American Economic Review 2024, 114(4): 1070–1106
https://doi.org/10.1257/aer.20211743

Redesigning the US Army's Branching Process: A Case Study in Minimalist Market Design[†]

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We present a proof-of-concept for minimalist market design (Sönmez 2023) as an effective methodology to enhance an institution based on stakeholders' desiderata with minimal interference. Four objectives—respecting merit, increasing retention, aligning talent, and enhancing trust—guided reforms to the US Army's centralized branching process of cadets to military specialties since 2006. USMA's mechanism for the class of 2020 exacerbated challenges in implementing these objectives. Formulating the Army's desiderata as rigorous axioms, we analyze their implications. Under our minimalist approach to institution redesign, the Army's objectives uniquely identify a branching mechanism. Our design is now adopted at USMA and ROTC. (*JEL* D47, H56, J45)

Consider an economic, political, or social institution that is deployed to fulfill a number of objectives. Typically, it has many components, each serving its own purposes and interacting with each other in various ways. For example, in the context of an auction design that involves equity considerations for minority-owned businesses, a component may be used to collect private information from the participants, a second component may be used to process this information, a third component may be used to determine the pricing of various outcomes, and a fourth component may be used to ensure a fair outcome. Now suppose that the institution fails in some of its objectives. Maybe some of its components are broken, or maybe there is an issue with the interface between various components. How can a design economist be helpful in addressing these failures?

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[†] Go to https://doi.org/10.1257/aer.20211743 to visit the article page for additional materials and author disclosure statements.

Before formulating a potential answer to this question, let us imagine how experts in other areas would respond to similar challenges. How would a surgeon address an analogous failure on a human body or a mechanic on a broken car? These experts would first identify the root causes of the problem, whether they have to do with a component itself or an interface between various components and directly address these failures. For example, a surgeon would remove diseased tissues or organs, repair body systems, or replace diseased organs with transplants. Similarly, a mechanic would directly repair or replace the worn part of the broken car.

Minimalist market design (Sönmez 2023) is a paradigm under which a design economist operates in a similar way. In this paradigm, the first task is to identify the primary objectives of the system operators or other stakeholders in designing the institution. In many cases, various components of an institution, interfaces between its components, or its mission have evolved over time. The history of an institution is often instructive in identifying the primary objectives of stakeholders in designing and deploying the institution. The second task under minimalist market design is to find out whether the current institution satisfies these primary objectives or not. If it doesn't, then there is potential for policy impact with a compelling alternative design. To turn this potential into a successful redesign, the root causes of the failures are identified. That is, akin to a surgeon or a mechanic, a design economist following the minimalist institution design paradigm identifies which components or interfaces are responsible for the failure. As the third task, the failures of the current institution are addressed by interfering only with its flawed components and interfaces, much like a surgeon performing a "minimally invasive" procedure. Hence, the adjective "minimalist" is the signature feature of this design paradigm.

Drawing on our more than a decade long integrated research and policy efforts on the US Army's process for matching cadets to military branches (henceforth referred to as the "branching" process),[1] the first contribution of this paper is to present a proof-of-concept for minimalist market design. Although this paradigm evolved through our experiences from earlier research and policy efforts in school choice and kidney exchange, the US Army's branching process is the first application where the minimalist market design paradigm was deliberately and systematically followed at all stages and eventually succeeded in changing an important institution.[2]

[1] Sönmez and Switzer (2013) and Sönmez (2013) mark the inception of these research and policy efforts at United States Military Academy and Reserve Officer Training Corps, respectively. This paper reports how these efforts finally reached successful completion in both institutions consistent with the initial prescriptions in Sönmez and Switzer (2013) and Sönmez (2013).

[2] Sönmez (2023) presents how the framework developed from research and policy efforts to reform student assignment systems (Balinski and Sönmez 1999; Abdulkadiroğlu and Sönmez 2003; Abdulkadiroğlu et al. 2005) along with establishing kidney exchange systems (Roth, Sönmez, and Ünver 2004, 2005) and describes how it has recently proven useful in other settings following its successful deployment for the US Army's branching process in fall 2020. These settings include the design of pandemic rationing schemes for scarce medical resources (Pathak et al. Forthcoming) and implementation of court-ordered Indian affirmative action schemes (Sönmez and Yenmez 2022a).

A. The Making of a Partnership between the US Army and Design Economists

A military specialty in the US Army, known as a *branch*, is an important factor in the career progression of cadets. Each year, the US Army assigns thousands of graduating cadets from the United States Military Academy (USMA) at West Point and the Reserve Officer Training Corps (ROTC) to a branch through two separate centralized mechanisms with distinct branch allocations. These *branching mechanisms* at West Point and ROTC determine the branch assignments for 70 percent of newly commissioned Army officers (Department of Defense 2020). Prior to 2006, positions at each branch were assigned purely based on a performance ranking called *order of merit list* (OML). Thus, the original mission of the branching system was to allocate the positions in a way that reflects the hierarchical structure of the Army.

In 2006, the US Army created an incentive scheme within its branching systems with the goal of increasing officer retention (Colarruso, Lyle, and Wardynski 2010). Under this incentive scheme, known as the *BRADSO program*, cadets receive heightened priority for a fraction of a branch's positions (henceforth, *flexible-price* positions) if they express a willingness to extend the length of their service commitment.[3] In Army terminology, a cadet who volunteers for this incentive scheme at a given branch b is said to *BRADSO* for branch b. The USMA leadership accordingly embedded the BRADSO incentive scheme into a new branching mechanism we call *USMA-2006*.[4] The adjustment of the mechanism reflected the changing mission of the branching system, which now included retention.

Although the Army's branching process is a natural application of the celebrated *matching with contracts* model by Hatfield and Milgrom (2005), it remains outside the scope of the original theory in that paper. USMA-2006 was designed at a time when the matching with contracts model was still being developed. Consequently, the connection between the Army's practical problem and the original theory had some missing pieces. These pieces were later completed by Hatfield and Kojima (2010), albeit in an abstract framework.[5] The connection between abstract theory and the Army's practical problem was subsequently discovered in Sönmez and Switzer (2013). In addition, Sönmez and Switzer (2013) also proposed an alternative mechanism for the Army by embedding the BRADSO incentive scheme directly within the *cumulative offer mechanism* by Hatfield and Milgrom (2005).

While this proposal had desirable theoretical properties, it required a more complex strategy space in which cadets have to rank branches and *contractual terms* (also referred to as *prices*) jointly. Under the USMA-2006 mechanism, cadets only rank branches and separately indicate their willingness to BRADSO for any branch. The Army considered the existing strategy space manageable compared to a more complex alternative and kept the USMA-2006 mechanism in the intervening years.

[3] ADSO is short for Active Duty Service Obligation. BRADSO stands for Branch of Choice Active Duty Service Obligation. BRADSO slots are 25 percent of the total branch allocations at USMA from the class of 2006 through 2020 and 35 percent for the class of 2021, and either 50 percent or 60 percent of total branch allocations at ROTC depending on the graduating class. USMA and ROTC cadets receive branches through separate centralized branching systems.

[4] The BRADSO incentive was also embedded into the branching mechanism at ROTC, albeit through a different mechanism (Sönmez 2013).

[5] Further elaboration is provided by Echenique (2012); Schlegel (2015); and Jagadeesan (2019).

In 2012, the US Army introduced the Talent-Based Branching program to develop a "talent market" where additional information about each cadet influences the priority a cadet receives at a branch (Colarusso et al. 2016). This program allowed branches and cadets to better align their interests and fit for one another. Under Talent-Based Branching, branches rated cadets into one of three tiers: high, medium, and low. These cadet ratings were originally a pilot initiative, but for the class of 2020, the US Army decided to use them to adjust the underlying OML-based prioritization, constructing priorities at each branch first by the tier and then by the OML within the tier.

The desire to use the branching system to improve talent alignment created a new objective for the system, thus changing its mission yet another time. Since the decision to integrate cadet ratings into the branching mechanism took place under an abbreviated timeline, the US Army maintained the same strategy space for the mechanism as in previous years and devised the USMA-2020 mechanism to accommodate heterogeneous branch priorities. In their design, the Army created two less-than-ideal theoretical possibilities in the USMA-2020 mechanism. First, a cadet could be charged BRADSO under the USMA-2020 mechanism even if she does not need heightened priority to receive a position at that branch. While this was also possible under USMA-2006, it was nearly four times as common under USMA-2020. Second, under USMA-2020, a cadet's willingness to BRADSO for a branch could improve her priorities even for the base price positions. Surveys of cadets designed by the military coauthor of this paper showed that these aspects potentially undermined trust in the branching system and led the Army to reconsider a refinement of the cumulative offer mechanism, despite its more complex strategy space. At that stage, the Army established a partnership with the two civilian coauthors of this paper after nearly a decade since Sönmez and Switzer (2013) was first brought to their attention.

As the second main contribution of our paper, we report the design and successful deployment of a new branching system for the class of 2021, the *multi-price cumulative offer mechanism*, a refinement of the cumulative offer mechanism (Hatfield and Milgrom 2005) that uses a specific choice rule for each branch that reflects the Army's objectives of retention and talent alignment. In our main formal result, Theorem 1, we show that the Army's objectives, when formulated through five axioms, uniquely give rise to the *multi-price cumulative offer mechanism*. In our setting, the cumulative offer process and a specific choice rule emerge from foundational axioms, even though branches are not assumed to be endowed with choice rules in our model. Therefore, the foundations for two major components of the branching system, the allocation procedure and the choice rules that feed into the procedure, are established jointly in our main formal result. This is a departure from earlier literature, where the foundations for these two parts are established separately.

The axioms that characterize the multi-price cumulative offer mechanism are as follows.

Individual Rationality: No cadet should be assigned an unacceptable branch-price pair.

Nonwastefulness: No position at a branch can be left idle while there is a cadet who is unassigned, unless she would rather remain unassigned than receive the position at the cheapest possible price.

No Priority Reversal: No cadet i should prefer the branch-price package (b,t) of another cadet j to her own assignment, even though she had a higher baseline priority for branch b.

Respect for the Price Responsiveness Scheme: No cadet i should prefer a branch-price package (b,t) to her own assignment while there is another cadet j who received a position at branch b at a different price $t' \neq t$, even though

1. cadet i has higher adjusted priority for a flexible-price position at branch b at price t than cadet j has at price t', and

2. it is feasible to award the flexible-price position at branch b to cadet i at price t instead of cadet j at price t'.

Strategy-Proofness: No cadet ever benefits from misrepresenting her preferences over branch-price pairs.

Of these axioms, only *respect for the price responsiveness scheme* is novel to our model and analysis. This condition formulates a key objective for the Army and is therefore critical for the broader mission of the branching system. Moreover, while our formal analysis is motivated by the Army's branching application, it can be directly applied for any extension of a priority-based indivisible goods allocation model (such as the *school choice* model by Abdulkadiroğlu and Sönmez 2003), where priorities of individuals can be increased with a costly action (such as paying higher tuition) at a fraction of the units of allocated goods.[6] In particular, our model has an additional direct application for the seat-purchasing policies at Chinese high schools, presented in Section F.1 of the online Appendix. Another promising application is the allocation of school seats with or without financial aid (e.g., Sönmez and Switzer 2013; Artemov, Che, and He 2020; Hassidim, Romm, and Shorrer 2021; Shorrer and Sóvágó forthcoming).

B. Organization of the Rest of the Paper

The rest of this paper is organized as follows. Section I describes the main elements of minimalist market design and relates the evolution of the US Army's branching system to this design paradigm. Section II introduces the formal model. This section also includes Section IID, which formalizes the Army's desiderata as rigorous axioms. Section III presents the USMA-2020 mechanism and its shortcomings, which convinced the Army to reconsider a version of the cumulative offer mechanism previously advocated by Sönmez and Switzer (2013). Section IV presents the multi-price cumulative offer mechanism and our main theoretical result characterizing it as the unique mechanism that satisfies the Army's desiderata. This section also includes Section IVC, which relates our main result to earlier

[6] It is already well-established that any such extension can be modeled as a special case of the matching with contracts model. However, the novel insight we offer is that the underlying choice rules for institutions can also be endogenously obtained in such extensions as a direct implication of natural axioms for these extensions. These choice rules have been exogenously given in earlier literature. This distinction is the sense in which our analysis unifies the design of two major components of a priority-based allocation system, i.e., the design of its allocation procedure for given choice rules and the design of the choice rules that feed into the allocation procedure.

literature. Section V describes how the Army used the new mechanism to refine the trade-off between retention and talent alignment by changing the main parameters of this mechanism. Section VI provides some general lessons for broader efforts in institution design and the role of theory in policy. All proofs, independence of the axioms in our main characterization result, an in-depth analysis of the USMA-2020 mechanism, additional data analysis, results from cadet surveys, and other potential applications are presented in the online Appendix.[7]

I. Minimalist Market Design

A. *Overview*

This section summarizes some essential elements of minimalist market design formulated in Sönmez (2023). This approach to institution design integrates research and policy efforts to influence the design of real-life allocation systems. We describe the framework to explain some of our modeling choices for the US Army branch assignment process and to elaborate on our strategy for convincing stakeholders to adopt a new system. We first review the general framework and then describe how it relates to the Army's branching system. While minimalist market design has evolved through other experiences in other resource allocation problems, our integrated research and policy efforts to reform the US Army's branching process is the first time it is directly tested and deployed systematically. That is, from its inception in Sönmez and Switzer (2013), the effort to influence the design of the US Army's branching process followed the framework and eventually succeeded.

