Risk-taking over the Life Cycle: Aggregate and Distributive Implications of Entrepreneurial Risk*

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Abstract

We study the risk-taking behavior over the life cycle of entrepreneurs subject to partial insurance against idiosyncratic shocks. The model quantitatively accounts for the aggregate and idiosyncratic risk premia, the life-cycle profiles of consumption and risk-taking, and the patterns of wealth inequality observed in the data. A reform that relaxes the risk constraints reduces the idiosyncratic risk premium and induces an investment boom. Consistent with a Kuznets curve, inequality increases in the short run and declines in the long run. The initial generation of entrepreneurs benefits from better insurance, but future generations will be worse off after the reform.

KEYWORDS: Entrepreneurship, risk-taking, risk premium, insurance, inequality

JEL CLASSIFICATION: G11, G51, E44.

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1 Introduction

Entrepreneurship is inherently a risky activity. Entrepreneurial risk is particularly important in developing economies, where the bulk of production occurs in privately owned businesses, and risk-sharing opportunities are limited. Given that entrepreneurs hold under-diversified portfolios, such risks have far-reaching implications.[1] At the individual level, entrepreneurial risk distorts investment and savings decisions. The importance of (uninsurable) business income is highly heterogeneous and varies over the life cycle, shaping wealth inequality patterns. At the macro level, imperfect insurance leads to an inefficient risk premium, which depresses the capital stock and hinders economic development. The pervasive effects of limited risk-sharing highlight the relevance of policy interventions that alleviate the consequences of entrepreneurs' lack of diversification.

This paper studies the aggregate and distributive implications of entrepreneurial risk in a quantitative life-cycle model with imperfect idiosyncratic insurance. We discipline the model using a rich dataset on small business in Thailand, which includes information on the returns of entrepreneurial activity and entrepreneurs' risk-taking and consumption.[2] The model captures several salient features of the data, such as the risk premium on business returns, the life-cycle profile of risk-taking and consumption, and the patterns of inequality between- and within-age groups. Combining a theory of the idiosyncratic risk premium with detailed information on portfolios and business returns, we assess the limits to risk sharing faced by entrepreneurs and, ultimately, the impact of relaxing these constraints on investment, inequality, and economic development.

We start by studying empirically the determinants of expected entrepreneurial returns. Expected returns are tightly connected to the marginal product of capital (MPK), so the required return on a project is potentially a key determinant of its scale.[3] Imperfect risk sharing implies that entrepreneurs require compensation for holding idiosyncratic risk. In contrast, expected returns should depend only on the exposure to aggregate risk under perfect risk sharing. To assess the importance of risk-sharing frictions, we consider a parsimonious two-factor model where expected returns depend on the exposure to aggregate and idiosyncratic risk. We find that entrepreneurs with higher exposure to either source of risk receive higher expected returns. Perhaps surprisingly, a two-factor model explains most of the observed variation in entrepreneurial returns. Differences in

[1] For evidence on the under-diversification of entrepreneurs see, e.g., Moskowitz and Vissing-Jørgensen (2002), and Herranz et al. (2009) for a study focused on small businesses.

[2] A large literature studies the macroeconomic and asset-pricing implications of firm-level risk (see, e.g., Christiano et al. 2014, Gârleanu et al. 2015, and Herskovic et al. 2018), but these studies usually rely on data of public companies to discipline their quantitative exercises.

[3] See David et al. (2022) on how differences in (aggregate) risk premium lead to dispersion in MPK.

idiosyncratic volatility and exposure to aggregate risk explain almost 70% of the cross-sectional variation in average returns. Given idiosyncratic volatility and aggregate betas are measured with error, this captures only a lower bound on how much these two factors explain the variation in returns.[4] Therefore, a risk-based explanation can account for nearly all cross-section differences in expected entrepreneurial returns.

Building on recent advances in empirical asset-pricing models, we also estimate a richer factor model where returns are driven by both observable and latent aggregate factors. We allow for expected returns to reflect not only compensation for the exposure to idiosyncratic risk and the multiple sources of aggregate risk, but also a residual component orthogonal to risk. We find only a limited role for the residual component. Hence, the risk-based explanation is robust to the inclusion of latent factors.

While more than 90% of the variance is explained by idiosyncratic shocks, less than half of expected returns are compensation for such shocks. Sharpe ratios then vary by source of risk. This is inconsistent with an autarky allocation, where only total variance would matter, and also inconsistent with perfect risk sharing, as the idiosyncratic risk is priced. This evidence suggests entrepreneurs partially insure idiosyncratic shocks.

Next, we consider the determinants of entrepreneurs' risk-taking decisions, as measured by the share of net worth invested in the business. Risk-taking depends not only on the project's risk and return but also on attitudes towards risk. Households' characteristics potentially shape their risk preferences. In particular, we find substantial variation in risk-taking over the life cycle. Young entrepreneurs are nearly 40% more exposed to the business than old entrepreneurs. Differences in risk-taking cannot be explained by differences in expected returns, suggesting heterogeneity in risk tolerance across age groups.

To capture these motivating facts, we propose a general equilibrium model with two main ingredients: limited idiosyncratic insurance and finite lives with imperfect altruism. Entrepreneurs operate a technology exposed to aggregate and idiosyncratic shocks. Due to a moral hazard problem, entrepreneurs must bear a fraction of the idiosyncratic risk in equilibrium. The moral hazard parameter controls the degree of partial insurance. We identify the moral hazard parameter by matching the decomposition of expected returns into aggregate and idiosyncratic components performed in the data. Intuitively, if entrepreneurs can diversify a significant fraction of the risk, expected returns would reflect primarily compensation for holding aggregate risk. We find that entrepreneurs can diversify roughly 40% of the idiosyncratic volatility. Therefore, entrepreneurs face relevant limitations in diversifying risks, significantly affecting their investment decisions.

[4] When we sort entrepreneurs by their exposure to aggregate and idiosyncratic risk, as is standard in empirical asset pricing, we obtain that nearly 90% of the variation can be explained by these two factors.

Importantly, we assume that entrepreneurial households also receive some labor income from household members working outside the business, consistent with what we observe in the data. This implies that entrepreneurs' effective risk aversion depends inversely on the ratio of human wealth (present discounted value of future labor income) to financial wealth (investment in safe assets and the business). The human-financial wealth ratio declines over the life cycle in the data. This mechanism endogenously creates heterogeneity in risk aversion, which is essential to replicate the empirical life-cycle profiles.[5]

Entrepreneurial risk has a significant effect on inequality. On average, wealth inequality initially increases with age and then declines later in life. Heterogeneity in returns drives the initial increase in inequality, while an increase in the marginal propensity to consume (MPC) as entrepreneurs approach the end of the life cycle brings inequality down. The model roughly captures the wealth inequality patterns observed in the data.

The model also replicates the heterogeneity in risk premia observed in the data. To capture variation in expected returns across entrepreneurs, we extend the model along two dimensions: heterogeneous idiosyncratic volatility and decreasing returns to scale (DRS) in production. DRS plays a central role in generating dispersion in expected returns. Without it, the capital-labor ratio is equalized across entrepreneurs, eliminating any systematic differences in expected returns despite differences in risk.

Generating the patterns of expected returns with standard ingredients, such as skill heterogeneity and collateral constraints, can be challenging. A version of the model with skill heterogeneity, but no differences in idiosyncratic risk, is unable to generate heterogeneity in expected returns. More productive entrepreneurs end up with larger business, while expected returns are determined by the risk exposure. Introducing collateral constraints severs the link between risk and return, so expected returns are instead driven by differences in net worth for constrained entrepreneurs. However, we do not find a significant relationship between expected returns and net worth empirically. Risk exposure actually explains a larger fraction of the variation in expected returns for entrepreneurs with relatively low net worth, who are more likely to be constrained. In combination, these results point towards a risk-based explanation of entrepreneurial returns.

Having shown the model captures the salient features of our data, we study a counterfactual where entrepreneurs have access to better idiosyncratic risk. Improving risk sharing leads to a decline in the idiosyncratic risk premium and an increase in the capital stock in the long run. The effect is quantitatively significant, where a reform that reduces the risk premium by 140 basis points leads to an increase of the capital stock of 13%.

[5]The mechanism is reminiscent of work on portfolio choice with labor income (see, e.g., Bodie et al. 1992 and Viceira 2001).

We also consider the dynamic implications of relaxing risk constraints. Improving idiosyncratic insurance leads to an investment boom that lasts for a decade, accompanied by a sharp increase in the value of private businesses in the short run. Inequality falls in the long run, as entrepreneurs are less exposed to risk after the reform. In contrast, inequality increases in the short run, as the increase in the value of the business benefits richer entrepreneurs, given their larger initial investments. It takes a long time for inequality to converge to its new long-run level. This pattern is consistent with Kuznets's (1955) hypothesis over the relationship between inequality and economic development.

Considering the transitional dynamics is relevant to assess the welfare implications. Entrepreneurs are worse off in the long run after the reform, despite the benefits of better diversification, as entrepreneurs accumulate less wealth over time. In contrast, the initial generation's welfare improves with the reform. They received their bequest before the intervention, and the value of their businesses increased substantially in the short run. Therefore, most of the gains of the reform are reaped by the initial generation of entrepreneurs and wage earners, who receive higher wages given the higher capital stock.

Related literature. This paper relates to several strands of literature in macroeconomics and finance. First, the work studying how firm-level uncertainty affects asset prices and the real economy (Herskovic et al. 2016, Iachan et al. 2021).[6] While this literature focuses primarily on business-cycle fluctuations, we study how firm-level uncertainty affects the economy in the long run. Second, the work on how the entrepreneurs' lack of diversification affects real investment (Panousi and Papanikolaou 2012), capital structure (Chen et al. 2010, Herranz et al. 2015), and risk-taking (Chen and Strebulaev 2019). This work is mainly in partial equilibrium and abstracts from aggregate implications.

We also contribute to work on heterogeneous returns and inequality. This literature documents substantial heterogeneity in portfolio returns (Fagereng et al. 2019, Bach et al. 2020); it finds that private businesses are one the main sources of wealth at the top (Smith et al. 2019, Smith et al. 2020), and that return heterogeneity is important to match the observed levels of inequality (Gomez 2017, Hubmer et al. 2021). We focus on how the idiosyncratic risk premium in private businesses affects inequality.

A related literature studies the extent to which households partially insure *labor income* shocks (Blundell et al. 2008 and Kaplan and Violante 2010). Krueger and Perri (2006) and Heathcote et al. (2014) show how the degree of partial insurance can be inferred with data on consumption and labor income. We focus on entrepreneurial risk and show that the

[6] Another strand of literature studies the asset-pricing implications of labor income risk in infinite-horizon (e.g. Constantinides and Duffie 1996) and life-cycle models (e.g. Storesletten et al. 2007).

degree of partial insurance can be identified the idiosyncratic risk premium.

An extensive micro-development literature studies risk sharing (Townsend 1994, Morduch 1995) and the risk and return of production activities in developing economies (Udry and Anagol 2006, De Mel et al. 2008). The macro-development literature studies the aggregate implications of credit constraints (Buera and Shin 2013, Midrigan and Xu 2014, Moll 2014). Our approach is complementary to theirs, as we focus instead on the role of risk constraints. A related literature studies the impact of risk on entrepreneurial activity in the context of developed economies (see, e.g., Tan 2018, Robinson, 2021, and Boar et al. 2022). Our work is closer to the original model of uninsurable investment risk by Angeletos (2007), which we extend to allow for partial idiosyncratic insurance, a rich demographic structure, and aggregate risk. These extensions are crucial to capture the patterns of consumption and risk-taking observed in the microdata and derive the dynamics of inequality in response to a relaxation of risk constraints.

Organization. The rest of the paper is organized as follows. Section 2 provides evidence on entrepreneurial activity. Section 3 presents our life-cycle model with partial insurance. Section 4 discuss the life-cycle implications. Section 5 considers an extension with heterogeneous risk premia. Section 6 studies the implications of changes in the degree of idiosyncratic insurance. Section 7 concludes.

2 Motivating evidence

In this section, we provide motivating evidence on entrepreneurial activity in the context of a developing country. First, we study the determinants of entrepreneurial returns. Second, we consider how risk-taking and consumption decisions evolve over the life cycle.

Data. We use data from the Townsend Thai Monthly Survey, an intensive monthly survey initiated in 1998 in four provinces of Thailand. Two provinces, Chachoengsao and Lopburi, are semi-urban in a more developed central region near the capital, Bangkok. The other two provinces are rural, Buriram and Srisaket, and are located in the less developed northeastern region by the border of Cambodia. In each province, the survey is conducted in four villages, chosen at random within a given township. A detailed discussion of the survey can be found in Samphantharak and Townsend (2010).

Our sample covers 710 households and 14 years of monthly data, starting in January 1999. These economies were subject to various aggregate and idiosyncratic shocks during this time. Rice cultivation is affected by seasonal variations in rainfall and temperature.

Restrictions on exports to the EU affected shrimp ponds. Milk cows' productivity varies substantially over time for a given animal and over the herd. The exposure to this rich set of shocks enable us to disentangle the role of aggregate and idiosyncratic shocks.

The data collected in the Townsend Thai Monthly Survey includes information on the net income generated by the business and household total assets and liabilities. The data is rich enough to construct a detailed balance sheet for these businesses. We can compute the return on assets (ROA), measured as net profits income over business assets.[7] We also measure the fraction of entrepreneurs' wealth invested in the business and the fraction invested in safe (real or financial) assets, providing us with a measure of risk-taking. Finally, the data includes information on labor income and consumption. This allows us to characterize the savings behavior and the importance of non-business income for entrepreneurial households. See Appendix B for a detailed description of these variables.[8]

2.1 The determinants of expected entrepreneurial returns

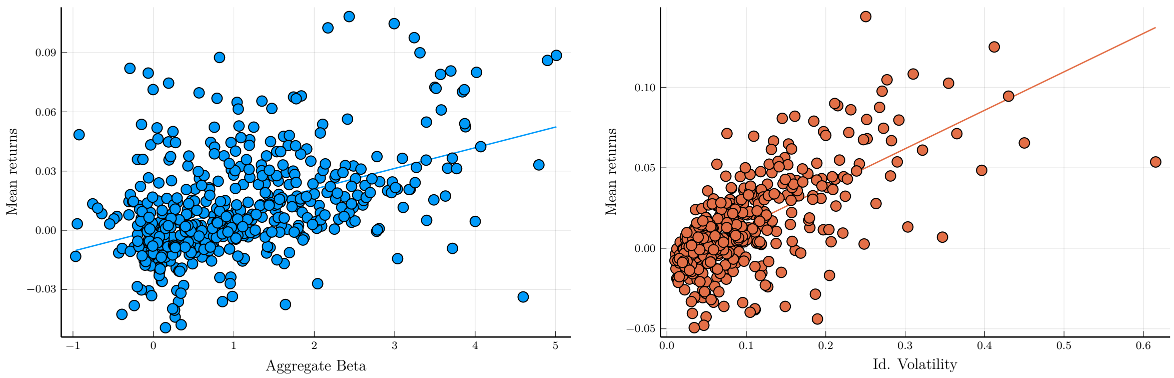
Whether and how much to invest in a business depends on its risk/return trade-off. We study this trade-off by considering the required risk compensation implicit in entrepreneurial returns. Perfect risk sharing provides a natural benchmark. Under this assumption, differences in expected returns are entirely driven by differences in the exposure to aggregate risk, as idiosyncratic shocks can be perfectly insured or diversified. On the other extreme, entrepreneurs can be in financial autarky, without access to any insurance. In this case, only the total amount of risk is relevant to entrepreneurs, causing expected returns to vary with both aggregate and idiosyncratic volatility. When idiosyncratic shocks can be partially insured or diversified, expected returns depend not only on the total variance of returns but also on the relative importance of each source of risk.

This discussion motivates using a two-factor model to explain the cross-section of expected entrepreneurial returns based on the exposure to aggregate and idiosyncratic shocks. This approach allows us to evaluate whether perfect risk sharing or autarky holds and, more importantly, whether a risk-based explanation can quantitatively account for the differences in entrepreneurial returns observed in the data. We estimate our factor model following the standard two-pass regression methodology developed by Fama and MacBeth (1973). In the first stage, we estimate entrepreneurs' exposure to aggregate risk by the slope of a time-series regression of returns for entrepreneur i, $R_{i,t}$, on the cross-

[7] Notice this measure does not capture unrealized capital gains. In Section 5, we show that differences in expected returns are driven by the dividend yield in our theory instead of capital gains.

[8] The household is the unit of measurement. We treat the household as a whole as unitary (see, e.g., Doepke and Tertilt (2016) for a discussion of unitary models of the household).

Figure 1: Average returns vs. aggregate and idiosyncratic risk



Note: The left (right) panel shows a scatter plot of average time-series returns for each entrepreneur against aggregate beta (idiosyncratic volatility). Aggregate beta is measured as the slope of the time-series regression of individual returns on the leave-one-out average return in the entrepreneur's province. Idiosyncratic volatility is calculated as the volatility of residuals from the same regression. To limit the influence of outliers, we trim 1% of the observations in the left and right tails.

sectional average return across all entrepreneurs in a given province, R_t^{agg}:

$$R_{i,t} = \alpha_i + \beta_i R_t^{agg} + \epsilon_{i,t}, \tag{1}$$

for each entrepreneur $i \in \{1, \ldots, N\}$. Notice that R_t^{agg} plays the role of the market portfolio in standard tests of the capital asset pricing model (CAPM). By averaging across entrepreneurs in a given region, idiosyncratic risk gets diversified, so β_i captures the exposure to aggregate risk for entrepreneur i.[9] We measure the exposure to idiosyncratic risk by the variance of the residuals in the above regression, $\sigma_i^2 = var[\epsilon_{i,t}]$.

Figure 1 shows how the exposure to each source of risk is related to entrepreneurs' average return. We find a strong association between average returns and exposure to aggregate and idiosyncratic risk. The left panel shows that entrepreneurs more exposed to aggregate risk have higher average returns. Similarly, the right panel shows a positive and significant relationship between average returns and idiosyncratic volatility.

Given that idiosyncratic volatility may be correlated with the aggregate beta, the positive association between idiosyncratic risk and expected returns could still be consistent with perfect risk sharing. To address this point, we run a second-stage cross-sectional regression of average return for entrepreneur i, $\overline{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$, on the estimated exposure to aggregate risk, $\hat{\beta}_i$, and idiosyncratic risk $\hat{\sigma}_i^2$:

$$\overline{R}_i = \lambda_0 + \lambda_{ag}\hat{\beta}_i + \lambda_{id}\hat{\sigma}_i^2 + u_i. \tag{2}$$

[9]Following Samphantharak and Townsend (2018), we adopt the province as the relevant geographic unit. Moreover, we compute a "leave-one-out" mean of the returns, so we regress the return of entrepreneur i on the average return across all entrepreneurs other than i on a given province.

Table 1: Cross-sectional regressions

Dependent Variable:	Mean ROA					
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
(Intercept)	-0.007	0.009***	-0.003	-0.004	0.000	-0.003***
	(0.004)	(0.002)	(0.004)	(0.002)	(0.001)	(0.001)
Beta	0.02***		0.012***	0.009**	0.009***	0.008**
	(0.004)		(0.004)	(0.003)	(0.001)	(0.001)
Id. Variance		0.125***	0.103***	0.213***		
		(0.028)	(0.029)	(0.072)		
Id. Variance (PCA)					0.103	0.248**
					(0.066)	(0.126)
Fit statistics						
Observations	541	541	541	24	541	24
R^2	0.344	0.578	0.685	0.892	0.703	0.876
Adjusted R^2	0.343	0.577	0.684	0.882	0.701	0.858

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: The left panel shows a cross-sectional regression of average returns on the aggregate beta or idiosyncratic variance (or both). Standard errors account for the uncertainty in estimating the regressors, and they are robust to contemporaneous correlations between error terms across entrepreneurs. Columns (5) and (6) show the results with the exposure to the latent factor as an additional control.

The coefficients λ_{ag} and λ_{id} correspond to the *price of aggregate and idiosyncratic risk*, respectively. They capture the required compensation for one unit of exposure to each source of risk. The standard errors for the prices of risk are obtained by embedding our two-stage procedure into a generalized method of moments (GMM) framework, where the uncertainty in the estimation of $\hat{\beta}_i$ and $\hat{\sigma}_i^2$ is accounted for.[10]

Table 1 shows the results for the second-stage regression. We find a positive and significant price of risk for β_i and σ_i^2. These two factors account for a substantial fraction of the cross-sectional variation in expected returns. Column 1 shows that the CAPM-inspired one-factor model explains a sizeable fraction of the variation in average portfolio returns, with an adjusted R^2 of 34%. Given the limited empirical success of the CAPM (see, e.g., Fama and French 1992), it is surprising that a single aggregate factor plays a significant role in explaining entrepreneurial returns. Column 2 shows that idiosyncratic risk explains an even larger fraction of the variation, with an adjusted R^2 of 58%. Together, these two factors explain most of the variation in expected returns, with an adjusted R^2 of 68%. To put this result in perspective, the R^2 for the three-factor Fama-French model in the cross-section of 25 equity portfolios sorted by size and book to market is 73%, comparable to the R^2 for our two-factor model in a cross-section of more than 500 entrepreneurs.

An important issue is that our risk exposure measures are potentially noisy estimates

[10]See Cochrane (2009) for a discussion on how to correct the standard errors in two-pass regressions.

of the actual beta and idiosyncratic volatility, so measurement error may bias our results. To deal with this issue, we follow the standard procedure in empirical asset pricing (see, e.g., Black et al. 1972) and group entrepreneurs into bins, or portfolios, according to a 5×5 double sort based on aggregate beta and idiosyncratic risk.[11] The group-level betas and idiosyncratic variance are arguably better measured than their individual-level counterparts, which limits the impact of measurement error.

Column 4 shows the results. We obtain a similar price of aggregate risk and a larger price of idiosyncratic risk, consistent with some attenuation bias in this case. The two risk factors account for 88% of the variation in group-level returns. Figure 2 shows the fit of the two-factor model by comparing predicted and realized average returns. The model successfully accounts for the substantial variation in expected returns observed in our data. Formally, we cannot reject that the data is generated by the two-factor model, and the remaining variation in the cross-sectional regression is due to sampling variability. Both factors are necessary to achieve this success. In particular, we can reject the hypothesis of a single-factor model based on either source of risk. Therefore, this evidence suggests that a two-factor model is necessary to explain the cross-section of entrepreneurial returns.

Dealing with omitted factors. The success of the two-factor model relies on our ability to capture all relevant aggregate factors. To the extent entrepreneurs are exposed to multiple aggregate factors, the error term in Eq. (1) would reflect not only idiosyncratic shocks but also the influence of omitted aggregate factors. The presence of omitted factors may bias the estimation of risk exposures and prices of risk.[12] Building on recent advances on the estimation of asset-pricing models, we discuss a method to address this concern.

Consider the following generalized model where entrepreneurial returns are exposed to a vector $f_t \in \mathbb{R}^K$ of aggregate factors. Excess returns for entrepreneur i is given by:

$$R_{i,t} = \mu_i + \beta_i^\top f_t + \epsilon_{i,t},$$

where $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$ and $f_t \perp \epsilon_{i,t}$. Factors have mean zero, so μ_i denotes expected excess returns. Expected excess returns are related to risk exposure as follows:

$$\mu_i = \lambda_0 + \lambda_{ag}^\top \beta_i + \lambda_{id} \sigma_i^2 + u_i. \quad (3)$$

The vector $\lambda_{ag} \in \mathbb{R}^K$ corresponds to the price of risk for the aggregate factors, so $\lambda_{ag}^\top \beta_i$

[11] Even though these portfolios are not tradeable, forming portfolios allows us to reduce the noise in the estimation and better capture the risk/return relationship. To avoid within-portfolio diversification, we assign to the portfolio the average idiosyncratic volatility instead of the volatility of the average.

[12] See, e.g., Burmeister and McElroy (1988) for a discussion of the biases caused by omitted factors.

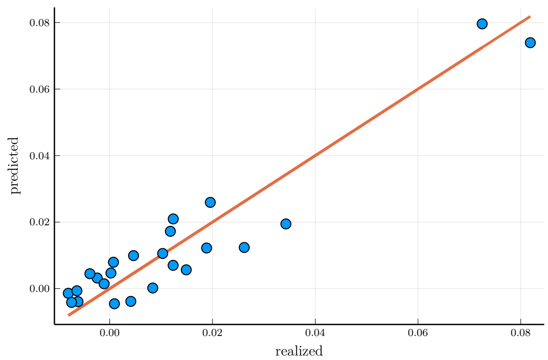


Figure 2: Realized vs. predicted returns

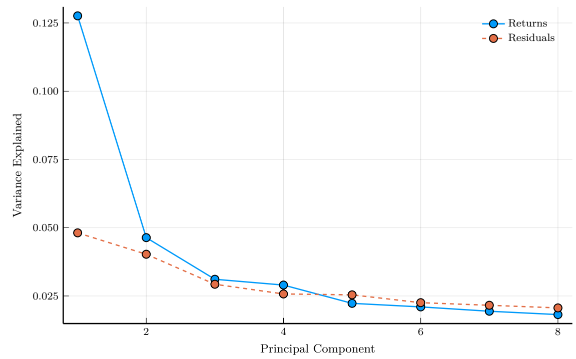


Figure 3: Scree plot

Note: The left panel shows the 45° degree line and a scatter plot of the predicted returns of the two-factor model from column (4) of Table 1 and actual average returns for the portfolio-level analysis. The right panel shows the scree plot of the eigenvalues of the covariance matrix of returns and the residuals of the first-stage regression.

represents compensation for exposure to aggregate risk. The coefficient $\lambda_{id} \in \mathbb{R}$ is the price of idiosyncratic risk. Finally, $u_i \sim \mathcal{N}(0, \sigma_u^2)$ captures any potential misspecification, absorbing the influence of potential non-risk based elements.

Importantly, a subset of factors may be latent, i.e., $f_t = [f_{o,t}^\top, f_{u,t}^\top]^\top$, where $f_{o,t}$ is observable and $f_{u,t}$ is unobservable. In this case, we can express realized returns as follows:

$$R_{i,t} = \mu_i + \beta_{i,o}^\top f_{o,t} + \tilde{\epsilon}_{i,t}, \tag{4}$$

where $\tilde{\epsilon}_{i,t} = \beta_{i,u}^\top f_{u,t} + \epsilon_{i,t}$. When $f_{u,t}$ is correlated with $f_{o,t}$, omitted factors bias the estimation of the aggregate risk exposure, $\beta_{i,o}$. Differences in residual variance, $Var[\tilde{\epsilon}_{i,t}]$, would reflect not only idiosyncratic shocks, but also exposure to latent factors. Finally, the estimation of the prices of risk in the second-stage regression would also be biased.

When omitted factors represent an important fraction of the variation in entrepreneurial returns, the residuals of regression (4) will have a factor structure. The omitted factors would drive correlation across residuals. Up to a rotation, the loadings on latent factors can be recovered using the principal components of the residuals. Given these loadings, we can control for the exposure to latent factors in the second-stage regression.

Based on this observation, Giglio and Xiu (2021) proposed a method to estimate factor models with omitted factors in a setting with large cross-sectional and time-series dimensions, i.e., as $N \to \infty$ and $T \to \infty$. They show that the prices of risk can be consistently estimated by running a three-pass regression. The first stage is the same as before, where we run a sequence of time-series regressions of returns on the observable factors. The second stage consists of applying a principal components analysis (PCA) to the residuals of the first-stage regression. The third stage is a cross-sectional regression of average returns on the estimated factor loadings for both observable and latent factors. We implement this method to assess the role of latent factors in our setting. We discuss the estimation in detail in Appendix B.5.

