10 Sustainable intervention policies and exchange rate dynamics

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1 Introduction

Recent work on stochastic exchange rate models has focused on the nonlinearities induced by intervention aimed at maintaining exchange rates within target zones and by exchange-rate regime shifts. In this literature, the stochastic process followed by the fundamental determinants of exchange rates is modeled as a combination of continuous developments, assumed exogenous, and infrequent shifts (either infinitesimal, as in Krugman, 1991, or of finite size, as in Flood and Garber, 1989) occurring on a measure-zero set of time points (or perhaps at one point in time only, as in some models of Froot and Obstfeld, 1989). Exchange rate determination has typically been modeled in terms of money market equilibrium conditions, and this has led to interpreting the infrequent fundamental shocks as (nonsterilized) intervention in the foreign exchange or money market. Since foreign exchange reserves impose obvious limits on the size of cumulative interventions, some attention has been devoted to the sustainability of such intervention schemes (see Delgado and Dumas, 1989).

This paper discusses these aspects and their relationship to each other. In a stylized probabilistic model of exchange rate fundamentals, we find that if the cumulation of infrequent fundamental movements has bounded support (which is necessary for the intervention policy to be sustainable in the long run), then not only is the long-run relationship between fundamentals and exchange rates the same that would be valid under a free-float exchange rate regime, but also the possibly very pronounced within-band nonlinearities in the exchange rate-fundamentals relationship induced by 'intervention' cancel each other out when weighed by their long-run probabilities.

The paper is organized as follows. Section 2 describes, under simplifying assumptions, the general structure of the class of models we study.

Section 3 interprets the technical assumptions in terms of exchange rates and fundamentals, and notes that sustainability issues arise when cumulative intervention has no role in determining the behaviour of fundamentals. Section 4 addresses those issues by allowing the probability of repeated, stochastic realignments to vary as a function of cumulative intervention; Section 5 characterizes exchange rate behaviour under such an intervention scheme; and Section 6 studies the long-run implications of sustainability for the relationship between exchange rates and their fundamental determinants. Section 7 concludes outlining directions for further research.

2 The probability structure of intervention and realignment models

We denote with $\{x_t\}$ the log-exchange rate process, and we assume the familiar asset-pricing relationship

$$x_t = f_t + \frac{a}{dt} E_t\{dx_t\} \tag{1}$$

where $E_t\{\cdot\}$ denotes the conditional expectation formed on the basis of relevant information available at time t, and $\{f_t\}$ denotes the process followed by the *fundamental* determinants of exchange rates, e.g. the variables appearing in the excess money demand function. Ruling out bubbles, we can integrate equation (1) between t and infinity to obtain

$$x_t = \frac{1}{a}\int_t^{\infty} E_t\{f_\tau\} e^{-(\tau-t)/a} d\tau \tag{2}$$

Without specifying the economic counterpart of f_t, we simply write $f_t = i_t + z_t$: by assumption, the levels of the $\{i_t\}$ and $\{z_t\}$ processes have identical roles in exchange rate determination, but their dynamic behaviour is different. To capture the expectational effects emphasized by the literature on target zones and on intervention, we assume continuous sample paths of infinite variation to the $\{z_t\}$ process, while $\{i_t\}$ is constant almost everywhere on the time line and increases or decreases by finite or infinitesimal amounts on a measure-zero set of time points.

We let the dynamics of $\{z_t\}$ be given by

$$dz_t = \sigma dW_t \tag{3}$$

and all movements of $\{i_t\}$ have equal absolute size Δf. If j_t denotes the *net* number of jumps up to an including time t (i.e. the number of positive jumps *minus* that of negative jumps), we then have

$$i_t = j_t \Delta f$$

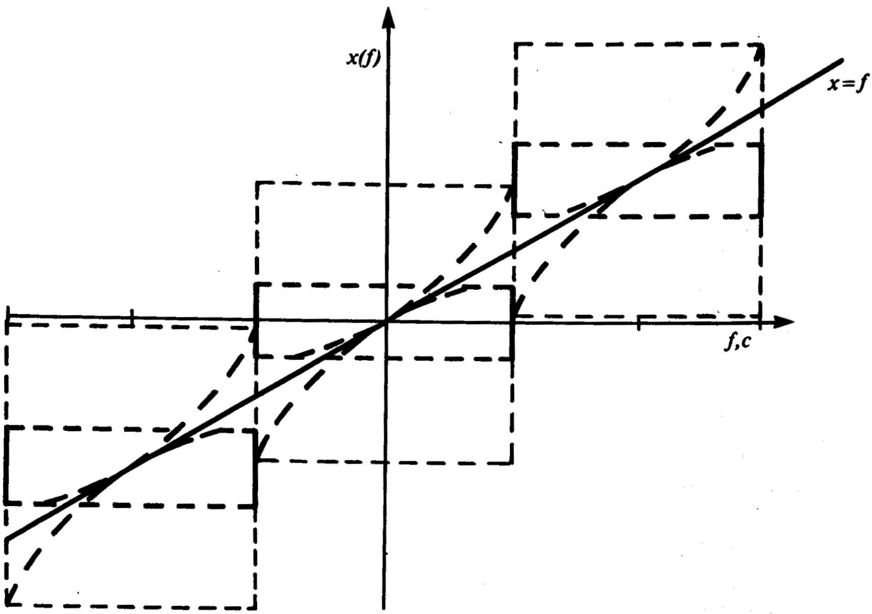

Figure 10.1 Exchange rate and fundamental fluctuation bands

Allowing for non-zero $\{z_t\}$ drift or for jumps of different sizes, while conceptually easy, would considerably complicate the notation and the algebraic derivations.

If $\Delta f = 0$, i.e. if no jumps ever occur in the fundamentals process, then

$$E_t\{f_\tau\} = E_t\{z_\tau\} = z_t = f_t \qquad \forall \tau \geq t$$

which, by equation (2), yields the exchange rate solution $x_t = f_t$ for all t, plotted in Figure 10.1 as the solid diagonal line.

When $\Delta f > 0$, the likelihood of a jump at every point in time has an essential role in determining the relationship between f_t and x_t. We denote with p_t^u the probability of a positive jump at time t, with p_t^d that of a negative jump, and with $1 - p_t^u - p_t^d \geq 0$ the probability of no jump at time t.

If the jump probabilities are *constant* or, more generally, independent of the exchange rate and fundamental processes, the relationship between fundamentals and exchange rates remains linear.[1] To obtain the nonlinear relationships emphasized in the target zone literature, it is necessary to allow the size and/or the probability of jumps to vary over time *in a way that is related to the position of the fundamentals.*

The simplest of the models proposed in our earlier paper (Bertola and

Caballero, 1990), for example, is obtained if the jump probabilities are specifed as

$$p_t^u = \begin{cases} p & \text{if } f_t = c_t + \Delta f \\ (1-p) & \text{if } f_t = c_t - \Delta f \\ 0 & \text{otherwise;} \end{cases} \qquad p_t^d = \begin{cases} (1-p) & \text{if } f_t = c_t + \Delta f \\ p & \text{if } f_t = c_t - \Delta f \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where the auxiliary 'central parity' process $\{c_t\}$ jumps up by $2\Delta f$ when $f_t = c_t + \Delta f$ and i_t jumps upwards, down by $2\Delta f$ when $f_t = c_t - \Delta f$ and i_t jumps downwards. This process has no direct effect on exchange rates, but its level does determine the probability of $\{i_t\}$ jumps.