The minimalist approach to institution design or reform can be particularly useful when there is no consensus for reform and the design economist is initially an outsider. When a consensus for reform exists and the design economist is commissioned to guide a reform, the cautious approach underlying the minimalist framework may not be necessary. In this case, stakeholders often commission the market designer to devise a system and delegate the critical design decisions to her expertise. To propose a design, the market designer can deploy the usual economic design tools, including game-theoretic models, constrained optimization approaches, simulations, lab experiments, or computational heuristics.

When stakeholders need convincing, however, a redesign is aspirational, and the minimalist approach helps. When a redesign effort starts with a criticism of an existing institution, stakeholders are often defensive, so gaining trust is imperative. Instead of challenging the main features of the existing institution, the premise of the minimalist approach is that the existing institution reflects the objectives and constraints of stakeholders. With multiple stakeholders, a range of views about objectives may exist. Some objectives may not even be consequentialist, and a single objective function in which everyone agrees may not exist, especially when the problem has incommensurable dimensions. Indeed, the involvement of the market designer itself may interfere with delicate balances across constituents, including those purposely left implicit.

[7] The code required to replicate all empirical results is published in Greenberg, Pathak, and Sönmez (2024). The data used in this project must be stored, accessed, and analyzed within USMA's information system.

The first step of minimalist market design is to identify the objectives that are key to stakeholders in designing their current institution. An in-depth understanding of the historical evolution of a system aids in determining these considerations. It is essential to recognize the multifaceted nature of these principles. Situations with delicate social and distributional elements may necessitate respecting a range of viewpoints (Hitzig 2020). Incomplete or superficial knowledge about the origin or rationale for certain institutional practices risks undermining the credibility of reform efforts. During the initial design of an institution, formal tools may not have been available to policymakers and system operators. Such designs may be improved with a rigorous formulation of the underlying objectives and constraints of the institution. In our experience, even when stakeholders are able to verbalize their objectives, they still need help with operationalizing them with a procedure. If the policy proposals of a design economist are in line with the stakeholder objectives, stakeholders may be open to reform. It is important to emphasize that a standard optimization framework may not be possible when principles are complex or conflicting. Therefore, mainstream approaches from traditional mechanism design may not receive a favorable reception.

The next step is to examine whether the existing institution satisfies stakeholders' fundamental objectives or if there is a discord between the intention and the practice. Here, the aim is to provide the stakeholders with a critique of the existing institution on the terms laid out by themselves rather than with a critique that is based on primary considerations in mainstream economics (such as preference utilitarianism).[8] The inconsistency between the aims of the institution and its practice in the field can take many forms and be more than just incentive and participation issues central to mechanism design. Identification of such an inconsistency creates an opening for the market designer. If some stakeholder objectives are not satisfied by the existing institution, then a strong case can be made for a reform, provided that the market designer advocates for an alternative institution that satisfies all objectives.

When a disconnect is identified between the intention and the practice of an institution, stakeholders are more likely to be receptive to a reform if it involves minimal interference with the existing system. To design such an alternative, it is imperative to find the root causes of the failures. Once the culprits are identified in the discord between the objectives and the practice of an institution, the aim is to correct those issues only and otherwise leave the rest of the system untouched. That is, any interference with the existing institution should target the root causes of the issues and aspire to a surgical fix. This allows stakeholders to view and position the reform as a relatively small tweak. It also reduces the risk of changing aspects of the institution in which the market designer might be unaware and upsetting implicit compromises between stakeholders. The aim is for stakeholders to see the proposal as representing what they wanted to do in the first place but did not have the technical know-how to formulate.

[8] Our approach is consistent with Chassang and Ortner (2023, p. 177), who make a similar point in the context of regulating collusion: "In addition, we try to do justice to the peculiarities of the legal system: modeling the courts as they are, rather than as economists think they should be, is essential for economic analysis to improve the way collusion is regulated."

The model's realism is important if criticizing an existing system because stakeholders are often wedded to the status quo. Models with abstractions, even those intentionally made to isolate specific intuitions, can be easily dismissed as being unrealistic. Furthermore, if particular design choices have normative implications, they should be made transparent. Li (2017) calls this maintaining *informed neutrality* between reasonable normative principles. For establishing trust and mitigating concerns about ulterior motives or hidden agendas, it is in the best interest of a market designer to elucidate implications for an aspired reform than take positions on trade-offs.[9] Beyond the pragmatic considerations, it is also a good practice to be completely transparent about various normative implications of any design. A minimalist market designer should aim to provide stakeholders with tools to examine the implications of particular design choices and help to facilitate an open and informed debate about their system.

Since the starting point of the aspired reform is to find a way to accommodate stakeholders' key principles, the axiomatic approach is a natural methodology (Moulin 1988, 2004; Thomson 2001, 2011). In the axiomatic methodology, the researcher formalizes principles as mathematical properties and examines their implications. In some cases, only a unique system or a family of systems satisfies all requirements, a result known as an axiomatic characterization. If such a characterization exists, it provides a natural candidate for practical implementation. To the extent it is technically possible, finding all systems that meet the objectives is the best practice because it describes the landscape of possibilities, including the identification of systems that may have unintended effects.

Since the minimalist framework starts with the existing institution, it is best suited for pursuing incremental changes within the system. Once a market designer has shown apparent deficits with the existing system, it may be possible to consider more substantial changes. At this stage, questions regarding the implications of taking some aspects of the problem as given or whether they can be modified are most fruitful. In an ideal scenario, the market designer partners with stakeholders and can jointly design the institution. Stakeholders can rely on the market designer for technical expertise and delegate any formal analysis of specific design changes based on their expertise. Through this iterative process, it may be possible to move from local changes to more substantial changes.[10]

B. *Minimalist Market Design to Reform the US Army's Branching Process*

This section provides background on cadet branch assignment in the US Army and frames it as a case study in minimalist market design.

For decades, the Army offered cadets choice over their branch assignment and used a cadet's performance ranking, known as the *order of merit list* (OML), as a factor in determining assignments.[11] Through the late 1970s, cadet assignment was an in-person process, where cadets convened in an auditorium. Cadet names

[9] Pro bono assistance also helps to build trust.

[10] This process of continuous improvement has been emphasized in other policy contexts, including the Duflo (2017) metaphor of plumbing in development economics.

[11] The OML was first formalized in 1818 when the Army's Secretary of War approved USMA's criteria. Army documents from that period describe the importance of respecting priority, stating that "the distribution of cadets,

were called in OML order, and each cadet selected their most preferred branch with available capacity.[12] Starting in the 1980s, cadets submitted preferences over the set of branches, and a branching board convened to match cadets to branches (United States Military Academy 1982). In the mechanism, the highest-OML cadet was assigned her most preferred branch, the second-highest-OML cadet was assigned her most preferred branch among branches with remaining positions, and so on. This mechanism, the *simple serial dictatorship induced by OML* (SSD-OML), established several foundational components of the assignment system and formed the basis for further reforms.

A new objective of encouraging retention arose due to declining junior officer retention rates during the late 1990s and early 2000s. The Army offered a menu of retention incentives to cadets at USMA and ROTC through the Officer Career Satisfaction Program, first implemented in 2006 (Colarruso, Lyle, and Wardynski 2010). The most popular incentive, which involved a reform of the branching mechanism, was the *BRADSO* program. The BRADSO program gives higher priority for a fraction of positions in each branch to cadets willing to extend their Active Duty Service Obligation (ADSO) by three years if assigned to that branch. We call these *flexible-price positions* and say a cadet who ranks such a position is willing to pay the increased price. By creating these new types of positions, the BRADSO program altered the role of the OML for these slots. To infer which cadets were willing to pay the increased price, USMA required cadets to report the set of branches in which they were willing to serve the additional years through a new message space under a new mechanism, the USMA-2006 mechanism.[13]

USMA-2006.—Sönmez and Switzer (2013) formulate and analyze the USMA-2006 mechanism. The USMA-2006 mechanism extends the SSD-OML to accommodate the treatment of the flexible-price positions. When a cadet expresses willingness to pay the increased price for the flexible-price positions at any branch, that cadet is given priority over any other cadet unwilling to pay the increased price at these positions. When two cadets are willing to pay the increased price, they are ordered according to their OML. If a cadet is assigned a base-price position, she is charged the base price. If a cadet is assigned one of the flexible-price positions, she is charged the increased price if she is willing to pay the increased price at the branch. Otherwise, she is charged the base price. Section C.1 in the online Appendix provides a formal definition of USMA-2006.

Whether the Army should override the OML to increase retention was subject to intense debate. Colarruso, Lyle, and Wardynski (2010, p. 63) write:

> Devoted supporters of the ROTC and West Point Order of Merit (OML) system for allocating branches and posts objected that low OML cadets could "buy" their branch or post of choice ahead of higher OML cadets. Since branch and post assignments represent a zero-sum game, the ability of

into the branches of the army, be made in accordance with their qualifications, talents, and without violating the principle of order of merit" (Topping 1989, p. 8).

[12] Atkinson (2009) provides a vivid account of the process for the West Point class of 1966.

[13] While the BRADSO program is adopted by both USMA and ROTC, prior to 2020, it had been implemented by two distinct mechanisms at these institutions.

cadets with lower OML ranking to displace those above them was viewed by some as unfair or as undermining the OML system.

This discussion illustrates that stakeholders had different views on the appropriate balance between retention incentives and merit, an issue subject to ongoing debate.

In USMA-2006, cadets only submit their preferences over branches alone and "signal" their willingness to pay the increased price at any branch rather than over branch-price pairs. A direct mechanism, in contrast, would solicit cadet preferences over branch-price pairs. Sönmez and Switzer (2013) describe two main failures of USMA-2006. First, cadet i can prefer cadet j's assignment to her own even though cadet i has a higher OML score than cadet j. We refer to this situation as a *priority reversal*. Computation of *all* priority reversals depends on knowledge of cadet preferences over all branch-price pairs. Detection of *some* priority reversals only requires information on cadet preference collected under the USMA-2006 mechanism. We denote these as *detectable priority reversals.* Second, under USMA-2006, a cadet who is assigned an increased-price position at branch b can potentially receive that position at a base price by declaring that she is unwilling to pay the increased price at the branch. We refer to this as a *failure of BRADSO-incentive compatibility*. After introducing the model in Section II, we define these concepts formally.

The issue with the message space and a disconnect between branch and price assignments are the two root causes of the problem with the USMA-2006 mechanism. When a cadet volunteers for BRADSO at her top choice, the mechanism cannot tell whether she prefers her first choice branch at an increased price over her second choice branch at its cheaper base price. Sönmez and Switzer (2013) proposed fixing the first issue by simply changing the message space of the mechanism. The disconnect between branch and price assignments were then addressed via the *cumulative offer process* (Hatfield and Milgrom 2005).

Despite the shortcomings of the USMA-2006, for many years the Army did not embrace the Sönmez and Switzer (2013) proposal. The Army did not change its mechanism for three main reasons:

1. The Army could manually correct a failure of BRADSO-incentive compatibility or a detectable priority reversal ex post. Both issues involve a cadet needlessly paying the increased price at her assigned branch. The Army could resolve either issue by manually reducing the charged price to the base price.

2. Even though the USMA-2006 mechanism allows for additional priority reversals, which cannot be manually corrected ex post, verifying any such theoretical failure relies on cadet preferences over branch-price pairs. Since USMA-2006 is not a direct mechanism, information on cadet preferences over branch-price pairs was unavailable.

3. Failures of BRADSO-incentive compatibility and detectable priority reversals had been relatively rare in practice.

The Army initially thought that the issues identified by Sönmez and Switzer (2013) are not significant enough to justify adopting a mechanism with a more complex message space. Any possible failure could either be manually corrected ex post or could not be verified with data solicited under the message space for the USMA-2006 mechanism. Therefore, the Army concluded that the failures identified by Sönmez and Switzer (2013) were not visible or significant enough to justify a change. The introduction of a new program aimed at improved talent alignment altered these trade-offs and triggered an adjustment in the mechanism, which we describe next.

Talent-Based Branching and the USMA-2020 Mechanism.—In 2012, the US Army introduced Talent-Based Branching (TBB) to develop a "talent market" where additional information about each cadet influences the priority a cadet receives at a branch (Colarusso et al. 2016). Instead of relying only on the OML, TBB allowed branches and cadets to align their interests and fit better with one another. Under TBB, branches prioritize cadets into three tiers: high, medium, and low. Before the class of 2020, these rating categories did not influence baseline branch priorities at USMA. The Army used these ratings as part of talent assessments to help cadets learn which branches would be a good fit for them. The Army also made rare, ex post adjustments to a cadet's branch assignment based on ratings. After several years and much debate, the Army decided to use TBB ratings to adjust the underlying OML-based prioritization for the class of 2020. The slow pace of reform was due partly to ongoing debates between a faction in favor of granting branches more power to directly influence branch assignments and another faction concerned about diluting the power of the OML (Garcia 2020).

Just as the introduction of the BRADSO program triggered a reform in the branching mechanism, the full integration of the TBB program with the branching process resulted in another adjustment. The Army operated under an abbreviated timeline, and their perspective focused on coming up with an algorithm rather than issues brought about by the new structure of claims for branches created by the TBB program. The *US Army News* suggests that the National Residency Matching Program inspired the design of the USMA-2020 mechanism.[14] The Army replaced the USMA-2006 mechanism with another quasi-direct mechanism based on the individual-proposing deferred acceptance algorithm,[15] where branches have heterogeneous baseline priorities over cadets according to a tiered price responsiveness scheme described in Section IIB. This procedure separated the assignment of branches from their pricing. After branch assignments were determined, a cadet's willingness to pay the increased price determined price charges. The Army charged the increased price to willing cadets in reverse priority order, stopping when 25 percent of cadets assigned to the branch were charged the increased price. For example, if 100 cadets are assigned to a branch and 50 of the cadets volunteer for the increased price, the Army would charge the increased price to the 25 lowest-priority cadets of the 50 willing to pay the increased price. We formally define this mechanism in Section III.