We start by selecting the number of latent factors. Figure 3 shows the scree plot —- which exhibits the share of variance explained by each principal component — for returns and residuals of the first-stage regression.[13] As before, the observable factor corresponds to the average return on the region. While returns show a clear factor structure, with the typical "elbow" pattern in the scree plot, the residuals do not. This is indicative that the observable factor accounts for most of the common variation in returns. Formally, we implement the test that the number of latent factors is equal to zero proposed by Onatski (2009). We cannot reject the null hypothesis that the number of latent factors is zero.

Nevertheless, we proceed by assuming a single latent factor to assess how sensitive our results are to the presence of omitted factors. Column 5 of Table 1 shows the results of the cross-sectional regression.[14] In this case, we regress average returns on the estimated factor loadings for both observable and latent factors (not shown in the table) as well as the variance of residuals of the PCA computed in the second stage. Therefore, our measure of idiosyncratic risk has been purged of the influence of latent factors. The results are very similar to the ones in column 3, consistent with the weak influence of latent factors. Column 6 shows the results for the portfolio-level analysis. The results are in line with the ones in column 4. As in that case, we cannot reject that all the variation in expected returns is explained by exposure to aggregate and idiosyncratic risk.

These results suggest that our measure of residual variance mostly captures exposure to idiosyncratic shocks, instead of reflecting omitted factors. Moreover, the fact that our factor model explains most of the variation in expected returns indicates that a non-risk based explanation, captured by u_i in Eq. (3), plays only a minor role in our setting.

Appendix B shows that our results are robust to a range of extensions. We show that our factor model can accommodate time variation in factor loadings and prices of risk. We discuss the impact of (classical) measurement error in returns. Finally, to deal with concerns related to overfitting, we show our results hold when we use shrinkage estimators for the idiosyncratic variance, betas, and expected returns.

Risk and return decomposition. To better grasp the role of each risk factor, we provide a decomposition of entrepreneurial returns into an aggregate and idiosyncratic component.

[13] Given we have an unbalanced panel, we apply an iterative procedure to compute the principal components (see, e.g., Stock and Watson 2002). Appendix B.5 discusses the procedure in detail.

[14] Following Giglio et al. (2021), we compute the standard errors of the cross-section regression in the three-pass procedure using a wild bootstrap method.

Table 2: Aggregate and idiosyncratic components of risk and return

	Risk premium	% of returns	Volatility	% of variance	Sharpe ratio
Total returns	4.4%	100%	21.0%	100%	0.21
Aggregate component	2.4%	54.7%	5.1%	6.0%	0.47
Idiosyncratic component	2.0%	45.3%	20.3%	94.0%	0.10

Note: The aggregate component is measured as the coefficient on the aggregate beta on the cross-sectional regression times the average beta across entrepreneurs. The idiosyncratic component is measured as the cross-sectional regression coefficient on idiosyncratic risk times the idiosyncratic variance averaged across entrepreneurs. The variance decomposition is computed at the individual level based on the results from the first-stage regression and then averaged across all entrepreneurs.

Table 2 decomposes the risk premium and variance of returns based on the results from Table 1. Given the limited evidence of latent factors, we focus on the results in column 4. Most risk is idiosyncratic, accounting for 94% of the variance, while idiosyncratic shocks account for nearly half of the risk premium. The Sharpe ratio of the aggregate component is then nearly five times larger than the Sharpe ratio of the idiosyncratic component.[15]

The results from Table 2 are inconsistent with the perfect risk sharing and autarky benchmarks. The fact that the idiosyncratic risk premium explains almost half of total expected returns indicates that limits to diversification are substantial, which allows us to reject the perfect risk sharing. Financial autarky is inconsistent with the observed pattern in the Sharpe ratio. As shown in Appendix E.3, the share of variance of the idiosyncratic component should coincide with its share of expected returns under autarky. Equivalently, the Sharpe ratio should be proportional to the volatility.[16] Hence, the Sharpe ratio for the idiosyncratic component should be four times *larger* than the one for the aggregate component, but we observe the opposite pattern in the data. These results suggest that partial insurance is relevant to explain the empirical patterns involving risk and return. We revisit this decomposition through the lenses of our structural model in Section 6.

2.2 Risk-taking and savings over the life cycle

Next, we consider entrepreneurs' risk-taking and savings decisions. The decision of how much to invest depends on the risk and return of the entrepreneurial activity and households' appetite for risk. Behavior towards risk is potentially shaped by households' characteristics, such as age and household size. We study how risk-taking and consumption

[15] Samphantharak and Townsend (2018) computes a similar decomposition based on entrepreneurs' total portfolio returns, including risky and safe assets. The decomposition of the portfolio's risky component, which is our focus, is the relevant one to map into the structural model in Section 3.

[16] In a two-factor model, the risk premium is given by $p^{ag}\sigma_{ag} + p^{id}\sigma_I$, where (p^{ag}, p^{id}) is the return per unit of risk (the prices of risk) and (σ_{ag}, σ_I) the volatilities. Perfect risk sharing corresponds to $p^{id} = 0$. In autarky, the prices of risk (or Sharpe ratio) are $p^{ag} = \gamma\sigma_{ag}$ and $p^{id} = \gamma\sigma_I$, so the risk premium is $\gamma(\sigma_{ag}^2 + \sigma_I^2)$.

Table 3: Risk-taking and consumption behavior over the life cycle

	Risk-taking			Consumption-wealth ratio		
	(1)	(2)	(3)	(4)	(5)	(6)
Age group: 1	0.30***	0.31***	0.27***	0.14***	0.14***	0.10***
	(0.01)	(0.01)	(0.02)	(0.009)	(0.008)	(0.007)
Age group: 2	0.27***	0.28***	0.25***	0.13***	0.13***	0.11***
	(0.01)	(0.01)	(0.01)	(0.009)	(0.007)	(0.007)
Age group: 3	0.27***	0.27***	0.25***	0.12***	0.12***	0.11***
	(0.01)	(0.01)	(0.01)	(0.008)	(0.007)	(0.007)
Age group: 4	0.23***	0.23***	0.22***	0.10***	0.10***	0.08***
	(0.02)	(0.01)	(0.01)	(0.007)	(0.006)	(0.006)
Age group: 5	0.22***	0.22***	0.21***	0.11***	0.11***	0.09***
	(0.01)	(0.01)	(0.01)	(0.008)	(0.008)	(0.007)
Year FE		Yes	Yes		Yes	Yes
Additional controls			Yes			Yes
N	8,680	8,680	6,499	8,548	8,548	6,495
Adjusted R^2	0.015	0.025	0.139	0.010	0.020	0.070

Clustered (year & household) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

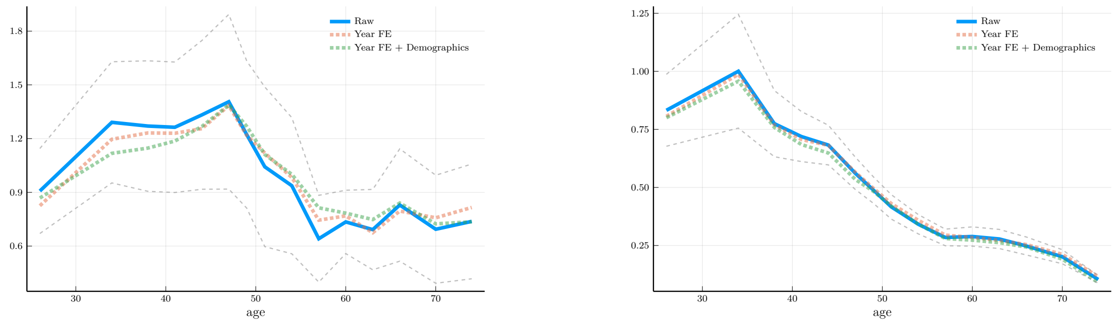
Note: Age effects correspond to the conditional expectation of the outcome variable evaluated at the mean of the continuous controls and averaged over the year, province, and sector fixed-effects (when included). Additional controls include dummies for the province, household size, the number of kids, a set of sector affiliation dummies, the entrepreneur's average return on the business and the exposure to aggregate and idiosyncratic risk.

decisions depend on expected returns, age, and a range of demographic controls.

To ensure stationarity, we consider the ratio of the business value to financial wealth, our measure of risk-taking, and the ratio of consumption to financial wealth. By looking at ratios instead of levels, we limit the influence of aggregate shocks and focus on more stable relationships. We consider five age groups, ranging from 25 to 80 years old, where group 1 is the youngest and group 5 is the oldest. The cutoffs for each group are chosen such that we have roughly the same number of households in each group. We cluster standard errors by household and year to account for shocks potentially correlated over time or across households. We aggregate the data to the annual level to better isolate the age effects. To limit the influence of outliers, we trim the data at the 1% level at both tails. The details of the empirical methodology are described in Appendix B.6.

We compute age effects controlling for year fixed-effects and additional controls following the methodology of Kaplan (2012). Table 3 presents the results. We find that risk-taking has a strong life-cycle pattern. Column 1, the case without any additional controls, shows that the amount invested in the business declines sharply with age, with the coefficient for the oldest group being roughly 30% smaller than for the youngest group. A similar pattern holds as we add more controls. Time fixed-effects do not change the

Figure 4: Labor Income and Human-Financial Ratio



(a) Labor Income

(b) Human-financial wealth ratio

Note: Labor income is normalized by the average labor income across all entrepreneurs. Human wealth is the present discounted value of future labor income, as measured by average income for each age. Grey dashed lines represent the 95% confidence interval for the raw estimates, i.e., the ones without any controls.

results, as shown in column 2, which suggests that aggregate shocks affect the amount invested in the business and financial wealth but leave their ratio roughly unchanged. Column 3 includes demographic, geographic controls, and risk/return controls, including the household size, the number of children, province, sector affiliation, average business return and risk exposure. Once again, we find substantial variation in risk-taking over the life cycle. Given these patterns are not driven by differences in risk or expected returns, the results suggests a role for differences in attitudes towards risk.

The consumption-wealth ratio also has a strong life-cycle pattern. However, instead of declining monotonically with age as our risk-taking measure, we find a U-shaped pattern: consumption-wealth ratio initially decreases with age but increases for the oldest group.

Both risk-taking and the consumption-wealth ratio are potentially affected by the presence of outside income. The left panel of Figure 4 shows the life-cycle profile of labor income received by members of an entrepreneurial household. The right panel shows the ratio of the household's human wealth — the present discounted value of future labor income — to financial wealth. The fact that the human-financial ratio declines with age will play an important role in rationalizing the patterns of risk-taking and consumption.

We derive three main conclusions from these results. First, the cross-sectional variation in risk-taking and consumption has a substantial life-cycle component. Second, the life-cycle patterns in risk-taking are mainly driven by differences in risk appetite instead of differences in risk or average return. Third, year fixed-effects do not significantly affect the life-cycle patterns. This suggests a relatively stationary environment despite the presence of aggregate shocks. These facts, in conjunction with the importance of partial insurance discussed in Section 2.1, motivate the ingredients of our theoretical model.

3 A life-cycle model of entrepreneurial risk taking

We consider a model of entrepreneurial activity with two main ingredients: i) imperfect idiosyncratic insurance and ii) finite lives with imperfect altruism. These two ingredients play a crucial role in capturing the empirical patterns from Section 2.

3.1 Environment

Time is continuous, and the economy is populated by entrepreneurs and wage earners. Population grows at rate g, and the share of entrepreneurs in the population is constant and given by χ_e. The set of entrepreneurs and wage earners alive at period t are denoted by $\mathcal{E}_t$ and $\mathcal{W}_t$, respectively. Entrepreneurs live for T periods and leave bequests to their offspring. For simplicity, we assume that wage earners have an infinite horizon. While all households receive labor income, only entrepreneurs can access a production technology. Production is exposed to both aggregate and idiosyncratic shocks. Households buy and sell aggregate insurance in a frictionless market, but they have access only to imperfect idiosyncratic insurance. Households borrow and lend at a riskless rate r_t. We now describe in detail the technology, preferences, and financial frictions both types of households face.

Technology. Entrepreneur i combines capital $k_{i,t}$ and hired labor $l_{i,t}$ to produce a final homogeneous good $\tilde{y}_{i,t}$, the numeraire in this economy, using the technology:

$$\tilde{y}_{i,t} = A_t k_{i,t}^{\alpha} l_{i,t}^{1-\alpha}, \tag{5}$$

and we denote scaled output by $y_t = \tilde{y}_t / A_t$. We adopt this convention throughout the paper: variables that grow with aggregate productivity A_t are denoted with a tilde, and the corresponding scaled variable is denoted without a tilde.

Productivity is subject to aggregate shocks and follows a geometric Brownian motion:

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dZ_t, \tag{6}$$

where Z_t is a standard Brownian motion.

Entrepreneurs can adjust their capital stock by investing in new capital or buying capital from another entrepreneur. The investment technology is risky and subject to adjustment costs. Given a total investment of $\iota_{i,t} A_t k_{i,t}$, capital evolves according to

$$\frac{dk_{i,t}}{k_{i,t}} = \left(\Phi(\iota_{i,t}) - \delta\right) dt + \sigma_I dZ_{i,t}. \tag{7}$$

$Z_{i,t}$ is an idiosyncratic Brownian motion for entrepreneur i, which is independent across entrepreneurs. The investment function $\Phi(\cdot)$ satisfies $\Phi(0) = 0$, $\Phi'(\cdot) > 0$, and $\Phi''(\cdot) < 0$. The concavity of $\Phi(\cdot)$ captures the presence of adjustment costs. Notably, investment is risky and subject to idiosyncratic shocks. Entrepreneurs can also adjust their capital stock by buying capital from other entrepreneurs at the price $\tilde{q}_t = q_t A_t$. The market value of the business is given by $\tilde{q}_t k_{i,t}$. In equilibrium, q_t will be non-stochastic, so the relative price of capital $\tilde{q}_t$ moves proportionally with aggregate productivity shocks.

The evolution of the aggregate capital stock, $k_t = \int_{\mathcal{E}_t} k_{i,t} di$, is not directly affected by idiosyncratic shocks, as these shocks get diversified in the aggregate:

$$dk_t = \left[\int_{\mathcal{E}_t} \left(\Phi(\iota_{i,t}) - \delta \right) k_{i,t} di \right] dt. \tag{8}$$

The return on investing in the project is the sum of the dividend yield and capital gains:

$$dR_{i,t} = \frac{\tilde{y}_{i,t} - \tilde{w}_t l_{i,t} - \iota_{i,t} A_t k_{i,t}}{\tilde{q}_t k_{i,t}} dt + \frac{d(\tilde{q}_t k_{i,t})}{\tilde{q}_t k_{i,t}} \equiv \mu^R_{i,t} dt + \sigma_A dZ_t + \sigma_I dZ_{i,t},$$

where $\tilde{w}_t = w_t A_t$ denotes the wage rate and $\mu^R_{i,t}$ is the expected return. Using Ito's lemma to compute expected capital gains, the expected return on the project is

$$\mu^R_{i,t} \equiv \frac{y_{i,t} - w_t l_{i,t} - \iota_{i,t} k_{i,t}}{q_t k_{i,t}} + \frac{\dot{q}_t}{q_t} + \mu_A + \Phi(\iota_{i,t}) - \delta. \tag{9}$$

Preferences. Entrepreneurs live for T periods. They have isoelastic preferences over consumption $\tilde{c}_{i,t}$ with curvature parameter γ and derive utility of leaving bequests:

$$\mathbb{E}_{s_i} \left[\int_{s_i}^{s_i+T} e^{-\rho(t-s_i)} \frac{\tilde{c}_{i,t}^{1-\gamma}}{1-\gamma} dt + e^{-\rho(T-s_i)} (1-\psi)^{\gamma} V^* \frac{\tilde{n}_{i,s_i+T}^{1-\gamma}}{1-\gamma} \right], \tag{10}$$

$\tilde{n}_{i,t}$ denotes financial wealth (or net worth), and s_i denotes the entrepreneur's birthdate.

The parameter ψ measures the strength of the bequest motive. If $\psi = 1$, entrepreneurs give no weight to their offspring. If $\psi = 0$, entrepreneurs behave as if they have infinite horizon.[17] The case $0 < \psi < 1$ captures imperfect altruism.

Entrepreneurial households receive labor income in addition to business income, consistent with the observation that households have multiple sources of income in our data. Labor supply is denoted by $\bar{l}_{i,t}$, and it varies deterministically over the life cycle.[18]

[17] V^*, given in Appendix A.1, equals the value-function coefficient for an infinite-horizon agent.

[18] Notice that $\bar{l}_{i,t}$ denotes labor exogenously supplied by the household, while $l_{i,t}$ denotes labor demand.

Financial friction. Entrepreneurs face a moral hazard problem, similar to the one in He and Krishnamurthy (2012) and Di Tella (2017). We follow Di Tella (2017) and restrict attention to short-term contracts. Aggregate shocks are perfectly observable by all households, while idiosyncratic shocks are only observed by the entrepreneur. An entrepreneur can divert capital, but a fraction $1-\phi$ of the diverted capital is lost in the process. The parameter $\phi \in (0,1)$ controls the severity of the moral hazard problem.

Following the literature on dynamic moral hazard, the solution to the contracting problem can be implemented by a market structure where entrepreneurs have access to both aggregate and idiosyncratic insurance as well as a riskless asset with return r_t. There is no limit on aggregate insurance, as aggregate shocks are perfectly observable. The quantity of idiosyncratic insurance is limited for incentive purposes. Formally, entrepreneur i pays $p_t^{ag}\tilde{\theta}_{i,t}^{ag}$ to reduce aggregate volatility by $\tilde{\theta}_{i,t}^{ag}$, where p_t^{ag} denotes the price of aggregate insurance. Entrepreneur i can buy idiosyncratic insurance $\tilde{\theta}_{i,t}^{id}$ at zero cost in equilibrium, as insurance providers perfectly diversify across entrepreneurs. However, the amount of idiosyncratic insurance is limited by the *skin-in-the-game* constraint:

$$\tilde{\theta}_{i,t}^{id} \leq (1-\phi)\tilde{q}_t k_{i,t}\sigma_I. \tag{11}$$

This particular market structure represents one possible implementation of the optimal contract allocation. For instance, instead of formal insurance contracts, this implementation may capture informal insurance arrangements. As documented by, e.g., Kinnan and Townsend (2012), kinship networks play an important role in allowing households to share idiosyncratic risk. The insurance constraint can also be interpreted as an equity constraint, where entrepreneurs are unable to freely sell claims on their business to a diversified set of investors, as in e.g. Chen et al. (2010) or Panousi and Papanikolaou (2012).

Constraint (11) is binding in equilibrium, so entrepreneurs are *insurance-constrained*. To focus on entrepreneurial risk, we initially abstract from borrowing constraints and discuss their role in Section 5.2. Households face only a natural borrowing limit:

$$\tilde{n}_{i,t} \geq -\tilde{h}_{i,t}, \tag{12}$$

Entrepreneurs' problem. Entrepreneur i with age a_i chooses the stochastic processes $(\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_i^{id}, k_i, l_i, \iota_i)$, taking prices $(\tilde{q}, \tilde{w}, r, p^{ag})$ as given, to solve the following problem:

$$\tilde{V}_t(\tilde{n}_i, a_i) = \max_{\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_i^{id}, k_i, l_i, \iota_i} \mathbb{E}_t \left[\int_0^{T-a_i} e^{-\rho z} \frac{\tilde{c}_{i,t+z}^{1-\gamma}}{1-\gamma} dz + e^{-\rho(T-a_i)}(1-\psi)^{\gamma} V^* \frac{\tilde{n}_{i,t+T-a_i}^{1-\gamma}}{1-\gamma} \right], \tag{13}$$

subject to (11), (12), non-negativity constraints $c_{i,t}, k_{i,t} \geq 0$, and the law of motion of $\tilde{n}_{i,t}$

$$d\tilde{n}_{i,t} = \left[(\tilde{n}_{i,t} - \tilde{q}_t k_{i,t})r_t + \tilde{q}_t k_{i,t}\mu^R_{i,t} - p^{ag}_t \tilde{\theta}^{ag}_{i,t} + \tilde{w}_t \bar{l}_{i,t} - \tilde{c}_{i,t}\right] dt + \left(\tilde{q}_t k_{i,t}\sigma_A - \tilde{\theta}^{ag}_{i,t}\right) dZ_t + \left(\tilde{q}_t k_{i,t}\sigma_I - \tilde{\theta}^{id}_{i,t}\right) dZ_{i,t},$$

given initial financial wealth $\tilde{n}_{i,t} = \tilde{n}_i > -\tilde{h}_{i,t}$.

The term in brackets in the expression above is the expected change in financial wealth. The entrepreneur invests $\tilde{n}_{i,t} - \tilde{q}_t k_{i,t}$ in the riskless asset, with rate of return r_t, and she invests the amount $\tilde{q}_t k_{i,t}$ in the risky business technology, with expected rate of return $\mu^R_{i,t}$. The cost of aggregate insurance is $p^{ag}_t \tilde{\theta}^{ag}_{i,t}$. The entrepreneur receives labor income $\tilde{w}_t \bar{l}_{i,t}$ and consumes $\tilde{c}_{i,t}$. The last two terms represent the exposure to aggregate and idiosyncratic risk, which equals the net of insurance risk exposure from the business.

Wage earners' problem. Wage earners do not have access to a production technology. To simplify exposition, we assume they have an infinite horizon. We show in Appendix E.2 that allowing for finite lives and a bequest motive, or allowing an endogenous choice to become an entrepreneur, does not change our main results. Wage earners and entrepreneurs share a per-period isoelastic utility function with curvature parameter γ.[19] As often assumed in models with heterogeneous returns, as e.g. Kiyotaki and Moore (1997), wage earners and entrepreneurs have different discount rates.

The problem of wage earner $j \in \mathcal{W}_t$ is given by

$$\tilde{V}^w_t(\tilde{n}_j) = \max_{\tilde{c}_j, \tilde{\theta}^{ag}_j} \mathbb{E}_t \left[\int_t^\infty e^{-\rho_w(z-t)} \frac{\tilde{c}^{1-\gamma}_{j,z}}{1-\gamma} dz\right], \tag{14}$$

subject to non-negativity constraint $\tilde{c}_{j,t} \geq 0$, $\tilde{n}_{j,t} \geq \tilde{h}_{j,t}$, where $\tilde{h}_{j,t}$ denotes their human wealth, and the law of motion of financial wealth $\tilde{n}_{j,t}$ is

$$d\tilde{n}_{j,t} = \left[\tilde{n}_{j,t} r_t - p^{ag}_t \tilde{\theta}^{ag}_{j,t} + \tilde{w}_t \bar{l}_{j,t} - \tilde{c}_{j,t}\right] dt - \tilde{\theta}^{ag}_{j,t} dZ_t,$$

given initial financial wealth $\tilde{n}_{j,t} = \tilde{n}_j > -\tilde{h}_{j,t}$. $\tilde{\theta}^{ag}_{j,t}$ can take positive or negative values, so wage earners can choose to either buy or provide aggregate insurance to entrepreneurs.[20]

Equilibrium. We provide below a definition of the competitive equilibrium.

[19] As shown by Swanson (2012), and consistent with Lemma 1 below, the coefficient of relative risk aversion in the presence of labor is affected by, but is not equal to, γ.

[20] Wage earners also provide idiosyncratic insurance. We have already imposed that they can diversify the exposure to idiosyncratic risk, so their financial wealth is only exposed to aggregate risk.

Definition 1. *The competitive equilibrium is a set of aggregate stochastic processes: the capital stock k, the interest rate r, the wage rate $\tilde{w}$, the relative price of capital $\tilde{q}$, and the price of aggregate insurance p^{ag}; a set of stochastic processes for each entrepreneur $i \in \mathcal{E}_t$ and wage earner $j \in \mathcal{W}_t$: consumption $\tilde{c}_i$, financial wealth $\tilde{n}_i$, capital k_i, labor l_i, aggregate insurance $\tilde{\theta}_i^{ag}$, and idiosyncratic insurance $\tilde{\theta}_i^{id}$ for $i \in \mathcal{E}_t$; consumption $\tilde{c}_j$ and aggregate insurance $\tilde{\theta}_j^{ag}$ for $j \in \mathcal{W}_t$ such that:*

(a) *Aggregate capital stock satisfies the law of motion (8), given the initial capital stock k_0.*

(b) $(\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_i^{id}, k_i, l_i, \iota_i)$ *solve entrepreneurs' problem (13), given* $(\tilde{q}, \tilde{w}, r, p^{ag})$.

(c) $(\tilde{c}_j, \tilde{\theta}_j^{ag})$ *solve wage earners' problem (14), given* $(\tilde{w}, r, p^{ag})$.

(d) *Markets clear for all $t \geq 0$:*

i. *Goods:* $\int_{\mathcal{E}_t} \tilde{c}_{i,t} di + \int_{\mathcal{W}_t} \tilde{c}_{j,t} dj + \int_{\mathcal{E}_t} \iota_{i,t} A_t k_{i,t} di = \int_{\mathcal{E}_t} \tilde{y}_{i,t} di.$

ii. *Capital and labor:* $\int_{\mathcal{E}_t} k_{i,t} di = k_t$ *and* $\int_{\mathcal{E}_t} l_{i,t} di = \int_{\mathcal{E}_t} \bar{l}_{i,t} di + \int_{\mathcal{W}_t} \bar{l}_{j,t} dj.$

iii. *Aggregate insurance:* $\int_{\mathcal{E}_t} \tilde{\theta}_{i,t}^{ag} di + \int_{\mathcal{W}_t} \tilde{\theta}_{j,t}^{ag} dj = 0$

iv. *Riskless bond:* $\int_{\mathcal{E}_t} [\tilde{n}_{i,t} - \tilde{q}_t k_{i,t}] di + \int_{\mathcal{W}_t} \tilde{n}_{j,t} dj = 0.$

3.2 Solution to entrepreneurs' problem

We describe next the solution to the entrepreneurs' problem. We focus on a stationary equilibrium, where scaled aggregate variables are constant, that is, $w_t = w$ and $q_t = q$.

Maximizing expected returns. As $(l_{i,t}, \iota_{i,t})$ enters entrepreneurs' maximization problem only through the expected return on the business, given in Equation (9), they choose these variables to maximize expected returns. Labor demand takes the usual form:

$$w = (1-\alpha)\left(\frac{k_{i,t}}{l_{i,t}}\right)^{\alpha}. \tag{15}$$

The capital-labor ratio is equalized across entrepreneurs and coincides with the aggregate capital-labor ratio $K_t \equiv k_t/\bar{l}_t$, where k_t denotes the aggregate capital stock and $\bar{l}_t$ denotes the aggregate labor supply. In a stationary equilibrium, capital grows at the same rate as labor supply, which grows with the population at a rate g.

The investment rate $\iota_{i,t}$ satisfies the optimality condition

$$\Phi'(\iota_{i,t}) = \frac{1}{q} \Rightarrow \iota_{i,t} = (\Phi')^{-1}\left(\frac{1}{q}\right) \equiv \iota(q), \tag{16}$$

where $\iota(q)$ is increasing in q, given the concavity of $\Phi(\cdot)$. Substituting Equations (15) and (16) into Equation (9), we obtain

$$\mu^R = \frac{\alpha K^{\alpha-1} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta, \tag{17}$$

where $\mu^R_{i,t} = \mu^R$ in a stationary equilibrium, so expected returns are equalized across entrepreneurs. Realized returns are, of course, still heterogeneous.

