theta_i \in [0,1] \). Preferences of a household \( j \) of type \( i \) are

\[
E_t \left[ \sum_{k=0}^\infty (\beta \theta_i)^k \left[ u(c_{ijt+k}) - v(\ell_{ijt+k}) \right] \right]
\]

where the expectation operator captures idiosyncratic mortality risk, and I will assume that preferences take the standard form \( u(c) = (c^{1-\gamma} - 1)/(1 - \gamma) \) and \( \nu(\ell) = \chi \ell^{1+\frac{1}{\phi}}/(1 + \frac{1}{\phi}) \). For each type \( i \) of household, the aggregate type budget constraint is exactly like (1), i.e.,

\[
c_{it} + b_{it} = \frac{\text{total income } e_{it}}{(1 - \tau_t)w_t \ell_t + \tau_t + d_t + \frac{1 + i_{b,t-1}}{1 + \pi_t} b_{it-1}}
\]

where the total income is

\[
e_{it} = \left\{ \begin{array}{ll}
\text{total income } e_{it} & \text{if } i_{b,t}\neq 0 \\
\end{array} \right.
\]

Cash-on-hand \( x_{it} \) is

\[
x_{it} = \left\{ \begin{array}{ll}
\text{cash-on-hand } x_{it} & \text{if } i_{b,t}\neq 0 \\
\end{array} \right.
\]

Note that all types receive the same income, but may differ in their consumption and wealth holdings. Within types, households trade in actuarially fair, risk-free nominal annuities, with
\[ \beta(1 + \bar{r}) = 1. \] Newborns furthermore receive a transfer that facilitates aggregation, exactly as in Angeletos et al. (2023). As argued in Woodford (1990) and Farhi & Werning (2019), this perpetual-youth set-up can be re-interpreted as a model of occasionally-binding borrowing constraints, with \( 1 - \theta_i \in [0, 1] \) giving the probability that the borrowing constraint will bind in any given time period. For \( N = 1 \), my model is identical to theirs, with all households subject to binding borrowing constraints with some common probability \( 1 - \theta \). For \( N > 1 \), borrowing constraints can in principle bind differentially often for different types.

Following the derivations of Angeletos et al., it can be shown that total consumption of households of type \( i \) satisfies the following linearized demand function:

\[
\tilde{c}_{it} = \left(1 - \frac{\theta_i}{1 + \bar{r}}\right) \cdot \left\{\tilde{x}_{it} + \sum_{k=1}^{\infty} \left(\frac{\theta_i}{1 + \bar{r}}\right)^k \tilde{c}_{it+k}\right\} - \sigma_i \sum_{k=0}^{\infty} \left(\frac{\theta_i}{1 + \bar{r}}\right)^k \left(\tilde{\mu}_{it+k} - \tilde{\mu}_{t+k+1}\right) \tag{12}
\]

where \( \sigma_i \) is a function of primitives. For the purposes of this paper, what matters most is the first part of (12)—the mapping from current and expected future income, including in particular transfer income, to consumer demand. Consistent with the borrowing-constraint interpretation of Farhi & Werning, we see that \( \theta_i < 1 \)—i.e., occasionally-binding constraints—has two important effects: first, the contemporaneous MPC \( 1 - \theta_i/(1 + \bar{r}) \) is elevated; and second, future income is subject to additional discounting relative to the steady-state rate of interest. These two features of (12) will loom large throughout the paper.

Combining the demand relation (12) with the budget constraint (11) yields a type-specific consumption function \( C^i(\bullet) \)—i.e., a mapping from perfect-foresight sequences of equilibrium aggregates \( \{w, \ell, \pi, d\} \) and policy variables \( \{\tau, i\} \) into sequences of type-specific consumption \( c_i \). Aggregating across types, we obtain the aggregate consumption function \( C(\bullet) \) in (2), and in particular

\[
C_q \equiv \sum_{i=1}^{N} \mu_i \cdot C^i_q, \quad q \in \{w, \ell, \pi, d, \tau, i\}. \tag{13}
\]

The main results in Sections 3 and 4 of this paper concern \( C_\tau \). The remainder of this section thus characterizes the type-specific matrix \( C^i_\tau \) under different assumptions on \( \theta_i \).

1. **Permanent-income consumer:** \( \theta_i = 1 \). The familiar case of \( \theta_i = 1 \) corresponds to standard permanent-income consumers, with borrowing constraints never binding. We see from (12) that, in this case, the MPC is simply equal to \( \frac{\bar{r}}{1 + \bar{r}} \)—in response to any lump-sum income gain, households increase their consumption by the annuity value of that gain at each date. This is true both for contemporaneous income gains as well as for future income gains.
gains, with those future income gains discounted at the steady-state rate of interest, but not any further. Combining (12) with (11) we arrive at the following expression for $C^R_\tau$ in the permanent-income case, denoted $C^R_\tau$ for future reference:

$$C^R_\tau = \begin{pmatrix} \frac{r}{1+\bar{r}} & \frac{r}{(1+\bar{r})^2} & \frac{r}{(1+\bar{r})^3} & \cdots \\ \frac{r}{1+\bar{r}} & \frac{r}{(1+\bar{r})^2} & \frac{r}{(1+\bar{r})^3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$  

(14)

We see that $C^R_\tau$ is rank-1, and in particular that any sequence of transfers with zero net present value does not affect household consumption at all—i.e., the familiar Ricardian equivalence result (Barro, 1974).

2. **Hand-to-mouth consumer:** $\theta_i = 0$. The case of $\theta_i = 0$ corresponds to perpetually-binding borrowing constraints—households immediately and fully consume any lump-sum income gain, i.e., they are hand-to-mouth. It is straightforward to see that $C_\tau$, for future reference here denoted as $C^H_\tau$, is given as

$$C^H_\tau = I$$  

(15)

where $I$ is the identity matrix. Any sequence of taxes and transfers is thus passed through one-to-one into consumption; in particular, this means that even transfer sequences with zero net present value will affect consumer demand.

3. **Occasionally-binding constraints:** $\theta_i \in (0, 1)$. The intermediate case $\theta_i \in (0, 1)$—which is the one studied, for example, in Farhi & Werning (2019) or Angeletos et al. (2023)—is one of occasionally-binding constraints, leading to elevated MPCs and partial discounting of future income. Appendix C.1 provides the full expression for $C_\tau$, denoted $C^{OLG}_\tau$. For the purposes of the discussion here it suffices to consider the following approximate expression, with the sense of the approximation made precise in Appendix C.1 (see Lemma C.1):

$$C^{OLG}_\tau \approx \begin{pmatrix} 1 & \frac{\theta_i}{1+\bar{r}} & \left(\frac{\theta_i}{1+\bar{r}}\right)^2 & \cdots \\ \theta_i & 1 & \frac{\theta_i}{1+\bar{r}} & \cdots \\ \theta_i^2 & \theta_i & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$  

(16)

As discussed above, if $\theta_i < 1$, then MPCs are elevated. The second term in (16) tells us
about the implied *intertemporal* consumer spending profile. First, because of current and anticipated future binding borrowing constraints, households frontload spending, decaying at rate $\theta_i$ (i.e., entries below the main diagonal of $C_{OLG}$). Second, borrowing constraints binding between now and the future also means that future income receipt only partially affects spending today, leading to anticipation effects (i.e., entries above the main diagonal of $C_{OLG}$) that similarly decay, now at rate $\theta_i/(1+\bar{r})$. It follows that even transfer sequences with zero present value affect consumer spending, unlike the permanent-income baseline; compared to the simple hand-to-mouth limit, however, the passthrough from transfers to demand is now not immediate (i.e., $C_{OLG} \neq I$), instead exhibiting the more complicated, geometrically decaying dynamics displayed in the far-right part of (16).