The horizontal axis in Figure 10.1 may then be divided into adjoining segments of length $2\Delta f$ ('fluctuation bands'), each centered at a value of c. In the interior of every band jumps have probability zero and $\{f_t\}$ has the dynamics of $\{z_t\}$ in equation (3). By (2), the exchange rate can, as a conditional expectation, be written as a function of f_t and c_t processes; and the usual stochastic calculus arguments imply that in the interior of regions where c_t and i_t are constant this function should satisfy the differential equation

$$x(f;c) = f + \frac{a\sigma^2}{2} x''(f;c) \tag{5}$$

where $x''(f;c) \equiv \partial^2 x(f;c)/\partial f^2$. It should also be the case that

$$\begin{aligned} (c + 2\Delta f;\, c + 2\Delta f) + (1-p)x(c;c) &= x(c + \Delta f;\, c) \\ (c - 2\Delta f;\, c - 2\Delta f) + (1-p)x(c;c) &= x(c - \Delta f;\, c) \end{aligned} \qquad \forall c \tag{6}$$

namely that the two possible exchange rates just after a jump be equal, when weighted by their respective probabilities, to the exchange rate prevailing at the instant before the jump. From the economic point of view, these boundary conditions rule out arbitrage opportunities; from the purely technical standpoint, they ensure that equation (1) is satisfied at the instant before a jump: $E_t\{dx_t\}$ would be larger than infinitesimal if (6) did not hold with equality, and no finite exchange rate could satisfy (1), where $E_t\{dx_t\}/dt \to \infty$.

All solutions to the differential equation in (5) have the form

$$x(f;c) = f + A_1(c)e^{\lambda f} + A_2(c)e^{-\lambda f}, \qquad \lambda \equiv \sqrt{\frac{2}{a\sigma^2}} \tag{7}$$

Using (7) in (6), we obtain

$$A_1(c) = -\frac{(1-2p)\Delta f}{e^{\lambda(\Delta f + c)} - e^{-\lambda(\Delta f - c)}} \qquad A_2(c) = -\frac{(1-2p)\Delta f}{e^{\lambda(\Delta f - c)} - e^{-\lambda(\Delta f + c)}}$$

The two $A(\cdot)$ functions have opposite signs for every c, and in the interior of every zone the relationship between exchange rates and fundamentals takes an S-shaped form like those plotted by dashed lines in Figure 10.1.

If $p = \frac{1}{2}$, then $E_t\{f_\tau\} = f_t$ for all $\tau \geq t$, and $x(f; c)$ once again lies on the $x = f$ line. If $p < \frac{1}{2}$, however, $x(f; c) \lesseqgtr f$ according to $f \gtreqless c$: because the jump in fundamentals at the upper (lower) boundary of a zone is expected to be negative (positive), $E_t\{f_\tau\} < f_t$ if f_t is close to $c + \Delta f$, and $E_t\{f_\tau\} > f_t$ if f_t is close to $c - \Delta f$. Symmetrically, if $p > \frac{1}{2}$ then $x(f; c) \lesseqgtr f$ according to $f \lesseqgtr c$.

3 Intervention, reserves and sustainability

Other specifications for the probability and size of i_t jumps may be recast in terms of Section 2's notation along similar lines. The full-credibility model in Krugman (1990) lets the jump component be infinitesimally small (and therefore unrelated to the width of the fundamental fluctuation band, $\bar{f}$), and assumes

$$p_t^u = \begin{cases} 1 & \text{if } f_t = c_t - \bar{f} \\ 0 & \text{otherwise;} \end{cases} \qquad p_t^d = \begin{cases} 1 & \text{if } f_t = c_t + \bar{f} \\ 0 & \text{otherwise,} \end{cases}$$

while Flood and Garber (1989) make similar assumptions on the probabilities and let jumps have finite size. Svensson (1989) considers both infinitesimal and discrete movements in the $\{i_t\}$ component of fundamentals, making assumptions similar to Krugman's on the former and letting the latter have constant probability and size, and Bertola and Svensson (1990) extend the model to allow for stochastic fluctuations in the probability and size of $\{i_t\}$ jumps.

Following these and other authors, this probabilistic framework can be interpreted as a stylized model of (adjustable) exchange rate bands. The Brownian motion component of fundamentals $\{z_t\}$ may be viewed as a money velocity shock, or perhaps as monetization of government deficits beyond the control of the monetary authorities, and the jump component $\{i_t\}$ may be taken to represent nonsterilized intervention in the foreign exchange market.

Intervention, of course, entails a variation in foreign exchange reserves; and if only the infrequent $\{i_t\}$ component is associated with interventions, then the variation of reserves corresponds to the variation in $\{i_t\}$. To illustrate this point and introduce sustainability issues, consider the simple probabilistic assumptions of equation (4), where the size Δf of the $\{i_t\}$ jumps coincides with a half-width of the fundamental fluctuation band, $\bar{f}$. These assumptions correspond to the economic framework of our earlier paper, where authorities *intervene* at prespecified, common knowledge points $c_t - \bar{f}$, $c_t + \bar{f}$: *either* defending the band and bringing f and x

back to the centre of the current band (c_i), *or* realigning the central parity and declaring a new fluctuation band, adjoining the current one, with centre $c_{t+} \equiv c_t + 2\bar{f}$ and unchanged width. By a normalization, set $z_0 = i_0 = 0$ and let reserves be equal to zero at time zero. Starting at zero, z_t takes a random, finite time to reach one of the boundaries and to trigger the first jump. Either boundary can be reached with probability $\frac{1}{2}$ since $\{f_t\}$ follows a driftless Brownian motion. Denoting by $\tau > 0$ the time at which the first jump occurs, we have:

$i_\tau = -\bar{f}$ if $\{z_t\}$ has drifted upwards to $\bar{f}$ and the band has been defended *or* if $\{z_t\}$ has drifted downwards to $-\bar{f}$ and a realignment has taken place: the combined probability of these events is $\frac{1}{2}(1-p) + \frac{1}{2}p = \frac{1}{2}$;

$i_\tau = +\bar{f}$ if $\{z_t\}$ has drifted downwards to $-\bar{f}$ and the band has been defended *or* if $\{z_t\}$ has drifted upwards to $\bar{f}$ and a realignment has taken place: the combined probability of these events is, again, $\frac{1}{2}p + \frac{1}{2}(1-p) = \frac{1}{2}$;

The derivation in Karatzas and Schreve (1988, especially pp. 99–100) can be adapted to show that the time interval between jumps is finite with probability one and has expectation $(\bar{f}/\sigma)^2$; its distribution is independent of the past history of the stochastic processes under study, which enjoy the strong Markov property. Thus, *regardless of the realignment probabilities*, $\{i_t\}$ follows a generalized random walk with variable time steps, constant state steps, and constant transition probabilities under our stylized assumptions; this is true, in particular, when the fluctuation band is fully credible ($p = 0$).[2]

Reserves therefore follow a random walk on a redefined time scale and, over an infinite time horizon, reach arbitrarily large and arbitrarily small levels with probability one, raising obvious issues of *sustainability* of the exchange rate regime.[3] To obtain a (stochastically) bounded reserve process when *intervention* is taken to occur only at the boundaries and is interpreted in terms of purchases/sales of foreign exchange, the probability and size of upwards and downwards jumps should be allowed to depend on reserve levels or, equivalently, on cumulative intervention up to date. For example, the probability structure of interventions might be assumed to be independent of reserves until these reach some prespecified boundary, and to change abruptly at that point, adapting to the problem at hand the assumptions of the literature on exchange rate regime collapses (Krugman, 1979; Flood and Garber, 1984). Delgado and Dumas (1989) analyse the case of infinitesimal interventions at the margin of fluctuation zones along these lines, supposing that exchange rates revert to a one-sided or two-sided float when reserves reach a nonrandom, prespecified limit.

