Online Appendix for: Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result

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This online appendix contains supplemental material for the article "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result". I provide (i) details for the various structural models used in the paper, (ii) supplementary theoretical results, and (iii) a detailed discussion of equivalence in terms of policy rules. The end of this appendix contains further proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded "B."—"E." refer to the main article.

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B Model details

This appendix contains supplementary model details. I begin in Appendix B.1 by discussing in more detail the price-NKPC (5). Appendices B.2 and B.3 provide further details regarding the consumption-savings models of Section 2.2, including in particular a discussion of labor supply. In Appendix B.4 I sketch an alternative model of non-Ricardian consumer behavior—a model with bonds in the utility function—, and in particular discuss a variant of that model with generalized Greenwood et al. preferences. Finally, Appendix B.5 presents the extended model with investment.

B.1 Sticky-price retailers

To derive the price-NKPC (5), I let $1-\alpha$ denote the elasticity of output with respect to total labor input at the steady state, $1-\theta_p \in (0,1)$ denote the probability of a price re-set, and ϵ_p the substitutability between different retail varieties in aggregation to the final good. We can then follow the standard derivations in Galí (2015) to arrive at the following log-linearized aggregate price-NKPC:

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}})}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}\left(\widehat{w}_t+\alpha\widehat{\ell}_t\right)+\beta\widehat{\pi}_{t+1} \tag{B.1}$$

(B.1) is a special case of (5).

B.2 Further details on the analytical model

I first provide some additional details on the household consumption-savings problem—i.e., the mapping from sequences of income and interest rates to consumer demand. I then discuss household labor supply decisions.

CONSUMPTION-SAVINGS DECISIONS. Consider an individual household type i. I write their steady-state consumption level as $\bar{c}_i$ and their steady-state wealth holdings as $\bar{b}_i$. Under my stated assumptions on transfers to newborns, the consumption-savings problem of type i is identical to the consumer demand block studied in Angeletos et al. (2023). In particular, the aggregate demand relation in their equation (11) is my linearized optimality relation (12), reproduced below for convenience:

$$\widehat{c}_{it} = \left(1 - \frac{\theta_i}{1+\bar{r}}\right) \cdot \left\{\widehat{x}_{it} + \sum_{k=1}^{\infty}\left(\frac{\theta_i}{1+\bar{r}}\right)^k \widehat{e}_{it+k}\right\} - \sigma_i \sum_{k=0}^{\infty}\left(\frac{\theta_i}{1+\bar{r}}\right)^k \left(\widehat{i}_{b,t+k} - \widehat{\pi}_{t+k+1}\right) \quad \text{(B.2)}$$

where $\sigma_i \equiv \beta\theta_i\gamma^{-1} - (1-\beta\theta_i)\beta\bar{b}_i/\bar{c}_i$. The second equation that is needed to characterize the consumption function $\mathcal{C}^i(\bullet)$ of a consumer type i is the budget constraint. Here we have

$$c_{it} + b_{it} = (1-\tau_\ell)w_t\ell_t + \tau_t + d_t + \frac{1+i_{b,t-1}}{1+\pi_t}b_{it-1} \quad \text{(B.3)}$$

Linearizing (B.3) and combining with (B.2), we obtain a mapping from $\{\boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{\pi}, \boldsymbol{d}, \boldsymbol{\tau}, \boldsymbol{i}_b\}$ to the type-i consumer demand sequence $\boldsymbol{c}_i$—i.e., the matrices $\{\mathcal{C}^i_w, \mathcal{C}^i_\ell, \mathcal{C}^i_\pi, \mathcal{C}^i_d, \mathcal{C}^i_\tau, \mathcal{C}^i_{i_b}\}$.

Labor supply. I begin by considering a model with a single household type i. I assume that labor is assigned so that all households work the same amount of hours, and furthermore that unions bargain as described in Auclert et al. (2018). Then, since all households have the same steady-state consumption level, we get the following standard aggregate log-linearized wage-NKPC, exactly as in Erceg et al. (2000):

$$\widehat{\pi}^w_t = \kappa_w \times \underbrace{\left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma\widehat{c}_t)\right]}_{\text{wedge in labor supply}} + \beta\widehat{\pi}^w_{t+1} \quad \text{(B.4)}$$

where κ_w is a function of model primitives, satisfying

$$\kappa_w = \frac{(1 - \frac{1}{1+\bar{r}}\phi_w)(1-\phi_w)}{\phi_w(\varepsilon_w\frac{1}{\varphi}+1)},$$

with ϕ_w indicating the degree of wage stickiness, and ε_w indicating the elasticity of substitution between different types of labor. As in the price-NKPC case, (B.4) is a special case of the general relation (6).

Matters are slightly more subtle in my most general multi-type model. There, if different types have different steady-state wealth holdings, then their steady-state consumption also invariably differs, implying that standard union wage bargaining does not exactly map into a representation like (B.4).[1] Type-specific transfers that equalize steady-state consumption

[1] For example, in a two-type spender-saver model with types R and H, the static wedge in labor supply

across types i would thus be needed to return the model to a standard wage-NKPC. I investigate the importance of labor supply wedges in empirically relevant models in Section 5.1.

B.3 Further details on the heterogeneous-household model

This section completes the description of the quantitative heterogeneous-agent model introduced in Section 2.2 and studied in Section 4. I first discuss my assumptions on union bargaining (which are revisited in Section 5.1) and then describe the model calibration.

Union bargaining. I assume that unions order aggregate consumption and employment streams according to "as-if" representative-agent preferences:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{B.5}$$

Given this particular choice of union preferences, we can yet again follow the same steps as in Erceg et al. (2000) or Auclert et al. (2018) to arrive at (B.4).

Note that, if unions instead maximized an equal-weighted average of household utility (i.e., the baseline specification of Auclert et al., 2018),

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 \left\{ \frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_{it}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} di = \sum_{t=0}^{\infty} \beta^t \int_0^1 \left\{ \frac{c_{it}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} di \tag{B.6}$$

then a weighted average of household marginal consumption utilities—rather than marginal consumption utility evaluated at the aggregate consumption level c_t—would enter the static labor wedge and thus (B.4). The model would thus be inconsistent with a wage-NKPC of my assumed form (6). I discuss this case further in Section 5.1 and Appendix C.4.

Model calibration. I first discuss the parameterization of the steady state. Recall that this is all that matters for the consumption function $\mathcal{C}(\bullet)$, and so in particular for $\mathcal{C}_\tau$.

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would be

$$\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma \widehat{c}_t) + \gamma \left[\mu_R \widehat{c}_{R,t} + \mu_H \widehat{c}_{H,t} - \widehat{c}_t\right],$$

where the last term in brackets is evidently equal to zero if $\bar{c}_R = \bar{c}_H = \bar{c}$, but not in general. Prior work in such models thus often assumes identical steady-state consumption shares, allowing straightforward aggregation to (B.4) (e.g., Bilbiie et al., 2021).

Parameter	Description	Value	Target	Value
Households				
ρ_ε, σ_ε	Income Risk	-	Kaplan et al. (2018)	-
$\underline{\varepsilon}^p$, χ_0, χ_1	Dividend Endowment	-	Illiquid Wealth Shares	-
β	Discount Rate	0.97	$\bar{b}/\bar{y}$	1.5
$\bar{r}$	Average Return	0.01	Annual Rate	0.04
ϱ	Death Rate	1/180	Average Age	45
γ	Preference Curvature	1	Standard	
φ	Labor Supply Elasticity	0.5	Standard	
ε_w	Labor Substitutability	10	Standard	
$\underline{b}$	Borrowing Limit	0	McKay et al. (2016)	
Firms				
$1-\alpha$	Returns to Scale	1	Standard	
ε_p	Goods Substitutability	16.67	Profit Share	0.06
Government				
τ_ℓ	Labor Tax	0.3	Average Labor Tax	0.30
$\bar{\tau}/\bar{y}$	Transfer Share	0.05	Transfer Share	0.05

Table B.1: HANK model, steady-state calibration.

The values of all parameters relevant for the model's deterministic steady state are displayed in Table B.1. For my quantitative analysis I slightly enrich the preferences displayed in (17) to allow for exogenous household death at rate ϱ. Preference parameters $\{\gamma, \varphi, \varrho\}$ as well as the labor substitutability ε_w are set to standard values. The average return on (liquid) assets is set in line with standard calibrations of business-cycle models, and the discount rate is then disciplined through the total amount of liquid wealth. As in McKay et al. (2016), I assume that households cannot borrow in the liquid asset. Next, for income risk, I adopt the 33-state specification of Kaplan et al. (2018), ported to discrete time. For share endowments, I assume that

$$d_{it} = \begin{cases} 0 & \text{if } \varepsilon_{it}^p \leq \underline{\varepsilon}^p \\ \chi_0(\varepsilon_{it}^p - \underline{\varepsilon}^p)^{\chi_1} \times d_t & \text{otherwise} \end{cases}$$

where ε_{it}^p is the permanent component of household i's labor productivity. I set the parameters $\{\underline{\varepsilon}^p, \chi_0, \chi_1\}$ as in Wolf (2020). On the firm side, I assume constant returns to scale in production, and set the substitutability between goods to a standard value. Finally, the average government tax take, transfers, and debt issuance are all set in line with direct empirical evidence. Relative to Definition 3, I slightly generalize the model to allow for non-zero government spending, giving the new market-clearing condition

$$y_t = c_t + g_t \tag{B.7}$$

Note that, for all experiments, I keep government expenditure fixed at $g_t = \bar{g}$, so its presence only matters for the steady-state fiscal tax-and-transfer system, and does not directly show up anywhere in equilibrium dynamics.

In the second step I set the remaining model parameters (which exclusively govern dynamics around the deterministic steady state). For the baseline interest rate-only policy studied in Section 4.2, I consider an interest rate rule of the form

$$\widehat{i}_{b,t} = \phi_\pi \widehat{\pi}_t + m_t$$

where m_t is the monetary shock, set to give the gradually decaying path of nominal rates displayed in Figure 3. In a slight generalization of Definition 2, I assume that the baseline interest rate-only policy is not financed through taxes and transfers adjusting period-by-period (as in (8)), but instead consider a more general fiscal financing rule of the form

$$\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \left[(1+\bar{r})\bar{b}(\widehat{i}_{b,t-1} - \widehat{\pi}_t) - \tau_\ell(\bar{w}\widehat{\ell}_t + \bar{\ell}\widehat{\ell}_t)\right], \tag{B.8}$$

and with $\rho_b \in (0,1)$. Total transfers adjust residually to balance the government budget. Since b_t evolves gradually over time, it follows that a nominal interest rate cut at time t only feeds through to higher transfers with a delay. While the financing rule in Definition 2 was conceptually simpler, the alternative fiscal rule (B.8) has the advantage that nominal interest rate movements are not accompanied by (counterfactual) large contemporaneous changes in transfers. This completes the specification of policy for the baseline monetary experiment. I present the rule parameterizations as well as all other model parameters in Table B.2.

Alternative calibrations. For my robustness checks in Appendix C.3 I consider two alternative model calibrations: one with less liquid wealth ($\bar{b}/\bar{y} = 0.5$, implying substantially larger MPCs, with $\omega = 0.64$), and one with more liquid wealth ($\bar{b}/\bar{y} = 7.5$, implying

Parameter	Description	Value
ϕ_p	Price Calvo Parameter	0.85
ϕ_w	Wage Calvo Parameter	0.70
ϕ_π	Taylor Rule Inflation	1.5
ρ_b	Financing Rule Persistence	0.85

Table B.2: HANK model, parameters governing dynamics.

substantially smaller MPCs, with $\omega = 0.12$).

B.4 Bond-in-utility models

While my main analysis considers (mixtures of) perpetual-youth overlapping-generations models as particularly convenient models of non-Ricardian consumer behavior, I emphasize that my results extend with very little change to an alternative popular model variant: bond-in-utility models, as considered in Michaillat & Saez (2018).

A BASELINE BOND-IN-UTILITY MODEL. Household preferences are now

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} - \psi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{B.9}$$

The budget constraint is still exactly as in Section 2.2. The log-linearized optimality conditions are then

$$\bar{c}\widehat{c}_t + \bar{b}\widehat{b}_t = (1-\tau_\ell)\bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{b}(\widehat{b}_{t-1} + \widehat{i}_{b,t-1} - \widehat{\pi}_t) + \bar{\tau}\widehat{\tau}_t + \bar{d}\widehat{d}_t \tag{B.10}$$

$$\widehat{c}_t = \beta(1+\bar{r})\widehat{c}_{t+1} + \frac{\eta}{\gamma}\left[1 - \beta(1+\bar{r})\right]\widehat{b}_t - \frac{1}{\gamma}\beta(1+\bar{r})\left(\widehat{i}_{b,t} - \widehat{\pi}_{t+1}\right) \tag{B.11}$$

where $\beta(1+\bar{r}) < 1$ as long as $\alpha, \eta > 0$. Together, the two relations (B.10) - (B.11) fully characterize the model-implied consumption derivative matrix $\mathcal{C}_\tau^{BiU}$. I provide a closed-form expression for its inverse $(\mathcal{C}_\tau^{BiU})^{-1}$ in Appendix C.1.

Finally I also note that, since there is a single representative household with separable preferences over consumption, wealth, and hours worked, union bargaining again gives (B.4). The analytical bond-in-utility model is thus also consistent with all of the high-level assumptions on labor supply made in Section 2.1.

Parameter	Description	Value
α	Wealth preference level	0.10
η	Wealth preference curvature	0.50
δ_R	Savers GHH+ coefficient	0.03
δ_H	Spenders GHH+ coefficient	0.95
μ	Share of spenders	0.25

Table B.3: Mixture model with heterogeneous wealth effects, parameterization of preferences.

GHH+ PREFERENCES. For my second investigation of the role of wealth effects in labor supply in Section 5.1, I consider a two-type model with generalized Greenwood et al. preferences, as originally proposed in Auclert et al. (2020). I assume that savers have bonds in their utility function; that is, their preferences are[2]

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(c_t - \psi_R \delta_R \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{b_{t-1}^{1-\eta} - 1}{1-\eta} - \psi_R (1-\delta_R) \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\} \tag{B.12}$$

while spenders have static per-period preferences

$$\frac{\left(c_t - \psi_H \delta_H \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{1-\gamma} - 1}{1-\gamma} - \psi_H (1-\delta_H) \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{B.13}$$

Here the two coefficients $\{\delta_R, \delta_H\}$ control the strength of wealth effects in labor, with $\delta = 0$ corresponding to standard separable preferences and $\delta = 1$ corresponding to GHH preferences. I calibrate the consumer part of the model to induce consumption behavior similar to my baseline HANK model—matching in particular $\omega = 0.3$—and MPEs consistent with Golosov et al. (2021)—an MPE of \$3.5 for savers and an MPE of \$1.8 for spenders. The model parameterization is reported in Table B.3. All other parameters are set exactly as in my baseline HANK model, as reported in Tables B.1 and B.2.[3] Detailed results on the

[2]The perpetual-youth model with generalized Greenwood et al. preferences is substantially less tractable, so I consider the bond-in-utility variant instead.