My general multi-type analytical model nests each of these canonical models as special cases, as well as arbitrary mixtures between them. The theoretical results in Section 3 will all apply to such general mixtures. Notable special examples include the spender-saver model of Campbell & Mankiw (1989) and the OLG-spender hybrid of Angeletos et al. (2023). The latter will in particular be important in my quantitative analysis of $C_\tau$ in Section 4.\footnote{My analytical results extend with rather little change to the most popular alternative model of tractable, non-Ricardian consumer behavior: bond-in-utility models, as considered for example in Michaillat & Saez (2018). I provide a discussion of such models in Appendices B.4 and C.1.}

A QUANTITATIVE “HANK”-TYPE MODEL. The set-up of Section 2.1 is similarly consistent with quantitative heterogeneous-agent models of the consumption-savings problem. In a canonical “HANK” model, a unit continuum of households $i \in [0, 1]$ has preferences

$$
E_t \left[ \sum_{k=0}^{\infty} \beta^k [u(c_{it+k}) - v(\ell_{it+k})] \right] 
$$

where expectations are now taken over idiosyncratic household productivity $\varepsilon_{it}$, with $\int_0^1 \varepsilon_{it} \, di = 1$ for all $t$. The individual household budget constraint is

$$
c_{it} + b_{it} = (1 - \tau_t) w_t \varepsilon_t \ell_t + \tau_t + d_t + \frac{1 + \bar{b}_{it-1}}{1 + \pi_t} b_{it-1}, \quad b_{it} \geq b
$$

where household borrowing is now subject to an-hoc (tight) borrowing constraint $b$.

The household consumption-savings decision is to choose sequences of consumption and savings, $\{c_i, b_i\}$ to maximize (17) subject to (18). Aggregating across all households, we yet
again obtain an aggregate consumption function (2). This consumption function—and in particular the derivative matrix $C_\tau$—now does not admit any closed-form characterization. However, these objects can be recovered straightforwardly using the computational methods developed by Boppart et al. (2018) and Auclert et al. (2019). The numerical explorations in Section 4 will study the properties and shape of $C_\tau$ in such models.

3 Interest rate cuts vs. stimulus checks

This section presents my core theoretical results on how stimulus checks can replicate nominal interest rate policy. I proceed in three steps. First, in Section 3.1, I present a general sufficient condition for policy equivalence. This general condition only uses the aggregate consumption function of Section 2.1. Second, in Section 3.2, I show that, in my analytical model, stimulus checks can \textit{generically} replicate arbitrary interest rate policy, in the sense that borrowing constraints only need to bind some of the time.

My analysis in Sections 3.1 and 3.2 is couched in terms of linearized perfect-foresight transition sequences. Section 3.3, building on discussions in Auclert et al. (2019) and McKay & Wolf (2022b), translates these conclusions to equivalence in terms of policy rules in models with aggregate risk, solved using standard first-order perturbation techniques.

3.1 A sufficient condition for aggregate policy equivalence

I begin with a preliminary definition: a high-level property of the consumption derivative map $C_\tau$ that I refer to as \textit{strong Ricardian non-equivalence}. This property will turn out to be a general sufficient condition for my core equivalence result.

\textbf{Definition 4.} A consumption function $C(\bullet)$ exhibits strong Ricardian non-equivalence if the linear map $C_\tau$ is invertible. I denote its inverse by $C_\tau^{-1}$.

Under the Barro (1974) definition of Ricardian equivalence, the time path of (lump-sum) taxes used to finance any given fiscal expenditure is completely irrelevant for consumption—only the present value matters. We already saw this in the expression for $C_\tau$ in the permanent-income model (see (14)): there $C_\tau$ is rank-1, and so in particular it is not invertible. With non-Ricardian households, on the other hand, the timing of transfers also begins to matter, thus increasing the rank of $C_\tau$; strong Ricardian non-equivalence corresponds to the limit case of invertibility. Section 3.2 will establish that perpetual-youth models satisfy this property as soon as $\theta < 1$—i.e., as soon as borrowing constraints bind at least occasionally.
The equivalence result. I am now in a position to state the policy equivalence result.

**Proposition 1.** Consider the model of Section 2.1, and let $\hat{c}$ be a path of household consumption with zero net present value, i.e., $\sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \hat{c}_t = 0$. Suppose that

$$
\hat{c} \in \text{image}(C_\tau) \iff \hat{c} \in \text{image}(\tilde{C}_i)
$$

(19)

Then $\tau$ and $i_b$ are macro-equivalent: any aggregate allocation that is implementable with interest rate-only policy is also implementable with transfer-only policy, and vice-versa.

An easy-to-interpret condition ensuring the direction “$\Leftarrow$” in (19) is strong Ricardian non-equivalence: if $C_\tau$ is invertible, then the space of aggregate allocations implementable using transfer-only policies is at least as large that implementable using rate-only policies.

**Proof sketch.** Key to the proof of Proposition 1 is the insight that both interest rate as well as transfer policies only directly perturb the model’s equilibrium conditions in two places: first, the left-hand side of the output market-clearing condition $C(\bullet) = y(\ell)$; and second, the sequence of government budget constraints (7).

In partial equilibrium—i.e., prior to general equilibrium price and quantity adjustments—a feasible monetary policy is simply a budget-neutral perturbation of relative intertemporal prices, inducing a path of net excess consumption demand

$$
\hat{c}^{PE}_{i_b} \equiv \tilde{C}_{i_b} \times \hat{i}_b
$$

Note that, since my definition of an interest rate policy includes its financing, this demand path necessarily has zero net present value. By (19), we can find some transfer sequence $\hat{\tau}(\hat{i}_b)$ that induces the exact same perturbation of net excess demand. Since the initial monetary policy was consistent with fiscal budget balance, and so since $\hat{c}^{PE}_{i_b}$ has zero net present value, it then follows from the household budget constraint (1) that the equivalent transfer also necessarily has zero net present value,

$$
\sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \hat{c}^{PE}_{i_b} = \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \hat{\tau}(\hat{i}_b) = 0
$$

Thus, prior to any general equilibrium feedback, $\hat{\tau}(\hat{i}_b)$ is also consistent with budget balance, in the sense that the induced debt path $\hat{b}$ satisfies $\lim_{t \to \infty} \hat{b}_t = 0$. Finally note that the argument also works in reverse: any transfer-only policy that is consistent with the government
budget constraint (7) (i.e., \( \lim_{t \to \infty} \delta_t = 0 \)) necessarily induces a perturbation of consumption demand with zero net present value, and so by (19) we can always find an equivalent interest rate-only policy. To summarize, this first step leverages the fact that both policies equally flexibly manipulate the same lever: the consumption-savings decision of households.

Given that the transfer and nominal interest rate policies are both budget-feasible and both perturb the output market-clearing condition by exactly the same amounts time period by time period, it then follows via the implicit function theorem that they must also induce the same general equilibrium paths of inflation, hours worked, wages, and dividends. The intuition is simple: if, for example, the excess demand path induced by some monetary policy is accommodated in general equilibrium through increases in inflation and hours worked, then the same inflation and hours worked paths are also consistent with agent optimality and market-clearing after the equivalent transfer policy. This proof strategy is a simple application of the Arrow & Debreu (1954) approach to general equilibrium characterization using state-by-state (here \( t \)-by-\( t \)) net excess demand functions. Finally, the proof also reveals that the two policies are revenue-equivalent in general equilibrium: e.g., if increases in economic activity and inflation lead to an additional budget surplus after a rate cut, then the exact same budget surplus also opens up after the equivalent transfer stimulus.

**Discussion of Assumptions.** The first step of my argument relied on two assumptions: (i) the existence of an aggregate consumption function \( C(\bullet) \), and (ii) the restriction that both policies operate only by manipulating that function. Many macroeconomic models induce such consumption functions—e.g., ranging from the models reviewed in Section 2.2 to those with behavioral biases (like Laibson et al., 2020; Lian, 2021)—, and so strong Ricardian non-equivalence is a useful, quite widely applicable sufficient condition. By (ii), however, my conclusions do not immediately extend to models in which monetary policy acts through multiple levers, e.g., firm investment. I discuss such extensions in Section 6.