Human and total wealth. The lemma below shows that the relevant notion of wealth is *total wealth*, $\omega_{i,t} \equiv n_{i,t} + h_{i,t}$, the sum of financial wealth and human wealth. In particular, the entrepreneur's value function depends on total wealth $\omega_{i,t}$ and on age $a_{i,t} = t - s_i$. All proofs are provided in Appendix A.

Lemma 1. *Suppose the economy is in a stationary equilibrium. Human wealth satisfies*

$$\frac{\partial h(a)}{\partial a} = \left(r + p^{ag}\sigma_A - \mu_A\right) h(a) - w\bar{l}(a), \tag{18}$$

given $h(T) = 0$. The (scaled) value function is given by[21]

$$V(n,a) = \zeta(a)^{-\frac{1}{\gamma}} \frac{(n + h(a))^{1-\gamma}}{1-\gamma}, \tag{19}$$

where $\zeta(a)$ is the consumption to total wealth ratio. The effective risk aversion is given by

$$-\frac{V_{nn} n}{V_n} = \frac{\gamma}{1 + \frac{h(a)}{n}}. \tag{20}$$

The first part of Lemma 1 implies that human wealth is the present discounted value of future labor income, where the discount rate incorporates the aggregate risk premium $p^{ag}\sigma_A$. This extra discount is required as wages move with aggregate productivity. Human wealth being a risky asset is consistent with Benzoni et al. (2007), which shows that human wealth becomes highly correlated with stocks when aggregate output and labor income are cointegrated, as in our model. As human wealth depends only on the entrepreneur's age, we drop the dependence on the household, i.e., $h_{i,t} = h(a_{i,t})$.

The second part of Lemma 1 gives the value function, an age-dependent CRRA function of total wealth. Notably, the entrepreneur's effective risk aversion depends on $h_{i,t}/n_{i,t}$, the *human-financial wealth* ratio, which varies substantially over the life cycle in the data.

[21] The scaled value function is related to the original value function by $\tilde{V}_t(\tilde{n}, a) = A_t^{1-\gamma} V\left(\frac{\tilde{n}}{A_t}, a\right)$.

Policy functions. The next proposition provides the entrepreneurs' policy functions.

Proposition 1. *Suppose the economy is in a stationary equilibrium. Then,*
i) *Demand for capital is given by*

$$\frac{qk_{i,t}}{n_{i,t}} = \frac{1+\frac{h_{i,t}}{n_{i,t}}}{\gamma}\frac{p^{id}}{\phi\sigma_I}, \tag{21}$$

where p^{id} is the shadow price of idiosyncratic insurance, which is given by

$$p^{id} = \frac{\mu^R - r - p^{ag}\sigma_A}{\phi\sigma_I}. \tag{22}$$

ii) *The price of aggregate insurance is $p^{ag} = \gamma\sigma_A$, and the quantity of aggregate insurance is*

$$\theta^{ag}_{i,t} = (qk_{i,t} - n_{i,t})\sigma_A. \tag{23}$$

iii) *The consumption-wealth ratio is given by*

$$\frac{c_{i,t}}{n_{i,t}} = \frac{\bar{r}}{1-\psi e^{-\bar{r}(T-a_i)}}\left(1+\frac{h_{i,t}}{n_{i,t}}\right), \tag{24}$$

where $\bar{r} \equiv \frac{1}{\gamma}\rho + \left(1-\frac{1}{\gamma}\right)\left[r + \frac{(p^{ag})^2+(p^{id})^2}{2\gamma}\right]$.

The demand for capital has three components: entrepreneur's effective risk aversion, $\gamma/(1+\frac{h_{i,t}}{n_{i,t}})$, the price of idiosyncratic insurance, p^{id}, and the quantity of non-diversified risk, $\phi\sigma_I$. The human-financial wealth ratio drives differences in effective risk aversion, creating cross-sectional dispersion in risk-taking.

Equation (22) shows that the shadow price of idiosyncratic insurance, the Lagrange multiplier on the skin-in-the-game constraint, corresponds to the return per unit of risk (the Sharpe ratio) of an investor who fully insures the project against aggregate risk. In equilibrium, this Sharpe ratio is positive, so the skin-in-the-game constraint is always binding, that is, $\theta^{id}_{i,t} = (1-\phi)q_t k_{i,t}\sigma_I$. Entrepreneurs purchase as much idiosyncratic insurance as possible, given it has zero cost in equilibrium.

Rearranging Equation (22), we can express expected excess returns as follows:

$$\mu^R - r = \underbrace{p^{ag}\sigma_A}_{\text{agg. risk premium}} + \underbrace{p^{id}\phi\sigma_I}_{\text{id. risk premium}}.$$

Hence, expected returns is comprised of compensation for aggregate and idiosyncratic risk, consistent with the evidence in Section 2. In the absence of aggregate risk, growth, and adjustment costs, this expression simplifies to $\alpha K^{\alpha-1} - r = p^{id}\phi\sigma_I$, so the MPK is not equal to the interest rate when $\phi > 0$, even though entrepreneurs face no borrowing constraints. Entrepreneurs do not expand the business, despite $\mu^R > r$, to limit their risk exposure. In equilibrium, the wedge $\mu^R - r$ depends on entrepreneurs' risk aversion and their ability to diversify these risks, as captured by ϕ.

The demand for capital depends on the price and quantity of *idiosyncratic* risk. Access to aggregate insurance leads to a separation between entrepreneurs' choice of business scale and the amount of aggregate risk they are willing to hold. Investing in the business becomes then a decision about how much idiosyncratic risk to bear.

Equation (23) gives the demand for aggregate insurance. Entrepreneurs who are too exposed to the business — $qk_{i,t} > n_{i,t}$ — end up buying insurance, while entrepreneurs with low exposure to the business — $qk_{i,t} < n_{i,t}$ — provide insurance. Hence, poor entrepreneurs buy aggregate insurance, $\theta^{ag}_{i,t} > 0$, while rich entrepreneurs provide insurance, $\theta^{ag}_{i,t} < 0$. This arrangement can be implemented by having richer entrepreneurs send transfers to poor entrepreneurs as an indemnity after a negative aggregate shock, with transfers in the opposite direction after a positive aggregate shock.

The consumption-wealth ratio is given in Equation (24). The first term $\bar{r}/(1 - \psi e^{-\bar{r}(T-a)})$ represents the marginal propensity to consume (MPC). It is increasing in age, as it is typical in finite-horizon problems. The bequest motive parameter ψ controls the strength of this effect. If $\psi = 0$, the MPC is constant. If $\psi = 1$, the MPC gets arbitrarily large as the entrepreneur approaches the end of life, so the stock of wealth is fully consumed at the final age T, as in Merton (1969). Finally, the consumption-wealth ratio depends on the level of the human-financial wealth ratio.

Taking stock. We derive three main lessons from the results in Proposition 1. First, entrepreneurs' risk-taking and consumption decisions depend on the human-financial ratio, $h_{i,t}/n_{i,t}$, which varies over the life cycle. Second, the expected excess return on the business is comprised of an aggregate risk premium and an idiosyncratic risk premium. Third, the idiosyncratic risk premium affects the business scale. These three facts play an important role in the model's ability to match the facts discussed in Section 2.

4 Life-cycle patterns

We consider next the quantitative implications of the model for the life-cycle behavior of entrepreneurs. We first describe the model's calibration and then the life-cycle profiles for risk-taking, consumption, and financial wealth.

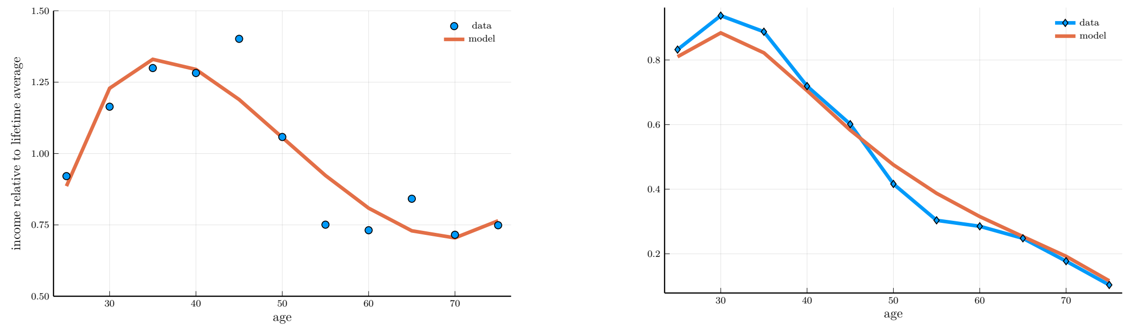
Table 4. Calibrated parameters

	Parameter	Choice
	Preferences	
ρ	Entrepreneur's rate of time preference	0.141
ρ_w	Wage earner's rate of time preference	0.127
ψ	Bequest motive	0.37
γ	Risk aversion	9.23
	Technology & financial friction	
μ_A	Average productivity growth rate	0.003
σ_A	Aggregate volatility	0.051
σ_{id}	Idiosyncratic volatility	0.203
ϕ	Moral hazard parameter	0.571
$\overline{\Phi}$	Adjustment cost parameter	0.90
α	Capital share in production function	0.33
δ	Depreciation rate	0.10
	Demographics	
g	Population growth	0.003
χ_e	Share of entrepreneurs in the population	0.46
T	Life span (adult life)	55

4.1 Risk-taking and savings over the life cycle

Technology, preferences, and demographics. We adopt the following calibration, summarized in Table 4. The capital share is set to $\alpha = 0.33$, and the depreciation rate is set to $\delta = 0.10$, common values found in the literature (see e.g. Campbell 1994). Expected productivity growth is set to $\mu = 0.003$, following the evidence in Jeong and Townsend (2007) for Thailand. The investment function assumes the functional form $\Phi(\iota) = \sqrt{\overline{\Phi}^2 + 2\iota} - \overline{\Phi}$. This corresponds to the case of quadratic adjustment costs, as the investment rate required for capital to grow at rate g is $\iota = \overline{\Phi}(g+\delta) + 0.5(g+\delta)^2$. The coefficient of the investment function is chosen to match a long-run relative price of capital q of one. The discount rate of wage earners is chosen to match a risk-free rate of $r = 3.5\%$, consistent with the average real rate for Thailand over the last two and half decades. The discount rate of entrepreneurs and the bequest motive parameters are chosen to match the consumption-wealth ratio at the beginning and end of life. The life horizon is set to $T = 55$, so it covers the life span from 25 to 80 years old, and the population growth is set to $g = 0.3\%$, the most recent value for population growth in Thailand. The parameter χ_e is chosen to match the average share of business wealth to financial wealth.

Figure 5: Life Cycle Profiles: Labor Income and Human-Financial Ratio



(a) Labor Income

(b) Human-financial wealth ratio

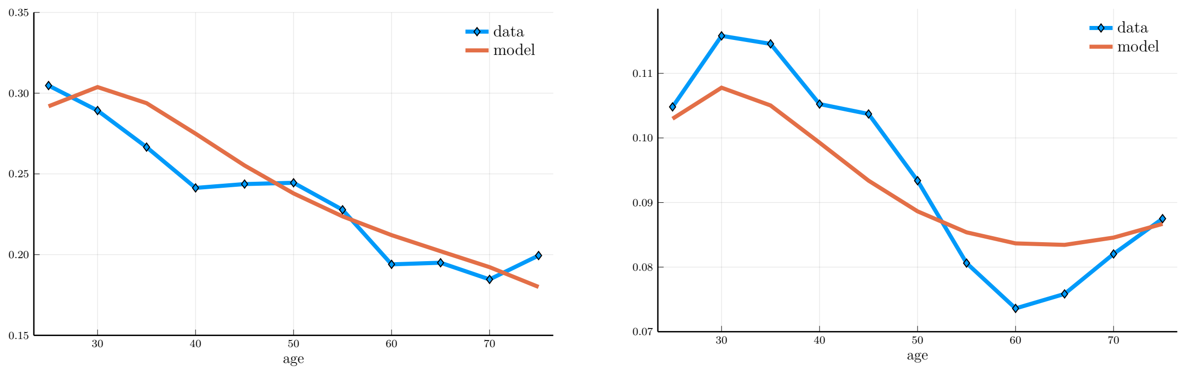
Risk, return, and the moral hazard parameter. The volatility parameters, σ_A and σ_I, are chosen to match the aggregate and idiosyncratic components of total volatility. The aggregate risk premium is given by $p^{ag}\sigma_A = \gamma\sigma_A^2$, so we choose γ to match the aggregate risk premium. The idiosyncratic risk premium is informative about ϕ. If $\phi = 0$, the idiosyncratic risk premium equals zero. As we raise ϕ, the importance of the idiosyncratic risk premium increases. Using expression (21) to solve for p^{id}, we obtain the idiosyncratic risk premium in terms of the moral hazard parameter ϕ and observable quantities, namely the level of idiosyncratic volatility and the exposure of entrepreneurs' total wealth to the private business.[22] Given these quantities, we can identify ϕ.

Measuring human wealth. We follow Gârleanu and Panageas (2015) and assume that $\bar{l}_{i,t}$ is a function of age: $\bar{l}_{i,t} = \sum_{l=1}^{L} \Gamma_l e^{\varphi_l a_i}$, given the normalization $\frac{\int_{\mathcal{E}_t} \bar{l}_{i,t} di}{\int_{\mathcal{E}_t} di} = 1$. This functional form is flexible enough to capture the empirical labor income dynamics while being analytically tractable. We estimate the parameters $(\Gamma_l, \varphi_l)_{l=1}^{L}$ by non-linear least squares and set the number of exponential terms to $L = 3$. The left panel of Figure 5 shows that the functional form approximates well the empirical labor income profile.

We consider next the human-financial wealth $h_{i,t}/n_{i,t}$. For simplicity, we use the empirical age effects without any additional controls. In Appendix B, we show that the empirical life-cycle profiles are very similar after we control for year fixed-effects or additional demographic variables. The right panel of Figure 5 shows that the human-financial wealth ratio declines over the life cycle. This is the result of labor income being relatively high at the beginning of the life cycle and that households have fewer years of future income as time goes by. Quantitatively, human wealth is nearly as important as financial wealth at the beginning of life, and nearly half financial wealth by age 50.

[22] See Section 6 for a detailed discussion of the determination of p^{id} in equilibrium.

Figure 6: Life Cycle Profiles: Risk-taking and Consumption-Wealth Ratio



(a) Share of wealth invested in the business **(b)** Consumption-wealth ratio

Risk-taking and savings. The left panel of Figure 6 shows that our measure of risk-taking, the share of wealth invested in the business, declines with age, consistent with the evidence in Table 3. The model generates this pattern by having the effective risk aversion decrease with the human-financial wealth ratio. Young entrepreneurs are endogenously less risk-averse than older entrepreneurs, so they invest a larger fraction of their wealth in the business. Notice that the ratio of the risk-taking measure at the beginning and the end of life is entirely determined by the human-financial wealth ratio, which was calibrated independently of any information on the cross-section of entrepreneurs' risk-taking.

The consumption-wealth ratio is roughly U-shaped as a function of age. This non-monotonic behavior is the result of two forces. First, the human-financial wealth ratio declines with age, which induces households to reduce consumption. Second, the MPC increases with age, which induces households to consume more. The first effect initially dominates, and the second effect dominates as the entrepreneur gets older.

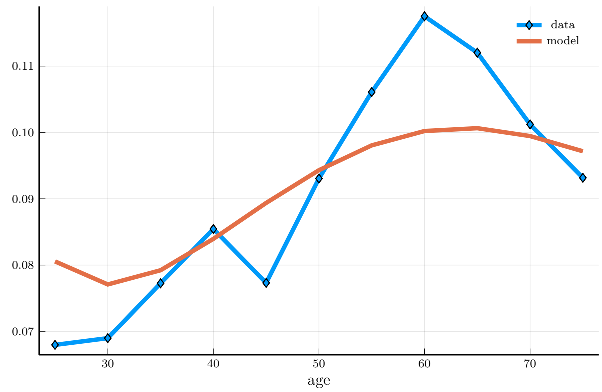
4.2 Distributive implications of entrepreneurial risk

We consider next how entrepreneurial risk affects wealth inequality. We focus on the joint distribution of (scaled) financial wealth and age, denoted by $f(n, a)$. Given this joint distribution, we obtain the average wealth by age, $n(a) = \int_{-h(a)}^{\infty} n f(n|a) dn$, and average entrepreneurial wealth, $n_e = \int_0^T n(a) f(a) da$, where $f(a)$ is the age distribution.

Between-group inequality. The next proposition characterizes average wealth by age.

Proposition 2. *Suppose the economy is in a stationary equilibrium. Then,*

Figure 7: Financial wealth distribution across age groups



i) Between-group inequality: *The wealth share of entrepreneurs of age a, $\frac{f(a)n(a)}{n_e}$, satisfies*

$$\log\frac{f(a)n(a)}{n_e} = \log\frac{f(0)n(0)}{n_e} + \underbrace{\log\left(\frac{1+\frac{h(0)}{n(0)}}{1+\frac{h(a)}{n(a)}}\right)}_{\substack{\text{human-to-financial}\\ \text{wealth effect}}} + \underbrace{\left[r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-(g+\mu_A)\right]a}_{\text{generalized "r-g" effect}} - \underbrace{\int_0^a \frac{\bar{r}}{1-\psi e^{-\bar{r}(T-a')}}da'}_{\text{average MPC effect}}. \tag{25}$$

where $n(0) = \left[e^{-\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-(g+\mu_A)-mpc_e\right)T}-1\right]^{-1} h(0)$, *and* $mpc_e = \frac{1}{T}\int_0^T \frac{\bar{r}}{1-\psi e^{-\bar{r}(T-a)}}da.$

ii) Average financial wealth: *The average financial wealth of entrepreneurs is given by*

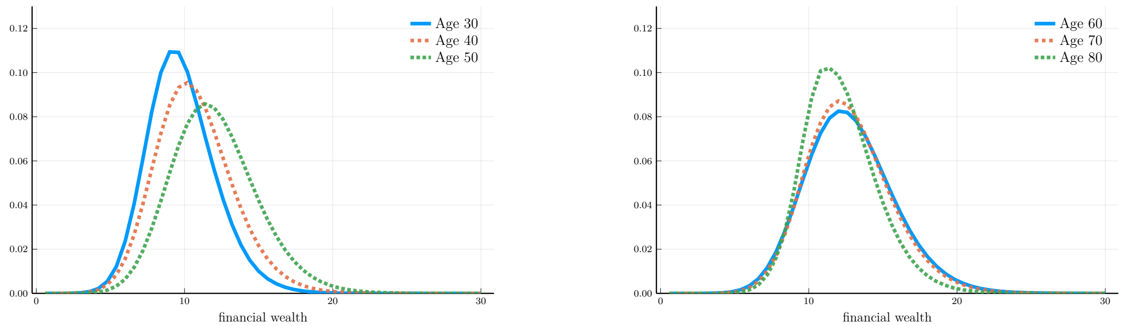
$$n_e = f(0)(n(0)+h(0))\int_0^T e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-(g+\mu_A)\right)a}\frac{e^{-\bar{r}a}-\psi e^{-\bar{r}T}}{1-\psi e^{-\bar{r}T}}da - h_e, \tag{26}$$

where $h_e = \int_0^T f(a)h(a)da$.

Proposition 2 decomposes the wealth distribution across age groups into three effects. The human-to-financial wealth effect, as labor income accelerates the accumulation of financial wealth. The generalized "$r-g$" effect, where the correct rate of return includes the aggregate and idiosyncratic risk premium. The average MPC effect, capturing how wealth accumulated at a given age depends on past consumption decisions. The second part of Proposition 2 characterizes average entrepreneurial wealth, which reflects the distribution of wealth between entrepreneurs and workers.

Figure 7 shows the share of wealth among entrepreneurs held by each age group. The model captures the inverted U-pattern of financial wealth observed in the data. The human-financial wealth effect and the "$r-g$" effect dominate the average MPC effect for

Figure 8: Stationary distribution of financial wealth conditional on age



(a) Ages 30, 40, and 50 **(b)** Ages 60, 70, and 80

young entrepreneurs, so the wealth share initially increases with age. The average MPC effect dominates later in life, bringing down the wealth share.

This hump-shaped behavior of wealth is typical of life-cycle models, see e.g. Gomes (2020). The fact that wealth accumulation responds to the price of idiosyncratic risk through the channels discussed above will be particularly important when we consider counterfactual changes in insurance in Section 6.

Within-group inequality. We characterize next the wealth distribution by age. Let $\mu_{n,t}(n,a)$ denote the expected change and $\sigma_{n,t}(n,a)$ the instantaneous volatility of financial wealth. The evolution of $f(n|a)$ is given by the Kolmogorov Forward Equation.

Lemma 2 (Kolmogorov Forward Equation)**.** *The conditional distribution of financial wealth $f_t(n|a)$ satisfies the partial differential equation*

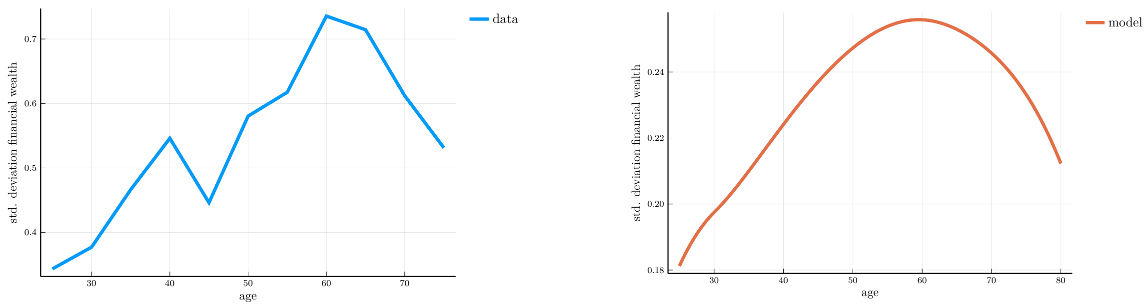
$$\frac{\partial f_t(n|a)}{\partial t} + \frac{\partial f_t(n|a)}{\partial a} = -\frac{\partial [f_t(n|a)\mu_{n,t}(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial^2 \left[f_t(n|a)\sigma^2_{n,t}(n,a)\right]}{\partial n^2}, \tag{27}$$

and the boundary condition $f_t(e^{-gT}n|0) = f_t(n|T)$, given an initial condition $f_0(n|a)$.

It is possible to solve for the conditional distribution of financial wealth in closed form for the special case where entrepreneurs leave no bequests, that is, $\psi = 1$.

Proposition 3 (Within-group inequality)**.** *Suppose $\psi = 1$ and $r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} > \mu_A$.*
i) Shifted log-normal distribution. *Conditional on age, financial wealth follows a shifted log-normal distribution with support $(-h(a), \infty)$, i.e., $n + h(a)$ follows a log-normal distribution.*

Figure 9: Standard deviation of financial wealth within age groups



(a) Within-group inequality: data

(b) Within-group inequality: model

Note: Panel (a) shows the standard deviation of financial wealth by age in the data. Panel (b) shows the standard deviation of financial wealth by age in the model economy. In both cases, we normalize the standard deviation by entrepreneurs' average wealth.

ii) Variance by age. *The variance of n conditional on age a is given by*

$$\mathbb{V}[n|a] = \left[e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1 \right] \left[h(0) e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A\right) a} \frac{e^{-\bar{r}a} - e^{-\bar{r}T}}{1 - e^{-\bar{r}T}} \right]^2 . \tag{28}$$

iii) Inverted-U shape of inequality over the life cycle. *There exists $0 < \hat{a} < T$ such that $\mathbb{V}[n|a]$ is increasing in a for $a < \hat{a}$ and decreasing for $a > \hat{a}$.*

Proposition 3 shows that wealth follows a *shifted log-normal* distribution, with an age-dependent shifter $-h(a)$. Expression (28) shows how the variance of wealth evolves over the life cycle. Wealth dispersion increases at the beginning of life, as some entrepreneurs receive a series of positive shocks while others suffer adverse shocks. The increasing MPC provides a countervailing force, bringing dispersion down at the end of the life cycle.

The results in Proposition 3 assume no bequests. In the general case, we must use numerical methods. Figure 8 shows the stationary distribution of financial wealth for selected ages. The mean and the dispersion of the distribution initially increase with age, then eventually both start to decline as entrepreneurs get older.

The same inverted U-pattern can be found in the data, as shown in Figure 9. The standard deviation of n/n_e increases sharply until roughly age 60 and then declines by the end of the life cycle. Quantitatively, the model generates a substantial increase in inequality, over 40% from early in life until the peak, even though idiosyncratic return shocks are the only source of heterogeneity across entrepreneurs. The increase in inequality is even more pronounced in the data. Introducing differences in preferences or labor income could potentially bring the within-group inequality closer to the one in the data.

Consumption inequality and risk sharing. As shown in Appendix A.4, the variance of log total wealth, and ultimately log consumption, increases with age: $\mathbb{V}[\log \omega_{i,t}|a] = \mathbb{V}[\log c_{i,t}|a] = \left(\frac{p^{id}}{\gamma}\right)^2 a$. The steepness of the variance life-cycle profile depends on the degree of risk sharing in the economy, in line with work using the consumption dispersion over the life cycle to infer how imperfect risk sharing is (see, e.g., Storesletten et al. 2004).

5 Heterogeneous risk premia

In the previous section, we showed that the model captures the main life-cycle patterns observed in our data. In this section, we show that an extension of the baseline model can also capture the heterogeneity in expected returns documented in Section 2.

5.1 Risk heterogeneity and decreasing returns to scale

Proposition 1 shows that the expected return on the business includes compensation for idiosyncratic risk, consistent with the evidence in Table 2. However, the risk premium is equalized across entrepreneurs, contrasting with the rich heterogeneity in expected returns observed in the data. This will be the case even if we introduce heterogeneity in idiosyncratic volatility. From Equation (15), the capital-labor ratio would be equalized across entrepreneurs, so they would still have the same expected return by Equation (17).

Introducing decreasing returns to scale. To capture heterogeneity in risk premia, the model needs a combination of heterogeneous exposure to idiosyncratic risk and *decreasing returns to scale* (DRS) in production. To see this fact, consider the following extension of the baseline model. Production depends not only on the amount of capital and labor but also on a fixed entrepreneurial ability e_i:

$$\tilde{y}_{i,t} = A_t k_{i,t}^{\alpha} l_{i,t}^{\beta} e_i^{1-\alpha-\beta}.$$

For simplicity, we focus initially on the case where e_i is common among entrepreneurs, and we normalize e_i to 1. This formulation captures a form of span-of-control problem (Lucas 1978). We also assume that entrepreneurs differ in their exposure to idiosyncratic risk, which is denoted by $\sigma_{I,i} \in \{\sigma_I^1, \ldots, \sigma_I^n\}$. Finally, to focus on the role of return heterogeneity, we abstract from the overlapping generations feature in this section, so we set $g = 0$ and $T \to \infty$. The rest of the environment is the same as discussed in Section 3.

