Tasks At Work: Comparative Advantage, TECHNOLOGY AND LABOR DEMAND^{*}

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Abstract

This chapter reviews recent advances in the task model and shows how this framework can be put to work to understand the major labor market trends of the last several decades. Production in each industry necessitates the completion of a range of tasks, which can be allocated to workers of different skill types or to capital. Factors of production have well-defined comparative advantage across tasks, which governs the pattern of substitution between skill groups. Technological change can: (1) augment a specific labor type—e.g., increase the productivity of labor in tasks it is already performing; (2) augment capital; (3) automate work by enabling capital to perform tasks previously allocated to labor; (4) create new tasks. The task model clarifies that these different types of technological changes have distinct effects on labor demand, factor shares and productivity, and their full impact depends on the pattern of substitution between different factors which arises endogenously in the task framework. We explore the implications of the task framework using reduced-form evidence, which highlights the central role of automation and new tasks in recent labor market trends. We also explain how general equilibrium effects ignored in these reduced-form approaches can be estimated structurally.

Keywords: automation, productivity, technology, inequality, wages, rents JEL Classification: J23, J31, O33.

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1 Introduction

The wage and occupational structures of the United States and other industrialized countries have experienced epochal changes over the last several decades. US wage inequality has soared, while the real wages of less educated workers have stagnated or even fallen and their employment rates have declined. Simultaneously, employment has shifted away from production and clerical occupations towards higher-paying managerial, professional and technical jobs and to various service occupations with lower pay. These trends have been accompanied by a lower labor share, especially in manufacturing, and lackluster produc-tivity growth.^{[1](#page-1-0)} Early research focused on the role of labor demand in these trends using a (reduced-form) approach based on an aggregate production function and technologies assumed to augment either skilled or unskilled labor.^{[2](#page-1-1)} In the canonical approach, labor demand changes were then combined with labor supply and institutional factors to account for the major trends.

A more recent strand departs from this approach and starts with a setup in which the production of goods and services necessitates the completion of a series of tasks and factors of production are allocated to perform these tasks.^{[3](#page-1-2)} For example, the production of a smartphone relies on a range of design and planning tasks, the manufacturing of the microchip, the battery, the camera, the speakers, the screen, numerous different types of sensors, and various other components, assembly of these components, and a series of nonproduction tasks, including various back-office activities, quality control, and inventory control. Additionally, for a smartphone to reach the consumer, a number of marketing, advertising, transport, wholesale and retail functions need to be completed. Each task needs to be assigned to one or multiple factors of production. For example, assembly can be performed by craft workers, low-skill workers, a combination of computerized equipment and human labor, or by robots. The assignment of tasks to factors is at the heart of the production process and is shaped by technology (e.g., whether the task is standardized so

¹For a summary of the wage and inequality trends, see [Goldin and Katz](#page-92-0) [\(2008\)](#page-92-0), [Acemoglu and Autor](#page-90-0) [\(2011\)](#page-90-0), [Acemoglu and Restrepo](#page-91-0) [\(2019\)](#page-91-0), [Autor](#page-91-1) [\(2019\)](#page-91-1), [Restrepo](#page-93-0) [\(2024\)](#page-93-0). [Karabarbounis and Neiman](#page-93-1) [\(2013\)](#page-93-1) documents the decline in the labor share in the United States and other industrialized countries, while [Acemoglu and Autor](#page-90-0) [\(2011\)](#page-90-0) and [Goos et al.](#page-92-1) [\(2014\)](#page-92-1) show correlated shifts in occupational structure across several OECD economies. For recent reviews of trends in the wage structure in European and OECD countries see, e.g., [Gornick](#page-92-2) [\(2024\)](#page-92-2).

²See, among others, [Bound and Johnson](#page-92-3) [\(1992\)](#page-92-3), [Katz and Murphy](#page-93-2) [\(1992\)](#page-93-2), [Berman et al.](#page-91-2) [\(1994\)](#page-91-2) and [Autor et al.](#page-91-3) [\(1998\)](#page-91-3). See [Acemoglu](#page-90-1) [\(2002\)](#page-90-1) for a review and extensions of these approaches.

³See [Autor et al.](#page-91-4) [\(2003\)](#page-91-4), [Acemoglu and Autor](#page-90-0) [\(2011\)](#page-90-0) and [Autor and Handel](#page-91-5) [\(2013\)](#page-91-5) for some of the early works using the task approach to study inequality. We discuss the evolution of this literature in more detail below.

that it can be performed by unskilled labor and whether technology permits the tasks to be performed by machines or algorithms). The task framework clarifies, for instance, that technological change can have a major impact on productivity and equilibrium factor prices by enabling new ways of completing tasks—most importantly, via automation, which means new equipment, robots, software or algorithms taking over tasks previously performed by labor. This framework is useful not only because of the greater descriptive realism it brings to the modeling of the production process and the effects of technology, but because it leads to a richer set of comparative statics with respect to technological advances—depending on what types of new technologies are being introduced—and enables a more flexible pattern of substitution between factors of production, as we explain next.

In this chapter, we review recent advances in the task framework and show how it can be a powerful tool for theoretical, reduced-form empirical and structural empirical research. We exposit the main ideas in a one-sector economy with a continuum of tasks and multiple types of labor and capital. We then extend this framework to include multiple sectors, which is useful for mapping our model to data and incorporating additional forms of technology, structural change, and sectoral reallocations.

The two most important distinguishing features of the task framework are:

Different technologies, different effects: The early literature in labor and macroeconomics dealing with wage inequality relied on a restrictive form of technological change essentially, augmenting one of the factors of production. This reliance was at the root of some of its major conclusions.^{[4](#page-2-0)} In reality, technologies take more variegated forms and have correspondingly richer effects on wages, inequality and productivity. New technologies can:

- increase the productivity of specific types of labor in certain tasks currently assigned to them (a better drill does not make workers better at other manual or non-manual tasks); this is a type of labor-augmenting change, except that it affects only some types of labor and in some tasks. Our framework shows that this kind of technological change tends to have relatively small effects on wages and inequality, and ambiguous impacts on the labor share of national income, but increases productivity.
- increase the productivity of capital in tasks already assigned to capital, which is a type of capital-augmenting change. A new and more powerful software system

⁴For example, an implication of the standard models, emphasized in [Acemoglu](#page-90-1) [\(2002\)](#page-90-1), is that skillbiased technological change always raises the real wages of low-skill workers, even as it increases inequality. See [Acemoglu and Autor](#page-90-0) [\(2011\)](#page-90-0) for other implications that follow from the earlier modeling assumptions.

that replaces an older system is an example of this kind of technological change, as it mainly affects productivity without changing the range of tasks performed by capital. This type of technological advance increases productivity and always pushes up real wages, but has ambiguous and small effects on the labor share of national income.

Beyond these types of technologies that have been the focus of the standard approach in the literature, new technologies can also:

- *automate work*. New technologies achieve this by enabling the use of capital equipment, software and algorithms in tasks previously performed by labor. Examples include software systems that can take over office tasks previously assigned to workers or robots that now perform various welding, cutting, painting and assembly tasks. This type of technological advance can have major distributional effects while its pro-ductivity impacts can be limited.^{[5](#page-3-0)} In particular, automation always reduces the labor share, increases the capital share and can depress the real wages of factors displaced from the tasks they used to perform.
- *create new tasks*. New tasks increase productivity by reorganizing production or by introducing a finer division of labor. New tasks that are assigned to labor ("laborintensive new tasks") tend to increase the real wages of all skill groups and the labor share of national income. Computer-assisted design tools and machinery that enables novel technical work as well as new programming, integration, and customer service tasks introduced by recent technologies are examples of new tasks.

In sum, the task framework clarifies the critical distinction between labor-augmenting technology and new tasks. Labor-augmenting technology affects the productivity of some worker types in the tasks that they are already performing, and as a result does not have a major impact on the labor share and typically generates small effects on wages. New tasks, in contrast, can have a much more pronounced impact on wages and can majorly increase the labor share of national income.^{[6](#page-3-1)}

⁵Limited productivity effects are related to the high (microeconomic) elasticity of substitution between factors within a task. This implies that even a small cost advantage for one factor will lead to a major reallocation of tasks from one factor to another, and such reallocation can be associated with significant distributional impacts, while leading to only small productivity gains.

 6 This distinction thus argues against the use of "augmenting technology" as a general term to capture all technologies that "complement" labor in some form.

Flexible substitution between factors depending on comparative advantage: The framework incorporates the assignment of different factors to different tasks according to comparative advantage, which then shapes the pattern of substitution between factors. The elasticity of substitution between different worker groups, such as college-educated and non-college-educated workers, varies. It is influenced by their comparative advantages, the tasks that have been automated, and the tasks assigned to other labor types. This elasticity also changes as tasks are redistributed among these groups. Furthermore, the task framework also showcases the ripple effects of demand-side or supply-side forces affecting a factor, which capture the reassignment of tasks in response to (endogenous) changes in factor prices.

A convenient feature is that these ripple effects can be, to a first-order, fully summarized by a propagation matrix, and our framework clarifies how entries of this matrix vary with model parameters and relate to (local) elasticities of substitution. As a byproduct, the propagation matrix further clarifies the difference between microeconomic and macroeconomic substitution: microeconomic substitution takes place within tasks (and tends to be on the high side), while macroeconomic substitution depends on comparative advantage, the ripple effects and the demand side elasticity of substitution between different goods.

In addition to its conceptual difference from the standard framework, the task model also enables a tractable characterization of equilibrium in which group-level wages depend on different types of technologies as well as on other labor demand factors, such as international trade, offshoring, structural change and product market characteristics, including markups. These can be further combined with institutional and supply-side factors. This tractable characterization leverages an important characteristic of the framework: the impact of any demand-side factor can be decomposed into its productivity effects and the direct or indirect reallocation of tasks it induces between factors, and the sectoral reallocation it triggers. For example, automation has its most major impact by reallocating tasks from labor to capital, while final goods imports influence labor demand mainly via sectoral reallocation. Beyond its simplicity, this characterization is particularly useful as it enables the estimation of the consequences of automation and new tasks for the economy, while simultaneously controlling for the influence of other demand-side and supply-side factors.

Our task model further clarifies how various (general) equilibrium effects are subsumed in the constant included in reduced-form analysis, and provides simple equations that can be estimated using the same data as the reduced-form analysis, combined with estimates of the elasticities of substitution between tasks and goods, and the aforementioned propagation matrix (and we also show how this matrix can be estimated). This structural exercise can then be used to generate estimates that are inclusive of full general equilibrium effects and to carry out counterfactual analysis.

We exposit the task framework, explain its distinguishing features, derive the wage equations and conduct a range of comparative statics. This part of the chapter largely builds on existing work, in particular [Acemoglu and Restrepo](#page-91-6) [\(2022\)](#page-91-6). The new element is drawing out the implications of new tasks for the wage and employment structure of the economy, which has not been the focus of past work.

We then show how the equations implied by this framework can be estimated using publicly-available US data. Namely, we use data for 500 groups of US workers, defined by age, gender, race, and native/foreign-born status, as our skill groups, and focus on changes from 1980 to 2016. This part of the chapter also draws on past work, but the estimation of the effects of new tasks is again original to this chapter.

We document that a 10% loss of tasks for a group due to automation during this period leads to a 12% relative wage decline and 8.2% reduction in hours worked per adult. We further introduce a measure of skill groups' exposure to new task creation and document that 10% new tasks for a group leads to a 8.5% increase in relative wage and 26% increase in hours worked per adult. Overall, in the reduced form, the change in the share of tasks across groups due to automation and new task creation account for 67%-84% of the changes in the group wage structure in the US during this period and 53%-68% of the changes in grouplevel employment. We also estimate the distributional effects of other factors, including sectoral reallocation, sectoral TFP trends, and changes in product market markups. We find that these factors have played a significantly smaller role in changing the betweengroup distribution of wages. For example, while automation and new task creation, jointly, explain about 67%-84% of the variation in between-group wage growth in the US from 1980 to 2016, standard factor-augmenting (skill-biased) technologies appear to explain no more than a few percentage points of these changes.

As already noted, the full real wage impacts of technology cannot be estimated using these reduced-form equations, because their productivity effects are absorbed by the constants in the reduced-form equations, and because potentially complex ripple effects are ignored. We derive a tractable structural approach for estimating the first-order effects of automation, new task creation, and other shocks, and then estimate these structural effects. Combining our measures of automation and new tasks, the propagation matrix, and estimates of the elasticities of substitution between industries and between tasks, we

quantify the full general equilibrium impacts of automation and new tasks. We also use this approach to obtain the general equilibrium effects of other technological developments, rising markups and structural changes in the economy.

Tasks: A Partial Review of the Literature

The microfoundations of the task model go back to [Zeira](#page-94-0) [\(1998\)](#page-94-0), who considers a model where aggregate output is produced from a continuum of product lines (similar to tasks here), which can be allocated to capital or labor. Economic growth is driven by innovations that reallocate product lines/tasks away from labor towards capital.

[Acemoglu and Zilibotti](#page-91-7) [\(2001\)](#page-91-7) build a model in which two types of labor have different comparative advantages across a continuum of tasks, and technology affects the task production functions. This model is used to study how new technologies developed in the industrialized world influences inequality and growth in these economies as well as in developing countries, and especially how the possibility that these technologies may be inappropriate for the needs of developing economies.

The first paper to use the task framework for a systematic analysis of inequality is [Autor et al.](#page-91-4) [\(2003\)](#page-91-4). This paper builds a model with three tasks—one that corresponds to nonroutine problem-solving and complex communication activities performed by skilled labor, one that corresponds to nonroutine manual work performed by unskilled labor, and one that is closely associated with routine cognitive and manual tasks. The authors argue that computers can substitute for workers engaged in routine cognitive and manual activities because they can cheaply perform routine tasks that are reducible to step-bystep, codifiable rules. Computers also, directly and indirectly, complement workers in nonroutine problem-solving and complex communications tasks. These authors develop a novel empirical mapping from these tasks to data and undertake the first comprehensive empirical analysis of the implications of the task model. [Autor and Handel](#page-91-5) [\(2013\)](#page-91-5) extend both the theoretical framework and the measurement of the task content of occupations of this earlier paper.

[Acemoglu and Autor](#page-90-0) [\(2011\)](#page-90-0) build a model that combines elements from the papers mentioned above and builds on the classic Ricardian trade framework of [Dornbusch et al.](#page-92-4) [\(1977\)](#page-92-4). In their model, there are three types of workers (low, middle and high skill) and a continuum of tasks and it is assumed that higher-skilled workers have a comparative advantage in higher-indexed (more complex) tasks. Technological change can augment one or multiple labor types, and enables the automation of some tasks using new equipment or

software. This paper clarifies both the distinction between standard (factor-augmenting) skill-biased technological change and automation—emphasizing how these technologies impact different parts of the earnings distribution and can have distinct effects on the level of real wages and on inequality. This work also highlights the connection between the task framework and the earlier assignment literature, for example, how the task approach builds on the competitive assignment setup of [Sattinger](#page-94-1) [\(1975\)](#page-94-1) and [Teulings](#page-94-2) [\(1995,](#page-94-2) [2005\)](#page-94-3) and the international trade literature focusing on offshoring of tasks, such as [Grossman](#page-93-3) [and Rossi-Hansberg](#page-93-3) [\(2008\)](#page-93-3), Rodríguez-Clare [\(2010\)](#page-94-4) and [Acemoglu et al.](#page-90-2) [\(2015\)](#page-90-2).

Our approach in this chapter builds more directly on recent work in task-based models. [Acemoglu and Restrepo](#page-90-3) [\(2018b\)](#page-90-3) develop a tractable task-based model and generalize this framework by introducing new tasks. This paper also demonstrates how the combination of automation and new task creation and capital accumulation can lead to economic growth, but for this growth to be balanced, the decline in the labor share and the contraction in the range of tasks induced by automation need to be compensated by the creation of new (labor-intensive) tasks. [Acemoglu and Restrepo](#page-91-8) [\(2020b\)](#page-91-8) extend this framework and draw the implications of automation and new task creation for wage inequality.

[Acemoglu and Restrepo](#page-91-9) [\(2020a\)](#page-91-9) use a task model to study the implications of the robot adoption in US manufacturing. Their work shows how simple estimating equations can be derived from the task model and estimates finds that robots had major impacts for wages and employment, especially for workers specializing in manual blue-collar tasks. It also clarifies how the aggregate effects of this type of automation can be determined by combining the productivity impacts of robots with reduced-form estimates of the displacement effects.[7](#page-7-0)

Our treatment in this chapter builds most closely on [Acemoglu and Restrepo](#page-91-6) [\(2022\)](#page-91-6). This paper introduces a general version of the task model with multiple skill groups and with a flexible pattern of comparative advantage. Despite the generality of the model, the paper shows that the equilibrium takes a simple form and enables the empirical exploration of the consequences of different technologies and their propagation. This paper further clarifies the distinction between capital-skill complementarity, which increases the quantity or quality of capital as discussed by [Griliches](#page-92-5) [\(1969\)](#page-92-5), [Berman et al.](#page-91-2) [\(1994\)](#page-91-2), and [Krusell et al.](#page-93-4) [\(2000\)](#page-93-4), and automation, which is driven by improvements in capital productivity for tasks previously performed by labor. While the former process affects inequality indirectly—by increasing

⁷For recent empirical work exploring the effects of industrial robots on firms and workers, see [Graetz](#page-92-6) [and Michaels](#page-92-6) [\(2018\)](#page-92-6), [Acemoglu et al.](#page-90-4) [\(2020\)](#page-90-4), [Humlum](#page-93-5) [\(2020\)](#page-93-5), [Bonfiglioli et al.](#page-91-10) [\(2020\)](#page-91-10), [Acemoglu et al.](#page-90-4) [\(2020\)](#page-90-4), [Dauth et al.](#page-92-7) [\(2021\)](#page-92-7), and [Acemoglu et al.](#page-90-5) [\(2023\)](#page-90-5). See [Restrepo](#page-93-0) [\(2024\)](#page-93-0) for a review of this literature.

the output of capital-intensive activities or sectors—automation impacts inequality directly, by displacing some groups of workers from the tasks they used to perform.

Other contributions exploring the implications of automation in task-based models include [Acemoglu and Restrepo](#page-90-6) [\(2018a\)](#page-90-6), [Acemoglu and Restrepo](#page-91-0) [\(2019\)](#page-91-0), [Aghion et al.](#page-91-11) [\(2018\)](#page-91-11), [Feng and Graetz](#page-92-8) [\(2020\)](#page-92-8), [Moll et al.](#page-93-6) [\(2022\)](#page-93-6), [Nakamura and Zeira](#page-93-7) [\(2024\)](#page-93-7), [Jones and Liu](#page-93-8) [\(2022\)](#page-93-8), [Hubmer and Restrepo](#page-93-9) [\(2021\)](#page-93-9) and [Acemoglu and Loebbing](#page-90-7) [\(2024\)](#page-90-7). Another branch of the literature proposes models of factor-eliminating technical change, where technology works by reducing the weight of a factor in the production of process (see, for example, [Zuleta,](#page-94-5) [2008;](#page-94-5) [Peretto and Seater,](#page-93-10) [2013\)](#page-93-10). We show below how the task framework provides a microfoundation for this form of technological progress.

The rest of this chapter is organized as follows. The next section introduces a one-sector economy with multiple types of skills, tasks and technologies, and defines and characterizes the competitive equilibrium in this economy. Section [3](#page-16-0) further specializes this environment to what we refer to as the "no-ripples economy" in order to provide a transparent exposition of the varying effects of different types of technologies. Section [4](#page-29-0) clarifies the distinction between microeconomic and macroeconomic elasticities of substitution and how the latter elasticity is shaped by competition for marginal tasks and comparative advantage schedules. Section [5](#page-34-0) introduces the propagation matrix, which summarizes the rich substitution patterns implied by the task framework and uses this matrix to provide a full characterization of equilibrium and the implications of different types of technologies in the one-sector case. Section [6](#page-46-0) extends this economy to a multi-sector setup, which is the basis of our measurement strategy and also introduces product market markups. Section [7](#page-54-0) shows how the equilibrium relations characterized in the previous sections can be estimated via reduced-form equations and presents results from this estimation strategy. Our results show the very different effects of automation technologies, of new tasks and of factor-augmenting technologies. We also provide a range of robustness results, which involve controlling for other forms of technological developments, structural change, rising markups, and supply-side factors. Section [8](#page-71-0) develops an approach for estimating the full general equilibrium effects of technologies in this framework and implements this approach. Section [9](#page-86-0) concludes, while the Appendix contains proofs and additional empirical results.

2 The Task Model: The One-Sector Case

This section introduces the task model, describes the equilibrium, and provides a first characterization of this equilibrium. We focus on the one-sector version of the model for simplicity, returning to the multi-sector economy in Section [6.](#page-46-0)

2.1 Environment

A (unique) final good y is produced by combining a set of complementary tasks $x \in \mathcal{T}$ with measure $M > 0$. This good is set as the numeraire, with price normalized to 1. Task quantities $y(x)$ are aggregated using a constant elasticity of substitution (CES) aggregator with elasticity $\lambda \in (0,1)$,

$$
y = \left(\frac{1}{M}\int_{\mathcal{T}} (M \cdot y(x))^{\frac{\lambda - 1}{\lambda}} dx\right)^{\frac{\lambda}{\lambda - 1}}.
$$

The set $\mathcal T$ is assumed measurable and "dx" denotes the Lebesgue integral. $\mathcal T$ could represent a continuum of tasks arranged along a line (as in [Acemoglu and Autor,](#page-90-0) [2011\)](#page-90-0), or could be a region of the plane or a multi-dimensional space.

The key economic decision in this model is how to produce all tasks in $\mathcal T$. The total quantity produced of task x is assumed to be

(1)
$$
y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_g A_g \cdot \psi_g(x) \cdot \ell_g(x).
$$

Intuitively, tasks can be produced by workers of different skill types, indexed by $g \in \mathbb{G}$ = $\{1, 2, \ldots, G\}$ or by (specialized) capital equipment. We denote the quantity of labor of skill type g used in task x by $\ell_g(x)$ and the amount of capital used in the production of task x by $k(x)$. Workers in skill group g have productivity $A_g \cdot \psi_g(x) \geq 0$ in task x, where the $\psi_g(x)$ schedule represents their comparative advantage across tasks. Capital has productivity $A_k \cdot \psi_k(x) \geq 0$ in task x, which is equal to zero for tasks where technology does not yet permit capital to substitute for workers. The A_k and A_g terms represent standard factor-augmenting technologies, which make factors uniformly more productive in all tasks.

Equation (1) imposes perfect substitutability of capital and the different groups at the task level. This feature of the model is a simplifying, but not implausible, assumption. Many of the new types of equipment and software, such as computer numerical control machinery and robots, can perform various tasks with little human involvement (while the programming, maintenance and service of such equipment correspond to other tasks). This feature is nonetheless a simplification, since some labor-intensive tasks require tools (e.g., hammers), but it does not affect the major implications of the framework.^{[8](#page-9-1)}

⁸It is straightforward to generalize this production function so that labor uses some tools and capital equipment needs operators. So long as the share of these factors is small, all of the implications of our framework continue to hold. See the discussion in the online appendix of [Acemoglu and Restrepo](#page-90-3) [\(2018b\)](#page-90-3).

Labor supply is assumed inelastic, with the total supply of group g denoted as ℓ_g , while the real wage of this group is denoted by w_g . We discuss elastic labor supply in Section [8.](#page-71-0)

To keep the model static, capital is treated as an intermediate good, produced using units of the final good and used up in the same period due to depreciation. Specifically, capital of type x, $k(x)$, is produced using the final good at a constant marginal cost normalized to 1. Changes in the productivity and cost of capital are subsumed into changes in the $\psi_k(x)$ schedules. Net output, which is equal to consumption, is therefore obtained by subtracting the production cost of capital goods from output:

$$
c = y - \int_{\mathcal{T}} k(x) \cdot dx.
$$

Following [Acemoglu and Restrepo](#page-91-6) [\(2022\)](#page-91-6), throughout we impose the following restrictions on the task space, which are sufficient for the existence of a unique equilibrium where all workers are assigned a positive measure of tasks and output is positive and finite. While these assumptions can be weakened, this would be at the cost of additional complication and we do not pursue this path here.

Assumption 1 (Restrictions on the task space)

• For each task $x \in \mathcal{T}$, there exists at least one $g \in \mathbb{G}$ such that $\psi_g(x) > 0$. Moreover, the integrals

$$
\int_{x:\psi_g(x)>0} \psi_g(x)^{\lambda-1} \cdot dx
$$

are finite.

- For each $g \in \mathbb{G}$, there is a positive measure of tasks x for which $\psi_g(x) > 0$, $\psi_{g'}(x) = 0$ for all other $g' \neq g$, and $\psi_k(x) = 0$.
- Comparative advantage is strict. For any two groups $g \neq g'$ and constant $a > 0$, the set of tasks such that $\psi_g(x)/\psi_{g'}(x)$ = a has measure zero. For any group g and onstant $a > 0$, the set of tasks such that $\psi_g(x)/\psi_k(x) = a$ has measure zero.

Part 1 of the assumption is a sufficient condition for positive output in the economy (otherwise, such an economy may only be able to produce zero output). Part 2 guarantees that all skill groups are necessary for production to take place and also implies that technological changes will not make any skill group completely redundant. These conditions also ensure that output is always finite (because it rules out the possibility that capital will perform all tasks). Part 3 of the assumption imposes strict comparative advantage. This removes any indeterminacy in the allocation of tasks to workers and ensures that ties (situations in which a task can be produced in a cost-minimizing way with more than one factor) occur only on measure zero sets. Throughout, we also adopt the (non-consequential) tie-breaking rule that whenever there is a tie, tasks are allocated to capital first and then to lower-indexed skill types ahead of higher-indexed skill types.

2.2 Equilibrium

A market equilibrium is defined by a positive vector of real wages $w = \{w_g\}_{g \in \mathbb{G}}$, an output level y, an allocation of tasks to worker groups $\{\mathcal{T}_g\}_{g\in\mathbb{G}}$ and capital \mathcal{T}_k , task prices $\{p(x)\}_{x\in\mathcal{T}}$, task labor demands $\{\ell_g(x)\}_{g\in\mathbb{G},x\in\mathcal{T}}$ and capital production levels $\{k(x)\}_{x\in\mathcal{T}}$ such that:

E1 Task prices are equal to the minimum unit cost of producing the task:

$$
p(x) = \min\left\{\frac{1}{A_k \psi_k(x)}, \left\{\frac{w_g}{A_g \psi_g(x)}\right\}_{g \in \mathbb{G}}\right\}
$$

.

E2 Tasks are produced in a cost-minimizing way, with tasks

$$
\mathcal{T}_g = \left\{ x : p(x) = \frac{w_g}{A_g \psi_g(x)} \right\}
$$

allocated to workers from skill group g , and tasks

$$
\mathcal{T}_k = \left\{ x : p(x) = \frac{1}{A_k \psi_k(x)} \right\}
$$

produced with capital.

E3 Task-level employment of labor and capital are given by

$$
\ell_g(x) = \begin{cases} y \cdot \frac{1}{M} \cdot A_g^{\lambda - 1} \cdot \psi_g(x)^{\lambda - 1} \cdot w_g^{-\lambda} & \text{for } x \in \mathcal{T}_g \\ 0 & \text{otherwise.} \end{cases}
$$

and

$$
k(x) = \begin{cases} y \cdot \frac{1}{M} \cdot A_k^{\lambda - 1} \cdot \psi_k(x)^{\lambda - 1} & \text{for } x \in \mathcal{T}_k \\ 0 & \text{otherwise.} \end{cases}
$$

E4 The labor market clears for all g :

$$
\int_{\mathcal{T}_g} \ell_g(x) \cdot dx = \ell_g.
$$

E5 The price of the final good is 1 and thus the ideal-price index condition,

$$
1 = \left(\frac{1}{M} \int_{\mathcal{T}} p(x)^{1-\lambda} \cdot dx\right)^{1/(1-\lambda)},
$$

holds.

Figure [1](#page-12-0) provides a graphical illustration of this equilibrium. The task space is represented as a subset of the plane, which is partitioned into $G + 1$ subsets, representing the \mathcal{T}_g 's and \mathcal{T}_k . We explicitly condition these sets on the wage rector w to emphasize that task allocations depend on wages. The fact that these sets are shown as connected is for simplicity. It can be seen from the figure why the boundaries of these sets, where a task can be produced in a cost-minimizing way by more than one factor, are of measure zero. These sets are determined by comparative advantage, factor-augmenting technologies and factor prices, which influence the costs of performing a task with a given factor.

FIGURE 1: EQUILIBRIUM TASK ASSIGNMENT AND TASK SHARES. The figure depicts the task space and illustrates the assignment of tasks to different groups of workers $(g \text{ and } g', \text{ in this})$ example) and capital (k) .

2.3 Equilibrium Representation in Terms of Task Shares

Following [Acemoglu and Restrepo](#page-91-6) [\(2022\)](#page-91-6), we represent and characterize the equilibrium in terms of task shares.

Let $\mathcal{T}_g(w)$ be the set of tasks that would be assigned to workers from skill group g at a given level of wages $w = \{w_g\}_{g \in \mathbb{G}}$. Aggregating the labor demand in E3 across tasks, we obtain the labor market-clearing condition

(2)
$$
\int_{\mathcal{T}_g(w)} y \cdot \frac{1}{M} \cdot A_g^{\lambda-1} \cdot \psi_g(x)^{\lambda-1} \cdot w_g^{-\lambda} \cdot dx = \ell_g.
$$

Inverting this equation yields the market-clearing wage for group q ,

$$
w_g = \left(\frac{y}{\ell_g}\right)^{1/\lambda} \cdot A_g^{1-1/\lambda} \cdot \Gamma_g(w)^{1/\lambda},
$$

where the task shares are defined as

$$
\Gamma_g(w) \equiv \frac{1}{M} \int_{\mathcal{T}_g(w)} \psi_g(x)^{\lambda - 1} \cdot dx \quad \text{and} \quad \Gamma_k(w) \equiv \frac{1}{M} \int_{\mathcal{T}_k(w)} \psi_k(x)^{\lambda - 1} \cdot dx.
$$

Task shares summarize how the market value of tasks assigned to the different groups of workers change as we vary wages. The assumption of strict comparative advantage guarantees that task shares are continuous and differentiable functions of factor prices and technology. Moreover, cost-minimization implies the symmetry property

(3)
$$
A_{g'}^{1-\lambda} \cdot w_{g'}^{\lambda} \cdot \frac{\partial \Gamma_g(w)}{\partial w_{g'}} = A_g^{1-\lambda} \cdot w_g^{\lambda} \cdot \frac{\partial \Gamma_{g'}(w)}{\partial w_g} \quad \text{for all } g' \neq g.
$$

This property says that the additional task share that g gains when wages for g' increase equals the additional task share that g' gains when wages for g increase.

Task shares encode all the relevant (local) information on comparative advantage. For example, if the task share of a group decreases by a small (large) amount when its wage increases, this implies that the group has a steep (shallow) comparative advantage at the tasks it currently performs, and cannot be (can be) easily substituted by other groups of workers. Additionally, the behavior of task shares when we increase all wages by the same amount is informative about the substitutability of different groups of workers for capital in marginal tasks.

PROPOSITION 1 (EQUILIBRIUM REPRESENTATION) The competitive equilibrium exists and

is unique. The wage vector w and output level y are given by

(4)
$$
w_g = \left(\frac{y}{\ell_g}\right)^{1/\lambda} \cdot A_g^{1-1/\lambda} \cdot \Gamma_g(w)^{1/\lambda} \text{ for } g \in \mathbb{G},
$$

(5)
$$
1 = \underbrace{\left(\Gamma_k(w) \cdot A_k^{\lambda-1} + \sum_g \Gamma_g(w) \cdot \left(\frac{w_g}{A_g}\right)^{1-\lambda}\right)^{1/(1-\lambda)}}_{\equiv \mathcal{C}(w)},
$$

where $\mathcal{C}(w)$ denotes the marginal cost of producing the final good given the wage vector w. The equilibrium level of output can be written as a CES aggregator of the different labor types and capital k, with the equilibrium task shares $\Gamma_g = \Gamma_g(w)$ and $\Gamma_k = \Gamma_k(w)$ appearing as endogenous weights:

(6)
$$
y = \left(\Gamma_k^{1/\lambda} \cdot (A_k \cdot k)^{1-1/\lambda} + \sum_g \Gamma_g^{1/\lambda} \cdot (A_g \cdot \ell_g)^{1-1/\lambda}\right)^{\lambda/(\lambda-1)}
$$

Like all proofs in this chapter, the proof of this proposition is provided in the Appendix.