[14] O'Connor (2019) states, "The cadets' branch rankings and the branches' cadet preferences will then determine a cadets' branch using a modified version of the National Resident Matching Program's algorithm, which won a Nobel Prize for Economics in 2012 and pairs medical school graduates with residency programs."

[15] Section C.2 in the online Appendix defines the individual-proposing deferred acceptance algorithm.

The Army created two less-than-ideal theoretical possibilities in the USMA-2020 mechanism in their design. First, a cadet can be charged BRADSO under the USMA-2020 mechanism even if she does not need heightened priority to receive a position at that branch. While this was also possible under USMA-2006, it was nearly four times as frequent under USMA-2020. Second, under USMA-2020, a cadet's willingness to BRADSO for a branch can improve priorities even for base-price positions. Surveys of cadets showed that these aspects potentially undermined trust in the branching system and led the Army to reconsider the cumulative offer mechanism, despite its more complex message space.

We next introduce our model before formally describing USMA-2020 and elaborating on its failures.

II. Model

A set of individuals I seek placement at one of a set of institutions B. Since our primary application is US Army branching, we refer to an individual as a **cadet** and an institution as a **branch**.[16] At any given branch $b \in B$, there are q_b identical positions. Each cadet wants at most one position and can be assigned a branch under multiple contractual terms. Let $T = \{t^0, t^1, \ldots, t^h\}$ denote a finite set of contractual terms or "prices," where

1. $t \in \mathbb{R}^+$ for each $t \in T$, and

2. $t^0 < t^1 < \ldots < t^h$.

Here, t^0 denotes the **base price**, and it represents the default arrangement. In our Army application, it corresponds to t^0 years of mandatory service upon completion of the USMA Military program or the ROTC program. Let $T^+ = T \setminus \{t^0\}$ denote the **set of increased prices**.[17] For the Army application, a single increased price corresponds to $t^h = t^1$ years of mandatory service through the BRADSO program.[18]

For any branch $b \in B$, at most $q_b^f \in [0, q_b]$ positions can be assigned to cadets at an increased price in T^+. We refer to these positions as **flexible-price** positions. Throughout Sections II–IV, the number of flexible-price positions q_b^f at any branch $b \in B$ is assumed to be fixed at values as a result of compromises between stakeholders. As such, the parameter q_b^f is not a design variable for any $b \in B$. For any branch $b \in B$, let $q_b^0 = (q_b - q_b^f)$ denote the number of **base-price** positions that can be assigned only at the default price of t^0.

[16] Section F in the online Appendix presents other direct applications of our model outside of the US Army context.

[17] We assume that the set of contractual terms T is a finite subset of real numbers due to the price interpretation of the contractual terms in our main application. Our entire analysis directly extends to any finite and strictly ordered set of contractual terms T.

[18] In our US Army application, the base price corresponds to three to five years of mandatory service, and the increased price corresponds to three additional years of mandatory service. USMA graduates incur a five-year service obligation upon graduation. ROTC graduates incur a three- or four-year service obligation upon graduation. Incurring the increased price through the BRADSO program extends the initial service obligation for USMA and ROTC cadets by three years (US Army 2017).

A. *Cadet Preferences and Baseline Branch Priorities*

Each cadet has a strict preference relation on branch-price pairs and remaining unmatched, represented by a linear order on $(B \times T) \cup \{\emptyset\}$. We assume that, for any cadet $i \in I$ and branch $b \in B$, cadet i strictly prefers a cheaper position at branch b to a more expensive position at branch b. Let $\mathcal{Q}$ denote the set of linear orders on $(B \times T) \cup \{\emptyset\}$ identified by this assumption. Therefore, for any cadet $i \in I$, preference relation $\succ_i \in \mathcal{Q}$, branch $b \in B$, and pair of prices $t, t' \in T$,

$$t < t' \Rightarrow (b,t) \succ_i (b,t').$$

For any strict preference relation $\succ_i \in \mathcal{Q}$, let $\succeq_i$ denote the induced weak preference relation.

Let Π denote the set of all linear orders on the set of cadets I. Each branch $b \in B$ has a strict priority order $\pi_b \in \Pi$ on the set of cadets I. We refer to π_b as the **baseline priority order** at branch b. The baseline priority order represents the "baseline claims" of cadets for positions at the branch.

B. *Price Responsiveness Schemes*

Given any branch $b \in B$, the overall claims of cadets for positions at the branch depend on both the baseline priority order π_b and how much cadets are willing to pay for a position at the branch. The Army policy fully specifies the scenarios under which the baseline priority order at a branch can be overturned due to cadets who are willing to incur higher prices. This trade-off is captured by a *price responsiveness scheme*, which specifies the priority advantage any given cadet gains against other cadets if she is willing to bear a higher price.[19]

Formally, for a given branch $b \in B$ and a baseline priority order $\pi_b \in \Pi$, a **price responsiveness scheme** is a linear order ω_b on $I \times T$ with the following two properties:

1. For any pair of cadets $i,j \in I$ and price $t \in T$,

$$(i,t) \; \omega_b \; (j,t) \Leftrightarrow i \; \pi_b \; j, \quad \text{and}$$

2. For any cadet $i \in I$ and price pair $t, t' \in T$,

$$t < t' \Rightarrow (i,t') \; \omega_b \; (i,t).$$

Under a price responsiveness scheme ω_b, (i) the relative priority order of cadets who are willing to pay the same price is the same as in their baseline priority order π_b, and (ii) any given cadet has a higher claim with a higher price compared to her claims at a lower price. The price responsiveness scheme is invoked at a branch only for its flexible-price positions. As in the case of the number of flexible-price positions, throughout Sections II–IV, the price responsiveness scheme ω_b at any branch $b \in B$

[19] A price responsiveness scheme in our model is similar to the marginal rates of substitution from price theory.

is assumed to be fixed at values resulting from compromises between stakeholders. As such, the function ω_b is not a design variable for any $b \in B$.

Let Ω_b be the set of all linear orders on $I \times T$ that satisfy these two conditions. The set Ω_b denotes the set of all price responsiveness schemes at branch b.

The advantage a price responsiveness scheme gives to cadets in securing a position at branch b due to their willingness to pay higher prices differs between distinct price responsiveness schemes. Given two distinct price responsiveness schemes $\omega_b, \nu_b \in \Omega_b$, scheme ν_b **is more responsive to a price increase** than scheme ω_b, if

$$\text{for any } i,j \in I \text{ and } t,t' \in T, \text{ with } t' > t, \ (i,t')\ \omega_b\,(j,t) \ \Rightarrow \ (i,t')\nu_b(j,t).$$

We next present three price responsiveness schemes used in practice.

Ultimate Price Responsiveness Scheme.—Given a branch $b \in B$ and a baseline priority order $\pi_b \in \Pi$, define the **ultimate price responsiveness scheme** $\bar{\omega}_b \in \Omega_b$ as one where willingness to pay any higher price overrides any differences in cadet ranking under the baseline priority order π_b at branch b. That is, for any pair of cadets $i,j \in I$ and pair of prices $t,t' \in T$,

$$t' > t \Rightarrow (i,t')\ \bar{\omega}_b\,(j,t).$$

As we have indicated earlier, the Army application has only one increased price. For the classes of 2006–2019, the USMA used the ultimate price responsiveness scheme. During these years, the USMA capped the positions that could be assigned at the increased price at 25 percent of total positions within each branch. At each branch $b \in B$, any cadet who is willing to pay the increased price for branch b had higher priority for the q_b^f flexible-price positions than any cadet unwilling to pay the increased price for branch b.

Tiered Price Responsiveness Schemes.—Fix a branch $b \in B$ and a baseline priority order $\pi_b \in \Pi$. To define our second price responsiveness scheme, partition cadets into n tiers $I_b^1, I_b^2, \ldots, I_b^n$ so that for any two tiers $\ell, m \in \{1,\ldots,n\}$ and pair of cadets $i,j \in I$,

$$\left.\begin{array}{r} \ell < m, \\ i \in I_b^\ell, \text{ and} \\ j \in I_b^m \end{array}\right\} \Rightarrow i\ \pi_b\ j.$$

In this partition, any cadet in tier I_b^ℓ has higher baseline priority at branch b than a cadet in tier I_b^m for $\ell < m$.

Under a **tiered price responsiveness scheme** $\omega_b^T \in \Omega$, for any tier $\ell \in \{1,\ldots,n\}$, triple of cadets $i,j,k \in I$, and pair of prices $t,t' \in T$ with $t' > t$,

$$\left.\begin{array}{r} i\ \pi_b\ k, \\ j\ \pi_b\ k, \text{ and} \\ i,j \in I_b^\ell \end{array}\right\} \Rightarrow \big((k,t')\omega_b^T(i,t) \Leftrightarrow (k,t')\omega_b^T(j,t)\big).$$

That is, given two cadets $i,j \in I$ in the same tier and a third cadet $k \in I$ with lower π_b-priority than both i and j, cadet k can gain priority over cadet i through willingness to pay a higher price if and only if cadet k can gain priority over cadet j through willingness to pay a higher price.

For the classes of 2020 and 2021, the USMA used two different tiered price responsiveness schemes. In both years, cadets were prioritized by each branch into one of three tiers, which we denote high, middle, and low.[20] Under the 2020 scheme, when a cadet expressed a willingness to pay the increased price, she had higher priority among cadets in the same tier. For example, a middle-tier cadet who was willing to pay the increased price would not obtain higher priority than a high-tier cadet who was unwilling to pay the increased price. Therefore, under the 2020 scheme, the willingness to pay the higher price overrides any difference in cadet ranking under π_b only among cadets in the same tier. The price responsiveness scheme for the class of 2021 granted cadets more advantage in securing a position. Under the 2021 scheme, when a cadet expressed a willingness to pay the increased price, she had higher priority over all other cadets if she was in the medium- or high-tier categories. Low-tier cadets who expressed a willingness to pay, in contrast, only received higher priority among other low-tier cadets. The ultimate price responsiveness scheme is more responsive to a price increase than the 2021 scheme, which is in turn more responsive to a price increase than the 2020 scheme.

Scoring-Based Price Responsiveness Schemes.—Our third price responsiveness scheme is defined for settings where the baseline priority ranking at any branch is based on an underlying score (such as from a standardized test). Under the **scoring-based price responsiveness scheme**, each level of increased price increases the total score by a given amount.

Given a branch $b \in B$ and individual $i \in I$, let $m_i^b \in \mathbb{R}^+$ denote the merit score of individual i at branch b.[21] The baseline priority order $\pi_b \in \Pi$ is such that, for any pair of individuals $i,j \in I$,

$$i \; \pi_b \; j \; \Leftrightarrow \; m_i^b > m_j^b.$$

Given a branch $b \in B$, let $S^b : T \rightarrow \mathbb{R}^+$ be a **scoring rule** such that

$$0 = S^b(t^0) < S^b(t^1) < \ldots < S^b(t^{h-1}) < S^b(t^h).$$

Under a **scoring-based price responsiveness scheme** $\omega_b^S \in \Omega$, for any two individual-price pairs $(i,t),(j,t') \in I \times T$,

$$(i,t)\omega_b^S(j,t') \; \Leftrightarrow \; m_i^b + S^b(t) > m_j^b + S^b(t').^{22}$$

[20] Branch rating categories are known to cadets and finalized before cadets submit their preferences for branches.
[21] Suppose that any ties between two distinct individuals are broken with a tie-breaking rule so that no two distinct individuals have the same merit score at any given branch.
[22] Suppose that any ties are broken with a given tie-breaking rule.

Drawing upon an analysis in Zhou and Wang (2021), in Section F.1 of the online Appendix, we present a real-world application of the scoring-based price responsiveness scheme for public high school admissions in China. Under this scheme, student merit scores receive a boost for a fraction of seats if they are willing to pay higher tuition.

C. *Formulation through the Matching with Contracts Model*

To introduce the outcome of an economy and some of the mechanisms analyzed in the paper, we use the following formulation through the *matching with contracts* model by Hatfield and Milgrom (2005).

For any $i \in I$, $b \in B$, and $t \in T$, the triple $x = (i, b, t)$ is called a **contract**. It represents a bilateral match between cadet i and branch b at the price of t. Let

$$\mathcal{X} = I \times B \times T$$

denote the set of all contracts. Given a contract $x \in \mathcal{X}$, let $i(x)$ denote the cadet, $b(x)$ denote the branch, and $t(x)$ denote the price of the contract x. That is, $x = (i(x), b(x), t(x))$.

For any cadet $i \in I$, let

$$\mathcal{X}_i = \{x \in \mathcal{X} : i(x) = i\}$$

denote the set of contracts that involve cadet i. Similarly, for any branch $b \in B$, let

$$\mathcal{X}_b = \{x \in \mathcal{X} : b(x) = b\}$$

denote the set of contracts that involve branch b. For any cadet $i \in I$, preferences $\succ_i \in \mathcal{Q}$ defined over $B \times T \cup \{\varnothing\}$ can be redefined over $\mathcal{X}_i \cup \{\varnothing\}$ (i.e., her contracts and remaining unmatched) by simply interpreting a branch-price pair $(b, t) \in B \times T$ in the original domain as a contract between cadet i and branch b at price t in the new domain.