Collapse models, however, have several shortcomings. On the one hand, there is no clear-cut upper or lower limit to cumulative exchange rate intervention in reality, since foreign exchange can in principle be borrowed in any amount (provided, of course, that principal and interest are surely repaid over the relevant, possibly infinite time horizon).[4] On the other hand, the assumption of *permanent* reversion to free float when the reserves limits (however specified) are reached is both unrealistic and formally questionable: no central bank ever operates without exchange rate reserves, thus exchange rates are never truly freely floating; and allowing for the free-float regime to be absorbing prevents any study of the long-run properties of the model. Historically, exchange rate crises have most often resulted in a realignment of central parities or in a redefinition of the exchange rate regime. While these points are not essential to the stylized models of Krugman (1979) and Flood and Garber (1984), the discrete time structure of the empirically oriented model of Blanco and Garber (1986) produces stochastic, recurring balance-of-payments crises.

4 A sustainable probability structure

In what follows, we address sustainability issues in the context of a stylized model of repeated, stochastic realignments (of the type studied in Miller and Weller, 1989 and in our earlier paper). To this end, we allow the probability and/or the size of upward and downward fundamental jumps to depend on accumulated interventions or, equivalently, on the net number j_t of interventions. When reserves are plentiful, downward jumps should be more likely (or larger) than upward ones if reserves are to display a tendency to return to normal levels; symmetrically, when reserves move towards minus infinity upward jumps must become more likely (or larger) than downward ones.

As above, let all jumps have absolute size $\bar{f}$, let jumps only occur when f_t is at the edges of a $(c_t - \bar{f},\ c_t + \bar{f})$ band, and let a jump that takes f_t beyond the limits of the current fluctuation band be accompanied by a jump in $\{c_t\}$ of the same sign and twice the absolute size: after every jump, the f_t process is always in the middle of its current fluctuation band. Still assuming jumps to have nonzero probability only when fundamentals are at the boundary of a band, we let the probability of an upwards j jump be decreasing in the size of cumulative intervention to date, capturing the qualitatively realistic idea that devaluations are more likely when reserves are low and, for simplicity, we let the probability of upward or downward jumps be the same at both boundaries of the band:

$$p_t^u = \begin{cases} p(j_t) & \text{if } f_t = c_t + \bar{f} \text{ or } c_t - \bar{f} \\ 0 & \text{otherwise;} \end{cases}$$
$$p_t^d = \begin{cases} 1 - p(j_t) & \text{if } f_t = c_t + \bar{f} \text{ or } c_t - \bar{f} \\ 0 & \text{otherwise,} \end{cases} \tag{8a}$$

where

$$\begin{aligned} 0 < p(j) < 1 \text{ for } \underline{j} < j < \bar{j} \\ p(j) = 1 \text{ for } j \leq \underline{j} \\ p(j) = 0 \text{ for } j \geq \bar{j} \end{aligned} \tag{8b}$$

By these assumptions and the arguments above, $\{j(t)\}$ follows a random walk with variable transition probabilities and random time steps over the $(\underline{j}, \bar{j})$ region; $\underline{j}$ and $\bar{j}$ may be, respectively, minus and plus infinity, in which case we require

$$\lim_{j \to -\infty} p(j) = 1, \qquad \lim_{j \to \infty} p(j) = 0$$

Since $0 < p(j) < 1$ in this range, all $\underline{j} \leq j \leq \bar{j}$ can be reached from each other with positive probability, and the corresponding states of the Markov chain are recurrent.[5] The unconditional distributions of the $\{j_t\}$ process, denoted $\phi(\cdot)$, is non-degenerate under our assumptions and can be computed by the invariance relationship

$$\phi(j) = p(j-1)\,\phi(j-1) + (1 - p(j+1))\,\phi(j+1) \tag{9a}$$

and summing-up constraint

$$\sum_{j=\underline{j}}^{\bar{j}} \phi(j) = 1 \tag{9b}$$

The $\phi(j)$ probability distribution has a simple analytic form for simple $p(\cdot)$ functions, and can be computed numerically for any $p(\cdot)$. Our assumptions guarantee that the probability of the absolute value of j ever exceeding arbitrarily large amounts over an infinite time horizon is zero, or vanishingly small. If the jump component of fundamentals is associated with official intervention in exchange rate markets and j_t corresponds to the cumulative variation of reserves, we can thus proceed to model exchange rate behaviour in a 'sustainable' exchange rate regime.

5 Exchange rate dynamics under sustainable intervention

The $\{f_t, c_t, j_t\}$ processes are jointly Markov under the assumptions in (8): the distribution of f_τ at all future times depends only on their current levels. The exchange rate is then, by equation (2), a function of the three driving processes, and this function must once again satisfy

$$x(f; c, j) = f + \frac{a\sigma^2}{2} x''(f; c, j) \tag{10}$$

when jumps are ruled out (i.e. in the interior of a fluctuation zone), as well as the no-expected-jump conditions

$$\begin{aligned} x(c + \bar{f}; c, j) &= p(j)x(c + 2\bar{f}; c + 2\bar{f}, j + 1) \\ &\quad + (1 - p(j))x(c; c, j - 1) \\ x(c - \bar{f}; c, j) &= (1 - p(j))x(c - 2\bar{f}; c - 2\bar{f}, j - 1) \\ &\quad + p(j)x(c; c, j + 1) \end{aligned} \qquad \forall c, \forall j$$

at the boundaries between fluctuation zones.

It is notationally convenient to define the transformed variables

$$\tilde{x}_t \equiv \frac{x_t - c_t}{\bar{f}} \qquad \tilde{f}_t \equiv \frac{f_t - c_t}{\bar{f}} \tag{11}$$

The assumptions of our simple model guarantee that $-1 \leq \tilde{f} \leq 1$, and that $\tilde{f}$ jumps to zero with probability one whenever it reaches one or minus one.

In terms of these normalized processes,(10) can be written

$$\tilde{x}(\tilde{f}; j) = \tilde{f} + \frac{a}{2} \tilde{x}''(\tilde{f}; j)(\sigma/\bar{f})^2 \tag{12}$$

and the no-expected-jump conditions are satisfied if

$$\begin{aligned} \tilde{x}(1; j) &= p(j)\big(\tilde{x}(0; j + 1) + 2\big) \\ &\quad + \big(1 - p(j)\big)\tilde{x}(0; j - 1) \\ \tilde{x}(-1; j) &= (1 - p(j))\big(\tilde{x}(0; j - 1) - 2\big) \\ &\quad + p(j)\tilde{x}(0; j + 1) \end{aligned} \qquad \forall c, \forall j \tag{13}$$

Subtraction of one condition from the other immediately yields $\tilde{x}(1; j) = \tilde{x}(-1; j) + 2$, for all j. Thus, the difference between the extremes of exchange rate fluctuation bands is independent of reserves, and the general solution of (10) can be written

$$\tilde{x}(\tilde{f}; j) = \tilde{f} + A(j)\big(e^{\tilde{\lambda}\tilde{f}} + e^{-\tilde{\lambda}\tilde{f}}\big), \qquad \tilde{\lambda} \equiv \frac{\bar{f}}{\sigma}\sqrt{2/a} \tag{14}$$