[3]Except for δ_H and δ_R and thus their wealth effects in labor supply, I treat spenders and savers entirely symmetrically, ensuring in particular identical steady-state consumption.

accuracy of my policy equivalence result in this alternative model environment are discussed in Appendix C.4.

B.5 Adding investment

All firms are identical, so I drop the j subscript. Analogously to the discussion in Section 2.1, we can summarize the solution to the firm problem with an investment demand function,

$$\mathbf{i} = \mathcal{I}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.14}$$

a production function,

$$\mathbf{y} = \mathcal{Y}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.15}$$

a labor demand function,

$$\boldsymbol{\ell} = \mathcal{L}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.16}$$

and a dividend function

$$\mathbf{d} = \mathcal{D}\left(\mathbf{w}, \mathbf{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \mathbf{i}_b\right) \tag{B.17}$$

where the dividend function aggregates over both intermediate goods producers and sticky-price retailers. Note that, since intermediate goods firms hire labor on a competitive spot market, and since all firms j are identical, two sequences $\boldsymbol{i_b}$ and $\boldsymbol{\tau}_f$ that induce the same paths of investment also invariably induce the same paths of output and labor hiring. However, since interest rates and investment subsidies enter the firm budget constraint differently, the implied dividend paths may be different.

Given investment subsidies to firms, the government budget constraint is adjusted to give

$$\frac{1+i_{b,t-1}}{1+\pi_t} b_{t-1} + \tau_t + \tau_{f,t}(\{i_{t-q}\}_{q=0}^{t}) = \tau_\ell w_t \ell_t + b_t \tag{B.18}$$

All other parts of the model are unchanged relative to Section 2.1. We thus arrive at the following equilibrium definition:

Definition 1. *An equilibrium is a set of government policies* $\{i_{b,t}, \tau_t, \tau_{f,t}, b_t\}_{t=0}^{\infty}$ *and a set of aggregates* $\{c_t, \ell_t, y_t, i_t, k_t, w_t, \pi_t, d_t, p_t^I\}_{t=0}^{\infty}$ *such that:*

1. *Consumption is consistent with the aggregate consumption function* (2).

2. *Aggregate investment, output, hours worked and dividends satisfy*

$$\begin{aligned}
\boldsymbol{i} &= \mathcal{I}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right) \\
\boldsymbol{y} &= \mathcal{Y}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right) \\
\boldsymbol{\ell} &= \mathcal{L}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right) \\
\boldsymbol{d} &= \mathcal{D}\left(\boldsymbol{w}, \boldsymbol{p}^I, \boldsymbol{\pi}; \boldsymbol{\tau}_f, \boldsymbol{i}_b\right)
\end{aligned}$$

3. *Wage inflation* $\{\pi_t^w\}_{t=0}^{\infty}$ *and* $\{\ell_t, c_t, w_t\}_{t=0}^{\infty}$ *are consistent with the wage-NKPC* (6).

4. *The paths* $\{\pi_t, p_t^I\}_{t=0}^{\infty}$ *are consistent with the adjusted aggregate price-NKPC* (28).

5. *The output market clears:* $y_t = c_t + i_t$ *for all* $t \geq 0$*, the government budget constraint* (B.18) *holds at all* t*, and* $\lim_{t\to\infty} b_t = \bar{b}$*. The bond market then clears by Walras' law.*

C Supplementary results

This section presents supplementary theoretical results. Appendices C.1 and C.2 characterize $\mathcal{C}_\tau$, $\mathcal{C}_\tau^{-1}$ as well as $\tilde{\mathcal{C}}_{i_b}$ in the analytical model of Section 2.2. Appendix C.3 elaborates on my sufficient statistics formula and discusses its accuracy in other models. Results supplementing my discussion of wealth effects in labor supply and non-equivalence at the household level are provided in Appendices C.4 and C.5. Finally Appendices C.6 and C.7 extend the policy equivalence result to some richer model environments and to targeted transfers.

C.1 $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ in analytical models

I here characterize the matrix $\mathcal{C}_\tau$ as well as its inverse $\mathcal{C}_\tau^{-1}$ in my analytical models of non-Ricardian consumption behavior. I first present results for a perpetual-youth consumer block (as in Section 2.2, with some general θ) and then consider a further extended model with an arbitrary contemporaneous MPC, thus allowing me to nest environments with $\beta(1+\bar{r}) \neq 1$ and/or behavioral frictions. Finally I sketch results for bond-in-utility models.

Characterizing $\mathcal{C}_\tau$. I begin with the shape of $\mathcal{C}_\tau^{OLG}$ as displayed in (16). From the discussion in Appendix B.2 it follows that the matrix $\mathcal{C}_\tau^{OLG}$ is fully characterized by the following pair of equations:

$$\begin{aligned} \widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} &= \widehat{\tau}_t, && \text{(C.1)} \\ \left[1-\theta(1-\beta\theta)\right]\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1-\beta\theta)(1-\theta)\frac{1}{\beta}\widehat{b}_{t-1} &= (1-\beta\theta)(1-\theta)\widehat{\tau}_t, && \text{(C.2)} \end{aligned}$$

where (C.2) is the Euler equation representation of the aggregate demand relation (B.2).

From this system we arrive at the following characterization of $\mathcal{C}_\tau^{OLG}$. First, it is straightforward to see that the first column and the first row of $\mathcal{C}_\tau^{OLG}$ are respectively given as

$$\mathcal{C}_\tau^{OLG}(\bullet, 1) = \left(1 - \frac{\theta}{1+\bar{r}}\right) \times \left\{1, \theta, \theta^2, \dots\right\}'$$

and

$$\mathcal{C}_\tau^{OLG}(1, \bullet) = \left(1 - \frac{\theta}{1+\bar{r}}\right) \times \left\{1, \frac{\theta}{1+\bar{r}}, \left(\frac{\theta}{1+\bar{r}}\right)^2, \dots\right\}$$

Second, all higher-order columns are given recursively as

$$\mathcal{C}_\tau^{OLG}(\bullet,h) = \mathcal{C}_\tau^{OLG}(1,h)\times\begin{pmatrix}1\\ -\mathcal{C}_\tau^{OLG}(\bullet,1)(1+\bar{r})\end{pmatrix} + \begin{pmatrix}0\\ \mathcal{C}_\tau^{OLG}(\bullet,h-1)\end{pmatrix}, \quad h=2,3,4,\ldots$$

This expression—which is straightforward to verify from (C.1) - (C.2)—reflects the intuition of the "fake-news" algorithm of Auclert et al. (2019): the first term is the response of households to a date-h income shock announced at date-0 but then reversed at date 1; the second term then undoes that date-1 reversal, ensuring that the sum gives us the actual response to a date-h income shock—that is, $\mathcal{C}_\tau^{OLG}(\bullet,h)$. This somewhat complicated exact shape of $\mathcal{C}_\tau^{OLG}$ corresponds to the approximate shape displayed in (16), as established in the following result.

Lemma C.1. *Consider the consumption-savings problem of a perpetual-youth household block, as described in Section 2.2. Then, for any $\ell > 0$ the impulse response path $\widehat{\mathbf{c}}_H$ to an income shock at time H satisfies*

$$\lim_{H\to\infty}\widehat{c}_{H,H} = const., \qquad \lim_{H\to\infty}\frac{\widehat{c}_{H+\ell,H}}{\widehat{c}_{H+\ell-1,H}} = \theta, \qquad \lim_{H\to\infty}\frac{\widehat{c}_{H-\ell-1,H}}{\widehat{c}_{H-\ell,H}} = \frac{\theta}{1+\bar{r}} \tag{C.3}$$

In words, for large H, the intertemporal spending profile in $\mathcal{C}_\tau$ looks as indicated in (16). This is the sense of the approximation $\approx$ in that relation.

Characterizing $\mathcal{C}_\tau^{-1}$. Given an arbitrary target consumption sequence $\widehat{\boldsymbol{c}}$, we can solve the system (C.1) - (C.2) for $\{\widehat{\boldsymbol{\tau}},\widehat{\boldsymbol{b}}\}$, with $\widehat{b}_{-1}=0$. This gives the solution displayed in (20). The detailed steps are provided in the proof of Lemma 1.

Extension to arbitrary MPCs. I next consider an even further-generalized aggregate demand relation of the following form:

$$\widehat{c}_t = M\cdot\left\{\widehat{x}_t + \sum_{k=1}^{\infty}\left(\frac{\theta}{1+\bar{r}}\right)^k \widehat{e}_{t+k}\right\} - \sigma\sum_{k=0}^{\infty}\left(\frac{\theta}{1+\bar{r}}\right)^k\left(\widehat{i}_{b,t+k} - \widehat{\pi}_{t+k+1}\right). \tag{C.4}$$

Relative to (B.2), (C.4) additionally disentangles the impact MPC M from the discounting factor θ applied to future income. This allows me to study two meaningful extensions of the baseline model. First, in an environment with $\beta(1+\bar{r})\neq 1$, and under the simplifying assumption of log preferences, (C.4) applies with $M = 1-\beta\theta$ (see Farhi & Werning, 2019, for the continuous-time analogue).[4] Second, in a model with cognitive discounting and occasionally-binding borrowing constraints, we have $M = 1 - \frac{\theta_1}{1+\bar{r}}$ and $\theta = \theta_1 \cdot \theta_2$, where θ_1 and θ_2 are the borrowing constraint and behavioral discounting coefficients, respectively.

The following result characterizes existence and shape of $\mathcal{C}_\tau^{-1}$ in this environment.

Lemma C.2. *Consider a variant of the analytical consumption-savings problem of Section 2.2 with generalized aggregate demand relation (C.4). Suppose that* $M \in \left[\frac{\bar{r}}{1+\bar{r}}, 1 - \frac{\theta}{1+\bar{r}}\right]$. *Then, if* $\theta < 1$, $\mathcal{C}_\tau$ *is invertible, with*

$$\mathcal{C}_\tau^{-1} = \frac{1}{M} \cdot \begin{pmatrix} \frac{1-M\theta}{1-\theta} & -\frac{1}{1+\bar{r}}\frac{\theta}{1-\theta} & 0 & \cdots \\ -\frac{(1-M)(1+\bar{r})}{1-\theta} & \frac{1+\theta(1-M)}{1-\theta} & -\frac{1}{1+\bar{r}}\frac{\theta}{1-\theta} & \cdots \\ 0 & -\frac{(1-M)(1+\bar{r})}{1-\theta} & \frac{1+\theta(1-M)}{1-\theta} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.5}$$

Note that Lemma C.2 imposes bounds on M. Here, the upper bound corresponds to the baseline perpetual-youth model—so all discounting of future income θ also increases contemporaneous MPCs—while the lower bound ensures that $\mathcal{C}_\tau$ is actually a bounded operator.[5] Importantly, Lemma C.2 allows me to substantiate two claims made in Section 3.2. First, for the special case of a perpetual-youth model with $\beta(1+\bar{r}) > 1$ and with log preferences, the requirement on M that $M \geq \frac{\bar{r}}{1+\bar{r}}$ becomes

$$\theta \leq \frac{1}{\beta(1+\bar{r})} < 1,$$

—a condition that is strictly tighter than my baseline perpetual-youth requirement of $\theta < 1$. In words, borrowing constraints now need to bind *often enough*. This is precisely what is needed to counteract the backloading implied by $\beta(1+\bar{r}) > 1$, implying that $\mathcal{C}_\tau$ is a bounded operator whose inverse takes the shape (C.5). Second, a standard model with behavioral discounting corresponds to the special case where $M = \frac{\bar{r}}{1+\bar{r}}$ as well as $\theta < 1$. By Lemma C.2, this is again sufficient to ensure invertibility of $\mathcal{C}_\tau$, further underscoring my claims about the generality of this property of aggregate consumption functions.

[4] Without log preferences, a consumption function like (C.4) still obtains, but the mapping into M is more complicated—we have $M = \gamma^{-1}[1-\beta\theta] + (1-\gamma^{-1})[1 - \frac{\theta}{1+\bar{r}}]$ (see Appendix 1.2 of Farhi & Werning, 2019). The characterization of $\mathcal{C}_\tau^{-1}$ in Lemma C.2 of course continues to apply, however, and so my conclusions are unchanged—invertibility obtains if MPCs are large enough and spending is front-loaded.

[5] To see this, consider for example the first column of $\mathcal{C}_\tau$. It is straightforward to see that its entries grow at rate $(1-M)(1+\bar{r})$. Thus, if $M < \frac{\bar{r}}{1+\bar{r}}$, the implied consumption path diverges.

Front-loaded spending and a counterexample. To even more clearly see the importance of front-loading in consumer spending it will prove useful to consider a particularly transparent case: a simple two-period OLG model in which households have log preferences and receive income (including transfers) only when young.[6] In that case we have

$$\mathcal{C}_\tau = \begin{pmatrix} \frac{1}{1+\beta} & 0 & 0 & \dots \\ \frac{\beta(1+\bar{r})}{1+\beta} & \frac{1}{1+\beta} & 0 & \dots \\ 0 & \frac{\beta(1+\bar{r})}{1+\beta} & \frac{1}{1+\beta} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Note that spending here is back-loaded if and only if $\beta(1+\bar{r}) > 1$. In that case, the majority of a dollar of income received at date t is spent at date $t+1$, not t.

Straightforward algebra reveals that, if $\mathcal{C}_\tau^{-1}$ exists, it is lower-triangular, with off-diagonal elements that (in absolute value) decay at rate $\beta(1+\bar{r})$. $\beta(1+\bar{r}) \leq 1$ is thus necessary (and here also sufficient) for invertibility. Intuitively, if households have a natural tendency to postpone their spending, then a transfer today designed to induce spending today will require ever-larger transfers in the future to offset it. As a result, for a given target sequence of consumption, it becomes impossible to find a bounded sequence of transfers that induces it. This instructive example reveals that $\mathcal{C}_\tau$ may well be *injective*—changing the timing of transfers invariably affects consumption—yet fail to be *surjective*—certain bounded sequences of demand cannot be induced via bounded sequences of transfers. Households front-loading their spending—which is ensured in my headline environment by occasionally-binding borrowing constraints, and which is a robust feature of actual consumer behavior—prevents the divergence that here is causing non-invertibility.[7]

Bond-in-utility model. Recall from the discussion in Appendix B.4 that the consumption derivative matrix $\mathcal{C}_\tau^{BiU}$ is fully characterized by the following pair of equations:

$$\widehat{c}_t + \widehat{b}_t - (1+\bar{r})\widehat{b}_{t-1} = \widehat{\tau}_t \tag{C.6}$$

$$\widehat{c}_t - \beta(1+\bar{r})\widehat{c}_{t+1} - \frac{\bar{c}}{\bar{b}}\frac{\eta}{\gamma}\left[1-\beta(1+\bar{r})\right]\widehat{b}_t = 0 \tag{C.7}$$

[6] I thank an anonymous referee for bringing this illuminating example to my attention.

[7] An elevated impact MPC, on the other hand, is not sufficient to rule out a linear map like $\mathcal{C}_\tau$. This is why, in Proposition 3, I highlight *both* the elevated MPC as well as the front-loaded spending profile, even though in my particular model of occasionally-binding borrowing constraints the two are interchangeable.