An **allocation** is a (possibly empty) set of contracts $X \subset \mathcal{X}$ such that

1. for any $i \in I$, $\left|\{x \in X : i(x) = i\}\right| \leq 1$,
2. for any $b \in B$, $\left|\{x \in X : b(x) = b\}\right| \leq q_b$, and
3. for any $b \in B$, $\left|\{x \in X : b(x) = b \text{ and } t(x) \in T^+\}\right| \leq q_b^f$.

That is, under an allocation X, no individual can appear in more than one contract, no branch b can appear in more contracts than the number of its positions q_b, and no branch b can appear in more contracts than q_b^f along with an increased price in T^+. Let $\mathcal{A}$ denote the set of all allocations.

For a given allocation $X \in \mathcal{A}$ and cadet $i \in I$, the **assignment** X_i of cadet i under allocation X is defined as

$$X_i = \begin{cases} (b, t), & \text{if } (i, b, t) \in X; \\ \varnothing, & \text{if } X \cap \mathcal{X}_i = \varnothing. \end{cases}$$

For the latter case, i.e., if $X_i = \emptyset$, we say that cadet i is **unmatched** under X.

Similarly, for a given allocation $X \in \mathcal{A}$ and branch b, define

$$X_b = \{(i,t) \in I \times T : (i,b,t) \in X\}.$$

Given an allocation $X \in \mathcal{A}$ and a cadet $i \in I$, with a slight abuse of the notation,[23] define the **branch assignment** $b(X_i)$ of cadet i as

$$b(X_i) = \begin{cases} b, & \text{if } (i,b,t) \in X; \\ \emptyset, & \text{if } X \cap \mathcal{X}_i = \emptyset. \end{cases}$$

Given an allocation $X \in \mathcal{A}$ and a cadet $i \in I$, with a slight abuse of the notation, define the **price assignment** $t(X_i)$ of cadet i as

$$t(X_i) = \begin{cases} t, & \text{if } (i,b,t) \in X; \\ \emptyset, & \text{if } X \cap \mathcal{X}_i = \emptyset. \end{cases}$$

A **mechanism** is a message space $\mathcal{S}_i$ for each cadet $i \in I$ along with an outcome function

$$\varphi : \prod_{i \in I} \mathcal{S}_i \rightarrow \mathcal{A}$$

that selects an allocation for each strategy profile. Let $\mathcal{S} = \prod_{i \in I} \mathcal{S}_i$.

A **direct mechanism** is a mechanism where $\mathcal{S}_i = \mathcal{Q}$ for each cadet $i \in I$. We denote a direct mechanism with its outcome function only, suppressing its message space, which is always $\mathcal{Q}^{|I|}$.

Given a mechanism $(\mathcal{S}, \varphi)$, the resulting **assignment function** $\varphi_i : \mathcal{S} \rightarrow B \times T \cup \{\emptyset\}$ for cadet $i \in I$ is defined as follows: for any $s \in \mathcal{S}$ and $X = \varphi(s)$,

$$\varphi_i(s) = X_i.$$

D. *Primary Desiderata for Allocations and Mechanisms*

The history of the cadet branch assignment process in Section IB describes some of the system's goals and origins. Using the notation introduced in the last section, we next formulate these goals as formal axioms.

Our first axiom is *individual rationality*. The Army cannot compel an assignment because a cadet always has the option to leave the Army. When cadets fail to complete their initial service obligation, they must reimburse the government's education cost according to Army Regulation 150-1 (United States Army 2021). For West Point graduates in the class of 2018, this cost was \$236,052 (United States Military Academy 2019). When a cadet voluntarily leaves the Army and pays the fine, we denote this outcome as unmatched. In the last two decades, between 5 and 10 percent of West

[23] The abuse of notation is due to the fact that while the argument of the functions $b(\cdot)$, $t(\cdot)$ is previously introduced as a contract, here, it is an assignment. Since a cadet and an assignment uniquely defines a (possibly empty) contract, the notational abuse is innocuous.

Point graduates have not fulfilled their commitment.[24] At ROTC, unmatched cadets are placed in reserve duty. For this application of our model, the unmatched outcome corresponds to reserve duty. For the classes of 2022 and 2023, about 10 percent of ROTC cadets remained unassigned and were placed in reserve duty.

DEFINITION 1: *An allocation $X \in \mathcal{A}$ satisfies* ***individual rationality*** *if, for any $i \in I$,*

$$X_i \succeq_i \varnothing.$$

Likewise, a mechanism $(\mathcal{S}, \varphi)$ satisfies ***individual rationality*** *if the allocation $\varphi(s)$ satisfies individual rationality for any strategy profile $s \in \mathcal{S}$.*

Each year, the Army carefully regulates the number of positions in each branch to ensure adequate staffing and effective deployment of the Army's human resources (United States Army 2019c). Given this, if a branch has a vacant slot, there shouldn't be an unassigned cadet who would like to have a position at the branch at the base price. The Army is keen not to waste valuable slots when a cadet is otherwise willing to take that assignment. This consideration leads to the next axiom.

DEFINITION 2: *An allocation $X \in \mathcal{A}$ satisfies* ***nonwastefulness*** *if, for any $b \in B$ and $i \in I$,*

$$\left.\begin{array}{r} \left|\{x \in X : b(x) = b\}\right| < q_b, \text{ and} \\ X_i = \varnothing \end{array}\right\} \Rightarrow \varnothing \succ_i (b, t^0).$$

Likewise, a mechanism $(\mathcal{S}, \varphi)$ satisfies ***nonwastefulness*** *if the allocation $\varphi(s)$ satisfies nonwastefulness for any strategy profile $s \in \mathcal{S}$.*

Nonwastefulness is a mild efficiency axiom that requires that no position remains unfilled while an unassigned cadet who would rather receive the position at the branch at the base price t^0 exists.

As we have described, a cadet's OML ranking forms the basis of a cadet's claims at a branch. If cadet *i* prefers the assignment of cadet *j* to her own assignment, then cadet *i* should not have higher priority at cadet *j*'s assigned branch. When the OML is the only source of prioritization, a cadet with a higher OML score should not prefer the assignment of a cadet with a lower OML. In that situation, the cadet's priority for a branch is reversed. Our next axiom formalizes this consideration.[25]

DEFINITION 3: *An allocation $X \in \mathcal{A}$ satisfies* ***no priority reversal*** *if, for any $i, j \in I$, and $b \in B$,*

$$\left.\begin{array}{r} b(X_j) = b, \text{ and} \\ X_j \succ_i X_i \end{array}\right\} \Rightarrow j \;\pi_b\; i.$$

[24] In some cases, like a medical or health issue, a cadet does not need to reimburse the Army for early separation.

[25] This axiom is called *fairness* in Sönmez and Switzer (2013). Here, we use the Army's terminology. See Section IIIA for an additional reason that justifies the use of a terminology different from *fairness*.

Likewise, a mechanism $(\mathcal{S}, \varphi)$ *satisfies* ***no priority reversal*** *if the allocation* $\varphi(s)$ *satisfies no priority reversals for any strategy profile* $s \in \mathcal{S}$.

This axiom captures the idea that once the price is fixed at $t \in T$, cadets with higher baseline priorities at any given branch $b \in B$ have higher claims for a position at branch b. Therefore, whenever cadet i strictly prefers another cadet j's assignment to her own assignment, cadet j must have higher baseline priority at her assigned branch than cadet i. Otherwise, if cadet i strictly prefers cadet j's assignment even though cadet j has lower baseline priority than cadet i, then we say that there is a **priority reversal**.

The axiom *no priority reversal* reduces to the axiom *no justified envy* in the simpler settings of Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003).[26] The essence of the axiom no justified envy has to do with the following two questions:

1. Is there envy?

2. If there is envy, is it justified?

A cadet can envy the assignment of another cadet if she prefers it over her own assignment. The envy is justified relative to the property rights structure, which describes which cadet's claims on any given assignment are most deserving. In the simplest case, when there is a single performance metric, like the OML, envy is justified if the envious cadet has a higher claim to the assignment based on this single performance metric.[27] The introduction of the BRADSO program in 2006 changed this basic structure of claims.

Under the BRADSO program, the structure of the property rights over positions at a branch does not merely depend on the OML but also on the price cadets are willing to pay for a position in the branch.[28]

To formulate these trade-offs rigorously, we consider whether a cadet may have a legitimate claim on a position awarded to other cadets, but rather than for a given price as in our previous axiom, this time with a different price due to the price responsiveness scheme. As a reminder, the price responsiveness scheme ω_b is exogenously specified by institutional actors, as we emphasized in Section IIB. In what follows, we break this down into two separate cases.

DEFINITION 4: *Let allocation* $X \in \mathcal{A}$ *and cadet* $i \in I$ *be such that* $X_i = (b,t) \in B \times T^+$. *Then, cadet* $j \in I \setminus \{i\}$ *has a* ***legitimate claim for a price-reduced version*** *of cadet i's assignment* X_i, *if there exists a lower price* $t' < t$ *such that*

$$(b,t') \succ_j X_j \quad \textit{and} \quad (j,t') \; \omega_b \; (i,t).$$

[26] The axiom *no justified envy* is called *fairness* in Balinski and Sönmez (1999) and *elimination of justified envy* in Abdulkadiroğlu and Sönmez (2003).

[27] The simplest form of axiom no justified envy by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003) is technically related to the lack of pairwise blocking in definitions of core stability in two-sided matching models. However, the conceptual justification for the axiom no justified envy is different because it is a completely normative axiom based on enforcing property rights rather than the traditional positive considerations related to core stability.

[28] According to Colarruso, Lyle, and Wardynski (2010), stakeholders have opposing views on whether and how often a lower OML-ranked cadet should be able to displace a higher OML-ranked cadet at a branch.

We say that cadet j's claim for a position at branch b at a lower price t' is *legitimate* because the price responsiveness scheme ω_b does not overturn her claim in favor of cadet i even when cadet i pays a higher price t.

DEFINITION 5: *Let allocation $X \in \mathcal{A}$ and cadet $i \in I$ be such that $X_i = (b,t) \in B \times T \backslash \{t^h\}$. Then, cadet $j \in I \backslash \{i\}$ has a* ***legitimate claim for a price-elevated version*** *of cadet i's assignment X_i, if there exists a higher price $t' > t$ such that*

$$(b,t') \succ_j X_j, \quad (j,t')\, \omega_b\, (i,t), \quad \text{and}$$

$$\left|\left\{(k,t^+) \in I \times T^+ : (k,b,t^+) \in X_b\right\}\right| < q_b^f.$$

Even if cadet i has a higher baseline priority at branch b than cadet j, cadet j's claim for a position at branch b is legitimate with the higher price t' because of the following:

1. The price responsiveness scheme ω_b overturns the baseline priority in favor of cadet j as long as cadet j pays the higher price t', and

2. Awarding the position originally given to cadet i instead to cadet j at the higher price t' is feasible and does not result in exceeding the cap q_b^f for flexible-price positions at branch b.[29]

Legitimate claims for price-reduced and price-elevated versions of another cadet's assignment are conceptually similar, but they have one technical difference due to the feasibility of changing a price of a position.[30] Given a pair of prices t, t' with $t > t'$, it is always feasible to replace the higher-price contract (i,b,t) of a cadet i with a lower-price contract (j,b,t') for another cadet j. In contrast, it is not always feasible to replace a lower-price contract (i,b,t) of a cadet i with a higher-price contract (j,b,t') for another cadet j. In particular, the replacement is not possible if $t = t^0$ and there are already q_b^f positions at branch b that are awarded at an increased price in T^+ under allocation X.

The absence of either type of legitimate claim defines the role of the price responsiveness scheme in our model, as we formulate next.

DEFINITION 6: *An allocation $X \in \mathcal{A}$* ***respects the price responsiveness scheme*** *if no cadet $j \in I$ has a legitimate claim for either a price-reduced version or a price-elevated version of the assignment X_i of another cadet $i \in I \backslash \{j\}$.*

Likewise, a mechanism $(\mathcal{S}, \varphi)$ ***respects the price responsiveness scheme*** *if the allocation $\varphi(s)$ respects the price responsiveness scheme for any strategy profile $s \in \mathcal{S}$.*

[29] Observe that awarding a position at a higher price can be potentially infeasible only when the original price is equal to the base price.

[30] This is why the last condition in Definition 5 is absent in Definition 4.

Since claims of individuals over positions at any given institution are represented with a baseline priority order in Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003), the only axiom in these papers that "enforces" the underlying basic structure of property rights is *no justified envy*. In our more complex setting, in contrast, a cadet may also increase her claims on a position at any given branch by paying a higher price than its base price. The role of our axiom *respect for the price responsiveness scheme* is to regulate how that happens in our setting. Taken together, our two axioms *no priority reversal* and *respect for the price responsiveness scheme* can be therefore interpreted as a generalization of the *no justified envy* axiom for our richer setting.

A mechanism vulnerable to "gaming" could erode cadets' trust in the Army's branching process. A mechanism that erodes trust is unlikely to persist in the US Army, where trust is seen as the foundation of their talent management strategy.[31] Maintaining trust is especially important since cadets may find themselves relying on other cadets for their own security in life-and-death combat situations. Perhaps unsurprisingly, when considering potential reforms to the USMA-2020 mechanism, the manager of the Talent-Based Branching program stated the Army prefers a mechanism that incentivizes honest preference submissions.[32]

Our next axiom is the gold standard for incentive compatibility in direct mechanisms.