Using (14) in (13), we obtain a second-order, variable-coefficients difference equation in $A(\cdot)$:

$$\begin{aligned} &(1 - p(j))A(j - 1) - \delta A(j) + p(j)A(j + 1) = (\tfrac{1}{2} - p(j)) \\ &\delta \equiv \frac{e^{-\tilde{\lambda}} + e^{\tilde{\lambda}}}{2} \end{aligned} \tag{15}$$

At the upper and lower boundaries of the allowable range of reserves, the no-expected-jump conditions and (15) imply that

$$\begin{aligned} \delta A(\underline{j}) - A(\underline{j}+1) &= \tfrac{1}{2} \\ \delta A(\bar{j}) - A(\bar{j}-1) &= -\tfrac{1}{2} \end{aligned} \tag{16}$$

The difference equation (15) and boundary conditions (16) form a system of linear equations, which is easily solved for any $p(j)$ function; simple analytical solutions are available for some $p(\cdot)$ functional forms.

Several $p(j)$ functions, the corresponding $A(j)$ and $\phi(j)$ sequences, and the $\tilde{x}(\bar{f}; j)$ they imply for a selection of j values are plotted in Figures 10.2–10.4. All the $p(j)$ functions considered in the Figures satisfy the assumptions in (8b) and, in all cases, the probability of reserves ever reaching unbounded levels in either direction is zero or vanishingly small. Thus, the exchange rate regime is 'sustainable'. In the Figures, we set $\bar{\lambda} = \sqrt{2}$, consistently with, for example, $a = 1$ (unitary semi-elasticity of the exchange rate to its own expected rate of change over a time unit) and $\sigma = \bar{f}$ (fluctuations bands and fundamental volatility are such that the expected time to hit either boundary is one time unit).

In Figures 10.2, 3 and 4 the assumed jump probability function is symmetric around zero (in the sense that $p(j) = 1 - p(-j)$), with $p(0) = \tfrac{1}{2}$. Then, $A(0) = 0$: when intervention is as likely to be positive as it is to be negative over all forecast horizons the relationship between the (normalized) exchange rate and the (normalized) fundamentals is linear, since $E_t\{f_\tau\} = f_t$ for $\tau > t$. The $p(j)$ function considered in Figure 10.2, which is constant throughout and moves sharply to zero or one when reserves reach their absolute limit, models a reversible collapse model. The $p(j)$ function in Figure 10.5 is asymmetric, in the sense that the probability of upward jumps increases faster as $j \to \underline{j}$ than it decreases as $j \to \bar{j}$: this may be taken to represent greater concern for unusually low levels of reserves than for unusually high ones, as might be appropriate for a small country, and yields a skewed long-run distribution of reserves.

The relationship between exchange rates, fundamentals, central parities and reserves can be recovered from the change-of-variable in equation (11). Figure 10.6 plots all possible $x(f; c, j)$ relationships in a three dimensional box, and Figure 10.7 plots the $x(f; c, j)$ relationships implied by the probability function of Figure 10.4 in the neighbourhood of $c = 0$, $j = 0$. When $c = 0$ and $j = 0$, exchange rates and fundamentals are driven by z_t fluctuations along the solid 45° line in Figure 10.7. If the upper boundary of the $(-\bar{f}, \bar{f})$ region is reached at point T^u, a jump is triggered in the fundamental process, in the reserves process, and (possibly) in the central parities process. The relevant $x(f; c, j)$ function is then one of those plotted by long dashes in Figure 10.7. With probability $\tfrac{1}{2}$, an