We see that the factor $\frac{\bar{c}}{\bar{b}}$ simply scales the last term in the Euler equation, so I will without loss of generality set this term equal to 1. Algebra similar to that in the proof of Lemma 1 then yields the following exact expression for $\mathcal{C}_\tau^{-1}$:

$$(\mathcal{C}_\tau^{BiU})^{-1} = \begin{pmatrix} 1+\frac{1}{\tilde{\omega}}\frac{1}{1+\bar{r}}\frac{\theta}{1-\beta\theta^2} & -\frac{1}{\tilde{\omega}}\frac{\beta\theta}{1-\beta\theta^2} & 0 & \dots \\ -\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2} & 1+\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2}\left[\beta(1+\bar{r})+\frac{1}{1+\bar{r}}\right] & -\frac{1}{\tilde{\omega}}\frac{\beta\theta}{1-\beta\theta^2} & \dots \\ 0 & -\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2} & 1+\frac{1}{\tilde{\omega}}\frac{\theta}{1-\beta\theta^2}\left[\beta(1+\bar{r})+\frac{1}{1+\bar{r}}\right] & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{C.8}$$

where $\omega = \mathcal{C}_\tau^{BiU}(1,1)$, $\theta = \mathcal{C}_\tau^{BiU}(2,1)/\mathcal{C}_\tau^{BiU}(1,1)$, and $\tilde{\omega}$ is given via

$$1 = \tilde{\omega}\left[\frac{(1+\bar{r})\beta\theta}{1-(1+\bar{r})\beta\theta} + \omega^{-1}\right]$$

We see that $(\mathcal{C}_\tau^{BiU})^{-1}$ has the same tridiagonal shape as $(\mathcal{C}_\tau^{OLG})^{-1}$, as claimed.

C.2 $\tilde{\mathcal{C}}_{i_b}$ in analytical models

This section offers additional results on the shape and properties of $\tilde{\mathcal{C}}_{i_b}$ in the one-type perpetual-youth OLG model. I proceed in two steps. First, I provide a closed-form expression for $\tilde{\mathcal{C}}_{i_b}$. Second, I establish that, if $\theta > 0$ (i.e., not pure spender behavior), then interest rate policy can similarly be used to induce any sequence of net excess consumption demand with zero net present value.

A closed-form expression for $\tilde{\mathcal{C}}_{i_b}$. Recall that the matrices $\mathcal{C}_\tau$ and $\tilde{\mathcal{C}}_{i_b}$ in the baseline (one-type) perpetual-youth model are characterized by the following pair of equations:

$$\widehat{c}_t + \widehat{b}_t - \frac{1}{\beta}\widehat{b}_{t-1} = \widehat{\tau}_t, \tag{C.9}$$

$$[1-\theta(1-\beta\theta)]\,\widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1-\beta\theta)(1-\theta)\frac{1}{\beta}\widehat{b}_{t-1} = (1-\beta\theta)(1-\theta)\widehat{\tau}_t - \gamma\beta\theta\widehat{i}_{b,t}. \tag{C.10}$$

From here it is straightforward to see that

$$\tilde{\mathcal{C}}_{i_b} = -\frac{1}{\gamma}(I-\mathcal{C}_\tau)\begin{pmatrix} 1 & 1 & 1 & \dots \\ 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{C.11}$$

To formally establish this relation, I will define $\widehat{c}_t^* \equiv \widehat{c}_t - \widehat{\tau}_t$. Now consider first an income shock at date 0. Plugging this into the optimality conditions and re-arranging, we see that the impulse response of c^* is identical to the impulse response of c to a date-0 interest rate change scaled by γ. That is, we have

$$\mathcal{C}_\tau^*(\bullet, 1) = \mathcal{C}_\tau(\bullet, 1) - e_1 = \gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 1)$$

where $e_1 = (1, 0, 0, \dots)'$. This gives the first column of (C.11):

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 1) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1))$$

Similarly, for date-1 shocks, we have

$$\mathcal{C}_\tau^*(\bullet, 2) = \mathcal{C}_\tau(\bullet, 2) - e_2 = -\gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 1) + \gamma\tilde{\mathcal{C}}_{i_b}(\bullet, 2)$$

where $e_2 = (0, 1, 0, \dots)'$. Thus we get

$$\tilde{\mathcal{C}}_{i_b}(\bullet, 2) = -\frac{1}{\gamma}(e_1 - \mathcal{C}_\tau(\bullet, 1) + e_2 - \mathcal{C}_\tau(\bullet, 2))$$

giving the second column of (C.11). All other columns follow analogously. We thus see that the perpetual-youth model admits a straightforward relation between $\mathcal{C}_\tau$ and $\tilde{\mathcal{C}}_{i_b}$, with the mapping between the two fully governed by γ.

Establishing equivalence. Consider an arbitrary consumption sequence $\widehat{\boldsymbol{c}}$ such that $\sum_{t=0}^\infty \left(\frac{1}{1+\bar{r}}\right)^t \widehat{c}_t = 0$—i.e., it has zero net present value. I will now provide a constructive argument showing that we can find a bounded sequence of interest rates $\widehat{\boldsymbol{i}}_b^*$ such that $\tilde{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b^* = \widehat{\boldsymbol{c}}$. For this I first of all note that, from (C.9), we must have that

$$\widehat{b}_t = \frac{1}{\beta}\widehat{b}_{t-1} - \widehat{c}_t$$

Since $\widehat{\boldsymbol{c}}$ has zero net present value, it follows that $\widehat{b}_t \to 0$. Next, from (C.10), it follows that it suffices to set

$$\widehat{i}_{b,t} = -\frac{1}{\gamma\beta\theta} \cdot \left([1 - \theta(1 - \beta\theta)]\, \widehat{c}_t - \beta\theta\widehat{c}_{t+1} - (1 - \beta\theta)(1 - \theta)\frac{1}{\beta}\widehat{b}_{t-1} \right)$$

Since $\widehat{\boldsymbol{c}}$ is bounded by construction and $\widehat{b}_t \to 0$ by the argument above, it follows that—if $\theta > 0$—we can find a bounded sequence $\widehat{\boldsymbol{i}}_b^*$ that induces net excess demand $\widehat{\boldsymbol{c}}$, as claimed.

C.3 The sufficient statistics formula and its accuracy

I begin with some additional details on the sufficient statistics formula. The formula maps the three observables $\{\omega, \theta, \bar{r}\}$ into the matrix $\mathcal{C}_\tau$ (and thus its inverse $\mathcal{C}_\tau^{-1}$). The formula proceeds in two steps.

First, given $\{\theta, \bar{r}\}$, I construct a matrix $\mathcal{C}_\tau^{(1)}$ that has the same shape as in a one-type perpetual-youth model. By the discussion in Appendix C.1, this means that

$$\mathcal{C}_\tau^{(1)}(\bullet, 1) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \theta, \theta^2, \dots\right\}'}_{\text{spending decay}}$$

and

$$\mathcal{C}_\tau^{(1)}(1, \bullet) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \frac{\theta}{1+\bar{r}}, \left(\frac{\theta}{1+\bar{r}}\right)^2, \dots\right\}}_{\text{anticipation effects}}$$

together with

$$\mathcal{C}_\tau^{(1)}(\bullet, h) = \mathcal{C}_\tau^{(1)}(1, h) \times \begin{pmatrix} 1 \\ -\mathcal{C}_\tau^{(1)}(\bullet, 1)(1+\bar{r}) \end{pmatrix} + \begin{pmatrix} 0 \\ \mathcal{C}_\tau^{(1)}(\bullet, h-1) \end{pmatrix}, \quad h = 2, 3, 4, \dots$$

This specifies the entire matrix $\mathcal{C}_\tau^{(1)}$ as a function only of $\{\theta, \bar{r}\}$.

Second, I add a margin of spenders to disentangle the MPC ω and the spending slope θ. Note that, in my construction of $\mathcal{C}_\tau^{(1)}$, the MPC is mechanically given as $\left(1 - \frac{\theta}{1+\bar{r}}\right)$. To match any desired arbitrary MPC ω I then simply set

$$\mathcal{C}_\tau = \frac{\theta - (1-\omega)(1+\bar{r})}{\theta} \times I + \frac{(1-\omega)(1+\bar{r})}{\theta} \times \mathcal{C}_\tau^{(1)}$$

It is straightforward to verify that the resulting $\mathcal{C}_\tau$ matches the desired MPC ω. I have thus mapped my three sufficient statistics $\{\omega, \theta, \bar{r}\}$ into a matrix $\mathcal{C}_\tau(\omega, \theta, \bar{r})$ that (i) matches the average MPC ω and spending slope θ, and (ii) is by construction consistent with lifetime household budget constraints. From here I can then also construct $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r})$. Note that this inverse exists as long as $\theta < 1$ and $\omega \geq 1 - \frac{\theta}{1+\bar{r}}$, by the proof Proposition 2.

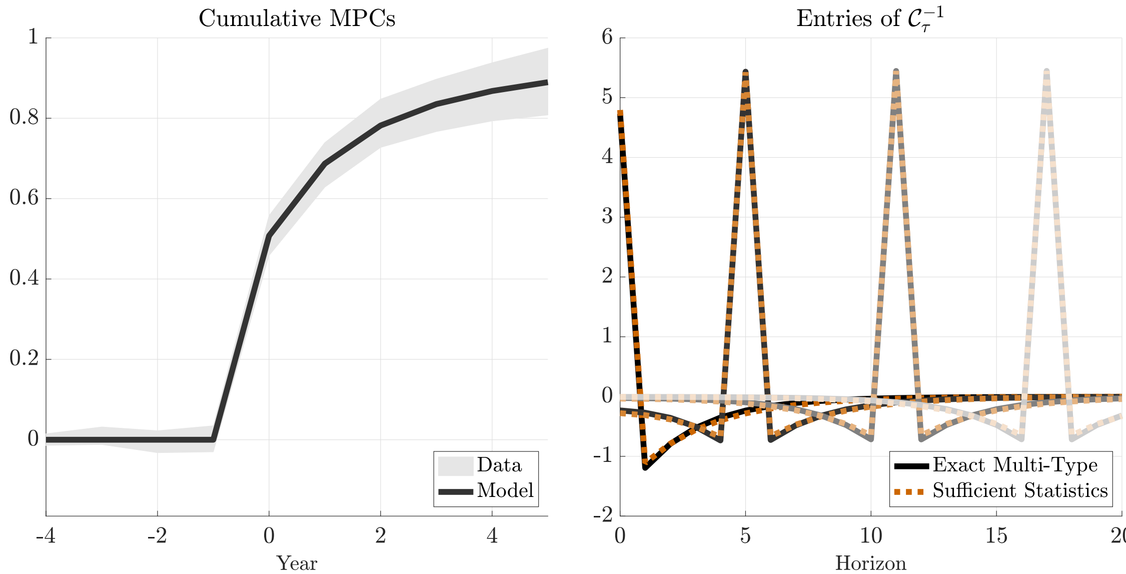

Figure C.1: Left panel: cumulative MPCs in the data (taken from Fagereng et al., 2018) and in the three-type model of Angeletos et al. (2023). Right panel: model-implied entries of $\mathcal{C}_\tau^{-1}$ (blue) vs. predicted values from my sufficient statistics formula (orange dashed).

The role of tail MPCs. The sufficient statistics formula imposes a constant rate of decay θ of intertemporal marginal propensities to consume. As discussed in Angeletos et al. (2023), empirical evidence on the other hand suggests that this rate of iMPC decay slows down in the far tails, at long horizons. To gauge whether this mismatch in the tails actually matters for the purposes of my results here, I consider the model used in Angeletos et al. to match empirical evidence on the entire intertemporal MPC profile—a hybrid model with *two* types of perpetual-youth consumers, together with a margin of spenders. The left panel of Figure C.1, taken from Angeletos et al., shows that this model indeed matches empirical evidence on far-ahead MPCs very well. The right panel then compares the true model-implied $\mathcal{C}_\tau^{-1}$ with the prediction from my sufficient statistics formula.[8] We see that the two are almost indistinguishable—i.e., for the purposes of the analysis here, a mild mismatch in the far-ahead tails is essentially irrelevant.

Perturbing the sufficient statistics. I here further substantiate my claim that, for empirically relevant values of the sufficient statistics, even moderately sized stimulus check

[8] I recover the coefficients for the sufficient statistics formula exactly as done for my quantitative HANK model: I set $\bar{r}$ to its true model-implied value, and then set ω and θ so that $\mathcal{C}_\tau(\omega, \theta, \bar{r})(1, 1)$ and $\mathcal{C}_\tau(\omega, \theta, \bar{r})(2, 1)$ match the extended hybrid model.

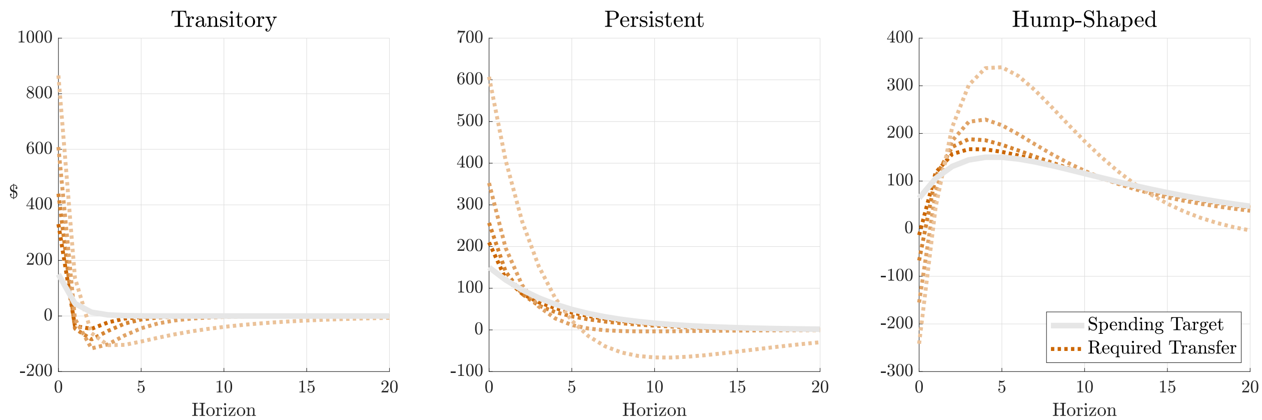

Figure C.2: Three desired net excess demand paths $\widehat{\boldsymbol{c}}^{PE}$ (grey) and the required sequences of uniform lump-sum taxes and transfers, $\mathcal{C}_\tau^{-1}(\omega, \theta, \bar{r}) \times \widehat{\boldsymbol{c}}^{PE}$ for a range of values of my sufficient statistics (orange dashed, see text), with lighter shades indicating lower MPCs ω.

policies suffice to close meaningful shortfalls in aggregate spending. To do so I repeat the exercise of Figure 2 for a range of values of my sufficient statistics $\{\omega, \theta, \bar{r}\}$. Specifically, I continue to fix $\bar{r} = 0.01$, consider $\omega \in \{0.2, 0.3, 0.4, 0.5\}$ and then pin down the slope θ by requiring the same ratio $\frac{\theta}{1-\omega}$ as in my quantitative heterogeneous-household model. The range of MPCs ω that I consider is chosen to contain and in fact go beyond the range of estimates available from the literature (e.g. Parker et al., 2013; Fagereng et al., 2018).