.

Equation (4) gives the market-clearing wage. This equation demonstrates that equilibrium wages depend on output per worker (y/ℓ_g) , factor-augmenting productivity terms (the A_g 's), and the task shares (the $\Gamma_g(w)$'s). Equation [\(5\)](#page-14-1) is the ideal-price index condition in E5, rewritten in terms of task shares. This system has a unique solution because task shares satisfy the gross-substitutes property: $\Gamma_g(w)$ is decreasing in w_g and increasing in $w_{g'}$ for all $g' \neq g$.

Equation (6) is a representation result. Once equilibrium wages and task shares are solved, they can be substituted back into the production function [\(1\)](#page-9-0) to obtain this form. It shows that the economy behaves as if output were produced using a CES aggregate production function, with the CES weights determined endogenously by equilibrium task shares. In this expression, $k = \int_{\mathcal{T}_k(w)} k(x) dx$ denotes the total amount of capital used in production.

Just as the CES weights govern the distribution of income in a model with a CES aggregate production function, task shares are the key objects governing the distribution of income in the task model. The share of skill group q in gross national income is:^{[9](#page-15-0)}

$$
s_g^y = \Gamma_g(w) \cdot \left(\frac{w_g}{A_g}\right)^{1-\lambda}
$$

.

The share of all labor in gross national income is therefore

$$
s_L^y = \sum_g \Gamma_g(w) \cdot \left(\frac{w_g}{A_g}\right)^{1-\lambda} = 1 - \Gamma_k(w) \cdot A_k^{\lambda-1},
$$

and the share of capital in gross national income is

(7)
$$
s_K^y = \Gamma_k(w) \cdot A_k^{\lambda - 1}.
$$

Two additional objects of interest are the capital-output ratio, given by

$$
\frac{k}{y} = \Gamma_k(w) \cdot A_k^{\lambda - 1},
$$

and the share of consumption in gross national income, which is

$$
\frac{c}{y} = 1 - \Gamma_k(w) \cdot A_k^{\lambda - 1}.
$$

2.4 Beyond CES

Proposition [1](#page-13-0) shows that the task model aggregates to an economy that behaves as if output were produced from a CES aggregator. In this aggregation, task shares determine the resulting CES weights. The fact that task shares are endogenous and depend both on technology and factor prices introduces the two key features that distinguish the task model from previous approaches that rely on CES production functions (or nested versions thereof) and that assume technology works by increasing factor productivity.

• Distinctive feature 1—different technologies, different effects: Technology operates by directly altering the task shares, and this enables us to incorporate the distinct impacts of different types of technologies. To see the significance of this feature, suppose we treated (6) as a standard CES production function. Then, the modal form of technology would be a labor-augmenting one, say an increase in A_g ,

⁹ "Gross" here refers to national income inclusive of payments to capital, while net output subtracts these payments.

and its effects could be obtained by modifying the first and second terms in the wage equation [\(4\)](#page-14-0). In this exercise, the elasticity of substitution and the weights would be held constant. In contrast, in our framework, even a change in A_g would have a third important effect because it would alter all task shares. More importantly, in the standard framework, we would be forced to think of automation—for example, the introduction of industrial robots—as increasing capital productivity, A_k (this is the only way in which capital can become more productive in that framework). This would have the unambiguous comparative static that it always raises real wages for all worker groups (see more on this below). Instead, in our framework automation operates entirely by changing task shares and output per worker (the first term), which, as we will see, will have very different consequences.^{[10](#page-16-1)}

• Distinctive feature 2—rich substitution patterns: Despite appearances, the task model does not force the elasticity of substitution across groups to equal λ the elasticity of substitution between tasks. This is because task shares respond to wages, capturing substitution in marginal tasks. The task model thus allows for richer substitution patterns than a standard CES model and implies that the resulting macroeconomic elasticities are linked to the pattern of comparative advantage and competition for marginal tasks.

Section [3](#page-16-0) introduces a special case of the framework here, which we will refer to as "the no-ripples economy", to explain the first distinctive feature, while Section [4](#page-29-0) discusses the second one and presents a number of simple examples where the influence of comparative advantage on the macroeconomic elasticity of substitution can be seen clearly. Section [5](#page-34-0) puts these elements together and characterizes the full implications of different types of technologies in the one-sector model.

3 Different Technology, Different Effects

The first distinctive feature of the task framework is its ability to differentiate between different types of technologies. This section describes the different classes of technology that can be modeled using the task framework and delineates the distinct mechanisms via which they affect labor demand and productivity. To facilitate the exposition, we focus on

¹⁰One could try to replicate the effects of automation by exogenously changing the weights of the CES production function, but this has the disadvantage of being highly reduced-form and one could not know ex ante which weights should be changed by how much.

a special case of our framework, the "no-ripples economy", in which there is no competition for marginal tasks.

3.1 The No-Ripples Economy

We have demonstrated that task shares are endogenous and depend not just on technology but also on factor prices. Consequently, a technological change will have both direct effects encoded in its impact on task shares given factor prices, and indirect effects that work through factor price changes. The latter channel is critical for our full framework, but it is possible to understand the effects of different types of technologies without this additional endogeneity, which motivates us to focus on what we call the no-ripples economy. This economy, also studied in [Acemoglu and Restrepo](#page-91-6) [\(2022\)](#page-91-6), imposes the following assumption which ensures that there are no marginal tasks—or no competition for marginal tasks.

ASSUMPTION 2 (NO RIPPLES) The task space can be partitioned into sets $\{\mathcal{T}_{g}^*\}_{g \in \mathcal{G}}$ and \mathcal{T}_{k}^* such that for each g, tasks \mathcal{T}_g^* can be produced only by workers in skill group g and tasks in \mathcal{T}_{k}^{*} can be produced only by capital.

This assumption ensures that there are no marginal tasks being contested between skill groups or between capital and labor, and thus task shares are pinned down by technology and can be written as

$$
\Gamma_g=\frac{1}{M}\int_{\mathcal{T}_g^*}\psi_g(x)^{\lambda-1}\cdot dx\text{ for all }g\in\mathbb{G},\quad \text{ and }\quad \Gamma_k=\frac{1}{M}\int_{\mathcal{T}_k^*}\psi_k(x)^{\lambda-1}\cdot dx.
$$

Because task shares do not depend on wages, one can readily obtain equilibrium wages and output from (4) and (6) . We maintain Assumption [2](#page-17-0) in this section, but do not impose it in any other part of the chapter.

3.2 Automation

Automation technologies are those that directly displace workers from the tasks they are performing. In terms of the example of iPhone production in the Introduction, the use of robots or computer numerical control machinery that take over various manufacturing and assembly tasks or new software systems that perform some of the back-office tasks would be examples of automation.

We can model automation technologies by allowing an increase in the productivity of capital in some of the tasks previously assigned to labor. Suppose in particular that new automation technologies become available in a set of tasks $\mathcal{A} \subset \cup_{g \in \mathbb{G}} \mathcal{T}_k^*$ and increase capital productivity in these tasks discretely, from $\psi_k(x) = 0$ in $x \in \mathcal{A}$ to $\psi_k^{\text{auto}}(x) > 0$. We assume that in the initial equilibrium $\frac{1}{A_k \cdot \psi_k^{\text{auto}}(x)} < \frac{w_g}{A_g \cdot \psi_g}$ $\frac{w_g}{A_g \cdot \psi_g(x)}$ for all $x \in \mathcal{A}$ and for any $g \in \mathbb{G}$ and that A has a small measure, which implies that automating these tasks will not have a large impact on wages and it is cost-minimizing to assign these tasks to capital.^{[11](#page-18-0)}

A convenient feature of the task framework is that the effects of any technology depend on its impact on task allocations and productivity. In the case of automation technologies, we can therefore summarize their effects via two sufficient statistics: the direct task displacement and the cost savings that these technologies generate.

Let us also denote the set of tasks that were previously perfomed by skill group g and are now being automated by $\mathcal{A}_g = \mathcal{A} \cap \mathcal{T}_g^*$. Direct task displacement from automation impacting group g can be written as

$$
d\ln\Gamma_g^{\text{auto}} = \frac{\int_{\mathcal{A}_g} \psi_g(x)^{\lambda-1} \cdot dx}{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda-1} \cdot dx} \ge 0.
$$

Here, recall that \mathcal{T}^*_g is the set of tasks performed by workers in group g before the arrival of the new automation opportunities. This expression clarifies that direct task displacement can be measured as the proportional reduction in group g's task share resulting from automation—the numerator is the share of tasks in the set \mathcal{A}_g , while the denominator is group g's task share in the initial equilibrium, before the arrival of new automation technologies. Note also that in all of these expressions the set of tasks assigned to a group does not depend on w because of Assumption 2—or, equivalently, because we are focusing on the no-ripples economy.

Cost savings from automating task x in A_g , are also evaluated at the initial equilibrium wages, and can thus be written as

(8)
$$
\pi^{\text{auto}}(x) = \frac{1}{1-\lambda} \cdot \left(1 - \left[\frac{w_g \cdot \psi_k^{\text{auto}}(x)}{\psi_g(x)}\right]^{ \lambda - 1}\right).
$$

This expression measures the decline in costs resulting from a discrete reduction in the price of task x. Cost savings are positive by assumption in this case. Average cost savings from

¹¹Notice that after this change, Assumption 2 no longer holds because tasks in $\mathcal A$ can be produced by more than one factor of production. In this economy, Assumption 2 holds in the initial equilibrium, before the change in technology.

automating tasks previously assigned to group g can then be computed as the employmentweighted average of $\pi^{\text{auto}}(x)$'s:

$$
\pi_g^{\text{auto}} = \frac{\int_{\mathcal{A}_g} \psi_g(x)^{\lambda - 1} \cdot \pi^{\text{auto}}(x) \cdot dx}{\int_{\mathcal{A}_g} \psi_g(x)^{\lambda - 1} \cdot dx} > 0.
$$

Figure [2](#page-19-0) illustrates the role of direct displacement effects from automation and the resulting cost savings diagrammatically in the case of two skill groups.

FIGURE 2: EFFECTS OF AUTOMATION ON THE ALLOCATION OF TASKS. The figure depicts the task space and illustrates an example of new automation technologies increasing the productivity of capital in tasks previously assigned to group g workers.

The objects $\{d\ln \Gamma_g^{\text{auto}}, \pi_g^{\text{auto}}\}_{g\in\mathbb{G}}$ summarize the capabilities of new technologies, the extent to which these capabilities outcompete workers of different skills, and the cost savings generated in the process. The next proposition shows how to compute the effects of automation in terms of these objects.

PROPOSITION 2 (EFFECTS OF AUTOMATION IN NO-RIPPLE ECONOMY) The effects of automation technologies, summarized by $\{d\ln \Gamma_g^{auto}, \pi_g^{auto}\}_{g\in \mathbb{G}}$, are given by the formulas

(9)
$$
d\ln w = (1/\lambda) \cdot (d\ln y - d\ln \Gamma^{auto})
$$

(10)
$$
\sum_{g} s_g^y \cdot d\ln w_g = \underbrace{\sum_{g} s_g^y \cdot d\ln \Gamma_g^{auto} \cdot \pi_g^{auto}}_{=d \ln t f p.}
$$

Equation (9) follows by differentiating (4) and using the fact that task shares are independent of wages in the no-ripples economy. It shows that the impact of automation on wages is given by the sum of two economic forces: the first term, representing the *produc*tivity effect from automation, and the second term, representing the *displacement* effect from automation—meaning the displacement of workers of group g from the tasks they previously performed. The displacement effect is proportional to $d\ln\Gamma_g^{\text{auto}}$ and is straightforward to compute given the initial equilibrium, as we showed. The productivity effect, on the other hand, depends on how much output increases.

The second equation, [\(10\)](#page-19-2), which is derived by differentiating [\(5\)](#page-14-1), can be used to compute the productivity effect and pins down the impact of automation on real wage levels.^{[12](#page-20-0)} This equation shows that the average increase in wages equals the TFP gains from automation, which can be computed with a logic identical to Hulten's theorem: $d\ln t f p = \sum_g s_g^y \cdot d\ln w_g$.^{[13](#page-20-1)}

The formula for the productivity gains from automation shows that these depend on π_g^{auto} , and by assumption capital produces the tasks in A more cheaply than labor, which implies that $\pi_g^{\text{auto}} > 0$ and that automation will increase TFP. But this contribution can be small—which will correspond to what was referred to as so-so *automation technologies* in [Acemoglu and Restrepo](#page-91-0) [\(2019\)](#page-91-0). This will be the case when labor is fairly productive in these tasks to start with or when capital can perform these tasks with moderate productivity (just high enough to outcompete labor but not so high as to yield very large cost savings). One important implication of the task framework, combined with so-so automation technologies, is that it naturally accounts for the possibility that productivity growth can be very slow even as there is significant investment in automation technologies and major changes in inequality.

Its possible distributional effects notwithstanding, equation [\(10\)](#page-19-2) also implies that automation increases the average wage—and does so by the same amount as TFP, as shown by the first equality in [\(10\)](#page-19-2). Intuitively, the change in TFP corresponds to how much the cost of producing the final good declines at given factor prices. Since this cost has to remain at 1, wages must increase, and if relative wages remained constant, all wages would have to increase by the same (proportionate) amount as TFP. This result is in turn a consequence

¹²Specifically, the productivity effect $d\ln y$ can be computed by solving equations [\(9\)](#page-19-1) and [\(10\)](#page-19-2). This system comprises $G + 1$ unknowns and $G + 1$ equations that can be solved together to determine the changes in the real wage of each group of workers and in output. An alternative and equivalent approach is to use the general result that $d\ln y = (1-s_k^y)^{-1} \cdot (d\ln tfp - ds_k^y)$, where s_k^y is the capital share in gross output to obtain this productivity effect and ds_k^y is the change in the capital share, obtained from [\(11\)](#page-21-0).

¹³Hulten's original result focuses on the effects of infinitesimal changes in technology. Here, we have a discrete jump in technology taking place over a small (infinitesimal) set of tasks, but this does not change the overall logic. The only difference is that, when computing $\pi_g^{\text{auto}}(x)$, we have to take into account the impact of this discrete jump on cost shares, which is the reason why the $1 - \lambda$ terms appear in [\(8\)](#page-18-1).

of the following three features: (i) capital is supplied fully elastically (see, for example, [Simon,](#page-94-6) [1965;](#page-94-6) [Caselli and Manning,](#page-92-9) [2019;](#page-92-9) [Moll et al.,](#page-93-6) [2022;](#page-93-6) [Acemoglu et al.,](#page-90-8) [2024\)](#page-90-8); (ii) all markets are competitive (see [Acemoglu and Restrepo,](#page-91-12) [2024,](#page-91-12) for the role of labor-market imperfections); and (iii) the production technology exhibits constant returns to scale.

However, equation [\(10\)](#page-19-2) does not guarantee that the real wage of all groups will increase. In particular, equation [\(9\)](#page-19-1) highlights that while the productivity effect raises wages, the displacement effect reduces the wage of the affected groups. Consequently, groups that are most impacted by automation may experience real wage declines. This can be seen in the simplest possible way by assuming that automation only impacts one group, g , and the new automation technologies are so-so $(\pi_g^{\text{auto}} \approx 0)$. In this case, group g's real wage will necessarily decline. We return to a detailed discussion of real wage consequences of automation in Section [5.](#page-34-0)

Finally, we can also use equation [\(7\)](#page-15-1) together with the expansion in the set of tasks performed by capital to conclude that the labor share of national income s_ℓ^y ℓ decreases—and equivalently, the capital share s_k^y $\frac{y}{k}$ increases—by

(11)
$$
ds_{\ell}^{y} = -\sum_{g} s_{g}^{y} \cdot d \ln \Gamma_{g}^{\text{auto}} \cdot (1 + (\lambda - 1) \cdot \pi_{g}) < 0
$$

This result is a direct consequence of the fact that automation's displaces workers from the tasks they used to perform and makes production more capital intensive.

Offshoring: The task framework can also be used to study the effects of offshoring, which are very similar to automation (see, for example, [Grossman and Rossi-Hansberg,](#page-93-3) [2008\)](#page-93-3). Offshoring corresponds to some tasks previously performed domestically by labor now being transferred to workers in another country. This can be most readily incorporated into the framework here by interpreting $k(x)$ to include imports of intermediates (or services) corresponding to task x . For example, the assembly of an iPhone can be performed by robots in the United States or components can be shipped and assembled in Vietnam. From the viewpoint of workers in the United States, these two will have identical effects.^{[14](#page-21-1)}

We can therefore model the arrival of new opportunities for offshoring as a jump in the capabilities of the technology used for organizing global supply chain for task x from

 14 This is provided that trade is balanced, so that a corresponding amount of the final good is transferred to the foreign country to pay for the offshored tasks. In the multi-sector studied in the next section, trade balance could be achieved by exporting goods produced in certain industries. If so, the effects of offshoring could differ from automation because they could also involve additional sectoral reallocation.

 $\psi_k(x) = 0$ to $\psi_k^{\text{offshore}}(x) > 0$. We define the direct task displacement from offshoring as $d\ln\Gamma_g^{\text{offshore}}$ and the cost savings from offshoring as π_g^{offshore} , perfectly analogously to the same expressions for automation.

The objects $\{d\ln \Gamma_g^{\text{offshore}}, \pi_g^{\text{offshore}}\}_{g\in\mathbb{G}}$ then fully summarize new offshoring opportunities. The effects of offshoring are the same as those in Proposition [2,](#page-19-3) except that ${d \ln \Gamma_g^{\text{offshore}}}, {\pi_g^{\text{offshore}}}_{g \in \mathbb{G}}$ replace ${d \ln \Gamma_g^{\text{auto}}}, {\pi_g^{\text{auto}}}_{g \in \mathbb{G}}$ throughout. This further emphasizes that the impact of offshoring operates via productivity and displacement effects as well. Just like automation, offshoring will have its most negative impact on a group when offshored tasks are unevenly distributed across groups and when their productivity benefits are limited.

3.3 New Tasks

The second class of technologies considered here are advances that enable the creation of new (labor-intensive) tasks. We emphasized in the Introduction the critical role that new tasks play in generating new opportunities and demand for labor—raising the labor share and counterbalancing the decline in labor share coming from automation. [Acemoglu and](#page-90-3) [Restrepo](#page-90-3) [\(2018b\)](#page-90-3) and [Autor et al.](#page-91-13) [\(2022\)](#page-91-13) suggest that a significant part of employment growth over the last six decades is accounted for by occupations in which we see a range of new tasks, such as various technical occupations, radiology, management consulting, design and programming of new devices and applications.

While some new tasks emerge as a result of non-homothetic preferences (e.g., sommeliers), most new tasks are enabled by advances in technology. For example, technical production occupations are created by new, more sophisticated technologies that require novel expertise, radiology became a major occupation because of advances in radiography technology, while management consulting and design occupations are dependent on a range of new communication and design tool innovations. In the context of the iPhone example from the Introduction, the production and design tasks associated with miniaturized microchips and touchscreens are enabled by innovations that led to these new components. The defining feature of these examples is that technology creates the demand for new specialized roles or endows workers with new capabilities to produce value and contribute to economic output.

We incorporate new task creation by assuming that there is a technological advance that enables the production of a set N of new tasks that did not exist in $\mathcal T$. We assume that the set of $\{\mathcal{N}_g\}_{g\in\mathcal{Z}}G$, has a small measure and that, at the initial equilibrium wages,

firms strictly prefer to produce tasks in \mathcal{N}_g with workers from skill group g. One can allow for new tasks produced by capital, but we do not do so to simplify the exposition.

The direct effects of new tasks can be summarized by two sufficient statistics, similar to their counterparts for automation: *direct task reinstatement* and *economic surplus* from new tasks. The direct task reinstatement from new task creation on group q is

$$
d\ln\Gamma_g^{\text{new}} = \frac{\int_{\mathcal{N}_g} \psi_g(x)^{\lambda-1} \cdot dx}{\int_{\mathcal{T}_g(w)} \psi_g(x)^{\lambda-1} \cdot dx} \ge 0
$$

and gives the percent increase in group g's task share resulting from the creation of tasks in $\mathcal N$. We will see that which group gains from the capabilities introduced by new tasks will depend crucially on $\{d\ln\Gamma_g^{\text{new}}\}_g$. We refer to this measure as task reinstatement, because it corresponds to the expansion of the set of tasks performed by workers in g and is thus the exact counterpart of the displacement caused by automation.

The economic surplus from new task x in \mathcal{N}_g , evaluated at the initial equilibrium wages, is defined analogously to the cost savings from automation as

$$
\pi^{\text{new}}(x) = \frac{1}{1-\lambda} \cdot \left(\left[\frac{w_g}{A_g \cdot \psi_g(x)} \right]^{\lambda-1} - 1 \right).
$$

The economic surplus from new tasks is positive if the cost of producing the task with labor $w_g/(A_g\psi_g(x))$ is below 1—which, recall, is the price of the final good and our choice of numeraire. We assume this is the case, so that new task x increases TFP and will be adopted. We also define average economic surplus from new tasks for group g as:

$$
\pi_g^{\text{new}} = \frac{\int_{\mathcal{N}_g} \psi_g(x)^{\lambda - 1} \cdot \pi^{\text{new}}(x) \cdot dx}{\int_{\mathcal{N}_g} \psi_g(x)^{\lambda - 1} \cdot dx} > 0.
$$

Figure [3](#page-24-0) illustrates the role of direct reinstatement effects from new task creation and the resulting cost savings.

The objects $\{d\ln\Gamma_g^{\text{new}}, \pi_g^{\text{new}}\}_{g\in\mathbb{G}}$ summarize the direct gains from new tasks. The next proposition shows how to compute the effects of new task creation in terms of these objects.

PROPOSITION 3 (EFFECTS OF NEW TASK CREATION IN NO-RIPPLE ECONOMY) The effects

FIGURE 3: EFFECTS OF NEW TASK CREATION ON THE ALLOCATION OF TASKS. This figure depicts the tasks space and illustrates a change in technology that introduces new tasks.

of new tasks, summarized by $\{d\ln \Gamma_g^{new}, \pi_g^{new}\}_{g \in \mathbb{G}}$, are given by the formulas

(12)
$$
d\ln w = (1/\lambda) \cdot (d\ln y - d\ln M + d\ln \Gamma^{new})
$$

(13)
$$
\sum_{g} s_g^y \cdot d\ln w_g = \underbrace{\sum_{g} s_g^y \cdot d\ln \Gamma_g^{new} \cdot \pi_g^{new}}_{=d \ln tfp}.
$$

As in Proposition [2,](#page-19-3) these two equations can be solved together to determine changes in the real wages of all demographic groups as well as the increase in output. Equation [\(12\)](#page-24-1) is the analogue of [\(9\)](#page-19-1) in the case of automation and describes the distributional effects of new tasks. Equation (13) is the analogue of (10) in the case of automation. It gives the TFP improvements due to new tasks and pins down their effects on wage levels.

The proposition shows that the wage and inequality consequences of new tasks are now given by a combination of a productivity effect, which is similar to the productivity effect from automation, and a new *reinstatement effect*, which can be thought as the converse of the displacement effect. While the displacement effect is the direct negative impact of automation technologies that push workers out of the tasks they used to perform, the reinstatement effect measures the beneficial (positive) impact from new tasks where workers will be employed. In addition, $d \ln M$ is included as a correction term because M, the measure of tasks in the economy, is in the denominator of (1) . The assumption that there is positive economic surplus from new task adoption is sufficient to ensure that average wages increase even after accounting for this correction.

Because both the productivity and reinstatement effects are positive, new tasks increase wages for affected groups. Moreover, in contrast to automation technologies, new tasks increase the labor share of national income, because they expand the set of tasks performed by labor, making the production process more labor-intensive.[15](#page-25-0)

3.4 Labor-Augmenting Technologies

The most common type of technological change studied in economic growth models and in analyses of inequality are those that are factor-augmenting—assumed to increase the productivity of a factor. In the task framework, there are two ways in which we can think about labor-augmenting technological changes. The first and the more plausible one is new technologies raising the productivity of a factor, say low-education male workers, in certain tasks, as exemplified by a better hammer that increases the effectiveness of workers in a few construction and assembly tasks. In terms of the iPhone example from the Introduction, new precision drills that help workers in the assembly and manufacturing of components would be an example. We refer to these as *narrow labor-augmenting technological change*, to emphasize that they only affect a narrow set of tasks—rather than all tasks—and represent them by increases in $\psi_q(x)$ in a subset of the tasks assigned to group g. Importantly, narrow labor-augmenting technological change always refers to the affected group becoming more productive in the tasks currently assigned to it. We therefore represent the effects of narrow labor-augmenting technologies on group q by

$$
d\ln\psi_g^{\text{intensive}} = \frac{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda-1} \cdot d\ln\psi_g(x) \cdot dx}{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda-1} \cdot dx}.
$$

This notation emphasizes that these are "intensive-margin" changes, affecting group q 's productivity in tasks it is already performing, as opposed to those that alter the extensive margin of task allocation, such as automation.^{[16](#page-25-1)}

The more common alternative in the literature is what we refer to as *uniformly labor*augmenting technological change, which increases the productivity of a factor in all the tasks in the economy, and can be represented by increases in the A_g terms. It is more difficult to find actual examples of uniformly labor-augmenting technologies that raise the

¹⁵We give the exact equations for labor share changes for this and other technologies in the Appendix to save space in the text.

¹⁶This discussion also clarifies that we could alternatively refer to narrow labor-augmenting technological change as "productivity deepening" to capture the fact that it deepens the comparative advantage that the group has for the tasks it is already performing (those in the set \mathcal{T}_g^*).

productivity of the factor in all tasks, but one possibility would be assistive technologies that improve the sight of visually-impaired workers. The distinction between narrow and uniformly labor-augmenting technologies is important in our general framework, though the next proposition shows that in the no-ripples economy, they have identical effects.

PROPOSITION 4 (LABOR-AUGMENTING TECHNOLOGIES IN THE NO-RIPPLE ECONOMY) The effects of labor-augmenting technologies are given by the formulas

(14)
$$
d\ln w = (1/\lambda) \cdot (d\ln y - (1-\lambda) \cdot d\ln A_g - (1-\lambda) \cdot d\ln \psi_g^{intensive})
$$

(15)
$$
\sum_{g} s_g^y \cdot d\ln w_g = \underbrace{\sum_{g} s_g^y \cdot (d\ln A_g + d\ln \psi_g^{intensive})}_{=d\ln tfp}.
$$

The proposition demonstrates that the two forms of labor-augmentation have the same implications for wages, output, and productivity. Both forms of augmentation affect wages via a productivity effect, captured in the proposition by $d \ln y$. In addition, both forms directly increase worker productivity one-to-one (by $d\ln A_g$ or by $d\ln \psi_g^{\text{intensive}}$), but this has to be weighed against a negative task-price effect, given by $(-1/\lambda) \cdot (d \ln A_g + d \ln \psi_g^{\text{intensive}})$. In the no-ripple economy, the task-price effect dominates the quantity expansion for both forms of augmentation in the empirically relevant case where tasks are gross complements $(\lambda < 1)$. Due to Uzawa's theorem, labor-augmenting technologies do not affect the labor share of national income in environments with elastic capital, such as the one we have.

That these two forms have identical effects in the no-ripples economy should not be surprising: the set of tasks performed by a factor, say skill group g , does not change in response to augmenting technologies. Hence a marginal increase in A_g only improves the productivity of this factor in the tasks it is performing, and is thus very similar to an increase in $d \ln \psi_g^{\text{intensive}}$.

It is useful to note how fundamentally different labor-augmenting technologies are from automation and new tasks—a feature that is particularly evident in the no-ripples economy. All of the effects of labor-augmenting technologies are at the intensive margin and there is no change in the allocation of tasks to factors. In contrast, both automation and new tasks work at the extensive margin—their main impacts are rooted in the changes in the allocation of tasks that they cause. This is also the reason why the balance between the distributional and productivity effects of these types of technologies are so different.

One way of illustrating this point is by comparing how big the direct distributional

consequences of labor-augmenting and automation technologies are relative to productivity effects. For narrow labor-augmenting technology, this ratio is

$$
-\frac{(1-\lambda)\cdot\psi_g^{\text{intensive}}}{\psi_g^{\text{intensive}}} = -(1-\lambda).
$$

The numerator is the impact via the combination of task-price and quantity effects, while the denominator is the increase in their productivity. The same ratio for automation is

$$
-\frac{d\ln\Gamma_g^{\rm auto}}{d\ln\Gamma_g^{\rm auto}\cdot\pi_g^{\rm auto}}=-\frac{1}{\pi_g^{\rm auto}},
$$

since $d\ln\Gamma_g^{\text{auto}}$ measures the direct test displacement, which is the source of the negative effects from automation, while the denominator is the productivity effect, now obtained by multiplying the range of affected tasks by average cost savings. The first of these expressions is positive when $\lambda > 1$ (because the quantity effects are larger than the price effects), and even when it is negative, it takes a finite value less than 1. In contrast, the second expression can be infinitely large, especially for the case of so-so automation technologies for which the productivity gains are small. This comparison thus provides one way of understanding the fundamental differences between labor-augmenting technologies and automation.

Labor-augmenting technologies are also very different from new tasks. While the former increases the quantity of goods and services that workers produce in existing tasks (and this comes at the expense of a reduction in the price of these tasks and services, putting downward pressure on their wages), new tasks reinstate workers into new activities, allowing them to spread their labor across a wider range of tasks. This is the reason why new tasks, by spreading out the labor hours of the affected group across a larger set of tasks, do not run into the same diminishing returns that labor-augmenting improvements—which increase production in a given set of tasks—do.

3.5 Capital-Augmenting Technologies

The analysis of capital-augmenting changes is similar to that of labor-augmenting ones. For narrow capital-augmenting technological change, we define

$$
d\ln\psi_k^{\text{intensive}} = \frac{\int_{\mathcal{T}_k} \psi_k(x)^{\lambda-1} \cdot d\ln\psi_k(x) \cdot dx}{\int_{\mathcal{T}_k} \psi_k(x)^{\lambda-1} \cdot dx}.
$$

as the increase in the productivity of capital in the tasks it is already performing. Uniformly capital-augmenting technological changes summarized by $d \ln A_k$, analogously to the previous subsection.

PROPOSITION 5 (CAPITAL-AUGMENTING TECHNOLOGIES IN THE NO-RIPPLE ECONOMY) The effects of capital-augmenting technologies are given by the formulas

(16)
$$
d\ln w = (1/\lambda) \cdot d\ln y
$$

(17)
$$
\sum_{g} s_g^y \cdot d\ln w_g = \underbrace{s_k^y \cdot (d\ln A_k + d\ln \psi_k^{intensive})}_{=d\ln tfp}.
$$

The proposition shows once more the equivalence between narrow and uniformly capitalaugmenting technologies in the no-ripple economy. One noteworthy point is that because, in the no-ripple economy, capital-augmenting technologies only change the productivity of already capital-intensive tasks, they do not create any adverse effects for labor, and thus always have a positive impact on wages of all groups of workers, proportional to the increase in productivity. Relatedly, when $\lambda < 1$, capital-augmenting technological change increases the labor share of national income.

This proposition reiterates that there is a fundamental difference between capitalaugmenting technologies and automation. As already noted, the latter acts at the extensive margin—by altering the allocation of tasks—while capital-augmenting technologies act primarily (and in the no-ripples economy entirely) at the intensive margin. In fact, while automation reduces the labor share and could reduce the real wage of affected groups, capital-augmenting technologies increase all worker wages uniformly and, in the plausible scenario where capital and labor are gross complements, they also increase the labor share. This distinction clarifies why it would be incorrect to think of the development of industrial robots or other automation technologies as augmenting existing capital.