Lemma 3 (Model with DRS)**.** *In a stationary equilibrium, the investment rate is given by* (16)*, the price of aggregate risk is $p^{ag} = \gamma\sigma_A$, and aggregate insurance is given by* (23)*. Labor demand is given by $w_t = \beta k_{i,t}^{\alpha} l_{i,t}^{\beta-1}$, and the conditional expectation of marginal returns is given by*

$$\mu_{i,t}^{R} = \frac{\alpha\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_{i,t}^{\frac{\alpha+\beta-1}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{29}$$

Given value function $V_i(\omega)$, the optimality conditions for consumption and capital are given by

$$c_{i,t}^{-\gamma} = V_{i,\omega}, \qquad\qquad qk_{i,t} = -\frac{V_{i,\omega}}{V_{i,\omega\omega}}\frac{p_{i,t}^{id}}{\phi\sigma_{I,i}}.$$

Lemma 3 describes the model solution for the case with DRS. Expected returns are *scale-dependent* when $\alpha + \beta < 1$, as they vary with capital. Notice that heterogeneity in expected returns is driven by differences in the dividend yield, given that expected capital gains are the same across entrepreneurs. This is consistent with how we measure returns in the data, which captures mainly differences in the dividend yield.

Scale-dependent returns, however, imply that there is no closed-form solution to the model. In this case, policy functions will not be linear in total wealth anymore. To obtain analytical results, we adapt the perturbation methods used by Viceira (2001). Consider the following log-linear approximation of the policy functions:

$$\log c_j(\omega_{i,t}) = \log \bar{c}_j + \psi_{c,\omega}^{j}\hat{\omega}_{i,t} + \mathcal{O}(\hat{\omega}_{i,t}^2), \qquad \log k_j(\omega_{i,t}) = \log \bar{k}_j + \psi_{k,\omega}^{j}\hat{\omega}_{i,t} + \mathcal{O}(\hat{\omega}_{i,t}^2),$$

where $\hat{\omega}_{i,t} = \log\omega_{i,t} - \log\overline{\omega}_j$, given $\overline{\omega}_j = \exp\mathbb{E}[\log\omega_{i,t}|\sigma_{I,i} = \sigma_I^j]$, denotes the deviations of log total wealth from its mean. Notice that the approximation point $\overline{\omega}_j$ and the intercepts $(\bar{c}_j, \bar{k}_j)$ are endogenous, as in a risky steady state approximation (see e.g. Coeurdacier et al. 2011). The following proposition provides the main result of this section.

Proposition 4 (Heterogeneous risk premia)**.** *The price of idiosyncratic risk is given by*

$$p_{i,t}^{id} = \overline{p}^{id} - \psi_{p^{id},\omega}^{j}\hat{\omega}_{i,t}, \tag{30}$$

where $\mathbb{E}[p_{i,t}^{id}] = \overline{p}^{id}$ is equalized across entrepreneurs, and $\psi_{p^{id},\omega}$ is a positive constant. Therefore, the unconditional expectation of returns is given by

$$\mathbb{E}[\mu_{i,t}^{R}] = r + p^{ag}\sigma_A + \overline{p}^{id}\phi\sigma_{I,i}. \tag{31}$$

Proposition 4 shows that risk premia are heterogeneous in the economy with DRS and differences in idiosyncratic volatility. Entrepreneurs operating riskier technologies have higher expected returns in equilibrium, consistent with the evidence in Section 2. From Equation (30), there is variation in the conditional Sharpe ratio, as entrepreneurs experience shocks to total wealth $\hat{\omega}_{i,t}$. Still, the unconditional Sharpe ratio is the same for everyone. Therefore, unconditional expected returns vary with idiosyncratic volatility.

Skill heterogeneity. We abstracted from skill heterogeneity to focus on differences in risk. Consider now the polar opposite assumptions: $e_i \in \{e^1, \ldots, e^n\}$ and $\sigma_{I,i} = \sigma_I$. The following proposition shows that there is no heterogeneity in risk premia in this case.

Proposition 5 (Skill heterogeneity)**.** *The price of idiosyncratic risk is still given by Equation* (30)*, so* $\mathbb{E}[p^{id}_{i,t}] = \overline{p}^{id}$ *is equalized across entrepreneurs. Capital is given by* $\overline{k}_j = e_j K$*. Moreover, the unconditional expectation of returns is equalized across entrepreneurs.*

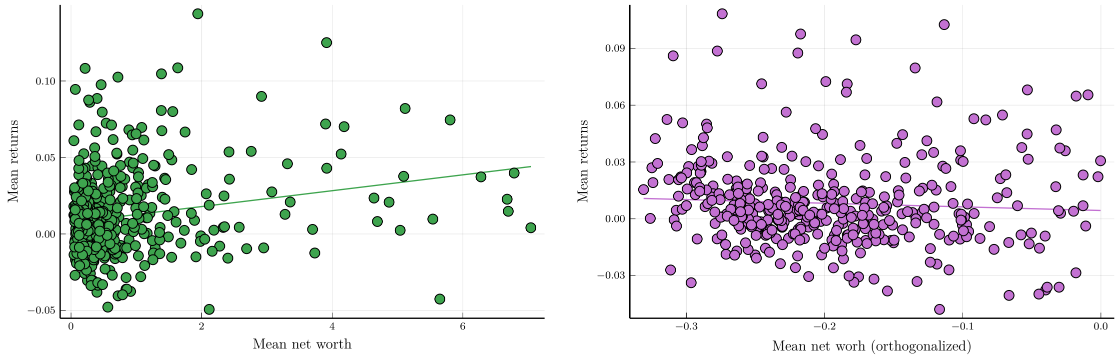
Proposition 5 shows that differences in skill alone cannot explain the heterogeneity in risk premia observed in the data. It is perhaps surprising that more skilled entrepreneurs do not experience higher returns on average. The explanation is reminiscent of the logic in Berk and Green (2004) who study how skill affects mutual fund returns. In a setting with DRS in investment returns, they argue that more skilled managers end up with *larger* funds, and expected returns are explained by their exposure to risk. The same intuition applies in our setting: more skilled entrepreneurs end up with larger businesses, but expected returns are driven by their exposure to risk, consistent with Equation (31).

5.2 The role of borrowing constraints

We have seen that differences in expected returns are entirely driven by differences in risk exposure. However, this result relies on the absence of borrowing constraints. Consider the case with limited pledgeability of physical capital, where entrepreneurs face collateral constraints. Differences in expected returns are then driven by differences in net worth among constrained entrepreneurs. Hence, if collateral constraints are a major factor limiting the entrepreneurial activity, we should observe a strong negative association between financial wealth and expected returns.

We test this prediction in our data. The left panel of Figure 10 shows the (cross-sectional) scatter plot of average returns and average net worth. We find a non-significant positive relationship between returns and net worth. However, the theory predicts the association between wealth and expected returns should hold only for entrepreneurs with

Figure 10: Average returns vs. average net worth



Note: The left panel shows a scatter plot of the average net worth against the average return for each entrepreneur. Net worth is normalized by its cross-sectional mean. The right panel shows a scatter plot of the residuals of a cross-sectional regression of average net worth on aggregate beta and idiosyncratic variance against average time-series returns for a sample of entrepreneurs with below-average orthogonalized net worth. To limit the influence of outliers, we trim 1% of the observations in the left and right tails.

relatively low net worth, after controlling for differences in risk exposure, so a sharper test would focus on poorer entrepreneurs. The right panel shows the relationship between average returns and the component of net worth that is orthogonal to aggregate beta and idiosyncratic volatility for entrepreneurs with below-average (orthogonalized) net worth. We observe a non-significant association between net worth and expected returns. In both cases, net worth explains a negligible fraction of the variation in the cross-section of entrepreneurial returns, with an adjusted R^2 of less than half a percentage point.

These findings are consistent with the results in Proposition 4, which predict no association between expected returns and net worth after controlling for risk.[23] In contrast, the model with leverage constraints predicts no association between expected returns and risk exposure for entrepreneurs with sufficiently low net worth. To test this prediction, we run the same cross-sectional regressions from Section 2, but restricted to a sample of entrepreneurs with below-average net worth. The association between risk and return is actually *stronger* for this subsample, where the R^2 of the cross-sectional regression is 0.82, compared to $R^2 = 0.68$ for the whole sample. These results suggest that collateral constraints are not the main driver of expected entrepreneurial returns in our sample.

6 Counterfactuals

Having shown that the model captures the salient features of our data, we study next the effects of counterfactual changes in idiosyncratic insurance. We find that financial devel-

[23]For instance, in the version with skill heterogeneity, a skilled entrepreneur has a larger net worth but the same expected return compared to a less skilled entrepreneur, conditional on the same risk exposure.

opment, captured by changes in the moral hazard parameter ϕ, is tightly linked to economic development. We study first the long-run implications of a reform that improves insurance, and then we consider the transitional dynamics associated with the reform.

6.1 Aggregate capital and idiosyncratic risk premium

We revert to the baseline case of constant returns to scale production and $\sigma_{I,i} = \sigma_I$. The interest rate is given by the standard condition $r = \rho_w + \gamma\mu_A - \frac{\gamma(\gamma+1)}{2}\sigma_A^2$, see Appendix C.1. From Equation (16), we obtain $q = \overline{\Phi} + g + \delta$, where we choose $\overline{\Phi}$ such that $q = 1$. From Proposition 1, we have $p^{ag} = \gamma\sigma_A$. From Equation (22), we obtain

$$r + p^{ag}\sigma_A + p^{id}\phi\sigma_I = \frac{\alpha K^{\alpha-1} - \iota(q)}{q} + g + \mu_A. \tag{32}$$

The left-hand side equals the required rate of return to invest in the business. The right-hand side gives the actual expected return, a function of the MPK. Equation (32) generalizes the standard textbook relation between MPK and interest rates, as $r = \alpha K^{\alpha-1} - \delta$ in the absence of growth and risk.

Expression (32) gives an inverse relation between the idiosyncratic risk premium $p^{id}\phi\sigma_I$ and the capital-labor ratio K. This downward-sloping relationship is represented by the solid blue line in the left panel of Figure 11, which we call the MPK schedule. This inverse relationship is analogous to the one in the model with DRS from Section 5, as there are DRS in production at the aggregate level, given the exogenous labor supply.

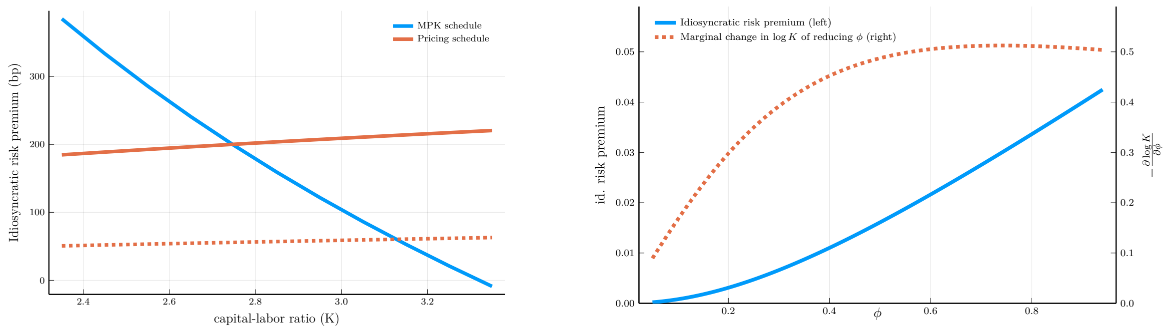
We need another condition relating K and p^{id}. Aggregating the demand for capital (21) across all entrepreneurs, we obtain

$$p^{id} = \underbrace{\gamma}_{\text{risk aversion}} \underbrace{\phi\sigma_I}_{\text{effective risk}} \underbrace{\frac{qK}{\chi_e(n_e + h_e)}}_{\text{id. risk exposure}}, \tag{33}$$

where n_e and h_e denote the average financial and human wealth of entrepreneurs.

The price of idiosyncratic risk depends on the risk aversion γ and the idiosyncratic risk (net of insurance) $\phi\sigma_I$. p^{id} also depends on the *idiosyncratic risk exposure*, that is, the ratio of physical assets to total wealth of entrepreneurs. Entrepreneurs require a higher idiosyncratic risk premium when they are more exposed to the business. Using Equation (26) to eliminate $n_e + h_e$, we obtain an implicit relationship between p^{id} and K. The left panel of Figure 11 plots this relationship as the solid upward-sloping curve, which we call the *pricing schedule*. The intersection of these schedules determines p^{id} and K.

Figure 11: Idiosyncratic Risk Premium and Capital Stock



(a) MPK schedule vs. Pricing schedule **(b)** Idiosyncratic risk premium and $-\frac{\partial \log K}{\partial \phi}$

Note: In the left panel, the solid (dashed) upward-sloping curve shows the pricing schedule in the initial (new) stationary equilibrium for $\phi_0 = 0.571$ ($\phi_1 = 0.286$). The right panel shows the equilibrium idiosyncratic risk premium and the marginal increase in the capital stock by reducing the moral hazard parameter for different values of ϕ.

Price of idiosyncratic risk in the data The model matches the risk premium and volatility for both aggregate and idiosyncratic risk given in Table 2. A striking fact is that the Sharpe ratio for aggregate risk is nearly five times larger than the one for idiosyncratic risk, despite risk being mostly idiosyncratic. Expression (33) sheds light on this pattern.

The Sharpe ratio of aggregate risk corresponds to $p^{ag} = \gamma\sigma_A$. The Sharpe ratio of idiosyncratic risk is given by $p^{id}\phi$, as the volatility computed in the data does not consider any insurance available to entrepreneurs. If we were to naively price the idiosyncratic risk by analogy with the aggregate risk, the Sharpe ratio would be $\gamma\sigma_I$, that is, more than four times larger than the one for aggregate risk. However, from the pricing equation,

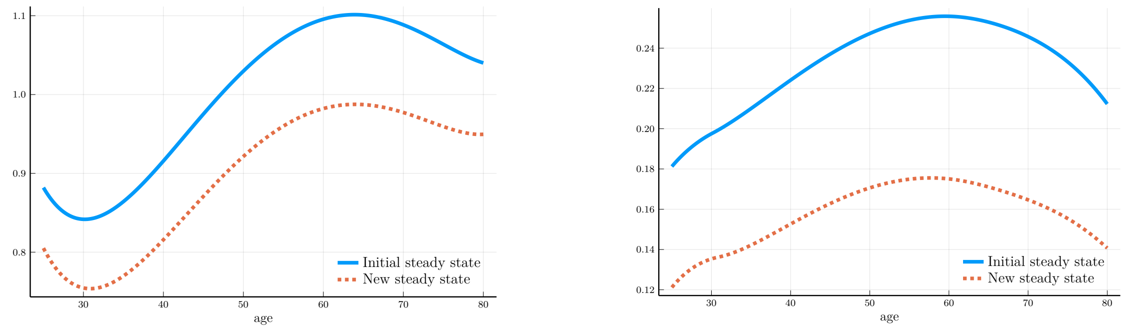
$$p^{id}\phi = \gamma\sigma_I\phi^2 \frac{qK/\chi_e}{n_e + h_e}.$$

The moral hazard parameter is $\phi = 0.57$, which reduces p^{id} by $1 - \phi^2 = 67\%$. The rest of the adjustment comes from the risk exposure factor: $qK/\chi_e(n_e + h_e) \approx 0.16$. The price of idiosyncratic is low either due to insurance mechanisms or because only a fraction of their wealth is exposed to this risk. Without human wealth and heterogeneous agents, one would incorrectly attribute the low Sharpe ratio to a high degree of insurance. The economy would appear to have better insurance than it has.

6.2 Long-run effects of relaxing insurance constraints

We consider next the aggregate implications of relaxing insurance constraints. High values of ϕ capture situations where access to insurance arrangements, formal or informal,

Figure 12: Financial development and inequality in the long-run



(a) Between-group inequality

(b) Within-group inequality

Note: Inequality in a stationary equilibrium for $\phi_0 = 0.571$ (initial steady state) and $\phi_1 = 0.286$ (new steady state). Panel (a) shows average financial wealth by age. Panel (b) shows the standard deviation of financial wealth by age. All variables are normalized by entrepreneurs' average wealth in the initial steady state.

is rather limited. As institutional arrangements improve, e.g., mechanisms to monitor entrepreneurs' activities, such frictions are expected to be reduced, and entrepreneurs would bear less risk. Financial development reduces the moral hazard parameter ϕ.

Panel (a) of Figure 11 shows the impact of reducing ϕ. The MPK schedule is unchanged, but the pricing schedule is shifted down. In the long run, the reduction in expected returns leads to a reduction in the MPK and higher capital stock. Reducing the moral hazard parameter in half, from $\phi_0 = 0.57$ to $\phi_1 = 0.29$, the idiosyncratic risk premium falls by about 140 basis point, raising the capital stock by roughly 13%.

Panel (b) of Figure 11 shows that the economy's response to changes in ϕ is highly nonlinear. The idiosyncratic risk premium (solid line) is convex in ϕ, so changes in the moral hazard parameter lead to stronger effects in economies with low financial development. The dashed line shows the marginal change in capital due to a reduction in the moral hazard parameter for different values of ϕ. The same reduction in ϕ leads to an increase in the capital stock that is five times larger in an economy with low financial development (high ϕ) compared to an economy with high financial development (low ϕ).

Financial development also has important implications for inequality. The left panel of Figure 12 shows the average financial wealth for each age group relative to average entrepreneurial wealth on the initial stationary equilibrium. Financial wealth falls for all age groups in the new equilibrium, as it is harder to accumulate wealth with lower expected returns. In the long run, inequality falls after a reduction in ϕ, as shown in the right panel of Figure 12, given that entrepreneurs are less exposed to risk.

6.3 Dynamic effects of relaxing risk constraints

So far, we have compared two different stationary equilibria. To compute the welfare implications of relaxing risk constraints, it is important to explicitly take into account what happens during the transition.[24] We turn next to the dynamic effects of the reform.

Computing the transitional dynamics. We consider a "small open economy" version of the model, where the interest rate stays at the level of the original stationary equilibrium.[25] This allows us to focus on the dynamic implications of fluctuations in the idiosyncratic risk premium. The next proposition characterizes the evolution of aggregate variables during the transition.

Proposition 6. *The price of aggregate risk is $p_t^{ag} = \gamma\sigma_A$. The evolution of $(q_t, K_t, \{\zeta_t(a), h_t(a), \omega_t(a)\})$ is characterized by a pair of ordinary differential equations (ODEs)*

$$\dot{K}_t = \left[\Phi(\iota(q_t)) - \delta - g\right] K_t$$
$$\dot{q}_t = \left[r + \gamma\sigma_A^2 + \gamma\phi^2\sigma_I^2 \frac{q_t K_t}{\chi_e \omega_{e,t}} + \delta - \mu_A - \Phi(\iota(q_t)) - \frac{\alpha K_t^{\alpha-1} - \iota(q_t)}{q_t}\right] q_t$$

and three partial differential equations (PDEs)

$$\frac{\partial \zeta_t(a)}{\partial t} = -\frac{\partial \zeta_t(a)}{\partial a} + \zeta_t^2(a) - \bar{r}_t \zeta_t(a)$$
$$\frac{\partial h_t(a)}{\partial t} = -\frac{\partial h_t(a)}{\partial a} + (r + \gamma\sigma_A^2 - \mu_A) h_t(a) - (1-\alpha) K_t^{\alpha} \bar{l}(a)$$
$$\frac{\partial \omega_t(a)}{\partial t} = -\frac{\partial \omega_t(a)}{\partial a} + \left[r + \gamma\sigma_A^2 - \mu_A + \gamma\phi^2\sigma_I^2 \left(\frac{q_t K_t}{\chi_e \omega_{e,t}}\right)^2 - \zeta_t(a)\right] \omega_t(a),$$

subject to the boundary conditions described in the appendix.

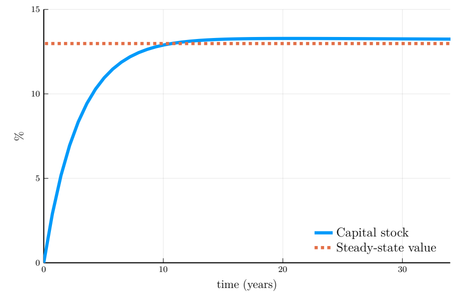
We obtain the first ODE by aggregating (7) and the second one by the time-dependent version of (32). The first PDE, for the consumption-wealth ratio $\zeta_t(a)$, comes from the HJB equation; the second one corresponds to the time-dependent version of (18), and the last PDE can be obtained by averaging the budget constraint of entrepreneurs.

The transitional dynamics for heterogeneous agents models are often computed using a shooting algorithm, as in, e.g., Guerrieri and Lorenzoni (2017) or Achdou et al. (2017).

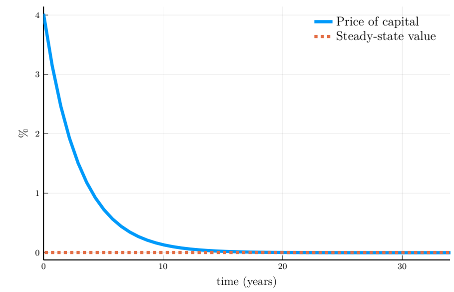
[24]Notice that while equilibrium variables are still exposed to aggregate shocks, we are able to compute the transitional dynamics for scaled variables. For instance, we can compute the transition for $q_t \equiv \tilde{q}_t / A_t$, while $\tilde{q}_t$ moves with the realization of A_t and the deterministic path of q_t.

[25]We show in Appendix D that the interest rate is constant during the transitional dynamics in a closed economy when wage earners have Epstein-Zin utility with linear intertemporal preferences.

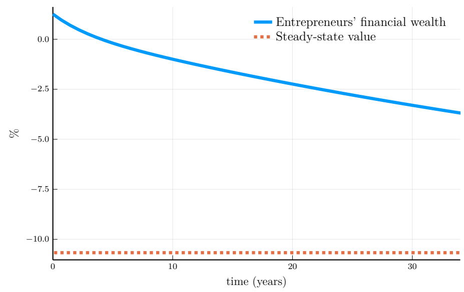
Figure 13: Transitional dynamics: aggregate variables



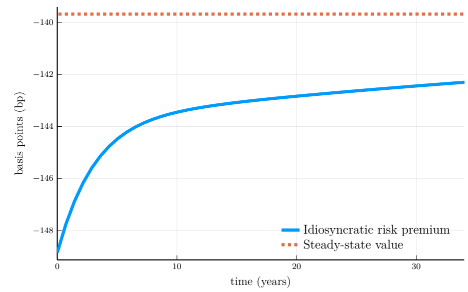
(a) Capital Stock



(b) Price of Capital



(c) Entrepreneurs' Financial Wealth



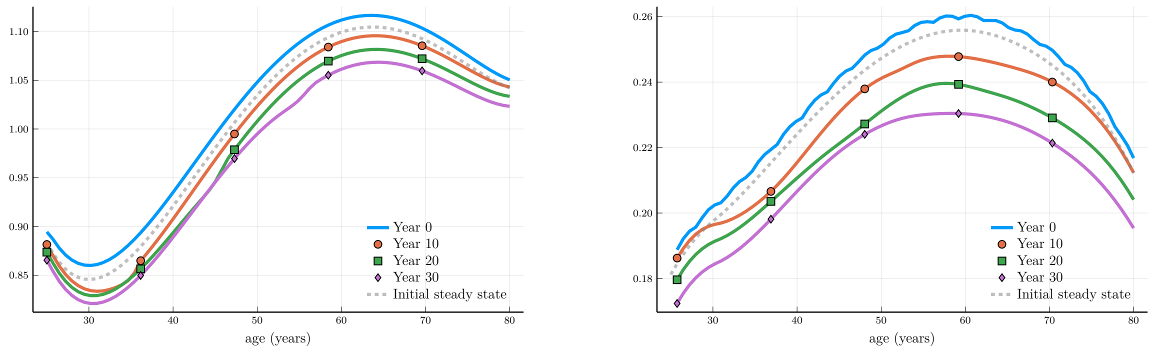
(d) Idiosyncratic Risk Premium

Note: Transitional dynamics from a stationary equilibrium with $\phi_0 = 0.571$ to a stationary equilibrium with $\phi_1 = 0.286$. Capital stock, the relative price of capital, and entrepreneurs' financial wealth are expressed as percentage deviations from the initial steady state. The idiosyncratic risk premium is expressed as absolute deviation from the initial steady state in basis points.

As such an algorithm is impractical in our setting, we adopt a combination of perturbation and finite-difference methods. First, we use finite differences to discretize the system of ODE/PDEs. We then linearize the system around the new stationary equilibrium. In contrast to the approach in e.g. Ahn et al. (2018), which linearizes around the economy without aggregate shocks, we do not assume that shocks are small. The economy continues to be exposed to aggregate shocks, but variables in levels are proportional to the aggregate shock, such as $\tilde{q}_t = q_t A_t$. By avoiding small-risk approximations, we capture time-varying risk premia and precautionary savings effects using this method.

Short-run dynamics: the overshooting effect. Figure 13 shows the transitional dynamics for aggregate variables. We consider a reform that reduces the moral hazard parameter by half, from $\phi_0 = 0.571$ under our calibration to $\phi_1 = 0.286$. We observe an investment boom and a sharp increase in business valuation that lasts for roughly a decade, as shown in Panels (a) and (b). The short-run response of q exceeds by a large margin its long-run level, i.e. there is an *overshooting* effect. As entrepreneurs bear less risk, they also require a smaller premium. In the long run, this leads to a larger capital stock. However, capital is fixed in the short run, so expected returns can only decrease through expected capital

Figure 14: Transitional dynamics: inequality



(a) Between-group inequality

(b) Within-group inequality

Note: Transitional dynamics from a stationary equilibrium with $\phi_0 = 0.571$ to a stationary equilibrium with $\phi_1 = 0.286$. Panel (a) shows average financial wealth by age. Panel (b) shows the standard deviation of financial wealth by age.

losses. The price of capital then jumps on impact and slowly reverts to its long-run level. This logic is reminiscent of Dornbusch's (1976) overshooting model, where exchange rates react more strongly to shocks in the short run to create expected capital losses to investors. The overshooting effect has important implications for wealth dynamics and inequality.

While entrepreneurs' wealth goes down in the long run, their wealth actually increases in the short run. Wealth jumps up on impact due to a revaluation effect. However, given lower expected returns, they now accumulate wealth at a slower pace. This effect takes a long time to materialize, as it is partially transmitted through lower bequests. Even thirty years after the shock, the reduction in financial wealth is only 25% of the long-run effect.

Similarly, the short-run response of the idiosyncratic risk premium exceeds its long-run level by nearly ten basis points, as shown in Panel (d). After a decade, the difference between the risk premium to the new steady state level is cut by more than half, and then the risk premium increases slowly as entrepreneurs' wealth declines.

Kuznets dynamics. The left panel of Figure 14 shows average financial wealth for each age group at different points in time, normalized by average entrepreneurial wealth in the initial equilibrium. Due to the revaluation effect, financial wealth increases on impact. The effect is stronger for younger entrepreneurs, as they are more exposed to the business. Over time, financial wealth goes down due to the reduction in expected returns.

The right panel on Figure 14 shows the standard deviation of financial wealth by age. Again the short-run and long-run responses are different. While wealth inequality goes down in the long run, wealth inequality increases in the short run. Rich entrepreneurs hold more capital, so wealthier entrepreneurs benefit the most from the reform.

Inequality interacts in interesting ways with the demographic structure. After the initial increase, wealth dispersion goes down for all age groups, but entrepreneurs starting their professional lives just after the reform are the ones most affected. Ten years after the reform, the drop in inequality is two times larger for 35-year-old entrepreneurs, who lived their entire professional life under the new regime, relative to 80-year-old entrepreneurs, who lived most of their lives under the old regime. Similarly, the drop in inequality is more pronounced for 35-year-old entrepreneurs than for 25-year-old entrepreneurs.