Results are displayed in Figure C.2. The figure illustrates my claim: across the range of empirically relevant values for my sufficient statistics, the stimulus check policies that close the indicated shortfalls in aggregate consumer spending are moderate in size. As discussed in Section 4.2, this finding is important to ensure that my theoretical results—which rely on linearization at the aggregate level—are actually practically relevant.

$\mathcal{C}_\tau^{-1}$ IN ALTERNATIVE HANK CALIBRATIONS. The analysis in Section 4.2 confirmed the accuracy of the sufficient statistics formula for $\mathcal{C}_\tau$ and $\mathcal{C}_\tau^{-1}$ in the baseline calibration of my heterogeneous-household model. I here repeat the same exercise for two materially different model calibrations: one with very low liquid wealth (implying a counterfactually large average MPC of $\omega = 0.64$) and one with a lot of liquid wealth (implying a counterfactually small average MPC of $\omega = 0.12$). Results are reported in Figures C.3 and C.4.

The takeaways from these figures are twofold. First, changing the model calibration materially affects the model-implied consumption map $\mathcal{C}_\tau$ and its inverse $\mathcal{C}_\tau^{-1}$. As expected, for low liquid wealth, the inverse $\mathcal{C}_\tau^{-1}$ looks closer to a simple spender-saver model and the required transfer stimulus policies are even smaller than in my baseline analysis. For

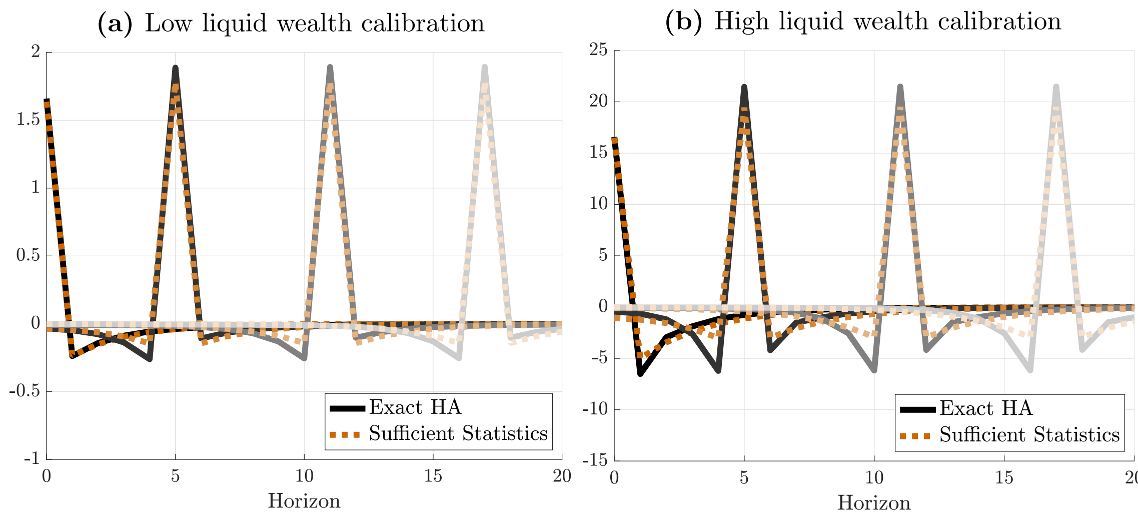

Figure C.3: See the caption of Figure 1.

high liquid wealth, the inverse $\mathcal{C}_\tau^{-1}$ looks closer to a perpetual-youth overlapping-generations model, and the transfer stimulus policies required to close a given shortfall in demand are now much larger. Second, even though $\mathcal{C}_\tau$ looks very different across calibrations, my sufficient statistics formula throughout approximates $\mathcal{C}_\tau^{-1}$ and thus the implied equivalent transfer stimulus policies very well.

A GENERALIZED SUFFICIENT STATISTICS FORMULA. My three-parameter sufficient statistics formula imposes that the same coefficient θ governs both the decay of intertemporal MPCs after spending receipt as well as the strength of anticipation effects. It is in principle straightforward to disentangle the two by allowing for an additional degree of freedom in the first row of $\mathcal{C}_\tau$. For example, one simple and natural choice would be to set[9]

$$\mathcal{C}_\tau^{(1)}(1, \bullet) = \underbrace{\left(1 - \frac{\theta}{1+\bar{r}}\right)}_{\text{MPC}} \times \underbrace{\left\{1, \psi \cdot \frac{\theta}{1+\bar{r}}, \psi \cdot \left(\frac{\theta}{1+\bar{r}}\right)^2, \ldots\right\}}_{\text{anticipation effects}}$$

where the coefficient ψ could be recovered from empirical evidence on the strength of anticipation effects in MPCs (e.g., Ganong & Noel, 2019). The rest of $\mathcal{C}_\tau$ would then be completed exactly as in the baseline sufficient statistics formula. Unsurprisingly, with this additional de-

[9]Here anticipation effects are additionally discounted by a constant factor ψ. An alternative—which I have found to be less accurate in my quantitative HANK models—is to discount future income receipts at some constant rate ψ (that is allowed to be different from θ).

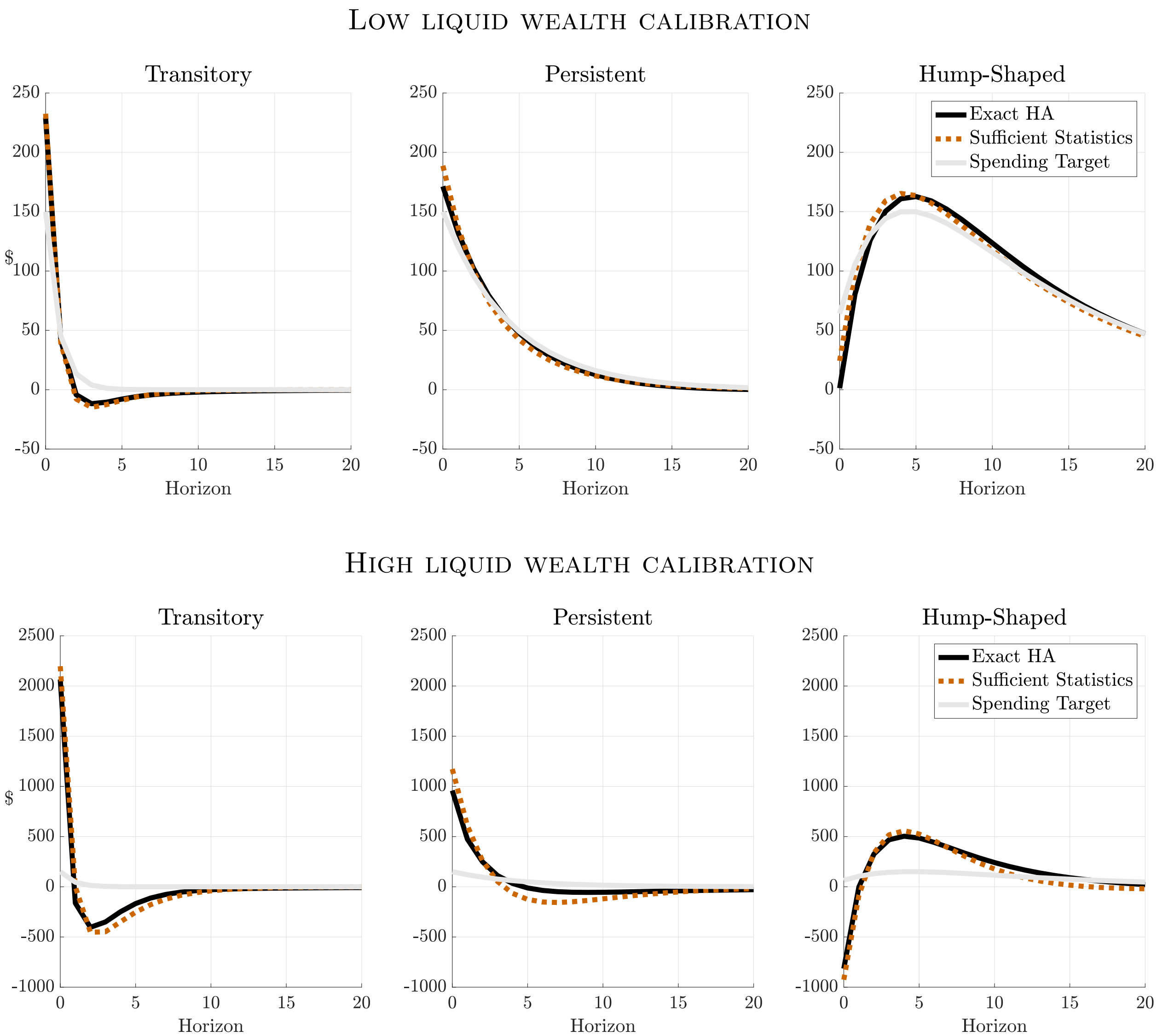

Figure C.4: See the caption of Figure 2.

gree of freedom, the approximation becomes even more accurate, with the difference between actual and approximate $\mathcal{C}_\tau$ now barely visible (figure available upon request). However, as argued in Section 4.2, my simpler three-coefficient formula already provides a very accurate approximation, so I focus on results from that simpler specification instead. Intuitively, at least in my HANK model, anticipation effects are not particularly far from being governed by the iMPC decay rate θ, and so the simpler three-parameter formula suffices.

C.4 Heterogeneous wealth effects in labor supply

This section elaborates on my discussion of the role of heterogeneity in wealth effects in labor supply across households (see Section 5.1). I first present results for an alternative union bargaining protocol and then consider an alternative model with preference heterogeneity, designed to match empirical evidence on heterogeneity in marginal propensities to earn (from Golosov et al., 2021). Results for both are reported in Figure C.5.

Alternative bargaining results. I return to my main quantitative HANK model, but with one twist: the wage-NKPC (B.4) is replaced by the alternative formulation

$$\widehat{\pi}_t^w = \kappa_w \times \left[\frac{1}{\varphi}\widehat{\ell}_t - (\widehat{w}_t - \gamma \widehat{c}_t^*)\right] + \beta \widehat{\pi}_{t+1}^w \tag{C.12}$$

where

$$c_t^* \equiv \left[\int_0^1 e_{it} c_{it}^{-\gamma} di\right]^{-\frac{1}{\gamma}} \tag{C.13}$$

This is the specification of the wage-NKPC originally derived in Auclert et al. (2018) and implied by the union objective (B.6). I note that, in this case, my policy equivalence result will not hold *exactly*: two nominal interest rate and stimulus check policies with identical direct effects on net excess demand (and so c_t) will not necessarily have identical direct effects on c_t^*, thus inducing different wedges in the economy's aggregate supply relation (C.12).

Are these differential labor supply effects likely to materially undermine the policy equivalence result? The left panel Figure C.5 suggests that the answer is "no". To construct the panel, I first compute impulse responses to a gradual monetary policy shock (with persistence 0.6), normalized to in general equilibrium increase consumption on impact by one per cent (grey). I then follow the steps in the proof of Proposition 1 to construct a stimulus check policy with identical effects on partial equilibrium consumer spending. The general equilibrium impulse response of consumption to this policy is displayed as the blue dashed line. The main takeaway is that the two lines are very close, with the stimulus check policy overall slightly more stimulative than the (not-quite-)equivalent interest rate cut.

The intuition for the results displayed in Figure C.5 is somewhat subtle. Both policies by design lead to a response of partial equilibrium consumption demand with zero present value—initially positive and then later on negative. Under my baseline wage-NKPC (6), this initial decrease and later increase in the average marginal utility of consumption leads to an initial decrease and later increase of union labor supply. With the alternative formu-

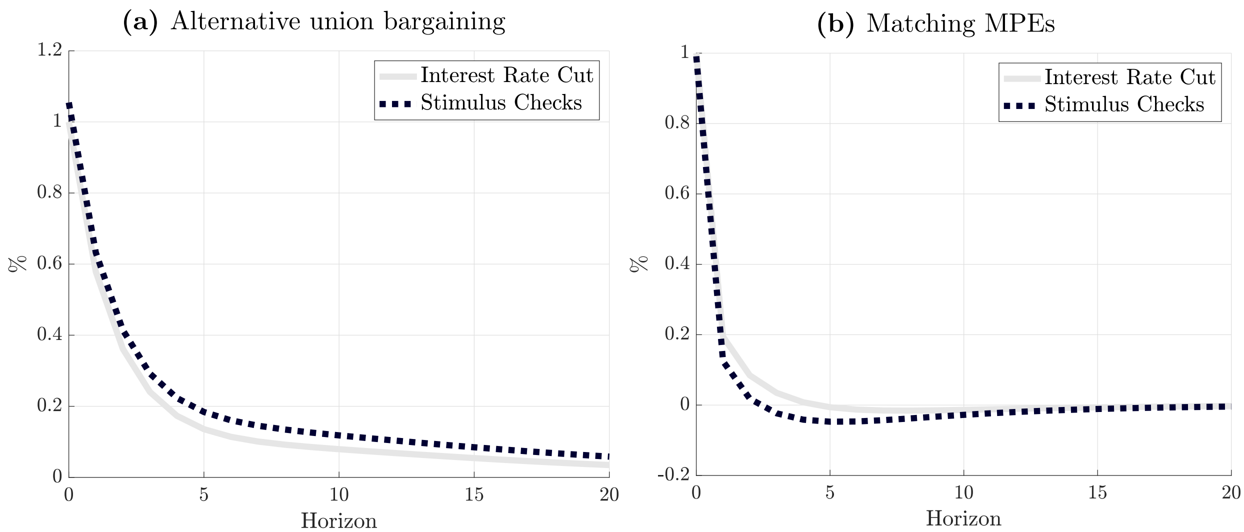

Figure C.5: Left panel: impulse response of consumption to a monetary policy shock with persistence $\rho_m = 0.6$ and peak effect of 1% (grey) and the "equivalent" stimulus check policy (blue dashed) in a HANK model with labor supply relation (C.12). Right panel: analogous figure for my hybrid spender-saver model described below.

lation (C.12), on the other hand, the labor supply response is governed by average marginal utility. Since most of the consumption adjustment following the transfer policy comes from high-MPC households (who tend to consume less), average marginal utility of consumption changes by more than after the equivalent rate cut—it drops by more initially and expands by more later on. It then follows that labor supply also contracts by more initially and expands by more later on, so the overall partial equilibrium net excess demand path (which equals consumption demand less labor supply) is more front-loaded after the stimulus check policy. But we know from Auclert et al. (2018) that more front-loaded excess demand paths lead to a bigger boom in general equilibrium, so stimulus check policy will stimulate output by more—exactly as we see in Figure C.5. This effect however is moderate in size, for two reasons. First, average marginal utility and marginal utility at the average are still not too different, so the desired labor supply response is also not too different. Second, any such transitory differences in desired labor supply matter even less in general equilibrium as long as wages are at least moderately sticky (Christiano, 2011), as is the case in my model.