DEFINITION 7: *A direct mechanism φ is* ***strategy-proof*** *if, for any $\succ \in \mathcal{Q}^{|I|}$, any $i \in I$, and any $\succ_i' \in \mathcal{Q}$,*

$$\varphi_i(\succ) \succeq_i \varphi_i(\succ_{-i}, \succ_i').$$

In a strategy-proof mechanism, truthful preference revelation is always in the best interests of the cadets.

III. USMA-2020 Mechanism and Its Shortcomings

During the first 15 years of the BRADSO program, the US Army did not use a direct mechanism for the branching process. Cadets do not submit their full preferences over branch-price pairs under USMA-2006 or USMA-2020. To describe and analyze these two mechanisms, we next introduce the class of *quasi-direct* mechanisms. A quasi-direct mechanism is defined for a version of the problem with a single increased price and thus, two prices in total. As such, throughout this section, we assume that $T = \{t^0, t^h\}$, or equivalently, $T^+ = \{t^h\}$.

[31] For example, in *The Army Profession*, the US Army's Training and Doctrine Command identifies trust as an essential characteristic that defines the Army as a profession (United States Army 2019b). The Army's People Strategy describes one of the Army's strategic outcomes as building a professional Army that retains the trust and confidence of the American people and its members (United States Army 2019a).

[32] Lieutenant Colonel Riley Post, the Talent-Based Branching Program Manager, said "cadets should be honest when submitting preferences for branches, instead of gaming the system" in a statement in West Point's official newspaper (Garcia 2020).

A. *Quasi-Direct Mechanisms and Their Desiderata*

A **quasi-direct mechanism** is a mechanism where the message space is $\mathcal{S}_i = \mathcal{P} \times 2^B$ for each cadet $i \in I$. For any strategy $s_i = (P_i, B_i) \in \mathcal{S}_i$ of cadet $i \in I$, the first component $P_i \in \mathcal{P}$ of the strategy is the cadet's preference ranking over branches (when they are awarded at the base price t^0) and remaining unmatched. The second component, $B_i \in 2^B$, is the set of branches for which the cadet is willing to pay the increased price t^h.

With the exception of *strategy-proofness*, which is only defined for direct mechanisms, all other axioms are well defined for quasi-direct mechanisms. However, a subtle issue arises in the verification of two of these axioms under quasi-direct mechanisms. Both *no priority reversals* and *respect for the price responsiveness scheme* rely on knowing if there is a cadet who prefers another cadet's assignment to her own assignment. Unlike a direct mechanism, this information is not fully solicited under a quasi-direct mechanism. Hence, for any given cadet, determining all priority reversals that adversely affect her legitimate claims for price-reduced versions of other cadets' assignments is not possible in a quasi-direct mechanism. However, some priority reversals can still be detected even under the restricted message space of quasi-direct mechanisms. This is the motivation for our next definition.

DEFINITION 8: *A quasi-direct mechanism φ has* ***no detectable priority reversal*** *if, for any $s = (P_j, B_j)_{j \in I} \in (\mathcal{P} \times 2^B)^{|I|}$, $b \in B$, and $i,j \in I$,*

$$\left.\begin{aligned} \varphi_j(s) &= (b, t^0), \text{ and} \\ \varphi_i(s) = (b, t^h) \quad &\text{or} \quad b\, P_i\, b(\varphi_i(s)) \end{aligned}\right\} \Rightarrow j \ \pi_b \ i.$$

Under this axiom, if cadet j is assigned a base-price position at branch b and another cadet i receives a less desired assignment by

(i) either receiving an increased-price position at the same branch, or

(ii) by receiving a position at a strictly less preferred (and possibly empty) branch based on cadet i's submitted preferences P_i on $B \cup \{\varnothing\}$,

then cadet j must have higher baseline priority under branch b than cadet i.

When a quasi-direct mechanism has detectable priority reversals, there is a cadet i who strictly prefers the assignment of another cadet j no matter what cadet i's preferences $\succ_i \in \mathcal{Q}$ over branch-price pairs are (provided that they are consistent with her submitted preferences $P_i \in \mathcal{P}$ over branches alone). For this reason, detectable priority reversals can be verified under a quasi-direct mechanism. Verification of the absence of all priority reversals, in contrast, requires knowledge of cadet i's preferences over branch-price pairs.

To study incentive compatibility of quasi-direct mechanisms, we can no longer consider *strategy-proofness* because that concept is only defined for direct mechanisms. We instead tailor variants of this axiom that accord with the quasi-direct message space. We formulate two incentive compatibility axioms that do not rely on preferences over branch-price pairs.

The Army created flexible price positions to allow some cadets to obtain priority over other cadets who may have a higher OML but are unwilling to extend their service commitment. Our next axiom captures the idea that a cadet should not be charged an increased price for a position when the price responsiveness scheme has not been pivotal in securing a branch.

DEFINITION 9: *A quasi-direct mechanism φ satisfies* ***BRADSO-incentive compatibility*** *(or* ***BRADSO-IC****) if, for any $s = (P_j, B_j)_{j \in I} \in (\mathcal{P} \times 2^B)^{|I|}$, $i \in I$, and $b \in B$,*

$$\varphi_i(s) = (b, t^h) \Rightarrow \varphi_i\big((P_i, B_i \setminus \{b\}), s_{-i}\big) \neq (b, t^0).$$

A cadet i who receives an increased-price position at branch b under φ should not be able to profit by receiving a position at the same branch at the base price by dropping branch b from the set of branches B_i for which she's willing to pay the increased price.

A cadet also should not benefit by declaring a willingness to pay the increased price to obtain an assignment at the branch at the base price. Failure of this desideratum undermines the idea behind the BRADSO system, which is to use information on the willingness to serve extended service commitments in exchange for priority. Our last axiom formulates this desideratum.

DEFINITION 10: *A quasi-direct mechanism φ is* ***immune to strategic BRADSO*** *if, for any $s = (P_j, B_j)_{j \in I} \in (\mathcal{P} \times 2^B)^{|I|}$, $i \in I$, and $b \in B$,*

$$\varphi_i(s) = (b, t^0) \Rightarrow \varphi_i\big((P_i, B_i \setminus \{b\}), s_{-i}\big) = (b, t^0).$$

A cadet i who receives a base-price position at branch b under φ should still do so upon dropping branch b from the set of branches B_i for which she has indicated willingness to pay the increased price (in case $b \in B_i$).[33] If this axiom fails, cadet i could strategically indicate a willingness to pay the increased price at branch b and receive an otherwise unattainable base-price position at this branch.

B. *USMA-2020 Mechanism*

The USMA-2020 mechanism is a quasi-direct mechanism with message space $\mathcal{S}_i^{2020} = \mathcal{P} \times 2^B$ for each cadet $i \in I$. Given a strategy profile $s = (P_i, B_i)_{i \in I}$, for any branch $b \in B$, construct the following adjusted priority order $\pi_b^+ \in \Pi$ on the set of cadets I. For any $i, j \in I$,

1. $b \in B_i$ and $b \in B_j \Rightarrow i \pi_b^+ j \Leftrightarrow i\ \pi_b\ j$,
2. $b \notin B_i$ and $b \notin B_j \Rightarrow i \pi_b^+ j \Leftrightarrow i \pi_b j$, and
3. $b \in B_i$ and $b \notin B_j \Rightarrow i \pi_b^+ j \Leftrightarrow (i, t^h)\ \omega_b\ (j, t^0)$.

[33] This statement holds vacuously if $b \notin B_i$.

Under the priority order π_b^+, any two cadets are rank ordered using the baseline priority order π_b if they have indicated the same willingness to pay the increased price for branch b, and using the price responsiveness scheme ω_b otherwise.[34]

For any strategy profile $s = \left(P_i, B_i\right)_{i \in I}$, let μ be the outcome of the *individual-proposing deferred acceptance algorithm* for submitted cadet preferences $\left(P_i\right)_{i \in I}$ and constructed branch priorities $\left(\pi_b^+\right)_{b \in B}$.

For any strategy profile $s = \left(P_i, B_i\right)_{i \in I}$, the outcome $\varphi^{2020}(s)$ of the ***USMA-2020 mechanism*** is given as follows: for any cadet $i \in I$,

$$\varphi_i^{2020}(s) = \begin{cases} \varnothing, & \text{if } \mu(i) = \varnothing; \\ \left(\mu(i), t^0\right), & \text{if } \mu(i) \notin B_i \text{ or } \left|\left\{j \in I : \mu(j) = \mu(i), \mu(j) \in B_j, \text{ and } i \, \pi_{\mu(i)} \, j\right\}\right| \geq q_{\mu(i)}^f; \\ \left(\mu(i), t^h\right), & \text{if } \mu(i) \in B_i \text{ and } \left|\left\{j \in I : \mu(j) = \mu(i), \mu(j) \in B_j, \text{ and } i \, \pi_{\mu(i)} \, j\right\}\right| < q_{\mu(i)}^f. \end{cases}$$

Under the USMA-2020 mechanism, each cadet $i \in I$ is asked to submit a preference relation $P_i \in \mathcal{P}$ along with a (possibly empty) set of branches $B_i \in 2^B$ for which she indicates her willingness to pay the increased price t^h to receive preferential admission. A priority order π_b^+ of cadets is constructed for each branch b by adjusting the baseline priority order π_b using the price responsiveness scheme ω_b whenever a pair of cadets submitted different willingness to pay the increased price t^h at branch b. Cadets' branch assignments are determined by the individual-proposing deferred acceptance algorithm using the submitted profile of cadet preferences $\left(P_i\right)_{i \in I}$ and the profile of adjusted priority rankings $\left(\pi_b^+\right)_{b \in B}$. A cadet pays the base price for her branch assignment if either she has not declared willingness to pay the increased price for her assigned branch or the capacity for the flexible-price positions of the branch is already filled with cadets who have lower baseline priorities. With the exception of those who remain unmatched, all other cadets pay the increased price for their branch assignments.

C. *Shortcomings of the USMA-2020 Mechanism*

USMA-2020 has perverse incentives in large part because it determines who is charged the increased price for their assignments only after the completion of branch assignments. We present an in-depth equilibrium analysis of the USMA-2020 mechanism in Section B of the online Appendix. Among our results in this analysis, Example 3 in Section B.2 of the online Appendix shows that the USMA-2020 mechanism fails BRADSO-IC and admits strategic BRADSO even at equilibrium. That example also illustrates the "knife-edge" aspects of equilibrium strategies in this mechanism. When there is a minor change in the underlying economy involving the lowest baseline priority cadet changing her preferences and this only affects her assignment, it nonetheless affects the equilibrium strategies of several higher-priority cadets. Example 1 further shows that the USMA-2020 mechanism can admit detectable priority reversals even under its Bayesian Nash equilibrium outcomes.

[34] When (i) the baseline priority order π_b is fixed as OML at each branch $b \in B$ and (ii) the price responsiveness scheme ω_b is fixed as the ultimate price responsiveness scheme $\bar{\omega}_b$ at each branch $b \in B$, this construction gives the same adjusted priority order constructed for the USMA-2006 mechanism.

EXAMPLE 1 (Detectable Priority Reversals at Bayesian Equilibria): *Suppose there is a single branch b with* $q_b^0 = q_b^f = 1$ *and three cadets* i_1, i_2, *and* i_3. *The baseline priority order* π_b *is such that*

$$i_1 \ \pi_b \ i_2 \ \pi_b \ i_3,$$

and the price responsiveness scheme ω_b *is the ultimate price responsiveness scheme* $\bar{\omega}_b$.

Each cadet has a utility function that is drawn from a distribution with the following two elements, u and v, where

$$u(b, t^0) = 10, \quad u(\varnothing) = 8, \quad u(b, t^h) = 0,$$

$$\text{and} \quad v(b, t^0) = 10, \quad v(b, t^h) = 8, \quad v(\varnothing) = 0.$$

Let us refer to cadets with a utility function $u(\cdot)$ *as type 1 and cadets with a utility function* $v(\cdot)$ *as type 2. All cadets have a utility of* 10 *for their first choice assignment of* (b, t^0), *a utility of* 8 *for their second choice assignment, and a utility of* 0 *for their last choice assignment. For type 1 cadets, the second choice is remaining unmatched, whereas for type 2 cadets, the second choice is receiving a position at the increased price* t^h. *Suppose each cadet can be of either type with a probability of* 50 *percent and they are all expected utility maximizers.*

The unique Bayesian Nash equilibrium s^* *under the incomplete information game induced by the USMA-2020 mechanism is, for any cadet* $i \in \{i_1, i_2, i_3\}$,

$$s_i^* = \begin{cases} \varnothing, & \text{if cadet } i \text{ is of type 1;} \\ b, & \text{if cadet } i \text{ is of type 2.} \end{cases}$$

That is, truth telling is the unique Bayesian Nash equilibrium strategy for each cadet. However, this unique Bayesian Nash equilibrium strategy results in detectable priority reversals whenever either

1. *cadet* i_1 *is of type 1 and cadets* i_2, i_3 *are of type 2, or*

2. *cadet* i_1 *is of type 2 and cadets* i_2, i_3 *are of type 1.*

While cadet i_2 *receives a position at the base price* t^0 *in both cases, the highest baseline priority cadet* i_1 *remains unassigned in the first case and receives a position at the increased price* t^h *in the second case.*

The fragility of the equilibrium strategies helps to understand some of the failures observed under the USMA-2020 mechanism in the field, which we describe next. These observations provide support for the relevance of the formal axioms we discussed in Section IIIA.