Taking stock. The reform initially leads to an investment boom and more wealth inequality. As the economy approaches its new output level, inequality starts to recede, reaching a lower level in the long run. Inequality increases as the economy enters a high growth phase, and inequality goes down as the economy reaches a higher level of development, consistent with a Kuznets's (1955) curve.[26]

Welfare implications. We turn next to the welfare implications of insurance constraints. Remember that the value function of an entrepreneur of age a at time t is given by $V_t(n,a) = \zeta_t^{-\frac{1}{\gamma}}(a)\frac{(n+h_t(a))^{1-\gamma}}{1-\gamma}$. The welfare of an entrepreneur depends then on financial wealth n, human wealth $h_t(a)$, and the consumption-wealth ratio $\zeta_t(a)$, which captures the path of expected future returns. We evaluate financial wealth at the average level of the age group, $n = n_t(a)$. Finally, we take a monotonic transformation of the value function to measure welfare in consumption units. Hence, our measure of welfare will be

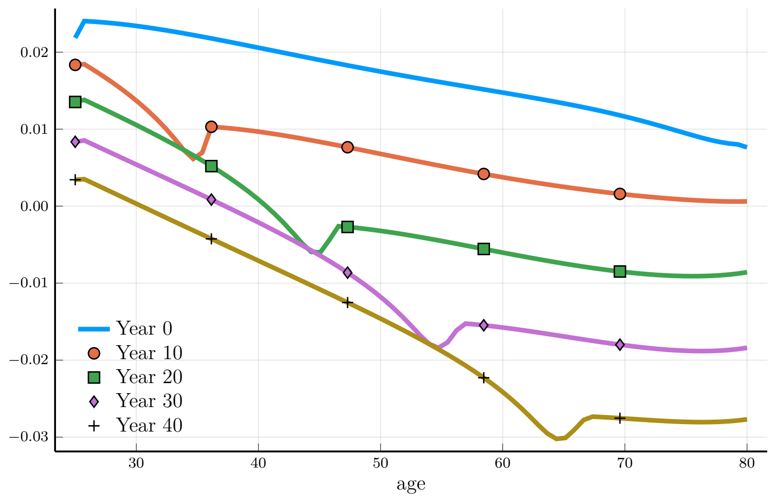
$$\mathcal{W}_t(a) = \log\left[u^{-1}\left(V_t(n_t(a),a)\right)\right] - \log\left[u^{-1}\left(V^*(n^*(a),a)\right)\right] = \frac{\hat{\zeta}_t(a)}{\gamma(\gamma-1)} + \hat{\omega}_t(a),$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and a hat denotes log deviations from the initial steady state.

Figure 15 shows the welfare gains for each age group at different points in time. The generation that is alive at the moment of the reform benefits the most, with gains concentrated on younger entrepreneurs. This is the result of the revaluation effect. However, the negative impact on wealth accumulation affects future generations of entrepreneurs. As they receive smaller bequests and it becomes harder to accumulate wealth, their welfare is adversely impacted. Figure 15 shows how demographics affect welfare gains. Ten years after the intervention, the welfare gains for entrepreneurs who started their professional life after the reform, the ones with age between 25 to 35 years old, have welfare gains that are smaller than the entrepreneurs of the same age at the time of the intervention. Thirty

[26] Moll (2012) derived a Kuznet's curve by showing the steady-state top wealth share is hump-shaped in financial development. In contrast, we focus on the transitional dynamics instead of long-run comparisons.

Figure 15: Transitional dynamics: welfare



years after the reform, entrepreneurs aged between 35 and 55 are worse off compared to an equilibrium without the reform, with larger welfare losses for the older entrepreneurs. Therefore, the initial generation of entrepreneurs reaps most of the benefits of the reform.

7 Conclusion

We study the aggregate and distributive implications of entrepreneurial risk. We propose a life-cycle model of entrepreneurship with aggregate and idiosyncratic risk under limited insurance. The model captures quantitatively the empirical patterns of risk-taking and savings over the life cycle, the inverted-U shape of wealth inequality, and the level of aggregate and idiosyncratic risk premia. An extension with DRS and differences in volatility also generates the heterogeneity in risk premia observed in the data.

Our findings highlight the importance of risk-sharing friction in explaining the level of economic development and the dynamics of wealth inequality. Financial innovations and policy interventions that alleviate the consequences of limited insurance have far reaching implications on how these economies develop and ultimately on welfare.

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Online Appendix

A Proofs

A.1 Proof of Lemma 1 and Proposition 1

Proof. We start by showing part (b) of Lemma 1, that is, we solve for $h_t(a)$ and its dynamics. Then, we proceed to solve for the entrepreneurs' value function and policy functions.

Pricing human wealth. Define the stochastic discount factor (SDF) for this economy as the process π_t satisfying the law of motion

$$\frac{d\pi_t}{\pi_t} = -r_t dt - p_t^{ag} dZ_t. \tag{A.1}$$

Without loss of generality, we assumed that the SDF is not exposed to idiosyncratic risk, as we only use the SDF to price human wealth which is not exposed to idiosyncratic risk. Integrating the process above, we obtain

$$\frac{\pi_z}{\pi_t} = \exp\left(-\int_t^z \left(r_u + \frac{(p_u^{ag})^2}{2}\right) du - \int_t^z p_u^{ag} dZ_u\right). \tag{A.2}$$

Similarly, integrating the process for A_t

$$\frac{A_z}{A_t} = \exp\left(\int_t^z \left(\mu_A - \frac{\sigma_A^2}{2}\right) du + \int_t^z \sigma_A dZ_u\right). \tag{A.3}$$

Hence, we can explicitly compute the following expectation

$$\begin{aligned}\mathbb{E}_t\left[\frac{\pi_z A_z}{\pi_t A_t}\right] &= \mathbb{E}_t\left[\exp\left(-\int_t^z \left(r_u - \mu_A + \frac{(p_u^{ag})^2 + \sigma_A^2}{2}\right) du - \int_t^z \left(p_u^{ag} - \sigma_A\right) dZ_u\right)\right] \\ &= \exp\left(-\int_t^z \left(r_u + p_u^{ag}\sigma_A - \mu_A\right) du\right),\end{aligned} \tag{A.4}$$

where we used Ito's isometry and the fact that p_t^{ag} is deterministic.

Human wealth is given by

$$h_t(a) = \mathbb{E}_t\left[\int_t^{t+T-a} \frac{\pi_z A_z}{\pi_t A_t} w_z \bar{l}(a+z-t)dz\right] = \int_t^{t+T-a} e^{-\int_t^z \left(r_u + p_u^{ag}\sigma_A - \mu_A\right)du} w_z \bar{l}(a+z-t)dz. \tag{A.5}$$

Consider the human wealth for someone born at date s, so $a = t - s$:

$$h_t(t-s) = \int_t^{s+T} e^{-\int_t^z \left(r_u + p_u^{ag}\sigma_A - \mu_A\right)du} w_z \bar{l}(z-s)dz. \tag{A.6}$$

Differentiating the expression above with respect to time yields

$$\frac{\partial h_t(a)}{\partial t} + \frac{\partial h_t(a)}{\partial a} = \left(r_t + p_t^{ag}\sigma_A - \mu_A\right) h_t(a) - w_t \bar{l}(a), \tag{A.7}$$

which gives (18) in a stationary equilibrium.

The HJB equation. The HJB equation for problem (13) is given by

$$\rho \tilde{V}_t(\tilde{n}, t-s; A_t) = \max_{\tilde{c}_t, \tilde{\theta}_t^{ag}, \tilde{\theta}_t^{id}, k_t, l_t, \iota_t} \left\{ \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} + \frac{\mathbb{E}_t\left[d\tilde{V}_{i,t}\right]}{dt} \right\}, \tag{A.8}$$

subject to (11) as well as the terminal and boundary conditions

$$\tilde{V}_t(\tilde{n}, T) = (1-\psi)^{\gamma} V^* \frac{\tilde{n}^{1-\gamma}}{1-\gamma}; \qquad \lim_{\tilde{n} \to -\tilde{h}_t(a)} \tilde{V}_t(\tilde{n}, a) = \begin{cases} 0, & \text{if } \gamma < 1 \\ -\infty, & \text{if } \gamma \geq 1 \end{cases}. \tag{A.9}$$

The terminal condition captures the effect of bequests and the boundary condition the fact that consumption is zero if the entrepreneur hits the natural borrowing limit.

Using Ito's lemma, the HJB reduces to a partial differential equation for $\tilde{V}_t(\tilde{n}, a; A_t)$:

$$\rho \tilde{V}_t = \max_{\tilde{c}_t, \tilde{\theta}_t^{ag}, \tilde{\theta}_t^{id}, k_t, l_t, \iota_t} \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} + \frac{\partial \tilde{V}_t}{\partial t} + \frac{\partial \tilde{V}_t}{\partial a} + \frac{\partial \tilde{V}_t}{\partial \tilde{n}} \mu_{\tilde{n},t} + \frac{\partial \tilde{V}_t}{\partial A_t} \mu_A A_t + \frac{1}{2} \frac{\partial^2 \tilde{V}_t}{\partial \tilde{n}^2} (\sigma_{ag,t}^2 + \sigma_{id,t}^2) + \frac{\partial^2 \tilde{V}_t}{\partial \tilde{n} \partial A_t} \sigma_{ag,t} \sigma_A A_t + \frac{1}{2} \frac{\partial \tilde{V}_i}{\partial A_t^2} \sigma_A^2 A_t^2, \tag{A.10}$$

subject to (11), where $(\mu_{\tilde{n},t}, \sigma_{ag,t}, \sigma_{id,t})$ are the drift and diffusion terms for $\tilde{n}_t$.

First, we verify that the following guess for the value function solves the PDE

$$\tilde{V}_t(\tilde{n}, a; A_t) = \zeta_t(a)^{-\gamma} \frac{\left(\tilde{n} + A_t h_t(a)\right)^{1-\gamma}}{1-\gamma}. \tag{A.11}$$

Plugging the derivatives of the equation above into the HJB equation, we obtain

$$\frac{\rho}{1-\gamma} = \max_{c_{i,t},k_{i,t},l_{i,t},\iota_{i,t},\theta^{ag}_{i,t},\theta^{id}_{i,t}} \Bigg\{ \frac{\zeta_t^{\gamma}(a)}{1-\gamma}\left(\frac{c_{i,t}}{\omega_{i,t}}\right)^{1-\gamma} - \frac{\gamma}{1-\gamma}\frac{1}{\zeta_t(a)}\left(\frac{\partial \zeta_t(a)}{\partial t} + \frac{\partial \zeta_t(a)}{\partial a}\right) + r_t + \frac{q_t k_{i,t}}{\omega_{i,t}}(\mu^R_{i,t} - r_t)$$
$$- \frac{p_t^{ag}\theta^{ag}_{i,t}}{\omega_{i,t}} + \frac{h_{i,t}}{\omega_{i,t}}\sigma_A p_t^{ag} - \frac{c_{i,t}}{\omega_{i,t}} - \frac{\gamma}{2}\left[\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right)^2 + \left(\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right)^2\right]$$
$$+ p_t^{id}\left[(1-\phi)\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right]\Bigg\}, \tag{A.12}$$

where $p^{id}_{i,t}$ denotes the Lagrange multiplier on the skin-in-the-game constraint.

From the expression above, it is immediate that the optimal value of $(l_{i,t}, \iota_{i,t})$ maximizes the expected return on the business. The first-order conditions for $(l_{i,t}, \iota_{i,t})$ are given in (15) and (16), respectively. The expected return on the business will be equalized, allowing us to write $\mu^R_{i,t} = \mu^R_t$.

Policy functions. The first order condition for $\theta^{id}_{i,t}$ is given by

$$\gamma\left[\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right] = p^{id}_{i,t}. \tag{A.13}$$

The equation above implies that the skin-in-the-game constraint is always binding, so $p^{id}_{i,t} > 0$ and $\theta^{id}_t = (1-\phi)q_t k_{i,t}\sigma_I$. If this was not the case, i.e. $p^{id}_{i,t} = 0$, then we would have $\theta^{id}_{i,t} = q_t k_{i,t}\sigma_I$, which violates the skin-in-the-game constraint.

The first-order conditions for capital and aggregate insurance are given by

$$\mu^R_t - r + p^{id}_{i,t}(1-\phi)\sigma_I = \gamma\left[\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right)\sigma_A + \left(\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right)\sigma_I\right]$$
$$p_t^{ag} = \gamma\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right). \tag{A.14}$$

Combining the expressions above, we obtain

$$p^{id}_{i,t} = \frac{\mu^R_t - r_t - p^{ag}_t\sigma_A}{\phi\sigma_I}, \tag{A.15}$$

which coincides with expression (22) after we write $p^{id}_{i,t} = p^{id}_t$.

The demand for capital can be written as

$$\frac{q_t k_{i,t}}{\omega_{i,t}} = \frac{p_t^{id}}{\gamma\phi\sigma_I}. \tag{A.16}$$

Multiplying by $\omega_{i,t}/n_{i,t}$, we obtain expression (21). Solving for $\theta_{i,t}^{ag}$ in the optimality condition for aggregate insurance we obtain (23).

The first-order condition for consumption gives

$$\frac{c_{i,t}}{\omega_{i,t}} = \zeta_t(a). \tag{A.17}$$

Plugging the expressions above back into the HJB, we obtain a PDE for $\zeta_t(a)$

$$\frac{\partial\zeta_t(a)}{\partial t} + \frac{\partial\zeta_t(a)}{\partial a} = \zeta_t^2(a) - \bar{r}_t\zeta_t(a), \tag{A.18}$$

where $\bar{r}_t \equiv \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right)\left[r + \frac{(p_t^{id})^2 + (p_t^{ag})^2}{2\gamma}\right]$.

Define $z_{s,t} \equiv \zeta_t^{-1}(t-s)$ as the wealth-consumption ratio for an entrepreneur born at date s. Differentiating with respect to t, we obtain

$$\dot{z}_{s,t} = -\frac{1}{\zeta_t^2(a)}\left[\frac{\partial\zeta_t}{\partial t} + \frac{\partial\zeta_t}{\partial a}\right] = \bar{r}_t z_{s,t} - 1. \tag{A.19}$$

Solving the above differential equation, we get

$$z_{s,t} = \int_t^{s+T} e^{-\int_t^u \bar{r}_z dz} du + e^{-\int_t^{s+T} \bar{r}_z dz} z_{s,s+T}, \tag{A.20}$$

or in terms of $\zeta_t(a)$, we have

$$\zeta_t(a) = \frac{1}{\int_t^{t+T-a} e^{-\int_t^u \bar{r}_z dz} du + e^{-\int_t^{t+T-a} \bar{r}_z dz}(1-\psi)(V^*)^{\frac{1}{\gamma}}}, \tag{A.21}$$

where we used the boundary condition $\zeta_t^{-1}(T) = (1-\psi)\,(V^*)^{\frac{1}{\gamma}}$.

Assuming $(V^*)^{\frac{1}{\gamma}} = \frac{1}{\bar{r}}$ and a stationary equilibrium, where $\bar{r}_t = \bar{r}$, we obtain

$$\zeta(a) = \frac{\bar{r}}{1 - \psi e^{-\bar{r}(T-a)}}. \tag{A.22}$$

$(V^*)^{\frac{1}{\gamma}} = \frac{1}{\bar{r}}$ ensures $\psi = 0$ gives the corresponding value in an infinite-horizon economy.

Price and quantity of aggregate insurance. The demand for aggregate insurance for entrepreneus is $\theta^{ag}_{i,t} = (q_t k_{i,t} + h_{i,t})\sigma_A - \omega_{i,t}\frac{p^{ag}_t}{\gamma}$. A similar expression holds for wage earners

$$\theta^{ag}_{j,t} = h_{j,t}\sigma_A - (n_{j,t} + h_{j,t})\frac{p^{ag}}{\gamma}, \tag{A.23}$$

Combining the demand for aggregate insurance for entrepreneurs and wage earners with the corresponding market-clearing condition, we obtain

$$\int_{\mathcal{E}_t}\left[(qk_{i,t} + h_{i,t})\sigma_A - (n_{i,t} + h_{i,t})\frac{p^{ag}}{\gamma}\right]di + \int_{\mathcal{W}_t}\left[h_{j,t}\sigma_A - (n_{j,t} + h_{j,t})\frac{p^{ag}}{\gamma}\right]di = 0. \tag{A.24}$$

Rearranging the expression above, we can solve for the price of aggregate insurance p^{ag}

$$p^{ag}_t = \frac{\int_{\mathcal{E}_t}(qk_{i,t} + h_{i,t})di + \int_{\mathcal{W}_t} h_{j,t}dj}{\int_{\mathcal{E}_t}(n_{i,t} + h_{i,t})di + \int_{\mathcal{W}_t}(n_{j,t} + h_{j,t})dj}\gamma\sigma_A = \gamma\sigma_A, \tag{A.25}$$

using the fact that $\int_{\mathcal{E}_t} n_{i,t}di + \int_{\mathcal{W}_t} n_{j,t}dj = \int_{\mathcal{E}_t} qk_{i,t}di$. The demand for aggregate insurance for entrepreneurs if then given by $\theta^{ag}_{j,t} = (q_t k_{i,t} - n_{i,t})\sigma_A$.

□

A.2 Proof of Proposition 2

Proof. We start by deriving the law of motion of financial wealth for an entrepreneur of a given age. Using the value of capital, $k_{i,t}$, aggregate and idiosyncratic insurance, $(\theta^{ag}_{i,t}, \theta^{id}_{i,t})$, and the definition of the price of idiosyncratic risk, p^{id}_t, given in Proposition 1, we can write the law of motion of financial wealth as follows

$$\begin{aligned} d\tilde{n}_{i,t} = &\left[r_t\tilde{\omega}_{i,t} + \frac{(p^{id}_t)^2}{\gamma}\tilde{\omega}_{i,t} + \frac{(p^{ag}_t)^2}{\gamma}\tilde{\omega}_{i,t} - \tilde{h}_{i,t}(r_t + p^{ag}_t\sigma_A) + \tilde{w}_t\bar{l}_{i,t} - \tilde{c}_{i,t}\right]dt \\ &+ \left(\tilde{\omega}_{i,t}\frac{p^{ag}_t}{\gamma} - \tilde{h}_{i,t}\sigma_A\right)dZ_t + \frac{p^{id}_t}{\gamma}\tilde{\omega}_{i,t}dZ_{i,t}, \end{aligned} \tag{A.26}$$

where $\tilde{\omega}_{i,t} = \tilde{n}_{i,t} + \tilde{h}_{i,t}$.

Using the fact that $p^{ag}_t = \gamma\sigma_A$ in equilibrium, we find that the aggregate risk exposure of entrepreneurs is given $\tilde{n}_{i,t}\sigma_A$. Hence, scaled financial wealth, $n_{i,t} = \tilde{n}_{i,t}/A_t$, does not respond to aggregate shocks. The evolution of $n_{i,t}$ can then be written as

$$dn_{i,t} = \mu_{n,t}(n_{i,t}, a)dt + \sigma_{n,t}(n_{i,t}, a)dZ_{i,t}, \tag{A.27}$$

where

$$\mu_{n,t}(n,a) = \left[r_t + \frac{(p_t^{id})^2}{\gamma} + \frac{(p_t^{ag})^2}{\gamma} - \mu_A - \zeta_t(a)\right](n + h_t(a)) - \mu_{h,t}(a) \tag{A.28}$$

$$\sigma_{n,t}(n,a) = \frac{p_t^{id}}{\gamma}(n + h_t(a)), \tag{A.29}$$

and $\mu_{h,t}(a)$ is the drift of $h_t(a)$.

Derivation of Equation (25). Notice that total wealth evolves according to

$$\frac{d\omega_{i,t}}{\omega_{i,t}} = \left[r_t + \frac{(p_t^{ag})^2}{\gamma} + \frac{(p_t^{id})^2}{\gamma} - \mu_A - \zeta_t(t - s_i)\right]dt + \frac{p_t^{id}}{\gamma}dZ_{i,t}, \tag{A.30}$$

where s_i is the birthdate of entrepreneur i.

Let $\overline{\omega}_{s,t} \equiv \frac{\int_{\mathcal{E}_t} \mathbb{1}_{s_i=s}\omega_{i,t}di}{\int_{\mathcal{E}_t} \mathbb{1}_{s_i=s}di}$ denote the average total wealth of entrepreneurs born at date s. The law of motion of $\overline{\omega}_{s,t}$ is given by

$$d\overline{\omega}_{s,t} = \left[r_t + \frac{(p_t^{ag})^2}{\gamma} + \frac{(p_t^{id})^2}{\gamma} - \mu_A - \zeta_t(t - s)\right]\overline{\omega}_{s,t}dt \tag{A.31}$$

where the idiosyncratic risk is diversified by averaging out across entrepreneurs of a given cohort.

It is convenient to express total wealth as a function of age instead of the entrepreneurs' birthdate. Let $\omega_t(a)$ denote the average total wealth of investors with age a at period t. Using the fact that $\overline{\omega}_{s,t} = \omega_t(t - s)$, we obtain the following PDE for $\omega_t(a)$:

$$\frac{\partial\omega_t(a)}{\partial t} + \frac{\partial\omega_t(a)}{\partial a} = \left[r_t + \frac{(p_t^{ag})^2}{\gamma} + \frac{(p_t^{id})^2}{\gamma} - \mu_A - \zeta_t(a)\right]\omega_t(a). \tag{A.32}$$

In a stationary equilibrium, $\omega_t(a)$ does not depend on calendar time t, which allow us to write

$$\frac{d\log\omega(a)}{da} = r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - \zeta(a). \tag{A.33}$$

Integrating the expression above, we obtain

$$\log\omega(a) = \log\omega(0) + \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A\right]a - \int_0^a \zeta(u)du. \tag{A.34}$$

Using the fact $\log\frac{\omega(a)}{\omega(0)} = \log\frac{f(a)\omega(a)}{f(0)\omega(0)} + ga$ and the identity $\omega(a) = n(a)\left(1+\frac{h(a)}{n(a)}\right)$, we obtain expression (25) after some rearrangement.

Initial wealth. The expression for $\omega(a)$ in levels can be written as

$$\omega(a) = \omega(0)e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A\right)a}\frac{e^{-\bar{r}a}-\psi e^{-\bar{r}T}}{1-\psi e^{-\bar{r}T}}. \tag{A.35}$$

Evaluating at $a = T$ gives

$$\omega(T) = \omega(0)e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A-mpc_e\right)T}, \tag{A.36}$$

where $mpc_e = \frac{1}{T}\int_0^T \zeta(a)da$.

The boundary condition at age T implies $\omega(0) = e^{-gT}\omega(T) + h(0)$, then

$$\omega(0) = \frac{h(0)}{1-e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A-g-mpc_e\right)T}}. \tag{A.37}$$

Using $\omega(0) = n(0) + h(0)$ and rearranging the resulting expression for $n(0)$.

Derivation of Equation (26). Multiplying Equation (A.35) by $f(a)$, integrating over age, and using the fact that $f(a) = e^{-ga}f(0)$, we obtain

$$n_e + h_e = f(0)\omega(0)\int_0^T e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-(g+\mu_A)\right)a}\frac{e^{-\bar{r}a}-\psi e^{-\bar{r}T}}{1-\psi e^{-\bar{r}T}}da, \tag{A.38}$$

which gives Equation (26) after some rearrangement .

□

A.3 Proof of Lemma 2

Proof. We derive the Kolmogorov Forward Equation as the limit of a discrete-time economy. The discrete-time approximation goes as follows. Time takes values on the discrete set $\{t^1,\ldots,t^L\}$, where $\Delta t = t^{l+1} - t^l$ is the constant time step. Scaled financial wealth $n_{i,t}$ takes values on a discrete grid, $n_{i,t} \in \{n^1, n^2, \ldots, n^J\}$ with a constant step size $\Delta n = n^{j+1} - n^j$. Age is also assumed to take values in a discrete grid $\{a^1,\ldots,a^K\}$, where $\Delta a = a^{k+1} - a^k$, $a^1 = 0$, and $a^K = T$. For simplicity, assume $\Delta a = \Delta t$. The probability of moving up, down, or staying at the same point of the grid is chosen to approximate (A.27) and are given, respectively, by

$$p_u(n^j, a^k) = \frac{1}{2}\left[\frac{\sigma_n(n^j, a^k)^2}{\overline{\sigma}^2} + \frac{\mu_n(n^j, a^k)}{\overline{\sigma}^2}\Delta n\right] \tag{A.39}$$

$$p_d(n^j, a^k) = \frac{1}{2}\left[\frac{\sigma_n(n^j, a^k)^2}{\overline{\sigma}^2} - \frac{\mu_n(n^j, a^k)}{\overline{\sigma}^2}\Delta n\right] \tag{A.40}$$

$$p_s(n^j, a^k) = 1 - \frac{\sigma_n(n^j, a^k)^2}{\overline{\sigma}^2}. \tag{A.41}$$

where $\overline{\sigma} = \max_{1\leq j\leq J, 1\leq k\leq K} \sigma_n(n^j, a^k)$, $\Delta n = \overline{\sigma}\sqrt{\Delta t}$, and $\Delta a = \Delta t$.

Notice that the expected change in $n_{i,t}$, where $n_{i,t} = n^j$ and $a_i = a^k$, is given by

$$\mathbb{E}\left[n_{i,t+1} - n_{i,t}\right] = p_u(n^j, a^k)\Delta n + p_d(n^j, a^k)(-\Delta n) = \mu_n(n^j, a^k)\Delta t, \tag{A.42}$$

and

$$\mathbb{E}\left[(n_{i,t+1} - n_{i,t})^2\right] = p_u(n^j, a^k)\Delta n^2 + p_d(n^j, a^k)(-\Delta n)^2 = \sigma_n(n^j, a^k)^2\Delta t. \tag{A.43}$$

Let $m(n^j, a^k, t^l)$ denote the mass of agents with financial wealth n^j, age a^k, at period t^l. Summing over n^j, we obtain the mass of agents with age a^k, $M_{k,l} \equiv \sum_{j=1}^{J} m(n^j, a^k, t^l) = e^{g(t^l-(k-1)\Delta t)}$. Summing over (n^j, a^k), we obtain the total population $M_l = \sum_{k=1}^{K} M_{k,l}$, so $M_{l+1} = e^{g\Delta t}M_l$. The law of motion of m, for $k > 1$ and $1 < j < J$, is given by

$$\begin{aligned} m(n^j, a^k, t^l + \Delta t) = {} & p_u(n^j - \Delta n, a^k - \Delta a)m(n^j - \Delta n, a^k - \Delta a, t^l) + p_s(n^j, a^k - \Delta a)m(n^j, a^k - \Delta a, t^l) \\ & + p_d(n^j + \Delta n, a^k - \Delta a)m(n^j + \Delta n, a^k - \Delta a, t^l). \end{aligned} \tag{A.44}$$

The boundary conditions are defined as follows. For $j = 1$ and $j = J$, we will assume a reflecting boundary, that is, if n moves up from n_J or down from n_1, it is immediately reflected back to its initial position

$$\begin{aligned} m(n^J, a^k, t^l + \Delta t) = {} & p_u(n^J - \Delta n, a^k - \Delta a)m(n^J - \Delta n, a^k - \Delta a, t^l) + p_s(n^J, a^k - \Delta a)m(n^J, a^k - \Delta a, t^l) \\ & + p_u(n^j, a^k - \Delta a)m(n^J, a^k - \Delta a, t^l), \end{aligned} \tag{A.45}$$

and analogously for $j = 1$.

