Deficits and Inflation:
HANK meets FTPL*

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Abstract

In the Fiscal Theory of the Price Level (FTPL), households are Ricardian, so fiscal deficits drive output and inflation only under hard-to-test assumptions about which policy authority is "active" or "dominant." In the Heterogeneous Agent New Keynesian (HANK) paradigm, households are instead non-Ricardian, so deficits drive aggregate demand, and thereby also inflation, through classical wealth or liquidity effects. *Because* of this difference, HANK is free of FTPL's fragilities and controversies. *Despite* this difference, HANK actually reproduces FTPL's core empirical predictions regarding the relation between deficits and inflation. This is true even for the most extreme FTPL scenario, where unfunded deficits are financed entirely by inflation-induced debt erosion. In practice, however, deficit-financed fiscal stimuli can help expand output and thus tax revenue, substituting for debt erosion and moderating the associated inflationary pressures.

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1 Introduction

Do fiscal deficits drive inflation? If so, how, and by how much? One answer is provided by the Fiscal Theory of the Price Level (FTPL): a higher deficit today not backed by commensurate surpluses in the future *must* be accompanied by higher inflation, so that the resulting erosion in the real value of government debt can substitute for the missing surpluses. This theory has received much attention following the recent inflationary episode (e.g., Cochrane, 2023; Barro and Bianchi, 2025), yet it remains controversial—both for theoretical reasons (Kocherlakota and Phelan, 1999; Buiter, 2002; Niepelt, 2004; Atkeson, Chari and Kehoe, 2010, among many others) and for its rejection of "monetary dominance," i.e., of the view that the monetary authority has ultimate control of inflation. A different answer is provided by mainstream Keynesian logic: fiscal deficits can stimulate aggregate demand via classical wealth or liquidity effects, and can therefore trigger inflationary booms, unless the monetary authority offsets the fiscal stimulus with large enough interest rate hikes. This mechanism is absent in the representative-agent New Keynesian model (RANK), because households there are Ricardian; it lies at the heart, however, of both the old IS-LM paradigm and the modern Heterogeneous Agent New Keynesian literature (known as HANK, per Kaplan, Moll and Violante, 2018).

This paper builds a bridge between these perspectives. We first establish an equivalence result: *despite* the different mechanism at work, HANK can reproduce FTPL's core predictions about the relation between deficits and inflation; in particular, when fiscal adjustment is sufficiently slow, HANK predicts as much inflation as the FTPL. We next show that, *because* of the difference in mechanism, these predictions are now grounded in compelling microeconomic evidence, are reconciled with monetary dominance, and are freed from the controversies surrounding the FTPL. In a nutshell, our paper shows that HANK's different "how" gives new credence to the FTPL's "how much."

We complement these theoretical lessons with a second, practical takeaway. The simplest FTPL arithmetic stipulates that unfunded deficits induce an exactly offsetting increase in prices. Although our equivalence result applies even to this extreme scenario, the empirically relevant case is not close to it: unfunded deficits also trigger a boom in real activity and thus tax revenue, leaving less scope for inflation-induced debt erosion. Indeed, in our post-Covid application, the inflationary pressure from "stimulus checks" is found to be less than one half of that implied by the baseline FTPL arithmetic.

Environment. For our main analysis, we consider an overlapping-generations version of the New Keynesian model. Households survive from one period to another with probability $\omega \in (0, 1]$. When $\omega = 1$, households have infinite horizons and our model reduces to RANK. When instead $\omega < 1$, our model emulates HANK: as in a recent complementary literature (Farhi and Werning, 2019; Aguiar, Amador and Arellano, 2024; Angeletos, Lian and Wolf, 2024; Rachel and Ravn, 2025), finite horizons proxy for liquidity constraints and so help align the theory with the microeconomic evidence on consumption (e.g., as in Parker et al., 2013; Fagereng, Holm and Natvik, 2021). In both cases, output is demand-determined and inflation is governed by the familiar New Keynesian Phillips curve (NKPC). Fiscal deficits can thus be inflationary if and only if they trigger a boom in equilibrium spending. Understanding when and how exactly such a boom obtains in RANK ($\omega = 1$) vs. in HANK ($\omega < 1$) will be a focal point of our analysis. Finally, government debt takes the form of nominal, one-period bonds, with long-term debt accommodated in an extension and in our quantitative evaluation.

A review of RANK-FTPL. We start with RANK ($\omega = 1$). In this model's conventional solution, as studied in standard textbook treatments and a vast applied literature (e.g., Galí, 2008; Christiano, Eichenbaum and Evans, 2005), the monetary authority alone regulates interest rates, output, and inflation, and fiscal deficits are non-inflationary. This conclusion is, however, reversed in that model's FTPL solution (e.g., Bianchi and Ilut, 2017; Cochrane, 2017, 2018): fiscal policy now becomes the key determinant of all these outcomes, with higher deficits triggering inflationary booms.

These starkly different predictions reflect opposite assumptions about which authority is "active" (or "dominant"). In particular, the FTPL equilibrium assumes a "passive" monetary authority that violates the Taylor principle, along with an "active" fiscal authority that *never* adjusts future taxes enough to pay for a higher deficit today. Something else must then substitute for the missing taxes. In the most familiar case, which we will refer to as the simple FTPL arithmetic, this "something else" is the nominal price level: the present value of future surpluses is kept fixed by assumption, so today's deficit must be financed entirely through nominal debt erosion. It follows that the price level must jump by an amount equal to the inverse of the debt-to-GDP ratio times the change in today's deficit.[1]

The FTPL's controversies and fragilities. The FTPL account of the deficits-inflation nexus has been the subject of long theoretical controversy. A large part of this controversy has centered on equilibrium selection and in particular on whether an active fiscal authority amounts to an off-equilibrium threat to "blow up the government budget".[2] Here, we zero in on a different issue, related to the fact that, in RANK, households are Ricardian—i.e., they have infinite horizons, access to complete markets, and rational expectations, just as in Barro (1974). In spite of this fact, Ricardian equivalence necessarily *fails* in the FTPL equilibrium: to generate an inflationary boom along the NKPC and make up for the missing tax adjustment, households *must* spend more in response to higher deficits.

What supports this higher spending in equilibrium? Since households are Ricardian, they under-

[1] Although this prediction is familiar from the classical, flexible-price version of the FTPL (Sims, 1994; Woodford, 1995; Cochrane, 2005), the key tasks for us will be to understand the precise mechanism that supports this prediction—and more generally any deficit-led inflation—in the prevailing, New Keynesian incarnation of this theory (as articulated, e.g., in Cochrane, 2017, 2018, 2023), and then to contrast with HANK.

[2] See Kocherlakota and Phelan (1999) and Buiter (2002) for this interpretation, Canzoneri, Cumby and Diba (2001) and Niepelt (2004) for additional criticisms, Bassetto (2002) and Cochrane (2005, 2011) for rebuttals, and Atkeson, Chari and Kehoe (2010), Angeletos and Lian (2023) and Neumeyer and Nicolini (2025) for further contributions.

stand that government bonds are not net wealth in equilibrium, and so the spending boom is not supported by classical wealth effects. Instead, it is entirely self-sustained: consumers spend more because they expect higher lifetime income, which in turn is true just because they spend more. This point explains why the FTPL equilibrium unravels with appropriate "noise" as in the global-games literature (Angeletos and Lian, 2023), as well as why it hinges on the *perpetual* absence of sufficient tax adjustment: if instead taxes adjust at any finite horizon, no matter how far in the future, Ricardian equivalence can be restored and deficits need not be inflationary.

HANK meets FTPL. The preceding discussion sets the stage for our main contribution—which is, not to criticize the FTPL's foundations, but instead to bolster its predictions. Once we move to HANK ($\omega < 1$), the economic mechanism at work (the "how") changes: deficits influence household spending and thus inflation via empirically verifiable non-Ricardian effects (i.e., that marginal propensities to consume, MPCs, are high), as opposed to subtle, hard-to-test assumptions about policies and beliefs at far-ahead horizons. This difference in mechanism directly explains our *robustness* result: the deficits-inflation mapping in HANK does not suffer from the fragilities discussed above.

The same logic also underlies our *equivalence* result. Because non-Ricardian households have finite horizons, fiscal adjustment that happens far in the future is *as if* it never happens. As a result, the price jump and the debt erosion predicted by HANK converge—monotonically and continuously—to their FTPL counterparts as fiscal adjustment gets delayed more and more. Intuitively, the more delayed fiscal adjustment, the larger the effective transfer to today's households, implying a bigger boom in output and thus also more inflation. If, as in the simple FTPL arithmetic, the present value of future surpluses is kept fixed—because the fiscal authority exogenously fixes future tax revenue, while the monetary authority stabilizes real interest rates—then the price jump keeps on smoothly increasing until it *fully* finances the initial fiscal deficit. If instead, and more plausibly, tax revenue endogenously responds to economic activity, then unfunded deficits partially finance themselves via their stimulating effect on tax revenue, as emphasized in Angeletos, Lian and Wolf (2024). This reduces the scope for debt erosion relative to the simple FTPL arithmetic, but our equivalence result continues to apply, now just compared to a variant of RANK-FTPL featuring the same tax revenue feedback.

Away from the limit of very delayed fiscal adjustment, HANK naturally predicts a smaller boom and thus also less inflation. Equivalence in that case re-emerges in terms of comparative statics: holding constant the size of the initial deficit and the speed of fiscal adjustment, a higher initial debt-to-GDP ratio translates to *less* inflation—and so does a longer debt maturity, in our extension with long-term debt. All in all, HANK produces the same set of predictions as the one that Barro and Bianchi (2025) associate with the FTPL and test against the post-Covid experience.[3]

[3] In our baseline analysis, which features one-period debt, our equivalence results concern the initial price jump. In the

The simplest and sharpest version of our equivalence results, as discussed in the prequel, is proven under the assumption that the monetary authority stabilizes the real interest rate. This assumption, however, is not strictly needed—even for exact equivalence it suffices that any interest rate hike is sufficiently muted or delayed. Intuitively, because households have short horizons, delayed monetary adjustment is again *as if* that adjustment never happens. HANK therefore not only addresses the fragilities noted earlier but also reconciles FTPL predictions with an "active" monetary authority—provided, of course, that the policy reaction to inflationary pressure is sufficiently delayed, e.g., as it appears to have been the case in the post-Covid episode.

The time profile of inflation. While our baseline HANK economy predicts the same limiting debt erosion and so the same date-0 price jump as the FTPL, the associated time path of inflation is different. Because non-Ricardian consumers spend any deficit-financed transfers relatively quickly, the inflationary boom in HANK is more front-loaded and hence more transitory than its FTPL counterpart. Short household horizons thus do two things: they not only deliver robustness, as discussed above, but also condense the inflation burst. An extension that accommodates realistic forms of household heterogeneity in wealth holdings only further reinforces these effects. Adding inflation inertia, e.g., through a hybrid NKPC, also does not change that picture: inflation is now back-loaded in both HANK and RANK-FTPL, but still *relatively* more transitory in the former.

Application to post-Covid. Although the various extensions that we discussed above—from endogenous tax revenue, to variable real interest rates, long-term debt, household heterogeneity, and inflation inertia—do not change the qualitative essence, they do affect the exact answer to our "how much" question. We thus close with a quantitative exercise, adapted to the post-Covid episode.

To this goal, our richer quantitative model features: intertemporal marginal propensities to consume (iMPCs) consistent with empirical evidence; plausible heterogeneity in fiscal transfer incidence and private wealth; an estimated hybrid NKPC; a realistic average maturity for government debt; and meaningful feedback from output to tax revenue. In this model, the cumulative inflation triggered by unfunded fiscal deficits is sizable, but still less than half that predicted by the simple FTPL arithmetic. Concretely, the household components of the CARES and ARP programs are predicted in our environment to produce a cumulative inflation of about 6 to 8%, compared to 18% implied by the simple FTPL arithmetic. The main reason behind this difference is the presence of a meaningful feedback from output to tax revenue, and so the resulting self-financing mechanism emphasized in our earlier work. Finally, the predicted inflation burst is more short-lived than its FTPL counterpart, for the reasons explained above and arguably consistent with the post-Covid experience.

extension with long-term debt, they instead concern an appropriate measure of *cumulative* inflation. Similarly to Barro and Bianchi (2025), this measure discounts future inflation at a rate that reflects the maturity structure of government debt, because this is what is relevant for debt erosion.

Related literature. The idea that fiscal deficits can stimulate aggregate demand is central to Keynesian thinking, culminating in recent work in HANK (e.g., Galí, López-Salido and Vallés, 2007; Kaplan, Moll and Violante, 2018; Auclert, Rognlie and Straub, 2024, 2025; Eichenbaum, Guerreiro and Obradovic, 2025). To the best of our knowledge, however, the link we draw between this view and the FTPL is new, as are our specific formal results on the deficits-inflation mapping. By establishing this link, we also offer a new angle on a literature that structurally estimates the inflationary contribution of fiscal deficits (e.g., Bianchi and Ilut, 2017; Smets and Wouters, 2024); in light of our results, the patterns that this literature often attributes to "fiscal dominance" could also be rationalized by a classical failure of Ricardian equivalence. Finally, although we share with Hagedorn (2016, 2024) the emphasis on non-Ricardian behavior, our equivalence and robustness results have no parallel in that work, and our HANK-meets-FTPL message ultimately stands in contrast to those papers' anti-FTPL theme.[4]

To deliver this contribution, we build on our earlier work in Angeletos, Lian and Wolf (2024): we harness, and then further extend, that paper's framework and insights to establish a new set of results about the relation between deficits and inflation in the presence of non-Ricardian consumers. At a high level, the novel contributions are thus the equivalence and robustness results reviewed above, as well as the quantitative evaluation. Complementary are also Aguiar, Amador and Arellano (2024) and Rachel and Ravn (2025), which use similar OLG frameworks to study Pareto-improving policies (in the first paper) and equilibrium determinacy and fiscal-monetary interactions (in the second).[5]

Finally, our paper adds to a topical literature on the post-Covid inflationary episode. While some research emphasized the connection to the FTPL (Barro and Bianchi, 2025; Bianchi, Faccini and Melosi, 2023; Anderson and Leeper, 2023; Kaplan, Nikolakoudis and Violante, 2023; Bigio, Caramp and Silva, 2024), much of the policy debate instead remained anchored in conventional Keynesian logic (e.g., see Blanchard, 2021; Summers, 2021; Bernanke and Blanchard, 2025). In this context, our contribution is threefold. First, we show that the gap between these two perspectives is smaller than previously thought. Second, we offer a quantitative evaluation of the inflationary effects of "stimulus checks," based on empirically-disciplined models. And third, we show that the empirical patterns identified in Barro and Bianchi (2025) are entirely consistent with the HANK paradigm.

Outline. Section 2 introduces the model. Section 3 briefly reviews RANK-FTPL. Section 4 then moves to HANK and develops our equivalence and robustness results. Section 5 discusses several extensions, setting the stage for the richer quantitative explorations in Section 6. Finally Section 7 concludes. Supplementary details and all proofs are relegated to the appendix.

[4] At the same time, we wish to clarify what our paper does *not* do: we abstract from the question of what escape clauses or nominal anchors guarantee global determinacy, as well as from flexible-price versions of the FTPL, where the price level can vary without a change in real spending and so the link between inflation and Ricardian equivalence is broken.

[5] Other works that study fiscal-monetary interactions in OLG settings, but do not share our lessons, include Leith and Wren-Lewis (2000), Bénassy (2007, 2008), Leith and von Thadden (2008), and Dupraz and Rogantini Picco (2025).

2 Environment

We consider a (log-linearized) perpetual-youth, overlapping-generations (OLG) version of the textbook New Keynesian model, where finite lives can also be interpreted as a proxy for liquidity frictions (Farhi and Werning, 2019; Angeletos, Lian and Wolf, 2024). The detailed micro-foundations and the steady state characterization, which follow from Angeletos, Lian and Wolf (2024), are delegated to Appendix A.1. Time is discrete and indexed by $t \in \{0, 1, \ldots\}$, uppercase variables denote levels, lowercase variables denote (log-)deviations from the steady state in which inflation is zero, real allocations are given by their flexible-price counterparts, and real government debt is fixed at some level $D^{ss} \geq 0$.[6]

2.1 Aggregate demand

The economy is populated by a unit continuum of households, where a household survives from one period to the next with probability $\omega \in (0, 1]$ and is replaced by a new one whenever it dies. Households have standard separable preferences over consumption and labor; as in Blanchard (1985), they can save and borrow through an actuarially fair, risk-free, nominal annuity backed by government bonds. To facilitate aggregation, we further assume that all households face the same wage, supply the same (union-intermediated) labor, receive the same dividend payments, and pay the same taxes. Finally, we abstract from the steady-state effects of finite lives and fiscal policy by assuming that all cohorts have the exact same wealth in steady state.[7]

Deriving the (log-linearized) consumption function of each household, and then aggregating across households, we obtain the following aggregate consumption function:

$$c_t = (1 - \beta\omega)\left(a_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\omega)^k (y_{t+k} - t_{t+k})\right]\right) - \beta\left(\sigma\omega - (1 - \beta\omega)\frac{A^{ss}}{Y^{ss}}\right)\mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\omega)^k r_{t+k}\right], \quad (1)$$

where c_t is aggregate consumption, a_t is real private wealth, y_t is real private income (labor income plus dividends), t_t is real tax payments, r_t is the expected real rate of interest, σ is the elasticity of intertemporal substitution, A^{ss}/Y^{ss} is the steady state wealth-to-income ratio, β is the discount factor (also the reciprocal of R^{ss}, the steady-state gross real interest rate), and $\mathbb{E}_t$ is the rational-expectations operator. Equation (1) generalizes the familiar infinite-horizon Permanent Income Hypothesis (PIH): the first term on the right-hand side captures financial wealth and permanent income, while the sec-

[6]We work exclusively with the log-linearized model, so the equilibria characterized below are local approximations of the nonlinear equilibria around the aforementioned steady state. The local uniqueness of this steady state is established in Appendix A.2. To accommodate the case of zero debt, all fiscal and household wealth variables are measured in absolute deviations from this steady state, scaled by steady-state output; all other variables are measured in log-deviations.

[7]This is achieved by assuming that older households make appropriate, time-invariant contributions to a social fund, with the proceeds of the fund distributed to the newborn households. This assumption makes sure that the flexible-price steady state is invariant to both ω and the real level of government debt—which in turn means that the point around which we log-linearize our economy remains the same as we vary either ω or the fiscal and monetary policies.

ond term captures the substitution and wealth effects of real interest rates.

Connection to HANK. As we move from $\omega = 1$ to $\omega < 1$, our model implies the following two properties of consumption behavior: (i) households discount future income and future taxes at a rate higher than the steady-state interest rate; (ii) relative to the permanent-income benchmark, households exhibit a higher marginal propensity to consume (MPC) out of "cash-in-hand" (i.e., current income plus current wealth). As will become clear, all of our conclusions regarding HANK derive from these two properties. While these properties are modeled here as a result of finite lives, they can also be framed as the outcome of liquidity constraints (see Farhi and Werning, 2019), and so they similarly emerge in a broad class of HANK models (Kaplan, Moll and Violante, 2018; Auclert, Rognlie and Straub, 2024; Wolf, 2025). An obvious limitation is that our model abstracts from heterogeneity in wealth, marginal propensities to consume, and exposure to fiscal transfers. However, as shown in Sections 5.2 and 6, these abstractions are orthogonal to our main results on robustness and equivalence.

2.2 Aggregate supply

The supply block of the economy follows the textbook New Keynesian model and reduces to the standard New Keynesian Phillips Curve (NKPC):[8]

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{2}$$

for some $\kappa > 0$ that captures the degree of price flexibility. Iterating this equation forward pins down the path of inflation as a function of the path of output:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t [y_{t+k}]. \tag{3}$$

This highlights the following important property, which also directly extends to alternative, more empirically relevant versions of the NKPC (see Section 5.3): fiscal deficits can be inflationary *only if* they also trigger a real boom. Put differently, a failure of Ricardian equivalence—i.e., equilibrium spending, employment, and output not being invariant to the time path of taxes and transfers—is *necessary* for deficits to drive inflation, irrespective of whether $\omega = 1$ (RANK) or $\omega < 1$ (HANK).

We stress that this link between inflation and Ricardian equivalence is absent in flexible-price versions of the FTPL (e.g., see Sims, 1994; Bassetto, 2002; Cochrane, 2005): in those models, the nominal price level can be a "free variable," disconnected from real economic activity. By contrast, this link

[8] The microfoundations behind (2) are standard and detailed in Appendix A.1. There is a unit-mass continuum of monopolistically competitive retailers, who set prices subject to the standard Calvo friction, hire labor on a spot market, produce according to a technology that is linear in labor, and then pay out all their profits as dividends back to the households. Together with our assumptions about union-intermediated labor supply and time-invariant tax distortions, this guarantees that equation (2) remains unchanged as we move from RANK ($\omega = 1$) to HANK ($\omega < 1$).

is at the heart of the modern, sticky-price version of the FTPL—and a focal point of our subsequent analysis. Indeed, one can already here readily see the tension that we will emphasize in Section 3.2: by employing RANK, the modern FTPL assumes that households are Ricardian, yet it ultimately requires that Ricardian equivalence fails.