After announcing the mechanism to cadets in fall 2019, USMA leadership recognized the possibility of detectable priority reversals under the USMA-2020 mechanism due to either failure of BRADSO-IC or presence of strategic BRADSO. In a typical year, the number of cadets willing to pay the increased price for traditionally oversubscribed branches like military intelligence greatly exceeds 25 percent of the branch's positions. Therefore, by volunteering to pay the increased price for an oversubscribed branch, some cadets could receive a priority upgrade even though they may not be charged for it, making detectable priority reversals possible. Moreover, unlike the detectable priority reversals under the USMA-2006 mechanism, some of these detectable priority reversals can affect cadet branch assignments, thereby making manual ex post adjustments infeasible.

Failures of BRADSO-IC, the possibility of strategic BRADSO, or the presence of detectable priority reversals, especially when not manually corrected ex post, could erode cadets' trust in the Army's branching process. Consider, for example, a comment from a cadet survey administered to the class of 2020:[35]

> I believe this system fundamentally does not trust cadets to make the best choice for themselves. It makes it so that we cannot choose what we want and have to play games to avoid force branching.

To mitigate these concerns, USMA leadership executed a simulation using preliminary cadet preferences to inform cadets of the potential cutoffs for each branch.[36] The goal of this simulation was to help cadets to optimize their submitted strategies. Army leadership was quoted as follows (O'Connor 2019):

> "We're going to tell all the cadets, we're going to show all of them, here's when the branch would have went out, here's the bucket you're in, here's the branch you would have received if this were for real. You have six days to go ahead and redo your preferences and look at if you want to BRADSO or not." Sunsdahl said. "I think it's good to be transparent. I just don't know what 21-year-olds will do with that information."

Several open-ended survey comments from USMA cadets in the class of 2020 mirrored USMA leadership's concern about the USMA-2020 mechanism. We present three additional comments articulating concerns related to the lack of BRADSO-IC, the presence of strategic BRADSO, and the difficulty of navigating a system with both shortcomings:

1. "Volunteering for BRADSO should only move you ahead of others if you are actually charged for BRADSO. By doing this, each branch will receive the most qualified people. Otherwise people who are lower in class rank will receive a branch over people that have a higher class rank which does not benefit the branch. Although those who BRADSO may be willing to serve longer, if they aren't charged then they can still leave after their 5 year commitment so it makes more sense to take the cadets with a higher OML."

[35] The survey was administered to the class of 2020 immediately before they submitted their preferences for branches under the USMA-2020 mechanism. The response rate to this survey was 98 percent. Section E.2 of the online Appendix contains specific questions and results.

[36] Cadets in the class of 2020 submitted preliminary preferences one month before submitting final preferences. USMA ran the USMA-2020 mechanism on these preliminary preferences to derive results for the simulation, which USMA provided to cadets six days prior to the deadline for submitting final preferences.

2. "I think it is still a little hard to comprehend how the branching process works. For example, I do not know if I put a BRADSO for my preferred branch that happens to be very competitive, am I at a significantly lower chance of getting my second preferred if it happens to be something like engineers? Do I have to BRADSO now if I want engineers??? Am I screwing myself over by going for this competitive branch now that every one is going to try to beat the system????"

3. "Releasing the simulation just created chaos and panicked cadets into adding a BRADSO who otherwise wouldn't have."

Empirical evidence on the extent of failures of these desiderata under USMA-2020 and how they compare with the failures under USMA-2006 is presented in Section D.1 of the online Appendix.

IV. Multi-price Cumulative Offer Mechanism and Its Characterization

The integrated research and policy strategy under minimalist market design by Sönmez (2023) revolves around the following three endeavors:

1. Identify key objectives for the stakeholders.

2. Establish whether the institution in place satisfies all these objectives.

3. If the current institution fails to meet some of these objectives, provide an alternative mechanism that satisfies them if possible.

In addition, the third endeavor should identify the root causes of the failures of the existing mechanism and address these root causes in designing an alternative institution.

In our application, the main stakeholders are Army officers in charge of the branching process and the cadets. In Sections IID and IIIA, we formulated the key objectives of these stakeholders as rigorous axioms. In Section IIIC and Section B of the online Appendix, we have shown that USMA-2020 fails three key desiderata and identified the following two culprits for these failures:

1. The message space is not rich enough to capture cadet preferences over branch-price pairs.

2. The pricing of flexible-price positions does not take whether volunteering to pay an increased price is pivotal for a cadet to secure a position or not into consideration.

All three failures under USMA-2020, the presence of priority reversals, the lack of BRADSO-IC, and the presence of strategic BRADSO, are directly tied to these flawed aspects of the mechanism. In this section, we formulate an alternative mechanism for the Army by directly addressing these two root causes of the failures of

USMA-2020 and leaving the rest of the system untouched. This is what makes our intervention minimalist.

A. *Changing the Message Space*

To resolve the problems with the USMA-2020 mechanism, most notably its failure of BRADSO-IC, the possibility of strategic BRADSO, and the resulting detectable priority reversals, the Army established a partnership with the two civilian coauthors of this paper to redesign their branching mechanism. Critical to achieving these objectives was the Army's decision to permit cadets in the class of 2021 to submit preferences over branch-price pairs, thus paving the way to adopt a direct mechanism.

This decision was aided by evidence from a cadet survey that mitigated concerns that rating branch-price pairs would be overly complex or unnecessary. Indeed, some of the cadets indicated the need for a system that would allow them to rank order branch-price pairs.[37] More generally, the survey revealed that more than twice as many cadets prefer a mechanism that allows them to submit preferences over branch-price pairs relative to a mechanism that requires them to submit preferences over branches and then separately indicate their willingness to pay an increased price for each branch as in the USMA-2006 and USMA-2020 mechanisms.[38]

B. *Coordinating the Branch Assignment and the Price Assignment*

The central mechanism in the matching with contracts literature is the *cumulative offer mechanism* (Hatfield and Milgrom 2005), a direct mechanism that is based on a procedure that involves a sequence of cadet proposals and branch responses. Cadet proposals simulated under this procedure are based on their submitted preferences. The **multi-price cumulative offer** (**MPCO**) mechanism is a refinement of the cumulative offer mechanism, where, for each branch $b \in B$, the branch response takes a specific form determined by the following choice rule $\mathcal{C}_b^{MP}$.[39]

[37] One cadet wrote, "I wish there was an option to pick your second choice over your first if your first choice mandated a branch detail." Resonating this sentiment, another cadet wrote,

> I am indifferent to the alternative or current bradso system. However, [...] I believe that DMI (Department of Military Instruction) could elicit a new type of ranking list. Within my proposed system, people could add to the list of 17 branches BRADSO slots and rank them within that list. For example: *'AV* (*Aviation*) $>$ *IN* (*Infantry*) $>$ *AV:B* (*Aviation with BRADSO*).' … BRADSO slots are considered almost different things.

[38] A question on the survey asked cadets whether they prefer a mechanism that allows them to submit preferences over branch-price pairs or a mechanism that requires them to submit preferences over branches alone while separately indicating willingness to pay an increased price, or BRADSO, for each branch. Section E.2 of the online Appendix shows that 50 percent of respondents preferred the mechanism that permitted ranking branch-price pairs, 21 percent preferred the mechanism without the option to rank branch-price pairs, 24 percent were indifferent, and 5 percent did not understand.

[39] The MPCO mechanism is a generalization of the COSM proposed by Sönmez and Switzer (2013) for the case of a single increased price along with the ultimate price responsiveness scheme $\bar{\omega}_b$ for each branch $b \in B$, and a refinement of the cumulative offer mechanism for the matching with slot-specific priorities model by Kominers and Sönmez (2016).

Multi-price Choice Rule $\mathcal{C}_b^{MP}$: Given $b \in B$ and $X \subset \mathcal{X}_b$, select (up to) q_b contracts with distinct cadets as follows:

Step 1 (Selection for the Base-Price Positions)**:** Let X^1 be the set of all base-price contracts in X if there are no more than q_b^0 base-price contracts in X, and the set of base-price contracts in X with q_b^0 highest π_b-priority cadets otherwise. Pick contracts in X^1 for the base-price positions, and proceed to Step 2.

Step 2 (Selection for the Flexible-Price Positions)**:** Construct the set of contracts Y from X by first removing the lower ω_b-priority contract of any cadet who has two contracts in X and next removing all contracts of any cadet who has a contract already selected in X^1. Let X^2 be the set of all contracts in Y if there are no more than q_b^f contracts in Y, and the set of q_b^f highest ω_b-priority contracts in Y otherwise. Pick contracts in X^2 for the flexible-price positions, and terminate the procedure.

The outcome of the multi-price choice rule is $\mathcal{C}_b^{MP}(X) = X^1 \cup X^2$.

Intuitively, the multi-price choice rule $\mathcal{C}_b^{MP}$ first allocates the base-price positions following the baseline priority order π_b and next allocates the flexible-price positions following the price responsiveness scheme ω_b.

We are ready to formally define the multi-price cumulative offer mechanism. Given a profile of baseline priority orders $(\pi_b)_{b \in B}$ and a profile of price responsiveness scheme $(\omega_b)_{b \in B}$, let $\mathcal{C}^{MP} = (\mathcal{C}_b^{MP})_{b \in B}$ denote the profile of multi-price choice rules defined above. Since the MPCO mechanism is a direct mechanism, the message space for each cadet $i \in I$ is $\mathcal{S}_i^{MPCO} = \mathcal{Q}$, where $\mathcal{Q}$ is the set of linear orders on $(B \times T) \cup \{\varnothing\}$. The second element of the MPCO mechanism, its outcome function ϕ^{MPCO}, is given by the following **multi-price cumulative offer** procedure, which is simply the cumulative offer procedure (Hatfield and Milgrom 2005) implemented with the MPCO choice rule for each branch.

Multi-price Cumulative Offer Procedure: Fix a linear order of cadets $\pi \in \Pi$.[40] For a given profile of cadet preferences $\succ = (\succ_i)_{i \in I} \in \mathcal{Q}^{|I|}$, cadets propose their acceptable contracts to branches in a sequence of steps $\ell = 1, 2, \ldots$:

Step 1: Let $i_1 \in I$ be the highest π-ranked cadet who has an acceptable contract. Cadet $i_1 \in I$ proposes her most preferred contract $x_1 \in \mathcal{X}_{i_1}$ to branch $b(x_1)$. Branch $b(x_1)$ holds x_1 if $x_1 \in \mathcal{C}_{b(x_1)}^{MP}(\{x_1\})$ and rejects x_1 otherwise. Set $A_{b(x_1)}^2 = \{x_1\}$ and set $A_{b'}^2 = \varnothing$ for each $b' \in B \setminus \{b(x_1)\}$; these are the sets of contracts available to branches at the beginning of Step 2.

Step ℓ ($\ell \geq 2$)**:** Let $i_\ell \in I$ be the highest π-ranked cadet for whom no contract is currently held by any branch, and let $x_\ell \in \mathcal{X}_{i_\ell}$ be her most preferred acceptable contract that has not yet been rejected. Cadet i_ℓ proposes contract x_ℓ to branch $b(x_\ell)$. Branch $b(x_\ell)$ holds the contracts in $\mathcal{C}_{b(x_\ell)}^{MP}\left(A_{b(x_\ell)}^\ell \cup \{x_\ell\}\right)$ and rejects all other contracts in

[40] By Kominers and Sönmez (2016), the outcome of this procedure is independent of this linear order.

$A^{\ell}_{b(x_\ell)} \cup \{x_\ell\}$. Set $A^{\ell+1}_{b(x_\ell)} = A^{\ell}_{b(x_\ell)} \cup \{x_\ell\}$, and set $A^{\ell+1}_{b'} = A^{\ell}_{b'}$ for each $b' \in B \setminus \{b(x_\ell)\}$; these are the sets of contracts available to branches at the beginning of Step $\ell + 1$.[41]

The procedure terminates at a step when either no cadet remains with an acceptable contract that has not been rejected or when no contract is rejected.

Given a profile of cadet preferences $\succ = (\succ_i)_{i \in I} \in \mathcal{Q}^{|I|}$, all the contracts on hold in the final step of the multi-price cumulative offer procedure are finalized as the outcome $\phi^{MPCO}(\succ)$ of the **multi-price cumulative offer** mechanism.[42]

Our main theoretical result shows that MPCO is the only direct mechanism that satisfies all five desiderata of the Army formulated in Section IID.

THEOREM 1: *Fix a profile of baseline priority orders $(\pi_b)_{b \in B} \in \Pi^{|B|}$ and a profile of price responsiveness schemes $(\omega_b)_{b \in B} \in \prod_{b \in B} \omega_b$. A direct mechanism φ respects the price responsiveness scheme, and it satisfies individual rationality, nonwastefulness, no priority reversal, and strategy-proofness if and only if it is the MPCO mechanism ϕ^{MPCO}.*

Apart from singling out the MPCO mechanism as the unique mechanism that satisfies the Army's desiderata, to the best of our knowledge, Theorem 1 is the first characterization of an allocation mechanism (i.e., the cumulative offer mechanism) that pins down a specific choice rule (i.e., the multi-price choice rule) endogenous to the policy objectives of the central planner. We next relate our characterization to earlier literature.

C. *Related Literature*

Without the BRADSO program, our model reduces to the standard priority-based and unit demand indivisible goods allocation problem. In this more basic version of the problem, the axiom *respect for the price responsiveness scheme* becomes vacuous, the axiom *no priority reversals* reduces to the axiom *no justified envy* in its most basic form, and the MPCO mechanism reduces to the *individual-proposing deferred acceptance mechanism* by Gale and Shapley (1962). As such, Theorem 1 is a generalization of the following well-known result.