Finally, for $k = 1$, we have

$$m(e^{-gT}n^j, a^1, t^l + \Delta t) = e^{gT}\left[p_u\left(n^j - \Delta n, a^K\right) m\left(n^j - \Delta n, a^K, t^l\right) + p_s\left(n^j, a^k\right) m\left(n^j, a^K, t^l\right) + p_d\left(n^j + \Delta n, a^K\right) m\left(n^j + \Delta n, a^K, t^l\right)\right]. \tag{A.46}$$

since each one of the e^{gT} heirs inherit $e^{-gT}n^j$, where we assumed $e^{-gT}n^j$ belongs to the grid.

Let $f(n^j, a^k, t^l) \equiv \frac{m(n^j, a^k, t^l)}{M_l}$ denote the share of agents in state (n^j, a^k) in period t^l. Dividing both sides of (A.44) by M_l and taking a Taylor expansion, we obtain

$$\begin{aligned}(1+g\Delta t)(f + f_t\Delta t) = \frac{1}{2}\left(\frac{\sigma_n^2 - (\sigma_n^2)_n\Delta n + 0.5(\sigma_n^2)_{nn}\Delta n^2 - (\sigma_n^2)_a\Delta t}{\overline{\sigma}^2} + \frac{\mu_n - (\mu_n)_n\Delta n}{\overline{\sigma}^2}\Delta n\right)\left(f - f_a\Delta t - f_n\Delta n + 0.5 f_{nn}\Delta n^2\right) \\ + \frac{1}{2}\left(\frac{\sigma_n^2 + (\sigma_n^2)_n\Delta n + 0.5(\sigma_n^2)_{nn}\Delta n^2 - (\sigma_n^2)_a\Delta t}{\overline{\sigma}^2} - \frac{\mu_n + (\mu_n)_n\Delta n}{\overline{\sigma}^2}\Delta n\right)\left(f - f_a\Delta t + f_n\Delta n + 0.5 f_{nn}\Delta n^2\right) \\ + \left(1 - \frac{\sigma_n^2 - (\sigma_n^2)_a\Delta t}{\overline{\sigma}^2}\right)(f - f_a\Delta t) + o(\Delta t).\end{aligned} \tag{A.47}$$

Simplifying the expression above and taking the limit $\Delta t \to 0$, we obtain

$$f_t + f_a + gf = \frac{1}{2}(\sigma_n^2)_{nn} f - (\mu_n)_n f + (\sigma_n^2)_n f_n - \mu_n f_n + \frac{1}{2}\sigma_n^2 f_{nn}, \tag{A.48}$$

or, more explicitly, we can write the expression as follows

$$\frac{\partial f(n,a,t)}{\partial t} + \frac{\partial f(n,a,t)}{\partial a} + g f(n,a,t) = -\frac{\partial[f(n,a,t)\mu_n(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial\left[f(n,a,t)\sigma_n^2(n,a)\right]}{\partial n^2}. \tag{A.49}$$

Let $f_t(n|a)$ denote the conditional density at date t, so $f_t(n,a) = f_t(n|a)f(a)$. We can write the Kolmogorov Forward Equation in terms of the conditional density:

$$f(a)\frac{\partial f_t(n|a)}{\partial t} + f(a)\frac{\partial f_t(n|a)}{\partial a} + f_t(n|a)f'(a) = -f(a)\frac{\partial[f_t(n|a)\mu_n(n,a)]}{\partial n} + f(a)\frac{1}{2}\frac{\partial\left[f_t(n|a)\sigma_n^2(n,a)\right]}{\partial n^2} - g f_t(n,a). \tag{A.50}$$

Dividing by $f(a)$ and using the fact that $f'(a) = -gf(a)$, we obtain

$$\frac{\partial f_t(n|a)}{\partial t} + \frac{\partial f_t(n|a)}{\partial a} = -\frac{\partial[f_t(n|a)\mu_n(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial\left[f_t(n|a)\sigma_n^2(n,a)\right]}{\partial n^2}. \tag{A.51}$$

In a stationary equilibrium, we can ignore the dependence on calendar time to obtain

$$\frac{\partial f(n|a)}{\partial a} = -\frac{\partial[f(n|a)\mu_n(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial\left[f(n|a)\sigma_n^2(n,a)\right]}{\partial n^2}. \tag{A.52}$$

□

A.4 Proof of Proposition 3

Proof. The law of motion of (log) total wealth is

$$d\log\omega_{i,t} = \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \frac{\bar{r}}{1-e^{-\bar{r}(T-(t-s))}} - \mu_A - \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2\right]dt + \frac{p^{id}}{\gamma}dZ_{i,t}, \tag{A.53}$$

where s denotes the birth date of entrepreneur i.

Integrating the expression above, we obtain

$$\log\omega_{i,t} = \log\omega_{i,s} + \int_s^t \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \frac{\bar{r}}{1-e^{-\bar{r}(T-(t'-s))}} - \mu_A - \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2\right]dt' + \frac{p^{id}}{\gamma}(Z_{i,t} - Z_{i,s}), \tag{A.54}$$

where $Z_{i,t} - Z_{i,s} \sim \mathcal{N}(0,a)$ and $a = t - s$.

Hence, $\log\omega_{i,t} \sim \mathcal{N}(m(a), v(a))$, where the mean and variance are given by

$$m(a) = \log h(0) + \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2 - \bar{r}\right]a + \log\frac{1-e^{-\bar{r}(T-a)}}{1-e^{-\bar{r}T}} \tag{A.55}$$

$$v(a) = \left(\frac{p^{id}}{\gamma}\right)^2 a, \tag{A.56}$$

using the fact that $\omega_{i,s_i} = h(0)$ when $\psi = 1$.

As the ratio of consumption to total wealth is the same for all entrepreneurs of the same age, the variance of log consumption is given by

$$\mathbb{V}[\log c_{i,t}|a] = \mathbb{V}[\log\omega_{i,t}|a] = \left(\frac{p^{id}}{\gamma}\right)^2 a. \tag{A.57}$$

Normalized financial wealth $n_{i,t} = \omega_{i,t} - h_{i,t}$ has a shifted log-normal distribution conditional on $s_i = s$, with support $(-h(a), \infty)$. The expected value and variance of $n_{i,t}$ are

$$\mathbb{E}[n|a] = h(0)e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A-\bar{r}\right)a}\frac{1-e^{-\bar{r}(T-a)}}{1-e^{-\bar{r}T}} - h(a) \tag{A.58}$$

$$\mathbb{V}[n|a] = \left[e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1\right]\left[h(0)e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A\right)a}\frac{e^{-\bar{r}a} - e^{-\bar{r}T}}{1-e^{-\bar{r}T}}\right]^2. \tag{A.59}$$

We show next that $\mathbb{V}[n|a]$ has an inverted U shape. Define the following functions:

$$v_1(a) = \left[e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1\right]^{\frac{1}{2}} e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A\right)a}; \qquad v_2(a) = \frac{e^{-\bar{r}a} - e^{-\bar{r}T}}{1-e^{-\bar{r}T}}. \tag{A.60}$$

The derivative of the product of $v_1(a)$ and $v_2(a)$ will be positive if

$$v_1'(a)v_2(a) + v_1(a)v_2'(a) > 0 \iff \frac{v_1'(a)}{v_1(a)} > -\frac{v_2'(a)}{v_2(a)}, \tag{A.61}$$

for $a \neq 0$ and $a \neq T$. Notice that $-v_2'(a)/v_2(a)$ is positive, monotonically increasing, and approaches ∞ as a approaches T:

$$-\frac{v_2'(a)}{v_2(a)} = \bar{r}\frac{1}{1 - e^{-\bar{r}(T-a)}}. \tag{A.62}$$

The term $v_1'(a)/v_1(a)$ is positive, monotonically decreasing, and approaches $+\infty$ as $a \to 0$:

$$\frac{v_1'(a)}{v_1(a)} = \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2 \frac{e^{\left(\frac{p^{id}}{\gamma}\right)^2 a}}{e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1} + r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A. \tag{A.63}$$

Hence, there exists a unique $0 < \hat{a} < T$ such that $v_1'(a)v_2(a) + v_1(a)v_2'(a) > 0$ for all $a < \hat{a}$ and $v_1'(a)v_2(a) + v_1(a)v_2'(a) < 0$ for all $a > \hat{a}$. Hence, $\mathbb{V}[n|a]$ is equal to zero at $a = 0$, it increases monotonically for $a < \hat{a}$, where it achieves the maximum, and it decreases towards zero for $\hat{a} < a \leq T$.

□

A.5 Proof of Lemma 3

Proof. The HJB for the entrepreneur's problem is given by

$$\rho \tilde{V}_t = \max_{\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_t^{id}, k_i, l_i, \iota_i} \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} + \frac{\mathbb{E}_t\left[d\tilde{V}_t\right]}{dt} \tag{A.64}$$

subject to (11). We guess-and-verify that the value function can be written as

$$\tilde{V}^j(\tilde{\omega}; A_t) = A_t^{1-\gamma} V^j\left(\frac{\tilde{\omega}}{A_t}\right). \tag{A.65}$$

where $V_j(\omega)$ is independent of A_t, and $j \in \{1, \ldots, n\}$ index the entrepreneur's type.

Let $\omega_{i,t} \equiv \tilde{\omega}_{i,t}/A_t$ denote scaled total wealth. From the law of motion of $h_{i,t} = \tilde{h}_{i,t}/A_t$ in Lemma 1 and the law of motion of $n_{i,t}$, we obtain

$$d\omega_{i,t} = \mu_{\omega_i,t}dt + \sigma_{i,t}^{ag}dZ_t + \sigma_{i,t}^{id}dZ_{i,t}, \tag{A.66}$$

where $\sigma^{ag}_{i,t} \equiv (qk_{i,t} + h_{i,t} - \omega_{i,t})\,\sigma_A - \theta^{ag}_{i,t}$, $\sigma^{id}_{i,t} \equiv qk_{i,t}\sigma_{I,i} - \theta^{id}_{i,t}$, and

$$\mu_{\omega_i,t} \equiv (r + p^{ag}\sigma_A - \mu_A)\omega_{i,t} + qk_{i,t}(\hat{\mu}^R_{i,t} - r - p^{ag}\sigma_A) + (p^{ag} - \sigma_A)\sigma^{ag}_{i,t} - c_{i,t}. \tag{A.67}$$

The HJB equation for the scaled value function can be written as

$$\hat{\rho}V^j = \max_{c_i,\theta^{ag}_i,\theta^{id}_t,k_i,l_i,\iota_i} \frac{c_t^{1-\gamma}}{1-\gamma} + V^j_a + V^j_\omega\left[\hat{r}\omega_{i,t} + qk_{i,t}(\hat{\mu}^R_{i,t} - r - p^{ag}\sigma_A) + \hat{p}^{ag}\sigma^{ag}_{i,t} - c_{i,t}\right] + \frac{1}{2}V^j_{\omega\omega}\left((\sigma^{ag}_{i,t})^2 + (\sigma^{id}_{i,t})^2\right). \tag{A.68}$$

subject to $\theta^{id}_{i,t} \le (1-\phi)qk_{i,t}\sigma_{I,i}$ and $\omega_{i,t} \ge 0$, where $\sigma_{I,i} = \sigma^j_I$ and

$$\hat{\rho} \equiv \rho - (1-\gamma)\left(\mu_A - \frac{\gamma\sigma_A^2}{2}\right), \qquad \hat{r} \equiv r + p^{ag}\sigma_A - \mu_A, \qquad \hat{p}^{ag} \equiv p^{ag} - \gamma\sigma_A. \tag{A.69}$$

It is optimal to choose $l_{i,t}$ and $\iota_{i,t}$ to maximize expected returns. The investment demand is then given by Equation (16) and the labor demand is given by $w = \beta k^{\alpha}_{i,t} l^{\beta-1}_{i,t}$.

The expected return for entrepreneur i is given by

$$\hat{\mu}^R_{i,t} = \frac{(1-\beta)\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_{i,t}^{\frac{\alpha+\beta-1}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{A.70}$$

The first-order condition for aggregate insurance is given by

$$\hat{p}^{ag}_t = -\frac{V^j_{\omega\omega}}{V^j_\omega}\sigma^{ag}_{i,t} \Rightarrow \theta^{ag}_{i,t} = (qk_{i,t} - n_{i,t})\sigma_A + \frac{V^j_\omega}{V^j_{\omega\omega}}\hat{p}^{ag}. \tag{A.71}$$

In equilibrium, we must have that average $\sigma^{ag}_{i,t}$ across entrepreneurs and workers must be equal to zero, such that scaled wealth is, on average, not exposed to aggregate risk. Then, we have $\hat{p}^{ag}_t = 0$, so $p^{ag}_t = \gamma\sigma_A$, as in the baseline model. The first-order conditions for $k_{i,t}$ and $\theta^{id}_{i,t}$ are given by

$$\mu^R_{i,t} - r - \gamma\sigma_A^2 + p^{id}_i(1-\phi)\sigma_{I,i} = -\frac{V^j_{\omega\omega}}{V^j_\omega}\left[\sigma^{ag}_{i,t}\sigma_A + \sigma^{id}_{i,t}\sigma_{I,i}\right], \qquad p^{id}_{i,t} = -\frac{V^j_{\omega\omega}}{V^j_\omega}\sigma^{id}_{i,t}, \tag{A.72}$$

where $\mu^R_{i,t}$ denotes the expected *marginal* return on the business:

$$\mu^R_{i,t} = \frac{\alpha\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_{i,t}^{\frac{\alpha+\beta-1}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{A.73}$$

$\mu^R_{i,t}$ corresponds to the expected profit from an additional unit of capital divided by the cost of one unit of capital, while $\hat{\mu}^R_{i,t}$ corresponds to the expected total profit divided by the cost of the entire capital stock. Under constant returns to scale, the two concepts coincide. With DRS, the marginal concept is the relevant determining the investment decision.

Given that $\sigma^{id}_{i,t} > 0$ by the insurance constraint, and given the concavity of the value function, $V^j_{\omega\omega} < 0$, we have that $p^{id}_{i,t} > 0$. Therefore, the insurance constraint is always binding, that is, $\theta_{i,t} = (1-\phi)qk_{i,t}\sigma_{I,i}$. Rearranging the expressions above, we obtain

$$p^{id}_{i,t} = \frac{\mu^R_{i,t} - r - \gamma\sigma^2_A}{\phi\sigma_{I,i}}, \qquad qk_{i,t} = -\frac{V^j_\omega}{V^j_{\omega\omega}}\frac{p^{id}_{i,t}}{\phi\sigma_{I,i}}. \tag{A.74}$$

Finally, the optimality condition for consumption is given by $c^{-\gamma}_{i,t} = V^j_\omega$. □

A.6 Proof of Propositions 4 and 5

Proof. We consider the case where entrepreneurs have different levels of skill, $e_i \in \{e^1, \ldots, e^n\}$, and different exposures to idiosyncratic risk $\sigma_{I,i} \in \{\sigma^1_I, \ldots, \sigma^n_I\}$. The share of entrepreneurs of type j is denoted by $\chi_{e,j}$. Consider the log-linear approximation of the policy functions:

$$\hat{c}_{i,t} = \psi^j_{c,\omega}\hat{\omega}_{i,t}, \qquad \hat{k}_{i,t} = \psi^j_{k,\omega}\hat{\omega}_{i,t}, \tag{A.75}$$

up to first order in $\hat{w}_{i,t} = \log\omega_{i,t} - \log\overline{\omega}_j$, where $\overline{\omega}_j \equiv \exp\mathbb{E}[\log\omega_{i,t}|\sigma^i_I = \sigma^j_I, e_i = e^j]$, $\hat{c}_{i,t} = \log c_{i,t} - \log c_j(\overline{\omega}_j)$, and $\hat{k}_{i,t} = \log k_{i,t} - \log k_j(\overline{\omega}_j)$.

Wages. Aggregating the labor demand condition, $w_t = \beta k^\alpha_{i,t} l^{\beta-1}_{i,t} e^{1-\alpha-\beta}_i$, we obtain

$$\int_{\mathcal{E}} l_{i,t}di = \left(\frac{\beta}{w_t}\right)^{\frac{1}{1-\beta}} \int_{\mathcal{E}} k^{\frac{\alpha}{1-\beta}}_{i,t} e^{\frac{1-\alpha-\beta}{1-\beta}}_{i,t} di = \left(\frac{\beta}{w_t}\right)^{\frac{1}{1-\beta}} \sum_{j=1}^n \tilde{\chi}_{e,j} e_i^{\frac{1-\alpha-\beta}{1-\beta}} \bar{k}_j^{\frac{\alpha}{1-\beta}} \left[\chi_e + \frac{\alpha\psi_{k,\omega}}{1-\beta}\frac{1}{\tilde{\chi}_{e,j}}\int_{\mathcal{E}_j}\hat{\omega}_{i,t}di\right]. \tag{A.76}$$

Using a law of large numbers to obtain $\int\hat{\omega}_{i,t}di = 0$, we obtain

$$w_t = \beta K^\alpha \left(\frac{\chi_e}{\bar{l}}\right)^{1-\alpha-\beta}, \tag{A.77}$$

where $K = \chi_e\left[\sum_{j=1}^n \tilde{\chi}_{e,j}(\bar{k}^j)^{\frac{\alpha}{1-\beta}}\right]^{\frac{1-\beta}{\alpha}}/\bar{l}$ is the aggregate capital-labor ratio, and $\tilde{\chi}_{e,j} \equiv \chi_{e,j}e_i^{\frac{1-\alpha-\beta}{1-\beta}}$. We choose the units of skill such that $\sum_{j=1}^n \tilde{\chi}_{e,j} = 1$. We also adopt the normalization $\bar{l} = \chi_e$, so $w_t = \beta K^\alpha$.

Expected returns. Expected returns are given by

$$\hat{\mu}^R_{i,t} = \overline{\hat{\mu}}^R + \psi_{\hat{\mu}^R,k}\hat{k}_{i,t}, \tag{A.78}$$

where $\overline{\hat{\mu}}^R \equiv \frac{(1-\beta)B\bar{k}^{\alpha-1}-\iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta$ and $\psi_{\hat{\mu}^R,k} \equiv -\frac{(1-\alpha-\beta)B\bar{k}^{\alpha-1}}{q}$, where $B_j = e_j^{\frac{1-\alpha-\beta}{1-\beta}}\left(\frac{\bar{k}_j}{K}\right)^{\frac{\alpha\beta}{1-\beta}}$. To ease notation, we dropped the subscript j from above.

Expected marginal returns. The expected marginal return on the business is given by

$$\mu^R_{i,t} = \overline{\mu}^R - \psi_{\mu^R,k}\hat{k}_t, \tag{A.79}$$

where $\overline{\mu}^R \equiv \frac{\alpha B\bar{k}^{\alpha-1}-\iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta$ and $\psi_{\mu^R,k} \equiv \frac{1-\alpha-\beta}{1-\alpha}\frac{\alpha B\bar{k}^{\alpha-1}}{q}$.

Log portfolio returns. Define the expected log portfolio returns as

$$r^p_{i,t} \equiv \hat{r} + z_{i,t}(\mu^R_{i,t} - r - \gamma\sigma_A^2) - \frac{1}{2}\left(z_{i,t}\phi\sigma_I\right)^2. \tag{A.80}$$

where $z_{i,t} \equiv \frac{qk_{i,t}}{\omega_{i,t}}$. Log-linearizing the expression above, we obtain

$$r^p_{i,t} = \bar{r}^p + \psi_{r^p,z}\hat{z}_{i,t} + \psi_{r^p,k}k_{i,t}, \tag{A.81}$$

$\bar{r}^p \equiv \hat{r} + \bar{z}(\overline{\hat{\mu}}^R - r - \gamma\sigma_A^2) - \frac{1}{2}\left(\bar{z}\phi\sigma_I\right)^2$, $\psi_{r^p,z} \equiv \bar{z}(\overline{\hat{\mu}}^R - r - \gamma\sigma_A^2) - \left(\bar{z}\phi\sigma_I\right)^2$, $\psi_{r^p,k} \equiv \bar{z}\psi_{\hat{\mu}^R,k}$.

Wealth dynamics. The log of total wealth for entrepreneur i evolves according to

$$d\log\omega_{i,t} = \left[r^p_{i,t} - \frac{c_{i,t}}{\omega_{i,t}}\right]dt + z_{i,t}\phi\sigma_I dZ_{i,t}, \tag{A.82}$$

Log-linearizing the law of motion of $\omega_{i,t}$, we obtain

$$d\hat{\omega}_{i,t} = \left[\psi_{\omega,0} + \psi_{\omega,\omega}\hat{\omega}_{i,t}\right]dt + \phi\sigma_I z_{i,t}dZ_{i,t}, \tag{A.83}$$

where $\psi_{\omega,0} = \bar{r}^p - \frac{\bar{c}}{\bar{\omega}}$, and $\psi_{\omega,\omega} = \psi_{r^p,z}(\psi_{k,\omega} - 1) + \psi_{r^p,k}\psi_{k,\omega} - \frac{\bar{c}}{\bar{\omega}}(\psi_{c,\omega} - 1)$.

Risk taking. The first-order condition for capital and consumption are given by

$$-V_{\omega\omega}\sigma^{id}_{i,t} = V_\omega p^{id}_{i,t}, \qquad V_\omega = c_{i,t}^{-\gamma}. \tag{A.84}$$

Using the fact that $dV_\omega dZ_{i,t} = V_{\omega\omega}\sigma^{id}_{i,t}dt$ and the expressions above, we can express the optimality condition for capital as follows:

$$p^{id}_{i,t}dt = \gamma \frac{dc_{i,t}}{c_{i,t}} dZ_{i,t}. \tag{A.85}$$

Up to first order, we have that $\frac{dc_{i,t}}{c_{i,t}} dZ_{i,t} = d\hat{c}_{i,t}dZ_{i,t} = \psi_{c,\omega} d\hat{\omega}_{i,t} dZ_{i,t}$. We can then write the expression above as follows:

$$\mu^R_{i,t} - r - \gamma\sigma^2_A = \gamma\psi_{c,\omega}(\phi\sigma_I)^2 z_{i,t}. \tag{A.86}$$

Evaluating the expression above at $\omega_{i,t} = \overline{\omega}$, we obtain

$$\frac{q\bar{k}}{\overline{\omega}} = \frac{\overline{\mu}^R - r - \gamma\sigma^2_A}{\gamma\psi_{c,\omega}(\phi\sigma_I)^2} = \frac{\overline{p}^{id}}{\gamma\psi_{c,\omega}\phi\sigma_I}. \tag{A.87}$$

Linearizing the optimality condition for capital and matching coefficients, we obtain

$$-\psi_{\mu^R,k}\psi_{k,\omega} = (\overline{\mu}^R - r - \gamma\sigma^2_A)(\psi_{k,\omega} - 1). \tag{A.88}$$

Rearranging the expression above, we obtain

$$\psi_{k,\omega} \equiv \frac{\overline{\mu}^R - r - \gamma\sigma^2_A}{\overline{\mu}^R - r - \gamma\sigma^2_A + \psi_{\mu^R,k}}. \tag{A.89}$$

Consumption. The envelope condition with respect to ω is given by

$$\hat{\rho}V_\omega = V_\omega \hat{r} + \frac{\mathbb{E}\left[dV_\omega\right]}{dt}. \tag{A.90}$$

Using Ito's lemma, we can write the expression above as follows

$$r = \rho + \gamma\left(\mu_A + \frac{1}{dt}\mathbb{E}[d\hat{c}_{i,t}]\right) - \frac{\gamma(\gamma+1)}{2}\sigma^2_A - \frac{\gamma^2}{2}\mathbb{E}[(d\hat{c}_{i,t})^2]. \tag{A.91}$$

Up to first order, the expression above can be written as

$$r = \rho + \gamma\left[\mu_A + \psi_{c,\omega}\left(\psi_{\omega,0} + \psi_{\omega,\omega}\hat{\omega}_{i,t}\right)\right] - \frac{\gamma(\gamma+1)}{2}\sigma^2_A - \frac{\gamma^2}{2}\psi^2_{c,\omega}(\phi\sigma_I)^2 z^2_{i,t}. \tag{A.92}$$

By matching coefficients, we obtain the condition

$$\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}-\frac{\overline{c}}{\overline{\omega}}(\psi_{c,\omega}-1)=\frac{(\overline{p}^{id})^2}{\gamma\psi_{c,\omega}}(\psi_{k,\omega}-1) \tag{A.93}$$

Rearranging the expression above, we obtain

$$\frac{\overline{c}}{\overline{\omega}}\psi_{c,\omega}^2-\psi_{c,\omega}\left[\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}+\frac{\overline{c}}{\overline{\omega}}\right]-\frac{(\overline{p}^{id})^2}{\gamma}(1-\psi_{k,\omega}). \tag{A.94}$$

Solving the quadratic equation, we obtain

$$\psi_{c,\omega}=\frac{\overline{\omega}}{2\overline{c}}\left[\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}+\frac{\overline{c}}{\overline{\omega}}+\sqrt{\left(\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}+\frac{\overline{c}}{\overline{\omega}}\right)^2+4\frac{(\overline{p}^{id})^2}{\gamma}(1-\psi_{k,\omega})\frac{\overline{c}}{\overline{\omega}}}\right]. \tag{A.95}$$

It remains to determine the intercept:

$$r=\rho+\gamma\left[\mu_A+\psi_{c,\omega}\left(\overline{r}^p-\frac{\overline{c}}{\overline{\omega}}\right)\right]-\frac{\gamma(\gamma+1)}{2}\sigma_A^2-\frac{\gamma^2}{2}\psi_{c,\omega}^2(\phi\sigma_I)^2\overline{z}^2. \tag{A.96}$$

We can solve for the consumption-wealth ratio:

$$\frac{\overline{c}}{\overline{\omega}}=\frac{\rho}{\gamma\psi_{c,\omega}}+\left(1-\frac{1}{\gamma\psi_{c,\omega}}\right)\left[r+\frac{(p^{ag})^2}{2\gamma}+\frac{(\overline{p}^{id})^2}{2\gamma\psi_{c,\omega}}\right]+\left[\frac{\gamma\sigma_A^2}{2}-\mu_A\right]\left(1-\psi_{c,\omega}^{-1}\right)+\overline{z}(\overline{\hat{\mu}}^R-\overline{\mu}^R). \tag{A.97}$$

The expression above reduces to $\frac{\overline{c}}{\overline{\omega}}=\overline{r}$ in the case with constant returns to scale.

Determining the approximation point. At the point $\hat{\omega}_{i,t}=0$, the drift of $\hat{\omega}_{i,t}$ is zero, which implies that $\overline{r}^p=\frac{\overline{c}}{\overline{\omega}}$. The triplet $(\overline{c}_j,\overline{k}_j,\overline{\omega}_j)$ satisfies the conditions:

$$r=\rho+\gamma\mu_A-\frac{\gamma(\gamma+1)}{2}\sigma_A^2-\frac{(\overline{p}_j^{id})^2}{2},\qquad \overline{z}_j=\frac{\overline{p}^{id}}{\gamma\psi_{c,\omega}\phi\sigma_I^j},\qquad \overline{r}_j^p=\frac{\overline{c}_j}{\overline{\omega}_j}, \tag{A.98}$$

where $\overline{r}^p=r+\gamma\sigma_A^2-\mu_A+\frac{(\overline{p}^{id})^2}{\gamma\psi_{c,\omega}}-\frac{1}{2}\left(\frac{\overline{p}^{id}}{\gamma\psi_{c,\omega}}\right)^2+\overline{z}(\overline{\hat{\mu}}^R-\overline{\mu}^R)$. These equations give the value of $(\overline{c},\overline{k},\overline{\omega})$ given $\psi_{c,\omega}$. We can then iterate between the computation of $\psi_{c,\omega}$ and $(\overline{c},\overline{k},\overline{\omega})$ until convergence. Using the fact that $r=\rho_w+\gamma\mu_A-\frac{\gamma(\gamma+1)}{2}\sigma_A^2$, we can solve for the price of idiosyncratic risk:

$$\overline{p}_j^{id}=\sqrt{2(\rho-\rho_w)}. \tag{A.99}$$

Hence, the unconditional price of idiosyncratic risk is equalized across entrepreneurs.