3.6 Microfoundation for Shifting Cobb Douglas Exponents

The no-ripples economy also provides a tractable microfoundation for a Cobb-Douglas aggregate production function where technology acts by changing its elasticities. To see this, consider the limit case with $\lambda \to 1$. Output in this economy can be represented as

$$
y = \mathcal{A} \cdot \left(\frac{k}{\Gamma_k}\right)^{\Gamma_k} \prod_g \left(\frac{\ell_g}{\Gamma_g}\right)^{\Gamma_g},\,
$$

where the exponents are simply given by the share of tasks in \mathcal{T}_{g}^{*} and \mathcal{T}_{k}^{*} , and $\ln A$ $\frac{1}{M} \cdot \int_{x \in \mathcal{T}_k^*} \ln(A_k \cdot \psi_g(x)) \cdot dx + \sum_g \frac{1}{M} \cdot \int_{x \in \mathcal{T}_g^*} \ln(A_g \cdot \psi_g(x)) \cdot dx.$

This example can be used to illustrate several of the conclusions of Propositions $2-5$ $2-5$. In particular, we can easily see how automation and new task creation can have sizable effects on the equilibrium by shifting the Cobb-Douglas exponents. In contrast, augmenting technologies work by increasing aggregate productivity A in a factor-neutral way.

This example also provides a microfoundation for models of factor-eliminating technologies, such as [Zuleta](#page-94-5) [\(2008\)](#page-94-5) and [Peretto and Seater](#page-93-10) [\(2013\)](#page-93-10). It shows that one can map automation to a reduction in the Cobb-Douglas exponent for skill groups whose tasks become automated and an increase in the exponent for capital, while new tasks increase the Cobb-Douglas exponent for the favored skill groups and reduce the exponent for capital.

3.7 Taking Stock

Several of the key messages discussed in the Introduction are clarified by Propositions [2](#page-19-3)[-5.](#page-28-0) Most importantly, these results show that new technologies affect equilibrium wages through three mechanisms: a *productivity effect* (any technology that increases productivity and expands output raises labor demand and wages); *displacement and reinstatement effects* (that work at the extensive margin by directly changing the allocation of tasks to factors of production); and task-price effects (factor-augmenting technologies increase the supplies of some tasks reduce their prices).

4 From Micro to Macro Elasticities

In this section, we focus on the second distinctive feature of the task framework: the endogenous determination of macroeconomic elasticities of substitution. The full richness of this substitution will be formalized in the next section. The purpose of this section is to define these elasticities and show how the pattern of comparative advantage shapes these elasticities, including in a number of tractable cases.

4.1 Macroeconomic Elasticities of Substitution

In the production function (1) , the elasticity of substitution between any two factors within tasks is infinite. In the no-ripples economy studied in the previous section, any substitution between factors comes only via the substitution between tasks—if high-skill workers become more abundant, the tasks they produce also become more abundant, driving down their price. In the general case, however, there is an additional substitution in that as one group of workers become cheaper, they will take tasks away from other factors of production and the extent of this effect will depend on their comparative advantage, in particular, how steep their comparative advantage is for marginal tasks.

To formally analyze these issues, let us define the macroeconomic elasticity of substitution between skill groups g and g' as

$$
\sigma_{gg'} = \frac{1}{s_{g'}^y} \cdot \frac{d\ln\ell_g}{d\ln w_{g'}}\Big|_{y \text{ constant}}.
$$

This elasticity measures by how much a proportional increase in the wage of skill group g' changes the demand for skill group g. In the task framework, for $g' \neq g$, this elasticity is

$$
\sigma_{gg'} = \lambda + \underbrace{\frac{1}{s_{g'}^y} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}}}_{\text{substitution between tasks}} + \underbrace{\frac{1}{s_{g'}^y} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}}}_{\text{substitution within marginal tasks}}
$$

.

With constant returns to scale, the elasticity is symmetric: $\sigma_{gg'} = \sigma_{g'g}$.^{[17](#page-30-0)}

The formula illustrates the two separate margins of substitution. First, we have substitution between tasks produced by different skill groups, and controlled by λ . This is similar to the substitution in the standard CES production function, and is the only margin of substitution in the no-ripples economy. Second, we have substitution of one skill type for another taking place in marginal tasks. This second source of substitution depends on the intensity of competition for marginal tasks and is shaped by the comparative advantage schedules. This term will be high when the two groups in question have similar comparative advantage schedules in marginal tasks, which in turn would imply that a small difference in costs of producing these marginal tasks can lead to a big shift from one group to the other.

Macroeconomic elasticities of substitution can be estimated from the data, but the exact source of variation being exploited is important. If one focuses on situations in which tasks

$$
\sigma_{gg'} = \frac{\mathcal{C}(w) \cdot \mathcal{C}_{gg'}(w)}{\mathcal{C}_g(w) \cdot \mathcal{C}_{g'}(w)},
$$

 17 The notion of elasticity of substitution used here is due to Allen-Uzawa. With constant returns to scale, the Allen-Uzawa elasticity can be expressed in terms of the cost function $\mathcal{C}(w)$ as

which is symmetric due to Young's theorem. Note that the symmetry of $\sigma_{gg'}$ is equivalent to the symmetry property in [\(3\)](#page-13-1), also proving that assertion.

cannot be or are not reassigned between factors of production, then one would estimate this elasticity as λ , because only the substitution between tasks would be allowed.

The elasticity of substitution between capital and skill group g can be similarly computed as:

$$
\sigma_{kg} = \frac{1}{s_g^y} \cdot \frac{\partial \ln k}{\partial \ln w_g} \Big|_{y \text{ constant}} = \frac{\lambda}{\frac{\partial \ln \Gamma_k(w)}{\partial \ln w_g}} + \frac{\frac{1}{s_g^y} \frac{\partial \ln \Gamma_k(w)}{\partial \ln w_g}}{\frac{\partial \ln \Gamma_k(w)}{\partial \ln w_g}}
$$

.

The two margins of substitution are present in this case as well, and play a central role in determining how advances in the productivity of capital in marginal tasks impacts workers (see [Acemoglu and Loebbing,](#page-90-7) [2024\)](#page-90-7).

4.2 Examples

In this subsection, we briefly illustrate how the macroeconomic elasticity of substitution is determined in a number of tractable cases, clarifying the role of comparative advantage.

Equilibrium with a Common Elasticity of Substitution Between Tasks: The simplest example of how the macroeconomic elasticity of substitution is determined by the pattern of comparative advantage is provided in [Acemoglu and Zilibotti](#page-91-7) [\(2001\)](#page-91-7), who analyze a task model with two types of labor: low-skill (with supply ℓ) and high-skill (with supply h). The task space is a line from [0, 1] (so that $M = 1$), tasks are combined with an elasticity of substitution $\lambda = 1$, and

$$
y(x) = A_{\ell} \cdot (1-x)^{1/\kappa} \cdot \ell(x) + A_h \cdot x^{1/\kappa} \cdot h(x), \text{ where } \kappa > 0.
$$

In this economy, task shares can be computed as

$$
\Gamma_{\ell}(w) = \frac{(w_h/A_h)^{\kappa}}{(w_h/A_h)^{\kappa} + (w_{\ell}/A_{\ell})^{\kappa}}, \qquad \Gamma_h(w) = \frac{(w_{\ell}/A_{\ell})^{\kappa}}{(w_h/A_h)^{\kappa} + (w_{\ell}/A_{\ell})^{\kappa}},
$$

and the macroeconomic elasticity of substitution between low and high-skill labor is constant and given by

$$
\sigma_{h\ell} = \underbrace{1}_{=\lambda} + \frac{1}{s_{\ell}^{y}} \cdot \frac{\partial \ln \Gamma_h(w)}{\partial \ln w_{\ell}} = 1 + \frac{1}{s_{\ell}^{y}} \cdot (1 - s_h^{y}) \cdot \kappa = 1 + \kappa.
$$

In fact, the equilibrium admits a representation that takes the following CES form:

$$
y = \left(\left(A_{\ell} \cdot \ell \right)^{\frac{\kappa}{1+\kappa}} + \left(A_{h} \cdot \ell \right)^{\frac{\kappa}{1+\kappa}} \right)^{\frac{1+\kappa}{\kappa}}
$$

We see in this example that the macroeconomic elasticity of substitution between low and high-skill is $1 + \kappa > 0$, different both from the (infinite) within-task elasticity of substitution and the elasticity of substitution between tasks (which is equal to λ). Intuitively, a greater value for κ makes the comparative advantage of high-skill labor relative to low-skill labor shallower in marginal tasks, facilitating the assignment of more tasks to the type of labor that is cheaper. In contrast, when κ is low, the productivity of high-skill labor relative to low-skill labor declines sharply as more tasks are assigned to high-skill workers.

Macroeconomic Elasticity of Substitution with Correlated Frechet Distributions: This example generalizes the previous one to a setting with multiple (> 2) skill groups. It is also an adaptation of the commonly-used parameterization of [Eaton and Ko](#page-92-10)[rtum](#page-92-10) [\(2002\)](#page-92-10) of the original [Dornbusch et al.](#page-92-4) [\(1977\)](#page-92-4) model, with skill groups taking the place of countries and no trade costs.[18](#page-32-0) This example helps illustrate how correlation and (lack of dispersion) in task-level productivities makes skill groups more substitutable in the aggregate.

Consider a version of the task model with multiple types of workers and no capital. The task space is a line from [0,1] (so that $M = 1$), tasks are combined with an elasticity of substitution $\lambda \in (0,1)$, and

$$
y(x) = \sum_{g} A_g \cdot \psi_g(x) \cdot \ell_g(x).
$$

Suppose that the distribution of productivities $\psi_g(x)$ over tasks is drawn from correlated Frechet distributions across workers:

$$
\Pr(\psi_1(x) \leq a_1, \ldots, \psi_G(x) \leq a_G) = \exp\left\{-\left[\sum_g a_g^{-\kappa/(1-\rho)}\right]^{1-\rho}\right\}.
$$

In this specification, $\rho \in [0, 1)$ measures correlation between the productivities of different groups of workers, and $\kappa > 0$ is an inverse measure of dispersion in productivities. The special case where $\rho = 0$ gives the commonly-used case of independent Frechet distributions.

¹⁸See [Lind and Ramondo](#page-93-11) [\(2023\)](#page-93-11) for work studying this parametrization in the trade context, and [Dvorkin](#page-92-11) [and Monge-Naranjo](#page-92-11) [\(2019\)](#page-92-11) and [Freund](#page-92-12) [\(2024\)](#page-92-12) for work using this parametrization in task models.

In this example, task shares can be computed as

$$
\Gamma_g(w) = \left(\frac{w_g}{A_g}\right)^{\lambda - 1 - \kappa/(1-\rho)} \cdot \left[\sum_{g'} \left(\frac{w_g}{A_g}\right)^{-\kappa/(1-\rho)}\right]^{\frac{\lambda - 1 - \kappa/(1-\rho)}{\kappa/(1-\rho)}}
$$

,

which implies a common macroeconomic elasticity of substitution between skill groups equal to

$$
\sigma_{gg'} = \lambda + \underbrace{\frac{1}{s_{g'}^y} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}}}_{\text{between tasks}} = \lambda + \left(\frac{\kappa}{1-\rho} - \lambda + 1\right).
$$

Equilibrium output again aggregates to a CES representation, this time with elasticity $1 + \kappa/(1 - \rho)$ and productivity level B (for some constant B):

$$
y = B \cdot \left(\sum_{g} (A_g \cdot \ell_g)^{\frac{\kappa}{1 - \rho + \kappa}} + (A_h \cdot \ell)^{\frac{\kappa}{1 + \kappa}} \right)^{\frac{1 - \rho + \kappa}{\kappa}}.
$$

The macroeconomic elasticity of substitution, $1 + \kappa/(1 - \rho)$, exceeds λ because it also accounts for substitution in marginal tasks. Note that when κ is larger, skills are less dispersed and comparative advantage across workers is shallower, translating into greater substitution between worker types. Substitution in marginal tasks also increases when ρ increases, because with greater correlation between productivities, there is more intense competition for marginal tasks.

The Macroeconomic Elasticity of Substitution between Capital and Labor The setup of [Hubmer and Restrepo](#page-93-9) [\(2021\)](#page-93-9) provides an example where tasks are complements but the macroeconomic elasticity of substitution between capital and labor becomes 1.

Suppose that there are two factors of production: labor ℓ and capital k. The task space is the line [0, 1] (so that $M = 1$) and tasks are combined with an elasticity $\lambda \in (0, 1)$. Suppose also that the productivities of capital and labor in task x are

$$
\psi_k(x) = x^{\frac{1-1/\gamma_k}{1-\lambda}} \cdot (1-x)^{\frac{1+1/\gamma_k}{1-\lambda}}
$$
 and $\psi_\ell(x) = x^{\frac{1+1/\gamma_\ell}{1-\lambda}} \cdot (1-x)^{\frac{1-1/\gamma_\ell}{1-\lambda}}$.

Equilibrium output now takes a Cobb-Douglas form

$$
y = \mathcal{A} \cdot k^{\frac{\gamma_k}{\gamma_k + \gamma_\ell}} \cdot \ell^{\frac{\gamma_\ell}{\gamma_k + \gamma_\ell}}
$$

and we can also see that the macroeconomic elasticity of substitution between capital and

labor is unity. This is because, in this case, the additional substitution coming from the comparative advantage schedules adds to the elasticity of substitution between tasks, $\lambda < 1$. The γ parameters determine the importance of capital and labor in this Cobb-Douglas aggregator.

5 Putting it All Together: Shocks and Propagation in the One-Sector Economy

In this section, we provide a characterization of the full equilibrium in the one-sector economy, bringing together the analysis of different types of technologies from Section [3](#page-16-0) and the macroeconomic patterns of substitution from Section [4.](#page-29-0) The main tool for this analysis is the propagation matrix, which we introduce in the next subsection. This tool will enable us to keep track of the rich pattern of substitution between factors, thus enabling us to go beyond the stylized examples of the previous section where the relevant macroeconomic elasticities were constant. We will also see that the effects of different types of technologies are richer in this case because of the substitution patterns that they initiate. Throughout, we focus on first-order approximations to the equilibrium effects of various changes, meaning that the formulas we present apply for small changes.

5.1 Equilibrium: Ripple Effects and the Propagation Matrix

In the no-ripple economy of Section [3,](#page-16-0) technology affected task shares directly. For example, Proposition [2](#page-19-3) showed how automation affecting workers from skill group g could reduce this group's relative wage and potentially its real wage via a displacement effect. More generally, however, once group g experiences a relative decline in its relative wage, it becomes more profitable for some firms to use this group of workers in tasks for which it was marginally less profitable to do so before. This competition for marginal tasks is the source of ripple effects, which capture the (indirect) consequences of the reallocation of tasks between groups.

Figure [4](#page-35-0) illustrates the role of ripple effects in an example where automation displaces workers from group g and new tasks are created for group g' . Both technological developments increase the relative wage of group g' , encouraging firms to substitute capital or workers from skill group g for those from group g' in marginal tasks. This endogenous reallocation of tasks is depicted by the dotted lines.

To understand ripple effects and their implications, consider a general shock z_g affecting group g, which could be automation, labor-augmenting technological change, new tasks

FIGURE 4: DIRECT EFFECTS OF TECHNOLOGY AND RIPPLE EFFECTS. The figure depicts the task space and shows the direct and the ripple effects caused by automation and new tasks.

or other changes. In the no-ripple economy, the impact of this shock on group g can be decomposed into its productivity and direct effects, $d \ln w_g = d \ln y / \lambda + z_g / \lambda$. In the general case we are considering here, as z_g alters group g's wage, there will be a first-round reassignment of marginal tasks. This first-round reassignment will then affect the wages of all groups, and this will induce a second-round reassignment and so on. To capture the full ripple effects, let us start by differentiating [\(4\)](#page-14-0) totally, which leads to

(18)
$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} \cdot z_g + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot d\ln w,
$$

where $d \ln w = (d \ln w_1, \dots, d \ln w_G)$ is the column vector of all wage changes. These wage changes affect the equilibrium wage of group g by reallocating marginal tasks, which is what the Jacobian $\partial \ln \Gamma_q(w)/\partial \ln w$, written as the row vector of marginal changes in this group's task share, represents.

Stacking (18) for all groups and collecting the terms involving $d \ln w$ on the left-hand side allows us to solve for the endogenous change in wages as a function of the vector (z_1, \ldots, z_g) . In what follows, we use the notation stack (a_g) to represent the $G \times 1$ column vector (a_1, a_2, \ldots, a_G) .

PROPOSITION 6 (EFFECTS OF TECHNOLOGY WITH RIPPLE EFFECTS) Consider a set of technological changes with direct effects stack(z_q), which jointly reduce the marginal cost of producing the final good by $\pi = -d \ln C(w)|_{w=constant} > 0$ holding all wages constant. The
effect of these technological changes on wages and output is given by

(19)
$$
d\ln w = \Theta \cdot \text{stack} (d\ln y - z_g)
$$

(20)
$$
\sum_{g} s_g^y \cdot d \ln w_g = \underbrace{\pi}_{=d \ln tfp},
$$

where

$$
\Theta = \frac{1}{\lambda} \cdot \left(1 - \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \right)^{-1}
$$

is the propagation matrix.

Equation [\(19\)](#page-36-0) is a generalization of the wage effects we have seen so far. Rather than focusing on a specific type of technology, such as automation causing displacement, we are now including all possible technology types, and hence, the vector dz could comprise a combination of different types of technological advances. Nevertheless, in line with the results in Propositions [2](#page-19-0)[-5,](#page-28-0) the implications of all of these technological developments depend on their productivity effect and direct effects (which include task displacement, task reinstatement, and task-price substitution effects). Equation (20) , on the other hand, is analogous to the TFP equation in these earlier propositions. These $G+1$ equations can again be solved together to obtain wage changes for the G groups of workers and the change in output for the unique final good. The latter equation additionally specifies that the change in TFP is equal to the change in average wages.

The major difference from the earlier results is that we are now allowing for ripple effects, and their implications are fully summarized by the propagation matrix, which premultiplies the productivity and direct effects in equation [\(18\)](#page-35-0).

When there is no competition for marginal tasks as in the case studied in the no-ripples economy, the propagation simplifies to

$$
\Theta = \left(\begin{array}{cccc} \frac{1}{\lambda} & 0 & \dots & 0 \\ 0 & \frac{1}{\lambda} & \dots & 0 \\ & & \dots & \\ 0 & 0 & \dots & \frac{1}{\lambda} \end{array} \right).
$$

However, in general, the propagation matrix is not diagonal and its off-diagonal entries provide information on how direct effects for one group impact others.

5.2 Properties of the Propagation Matrix

The Appendix establishes that the propagation matrix is well defined and has non-negative entries. The entry $\theta_{gg'} \geq 0$ of this $G \times G$ matrix captures the extent to which group g ′ competes for marginal tasks with workers in group g. Such competition is typically the result of these two groups having similar comparative advantage in marginal tasks. Our analysis, and in particular the form of Θ , highlights that task shares can also change indirectly, for example, because group g' is a close substitute to some other group g'' , and it is group g'' that is closely substitutable to group g , and so on. These ripple effects operating via other groups are the reason why the propagation matrix takes the form of a Leontief inverse, accumulating second and higher-order indirect consequences. This Leontief inverse characterization also underscores that, as opposed to the no-ripples economy in Section [3,](#page-16-0) equilibrium effects necessitate a solution to a fixed point problem—the changes in wages have to be such that the task share changes inducing these wage changes are exactly in line with cost-minimizing reallocations of tasks given those wage changes.

The propagation matrix has several properties that are worth noting:

- 1. **Dampening:** All eigenvalues of Θ are real and in the $[0,1/\lambda]$ interval. This means that ripple effects dampen the distributional consequences of a shock. Intuitively, once a group is able to compete for and take over marginal tasks from others, the burden of the direct shocks it suffers will be lessened. This exhibits itself by the diagonal element of Θ corresponding to group g being less than $1/\lambda$ (recalling that the direct effect of a shock is $(1/\lambda) \cdot z_g$.
- 2. **Monotonicity:** for all $g' \neq g$, we have

$$
\theta_{gg} > \theta_{g'g},
$$

so that the maximum entry along a column of the propagation matrix is in the diagonal. This implies that a shock directly raising (reducing) demand for q cannot increase (decrease) the wage of group g' by more than g 's wage. It is this monotonicity property that ensures that relative demand curves for skill groups are always downward sloping in this framework.

3. Row sums: Row sums of the propagation matrix are

$$
\rho_g = \sum_{g'} \theta_{gg'} = \frac{1}{\lambda} \cdot \left[1 + s_k^y \cdot \left(\frac{\bar{\sigma}_{kg}}{\lambda} - 1 \right) \right]^{-1} \quad \text{for all} \quad g \in \mathbb{G},
$$

where $\bar{\sigma}_{kg} = \sum_{g'} (\theta_{gg'}/\rho_g) \cdot \sigma_{kg'}$ and s_k^y $\frac{y}{k}$ is the share of capital in national income. In the special case where there is no capital, so that s_k^y $\mathcal{L}_k^y = 0$, this simplifies to $\rho_g = \sum_{g'} \theta_{gg'} =$ $1/\lambda$ for all groups. Another noteworthy special case is when all groups are equally substitutable with capital, i.e., $\sigma_{kg} = \sigma_k$, in which case we have

$$
\rho_g = \sum_{g'} \theta_{gg'} = \frac{1}{\lambda} \cdot \left[1 + s_k^y \cdot \left(\frac{\sigma_k}{\lambda} - 1 \right) \right]^{-1} \quad \text{ for all } \quad g \in \mathbb{G}.
$$

The comparison of these two expressions shows that skill groups that are more substitutable for capital tend to have lower row sums (and we will see that this is going to influence whether a type of labor is a substitute or a complements to capital).

4. Propagation and substitution: The propagation matrix Θ is related to the matrix of elasticities of substitution $\Sigma = {\{\sigma_{gg'}\}}_{g,g'\in\mathbb{G}}$ via the identity

$$
\Theta = \text{diag}\left(\frac{1}{s^y}\right) \cdot \left(\lambda - \Sigma\right)^{-1},
$$

where diag($1/s^y$) is a diagonal matrix with entries $(1/s^y₁)$ $y_1^y, \ldots, 1/s_\mathcal{C}^y$ $_{G}^{y}$). This equation thus clarifies the tight connection between ripple effects and substitutability between labor types—greater substitution generates more substantial ripple effects.

5. **Symmetry:** The propagation matrix satisfies the symmetry property $\theta_{gg'}/s_q^y$ $\frac{y}{g'}$ = $\theta_{g'g}/s_g^y$ —a corollary of the symmetry of task shares and elasticities of substitution.

To illustrate these properties, we can return to the examples introduced above. In the Frechet example, the propagation matrix becomes

$$
\Theta = \left(\begin{array}{cccccc} \frac{1}{\kappa/(1-\rho)+1}+\frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_1^y & \frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_2^y & \dots & \frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_G^y \\ \frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_1^y & \frac{1}{\kappa/(1-\rho)+1}+\frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_2^y & \dots & \frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_G^y \\ & & & & \dots & \dots \\ \frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_1^y & \frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_2^y & \dots & \frac{1}{\kappa/(1-\rho)+1}+\frac{\kappa/(1-\rho)+1-\lambda}{(\kappa/(1-\rho)+1)\cdot\lambda}\cdot s_G^y \end{array}\right).
$$

With the Frechet parameterization, ripple effects are uniform—so that a shock to group g creates the same wage effect across all other groups. All eigenvalues of this matrix are equal to $1/(\kappa/(1-\rho)+1)$, and all shocks are dampened by an amount $\lambda/(\kappa/(1-\rho)+1)$. Naturally, the task framework is more general and allows for richer (and less restrictive) propagation patterns.

In the rest of this section, we study how different types of technological and factor

supply changes impact the economy via their direct effects and their indirect effects working through the propagation matrix.

5.3 Automation

We first use Proposition [6](#page-35-1) to study the implications of automation technologies.

Consider new technologies leading to the automation of the set of tasks $A = \bigcup_{g} A_g$ (with the same convention as before that \mathcal{A}_g comprises tasks previously performed by skill group g). Let us also assume that, for each g, \mathcal{A}_g is in the interior of the set of tasks performed by this group, \mathcal{T}_g . Then we can again summarize the share of tasks lost to automation for each skill group by $\{d \ln \Gamma_g^{\text{auto}}\}_g$, and cost savings from automation can be written as $\pi = \sum_g s_g^y \cdot d \ln \Gamma_g^{\text{auto}} \cdot \pi_g^{\text{auto}}$, where π_g^{auto} is the average cost savings from automating tasks previously performed by skill group g.

Proposition [6](#page-35-1) implies that the implications of new automation technologies with associated displacement effects and cost savings $\{d\ln\Gamma_g^{\text{ auto}}, \pi_g^{\text{auto}}\}_{g\in\mathbb{G}}$ are given by

(21)
$$
d\ln w = \Theta \cdot \text{stack} \left(d\ln y - d\ln \Gamma_g^{\text{auto}}\right)
$$

(22)
$$
\sum_{g} s_g^y \cdot d\ln w_g = \underbrace{\sum_{g} s_g^y \cdot d\ln \Gamma_g^{auto} \cdot \pi_g^{auto}}_{=d \ln tfp}.
$$

Equation (9) from the no-ripples economy is a special case of (21) , with the propagation matrix replaced by a matrix with $1/\lambda$ on the diagonal. All discussion of that equation applies in this case as well: automation again works via the productivity effect captured in the increase in output and the displacement effects summarized by $d\ln\Gamma_g^{\text{auto}}$. The configuration with significant distributional consequences but small productivity benefits from automation technologies is again a distinct possibility here.

Importantly, however, the full distributional effects of automation differ from those in the special case with no ripples. In the general case, groups of workers displaced from their tasks by automation intensify the competition for marginal tasks against groups with whom they are highly substitutable. This competition mitigates the adverse effects of automation on exposed groups by spreading the incidence of this shock more broadly. The formula for wages in [\(21\)](#page-39-0) shows that, in equilibrium, the downward wage pressure exerted by automation on a group not only depends on the displacement it experiences directly, as in the no ripple case, but also on whether groups competing for marginal tasks are being displaced, and groups competing against these groups are being displaced, and so on, as accounted for by the propagation matrix.

The TFP impact of automation in equation [\(22\)](#page-39-1) are identical between the economies with and without ripples. This is because the economy is competitive, and hence any marginal reallocation induced by the original changes has second-order effects on TFP via a standard envelope theorem logic. The reason why the automation shock itself has an impact on TFP is that it is not second-order—it corresponds to a discrete increase in the productivity of capital in a small set of tasks.

When Automation Reduces Real Wages As we have seen, the combination of competitive markets, constant returns to scale production possibilities and constant cost of capital ensure that automation always increases average real wages. This holds both in the full economy with ripples and in the no-ripple economy. However, and as anticipated above, this positive average-wage effect can coexist with significant negative impacts on some groups of workers. Proposition [6](#page-35-1) allows for a sharper characterization of the conditions under which such negative real-wage effects can arise.

From Proposition [6,](#page-35-1) the full impact of automation technologies on group q is

$$
d\ln w_g = \rho_g \cdot d\ln y - \sum_{g'} \theta_{gg'} \cdot d\ln \Gamma_{g'}^{\text{auto}},
$$

where ρ_g is the gth row sum of the entries of Θ .

First, consider the case in which $d\ln\Gamma_g^{\text{auto}} > 0$ and $d\ln\Gamma_{g'}^{\text{auto}} = 0$ for all other groups. In this case, a negative effect on group g is more likely (i) when ρ_g is small, (ii) when θ_{gg} is large, and (iii) when π_g is small. The last condition is the one we have already discussed: so-so automation technologies that bring little productivity benefits but impose large displacement effects can lead to real wage losses for affected groups. The first two conditions also highlight that this is more likely to be the case when this group has a low macroeconomic elasticity of substitution with other types of labor—because this would prevent it from competing for marginal tasks.

Second, suppose that group g itself is not affected by automation directly, $d\ln\Gamma_g^{\text{auto}} = 0$, but $d \ln \Gamma_{g'}^{\text{auto}} > 0$ for another skill group. Even in this case, the real wage level of group g could decline if there is high substitution between the two groups. This is again more likely to be the case when $d \ln y$ is small.

Third, now suppose that we have both $d\ln\Gamma_g^{\text{auto}} > 0$ and $d\ln\Gamma_{g'}^{\text{auto}} > 0$ for other groups.

This creates two sorts of effects: on the one hand, group g' workers displaced from their tasks will compete for marginal tasks currently performed by workers from group g. On the other hand, automation of tasks previously held by group g' generates a productivity effect, from which group g benefits. Notice, however, that in the case where all groups are equally impacted by automation, $(d \ln \Gamma_g^{\text{auto}} = d \ln \Gamma^{\text{auto}}$ for all g), we have $d \ln w_g =$ $\rho_g \cdot (d \ln y - d \ln \Gamma^{\text{auto}})$ and from the fact that average wages must increase following any technological advance, we can conclude that $d \ln y - d \ln \Gamma^{\text{auto}} > 0$ and thus no group can experience a real wage decline. This result implies that an uneven automation shock, impacting only or primarily a few groups among several, is more likely to have negative real wage effects.

Finally, again highlighting the importance of an uneven distribution of the impacts of automation, consider the special case where comparative advantage schedules of all groups are very similar. In the limit, we can have all groups having exactly the same comparative advantage, in which case the propagation matrix Θ converges to

$$
\Theta = \begin{pmatrix} s_1^y & s_2^y & \dots & s_G^y \\ s_1^y & s_2^y & \dots & s_G^y \\ & \dots & & \\ s_1^y & s_2^y & \dots & s_G^y \end{pmatrix}.
$$

When the propagation matrix has equal rows, then $d \ln w_g = d \ln \bar{w} > 0$, where the positive effect is a consequence of the fact that average wages increase following a technological improvement.

In sum, an uneven distribution of the burden of automation is essential for automation technologies to have a negative impact on any group. In addition, so-so automation technologies make negative effects more likely, and a group is also more likely to suffer negative consequences when the groups to which it is highly substitutable are also subject to displacement and it has low substitutability with groups that are not impacted by automation.

5.4 New Tasks

Proposition [6](#page-35-1) generalizes Proposition [3](#page-23-0) in the case of new tasks. The direct effects from new task creation are given by the task reinstatement terms $d\ln\Gamma_g^{\text{new}}$, and the contribution of this changes to TFP is $\pi = \sum_g s_g^y \cdot d \ln \Gamma_g^{\text{new}} \cdot \pi_g^{\text{new}}$, where π_g^{new} is the economic surplus created by new tasks. Proposition 6 shows that the effects of new task creation on wages and output is now given by

$$
d\ln w = \Theta \cdot \text{stack} \left(d\ln y - d\ln M + d\ln \Gamma_g^{\text{new}} \right)
$$

$$
d\ln t f p = \sum_g s_g^y \cdot d\ln w_g = \sum_g s_g^y \cdot d\ln \Gamma_g^{\text{new}} \cdot \pi_g^{\text{new}}.
$$

The content of these equations is familiar from Proposition [3.](#page-23-0) The equation shows that wages depend on a productivity effect, a task reinstatement effect, and ripples, which account for the propagation of the shock across workers due to the endogenous reassignment of tasks. Note that here, this reassignment generates a positive impact on other groups, even if they do not benefit from new tasks directly. This is because skill groups that obtain new tasks become less competitive for previously marginal tasks, increasing the demand for other skill groups in those tasks. The reason why the TFP equation in this case is identical to that in Proposition [3](#page-23-0) is the same we discussed for automation—only first-order effects, and not the induced reallocations, matter for TFP.

5.5 Labor-Augmenting Technology

As explained above, the task model distinguishes between narrow and uniformly laboraugmenting technologies. While we saw in Proposition [4](#page-26-0) that these had identical effects in the no-ripples economy, this is no longer true in the general case because the two forms of technological progress have different direct effects. In particular, from Proposition [6,](#page-35-1) we conclude that the effects of these technologies are now given by

(23)
$$
d\ln w = \Theta \cdot (d\ln y - (1 - \lambda) \cdot \text{stack}(d\ln A_g + d\ln \psi_g^{\text{intensive}})) + (1 - \Theta \lambda) \cdot \text{stack}(d\ln A_g),
$$

negative task-price decline
from no-riple case
uniform improvements

and the contribution of these technologies to TFP (which pins their effect on wage levels) remains unchanged. As before, narrow labor-augmenting technologies affect wages via a productivity effect, as they increase the production of some of the tasks that were previously assigned to the factor in question, and via the same adverse task-price declines we saw in Proposition [4](#page-26-0) for the no-ripples economy.