The second (general equilibrium) step of my proof requires that two policies with identical effects on partial equilibrium net excess consumer demand are accommodated in the same way in general equilibrium. The shape of the wage-NKPC (6) plays an important role in ensuring that this is indeed the case. While this shape is entirely standard for representative-agent models and for those in particular nests the limiting special case of flexible labor supply, it requires additional assumptions in the case of cross-sectional household heterogeneity (e.g., as in quantitative HANK models). I will return to this point in Section 5.1.
3.2 Strong Ricardian non-equivalence in analytical models

I now investigate whether the high-level condition stated in Proposition 1 holds in particular models of household consumption behavior. The key takeaway will be that, in the analytical model of occasionally-binding borrowing constraints of Section 2.2, uniform lump-sum taxes and transfers can generically be used to replicate arbitrary interest rate policy, in the precise sense that borrowing constraints only need to bind some of the time.

\[ C_\tau \text{ when borrowing constraints bind.} \]

I begin by studying \( C_\tau \) for an individual type \( i \). My first result is that strong Ricardian non-equivalence holds—that is, \( C_\tau \) is invertible—if and only if \( \theta_i < 1 \), i.e., type \( i \) is subject to occasionally-binding borrowing constraints. The proof is constructive, involving a direct characterization \((C_\tau^i)^{-1}\).

**Lemma 1.** Consider the analytical model of occasionally-binding borrowing constraints of Section 2.2. If \( \theta_i < 1 \), then the transfer derivative matrix \( C_\tau^i \) satisfies strong Ricardian non-equivalence, with its inverse given as

\[
(C_\tau^i)^{-1} = \frac{1}{1 - \frac{\theta_i}{1 + \bar{r}}} \begin{pmatrix}
1 - \theta_i (1 - \frac{\theta_i}{1 + \bar{r}}) & -\frac{1}{1 + \bar{r}} & \theta_i & 0 & \cdots \\
-\frac{\theta_i}{1 - \theta_i} & \frac{1 + \bar{r}}{1 - \theta_i} & -\frac{1}{1 + \bar{r}} & \theta_i & \cdots \\
0 & -\frac{\theta_i}{1 - \theta_i} & \frac{1 + \bar{r}}{1 - \theta_i} & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots
\end{pmatrix}
\]  

(20)

Note that the expression in (20) is exact, and not approximate as in (16)—characterizing \((C_\tau^i)^{-1}\) is actually easier than characterizing \( C_\tau^i \) itself. The particular shape of \((C_\tau^i)^{-1}\) is also intuitive. First, as \( \theta_i \to 1 \), we see that its entries diverge, reflecting the lack of invertibility in the permanent-income limit. Second, the constant probability of binding borrowing constraints—which led to the simple geometric shape of intertemporal spending displayed in (16)—means that the inverse also has a simple shape. Intuitively, to engineer one dollar of additional spending at some date \( t \), a policymaker would need to give money to households at date \( t \) and then take money away at dates \( t - 1 \) and \( t + 1 \), exactly offsetting the leakage of date-\( t \) income to demand in adjacent periods.\(^{12}\) Finally, if \( \theta_i = 0 \) (i.e., hand-to-mouth spending), then there is no cross-period leakage, so we just have \((C_\tau^i)^{-1} = I\), as expected.

Lemma 1 reveals that occasionally-binding borrowing constraints suffice to ensure that

\(^{12}\)As I show in Appendix C.1, \( C_\tau^{-1} \) has (almost) the exact same shape in bond-in-utility models.
(bounded) sequences of taxes and transfers can be used to induce arbitrarily complicated (bounded) paths of consumer demand. It thus follows that the popular models of occasionally-binding constraints considered in previous work—e.g., Farhi & Werning (2019) and Angeletos et al. (2023), which correspond to a single-type, $N = 1$ case of my general model—indeed satisfy policy equivalence. I now go one step further and return to the general model.

**Proposition 2.** Consider the analytical model of occasionally-binding borrowing constraints of Section 2.2, and let $\tilde{c}$ be a square-summable path of consumption with zero net present value. Then, if $\theta_i < 1$ for all $i$, and if $\bar{r}$ is sufficiently close to 0,

$$\tilde{c} \in \text{image}(C_\tau),$$

and any aggregate allocation that is implementable with interest rate-only policy is also implementable with transfer-only policy.

The proof of Proposition 2 reveals that, for uniform transfer policy to be able to engineer any possible (square-summable, zero net present value) path of aggregate demand, it suffices if borrowing constraints bind some of the time for some households—i.e., that $\theta_i < 1$ for some $i$ with $\mu_i > 0$. The restriction that $\theta_i$ is (at least marginally) below 1 for all types $i$ is then just added for technical reasons, to ensure that the monetary policy-induced net excess demand path $\tilde{c}_n^{PE}$ is in fact also square-summable.\(^{13}\) Intuitively, as soon as there is at least one non-Ricardian type, consumer spending at date $t$ becomes anchored to consumer income at date $t$, moving $C_\tau$ away from the flat permanent-income shape and towards an identity matrix. As a result, the policymaker has all the leeway needed to flexibly manipulate excess demand through transfers, as required for my equivalence results.

**Connection to empirical evidence.** Proposition 2 has established that the high-level condition underlying Proposition 1 is a generic feature of a particular class of popular models. It turns out to also be empirically relevant. To show this I now map the conditions underlying Proposition 2 into empirically testable properties of the matrix $C_\tau$.

**Proposition 3.** Consider the analytical model of occasionally-binding borrowing constraints of Section 2.2. Then the condition in (21) is equivalent to either of the following two (interchangeable) properties of $C_\tau$:

\(^{13}\)In the presence of permanent-income consumers, the net excess demand path induced by a given target monetary policy is generally not square-summable, by the usual unit-root property. This limitation, however, is of only limited practical relevance, as even a marginal deviation from permanent-income behavior (i.e., $\theta_i < 1$) breaks the unit-root property and restores square-summability.
1. The average contemporaneous MPC is strictly above the permanent-income level, i.e.,

\[ C_r(1, 1) > \frac{\bar{r}}{1 + \bar{r}} \]  \hspace{1cm} (22)

2. The average consumer spending response to a lump-sum income gain is front-loaded, i.e.,

\[ C_r(1, 1) > C_r(2, 1) \]  \hspace{1cm} (23)

The two conditions in Proposition 3 capture the same intuition. In the permanent-income limit, the impact MPC \( C_r(1, 1) \) is equal to the annuity value of the income gain, and the dynamic MPC profile is flat. Binding borrowing constraints elevate the impact MPC beyond the annuity value—condition (22)—and thus at the same time front-load overall consumption demand—condition (23). Importantly, both of these conditions are testable using evidence on how consumers spend lump-sum income gains (e.g., see Parker et al., 2013; Fagereng et al., 2018; Borusyak et al., 2021). A consistent finding here is that impact MPCs are elevated and that dynamic spending profiles are front-loaded and peak at income receipt. The policy equivalence result is thus not just theoretically general but also empirically relevant.

**Extensions and Limitations.** In Appendix C.1 I extend the analysis to an even richer consumption function. Relative to (12), this alternative specification of consumer demand is more general in that it disentangles the impact MPC from the dynamic discounting factor \( \theta \). This extension is economically interesting for at least two reasons.

First, it nests the case \( \beta(1 + \bar{r}) \neq 1 \).\(^{14}\) If \( \beta(1 + \bar{r}) < 1 \) (e.g., as is the case in models with precautionary savings), then relative household impatience increases MPCs and front-loads consumer spending even more, and so my conclusions about invertibility of \( C_r \) are entirely unaffected. If instead \( \beta(1 + \bar{r}) > 1 \), then invertibility of \( C_r \) requires that borrowing constraints are binding enough; e.g., with log preferences, the condition becomes

\[ \theta_i \leq \frac{1}{\beta(1 + \bar{r})} < 1 \]  \hspace{1cm} (24)

Intuitively, \( \beta(1 + \bar{r}) > 1 \) leads households to back-load their spending. This is problematic for my invertibility requirement, for the following reason. Suppose a policymaker wishes to

\(^{14}\)With \( \beta(1 + \bar{r}) \neq 1 \) the analytical model need not have a well-defined steady-state level of consumption. I thus simply consider a first-order approximation to the partial equilibrium consumption-savings problem, characterize the implied \( C_r \), and ask when it is invertible (see Farhi & Werning, 2019).
increase spending by $1 today. If consumers postpone their spending, then the transfer that suitably increases demand today at the same time increases demand by even more tomorrow, thus necessitating further—and larger—transfer movements tomorrow. In Appendix C.1 I discuss a specific example in which this iteration diverges, breaking invertibility even though variations in the timing of transfers do invariably impact spending. Borrowing constraints binding often enough—as ensured by the bound on $θ_i$ in (24)—counteracts this effect, front-loads spending, and thus returns us to invertibility. Importantly, (24) is implied by either of the two testable conditions in Proposition 3, thus further reinforcing the practical relevance of the equivalence result.