2.3 Fiscal policy

The government issues non-contingent, short-term, nominal debt; the extension to long-term debt is provided in Section 5.1. Let B_t denote the level of nominal public debt outstanding at the beginning of period t, P_t the nominal price level, and $D_t \equiv B_t/P_t$ the real value of public debt. In nominal terms, the government's flow budget constraint is $B_{t+1} = I_t(B_t - P_t T_t)$, where T_t is real tax revenue (and also, under our assumptions, the real primary surplus) in date t, and I_t is the gross nominal rate between dates t and $t+1$. Rewriting this flow constraint in real, log-linearized terms, it follows that the real value of government debt at any time $t+1$ satisfies

$$d_{t+1} = \frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t - \frac{D^{ss}}{Y^{ss}}(\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}]), \tag{4}$$

where $r_t = i_t - \mathbb{E}_t[\pi_{t+1}]$ is the expected real rate and D^{ss}/Y^{ss} is the steady-state debt-to-GDP ratio, which, by asset-market clearing, also equals the steady state wealth-to-income ratio A^{ss}/Y^{ss}. As we assume that the economy starts in steady state (and hence $x_{-1} = 0$ for any variable x), we also have the following initial condition:

$$d_0 = -\frac{D^{ss}}{Y^{ss}} \pi_0. \tag{5}$$

Equation (4), together with the usual no-Ponzi condition, implies that, for any date $t \geq 0$, the real value of government debt must equal the discounted present value of surpluses:

$$d_t = \mathbb{E}_t\left[\sum_{k=0}^{\infty} \beta^k \left(t_{t+k} - \beta \frac{D^{ss}}{Y^{ss}} r_{t+k}\right)\right]. \tag{6}$$

This equation is known as the government's intertemporal budget constraint (Barro, 1974) or as the valuation equation for government debt (Cochrane, 2005).[9] Had government debt been in *real* one-period bonds (as in Barro, 1974), its real value d_t would have been predetermined from the previous period. Here, instead, debt is in *nominal* one-period bonds, so its real value can jump in proportion to a jump in the price level: from equations (4) and (5) we have that, for any date $t \geq 0$,

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]). \tag{7}$$

[9]The literature has debated whether (6) is a "constraint" that must hold both on and off equilibrium, or a "valuation equation" that must hold only in equilibrium. However, *both* sides of the debate agree that (6) is an equilibrium restriction, and our subsequent analysis will only leverage this fact.

This equation captures the debt-erosion channel at the heart of the FTPL: the innovation in the real value of government debt equals the negative of the concurrent surprise in the price level times the steady-state debt-to-GDP ratio. Together, equations (6) and (7) identify equilibrium restrictions that must hold under *any* fiscal policy—whether "passive" or "active"—and show precisely how inflation surprises can substitute for future tax hikes in financing current deficits.

Tax rule. We close the fiscal block of the model by assuming that the fiscal authority sets tax revenue according to the following rule, for some $\tau_d, \tau_y \in [0,1)$:

$$t_t = \underbrace{-\varepsilon_t}_{\text{deficit shock}} + \underbrace{\tau_d (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} + \underbrace{\tau_y y_t}_{\text{tax base}} . \tag{8}$$

This rule mirrors those commonly used in applied work. Its first component, ε_t, is the exogenous fiscal deficit shock. For concreteness, we interpret ε_t as an unexpected, one-off, lump-sum transfer (e.g., a surprise issuance of stimulus checks). We assume that this shock is independently distributed over time and, for technical reasons, has bounded support and ceases to occur after some finite date T.[10] The second component captures how much taxes adjust over time in response to accumulated debt, conditional on aggregate income. For simplicity, and in line with the FTPL literature, this adjustment is assumed to be non-distortionary, i.e., it takes the form of lump-sum tax hikes. Similarly to Leeper (1991), the coefficient τ_d parameterizes the speed of fiscal adjustment: taxes adjust with greater delay as τ_d falls, and *never* adjust if $\tau_d = 0$. An important policy question, and one central to the remainder of our analysis, is which values of τ_d are consistent with the requirement that "government debt does not explode," in the sense that the government satisfies its no-Ponzi condition. Finally, the third term indicates how much tax revenue covaries with aggregate income (the "tax-base channel"), arising from a time-invariant, proportional tax on total household income at rate τ_y—the automatic feedback from economic activity to tax revenue stressed in Angeletos, Lian and Wolf (2024).[11]

2.4 Monetary policy

We abstract from the zero lower bound and let the monetary authority set i_t, the nominal interest rate between dates t and $t+1$, according to the following Taylor rule:

$$i_t = \mathbb{E}_t[\pi_{t+1}] + \phi y_t, \tag{9}$$

[10]The sole purpose of the latter assumption is to ensure that RANK's FTPL equilibrium (characterized in Proposition 1) remains bounded even in the case with fixed real rates ($\phi = 0$), which otherwise induces a random walk.

[11]By assuming that the proportional tax τ_y is time-invariant and that tax hikes are lump-sum, we abstract from time-varying distortions that would otherwise appear as cost-push shocks in the NKPC, isolating the failure of Ricardian equivalence on the demand side of the economy. That said, since our HANK-FTPL equivalence result concerns the limit where tax hikes vanish ($\tau_d = 0$), the assumption of non-distortionary tax hikes is without any loss of generality.

for some $\phi \in \mathbb{R}$. Re-writing this in terms of the (expected) *real* rate, we have

$$r_t = \phi y_t. \tag{10}$$

Monetary policy is thus parameterized by whether it implements lower, constant, or higher real rates in response to any boom in output and thus inflation. We allow both $\phi \leq 0$ and $\phi > 0$, to accommodate "passive" and "active" monetary policies, though we restrict $\phi > \underline{\phi} \equiv -\frac{1}{\sigma}$ for technical reasons.[12]

2.5 Equilibrium definition

Definition 1. An equilibrium is a stochastic path $\{y_t, \pi_t, c_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ for output, inflation, consumption, the real values of household wealth and government debt, tax revenue, and real interest rates that satisfies all of the following: the aggregate consumption function (1) and the NKPC (2); market clearing $c_t = y_t$ and $a_t = d_t$; the government's intertemporal budget constraint and spanning restriction (6) and (7); the fiscal and monetary policy rules (8) and (10); and boundedness of y_t.[13]

Note that, unlike Leeper (1991), we do not *a priori* require that d_t be bounded; instead, we impose only the government's intertemporal budget constraint (6), which in particular embeds the no-Ponzi condition. This eliminates a small discrepancy between the notions of "passive" and "active" fiscal policy found in Leeper (1991) and those found in much of the FTPL literature (and here). In particular, we define a passive fiscal policy as one that guarantees that the no-Ponzi condition is satisfied regardless of the paths of output, inflation and interest rates, and an active fiscal policy as one for which this happens only for a particular combination of such paths. Under the policy rule (8), these definitions translate to $\tau_d > 0$ for passive fiscal policy and $\tau_d = 0$ for active fiscal policy. Our definitions thus agree with the textbook treatment of the FTPL in Cochrane (2023), to which we will relate our analysis. Further details are made clear in the next section.

3 A review of RANK-FTPL

In this section, we study RANK ($\omega = 1$). Section 3.1 reviews RANK's conventional and FTPL equilibria and contrasts their predictions regarding the deficits-inflation nexus. Section 3.2 highlights that the

[12] $\phi > \underline{\phi} \equiv -\frac{1}{\sigma}$ rules out oscillatory impulse responses—a familiar and, for our purposes, immaterial nuisance. We also depart slightly from the common practice of specifying monetary policy as $i_t = \psi \pi_t$. As a result, the Taylor principle translates to $\phi > 0$ rather than $\psi > 1$.

[13] By boundedness for a variable x, we mean that there exists $M > 0$ such that $|x_t| < M$ for all dates t and all realizations of uncertainty. As usual, equilibria in the log-linearized economy that violate the assumed boundedness of y_t may translate in the non-linear economy to proper equilibria featuring either speculative hyperinflation or a self-fulfilling trap at the zero lower bound. The literature has discussed various "escape clauses" that may help rule out such unbounded equilibria (Atkeson, Chari and Kehoe, 2010; Benhabib, Schmitt-Grohé and Uribe, 2002; Obstfeld and Rogoff, 1983, 2021).

FTPL equilibrium requires a failure of Ricardian equivalence, even though households are Ricardian in the classical sense of Barro (1974), and illustrates the fragility of this mechanism.

3.1 RANK's conventional and FTPL equilibria

When $\omega = 1$ (i.e., RANK), our economy reduces to two familiar systems of equations. The first system collects the well-known three equations of the textbook NK model (i.e., Galí, 2008)—the Euler equation, the NKPC, and the monetary-policy rule:[14]

$$y_t = -\sigma r_t + \mathbb{E}_t\left[y_{t+1}\right], \qquad \pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right], \qquad r_t = \phi y_t. \tag{11}$$

The second system is the "fiscal block": equations (6) and (7) together with the fiscal rule (8).

Standard practice (e.g., Woodford, 2003; Galí, 2008) drops the fiscal block by assuming, explicitly or implicitly, that fiscal policy is "passive" (i.e., that $\tau_d > 0$). This ensures that the government's intertemporal budget constraint is satisfied *regardless* of the paths of output, inflation, and interest rates, and so a path for these variables is part of an equilibrium if and only if it solves (11). RANK's conventional solution is then completed by letting monetary policy be "active" (i.e, by imposing the Taylor principle, here $\phi > 0$). This ensures that $y_t = \pi_t = r_t = 0$ is the unique solution to (11) in which y_t is bounded, and hence it is also the unique equilibrium per Definition 1. As a result, RANK's conventional solution rules out any effect of fiscal deficits on output and inflation.

RANK's FTPL solution instead assumes the opposite policy mix: monetary policy is now "passive" (i.e., $\phi \leq 0$), allowing (11) to admit a continuum of bounded solutions, and fiscal policy is "active" (i.e., $\tau_d = 0$), ruling out fiscal adjustment at *any* horizon. Under these assumptions, $\pi_t = y_t = r_t = 0$ continues to solve (11), but now is no longer an equilibrium (again per Definition 1). Intuitively, when $\tau_d = 0$, the government's intertemporal budget constraint cannot be satisfied unless appropriate adjustments in equilibrium output, inflation, and real rates substitute for the missing tax hikes. Accordingly, equilibrium is now given by a *different* solution to (11)—namely the unique one in which output, inflation, and real rates move to satisfy the government's intertemporal budget constraint under $\tau_d = 0$.

We summarize these familiar lessons in the next result.[15]

Proposition 1. *Suppose that* $\omega = 1$*. Then:*

1. *If* $\phi > 0$ *and* $\tau_d > 0$*, there is a unique equilibrium, referred to as* RANK's conventional equilibrium. *In this equilibrium, fiscal deficits have no effect on output and inflation:* $\pi_t = y_t = 0$ *always.*

[14] The Euler equation follows from a recursive version of aggregate demand (1) when $\omega = 1$, together with market clearing conditions $c_t = y_t$ and $a_t = d_t$, and the government's intertemporal budget constraint (6).

[15] Proposition 1 echoes Leeper (1991), except for the difference mentioned earlier—that we accommodate unbounded government debt. Had we required that d_t be bounded, Proposition 1 would have applied with passive fiscal policy redefined as $\tau_d \geq 1-\beta$ and active fiscal policy redefined as $\tau_d \in [0, 1-\beta)$, exactly as in Leeper (1991).

2. *If instead $\phi \leq 0$ and $\tau_d = 0$, there is a (different) unique equilibrium, henceforth referred to as* the FTPL equilibrium. *In this equilibrium, fiscal deficits trigger output booms and inflation: y_t and π_t increase with ε_t. More specifically, in response to a fiscal deficit shock, the price level jumps by the following amount:*

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi^{FTPL}_{\varepsilon,0} \cdot \varepsilon_t \qquad \textit{with} \qquad \pi^{FTPL}_{\varepsilon,0} \equiv \frac{\kappa}{\tau_y + (\kappa - \beta\phi)\frac{D^{ss}}{Y^{ss}}} > 0 \tag{12}$$

Equation (12) gives the FTPL's answer to the "how much" question—i.e., the size of the inflation surprise triggered by a fiscal deficit shock. We first further elaborate on this, before then turning to the "how" (i.e., the mechanism) behind the "how much."

The FTPL arithmetic. The defining feature of the FTPL equilibrium is that output, inflation, and interest rates jointly adjust by whatever amount is necessary to finance any given deficit shock. This is implicit in the size of the inflation response in (12), and is most transparently seen when $\phi = \tau_y = 0$, i.e., with constant real rates and no feedback from aggregate income to tax revenue. We refer to this case as the "simple FTPL arithmetic" because it eliminates every margin of adjustment other than debt erosion. This case thus imposes

$$\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = \varepsilon_t, \tag{13}$$

which is nested in equation (12) when $\phi = \tau_y = 0$. In words, the real value of public debt must drop by the same amount as the increase in the fiscal deficit. Equivalently, the price jump per unit of deficit must equal the reciprocal of the debt-to-GDP ratio. Intuitively, the higher debt-to-GDP, the smaller the price jump necessary to erode the real value of debt by a given amount. To then generate this required price jump via the NKPC, real output must itself jump by an appropriate amount.

This logic readily extends to $\tau_y > 0$ and $\phi < 0$. In this more general case, a fiscal deficit shock may be financed not only by debt erosion, but also by an expansion in the tax base (when $\tau_y > 0$) and a drop in interest rate costs (when $\phi < 0$). This reduces the requisite output and price jumps—accordingly, equation (13) generalizes to equation (12), with the inflation response decreasing in τ_y and increasing in ϕ—but does not otherwise change the economic essence. And importantly, the exact same logic continues to apply even if prices are rigid ($\kappa = 0$, or equivalently government debt were real). In that case the debt erosion margin is of course absent, but the FTPL equilibrium identified in Proposition 1 remains: a deficit is now financed by a boom in real spending and output, which translates to higher tax revenue (via $\tau_y > 0$) and lower interest rate costs (via $\phi < 0$). In all these cases, the FTPL equilibrium exists because, and only because, fiscal deficits trigger a boom in real spending and output, thus activating the alternative financing margins and substituting for the missing fiscal adjustment.[16]

[16]These arguments extend to the case of partially funded fiscal shocks (e.g., Cochrane, 2023; Smets and Wouters, 2024).

3.2 FTPL's "how" behind the "how much", and its fragility

We now dig deeper into *how* deficits drive inflation in the FTPL equilibrium characterized above. We begin by pointing out a tension: in this equilibrium, Ricardian equivalence fails even though households are Ricardian. We next show how this tension manifests in fragility, connect to the existing debate about the FTPL, and set the stage for our main contribution—which will be to show that HANK's different "how" avoids the fragility and ultimately offers new credence to the FTPL's "how much."

Breaking Ricardian equivalence with Ricardian households. In RANK, households are Ricardian, just as in Barro (1974): they have infinite horizons, access to complete markets, and rational expectations, implying that they understand that government bonds are not net wealth in *any* equilibrium. And yet, in the FTPL equilibrium, Ricardian equivalence necessarily fails: households spend more in response to higher fiscal deficits, leading to inflation.

To understand what supports such higher spending in equilibrium, it is again useful—and without loss of generality for our purposes—to temporarily let $\phi = 0$ (constant real rates) along with $\omega = 1$ (Ricardian households). In this case, the aggregate consumption function (1) reduces to

$$c_t = \underbrace{(1-\beta)\, z_t}_{\text{wealth effect of fiscal policy}} + \underbrace{(1-\beta)\,\mathbb{E}_t\left[\sum_{k=0}^{\infty}\beta^k y_{t+k}\right]}_{\text{permanent income}}, \tag{14}$$

where $z_t \equiv a_t - \sum_{k=0}^{\infty}\beta^k \mathbb{E}_t[t_{t+k}]$ measures private wealth net of tax obligations. Next, note that, regardless of the policy regime, equation (6) together with asset market clearing ($a_t = d_t$) implies that $z_t = 0$; in words, government bonds are not net wealth in equilibrium. Using this fact in equation (14), we conclude that the following is also true in equilibrium: consumption, c_t, can vary with the deficit shock, ε_t, if and only if permanent income, $\mathbb{E}_t\left[\sum_{k=0}^{\infty}\beta^k y_{t+k}\right]$, varies with ε_t by the exact same amount. But since income coincides with spending ($y_{t+k} = c_{t+k}$), we conclude any fiscally-led boom is entirely self-sustained: Ricardian consumers spend more along the FTPL equilibrium only because they expect higher income, which in turn is the case only because other consumers spend more.[17]

Fragility. The tension identified above results in fragility: the FTPL equilibrium is non-robust to small and hard-to-test changes in various auxiliary assumptions. Here, we illustrate a fragility with respect to fiscal adjustment at far-ahead horizons.[18] Consider the following modification to the policy

In this context, following any deficit shock ε_t, the fiscal authority adjusts the discounted present value of future surpluses (inclusive of interest payments) by a fraction λ of ε_t, for some $\lambda \in [0,1)$. This again induces an equilibrium in which output and prices jump in response to ε_t, now by the amount necessary for the resulting debt erosion to cover $(1-\lambda)\varepsilon_t$, the "unfunded" portion of the deficit. In the same spirit, models such as Bianchi and Ilut (2017) and Bianchi, Faccini and Melosi (2023) can be understood as involving a time-varying, and also possibly shock-specific, λ.

[17] This interpretation is further corroborated in an upcoming companion paper (Angeletos et al., 2025).

[18] The fragility documented here therefore is related to the analysis of Canzoneri, Cumby and Diba (2001), which also perturbs assumptions about policy, but does not share our paper's other insights about either RANK or HANK. Angeletos

mix that supports the FTPL equilibrium. Before some date H, the monetary and fiscal authorities follow our baseline policy rules (8) and (10) (i.e., they can follow the FTPL rules $\phi \leq 0$ and $\tau_d = 0$ with an active fiscal authority and a passive monetary authority). After that date, there is a switch: the fiscal authority turns passive, adjusting taxes to ensure that government debt returns to its original, pre-shock value ($d_t = 0$ for all $t > H$), and the monetary authority becomes active, leaning against any inflationary boom after H. No matter how far in the future H is, this modification guarantees that fiscal deficits have no effect on output and inflation, not just after H, but also before H.

Proposition 2. *Suppose $\omega = 1$ and let $H \geq 1$ be any finite date. Next, suppose that the fiscal and monetary authorities follow the rules* (8) *and* (10) *for $t < H$ but switch to, respectively,*

$$t_t = d_t + \beta \frac{D^{ss}}{Y^{ss}} r_t \quad \text{and} \quad r_t = \phi' y_t \quad \text{for } t \geq H, \quad \text{with } \phi' > 0. \tag{15}$$

Then, there exists a unique equilibrium, and it is such that $y_t = \pi_t = 0$ for all t and all realizations of uncertainty.

The proof is straightforward, yet revealing. The policy switch at date H guarantees that $y_t = 0$ for all $t \geq H$. Starting from $y_H = 0$ and iterating the Euler equation (11) backwards yields $y_t = 0$ also for $t < H$. By the NKPC, it then follows that $\pi_t = 0$ as well, for all t. Crucially, this argument is valid no matter how large H is, unless it is *literally* infinity. Put differently, if today's fiscal deficits cease to influence output and inflation at any finite date, they can never matter to start with.

At the heart of this fragility lies the basic tension reviewed above: because households are Ricardian, fiscal policy has no wealth effect in equilibrium ($z_t = 0$) and so a fiscally-led boom can be supported today only if households expect aggregate spending and thus their income to be elevated—and in fact elevated forever after. Otherwise, backward induction from a finite date unravels these expectations, recovers Ricardian equivalence, and rules out the FTPL equilibrium.

Takeaways. As mentioned in the Introduction, a large part of the existing debate about the FTPL has centered on the question of whether the FTPL relies on an off-equilibrium threat to "blow up the government's budget" to select a particular equilibrium (Kocherlakota and Phelan, 1999; Buiter, 2002; Bassetto, 2002; Atkeson, Chari and Kehoe, 2010). Although the specific points we made above are distinct, they ultimately point in the same direction: the prevailing formulation of the FTPL relies on controversial and hard-to-test assumptions about policy and beliefs at far-ahead horizons. One could of course counter-argue that the Taylor principle and the policy mix in Proposition 2 are equally hard to test. But our point here is not to add to the long-standing debate about the FTPL, but rather

and Lian (2023) document a different, but related fragility with respect to the information structure: adding small but appropriate noise, as in the global-games literature, makes sure that output and inflation are invariant to fiscal deficits regardless of whether the Taylor principle holds or not.

to provide a constructive way out of it: in the next section, we show that HANK's different "how" naturally avoids these fragilities and controversies, while at the same time reproducing FTPL's "how much" when fiscal adjustment is slow.

4 HANK meets FTPL

We now consider the HANK version of our model ($\omega < 1$). We begin in Section 4.1 by first delineating the HANK mechanism from its FTPL counterpart, and then characterizing HANK's equilibrium. We next show that, *despite* the difference in mechanism, HANK reproduces FTPL's predictions. Section 4.2 establishes this equivalence result for the special case of fixed real rates, while Section 4.3 extends to more general monetary policies. Sections 4.4 and 4.5 turn to differences between the two theories. In particular we establish that, *because* of the difference in mechanism, HANK avoids FTPL's fragilities.