Uniform labor-augmenting technologies additionally allow groups becoming more productive to outcompete others for marginal tasks, increasing their task shares. This reallocation is also governed by the propagation matrix, which explains the extra term $(1 - \Theta \lambda) \cdot$ stack $(d \ln A_q)$ in the equation. This is always beneficial for own wages because 1−Θλ has a positive diagonal (and also negative off-diagonals, which correspond to marginal tasks being lost to other groups that have become more productive). This positive benefit dominates the adverse price declines at the intensive margin if θ_{gg} is below one, meaning that, group g has a sufficiently high macroeconomic elasticity of substitution with other skill groups.

This discussion further clarifies the difference between (uniform) factor-augmenting technological change—the form of technological progress typically emphasized in the literature on skill-biased technical change building on [Katz and Murphy](#page-93-0) [\(1992\)](#page-93-0)—and automation, as analyzed in [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0). As shown in equation [\(23\)](#page-42-0), the distributional effects of factor-augmenting improvements in technology are fully mediated by the macroeconomic elasticities of substitution, summarized by the propagation matrix. If macroeconomic elasticities are not far from unity, as many available estimates suggest, factor-augmenting technologies will have modest distributional effects (or put differently, with macroeconomic elasticities close to unity, one would need very large increases in group-level productivities to generate a meaningful divergence in wages across groups). In contrast, automation works at the extensive margin, and if it displaces low-education groups from the tasks they were previously performing, its direct impacts could be much larger—regardless of the macroeconomic elasticities of substitution since its main impact is by changing task shares given wages. This explains why automation can have sizable distributional consequences, even when different factors of production have macroeconomic elasticities of substitution near one.[19](#page-43-0) We return to this issue in Section [8,](#page-71-0) where we show that this difference has major quantitative implications about how models with automation and factor-augmenting technologies can account for large changes in inequality (see also the discussion in [Acemoglu and Restrepo,](#page-91-1) [2020b\)](#page-91-1).

5.6 Capital-Augmenting Technologies

We can again distinguish *narrow capital-augmenting technological change* and *uniformly* capital-augmenting technological change, whereby the former involves the productivity of capital changing only in tasks that are already assigned to capital, while the latter raises

¹⁹A related distinction explained in [Acemoglu](#page-90-0) [\(2002\)](#page-90-0) and [Acemoglu and Autor](#page-90-1) [\(2011\)](#page-90-1) is that, in canonical models of skill-biased technical change with two skill groups, technological progress making highlyeducated workers more productive always raises wages for the low-education group (an implication of q−complementarity with two production factors and constant returns to scale). In contradistinction, models of automation can generate large wage declines for exposed groups.

the productivity of capital in all tasks, including marginal ones. Proposition [6](#page-35-1) implies that the effects of these technologies on wages is given by

$$
d\ln w_g = \rho_g \cdot d\ln y - (1 - \lambda \cdot \rho_g) \cdot d\ln A_k
$$

while the effects on TFP are identical to those in the no-ripple economy. In this expression, $\rho_g \in [0, 1/\lambda]$ are the row sums of the propagation matrix. As in Proposition [5,](#page-28-0) narrow capital-augmenting technologies benefit all worker groups because they make capital more productive, generating a productivity effect, but they do not make capital more competitive in any marginal tasks. In contrast, the implications of uniformly capital-augmenting technologies differ, because, as opposed to the no-ripples economy, this makes capital more competitive in marginal tasks. This extra competition is captured by the negative term $(1 - \lambda \cdot \rho_g) \cdot d \ln A_k$, where a larger difference between 1 and $\lambda \cdot \rho_g$ signifies that group g is more substitutable for capital in marginal tasks.

As with uniform labor augmenting technologies, we see here that the distributional effects of uniform capital augmenting technologies is entirely determined by the macro elasticities of substitution between capital and labor, which are subsumed in the row sums of the propagation matrix. If these elasticities are not far from unity, uniform advances in capital, as those considered in [Krusell et al.](#page-93-1) [\(2000\)](#page-93-1) and the literature on investmentspecific technical change won't generate sizable distributional effects. Moreover, if these macro elasticities are below one, uniform advances in capital cannot generate the observed decline of the labor share. The direct effects of automation on the wage distribution and factor shares, on the other hand, are decoupled from these macro elasticities because these technologies operate at the extensive margin.

The task framework can also be used to provide a different microfoundation for skillspecific elasticities of substitution between capital and labor (a possibility first considered by [Griliches,](#page-92-0) [1969\)](#page-92-0). Consider for example, an economy with two types of labor, low-skill and high-skill, and suppose that high-skill labor has a very steep comparative advantage schedule in tasks that are marginal between itself and capital, while low-skill labor has flatter comparative advantage. Then, uniformly capital-augmenting technological change will increase inequality, because it *de facto* complements high-skill labor, while creating a more intense competition against low-skill labor.

5.7 Factor Supply Shocks

We have seen that the propagation matrix, which itself depends on comparative advantage, determines the equilibrium effects of any technological shock in our framework. The following proposition shows that the propagation matrix also mediates the effect of labor supply shocks.

PROPOSITION 7 (EFFECTS OF EXOGENOUS CHANGES IN LABOR SUPPLY) The effects of exogenous changes in $\{\ell_g\}_{g\in\mathbb{G}}$ are given by

(24)
$$
d\ln w = \Theta \cdot \text{stack}(d\ln y - d\ln \ell)
$$

where $d\ln y$ is pinned down by $\sum_{g} s_{g}^{y} \cdot d\ln w_{g} = 0$.

Labor supply changes also affect the wage structure through the propagation matrix because a labor supply expansion generates competition for marginal tasks from the expanding groups. This competition then determines the impact on the wages of both the expanding group and others. The propagation matrix summarizes these cross-group elasticities as well as the demand elasticity for the affected group. The substitution patterns summarized in the propagation matrix also point to the possibility that a particular group (say, domestic low-education workers) may suffer lower wages because of the increase in the supply of another group that is highly substitutable to them (such as immigrant workers).^{[20](#page-45-0)}

This proposition also provides guidance on how to account for the effects of exogenous labor supply changes on the wage structure, generalizing the approach in [Katz and Murphy](#page-93-0) [\(1992\)](#page-93-0) and [Card and Lemieux](#page-92-1) [\(2001\)](#page-92-1), who assume substitution patterns are given by a nested CES.

 20 In this case, we would have that the two groups are q-substitutes (as opposed to the more standard notion of q-complementarity). The propagation matrix contains all relevant information on whether different skill groups are q−complements or q−substitutes. Consider, for example, a case with no capital. An increase in the supply of skill group g increases output by $d\ln y = s_g^y \cdot d\ln \ell_g$ and reduces this group's wages by $\theta_{gg} \cdot (1-s_g^y)$. The diagonal terms in the propagation matrix thus specifies the slope of the aggregate elasticity of demand for group g. The supply shift alters other groups' wages by $d\ln w_{g'} = (\frac{1}{\lambda} \cdot s_g^y - \theta_{g'g}) \cdot d\ln \ell_g$ and we can see that g and g' are q-complements if $\frac{1}{\lambda} > \frac{1}{s_g^y} \cdot \theta_{g'g}$ (or equivalently, from symmetry $\frac{1}{\lambda} > \frac{1}{s_{g'}^y} \cdot \theta_{gg'}$). Pairs of groups with large corresponding off-diagonal entries can be q-substitutes. With the standard CES aggregate production function (with a common elasticity of substitution), all groups are q-complements.

6 The Multi-Sector Economy

In this section, we generalize the results from our analysis of the one-sector case to a multi-sector economy, and we also introduce markups at the sectoral level. The multisector extension is important for several reasons. First, the way we measure direct task displacement in the rest of the paper relies on this extension, since, in reality, the rate at which tasks are automated varies substantially across sectors. Second, the multi-sector economy enables us to incorporate the consequences of a richer menu of competing technological effects—including those that work through sectoral productivity changes—and the implications of changes in markups.

6.1 Environment

We now describe the environment, maximizing the similarity between our one-sector setup and this multi-sector environment.

A (unique) final good y is produced by combining the output y_i of a finite number of industries, indexed by $i \in \mathbb{I} = \{1, 2, ..., I\}$, via a constant returns to scale function $y = f(y_1, \ldots, y_I)$. We denote the unit-cost function for the final good by $c^f(p)$, where $p = (p_1, \ldots, p_I)$ is the vector of sector prices. We also denote the share of industry i in the economy by s_i^y $i^y(p) = \frac{\partial \ln c^f(p)}{\partial \ln p_i}$ $\frac{\ln c'(p)}{\partial \ln p_i}$, which naturally depends on the vector of sector prices (where the equality is a consequence of Shephard's lemma). We continue to set the final goal as the numeraire.

Production in each sector y_i requires the completion of the tasks in the set \mathcal{T}_i , where \mathcal{T}_i has positive measures given by $M_i > 0$. We assume without loss of generality that the sets $\{\mathcal{T}_i\}_{i\in\mathbb{I}}$ are disjoint and denote their union by \mathcal{T} , which makes up the tasks space of the entire economy.^{[21](#page-46-0)} As in our one-sector setup, task quantities $y(x)$ are aggregated using a constant elasticity of substitution (CES) aggregator with elasticity $\lambda \in (0,1)$:

$$
y_i = A_i \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_i} (M_i \cdot y(x))^{\frac{\lambda-1}{\lambda}} dx\right)^{\frac{\lambda}{\lambda-1}},
$$

where the new term, A_i , is a Hicks-neutral sector-specific productivity term.

An additional new element is that we allow for (exogenous) sector-specific markups, denoted by $\mu_i \geq 1$. This means that there will be a wedge between the marginal cost

 21 It is straightforward to allow for the same tasks to be performed in different industries, and whether we do so or not has no relevance for any of the results below.

of production in sector i and the sector's price, p_i , as we explain in the next subsection. The role of this assumption is to allow us to model labor market implications of changing markups within the US economy (as studied, for example, in [De Loecker et al.,](#page-92-2) [2020\)](#page-92-2). The case with $\mu_i = 1$ for all $i \in \mathbb{I}$ is a special case corresponding to a competitive economy.

As in the one-sector model, tasks are produced according to [\(1\)](#page-9-0). We continue to assume that labor is inelastically supplied while the capital needed for any task $x \in \mathcal{T}$ is produced from the final good at a constant marginal cost of 1.

We also continue to impose Assumption 1 from the one-sector model, except that the finite integrals and strict comparative advantage are now imposed sector by sector.

6.2 Equilibrium

We now define an equilibrium in this extended environment, which takes into account cost minimization by sectoral producers as well as the markups.

A market equilibrium is given by a positive vector of real wages $w = \{w_g\}_{g \in \mathbb{G}}$, a positive vector of sectoral prices $p = \{p_i\}_{i \in \mathbb{I}}$, an aggregate output level y, an allocation of tasks to skill groups $\{\mathcal{T}_{gi}\}_{g \in \mathbb{G}, i \in \mathbb{I}}$ and capital $\{\mathcal{T}_{ki}\}_{i \in \mathbb{I}}$ in each industry, task prices $\{p(x)\}_{x \in \mathcal{T}}$, task labor demands $\{\ell_g(x)\}_{g\in\mathbb{G},x\in\mathcal{T}}$ and capital production levels $\{k(x)\}_{x\in\mathcal{T}}$ such that:

E1 Task prices are equal to the minimum unit cost of producing the task:

$$
p(x) = \min \left\{ \frac{1}{A_k \psi_k(x)}, \left\{ \frac{w_g}{A_g \psi_g(x)} \right\}_{g \in \mathbb{G}} \right\}.
$$

E2 Tasks are produced in a cost-minimizing way, which means that for each sector $i \in \mathbb{I}$, the set of tasks

$$
\mathcal{T}_{gi}(w) = \left\{ x : p(x) = \frac{w_g}{A_g \psi_g(x)} \right\}
$$

is allocated to workers from skill group $q \in \mathbb{G}$, and the set of tasks

$$
\mathcal{T}_{ki}(w) = \left\{ x : p(x) = \frac{1}{A_k \psi_k(x)} \right\}
$$

is produced with capital (where we condition on the vector of wages for later reference).

E3 Task-level demands for labor (for any $g \in \mathbb{G}$) and capital are given by

$$
\ell_g(x) = \begin{cases} y_i \cdot p_i^{\lambda} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda - 1} \cdot \frac{1}{M_i} \cdot A_g^{\lambda - 1} \cdot \psi_g(x)^{\lambda - 1} \cdot w_g^{-\lambda} & \text{for } x \in \mathcal{T}_{gi}(w) \\ 0 & \text{otherwise.} \end{cases}
$$

and

$$
k(x) = \begin{cases} y_i \cdot p_i^{\lambda} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda - 1} \cdot \frac{1}{M_i} \cdot A_k^{\lambda - 1} \cdot \psi_k(x)^{\lambda - 1} & \text{for } x \in \mathcal{T}_{ki}(w) \\ 0 & \text{otherwise.} \end{cases}
$$

E4 The labor market clears for all q .

$$
\sum_i \int_{\mathcal{T}_{gi}} \ell_g(x) \cdot dx = \ell_g.
$$

E5 Sector *i*'s price is given by its marginal cost times markup μ_i :

$$
p_i = \mu_i \cdot \frac{1}{A_i} \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_i} p(x)^{1-\lambda} \cdot dx\right)^{1/(1-\lambda)}.
$$

E6 The price of the final good is 1, which implies

$$
1 = c^f(p).
$$

In addition, as in the one-sector model, we use the tie-breaking rule that when a task can be performed at equal cost by multiple factors, it is first assigned to capital and then to lower-indexed skill groups ahead of higher-indexed groups. Strict comparative advantage again ensures that such ties can occur only on a set of measures zero, and thus this tiebreaking rule has no substantive implications.

Figure [5](#page-49-0) provides a graphical illustration of the equilibrium, emphasizing the allocation of the tasks in each industry to different factors and their aggregation to the production of the unique final good.

Most of these equilibrium conditions are familiar from the one-sector model. E1-E2 are identical to before, except for the indexing by industry, and leverage cost-minimization. E3 and E5 are different from before because of the presence of markups: the latter condition imposes that price is a markup over marginal cost and the former adjusts factor demands for the presence of markups—higher markups translate into lower factor demands. E4 aggregates the demand for labor across industries, while E6 is again the numeraire condition.

FIGURE 5: EQUILIBRIUM TASK ASSIGNMENT AND TASK SHARES. The figure depicts the tasks space of a multi-sector economy and shows automation and new task creation taking place in industry i.

As before, we can represent the equilibrium in terms of task shares, but now defined separately by sector $i \in \mathbb{I}$:

$$
\Gamma_{gi}(w) \equiv \frac{1}{M_i} \int_{\mathcal{T}_{gi}(w)} \psi_g(x)^{\lambda - 1} \cdot dx \text{ for all } i \in \mathbb{I} \text{ and } g \in \mathbb{G}
$$

$$
\Gamma_{ki}(w) \equiv \frac{1}{M_i} \int_{\mathcal{T}_{ki}(w)} \psi_k(x)^{\lambda - 1} \cdot dx \text{ for all } i \in \mathbb{I}.
$$

PROPOSITION 8 (EQUILIBRIUM REPRESENTATION) Equilibrium wages w, industry prices p, and level of output y, solve the system of equations

(25)
$$
w_g = \left(\frac{y}{\ell_g}\right)^{1/\lambda} \cdot A_g^{1-1/\lambda} \cdot \left[\sum_i s_i^y(p) \cdot p_i^{\lambda-1} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda-1} \cdot \Gamma_{gi}(w)\right]^{1/\lambda} \text{ for } g \in \mathbb{G},
$$

(26)
$$
p_i = \mu_i \cdot \underbrace{\frac{1}{A_i} \cdot \left(\Gamma_{ki}(w) \cdot A_k^{\lambda-1} + \sum_g \Gamma_{gi}(w) \cdot \left(\frac{w_g}{A_g} \right)^{1-\lambda} \right)^{1/(1-\lambda)}}_{\equiv C_i(w)} \text{ for } i in \mathbb{I},
$$

$$
(27) \t 1 = c_f(p),
$$

where $C_i(w)$ denotes the marginal cost of producing output of sector i.

This characterization is analogous to the one in Proposition [1](#page-13-0) for the one-sector model, except that we now also have an additional equilibrium condition for sectoral prices. Two new economic forces are apparent from the wage equation. First, the summation over the sectoral value-added shares, the s_i^y $i(p)$'s, represents the role of sectoral composition in the labor market equilibrium. Intuitively, technology, price, markup and task share variables from sectors with high s_i^y $i(p)$ will matter more for wages. We will see the implications of this first force in more detail below. Second, higher markups reduce wages, for reasons we have already mentioned.

6.3 Effects of Technology in the Multi-Sector Economy

We can use the characterization in Proposition [8](#page-49-1) to derive the effects of different types of technologies on the equilibrium wage structure. To do this, we again use the propagation matrix, which in this case can be written as

$$
\Theta = \frac{1}{\lambda} \cdot \left(1 - \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma(w)}{\partial \ln w} \right)^{-1},
$$

where the Jacobian $\partial \ln \Gamma(w)/\partial \ln w$ is now the $G \times G$ matrix with its gg'th entry

$$
\sum_{i} \omega_{gi} \cdot \frac{\partial \ln \Gamma_{gi}(w)}{\partial \ln w_{g'}},
$$

where ω_{gi} denotes wage payments received by group g in industry i as a share of total group wage payments. This matrix summarizes how changes in the wage of group g' affects group g by summing over the effects taking place in different industries.

As in the previous section, we start with the direct effects of new technologies, represented by the vector z , on the demand for skill group g . These effects are defined as the percent change in the right-hand side of [\(25\)](#page-49-2) (holding wages, output, and sectoral prices constant). For notational convenience, we normalize direct effects by $1/\lambda$, so that $(1/\lambda) \cdot z$ is the change in the right side of [\(25\)](#page-49-2). We also define the productivity gains at the sectoral level as $\pi_i = -d \ln C_i(w)|_{w=\text{constant}} > 0$, which gives the TFP change for sector *i*. Finally, and to simplify the exposition, we assume industries are combined into the final good with a constant elasticity of substitution η , though this can be relaxed.

PROPOSITION 9 (EFFECTS OF TECHNOLOGY IN THE MULTI-SECTOR ECONOMY) Consider a change in technology with direct effect $\{z_{gi}\}_{g \in \mathbb{G}, i \in \mathbb{I}}$ on task shares and productivity gains

 $\{\pi_{gi}\}_{g \in \mathbb{G}, i\in \mathbb{I}}$. The effect of this technology on wages, sectoral prices, and output is given by

(28)
$$
d\ln w = \Theta \cdot \text{stack}\left(d\ln y + \sum_{i} \omega_{gi} \cdot z_{gi} + (\lambda - \eta) \cdot \sum_{i} \omega_{gi} \cdot d\ln p_i\right)
$$

(29)
$$
d\ln p_i = \sum_g s_g^{y_i} \cdot d\ln w_g - \pi_i \text{ (for all } i \in \mathbb{I})
$$

(30)
$$
0 = \sum_{i} s_i \cdot d \ln p_i.
$$

Here s_g^y is the share of payments to skill group g in value-added in industry i, s_i is the share of industry i in total costs, and ω_{gi} denotes wage payments received by group g in industry i as a share of total group wages.

The proposition shows that technological change affects the wage structure via four distinct channels—three of which were already present in the one-sector model (recall Propo-sition [6\)](#page-35-1). The first is the productivity effect, represented by $d\ln y$. The second comprises the usual direct effects of technology, the z_{gi} 's, except that these are now at the industry level and have to be aggregated. The third is captured by the propagation matrix, Θ, pre-multiplying the vector on the right-hand side of equation [\(28\)](#page-51-0), which again summarizes the ripple effects of the technology terms.

The fourth and new element is the last term on the right side of [\(28\)](#page-51-0). This corresponds to changes in the sectoral composition of the economy, which can be non-neutral if expanding sectors substantially differ from contracting ones in their factor demands. Conversely, these changes are neutral when all sectors employ the same mix of workers. More generally, this term captures two forces. On the one hand, a reduction in the price of sector i increases its quantity, raising its demand for labor. This sectoral-demand effect depends on the elasticity of substitution between sectors η . On the other hand, a reduction in the price of sector i reduces the value of marginal product of workers and the demand for their services with an elasticity λ . When $\lambda > \eta$, the first effect dominates and sectoral shifts benefit workers in sectors experiencing less productivity growth. This captures the same economic mechanism as in the celebrated Baumol effect [\(Baumol et al.,](#page-91-2) [2012\)](#page-91-2): workers specializing in sectors with lower (technological) productivity growth, such as healthcare, tend to benefit because the relative prices of these sectors increase.

Finally, the exact equilibrium changes in sectoral prices can be obtained from [\(29\)](#page-51-1), while equation [\(30\)](#page-51-2) pins down the change in the output level.

It is useful to illustrate the results of Proposition [9](#page-50-0) for automation technologies. The effects of automation on wages are now given by

(31)
$$
d\ln w = \Theta \cdot \operatorname{stack}\left(d\ln y - d\ln \Gamma_g^{\text{auto}} + (\lambda - \eta) \cdot \sum_i \omega_{gi} \cdot d\ln p_i\right),
$$

with $d\ln\Gamma_g^{\text{auto}}$ the total direct task displacement due to automation experienced by g,

(32)
$$
d\ln\Gamma_g^{\text{auto}} = \sum_i \omega_{gi} \cdot d\ln\Gamma_{gi}^{\text{auto}}.
$$

This is obtained by summing the direct task displacement from automation experienced by group g in industry i, $d \ln \Gamma_{gi}^{\text{auto}}$, across industries. The summation weights are given by the shares of wage payments from industry i in group g 's total wage payments. The wage equation [\(31\)](#page-52-0) again contains the usual productivity and displacement effects of automation, as well as the ripples via the propagation matrix.

The new element here relative to the single-sector economy is the indirect effect of automation working via its impact on sectoral prices, which shift the composition of the economy. These effects depend on the contribution of automation to the TFP of the different sectors, which is given by $\pi_i = \sum_g s_g^{y_i} \cdot d \ln \Gamma_{gi}^{\text{auto}} \cdot \pi_{gi}^{\text{auto}},$ where the π_{gi}^{auto} 's are the average cost savings from automation in sector i. For $\lambda > \eta$, which is the case we consider in our quantitative exercise, automation reallocates labor demand away from sectors that automate at a higher rate, reducing the relative wages of workers in these industries.

Observe that the equilibrium here is not competitive because of the presence of markups. But when there are no markups or when markups are uniform across sectors $(\mu_i = \mu)$, the economy is again efficient. In that case, equations [\(29\)](#page-51-1) and [\(30\)](#page-51-2) imply that average wage changes from automation are

$$
\sum_{g} s_g^y \cdot d\ln w_g = \underbrace{\sum_{i} s_i \sum_{g} s_g^{y_i} \cdot d\ln \Gamma_{gi}^{\text{auto}} \cdot \pi_{gi}^{\text{auto}}}_{=d\ln tfp},
$$

where the term on the right-hand side is aggregate TFP, obtained by summing the cost savings due to automation in different industries. As in the single-sector model, we see that the effect of automation on wage levels depends on its contribution to TFP, and could be large or small depending on how big the cost savings due to this technology are.

6.4 Sectoral TFP and Markups

Proposition [8](#page-49-1) also enables us to determine the labor market implications of sector-specific (Hicks-neutral) technological advances and changes in markups. Let us start with sectorspecific technologies, which are important drivers of structural change in the economy (see [Ngai and Pissarides,](#page-93-2) [2007;](#page-93-2) [Buera et al.,](#page-92-3) [2021\)](#page-92-3). From Proposition [9,](#page-50-0) the effect of these technologies satisfies

$$
d\ln w = \Theta \cdot \text{stack}\left(d\ln y - (1 - \lambda) \cdot \sum_{i} \omega_{gi} \cdot d\ln A_i + (\lambda - \eta) \cdot \sum_{i} \omega_{gi} \cdot d\ln p_i\right)
$$

$$
d\ln p_i = \sum_{g} s_g^{y_i} \cdot d\ln w_g - d\ln A_i \text{ (for all } i \in \mathbb{I})
$$

$$
0 = \sum_{i} s_i \cdot d\ln p_i.
$$

Hicks-neutral increases in sectoral TFP affect the wage structure via the four channels identified in Proposition [9.](#page-50-0) The first is the usual productivity effect, which corresponds to the expansion of aggregate output, $d \ln y$. The second is through the reduction of task prices in the sectors that become more productive. These task-price effects are also aggregated according to the exposure of different skill groups to the industry in question, as measured by the wage-bill shares ω_{gi} . The third channel is via the ripple effects, encoded in the propagation matrix Θ. The fourth is through sectoral price changes, as captured by the last term on the right-hand side of the wage equation.

The comparison of this wage equation to [\(31\)](#page-52-0) shows the differences between sectoral TFP improvements and automation. While the latter works via the extensive-margin of task reallocation taking place within sectors, there is no equivalent of such direct effects in the case of sectoral TFP, which work exclusively by reallocating labor demand across sectors.

Finally, we can derive the effects of changes in markups. This follows from our characterization of the equilibrium in Proposition [8](#page-49-1) and is presented next.

PROPOSITION 10 (EFFECTS OF MARKUPS IN THE MULTI-SECTOR ECONOMY) Consider an exogenous change in sectoral markups $\{d \ln \mu_i\}_{i \in \mathbb{I}}$. The impact on wages, sectoral prices, and

output is given by

(33)
$$
d\ln w = \Theta \cdot \text{stack}\left(d\ln y - \lambda \cdot \sum_{i} \omega_{gi} \cdot d\ln \mu_i + (\lambda - \eta) \cdot \sum_{i} \omega_{gi} \cdot d\ln p_i\right)
$$

(34)
$$
d\ln p_i = \sum_g s_g^{y_i} \cdot d\ln w_g + d\ln \mu_i \text{ (for all } i \in \mathbb{I})
$$

(35)
$$
0 = \sum_{i} s_i \cdot d \ln p_i.
$$

This proposition shows that markups affect the wage structure via the same four chan-nels identified in Proposition [9.](#page-50-0) The first is the productivity effect, given by $d\ln y$, which results from the fact that increases in markups can reduce output. The second is the direct effects of the changes in markups, which are aggregated using wage-bill shares. This effect is negative because markups reduce production in the affected sectors. The third is through the ripple effects that these changes create and is summarized by the propagation matrix, Θ. The fourth channel are the shifts in the sectoral composition of the economy due to price changes.

Just like the sector-specific technology terms discussed above, markups' impact all workers in an industry uniformly. This is why their distributional effects work through shifts in labor demand across sectors—and they do not generate any type of displacement or reinstatement. The distributional effects of this reallocation across sectors will be muted if expanding and contracting sectors do not differ substantially in their skill mixes. This is the reason why we expect, from a theoretical point of view, these effects to be less pronounced than those coming from automation and new tasks, and this is indeed what we document in the next sections.

7 Reduced-Form Evidence

In this section, we estimate the reduced-form equations implied by the task framework. We focus on US labor market trends between 1980 and 2016. The evidence we present supports the key prediction of the task framework—that extensive-margin changes in the allocation of tasks to factors, driven by automation and new tasks, have first-order effects on wage inequality, and in fact, appear to have impacts that are much larger than proxies for other technologies that either work mainly by increasing the productivity of highly-educated workers or induce sectoral reallocations. Consistent with the expectation that automation and new tasks, via their influence on labor demand, should also affect employment, we find that these forces have had large impacts on employment outcomes as well.

We start by exhibiting some of the key inequality and employment trends in the US data. We then derive and discuss the specification we use, which is based on Proposition [9.](#page-50-0) Before presenting our main reduced-form results, we describe in detail how displacement due to automation and reinstatement due to new tasks are measured.

7.1 US Labor Market Trends

FIGURE 6: Cumulative growth in real hourly wages for men and women by education level (GTC: postcollege degree, CLG: college degree, SCL: some college, HSG: high school degree, HSD: high school dropout), 1960-2022. Diamonds: data from the US Census and the American Community Survey. Connected line: data from the Current Population Survey. Wages deflated using the personal consumption expenditure index from the Bureau of Economic Analysis.

Figure [6](#page-55-0) summarizes the major inequality trends in the US data. It plots cumulative real hourly wage growth since 1960 by gender (separately in the left and the right panels) and by education (different series in the same panel). We show both data from the CPS (with connected dots) and the decennial Censuses and the ACS (with diamonds), since there are some minor differences in the implied trends in these two data sources. The main message is quite clear: in the 1960s and 70s, hourly wages grew by about 1.5%-2% per annum for all groups, and the average rate of real wage growth was very similar to the growth rate of labor productivity, implying that the labor share of national income remained stable during this period. From around 1980 to 2016, we see a strikingly different pattern: hourly

real wages continue to grow for workers with a college degree, and especially those with a postcollege degree, while wages for noncollege workers stagnated and, for men with a high school degree or less, even declined by a nontrivial amount.^{[22](#page-56-0)} In line with the sluggish wage growth observed during this period, the labor share of national income declined markedly since 1980, as shown in Figure [7,](#page-56-1) especially in manufacturing and retail.

FIGURE 7: The evolution of labor shares in the US economy, manufacturing, wholesale, retail, utilities, and transportation. Data from the BEA-BLS Integrated Industry Accounts, 1963-2016.

Figure [8](#page-57-0) shows that these unequal wage trends coincided with a divergence in employment patterns. Since 1980, employment rates for men without a college degree have declined (though the beginning of this trend for some groups can be seen in the 1970s), while employment rates among women with a high school degree and associate degree, which were increasing rapidly before, start a mild decline.

For our analysis, we organize the data at a more granular level than these two figures, studying the evolution of wages and employment for 500 demographic groups, which proxy for the skill groups our theory. These demographic groups are defined by the same five education groups, five age groups (16–25 years of age, 26–35, 36–45, 46–55, 56–65), gender, ethnicity (White, Black, Hispanic, and Asian), and native vs. foreign-born status. For each

 22 There is an uptick in the wage growth of workers with high school degree or less at the very end of our sample. [Autor et al.](#page-91-3) [\(2024\)](#page-91-3) show that this pattern continues after 2017 and explore its causes.

FIGURE 8: Employment rates for men and women by education level (GTC: postcollege degree, CLG: college degree, SCL: some college, HSG: high school degree, HSD: less than high school), 1960-2022. Data from the US Census and the American Community Survey are shown as diamonds, and data from the Current Population Survey are shown as the connected lines.

group, we compute the change in log hourly wages and the change in log hours worked from 1980 to 2016 using the 1980 Census and pooling five years of the American Community Survey (ACS) between 2014 and 2018.

7.2 Specification

Our reduced-form specification relates the wage changes experienced by US worker groups between 1980 and 2016 to proxies of automation, new task creation, sectoral TFP growth and markups. To motivate our choice of specification, let us rewrite the wage equation derived in the previous section, [\(28\)](#page-51-0), in the following form:

(36)
\n
$$
d\ln w_g = \theta_{gg} \cdot d\ln y - \theta_{gg} \cdot d\ln \Gamma_g^{\text{auto}} + \theta_{gg} \cdot d\ln \Gamma_g^{\text{new}} - \theta_{gg} \cdot (1 - \lambda) \cdot d\ln \psi_g^{\text{int}} + (1 - \theta_{gg}) \cdot d\ln A_g - \theta_{gg} \cdot (1 - \lambda) \sum_i \omega_{gi} \cdot d\ln A_i - \theta_{gg} \cdot \lambda \sum_i \omega_{gi} \cdot d\ln \mu_i + \theta_{gg} \cdot (\lambda - \eta) \sum_i \omega_{gi} \cdot d\ln p_i + u_g.
$$

Here, θ_{gg} is the gth diagonal entry of the propagation matrix. In addition, $d\ln\Gamma_g^{\text{auto}}$ is total direct task displacement for group g , defined in (32) as the summation of the industry-level task displacements, the $d\ln\Gamma_{gi}^{\text{auto}}$'s across industries. $d\ln\Gamma_{g}^{\text{new}}$ is defined analogously as total

direct task reinstatement experienced by group q:

$$
d\ln\Gamma_g^{\rm new}=\sum_i\omega_{gi}\cdot d\ln\Gamma_{gi}^{\rm new},
$$

with ω_{gi} being the share of wage payments received by group g in industry i in the total wage payments of the group. Finally, as before, the $d \ln A_g$'s are the labor-augmenting terms, the $d\ln A_i$'s are the sectoral TFP terms, the $d\ln \mu_i$'s denote changes in sectoral markups, and $d\ln p_i$'s are changes in sectoral prices. Notice that in writing this equation, we have ignored effects working through the off-diagonal terms of the propagation matrix (ripple effects and capital-augmenting technologies), which are therefore included in the error term, u_q .