3.3 From perfect foresight transitions to policy rules

The equivalence result in Proposition 1 was phrased in terms of perfect foresight transition paths, or equivalently in terms of impulse responses to policy shocks in a linearized model with aggregate risk. By the results in McKay & Wolf (2022b), such equivalence in terms of responses to policy shocks also implies equivalence in terms of policy rules: in response to any set of non-policy shocks, the aggregate outcomes implied by any given nominal interest rate rule can equivalently be implemented using a transfer-only rule. This subsection briefly elaborates on this observation, with further details provided in Appendix D.

A sketch of policy rule equivalence. Consider an extended version of the baseline model that features a rich menu of non-policy shocks—wedges to the aggregate consumption function (2) as well as the two Phillips curves (5) - (6), corresponding to simple reduced-form representations of canonical demand and supply shocks. Given such a menu of shocks, I now ask whether the space of aggregate allocations that the policymaker can implement through commitment to policy rules is affected by constraints on nominal rates $i_{b,t}$ (e.g., a binding ELB). Under the conditions of Proposition 1, the answer turns out to be “no”.

The core intuition for this policy rule equivalence result is straightforward and follows from McKay & Wolf (2022b). In a general linearized system, a policy rule is nothing but a mapping from lagged, current, and future expected macroeconomic outcomes to current and expected future values of the policy instrument. Under the conditions of Proposition 1, for every path of current and expected nominal interest rates, we can find a path of current and expected future values of the policy instrument that the policymaker can implement through commitment to policy rules.

Formally, in this example, the linear map $C_τ$ is injective—variations in the timing of transfers affect consumption—but not surjective—there exist bounded paths of consumption that cannot be induced through bounded paths of transfers. I thank an anonymous referee for suggesting the insightful counterexample.
expected future transfers that perturb all equilibrium conditions by the same amount, thus ensuring that current and expected future values of all macroeconomic outcomes—including in particular output and inflation—are identical. For a concrete, canonical example, consider the “forecast targeting rule” of a textbook dual-mandate policymaker (Woodford, 2011),

$$\hat{\pi}_t + \lambda(\hat{y}_t - \hat{y}_{t-1}) = 0, \quad t = 0, 1, 2, \ldots$$

(25)

where \(\lambda\) is a function of policymaker preferences and model primitives. Under the conditions of Proposition 1, if the rule (25) induces a unique equilibrium when implemented through an interest rate-only policy, then (25) implemented through a transfer-only policy also induces a unique equilibrium, featuring the exact same aggregate allocation.\(^{16}\) Appendix D substantiates the claims made here; in particular, I there show how transfers can replicate a general set of interest rate-only rules, including both implicit rules—like the forecast targeting criterion (25)—and explicit rules—like Taylor-type rules.

I note that my arguments on mapping one policy rule into an equivalent alternative policy rule sensitively rely on the invariance of the aggregate consumption function—including in particular \(C_r\)—with respect to changes in policy rules. This model property is an immediate implication of linearity: I study the response of the economy to “small” incremental business-cycle shocks, linearizing around a deterministic steady state that is not affected by the change from one policy rule to the other. Naturally, focusing on such linearized dynamics could be misleading if my policy equivalence construction maps a given (moderately sized) nominal interest rate path into very large tax-and-transfer movements—i.e., policy changes that will plausibly move households far away from or meaningfully towards binding borrowing constraints, and are thus likely to materially change \(C_r\). This could happen if households are almost Ricardian: for example, if the economy is populated by just one consumer type, and if that consumer type has \(\theta\) below but close to 1, then \(C_r^{-1}\) takes the form displayed in (20), with its entries diverging as \(\theta \rightarrow 1\). As a result, to move consumer demand by a given target

\(^{16}\)I picked (25) as my example because of its clear practical relevance. Notably, Bernanke (2015) summarizes the salience of the implicit targeting perspective embedded in (25) for Federal Reserve policymaking:

“The Fed has a rule. The Fed’s rule is that we will go for a 2 percent inflation rate. We will go for the natural rate of unemployment. We put equal weight on those two things. We will give you information about our projections about our interest rates. That is a rule and that is a framework that should clarify exactly what the Fed is doing.”

My main result simply states that, under my assumptions, this rule can equivalently be implemented using a different policy instrument, giving exactly the same equilibrium outcomes of inflation and output.
amount, impractically large changes in lump-sum transfers would be required, invalidating my linear approximations. I will return to this important point in Section 4.

Taking stock. The analysis in this section has argued that my policy equivalence results are quite general: they hold under economically weak and empirically relevant assumptions, and they suffice for equivalence in the strong sense of delivering identical aggregate outcomes for an arbitrary menu of shocks hitting the economy.

Where the analysis in this section has fallen short, however, is on characterization of the equivalent stimulus check policies. In particular, while existence of the inverse $C^{-1}$ may be general, a similarly important piece of information for a policymaker is the shape of that inverse—only with this knowledge can she figure out the particular policy that provides the desired macroeconomic stabilization. Section 4 will provide that characterization.

4 Quantitative analysis and policy characterization

The objective of this section is to provide a simple yet empirically relevant characterization of $C^{-1}$—the potentially complicated infinite-dimensional object that governs the mapping from any possible shortfall in consumer spending to the transfer policy that would offset it. To this end I study the properties of $C^{-1}$ in a quantitative HANK model.

Overall, this section establishes three main results. First, I verify numerically that $C$ in my HANK model is indeed invertible, consistent with the theoretical discussion surrounding Proposition 2. Second, I show that $C^{-1}$ takes a simple form, being robustly well-characterized by a small number of measurable sufficient statistics. Both results hold across a range of model parameterizations. Third, for empirically relevant values of the sufficient statistics, I find that moderately sized transfers already suffice for meaningful aggregate stabilization.

4.1 Heterogeneous-household model

My analysis relies on a relatively standard one-asset HANK model. This section presents the parameterization and discusses the model-implied consumption behavior.

Parameterization. I consider a calibrated version of the model of Section 2.1, with the aggregate consumption function coming from the heterogeneous-household consumption-savings problem described in Section 2.2. I only present a very brief overview of the (standard) parameterization here, and relegate further details to Appendix B.3.
Households face the same income process as in Kaplan et al. (2018), and can self-insure by saving, but not borrowing. I calibrate total liquid bond holdings to the amount of liquid wealth in the U.S. economy; corporate wealth, instead, is perfectly illiquid, with households receiving dividend payments as a function of their labor productivity (so that \( d_i \) is actually \( i \)-specific). For my robustness exercises, I consider alternative—less empirically relevant—parameterizations with materially higher or lower wealth holdings, thus delivering materially lower or higher average marginal propensities to consume, respectively.

The economy is closed with a constant-returns-to-scale production function as well as conventional degrees of nominal wage and price stickiness. This general equilibrium closure is irrelevant for all of the main results of this section (which only concern partial equilibrium consumer behavior, i.e. \( C_\tau \)); rather, the model closure will only start to matter once I report general equilibrium experiments (as I do at the end of Section 4.2 as well as in Section 5).