4.1 Classical, non-Ricardian effects in HANK

As we move from $\omega = 1$ to $\omega < 1$, the only—but crucial—change in the economics is that fiscal deficits now *do* have wealth effects in equilibrium, simply because households are non-Ricardian (in the classical sense of Barro, 1974). To see this clearly, and to understand how these wealth effects depend on the speed of fiscal adjustment, we will again temporarily focus on the special case of constant real rates ($\phi = 0$). The aggregate consumption function (1) then simplifies to

$$c_t = (1-\beta\omega)\, z_t + (1-\beta\omega)\,\mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\omega)^k\, y_{t+k}\right], \tag{16}$$

where z_t is now redefined as $z_t \equiv a_t - \sum_{k=0}^{\infty} (\beta\omega)^k\, \mathbb{E}_t\,[t_{t+k}]$ and still measures the extent to which government bonds are net wealth in equilibrium. Next, we use asset market clearing ($a_t = d_t$) and the government's intertemporal budget constraint (6) to arrive at the following expression:

$$z_t = \mathbb{E}_t\left[\sum_{k=0}^{\infty} \beta^k\, t_{t+k} - \sum_{k=0}^{\infty} (\beta\omega)^k\, t_{t+k}\right].$$

When $\omega = 1$, z_t is identically zero, recalling our earlier discussion of how fiscal policy has no wealth effects in RANK. When instead $\omega < 1$, z_t increases with the deficit shock ε_t, because non-Ricardian consumers discount their future tax obligations t_{t+k} at a higher rate than the interest rate faced by the government. In the literal interpretation of our model, this extra discounting is due to finite lives (shifting the tax burden to future generations); more generally, it can result from liquidity constraints, or even from consumers' bounded rationality. Importantly, unlike its FTPL counterpart, this mechanism is now grounded in a large empirical literature documenting the causal effects of fiscal transfers at the micro level, consistent with non-Ricardian consumer behavior (e.g., Parker et al., 2013).

Equilibrium characterization. We now combine the consumer spending relation (16) with the other model relations. Because of the non-Ricardian demand channel present in (16), a fiscal deficit shock $\varepsilon_t > 0$ that is not accompanied by immediate tax adjustment boosts consumer demand. In particular, the more delayed the fiscal adjustment, the larger the short-run stimulative effects of any initial lump-sum transfer. In general equilibrium, this demand increase stimulates income, thus further increasing demand—the standard amplification of the "Intertemporal Keynesian Cross" (Auclert, Rognlie and Straub, 2024), visible in the second part of the spending relation (16). Furthermore, insofar as $\tau_y > 0$ and $\kappa > 0$, the Keynesian boom in output and prices feeds back into higher tax revenue and greater debt erosion, stabilizing government debt and lessening the need for fiscal adjustment, as emphasized previously in Angeletos, Lian and Wolf (2024). The next proposition completes the picture by characterizing how this two-way feedback plays out in general equilibrium.[19]

Proposition 3. *Suppose that $\omega < 1$, $\tau_y > 0$, and $\phi = 0$. Then:*

1. *There exists a unique equilibrium, henceforth referred to as* the HANK equilibrium*, and it is such that*
$$y_t = \chi\,(d_t + \varepsilon_t) \qquad \textit{and} \qquad \mathbb{E}_t\,[d_{t+1}] = \rho_d\,(d_t + \varepsilon_t)\,, \tag{17}$$
for some scalars $\chi > 0$ and $\rho_d \in (0,1)$ that are continuous functions of $(\beta, \omega, \tau_y, \tau_d)$.

2. *In this equilibrium, the inflation surprise in response to a deficit shock—or the price jump causing debt erosion—is given by*
$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi^{HANK}_{\varepsilon,0} \cdot \varepsilon_t \qquad \textit{with} \qquad \pi^{HANK}_{\varepsilon,0} \equiv \frac{\kappa\chi}{1 - \beta\rho_d + \kappa\chi\frac{D^{ss}}{Y^{ss}}}. \tag{18}$$

The first part of the proposition, which is borrowed from Angeletos, Lian and Wolf (2024), verifies that the aforementioned two-way feedback induces a unique equilibrium, and then characterizes the resulting dynamics of output and public debt. Intuitively, because households here are non-Ricardian ($\omega < 1$), deficit-financed transfers naturally increase aggregate spending, thereby boosting output and inflation ($\chi > 0$), which in turn helps stabilize government debt ($\rho_d < 1$) through both tax base expansion and debt erosion. The second part of the proposition then spells out the prediction of interest: the inflation surprise, or price jump, triggered by a fiscal deficit shock. In the next section we will compare this prediction to the RANK-FTPL counterpart of our model.

Proposition 3 already reveals a key difference in terms of how equilibria in RANK and HANK vary with assumptions on policy. Recall that, in RANK, the equilibrium set was (right-)discontinuous at

[19] The proposition—as well as all our subsequent results—assumes that $\tau_y > 0$. If instead we let $\tau_y = 0$, then we may lose local determinacy. However, our main lessons on equivalence and robustness (as discussed in the next subsections) extend: inflation in the particular equilibrium that is selected by the refinement of Proposition 7—i.e., that the economy returns to steady state in finite time—still converges to its FTPL counterpart.

$\tau_d = 0$: Ricardian equivalence could be preserved for any $\tau_d \in (0,1)$, but it had to fail at $\tau_d = 0$ in order to reconcile the absence of fiscal adjustment with equilibrium existence. In HANK, instead, Ricardian equivalence fails naturally, regardless of the degree of fiscal adjustment, and the equilibrium is continuous for all $\tau_d \in [0,1)$. In other words, there is no longer a material difference between "adjusting taxes very slowly" ($\tau_d > 0$ but small) and "never adjusting taxes" ($\tau_d = 0$). We will later show that a similar continuity applies with respect to ϕ: there is no material difference between a monetary policy that stabilizes real rates ($\phi = 0$), one that leans against a fiscally-led boom by hiking real rates ($\phi > 0$), and one that amplifies the boom or eases the government's cost of borrowing by letting real rates fall ($\phi < 0$). These continuity properties, and the related robustness of HANK that we will document in Section 4.5, are all manifestations of the different mechanism at work.

4.2 HANK meets FTPL

Our headline result is that, despite the difference in mechanism, the HANK equilibrium replicates the FTPL's core empirical predictions on the deficits-inflation nexus.

Theorem 1. *Let $\omega < 1$, $\tau_y > 0$, and $\phi = 0$, and consider $\pi_{\varepsilon,0}^{HANK}$, the initial price jump in response to a deficit shock in the HANK equilibrium.*

1. *$\pi_{\varepsilon,0}^{HANK}$ is a decreasing and continuous function of $\tau_d \in [0,1)$. That is, slower fiscal adjustment implies a larger price jump (and hence more fiscally induced debt erosion).*

2. *As fiscal adjustment gets slower and slower ($\tau_d \to 0^+$), the price jump converges from below to its FTPL counterpart, and the limit is attained at $\tau_d = 0$:*

$$\lim_{\tau_d \to 0^+} \pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{HANK}\Big|_{\tau_d = 0} = \frac{\kappa}{\tau_y + \frac{D^{ss}}{Y^{ss}}\kappa} = \pi_{\varepsilon,0}^{FTPL}. \tag{19}$$

3. *For any $\tau_d \in [0,1)$, $\pi_{\varepsilon,0}^{HANK}$ decreases with $\frac{D^{ss}}{Y^{ss}}$, decreases with τ_y, and increases with κ. That is, the price jump in HANK inherits the comparative statics of its FTPL counterpart in (12), and extends them from the $\tau_d = 0$ extreme to the general case with arbitrary fiscal adjustment.*

The first part of the theorem highlights that, just like the underlying real boom, the price jump in HANK in response to a deficit shock grows larger as fiscal adjustment becomes slower. Intuitively, non-Ricardian households discount future tax hikes more aggressively—and thus they spend more—the further into the future the eventual tax hikes occur. The second part gives our main HANK-FTPL equivalence result: despite the difference in mechanism, HANK predicts *exactly* the same price jump as FTPL as fiscal adjustment gets slower and slower (i.e., as $\tau_d \to 0^+$). The third part then adds a complementary lesson: away from that limit, HANK naturally predicts a smaller inflation surprise than its FTPL counterpart, yet it preserves the latter's comparative statics with respect to the debt-to-GDP ratio, the strength of the tax-base margin, and the slope of the NKPC. We conclude that the empirical predictions associated with the FTPL (as for example emphasized in Barro and Bianchi, 2025) are also entirely consistent with traditional Keynesian logic, provided one accommodates realistic non-Ricardian consumer spending effects in the way we have done here.

Our HANK-FTPL equivalence result holds independently of the strength of the tax-base channel (τ_y), and so it in particular also applies to the famous "FTPL arithmetic" of prices jumping to entirely finance the deficit. As is evident from equation (19), the common price jump in HANK and FTPL decreases with the relative strength of the tax base channel (i.e., it decreases with τ_y and it increases with κ). However, if this channel is absent ($\tau_y \to 0$), or if prices are very flexible ($\kappa \to \infty$), then the jump in prices *entirely* finances the deficit shock, just as in the famous simple FTPL arithmetic.

Corollary 1. *Let $\omega < 1$, $\phi = 0$ and $\tau_d = 0$. If $\tau_y \to 0$ or $\kappa \to \infty$, then the price jump in response to a deficit shock in the HANK equilibrium converges to that predicted by the simple FTPL arithmetic:*

$$\pi_{\varepsilon,0}^{HANK} \to \left(\frac{D^{ss}}{Y^{ss}}\right)^{-1} \tag{20}$$

Figure 1 provides a visual illustration of Theorem 1 as well as Corollary 1.[20] For any $\tau_y > 0$, the price jump triggered by a fiscal deficit shock decreases with the speed of fiscal adjustment, converging to the FTPL limit as $\tau_d \to 0$. When $\tau_y \to 0$, this limit corresponds to the simple FTPL arithmetic: prices jump by exactly enough to fully finance the deficit. Otherwise, the price jump is strictly smaller, by an amount that increases with τ_y. Intuitively, this is so because the automatic increase in tax revenue partially offsets the initial fiscal stimulus, thereby also arresting the fiscally led boom in output and prices, while at the same time helping stabilize public debt. The remainder of this section digs deeper into the economic intuition for the deficit-inflation nexus in HANK, and its connection to FTPL.

Understanding equivalence for $\tau_d \to 0^+$. Why does our HANK economy, as fiscal adjustment is delayed further and further, predict the same inflation response and thus debt erosion as RANK-FTPL? We will provide two complementary perspectives of this result.

The first perspective echoes standard FTPL analysis, and begins with the government's intertemporal budget constraint. By continuity of the HANK equilibrium in τ_d, we can evaluate that constraint at $\tau_d = 0$, and note that the conclusions will also be informative for what happens when $\tau_d \to 0^+$. With fixed real rates, the government budget (6) reduces to $d_0 = \mathbb{E}_0\left[\sum_{k=0}^{\infty} \beta^k t_k\right]$. Substituting t_k from the policy rule (8) and setting $\tau_d = 0$, we obtain $d_0 = -\varepsilon_0 + \tau_y \mathbb{E}_0\left[\sum_{k=0}^{\infty} \beta^k y_k\right]$. Finally, combining this with the initial condition (5), we obtain a relationship between the initial deficit shock, the impact inflation

[20] For this illustration, we set $\omega = 0.8$ and $\kappa = 0.1$, representing a meaningful failure of Ricardian equivalence and a relatively steep NKPC. For our later quantitative analysis, we will consider empirically disciplined variants of our model.

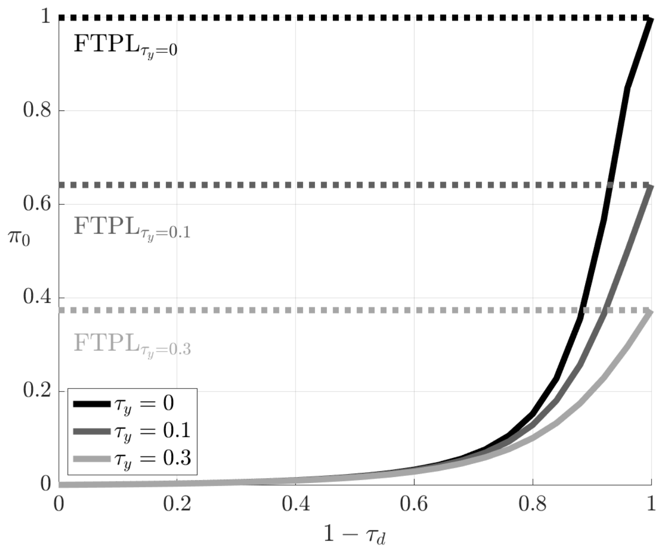

Figure 1: Date-0 inflation response to a fiscal deficit shock in HANK (solid), for different τ_d and τ_y. The dashed lines show the corresponding inflation response in the FTPL equilibrium. The size of the shock is normalized to give a date-0 FTPL inflation response of 1 percent for $\tau_y = 0$.

jump, and the cumulative output response:

$$\underbrace{\varepsilon_0}_{\text{deficit shock}} = \underbrace{\frac{D^{ss}}{Y^{ss}}\pi_0}_{\text{debt erosion}} + \underbrace{\tau_y \mathbb{E}_0\left[\sum_{k=0}^{\infty}\beta^k y_k\right]}_{\text{tax base expansion}}.$$

In words, since fiscal adjustment has been ruled out (i.e., $\tau_d = 0$), the initial fiscal deficit must be financed by inflation and its induced debt erosion, by an expansion in the tax base, or by a mixture of both. Furthermore, this equation must hold in *both* our HANK economy and its RANK-FTPL counterpart. Therefore, the *sum* of the two terms on the right-hand side of this equation must be the same in both economies. Finally, because inflation in both economies follows the NKPC and hence (3), the *ratio* of these two terms is also the same and is given by

$$\frac{\frac{D^{ss}}{Y^{ss}}\pi_0}{\tau_y \mathbb{E}_0\left[\sum_{k=0}^{\infty}\beta^k y_k\right]} = \frac{D^{ss}}{Y^{ss}}\frac{\kappa}{\tau_y}.$$

If both the sum and the ratio of these two terms are the same, then each term itself must also be the same, and so RANK-FTPL and our limit HANK economy must deliver the exact same debt erosion, and hence the same initial price jump.

This argument, which—just like RANK-FTPL—simply leverages government budget arithmetic together with the NKPC, is however silent on the underlying economic mechanism, and in particular does not allow us to understand the continuity and monotonicity in τ_d of the equilibrium inflation response. Returning to the simple non-Ricardian spending mechanism that we described in Section 4.1 instead allows us to shed light on these properties. If fiscal adjustment is fast, then the "net wealth" of today's non-Ricardian consumers, z_t, has not changed much, so they increase their spending by very little. As adjustment gets delayed, the initial transfer is increasingly seen as a pure transfer from future to current consumers, so now short-run demand increases almost one-to-one with the fiscal deficit. In general equilibrium, this increase in demand leads to a boom in output (leading to even more spending) and prices (moderating spending, since household wealth is nominal). At one extreme, if prices are very flexible (i.e., if $\kappa \to \infty$), more delays in fiscal adjustment thus lead to greater and greater price pressure, smoothly approaching the limit where prices jump to fully offset the initial increase in demand—and the price jump that does so is $\left(\frac{D^{ss}}{Y^{ss}}\right)^{-1}$. If instead prices are partially rigid, then the general equilibrium feedback loop features both prices increasing and output booming, with the two adjusting in tandem to accommodate the short-run increase in consumer demand. This logic transparently explains the key components of Theorem 1: the continuity, the monotonicity, and the limit. And in particular, it reveals that all of these components of our equivalence result are intimately tied to the short horizons of households in HANK. In Section 4.5 we will show that our second main result—robustness—is similarly rooted in these short horizons, making our two headline takeaways two sides of the same coin.

HANK's comparative statics. Away from the limit of very delayed fiscal adjustment, HANK produces strictly less inflation than RANK-FTPL. Nonetheless, the FTPL's familiar comparative statics are preserved, as summarized in the third part of Theorem 1. Not surprisingly, a higher slope of the Phillips curve (κ) always leads to a higher inflation surprise in response to the fiscal deficit shock, because any given demand boom becomes more inflationary. Conversely, a higher debt-to-GDP ratio ($\frac{D^{ss}}{Y^{ss}}$) leads to a lower inflation surprise, as now a given size of the deficit-driven boom generates more debt erosion, arresting the boom and thus the associated inflation surprise. Finally, and by the same token, a stronger tax base channel (τ_y) similarly lowers the inflation surprise, as now a given size of the deficit-driven boom generates more tax revenue, again arresting the boom. In RANK-FTPL, all these comparative statics hold *only* for $\tau_d = 0$, and they derive from the government's budget arithmetic. In HANK, they instead hold for all $\tau_d \in [0,1)$ and they derive from the natural two-way feedback between real spending and debt erosion described above.

4.3 HANK meets FTPL, with interest rate feedback

We now relax the restriction $\phi = 0$; that is, we allow fiscal deficits to trigger a change in (expected) real rates via the monetary authority's endogenous response. We first of all clarify the conditions under which the HANK equilibrium characterized in Proposition 3 continues to exist for $\phi \neq 0$, before then extending our HANK-FTPL equivalence result to this more general case.

The HANK equilibrium with $\phi \neq 0$. We continue to assume that $\omega < 1$, but now let $\phi \neq 0$ and ask the following question: what are the values of ϕ such that an equilibrium of the same form—and same economics—as that in Proposition 3 continues to exist for *all* values of τ_d, including $\tau_d = 0$?

Proposition 4. *Suppose that $\omega < 1$ and $\tau_y > 0$. There exists a threshold $\bar{\phi} > 0$ such that: if $\phi < \bar{\phi}$ then for all $\tau_d \in [0,1)$, an equilibrium of the form (17) exists and is unique. The equilibrium coefficients χ and ρ_d, and the resulting inflation impulse responses, are all continuous in $(\beta, \omega, \tau_y, \tau_d, \phi)$.*[21]

Intuitively, if the monetary authority raises interest rates sufficiently aggressively in response to the fiscally-led boom (namely, if $\phi > \bar{\phi}$), then it both arrests the boom and raises the government's cost of borrowing. Fiscal adjustment must then be fast enough (i.e., τ_d must be sufficiently higher than 0), or else public debt will not be stabilized. It follows that, naturally, our HANK equilibrium continues to exist for $\phi > \bar{\phi}$ only insofar as τ_d is sufficiently high.[22] But if instead the rate hikes are modest (i.e., if $0 < \phi < \bar{\phi}$), then they only partially offset the aforementioned two-way feedback between fiscal conditions and economic activity, making it possible to sustain an equilibrium for all $\tau_d \in [0,1)$, similar to the case of $\phi = 0$.[23] If monetary policy lets real rates *fall* in response to the fiscally-led boom (i.e., if $\phi < 0$), then this only speeds up the boom and lowers the government's cost of borrowing, and so public debt is again stabilized for all $\tau_d \in [0,1)$. It follows that, as stated in Proposition 4, an equilibrium with $\tau_d = 0$ exists in our HANK economy on both sides of $\phi = 0$, and this equilibrium is furthermore continuous in ϕ. This verifies the earlier claim that there is no material difference between the different types of monetary policy—unless, of course, monetary policy is sufficiently aggressive to necessitate $\tau_d > 0$. Finally, the equilibrium is again continuous in τ_d, as in the baseline case in Proposition 3, again reflecting the simple non-Ricardian spending mechanism at play.

HANK meets FTPL, again. Pick any $\phi < \bar{\phi}$ and consider the HANK equilibrium obtained for $\tau_d = 0$. We now ask whether this equilibrium predicts the same impact price jump and thus debt erosion as a properly defined FTPL counterpart. In defining such a counterpart, we must deal with two challenges. First, while our HANK equilibrium exists for both $\phi > 0$ and $\phi < 0$, the FTPL equilibrium ceases to exist for $\phi > 0$. Second, even if we restrict to $\phi < 0$, HANK and FTPL are not directly comparable because the same monetary policy rule does not necessarily translate to the same equilibrium paths for real interest rates, which in turn affect both aggregate demand and the government budget. We address

[21] Note that, as stated in Section 2, we also restrict $\phi > \underline{\phi} \equiv -\frac{1}{\sigma}$. The lower bound $\underline{\phi}$ has the following property in HANK: as monetary policy becomes increasingly accommodative ($\phi \to \underline{\phi}^+$), the deficit-induced boom becomes so large that debt is stabilized immediately ($\rho_d \to 0^+$).

[22] From (17), because $\chi > 0$ with non-Ricardian households, an unbounded public debt d_t will lead to an unbounded output y_t in HANK, violating the equilibrium definition in Definition 1.

[23] Consistent with this intuition, $\bar{\phi}$ increases with both τ_y and $1 - \omega$: if the feedback is strong, then even very aggressive monetary reactions are consistent with stable public debt in the absence of fiscal adjustment ($\tau_d = 0$).

these challenges and provide the natural "apples-to-apples" comparison as follows: for any $\phi < \bar{\phi}$, we first take the HANK equilibrium that occurs for $\tau_d = 0$; we then identify the FTPL equilibrium that occurs in RANK under a *modified* monetary policy, which induces the same path of (expected) real interest rates as in our HANK equilibrium; and finally, we compare the inflation predictions of these two equilibria. Proposition 5 summarizes the results of this exercise.[24]

Proposition 5. *Suppose that* $\omega < 1$, $\tau_y > 0$, *and* $\phi < \bar{\phi}$, *and consider the HANK equilibrium that obtains when* $\tau_d = 0$. *Select any realization of the initial fiscal shock* ε_0, *abstract from any future shocks, and let* $\{r_t^{HANK}\}_{t=0}^{\infty}$ *be the equilibrium path of the (expected) real rate obtained in this equilibrium. Now consider an analogous RANK-FTPL economy in which* $\omega = 1$, *fiscal policy follows the same rule as in our HANK economy (with* $\tau_d = 0$*), and monetary policy follows the passive rule* $r_t = r_t^{HANK}$. *Then, similar to Theorem 1, the two economies continue to produce the same inflation surprise in response to a fiscal deficit shock:*

$$\pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{FTPL}.$$

Intuitively, once we equate the impulse response function of real interest rates to deficit shocks in the two economies, we also equate the government's interest rate costs of servicing its outstanding debt. This ensures that the sum of debt erosion and tax-base expansion remains equal across the two economies, exactly as in our original equivalence result. And since the ratio of these two forms of financing is pinned down by the NKPC, we conclude that the two economies must once again share the same debt erosion and the same inflation surprise.