Our estimation equation is derived from [\(36\)](#page-57-1) and takes the form:

$$
\Delta \ln w_g = \text{constant} + \beta^{\text{auto}} \cdot \text{Task displacement from automation}_g^{1980-2016} \n+ \beta^{\text{new}} \cdot \text{Task reinstantment from new tasks}_g^{1980-2016} \n+ Dummies for education level + Dummies for gender \n+ \beta^{\text{sector}} \cdot \text{Section} \text{ shifts}_g + \nu_g.
$$

This equation is derived with the following steps. First, we include the productivity effect, $d \ln y$, in the constant. Second, instead of the infinitesimal changes we focus on changes between 1980 and 2016, and thus we replace $d\ln\Gamma_g^{\text{auto}}$ and $d\ln\Gamma_g^{\text{new}}$ with their empirical counterparts, whose construction we discuss below. Third, we adopt specific parameterizations of the sectoral technology and markup variables (also provided below). Fourth, we include education and gender dummies to control for differential trends by education and gender, including those driven by labor-augmenting technologies. Fourth, we subsume the ripple effects, as well as the effects of capital-augmenting technologies, if any, in the error term v_q .

If ripple effects—terms that multiply the off-diagonal terms left in the error term—are large, this reduced-form will not estimate unbiased direct effects of task displacement and reinstatement. Hence, initially, we assume that the off-diagonal terms of the propagation matrix are small, which also implies that the θ_{gg} 's are close to $1/\lambda$, and this estimating equation has constant coefficients.

To see that the education and gender dummies also control for various forms of laboraugmenting technologies, first suppose that $d\ln\psi_g^{\text{intensive}} = 0$, and assume that the uni-

formly labor-augmenting technology can be written as $d\ln A_g = \delta_{\text{eduction}_g} + \delta_{\text{gender}_g} + \tilde{u}_g$, where $\delta_{\text{eduction}_q}$ denotes a set of dummies corresponding to the education level of group g, δ_{gender_g} is a dummy designating the gender of group g, and \tilde{u}_g is the residual assumed to be iid. This assumption thus imposes that any two demographic groups that have the same education level and gender have the same labor-augmenting technological effects, except an iid residual. When narrow labor-augmenting technology terms, the $d\ln\psi_g^{\text{ intensive's}},$ are present, these can also be subsumed into the same dummies. Note also that this strategy could overestimate the importance of labor-augmenting technologies if there are non-technological factors correlated with education or gender.^{[23](#page-59-0)}

Finally, the sectoral shifts in the last line represent the effects of sectoral reallocation, TFP and markups in the second line of [\(36\)](#page-57-1). We adopt two strategies to deal with sectoral shifts. The first one follows [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0) and proxies for the influence of sectoral shifts using observed changes in value-added.

$$
Section 1 value-added sharesg = \sum_{i} \omega_{gi} \cdot \Delta \ln Value-added sharei,
$$

where ω_{gi} is the share of wages group g received from industry i (computed using the 1980 Census) and $\Delta \ln$ Value-added_i is the change in industry value-added over 1980–2016 (computed for 50 industries using the BEA-BLS Integrated Industry Accounts, which are then matched to the 1980 Census). In this case, the regression equation should be interpreted as estimating the impacts of automation and new task creation holding sectoral shares constant, and for this reason, it might underestimate the total impact of automation and new tasks, which induce sectoral reallocations (especially if they are unevenly distributed across sectors).

Our second strategy is to directly use measures of sectoral TFP and markups in the regression. Our analysis in the previous section clarifies that both of these trends should only affect wages via their sectoral implications, which motivates this strategy. These controls are parameterized as

$$
\text{Sectoral TFP}_g = \sum_i \omega_{gi} \cdot \Delta \ln \text{ Multifactor TFP}_i,
$$

$$
\text{Sectoral markups}_g = \sum_i \omega_{gi} \cdot \Delta \ln \text{ Markups}_i.
$$

This strategy thus controls for (some of the) determinants of sectoral composition. In our

 23 Possible factors that could be absorbed by these dummies include changes in gender discrimination, quality of education, and compositional changes.

empirical models, these two strategies yield fairly similar results, which we find reassuring.

These considerations as well as the fact that any indirect effects are in the residual imply that the reduced-form estimates should be interpreted with caution. Moreover, without estimates of ripple and productivity effects, this approach cannot reveal the full equilibrium impacts of automation and new tasks, which is what motivates our more structural approach in the next section.

In addition to [\(37\)](#page-58-0), we estimate identical equations with changes in log hours per (adult) person on the left-hand side. Since the technology terms on the right-hand side of [\(37\)](#page-58-0) shift labor demand, we expect that they should also impact employment, provided that labor-leisure tradeoffs or labor market imperfections lead to upward-sloping (quasi-)labor supply curves (see Section [8](#page-71-0) for more on this).

7.3 Measuring Automation and New Tasks

As in [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0), we measure task displacement due to automation using automation-induced industry labor share changes and information on which types of workers within an industry are most likely to be impacted by automation. In particular, as in that paper, we assume that automation in an industry only displaces workers in routine occupations and that such displacement takes place at equal rates for workers in these occupations, regardless of their demographic groups. This means that if there are workers from two demographic groups g and g' in a routine occupation undergoing automation, then the same proportion of workers from these two demographic groups in this occupation will be displaced.

Under these assumptions, we show in the Appendix that task displacement due to automation in industry i is

(38)
$$
d\ln\Gamma_{gi}^{\text{auto}} = \text{RCA routine}_{gi} \cdot (-\Delta \ln s_{\ell}^{y_i,\text{auto}}),
$$

and in equation [\(32\)](#page-52-1), total task displacement can be computed as

(39) Task displacement from automation¹⁹⁸⁰⁻²⁰¹⁶
$$
= d \ln \Gamma_g^{\text{auto}} = \sum_i \omega_{gi} \cdot d \ln \Gamma_{gi}^{\text{auto}}.
$$

Overall, total task displacement experienced by group q depends on three terms:

• Group g's "revealed comparative advantage" in routine tasks in industry i, RCA routine_{gi}. This term adjusts for the incidence of automation across workers in an industry. Intuitively, if group q performs all routine tasks in industry i , then an increase in automation in that industry will displace group g only. If multiple groups perform routine tasks in the industry, then an increase in automation in that industry will displace them in proportion to the share of routine tasks they perform in that industry (which our revealed comparative advantage captures). This term is computed from the 1980 Census as the ratio of wages earned by group q in routine jobs in industry i over all wage payments in routine jobs in the industry. We define routine jobs as the top one-third of occupations with the highest routine content, using the measure of routine tasks from ONET from [Acemoglu and Autor](#page-90-1) [\(2011\)](#page-90-1).

• The extent of automation in industry i , measured by the percent reduction in industry i's labor share due to automation, $-\Delta \ln s_\ell^{y_i, \text{auto}}$ ℓ ^{*y_i*, auto}. This term gives the total share of tasks lost to automation among all workers in the industry under the assumption that $\lambda = 1$. The general case with non-unitary elasticity of substitution between tasks includes an additional adjustment term, but does not appreciably change the results, as we further discuss in the next section.

We measure this quantity in two steps. In a first step, we run a regression of the observed percent decline in industry labor shares from 1987 to 2016 from the BEA-BLS integrated industry accounts against three proxies of automation. These proxies include the adjusted penetration of robots over 1993–2007, which uses data from European countries that are ahead of the US in terms of robot adoption and with the adjustment discussed in [Acemoglu and Restrepo](#page-91-4) [\(2020a\)](#page-91-4); the change in cost share of dedicated machinery and specialized software from 1987 to 2016 (both from the BLS detailed capital tables). These regressions are reported in [Acemoglu and](#page-91-0) [Restrepo](#page-91-0) [\(2022\)](#page-91-0) and show that these three proxies account for 50% of the crossindustry variation in labor shares. In a second step, we take the predicted labor share change from this cross-industry regression and use it as a measure of the labor share decline driven by automation.

Figure [9](#page-62-0) summarizes the results of this measurement exercise. It depicts both the observed labor share declines and the predicted declines driven by automation (both in percent terms, and the former in blue and the latter in orange). Observed labor share declines and those driven by automation are highly correlated, but there are also some notable exceptions. Several industries that are part of the transport sector have large overall declines in labor share, but only moderate predicted declines due to automation—because they have relatively low levels of robot penetration and dedicated machinery and specialized software expenditures. Several other industries, including automobile manufacturing, show both sizable observed declines and predicted declines due to automation, because they also exhibit large investments in automation technologies.[24](#page-62-1)

• The group's exposure to the industry ω_{qi} , used as weight in summing across industrylevel task displacements. Intuitively, this term measures the importance of tasks performed in industry i for the total labor demand for group g , and is computed from the 1980 Census for 50 industries that we can track consistently in the BEA-BLS integrated industry accounts.[25](#page-62-2)

FIGURE 9: Percent decline in industry labor shares (in blue) and the predicted declines are based on industry investments in automation technologies (in orange). The observed declines are computed from the BEA-BLS Integrated Industry Accounts. The predicted declines are from a regression of the observed declines on the adjusted penetration of robots (from [Acemoglu](#page-91-4) [and Restrepo,](#page-91-4) [2020a\)](#page-91-4), and the increase in expenditures in dedicated machinery and specialized software (both from the BLS Detailed Capital Tables).

Our measure of task reinstatement due to new tasks uses data from [Lin](#page-93-3) [\(2011\)](#page-93-3), previously analyzed in [Acemoglu and Restrepo](#page-90-2) [\(2018b\)](#page-90-2). These data, in turn, rely on new job titles from the Dictionary of Occupational of Titles (DOT) in 1977 and 1991 and from the

²⁴One could use these proxies directly as regressors or instruments, and we do this in [Acemoglu and](#page-91-0) [Restrepo](#page-91-0) [\(2022\)](#page-91-0). Projecting these measures on the labor share decline is helpful because it converts them into units of "tasks lost" to automation and allows us to summarize their effects in a single variable representing the task displacement associated with these technologies.

²⁵The list of these industries and those for consistent occupations and aggregated job categories we use below can be found in the Replication Package.

2000 Census. Using these data, we construct task reinstatement for group g in industry i as

$$
d \ln \Gamma_{gi}^{\text{new}} = \sum_{o} \omega_{gio}^{1980} \cdot \text{Share new job titles DOT 1977}
$$

$$
+ \sum_{o} \omega_{gio}^{1990} \cdot \text{Share new job titles DOT 1991}
$$

$$
+ \sum_{o} \omega_{gio}^{2000} \cdot \text{Share new job titles Census 2000},
$$

where ω_{qio} denotes the share of total wage payments to group g in industry i that come from occupation o. Analogously with the total task displacement measure, total task reinstatement for group q is computed as

(40) Task reinstatement from new tasks¹⁹⁸⁰⁻²⁰¹⁶ =
$$
d \ln \Gamma_{gi}^{\text{new}} = \sum_{i} \omega_{gi} \cdot d \ln \Gamma_{gi}^{\text{new}}
$$
.

The underlying assumption in computing task reinstatement is that new job titles proxy for new tasks (and are not just a relabeling of existing jobs), that each new job title has the same positive impact on new tasks, and that these new tasks will be proportionately spread between workers that are currently performing other tasks in the same occupations. Therefore, this measurement strategy implies that if an occupation experiences 10% new job titles and another one experiences 20% new job titles, labor demand should increase twice more in the latter than the former, and as a result, demographic groups in the latter occupation should experience twice the proportionate increase in tasks. These considerations also motivate the use of the wage-bill share of different demographic groups in the occupation in the base period. We compute this measure using data for 300 detailed occupations that we can trace consistently over time and across Censuses and different waves of the ACS.[26](#page-63-0)

Before describing the group-level measure, we show in Figure [10](#page-64-0) that, at the occupational level, there is a strong negative association between new task creation (summed over 1977, 1991 and 2000 measures) and labor demand. A 10 pp increase in job titles over this time window is associated with a 0.4 pp higher yearly growth rate of wage payments in that occupation from 1980 to 2016. This reproduces and extends the results reported in [Acemoglu and Restrepo](#page-90-2) [\(2018b\)](#page-90-2).

 26 Notice that this is different from the measurement strategy of our baseline automation measure, which uses beginning of sample (1980) weights. This difference stems from the fact that, in the theory, new tasks benefit workers who end up taking over these tasks, while automation affects workers who used to work in the now-automated tasks. Tables [A1](#page-121-0) and [A2](#page-122-0) in the Appendix show that our reduced-form results are robust if we compute the new task measures using occupational shares fixed in 1980.

Figure 10: Changes in log total wage bill in an occupation, 1980–2016 against new job titles introduced in each occupation (from DOT 1977, DOT 1991, and Census 2000). Data are for 300 detailed occupations.

Finally, Figure [11](#page-65-0) provides a first comparison of our measures of task displacement from automation and task reinstatement due to new task creation. The figure plots both variables against group-level hourly wages in 1980, which is useful to indicate where in the wage distribution the effects of displacement and reinstatement are most likely to be felt. The left panel of the figure shows that, on average, US workers experienced a reduction in task shares of 17% during this period, but this was very unevenly distributed in the population. While noncollege workers saw task share declines in the range of 20–30%, college and postcollege workers were mostly shielded from task displacement.^{[27](#page-64-1)} The right panel, on the other hand, indicates that, on average, US workers benefited from a 23% expansion in their task shares due to new task creation, and in contrast to automation, reinstatement effects are typically higher for more highly-educated workers.

 27 Because this measure is based on predicted labor share declines over 1987–2016, we re-scale it to a 37-year equivalent change that matches the length of time used for the dependent variables (1980-2016).

Figure 11: Left panel shows (total) direct task displacement due to automation, 1980–2016 for 500 groups of US workers, and the right panel shows (total) task reinstatement due to new job titles for these groups. Both panels plot these data against group average hourly wages in 1980, from the Census. Marker sizes are proportional to hours worked in 1980. Marker colors distinguish groups with different education levels.

7.4 Main Results

The top two panels in Figure [12](#page-95-0) provide bivariate scatter plots of the change in group wages from 1980–2016 (top left panel) and log hours per person (top right panel) against our measure of task displacement due to automation for this period. The bottom panel provides residual scatter plots that partial out education and gender dummies and sectoral value-added shares.

The figure shows a negative association between task displacement due to automation and (relative) wage and employment changes. The associations are stable regardless of whether we do or do not control for covariates. The estimated effects are also sizable: in the bottom panels, a 10 pp increase in task displacement for a skill group is associated with a 16.5% decline in (relative) wage and a 22.5% decline in (relative) hours worked.

Figure [13](#page-96-0) presents the analogous specifications for new tasks—with the top panels depicting the bivariate relationships and the bottom panels showing estimates that partial out the effects of education and gender dummies and sectoral value-added shares.

New tasks are correlated with higher (relative) wage and employment growth. In the bottom-panel estimates that control for covariates, we find that a 10 pp increase in task reinstatement due to new job titles is associated with a 23.2% increase in wages and a 36.2% increase in hours worked.

The figures support the key implications of the task framework: task displacement from automation is associated with negative wage consequences for exposed workers relative to others, while reinstatement due to new tasks is associated with positive wage effects. These technologies also have commensurate effects on employment—groups experiencing more task displacement now have (relatively) lower hours worked, while the pattern is the opposite for those enjoying greater task reinstatement.

Tables [1](#page-67-0) provides the estimates for log hourly wages from the specifications in the figures as well as several variations. Column 1 in Panels A and B reports estimates of the bivariate relationships shown in the top-left panels of Figures [12](#page-95-0) and [13.](#page-96-0) The regression coefficient for task displacement is -1.65 (standard error $= 0.10$), while the coefficient for task reinstatement due to new tasks is 2.32 (standard error $= 0.19$).

Column 1 in Panel C includes both explanatory variables together. The coefficient for task displacement due to automation is now -1.19 (standard error $= 0.23$), and the coefficient for tasks reinstatement is 0.85 (standard error $= 0.33$). The point estimates are attenuated compared to Panels A and B, especially for new tasks, reflecting the fact that these two measures are negatively correlated, and including them one at a time exaggerates their roles. Nevertheless, these two variables jointly explain a remarkable 67% of the observed wage changes across worker groups in the US between 1980 and 2016, with automation accounting for 46% and new task creation for the remaining 20% .^{[28](#page-66-0)} The parameter estimates also imply sizable effects from both variables. A 10 pp increase in task displacement due to automation for a demographic is associated with 11.9% lower (relative) wages, while a 10 pp increase in new tasks reinstatement is associated with 8.5% higher (relative) wages.

Column 1 in Panel D leverages the fact that task displacement and reinstatement are predicted to impact wages with the same coefficient but with opposite signs. It combines them into a single explanatory variable, "net task change,"constructed as the difference between task reinstatement and displacement. Consistent with theory, this variable has a positive and precisely estimated coefficient, 1.05 (standard error $= 0.07$). Interestingly, this

$$
R^2 \text{ from } x_j = \beta_j \cdot \frac{\text{cov}(x_j, y)}{\text{var}(y)}
$$

By construction, these sum up to the model's total R^2 when added across all variables (subject to rounding).

²⁸Throughout this section, we follow Klenow and Rodríguez-Clare [\(1997\)](#page-93-4) and decompose the total R^2 into contributions from subsets of the variables by equally distributing the covariance terms between these subsets. This means that the contribution of a covariate x_j to the explanatory power of a model of the form $y = \sum \beta_j x_j + u$ is

Table 1: Reduced-form evidence: changes in real hourly wages regressed on automation and new task creation, 1980-2016.

Notes: This table presents estimates of the relationship between automation, new task creation, and the change in hourly wages across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in log hourly wages for each group between 1980 and 2016. Panel A reports results using only our task displacement measure. Panel B only uses our task reinstatement measure. Panel C includes both task displacement and task reinstatement on the right-hand side. Panel D combines task displacement and reinstatement into a single next task change measure. The bottom rows list additional covariates included in each specification. As in [Acemoglu and](#page-91-0) [Restrepo](#page-91-0) [\(2022\)](#page-91-0), we instrument changes in labor supply in columns 6 and 7 using trends in total hours worked by group from 1970 to 1980. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

restriction only leads to a small reduction in the explanatory power of automation and new tasks, which, together, still account for approximately 67% of the total variation in wage trends between demographic groups. This estimate implies that a 10 pp increase (decrease) in task shares is associated with a 10.5% increase (decrease) in relative wages.

Table 2: Reduced-form evidence: changes in hours worked per adult regressed on automation and new task creation, 1980-2016.

Notes: This table presents estimates of the relationship between automation, new task creation, and the change in hours worked per adult across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in log hours per adult for each group between 1980 and 2016. Panel A reports results using only our task displacement measure. Panel B only uses our task reinstatement measure. Panel C includes both task displacement and task reinstatement on the right-hand side. Panel D combines task displacement and reinstatement into a single next task change measure. The bottom rows list additional covariates included in each specification. As in [Acemoglu and](#page-91-0) [Restrepo](#page-91-0) [\(2022\)](#page-91-0), we instrument changes in labor supply in columns 6 and 7 using trends in total hours worked by group from 1970 to 1980. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

The remaining columns in Table [1](#page-67-0) explore the robustness of these reduced-form relationships to the inclusion of various covariates. Column 2 adds the sectoral value-added shares, with little effect on the coefficient estimates for task displacement and reinstatement. Column 3 adopts our second strategy for controlling for sectoral trends, and includes sectoral TFP and sectoral markups. The results are very similar once more, suggesting that automation and new task creation are distinct from these sectoral trends.

More importantly, column 4 includes the education and gender dummies as well as the sectoral value-added control, corresponding to the specifications in the left-bottom panels of Figures [12](#page-95-0) and [13.](#page-96-0) Column 5 includes the education and gender dummies together with sectoral TFP and markup controls. In both specifications we continue to estimate a sizable negative association between group outcomes and automation and a sizable positive association with new task creation. In fact, the parameter estimates are quite similar to those in column 1. Recall that education dummies account for the role of several forces previously emphasized in the literature, including skill-biased (factor-augmenting) technologies or improvements in capital equipment benefiting more educated workers. We interpret the results from these specifications as saying that automation and new task creation are distinct from these other forms of technological progress emphasized in previous literature. Moreover, the $R²$ decomposition in these columns indicates that the explanatory power of the sectoral value-added term and the education and gender dummies is quite limited. The results in column 4 of Panel C, for example, show that the overall explanatory power of this model for between-group wage changes is 84%, but only 6% of this 84% comes from the sectoral and education and gender dummy variables, while 50% is due to automation and the remaining 28% is from new tasks. The results from these decomposition therefore suggest that the extensive-margin changes associated with task displacement and reinstatement are more important drivers of wage trends between groups (at least when measured in terms of their reduced-form explanatory power) than the forces commonly emphasized in the literature and captured by the educational dummies and sectoral controls.

Finally columns 6 and 7 control for labor supply changes, incorporating the supply-side forces emphasized in [Katz and Murphy](#page-93-0) [\(1992\)](#page-93-0) and [Card and Lemieux](#page-92-1) [\(2001\)](#page-92-1). These supply terms are measured by the total increase in hours worked per group and instrumented using pre-existing trends in hours during 1970–1980. This strategy isolates the variation in hours due to demographic trends and trends in educational attainment. Controlling for changes in labor supply does not change the qualitative picture, but raises the explanatory power of our task displacement and reinstatement measures. For example, in column 7, Panel C, automation accounts for 66% of variation in between-group wage changes, and new tasks contribute another 37%, while the other variables have a negative contribution. This reflects the fact that demographic trends, especially in educational attainment, have gone in favor of groups experiencing more task displacement and less reinstatement during our sample period, and thus, according to our estimated model, without the task displacement and reinstatement developments, these groups would have experienced higher—rather than lower—relative wage growth.

Table [2](#page-68-0) turns to analogous specifications for hours worked. Column 1 in Panels A and B report estimates of the bivariate relationship shown in the top-right panels of Figures [12](#page-95-0) and [13.](#page-96-0) In Panel A, the coefficient estimate for task displacement is -2.25 (standard error $= 0.30$), and in Panel B, the coefficient estimate for task reinstatement is 3.62 (standard $error = 0.49$. Panel C includes both explanatory variables together, with the corresponding coefficients being, respectively, -0.82 (standard error = 0.39) and 2.61 (standard error = 0.71). In this specification, our measures of task changes due to automation and new task creation explain 53% of the variation in changes in hours worked across demographic groups between 1980 and $2016²⁹$ $2016²⁹$ $2016²⁹$ The remaining columns show that the employment effects are also fairly unchanged when we control for different measures of sectoral reallocation, education and gender dummies, and supply-side factors.

7.5 Robustness

The patterns documented above are robust. [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0) documented the robustness of the automation results to several other specifications, including those that control for exposure to imports from China and offshoring, exposure to routine jobs and industries experiencing labor share declines (the two constituent components in our task displacement measure) and exposure to minimum wages and union coverage. Similar results were also obtained in stacked-differences models and when exploiting variation across US regions.

In the Appendix, we show that the results reported here are very similar if we construct the reinstatement due to new tasks only using wage-bill variation from 1980 (see Tables [A1](#page-121-0) and [A2\)](#page-122-0). We also show in Table [A4](#page-124-0) that the coefficients on task displacement and task reinstatement variables are comparable when we estimate the models separately for workers with a college degree and workers without a college degree. This exercise shows that the benefits from new task creation and the costs of automation are visible even when focusing on these specific segments of the labor force.

 29 Table [A3](#page-123-0) decomposes these effects into an extensive and intensive margin changes. While the task displacement from automation has a robust negative association with both margins, new task creation is more strongly associated with increases in employment at the extensive margin.

7.6 Taking Stock

Overall, the results in this section confirm the main implications of the task framework: task displacement due to automation has a sizable negative effect on the relative wages of exposed groups, and reinstatement driven by new tasks has a sizable positive effect on relative wages. These two variables explain at least 62% of the total variation in between-group wage changes between 1980 and 2016 across different specifications. Consistent with the expectation that these technology measures shift the relative demand for labor from different skill groups, we also find that they have commensurate effects on employment. The two measures together account for approximately 53% of the variation in the changes in hours worked for the same time period. Moreover, in line with our theory, the estimates suggest that technologies that cause extensive-margin changes (thus reallocating tasks from one factor to another) explain the bulk of variation in the changes in the wage and employment structure, and have much greater explanatory power than proxies for factor-augmenting and sectoral technology variables.

It is important to exercise caution in interpreting these reduced-form results. First, our proxies for factor-augmenting and sectoral changes are imperfect. The education dummies may capture other trends as well as factor-augmenting technologies, while the reducedform estimates of the contribution of sectoral variables may be attenuated. Second, we are ignoring ripple effects, which link the wages of a skill group to the task displacement experienced by other groups of workers—especially when there are high levels of substitutability between the groups in question. Third, productivity effects are subsumed into the constant. All of these considerations motivate our approach in the next section, which further leverages the structure of the model to estimate the propagation matrix and productivity implications of different types of technologies, and performs counterfactual exercises to measure their contribution to the changes in wage inequality since 1980.

8 Estimation of General Equilibrium Effects and Counterfactuals

This section uses the task model to study the full equilibrium effects of different technologies on the US wage structure. We use the equations characterizing the impacts of technology, inclusive of the ripple effects. We implement these equations using the measures of the direct effects of different technologies introduced in the reduced-form section, and combine them with external information on a number of key elasticities of substitution and our estimates of the propagation matrix.
This exercise adds to the reduced-form findings in three ways. First, it accounts for the effects of technology on wage levels working via the productivity effect. As explained above, the reduced-form evidence is informative of the relative change in wages for exposed groups—but not of the effects of different technologies on wage levels. Second, we account for ripple effects using estimates of the propagation matrix. Finally, this exercise incorporates the effects of technology working through changes in sectoral composition. The reduced-form controlled for sectoral shifts, but did not account for the impact of each technology on the sectoral composition of the economy. Through this exercise, we are able to more fully explore the quantitative importance of different types of technologies. Our results from this structural exercise reinforce the reduced-form finding that automation and new task creation are the major drivers of the changes in the US wage structure.

8.1 General Equilibrium Effects of Technology and Markups

Our objective is to decompose observed changes in hourly wages studied in the reduced form into the separate effects of automation, new task creation, Hicks-neutral sectoral productivity (TFP) shifters and markups. We return to the contribution of factor-augmenting technologies later. The analysis can be expanded to include other factors, but we do not do so to keep the chapter focused on the consequences of technology trends.

Ignoring factor-augmenting technologies for now, from Propositions [9](#page-50-0) and [10,](#page-53-0) the change in group wages can be written as

$$
d\ln w = \Theta \cdot \text{stack}\left(d\ln y - d\ln M - d\ln \Gamma_g^{\text{auto}} + d\ln \Gamma_g^{\text{new}}\right)
$$

$$
-(1-\lambda)\cdot \sum_{i} \omega_{gi} \cdot d\ln A_i - \lambda \cdot \sum_{i} \omega_{gi} \cdot d\ln \mu_i + (\lambda - \eta) \cdot \sum_{i} \omega_{gi} \cdot d\ln p_i\right) + \upsilon.
$$

In this equation, v is an error term subsuming all other forces shaping the wage structure. The endogenous price changes $\{d \ln p_i\}_i$ associated with these shocks satisfy

(41)
$$
d\ln p_i = \sum_g s_g^{y_i} \cdot d\ln w_g - \sum_g s_g^{y_i} \cdot d\ln \Gamma_{gi}^{\text{auto}} \cdot \pi_{gi}^{\text{auto}} - \sum_g s_g^{y_i} \cdot d\ln \Gamma_{gi}^{\text{new}} \cdot \pi_{gi}^{\text{new}} - d\ln A_i + d\ln \mu_i.
$$

To determine the effects of these technologies on output, we simplify the analysis by assuming that, initially, $\mu_i = 1$ for all i. This assumption implies that the sectoral value-added shares are equal to sectoral cost shares, which would not otherwise be the case and we would have to make additional assumptions to proxy for these cost shares. An implication

of this assumption is that, as in Section [5,](#page-34-0) the change in aggregate output, $d\ln y$, is determined by the following equation, which relates average wage changes to changes in TFP and markups:

$$
(42)\ \sum_{g} s_g^y \cdot d\ln w_g = \sum_{i} s_i \cdot \left[\sum_{g} s_g^{y_i} \cdot d\ln \Gamma_{gi}^{\text{auto}} \cdot \pi_{gi}^{\text{auto}} + \sum_{g} s_g^{y_i} \cdot d\ln \Gamma_{gi}^{\text{new}} \cdot \pi_{gi}^{\text{new}} + d\ln A_i - d\ln \mu_i \right].
$$

Because we are looking at first-order approximations, these three equations provide an additive decomposition of the contribution of technologies and markups.

To implement this decomposition, we need estimates of (i) initial factor shares; (ii) the elasticities $\{\lambda, \eta\}$; (iii) the direct effects of task displacement and reinstatement, $\{d \ln \Gamma_g^{\text{auto}}, d \ln \Gamma_g^{\text{new}}\}_g$; (iv) sectoral TFP growth, $\{d \ln A_i\}_i$, and sectoral markup changes $\{d \ln \mu_i\}_i$; and (v) the propagation matrix, Θ.

For (i), we take factor shares directly from the Census data matched to the BEA-BLS industry accounts.

For (ii), we set $\lambda = 0.5$ and $\eta = 0.3$. Our estimate of the task-elasticity of substitution λ comes from [Humlum](#page-93-0) [\(2020\)](#page-93-0), who obtains it from Danish manufacturing data. Our estimate of the sectoral elasticity of substitution is from [Buera et al.](#page-92-0) [\(2021\)](#page-92-0) and is a standard value used in the structural transformation literature.

For (iii), we continue to use the measure of new task reinstatement in [\(40\)](#page-63-0), but a slightly different measure for task displacement due to automation, given by

(43)
$$
d\ln\Gamma_{gi}^{\text{auto}} = \text{RCA routine}_{gi} \cdot \frac{-\Delta\ln s_{\ell}^{y_i,\text{auto}}}{1 + (\lambda - 1) \cdot s_{\ell}^{y_i} \cdot \pi_i^{\text{auto}}}
$$

for group g in industry i. This expression differs from the measure used in the reduced-form analysis, in equation [\(38\)](#page-60-0), because of the term $(\lambda - 1) \cdot s_\ell^{y_i}$ $\ell^{y_i} \cdot \pi_i^{\text{auto}}$ in the denominator, which adjusts for the effect of automation on the labor share working via substitution towards the cheaper newly-automated tasks. The earlier expression obtains when $\lambda = 1$. We used this restriction in our reduced-form analysis to simplify the exposition. Here, we construct the adjustment term using $\lambda = 0.5$ and $\pi_i^{\text{auto}} = 30\%$. Total task displacement due to automation $d\ln\Gamma_g^{\text{auto}}$ is computed by aggregating the new measures for $d\ln\Gamma_{gi}^{\text{auto}}$ across industries, as in equation (39) .^{[30](#page-73-0)}

To obtain cost savings from these technologies, we follow [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0)

 30 The reduced-form results are very similar with the adjusted measure shown here and other variants, and are presented in the Appendix of [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0).

and set $\pi_{gi} = 30\%$. This choice is motivated by available estimates of cost savings due to the adoption of industrial robots in US manufacturing. This choice assumes the same savings for automation technologies in other sectors, though this is something that can be improved with additional data. For new tasks, we set $\pi_{gi}^{new} = 30\%$ for symmetry, especially since we do not have direct estimates of the surplus generated by new tasks. This number implies that a 10% increase in new tasks for all worker groups would raise TFP by 3%, which is a reasonable number.[31](#page-74-0)

For (iv), we estimate the sectoral Hicks-neutral productivity shifters $\{d \ln A_i\}_i$'s by subtracting the implied TFP gains due to automation and new task creation from observed industry TFP changes. The left panel in Figure [14](#page-97-0) depicts observed industry TFP changes together with the implied estimates for the $d \ln A_i$'s.^{[32](#page-74-1)} Computers and electronics and transportation pipelines experienced the largest sectoral productivity increases, while legal services and transportation services experienced the least. Overall, the two series are highly correlated, but there are some notable exceptions, such as motor vehicles, where observed TFP exceeds our estimate for $d \ln A_i$ by a sizable amount. This is the industry that has had the largest investment in automation technology during this period, explaining why a major portion of its observed TFP growth is accounted for by automation.