**Properties of \( C_\tau \).** In an important contribution, Auclert et al. (2018) establish that the first column of \( C_\tau \) in HANK models—i.e., the response of consumption demand over time to a lump-sum income gain today—is consistent with empirical evidence on consumer spending behavior (e.g., see Parker et al., 2013; Fagereng et al., 2018). They furthermore show that, in both model and data, this first column actually has a rather simple shape: the impact MPC—say, within quarter of income receipt—is elevated; the MPC in the following period is then meaningfully smaller; and finally, from thereon out, intertemporal MPCs approximately decline at a constant, geometric rate.\(^{17}\) This second observation suggests that the first column of \( C_\tau \) in quantitatively relevant HANK-type models may be parameterized with three “sufficient statistics”: the impact MPC \( \omega \); the dynamic rate of decay \( \theta \) from horizon \( t = 2 \) onwards; and finally the interest rate \( \bar{r} \) which—via the fact that discounted lifetime MPCs necessarily add up to 1—delivers the delayed date-1 MPC. The sufficient statistics formula developed in the next section shows that these three numbers in fact suffice to provide an accurate characterization of the entirety of \( C_\tau \) in HANK, and thus also of \( C_\tau^{-1} \).

### 4.2 A three-coefficient sufficient statistics approximation

The analysis in this section proceeds in three steps. First, I propose a simple three-parameter sufficient statistics formula for \( C_\tau^{-1} \), leveraging the analytical models of Section 3.2. Second,

\(^{17}\)Empirical evidence and some quantitative HANK modeling suggest that this rate of decay of intertemporal MPCs may slow down at longer horizons, in the tail. I investigate this point in Appendix C.3 and find that those tail MPCs are essentially irrelevant for my characterization of \( C_\tau^{-1} \).
I document the accuracy of that formula in my quantitative HANK model. Third, I discuss some practical policy implications.

The sufficient statistics formula. The proposed sufficient statistics formula leverages a special case of the general analytical model of Section 2.2—a simple two-type version that is rich enough to agree with both empirical evidence and quantitative HANK models on the shape of the first column of $C_\tau$. Specifically, I consider a model populated by perpetual-youth consumers (i.e., with $\theta \in (0, 1)$) together with a residual margin $\mu$ of spenders. By the discussion in Section 2.2 and Appendix C.1, the first column of $C_\tau$ in such a simple mixture model satisfies

$$
C_\tau(\bullet, 1) = \mu \cdot \left( 1 - \frac{\theta}{1 + \bar{r}} \right) \cdot \begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \vdots \\ \theta^n \\ \vdots \end{pmatrix} + (1 - \mu) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}
$$

(26)

We see from (26) that the two-type hybrid model is rich enough to match arbitrary values of the three statistics $\{\omega, \theta, \bar{r}\}$—$\theta$ and $\bar{r}$ are given directly, and $\mu$ is then recovered residually to match the impact MPC $\omega$. Importantly, however, in my hybrid model, these three statistics then characterize the entirety of $C_\tau \equiv (1 - \mu) \cdot C^{OLG}_\tau(\theta, \bar{r}) + \mu \cdot I$ and thus also $C^{-1}_\tau$. This is my proposed three-parameter sufficient statistics approximation $C_\tau(\omega, \theta, \bar{r})^{-1}$.

The inverse $C_\tau(\omega, \theta, \bar{r})^{-1}$ has a simple and intuitive shape, displayed as the orange dashed lines in Figure 1. Recall from the discussion in Section 3.2 that, for a single perpetual-youth consumer type, $C^{-1}_\tau$ is tridiagonal—to engineer a dollar of excess demand at some arbitrary date $t$, it suffices to hand out a transfer at date $t$ and then reduce transfers at the adjacent dates $t - 1$ and $t + 1$, to prevent any cross-period leakage. In the two-type model underlying my sufficient statistics formula the shape of $C_\tau(\omega, \theta, \bar{r})^{-1}$ is similar: it has peaks along the main diagonal, just now the off-diagonal entries decay to zero more gradually, simply because the cross-period spending leakage takes a more complicated (i.e., non-geometric) form.

Approximation accuracy. The main result of this section is that the proposed three-dimensional approximation $C_\tau(\omega, \theta, \bar{r})^{-1}$ actually provides a quite accurate description of $C^{-1}_\tau$ in quantitative HANK-type models. This conclusion closely echoes the findings of Auclert et al. (2018), who show that a mixture of spender-saver and bond-in-utility models can match
Figure 1: Entries of $C^{-1}_\tau$ in the quantitative heterogeneous-agent model (shades of grey) and in the sufficient statistics approximation $C_\tau(\omega, \theta, \bar{r})^{-1}$ (shades of orange, dashed). Here $\bar{r}$ is set as in the heterogeneous-agent model, and $\{\omega, \theta\}$ are set to match $C_\tau(1, 1)$ and $C_\tau(2, 1)$. The lines correspond to columns $\{1, 6, 11, 16\}$, with lighter shades indicating farther-out columns.

HANK reasonably well on the entirety of $C_\tau$.\(^{18}\) While the analysis in this section will only look at the preferred calibration of my HANK model, I in Appendix C.3 document the robustness of my conclusions by also looking at alternative, materially different parameterizations.

Figure 1 begins by displaying several individual columns of $C^{-1}_\tau$ taken from both (i) the full heterogeneous-agent model (shades of grey) and (ii) the sufficient statistics approximation (orange dashed), with $\{\omega, \theta, \bar{r}\}$ set to agree with the heterogeneous-agent model: $\omega = 0.30$, $\theta = 0.82$, and $\bar{r} = 0.01$—all values that are broadly consistent with the empirical evidence.\(^{19}\) The key takeaway from the figure is that the orange lines are throughout quite close to the grey ones.\(^{20}\) The intuition is that, in the HANK model, some households are up against the

\(^{18}\)Relative to Auclert et al., the further value-added of my analysis here is to provide an explicit mapping from $\{\omega, \theta, \bar{r}\}$ into $C^{-1}_\tau$, and to discuss how these three statistics affect the shape of $C^{-1}_\tau$.

\(^{19}\)To be precise, I set $\bar{r}$ to its value in the heterogeneous-agent model, $\omega = C_\tau(1, 1)$, and finally $\theta$ is set to ensure that $C_\tau(\omega, \theta, \bar{r})(2, 1) = C_\tau(2, 1)$.

\(^{20}\)The figure reveals that there is some inaccuracy above the main diagonal of $C^{-1}_\tau$, reflecting the fact that anticipation effects in the hybrid model somewhat differ from those in HANK. In principle, a generalized four-parameter sufficient statistics formula could better capture these effects, as discussed in Appendix C.3. In practice, however, the simpler three-parameter formula already provides an excellent approximation, as in particular is visible in my policy applications in Figure 2.
borrowing constraint (so they act like spenders), while others will be up against it in the future with positive probability, so they act like perpetual-youth consumers with intermediate $\theta$. Importantly, three simple (and measurable!) parameters are already enough to capture these spending dynamics—and their implications for $C_{\tau}^{-1}$—quite well.

Next, Figure 2 shows several different “typical” target paths of net excess demand that a policymaker may wish to implement (grey) together with the sequences of transfers and taxes that do so (black and orange dashed). More precisely, given the three distinct paths of desired net excess demand $\hat{c}_{\tau}^{PE}$ (grey), the figure plots

$$\hat{\tau}(\hat{c}_{\tau}^{PE}) \equiv C_{\tau}^{-1} \times \hat{c}_{\tau}^{PE},$$

where $C_{\tau}^{-1}$ is either taken from the full quantitative HANK model (black) or from my simple sufficient statistics approximation (orange dashed). The three time paths of desired spending that I consider all have a peak of one per cent of steady-state consumption, but they are quite distinct in shape: short-lived in the left panel, more persistent in the middle panel, and hump-shaped in the right panel, capturing a range of policy-relevant scenarios. As expected in light of Figure 1, I find that, for all three paths, the actual required transfer sequence and the sufficient statistics prediction are close. I will return to the magnitudes of the required stimulus check policies when discussing practical policy implications.

Appendix C.3 repeats the exercises of Figures 1 and 2 for two possible alternative calibrations of the HANK model: one with much lower household wealth and thus higher MPCs,
and the other one with very high wealth and so low MPCs. The matrix $C_{\tau}^{-1}$ and so the time paths $\tau(c_{\tau}^{PE})$ materially differ across all of these models, but crucially my three-parameter sufficient statistics approximation remains accurate for all of them.