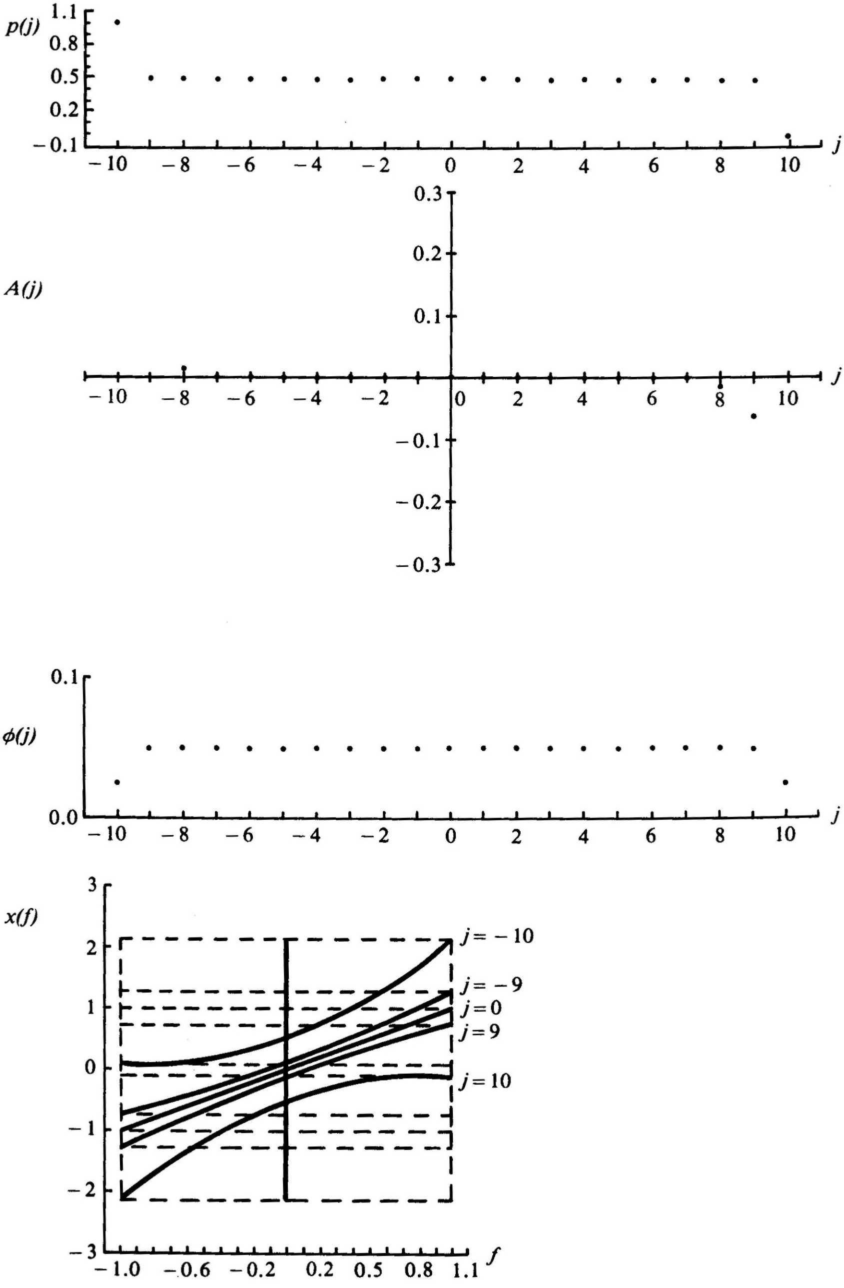

Figure 10.2 (Almost) constant realignment probability

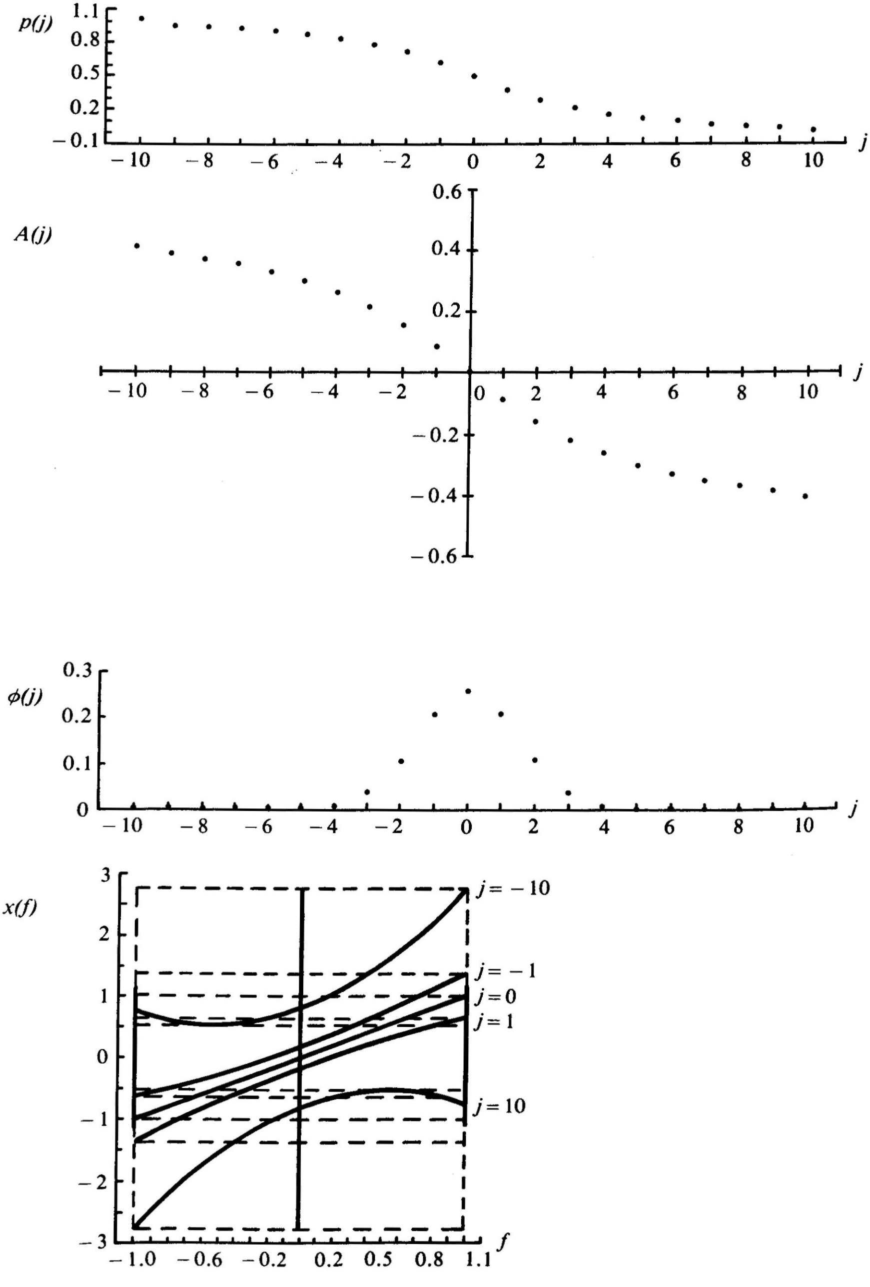

Figure 10.3 Linear realignment probability

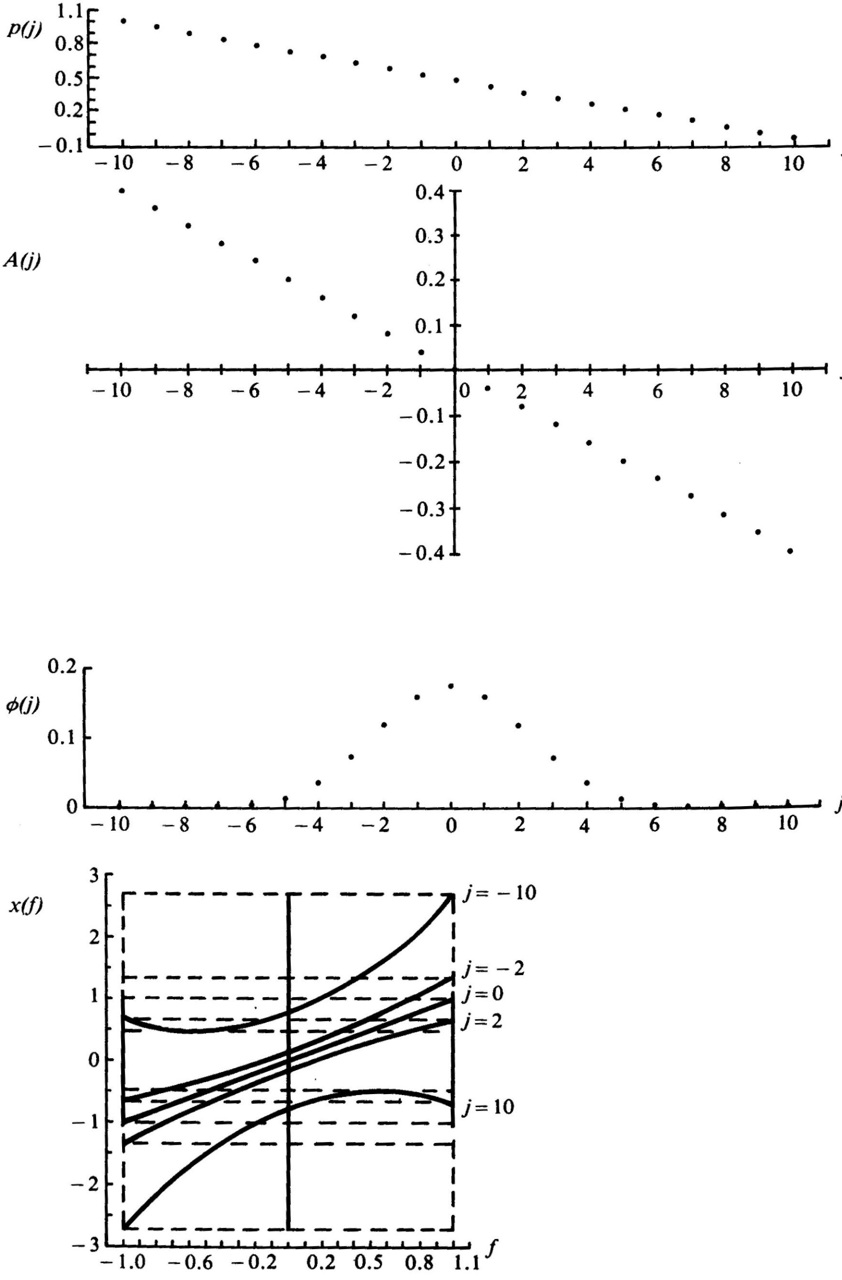

Figure 10.4 Symmetric realignment probability

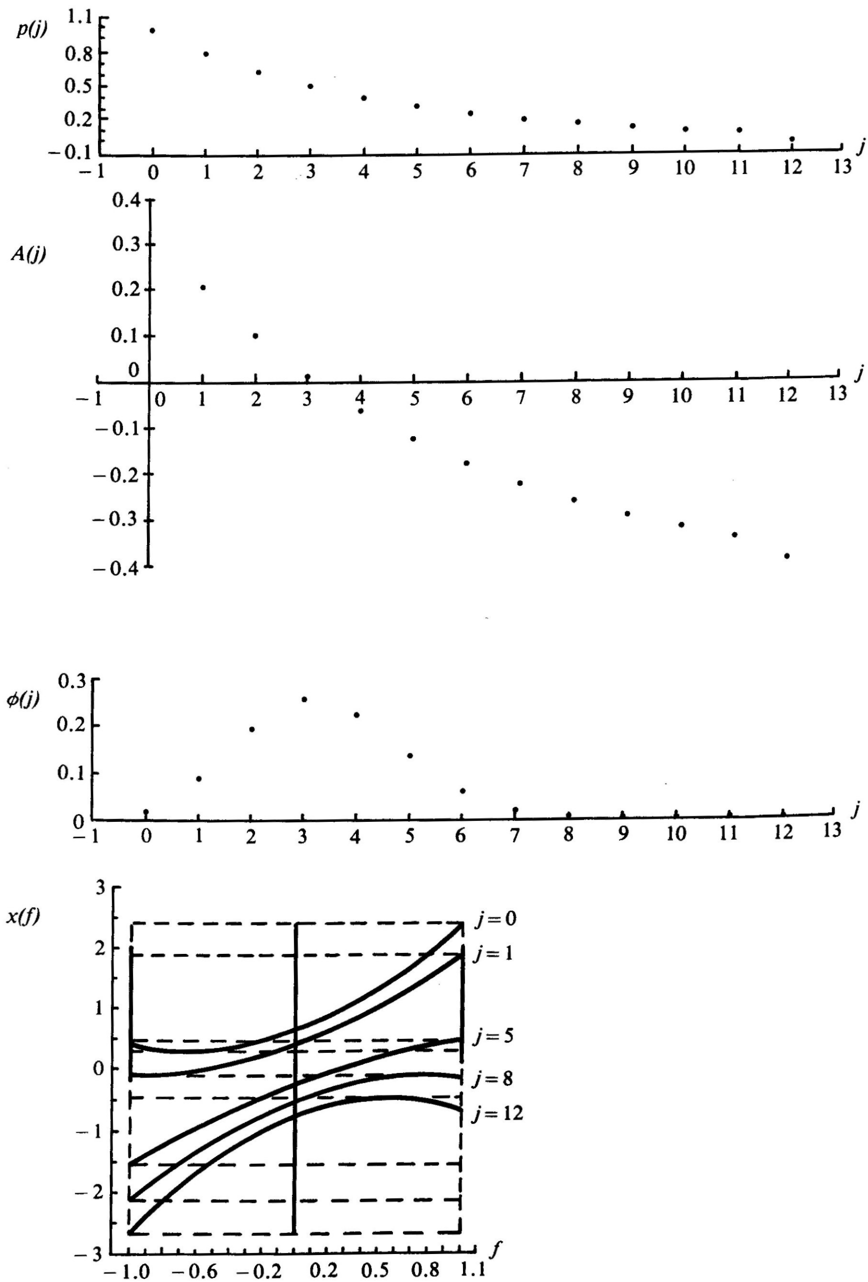

Figure 10.5 Asymmetric realignment probability

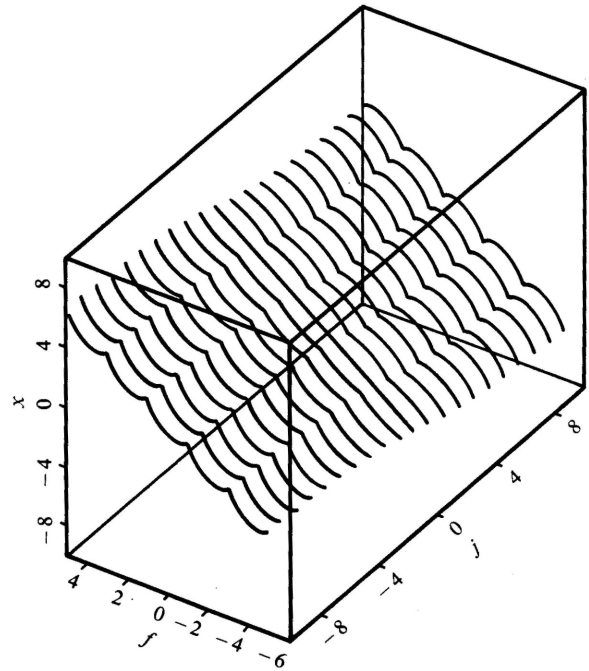

Figure 10.6 Exchange rates, fundamentals and reserves

upward jump in fundamentals and central parities occurs, accompanied by an upward jump in reserves: the exchange rate jumps to B, and starts fluctuating along the (concave) ABC line. With probability $\frac{1}{2}$, fundamentals and reserves jump down, while c remains constant at zero: the exchange rate then jumps to point B' and starts fluctuating along the (convex) $A'\,B'\,C'$ line. If the lower boundary of $(-\bar{f}, \bar{f})$ is reached first (at point T'), the exchange rate symmetrically shifts to points on the lines plotted by short dashes.