For markups, we use the estimates from [Hubmer and Restrepo](#page-93-1) [\(2021\)](#page-93-1), who obtain these from the same production function approach and Compustat data as [De Loecker et al.](#page-92-1) [\(2020\)](#page-92-1), but allow firm-level output elasticities to vary by size, and also aggregate these markups using their sales-weighted harmonic mean to obtain aggregate industry markups. These estimates are depicted in the right Panel of Figure [14.](#page-97-0)

We explain below how we additionally incorporate the possible contribution of factoraugmenting changes. Finally, the estimation of the propagation matrix is discussed next.

³¹Our prior is that this number should be bigger, since new tasks can significantly reorganize the production process and bring various efficiency improvements. Nevertheless, we choose 30% to be on the conservative side. A larger number would increase the wage gains from new tasks and leave less of the residual TFP to be explained by Hicks-neutral sectoral technologies and factor-augmenting changes.

³²For simplicity, our theory used value-added production functions at the industry level (with material inputs solved out). To match this choice, we use measures of value-added TFP instead of gross-output TFP. While it would be preferable to use measures of TFP for gross output (so that they can be readily interpreted as technology), this would require modeling input-output linkages across industries, which we do not pursue for this chapter.

8.2 Estimating the Propagation Matrix

The wage equation in the multi-sector model, [\(28\)](#page-51-0), can be rewritten as

(44)
$$
\Delta \ln w_g = \frac{1}{\lambda} \cdot (d \ln \Gamma_g^{\text{new}} - d \ln \Gamma_g^{\text{auto}}) + \beta \cdot X_g + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g}{\partial \ln w} \cdot \text{stack}(\Delta \ln w_{g'}) + u_g,
$$

where X_g is a vector containing sectoral shifts and education and gender dummies, proxying for other technological trends. Rather than solving out for the vector of wage effects using the propagation matrix as in [\(28\)](#page-51-0), here we include the vector of wage changes for other demographic groups on the right-hand side, highlighting that these will impact the wage of group g via the gth row of the task-shares Jacobian matrix, $\frac{\partial \ln \Gamma}{\partial \ln w}$. The error term u_g contains all unobserved labor demand and supply shocks impacting demographic group g.

Our strategy is to estimate the Jacobian using GMM (Generalized Method of Moments). In this estimation, we impose external values for λ and use the orthogonality conditions

$$
d\ln\Gamma_g^{\text{auto}}, d\ln\Gamma_g^{\text{new}}, X_g \perp u_{g'}
$$
 for all $g, g' \in \mathbb{G}$,

which impose that task displacement and reinstatement terms as well as the education and gender dummies and sectoral shifters in X_g are orthogonal to the error term. This orthogonality assumption was implicit in the reduced-form models we estimated in the previous section. Once the Jacobian matrix is estimated, the propagation matrix can be obtained as $\Theta = \frac{1}{\lambda}$ $\frac{1}{\lambda} \cdot \left(1 - \frac{1}{\lambda}\right)$ $\frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma}{\partial \ln w}$ $\frac{\partial \ln \Gamma}{\partial \ln w}\Big)^{-1}.$

The Jacobian is $G\times G$, and hence it would be impossible to estimate all of its entries in an unrestricted fashion. Instead, we follow [Acemoglu and Restrepo](#page-91-1) [\(2024\)](#page-91-1) and parameterize the entries of the Jacobian matrix in terms of similarities between groups.^{[33](#page-75-0)} This approach operationalizes the intuitive idea that the Jacobian matrix is informative about the extent of substitutability between groups, and such substitutability should depend on how similar the groups are. We assume that the off-diagonal terms of the Jacobian (for $g' \neq g$) can be parameterized as

$$
\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} = s_{g'}^y \cdot \varphi + \sum_n \omega_{gn} \cdot s_{g'}^n \cdot \left[\gamma + \gamma_{job} \cdot \text{job similarity}_{gg'} + \gamma_{edu\text{-age}} \cdot \text{edu\text{-age similarity}_{gg'} \right],
$$

³³In [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0), we directly parameterized and estimated the propagation matrix. We prefer the current approach because it is easier to develop intuitions about the entries of the Jacobian, which correspond to first-round ripples encoded (rather than the Leontief inverse of this matrix, which depends on higher-round ripples).

while the diagonal terms take the form

$$
\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} = (s_g^y - 1) \cdot \varphi - \sum_n \sum_{g' \neq g} \omega_{gn} \cdot s_{g'}^n \cdot \left[\gamma + \gamma_{job} \cdot \text{job similarity}_{gg'} + \gamma_{edu\text{-age}} \cdot \text{edu\text{-age similarity}}_{gg'} \right].
$$

This parameterization implies that competition for marginal tasks between skill groups takes place within job categories, denoted by n . In the data, we assume that there are 96 job categories, given by combinations of 16 aggregated industries and six aggregated occupations. The summation terms indicate that competition from demographic group g' on q in category n depends on the importance of this category for group q , summarized by the share of category n in the total wage payments for group $g(\omega_{gn})$, and the share of wage payments in job category n accruing to group $g'(s_{g'}^n)$. Both of these objects are computed from the 1980 Census. Intuitively, groups with greater wage shares should generate more competitive pressure on other groups in the same job category, as implied, for example, by the Frechet parameterization of comparative advantage in Section [4.](#page-29-0) In addition, in the square brackets, we parameterize competition between groups via three terms. The first, represented by $\gamma \geq 0$, corresponds to the component of competition that is common to all workers in a job category. The second, with coefficient $\gamma_{job} \geq 0$, is from the similarity of the jobs performed by the two demographic groups. In particular, we use the cosine similarity of job categories performed by groups g' and g in the 1980 Census. This functional form is also motivated by the Frechet example, where a higher correlation in task-level productivities results in higher substitutability. The third term, with coefficient $\gamma_{\text{edu-age}} \geq 0$, parameterizes the extent to which competition for tasks is stronger for workers of similar education and experience, as in [Card and Lemieux](#page-92-2) [\(2001\)](#page-92-2). We compute this similarity measure as follows: we run a Mincer wage equation for log hourly wages in 1980, as a function of age and education dummies, and then construct the education-age similarity between two groups as the inverse distance between the predicted wage level of groups g and g' in 1980. This procedure captures how similar the two groups are in terms of their education and age, with each of these dimensions weighted by their Mincer coefficients.

Finally, the parameter $\varphi \geq 0$ controls the extent of competition between capital and workers for marginal tasks, which is assumed to be the same for all worker groups. Our parametrization implies that the row sums of the Jacobian are equal to $-s_k \cdot \varphi$. Using the definitions in Section [4,](#page-29-0) we see that the macroeconomic elasticity of substitution between

capital and labor is common across groups and equal to $\sigma_k = \lambda + \varphi^{34}$ $\sigma_k = \lambda + \varphi^{34}$ $\sigma_k = \lambda + \varphi^{34}$. We set $\varphi = 0.1$, so that σ_k matches estimates of the elasticity of substitution between capital and labor in [Oberfield](#page-93-2) [and Raval](#page-93-2) [\(2020\)](#page-93-2) of around 0.6. This parameterization therefore fixes the row sums of the Jacobian, $\frac{\partial \ln \Gamma}{\partial \ln w}$, and allows the data to determine the γ coefficients, which determine the strength of competition for marginal tasks between different groups.

Table [3](#page-78-0) reports our estimates for the γ 's obtained from equation [\(44\)](#page-75-1). For these estimates, we impose the restriction that γ , γ_{job} , $\gamma_{edu\text{-}age} \geq 0$. When entered individually, we detect spillovers across each of these dimensions as the different columns show. However, when we include all three terms simultaneously, the first two are estimated to have zero coefficients (given our no negativity constraint) and the spillover patterns are explained by the education-age similarity measure, exactly as in columns 3 and 6. In what follows, we take column 3— $\gamma = 0$, $\gamma_{job} = 0$, and $\gamma_{edu\text{-age}} = 0.8$ —as our preferred specification.

The estimated propagation matrix has an average diagonal of 0.84, and the row sum of the off-diagonal terms is about 1. This implies that workers from group g bear about 45% of the incidence of a direct shock reducing their labor demand, with the rest being shifted to other groups via competition for marginal tasks.

Another way to illustrate the structure of the estimated propagation matrix is by looking at the implied elasticity of substitution between skill groups. Figure [15](#page-97-1) provides this information by aggregating pairwise elasticities of substitution with a simple average by education and age. The left panel shows that the average elasticity of substitution between groups with a college and postgraduate degree is 2. Instead, the average elasticity of substitution between groups with a college degree and those with no completed high school is 0.82.

8.3 Decompositions

We first illustrate the effects of each type of technological change, highlighting the different pathways via which they affect labor demand.

Figure [16](#page-98-0) depicts the effects of automation. The panels plot estimates of the different mechanisms, which we accumulate from left to right, with the rightmost panel corresponding to the total effect of the technology in question. Throughout, the vertical axes show the model estimates (in units of change in hourly wages from 1980 to 2016), while the

³⁴Recall that due to symmetry, $\sigma_{kg} = \sigma_{gk}$. Moreover, we can write $\sigma_{gk} = \lambda + \frac{1}{s_k^y}(-\sum_{g'} \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}})$, since a change in the cost of capital is isomorphic to an increase in all wages. This implies $\sigma_{kg} = \sigma_{gk} = \lambda + \varphi$.

	DEPENDENT VARIABLES: CHANGE IN LOG HOURLY WAGES, 1980-2016							
	(1)	$\left(2\right)$	(3)	(4)	(5)	(6)		
Baseline	0.39			0.33				
competition γ	(0.13)	(0.16)						
Job-similarity		0.74			0.68			
competition $\gamma_{\rm ib}$		(0.21)			(0.25)			
Education-age			0.80			0.84		
competition $\gamma_{\text{edu-age}}$			(0.22)			(0.31)		
Observations	500	500	500	500	500	500		
Covariates:								
Gender and	\checkmark	✓	\checkmark	✓				
education dummies								
Sectoral value added control	✓	✓	✓					
Sectoral TFP and markups				\checkmark	\checkmark	\checkmark		

Table 3: Estimates of the task-shares Jacobian.

Notes: This table presents estimates of the (task share) Jacobian, using the parameterization in Section [8.](#page-71-0) The estimation equation can be written as $\sigma \Delta \ln w_g + d \ln \Gamma_g^{\text{auto}} - d \ln \Gamma_g^{\text{new}} = \tilde{\beta} X_g + \gamma \cdot \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n$. $(\Delta \ln w_{g'} - \Delta \ln w_g) + \gamma_{\text{job}} \cdot \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot \text{job similarity}_{gg'} \cdot (\Delta \ln w_{g'} - \Delta \ln w_g) + \gamma_{\text{edu-age}} \cdot \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n$ $s_{g'}^n$ · edu-age similarity_{gg}' · ($\Delta \ln w_{g'} - \Delta \ln w_g$) + $\tilde{\nu}$, where $\tilde{\beta}$ and $\tilde{\nu}$ are linear transformations of β and ν respectively. The ripple terms are instrumented using $\sum_{g'} \sum_{n} \omega_{gn} \cdot s_{g'}^n \cdot (\Delta \ln \hat{w}_{g'} - \Delta \ln \hat{w}_g)$, $\sum_{g'} \sum_{n} \omega_{gn} \cdot s_{g'}^n$ $s_{g'}^n$ · job similarity_{gg'} · ($\Delta \ln \hat{w}_{g'} - \Delta \ln \hat{w}_g$) and $\sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n$ · edu-age similarity_{gg'} · ($\Delta \ln \hat{w}_{g'} - \Delta \ln \hat{w}_g$), respectively, where $\Delta \ln \hat{w}_g$ is the predicted wage change based on task displacement, task reinstatement and the covariates. Columns 1 and 4 present estimates for γ excluding the other two spillover terms. Columns 2 and 5 present estimates for γ_{job} excluding the other two spillover terms. Columns 3 and 6 present estimates for $\gamma_{\text{edu-age}}$ excluding the other two spillover terms. When all three measures of competition are included and the restriction that they have to have no negative coefficient is imposed, the first two are estimated to have zero effects, and thus the results are identical to those reported in columns 3 and 6. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

horizontal axis is always for hourly wage in 1980. Panel A starts with the productivity gains from automation, $(1/\lambda) \cdot d\ln y$. We see here that automation increased output by 20% over this period, which raised the demand for labor and wages in all tasks by 40%.

Panel B adds the effects of automation working through changes in the sectoral composition of the economy, by plotting $(1/\lambda) \cdot (d \ln y + (\lambda - \eta) \sum_i \omega_{gi} \cdot d \ln p_i)$. Note that here we only account for the change in sectoral prices due to automation, computed according to equation [\(41\)](#page-72-0). While in principle the sectoral effects could differ across groups, in practice they are fairly uniform and do not generate much variation in terms of relative wage changes. This is because the skill composition of the sectors expanding due to automation is relatively similar to the rest.

Panel C adds the direct task displacement due to automation and plots $(1/\lambda) \cdot (d\ln y$ $d\ln\Gamma_g^{\text{auto}} + (\lambda - \eta) \sum_i \omega_{gi} \cdot d\ln p_i$. The uneven impacts are clearly visible. For example, the direct effects reduce the wages for some groups by as much as 30%, while the real wages of highly-educated groups shielded from automation increase by more than 40%. This panel confirms that automation works primarily by displacing workers from their tasks, shifting labor demand within sectors—rather than by shifting the sectoral composition of the economy, as in Panel B.

Panel D adds the ripple effects generated by automation. We see here that ripples play an equalizing role, consistent with our discussion in Section [5.](#page-34-0) This is because groups that experience a large reduction in their task share due to automation are able to compete for marginal tasks previously performed by other groups. This reallocation transfers the negative incidence of automation to other groups and mitigates the adverse direct effects on exposed groups. Our estimates imply that high school graduate groups experienced, on average, 4.3% wage declines due to automation and groups with less than high school experienced even steeper declines of about 8.1%, while college graduates and postgraduates enjoyed, respectively 17.6% and 22.9% wage increases from automation. Underscoring the equalizing role of the ripple effects, the declines in the real wages of high school graduate and less than high school groups would have been, respectively 10.1% and 16.2%, if these groups had not been able to compete for marginal tasks and shift some of the burden of task displacement two other skill groups.

Figure [17](#page-98-1) depicts the effects of new task creation on wages from 1980 to 2016. The panels have the same structure and interpretation as before. Our estimates imply that new task creation reduces output by a small amount. This does not mean that the economy is made less productive by new tasks. In fact, new tasks increase TFP by 5%, and average wages and aggregate consumption by 7%. The reason why output declines is because new tasks make the production process less capital intensive and as a result the share of capital and investment decrease (recall the relationship between TFP change and outputs change in footnote [12\)](#page-20-0).

We also see here that new tasks benefited all groups, but generated more pronounced gains for highly-educated and initially more highly-paid groups, thus further contributing to rising inequality, even if by a much smaller amount than automation. The overall wage implications of new tasks range from 3.83% for groups with less than high school to 11.6% for college workers in Panel C. This heterogeneity is, as usual, further compressed by the

ripple effects in Panel D.[35](#page-80-0)

Figures [18](#page-99-0) and [19](#page-99-1) plot the results for sectoral TFP changes and markups, both with modest distributional implications. Changes in sectoral TFP increase wages for all groups by about 22%. Due to the fact that $\eta < 1$, they also reallocate labor away from a handful of manufacturing sectors with high productivity growth, but these industries do not differ from the rest much in terms of their skill intensity and thus this reallocation does not have sizable distributional consequences.

Markups reduce output and real wages, but affect groups almost uniformly, because the sectors experiencing the most pronounced increase in markups are also similar to the rest in terms of the skill and demographic composition of their workforces.^{[36](#page-80-1)}

We now juxtapose the effects of these different shocks to understand their contributions to the observed wage and employment trends. Figure [20](#page-100-0) shows this by aggregating the data by gender and education and plots the observed hourly wage changes for each group. It also reports the model-implied wage changes due to automation, new task creation, sectoral TFP changes, and markups, combined. These trends together account for about 50% of between-group wage changes from 1980 to 2016.

The contribution of the different technologies studied here and sectoral markups to the observed wage changes is reported in Table [4.](#page-81-0) We can see that new automation technologies introduced since 1980 account for 41% of the observed wage trends across worker groups. New task creation contributed 7%, as it favored highly-educated workers the most. Sectoral shocks, such as measured changes in sectoral TFP and markups, on the other hand, had minor effects. The second column reports predicted average wage growth coming from each one of the sources. It shows that despite generating large distributional effects, automation brought a modest increase in average wages of about 4%. The opposite holds for Hicks' neutral sectoral TFP improvements, which increased average wages by 22% and had minor distributional effects. Overall, predicted wage growth from the model exceeds the composition-adjusted real wage growth in the US economy over the same time period, which is about 5%. This may be because there are other factors (for example, related to non-competitive elements in the labor market discussed below) which may put additional

³⁵Observe also that new task creation increases M by $d\ln M = (1 - (\lambda - 1) \cdot \pi_g^{new}) \cdot \sum_{g \in \mathcal{G}} s_g^L \cdot d\ln \Gamma_g^{new}$. This effect is common to all workers and is included as part of Panel C. This panel plots the net effect of task reinstatement once we account for the fact that new tasks dilute old ones. The reason why the net task reinstatement is positive for most groups is that the bulk of the dilution loads on capital.

³⁶The correlation between exposure to sectoral TFP growth and baseline group wages across our 500 demographic groups is 0.1, while the correlation between exposure to sectoral markups and baseline group wages is 0.26.

downward pressure on wages.

Table 4: Share of variance in wage trends across groups explained by different technologies and markups.

Notes: Column 1 reports the contribution of the indicated technology term to observed wage changes across 500 demographic groups between 1980 and 2016. Column 2 reports predicted average (real) wage growth between 1980 and 2016 from the indicated technology term.

Figure [21](#page-100-1) provides additional details and context on the impacts of different types of technologies on the wage structure. It depicts the contribution of the same four factors to the the wage premium between college groups and those with high school or less, to the college-some college premium, and to the postcollege-college premium. Automation emerges as the most important determinant of the changes in all three measures of inequality, while new tasks are the second most important contributor, though with a significantly smaller role, as also indicated in Table [4.](#page-81-0) One interesting pattern is that while sectoral TFP terms have played an equalizing role for the college-noncollege premia, they have contributed to the widening postcollege-college premium, partly because a few sectors, such as legal services and health care, that have low productivity growth disproportionately employ postgraduates and have consequently expanded their factor demands (because of the Baumol effect discussed above, since $\eta < \lambda$).

8.4 Limited Distributional Impacts of Labor-Augmenting Technologies

Our decomposition exercise so far has ignored the role of labor-augmenting changes, because we have no direct measures of such technologies. In this subsection, we perform a simple bounding exercise to indicate that these technologies are unlikely to be an important driver of the changes in the US wage structure between 1980 and 2016 and contrast this with automation.

We consider three types of technological changes: automation, uniformly labor-augmenting technologies, and narrow labor-augmenting technologies. For each technology, we consider the a shock that generates a 1% increase in TFP and then trace its contribution to inequality. Because each of the shocks we are considering is increasing TFP by 1%, we know from theory that their impact on average wages is $\sum_{g} s_{g}^{y} \cdot d \ln w_{g} = d \ln t f p = 0.01$.

In the top panel of Table [5,](#page-82-0) we investigate how large the distributional effects of automation are relative to the TFP impact. Specifically, we focus on automation changes equally affecting all skill groups with the same education level (as in our reduced-form models where we focused on education dummies to control for such labor-augmenting trends). In this case, of course, the restriction that the overall TFP increases 1% amounts to imposing $d\ln t f p = s_g^y \cdot d\ln \Gamma_g^{new} \cdot \pi_g^{auto}$. We depict the impact of these changes on the own group and the other education groups (in each case averaged across demographic groups with the same level of education).

	EFFECTS ON AVERAGE REAL HOURLY WAGES:								
Shock to		High School Dropout High School Graduate Some College College Postgraduate							
	PANEL A. AUTOMATION								
High School Dropout	-21.88	4.25	5.98	8.07	8.86				
High School Graduate	4.15	-8.57	5.38	7.06	8.05				
Some College	5.8	5.26	-13.41	5.9	6.68				
College	7.96	6.92	5.71	-27.65	3.13				
Postgraduate	9.12	8.15	6.47	2.63	-27.86				
	PANEL B. UNIFORM FACTOR-AUGMENTING								
High School Dropout	2.02	0.84	1.34	1.94	2.16				
High School Graduate	0.81	1.85	1.16	1.65	1.93				
Some College	1.29	1.13	2.29	1.32	1.54				
College	1.91	1.61	1.26	2.30	0.51				
Postgraduate	2.24	1.96	1.48	0.36	0.84				
	PANEL C. NARROW FACTOR-AUGMENTING								
High School Dropout	-2.11	1.88	2.20	2.56	2.75				
High School Graduate	1.85	-0.03	2.09	2.38	2.58				
Some College	2.16	2.06	-0.74	2.17	2.32				
College	2.54	2.35	2.14	-2.91	1.67				
Postgraduate	2.78	2.59	2.27	1.60	-3.16				

Table 5: Effects on Average Wages Due to a 1% Increase in TFP by Demographic Group

Notes: This table shows the effects on average wages in demographic groups due to a rise in factor-augmenting technologies that result in a 1% increase in TFP. The detailed breakdown by panel facilitates understanding of the differential impact across various scenarios of technological advancement and educational strata.

Panel A shows that automation has significant distributional effects. For instance, a (uniform) automation shock impacting all groups with less than high school reduces these groups' own wage by, on average, -21.88%. The impact on other demographic groups, operating via the productivity and ripple effects, is positive in this case. For example, the effect on college-graduate groups is an 8.07% increase. This implies that automation affecting workers with less than high school is increasing inequality between this group and college graduates by about 30%.

Panel B shows positive but comparatively much smaller effects on own group wages from uniformly labor-augmenting technologies, which reflects the fact that the macroeconomic elasticities between groups (taking into account the ripple effects) are close to 1. For example, a technological improvement raising the productivity of less than high school groups uniformly increases their wages by about 2%, and has a very similar impact on groups with college or more. The least positive effect is on high school graduates—an increase of 0.8% —and thus the rising inequality resulting from this shock is about a 1% widening of the gap between groups with or without a high school degree. The quantitative pattern in the other rows is similar. This implies that uniformly labor-augmenting technologies have about one-thirtieth of the impact of automation technologies.

Panel C of Table [5](#page-82-0) repeats this exercise for narrow labor-augmenting changes. As highlighted in Proposition 6 , these technologies will have a more negative impact on the own group, because they do not generate the same beneficial impact via competition for marginal tasks, and this is reflected in the numbers in the Panel C, where we see that the own-group effects are negative, though again distributionally small. For example, a narrow labor-augmenting technology benefiting groups with less than high school reduces their wages by about 2.11% and increases the wages of other groups by 1.88%-2.75%, thus amounting to a 4.5% widening of between-group wages. This quantitative impact is almost an order of magnitude smaller than the distributional implications of automation technologies.

In sum, factor-augmenting technologies have fairly limited distributional effects in this framework, and thus their contribution to the decompositions in Table [4](#page-81-0) and Figure [21](#page-100-1) is unlikely to be sizable.

We have so far emphasized the success of the task framework in accounting for various recent labor market trends. We conclude this section by highlighting two puzzles that this framework generates, which require further work.

8.5 Puzzle 1: Missing Technologies

Our decomposition exercise focused on accounting for wage changes across skill groups. A related but distinct exercise is to explore the contribution of different technological trends to total demand shifts. Since 1980, the US workforce has become significantly more educated, which translates into large changes in the size of more educated skill groups. As emphasized in [Katz and Murphy](#page-93-3) [\(1992\)](#page-93-3), all else equal, this demographic shift should have raised the relative wages of less educated workers. From the viewpoint of a supply-demand framework, this implies that the relative demand changes have been even larger and favoring the more educated groups.

Following [Katz and Murphy](#page-93-3) [\(1992\)](#page-93-3), we can use the framework here to quantify the extent of these demand shifts. In particular, given the propagation matrix Θ , which summarizes all the relevant elasticities, the demand shifts across demographic groups since 1980 can be computed as

$$
demand shift_g = \Delta \ln w_g + \Theta_g \cdot stack(\Delta \ln population_g + \Delta \ln \ell_g),
$$

where $\Delta \ln {\rm population}_g$ are changes in log group size and $\Delta \ln \ell_g$ denotes changes in log hours per capita. This expression leverages the fact that the propagation matrix also controls how changes in the supply of skills affect wages, as discussed in Proposition [7.](#page-45-0)

Figure [22](#page-101-0) compares the measured demand shifts with observed wage changes and underscores the point we made above: demand shifts are more pronounced than wage movements because supply shifts have favored low-education and low-pay groups. But then what explains these demand shifts? Given the estimates of the propagation matrix, the absolute values of the contribution of the four factors we are considering do not change much. As a result, now automation explains about 9.8% of the total demand shifts, while new tasks explain about 1.5%, and sectoral TFP and markups explain, in total, about 0.8%. This implies that about 88% of relative demand shifts are unexplained. Since, as we have just argued, factor-augmenting technologies are unlikely to contribute much to these betweengroup shifts, our framework highlights a puzzle: there is a big chunk of relative demand shifts in the US economy since 1980 that remain unexplained.

At some level, this is not a new problem. It was present in the existing literature but was hidden, because the standard framework assumed that there could be sufficient skill-biased technological changes to account for these demand shifts. By decomposing the contributions of different technologies and highlighting why factor-augmenting technologies

are unlikely to be a major contributor, our framework highlights this puzzle.

We return to what might account for this missing demand shifts after we introduce the next puzzle.

8.6 Puzzle 2: Too Large Employment Responses

In Section [7,](#page-54-0) we showed sizable effects of automation and new tasks on the employment structure as well. A natural way to think about these employment effects is to introduce an endogenous labor supply margin into the model, so that demand shifts induce moves along upward-sloping labor supply curve. This is straightforward to do, for example, by positing that the quantity of labor from skill group g is determined according to the labor supply schedule

$$
\ell_g = \chi_g \cdot w_g^{\varepsilon},
$$

where $\varepsilon \geq 0$ is the net elasticity of labor supply (inclusive of income effects) and m_g a supply shifter. The case of inelastic labor supply studied so far is obtained when $\varepsilon = 0$. This labor supply curve can be the result of frictions (as in [Kim and Vogel,](#page-93-4) [2021\)](#page-93-4) or derived from household optimization with quasi-linear preferences (as in [Acemoglu and Restrepo,](#page-91-0) [2022\)](#page-91-0).

Proposition [9](#page-50-0) extends to this environment, but now

$$
d\ln w = \Theta^* \cdot \text{stack}\left(d\ln y + \sum_i \omega_{gi} \cdot z_{gi} + (\lambda - \eta) \cdot \sum_i \omega_{gi} \cdot d\ln p_i\right)
$$

where the propagation matrix inclusive of endogenous supply responses takes the form

$$
\Theta^* = \frac{1}{\lambda + \varepsilon} \cdot \left(1 - \frac{1}{\lambda + \varepsilon} \cdot \frac{\partial \ln \Gamma(w)}{\partial \ln w} \right)^{-1}
$$

The key difference with the previous matrix is that in place of λ , we have $\lambda + \varepsilon$: wage effects are less pronounced when labor supply is elastic, since more of the adjustment takes place via quantities. Endogenous labor supply responses also weaken ripple effects, as lower hours worked for (negatively) affected groups means less competition for marginal tasks.

Conversely, the effect of demand shifts on employment becomes

$$
d\ln \ell_g = \varepsilon \cdot \Theta^* \cdot \text{stack}\left(d\ln y + \sum_i \omega_{gi} \cdot z_{gi} + (\lambda - \eta) \cdot \sum_i \omega_{gi} \cdot d\ln p_i\right),\,
$$

which implies that technology and markups affect employment through the same channels

emphasized above, as documented in our reduced-form analysis.

The puzzle in this case is that for realistic values of the labor supply elasticity, it is not possible to simultaneously account for the observed wage and employment changes. One way of seeing this is to note that, with the same derivations as in Section [2,](#page-8-0) the row sums of Θ^* should be less than or equal to $1/(\lambda + \varepsilon)$. This places an upper bound on the absolute magnitude of the coefficients for automation and new task creation in the reducedform specification studied in Section [7.](#page-54-0) For example, using the model estimates from our analysis there, we would need $\lambda + \varepsilon \leq 0.75$. Given the value of $\lambda = 0.5$ from [Humlum](#page-93-0) [\(2020\)](#page-93-0) that we have used so far, this implies $\varepsilon \leq 0.25$, which is much smaller than the estimates of about of $\varepsilon = 0.5$ reported in [Chetty et al.](#page-92-3) [\(2011\)](#page-92-3). This is neither a technical problem nor an entirely new one. Rather, it reflects the fact that with fairly elastic labor supply responses, it becomes impossible to generate large wage changes in general and sizable ripple effects in our setting—because automation, new tasks and other technology shifts induce large labor supply responses and not sufficiently large wage effects.

We conjecture that both of these puzzles are related to the assumption that labor markets are fully competitive, and introducing non-competitive elements would provide at least a partial solutions to be both puzzles. For example, the presence of rents (aboveopportunity cost wages) for some groups, for example as in [Acemoglu and Restrepo](#page-91-1) [\(2024\)](#page-91-1), would further amplify the effects of automation on wages but also shift the economy off the labor supply curve. Such non-competitive elements could also increase the extent of automation, because additional automation may be motivated by a desire to dissipate rents from certain groups of workers.

9 CONCLUSION

This paper has reviewed and extended the recent literature on the task framework, where the production process is explicitly modeled as being based on the allocation of a range of tasks to different factors of production.

The task model provides an attractive tool for studying the labor market transformations ongoing in the United States and other industrialized nations for several reasons. To start with, an essential aspect of these transformations appears to be related to large changes in the nature of tasks—and occupations—that different types of workers perform in the labor market. Moreover, both the wage and occupational changes appear to be related to the rollout of new automation technologies that have substituted capital equipment and algorithms for tasks previously performed by some worker groups [\(Autor et al.,](#page-91-2) [2003;](#page-91-2) [Acemoglu and Autor,](#page-90-0) [2011;](#page-90-0) [Acemoglu and Restrepo,](#page-91-0) [2022\)](#page-91-0). Less appreciated but equally important are the effects of new technologies that have introduced new tasks for certain worker groups, ranging from new technical occupations to those based on digital tools, such as programming, design, integration functions and related service responsibilities [\(Lin,](#page-93-5) [2011;](#page-93-5) [Acemoglu and Restrepo,](#page-90-1) [2018b;](#page-90-1) [Autor et al.,](#page-91-3) [2022\)](#page-91-3). Automation and the introduction of new tasks cannot be easily studied in existing frameworks, which typically focus on factor-augmenting technological advances and do not distinguish the effects of different types of technologies.

The task framework not only adds descriptive realism to the modeling of the production process and the labor market, but leads to new comparative statics concerning the effects of technologies on the labor market. These new results are rooted in the extensive-margin effects of new technologies—meaning that they reallocate tasks away from certain worker groups and in the case of new tasks, toward some worker groups—at given wages. We represent these extensive-margin influences via (direct) task displacement caused by automation and reinstatement generated by new tasks, and theoretically establish that they are very different than the consequences of technologies that make workers more productive in the tasks they are already performing or general factor-augmenting technologies that make factors uniformly more productive in all tasks.

The theoretical analysis in this chapter also builds a natural bridge between theory and empirics, and we exposited and utilized this bridge at two different levels. The first is via a set of reduced-form equations that can be estimated to link relative wage (and employment) changes at the level of skill groups (e.g., groups distinguished by education, gender, age, ethnicity, etc.) to empirical measures of direct task displacement and reinstatement as well as proxies for factor-augmenting technologies and sectoral reallocations. When estimated, this empirical framework points to a very significant role for task displacement and reinstatement in accounting for the changes in the US wage and employment structure—in all cases explaining more than 50% of the variation between 1980 and 2016. In contrast, our proxies for other technological factors appear much less important in the distributional changes observed since 1980. This reduced-form evidence thus suggests that the extensive-margin effects of new technologies, typically ignored or bundled with other factors in standard approaches, should be the main focus when exploring the determinants of the recent evolution of the wage structure in the US and other industrialized economies.