**Practical policy implications.** So far I have emphasized that my sufficient statistics formula yields an approximation $C_{\tau}(\omega, \theta, \bar{r})^{-1}$ that accurately reflects consumer behavior in HANK models, across a range of parameterizations. A second important point is that, for empirically relevant values of the “sufficient statistics” $\{\omega, \theta, \bar{r}\}$, the entries of this inverse are moderate in size. We can see this in Figure 1, with the diagonal entries of $C_{\tau}(\omega, \theta, \bar{r})^{-1}$ somewhat larger than $\omega^{-1}$, and the off-diagonal entries relatively small and quickly converging to zero away from the main diagonal. Given that empirical evidence suggests elevated average MPCs $\omega$, it follows that even moderately sized transfer stimulus suffices to close meaningful aggregate spending shortfalls. We also see this in Figure 2, which reveals that stimulus checks of the magnitudes observed in practice (e.g., around $600, as in 2008), are predicted to close aggregate demand shortfalls of around one per cent.\(^{21}\)

Figure 3 provides a final illustration by studying stabilization policy in response to a contractionary demand shock—that is, a shock that temporarily depresses partial equilibrium consumer spending, here with a peak effect of -1 per cent. By the classical divine coincidence

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\(^{21}\)Appendix C.3 elaborates further: there, I repeat the exercises in Figure 2 for a range of credible values of my sufficient statistics, and find that moderately sized checks throughout deliver meaningful stabilization.
logic, it is possible to perfectly stabilize inflation and output in the face of such a shock. To do so, policy needs to increase consumer spending to offset the 1 per cent contraction in demand (= $150 per household), shown in the middle (grey). The usual policy prescription would be to do so through a cut in rates (left panel, green); in my heterogeneous-agent model, this requires a relatively short-lived rate cut with a cumulative total of around 150 basis points.\footnote{This implied direct mapping from interest rates to consumer net excess demand is broadly consistent with recent empirical evidence (e.g., see Table 4 in Crawley & Kuchler, 2021).} The black lines indicate how to achieve the same stabilization instead using lump-sum tax-and-transfer policy alone: transfers initially go up (here by around $600), before then being financed through higher taxes down the line. Unsurprisingly (in light of Figures 1 and 2), this time path is again predicted almost perfectly by my sufficient statistics formula (orange, dashed). Finally, the right panel shows that the moderate stimulus check policy brings with it a moderate, transitory increase in government debt. Cyclical stabilization through stimulus check policy thus here requires no implausibly large or erratic fluctuations in taxes, deficits, or aggregate government debt. This is important: had meaningfully larger stimulus check policies been required, then the linearity assumption underlying my theoretical derivations in Section 3 would have been much less plausible.

Finally, while my experiments here throughout assumed uniform transfers, I emphasize that all results extend without any change if transfers are instead targeted at particular sub-populations of households, more similar to the more recent rounds of stimulus check policies observed in U.S. policy practice. A detailed discussion is provided in Appendix C.7.

## 5 The role of microeconomic heterogeneity

My analysis so far has been concerned exclusively with policy instrument equivalence at the aggregate level. Microeconomic heterogeneity played a role only to the extent that popular heterogeneous-agent consumption models are a natural (and empirically relevant) candidate to satisfy my sufficient condition of strong Ricardian non-equivalence.

This section sheds further light on the scope and the limitations of the policy equivalence result in the face of microeconomic heterogeneity. First, in Section 5.1, I gauge the extent to which microeconomic heterogeneity in labor supply decisions can break the equivalence result. I argue that, while this is possible in theory, it is unlikely to matter too much in practice. Second, in Section 5.2, I then note that equivalence at the aggregate level does not necessarily imply equivalence household-by-household.
5.1 Wealth effects and household labor supply

To understand how household heterogeneity and wealth effects in labor supply can in principle challenge my equivalence result, it will be useful to recall the proof sketch of Proposition 1. As the first step of the argument, I consider nominal interest rate and transfer policies that induce identical paths of spending. This however is in general not enough to ensure equivalence in general equilibrium—the two policies also need to induce identical responses of total household labor supply. In my environment, the specific wage-NKPC (6)—which importantly depends only on aggregate consumption $c_t$—ensures that interest rate and stimulus check policies with identical direct effects on consumer spending indeed also induce identical labor supply responses, as required.

I conclude from this discussion that it is not wealth effects in labor supply *per se* that threaten the equivalence result; rather, a potential challenge are *heterogeneous* wealth effects, which would allow two policies with identical effects on total spending to potentially lead to different responses of total labor supply. I here use two experiments to argue that the scope for such heterogeneity to materially threaten the equivalence result is likely to be limited, at least for stimulus check policies of the size considered in my applications.

**An alternative union bargaining protocol.** Some form of nominal wage rigidity is widely argued to be necessary to match business-cycle dynamics in general (Christiano et al., 2005; Smets & Wouters, 2007) and consumption responses to macro shocks in particular (Auclert et al., 2020; Broer et al., 2020); importantly, it is also consistent with microeconomic evidence (Grigsby et al., 2019). The derivation of my wage-NKPC (6) relies on one particular union bargaining protocol that only responds to changes in aggregate consumption, thus ensuring that my two candidate interest rate and stimulus check policies also lead to identical responses of labor supply. The exact same bargaining protocol has also been used in other recent contributions to the HANK literature (Auclert et al., 2021; Aggarwal et al., 2022; McKay & Wolf, 2022a). An alternative union bargaining protocol (used e.g. in Auclert et al., 2018) instead has the union respond to a weighted average of individual household marginal utilities (rather than marginal utility at the average, as in my protocol). Appendix C.4 extends my HANK model environment to such an alternative protocol.

The main takeaway from my analysis in Appendix C.4 is that the equivalence result continues to hold *almost exactly* even under the alternative protocol. Intuitively, equivalence is now not exact because the interest rate and transfer policies that induce identical responses of consumer spending will generically *not* induce identical changes in the weighted average
of consumer marginal utilities that enters the union problem. As a result, the adjustments in labor supply—$\tilde{\ell}_b^{PE}$ and $\tilde{\ell}_r^{PE}$—are not the same, and equivalence fails. However, marginal utility at the average and average marginal utility still co-move closely, so $\tilde{\ell}_b^{PE}$ and $\tilde{\ell}_r^{PE}$ remain quite similar, and aggregate outcomes are still nearly identical under the two policies.\footnote{Furthermore, even if the two policies were to induce quite heterogeneous wealth effects in labor supply, nominal wage rigidity of the degree usually assumed in quantitative work would mean that this heterogeneity would matter little in general equilibrium, at least for transitory policy shocks (Christiano, 2011).}

**Matching empirical evidence on labor supply responses.** Empirical evidence on household labor supply suggests that *marginal propensities to earn* (MPE)—that is, the response of earned income to a one-time, unexpected lump-sum transfer—are moderate in size, ranging from around 1% - 3% (Cesarini et al., 2017; Golosov et al., 2021), and somewhat increasing in household income, roughly doubling from the lowest to the highest income quartile (Golosov et al., 2021). Standard heterogeneous-household models with flexible labor supply struggle with both observations (Auclert et al., 2020): MPEs are predicted to be of the same order of magnitude as MPCs, and MPEs tend to be highest for high-MPC households. My solution is to adjust household preferences: I consider a hybrid of standard separable and Greenwood et al. (1988) GHH preferences (as originally proposed by Auclert et al., 2020) to match average MPEs, and then allow for preference heterogeneity across households to also match the cross-sectional MPE gradient. To make the heterogeneity particularly stark I consider a simple two-type model with low-MPC savers and high-MPC spenders, with their MPEs matched to the first and fourth quartiles of income reported in Golosov et al.. Full results are again reported in Appendix C.4.