Capital for type j satsifies the condition:

$$r + p^{ag}\sigma_A + \overline{p}_j^{id}\phi\sigma_I^j = \frac{\alpha K^{-\frac{\alpha\beta}{1-\beta}}\overline{k}_j^{\frac{\alpha}{1-\beta}-1} e_i^{\frac{1-\alpha-\beta}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{A.100}$$

We can rearrange the expression above as follows:

$$\zeta_j = K^{-\frac{\alpha\beta}{1-\beta}}\overline{k}_j^{\frac{\alpha+\beta-1}{1-\beta}} e_j^{\frac{1-\alpha-\beta}{1-\beta}} \Rightarrow \sum_{j=1}^{n} \tilde{\chi}_{e,j} e_j \zeta_j^{\frac{\alpha}{\alpha+\beta-1}} = K^{-\frac{\alpha\beta}{1-\beta}\frac{\alpha}{\alpha+\beta-1}} \sum_{j=1}^{n} \tilde{\chi}_{e,j}\overline{k}_j^{\frac{\alpha}{1-\beta}} e_j^{\frac{1-\alpha-\beta}{1-\beta}} \tag{A.101}$$

where $\zeta_j \equiv \alpha^{-1}\left\{q\left[r + p^{ag}\sigma_A + \overline{p}^{id}\phi\sigma_I^j - (\mu_A + \Phi(\iota(q)) - \delta)\right] + \iota(q)\right\}$.

Rearranging the expression above, we obtain

$$\zeta = K^{\alpha-1}, \tag{A.102}$$

where $\zeta = \left[\sum_{j=1}^{n} \tilde{\chi}_{e,j} e_j \zeta_j^{\frac{\alpha}{\alpha+\beta-1}}\right]^{\frac{\alpha+\beta-1}{\alpha}}$. The value of $\overline{k}_j$ is then given by $\overline{k}_j = e_j K \left(\frac{\zeta_j}{\zeta}\right)^{\frac{1-\beta}{\alpha+\beta-1}}$.

Price of idiosyncratic risk. The price of idiosyncratic risk is given by

$$p_{i,t}^{id} = \frac{\mu_{i,t}^R - r - \gamma\sigma_A^2}{\phi\sigma_{I,i}} = \overline{p}^{id} - \frac{\psi^j_{\mu^R,k}}{\phi\sigma_I^j}\psi^j_{k,\omega}\hat{\omega}_{i,t}. \tag{A.103}$$

Using the fact that $\mathbb{E}[\hat{\omega}_{i,t}]$ by definition of the approximation point, we obtain $\mathbb{E}[p_{i,t}^{id}] = \overline{p}^{id}$. Unconditional expected marginal returns are then given by

$$\mathbb{E}[\mu_{i,t}^R] = r + p^{ag}\sigma_A + \overline{p}^{id}\phi\sigma_{I,i}. \tag{A.104}$$

□

A.7 Proof of Proposition 6

Proof. Aggregating Equation (7) and using the fact that labor supply grows at rate g, we obtain

$$\dot{K}_t = [\Phi(\iota(q_t)) - \delta - g]\, K_t, \tag{A.105}$$

given the initial condition $K_0 = K^*$. From (A.15), we obtain the expression

$$r + p_t^{ag}\sigma_A + p_t^{id}\phi\sigma_I = \frac{\alpha K_t^{\alpha-1} - \iota(q_t)}{q_t} + \frac{\dot{q}_t}{q_t} + \Phi(\iota(q_t)) - \delta + \mu_A. \tag{A.106}$$

Using $p_t^{ag} = \gamma\sigma_A$ and $p^{id} = \gamma\phi\sigma_I \frac{q_t K_t}{\chi_e \omega_{e,t}}$ and solving for $\dot{q}_t$, we obtain

$$\dot{q}_t = \left[r + \gamma\sigma_A^2 + \gamma\phi^2\sigma_I^2 \frac{q_t K_t}{\chi_e \omega_{e,t}} + \delta - \mu_A - \Phi(\iota(q_t)) - \frac{\alpha K_t^{\alpha-1} - \iota(q_t)}{q_t} \right] q_t, \tag{A.107}$$

where $\omega_{e,t} = n_{e,t} + h_{e,t}$.

The ODE above is subject to the terminal condition

$$\lim_{t\to\infty} q_t = q, \tag{A.108}$$

where q is the value of q_t in the new stationary equilibrium.

The first PDE was derived in the proof of Lemma 1 and it was given in (A.18). The boundary conditions are $\zeta_t(T) = (1-\psi)^{-1} (V^*)^{-\frac{1}{\gamma}}$ and

$$\lim_{t\to\infty} \zeta_t(a) = \zeta(a), \tag{A.109}$$

where $\zeta(a)$ is the value in the new stationary equilibrium.

The PDE for the human wealth was given in (A.7). The boundary conditions are $h_t(T) = 0$ and

$$\lim_{t\to\infty} h_t(a) = h(a), \tag{A.110}$$

where $h(a)$ is the value in the new stationary equilibrium.

The PDE for total wealth was derived in the proof of Proposition 2 and it was given in (A.32). The first boundary condition is $\omega_t(0) = e^{-gT}\omega_t(T) + h_t(0)$. The initial condition for $\omega_t(a)$ is given by

$$\omega_0(a) = n^*(a) + (q_0 - q^*)k^*(a) + h_0(a), \tag{A.111}$$

where variables with an asterisk denote values before the change in ϕ.

□

B Data

We discuss next our empirical measures in more detail. For an extensive discussion of the Townsend Thai Monthly Survey and the derivation of entrepreneurs' balance sheet information from the survey questionnaire, see Samphantharak and Townsend (2010).

B.1 Sample selection and variable definition

The dataset includes information on both economic and demographic variables. Economic variables include households' assets and liabilities, financial wealth, business and labor income, and consumption. Demographic and geographic variables consist of the age of the household's head, year, number of household members, number of children in the household, and province in which the household is located. We focus on a sample of households from age 25 to 80. We drop observations for households without information on age or financial wealth, resulting in an unbalanced panel of 710 households over 14 years, from 1999 to 2012.

Business exposure. The Townsend Thai Monthly Survey contains detailed information on entrepreneurs' assets, including fixed assets, inventories, and financial assets. We classify these assets into business assets and non-business assets, which we treat as safe. The value of the business includes inventories, livestock, agricultural assets, business assets, and household assets. We follow Samphantharak and Townsend (2018) and include the value of household assets (cars, pick-up trucks, fishing boats, and so on) as part of the business. The motivation for this choice is that many of these assets are also used by households in their production activities. The value of safe assets includes cash in hand, account receivables, deposits at financial institutions, ROSCA, other lending, prepaid insurance, and land. Given this breakdown, we can compute the fraction of financial wealth (total asset net of liabilities) invested in the business, which we use as our measure of risk-taking.

Return on business activity. Given estimates for the value of entrepreneurs' businesses and the flow of business income generated in a given period, we can compute the return on assets (ROA) as the ratio of business income over the value of business assets. ROA is a common accounting measure used to capture the profitability of business activities. We compute a time series of returns for a subsample of 541 households with available information on assets and income.

Table B.1: Descriptive Statistics of Households Characteristics

Variable	Q1	Median	Mean	Q3	Count
Age	43.00	53.00	53.49	64.00	8858
Household size	3.00	4.00	3.86	5.00	7846
Number of children	0.00	1.00	0.94	2.00	7846
Consumption-wealth ratio	0.03	0.07	0.12	0.14	8858
Business exposure	0.09	0.19	0.24	0.35	8858
Human-financial wealth ratio	0.11	0.30	0.76	0.67	8858
Net worth (normalized)	0.18	0.38	1.00	0.84	8858

Note: The columns $Q1$ and $Q3$ refer to the first and third quartiles of the distribution, respectively. Business exposure is the ratio of the value of the business to the net worth. Human-financial wealth ratio is the ratio of human wealth to financial wealth. The normalized net worth for each household corresponds to the ratio of their net worth to the average net worth in that year.

Human wealth. We compute our empirical measure of human wealth in a way analogous to its counterpart in the model as the present discounted value of future expected labor income. This requires us to specify the discount rate and the expected value of future labor. We use the same discount rate used in the model, as shown in Equation (**??**). The expected value of future labor income is age-dependent and computed as the average labor income for households of that age. As in the construction of life-cycle profiles discussed below, we use trimmed means to limit the influence of outliers. Consistent with the model's assumption, human wealth has no idiosyncratic component, as it corresponds to the present discounted value of future labor income across different households conditional on age.

B.2 Descriptive statistics

We present next the descriptive statistics of the main variables used in the analysis. Table B.1 reports households' demographic characteristics, such as age, household size, and number of children in the household. It also reports the consumption-wealth ratio, business exposure (the ratio of the value of the business to the net worth), the human-financial wealth ratio, and the household's net worth (normalized by the average net worth on that year). Table B.2 reports the percentage of revenue on each activity by province.

B.3 Shrinkage estimators.

To perform the cross-sectional regression in the second stage, we must first estimate the idiosyncratic variance, the aggregate factor loadings, and the individual return means for each entrepreneur. The large number of parameters to estimate can lead to concerns of *overfitting*. We consider next a range of shrinkage estimators to address this concern.

Table B.2: Production Activity by Province

Region:	Central		Northeast	
Province:	**Chachoengsao**	**Lopburi**	**Buriram**	**Srisaket**
Production activity (%):				
Cultivation	13.30	44.10	16.90	40.30
Livestock	27.70	21.90	1.10	1.20
Fish and shrimp	18.60	0.00	0.30	1.80
Nonfarm business	27.40	23.40	62.90	31.50
Wage earning	12.90	10.50	18.80	25.20

Ledoit-Wolf shrinkage estimator. We start with the estimation of the covariance matrix of returns using the Ledoit-Wolf shrinkage estimator. Consider the residuals of the first-stage regressions, $\hat{\epsilon}_{i,t} = R_{i,t} - \hat{\alpha}_i - \hat{\beta}_i R_t^{agg}$ for $i \in \{1, \ldots, N\}$, and the $N \times N$ sample covariance matrix of residuals $\hat{\Sigma}$. The diagonal elements of $\hat{\Sigma}$ are the idiosyncratic variances, $\hat{\sigma}_i^2$. The Ledoit-Wolf shrinkage estimator is given by

$$\hat{\Sigma}_{LW} = (1 - \rho_{LW})\hat{\Sigma} + \rho_{LW}\overline{\sigma}^2 I_N, \tag{B.1}$$

where ρ_{LW} is the shrinkage coefficient and $\overline{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}\hat{\sigma}_i^2$ is the average of the diagonal elements of $\hat{\Sigma}$. The diagonal elements of $\hat{\Sigma}_{LW}$ are the shrinkage-adjusted idiosyncratic variances, $\hat{\sigma}_{i,LW}^2$. We use the Ledoit-Wolf formula for the shrinkage coefficient ρ_{LW}.

James-Stein estimator for the mean. The analogous shrinkage estimator for the mean is given by

$$\hat{\mu}_{JS} = (1 - \rho_{JS})\hat{\mu} + \rho_{JS}\overline{\mu}, \tag{B.2}$$

where ρ_{JS} is the shrinkage coefficient, $\hat{\mu} = [\hat{\mu}_1, \ldots, \hat{\mu}_N]^\top$ is the vector of sample means, and $\overline{\mu} = \frac{1}{N}\sum_{i=1}^{N}\hat{\mu}_i$ is the cross-sectional average of the time-series means. We use the James-Stein formula for the shrinkage coefficient ρ_{JS}.

Shrinkage estimator for factor loadings. In the two cases above, we shrinked the individual sample estimate towards the group mean. We can apply the same idea to the factor loadings by considering a mixed effect model where the coefficients on the aggregate factor. We can express the first-stage regression as

$$R_{i,t}|\beta_i, R_t^{agg} \sim \mathcal{N}\left(\alpha_i + \beta_i R_t^{agg}, \sigma_i^2\right). \tag{B.3}$$

Table B.3: Cross-sectional regressions with shrinkage estimators

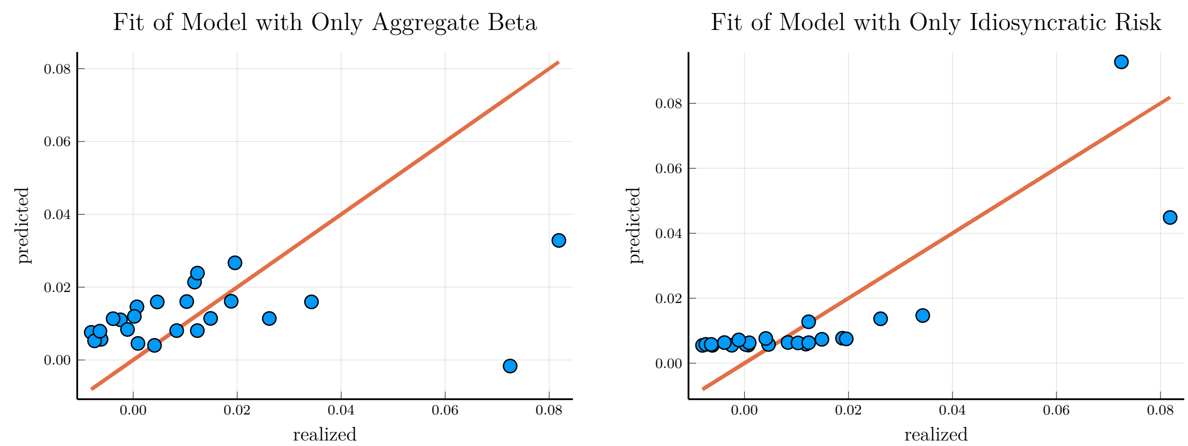
	Mean ROA		Mean ROA (JS)		Mean ROA		Mean ROA (JS)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Beta	0.352***	0.352***	0.352***	0.352***				
	(0.026)	(0.026)	(0.026)	(0.026)				
Id. Variance	0.629***		0.629***					
	(0.026)		(0.026)					
Id. Variance (LW)		0.629***		0.629***				
		(0.026)		(0.026)				
Beta (RE)					0.446***	0.446***	0.446***	0.446***
					(0.025)	(0.025)	(0.025)	(0.025)
Id. Variance (RE)					0.554***		0.554***	
					(0.025)		(0.025)	
Id. Variance (RE/LW)						0.554***		0.554***
						(0.025)		(0.025)
Observations	541	541	541	541	541	541	541	541
R^2	0.685	0.685	0.685	0.685	0.734	0.734	0.734	0.734
Adjusted R^2	0.684	0.684	0.684	0.684	0.733	0.733	0.733	0.733

Note: All variables are standardized to have zero mean and unit variance. Id. Variance (LW) refers to the Ledoit-Wolf estimate for the idiosyncratic variance. Id. Variance (RE) refers to the idiosyncratic variance of the residuals from the random effects regression. Id. Variance (RE/LW) refers to the Ledoit-Wolf estimate for the idiosyncratic variance of the residuals from the random effects regression. Mean ROA (JS) refers to the James-Stein estimate for the mean.

Importantly, β_i is a random effect, that is, the vector of coefficients $\beta = [\beta_1, \ldots, \beta_N]^\top$ follows a multivariate normal distribution $\beta \sim \mathcal{N}(\overline{\beta}\mathbf{1}_N, \Sigma_\beta)$, where the scalar $\overline{\beta}$ is the cross-sectional average of the coefficients and Σ_β is the covariance matrix of the coefficients. This model effectively shrinks the slope coefficients on the aggregate factor towards the cross-sectional average $\overline{\beta}$, in the same way as discussed above. We can estimate the parameters of the model by maximum likelihood.

Cross-sectional regressions. Given the estimates of the parameters above, we can then run our second-stage cross-sectional regressions. To facilitate the comparison across the different cases, we standardize all variables to have zero mean and unit variance. Table B.3 shows the results of the cross-sectional regressions. The first column replicates the results of column 3 in Table 1, the only difference is that the variables are standardized. Column (2) shows that the results are essentially unchaged if we use the Ledoit-Wolf estimator for the idiosyncratic variance. Columns (3) and (4) show we find again virtually identical results when we use the James-Stein estimator for the mean. In Columns (1) to (4), we use the OLS estimates for the betas. Column (5) shows the results when beta is estimated using the random effects model. The idiosyncratic variance is corresponds to the variance of the residuals of the random effects regression. In this case, we find a

Figure B.1: Realized vs. predicted returns: one-factor models



Note: The left (right) panel shows a scatter plot of predicted returns of the single factor with aggregate beta (idiosyncratic variance) as a factor against the realized returns for the portfolio-level analysis.

slightly higher coefficient for the aggregate beta, and smaller coefficient for the idiosyncratic variance. However, the estimates are in the same ballpark as our baseline results. Columns (6) applies the Ledoit-Wolf shrinkage estimator for the idiosyncratic variance of the residuals from the random effects regression, and columns (7) and (8) apply the James-Stein estimator for the mean. The results for Columns (6) to (8) are similar to the ones in Column (4).

B.4 Fit of single-factor models.

Figure 2 in Section 2 shows the fit of the two-factor model. We consider here the performance of one-factor models using either aggregate beta or idiosyncratic variance as the factor. Figure B.1 shows that both models struggle to properly account for the dispersion in entrepreneurial returns, showing the importance of considering a two-factor model.

B.5 Model with latent factors

We consider next the model with latent factors presented in Section 2.1 in more detail. First, we show how to accommodate time-varying factor loadings and measurement error in the model. Second, we discuss the estimation and construction of standard errors in more detail, including how we handled missing values.

Time-varying loadings. Consider a version of the latent factor model where the factor loadings are time-varying. The case with time-varying prices of risk can be handled analogously. Excess returns for household i in period t are given by

$$R_{i,t+1} = \mu_{i,t} + \beta_{i,t}^{\top} f_{t+1} + \epsilon_{i,t+1}. \tag{B.4}$$

Following, e.g., Lettau and Ludvigson (2001), the factor loadings are a function of an aggregate variable $z_t \in \mathbb{R}$:

$$\beta_{i,t} = \eta_{i,0} + \eta_{i,z} z_t, \tag{B.5}$$

where without loss of generality we assume that $\mathbb{E}[z_t] = 0$.

Conditional expected returns are given by

$$\mu_{i,t} = \lambda_0 + \lambda_{ag}^{\top} \beta_{i,t} + \lambda_{id} \sigma_i^2. \tag{B.6}$$

Combining the previous three expressions, we obtain

$$R_{i,t+1} = \mu_i + \eta_{i,0}^{\top} f_{t+1} + \eta_{i,z}^{\top} \left(z_t f_{t+1} - \mathbb{E}[z_t f_{t+1}] \right) + \lambda_{ag}^{\top} \eta_{i,z} z_t + \epsilon_{i,t+1}, \tag{B.7}$$

where μ_i denotes the unconditional expected return:

$$\mu_i \equiv \lambda_0 + \lambda_{ag}^{\top} \eta_{i,0} + \mathbb{E}[z_t f_t]^{\top} \eta_{i,z} + \lambda_{id} \sigma_i^2. \tag{B.8}$$

Define the new vector of aggregate factors as $\tilde{f}_t \equiv [f_t^{\top}, (z_t f_t - \mathbb{E}[z_t f_t])^{\top}, z_t]^{\top}$, factor loadings $\tilde{\beta}_i \equiv [\eta_{i,0}^{\top}, \eta_{i,z}^{\top}, \lambda_{ag}^{\top} \eta_{i,z}]^{\top}$, and the new vector of prices of risk $\tilde{\lambda}_{ag} \equiv [\lambda_{ag}^{\top}, \mathbb{E}[z_t f_t]^{\top}, 0]^{\top}$. Then, the model can be written in the familiar form:

$$R_{i,t} = \mu_i + \tilde{\beta}_i^{\top} \tilde{f}_t + \epsilon_{i,t}, \tag{B.9}$$

where $\mu_i = \lambda_0 + \tilde{\lambda}_{ag}^{\top} \tilde{\beta}_i + \lambda_{id} \sigma_i^2$. Hence, our formulation with multiple latent factors accommodates the case of time-varying factor loadings.

Measurement error. Suppose that returns are measured with error, i.e., observed returns are given by $\tilde{R}_{i,t} = R_{i,t} + v_{i,t}$, where $v_{i,t} \sim \mathcal{N}(0, \sigma_v^2)$. The measurement error $v_{i,t}$ is independent of the idiosyncratic shock $\epsilon_{i,t}$ and aggregate factors f_t. The residual variance now incorporates both idiosyncratic shocks and measurement error: $\tilde{\sigma}_i^2 = \sigma_i^2 + \sigma_v^2$. Expected returns, however, reflect only the compensation for idiosyncratic shocks:

$$\mu_i = \lambda_{ag}^{\top} \beta_i + \lambda_{id} \sigma_i^2. \tag{B.10}$$

We can write observed returns as follows:

$$\tilde{R}_{i,t} = \tilde{\lambda}_0 + \lambda_{ag}^\top \beta_i + \lambda_{id} \tilde{\sigma}_i^2 + \beta_i^\top f_t + \tilde{\epsilon}_{i,t}, \tag{B.11}$$

where $\tilde{\lambda}_0 = -\lambda_{id}\sigma_v^2$. This shows that classical measurement error affects only the intercept of the pricing equation, not the prices of risk. The intercept of the cross-sectional regression will then be downward biased. This bias may explain the negative, but often insignificant, intercepts found in Table 1.

Estimation. We estimate the model with latent factors following the three pass estimator of Giglio et al. (2021). In the first stage, we run time-series regressions of returns on the observable aggregate factor:

$$R_{i,t} = \mu_i + \beta_{i,o}^\top f_{o,t} + \tilde{\epsilon}_{i,t}, \tag{B.12}$$

where $\tilde{\epsilon}_{i,t} = \beta_{i,u}^\top f_{u,t} + \epsilon_{i,t}$. The second stage consists of applying PCA to the residuals from the first stage, which will give us estimates of $\beta_{i,u}$, the loadings on the latent factors. The third stage is a cross-sectional regression of average returns on the factor loadings on observable factors estimated in the first stage, the loadings on latent factors estimated in the second stage, and the variance of the PCA residuals from the second stage. Notice that we do not use the variance of the residuals from the first stage regressions, which includes not only the idiosyncratic shocks but also exposure to the latent factors.

Importantly, the estimate from the first stage $\hat{\beta}_{i,o}$ may not be a consistent estimator of $\beta_{i,o}$, to the extent that the observable factors are correlated with the latent facors. However, Giglio et al. (2021) show that $(\hat{\beta}_{i,o}, \hat{\beta}_{i,u})$ converges in probability to a rotation of the true factor loadings, $H\beta_i$, where H is a rotation matrix. Therefore, we are able to consistently estimate the aggregate risk premium $\lambda_{ag}^\top \beta_i = (H\lambda_{ag})^\top (H\beta_i)$, which is independent of the rotation matrix H.

Missing values. Standard PCA methods require complete data. Given we have an unbalanced panel, we adopt an iterative procedure to compute the principal components. PCA can be obtained from the single-value decomposition (SVD) of the $N \times T$ matrix of returns R. In our setting, this matrix will have missing values. To initialize the procedure, we impute the missing values with a simple method, such as the sample mean return. We then apply the SVD decomposition to the matrix of returns with imputed values and obtain a low-rank approximation of the matrix based on the first k singular values. We then replace the imputed values by new estimates based on the low-rank approximation.

We repeat this procedure until convergence. Giglio et al. (2021) show that the three-pass estimator is still consistent even when PCA requires this matrix completion step.[27]

Standard errors. Following Giglio et al. (2021), we compute standard errors for the three-pass estimator using a wild bootstrap procedure. Formally, we generate B bootstrap samples as follows:

$$R^b_{i,t} = \hat{\mu}_i + \hat{\beta}^\top_{i,o} f_{o,t} + \hat{\beta}^\top_{i,u} f_{u,t} + \hat{\epsilon}^b_{i,t}, \tag{B.13}$$

for $b = 1, \ldots, B$. Here, $\hat{\mu}_i = \hat{\lambda}_0 + \hat{\lambda}^\top_{ag}\hat{\beta}_i + \hat{\lambda}_{id}\hat{\sigma}^2_i$ is the predicted average return from the cross-sectional regression, $\hat{\beta}_{i,o}$ and $\hat{\beta}_{i,u}$ are the factor loadings on observable and latent factors, respectively, and $f_{u,t}$ is the vector of latent factors estimated in the second stage. $\epsilon^b_{i,t} = \hat{\epsilon}_{i,t} w^b_{i,t}$ is the resampled bootstrap residuals, where $\hat{\epsilon}_{i,t}$ is the residual from the PCA step, and $w^b_{i,t}$ satisfies $\mathbb{E}[w^b_{i,t}] = 0$ and $Var[w^b_{i,t}] = 1$. We follow the recommendation of Mammen (1993) and use $w^b_{i,t} = \frac{1}{\sqrt{2}}\eta^b_{i,t} + \frac{1}{2}((\gamma^b_{i,t})^2 - 1)$, where $\eta^b_{i,t}$ and $\eta^{b*}_{i,t}$ are independent draws from a standard normal distribution. Given the resampled residuals, we compute B estimates of the prices of risk $(\hat{\lambda}^b_0, \hat{\lambda}^b_{ag}, \hat{\lambda}^b_{id})$ and the standard errors correspond to the standard deviation of the estimates across simulations.

Testing the number of latent factors. We test the number of latent factors using the procedure proposed by Onatski (2009). In particular, we test the null hypothesis that the number of latent factors in the residuals of the first-stage regression is equal to zero. The test statistic is given by -0.32 and the critical value at the 5% significance level based on the Tracy-Widom distribution is 1.27, so we cannot reject the hypothesis that the number of latent factors is zero.

Testing the risk-based model. We can formally assess whether the risk-based model, where differences in expected returns can be entirely attributed to differences in risk exposure, is a good approximation of the data. The expected return for entrepreneur i is given by

$$\mu_i = \lambda_0 + \lambda^\top_{ag}\beta_{i,o} + \lambda_{id}\sigma^2_i + u_i. \tag{B.14}$$

The residual u_i captures the impact on expected entrepreneurial returns of variables orthogonal to exposure to risk. We test whether the residuals are simultaneously equal to

[27] They also show that a correction is necessary for testing hypotheses about the residuals of cross-sectional regression in the presence of missing values. In our application, we have roughly only 1.5% of the observations missing, so the found this correction to be negligible.

zero for all entrepreneurs. Formally, we compute the bootstrap p-value for the test statistic $T_u = \sum_{i=1}^{N} u_i^2$. We focus on the case of portfolio-level analysis, which minimizes the impact of measurement error on the estimation of factor loadings. For the version of the model with latent factors, we find that the p-value is 0.082 and for the version with only observable factors, the p-value is 0.214. In both cases, we cannot reject the hypothesis that the residuals are equal to zero at the 5% significance level.