COROLLARY 1 (*Alcalde and Barberà 1994; Balinski and Sönmez 1999*): *Fix a profile of baseline priority orders $(\pi_b)_{b \in B} \in \Pi^{|B|}$. A direct mechanism φ satisfies*

[41] If branch choice rules satisfy the bilateral substitutes condition by Hatfield and Kojima (2010), then it is immaterial at any step whether a branch makes selections from all contracts proposed so far or from all contracts it has been holding from the previous step along with the new contracts it received. This fact is the basis of the "cumulative offer" terminology, coined by Hatfield and Milgrom (2005) under a stronger *substitutes* condition. The multi-price choice rule satisfies the bilateral substitutes condition by Kominers and Sönmez (2016), and therefore,

$$A^{\ell+1}_{b(x_\ell)} = A^{\ell}_{b(x_\ell)} \cup \{x_\ell\} = \mathcal{C}^{MP}_{b(x_\ell)}\left(A^{\ell}_{b(x_\ell)} \cup \{x_\ell\}\right).$$

[42] As it is customary in the literature, we denote a direct mechanism with its outcome function and use ϕ^{MPCO} to denote both the outcome function and the resulting direct mechanism.

individual rationality, *nonwastefulness*, *no justified envy, and strategy-proofness if and only if it is the individual-proposing deferred acceptance mechanism.*

A choice rule is a single-institution solution concept that regulates who deserves positions at the institution. Under some technical conditions, this solution concept easily integrates with the individual-proposing deferred acceptance mechanism and its cumulative offer mechanism generalization, thus extending its scope for multi-institution settings.[43] In relation to minimalist market design, a natural interface between two major components of a resource allocation system becomes available in these settings. Consequently, the adoption of the individual-proposing deferred acceptance mechanism as a plausible mechanism for school choice in the mid-2000s (Abdulkadiroğlu and Sönmez 2003; Abdulkadiroğlu et al. 2005) resulted in a rich literature on analysis of choice rules that implement various social policies. Papers in this literature include Pycia (2012); Hafalir, Yenmez, and Yildirim (2013); Echenique and Yenmez (2015); Kominers and Sönmez (2016); Doğan (2017); Dur et al. (2018); Kojima, Tamura, and Yokoo (2018); Erdil and Kumano (2019); Dur, Pathak, and Sönmez (2020); Imamura (2020); Pathak, Rees-Jones, and Sönmez (Forthcoming); Aygün and Bó (2021); Doğan and Yildiz (2023); Sönmez and Yenmez (2022a, b); and Sönmez and Ünver (2022). Some of these papers assume a single institution. Others develop the foundations for various choice rules assuming that the underlying allocation mechanism is either the individual-proposing deferred acceptance mechanism or the cumulative offer mechanism.[44] Our paper, in contrast, establishes the foundations for both parts of the mechanism together from the primitives of the problem.

As in the case of the individual-proposing deferred acceptance mechanism, prior axiomatic characterizations for the cumulative offer mechanism also exist in the literature. Most related to Theorem 1 are Hirata and Kasuya (2017) and Hatfield, Kominers, and Westkamp (2021), who present axiomatic characterizations of the cumulative offer mechanism based on conceptually relevant axioms. Our characterization, however, differs from theirs in one fundamental aspect. In both Hirata and Kasuya (2017) and Hatfield, Kominers, and Westkamp (2021), institutions are each endowed with an exogenously given choice rule that satisfies various technical conditions. In our characterization, in contrast, the multi-price choice rule—one of the two pillars of the MPCO mechanism—emerges endogenously from the Army's policy objectives formulated by our desiderata.

V. Iterative Design: Trading Off Talent Alignment and Retention

In this section, drawing on our experience with the US Army's branching reform, we present an example of iteration in the design after a partnership is formed with the system operators. At this stage in the reform process, the market designer is no

[43] Two technical conditions on choice rules that enable this integration are the *substitutes condition* (Kelso and Crawford 1982; Hatfield and Milgrom 2005) and *independence of rejected individuals* (Aygün and Sönmez 2013).

[44] An exception is Sönmez and Yenmez (2022b), which follows our research strategy and establishes the foundations for the individual-proposing deferred acceptance mechanism along with a choice rule formulated in Sönmez and Yenmez (2022a).

longer an outsider and therefore has more flexibility to tinker with various aspects of the design.

After adopting the USMA-2020 mechanism, Army and USMA leadership had several discussions about the potential price responsiveness scheme for the class of 2021 and possibly increasing the share of flexible-price positions. As described in the excerpt below from a news article describing an interview with the Talent-Based Branching Program Manager, selecting these parameters presented the Army with a trade-off between retention and talent alignment (Garcia 2020):

> A key question the Army considered when designing this year's mechanism was how much influence to give cadets who are willing to BRADSO. If every cadet who volunteers to BRADSO can gain priority, or "jump" above, every cadet who did not volunteer to BRADSO, then that could improve Army retention through more cadets serving an additional three years, but it could also result in more cadets being assigned to branches that do not prefer them.

It is possible to formally analyze this trade-off by focusing on the choice rule $\mathcal{C}_b^{MP}$ in the new mechanism. For a given number of total positions, if the number of flexible-price positions increases, then the baseline priority order π_b is used for fewer positions and the price responsiveness scheme ω_b is used for more positions. Likewise, when a price responsiveness scheme becomes more responsive to a price increase, increased-price contracts receive weakly higher priorities. Under both of these scenarios, the number of increased-price contracts selected by the choice rule $\mathcal{C}_b^{MP}$ weakly increases. We collect these two straightforward observations in the following result.

PROPOSITION 1: *Fix a branch $b \in B$, the total number of branch-b positions at q_b, and a set of branch-b contracts $X \subset \mathcal{X}_b$. Then,*

1. *the number of price-elevated contracts selected under $\mathcal{C}_b^{MP}(X)$ weakly increases as the number of flexible-price positions q_b^f increases, and*

2. *the number of price-elevated contracts selected under $\mathcal{C}_b^{MP}(X)$ weakly increases as the price responsiveness scheme ω_b gets more responsive to a price increase.*

While the results on the BRADSO collected (i.e., the flexible-price positions awarded at elevated prices) given in Proposition 1 hold for a given branch under the multi-price choice rule, in theory, this result may not hold in aggregate across all branches under the MPCO mechanism.[45] However, as we present next and illustrate in Figure 1, the comparative static properties do hold in our simulations with the class of 2021 data for several price responsiveness schemes.

[45] The fact that a global comparative static result does not hold in matching models with slot-specific priorities has been explored in other work, including Dur et al. (2018) and Dur, Pathak, and Sönmez (2020). Both papers contain examples showing how a comparative static across all branches need not hold. However, the two papers also show empirically that these theoretical cases do not apply in their applications.

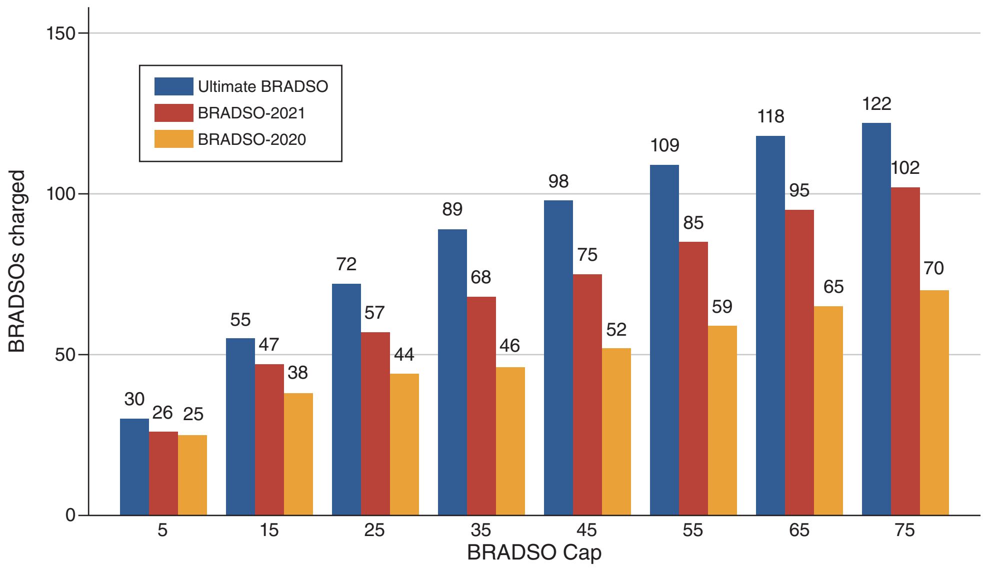

Figure 1. Number of BRADSOs Charged across Price Responsiveness Schemes and Cap Sizes

Notes: This figure reports on the number of BRADSOs charged for three price responsiveness schemes: Ultimate price responsiveness scheme, BRADSO-2020 scheme, and BRADSO-2021 scheme using data from the class of 2021. The BRADSO cap ranges from 5 percent to 75 percent of slots at each branch. Each outcome is computed by running MPCO mechanism given stated cadet preferences under different price responsiveness schemes and cap sizes.

The Army considered three price responsiveness schemes: the ultimate price responsiveness scheme and two tiered price responsiveness schemes. Under the BRADSO-2020 scheme, a cadet who expressed a willingness to sign a BRADSO contract only obtained priority over other cadets who had the same categorical branch rating. Under the BRADSO-2021 scheme, a cadet who expressed a willingness to sign a BRADSO contract obtained higher priority over all other cadets if she was in the medium or high category. To illustrate the trade-off between talent alignment and retention, Figure 1 uses preferences from the class of 2021 and reruns the MPCO mechanism under these three price responsiveness schemes for different levels of flexible-price positions q_b^f, where q_b^f is expressed as a percentage of the total number of positions for branch b.

To measure the effects of price responsiveness scheme on BRADSOs collected, Figure 1 shows how the number of BRADSOs charged increases with q_b^f and with the "closeness" of the price responsiveness scheme to the ultimate price responsiveness scheme. That is, for a given q_b^f, the BRADSO-2021 scheme results in more BRADSOs charged than the BRADSO-2020 scheme but fewer BRADSOs charged than the ultimate price responsiveness scheme. When the fraction of the flexible-price positions is small, there is relatively little difference between price responsiveness schemes. For example, when the fraction of the flexible-price positions is 15 percent of all positions, 55 BRADSOs are charged under the ultimate price responsiveness scheme, 47 BRADSOs are charged under the BRADSO-2021 scheme, and 38 BRADSOs are charged under the BRADSO-2020 scheme. When the fraction of the flexible-price positions is larger, the price responsiveness scheme has a larger effect on BRADSOs collected. When the fraction of the flexible-price positions is 65 percent, 118 BRADSOs are charged under the ultimate price responsiveness scheme, 95 BRADSOs are charged under the BRADSO-2021 scheme, and 65 BRADSOs are charged under the BRADSO-2020 scheme.

The ability to run this analysis on the effects of price responsiveness scheme is an important benefit of a strategy-proof mechanism and illustrates the iterative step in minimalist market design. At the request of the Army, we conducted a similar analysis using data from the class of 2020, but this analysis required stronger assumptions on cadet preferences.[46] As a result of this analysis, the Army decided to adopt the BRADSO-2021 scheme and increase the fraction of the flexible-price positions from 25 to 35 percent. These are both policies that increase the power of BRADSO. However, USMA decided against adopting the ultimate price responsiveness scheme because branches remained opposed to giving more BRADSO power to low-tier cadets.

VI. Conclusions

This paper presents a proof-of-concept for minimalist market design drawing on our longstanding research and policy effort for the US Army's branching process. Although the new design was initially intended only for USMA, the Army adopted the same assignment mechanism to assign more than 3,000 ROTC cadets the following year. After the Army's branching reforms at USMA and ROTC, minimalist market design also facilitated the design of pandemic medical resource allocation rules in several jurisdictions of the United States during the COVID-19 pandemic (Pathak et al. Forthcoming) and a living-donor liver exchange system in Turkey (Yilmaz et al. 2023).

We conclude by reflecting on the nature of the interaction between theory and applications. Stokes (1997) famously challenged the dichotomy between basic and applied science. He coins the concept of Pasteur's Quadrant, which is research that seeks fundamental understanding while also being inspired by practical uses. Watts (2017) advocates for social science to prioritize this type of research. In his view, social science faces an "incoherency problem," which has been "perpetuated by a historical emphasis in social science on the advancement of theories over the solution of practical problems." Watts (2017) suggests a superior path for social science to advance theory is by crafting solutions to real-life problems. Our partnership with the Army was motivated by solving the real-life problem of branch assignment, but it also led to new theoretical developments. More generally, applications of minimalist market design are primarily about solving real-world problems, but they have also proved to be valuable in advancing fundamental theory.

[46] Because cadets in the class of 2020 did not submit preferences over branch-price pairs, we assumed that all BRADSOs are consecutive and also considered different assumptions on the prevalence of nonconsecutive BRADSOs. These assumptions are not needed when cadets can rank branch-price pairs in a strategy-proof mechanism.

REFERENCES

Abdulkadiroğlu, Atila, and Tayfun Sönmez. 2003. "School Choice: A Mechanism Design Approach." *American Economic Review* 93 (3): 729–47.

Abdulkadiroğlu, Atila, Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez. 2005. "The Boston Public School Match." *American Economic Review, Papers and Proceedings* 95 (2): 368–71.