4.4 Short horizons and the time profile of inflation

Our analysis so far has emphasized that, despite the difference in underlying mechanism, HANK and RANK-FTPL can have equivalent predictions for the initial price jump—and so debt erosion—induced by fiscal deficit shocks. The next two sections instead show where the difference in mechanism causes material differences in outcomes.

The main result of this section is that, in HANK, fiscally induced inflation bursts are necessarily more front-loaded and more short-lived than in RANK-FTPL. This is straightforward to see when $\phi = 0$ (i.e., fixed real rates). In this case, RANK-FTPL produces a random walk for output and thus also inflation (by (11)), while HANK implies a mean-reverting process for both variables (by Proposition 3). To establish this point more generally, we consider the following measure of the "front-loadedness"

[24] In the interest of parsimony, Proposition 5 focuses on $\tau_d = 0$ and does not repeat the continuity and monotonicity of $\pi_{\varepsilon,0}^{HANK}$ in $\tau_d \in [0,1)$, although these properties continue to hold. See the proof of Proposition 4 for details.

of the inflation response to a fiscal deficit shock:

$$\pi^{\dagger} \equiv \frac{\pi_{\varepsilon,0}}{\sum_{k=0}^{\infty} \beta^k \pi_{\varepsilon,k}}, \tag{21}$$

where $\pi_{\varepsilon,k} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t}$ is the response of inflation to a deficit shock k periods earlier, i.e., $\pi^{\dagger}$ is the initial impact relative to the cumulative inflation response. We next show how this object in HANK compares to the appropriate FTPL counterpart.

Proposition 6. *Let $\omega < 1$, $\tau_y > 0$, and $\phi < \bar{\phi}$. The inflation impulse response to a fiscal deficit shock in the HANK equilibrium is more front-loaded when households are less Ricardian, i.e., $\pi^{\dagger,HANK}$ increases when ω is lower. Furthermore, $\pi^{\dagger}$ is bounded from below by its FTPL counterpart:*

$$\pi^{\dagger,HANK} \;>\; \pi^{\dagger,FTPL}. \tag{22}$$

This front-loadedness is a natural and immediate implication of the difference in mechanism underlying the RANK-FTPL and HANK equilibria. The short household horizons of HANK ($\omega < 1$) imply not only that future tax hikes are discounted (as stressed in Section 4.1), but also that the non-Ricardian consumers spend the initial transfer quickly, rather than smoothing it out over an infinite lifetime (like their Ricardian counterparts). It follows that the induced demand boom—and hence the resulting inflationary pressure—is necessarily more short-lived in HANK than in RANK-FTPL.

In Section 5 we will consider several extensions of our baseline HANK environment. We will see that, because of the front-loading force discussed here, these extensions can induce some interesting departures from the exact HANK and RANK-FTPL equivalence that we established in this section.[25]

4.5 The robustness of HANK

We conclude our analysis in this section with the second—and most important—difference between the two theories of the deficits-inflation nexus: HANK's ability to sidestep the fragilities and controversies surrounding the FTPL. Our main result is that, unlike its FTPL counterpart, HANK is robust to changes in assumptions about fiscal-monetary policy in the far-ahead future and, relatedly, to allowing the economy to return to steady state in finite time.

Proposition 7. *Suppose that $\omega < 1$, $\tau_y > 0$, and $\phi < \bar{\phi}$, and consider the same policy switch as in Proposition 2: for some $H \geq 1$, the fiscal and monetary authorities follow the rules* (8) *and* (10) *for $t < H$ but*

[25] The front-loadedness property also suggests that it may in principle be possible to distinguish between HANK and RANK-FTPL on the basis of *macroeconomic* time series. However, for this idea to be operationalized in practice, one would have to separately account for all other forces that may affect the persistence of fiscally-led fluctuations. Since *microeconomic* evidence overwhelmingly favors HANK models of consumption anyway, we in our quantitative explorations in Section 6 instead take a bottom-up approach: we discipline the model with relevant microeconomic evidence, and then quantify the degree of front-loading that this induces.

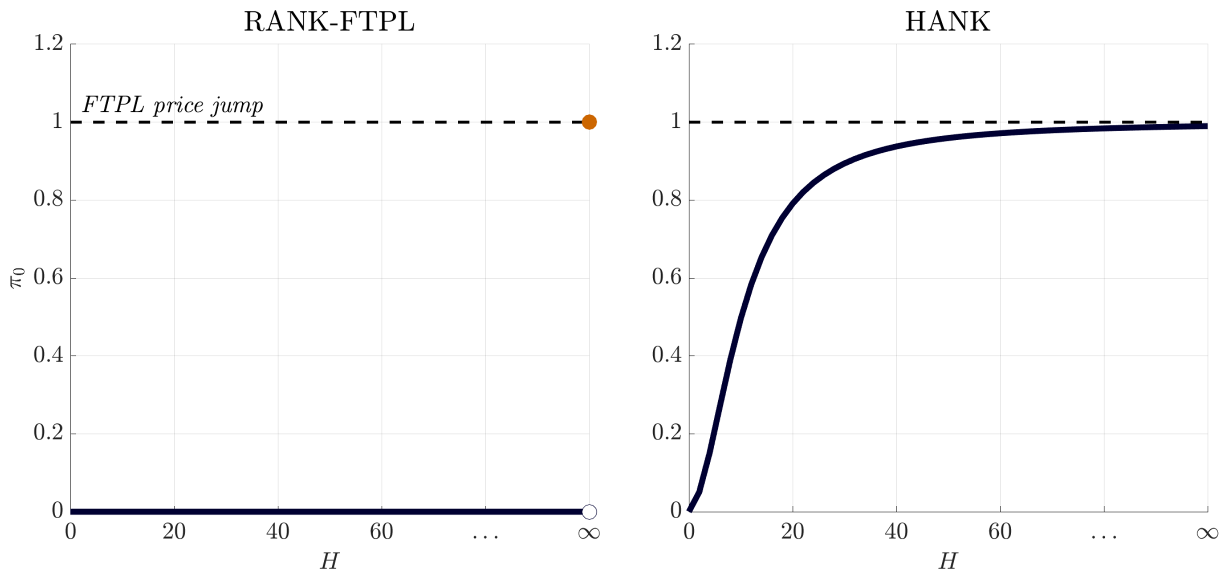

Figure 2: Date-0 inflation response to a deficit shock in RANK (left panel) and HANK (right panel) for different H. The size of the shock is normalized so that the FTPL price jump is 1 percent.

switch to (15) *for* $t \geq H$. *Then, there is a unique equilibrium, and it is such that, for all* $t < H$, *output and inflation continue to co-move with fiscal deficits. Furthermore, for any* $T > 0$ *and any realization of uncertainty,* $\{y_t, \pi_t\}_{t=0}^{T}$ *converges to its counterpart in Proposition* 4 *as* $H \to \infty$.

The proposition states that, in HANK, as the date H of the policy switch is increased, this switch ceases to matter for short-run inflation dynamics—in other words, what happens in the short run is invariant to what happens with far-ahead policy, unlike in RANK-FTPL. A visual illustration of this result is provided in Figure 2, which focuses on the benchmark case with $\phi = \tau_d = 0$ and then asks how the period-0 inflation response to a fiscal deficit shock varies with H. The left panel corresponds to RANK-FTPL. Consistent with the discussion in Section 3.2, we see that, for any finite H, the deficit shock has no effect on output and inflation; it is only when $H = \infty$ (the literal absence of fiscal adjustment, forever) that real spending and prices jump in response to the shock. The right panel then turns to HANK. The effect of the deficit shock is now positive throughout, increases with H, and converges to its FTPL counterpart as $H \to \infty$, with no discontinuity between large but finite H and $H = \infty$.

Understanding the robustness. The stark differences in robustness to seemingly innocuous changes in far-ahead policy follow from the difference in economic mechanism between the RANK-FTPL and HANK. The date-H policy switch guarantees that the economy returns to steady state in finite time, rather than asymptotically. In RANK, because fiscal policy has no wealth effects in equilibrium, this is enough to rule out FTPL outcomes, regardless of ϕ: if fiscal deficits cannot drive aggregate spending and inflation *forever*, then they cannot affect them at all, as discussed in Section 3.2.

In HANK, by contrast, fiscal deficits continue to drive output and thereby inflation for any $t < H$, simply because of their always-present wealth effects on consumer spending. Furthermore, because the non-Ricardian consumers discount the future, households spend their initial transfer in the short run (with any future tax hikes having a vanishing effect), so the outcomes converge smoothly to those characterized in Proposition 4 as H increases. In short, HANK is robust to inherently untestable assumptions about far-ahead policy, unlike its FTPL counterpart.[26]

The bottom line. The prevailing sticky-price formalization of the FTPL has an appealing empirical essence (e.g., see the analysis in Barro and Bianchi, 2025), but it has long been subject to theoretical controversies. In Section 3, we connected these controversies to the following tension: breaking Ricardian equivalence while assuming that households are Ricardian. In the present section, we then showed that HANK delivers those same appealing predictions while insulating them from the controversies and fragilities of the FTPL, precisely by avoiding this fundamental tension. While our discussion focused on the importance of assumptions regarding far-ahead policy, it is immediate from our results that HANK similarly sidesteps the long-standing debate about the government's ability to commit to a lack of fiscal adjustment (as in Kocherlakota and Phelan, 1999; Buiter, 2002)—HANK recovers the FTPL's predictions for large enough but still finite H, i.e., even for "passive" fiscal policies.

The remainder of the paper achieves two further objectives. First, in Section 5, we expand the scope and thus generality of our equivalence and robustness results. Second, in Section 6 we provide an empirically disciplined, quantitative account of the deficit-inflation nexus in HANK, and how it relates to the famous FTPL arithmetic.

5 Extensions

The preceding analysis focused on two tasks: (i) to establish that HANK can produce FTPL-like outcomes; and (ii) to clarify the difference in the underlying economic mechanisms, and the implications of this difference in terms of robustness and front-loading. To accomplish these tasks as transparently as possible, we used a highly tractable model. We now discuss how our results extend in three dimensions of practical relevance: long-term government debt (Section 5.1); heterogeneity in household bond holdings and transfer receipts (Section 5.2); and a hybrid NKPC (Section 5.3). All of these extensions will feature prominently in our quantitative analysis in Section 6.

[26]This discussion also verifies that the mechanism behind our HANK equilibrium is driven *exclusively* by the short-run wealth effects we have emphasized throughout, as opposed to any subtleties regarding beliefs at infinity.

5.1 Long-term government debt

We allow for government debt to be long-term. In keeping with much of the FTPL literature, we consider the analytically tractable case of a geometric maturity structure (e.g., Cochrane, 2001).

Environment. Nominal public debt is long-term, with its maturity parameterized by $\delta \in [0,1]$; the baseline case of short-term debt is nested as $\delta = 0$. The government flow budget now becomes

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\mathbb{E}_t[d_{t+1}]} - \frac{D^{ss}}{Y^{ss}}\left(\pi^{\delta}_{t+1} - \mathbb{E}_t\left[\pi^{\delta}_{t+1}\right]\right) - \frac{D^{ss}}{Y^{ss}}\left(r^{\delta}_{t+1} - \mathbb{E}_t\left[r^{\delta}_{t+1}\right]\right) \tag{23}$$

where

$$\pi^{\delta}_t \equiv \mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\delta)^k \pi_{t+k}\right] \quad \text{and} \quad r^{\delta}_t \equiv \mathbb{E}_t\left[\sum_{k=0}^{\infty} (\beta\delta)^{k+1} r_{t+k}\right] \tag{24}$$

A derivation of (23) is provided in Appendix A.4, but the logic is straightforward: $d_{t+1} - \mathbb{E}_t[d_{t+1}]$, the innovation in the real market value of the government debt, is proportional to the innovations in cumulative inflation as well as real rates over the duration of the public debt—i.e., debt erosion due to the inflation surprise plus debt revaluation due to real rate surprises. The remainder of the model is exactly as in Section 2. The essential structure of the HANK equilibrium furthermore remains exactly the same, in the sense that (17) in Proposition 3 continues to hold with the same values of χ and ρ_d. The only relevant change is the equilibrium size of the deficit-led boom, and how this boom translates to deficit-relevant inflation and real rate surprises, as explained next.

HANK meets FTPL. The comparison of inflation in HANK and FTPL now concerns the maturity-adjusted *cumulative* inflation response $\pi^{\delta}_{\varepsilon} \equiv \frac{d\pi^{\delta}_t}{d\varepsilon_t}$—i.e., the summary statistic of the impact of the inflation surprise on the government budget in (23). This object has received much attention in the FTPL literature (Barro and Bianchi, 2025), precisely because it is the object for which the FTPL makes the starkest prediction. We also note that, for the empirically relevant case of δ close to 1, it is very similar to the full cumulative inflation impulse response, an object customarily studied in the monetary economics literature (e.g., see Alvarez, Le Bihan and Lippi, 2016). For this object, we now find a weaker form of equivalence, as summarized in Proposition 8: still exact equivalence if the tax base channel is absent, but otherwise, RANK-FTPL now serves as an *upper bound.* For simplicity the proposition restricts attention to the special case of fixed (expected) real rates, with the straightforward extension to interest rate feedback relegated to Appendix A.4.

Proposition 8. *Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\delta > 0$, and $\phi = 0$.*[27] *There exists a unique equilibrium in the HANK economy. The quantity $\pi^{\delta}_{\varepsilon}$, which measures the degree of debt erosion or, equivalently, the*

[27] As in Proposition 5, Proposition 8 focuses on $\tau_d = 0$ and does not repeat the continuity and monotonicity of $\pi^{\delta,HANK}_{\varepsilon}$ in $\tau_d \in [0,1)$, although these properties continue to hold. See the proof of Proposition 8 for details.

maturity-adjusted cumulative inflation response to a fiscal deficit shock, is strictly lower in the HANK economy than in its FTPL counterpart:

$$\pi_{\varepsilon}^{\delta,HANK} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}(1-\beta\delta\rho_d)} < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}(1-\beta\delta)} = \pi_{\varepsilon}^{\delta,FTPL}, \tag{25}$$

with the distance between the two vanishing when $\tau_y \to 0$ or $\kappa \to \infty$ (no tax-base self-financing channel) or when $\delta \to 0$ (short term debt).

The intuition for why the (cumulative) inflation response in HANK is now *smaller* than in RANK-FTPL reflects the interaction of long-term debt with the inflation front-loading that is implied by HANK. Since inflation is at all dates proportional to the present discounted value of future output responses, making any given output boom more front-loaded (while holding its present value fixed) will leave the impact inflation unchanged, but lower the subsequent inflation responses. This then reduces the scope for debt erosion—and thus also the cumulative inflation response, discounted by δ—in HANK relative to RANK-FTPL. The interaction of front-loading and long-term debt is evident in equation (25), with ρ_d and δ entering $\pi_{\varepsilon}^{\delta,HANK}$ only via the product $\delta\rho_d$.

Though lower in overall magnitude because of front-loading, the cumulative inflation response in HANK continues to share all of the comparative statics of its RANK-FTPL counterpart, not only with respect to $\frac{D^{ss}}{Y^{ss}}$, τ_y and κ (as already stressed in Theorem 1), but also for debt maturity δ.

Proposition 9. *Let $\omega < 1$, $\tau_y > 0$, $\delta > 0$, and $\phi = 0$. $\pi_{\varepsilon}^{\delta,HANK}$ decreases with $\frac{D^{ss}}{Y^{ss}}$, decreases with τ_y, increases with κ, and increases with δ for any $\tau_d \in [0,1)$.*

Proposition 9 connects directly with the empirical findings of Barro and Bianchi (2025). That paper provides cross-country evidence that a country's debt-to-GDP ratio and its average debt maturity both help predict how much that country's inflation co-varied with government spending during the post-Covid period. The proposition shows that HANK delivers comparative statics consistent with these results (just like RANK-FTPL), with the cumulative inflation response decreasing with the level of government debt and increasing with its maturity.[28]

5.2 Heterogeneous distributional incidence

While the simple OLG model that we studied thus far captures the key feature of richer HANK models that is essential for our purposes—namely the classical failure of Ricardian equivalence—it abstracts from across-household heterogeneity and, consequently, from all of the distributional effects

[28] Furthermore, if the tax base channel is weak (because either $\tau_y \to 0$ or $\kappa \to \infty$), then we again converge to equality in (25)—i.e., the maturity-adjusted cumulative inflation in HANK also converges to the inverse of the debt-to-GDP ratio, as in the simple RANK-FTPL arithmetic, which is the theoretical benchmark in Barro and Bianchi (2025).

of inflation. Specifically, by eroding the real value of government bonds (or other nominal assets), fiscally-induced inflation will necessarily redistribute real wealth from households with large savings in such assets to households with small savings (or with debt). In complementary work, Kaplan, Nikolakoudis and Violante (2023) emphasize this channel in a flexible-price heterogeneous-agent model. Here, we ask whether and how this channel matters for the propagation of fiscal deficit shocks in the New Keynesian framework and in particular how it affects our HANK-FTPL equivalence. To address this question, we consider a tractable extension of our baseline model, featuring two types of non-Ricardian households—rich, low-MPC households and poor, high-MPC households.

Environment. We study a hybrid model that combines our baseline OLG block with a margin of hand-to-mouth spenders, with $\mu \in (0,1)$ denoting the share of spenders. From Auclert, Rognlie and Straub (2024) and Wolf (2025), we know that such models can fit well both the available microeconomic evidence on consumer responses to transfers as well as the overall time profile of iMPCs generated by fully-fledged quantitative HANK models. Furthermore, since spenders do not hold any assets, such a model can also capture—albeit in a crude way—the redistributional effects mentioned above.

In this extension, the aggregate consumption function (1) generalizes to

$$c_t = \left(1-\beta\omega\right) a_t + \left(\mu + (1-\mu)(1-\beta\omega)\right)\left(\left(y_t - t_t\right) + \frac{(1-\mu)(1-\beta\omega)}{\mu + (1-\mu)(1-\beta\omega)}\mathbb{E}_t\left[\sum_{k=1}^{\infty}\left(\beta\omega\right)^k\left(y_{t+k} - t_{t+k}\right)\right]\right), \tag{26}$$

where we have for simplicity already imposed the assumption of a neutral monetary policy, i.e. $\phi = 0$ in (10). The remainder of the model is as in Section 2, except that we will allow for long-term debt.

HANK meets FTPL. Even in this generalized model variant we continue to obtain similar comparison results between HANK and FTPL; there is exact equivalence when $\delta = 0$, and it takes the form of an upper bound when $\delta > 0$, mirroring Propositions 1 and 8.

Proposition 10. *Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\phi = 0$, and $\mu \in (0,1)$. There exists a unique equilibrium, and it has the following properties.*

1. *If $\delta = 0$, the initial price jump in response to a deficit shock is exactly the same as its counterparts in our baseline HANK economy and in FTPL:*

$$\pi_{\varepsilon,0} = \pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{FTPL}.$$

2. *If $\delta > 0$, the maturity-adjusted cumulative inflation response to a deficit shock is bounded from above by its analogue in our baseline HANK economy and hence also by FTPL:*

$$\pi_{\varepsilon}^{\delta} < \pi_{\varepsilon}^{\delta,HANK} < \pi_{\varepsilon}^{\delta,FTPL}. \tag{27}$$

By triggering inflation and eroding the real value of the government bonds held by rich, low-MPC households, fiscal deficit redistribute from these households to poor, high-MPC households (i.e., the hand-to-mouth households). This additional impetus to demand front-loads the fiscally-led boom even more. If government debt is short-term ($\delta = 0$), then this additional front-loading is immaterial for the initial price jump and so the overall debt erosion obtained when $\tau_d = 0$; if instead debt is long-term ($\delta > 0$), then the additional front-loading further dampens inflationary pressures, by exactly the same reasoning as that behind Proposition 8.[29]

Even more general aggregate demand. While formally proved only for our benchmark OLG setting and the two-type extension of this section, our equivalence and robustness results are materially more general, and in particular extend to richer, numerically solved HANK-type environments.

As emphasized throughout, the two properties of consumer behavior driving our conclusions are (i) that households discount the future at a higher rate than the interest rate on government bonds, and (ii) that they spend any additional income faster than in the permanent-income benchmark, leading to a transitory boom.[30] Provided these properties hold, any initial fiscal deficit will pass through the consumer demand block and so the NKPC to boost output and prices by an amount that increases with the delay in fiscal adjustment; and as this delay grows larger, the initial price jump invariably approaches its FTPL counterpart, by the same logic as that discussed in Section 4, and thus also with the same robustness properties. Our quantitative analysis in Section 6—which contains a fully-fledged HANK model, as well as several other demand structures— will further illustrate this discussion.

5.3 Hybrid NKPC

Our analysis thus far has featured the textbook NKPC. We now ask how our results change with generalized Phillips curves, such as those implied by price indexation (e.g., Christiano, Eichenbaum and Evans, 2005), menu costs (e.g., Auclert, Rognlie and Straub, 2023), or bounded rationality (e.g., Angeletos and Huo, 2021). All these cases boil down to replacing (2) with a more flexible mapping from the path y_t to the path of π_t.

We begin with some preliminary observations. First, the main conceptual point of Section 3 clearly extends to *any* such mapping: the basic RANK-FTPL tension of needing to break Ricardian equivalence even though households are Ricardian is invariant to how precisely prices adjust to demand pressure. Second, and similarly, the core of our HANK analysis in Section 4 does not depend on the

[29] A complementary analysis is provided in Diamond, Landvoigt and Sánchez Sánchez (2025), who instead focus on the role of mortgage debt in the propagation of fiscal inflation.