In the model with empirically relevant and heterogeneous MPEs, I find that the policy equivalence result again holds almost exactly. The intuition is simply that heterogeneity in MPEs is small relative to the *level* of the average MPC. For example, with an MPC of 30% (an empirically relevant number for checks of the size studied in Section 4) and MPEs of 2% for spenders and 4% for savers (in line with Golosov et al.), the direct demand stimulus associated with either transfers or the equivalent rate cut is an order of magnitude larger than the difference in the labor supply response across households. Aggregate equilibrium dynamics are thus still dominated by the demand effects that are at the heart of Proposition 1.

**Takeaways & qualifiers.** The analysis in this section suggests that dispersion in wealth effects in labor supply is unlikely to materially affect the aggregate policy equivalence result.
However, it is important to acknowledge that this conclusion is sensitively tied to the fact that throughout I am looking at stimulus check policies that are relatively moderate in size and transitory. For larger stimulus checks the cross-sectional heterogeneity in labor supply responses would likely become larger relative to the size of the demand stimulus (i.e., bigger MPE relative to MPC), thus moving the economy further away from policy equivalence.

5.2 Non-equivalence at the household level

It is important to note that the equivalence result of this paper applies to macroeconomic aggregates, but does not necessarily hold household-by-household.

**Household-level consumption.** Consider the general heterogeneous-household environment described in Section 2.2. Consumption of an individual household $i$ is given as

$$c_i = C_i(w, \ell, \pi, d; \tau, i_b)$$

where the individual consumption function $C_i(\bullet)$ is the solution to individual $i$’s consumption-savings problem, indexed by that individual’s initial asset holdings and productivity. Now consider two macro-equivalent interest rate and stimulus check policies $\hat{i}_b$ and $\hat{\tau}(i_b)$, constructed as in the proof of Proposition 1, and let $\Delta_i(\hat{i}_b)$ denote the difference in household $i$’s consumption under the two policies; that is, let

$$\Delta_i(\hat{i}_b) \equiv \hat{c}_{i,\hat{i}_b} - \hat{c}_{i,\hat{\tau}(i_b)}$$

where $\hat{i}_b$ and $\hat{\tau}$ subscripts indicate transition paths corresponding to interest rate and transfer policy, respectively. Since by construction both policies induce the same general equilibrium price and quantity responses (notably $\{w, \ell, \pi, d\}$), we find that this difference satisfies

$$\Delta_i(\hat{i}_b) = \hat{C}_{i,\hat{i}_b} - \hat{C}_{i,\hat{\tau}(i_b)}$$

where $\hat{C}_{i,\hat{i}_b}$ and $\hat{C}_{i,\hat{\tau}}$ are defined like $\hat{C}_{i_b}$ and $\hat{C}_\tau$, just now for each $i$. In words, the two policies may result in differences in consumption household-by-household because of potentially differential direct effects on consumer spending. Intuitively, while the two policies by design induce the same total direct stimulus (i.e., $\hat{C}_{i_b}\hat{i}_b = \hat{C}_\tau\hat{\tau}(\hat{i}_b)$ and so $\int_0^1 \Delta_i(\hat{i}_b)di = 0$), they may do so by affecting consumption at different points in the cross-section of households (i.e., we can have $\hat{C}_{i,\hat{i}_b} \neq \hat{C}_{i,\hat{\tau}(i_b)}$ and so $\Delta_i(\hat{i}_b) \neq 0$ for individual $i$).
Distributional outcomes in quantitative HANK models. I provide a numerical illustration of this non-equivalence at the household level by returning to the quantitative HANK model of Section 4. In this environment I compute the evolution of consumption along the household wealth distribution in response to the macro-equivalent nominal interest rate and stimulus check policies displayed in Figure 3. I only briefly discuss the main results here, with all further details relegated to Appendix C.5.

The headline result is that, while macro-equivalent, interest rate and transfer stimulus policies can have quite materially different effects in the cross-section of households. As discussed above, both policies induce the same general equilibrium feedback effects, household-by-household. The direct effects, however, are heterogeneous. On the one hand, rate cuts work mostly by directly stimulating the consumption of the rich. This is not surprising: wealthy households substitute intertemporally, while poor households are close to their borrowing constraint and so do not. On the other hand, the equivalent lump-sum transfer policy mostly acts at the bottom: it relaxes borrowing constraints and so stimulates consumption of the poor, while rich households barely respond (since the policy has zero present value). Relative to a given rate cut, stimulus checks that deliver the same aggregate stabilization thus do so at strictly smaller cross-sectional consumption dispersion. The normative implications of this positive observation are explored in McKay & Wolf (2022a).

6 Extension to investment

As the final step in my argument, I show that the policy equivalence result extends straightforwardly to a richer environment with investment if stimulus checks are complemented by a second, similarly standard fiscal tool: bonus depreciation stimulus.

6.1 A brief sketch of the environment

I augment the model of Section 2.1 to allow for productive capital. The firm block in this extended environment closely follows the tradition of standard business-cycle modeling (e.g., Smets & Wouters, 2007; Justiniano et al., 2010). I provide a brief sketch of this familiar model here, and relegate further details to Appendix B.5.

Production. A unit continuum of identical, perfectly competitive firms \( j \in [0, 1] \) produces a homogeneous intermediate good, sold at real relative price \( p^I_t \). The problem of firm \( j \) along
the perfect foresight transition path is to
\[
\max_{\{d_{jt}, \ell_{jt}, k_{jt}, b_{jt}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left( \prod_{q=0}^{t-1} \frac{1 + \pi_q - 1}{1 + \lambda b_{t-q}} \right) d_{jt}
\]
subject to the flow budget constraint
\[
d_{jt} = p_t^I y(\ell_{jt}, k_{jt-1}) - w_t \ell_{jt} - \left[ k_{jt} - (1 - \delta) k_{jt-1} \right] + \tau_{f,t}(\{i_{jt-q}\}_{q=0}^t) - \phi(k_{jt}, k_{jt-1}, i_{jt}, i_{jt-1}) \tag{27}
\]
where \(\phi(\cdot)\) captures costs on either stock (capital) or flow (investment) adjustment. Intermediate goods producers thus hire labor on spot markets, invest, and pay out dividends. My only twist to this familiar production block is that I allow for a general fiscal investment stimulus policy \(\tau_{f,t}(\cdot)\), mapping investment today into future payments to the firm. Importantly, this set-up nests the popular bonus depreciation stimulus policy, in which investment today reduces tax liabilities in the future (Zwick & Mahon, 2017; Koby & Wolf, 2020). I will index time-\(t\) investment stimulus policies by a single parameter \(\tau_{f,t}\).

Proceeding exactly as in Section 2.1, we can define an aggregate investment function \(I(\cdot)\); I relegate a discussion of the arguments of this function to Appendix B.5, as it is not essential here. For my purposes, the only important consideration is that fiscal stimulus is one of those arguments, with the direct effects of such stimulus summarized by the following derivative matrix:
\[
I_{\tau_f} \equiv \frac{\partial I(\cdot)}{\partial \tau_f}
\]
Statements about the degree to which conventional fiscal instruments can be used to replicate monetary stimulus will be statements about the properties of \(I_{\tau_f}\) (and of course \(C_\tau\), as before).

REST OF THE ECONOMY. The intermediate good is sold to monopolistically competitive retailers subject to nominal rigidities, summarized again with a general price-NKPC:
\[
\hat{\pi}_t = \kappa_p \times \hat{p}_t^I + \beta \hat{\pi}_{t+1}
\tag{28}
\]
The remainder of the model is unchanged. The extension of the equilibrium definition in Definition 3 is then straightforward, and provided in Appendix B.5.
6.2 Policy equivalence with conventional fiscal instruments

In this extended model, monetary policy operates through two channels: first, as before, it affects household consumption demand, and second, it changes firm investment and so labor hiring as well as intermediate goods production. Thus, transfer stimulus policy alone is now insufficient to replicate the effects of (infeasible) conventional monetary policy—exactly as in Correia et al. (2013), an additional instrument is needed. In a straightforward generalization of Proposition 1, Proposition 4 shows that invertibility of $C_\tau$ and $I_{\tau_f}$ is sufficient to leave the space of implementable output-inflation allocations unchanged.