Despite their simplicity and tight connection to theory, reduced-form equations have

important limitations. First, they ignore the ripple effects that result from the technologies impacting one skill group, which then spread to the rest of the workforce, as affected groups become more or less competitive for marginal tasks and thus transmit their wage effects to other groups. Second, reduced-form models are only informative about relative wage changes, because productivity effects that determine the common component of wage changes across groups are subsumed into the constant term of the regression. Third, while the task displacement and reinstatement terms can be reasonably well approximated with the data we have available, our proxies for other technological influences may be less reliable. These shortcomings are rectified by a more structural approach that the task framework also enables—and we derive systematically from the multi-sector version of the model.

Specifically, the framework shows that the full effects of technological developments can be summarized by the following channels: a productivity effect, the direct extensivemargin effects on task allocations, task-price substitution effects (as tasks produced by factors becoming more productive get cheaper), sectoral reallocations triggered by the uneven incidence of the technology in question across sectors, and the ripple effects. The ripple effects can be economically summarized by a propagation matrix, which we develop and estimate via GMM from the same wage and task displacement and reinstatements data. The remaining effects can be disciplined with external information on the elasticity of substitution between tasks within a sector and the elasticity of substitution between different sectoral goods in the production of the final good.

Using this structural approach, our estimates of the propagation matrix and external estimates on the relevant elasticities, we carry out a full general equilibrium decomposition of the contribution of different technologies. We once again conclude that more than 50% of the changes in the US wage structure between 1980 and 2016 are driven by automation and new tasks.

One of the attractive features of the task framework is its flexibility, which we illustrated by showing how relatively complex economic interactions can be modeled simply within this framework. There are several other directions for future work, which we hope our chapter will encourage:

• In this chapter, we focused on competitive models, with the exception of the exogenous sectoral markups which were introduced in the multi-sector model. The task framework naturally allows for the modeling of various imperfections. For example, the allocation of tasks to factors can be frictional due to search and matching considerations, discrimination against some groups in certain tasks, or licensing. Additionally,

the task model allows for efficiency-wage type considerations, rent-sharing or explicit bargaining at the task level (e.g., [Acemoglu and Restrepo,](#page-91-1) [2024\)](#page-91-1). Such frictions not only cause inefficient assignment of tasks to factors, but also significantly enrich the effects of automation-type technologies, because these now have the additional role of dissipating rents and can take place inefficiently as a result of an effort to dissipate rents (meaning to avoid paying above-opportunity cost wages to some factors). As mentioned above, non-competitive approaches can also hold the key to resolving the two puzzles we highlighted.

- More general preference structures, for example including non-homothetic utility over different goods and services, can be easily incorporated into this framework in order to study the process of structural change in the economy and its implications for the labor market. Such an extension can enable a more holistic analysis of the joint process of structural transformation and inequality following from different types of technological influences.
- The task framework is ideally suited to study the implications of trade in goods and services, offshoring and reshoring, and can be developed in the context of a multicountry setup in a relatively tractable form [\(Kikuchi,](#page-93-6) [2024\)](#page-93-6).
- The task framework can be useful for studying immigration and related changes on the supply side, making explicit how the effects of these developments depend on which tasks new labor groups compete for. For example, the framework suggests that the implications of an immigration shock should be very different when immigrants perform complementary tasks to natives; when they compete against machines; and when they compete for the tasks that certain native skill groups were previously performing.
- A major economic transformation will likely result from the rollout of new artificial intelligence (AI) tools in the coming decades. To the extent that AI can automate some tasks, can create new tasks for some types of workers, can increase the productivity or expertise of certain types of workers in existing tasks, and can change product market competition and markups, its variegated effects are ideally suited to be studied in the task framework (see, for example, [Acemoglu,](#page-90-2) [2024;](#page-90-2) [Acemoglu et al.,](#page-90-3) [2022;](#page-90-3) [Babina et al.,](#page-91-4) [2024\)](#page-91-4).
- The empirical work reported in this chapter uses publicly-available data, though we also mentioned an emerging literature using firm-level data. There is much more to

be done with firm-level data and matched firm-worker data to investigate how task displacement and reinstatements take place and how this triggers a series of indirect effects, as not just factors of production but also as firms compete with each other following the uneven adoption of various technologies.

- This chapter highlighted the importance of new tasks, which are challenging to measure in practice, and future empirical work on the measurement of new tasks and their effects on different labor groups is an important direction (see [Autor et al.,](#page-91-3) [2022,](#page-91-3) for recent work on this).
- Finally, it would be useful to extend the theoretical and empirical approaches reviewed in this chapter, which relied on first-order approximations, and incorporate the higherorder, nonlinear effects from large changes in technology or supplies.

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Figure 12: Reduced-form relationship between change in log hourly wages and change in log hours worked per adult labor-market outcomes vs. task displacement due to automation, 1980– 2016. The top panel presents bivariate scatter plots. The bottom panels present residual plots partially out gender and education dummies and changes in sectoral value-added shares. Marker sizes are proportional to hours worked in 1980. Marker colors distinguish groups with different education levels.

Figure 13: Reduced-form relationship between change in log hourly wages and change in log hours worked per adult labor-market outcomes vs. task reinstatement due to new tasks, 1980– 2016. The top panel presents bivariate scatter plots. The bottom panels present residual plots partially out gender and education dummies and changes in sectoral value-added shares. Marker sizes are proportional to hours worked in 1980. Marker colors distinguish groups with different education levels.

Figure 14: Percent change in Hicks-neutral technology (in orange) and TFP (in blue) and percent change in markups (in blue).

Figure 15: Elasticities of substitution between educational and age groups (based on propagation matrix estimates).

Figure 16: This figure decomposes the effects of automation on hourly wages between 1980 and 2016 into four components. Panels sequentially add productivity effects, industry shifts, the direct effects of task displacement from automation, and ripple effects. Marker sizes are proportional to hours worked in 1980, and marker colors distinguish groups with different education levels.

Figure 17: This figure decomposes the effects of new tasks on hourly wages between 1980 and 2016 into four components. Panels sequentially add productivity effects, industry shifts, the direct effects of task reinstatement from new tasks, and ripple effects. Marker sizes are proportional to hours worked in 1980, and marker colors distinguish groups with different education levels.

Figure 18: This figure decomposes the effects of sectoral TFP changes on hourly wages between 1980 and 2016 into four components. Panels sequentially add productivity effects, industry shifts, direct effects of sectoral TFP changes, and ripple effects. Marker sizes are proportional to hours worked in 1980, and marker colors distinguish groups with different education levels.

Figure 19: This figure decomposes the effects of sectoral markups on hourly wages between 1980 and 2016 into four components. Panels sequentially add productivity effects, industry shifts, the direct effects of changes in sectoral markups, and ripple effects. Marker sizes are proportional to hours worked in 1980, and marker colors distinguish groups with different education levels.

Figure 20: Observed changes in (real) hourly wages, 1980-2016, vs. predicted changes based on the combined effects of automation, new tasks, sectoral TFP changes and sectoral markup changes.

Figure 21: Contribution of technology and markups to the changes in various educational premia, 1980–2016.

Figure 22: This figure plots the demand shifts implied by the observed evolution of wages and supplies between 1980 and 2016 for 500 demographic groups. Marker sizes are proportional to hours worked in 1980, and marker colors indicate education levels.

: "Tasks At Work: Comparative Advantage, Technology and Labor Demand"

Daron Acemoglu, Fredric Kong, Pascual Restrepo August 13, 2024

A Equilibrium Existence and Uniqueness

This section proves Proposition [1.](#page-13-0) It also provides an extension that can be used to study the equilibrium when tasks are combined a-la Cobb-Douglas.

We first derive the equilibrium conditions in the text and provide a lemma for the Jacobian of task shares that will be used to establish the uniqueness of the equilibrium.

Preliminaries: This section derives the equilibrium conditions E1-E5. E1 and E2 follow from cost minimization. For E3, the production of the final good is competitive, so task prices equal their marginal product $p(x) = M^{-1/\lambda} \cdot \left(\frac{y}{y(x)}\right)$ $\frac{y}{y(x)}\right)^{1/\lambda}$, and

$$
(A45) \t\t y(x) = \frac{1}{M} \cdot y \cdot p(x)^{-\lambda}.
$$

For tasks in $\mathcal{T}_g(w)$, equation [\(A45\)](#page-102-0) implies

$$
\underbrace{A_g \cdot \psi_g(x) \cdot \ell_g(x)}_{y(x)} = \frac{1}{M} \cdot y \cdot \underbrace{\left(\frac{w_g}{A_g \psi_g(x)}\right)^{-\lambda}}_{p(x)},
$$

which can be rearranged into E3. The same steps establish the corresponding equation for capital.

E4 imposes labor market clearing.

For E5, we multiply equation $(A45)$ by p_x and integrate

$$
\underbrace{\int y(x) \cdot p(x) \cdot dx}_{y} = \int_{\mathcal{T}} p_x \cdot y_x \cdot dx = \frac{1}{M} \cdot y \cdot \int_{\mathcal{T}} p_x^{1-\lambda} \cdot dx.
$$

Canceling y on both sides yields the ideal-price index equation E5.

Jacobian lemma: The following lemma will be used in our proofs.

LEMMA A1 Let $\mathcal{H} = \mathbb{1} - \frac{1}{\lambda}$ λ $\partial \ln \Gamma(w)$ $\frac{\ln \Gamma(w)}{\partial \ln w}$. For all wage vectors w, the matrix Σ is non-singular. Moreover, H is a P-matrix of the Leontief type (i.e., with non-positive off-diagonal entries) whose inverse has all entries non-negative.

PROOF. Assumption [1](#page-10-0) ensures that task shares are a continuous and differentiable function of wages. We now establish the properties of H .

First, because $\partial \Gamma_g(w) / \partial w_{g'} \ge 0$ for $g' \ne g$, $\mathcal H$ is a Z-matrix (it has negative off diagonals).

Second, H has a positive dominant diagonal. This follows from the fact that \mathcal{H}_{gg} = $1-\frac{1}{\lambda}$ λ $\partial \ln \Gamma_g(w)$ $\frac{\ln\Gamma_g(w)}{\partial\ln w_g} > 0$, and $\mathcal{H}_{gg} - \sum_{g'\neq g} |\mathcal{H}_{gg'}| = 1 - \sum_{g'} \frac{1}{\lambda}$ λ $\partial \ln \Gamma_g(w)$ $\frac{\text{Im }u_g(w)}{\text{Im }w_{g'}} > 1$. This last inequality follows because $\sum_{g'} \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}}$ $\frac{\ln \Gamma_g(w)}{\partial \ln w_{g'}} \leq 0$: when all wages rise by the same amount, workers lose tasks to capital but do not experience task reallocation among them.

Third, all eigenvalues of $\mathcal H$ have real parts that exceed 1. This follows from Gershgorin's circle theorem: for each eigenvalue ζ of \mathcal{H} , we can find a dimension g such that $\|\zeta - \mathcal{H}_{gg}\|$ $\sum_{g' \neq g} |\mathcal{H}_{gg'}|$. This inequality implies $\Re(\zeta) \in [\mathcal{H}_{gg} - \sum_{g' \neq g} |\mathcal{H}_{gg'}|, \mathcal{H}_{gg} + \sum_{g' \neq g} |\mathcal{H}_{gg'}|]$. Because \mathcal{H}_{gg} – $\sum_{g'\neq g}|\mathcal{H}_{gg'}|>1$ for all g, all eigenvalues of H have real parts greater than 1.

Fourth, since $\mathcal H$ is a Z-matrix whose eigenvalues have positive real part, it is also an M−matrix and a P−matrix of the Leontief type. The inverse of such matrices exists and has non-negative real entries. \blacksquare

Proof of Proposition [1.](#page-13-0)

The derivations for the market-clearing wage in [\(4\)](#page-14-0) were presented in the text.

The numeraire condition in [\(5\)](#page-14-1) is obtained by plugging the expression for prices in E1 into the ideal-price index in E5.

We now turn to existence and uniqueness. To show (4) and (5) admit a unique solution, we first show that, given a level for output y, there is a unique set of wages $\{w_g(y)\}_g$ that satisfies the market clearing conditions in [2.](#page-13-1) We then show there is a unique level of output that satisfies [\(5\)](#page-14-1) evaluated at $\{w_g(y)\}_g$.

For the first step, Assumption [1](#page-10-0) implies that $\Gamma_g(w)$ lies in a compact set $[\Gamma, \overline{\Gamma}]$. $\mathbb{T}: w \to$ $(\mathbb{T}w_1,\ldots,\mathbb{T}w_G)'$ defined by $\mathbb{T}w_g = \left(\frac{y}{\ell}\right)$ $\frac{y}{\ell_g}$ $\frac{1}{\lambda} \cdot A_g^{1-1/\lambda} \cdot \Gamma_g(w)^\frac{1}{\lambda}$ for $g = 1, 2, \ldots, G$ is a continuous mapping from the compact convex set $\mathbb{X} = \prod_{g=1}^G \left[(y/\ell_g)^{\frac{1}{\lambda}} \cdot A_g^{1-1/\lambda} \cdot \underline{\Gamma}^{\frac{1}{\lambda}}, (y/\ell_g)^{\frac{1}{\lambda}} \cdot A_g^{1-1/\lambda} \cdot \overline{\Gamma}^{\frac{1}{\lambda}} \right]$ onto itself. The existence of a positive wage vector $\{w_g(y)\}_g$ solving this fixed-point problem follows from Brouwer's fixed point theorem.

We now turn to uniqueness of $\{w_g(y)\}_g$. We can rewrite the system of equations $\{w_g(y)\}_g$ defining $\{w_g(y)\}_g$ in logs as $F(x) = \frac{1}{\lambda}$ $\frac{1}{\lambda}$ ·stack(ln y-ln ℓ_g), where $x = (\ln w_1, \ldots, \ln w_G)$ and $F(x) = (f_1(x), \ldots, f_G(x))$ with $f_g(x) = x_g - \frac{1}{\lambda}$ $\frac{1}{\lambda} \cdot \ln \Gamma_g(x) - (1 - \frac{1}{\lambda})$ $rac{1}{\lambda}$) · $d \ln A_g$.

The Jacobian of F is given by the M−matrix H . Theorem 5 from [Gale and Nikaido](#page-92-4) [\(1965\)](#page-92-4) shows that the solution to the system $F(x) = a$ is unique if the Jacobian of F is a P-matrix of the Leontief type. The theorem also shows that the unique solution $x(a)$ is increasing in a. As a result, the unique solution to the system of equations in (4) is $\{w_q(y)\}_q$ with $w_g(y)$ strictly increasing in y. We also note that $(y/\ell_g)^{1/\lambda} \cdot \Gamma^{1/\lambda} \leq w_g(y) \leq (y/\ell_g)^{1/\lambda} \cdot \bar{\Gamma}^{1/\lambda}$, so that $w_g(y) \to \infty$ as $y \to \infty$, and $w_g(y) \to 0$ as $y \to 0$.

To conclude, we show that there is a unique y that satisfies the ideal-price index equation (5) . This condition can be written as $F(y) = 1$, where

$$
F(y) = \left(\frac{1}{M} \int_{\mathcal{T}} \left[\min\left\{\min_{g} \left\{\frac{w_g(y)}{A_g \cdot \psi_g(x)}\right\}, \frac{1}{A_k \cdot \psi_k(x)}\right\} \right]^{1-\lambda} dx \right)^{1/(1-\lambda)}
$$

.

.

Because wages are increasing in y, $F(y)$ increases in y. Assumption [1](#page-10-0) also ensures that a positive mass of tasks must be allocated to labor at any wage level, which implies that $F(y)$ is increasing in y. The function $F(y)$ can be written as

$$
F(y) = \left(\left(A_k^{\lambda-1} \cdot \Gamma_k(w(y)) + \sum_g A_g^{\lambda-1} \cdot \Gamma_g(w(y)) \cdot w_g(y)^{1-\lambda} \right)^{1/(1-\lambda)}
$$

As $y \to \infty$, $\Gamma_g(w) \cdot \mu_g(w) \cdot w_g(y)^{1-\lambda} \to \infty$ (since $\Gamma_g(w)$ is bounded from below and $\lambda < 1$) and $\Gamma_k(w(y)) \geq 0$. This implies $F(y) \to \infty$. Moreover, as $y \to 0$, $\Gamma_g(w) \cdot \mu_g(w) \cdot w_g(y)^{1-\lambda} \to 0$ (since $\lambda < 1$) and $\Gamma_k(w(y)) = 0$ (since, by Assumption [1,](#page-10-0) all tasks can be produced by at least one type of worker). This implies $F(y) \rightarrow 0$.

Because $F(y)$ is increasing in y, there is a unique $y \in (0, \infty)$ for which $F(y) = 1$ and, therefore, a unique equilibrium with wages $w_g = w_g(y)$. The equilibrium wages and the tiebreaking rule for tasks where there is indifference uniquely determine the task allocation.

Our argument for uniqueness also shows that, under Assumption [1,](#page-10-0) the unique equilibrium features finite output, positive wages, and positive task shares for all workers. Moreover, from $F(y) = 1$, we obtain that, in equilibrium, $1 - A_k^{\lambda-1} \cdot \Gamma_k(w) > 0$.

B Effects of technology in the no-ripple economy

This section derives the formulas for the effects of technology in the no-ripple economy in Propositions [2,](#page-19-0) [3,](#page-23-0) [4,](#page-26-0) and [5.](#page-28-0) In addition, the derivations here provide formulas for the effects of these technologies on the labor share and output. Throughout this section, we use Γ_g and Γ_k (without wage arguments) to denote equilibrium task shares and \mathcal{T}_g^* and \mathcal{T}_k^* to denote equilibrium task assignments.

Proof of Proposition [2.](#page-19-0) Consider a new technology that automates tasks in A_g . We denote the measure of \mathcal{A}_g by $m(\mathcal{A}_g)$. The derivations below provide a first-order approximation to the effects of automation in $\max_{g} m(\mathcal{A}_g)$.

To derive equation [\(9\)](#page-19-1), we start from [\(4\)](#page-14-0) and consider the log change in this equation following the automation of tasks in \mathcal{A}_q . The log change in both sides of this equation is

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} d\ln \Gamma_g.
$$

By assumption, all tasks in \mathcal{A}_g become automated, which implies

$$
d\ln\Gamma_g=\ln\left(\int_{\mathcal{T}_g^*\diagdown\mathcal{A}_g}\psi_g(x)^{\lambda-1}\cdot dx\right)-\ln\left(\int_{\mathcal{T}_g^*}\psi_g(x)^{\lambda-1}\cdot dx\right)=-\frac{\int_{\mathcal{A}_g}\psi_g(x)^{\lambda-1}\cdot dx}{\int_{\mathcal{T}_g^*}\psi_g(x)^{\lambda-1}\cdot dx},
$$

where the second equality is a first-order Taylor expansion of log changes into percent changes of order $m(\mathcal{A}_g)$. Implicitly here, automation does not change the task productivities. Together, this shows that $d\ln\Gamma_g = -d\ln\Gamma_g^{\text{auto}}$ (to a first-order approximation), establishing [\(9\)](#page-19-1).

To derive equation (10) , we start from the definition of the cost function on the right-side of [\(5\)](#page-14-1). The change in log cost is

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g + \frac{1}{1-\lambda} \cdot s_k^y \cdot d\ln\Gamma_k + \frac{1}{1-\lambda} \cdot \sum_g s_g^y \cdot d\ln\Gamma_g.
$$

This expression uses the fact that the elasticity of the cost function with respect to a change in factor prices equals cost shares (an implication of Shephard's lemma). A firstorder Taylor expansion of log changes into percent changes of order $\max_g m(\mathcal{A}_g)$ yields

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g + \sum_{g} \frac{1}{1-\lambda} \cdot \frac{1}{M} \cdot \left[\frac{s_k^y}{\Gamma_k} \cdot \int_{\mathcal{A}_g} \psi_k^{\text{auto}}(x)^{\lambda-1} \cdot dx - \frac{s_g^y}{\Gamma_g} \cdot \int_{\mathcal{A}_g} \psi_g(x)^{\lambda-1} \cdot dx \right].
$$

Using the fact that s_k^y $\int_k^y = \Gamma_k \cdot A_k^{\lambda-1}$ and $s_g^y = \Gamma_g \cdot A_g^{\lambda-1} \cdot w_g^{1-\lambda}$, the change in costs can be rewritten as

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g + \sum_{g} \frac{1}{1-\lambda} \cdot \frac{1}{M} \cdot \left[\int_{\mathcal{A}_g} A_k^{\lambda-1} \cdot \psi_k^{auto}(x)^{\lambda-1} \cdot dx - \int_{\mathcal{A}_g} A_g^{\lambda-1} \cdot \psi_g(x)^{\lambda-1} \cdot w_g^{1-\lambda} \cdot dx \right]
$$

$$
= \sum_{g} s_g^y \cdot d\ln w_g - \sum_{g} A_g^{\lambda-1} \cdot w_g^{1-\lambda} \cdot \frac{1}{M} \int_{\mathcal{A}_g} \psi_g(x)^{\lambda-1} \cdot \pi^{auto}(x) \cdot dx
$$

$$
= \sum_{g} s_g^y \cdot d\ln w_g + \sum_{g} \frac{\Gamma_g \cdot A_g^{\lambda-1} \cdot w_g^{1-\lambda}}{\prod_{g} s_g^y} \cdot \frac{\int_{\mathcal{A}_g} \psi_g(x)^{\lambda-1} \cdot dx}{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda-1} \cdot dx} \cdot \frac{\int_{\mathcal{A}_g} \psi_g(x)^{\lambda-1} \cdot \pi^{auto}(x) \cdot dx}{\int_{\mathcal{A}_g} \psi_g(x)^{\lambda-1} \cdot dx},
$$

which shows that $d\ln\mathcal{C}(w) = \sum_g s_g^y \cdot d\ln w_g - \sum_g s_g^y \cdot d\ln \Gamma_g^{\text{auto}} \cdot \pi_g$. In equilibrium, $d\ln\mathcal{C}(w) = 0$, which establishes (10) .

We now provide expressions for output and the labor share. Solving for output from (9) and (10) , we obtain

$$
d\ln y = \sum_g \frac{s^y_g}{s^y_\ell} \cdot d\ln \Gamma_g^{auto} \cdot \big(1 + \lambda \cdot \pi_g^{auto}\big).
$$

The change in the labor share can then be computed from $d\ln s_\ell^y$ $\frac{y}{\ell} = \frac{1}{s^2}$ $\frac{1}{s_\ell^y} \sum_g s_g^y \cdot d\ln w_g - d\ln y$ as

$$
d\ln s_{\ell}^y = -\sum_{g} \frac{s_g^y}{s_{\ell}^y} \cdot (1 - (1 - \lambda) \cdot \pi_g^{auto}) \cdot d\ln \Gamma_g^{auto}.
$$

Finally, the capital share can be computed from $d\ln s_k^y$ $\frac{y}{k} = \frac{-ds_{\ell}^{y}}{s_{k}^{y}} = -\frac{s_{\ell}^{y}}{s_{k}^{y}} \cdot d\ln s_{\ell}^{y}$ $_{\ell}^{y}$ as

$$
d\ln s_k^y = \sum_g \frac{s_g^y}{s_k^y} \cdot \left(1 - (1 - \lambda) \cdot \pi_g^{auto}\right) \cdot d\ln \Gamma_g^{auto}.
$$

Remark: The proof uses the fact that all tasks in \mathcal{A}_g become automated. The assumption that $\pi^{\text{auto}} > 0$ ensures this, because, at the initial equilibrium wages, producing these tasks with capital is cheaper than assigning them to labor, and moreover because $m(\mathcal{A}_g) \to 0$, as we assumed here, $d \ln w_g \to 0$ and the same remains true as more tasks are automated. Note that this logic can fail for large automation shocks, in which case only a subset of tasks in \mathcal{A}_g may become automated in equilibrium.

Proof of Proposition [3.](#page-23-0) Consider the arrival of a new technology that creates new

labor-intensive tasks in \mathcal{N}_g . We denote the measure of \mathcal{N}_g by $m(\mathcal{N}_g)$. The derivations below provide a first-order approximation to the effects of automation, with the approximation error being of the order $\max_q m(\mathcal{N}_q)$.

To derive equation [\(12\)](#page-24-0), we start from [\(4\)](#page-14-0) and consider the log change in this equation following the creation of tasks in \mathcal{N}_g . The log change in both sides of this equation is

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} d\ln \Gamma_g.
$$

By assumption, all tasks in \mathcal{N}_g are assigned to g, which implies

$$
d\ln\Gamma_g=\ln\left(\int_{\mathcal{T}_g^*\cup\mathcal{N}_g}\psi_g(x)^{\lambda-1}\cdot dx\right)-\ln\left(\int_{\mathcal{T}_g^*}\psi_g(x)^{\lambda-1}\cdot dx\right)-d\ln M=\frac{\int_{\mathcal{N}_g}\psi_g(x)^{\lambda-1}\cdot dx}{\int_{\mathcal{T}_g^*}\psi_g(x)^{\lambda-1}}-d\ln M,
$$

where the second equality is a first-order Taylor expansion of log changes into percent changes of order $m(\mathcal{N}_g)$. This shows that $d\ln\Gamma_g = d\ln\Gamma_g^{\text{new}} - d\ln M$ (to a first-order approximation), establishing [\(12\)](#page-24-0).

To derive equation [\(13\)](#page-24-1), we start from the definition of the cost function on the right-side of [\(5\)](#page-14-1). As before, the change in log cost is

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g + \frac{1}{1-\lambda} \cdot s_k^y \cdot d\ln\Gamma_k + \frac{1}{1-\lambda} \cdot \sum_g s_g^y \cdot d\ln\Gamma_g.
$$

A first-order Taylor expansion of log changes into percent changes yields

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g + \sum_{g} \frac{1}{1-\lambda} \cdot \frac{1}{M} \left[\frac{s_g^y}{\Gamma_g} \cdot \int_{\mathcal{N}_g} \psi_g(x)^{\lambda-1} \cdot dx - \int_{\mathcal{N}_g} dx \right],
$$

where we used the fact that $d\ln M = \frac{1}{M} \sum_g \int_{\mathcal{N}_g} dx$ (to a first order) and the omitted terms are max_g $m(\mathcal{N}_g)$. Using the fact that $s_g^y = \Gamma_g \cdot A_g^{\lambda-1} \cdot w_g^{1-\lambda}$, the change in costs can be rewritten as

$$
d\ln \mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g + \sum_{g} \frac{1}{1-\lambda} \cdot \frac{1}{M} \cdot \left[\int_{\mathcal{N}_g} A_g^{\lambda-1} \cdot \psi_g(x)^{\lambda-1} \cdot w_g^{1-\lambda} \cdot dx - \int_{\mathcal{N}_g} dx \right]
$$

$$
= \sum_{g} s_g^y \cdot d\ln w_g - \sum_{g} \frac{1}{M} \cdot A_g^{\lambda-1} \cdot w_g^{1-\lambda} \cdot \int_{\mathcal{N}_g} \psi_g(x)^{\lambda-1} \cdot \pi^{\text{new}}(x) \cdot dx
$$

$$
= \sum_{g} s_g^y \cdot d\ln w_g - \sum_{g} \frac{\Gamma_g \cdot A_g^{\lambda-1} \cdot w_g^{1-\lambda} \cdot \frac{\int_{\mathcal{N}_g} \psi_g(x)^{\lambda-1} \cdot dx}{\int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} \cdot dx} \cdot \frac{\int_{\mathcal{N}_g} \psi_g(x)^{\lambda-1} \cdot \pi^{\text{new}}(x) \cdot dx}{\int_{\mathcal{N}_g} \psi_g(x)^{\lambda-1} \cdot dx}
$$
which shows that $d\ln \mathcal{C}(w) = \sum_g s_g^y \cdot d\ln w_g - \sum_g s_g^y \cdot d\ln \Gamma_g^{\text{new}} \cdot \pi_g$. In equilibrium, $d\ln \mathcal{C}(w) = 0$, which establishes (13) .

We now provide expressions for output and the labor share. Solving for output from (12) and (13) , we obtain

$$
d\ln y = \sum_{g} s_g^y \cdot d\ln \Gamma_g^{new} \cdot \left[1 - \frac{1}{s_\ell^y} + \left((1-\lambda) + \frac{1}{s_\ell^y} \cdot \lambda\right) \cdot \pi_g^{new}\right].
$$

The change in the labor share can then be computed from $d\ln s_\ell^y$ $\frac{y}{\ell} = \frac{1}{s_a^3}$ $\frac{1}{s_\ell^y} \sum_g s_g^y \cdot d\ln w_g - d\ln y$ as

$$
d\ln s^y_\ell=\frac{s^y_k}{s^y_\ell}\cdot \sum_g s^y_g\cdot d\ln\Gamma^{new}_g\cdot\left(1+\left(1-\lambda\right)\cdot\pi^{new}_g\right)
$$

Finally, the capital share can be computed from $d\ln s_k^y$ $\frac{y}{k} = \frac{-ds_{\ell}^{y}}{s_{k}^{y}} = -\frac{s_{\ell}^{y}}{s_{k}^{y}} \cdot d\ln s_{\ell}^{y}$ $_{\ell}^{y}$ as

$$
d\ln s_k^y = -\sum_g s_g^y \cdot d\ln \Gamma_g^{new} \cdot (1 + (1 - \lambda) \cdot \pi_g^{new}).
$$

Proof of Propositions [4.](#page-26-0) Differentiating equation (4) establishes (14) :

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + (1 - 1/\lambda) \cdot d\ln A_g + (1 - 1/\lambda) \cdot \underbrace{\frac{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda - 1} \cdot d\ln \psi_g(x) \cdot dx}{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda - 1} \cdot dx}}_{d\ln \psi_g^{\text{intensive}}}.
$$

Total differentiation of the cost function $\mathcal{C}(w)$ in the right of (5) implies

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g - \sum_{g} s_g^y \cdot d\ln A_g + \sum_{g} s_g^y \cdot \underbrace{\frac{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda-1} \cdot d\ln \psi_g(x) \cdot dx}{\int_{\mathcal{T}_g^*} \psi_g(x)^{\lambda-1} \cdot dx}}_{d\ln \psi_g^{\text{intensive}}},
$$

establishing [\(15\)](#page-26-2).

We now provide expressions for output and the labor share. Solving for output from (14) and (15) , we obtain

$$
d\ln y = \sum_{g} \frac{s_g^y}{s_\ell^y} \cdot (d\ln A_g + d\ln \psi_g^{intensive}).
$$

In this case, the labor share and the capital share remain unchanged (a corollary of Uzawa's theorem, since technology augments labor and capital is assumed elastic).

Proof of Propositions [5.](#page-28-0) Total differentiation of equation [\(4\)](#page-14-0) implies

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y,
$$

establishing [\(16\)](#page-28-1).

Total differentiation of the cost function $\mathcal{C}(w)$ in the right of [\(5\)](#page-14-1) implies

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g - s_k^y \cdot d\ln A_k + s_k^y \cdot \underbrace{\frac{\int_{\mathcal{T}_k^*} \psi_k(x)^{\lambda-1} \cdot d\ln \psi_k(x) \cdot dx}{\int_{\mathcal{T}_k^*} \psi_k(x)^{\lambda-1} \cdot dx}}_{d\ln \psi_k^{\text{intensive}}},
$$

establishing [\(17\)](#page-28-2).

We now provide expressions for output and the labor share. Solving for output from (16) and (17) , we obtain

$$
d\ln y = \lambda \cdot \frac{s_k^y}{s_\ell^y} \cdot \left(d\ln A_k + d\ln \psi_k^{intensive}\right).
$$

The change in the labor share can then be computed from $d\ln s_\ell^y$ $y_{\ell}^{y} = \frac{1}{s_{\ell}} \sum_{g} s_{g}^{y} \cdot d \ln w_{g} - d \ln y$ as

$$
d\ln s_{\ell}^y = (1 - \lambda) \cdot \frac{s_k^y}{s_{\ell}^y} \cdot (d\ln A_k + d\ln \psi_k^{intensive}).
$$

Finally, the capital share can be computed from $d\ln s_k^y$ $y_k = \frac{-ds_\ell^y}{s_k^y} = -\frac{s_\ell^y}{s_k^y} \cdot d\ln s_\ell^y$ $_{\ell}^{y}$ as

$$
d\ln s_k^y = -(1-\lambda) \cdot (d\ln A_k + d\ln \psi_k^{intensive}).
$$

П

C Effects of Technology in the Full Equilibrium (with Ripples)

This section proves Proposition [6](#page-35-0) and explains the details of how we apply it to characterize the effects of the different technologies. We then prove Proposition [7.](#page-45-0)

Proof of Proposition [6.](#page-35-0)

For augmenting technologies, we can totally differentiate [\(4\)](#page-14-0) to obtain [\(18\)](#page-35-1) in the main text. For automation and new task creation the same equation holds but the argument is more involved, since we are approximating the change in task shares following the automation of tasks in a small set (or the creation of a small mass of tasks).