Ajzenman, Nicolás, Eleonora Bertoni, Gregory Elacqua, Luana Marotta, and Carolina Mendez Vargas. 2020. "Altruism or Money? Reducing Teacher Sorting Using Behavioral Strategies in Peru." Unpublished.

Alcalde, José, and Salvador Barberà. 1994. "Top Dominance and the Possibility of Strategy-Proof Stable Solutions to Matching Problems." *Economic Theory* 4 (3): 417–35.

Armacost, Andrew P., and James K. Lowe. 2005. "Decision Support for the Career Field Selection Process at the US Air Force Academy." *European Journal of Operational Research* 160 (3): 839–50.

Artemov, Georgy, Yeon-Koo Che, and Yinghua He. 2020. "Strategic Mistakes: Implications for Market Design Research." Unpublished.

Atkinson, Rick. 2009. *The Long Gray Line: The American Journey of West Point's Class of 1966*. New York, NY: Henry Holt and Company.

Aygün, Orhan, and Inácio Bó. 2021. "College Admission with Multidimensional Privileges: The Brazilian Affirmative Action Case." *American Economic Journal: Microeconomics* 13 (3): 1–28.

Aygün, Orhan, and Tayfun Sönmez. 2013. "Matching with Contracts: Comment." *American Economic Review* 103 (5): 2050–51.

Balinski, Michel, and Tayfun Sönmez. 1999. "A Tale of Two Mechanisms: Student Placement." *Journal of Economic Theory* 84 (1): 73–94.

Bar, Bocar, Patrick Bayer, Nayoung Rim, Roman Rivera, and Modibo Sidibe. 2021. "Police Officer Assignment and Neighborhood Crime." NBER Working Paper 29243.

Bertoni, Eleonaro, Gregory Elacqua, Dinia Hincapié, Carolina Méndez, and Diana Paredes. 2023. "Teachers' Preferences for Proximity and the Implications for Staffing Schools: Evidence from Peru." *Education Finance and Policy* 18 (2): 181–212.

Chassang, Sylvain, and Juan Ortner. 2023. "Regulating Collusion." *Annual Review of Economics* 15 (1): 177–204.

Colarusso, Michael J., Kenneth G. Heckel, David S. Lyle, and William L. Skimmyhorn. 2016. "Starting Strong: Talent-Based Branching of Newly Commissioned US Army Officers (Officer Corps Strategy, Volume 9)." Carlisle, PA: Army War College-Strategic Studies Institute.

Colarruso, Michael, David S. Lyle, and Casey Wardynski. 2010. "Towards a U.S. Army Officer Corps Strategy for Success: Retaining Talent." Unpublished.

Combe, Julien, Umut Dur, Olivier Tercieux, Camille Terrier, and M. Utku Ünver. 2022. "Market Design for Distributional Objectives in (Re)assignment: An Application to Improve the Distribution of Teachers in Schools." Unpublished.

Combe, Julien, Olivier Tercieux, and Camille Terrier. 2022. "The Design of Teacher Assignment: Theory and Evidence." *Review of Economic Studies* 89 (6): 3154–3222.

Davis, Jonathan M. V., Kyle Greenberg, and Damon Jones. 2023. "An Experimental Evaluation of Deferred Acceptance: Evidence from Over 100 Army Officer Labor Markets." NBER Working Paper 31612.

Department of Defense. 2020. "Population Representation in the Military Services: Fiscal Year 2018." Washington, DC: Office of the Under Secretary of Defense.

Doğan, Battal. 2017. "How to Control Controlled School Choice: Comment." *American Economic Review* 107 (4): 1362–64.

Doğan, Battal, and Kemal Yildiz. 2023. "Choice with Affirmative Action." *Management Science* 69 (4): 2284–96.

Duflo, Esther. 2017. "The Economist as Plumber." *American Economic Review* 107 (5): 1–26.

Dur, Umut, Scott Duke Kominers, Parag A. Pathak, and Tayfun Sönmez. 2018. "Reserve Design: Unintended Consequences and the Demise of Boston's Walk Zones." *Journal of Political Economy* 126 (6): 2457–79.

Dur, Umut, Parag A. Pathak, and Tayfun Sönmez. 2020. "Explicit vs. Statistical Preferential Treatment in Affirmative Action: Theory and Evidence from Chicago's Exam Schools." *Journal of Economic Theory* 187: 104996.

Echenique, Federico. 2012. "Contracts vs. Salaries in Matching." *American Economic Review* 102 (1): 594–601.

Echenique, Federico, and Bumin Yenmez. 2015. "How to Control Controlled School Choice?" *American Economic Review* 105 (8): 2679–94.

Erdil, Aytek, and Taro Kumano. 2019. "Efficiency and Stability under Substitutable Priorities with Ties." *Journal of Economic Theory* 184: 104950.

Gale, David, and Lloyd S. Shapley. 1962. "College Admissions and the Stability of Marriage." *American Mathematical Monthly* 69 (1): 9–15.

Garcia, Jorge. 2020. "New Innovations Improve Branching Process for Cadets, Branches." *U.S. Army*, September 17. https://www.army.mil/article/239119/new_innovations_improve_branching_process_for_cadets_branches.

Greenberg, Kyle, Mark Crow, and Carl Wojtaszek. 2020. *Winning in the Marketplace: How Officers and Units Can Get the Most Out of the Army Talent Alignment Process*. West Point, NY: Modern War Institute.

Greenberg, Kyle, Parag A. Pathak, and Tayfun Sönmez. 2021. "Redesigning the US Army's Branching Process: A Case Study in Minimalist Market Design." NBER Working Paper 28911.

Greenberg, Kyle, Parag A. Pathak, and Tayfun Sönmez. 2024. "Replication data for: Redesigning the US Army's Branching Process: A Case Study in Minimalist Market Design." American Economic Association [publisher], Inter-university Consortium for Political and Social Research [distributor]. https://doi.org/10.3886/E194504V1.

Hafalir, Isa E., M. Bumin Yenmez, and Muhammed A. Yildirim. 2013. "Effective Affirmative Action in School Choice." *Theoretical Economics* 8 (2): 325–63.

Hassidim, Avinatan, Assaf Romm, and Ran I. Shorrer. 2021. "The Limits of Incentives in Economic Matching Procedures." *Management Science* 67 (2): 951–63.

Hatfield, John William, and Fuhito Kojima. 2010. "Substitutes and Stability for Matching with Contracts." *Journal of Economic Theory* 145 (5): 1704–23.

Hatfield, John William, Scott Duke Kominers, and Alexander Westkamp. 2021. "Stability, Strategy-proofness, and Cumulative Offer Mechanisms." *Review of Economic Studies* 88 (3): 1457–1502.

Hatfield, John William, and Paul R. Milgrom. 2005. "Matching with Contracts." *American Economic Review* 95 (4): 913–35.

Hirata, Daisuke, and Yusuke Kasuya. 2017. "On Stable and Strategy-Proof Rules in Matching Markets with Contracts." *Journal of Economic Theory* 168: 27–43.

Hitzig, Zoë. 2020. "The Normative Gap: Mechanism Design and Ideal Theories of Justice." *Economics and Philosophy* 36 (3): 407–34.

Imamura, Kenzo. 2020. "Meritocracy vs. Diversity." Unpublished.

Jagadeesan, Ravi. 2019. "Cadet-Branch Matching in a Kelso-Crawford Economy." *American Economic Journal: Microeconomics* 11 (3): 191–224.

Kelso, Alexander, and Vincent Crawford. 1982. "Job Matching, Coalition Formation, and Gross Substitutes." *Econometrica* 50 (6): 1483–1504.

Khan, Adnan Q., Asim Khwaja, and Benjamin Olken. 2019. "Making Moves Matter: Experimental Evidence on Incentivizing Bureaucrats through Performance-Based Postings." *American Economic Review* 109 (1): 237–70.

Kojima, Fuhito, Akihisa Tamura, and Makoto Yokoo. 2018. "Designing Matching Mechanisms under Constraints: An Approach from Discrete Convex Analysis." *Journal of Economic Theory* 176: 803–33.

Kominers, Scott, and Tayfun Sönmez. 2016. "Matching with Slot-Specific Priorities: Theory." *Theoretical Economics* 11 (2): 683–710.

Li, Shengwu. 2017. "Ethics and Market Design." *Oxford Review of Economic Policy* 33 (4): 705–20.

Moulin, Hervé. 1988. *Axioms of Cooperative Decision Making*. Cambridge: Cambridge University Press.

Moulin, Hervé. 2004. *Fair Division and Collective Welfare*. Cambridge, MA: MIT Press.

NASEM. 2021. "Strengthening U.S. Air Force Human Capital Management: A Flight Plan for 2020-2030." Washington, DC: National Academies Press.

O'Connor, Brandon. 2019. "Assigning branches to cadets takes on new system." *U.S. Army,* September 12. *https://www.army.mil/article/227124/assigning.*

Pathak, Parag A., Alex Rees-Jones, and Tayfun Sönmez. Forthcoming. "Immigration Lottery Design: Engineered and Coincidental Consequences of H-1B Reforms." *Review of Economics and Statistics*.

Pathak, Parag A., Tayfun Sönmez, M. Utku Ünver, and Bumin Yenmez. Forthcoming. "Fair Allocation of Vaccines, Ventilators and Antiviral Treatments: Leaving No Ethical Value Behind in Health Care Rationing." *Management Science***.**

Pycia, Marek. 2012. "Stability and Preference Alignment in Matching and Coalition Formation." *Econometrica* 80 (1): 323–62.

Roth, Alvin E., Tayfun Sönmez, and Utku Ünver. 2004. "Kidney Exchange." *Quarterly Journal of Economics* 119 (2): 457–88.

Roth, Alvin E., Tayfun Sönmez, and Utku Ünver. 2005. "Pairwise Kidney Exchange." *Journal of Economic Theory* 125 (2): 151–88.

Schlegel, Jan Christoph. 2015. "Contracts versus Salaries in Matching: A General Result." *Journal of Economic Theory* 159 (A): 552–73.

Shorrer, Ran I., and Sándor Sóvágó. Forthcoming. "Choice with Affirmative Action." *Journal of Political Economy: Microeconomics*.

Sönmez, Tayfun. 2013. "Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism." *Journal of Political Economy* 121 (1): 186–219.

Sönmez, Tayfun. 2023. "Minimalist Market Design: A Framework for Economists with Policy Aspirations." Unpublished.

Sönmez, Tayfun, and M. Bumin Yenmez. 2022a. "Affirmative Action in India via Vertical, Horizontal, and Overlapping Reservations." *Econometrica* 90 (3): 1143–76.

Sönmez, Tayfun, and M. Bumin Yenmez. 2022b. "Constitutional Implementation of Affirmative Action Policies in India." Unpublished.

Sönmez, Tayfun, and Tobias B. Switzer. 2013. "Matching with (Branch-of-Choice) Contracts at the United States Military Academy." *Econometrica* 81 (2): 451–88.

Sönmez, Tayfun, and M. Utku Ünver. 2022. "Market Design for Social Justice: A Case Study on a Constitutional Crisis in India." Unpublished.

State Department. 2019. "Becoming a Foreign Service Officer." https://careers.state.gov/wp-content/uploads/2019/05/Becoming-a-Foreign-Service-Officer-Specialist.pdf.

Stokes, Donald E. 1997. *Pasteur's Quadrant: Basic Science and Technological Innovation*. Washington, DC: Brookings Institution Press.

Thomson, William. 2001. "On the Axiomatic Method and its Recent Applications to Game Theory and Resource Allocation." *Social Choice and Welfare* 18 (2): 327–86.

Thomson, William. 2011. "Fair Allocation Rules." In *Handbook of Social Choice and Welfare*, Vol. 2, edited by Kenneth J. Arrow, Amartya Sen, and Kotaro Suzumura, 393–506. Amsterdam: Elsevier.

Topping, Kimetha G. 1989. "The Order of Merit." Eisenhower Leader Development Program Papers.

United States Army. 2017. *Army Regulation 350-100: Officer Active Duty Service Obligations*. Washington, DC: Department of the Army.

United States Army. 2019a. *The Army People Strategy*. Washington, DC: Department of the Army.

United States Army. 2019b. *The Army Profession*. Washington, DC: Department of the Army.

United States Army. 2019c. *Army Regulation 614-100: Officer Assignment Policies, Details, and Transfers*. Washington, DC: Department of the Army.

United States Army. 2021. *Army Regulation 150-1: United States Military Academy Organization, Administration, and Operation*. Washington, DC: Department of the Army.

United States Marine Corps. 2021. "Talent Management 2030: Department of the Navy." Arlington County, VA: United States Marine Corps.

United States Military Academy. 1982. "Branch and Specialty Selection Results." *Assembly* 40 (4).

United States Military Academy. 2019. *General Scholarship, College Plans/Bonds and Cost of Education Information*. West Point, NY: United States Military Academy.

Watts, Duncan J. 2017. "Should Social Science Be More Solution-Oriented?" *Nature Human Behavior* 1: 0015.

Yilmaz, Sezai, Tayfun Sönmez, M. Utku Ünver, Volkan Ince, Sami Akbulut, Burak Isik, and Sukru Emre. 2023. "The first 4-way liver paired exchange from an interdisciplinary collaboration between health care professionals and design economists." *American Journal of Transplantation* 23 (10): 1612–21.

Zhou, Congyi, and Tong Wang. 2021. "Purchasing Seats for High School Admission and Inequality." Unpublished.