Mixture model results. My second exercise is designed to speak as closely as possible to the empirical evidence reported in Golosov et al. (2021). Those authors report marginal propensities to earn (MPEs)—defined as the response of labor income to an unearned lump-sum wealth gain—of up to \$3 per additional \$100 in wealth, with the response roughly two times larger for the highest-income households compared to the lowest-income ones (see their Table 3.2). These estimates are roughly twice as large as those reported in prior work, notably Cesarini et al. (2017) (see Table J.1 of Golosov et al.). While it is straightforward to match such *average* MPEs in heterogeneous-agent models (see Auclert et al., 2020), it is much harder to match the *cross-sectional dispersion* in MPEs (which is what matters for my policy equivalence result). Intuitively, the challenge is that, in standard models of household consumption and labor supply, MPEs are increasing (in absolute value) with MPCs.[10] Since poorer households tend to have higher MPCs, this would also imply that they have higher MPEs (in absolute value), inconsistent with the empirical evidence reviewed above.

My solution is to consider a two-type spender-saver model with preference heterogeneity chosen to ensure higher MPEs for low-MPC households—i.e., the model sketched in Appendix B.4. As discussed there, that model is calibrated to be consistent with empirical evidence on household MPEs. Importantly, since the model features cross-sectional heterogeneity in wealth effects in labor supply, the policy equivalence result will not hold exactly. The right panel of Figure C.5 however reveals that it continues to approximately hold. I already in the main text gave the intuition for why the *magnitude* of the inaccuracy is so small (recall Section 5.1). I here instead focus on the *direction* of the error. The intuition is exactly opposite to that of the adjusted HANK model studied above. Savers have a larger MPE, so labor supply initially contracts by relatively more after an interest rate cut, and later on increases by relatively more. The total implied net excess demand path is thus more frontloaded after the interest rate policy, and so now the interest rate cut is slightly more expansionary, as seen in the right panel of Figure C.5.

Summary. My conclusion from the previous two experiments is that cross-sectional heterogeneity in wealth effects in labor supply is unlikely to materially threaten my headline policy equivalence result. However, it is important to note that this takeaway hinges on the equivalent stimulus check policy being moderate in size: by the evidence in Golosov et al. (2021), for very large transfers, we would expect the cross-sectional heterogeneity in labor supply responses to become larger relative to the demand stimulus of the policy (i.e., MPEs are larger relative to MPCs). The fact that equivalent stimulus check policies are moderate in size—the key takeaway of Section 4—is thus an integral part of my argument.

[10] This follows straightforwardly from the standard labor supply optimality condition with separable preferences over consumption and labor supply. See Auclert et al. (2020) for details.

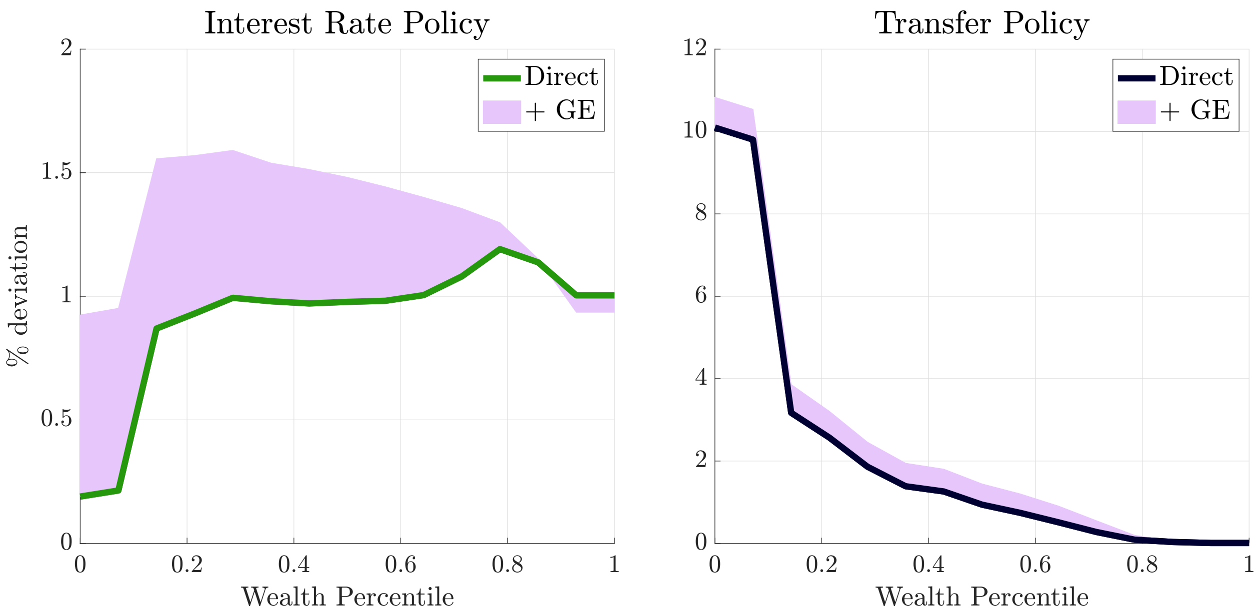

Figure C.6: Left panel: The green line shows the time-0 *direct* household spending response by liquid wealth percentile to the nominal interest rate policy of Figure 3. The shaded purple area gives the corresponding *indirect* general equilibrium effects. Right panel: Analogous figure for the equivalent uniform transfer stimulus (dark blue and shaded purple).

C.5 Non-equivalence at the household level

As discussed in Section 5.2, macro-equivalent interest rate and transfer stimulus policies need not and generally will not be equivalent in the cross-section of households. Figure C.6 provides an illustration.

The figure shows the evolution of consumption along the household wealth distribution in response to the macro-equivalent nominal interest rate and stimulus check policies displayed in Figure 3. The impact consumption response is split by household liquid wealth percentile (x-axis) into (a) the *direct* effects of the policy instrument (green and blue)—defined as the response of consumption demand to the policy instruments $\{\boldsymbol{i}_b, \boldsymbol{\tau}\}$ alone, fixing all non-policy variables at their steady state values forever—and (b) the residual *indirect* effects (shaded purple) coming from general equilibrium feedback. We see that the direct and thus overall effects are very heterogeneous in the cross-section of households.

C.6 Other model extensions

I consider two further model extensions: durable consumption, and a richer network production structure.

Durable goods. My results extend without change to a model with durables as long as durables and non-durables can be produced costlessly out of some common final good; that is, if real relative prices of the two goods are always one and we can write the aggregate resource constraint as

$$y_t = \underbrace{c_t + d_t^h - (1-\delta)d_t^h}_{e_t} \tag{C.14}$$

where e_t is total household expenditure, c_t is non-durables consumption, d_t^h is the stock of durables, and δ is the depreciation rate. Letting $\mathcal{E}_\tau$ denote the analogous derivative map for the response of total spending to lump-sum income, the key condition for my results to extend to this model is that $\mathcal{E}_\tau$ is invertible—strong Ricardian non-equivalence now applied to total spending. The details of the argument are straightforward and thus omitted: interest rate and transfer policies can perturb net excess demand for the common final good equally flexibly and are thus also equivalent in general equilibrium, by exactly the same argument as in the proof of Proposition 1.[11]

I emphasize that the assumptions underlying this extended equivalence result are empirically relevant: relative durable goods prices tend to not respond much to standard business-cycle fluctuations (House & Shapiro, 2008; McKay & Wieland, 2019; Beraja & Wolf, 2020), suggesting that the aggregation to a common aggregate resource constraint (C.14) is sensible. It is furthermore also an assumption made in recent quantitative structural explorations of durable goods spending (e.g., Berger & Vavra, 2015).

Network production. The policy equivalence result leverages properties of consumer spending behavior and as such is robust to many different possible model extensions on the production side of the economy. I here provide one illustration using a simple model of roundabout production (e.g., see Phaneuf et al., 2018).

Differently from my baseline model, intermediate goods firms now produce using both labor as well as the intermediate good itself, with production function

$$y_t = q_t^{\phi} \ell_t^{(1-\alpha)(1-\phi)}$$

[11] If non-durables and durables were not produced out of a common final good (and so their relative prices could fluctuate), then it would of course still be possible to engineer a sequence of transfers that mimics a given interest rate policy's effect on *total* spending. Nothing guarantees, however, that the *composition* of that spending would be the same. If relative prices can move then the composition will matter in general equilibrium, thus breaking equivalence.

where $\phi \in [0, 1)$ denotes the share of intermediates in production. A standard cost minimization problem gives marginal costs as

$$mc_t = \left(\frac{1}{(1-\alpha)(1-\phi)\phi^{\frac{\phi}{1-\phi}}}\right)^{1-\phi} w_t^{1-\phi}\ell_t^{\alpha(1-\phi)}$$

and so, in log deviations,

$$\widehat{mc}_t = (1-\phi)\left(\widehat{w}_t + \alpha\widehat{\ell}_t\right)$$

Following the same steps as in the derivation of (B.1) we thus find that

$$\widehat{\pi}_t = \frac{(1-\theta_p)(1-\frac{\theta_p}{1+\bar{r}})}{\theta_p}\frac{(1-\alpha)(1-\phi)}{1-\alpha+\alpha\varepsilon_p}\left(\widehat{w}_t + \alpha\widehat{\ell}_t\right) + \beta\widehat{\pi}_{t+1} \tag{C.15}$$

The only effect of roundabout production is thus to flatten the price-NKPC, leaving the headline policy equivalence result entirely unchanged.

C.7 Targeted transfers

My analysis throughout was focussed on *uniform* lump-sum taxes and transfers. This was by design: my objective was to establish that, in standard models of non-Ricardian consumption behavior, manipulating taxes and transfers *over time* can manipulate spending just like changes in intertemporal prices—that is, stimulus checks are stimulative even without any redistribution. In models with microeconomic heterogeneity (like HANK), it is of course also possible to consider transfer policies aimed at sub-populations of households and thus (in part) operational through redistribution. My results extend with little change to such alternative policy experiments.

Recall from the proof of Proposition 1 that the key requirement for policy equivalence is that, for any excess demand sequence $\widehat{\boldsymbol{c}}$ with zero net present value, we can find a transfer policy that induces a net excess demand path of $\widehat{\boldsymbol{c}}$. To see how this can be done using targeted transfers, consider a transfer targeted at some subgroup of households (group a) and financed using taxes on another subgroup (group b). I denote the transfer to group a by $\widehat{\boldsymbol{\tau}}^x$ and write the corresponding tax financing as $\widehat{\boldsymbol{\tau}}^e \equiv \mathcal{T}_\tau\widehat{\boldsymbol{\tau}}^x$, where $\mathcal{T}_\tau$ is such that

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t(\widehat{\tau}_t^x + \widehat{\tau}_t^e(\widehat{\boldsymbol{\tau}}^x)) = 0$$

Letting $\mathcal{C}_\tau^{(a)}$ and $\mathcal{C}_\tau^{(b)}$ denote the consumption derivative matrices for subgroups a and b, respectively, the effect of any given transfer policy $\widehat{\boldsymbol{\tau}}^x$ on net excess demand is given as

$$\underbrace{\left(\mathcal{C}_\tau^{(a)} + \mathcal{C}_\tau^{(b)}\mathcal{T}_\tau\right)}_{\equiv \mathcal{C}_\tau^x}\widehat{\boldsymbol{\tau}}^x$$

Analogously to the proof of Proposition 1, a sufficient condition for policy equivalence is now simply that every net excess demand path with zero net present value lies in the image of $\mathcal{C}_\tau^x$. Differently from my main analysis, characterizing $\mathcal{C}_\tau^x$ does not require MPCs averaged across the entire household cross-section, but MPCs averaged across the subgroups a and b.

For practical policy purposes, there are two key differences between my uniform policies and such targeted policies. First, the latter also work explicitly through redistribution across households, and thus in particular can affect net excess demand even with period-by-period budget balance and without any fluctuations in aggregate government debt. Second, it is unclear ex ante whether targeted transfers need to be larger or smaller in per capita terms. On the one hand, to engineer a given spending response by targeting a smaller group of households, the required transfer size per capita increases mechanically. On the other hand, if targeted households have larger MPCs, the required transfer decreases in size. I leave a detailed characterization of such macro-equivalent targeted transfers to future work.

D Equivalence in terms of policy rules

This appendix elaborates on the implications of my equivalence results for systematic policy *rules*. Appendix D.1 begins by formalizing the claims made in Section 3.3 from a sequence-space perspective. Appendix D.2 then translates all arguments to recursive notation. Finally, in Appendix D.3, I provide a worked-out example, deriving the transfer rule that replicates a standard Taylor rule in the context of the perpetual-youth consumption-savings model.

D.1 From policy paths to rules

I augment the baseline model of Section 2.1 to additionally feature wedges $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$ to the aggregate consumption function (2) as well as the Phillips curves (5) - (6), corresponding to reduced-form representations of canonical demand and supply shocks. Given a specification of policy in the form of policy rules, a bounded perfect-foresight transition path in response to any of these wedges corresponds to impulse response functions in the analogous linearized economy with aggregate risk (Boppart et al., 2018; Auclert et al., 2019). I will argue that, for any interest rate-only policy rule, there exists a transfer-only policy rule that implies the exact same impulse response of macroeconomic aggregates, including in particular aggregate output and inflation.

I will present my equivalence results for two particular kinds of interest rate policy rules: implicit targeting rules and explicit instrument rules (Giannoni & Woodford, 2002).

Implicit rules. A classical *implicit* targeting rule specifies a relationship between policy targets. For a standard dual-mandate policymaker, and written in perfect-foresight notation (e.g., see McKay & Wolf, 2022), such a rule takes the general form

$$\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} = \mathbf{0} \tag{D.1}$$

(D.1) specifies a relationship between inflation and output along the perfect-foresight transition path. It nests as special cases strict inflation targeting ($\widehat{\pi}_t = 0$ and so $\mathcal{B}_\pi = I$, $\mathcal{B}_y = \mathbf{0}$), strict output targeting ($\widehat{y}_t = 0$ and so $\mathcal{B}_\pi = \mathbf{0}$, $\mathcal{B}_y = I$), as well as the canonical optimal implicit targeting rule of a dual-mandate policymaker, (25), mentioned in the main text. Strong Ricardian non-equivalence is sufficient to ensure that, if a rule of the general form (D.1) can be (uniquely) implemented using an interest rate-only policy (i.e., with a policy as in Definition 2), then the same is true for a transfer-only policy.

Corollary D.1. *Suppose that, given a sequence of shocks $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$, the implicit targeting rule* (D.1) *implemented through an interest rate-only policy induces a unique equilibrium. Then, under the conditions of Proposition 1 and strong Ricardian non-equivalence, the rule* (D.1) *implemented through a transfer-only policy also induces a unique equilibrium featuring the same aggregate allocation.*

When implementing the targeting rule (D.1) through interest rate policy, the policymaker in the background sets nominal interest rates so that aggregate demand is consistent with output and inflation sequences satisfying (D.1). By my high-level assumption of strong Ricardian non-equivalence, she can engineer that exact same required time path of aggregate excess demand through transfers—she simply needs to set transfers equal to

$$\widehat{\boldsymbol{\tau}} \;=\; \mathcal{C}_{\tau}^{-1} \;\times\; \text{demand target}$$

The proof of Corollary D.1 formalizes this argument.