6 Short-run nonlinearities and the long run

In all the cases considered, $p(j)$ is monotonic and $A(j)$ is decreasing in j; when $p(j)$ is linear (Figure 10.3), so is $A(j)$. As j moves downwards, therefore, the $\tilde{x}(\tilde{f})$ function drifts upwards and becomes increasingly convex: intervention becomes more likely to increase than to decrease reserves as they become scarce, to slow down and eventually stop their fall. Since an increase in reserves entails an upward jump in fundamentals, the expected level of f_τ at all future time is *higher* than it would be if

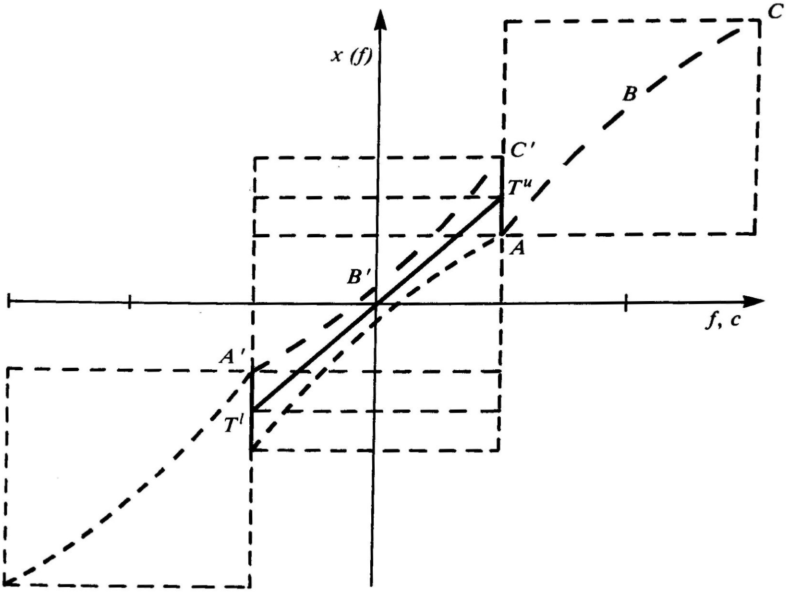

Figure 10.7 Intervention and exchange rates when reserves matter

intervention were never to occur, and the exchange rate is, correspondingly, higher (more depreciated) for every level of fundamentals.