[30] In sequence-space terms, such discounting translates to the off-diagonal elements of the intertemporal MPC matrix decaying to zero sufficiently quickly. See Section 5.2 and in particular Appendix E.1. of Angeletos, Lian and Wolf (2024) for a detailed discussion.

specific form of the Phillips curve: fiscal policy still influences aggregate demand through its wealth effect term z_t present in (16). However, things can change *quantitatively*, as the specification of the NKPC will in general affect the relative contributions of debt erosion and tax-base expansion in deficit financing, thus affecting the precise HANK-FTPL equivalence.

The remainder of this section elaborates on this last observation. We focus on the empirically relevant case of a Hybrid NKPC that allows price-setting to be partially backward-looking, thus capturing the sluggishness of inflation observed in the data.

Environment. Following the above discussion, we replace (2) with a standard Hybrid NKPC:[31]

$$\pi_t = \kappa y_t + \xi \beta \pi_{t-1} + (1-\xi)\beta \mathbb{E}_t[\pi_{t+1}], \tag{28}$$

where $\xi \in (0,1)$ parameterizes the degree of backward-lookingness in price-setting. The remainder of the model is exactly as in Section 2; in particular, we restrict attention to the case of short-term government debt, for reasons that will become clear shortly.

HANK meets FTPL. With a hybrid NKPC, the exact equivalence between HANK and FTPL continues to obtain in the absence of the tax base channel. If this channel is present, however, then the short-run inflationary pressures are *larger* in HANK than in FTPL—exactly the opposite of the case with long-term government debt discussed earlier.

Proposition 11. *Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\delta = 0$, and $\phi = 0$, and let inflation now follow the hybrid NKPC* (28) *with any $\xi \in (0,1)$. There exists a unique equilibrium in the HANK economy, of the same form as in Proposition 3. The initial price jump in response to a fiscal deficit shock is strictly higher than the FTPL counterpart with the same hybrid NKPC:*

$$\pi_{\varepsilon,0}^{HANK} > \pi_{\varepsilon,0}^{FTPL},$$

with the distance between the two vanishing when $\tau_y \to 0$ or $\kappa \to \infty$.

The intuition is as follows. Compared to the textbook NKPC, its hybrid generalization (28) is less forward-looking, so current inflation depends more heavily on output in the near future. But since the output boom itself is more front-loaded in HANK than in RANK-FTPL, this means that the initial inflation increase in the former is *larger* than in the latter.

Note that this result assumes short-term government debt to isolate how the front-loading of the output response, due to finite household horizons, interacts with inflation inertia, due to the hybrid NKPC. However, we have already shown that the same front-loading in output, when combined with

[31] The conventional micro-foundation of (28) is price indexation (Christiano, Eichenbaum and Evans, 2005), while an empirically plausible alternative is incomplete information or bounded rationality (Angeletos and Huo, 2021). In either case, the appeal of (28) lies in its ability to better account for the inflation dynamics observed in the data.

long-term debt, moves inflation in the opposite direction. It follows that the precise relationship between HANK and FTPL becomes ambiguous in the general case, which features both long-term debt and inflation inertia. Our quantitative analysis in the next section will combine all of the model ingredients considered here—and more—to shed light on the empirically relevant scenario.

6 Quantitative analysis

We finally complement our theoretical results with a quantitative analysis of the deficit-inflation nexus in HANK. Our results so far suggest that even the predictions of the textbook extreme version of the FTPL—in which current deficits are financed *entirely* through a commensurate jump in prices—can emerge in HANK economies. The main takeaway of this section, however, is that, in practice, deficits are likely to be much less inflationary than predicted by the simple FTPL arithmetic.

To this end, we study the mapping from deficits to inflation in a version of our HANK model that is disciplined through direct evidence on the key ingredients of our theory. Section 6.1 describes model and calibration, Sections 6.2 and 6.3 contain the main results, and Section 6.4 closes with an application to post-Covid inflation dynamics.

6.1 Extended HANK model and calibration

We consider a variant of the model in Section 2, with three additions, following our discussion in Section 5. First, government debt is now long-term. Second, we allow for moderate household heterogeneity, with three types of households i, indexed by heterogeneous survival probabilities ω_i. This extension will allow the model to be simultaneously consistent with empirical evidence on (i) intertemporal marginal propensities to consume (e.g., Auclert, Rognlie and Straub, 2024) as well as (ii) household wealth holdings and transfer receipts. Third, we consider a hybrid NKPC, yielding more realistic inflation dynamics. The remainder of this section presents calibration details for all model blocks, with a summary provided in Table 1. We will study several further model variants—including a full-fledged HANK model—in Section 6.3, with details provided in Appendix B.1.

Throughout this section, and as in Sections 2 - 5, the policy experiment that we consider is a one-off, surprise fiscal deficit increase at date 0 (i.e., a tax cut), equal to one percent of steady-state GDP.

Consumers. We extend the consumer block of Section 2.1 to allow for three types of households i, with respective population shares χ_i. Households differ in their survival probabilities ω_i—or, less literally, in their probability of being subject to a binding borrowing constraint—, steady-state wealth shares A_i^{ss}/A^{ss}, and exposure to the fiscal deficit shock (i.e., transfer receipts). We choose population shares and death probabilities to match empirical evidence on average intertemporal marginal

Parameter	Description	Value	Target
Demand Block			
χ_i	Population shares	$\{0.218, 0.629, 0.153\}$	Fagereng, Holm and Natvik
ω_i	Survival rates	$\{0.972, 0.833, 0\}$	Fagereng, Holm and Natvik
A_i^{ss}/A^{ss}	Wealth shares	$\{0.6, 0.4, 0\}$	See text
ε_i	Transfer receipt	$\{0.122, 0.706, 0.172\} \times \varepsilon$	See text
σ	EIS	1	Standard
β	Discount factor	0.998	Annual real rate
Supply Block			
κ	Slope of Hybrid NKPC	$\{0.006, 0.019\}$	Hazell et al.; Cerrato and Gitti
ξ	Backward-lookingness	0.288	Barnichon and Mesters
Policy			
τ_y	Tax rate	0.33	Average Labor Tax
D^{ss}/Y^{ss}	Gov't debt level	1.79	See text
δ	Gov't debt maturity	0.95	Av'g debt maturity
τ_d	Tax feedback	0	Anderson and Leeper
ϕ	Inflation feedback	0	See text

Table 1: Quantitative model, calibration.

propensities to consume, from Fagereng, Holm and Natvik (2021). Wealth shares are set to roughly replicate the skewness of the U.S. wealth distribution (e.g., see Kaplan, Moll and Violante, 2018; Hagedorn, Manovskii and Mitman, 2019), with the bottom 15 percent holding no wealth, and the top quantile holding 60 percent of all wealth. Finally, consistent with U.S. policy practice, transfer receipts are somewhat more concentrated at the bottom. We also set $\sigma = 1$ (giving log preferences), and back out β to hit a steady-state real rate of interest of 1% (annual). Our model variants in Section 6.3 will consider several alternative assumptions on the departure from Ricardian equivalence, wealth shares, and transfer receipts, including a full HANK model.

Nominal rigidities. We assume a hybrid NKPC, as discussed in Section 5.3. For the slope κ we consider two headline values: the shallow slope estimated by Hazell et al. (2022); and a three-times steepening of that NKPC, as estimated in Cerrato and Gitti (2022) for the post-Covid inflationary period. Finally, for the backward-forward split (ξ vs. $1-\xi$), we take the headline point estimates reported in Barnichon and Mesters (2020).

In our main quantitative analysis we will furthermore report results for an entire (and wide) range of κ's. The alternative model variants studied in Section 6.3 will also feature alternative assumptions on the backward-forward split in the NKPC.

Policy. We set $\tau_y = 0.33$, implying meaningful—and empirically realistic—feedback from economic activity to primary surpluses. Government debt, D^{ss}, is set to match the total amount of domestically, privately held U.S. government debt, and $\delta = 0.95$ gives an average debt maturity of five years.[32] Consistent with legislative evidence on the post-Covid fiscal stimulus (e.g., see the detailed discussion Anderson and Leeper, 2023), we consider an "unbacked" fiscal expansion, so $\tau_d = 0$. Finally, as in our main analysis, we set $\phi = 0$, corresponding again to a fixed real-rate rule. We do so for two reasons. First, in that case, our simulations will be informative about the pure effect of the deficit, without any direct monetary offset or accommodation. Second, as discussed in Angeletos, Lian and Wolf (2024), this case is actually a quite reasonable approximation to many past fiscal stimulus episodes.

For our alternative model variants and the quantitative post-Covid application we will pay particular attention to what happens under alternative assumptions on fiscal adjustment (τ_d) and on the monetary policy reaction (ϕ).

6.2 Benchmark specification

We study how, in our quantitative model, fiscal deficits transmit to inflation. Figure 3 shows impulse responses of aggregate output and inflation to a deficit shock that, according to the simple FTPL arithmetic, would move cumulative (maturity-adjusted) inflation by 1 percent (left and middle panel), for our two headline values of κ (shades of gray). The right panel then displays the cumulative (maturity-adjusted) inflation response π_ε^δ as a function of κ, over a large range.

The main takeaway from the figure is that the inflationary pressures associated with the unfunded fiscal deficit shock are—while material—quite substantially weaker than predicted by the simplest textbook FTPL arithmetic. The key panel is the right one, which shows the cumulative inflation response as a function of κ, relative to the simple FTPL arithmetic prediction (dashed line). We see that, even for an NKPC three-times as steep as the pre-Covid estimates of Hazell et al. (2022), the cumulative inflation response is actually only around half of the simple FTPL prediction. The left panel provides the answer for why: output booms with a cumulative multiplier around 1.37 - 1.94 (for our two headline values of κ), generating meaningful tax revenue through the feedback from economic activity to primary surpluses (with $\tau_y = 0.33$). Such a tax base expansion substitutes for the cumulative inflation response (and its induced debt erosion) to finance the deficit shock. Finally, the middle

[32] We target government debt—rather than household liquid wealth, as in Kaplan, Moll and Violante (2018)—since the government debt-to-GDP ratio is what matters for the FTPL arithmetic. With a share of around 42 percent of U.S. government debt being held by private, domestic entities (Department of the Treasury, 2024, p.50), the pre-Covid (2020:Q1) quarterly debt-to-GDP ratio of 4.28 gives $D^{ss}/Y^{ss} = 1.79$.

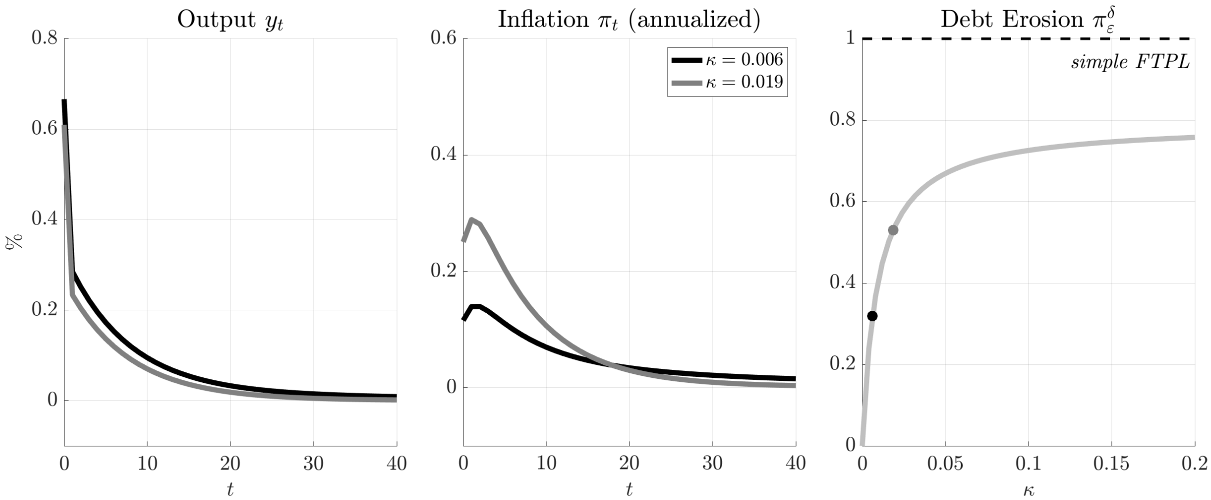

Figure 3: Output and inflation impulse responses to a date-0 deficit shock of size D^{ss}/Y^{ss} for different values of κ (left and middle), and π_ε^δ as a function of κ (right).

panel shows the time profile of the inflation response: consistent with our theoretical results, the inflation that does occur is front-loaded and relatively short-lived, with around a quarter of the entire inflation response already occurring over the first year. Given that government debt is long-term, this front-loading—which is further reinforced by the fiscal shock's distributional incidence—is also part of the dampening of the overall cumulative inflation response visible in the right panel.

The remainder of this section extends our analysis in two ways. First, in Section 6.3, we go beyond the benchmark model parameterization and explore the effects of various possible model alterations. Second, in Section 6.4, we discuss implications of our results for the post-Covid inflationary episode.

6.3 Model variants

We now study the deficit-inflation mapping in several alternative variants of our quantitative model, allowing us to shed light both on the broader relevance of our conclusions as well as on the role played by the various model ingredients. Details for all variants are provided in Appendix B.

- *Consumers.* For a first set of experiments, we alter our empirically disciplined consumer block to feature no heterogeneity in bond holdings and transfer receipts ("iMPC"), heterogeneity only in bond holdings ("Het. B"), and heterogeneity only in transfer receipts ("Target"). Second, we consider what happens if households adjust their expectations of future income slowly, as in particular the sticky-information specification of Auclert, Rognlie and Straub (2020) ("Behavioral"). Third, we replace our consumer block by the one-type OLG structure of Section 2 ("OLG") and by a full-blown HANK structure ("HANK").

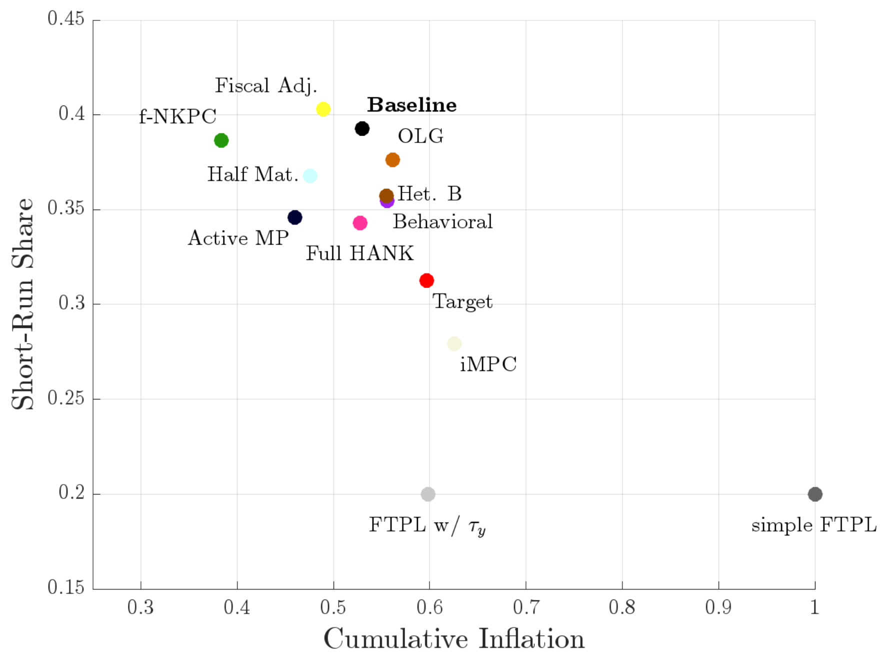

Figure 4: Cumulative inflation response and short-run response share to a date-0 deficit shock of size D^{ss}/Y^{ss}, for different model variants, indicated by dots.

- *Nominal rigidities.* Our analysis in Section 6.2 already shed light on the role of NKPC slope κ. We here additionally consider what happens if our empirically disciplined hybrid NKPC is replaced by a simple textbook forward-looking one ("f-NKPC").

- *Policy.* To further illustrate our "robustness" discussion of Section 4.5, we also investigate what happens with gradual fiscal adjustment ("Fiscal Adjustment", $\tau_d = 0.02$) and with active monetary policy ("Active MP", $\phi = 0.25$, together with fiscal adjustment, $\tau_d = 0.02$). We further consider a model variant in which the government debt maturity is halved ("Half Mat.", $\delta = 0.9$).

Our results are reported in Figures 4 and 5. Figure 4 shows the cumulative (maturity-adjusted) inflation response $\pi^{\delta}_{\varepsilon}$ (in the x-axis) and the short-run inflation share (defined as the share of inflation in the first year relative to the first five years, in the y-axis), under various model specifications. The simple FTPL arithmetic is in the bottom right ("simple FTPL"), with the cumulative inflation response normalized to 1. Starting from this reference point and then adding tax-base self-financing ("FTPL w/ τ_y") does not affect the persistence of the inflation burst, but dampens its magnitude. Moving to our HANK model ("baseline") reduces cumulative inflation a bit more while materially increasing the short-run inflation share. This increase simply reflects the front-loading property, while the reduction in cumulative inflation is governed by the interaction of front-loading with long-term debt and the hybrid NKPC. Long-term debt significantly dampens the inflation response, with the hybrid NKPC

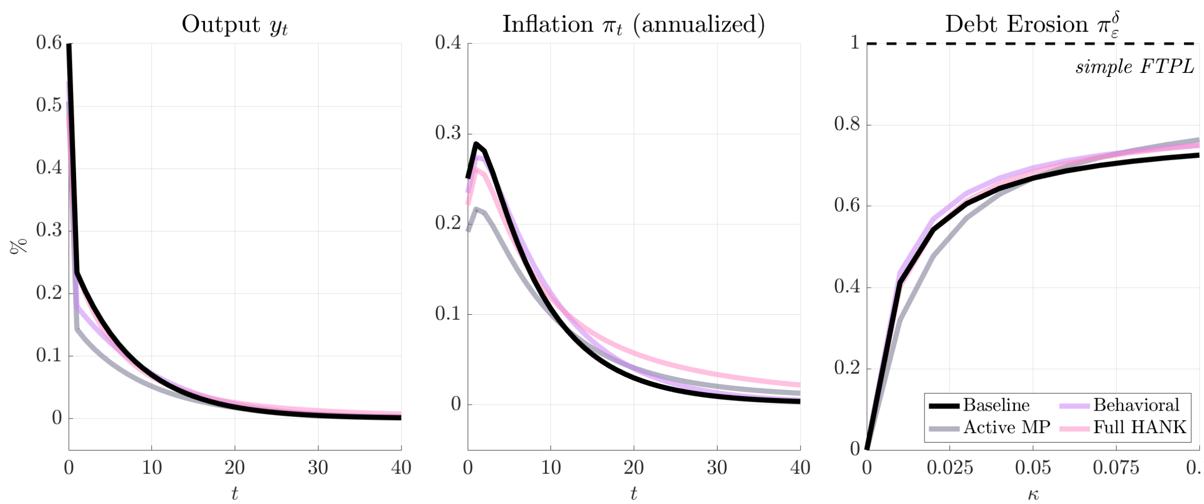

Figure 5: Output and inflation impulse responses to a date-0 deficit shock of size D^{ss}/Y^{ss} (left and middle) and π_ε^δ as a function of κ (right), for different model variants.

only partially offsetting this effect (cf. the "Baseline" and "f-NKPC" dots). Finally, *all* other HANK variants (all other dots) remain in the top left of the figure: while changing model parameterization details affects the precise magnitudes, it does not alter the core finding that inflation responses are substantially smaller and more front-loaded than in the simple FTPL benchmark.

Figure 5 shows full impulse responses for selected model variants, allowing us to dig deeper into the role played by the various model alterations. First, with a more aggressive monetary policy, the inflation response is—as expected—dampened, but of course remains present, illustrating our theoretical results on the robustness of the deficits-inflation mapping in HANK-type models. Second, in the less forward-looking behavioral model, the intertemporal Keynesian cross underlying the deficit-inflation mechanism in HANK plays out more slowly, and so the inflation burst is slightly more persistent. And third, moving to a full-blown HANK model has very limited effect on our results, consistent with prior work establishing that analytical models of the sort provided here provide an excellent approximation to aggregate output and inflation dynamics in HANK.

Finally, we also note that, while the results in Figure 4 assume a fixed real rate (except, of course, for the model variant with an active monetary rule), our results do not hinge on that assumption. Specifically, Appendix B.2 repeats our analysis for a fiscal stimulus accompanied by monetary accommodation (lower real rates). In that case, the standard FTPL also features a front-loaded inflation response, since the real rate cut encourages households to front-load consumption. Crucially, however, in our HANK model variants, and for the same real rate path, the inflation response is *even more* front-loaded (again because of discounting), thus overall delivering the same picture as in Figure 4.