Explicit rules. The same logic as above extends to *explicit* instrument rules—that is, rules that explicitly specify the value of the policy instrument as a function of observables. Again written in linearized perfect-foresight notation, a typical explicit interest rate rule takes the general form

$$\widehat{\boldsymbol{i}}_b \;=\; \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}} \tag{D.2}$$

(D.2) here specifies a mapping from inflation and output into nominal interest rates along the perfect-foresight transition path. For example, a simple Taylor rule would take the form

$$\widehat{i}_{b,t} \;=\; \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t, \quad t = 0, 1, 2, \ldots$$

and so $\mathcal{B}_\pi = \phi_\pi \times I$, $\mathcal{B}_y = \phi_y \times I$. With interest rates set according to (D.2), taxes under my definition of an interest rate-only policy rule adjust in the background to ensure a balanced government budget (recall Definition 2). In particular, for my environment in Section 2.1, taxes by (8) follow

$$\widehat{\boldsymbol{\tau}} \;=\; \tau_\ell \bar{w} \bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) - \bar{b}\widehat{\boldsymbol{i}}_{b,-1} + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}} \tag{D.3}$$

As before, strong Ricardian non-equivalence is sufficient to ensure that the equilibrium dynamics induced by a rule of the form (D.2)-(D.3) can equivalently be implemented uniquely through an explicit transfer-only policy rule.

Corollary D.2. *Suppose that, given a sequence of shocks $\{\boldsymbol{\varepsilon}^c, \boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\}$, the explicit interest rate rule* (D.2)-(D.3) *induces a unique equilibrium. Then, under the conditions of* Proposition 1 *and strong Ricardian non-equivalence, the transfer-only policy rule*

$$\widehat{\boldsymbol{\tau}} = \underbrace{\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}} + \widehat{\boldsymbol{\ell}}) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}}_{\text{deficit response}} + \underbrace{\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}\right)}_{\text{active demand management}} \tag{D.4}$$

together with $\widehat{\boldsymbol{i}}_b = \mathbf{0}$ uniquely implements the exact same aggregate allocation.

(D.4) is an explicit instrument rule for taxes and transfers. Just like (D.2) did for interest rates, (D.4) is a rule that gives the time path of taxes and transfers as a function of time paths of inflation and output (as well as the deficit). Intuitively, the rules (D.2) and (D.4) are equivalent because they both imply the same mapping from macroeconomic aggregates—output and inflation—into aggregate demand. The only difference is the instrument that is used to achieve that mapping.

D.2 A recursive aggregate-risk perspective

The equivalent policy rules characterized in Appendix D.1 were written in sequence-space perfect-foresight notation. Here I discuss how to interpret such rules in the analogous linearized economy with aggregate risk. For implicit rules no further arguments are needed, simply because the policy rule does not directly involve the instrument. For example, written in recursive aggregate-risk notation, suppose the economy was closed with either the interest rate-only rule

$$\begin{aligned} \widehat{\pi}_t + \lambda(\widehat{y}_t - \widehat{y}_{t-1}) &= 0 \\ \widehat{\tau}_t &= (1+\bar{r})\bar{b}\widehat{\pi}_t + \tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + \widehat{b}_t \end{aligned}$$

or the transfer-only rule

$$\begin{aligned} \widehat{\pi}_t + \lambda(\widehat{y}_t - \widehat{y}_{t-1}) &= 0 \\ \widehat{i}_{b,t} &= 0. \end{aligned}$$

Combining Corollary D.1 with the equivalence of linearized perfect-foresight solutions and linearized shock impulse responses (e.g., Boppart et al., 2018; Auclert et al., 2019), we can conclude that aggregate outcomes in the stochastic linearized economy with aggregate risk would be exactly the same under the two specifications of policy given above. I will thus from now on focus on explicit rules, where additional arguments are needed.

THE INTEREST RATE RULE. Consider first the general perfect-foresight explicit interest rate rule (D.2), re-stated here for convenience:

$$\widehat{\boldsymbol{i}}_b \;=\; \mathcal{B}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{B}_y \widehat{\boldsymbol{y}}$$

This rule specifies a relationship between sequences of interest rates—the policy instrument—and sequences of output and inflation—the arguments of the policy rule—along the perfect-foresight transition path. My objective now is to provide an interpretation of that rule in the analogous linearized economy with aggregate risk.

The key building block result is yet again that linearized perfect-foresight transition paths are identical to *shock impulse responses*—i.e., to conditional expectations—in analogous linearized economies with aggregate risk. Specifically, begin by considering the analogous linearized economy with aggregate risk at its initial date 0, subject to some initial date-0 shocks. The explicit nominal interest rate rule (D.2) then simply says that current and expected future rates at date 0 satisfy

$$\mathbb{E}_0\left(\widehat{\boldsymbol{i}}_b^0\right) \;=\; \mathbb{E}_0\left[\mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^0 + \mathcal{B}_y \widehat{\boldsymbol{y}}^0\right] \tag{D.5}$$

where the notation $\boldsymbol{x}^t = (x_t, x_{t+1}, \dots)'$ indicates time paths from date t onwards. That is, current and future expected interest rates are given as a simple function of date-0 expectations of current and future output. At date 1 additional shocks hit the economy; adding up impulse responses to the initial date-0 shocks and the new date-1 shocks, we see that interest rates at date 1 satisfy

$$\mathbb{E}_1\left(\widehat{\boldsymbol{i}}_b^1\right) \;=\; \underbrace{\mathbb{E}_0\left(\widehat{\boldsymbol{i}}_b^1\right)}_{\text{response to date-0 shocks}} \;+\; \underbrace{\mathcal{B}_\pi \times \left[\mathbb{E}_1\left(\widehat{\boldsymbol{\pi}}^1\right) - \mathbb{E}_0\left(\widehat{\boldsymbol{\pi}}^1\right)\right] + \mathcal{B}_y \times \left[\mathbb{E}_1\left(\widehat{\boldsymbol{y}}^1\right) - \mathbb{E}_0\left(\widehat{\boldsymbol{y}}^1\right)\right]}_{\text{response to date-1 shocks}} \tag{D.6}$$

or more compactly

$$\hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{i}}_b^1\right) \;=\; \mathcal{B}_\pi \times \hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{\pi}}^1\right) \;+\; \mathcal{B}_y \times \hat{\mathbb{E}}_{1,0}\left(\widehat{\boldsymbol{y}}^1\right) \tag{D.7}$$

where $\hat{\mathbb{E}}_{t,t-1}$ denotes the *change* in expectations between t and $t-1$. Continuing recursively, we in general find that interest rates in the linearized economy with aggregate risk satisfy the recursion

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{i}}_b^t\right) \;=\; \mathcal{B}_\pi \;\times\; \hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\pi}}^t\right) \;+\; \mathcal{B}_y \;\times\; \hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{y}}^t\right) \tag{D.8}$$

In words, at each t, the policymaker revises current and expected future paths of nominal interest rates in line with revisions about expectations of future inflation and output. This is an explicit instrument rule in the sense of Giannoni & Woodford (2002): it specifies, at each date t, the current and expected future values of the policy instrument as a function of lagged, current, and expected future values of macro aggregates. Note that the dependence on lagged aggregates is here encoded in the lagged instrument term $\mathbb{E}_{t-1}\left(\widehat{\boldsymbol{i}}_b^t\right)$.

The preceding discussion applies for arbitrarily complicated matrices $\{\mathcal{B}_\pi, \mathcal{B}_y\}$ specifying the mapping from expectations of macro aggregates to expectations of interest rates. Canonical recursive policy rules—like textbook Taylor rules—on the other hand restrict this mapping to have a particularly simple form. A standard Taylor rule maps into the general explicit rule form (D.2) with $\mathcal{B}_\pi = \phi_\pi \times I$ and $\mathcal{B}_y = \phi_y \times I$, where I denotes the identity map. If $\{\mathcal{B}_\pi, \mathcal{B}_y\}$ take such a simple diagonal form, then the in principle very complicated expectational revisions embedded in (D.8) are equivalent to one simple static equation—the familiar relation

$$\widehat{i}_{b,t} \;=\; \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t \tag{D.9}$$

Both (D.8) and (D.9) are valid explicit rules: at each date t, they specify a mapping from lagged, current, and expected future inflation and output into current and expected future policy instruments. The only difference is that in one case this mapping is restricted to have a very simple form, while in the other it is allowed to be much more general.

The equivalent transfer rule. Now consider the macro-equivalent transfer rule, written in perfect-foresight sequence-space notation as (D.4). By exactly the same arguments as above, it corresponds to the following recursive formulation in the analogous linearized economy with aggregate risk:

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) \;=\; \hat{\mathbb{E}}_{t,t-1}\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^t + \widehat{\boldsymbol{\ell}}^t) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^t \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\mathcal{B}_\pi\widehat{\boldsymbol{\pi}}^t + \mathcal{B}_y\widehat{\boldsymbol{y}}^t\right)\right] \tag{D.10}$$

Equation (D.10) is an explicit instrument rule in exactly the same way as (D.8): it specifies, at each date t, the current and expected future values of the policy instrument—here transfers—as a function of lagged, current, and expected future values of macro aggregates. Of course, since at this point I am imposing no further restrictions on the product $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$, a rule that may be "simple" in interest rate space—like a conventional Taylor rule, as discussed above—

may be complicated in transfer space, in the sense that the general set of restrictions (D.10) cannot be reduced to a single static relation like (D.9). The next subsection provides an explicit worked-out example in the special case of a one-type perpetual-youth economy. In that particular setting, policy rules that are simple in interest rate space also turn out to be simple in transfer space, and vice-versa, simply because the matrix product $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$ takes a very simple form.

D.3 A worked-out example

I consider a special case of my economy in Section 2.1 where the aggregate consumption function $\mathcal{C}(\bullet)$ is that implied by a one-type perpetual-youth consumer demand structure, as discussed in Section 2.2. I suppose that the monetary policymaker wishes to replicate the outcomes implied by the Taylor-type rule (D.9) by relying on transfers instead. By (D.10), the recursively written explicit transfer rule that does so is

$$\hat{\mathbb{E}}_{t,t-1}\left(\widehat{\boldsymbol{\tau}}^t\right) \;=\; \hat{\mathbb{E}}_{t,t-1}\left[\tau_\ell \bar{w}\bar{\ell}(\widehat{\boldsymbol{w}}^t + \widehat{\boldsymbol{\ell}}^t) + (1+\bar{r})\bar{b}\widehat{\boldsymbol{\pi}}^t \;+\; \mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}\left(\phi_\pi \widehat{\boldsymbol{\pi}}^t + \phi_y \widehat{\boldsymbol{y}}^t\right)\right] \tag{D.11}$$

The particular one-type perpetual-youth structure allows us to now further simplify the term $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$. Putting together the results from Appendices C.1 and C.2, we obtain

$$\mathcal{C}_\tau^{-1} \times \tilde{\mathcal{C}}_{i_b} \;=\; \frac{1}{\gamma}\frac{1}{1-\frac{\theta}{1+\bar{r}}}\begin{pmatrix} -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & 0 & 0 & \cdots \\ \frac{\theta}{1-\theta} & -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & 0 & \cdots \\ 0 & \frac{\theta}{1-\theta} & -\frac{\frac{\theta}{1+\bar{r}}}{1-\theta} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{D.12}$$

Plugging (D.12) into (D.11), we see that mapping the simple Taylor rule (D.9) into transfer space does indeed result in an almost equally simple explicit transfer-only rule:

$$\widehat{\tau}_t = \underbrace{\tau_\ell \bar{w}\bar{\ell}(\widehat{w}_t + \widehat{\ell}_t) + (1+\bar{r})\bar{\pi}\widehat{\pi}_t}_{\text{deficit response}} + \underbrace{\frac{1}{\gamma\left(1-\frac{\theta}{1+\bar{r}}\right)}\left[-\frac{\frac{\theta}{1+\bar{r}}}{1-\theta}\left(\phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t\right) + \frac{\theta}{1-\theta}\left(\phi_\pi\widehat{\pi}_{t-1} + \phi_y\widehat{y}_{t-1}\right)\right]}_{\text{active demand management}} \tag{D.13}$$

If transfers are set according to this simple policy rule, then they in the linearized equilibrium with aggregate risk indeed satisfy the general recursion (D.11). Thus, in this particular environment, an interest rate policy that responds to contemporaneous macro aggregates is equivalent to a still quite simple transfer-only policy that responds to current and one-period-lagged aggregates, with the response coefficients given from (D.12). The rule representation here is so simple because the map $\mathcal{C}_\tau^{-1}\tilde{\mathcal{C}}_{i_b}$—while not proportional to an identity matrix, like $\{\mathcal{B}_\pi, \mathcal{B}_y\}$—is nevertheless quite special: it is tridiagonal with repeating rows, and so we can summarize the potentially very complicated expectation revisions in (D.11) with just one simple equation, (D.13). This is exactly analogous to the Taylor rule (D.9) being equivalent to the more complicated general expression (D.8) when $\mathcal{B}_\pi = \phi_\pi \times I$ and $\mathcal{B}_y = \phi_y \times I$.

E Proofs and auxiliary lemmas

E.1 Proof of Lemma 1

Re-arranging (C.2) we obtain

$$\widehat{\tau}_t + \frac{1}{\beta}\widehat{b}_{t-1} = \frac{[1-\theta(1-\beta\theta)]\widehat{c}_t - \beta\theta\widehat{c}_{t+1}}{(1-\beta\theta)(1-\theta)}.$$

From the budget constraint (C.1) it follows that the left-hand side equals $\widehat{c}_t + \widehat{b}_t$. Re-arranging, we thus obtain

$$\widehat{b}_t = \frac{\beta\theta}{(1-\beta\theta)(1-\theta)}(\widehat{c}_t - \widehat{c}_{t+1})$$

and so, from the Euler equation (C.2),

$$\widehat{\tau}_t = \frac{[1-\theta(1-\beta\theta)]\widehat{c}_t - \beta\theta\widehat{c}_{t+1}}{(1-\beta\theta)(1-\theta)} - \frac{\theta}{(1-\beta\theta)(1-\theta)}(\widehat{c}_{t-1} - \widehat{c}_t).$$

Stacking these coefficients as the matrix $\mathcal{C}_\tau^{-1}$ (with $\beta(1+\bar{r}) = 1$), we obtain (20). □

E.2 Proof of Proposition 2

Key to the proof is the following auxiliary lemma.