**Proposition 4.** Consider the extended model of Section 6.1. Suppose that $C_\tau$ and $I_{\tau_f}$ are both invertible, and consider an allocation $\{\pi^*_t, y^*_t\}_{t=0}^\infty$ that is implementable using an interest rate-only policy. Then it is similarly implementable through time-varying uniform transfer and bonus depreciation policies alone.

Transfer stimulus now only replicates the consumption channel of monetary policy transmission. If additionally $I_{\tau_f}$ is invertible, then the general form of investment stimulus considered in (27) suffices to replicate the investment channel, and so leave the set of implementable aggregate allocations unchanged. The final step of my argument is to ascertain that bonus depreciation stimulus can indeed perturb firm investment demand over time as required in Proposition 4. Unlike the stimulus check case, however, this logic is now entirely straightforward and in particular can closely follow the work of Correia et al. (2013). Since my model features no firm-level financial frictions, it is straightforward to see that bonus depreciation is equivalent to a standard investment subsidy (e.g., see the discussion in Winberry, 2021; Koby & Wolf, 2020). The analysis of Correia et al. (2013) now applies unchanged: interest rates $i_{b,t}$ and the subsidy $\tau_{f,t}$ both enter investment optimality conditions as wedges, and so the investment channel of monetary policy can be replicated by matching those wedges.

**Taking stock.** Taken together, the results in this section as well as in Section 4 give the headline practical takeaway of my paper: even in quite large-scale quantitative business-cycle models, a conventional mix of fiscal instruments—lump-sum transfers together with bonus depreciation tax stimulus—suffice to replicate the effects of any desired monetary policy on macroeconomic aggregates. The investment side of the argument is straightforward: bonus depreciation as the standard fiscal tool is sufficiently close to an investment subsidy that I was able to adapt the original results of Correia et al. with little change. On the consumption
side, on the other hand, an entirely different argument was needed—one that I provided in Sections 3 and 4, constituting the main contribution of the paper.

7 Conclusion

Over the past decade, much academic and applied policy interest has centered on the question of how to replicate monetary stimulus when nominal interest rates are constrained.

The central contribution of this paper is to show that, in business-cycle models that are entirely standard except for the presence of non-Ricardian consumers, a conventional mix of fiscal instruments—in particular including uniform lump-sum stimulus checks—suffices to replicate the aggregate effects of an arbitrary interest rate policy. The core insight is a formalization of the notion that interest rate and stimulus check policies can manipulate consumer demand “equally flexibly.” My main theoretical result is to establish that, in a rich but analytically tractable model of occasionally-binding borrowing constraints, this condition holds generically. As a secondary contribution, I provide an explicit characterization of the transfer policy that is needed to close any given shortfall in demand; in particular I show that, even in state-of-the-art quantitative heterogeneous-household models, this transfer policy is well-characterized by a very small number of measurable sufficient statistics.

I leave several important extensions for future work. First, it would be interesting to compare interest rate and stimulus check policies in a full Ramsey problem. Steps in this direction are taken in McKay & Wolf (2022a). Second, to the extent that the linear map $C_\tau$ changes over the cycle, the required transfer stimulus will also depend on the aggregate state of the economy. Future empirical work should try to better measure that state dependence.
A Appendix

A.1 Proof of Proposition 1

Linearizing the government budget constraint (7), we find

$$\tilde{b}_{t, t-1} - (1 + \tilde{r})\tilde{b}_{t} + (1 + \tilde{r})\tilde{b}_{t-1} + \tilde{\tau}_t = \tau_t \tilde{w} \tilde{\ell}(\tilde{w}_t + \tilde{\ell}_t) + \tilde{b}_t$$ (A.1)

Using (A.1), I will decompose total transfers into two parts: an endogenous “general equilibrium” component related to labor tax revenue and inflation debt servicing costs,

$$\tilde{\tau}_t^e \equiv \tau_t \tilde{w} \tilde{\ell}(\tilde{w}_t + \tilde{\ell}_t) + (1 + \tilde{r})\tilde{b}_t$$ (A.2)

and an exogenous “policy” component

$$\tilde{\tau}_t^p \equiv \tilde{\tau}_t - \tilde{\tau}_t^e$$ (A.3)

I now present a constructive proof of Proposition 1: leveraging (19) I will show how to construct a transfer-only policy replicating any interest rate-only policy, and vice-versa. The decomposition in (A.2) and (A.3) will prove useful in this constructive proof.

1. An interest rate-only policy is a tuple \{\(i_{b,t}, \tau_t\)\}_{t=0}^\infty with

$$\tilde{\tau}_t = \tilde{\tau}_t^e - \tilde{b}_{t, t-1}$$

so that \(\tilde{\tau}_t^e = -\tilde{b}_{t, t-1}\). By (19), there exists a path of transfers \{\(\tau_t^*\)\}_{t=0}^\infty such that

$$\mathcal{C}_\tau \times \tilde{\tau}^* = C_{i_b} \times \tilde{\tau}^*_b + C_{\tau} \times \tilde{\tau}^p$$ (A.4)

Since the interest rate-only policy by construction has zero net present value, it follows that the transfer-only policy \(\tilde{\tau}^*\)—which induces the exact same (zero-NPV) consumption sequence—also has zero NPV:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1 + \tilde{r}}\right)^t \tilde{\tau}_t^* = 0$$ (A.5)

Now consider the transfer-only policy tuple \{\(i_b, \tilde{\tau} + \tilde{\tau}_t^e + \tilde{\tau}_t^p\)\}_{t=0}^\infty. I will verify that, at the initial \{\(c_t, \ell_t, y_t, w_t, \pi_t, d_t\)\}_{t=0}^\infty, all markets still clear and all agents still behave optimally. First, by (A.5) and the definition of \(\tilde{\tau}_t^e\), we still have that \(\lim_{t \to \infty} \tilde{b}_t = 0\). Second, by construction of \(\tilde{\tau}_t^e\) in (A.4), the path \(\tilde{\tau}\) is still consistent with optimal household behavior given \{\(w_t, \ell_t, \pi_t, d_t; i_b, \tilde{\tau} + \tilde{\tau}_t^e + \tilde{\tau}_t^p\)\}_{t=0}^\infty. Finally, all other model equations are unaffected, so the guess is verified, because
the initial allocation was an equilibrium.

2. A transfer-only policy is a tuple $\{i_b, \tau_t\}_{t=0}^\infty$ with

$$\sum_{t=0}^\infty \left(\frac{1}{1+r}\right)^t \tau_t^* = 0$$

By (19), there exists a path of interest rates $\{i^*_b, \tau_t\}_{t=0}^\infty$ with $\tau_t^* = -b_{i_b, t-1}$ such that

$$C_r \times \tau^* = C_{i_b} \times \tau^*_b = C_{i_b} \times \tau^*_b + C_r \times \tau^*$$

(A.6)

and where by construction $\{\hat{i}_b, \tau^*\}$ has zero NPV:

$$\sum_{t=0}^\infty \left(\frac{1}{1+r}\right)^t \tau_t^* + \sum_{t=0}^\infty \left(\frac{1}{1+r}\right)^t b_{i_b, t-1} = 0$$

(A.7)

Now consider the interest rate-only policy tuple $\{i^*_b, \tau, \tau^*_b, \tau^*_t\}_{t=0}^\infty$. As before I will verify that, at the initial $\{c_t, \ell_t, y_t, w_t, \pi_t, d_t\}_{t=0}^\infty$, all markets still clear and all agents still behave optimally. First, by (A.7) and the definition of $\tau^*_t$, we have that $\hat{b}_t = 0$ for all $t$, so indeed the policy is a valid interest rate-only policy. Second, by construction of $\hat{i}_b^*$ in (A.6), the path $\hat{c}$ is still consistent with optimal household behavior given $\{w_t, \ell_t, \pi_t, d_t; i^*_b, \tau + \tau^*_b + \tau^*_t\}_{t=0}^\infty$. Finally, all other model equations are unaffected, so the guess is verified, because the initial allocation was an equilibrium.

$\square$
References