Consider the automation of tasks in \mathcal{A}_g first. As explained in the text, we assume that \mathcal{A}_g is in the interior of \mathcal{T}_g . Let $\Gamma_g = \Gamma_g(w)$ denote the initial task share of g and $\Gamma_g^{\text{auto}}(w+d\ln w)$ denote the new task share after these tasks are automated, evaluated at the new equilibrium wages $w+d\ln w$. Because we assumed the new automation technologies strictly dominate labor in A_q , we obtain

$$
d\Gamma_g = \Gamma_g^{\text{auto}}(w + d\ln w) - \Gamma_g(w).
$$

A first-order Taylor expansion of the first term gives

$$
d\Gamma_g = \Gamma_g^{\text{auto}}(w) \cdot \frac{\partial \ln \Gamma_g^{\text{auto}}(w)}{\partial \ln w} \cdot d\ln w + \Gamma_g^{\text{auto}}(w) - \Gamma_g(w).
$$

Note that, $\frac{\partial \ln \Gamma_g^{auto}(w)}{\partial \ln w}$ $\frac{\Gamma_g^{auto}(w)}{\partial \ln w} = \frac{\partial \ln \Gamma_g(w)}{\partial \ln w}$ $\frac{n_1 g(w)}{\partial \ln w}$ since all automated tasks are in the interior of \mathcal{T}_g and inframarginal. Moreover, $\Gamma_g^{\text{auto}}(w) \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w}$ $\frac{\ln\Gamma_g(w)}{\partial\ln w}\cdot d\ln w$ can be approximated as $\Gamma_g(w)\cdot \frac{\partial\ln\Gamma_g(w)}{\partial\ln w}$ $\frac{\ln\Gamma_g(w)}{\partial\ln w}\cdot d\ln w$ to a first order since the product of the change in wages and $\Gamma_g^{\text{auto}}(w) - \Gamma_g(w)$ is second order. Finally, $\Gamma_g^{\text{auto}}(w) - \Gamma_g(w) = -\Gamma_g \cdot d \ln \Gamma_g^{\text{auto}}$ from the definition of task displacement. Using these observations, we obtain

$$
d\ln\Gamma_g = -d\ln\Gamma_g^{\text{auto}} + \frac{\partial\ln\Gamma_g(w)}{\partial\ln w} \cdot d\ln w,
$$

which establishes [\(18\)](#page-35-1) for automation technologies. The argument for new tasks follows the same steps. Stacking [\(18\)](#page-35-1) and solving for wages gives [\(19\)](#page-36-0).

Equation [\(20\)](#page-36-1) follows from the fact that $d\ln C(w) = \sum_g s_g^y \cdot d\ln w - \pi$, again from Shephard's lemma.

Remark 1: Proposition [6](#page-35-0) can be readily used to compute the effects of the different technologies accounting for Ripple effects. In the applications we presented, we computed $\pi = -d \ln \mathcal{C}(w)|_{w\text{constant}}$ following the exact same steps as those outlined in the proofs of Propositions [2](#page-19-0) for automation, [3](#page-23-0) for new tasks, [4](#page-26-0) for labor-augmenting technologies, and [5](#page-28-0) for capital augmenting technologies.

Remark 2: The calculation of the effects of uniform-augmenting technologies in terms of the propagation matrix requires some further explanation. For uniform-labor augmenting improvements, differentiating [\(4\)](#page-14-0) yields

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \underbrace{(1 - 1/\lambda) \cdot d\ln A_g - \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot d\ln A}_{z_g} + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot d\ln w,
$$

where $d \ln A = (d \ln A_1, \dots, d \ln A_G)$ and we used the fact that an increase in A_g generates an equal task reassignment as a commensurate decrease in w_g . Solving for $d \ln w$ yields $d \ln w = \Theta \cdot d \ln y + (1 - \Theta) \cdot d \ln A$, which is equivalent to the formula used in the text.

For uniform-capital augmenting improvements, differentiating [\(4\)](#page-14-0) yields

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \underbrace{\sum_{g'} \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}} \cdot d\ln A_k}_{z_g} + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot d\ln w,
$$

where this expression uses the fact that an increase in A_k generates the same reallocation of tasks as an increase in all wages of the same magnitude. Solving for $d \ln w$ yields $d \ln w$ = $\Theta \cdot (d \ln y + \lambda \cdot d \ln A_k) - d \ln A_k$, or equivalently $d \ln w_g = \rho_g \cdot d \ln y - (1 - \rho_g \cdot \lambda) \cdot d \ln A_k$ as claimed in the text.

Proof of Proposition [7.](#page-45-0) The expression for the change in wages in (24) follows from differentiating equation [\(4\)](#page-14-0):

$$
d\ln w_g=\frac{1}{\lambda}\cdot d\ln y-\frac{1}{\lambda}\cdot d\ln \ell_g+\frac{1}{\lambda}\cdot \frac{\partial\ln\Gamma_g(w)}{\partial\ln w}\cdot d\ln w.
$$

Stacking across groups and solving for $d \ln w_g$ yields [\(24\)](#page-45-1).

The fact that there are no average wage changes follows from differentiating the cost function in [\(5\)](#page-14-1). Because technology does not change, we obtain

$$
d\ln\mathcal{C}(w) = \sum_{g} s_g^y \cdot d\ln w_g = 0.
$$

D EQUILIBRIUM IN THE MULTI-SECTOR ECONOMY

This section provides details and proofs for the multi-sector economy.

Preliminaries: we first derive the equilibrium conditions E1-E6.

E1 and E2 follow from cost minimization.

For E3, because producers in sector *i* face an exogenous markup μ_i , they use task $x \in \mathcal{T}_i$ until $p_i \cdot M_i^{-1/\lambda}$ $i^{-1/\lambda} \cdot A_i^{1-1/\lambda}$ $\frac{1-1}{\mathrm{i}}$ \cdot $\left(\frac{y_i}{y(x_i)}\right)$ $\left(\frac{y_i}{y(x)}\right)^{1/\lambda} = \mu_i \cdot p(x)$, so that the value of the task marginal product (on the left) exceeds its marginal cost (on the right) by μ_i . The quantity of task $x \in \mathcal{T}_i$ used is then

(A46)
$$
y(x) = y_i \cdot p_i^{\lambda} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot p(x)^{-\lambda}.
$$

For tasks in $\mathcal{T}_{gi}(w)$, equation [\(A46\)](#page-112-0) implies

$$
\underbrace{A_g \cdot \psi_g(x) \cdot \ell_g(x)}_{y(x)} = y_i \cdot p_i^{\lambda} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot \underbrace{\left(\frac{w_g}{A_g \psi_g(x)}\right)^{-\lambda}}_{p(x)},
$$

which explains E3. The same steps establish the corresponding equation for capital.

E4 imposes labor market clearing, now adding labor demand across all sectors.

For E5, multiply equation $(A46)$ by $\mu_i \cdot p_x$ and integrate

$$
\underbrace{\mu_i \cdot \int y(x) \cdot p(x) \cdot dx}_{y_i \cdot p_i} = \int_{\mathcal{T}_i} y_i \cdot p_i^{\lambda} \cdot \mu_i^{1-\lambda} \cdot A_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot p(x)^{1-\lambda} \cdot dx.
$$

Canceling y_i on both sides and solving for p_i index equation E5.

Equation E6 requires the price of the final good to be 1—the numeraire condition.

Proofs for multi-sector model propositions: We now prove Proposition [8](#page-49-0) describing the equilibrium in the multi-sector economy and then turn to Propositions [9](#page-50-0) and [10](#page-53-0) characterizing the impact of technology and markups, respectively.

Proof of Proposition [8.](#page-49-0) We first derive the expression for the market-clearing wage in Equation [\(25\)](#page-49-1). Aggregating E3 across all tasks assigned to group g in all sectors, and using the definition of $\Gamma_{gi}(w)$, we can write the labor-market clearing condition as

$$
y \cdot A_g^{\lambda-1} \cdot w_g^{-\lambda} \cdot \left[\sum_i s_i^y(p) \cdot p_i^{\lambda-1} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda-1} \cdot \Gamma_{gi}(w) \right] \cdot = \ell_g.
$$

Isolating w_g from this equation yields (25) .

The formula for sectoral prices in terms of task shares in [\(26\)](#page-49-2) is obtained by plugging

the expression for prices in E1 into the price index formula in E5.

The final equilibrium equation in (27) is E6. \blacksquare

Proof of Proposition [9.](#page-50-0) For augmenting technologies, or Hicks' neutral TFP improvements across sectors, we can totally differentiate [\(25\)](#page-49-1) to obtain

(A47)
$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} \cdot \sum_i \omega_{gi} \cdot z_{gi} + \frac{1}{\lambda} \cdot (\lambda - \eta) \cdot \sum_i \omega_{gi} \cdot d\ln p_i.
$$

As in the single sector model, the decomposition in [\(A47\)](#page-113-0) also holds for new task creation and automation, with the direct effects defined as $z_{gi} = -d\ln\Gamma_{gi}^{\text{auto}}$ and $z_{gi} = d\ln\Gamma_{gi}^{\text{new}}$, respectively. The argument follows the exact same steps outlined in the proof of Proposition [6](#page-35-0) and so we do not reproduce it here.

Stacking [\(A47\)](#page-113-0) and solving for wages gives [\(28\)](#page-51-0).

Equation [\(29\)](#page-51-1) follows from the fact that $d\ln p_i = d\ln \mathcal{C}_i(w) = \sum_g s_g^{y_i} \cdot d\ln w - \pi$, again from Shephard's lemma.

Finally, equation [\(30\)](#page-51-2) follows from the fact that $0 = d \ln c^f(p) = \sum_i s_i \cdot d \ln p_i$, again from Shephard's lemma, but applied to the production of the final good. \blacksquare

Proof of Proposition [10.](#page-53-0) Totally differentiating (25) , we obtain

$$
d\ln w_g = \frac{1}{\lambda} \cdot d\ln y - \sum_i \omega_{gi} \cdot d\ln \mu_i + \frac{1}{\lambda} \cdot (\lambda - \eta) \cdot \sum_i \omega_{gi} \cdot d\ln p_i.
$$

Stacking these equations for all groups and solving for wages gives [\(33\)](#page-54-0).

Equation [\(34\)](#page-54-1) follows from the fact that $d\ln p_i = d\ln \mathcal{C}_i(w) = \sum_g s_g^{y_i} \cdot d\ln w + d\ln \mu_i$, again from Shephard's lemma.

Finally, equation [\(35\)](#page-54-2) follows from the fact that $0 = d \ln c^f(p) = \sum_i s_i \cdot d \ln p_i$, again from Shephard's lemma, but applied to the production of the final good. \blacksquare

E Endogenous Labor Supply

The following proposition extends our analysis to a multi-sector economy with endogenous labor supply. For this proposition, we assume labor supply is given by $\ell_g = \chi_g \cdot w_w^{\varepsilon}$.

PROPOSITION A1 (EFFECTS OF TECHNOLOGY IN THE MULTI-SECTOR ECONOMY) With an endogenous labor supply, equilibrium wages w, industry prices p, and the level of output y, solve the system of equations

(A48)
$$
w_g = \left(\frac{y}{\chi_g}\right)^{1/(\lambda+\varepsilon)} \cdot A_g^{(\lambda-1)/(\lambda+\varepsilon)} \cdot \left[\sum_i s_i^y(p) \cdot p_i^{\lambda-1} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda-1} \cdot \Gamma_{gi}(w)\right]^{1/(\lambda+\varepsilon)} \text{ for } g \in \mathbb{G},
$$

(A49)
$$
p_i = \mu_i \cdot \frac{1}{A_i} \cdot \left(\Gamma_{ki}(w) \cdot A_k^{\lambda-1} + \sum_g \Gamma_{gi}(w) \cdot \left(\frac{w_g}{A_g}\right)^{1-\lambda}\right)^{1/(1-\lambda)} \text{ for } i \in \mathbb{I},
$$

 $(A50)$ 1 = $c_f(p)$,

where $C_i(w)$ denotes the marginal cost of producing output of sector i.

 $\equiv \mathcal{C}_i(w)$

In addition, the effect of a change in technology with direct effect $\{z_{gi}\}_{g \in \mathbb{G}, i \in \mathbb{I}}$ and productivity gains $\{\pi_{gi}\}_{g \in \mathbb{G}, i \in \mathbb{I}}$ on wages, sectoral prices, and output is given by the formulas in Proposition [9,](#page-50-0) but with the propagation matrix redefined as

$$
\Theta^* = \frac{1}{\lambda + \varepsilon} \cdot \left(1 - \frac{1}{\lambda + \varepsilon} \cdot \frac{\partial \ln \Gamma(w)}{\partial \ln w} \right)^{-1},
$$

and direct effect re-scaled by $\lambda + \varepsilon$ (so that direct effect are $(1/(\lambda + \varepsilon)) \cdot z_{gi}$).

PROOF. The equilibrium conditions in this case are still given by E1–E6. The only difference is that the market clearing condition in E4 is now

$$
\sum_{i} \int_{\mathcal{T}_{gi}} \ell_g(x) \cdot dx = \chi_g \cdot w_g^{\varepsilon}.
$$

Following the same steps as in the proof of Proposition [8,](#page-49-0) we can write this condition as

$$
y \cdot A_g^{\lambda-1} \cdot w_g^{-\lambda} \cdot \left[\sum_i s_i^y(p) \cdot p_i^{\lambda-1} \cdot \mu_i^{-\lambda} \cdot A_i^{\lambda-1} \cdot \Gamma_{gi}(w) \right] \cdot = \chi_g \cdot w_g^{\varepsilon}.
$$

Isolating w_g from this equation yields [\(A48\)](#page-114-0).

The formula for sectoral prices in terms of task shares in [\(A49\)](#page-114-1) is obtained by plugging the expression for prices in E1 into the price index formula in E5.

The final equilibrium equation in [\(A50\)](#page-114-2) is E6.

The redefinition of the propagation matrix follows from totally differentiating [\(A48\)](#page-114-0),

which yields

$$
(A51) \t d\ln w_g = \frac{1}{\lambda + \varepsilon} d\ln y + \frac{1}{\lambda + \varepsilon} \sum_i \omega_{gi} \cdot z_{gi} + \frac{(\lambda - \eta)}{\lambda + \varepsilon} \cdot \sum_i \omega_{gi} \cdot d\ln p_i.
$$

As before, the decomposition in [\(A47\)](#page-113-0) also holds for new task creation and automation, with direct effects defined as $z_{gi} = -d \ln \Gamma_{gi}^{\text{auto}}$ and $z_{gi} = d \ln \Gamma_{gi}^{\text{new}}$, respectively.

Stacking these equations and solving for wages, we obtain

$$
d\ln w = \Theta^* \cdot \text{stack}\left(d\ln y + \sum_i \omega_{gi} \cdot z_{gi} + (\lambda - \eta) \cdot \sum_i \omega_{gi} \cdot d\ln p_i\right),\,
$$

establishing that the formulas coincide with those in Proposition [9](#page-50-0) with Θ^* in place of Θ . Г

F Derivations for the Allen-Uzawa elasticities of substitution and properties of the Propagation Matrix

This section proves several properties of task shares, elasticities of substitution, and the propagation matrix mentioned in the text.

Symmetry of the task-share Jacobian: Equation [\(3\)](#page-13-0) shows that the task-share Jacobian satisfies a symmetry property. To prove this, consider a proportional increase in w_g by $\Delta w_g = w_g \cdot \epsilon$ for some $\epsilon > 0$, a set $\mathcal{M}(\epsilon)$ of these tasks are assigned to g' and increase g' task share by $\Delta\Gamma_{g'} = \int_{\mathcal{M}(\epsilon)} \psi_{g'}(x)^{\lambda-1} \cdot dx$. Therefore,

$$
\frac{\partial \Gamma_{g'}(w)}{\partial w_g} = \lim_{\epsilon \to 0} \frac{\int_{\mathcal{M}(\epsilon)} \psi_{g'}(x)^{\lambda - 1} \cdot dx}{w_g \cdot \epsilon}.
$$

Now, suppose that $w_{g'}$ decreases proportionally by $\Delta w_{g'} = -w_{g'} \cdot \epsilon$ for some $\epsilon > 0$. The same set $\mathcal{M}(\epsilon)$ of tasks switch to g' and decrease skill group g's task share by $\Delta\Gamma_g$ = $-\int_{\mathcal{M}(\epsilon)} \psi_g(x)^{\lambda-1} \cdot dx$. Now noting that for marginal tasks we have $\frac{w_g}{A_g \cdot \psi_g(x)} = \frac{w'_g}{A_{g'} \cdot \psi_{g'}(x)}$, we can conclude

$$
\frac{\partial \Gamma_g(w)}{\partial w_{g'}} = \lim_{\varepsilon \to 0} \frac{\int_{\mathcal{M}(\varepsilon)} \psi_{g'}(x)^{\lambda-1} \cdot \left(\frac{w_g}{w'_g}\right)^{\lambda-1} \cdot \left(\frac{A_{g'}}{A_g}\right)^{\lambda-1} \cdot dx}{w_{g'} \cdot \varepsilon} = \left(\frac{w_g}{w'_g}\right)^{\lambda} \cdot \left(\frac{A_{g'}}{A_g}\right)^{\lambda-1} \cdot \frac{\partial \Gamma_{g'}(w)}{\partial w_g}.
$$

Properties of the propagation matrix: We now prove the properties of the propagation matrix mentioned for the one-sector economy.

I. Dampening: Gershgorin's circle theorem in the proof of Lemma [A1](#page-103-0) already implied that the real part of all eigenvalues of $\mathcal H$ are above 1. We now show that all eigenvalues of H are real. To show this, first note that $\text{diag}(s^y) \mathcal{H} = \mathcal{H}_{sym}$ is a symmetric matrix with off-diagonal entry gg' given by $-\frac{1}{\lambda}$ $\frac{1}{\lambda} \cdot s_{g}^{y} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_i}$ $\frac{\ln \Gamma_g(w)}{\partial \ln w_j}$ and entry $g'g$ given by $-\frac{1}{\lambda}$ $\frac{1}{\lambda} \cdot s_g^y$ $\frac{y}{g'} \cdot \frac{\partial \ln \Gamma_{g'}(w)}{\partial \ln w_g}$ $\frac{\sin q \cdot (w)}{\partial \ln w_g},$ which are equal due to the symmetry property of the Jacobian. Suppose ζ is an eigenvalue of $\mathcal H$ with eigenvector v. Using upper bars to denote complex conjugates and superscript T to denote the transpose operation, we obtain

$$
\zeta \cdot \bar{v}^T \cdot \text{diag}(s^y) \cdot v = \bar{v}^T \cdot (\text{diag}(s^y) \cdot \zeta \cdot v)
$$

\n
$$
= \bar{v}^T \cdot (\text{diag}(s^y) \cdot \mathcal{H} \cdot v)
$$

\n
$$
= \bar{v}^T \cdot (\mathcal{H}_{sym} \cdot v)
$$

\n
$$
= (\mathcal{H}_{sym} \cdot \bar{v})^T \cdot v
$$

\n
$$
= (\bar{H}_{sym} \cdot \bar{v})^T \cdot v
$$

\n
$$
= (\bar{G} \cdot \text{diag}(s^y) \cdot \bar{v})^T \cdot v
$$

\n
$$
= (\bar{\zeta} \cdot \text{diag}(s^y) \cdot \bar{v})^T \cdot v
$$

\n
$$
= \bar{\zeta} \cdot \bar{v}^T \cdot \text{diag}(s^y) \cdot v.
$$

This series of identities implies that ζ equals its complex conjugate $\bar{\zeta}$ (since $v^T \cdot diag(s^y) \cdot v$ is a weighted vector norm, which must be positive) and must therefore be real. The justification for the steps involved is as follows. The first line uses the fact that ζ is a scalar. The second line uses the fact that ζ is an eigenvalue with eigenvector v. The third lime uses the definition of \mathcal{H}_{sym} . The fourth line applies the transpose operator and uses the symmetry of \mathcal{H}_{sym} . The fifth line uses the fact that \mathcal{H}_{sym} is real. The sixth line uses once more the definition of \mathcal{H}_{sym} . The seventh line uses the fact that $\bar{\zeta}$ is also an eigenvalue of \mathcal{H}_{sym} with eigenvector \bar{v} . The last line applies the transpose operator once more. The idea behind the claim is intuitive: \mathcal{H} is an stretched version of a real symmetric matrix (which must therefore have all real eigenvalues and eigenvectors), and such stretching should not introduce complex eigenvalues.

The above derivations then show that all eigenvalues of H are real and in $(1,\infty)$ This implies that all eigenvalues of $\Theta = \frac{1}{\lambda}$ $\frac{1}{\lambda} \cdot \mathcal{H}^{-1}$ are also real and in $[0, 1/\lambda]$.

II. Monotonicity: We now turn to the monotonicity property, which says that $\theta_{gg} > \theta_{g'g}$ along a column. Suppose to obtain a contradiction that $\theta_{g'g} \ge \theta_{gg}$ and let $g' = \arg \max \theta_{g'g}$ be the index for the maximum along column g. We have that $\mathcal{H} \cdot \Theta = \frac{1}{\lambda}$ $\frac{1}{\lambda}$. This requires entry g ′g in this product to be zero or

$$
\left(1-\frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_{g'}(w)}{\partial\ln w_{g'}}\right)\cdot\theta_{g'g}=\sum_{j\neq g',g}\frac{\partial\ln\Gamma_{g'}(w)}{\partial\ln w_j}\cdot\theta_{jg}+\frac{\partial\ln\Gamma_{g'}(w)}{\partial\ln w_g}\cdot\theta_{gg}.
$$

By assumption, θ_{jg} and θ_{gg} are all less than or equal to $\theta_{g'g}$. This implies

$$
\Big(1-\frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_{g'}(w)}{\partial\ln w_{g'}}\Big)\cdot \theta_{g'g} \leq \sum_{j\neq g',g}\frac{\partial\ln\Gamma_{g'}(w)}{\partial\ln w_j}\cdot \theta_{g'g} + \frac{\partial\ln\Gamma_{g'}(w)}{\partial\ln w_g}\cdot \theta_{g'g},
$$

dividing by $\theta_{g'g}$ and rearranging, we see that this yields

$$
1 \le \sum_{j} \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_{g'}(w)}{\partial \ln w_j},
$$

which is a contradiction since the sums $\sum_j \frac{1}{\lambda}$ $\frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_{g'}(w)}{\partial \ln w_i}$ $\frac{\ln 1}{\partial \ln w_j}$ are 0 or negative (a common increase in wages causes all workers to loose tasks to capital).

III. Row sums: We now turn to the properties of the row sums of the propagation matrix, denoted by ρ_g . First, note that the elasticity of substitution between capital and group g can also be written in symmetrical form as

$$
\sigma_{kg} = \sigma_{gk} = \lambda - \frac{1}{s_k^y} \cdot \sum_{g'} \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}},
$$

since a percent increase in the user cost of capital generates the same substitution patterns as a commensurate percent reduction in all wages. This identity can be written in matrix form as

$$
-\frac{1}{\lambda} \frac{\partial \ln \Gamma(w)}{\partial \ln w} \cdot \text{stack}(1) = \text{stack}(s_k^y \cdot (\frac{\sigma_{kg}}{\lambda} - 1),
$$

or equivalently

$$
\mathcal{H}\cdot\mathrm{stack}(1)=\mathrm{stack}(1+s_k^y\cdot\big(\frac{\sigma_{kg}}{\lambda}-1\big)\big).
$$

Multiplying by Θ on the left of both sides yields

$$
\frac{1}{\lambda} \cdot \mathrm{stack}(1) = \Theta \cdot \mathrm{stack}(1+s_k^y \cdot (\frac{\sigma_{kg}}{\lambda}-1)).
$$

Comparing row g on both sides, we get

$$
\rho_g + s_k^y \cdot \sum_{g'} \theta_{gg'} \cdot \left(\frac{\sigma_{kg'}}{\lambda} - 1\right) = \frac{1}{\lambda},
$$

or equivalently

$$
\rho_g = \frac{1}{\lambda} \cdot \left[1 + s_k^y \cdot \left(\frac{\bar{\sigma}_{kg}}{\lambda} - 1 \right) \right]^{-1},
$$

which gives the formula in the main text. Note that this formula implies that $\rho_g \in (0, 1/\lambda],$ as also claimed in the main text.

IV. Relationship to elasticities of substitution: we now derive the expression that relates the propagation matrix to the matrix of elasticities of substitution Σ . First,

$$
\sigma_{gg} = \frac{1}{s_g^y} \cdot \frac{d\ln \ell_g}{d\ln w_g} \Big|_{y \text{ constant}} = \lambda - \frac{\lambda}{s_g^y} + \frac{1}{s_g^y} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_g},
$$

$$
\sigma_{gg'} = \frac{1}{s_{g'}^y} \cdot \frac{d\ln \ell_g}{d\ln w_{g'}} \Big|_{y \text{ constant}} = \lambda + \frac{1}{s_{g'}^y} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}},
$$

We can then write

$$
\Sigma = \lambda - \lambda \cdot \text{diag}\left(\frac{1}{s^y}\right) + \frac{\partial \ln \Gamma}{\partial \ln w} \cdot \text{diag}\left(\frac{1}{s^y}\right).
$$

Rearranging this yields

$$
\mathcal{H} \cdot \lambda \cdot \text{diag}\left(\frac{1}{s^y}\right) = \lambda - \Sigma.
$$

Pre-multiplying by Θ on both sides yields

$$
\operatorname{diag}\left(\frac{1}{s^y}\right)=\Theta\cdot\big(\lambda-\Sigma\big),
$$

and solving for Θ yields the relationship outlined in the text

$$
\Theta = \text{diag}\left(\frac{1}{s^y}\right) \cdot (\lambda - \Sigma)^{-1}.
$$

V Symmetry: The above identity also guarantees that diag $(s^y) \cdot \Theta = (\lambda - \Sigma)^{-1}$ is symmetric, which implies $\theta_{gg'}/s_q^y$ $y_g^y = \theta_{g'g}/s_g^y$.

G Additional Empirical Results

G.1 Robustness Checks

The tables in this part of the Appendix report a series of robustness checks on our reducedform analysis.

• Table [A1](#page-121-0) reports the same specifications shown in Table [1](#page-67-0) for wages in the main text, but measures new task creation as

$$
d\ln\Gamma_g^{\text{new}} = \sum_{o} \omega_{go}^{1980}
$$

$$
\cdot \text{Share new job titles DOT 1971}
$$

$$
+ \sum_{o} \omega_{go}^{1980}
$$

$$
\cdot \text{Share new job titles DOT 1991}
$$

$$
+ \sum_{o} \omega_{go}^{1980}
$$

$$
\cdot \text{Share new job titles Census 2000.}
$$

This measure apportions new tasks across groups based on 1980 employment shares.

- Table [A2](#page-122-0) reports the same specifications shown in Table [2](#page-68-0) for hours worked per person in the main text, but apportions new tasks across groups based on 1980 employment shares.
- Table [A3](#page-123-0) decomposes the effects of automation and new tasks into an extensive and intensive margin of employment.
- Table [A4](#page-124-0) reports estimates for wages and hours worked separately for workers with no college degree and those with a college degree.

G.2 Estimating the Propagation Matrix

Once we impose our parameterization of the Jacobian, we can rewrite the estimating equation in (44) as

$$
\sigma \Delta \ln w_g + d \ln \Gamma_g^{\text{auto}} - d \ln \Gamma_g^{\text{new}}
$$

= $\tilde{\beta} X_g + \gamma \cdot \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot (\Delta \ln w_{g'} - \Delta \ln w_g)$
+ $\gamma_{\text{job}} \cdot \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot \text{job similarity}_{gg'} \cdot (\Delta \ln w_{g'} - \Delta \ln w_g)$
+ $\gamma_{\text{edu-age}} \cdot \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot \text{edu-age similarity}_{gg'} \cdot (\Delta \ln w_{g'} - \Delta \ln w_g) + \tilde{\nu},$

where $\tilde{\beta}$ and $\tilde{\nu}$ are linear transformations of β and ν respectively.

This equation can be estimated via GMM/2SLS after imposing $\sigma = \lambda + \varphi = 0.6$ (as discussed in the text). Our estimation imposes the restriction that γ , γ_{job} , $\gamma_{edu\text{-age}} \geq 0$.

The ripple terms on the right hand side are instrumented using

$$
Z_g = \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot (\Delta \ln \hat{w}_{g'} - \Delta \ln \hat{w}_g)
$$

\n
$$
Z_{job,g} = \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot \text{job similarity}_{gg'} \cdot (\Delta \ln \hat{w}_{g'} - \Delta \ln \hat{w}_g)
$$

\n
$$
Z_{edu\text{-age},g} = \sum_{g'} \sum_n \omega_{gn} \cdot s_{g'}^n \cdot \text{edu\text{-age similarity}}_{gg'} \cdot (\Delta \ln \hat{w}_{g'} - \Delta \ln \hat{w}_g),
$$

respectively. Here $\Delta \ln \hat{w}_g$ is the predicted wage change based on groups experienced task displacement from automation, exposure to new tasks, and the exogenous covariates in the model. We get very similar results if we instead use $\Delta \ln \hat{w}_g = d \ln \Gamma_g^{\text{new}} - d \ln \Gamma_g^{\text{auto}}$ to form these instruments.

Table A1: Reduced-form evidence: changes in real hourly wages regressed on automation and new task creation, 1980-2016. Robustness check using alternative measure of new task creation.

Notes: This table presents estimates of the relationship between automation, new task creation, and the change in hourly wages across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The specifications are the same as in Table [1.](#page-67-0) The difference is that we use a measure of new task creation that holds occupational shares fixed in 1980. The dependent variable is the change in log hourly wages for each group between 1980 and 2016. Panel A reports results using only our task displacement measure. Panel B only uses our task reinstatement measure. Panel C includes both task displacement and task reinstatement on the right-hand side. Panel D combines task displacement and reinstatement into a single next task change measure. The bottom rows list additional covariates included in each specification. As in [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0), we instrument changes in labor supply in columns 6 and 7 using trends in total hours worked by group from 1970 to 1980. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

Table A2: Reduced-form evidence: changes in hours worked per adult regressed on automation and new task creation, 1980-2016. Robustness check using alternative measure of new task creation.

Notes: This table presents estimates of the relationship between automation, new task creation, and the change in hours worked per adult across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The specifications are the same as in Table [2.](#page-68-0) The difference is that we use a measure of new task creation that holds occupational shares fixed in 1980. The dependent variable is the change in log hours per adult for each group between 1980 and 2016. Panel A reports results using only our task displacement measure. Panel B only uses our task reinstatement measure. Panel C includes both task displacement and task reinstatement on the right-hand side. Panel D combines task displacement and reinstatement into a single next task change measure. The bottom rows list additional covariates included in each specification. As in [Acemoglu and Restrepo](#page-91-0) [\(2022\)](#page-91-0), we instrument changes in labor supply in columns 6 and 7 using trends in total hours worked by group from 1970 to 1980. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

Table A3: Reduced-form evidence: changes in hours intensive and extensive margin regressed on automation and new task creation, 1980-2016.

Notes: This table presents estimates of the relationship between automation, new task creation, and the change in hours worked per adult across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in (log) hours per worker (columns 1 and 2) and the change in (log) employment to population for each group between 1980 and 2016. Panel A reports results using only our task displacement measure. Panel B only uses our task reinstatement measure. Panel C includes both task displacement and task reinstatement on the right-hand side. Panel D combines task displacement and reinstatement into a single next task change measure. The bottom rows list additional covariates included in each specification. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

Table A4: Reduced-form evidence: changes in real hourly wages and hours worked regressed on automation and new task creation, 1980-2016. Robustness check reporting estimates for groups with and without a college DEGREE.

Notes: This table presents estimates of the relationship between automation, new task creation, and the change in hourly wages and hours worked per adult across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in (log) hourly wages (columns 1–3) and the change in (log) hours worked (columns 4–6) from 1980 and 2016. Panel A provides estimates for groups of workers with no college degree. Panel B provides estimates for groups of workers with a college degree. The bottom rows list additional covariates included in each specification. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.