Lemma E.1. *If $\theta_i < 1$, then, for $\bar{r}$ is sufficiently close to (but weakly above) zero, $\mathcal{C}_\tau^i$ is a positive operator (i.e., $\boldsymbol{\tau}'\mathcal{C}_\tau^i\boldsymbol{\tau} > 0$ for any $\boldsymbol{\tau} \neq \mathbf{0}$).*

Proof. $\mathcal{C}_\tau^i$ is positive if and only if its inverse is positive, so I will instead establish that $(\mathcal{C}_\tau^i)^{-1}$ is positive. Recall that

$$(\mathcal{C}_\tau^i)^{-1} = \begin{pmatrix} a_i & c_i & 0 & 0 & \dots \\ b_i & d_i & c_i & 0 & \dots \\ 0 & b_i & d_i & c_i & \dots \\ 0 & 0 & b_i & d_i & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where

$$a_i = \frac{1-\theta_i\left(1-\frac{\theta_i}{1+\bar{r}}\right)}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)}$$

$$
\begin{aligned}
c_i &= -\frac{\frac{\theta_i}{1+\bar{r}}}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)} \\
b_i &= -\frac{\theta_i}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)} \\
d_i &= \frac{1+\frac{\theta_i^2}{1+\bar{r}}}{\left(1-\frac{\theta_i}{1+\bar{r}}\right)(1-\theta_i)}
\end{aligned}
$$

From now on, to simplify notation, I will suppress all i subscripts. To prove that $\mathcal{C}_\tau^{-1}$ is positive I will decompose $\tilde{\mathcal{C}}_\tau^{-1} \equiv \frac{1}{2}\left(\mathcal{C}_\tau^{-1} + \mathcal{C}_\tau^{-1\prime}\right)$ or

$$
\tilde{\mathcal{C}}_\tau^{-1} = \begin{pmatrix}
a & \frac{b+c}{2} & 0 & 0 & \dots \\
\frac{b+c}{2} & d & \frac{b+c}{2} & 0 & \dots \\
0 & \frac{b+c}{2} & d & \frac{b+c}{2} & \dots \\
0 & 0 & \frac{b+c}{2} & d & \dots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

as $\tilde{\mathcal{C}}_\tau^{-1} = \mathcal{L}'\mathcal{L}$, where $\mathcal{L}$ is upper-triangular with real and positive diagonal entries. If I can find such an operator then $\tilde{\mathcal{C}}_\tau^{-1}$ (and so $\mathcal{C}_\tau^{-1}$) is positive. For this consider the candidate

$$
\mathcal{L} = \begin{pmatrix}
\sqrt{\delta_1} & \frac{\alpha}{\sqrt{\delta_1}} & 0 & 0 & \dots \\
0 & \sqrt{\delta_2} & \frac{\alpha}{\sqrt{\delta_2}} & 0 & \dots \\
0 & 0 & \sqrt{\delta_3} & \frac{\alpha}{\sqrt{\delta_3}} & \dots \\
\vdots & \vdots & \vdots & \ddots &
\end{pmatrix}
$$

where $\alpha \equiv \frac{b+c}{2}$ and δ_j follows the recursion

$$
\delta_j = d - \frac{\alpha^2}{\delta_{j-1}}
$$

with initial condition $\delta_1 = a > 0$. If $\delta_i > 0$ for all i and if the δ_i's are bounded, then it is straightforward to verify that $\tilde{\mathcal{C}}_\tau^{-1} = \mathcal{L}'\mathcal{L}$, and so that $\tilde{\mathcal{C}}_\tau^{-1}$ as well as $\mathcal{C}_\tau^{-1}$ (and so $\mathcal{C}_\tau$) are positive, bounded operators. To establish these properties of the δ_i's, note first of all that

$$
\alpha = \frac{b+c}{2} = -\frac{1}{2}\frac{\theta + \frac{\theta}{1+\bar{r}}}{\left(1-\frac{\theta}{1+\bar{r}}\right)(1-\theta)} < 0
$$

is a well-defined, finite number. Now consider the recursion for δ_j. Write $\delta_j = f(\delta_{j-1})$ and note that, for $\delta_{j-1} > 0$, $f(\bullet)$ is a strictly increasing, strictly concave function. The fixed points $\bar{\delta}$ and $\underline{\delta}$ satisfy

$$\bar{\delta} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - \alpha^2}, \quad \underline{\delta} = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \alpha^2}$$

Let me first establish that the argument under the square root is indeed strictly positive. For this it suffices to show that $d > |b + c|$, or that

$$1 + \frac{\theta^2}{1+\bar{r}} > \theta + \frac{\theta}{1+\bar{r}}$$

which holds for $\bar{r} \geq 0$ and $\theta < 1$. Next I will argue that $a > \underline{\delta}$. This requires

$$1 - \theta\left(1 - \frac{\theta}{1+\bar{r}}\right) > \frac{1}{2}\left(1 + \frac{\theta^2}{1+\bar{r}}\right) - \frac{1}{2}\sqrt{\left(1 + \frac{\theta^2}{1+\bar{r}}\right)^2 - \left(\theta + \frac{\theta}{1+\bar{r}}\right)^2}$$

Note that, for $\bar{r} = 0$, this becomes

$$1 - \theta(1-\theta) > \frac{1}{2}(1+\theta^2) - \frac{1}{2}\sqrt{(1+\theta^2)^2 - 4\theta^2}$$

which holds for $\theta < 1$. Thus $a > \underline{\delta}$, and so this also holds for $\bar{r} \in (0, r^*)$ for some upper bound r^*. Proceeding identically we can show that $a < \bar{\delta}$ for $\bar{r}$ sufficiently close to zero. But then it follows from the properties of $f(\bullet)$ that the sequence $\{\delta_j\}$ will converge monotonically from $\delta_1 = a$ to $\bar{\delta} > a > 0$. Thus the entire sequence consists of well-defined, finite, strictly positive numbers. We conclude that the candidate operator $\mathcal{L}$ exists and is well-defined. It follows that $\mathcal{C}_\tau^{-1}$ and so $\mathcal{C}_\tau$ are positive, as claimed. $\square$

The proof of Proposition 2 leverages Lemma E.1.[12] I will first of all show that the presence of *some* constrained households (i.e., $\theta_i < 1$ for at least some i with $\mu_i > 0$) suffices to ensure that transfer policy can induce any square-summable $\widehat{\boldsymbol{c}}$. I decompose

$$\mathcal{C}_\tau = \underbrace{\sum_{i=1}^{N} \mu_i \mathcal{C}_\tau^i}_{\equiv |\mathcal{I}| \times \mathcal{C}_\tau^{\mathcal{I}}} + (1 - |\mathcal{I}|)\mathcal{C}_\tau^R$$

where $\mathcal{I}$ denotes the set of groups with $\theta_i < 1$, and where $|\mathcal{I}| \equiv \sum_{i \in \mathcal{I}} \mu_i > 0$ by assumption. I

[12] I thank an anonymous referee for several suggestions that helped fix technical issues with the proof.

begin by studying $\mathcal{C}_\tau^{\mathcal{I}}$. This operator maps square-summable sequences into square-summable sequences, so from now on I will operate in ℓ^2, not ℓ^∞.[13] First suppose that $\bar{r}=0$. $\mathcal{C}_\tau^{\mathcal{I}}$ is then the sum of linear operators that are, by Lemmas 1 and E.1, both invertible and positive. Since $\bar{r}=0$ they are also symmetric, and so we can conclude from Proposition 1.5 in Mortad (2020) that the sum $\mathcal{C}_\tau^{\mathcal{I}}$ is also invertible. Next, by the representation of $\mathcal{C}_\tau^i$ in Appendix C.1, $\mathcal{C}_\tau^{\mathcal{I}}$ is continuous in $\bar{r}$. Since in any Banach space the set of invertible operators is open, it follows that $\mathcal{C}_\tau^{\mathcal{I}}$ is also invertible for $\bar{r}$ sufficiently close to (but above) 0.

Now return to $\mathcal{C}_\tau$. It remains to establish that any square-summable sequence $\widehat{\boldsymbol{c}}$ with zero net present value lies in the image of $\mathcal{C}_\tau$. By the previous results it follows that we can find a square-summable $\widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})$ such that

$$\mathcal{C}_\tau^{\mathcal{I}} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}}) = \widehat{\boldsymbol{c}}$$

Note that $\widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})$ necessarily has zero net present value, since $\mathcal{C}_\tau^{\mathcal{I}}$ embeds all individual agents' budget constraints (C.1). Now consider setting

$$\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}) = \frac{1}{|\mathcal{I}|} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})$$

Then, since $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}})$ also has zero net present value, it follows from the properties of $\mathcal{C}_\tau^R$ that

$$\mathcal{C}_\tau \cdot \widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{c}}) = \underbrace{|\mathcal{I}|\mathcal{C}_\tau^{\mathcal{I}} \cdot \frac{1}{|\mathcal{I}|} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})}_{=\widehat{\boldsymbol{c}}} + \underbrace{(1-|\mathcal{I}|)\mathcal{C}_\tau^R \cdot \frac{1}{|\mathcal{I}|} \cdot \widehat{\boldsymbol{\tau}}_{\mathcal{I}}(\widehat{\boldsymbol{c}})}_{=\boldsymbol{0}} = \widehat{\boldsymbol{c}}.$$

It finally remains to note that, if $\theta_i < 1$, then $\tilde{\mathcal{C}}_{i_b}^i \cdot \widehat{\boldsymbol{i}}_b$ is square-summable (for square-summable $\widehat{\boldsymbol{i}}_b$), and thus so is $\widehat{\boldsymbol{c}} = \widehat{\boldsymbol{c}}_{i_b}^{PE} = \tilde{\mathcal{C}}_{i_b}\widehat{\boldsymbol{i}}_b$ (if $\theta_i < 1 \; \forall i$).[14] This completes the argument. □

E.3 Proof of Proposition 3

It follows from the discussion in Appendix C.1 that

$$\mathcal{C}_\tau(1,1) \;=\; \sum_{i=1}^N \mu_i \left(1 - \frac{\theta_i}{1+\bar{r}}\right)$$

[13] With the entries of $\mathcal{C}_\tau^i$ decaying exponentially in rows and columns away from the main diagonal (recall Lemma C.1 with $\theta_i < 1$), $\mathcal{C}_\tau^i$ is a bounded linear operator on ℓ^2; see also Auclert et al. (2023, Section 3.3).

[14] By (C.11), the off-diagonal entries of $\tilde{\mathcal{C}}_{i_b}^i$ also decay exponentially in rows and columns away from the main diagonal, like $\mathcal{C}_\tau^i$.

$$\mathcal{C}_\tau(2,1) \;=\; \sum_{i=1}^{N} \mu_i \theta_i \left(1 - \frac{\theta_i}{1+\bar{r}}\right)$$

We thus see that $\theta_i < 1$ for at least one i with $\mu_i > 0$ suffices to ensure that both $\mathcal{C}_\tau(1,1) > \frac{\bar{r}}{1+\bar{r}}$ and $\mathcal{C}_\tau(1,1) > \mathcal{C}_\tau(2,1)$, as claimed. □

E.4 Proof of Proposition 4

By assumption, the allocation $\{\pi_t^*, y_t^*\}_{t=0}^{\infty}$ is implementable using an interest rate-only policy tuple $\{i_{b,t}^*, \tau_t^*, 0\}_{t=0}^{\infty}$. Now consider the alternative policy tuple $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \widehat{\tau}_{f,t}^\dagger\}_{t=0}^{\infty}$ where

$$\widehat{\boldsymbol{\tau}}_f^\dagger = \mathcal{I}_{\tau_f}^{-1} \mathcal{I}_{i_b} \widehat{\mathbf{i}}_b^* \tag{E.1}$$

and

$$\widehat{\boldsymbol{\tau}}^\dagger = \mathcal{C}_\tau^{-1} \left[\mathcal{C}_{i_b} \widehat{\mathbf{i}}_b^* + \mathcal{C}_d \left(\mathcal{D}_{i_b} \widehat{\mathbf{i}}_b^* - \mathcal{D}_{\tau_f} \widehat{\boldsymbol{\tau}}_f^\dagger\right)\right] \tag{E.2}$$

I now claim that this policy tuple similarly engineers the allocation $\{\pi_t^*, y_t^*\}_{t=0}^{\infty}$. First, with $\widehat{\boldsymbol{\tau}}_f^\dagger$ set as in (E.1), the investment, output and labor demand paths are unchanged; however, as remarked in Appendix B.5, the dividend paths may be different. The transfer path $\widehat{\boldsymbol{\tau}}^\dagger$ is constructed to offset both the missing monetary stimulus as well as neutralize any potential dividend-related effects: to see this, note that we have

$$\begin{aligned}
\widehat{\mathbf{c}}^\dagger \;&=\; \mathcal{C}_\tau(\widehat{\boldsymbol{\tau}}^* + \widehat{\boldsymbol{\tau}}^\dagger) + \mathcal{C}_d \mathcal{D}_{\tau_f} \widehat{\boldsymbol{\tau}}_f^\dagger + \text{non-policy terms} \\
&=\; \mathcal{C}_\tau \widehat{\boldsymbol{\tau}}^* + \mathcal{C}_{i_b} \widehat{\mathbf{i}}_b^* + \mathcal{C}_d \mathcal{D}_{i_b} \widehat{\mathbf{i}}_b^* + \text{non-policy terms} = \widehat{\mathbf{c}}^*
\end{aligned}$$

Next note that, since they induce the same paths of consumption, investment, hours worked and production, and since by assumption wages are unchanged, the initial policy $\{i_{b,t}^*, \tau_t^*, 0\}_{t=0}^{\infty}$ and the new policy $\{\bar{i}_b, \tau_t^* + \widehat{\tau}_t^\dagger, \widehat{\tau}_{f,t}^\dagger\}_{t=0}^{\infty}$ have the same present value in the augmented government budget constraint (B.18), exactly as in the proof of Proposition 1. With $\lim_{t\to\infty} \widehat{b}_t = 0$ in the initial equilibrium, it then follows that we must also have $\lim_{t\to\infty} \widehat{b}_t = 0$ in the new one, as required. All other model equations are unaffected, so the guess is verified. □

E.5 Proof of Lemma C.1

I will guess and verify that lagged wealth is the only endogenous state, so the decision rules take the general form

$$\begin{aligned}
\widehat{c}_t &= \vartheta_{cb}\widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{cyh}\widehat{\tau}_{t-h} \\
\widehat{b}_t &= \vartheta_{bb}\widehat{b}_{t-1} + \sum_{h=0}^{H} \vartheta_{byh}\widehat{\tau}_{t-h}
\end{aligned}$$

Plugging into the optimality conditions (C.1) - (C.2) and matching coefficients, we get the following system of equations characterizing behavior in response to an anticipated income shock H periods into the future:

$$\vartheta_{cb} + \vartheta_{bb} - \frac{1}{\beta} = 0 \tag{E.3}$$

$$\vartheta_{c\tau h} + \vartheta_{b\tau h} = 0, \quad h = 0, 1, \ldots, H-1 \tag{E.4}$$

$$\vartheta_{c\tau H} + \vartheta_{b\tau H} = 1 \tag{E.5}$$

$$[1 - \theta(1 - \beta\theta)]\, \vartheta_{cb} - \beta\theta\vartheta_{cb}\vartheta_{bb} - (1 - \beta\theta)(1 - \theta)\frac{1}{\beta} = 0 \tag{E.6}$$

$$[1 - \theta(1 - \beta\theta)]\, \vartheta_{c\tau h} - \beta\theta\, [\vartheta_{cb}\vartheta_{b\tau h} + \vartheta_{c\tau h+1}] = 0, \quad h = 0, 1, \ldots, H-1 \tag{E.7}$$

$$[1 - \theta(1 - \beta\theta)]\, \vartheta_{c\tau H} - \beta\theta\vartheta_{cb}\vartheta_{b\tau H} = (1 - \beta\theta)(1 - \theta) \tag{E.8}$$

I will begin by characterizing the solution of this system. From (E.3) and (E.6) we have

$$\begin{aligned}
\vartheta_{bb} &= \theta \\
\vartheta_{cb} &= \frac{1}{\beta} - \theta
\end{aligned}$$

Next, from (E.5) and (E.8), we have that

$$\begin{aligned}
\vartheta_{c\tau H} &= 1 - \beta\theta \\
\vartheta_{b\tau H} &= \beta\theta
\end{aligned}$$

Finally, from (E.4) and (E.7),

$$\vartheta_{c\tau h} = \beta\theta\vartheta_{c\tau h+1} = (\beta\theta)^{H-h}(1 - \beta\theta)$$

$$\vartheta_{b\tau h} \quad = \quad -\beta\theta\vartheta_{c\tau h+1} = -(\beta\theta)^{H-h}(1-\beta\theta)$$

This characterizes the full solution.