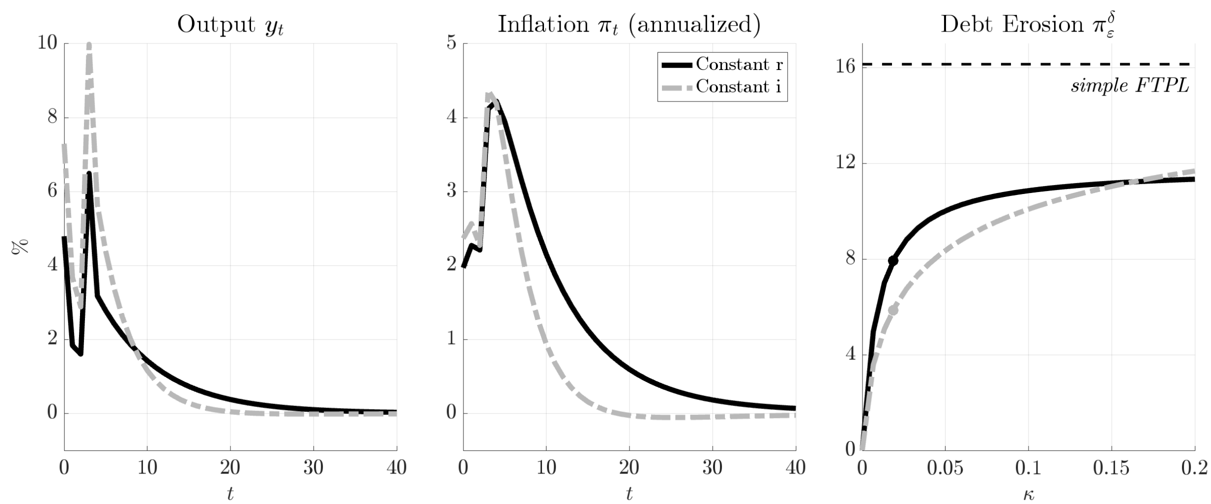

Figure 6: Output and inflation impulse responses (left and middle) to the post-Covid fiscal deficit shock (see text) and π_ε^δ (right) as a function of κ, under two different assumptions on the monetary policy reaction: fixed real rates (black) and fixed nominal rates (gray, dashed).

6.4 Application to post-Covid inflation dynamics

Finally, we use our quantitative model for an application to post-Covid inflation dynamics. Results are reported in Figure 6, which shows output and inflation impulse responses as well as the discounted cumulative inflation response under different assumptions on policy.

Policy experiments. We consider a two-step fiscal deficit shock: first, at $t = 0$, there is a shock equal to \$0.795tr (payments to households as part of the CARES Act), and second, at $t = 3$, there is a shock equal to \$0.844tr (payments to households as part of the ARP Act). We restrict attention to payments to households because our theoretical analysis only directly speaks to the propagation of this kind of fiscal deficit increase. We then furthermore assume that there is no fiscal adjustment (i.e., we set $\tau_d = 0$), consistent with actual legislation so far (e.g., see the review in Anderson and Leeper, 2023).[33]

We study impulse responses to this fiscal deficit shock under two different assumptions on the monetary policy reaction. First, we keep real rates fixed. The resulting impulse responses will identify the causal effect of the fiscal expansion *in isolation*; i.e., what is the incremental impetus to inflation, keeping the monetary policy stance—in terms of real rates—exactly as observed in the data? Second, we keep nominal rates fixed. This counterfactual keeps the monetary stance in policy instrument space as in the data, and thus—since the fiscal deficit will be inflationary—embeds the effects of additional monetary accommodation, i.e., a decline in real interest rates.

[33]We assume that the two stimulus packages are surprises. We obtain very similar results under the opposite extreme of perfect foresight, see Appendix B.3. The precise numbers for the payments to households in our policy experiment are taken from Committee for a Responsible Federal Budget (2025).

Results. The results from our policy experiments are reported as the black and gray lines in Figure 6. Consider first the overall magnitudes. Given the size of the deficit shock, the simple textbook FTPL accounting would predict a cumulative discounted inflation response of around 16%. We see that both policy experiments in our setting predict material dampening relative to that upper bound, consistent with our results in Sections 6.2 - 6.3. The burst in inflation is furthermore, in both cases, concentrated in the first couple of years after the fiscal deficit shock.

We next investigate further the role of the monetary policy response by contrasting the two sets of impulse responses. The counterfactual of nominal interest rates kept as in the data corresponds to additional monetary accommodation, and thus leads to a larger and more front-loaded demand boom, together with a reduction in government borrowing costs. Taken together, stronger front-loading as well as reduced borrowing costs (by the flip-side of the classical "stepping-on-a-rake" effect, as studied in Sims, 2011) *lower* the overall cumulative inflation response.

7 Conclusion

How, and by how much, do fiscal deficits drive inflation? We addressed these questions in the New Keynesian framework, comparing and contrasting the FTPL and HANK. While the two theories differ on the "how," they can actually align on the "how much," with the deficit-induced inflation surprise in HANK smoothly converging to its FTPL counterpart as fiscal adjustment gets delayed more and more. Crucially, however, because HANK instead roots the deficits-inflation nexus in a classical and empirically-measurable failure of Ricardian equivalence, this theory sidesteps the controversies that have long plagued the FTPL. The upshot of our paper is thus to move focus away from these controversies and redirect research toward the simpler, more tangible question of how quickly fiscal adjustment and monetary policy reactions take effect.

Our contribution concluded with a quantitative evaluation of just how inflationary fiscal deficits are actually likely to be in practice. We benchmarked our results against the simple FTPL arithmetic, which posits that prices jump enough to fully finance any fiscal deficit shock. Our main result was that the tax base channel—neglected in much of the FTPL literature—together with the interaction of long-term debt and inflation front-loading—a new channel we uncovered here—are likely to quite materially dampen the inflationary effects of unfunded fiscal deficits, to about half of what the simple FTPL arithmetic would predict.

Our analysis suggests at least three avenues for future research. First, our quantitative findings were model-based, with empirical discipline applied indirectly through evidence on individual model components; it would be valuable to confront the theory with more direct evidence on the deficits-inflation relationship (e.g., along the lines of Hazell and Hobler, 2025). Second, our analysis assumed rational expectations, abstracting from the possibility that private agents may perceive a different deficits-inflation relationship than the actual one (e.g., as in Bigio, Caramp and Silva, 2024) or may be learning about this relationship from past data (e.g., as in Eusepi and Preston, 2018); extending the analysis to account for these possibilities is another open question. Finally, a large literature estimates fiscal influences on inflation through the lens of RANK-FTPL (e.g., Bianchi and Ilut, 2017; Smets and Wouters, 2024); our work suggests revisiting these estimates through the lens of HANK.

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Appendices for:
Deficits and Inflation: HANK meets FTPL

This Appendix contains further material for the article “Deficits and Inflation: HANK meets FTPL”. We provide: (i) supplementary details for our baseline model environment (Section 2) and its various extensions (Section 5); (ii) supplementary model details, additional analysis, and alternative results for our quantitative investigations in Section 6; and (iii) all proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by “A.”—“C.” refer to the main article.

A Supplementary theoretical details and extensions

Appendix A.1 provides further details for the headline environment of Section 2, with Appendix A.2 zeroing in on the uniqueness of the steady state around which we (log-)linearize. Appendix A.4 similarly provides further details for the extended model with long-term debt (see Section 5.1), including a version of our HANK-FTPL equivalence result for general monetary policy.

Throughout, we will use uppercase variables to indicate levels; unless indicated otherwise, lowercase variables denote log-deviations from the economy's deterministic steady state. We log-linearize around a deterministic steady state in which inflation is zero ($\Pi^{ss} = 1$), real allocations are given by their flexible-price counterparts (e.g., Y^{ss} equals flexible-price output), and the real debt burden is constant at some given level $D^{ss} \geq 0$. As discussed below, our assumptions on annuities and the social fund ensure that $R^{ss} = \frac{1}{\beta} > 1$, and that steady-state taxes satisfy $T^{ss} = (1-\beta)D^{ss}$. While we will throughout focus on the empirically relevant scenario with $D^{ss} > 0$, we wish to also accommodate $D^{ss} = 0$, so we let $d_t \equiv (D_t - D^{ss})/Y^{ss}$, $t_t \equiv (T_t - T^{ss})/Y^{ss}$, and $a_{i,t} = \left(A_{i,t} - A^{ss}\right)/Y^{ss}$—i.e., we measure fiscal variables (and so also household wealth) in terms of absolute deviations (rather than log-deviations) from steady state, scaled by steady-state output. Otherwise, lowercase variables denote (log-)deviations from the steady state.

A.1 Environment

We here state the non-linear versions of all model equations.

Aggregate demand. The household block is the same as in Angeletos, Lian and Wolf (2024), which is restated here for completeness. The economy is populated by a unit continuum of households. A household survives from one period to the next with probability $\omega \in (0,1]$ and is replaced by a new one whenever it dies. Households have standard separable preferences regarding consumption and labor, and do not consider the utility of future households that replace them. The expected utility of any (alive) household i in period $t \in \{0,1,\ldots\}$ is hence

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left[u(C_{i,t+k}) - v(L_{i,t+k})\right]\right], \tag{29}$$

where $C_{i,t+k}$ and $L_{i,t+k}$ denote household i's consumption and labor supply in period $t+k$ (conditional on survival), $u(C) \equiv \frac{C^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$, $v(L) = \iota\frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Households can save and borrow through an actuarially fair, risk-free, nominal annuity, backed by government bonds. Conditional on survival, households receive a nominal return I_t/ω, where I_t is the nominal return on government bonds. Households furthermore receive labor income and dividend income $W_t L_{i,t}$ and $Q_{i,t}$ (both in real terms), and pay taxes. The real tax payment $T_{i,t}$ depends on both the individual's income and aggregate fiscal conditions:

$$T_{i,t} = \tau_y Y_{i,t} + \bar{T} - \mathscr{E}_t + \tau_d \left(D_t - D^{ss} + \mathscr{E}_t\right), \tag{30}$$

where $Y_{i,t} \equiv W_t L_{i,t} + Q_{i,t}$ is the household's total real income, $\tau_y \in [0,1)$ captures the rate of a proportional tax on her total income, $\bar{T} = T^{ss} - \tau_y Y^{ss}$ is set to guarantee budget balance at steady state, $\mathscr{E}_t$ is a mean-zero and i.i.d. deficit shock (e.g., issuance of stimulus checks), and $\tau_d \in [0,1)$ is a scalar that parameterizes the speed of fiscal adjustment.[34]

Finally, old households make contributions to a "social fund" whose proceeds are distributed to newborn households. We use $S_{i,t}$ to denote the transfer from or contribution to the fund, with $S_{i,t} = S^{\text{new}} = D^{ss} > 0$ for newborns and $S_{i,t} = S^{\text{old}} = -\frac{1-\omega}{\omega} D^{ss} < 0$ for old households. This guarantees $(1-\omega)S^{\text{new}} + \omega S^{\text{old}} = 0$, ensuring that the fund is balanced. The fund thus ensures that all cohorts, regardless of their age, enjoy the same wealth and hence consumption in steady state. This simplifies aggregation and implies that the steady state of our model is the same as its RANK counterpart. In particular, the social fund guarantees—together with the annuities, which offset mortality risk—that the steady-state rate of interest (in the steady state around which we log-linearize) is β^{-1} (thus "$r > g$").

Putting everything together, the date-t budget constraint of household i is given as

$$A^{\text{nominal}}_{i,t+1} = \underbrace{\frac{I_t}{\omega}}_{\text{annuity}} \left(A^{\text{nominal}}_{i,t} + P_t \cdot \left(\underbrace{W_t L_{i,t} + Q_{i,t}}_{Y_{i,t}} - C_{i,t} - T_{i,t} + S_{i,t}\right)\right), \tag{31}$$

where $A^{\text{nominal}}_{i,t}$ denotes household i's nominal wealth at the beginning of date t (exclusive of social fund payments) and P_t is the date-t price level. We use $A_{i,t} = \frac{A^{\text{nominal}}_{i,t}}{P_t} + S_{i,t}$ to denote the household's real wealth (inclusive of social fund payments). We furthermore assume that all households receive identical shares of dividends, and abstract from heterogeneity in labor supply, with labor supply intermediated by labor unions that demand identical hours worked from all households $L_{i,t} = L_t$.[35] The unions bargain on behalf of those households, equalizing the (post-tax) real wage and the average marginal rate of substitution between consumption and labor supply; i.e., we have that

$$(1-\tau_y)W_t = \frac{\iota L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-\frac{1}{\sigma}} di}. \tag{32}$$

Together, all households receive the same income and face the same taxes, $Y_{i,t} = Y_t$ and $T_{i,t} = T_t$.

[34] After (log-)linearization and aggregation, (30) becomes the tax rule (8) in the main text, where $\varepsilon_t \equiv \mathscr{E}_t / Y^{ss}$.

[35] This assumption simplifies the analysis by avoiding deficit-driven heterogeneity in the labor supply and income of different generations, without changing the essence of our results.

Aggregate supply. Log-linearizing (32),

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t. \tag{33}$$

Together with market clearing ($c_t = y_t$) and technology ($y_t = \ell_t$), this pins down the real wage as $w_t = \left(\frac{1}{\varphi}+\frac{1}{\sigma}\right)y_t$. Firm optimality pins down the optimal reset price as a function of current and expected future real marginal costs (wages), and thus also inflation. Together, the aggregate supply of the economy can be summarized by the familiar NKPC (2), where $\kappa = \frac{(1-\theta)(1-\beta\theta)\left(\frac{1}{\varphi}+\frac{1}{\sigma}\right)}{\theta} \geq 0$ and $1-\theta$ is the Calvo reset probability.

Fiscal policy. The government issues non-contingent, short-term, nominal debt. Let B_t denote the level of nominal public debt outstanding at the beginning of period t and P_t the nominal price level. In levels, the government's flow budget constraint is

$$B_{t+1} = I_t\left(B_t - P_t T_t\right),$$

where $T_t \equiv \int T_{i,t}di$ is real tax revenue (also, the real primary surplus) in date t and I_t is the gross nominal rate between dates t and $t+1$. We let $D_t \equiv B_t/P_t$ denote the *real* value of public debt, $\Pi_{t+1} \equiv P_{t+1}/P_t$ be the realized inflation between t and $t+1$, and $R_t \equiv I_t/\mathbb{E}_t\left[\Pi_{t+1}\right]$ be the (expected) real interest rate at t. We can rewrite the government budget in *real* terms as

$$D_{t+1} = R_t\left(D_t - T_t\right)\left(\frac{\mathbb{E}_t\left[\Pi_{t+1}\right]}{\Pi_{t+1}}\right).$$

This underscores how an inflation surprise between t and $t+1$ erodes the real value of the outstanding nominal debt, thus reducing the tax revenue needed to balance the government budget. Rewriting in log-linearized terms yields (4). Finally we assume that the government also needs to satisfy a non-Ponzi condition: $\lim_{k\to\infty}\mathbb{E}_t\left[\frac{D_{t+k+1}}{\prod_{l=0}^{k}(I_{t+l}/\Pi_{t+l+1})}\right] = 0$.

Total tax revenue T_t is determined as a function of exogenous shocks and endogenous outcomes. For each household i, the tax payment $T_{i,t}$, given by (30), consists of two components. First, there is a proportional tax $\tau_y \in [0,1)$ on household total income. This tax is distortionary but time-invariant. Second, there is a time-varying lump-sum component, which includes any initial fiscal stimulus (i.e., the exogenous deficit shock $\mathscr{E}_t$), subsequent tax hikes used to help return government debt to steady state (i.e., $\tau_d\left(D_t - D^{ss} + \mathscr{E}_t\right)$, where $\tau_d \in (0,1)$ parameterizes the speed of fiscal adjustment), and $\bar{T} = T^{ss} - \tau_y Y^{ss}$, which guarantees budget balance at steady state. Putting everything together and aggregating, total taxes are set as follows:

$$T_t = \tau_y Y_t + \bar{T} - \mathscr{E}_t + \tau_d\left(D_t - D^{ss} + \mathscr{E}_t\right). \tag{34}$$

Rewriting in log-linearized terms yields (8).

Monetary policy. The monetary authority sets I_t, the nominal rate of interest, according to the following policy rule:

$$\frac{I_t}{\Pi^{ss}} = R^{ss}\mathbb{E}_t\left[\frac{\Pi_{t+1}}{\Pi^{ss}}\right]\left(\frac{Y_t}{Y^{ss}}\right)^{\phi}, \tag{35}$$

for some $\phi \in \mathbb{R}$. We abstract from the zero lower bound. Rewriting in log-linearized terms yields (9). For our main analysis in Section 4, we let monetary policy be "neutral" in the sense that $\phi = 0$; that is, the expected real rate is kept fixed. In Section 5, we extend our analysis to allow for interest rate feedback with $\phi \neq 0$.

A.2 Steady state

This section discusses conditions under which the deterministic of our model (around which we log-linearize) is unique. We note that, as usual, validity of our log-linearized analysis requires *local* uniqueness of this deterministic steady state.

Steady state definition. We begin by stating the equations that characterize deterministic steady states in our model setting. In the interest of generality we for now do so using our extended monetary policy rule of (allowing $\phi \neq 0$ in (35)), which nests the baseline one ($\phi = 0$ in (35)).

In a deterministic steady state the fiscal shock is equal to zero at all times, $\mathcal{E}_t = 0$ for all t, and we denote steady-state quantities and prices by $\{Y^*, C^*, \Pi^*, R^*, I^*, T^*, D^*, A^*\}$ representing aggregate output, consumption, inflation, real and nominal interest rates, the real value of public debt, real tax revenue, and real household saving at the steady state. This steady state can potentially differ from the conventional steady state around which we log-linearize, which is indexed by the superscript "ss" and features zero inflation ($\Pi^{ss} = 1$), real allocations equal to their flexible-price counterparts, a real debt burden constant at a given level $D^{ss} \geq 0$, and the gross real rate given by $R^{ss} = \frac{1}{\beta}$, where $\beta \in (0,1)$ is the household discount factor. We drop the expectations operator and focus on the case of perfect foresight because we analyze a deterministic steady state.

For any deterministic steady state, consumer optimality implies

$$C_{i,t} = \left(\beta R^*\right)^{-\sigma} C_{i,t+1},$$

Together with household's budget (31), we have

$$C_{i,t} = \frac{A_{i,t} + \sum_{k=0}^{\infty}\left((R^*)^{-1}\omega\right)^k\left(Y_{t+k} - T_{t+k} + S_{i,t+k}\right)}{1 + \sum_{k=1}^{\infty}\left(\left(\beta R^*\right)^{\sigma}(R^*)^{-1}\omega\right)^k}. \tag{36}$$

Note that, for all households i alive at period t, their average social fund transfer at t is zero, because a fraction $1-\omega$ is newborn at t receives D^{ss}, while the remaining fraction, ω, were born before t and pays $\frac{1-\omega}{\omega}D^{ss}$. Their average social fund transfer at $t+k$ for $k \geq 1$ is $-\frac{1-\omega}{\omega}D^{ss}$, as all households alive at period t pay $\frac{1-\omega}{\omega}D^{ss}$ at $t+k$. Using this property and aggregating, we arrive at the aggregate demand relation. At the steady state, it is given by

$$C^* = \left(1-\beta^\sigma \left(R^*\right)^{\sigma-1}\omega\right)\left(A^* + \frac{Y^*-T^*}{1-\omega\left(R^*\right)^{-1}} - \frac{(1-\omega)\left(R^*\right)^{-1}}{1-\omega\left(R^*\right)^{-1}}D^{ss}\right). \tag{37}$$

We are now ready to state the formal definition of a deterministic steady state of this model.

Definition 2. A tuple $\{Y^*, C^*, \Pi^*, R^*, I^*, T^*, D^*, A^*\}$ is a deterministic steady if and only if the following conditions are satisfied:

1. Aggregate demand is given by (37).

2. Aggregate supply is given by
$$\Pi^* = \mathscr{P}\left(Y^*\right), \tag{38}$$
where $\mathscr{P}(\cdot)$ strictly increases in Y^* and satisfies $\mathscr{P}(Y^{ss}) = \Pi^{ss} = 1$, with Y^{ss} equal to the flexible-price output level.

3. The goods and asset markets clear, i.e.,
$$Y^* = C^* \quad \text{and} \quad A^* = D^*. \tag{39}$$

4. Monetary policy satisfies (35), i.e.,
$$\frac{I^*}{\Pi^*} = R^* = \frac{1}{\beta}\left(\frac{Y^*}{Y^{ss}}\right)^\phi. \tag{40}$$

5. The government budget satisfies
$$D^* = R^*\left(D^* - T^*\right). \tag{41}$$

6. Fiscal policy satisfies (34), i.e.,
$$T^* = \tau_y Y^* + T^{ss} - \tau_y Y^{ss} + \tau_d\left(D^* - D^{ss}\right). \tag{42}$$

Zero-inflation steady state. We begin our analysis by first of all constructing the conventional, zero-inflation steady-state $\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\}$ (i.e., the steady state around which we log-linearize) and verify that it indeed satisfies Definition 2. In this steady state, inflation and interest rates are given by $\Pi^{ss} = 1$ and $R^{ss} = I^{ss} = 1/\beta$, respectively. The real value of public debt D^{ss} is exogenously given, real tax revenue is $T^{ss} = \left(1-\beta\right)D^{ss}$, and finally output is equal to its flexible-price counterpart Y^{ss}. From labor supply (32), Y^{ss} is given by

$$\left(1-\tau_y\right)\frac{\epsilon^{CES}-1}{\epsilon^{CES}} = \frac{\iota\left(Y^{ss}\right)^{\frac{1}{\varphi}}}{\left(Y^{ss}\right)^{-\frac{1}{\sigma}}} \Longrightarrow Y^{ss} = \left(\frac{\left(1-\tau_y\right)\left(\epsilon^{CES}-1\right)}{\iota\epsilon^{CES}}\right)^{\frac{\sigma\varphi}{\sigma+\varphi}},$$

where we use the fact that $W_{i,t} = W^{ss} = \frac{\epsilon^{CES}-1}{\epsilon^{CES}}$ ($\epsilon^{CES} > 1$ is the elasticity of substitution among differentiated goods producers), $L_t = Y_t$ from linear technology, and $C_{i,t} = Y^{ss}$ from market clearing. Consumption and real household saving are given by $C^{ss} = Y^{ss}$ and $A^{ss} = D^{ss}$. As constructed, the tuple $\left\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\right\}$ satisfies (37) – (42) and constitutes a deterministic steady state of our model, as claimed.