B.6 Life-cycle profiles

When computing life-cycle profiles for a given variable, we aggregate households into 15 age groups.[28] The thresholds determining each group are chosen such that groups have roughly the same number of households. To limit the influence of outliers and reduce the noise in the estimation, we compute trimmed means with a trimming parameter of 7.5% on each side. We trim the data in a similar manner before running the regressions.

The life-cycle profiles presented in the main text are computed without any controls, which we denote by *raw* moments. We show next that controlling for year fixed-effects or demographics variables maintains our results essentially unchanged.

Let $z_{i,k,t}$ denote variable z for household i in age group k at year t. Consider the following process for $z_{i,k,t}$:

$$z_{i,k,t} = \alpha_t + age_k + \delta' x_{i,k,t} + u_{i,k,t}, \tag{B.15}$$

where α_t represents the year fixed effect, age_k is the age-group effect, and $x_{i,k,t}$ is a vector of demographic and geographic controls, which includes the size of the household, the number of children in the household, province dummies, and a set of sector dummies.

The raw age-group effect is given by

$$age_k^{raw} = \mathbb{E}\left[z_{i,k',t} | k' = k\right]. \tag{B.16}$$

The raw age-group effect can be estimated by taking averages by age or by regressing $z_{i,k,t}$ on a set of dummies for age groups.

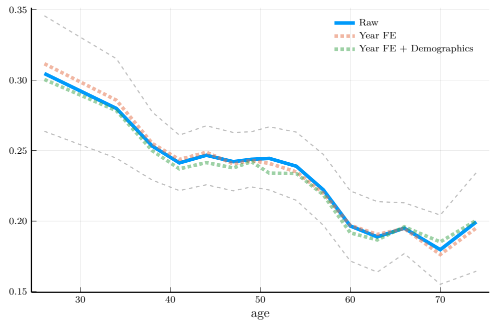
We follow Kaplan (2012) and define the age-group effect controlling for year fixed-effects as follows:

$$age_k^{year-FE} = \frac{1}{\overline{T}} \sum_{t=1}^{\overline{T}} \mathbb{E}\left[z_{i,k',t'} | k' = k, t' = t\right]. \tag{B.17}$$

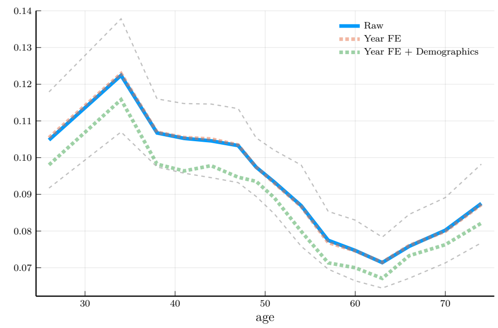
We can estimate $age_k^{year-FE}$ by running a regression on a set of dummy of age-group and year fixed-effects and computing the predicted value of the regression evaluated at age

[28] Table 3 shows estimates of the life-cycle profiles for the case of only 5 age groups.

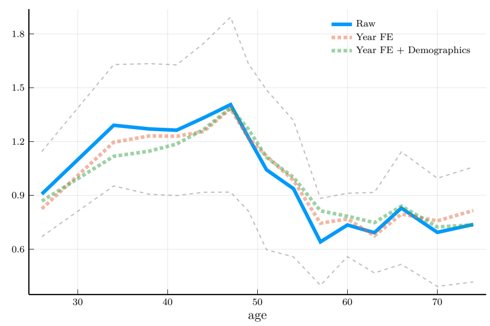
Figure B.2: Life-cycle profiles: raw and additional controls



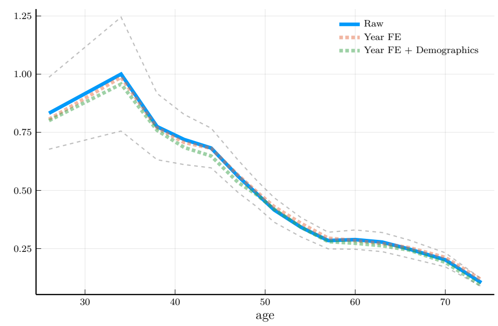
(a) Business exposure



(b) Consumption-wealth ratio



(c) Labor income



(d) Human-financial wealth ratio

k and at an "average" year. Similarly, we define the age-group effect controlling for year fixed-effects and demographic/geographic controls as follows:

$$age_k^{year-FE+dem} = \frac{1}{\overline{T}} \sum_{t=1}^{\overline{T}} \mathbb{E}\left[z_{i,k',t'} | k' = k, t' = t, x_{i,k',t'} = \overline{x}_{k,t}\right], \tag{B.18}$$

where $\overline{x}_{k,t}$ denotes the average value of the controls $x_{i,k,t}$ conditional on age group k and year t. We can estimate $age_k^{year-FE+dem}$ from the full regression with year and age-group fixed-effects and demographic controls.

Figure B.2 shows our estimates of the different age-group effects for the variables of interest. The grey dashed lines show the 95% confidence interval for the raw estimates. We cluster standard errors by household and year. The life-cycle patterns obtained under the raw measure are very similar to the ones obtained after controlling for year fixed-effects and demographic variables. Moreover, the estimates with additional controls are statistically indistinguishable from the raw estimates.

C Derivations

C.1 Equilibrium prices and capital stock in a stationary equilibrium

Interest rate. The financial wealth of wage earners evolves according to

$$d\tilde{n}_{j,t} = \left[(r + \gamma\sigma_A^2)\tilde{n}_{j,t} + \tilde{w}_t \bar{l}_{j,t} - \tilde{c}_{j,t}\right] dt + \tilde{n}_{j,t}\sigma_A dZ_t. \tag{C.1}$$

using the demand for aggregate insurance $\theta_{j,t} = -n_{j,t}\sigma_A$.

Combining the expression above with the law of motion for human wealth, we obtain the law of motion of total wealth:

$$\frac{d\tilde{\omega}_{i,t}}{\tilde{\omega}_{i,t}} = \left[r + \gamma\sigma_A^2 - \frac{c_{j,t}}{n_{j,t} + h_{j,t}}\right] dt + \sigma_A dZ_t. \tag{C.2}$$

In a stationary equilibrium, wage earners' scaled total wealth, $\omega_{j,t} = \tilde{\omega}_{j,t}/A_t$, is constant. Therefore, the drift of $\omega_{j,t}$ must be zero. Using Ito's lemma, we obtain:

$$\frac{d\omega_{i,t}}{\omega_{i,t}} = \frac{d\tilde{\omega}_{i,t}}{\tilde{\omega}_{i,t}} - \frac{dA_t}{A_t} + \left(\frac{dA_t}{A_t}\right)^2 - \frac{dA_t}{A_t}\frac{d\tilde{\omega}_{i,t}}{\tilde{\omega}_{i,t}} \tag{C.3}$$

$$= \left[r + \gamma\sigma_A^2 - \frac{c_{j,t}}{n_{j,t} + h_{j,t}} - \mu_A\right] dt. \tag{C.4}$$

The interest rate must then satisfy the condition

$$r + \gamma\sigma_A^2 - \left[\frac{1}{\gamma}\rho_w + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{\gamma\sigma_A^2}{2}\right)\right] - \mu_A = 0, \tag{C.5}$$

using the fact that the consumption-wealth ratio is given by

$$\frac{c_{j,t}}{n_{j,t} + h_{j,t}} = \frac{1}{\gamma}\rho_w + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{(p^{ag})^2}{2\gamma}\right), \tag{C.6}$$

which is a special case of (24), as we set $p_t^{id} = 0$ and $T \to \infty$.

Rearranging expression (C.5), we obtain

$$r = \rho_w + \gamma\mu_A - \gamma(\gamma + 1)\frac{\sigma_A^2}{2}. \tag{C.7}$$

Relative price of capital. Plugging the expression for $\Phi'(\iota)$ into the first-order condition for

ι in equation (16), we obtain

$$\frac{1}{\sqrt{\overline{\Phi}^2 + 2\iota}} = \frac{1}{q} \Rightarrow \iota = \frac{q^2 - \overline{\Phi}^2}{2}. \tag{C.8}$$

In a stationary equilibrium, the capital-labor ratio is constant. Thus, capital grows at the population rate g, which gives us the condition

$$\Phi(\iota) - \delta = g \Rightarrow q = \overline{\Phi} + (g + \delta), \tag{C.9}$$

obtained by plugging the expression for ι into the functional form for $\Phi(\iota)$.

C.1.1 Capital stock and idiosyncratic risk premium

Rearranging the expression for the shadow price of idiosyncratic risk (22), and using the definition of expected returns (17), we obtain the MPK schedule

$$r + p^{ag}\sigma_A + p^{id}\phi\sigma_I = \frac{\alpha K^{\alpha-1} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{C.10}$$

where r, p^{ag}, and q are functions of parameters, as derived above.

Integrating condition (21) across all entrepreneurs, we obtain

$$p^{id} = \gamma\phi\sigma_I \frac{qK}{\chi_e \omega_e}, \tag{C.11}$$

where χ_e is the fraction of entrepreneurs in the population.

D Transitional dynamics

In this section, we describe the computation of the transitional dynamics. To ensure that that the interest rate is constant, we assume that wage earners have Epstein-Zin preferences and take the limit as the elasticity of intertemporal substitution goes to infinity, that is, workers have linear intertemporal preferences.[29]

[29] This assumption is meant to capture, in an extreme form, the essence of the macro-finance literature that assumes a high EIS (see, e.g., Bansal and Yaron 2004 and Barro 2009). In these models, the high EIS dampens movements in interest rates, so risk premia accounts for most of the variation in discount rates.

D.1 Wage earners with Epstein-Zin preferences

Wage earners have the continuous-time analog of Epstein-Zin preferences with EIS ψ_w and risk aversion γ. The wage earner's problem is given by

$$\tilde{V}_t^w(\tilde{n}_j) = \max_{\tilde{c}_j, \tilde{\theta}_j^{ag}} \mathbb{E}_t \left[\int_t^\infty f_w(\tilde{c}_{j,z}, \tilde{V}_z) dz \right], \tag{D.1}$$

subject to $\tilde{n}_{j,t} \geq \tilde{h}_{j,t}$, where $\tilde{h}_{j,t}$ denotes wage earner j's human wealth, non-negativity constraint $\tilde{c}_{j,t} \geq 0$, and the law of motion of financial wealth $\tilde{n}_{j,t}$

$$d\tilde{n}_{j,t} = \left[\tilde{n}_{j,t} r_t - p_t^{ag} \tilde{\theta}_{j,t}^{ag} + \tilde{w}_t \bar{l}_{j,t} - \tilde{c}_{j,t} \right] dt - \tilde{\theta}_{j,t}^{ag} dZ_t,$$

where $f_w(\tilde{c}, V)$ is the aggregator given by

$$f_w(\tilde{c}, \tilde{V}) = \rho_w \frac{(1-\gamma)\tilde{V}}{1-\psi_w^{-1}} \left\{ \left(\frac{\tilde{c}}{((1-\gamma)\tilde{V})^{\frac{1}{1-\gamma}}} \right)^{1-\psi_w^{-1}} - 1 \right\}. \tag{D.2}$$

It is convenient to work with the scaled value function $V_t^w(n)$, which satisfies the condition $\tilde{V}_t^w(\tilde{n}) = A_t^{1-\gamma} V_t^w \left(\frac{\tilde{n}_t}{A_t} \right)$, where $V_t^w(\cdot)$ is independent of A_t. The HJB equation in terms of the scaled value function is given by

$$\tilde{\rho}_w \frac{(1-\gamma)V^w}{1-\psi_w^{-1}} = \max_{\tilde{c}, \tilde{\theta}^{ag}} \rho_w \frac{(1-\gamma)V^w}{1-\psi_w^{-1}} \left(\frac{c}{((1-\gamma)V^w)^{\frac{1}{1-\gamma}}} \right)^{1-\psi_w^{-1}} + V_t^w + V_n^w \left[\tilde{r} n + (\gamma\sigma_A - p^{ag})\theta^{ag} + w\bar{l} - c \right]$$
$$+ \frac{1}{2} V_{nn}^w \left(\theta^{ag} + n\sigma_A \right)^2. \tag{D.3}$$

where $\tilde{\rho}_w \equiv \rho_w - (1-\psi_w^{-1}) \left(\mu_A - \frac{\gamma\sigma_A^2}{2} \right)$ and $\tilde{r}_t \equiv r_t + \gamma\sigma_A^2 - \mu_A$.

Policy functions. The first-order conditions for this problem are given by

$$\rho_w \left((1-\gamma)V^w \right)^{\frac{\psi_w^{-1}-\gamma}{1-\gamma}} c^{-\psi_w^{-1}} = V_n^w, \qquad \gamma\sigma_A - p^{ag} = -\frac{V_{nn}^w}{V_n^w} \left(\theta^{ag} + n\sigma_A \right). \tag{D.4}$$

We will guess and verify that the scaled value function takes the form

$$V_t^w(n_{j,t}) = \left(\frac{\zeta_{w,t}}{\rho_w^{\psi_w}} \right)^{\frac{1-\gamma}{1-\psi_w}} \frac{(n_{j,t} + h_{j,t})^{1-\gamma}}{1-\gamma}, \tag{D.5}$$

where $\zeta_{w,t}$ and $h_{j,t}$ are potentially time-varying, but they are non-stochastic.

Using the expression for the value function above, we obtain the policy functions:

$$\frac{c_{j,t}}{n_{j,t}+h_{j,t}} = \zeta_{w,t}, \qquad \theta^{ag}_{j,t} = \sigma_A h_{j,t} - \frac{p^{ag}_t}{\gamma}(n_{j,t}+h_{j,t}). \tag{D.6}$$

Inserting the policy functions derived above back into the HJB equation, we obtain

$$\frac{\tilde{\rho}_w}{1-\psi_w^{-1}} = \frac{\psi_w^{-1}}{1-\psi_w^{-1}}\zeta_{w,t} + \frac{1}{1-\psi_w}\frac{\dot{\zeta}_{w,t}}{\zeta_{w,t}} + \tilde{r}_t. \tag{D.7}$$

Rearranging the expression above for the case of a stationary equilibrium, so $\dot{\zeta}_{w,t} = 0$, we obtain the consumption-wealth ratio:

$$\zeta_{w,t} = \psi_w \rho_w + (1-\psi_w)\left(r + \frac{\gamma\sigma_A^2}{2}\right), \tag{D.8}$$

which coincides with the expression for the consumption-wealth ratio for wage earners given in (C.6) in the special case of CRRA preferences, i.e., $\psi_w^{-1} = \gamma$.

Price of aggregate risk and interest rate. The demand for aggregate insurance derived above coincides with the expression for aggregate insurance in the CRRA case (see Equation A.23). Therefore, the same argument used in Section C.1 to solve for the price of aggregate risk can be applied in the case of Epstein-Zin preferences in a non-stationary setting. This implies that the price of aggregate risk is constant and given by $p^{ag}_t = \gamma\sigma_A$ during the transitional dynamics.

We consider next the behavior of the interest rate. A derivation analogous to the one in Section C.1 shows that the interest rate in a stationary equilibrium is given by

$$r = \rho_w + \psi_w^{-1}\mu_A - (1+\psi_w^{-1})\frac{\gamma\sigma_A^2}{2}, \tag{D.9}$$

which coincides with the expression in Section 6 when $\psi_w^{-1} = \gamma$.

Taking the limit of (D.7) as $\psi_w \to \infty$, we obtain r_t in the case of a non-stationary equilibrium:

$$r_t + \gamma\sigma_A^2 - \mu_A = \rho_w + \frac{\gamma\sigma_A^2}{2} - \mu_A \Rightarrow r_t = \rho_w - \frac{\gamma\sigma_A^2}{2}. \tag{D.10}$$

Therefore, the interest rate is constant when wage earners have linear intertemporal preferences. Moreover, the expression above coincides with the one for the interest rate in the stationary equilibrium (D.9) when specialized to $\psi_w^{-1} = 0$.

E Extensions

E.1 Limited pledgeability and heterogeneous expected returns

In this subsection, we consider a version of the model with limited pledgeability of physical assets and heterogeneous idiosyncratic volatility.

Limited pleadgeability of capital. Let $b_{i,t} \equiv n_{i,t} - q_t k_t$ denote the amount of safe assets held by entrepreneur i. The natural borrowing limit can be written as

$$-b_{i,t} \leq h_{i,t} + qk_{i,t}. \tag{E.1}$$

Entrepreneur can borrow freely against physical assets or human wealth. Let's now assume that there is limited pledgeability of physical assets, that is, entrepreneurs can only borrow a fraction of $1 - \lambda^{-1}$ of the value of physical assets, a form of collateral constraint:

$$-b_{i,t} \geq h_{i,t} + (1 - \lambda^{-1})qk_{i,t} \Rightarrow qk_{i,t} \leq \lambda \omega_{i,t}, \tag{E.2}$$

where $\lambda \geq 1$. Hence, the entrepreneur faces a portfolio problem subject to leverage constraints. The HJB for an entrepreneur can be written as[30]

$$\frac{\rho}{1-\gamma} = \max_{c_{i,t}, k_{i,t}, l_{i,t}, \iota_{i,t}, \theta^{ag}_{i,t}, \theta^{id}_{i,t}} \left\{ \frac{\zeta^\gamma(a)}{1-\gamma} \left(\frac{c_{i,t}}{\omega_{i,t}} \right)^{1-\gamma} + r + \frac{q_t k_{i,t}}{\omega_{i,t}} (\mu^R_{i,t} - r_t) - \frac{p^{ag} \theta^{ag}_{i,t}}{\omega_{i,t}} + \frac{h_{i,t}}{\omega_{i,t}} \sigma_A p^{ag} - \frac{c_{i,t}}{\omega_{i,t}} \right.$$
$$\left. - \frac{\gamma}{1-\gamma} \frac{1}{\zeta(a)} \frac{\partial \zeta(a)}{\partial a} - \frac{\gamma}{2} \left[\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}} \sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}} \right)^2 + \left(\frac{q_t k_{i,t}}{\omega_{i,t}} \sigma_{I,i} - \frac{\theta^{id}_{i,t}}{\omega_{i,t}} \right)^2 \right] \right\},$$

subject to leverage and insurance constraints: $q_t k_{i,t} \leq \lambda \omega_{i,t}$ and $\theta^{id}_{i,t} \leq (1 - \phi) q_t k_{i,t} \sigma_{I,i}$.

The optimal capital demand is given by

$$\frac{qk_{i,t}}{\omega_{i,t}} = \frac{\varphi^{id}_i}{\gamma \phi \sigma_{I,i}}, \tag{E.3}$$

[30]A similar result on the optimal portfolio share with leverage constraints can be found for investors without labor income in, for instance, Grossman and Vila (1992) and Detemple and Murthy (1997).

where $\varphi_{i,t}^{id}$ satisfies the condition:

$$\varphi_i^{id} = \min\left\{\frac{\mu_i^R - r - p^{ag}\sigma_A}{\phi\sigma_{I,i}}, \gamma\phi\sigma_{I,i}\lambda\right\}.$$

φ_i^{id} corresponds to the Lagrange multiplier on the insurance constraint. In this case, this multiplier may not coincide with the price of idiosyncratic risk $p_i^{id} \equiv \frac{\mu_i^R - r - p^{ag}\sigma_A}{\phi\sigma_{I,i}}$.

Equilibrium implications. We only sketch the equilibrium implications. Let $\bar{k}_j$, $\overline{\omega}_j$, and $\overline{\mu}_j^R$ denote the long-run values of capital, total wealth, and expected marginal return for an entrepreneur of type j. We have seen in Section 5 that expected returns are decreasing in the capital stock with DRS, so we can write $\overline{\mu}_j^R = f(\bar{k}_j)$, where $f'(\cdot) < 0$. By analogy with the result in Proposition 2, we can assume that $\overline{\omega}_j$ is an increasing function of the return $\overline{\mu}_j^R$, $\omega_j = g(\overline{\mu}_j^R)$, where $g'(\cdot) > 0$. For a constrained investor, we have $q\bar{k}_j = \lambda\overline{\omega}_j$, so $\bar{k}_j = \frac{\lambda}{q} g_j(\overline{\mu}_j^R)$ The capital stock would be determined by the intersection of $f(\bar{k}_j)$ and $g_j^{-1}(q\bar{k}_j/\lambda)$. In this case, variations in wealth, associated with shifts of the curve $g_j(\cdot)$, will lead to variations in the capital stock and ultimately expected returns. This argument indicates that cross-sectional differences in expected returns, controlling for differences in risk, are driven by differences in net worth for constrained entrepreneurs.

E.2 Endogenous occupational choice

In this subsection, we introduce an occupational choice into the households' problem. Moreover, we assume that wage earners have a finite horizon and imperfect altruism in the same way as entrepreneurs.

E.2.1 The occupational choice

At the beginning of life, a household can choose to become an entrepreneur or a wage earner. To become an entrepreneur, household i must pay a fixed cost $\varphi_i \tilde{y}_{i,t}$, where φ_i is a cost parameter draw from a distribution $F_\varphi(\cdot)$ with support $[\underline{\varphi}, \overline{\varphi}]$. Let $\tilde{V}_t(\tilde{n}_i, a)$ denote the value function of a household that chose to become an entrepreneur and $\tilde{V}_t^w(\tilde{n}_i, a)$ the value function of a household who chose to become a wage earner. In contrast to the model from Section 3, a wage earner lives for T periods and derives the same utility of bequests as entrepreneurs.

A household that inherits financial wealth $\tilde{n}_i$ will choose to become an entrepreneur if

$$\tilde{V}_t(\tilde{n}_i - \varphi_i \tilde{y}_t, 0) > \tilde{V}_t^w(\tilde{n}_i, 0). \tag{E.4}$$

The value function of an entrepreneur can be written, after normalization, as $V(n,a) = \zeta(a)^{-\frac{1}{\gamma}} \frac{(n+h(a))^{1-\gamma}}{1-\gamma}$. Similarly, the value function of a wage earner can be written as $V^w(n,a) = \zeta_w(a)^{-\frac{1}{\gamma}} \frac{(n+h_w(a))^{1-\gamma}}{1-\gamma}$.

The condition for becoming an entrepreneur can then be written as

$$\zeta(0)^{\frac{1}{\gamma(\gamma-1)}} (n_i + h(0) - \varphi_i y) > \zeta_w(0)^{\frac{1}{\gamma(\gamma-1)}} (n_i + h_w(0)). \tag{E.5}$$

Rearranging the expression above, we obtain that a household becomes an entrepreneur if $\varphi_i < \varphi^*(n_i)$, where the threshold $\varphi^*(n_i)$ is given by

$$\varphi^*(n_i) \equiv \frac{1}{y}\left[\left(\left(\frac{\zeta(0)}{\zeta_w(0)}\right)^{\frac{1}{\gamma(\gamma-1)}} - 1\right) n_i + \left(\frac{\zeta(0)}{\zeta_w(0)}\right)^{\frac{1}{\gamma(\gamma-1)}} h(0) - h_w(0)\right]. \tag{E.6}$$

It can be shown that $\zeta(0) > \zeta_\omega(0)$ for $\gamma > 1$, so households who received larger bequests are more likely to become entrepreneurs. The difference between $\zeta(0)$ and $\zeta_\omega(0)$ is increasing in p^{id}, the shadow price of idiosyncratic risk.

As the cost parameter is drawn independently of the bequest a household receives, then the mass of entrepreneurs in a stationary equilibrium is given by

$$\chi_e = F_\varphi\left(\varphi^*(n(0))\right), \tag{E.7}$$

where $n(0)$ is the average financial wealth of newborn entrepreneurs.

In a stationary equilibrium, the mass of entrepreneurs is constant. As $\zeta(0)$, $\zeta_w(0)$, and $n(0)$ depend on the interest rate and the aggregate and idiosyncratic risk premia, then the share of entrepreneurs in the economy depends on the equilibrium expected returns.

E.2.2 Wage earners' problem and equilibrium determination

The optimal consumption-wealth ratio and demand for insurance for wage earners are now given by

$$\frac{c_{j,t}}{\omega_{j,t}} = \zeta_w(a) = \frac{\bar{r}_w}{1 - \psi e^{-\bar{r}_w(T-a_j)}}, \qquad \theta_{j,t}^{ag} = h_{j,t}\sigma_A - \frac{p^{ag}}{\gamma}\omega_{j,t}, \tag{E.8}$$

where

$$\bar{r}_w = \frac{1}{\gamma}\rho_w + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{(p^{ag})^2}{2\gamma}\right). \tag{E.9}$$

The price of aggregate insurance, wages, and the relative price of capital are the same as in the baseline model:

$$p^{ag} = \gamma\sigma_A, \qquad w = (1-\alpha)K^\alpha, \qquad q = \Phi_0 + \Phi_1(g+\delta). \tag{E.10}$$

Finite lives for wage earners change the determination of the interest rate. The interest rate is now jointly determined with the capital-labor ratio and the price of idiosyncratic risk by conditions (32), (33), and the market clearing condition for consumption

$$\int_0^T \frac{\bar{r}\omega(a)}{1-\psi e^{-\bar{r}(T-a)}} f(a)da + \int_0^T \frac{\bar{r}_w\omega_w(a)}{1-\psi e^{-\bar{r}_w(T-a)}} f(a)da = \alpha K^\alpha - \iota K, \tag{E.11}$$

where

$$\omega(a) = \omega(0)e^{\left(r+\gamma\sigma_A^2+\frac{(p^{id})^2}{\gamma}-\mu_A-\bar{r}\right)a}\frac{1-\psi e^{-\bar{r}(T-a)}}{1-\psi e^{-\bar{r}T}}, \qquad \omega_w(a) = \omega_w(0)e^{\left(r+\gamma\sigma_A^2-\mu_A-\bar{r}_w\right)a}\frac{1-\psi e^{-\bar{r}_w(T-a)}}{1-\psi e^{-\bar{r}_w T}}.$$

Assuming finite lives for wage earners would change the calibration of ρ_w but otherwise would not affect our main results.

E.3 Financial autarky

We consider next the case of financial autarky, where entrepreneurs have no access to either aggregate or idiosyncratic insurance. To shut down idiosyncratic insurance, we must set $\phi = 1$. To capture the absence of aggregate insurance, we will focus on the special case where there is no demand for aggregate insurance, so the solution would coincide with the case where entrepreneurs have no access to insurance.

Suppose that $h_{i,t} = 0$, so entrepreneurs have no labor income, and assume that $n_{j,t} = 0$ for $j \in \mathcal{W}_t$, so wage earners have no financial wealth. The first assumption implies that $\frac{qk_{i,t}}{n_{i,t}}$ is equalized across entrepreneurs, and the second assumption implies that $qk_{i,t} = n_{i,t}$. As the demand for aggregate insurance is given by $\theta^{ag}_{i,t} = (qk_{i,t} - n_{i,t})\sigma_A$, we obtain that entrepreneurs do not demand aggregate insurance. The solution will then coincide with the case where aggregate insurance is unavailable.

Under these assumptions, the price of idiosyncratic risk, given in Equation (33), specializes to $p^{id} = \gamma\sigma_I$. The risk premium is then given by

$$p^{ag}\sigma_A + p^{id}\sigma_I = \gamma\left[\sigma_A^2 + \sigma_I^2\right]. \tag{E.12}$$

The aggregate Sharpe ratio relative to the idiosyncratic Sharpe ratio is $p^{ag}/p^{id} = \sigma_A/\sigma_I$.