I can now prove the various asymptotic statements of Lemma C.1. First we have that

$$\widehat{c}_{H,H} = \vartheta_{cb}\sum_{\ell=0}^{H-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H}$$

Plugging in from the closed-form expressions above and simplifying:

$$\lim_{H\to\infty} c_{H,H} = \frac{(1-\theta)(1-\beta\theta)}{1-\beta\theta^2} = \text{const.}$$

Next looking below the main diagonal:

$$\widehat{c}_{H+1,H} = \vartheta_{cb}\sum_{\ell=0}^{H}\vartheta_{bb}^{\ell}\vartheta_{d\tau H-\ell}$$

and so

$$\frac{1}{\theta}\widehat{c}_{H+1,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}(\vartheta_{bb}\widehat{b}_{H-1,H} + \vartheta_{b\tau H}) = \vartheta_{cb}\widehat{b}_{H-1,H} + (1-\theta\beta) = \widehat{c}_{H,H}$$

Similarly

$$\frac{1}{\theta}\widehat{c}_{H+\ell,H} = \frac{\vartheta_{cb}}{\vartheta_{bb}}\vartheta_{bb}\widehat{b}_{H+\ell-2,H} = \widehat{c}_{H+\ell-1,H}$$

The proof reveals that the result holds for any H, not just $H\to\infty$.

Finally I look above the main diagonal. Here we have

$$\widehat{c}_{H-1,H} = \vartheta_{cb}\sum_{\ell=0}^{H-2}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-2} + \vartheta_{cyH-1}$$

We thus have that

$$\frac{1}{\beta\theta}\widehat{c}_{H-1,H} = \vartheta_{cb}\sum_{\ell=0}^{H-2}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H} = \widehat{c}_{H,H} - \vartheta_{cb}\vartheta_{bb}^{H-1}\vartheta_{b\tau 0}$$

The last term goes to zero as $H\to\infty$. Similarly we have

$$\widehat{c}_{H-\ell,H} = \vartheta_{cb}\sum_{\ell=0}^{H-\ell-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-2} + \vartheta_{c\tau H-\ell}$$

and so

$$\frac{1}{\beta\theta}\widehat{c}_{H-\ell,H} = \vartheta_{cb}\sum_{\ell=0}^{H-\ell-1}\vartheta_{bb}^{\ell}\vartheta_{b\tau H-\ell-1} + \vartheta_{c\tau H-\ell+1} = \widehat{c}_{H-\ell+1,H} - \vartheta_{cb}\vartheta_{bb}^{H-\ell}\vartheta_{b\tau 0}$$

where the last term again goes to zero as $H \to \infty$, completing the argument. □

E.6 Proof of Lemma C.2

I begin with the budget constraint and the Euler equation. For notational convenience I suppress all i subscripts. The budget constraint is

$$\widehat{c}_t + \widehat{b}_t - (1+\bar{r})\widehat{b}_{t-1} = \widehat{\tau}_t. \tag{E.9}$$

Combining budget constraint and the sequential formulation of aggregate demand in (C.4), we obtain the following Euler equation:

$$(1-M\theta)\widehat{c}_t - \frac{\theta}{1+\bar{r}}\widehat{c}_{t+1} - M(1-\theta)(1+\bar{r})\widehat{b}_{t-1} = M(1-\theta)\widehat{\tau}_t. \tag{E.10}$$

I now proceed by combining (C.1) and (E.10) as in the proof of Lemma 1. First, we have

$$\widehat{\tau}_t + (1+\bar{r})\widehat{b}_{t-1} = \frac{(1-M\theta)\widehat{c}_t - \frac{\theta}{1+\bar{r}}\widehat{c}_{t+1}}{M(1-\theta)}$$

which again also equals $\widehat{c}_t + \widehat{b}_t$, from the budget constraint. Thus

$$\widehat{b}_t = \frac{1-M}{M(1-\theta)}\widehat{c}_t - \frac{\frac{\theta}{1+\bar{r}}}{M(1-\theta)}\widehat{c}_{t+1}$$

and so

$$\widehat{\tau}_t = \frac{(1-M\theta)\widehat{c}_t - \frac{\theta}{1+\bar{r}}\widehat{c}_{t+1}}{M(1-\theta)} - \frac{(1+\bar{r})(1-M)}{M(1-\theta)}\widehat{c}_{t-1} - \frac{\theta}{M(1-\theta)}\widehat{c}_t.$$

Stacking, we obtain (C.5).

E.7 Proof of Corollary D.1

I begin with some preliminary simplifications. The non-policy block of the economy can be summarized by the following system of equations, now written in compact sequence-space notation (as in Auclert et al., 2019). First, the price-NKPC,

$$\widehat{\boldsymbol{\pi}} = \Pi_w \widehat{\boldsymbol{w}} + \Pi_\ell \widehat{\boldsymbol{\ell}} + \beta \widehat{\boldsymbol{\pi}}_{+1} + \boldsymbol{\varepsilon}^p$$

Second, the production function

$$\widehat{\boldsymbol{y}} = \mathcal{Y}_\ell \widehat{\boldsymbol{\ell}}$$

Third, firm dividends,

$$\widehat{\boldsymbol{d}} = \mathcal{D}_y \widehat{\boldsymbol{y}} + \mathcal{D}_w \widehat{\boldsymbol{w}} + \mathcal{D}_\ell \widehat{\boldsymbol{\ell}}$$

Fourth, consumer demand

$$\widehat{\boldsymbol{c}} = \mathcal{C}_w \widehat{\boldsymbol{w}} + \mathcal{C}_\ell \widehat{\boldsymbol{\ell}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{C}_d \widehat{\boldsymbol{d}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}} + \mathcal{C}_{i_b} \widehat{\boldsymbol{i}}_b + \mathcal{C}_c \boldsymbol{\varepsilon}^c$$

Fifth, the wage-NKPC,

$$\widehat{\boldsymbol{\ell}} = \mathcal{L}_w \widehat{\boldsymbol{w}} + \mathcal{L}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{L}_c \widehat{\boldsymbol{c}} + \boldsymbol{\varepsilon}^w$$

And sixth, the output market-clearing condition

$$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{c}}$$

Using the price-NKPC, the production function, the equation for firm dividends, and the output market-clearing condition, we can substitute out $\{\boldsymbol{c}, \boldsymbol{w}, \boldsymbol{\ell}, \boldsymbol{d}\}$ in the consumer demand relation. This gives

$$\bar{\mathcal{C}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b + \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}} + \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \tag{E.11}$$

where $\bar{\mathcal{C}}_{i_b} = \mathcal{C}_{i_b}$, $\bar{\mathcal{C}}_\tau = \mathcal{C}_\tau$, $\bar{\mathcal{C}}_c = \mathcal{C}_c$ and $\{\bar{\mathcal{C}}_y, \bar{\mathcal{C}}_\pi\}$ are functions of model primitives. Similarly, using the price-NKPC as well as the production function and the output market-clearing condition, we can substitute out $\{\boldsymbol{\ell}, \boldsymbol{w}, \boldsymbol{c}\}$ in the wage-NKPC to write it as

$$\bar{\mathcal{L}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{L}}_\pi \widehat{\boldsymbol{\pi}} = \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \tag{E.12}$$

where $\{\bar{\mathcal{L}}_y, \bar{\mathcal{L}}_\pi, \bar{\mathcal{L}}_p, \bar{\mathcal{L}}_w\}$ are functions of model primitives.

Now consider first the case where the desired implicit targeting rule (D.1) is implemented using an interest rate-only policy. Plugging the price-NKPC and the production function into the financing rule (8), we can write the financing rule compactly as

$$\widehat{\boldsymbol{\tau}} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}} + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}} + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b \tag{E.13}$$

where $\{\bar{\mathcal{T}}_y, \bar{\mathcal{T}}_\pi, \bar{\mathcal{T}}_{i_b}\}$ are functions of the financing rule (8) and model primitives. Using the simplifications in (E.11), (E.12) and (E.13), we can write the equilibrium system as the following stacked linear system:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_y & \mathcal{B}_\pi & \mathbf{0} & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.14}$$

By assumption, the system (E.14) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the question of whether the same implicit targeting rule can be implemented using a transfer-only policy. Using the simplifications from above, we can write the equilibrium system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_y & \mathcal{B}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.15}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (E.15). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where $\widehat{\boldsymbol{\tau}}^{**}$ solves

$$\left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* + \bar{\mathcal{C}}_\tau \left(\bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^*\right) = \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**}$$

We know by the conditions of Proposition 1 (which recall are assumed for Corollary D.1) that such a $\widehat{\boldsymbol{\tau}}^{**}$ exists. Plugging into (E.11), we get

$$\begin{aligned} & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \Leftrightarrow \quad & \left(\bar{\mathcal{C}}_y - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y\right) \widehat{\boldsymbol{y}}^* + \left(\bar{\mathcal{C}}_\pi - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi\right) \widehat{\boldsymbol{\pi}}^* - \left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \end{aligned}$$

Thus (E.11) still holds. It is immediate that all other relations in (E.15) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (E.15).

To show uniqueness, suppose for a contraction that (E.15) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. By the assumptions of Proposition 1 we can thus find a bounded tuple $\{\widehat{\boldsymbol{y}}^{\dagger}, \widehat{\boldsymbol{\pi}}^{\dagger}, \widehat{\boldsymbol{i}}_b^{\dagger}, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^{\dagger} + \bar{\mathcal{C}}_\tau \left(\bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^{\dagger} + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^{\dagger}\right) = \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{\dagger}$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^{\dagger} + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^{\dagger} + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^{\dagger}$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^{\dagger}, \widehat{\boldsymbol{\pi}}^{\dagger}, \widehat{\boldsymbol{i}}_b^{\dagger}, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (E.14). Contradiction. □

E.8 Proof of Corollary D.2

Proceeding as in the proof of Corollary D.1, we arrive at the following equilibrium system for the explicit interest rate rule:

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ -\mathcal{B}_y & -\mathcal{B}_\pi & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y & -\bar{\mathcal{T}}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.16}$$

By assumption, the system (E.16) has a unique, bounded solution. Denote that solution by $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{i}}_b^*, \widehat{\boldsymbol{\tau}}^*\}$.

Now consider the equilibrium system corresponding to the proposed transfer-only rule (D.4). Using the simplifications from above, we can write that system as

$$\begin{pmatrix} \bar{\mathcal{C}}_y & \bar{\mathcal{C}}_\pi & -\bar{\mathcal{C}}_{i_b} & -\bar{\mathcal{C}}_\tau \\ \bar{\mathcal{L}}_y & \bar{\mathcal{L}}_\pi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} \\ -\bar{\mathcal{T}}_y - \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_y & -\bar{\mathcal{T}}_\pi - \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b}\mathcal{B}_\pi & -\bar{\mathcal{T}}_{i_b} & I \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{y}} \\ \widehat{\boldsymbol{\pi}} \\ \widehat{\boldsymbol{i}}_b \\ \widehat{\boldsymbol{\tau}} \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\ \bar{\mathcal{L}}_p \boldsymbol{\varepsilon}^p + \bar{\mathcal{L}}_w \boldsymbol{\varepsilon}^w \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{E.17}$$

It remains to show that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*\}$ are also part of the unique bounded solution of (E.17). To see this, consider first the candidate solution $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ where

$$\widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^* + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^* + \mathcal{C}_\tau^{-1}\mathcal{C}_{i_b} \widehat{\boldsymbol{i}}_b^*$$

Plugging the candidate solution into the consumer demand function (E.11), we get

$$
\begin{aligned}
& \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \mathbf{0} - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^{**} = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\
\Leftrightarrow \quad & \left(\bar{\mathcal{C}}_y - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_y\right) \widehat{\boldsymbol{y}}^* + \left(\bar{\mathcal{C}}_\pi - \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_\pi\right) \widehat{\boldsymbol{\pi}}^* - \left(\bar{\mathcal{C}}_{i_b} + \bar{\mathcal{C}}_\tau \bar{\mathcal{T}}_{i_b}\right) \widehat{\boldsymbol{i}}_b^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c \\
\Leftrightarrow \quad & \bar{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* + \bar{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \bar{\mathcal{C}}_{i_b} \widehat{\boldsymbol{i}}_b^* - \bar{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}^* = \bar{\mathcal{C}}_c \boldsymbol{\varepsilon}^c
\end{aligned}
$$

Thus (E.11) still holds. It is immediate that all other relations in (E.17) hold, so we can conclude that $\{\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*, \mathbf{0}, \widehat{\boldsymbol{\tau}}^{**}\}$ is indeed a solution of (E.17).

To show uniqueness, suppose for a contraction that (E.17) has a distinct bounded solution $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \mathbf{0}, \widehat{\boldsymbol{\tau}}^\dagger\}$ with $\widehat{\boldsymbol{y}}^\dagger \neq \widehat{\boldsymbol{y}}^*$ and/or $\widehat{\boldsymbol{\pi}}^\dagger \neq \widehat{\boldsymbol{\pi}}^*$. Now consider the tuple $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ where

$$\widehat{\boldsymbol{i}}_b^\dagger = \mathcal{B}_y \widehat{\boldsymbol{y}}^\dagger + \mathcal{B}_\pi \widehat{\boldsymbol{\pi}}^\dagger$$

and

$$\widehat{\boldsymbol{\tau}}^{\dagger\dagger} = \bar{\mathcal{T}}_y \widehat{\boldsymbol{y}}^\dagger + \bar{\mathcal{T}}_\pi \widehat{\boldsymbol{\pi}}^\dagger + \bar{\mathcal{T}}_{i_b} \widehat{\boldsymbol{i}}_b^\dagger$$

Then, following the same steps as above but in reverse, we can conclude that $\{\widehat{\boldsymbol{y}}^\dagger, \widehat{\boldsymbol{\pi}}^\dagger, \widehat{\boldsymbol{i}}_b^\dagger, \widehat{\boldsymbol{\tau}}^{\dagger\dagger}\}$ is a bounded solution of (E.16). Contradiction. $\square$

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