It is apparent from the Figures that the relationship between exchange rates and fundamentals depends on the level of reserves, $j\bar{f}$. Not all values of j are equally likely, of course: in particular, our assumptions in (8b) ensure that very large or very small values of j are seldom or never observed.

Taking an expectation over all possible j values, we then obtain the *unconditional* relationship between exchange rates and fundamentals,

$$E\{\tilde{x}(\tilde{f}; j)\,|\tilde{f}\} = \sum_{j=\underline{j}}^{\bar{j}} \phi(j)\,\tilde{x}(\tilde{f}; j)$$

$$= \tilde{f} + (e^{\tilde{\lambda}\tilde{f}} + e^{-\tilde{\lambda}\tilde{f}}) \sum_{j=\underline{j}}^{\bar{j}} \phi(j)\,A(j) \qquad (17)$$

The $\phi(j)$ probability distribution can be computed by (9), and is plotted in the third panel of Figures 10.2–10.4 for the cases we consider.

The Appendix shows that if the $p(j)$ function satisfies the assumptions in (8a), then

$$\sum_{j=\underline{j}}^{\bar{j}} \phi(j)\,A(j) = 0 \qquad (18)$$

to imply that when no information as to the level of reserves is available exchange rate should be expected to respond *linearly* to movements in fundamentals, as in the case of flexible exchange rates.

Thus, even though for particular reserve levels exchange rates may be a very nonlinear function of 'fundamentals' (reflecting the changing expectations of intervention in either direction), these nonlinearities cancel each other out in the long run if reserves have a well-defined probability distribution.

7 Directions for further research

The tendency of nonlinearities to cancel each other out in the long run (or when no information about 'reserves' is available) is more general than the specific simple model proposed above.[6] At the cost of notational and analytical complications, the simple model outlined above could be extended in several realistic directions, still assuming the only intervention to occur at the edges of prespecified fluctuation bands. The $\{z_t\}$ process driving within-band fluctuations could be allowed to have non-zero drift, or to display mean-reversion; and the probabilities and size of jumps in fundamentals and reserves could be modified so as to allow an interpretation of intervention as a 'defence' or 'realignment' of a given target zone.

For 'infrequent' intervention to be sustainable, the probability and size of jumps in either direction should still be allowed to depend on cumulative intervention, so as to obtain stationarity of the reserves process. If exchange rate bands are more likely to be defended than to be realigned, realignments will generally need to be large to keep cumulative intervention bounded; since these two elements interact in determining the shape of the relationship between exchange rates and 'fundamentals' (see our earlier paper), the concavity or convexity effects of infrequent intervention schemes would still be ambiguous in such an extended model, and would still tend to offset each other out as longer and longer periods of time are considered.

It may be more fruitful, however, to relax the assumption that interventions occur only at the boundaries of fluctuation bands. The behaviour of EMS exchange rates suggests that in 1979–87 fundamentals followed within-band processes with small drifts and low variability, and that the probability of large realignments in the direction of a Deutsche Mark appreciation was very close to one at the boundary of the target zone (see our earlier paper). If intervention were indeed occurring only at the boundaries of exchange rate bands, these facts would unrealistically imply a tendency of reserves to *increase* in 'weak' currency countries. In fact, *sustainability* of intervention need not be related to its *infrequency*, a purely technical device. Central banks may well monitor and control exchange rates in the interior of target zones, expending reserves, and reconstitute reserves by infrequent realignments: to model this realistic feature in a probabilistic framework similar to that outlined above, the probability of realignments and interventions should be allowed to depend on $\{z_t\}$ as well as on $\{i_t\}$.

Appendix:[7] The long-run relationship between exchange rates and fundamentals

The recursions in (9a–b) and in (15–16) can be written in matrix form as

$$(I - T)\phi = o \quad \Rightarrow \quad \phi = T\phi$$

$$\phi' l = 1$$

$$(T' - \delta I)A = \tfrac{1}{2}l - p \quad \Rightarrow \quad T'A = \delta A + \tfrac{1}{2}l - p$$

where $\delta = (e^{-\tilde{\lambda}} + e^{\tilde{\lambda}})/2 > 1$, l is a vector of ones, o is a vector of zeros, I is an identity matrix, and

$$T = \begin{pmatrix} 0 & 1 - p(j+1) & 0 & \cdots & 0 & 0 \\ 1 & 0 & 1 - p(j+2) & \cdots & 0 & 0 \\ 0 & p(j+1) & 0 & \cdots & 0 & 0 \\ \vdots & 0 & \ddots & & \vdots & \vdots \\ 0 & \vdots & & \cdots & 1 - p(\bar{j}-1) & 0 \\ 0 & 0 & & \cdots & 0 & 1 \\ 0 & 0 & & \cdots & p(\bar{j}-1) & 0 \end{pmatrix}$$

$$p = \begin{pmatrix} 1 \\ p(\underline{j}+1) \\ \vdots \\ p(\underline{j}+1) \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} A(\underline{j}) \\ A(\underline{j}+1) \\ \vdots \\ A(\bar{j}-1) \\ A(\bar{j}) \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi(\underline{j}) \\ \phi(\underline{j}+1) \\ \vdots \\ \phi(\bar{j}-1) \\ \phi(\bar{j}) \end{pmatrix}$$

Thus

$$\sum_{j=\underline{j}}^{\bar{j}} \phi(j)A(j) = \phi' A = \mathrm{tr}((T\phi)'A) = \mathrm{tr}(\phi'(T'A))$$

$$= \phi'(\delta A + \tfrac{1}{2}l - p) = \delta\phi' A + \phi'(\tfrac{1}{2}l - p)$$

to imply that

$$(1 - \delta)\phi' A = \phi'(\tfrac{1}{2}l - p)$$

Then (noting that $\phi l = 1$)

$$\sum_{j=\underline{j}}^{\bar{j}} \phi(j) A(j) = 0 \quad \Leftrightarrow \quad \phi' p = \tfrac{1}{2}$$

We can prove that, in fact, $\phi' p = \frac{1}{2}$. Rearranging (9a),

$$p(j+1)\phi(j+1) + \phi(j) = p(j-1)\phi(j-1) + \phi(j+1)$$

Summing the two sides of this relationship up from $j = \underline{j}$, we obtain

$$\sum_{k=\underline{j}}^{j} p(k+1)\phi(k+1) + \sum_{k=\underline{j}}^{j} \phi(k) = \sum_{k=\underline{j}}^{j} p(k-1)\phi(k-1) + \sum_{k=\underline{j}}^{j} \phi(k+1)$$

or

$$\sum_{k=\underline{j}}^{j} p(k+1)\phi(k+1) - \sum_{k=\underline{j}}^{j} p(k-1)\phi(k-1) = \sum_{k=\underline{j}}^{j} \phi(k+1) - \sum_{k=\underline{j}}^{j} \phi(k)$$

which simplifies to

$$p(j+1)\phi(j+1) + p(j)\phi(j) - \phi(j) = \phi(j+1) - \phi(\underline{j})$$

Summing over j on each side, and noting that $\phi(j) = 0$ for $j < \underline{j}$ and $j > \bar{j}$, we obtain

$$2\sum_{j=\underline{j}}^{\bar{j}} p(j)\phi(j) - \phi(\underline{j}) = \sum_{j=\underline{j}+1}^{\bar{j}} \phi(j) = 1$$

to imply that $\phi' p = \frac{1}{2}$ as was to be shown.