It is furthermore straightforward to see that this conventional steady state is the unique steady state with zero inflation. First note that for any steady state with $\Pi^* = 1$, the NKPC (38) implies that $Y^* = Y^{ss}$, and the monetary policy rule (40) implies $R^* = R^{ss}$. Aggregate demand (37), together with market clearing (39), then simplifies to $0 = \left(1-\beta\omega\right) D^* - T^* - \beta\left(1-\omega\right) D^{ss}$. Next, from the government budget (41), we have $T^* = \left(1-\beta\right) D^*$. We thus know that $D^* = D^{ss}$ and $T^* = T^{ss}$. Finally, from monetary policy (40) and market clearing (39), we know that $A^* = A^{ss}$, $I^* = I^{ss}$, and $C^* = C^{ss}$. It follows that indeed $\left\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\right\}$ is the only steady state with zero inflation.

As mentioned above, for our log-linearization around this conventional steady state and the accompanying local analysis to be valid, we need to show that this zero-inflation steady state is locally unique. In fact, for the baseline case with neutral monetary policy ($\phi = 0$ in (35)), we will establish something stronger: the conventional steady state is even globally unique. Afterwards we establish that, for the general case with monetary policy feedback ($\phi \neq 0$ in (35)), the conventional steady state is still locally unique, as required. We also discuss how global uniqueness can be ensured.

Baseline monetary policy ($\phi = 0$ in (35)). For our main analysis in Section 4, we let monetary policy be "neutral" in the sense that $\phi = 0$. In this case, it is straightforward to establish that the conventional steady state $\left\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\right\}$ is in fact globally unique.

Specifically, from (40), we know that $R^* = \frac{1}{\beta} = R^{ss}$. Aggregate demand (37), together with market clearing (39), then simplifies to $0 = \left(1-\beta\omega\right) D^* - T^* - \beta\left(1-\omega\right) D^{ss}$. Next, from the government budget (41), we have $T^* = \left(1-\beta\right) D^*$. Together, we know that $D^* = D^{ss}$ and $T^* = T^{ss}$. Together with fiscal policy (42), we have $Y^* = Y^{ss}$. From the NKPC (38), we have $\Pi^* = \Pi^{ss} = 1$. From monetary policy (40) and market clearing (39), we know that $A^* = A^{ss}$, $I^* = I^{ss}$,and $C^* = C^{ss}$. This proves that the conventional steady state $\left\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\right\}$ is globally unique, as claimed.

General monetary policy feedback ($\phi \neq 0$ in (35)). We now establish the local uniqueness of the conventional steady state in the general case with a monetary feedback rule that features $\phi \neq 0$. From the government budget (41), we have

$$T^* = \left(1 - \left(R^*\right)^{-1}\right) D^*. \tag{43}$$

Together with aggregate demand (37) and market clearing (39), we have

$$Y^* = \left(1-\beta^\sigma \left(R^*\right)^{\sigma-1}\omega\right)\left(D^* + \frac{Y^* - \left(1-(R^*)^{-1}\right)D^*}{1-\omega\,(R^*)^{-1}} - \frac{(1-\omega)\,(R^*)^{-1}}{1-\omega\,(R^*)^{-1}}D^{ss}\right),$$

which can be rewritten as

$$D^* = D^{ss} + \frac{\left(\beta^\sigma\,(R^*)^\sigma - 1\right)\omega}{(1-\omega)\left(1-\beta^\sigma\,(R^*)^{\sigma-1}\omega\right)}Y^*. \tag{44}$$

From (43) and the fiscal policy (42), we have

$$\tau_y\left(Y^* - Y^{ss}\right) + (1-\beta)\,D^{ss} + \tau_d\left(D^* - D^{ss}\right) = \left(1-\left(R^*\right)^{-1}\right)D^*,$$

which can be rewritten as

$$\left(1-\tau_d - \left(R^*\right)^{-1}\right)D^* = \tau_y\left(Y^* - Y^{ss}\right) + \left(1-\tau_d-\beta\right)D^{ss}. \tag{45}$$

Using (44), (45), and monetary policy (40), we arrive at the following equation that characterizes the steady-state real interest rate R^*:

$$g\left(R^*\right) \equiv \tau_y\left(\left(\beta R^*\right)^{1/\phi} - 1\right)Y^{ss} - \frac{\beta R^* - 1}{R^*}D^{ss} - \frac{\left(1-\tau_d-(R^*)^{-1}\right)\left(\beta^\sigma\,(R^*)^\sigma - 1\right)\omega}{(1-\omega)\left(1-\beta^\sigma\,(R^*)^{\sigma-1}\omega\right)}\left(\beta R^*\right)^{1/\phi}Y^{ss} = 0. \tag{46}$$

At R^{ss}, we have

$$g'\left(R^{ss}\right) = g'\left(\frac{1}{\beta}\right) = \beta\left(\frac{\tau_y}{\phi} - \frac{\omega\sigma\left(1-\tau_d-\beta\right)}{(1-\omega)\left(1-\beta\omega\right)}\right)Y^{ss} - \beta^2 D^{ss}. \tag{47}$$

It follows that, as long as ϕ satisfies $\phi \neq \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}} + \sigma(1-\tau_d-\beta)}$, $g'\left(R^{ss}\right)$ in (47) is non-zero and so the steady state $\left\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\right\}$ is locally unique. In other words, the conventional steady state is *generically* locally unique; the only exception is a knife-edge combination of ϕ and τ_d so that the above display equals zero.

For our theoretical result with general monetary policy feedback in Proposition 5, we restrict $\phi \in \left(\underline{\phi}, \bar{\phi}\right)$, where the thresholds are defined in (74), as well as $\tau_d = 0$. In this case, for $\phi < 0$, $g'\left(R^{ss}\right) < 0$ because $\tau_d = 0$. For $\phi \in \left(0, \bar{\phi}\right)$, from the threshold $\bar{\phi}$ in (74),

$$\begin{aligned}
g'\left(R^{ss}\right) &> \frac{\beta\omega}{\left(1-\beta\omega\right)(1-\omega)}\left(\sigma\left(1-\beta\right) + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}} - \sigma\left(1-\tau_d-\beta\right)\right)Y^{ss} - \beta^2 D^{ss} \\
&= \frac{\beta\omega}{\left(1-\beta\omega\right)(1-\omega)}\left(\sigma\tau_d + \frac{\left(1-\beta\omega\right)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}\right)Y^{ss} - \beta^2 D^{ss} \\
&= \frac{\sigma\beta\omega\tau_d}{\left(1-\beta\omega\right)(1-\omega)}Y^{ss} = 0.
\end{aligned}$$

We have thus established that, for the environment relevant for Proposition 5, $g'\left(R^{ss}\right)$ in (47) is guaranteed to be non-zero and so the steady state $\left\{Y^{ss}, C^{ss}, \Pi^{ss}, R^{ss}, I^{ss}, T^{ss}, D^{ss}, A^{ss}\right\}$ is locally unique (and not just generically), as claimed.

Finally, we note that global steady state uniqueness with general monetary policy feedback can be easily achieved through slight changes in our assumptions on policy (without changing the log-linearized relation (10)). Perhaps most transparently, if the monetary authority commits to the steady-state real interest rate R^{ss} and only responds to output deviations from steady state, then the steady state is again globally unique. The argument follows because the proof above for global steady-state uniqueness under the baseline monetary policy applies verbatim as long as the steady real interest rate is given by R^{ss}. And since the log-linearized real interest rate is still characterized by (10), our local analysis remains unaffected.

A.3 Derivation of the aggregate consumption function (1)

This section provides detailed derivations of the aggregate consumption function (1). Consider any (alive) household i in period $t \in \{0,1,\dots\}$, and let $C_{i,t+k}$ and $A_{i,t+k} = \frac{A^{\text{nominal}}_{i,t+k}}{P_t} + S_{i,t+k}$ denote household i's consumption and real wealth (measured at the beginning of the period, inclusive of social fund payments) in period $t+k$ (conditional on survival) for $k \geq 0$. We can then rewrite the household's budget in (31) as, for $k \geq 0$,

$$A_{i,t+k+1} = \frac{1}{\Pi_{t+1}} \frac{I_t}{\omega} \left(A_{i,t+k} + Y_{t+k} - C_{i,t+k} - T_{t+k}\right) - \frac{1-\omega}{\omega} D^{ss},$$

where we use $Y_{i,t+k} = Y_{t+k}$, $T_{i,t+k} = T_{t+k}$, and $S_{i,t+k+1} = -\frac{1-\omega}{\omega} D^{ss} < 0$. Log-linearizing the household budget yields, for $k \geq 0$,

$$a_{i,t+k+1} = \frac{1}{\beta\omega} \left(a_{i,t+k} + y_{t+k} - c_{i,t+k} - t_{t+k} + \beta \frac{A^{ss}}{Y^{ss}} \left(r_{t+k} + \left(\mathbb{E}_{t+k}\left[\pi_{t+k+1}\right] - \pi_{t+k+1}\right)\right)\right),$$

where we use the fact that steady-state real household wealth A^{ss} equals the steady-state real value of government debt D^{ss}.

Together with the household's transversality condition, we can express the household's budget in present-value form:

$$\sum_{k=0}^{\infty} (\beta\omega)^k \mathbb{E}_t\left[c_{i,t+k}\right] = a_{i,t} + \sum_{k=0}^{\infty} (\beta\omega)^k \mathbb{E}_t\left[y_{t+k} - t_{t+k}\right] + \beta \frac{A^{ss}}{Y^{ss}} \sum_{k=0}^{\infty} (\beta\omega)^k \mathbb{E}_t\left[r_{t+k}\right]. \tag{48}$$

Household i's optimal consumption in period $t+k$ for $k \geq 0$ implies

$$u'\left(C_{i,t+k}\right) = \mathbb{E}_t\left[\beta\omega \frac{I_{t+k}}{\omega \Pi_{t+k+1}} u'\left(C_{i,t+k+1}\right)\right], \tag{49}$$

which, after log-linearization, becomes

$$c_{i,t+k} = -\sigma r_{t+k} + \mathbb{E}_t\left[c_{i,t+k+1}\right]. \tag{50}$$

Together with the intertemporal budget (48), we have

$$c_{i,t} = (1-\beta\omega)\left(a_{i,t} + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right) - \beta\left(\sigma\omega - (1-\beta\omega)\frac{A^{ss}}{Y^{ss}}\right)\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right]. \tag{51}$$

Aggregating across households yields the aggregate consumption function (1) in the main text.

A.4 Long-term government debt and interest rate feedback

We first provide the missing details for our extended environment with long-term government debt. We then discuss how our "HANK-meets-FTPL" results here extend to the case with interest rate feedback (i.e., $\phi \neq 0$).

Details about the environment with long-term bonds. The fiscal authority issues nominal government bonds, whose maturity is parameterized by $\delta \in [0,1]$. Each unit of government debt outstanding at t pays \$1 at t, and $\$\delta^k$ at $t+k$ for all $k \geq 1$. We use J_t to denote the units of government debt outstanding at the start of period t, and Q_t to denote the post-coupon dollar price at the end of period t for a unit of government debt that pays \$1 at $t+1$ and $\$\delta^{k+1}$ at $t+k+1$. As a result, $B_t = J_t(1+\delta Q_t)$ captures the nominal value of government debt outstanding at the beginning of period t. The government budget constraint (in levels) can then be written as

$$J_{t+1} = \left(\frac{J_t \times 1 - P_t T_t}{Q_t} + \delta J_t\right), \tag{52}$$

where P_t is the price level at t and T_t is total tax revenue at t. Rewriting (52) in terms of the nominal value of government debt B_t, we have

$$B_{t+1} = \left(\frac{1+\delta Q_{t+1}}{Q_t}\right)(B_t - P_t T_t), \tag{53}$$

where $I^g_{t+1} = \left(\frac{1+\delta Q_{t+1}}{Q_t}\right)$ is the realized nominal rate of return on government bonds between dates t and $t+1$. Finally, the monetary authority sets the date-t expected nominal rate of return on government debt as $I_t = \mathbb{E}_t\left[I^g_{t+1}\right]$.

We use $R_t \equiv \mathbb{E}_t\left[\frac{1+\delta Q_{t+1}}{Q_t \Pi_{t+1}}\right]$ to denote the expected real return on government bonds, $\Pi_{t+1} = P_{t+1}/P_t$ to denote the inflation from t to $t+1$, and $D_t \equiv B_t/P_t$ to denote the *real* value of total public debt that is outstanding at the beginning of period t, which by market-clearing equals total real household saving A_t. Re-writing (53) in real terms, log-linearizing, iterating forward, and imposing the household transversality condition, we have that the nominal price of the long-term bond is given by the negative of the present value of nominal short-term rates (or equivalently inflation plus real short-term rates), discounted by $\beta\delta$:

$$q_t = -\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\delta)^k(\pi_{t+k+1} + r_{t+k})\right]. \tag{54}$$

Next, re-writing (53) in real terms and linearizing, we obtain

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\text{expected debt burden tomorrow } \mathbb{E}_t[d_{t+1}]} \underbrace{- \frac{D^{ss}}{Y^{ss}}\left(\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}] - \beta\delta\left(q_{t+1} - \mathbb{E}_t\left[q_{t+1}\right]\right)\right)}_{\text{debt erosion due to inflation and bond price surprise}} \tag{55}$$

together with

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 + \beta\delta\frac{D^{ss}}{Y^{ss}} q_0. \tag{56}$$

Finally, using (54) in (55) and (56), we arrive at (23) and

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0^{\delta} - \frac{D^{ss}}{Y^{ss}} r_0^{\delta}, \tag{57}$$

where $\{\pi_t^{\delta}, r_t^{\delta}\}_{t=0}^{\infty}$ is defined in (24). Similar to the baseline analysis, the flow budget constraint (56) and (57) together with the government's no-Ponzi condition imply the corresponding government intertemporal budget constraint (6).

HANK meets FTPL with interest rate feedback. Our "HANK-meets-FTPL" result extends naturally to the case with interest rate feedback, as in our baseline analysis with short-term government debt. Proposition 12 states the formal result, with the proof provided in Appendix C.

Proposition 12. *Suppose that $\omega < 1$, $\tau_y > 0$, $\delta > 0$, and $\phi < \bar{\phi}$, and consider the HANK equilibrium that obtains when $\tau_d = 0$. Select any realization of the initial fiscal shock ε_0, abstract from any future shocks, and let $\{r_t^{HANK}\}_{t=0}^{\infty}$ be the equilibrium path of the (expected) real rate obtained in this equilibrium. Finally, consider an analogous RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), and monetary policy follows the passive rule $r_t = r_t^{HANK}$. Then, the comparison established in Proposition 8 continues to hold, i.e.,*

$$\pi_{\varepsilon}^{\delta,HANK} < \pi_{\varepsilon}^{\delta,FTPL}. \tag{58}$$

B Additional results for quantitative analysis

In Appendix B.1 we provide supplementary details for the alternative model variants analyzed in Section 6.3. In Appendices B.2 and B.3 we then report results from two further sets of experiments, supplementing our main analysis in Section 6.

B.1 Further model details

The model variants discussed in Section 6 alter the baseline environment along three margins: consumers, nominal rigidities, and policy. Our alterations along the pricing and policy margins were already described in detail in the main text, so we here just provide the missing details for the consumer block of the model.

The model variants with no cross-sectional heterogeneity in bond holdings or transfer receipts (or both) are self-explanatory: we set $A_i^{SS} = A^{SS}$ and $\varepsilon_i = \varepsilon$ for all groups i. For the behavioral model variant, we add a sticky information friction, modeled as in Angeletos, Lian and Wolf (2024, Appendix B.2), the behavioral coefficient set to $\theta = 0.95$. For the single-type OLG model variant, we set $\omega = 0.8/\beta$. Finally, for the HANK variant, we consider the exact same heterogeneous-agent block as in Angeletos, Lian and Wolf (2024, Appendix E.6.1), but with one important change in the model calibration: we set total liquid household wealth to $A^{SS} = D^{SS} = 1.79$, exactly as in our benchmark model. Even with this slightly elevated liquid wealth level we still obtain a large quarterly MPC of around 0.24.

B.2 Deficits and inflation with real rate response

To construct Figure 4 we assumed a fixed real rate path, i.e., $\phi = 0$. We now ask what happens if instead (expected) real interest rates follow the exogenous path

$$r_t = \rho^t r_0,$$

i.e., the fiscal stimulus is accompanied by a particular movement in real rates. Specifically, we consider a one percent fiscal deficit shock, and then set $r_0 = -0.15$ and $\rho = 0.6$—a meaningful and persistent monetary easing. In particular, relative to our baseline exercise, this almost doubles the size of the initial fiscal boom, and because of intertemporal substitution makes it front-loaded also in FTPL. Results are reported in Figure 7.

The main takeaway from the figure is that our headline results are unchanged relative to Figure 4. A real rate cut now makes the inflation burst front-loaded also in FTPL, but it remains *more* front-loaded in our HANK model variants.

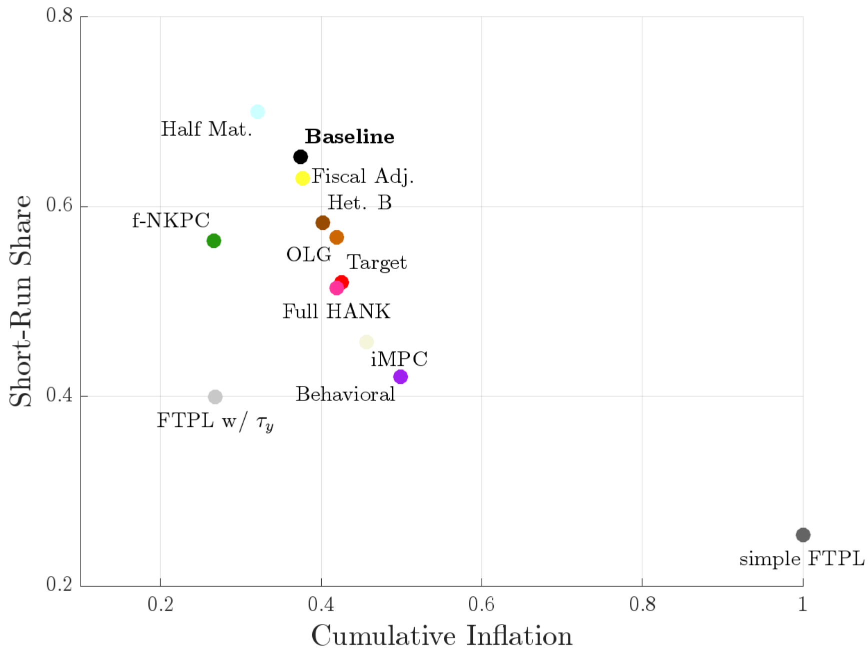

Figure 7: Cumulative inflation response and short-run response share to a date-0 deficit shock of size D^{ss}/Y^{ss} accompanied by a transitory real rate cut, for different model variants, indicated by dots.

B.3 Perfectly anticipated Covid stimulus

For our baseline exercise in Figure 6 we assumed that the two parts of the fiscal stimulus—reflecting the CARES and ARP Acts, respectively—arrived as surprises. Here we ask what happens if instead the ARP Act was perfectly anticipated at the time of CARES Act.

Results are displayed in Figure 8. We say that our conclusions are qualitatively and quantitatively robust to alternative assumptions on household expectations.

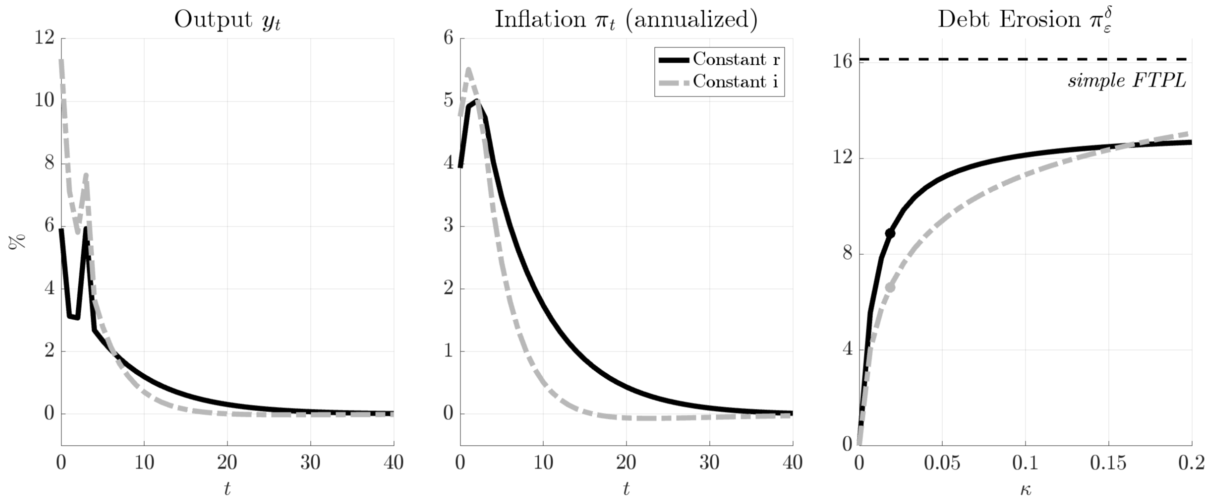

Figure 8: Output and inflation impulse responses (left and middle) to the post-Covid fiscal deficit shock (see text) and π_ε^δ (right) as a function of κ, with perfect foresight and under two different assumptions on the monetary policy reaction: fixed real rates (black) and fixed nominal rates (grey).