NOTES

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1 To see this, consider the simple case of constant, symmetric jump probabilities $p_t^u = p_t^d = p$, for all t. The $\{i_t\}$ process then has no moments unless either p or $\bar{f}$ is infinitesimally small; we consider these two possibilities in turn. If $\bar{f}$ is larger than infinitesimal, let one downwards (or one upwards) jump have probability $p \approx 1 - e^{-\gamma\Delta t}$ over a Δt time increment. In the continuous time limit, $\{i_t\}$ is the sum of two Poisson process with equal probability intensity γ and increments of equal absolute size and opposite sign. Alternatively, we may let the jump probabilities remain finite and normalize the size of jumps. If we let $\bar{f} = \gamma\sqrt{\Delta t}$, as $\Delta t \to 0$ the $\{i_t\}$ process converges to Brownian motion with variance γ^2 per unit time. In both cases, $E_t\{f_\tau\} = f_t$ for all $\tau \geq t$, and $x_t = f_t$. More general cases can be analysed along similar lines.

2 If its drift were positive, the $\{z_t\}$ process would be more likely to hit $\bar{f}$ first rather than $-\bar{f}$; the probability that a Brownian motion process with drift ϑ and standard deviation σ hits $\bar{f}$ before $-\bar{f}$, starting at zero, is

$$q(\vartheta, \sigma) = (1 - e^{-\frac{2\vartheta\bar{f}}{\sigma^2}})/(e^{\frac{2\vartheta\bar{f}}{\sigma^2}} - e^{-\frac{2\vartheta\bar{f}}{\sigma^2}})$$

if $\vartheta \neq 0$. The $\{i_t\}$ 'reserves' process would then follow a random walk with drift. These and other equally straightforward extensions are omitted for simplicity.

3 The model we propose, although phrased in terms of a single exchange rate and a single set of fundamentals, can easily be interpreted in a two-country framework. If the sum of the two countries' reserves is constant, which might be appropriate in a model of the gold standard, then unboundedly positive levels of j are just as unsustainable as unboundedly negative ones. Even more stringently, reserves should sum up to zero in a model of fiat money creation, where one central bank's assets must be offset by another's liabilities. Delgado and Dumas (1989) somewhat unconvincingly assume instead that reserves consist of assets in infinitely elastic supply.

4 The assumption of hard limits on reserve decumulation is particularly unrealistic in the context of the exchange rate mechanism of the European Monetary System: central banks often intervene to prevent excessive *appreciation* of their currency, thus accumulating reserves, and when intervening to prevent excessive depreciation they have statutory access to – in principle – unbounded credit from the issuer of the appreciating currency.

5 Once again, we might want to allow for a drift in the process driving the within-band fluctuations. The transition probabilities of the $\{j_t\}$ process would then have to be modified by the $q(\vartheta, \sigma)$ function of note 2.

6 Nonlinearities cancel in the long run of collapse models as well: the expectational effects emphasized by these models disappear in the long run if their assumption of reversion to perpetual float is to be taken literally.

7 We are very much indebted to Hyeng Keun Koo for decisive help on this proof.

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Discussion

LARS E.O. SVENSSON

This is an excellent paper on the short- and long-run dynamics of an exchange rate target zone when the probability and direction of realignments depends on the level of reserves. Different approaches to modeling realignments are clarified in an admirable way, and the paper contains several interesting results. The most striking is that the long-run relation between the exchange rate and fundamentals is linear, in contrast to the short-run nonlinear relation that has received a lot of interest. The long-run relation is similar to the relation between the exchange rate and fundamentals that would occur under a free float: so target zones do not matter in the long run! This is indeed a striking result; but we should realize that an important reason for it is that the parameters of the underlying exogenous fundamentals (denoted z_t in the paper) are assumed to be unaffected by the existence of a target zone (as in most or even all of the recent target zone literature).

Long-run sustainability of the target zone is assured by assuming that the probability of devaluations decreases, and the probability of revaluations increases, with the level of reserves. Hence, if reserves are low, on average they will be increased through devaluations; if they are high, on average they will be reduced through revaluations. We should note, however, that there are situations when the sustainability of the target zone is fairly clear, namely if monetary policy is exclusively focused on maintaining the target zone. Consider, for example, a cooperative bilateral target zone, in which case one can interpret the intervention variable i_t as $i_t = m_t - m_t^*$, the difference between domestic and foreign (log) money supplies m_t and m_t^*. Here there is a trivial mode of cooperation that will sustain the target zone indefinitely: let the domestic central bank carry out all interventions at the strong edge of the exchange rate band, and let the foreign central bank carry out all interventions at the weak edge of the exchange rate band. The reserves of the two central banks will never decrease, only increase, and the target zone can be maintained indefinitely, even without any realignments at all. (Of course, the target zone could be associated with a high rate of inflation in both countries.)

Even a unilateral target zone can be maintained indefinitely, if that is the sole focus of monetary policy. If the central bank is losing reserves, it can always reduce domestic credit so as to stop the capital outflow and even regain the lost reserves. Put differently, there is always a level of domestic interest rates that is sufficient to compensate for any devaluation risk and make investors indifferent between domestic-currency and foreign-currency denominated assets. This way capital flows can be balanced, and realignments are not needed.

Hence, a target zone can always be sustained indefinitely, if that is the sole focus of monetary policy. Sustainability of the target zone becomes a nontrivial issue only in situations when there are constraints on monetary policy that preclude its sole focus on defending the target zone. Such constraints may arise for instance if there is a politically motivated, explicit or implicit, ceiling for domestic interest rates, or if monetary policy is conducted with concern for its short-run effect on employment, or its effect on inflation. In cases when there is a ceiling on domestic interest rates, it appears that reserves are likely to be one of the factors affecting realignments. However, with other constraints and concerns for monetary policy, it is not clear that the level of reserves will be the crucial factor. Instead it may be other macro variables, like relative inflation rates, real exchange rates, current accounts or unemployment that determine the likelihood of realignment. It would indeed be very interesting to see empirical studies of what the principal determinants of realignments actually are; my conjecture is that for industrialized countries it will often be macro variables other than the level of reserves.

Incidentally, Bertola and Svensson (1990) demonstrate a method for extracting the time series of implicit devaluation risk from target zone data on exchange rates and interest rate differentials. In principle the devaluation risk's dependence on reserves and other macro variables can be examined empirically, in order to evaluate the relative importance of reserves in determining realignments.

In the model of Bertola and Caballero, realignments take place only at the edges of the exchange rate band. An alternative approach is to allow realignments to occur with positive probability even when the exchange rate is in the interior of the band. In fact, daily data reveal that few of the realignments in ERM occurred when the exchange rates were exactly at the edge, and sometimes the exchange rates were quite far away. This is also the case for devaluations in the Nordic countries outside the ERM. A concrete alternative is to have the probability intensity of a devaluation to be independent of the exchange rate's position in the band but decreasing in the level of reserves. It seems that the main result of the paper about the long-run linearity between exchange rates and fundamentals would still follow under this alternative specification. In all likelihood, the main result will be rather robust to different specifications of the devaluation risk.

As a final comment, we note that it is easy to derive very specific implications about the interest rate differentials in the Bertola–Caballero model, and in particular how these depend on the parameters of the model and the level of the reserves. The implications for the interest rate differentials could also be very useful in trying to distinguish empirically this particular model of realignments from other models.

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