C Proofs

C.1 Proof of Proposition 1

1. From (11), we have
$$y_t = -\sigma\phi y_t + \mathbb{E}_t\left[y_{t+1}\right]. \tag{59}$$
For any $\phi > 0$ ("active monetary policy"), the unique bounded solution to equation (59) is $y_t = 0$ for $t \geq 0$. From the NKPC (3), we can find that $\pi_t = 0$ for $t \geq 0$. One can then find $\{d_t, r_t, t_t\}_{t=0}^{\infty}$ from the policy block (4), (5), (8), and (10). From (4), (8), and (10), we know that, for $t \geq 0$ and $k \geq 1$,
$$\mathbb{E}_t\left[\beta^k d_{t+k}\right] = (1-\tau_d)^k (d_t + \varepsilon_t).$$
As a result, $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$. Together with (4), we know that (6) holds. We can find $\{c_t, a_t\}_{t=0}^{\infty}$ from goods and asset market clearing and verify that (1) holds. As a result, $\left\{y_t, d_t, \pi_t, c_t, a_t, t_t, r_t\right\}_{t=0}^{\infty}$ is an equilibrium according to Definition 1.

2. From (11), we have (59). Any bounded solution $\left\{y_t\right\}_{t=0}^{\infty}$ of (59) must satisfy
$$y_t = \varrho y_{t-1} + \eta_t, \tag{60}$$
with $\varrho \equiv 1 + \sigma\phi \in (0,1]$, where η_t is bounded and $\mathbb{E}_{t-1}\left[\eta_t\right] = 0$. From (4), (8), and (10), we know that, for $t \geq 0$ and $k \geq 1$,
$$\mathbb{E}_t\left[\beta^k d_{t+k}\right] = d_t + \varepsilon_t - \left(\tau_y - \beta\frac{D^{ss}}{Y^{ss}}\phi\right)\frac{1-\beta^k\varrho^k}{1-\beta\varrho} y_t. \tag{61}$$
Note that, in any equilibrium, (4) and (6) imply $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$. Because $\varrho \in (0,1]$ and $\beta \in (0,1)$, (61) then implies $d_t + \varepsilon_t = \frac{\tau_y - \beta\frac{D^{ss}}{Y^{ss}}\phi}{1-\beta\varrho} y_t$. As a result,
$$d_t - \mathbb{E}_{t-1}\left[d_t\right] + \varepsilon_t = \frac{\tau_y - \beta\frac{D^{ss}}{Y^{ss}}\phi}{1-\beta\varrho}\left(y_t - \mathbb{E}_{t-1}\left[y_t\right]\right). \tag{62}$$
From (3), (4), (5), and (60), we know,
$$d_t - \mathbb{E}_{t-1}\left[d_t\right] = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{1-\beta\varrho}\eta_t \quad \text{and} \quad y_t - \mathbb{E}_{t-1}\left[y_t\right] = \eta_t.$$
Together with (62), we know that,
$$\eta_t = \frac{1-\beta\left(1+\sigma\phi\right)}{\tau_y + \left(\kappa - \beta\phi\right)\frac{D^{ss}}{Y^{ss}}}\varepsilon_t.$$
From (3),
$$\pi_t - \mathbb{E}_{t-1}\left[\pi_t\right] = \pi_{\varepsilon,0}^{FTPL}\cdot\varepsilon_t \qquad \text{with} \qquad \pi_{\varepsilon,0}^{FTPL} \equiv \frac{\kappa}{\tau_y + \left(\kappa-\beta\phi\right)\frac{D^{ss}}{Y^{ss}}},$$

which is (12). One can construct the rest of $\{y_t, d_t, \pi_t, c_t, a_t, t_t, r_t\}_{t=0}^{\infty}$ uniquely and verify it is an equilibrium by Definition (1): $\{d_t, r_t, t_t\}_{t=0}^{\infty}$ from the policy block (4), (5), (8), and (10) and $\{c_t, a_t\}_{t=0}^{\infty}$ from goods and asset market clearing.

C.2 Proof of Proposition 2

Given 11 and (15), we know that, for $t \geq H$,

$$y_t = -\sigma\phi' y_t + \mathbb{E}_t\left[y_{t+1}\right]. \tag{63}$$

Similar to Part 1 of Proposition 1, there exists a unique equilibrium. In this equilibrium, $y_t = \pi_t = 0$ for $t \geq H$. Backward induction based (8) and (10) for $t < H$ implies that $y_t = \pi_t = 0$ for all t and all realizations of uncertainty.

C.3 Proof of Proposition 3

Note that we restrict that $\omega \in (0,1)$, $\tau_y \in (0,1)$, and $\tau_d \in [0,1)$. Imposing $r_t = 0$ (fixed real rates), $y_t = c_t$ (goods market clearing), and $a_t = d_t$ (asset market clearing) in (1), we have, for all $t \geq 0$,

$$y_t = \left(1-\beta\omega\right)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}\left(\beta\omega\right)^k\left(y_{t+k} - t_{t+k}\right)\right]\right).$$

We now write it recursively using the government's flow budget (4). For all $t \geq 0$,

$$\begin{aligned}
y_t &= \left(1-\beta\omega\right)\left(y_t + d_t - t_t\right) + \beta\omega\mathbb{E}_t\left[y_{t+1} - \left(1-\beta\omega\right)\cdot d_{t+1}\right] \\
&= \left(1-\beta\omega\right)\left(y_t + d_t - t_t\right) + \beta\omega\mathbb{E}_t\left[y_{t+1} - \frac{1-\beta\omega}{\beta}\left(d_t - t_t\right)\right] \\
&= \frac{\left(1-\beta\omega\right)\left(1-\omega\right)}{\beta\omega}\left(d_t - t_t\right) + \mathbb{E}_t\left[y_{t+1}\right].
\end{aligned}$$

Applying the fiscal rule (8), we have, for all $t \geq 0$,

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(1-\tau_d\right)}{1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\left(d_t + \varepsilon_t\right) + \frac{1}{1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying period-t expectations $\mathbb{E}_t\left[\cdot\right]$ to (4), we have, for all $t \geq 0$,

$$\begin{pmatrix} \mathbb{E}_t\left[d_{t+1}\right] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1 + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y \end{pmatrix}\begin{pmatrix} d_t + \varepsilon_t \\ y_t \end{pmatrix} \tag{64}$$

The two eigenvalues of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}\left(1-\tau_d\right) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y\left(1-\omega\right)\right) + \frac{1}{\beta}\left(1-\tau_d\right) = 0,$$

with

$$\lambda_1 = \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2}$$
$$= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \tag{65}$$
$$> \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1$$

and

$$\lambda_2 = \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2}$$
$$= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \tag{66}$$
$$< \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\left|\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2} \leq 1,$$

with $\lambda_2 > 0$ too since $\lambda_1\lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $(1,\chi_1)'$ and $(1,\chi_2)'$ denote the eigenvectors[36] associated with $\lambda_1 > 1$ and $\lambda_2 \in (0,1)$, where $\chi_1 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_1} \neq 0$ and

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_2} > 0. \tag{67}$$

This means that any bounded path of $\{y_t\}_{t=0}^{+\infty}$ that satisfies (64) must the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\varepsilon_t),$$

where χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1), \tag{68}$$

and are continuous functions of $(\beta,\omega,\tau_y,\tau_d)$. In other words, any equilibrium must take the form of (17).

From (3) and (5), we can find d_0 as a function of the deficit shock ε_0:

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0[y_k] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0. \tag{69}$$

[36] Note that $\tau_y > 0$ implies that the first elements of the eigenvectors associated with λ_1 and λ_2 are non-zero, so we can normalize them to 1.

Similarly, for $t \geq 1$, from (3) and (4),

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t\left[y_{t+k}\right] - \mathbb{E}_{t-1}\left[y_{t+k}\right]\right)$$

$$= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_t - \mathbb{E}_{t-1}[d_t] + \varepsilon_t) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_t. \tag{70}$$

Together with (3) and (17), we find an equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$. In particular,

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{HANK}\cdot\varepsilon_t \qquad \text{with} \qquad \pi_{\varepsilon,0}^{HANK} \equiv \frac{\kappa\chi}{1-\beta\rho_d + \kappa\chi\frac{D^{ss}}{Y^{ss}}}.$$

We can then find $c_t = y_t$, $a_t = d_t$, and t_t from the fiscal rule (8), and the entire equilibrium path $\{y_t, d_t, \pi_t, c_t, a_t, t_t, r_t\}_{t=0}^{\infty}$ satisfying Definition 1. The uniqueness comes from the fact that χ and ρ_d are uniquely pinned down by (68). Finally, from (3) and (17), for all $k \geq 0$,

$$\pi_{\varepsilon,k}^{HANK} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t} = \rho_d^k\pi_{\varepsilon,0}^{HANK}. \tag{71}$$

C.4 Proof of Theorem 1

From (66), (67), and (68), we know that τ_d and χ are continuous in $\tau_d \in [0,1)$. From the second part of (17), we know that

$$\frac{\chi}{1-\beta\rho_d} = \frac{\chi}{\tau_d + \tau_y\chi}. \tag{72}$$

From (66) and (68), we know

$$\rho_d = \lambda_2 = f(a,b) \equiv \frac{a+b+1-\sqrt{(a+b-1)^2+4b}}{2} \tag{73}$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}} > 0$, we know that ρ_d is a decreasing and continuous function of $\tau_d \in [0,1)$. From (67) and (68), we then know $\chi = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y - \rho_d}$ is a decreasing and continuous function of $\tau_d \in [0,1)$. From (18), we know $\pi_{\varepsilon,0}^{HANK}$ is a decreasing and continuous function of $\tau_d \in [0,1)$. In particular,

$$\lim_{\tau_d\to 0^+}\pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{HANK}\Big|_{\tau_d=0}.$$

When $\tau_d = 0$, again using (67), and (68), we have

$$\left.\frac{\chi}{1-\beta\rho_d}\right|_{\tau_d=0} = \frac{1}{\tau_y} \quad \text{and} \quad \pi_{\varepsilon,0}^{HANK}\Big|_{\tau_d=0} = \frac{\kappa}{\tau_y + \kappa\frac{D^{ss}}{Y^{ss}}} = \pi_{\varepsilon,0}^{\text{FTPL}}.$$

This proves Parts 1 and 2 of Theorem 1. To prove Part 3, note that (67) and (68) imply that χ and ρ_d are independent of κ and $\frac{D^{ss}}{Y^{ss}}$. Therefore, (18) implies that $\pi_{\varepsilon,0}^{HANK}$ increases with κ and decreases with

$\frac{D^{ss}}{Y^{ss}}$. From (73), we know

$$\rho_d = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

which decreases with $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$, and hence decreases with τ_y. From the second part of (67), we then know χ decreases with τ_y. Therefore, (18) implies that $\pi_{\varepsilon,0}^{HANK}$decreases with τ_y.

C.5 Proof of Corollary 1

This follows directly from (19).

C.6 Proof of Proposition 4

Note that we restrict $\phi \in \left(\underline{\phi}, \bar{\phi}\right)$, where the thresholds are given by

$$\underline{\phi} \equiv -\frac{1}{\sigma} \quad \text{and} \quad \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma\left(1-\beta\right)+\frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}} < \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}. \tag{74}$$

Aggregating the individual demand relation (1), together with the government budget (4), and goods and asset market clearing, leads to the following recursive aggregate demand relation for all $t \geq 0$:

$$\begin{aligned} y_t &= \left(1-\beta\omega\right)\left(\mathbb{E}_t\left[\sum_{k=0}^{\infty}\left(\beta\omega\right)^k y_{t+k}\right]\right) - \beta\sigma\omega\mathbb{E}_t\left[\sum_{k=0}^{\infty}\left(\beta\omega\right)^k r_{t+k}\right] \\ &+ \left(1-\beta\omega\right)\left(\mathbb{E}_t\left[\sum_{k=0}^{\infty}\beta^k\left(1-\omega^k\right)\left(t_{t+k}-\beta\frac{D^{ss}}{Y^{ss}}r_{t+k}\right)\right]\right) \\ &= -\sigma r_t + \frac{\left(1-\beta\omega\right)\left(1-\omega\right)}{\beta\omega}\left(d_t - t_t + \beta\frac{D^{ss}}{Y^{ss}}r_t\right) + \mathbb{E}_t\left[y_{t+1}\right] \end{aligned} \tag{75}$$

Together with the baseline fiscal policy (8) and monetary policy (10), we arrive at the following aggregate demand relation for all $t \geq 0$:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(1-\tau_d\right)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\left(d_t+\varepsilon_t\right) + \frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying the period-t expectation operator $\mathbb{E}_t\left[\cdot\right]$ to (4), we have, for all $t \geq 0$,

$$\begin{pmatrix} \mathbb{E}_t\left[d_{t+1}\right] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega} \end{pmatrix}\begin{pmatrix} d_t+\varepsilon_t \\ y_t \end{pmatrix} \tag{76}$$

The two eigenvalues are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{\left(1-\beta\omega\right)\left(1-\omega\right)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta} = 0. \tag{77}$$

Because $\phi \in \left(-\frac{1}{\sigma}, \frac{\tau_y}{\beta \frac{D^{ss}}{Y^{ss}}}\right)$ and $\tau_d \in [0,1)$, we know that $\lambda_1 + \lambda_2 \geq 0$ and $\lambda_1 \lambda_2 \geq 0$, so $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. Moreover,

$$\begin{aligned}\lambda_1 &= \frac{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \sqrt{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 - 4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \\ &= \frac{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \sqrt{\left(1 + \sigma\phi - \frac{1-\tau_d}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 + 4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2},\end{aligned} \tag{78}$$

and

$$\begin{aligned}\lambda_2 &= \frac{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) - \sqrt{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 - 4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \\ &= \frac{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) - \sqrt{\left(1 + \sigma\phi - \frac{1-\tau_d}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 + 4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}.\end{aligned} \tag{79}$$

Moreover, for $\phi \in \left(-\frac{1}{\sigma}, \bar{\phi}\right)$,

$$\lambda_1 \geq \frac{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \left|\frac{1-\tau_d}{\beta} + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) - 1 - \sigma\phi\right|}{2} > \frac{1-\tau_d}{\beta} \tag{80}$$

When $\phi \in \left(-\frac{1}{\sigma}, 0\right)$, from (78) and (79),

$$\lambda_2 \leq \frac{\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) - \left|1 + \sigma\phi - \frac{1-\tau_d}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \leq 1 + \sigma\phi < 1$$

When $\phi \in [0, \bar{\phi})$, from (74), we have

$$\frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1}{\beta} - 1\right). \tag{81}$$

Hence

$$\lambda_2 = \frac{2\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}{\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}} \tag{82}$$

$$< \frac{2\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}{\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}$$

$$= \frac{\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \leq 1.$$

The last step is from the fact that

$$\frac{\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\frac{\sigma\phi}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}}{2} \leq 1 \Longleftrightarrow \frac{1-\beta-\tau_d+\sigma\phi}{\beta} \leq \sqrt{\left(\frac{1-\tau_d+\sigma\phi+\beta}{\beta}\right)^2-4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta}}$$

$$\Longleftrightarrow 4\frac{(1+\sigma\phi)(1-\tau_d)}{\beta} \leq \left(\frac{1-\tau_d+\sigma\phi+\beta}{\beta}\right)^2-\left(\frac{1-\beta-\tau_d+\sigma\phi}{\beta}\right)^2$$

$$\Longleftrightarrow (1+\sigma\phi)(1-\tau_d) \leq (1-\tau_d+\sigma\phi) \Longleftrightarrow 0 \leq \phi.$$

Let $(1,\chi_1)'$ and $(1,\chi_2)'$ denote the eigenvector associated with λ_1 and λ_2, where $\chi_1 = \frac{1-\tau_d-\beta\lambda_1}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}} < 0$

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)-\lambda_2} > 0. \tag{83}$$

Consider any equilibrium that takes the form of (17) for some scalars $\chi > 0$ and $\rho_d \in (0,1)$. Because $\chi_1 < 0$, we know that χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1), \tag{84}$$

which are continuous in $(\beta,\omega,\tau_y,\tau_d,\phi)$ and, in particular, in $\tau_d \in [0,1)$.[37] Furthermore, from (79), we know

$$\rho_d = \lambda_2 = f(a,b) \equiv \frac{a+b+1+\sigma\phi-\sqrt{(a+b-1-\sigma\phi)^2+4b(1+\sigma\phi)}}{2} \tag{85}$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1-\sigma\phi)}{2\sqrt{(a+b-1-\sigma\phi)^2+4b(1+\sigma\phi)}} > 0$, we know that ρ_d decreases with τ_d. From (83), we then know χ also decreases in τ_d.

Note that (69) and (70) remain to be true. We can then find a equilibrium path of $\{\pi_t,d_t,y_t\}_{t=0}^{+\infty}$

[37] Here, we have proved that, if $\phi \in (\underline{\phi},\bar{\phi})$, for all $\tau_d \in [0,1)$, an equilibrium of the form (17) exists and is unique. It is possible that equilibria of other forms exist.

where

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{HANK} \cdot \varepsilon_t \qquad \text{with} \qquad \pi_{\varepsilon,0}^{HANK} \equiv \frac{\kappa\chi}{1-\beta\rho_d+\kappa\chi\frac{D^{ss}}{Y^{ss}}},$$

where $\pi_{\varepsilon,0}^{HANK}$ is continuous in $(\beta,\omega,\tau_y,\tau_d,\phi)$ and, in particular, in $\tau_d \in [0,1)$. Moreover, $\pi_{\varepsilon,0}^{HANK}$ decreases in $\tau_d \in [0,1)$.

We can then find $c_t = y_t$, $a_t = d_t$, and t_t from the fiscal rule (8), and the entire equilibrium path $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ satisfying Definition 1. The uniqueness comes from the fact that χ and ρ_d are uniquely pinned down by (68). Finally, from (3) and (17), for all $k \geq 0$,

$$\pi_{\varepsilon,k}^{HANK} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t} = \rho_d^k \pi_{\varepsilon,0}^{HANK}, \tag{86}$$

continuous in $(\beta,\omega,\tau_y,\tau_d,\phi)$ and, in particular, in $\tau_d \in [0,1)$.

C.7 Proof of Proposition 5

In this proof, objects without superscripts, such as $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ and (ρ_d,χ) capture relevant objects in the HANK economy characterized in Proposition 4. Objects with the superscript FTPL, such as $\{\pi_t^{FTPL}, d_t^{FTPL}, y_t^{FTPL}\}_{t=0}^{+\infty}$ and $(\rho_d^{FTPL}, \chi^{FTPL})$ capture the corresponding objects in the RANK-FTPL economy which shares the same path of (expected) real interest rates as the HANK economy.

Consider the HANK economy with $\tau_d = 0$. From (5), the government's intertemporal budget constraint (6) at $t = 0$, and (8), we have

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \beta^{t+1} r_t = \frac{D^{ss}}{Y^{ss}} \pi_0 + \tau_y \sum_{t=0}^{+\infty} \beta^t y_t, \tag{87}$$

where we drop the expectation operator because we abstract from any future shocks after the initial shock ε_0.

Now we feed the equilibrium path of the (expected) real rate obtained in the HANK equilibrium $\{r_t\}_{t=0}^{\infty}$ into the RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), and monetary policy follows the passive rule $r_t^{FTPL} = r_t$. From (5), the government's intertemporal budget constraint (6) at $t = 0$, and (8), we have

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \beta^{t+1} r_t = \frac{D^{ss}}{Y^{ss}} \pi_0^{FTPL} + \tau_y \sum_{t=0}^{+\infty} \beta^t y_t^{FTPL}. \tag{88}$$

Together with (3), we know that

$$\pi_0 = \pi_0^{FTPL} = \frac{\kappa}{\tau_y + \frac{D^{ss}}{Y^{ss}}\kappa} \left(\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \beta^{t+1} r_t \right).$$

As a result, $\pi_{\varepsilon,0}^{HANK} = \pi_{\varepsilon,0}^{FTPL}$.

C.8 Proof of Proposition 6

In this proof, objects without superscripts, such as $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ and (ρ_d, χ) capture relevant objects in the HANK economy characterized in Proposition 4. Objects with the superscript FTPL, such as $\{\pi_t^{FTPL}, d_t^{FTPL}, y_t^{FTPL}\}_{t=0}^{+\infty}$ and $(\rho_d^{FTPL}, \chi^{FTPL})$ capture the corresponding objects in the RANK-FTPL economy which shares the same path of (expected) real interest rates as the HANK economy.

From (86), $\pi^{\dagger} = 1 - \beta\rho_d$. From (85), we know

$$\rho_d = \frac{(a+b+1+\sigma\phi) - \sqrt{(a+b+1+\sigma\phi)^2 - 4(1+\sigma\phi)\,a}}{2} = \frac{2(1+\sigma\phi)\,a}{(a+b+1+\sigma\phi) + \sqrt{(a+b+1+\sigma\phi)^2 - 4(1+\sigma\phi)\,a}}$$

where $a = \frac{1-\tau_d}{\beta} > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)(1-\omega) > 0$. From the second part of the equation, we know that ρ_d decreases in b and increases in ω. As a result, $\pi^{\dagger,HANK}$ decreases in ω.

To prove $\pi^{\dagger,HANK} > \pi^{\dagger,FTPL}$, we first need to establish some additional property of the HANK economy characterized in Proposition 4. From (10) and (17), we know that, for all $t \geq 0$,

$$r_t = \rho_d^t r_0 = \phi\rho_d^t y_0. \tag{89}$$

From the recursive demand relation (75) and the government budget (4), for $t \geq 0$,

$$y_t = -\sigma r_t + \frac{(1-\beta\omega)(1-\omega)}{\omega}\rho_d(d_t + \varepsilon_t) + y_{t+1},$$

where $\varepsilon_t = 0$ for all $t \neq 0$. Because $\rho_d \in (0,1)$ so $\lim_{t\to\infty} y_t = 0$ in the HANK equilibrium. We have, for $t \geq 0$,

$$\begin{aligned} y_t &= -\frac{\sigma}{1-\rho_d} r_t + \frac{(1-\beta\omega)(1-\omega)}{\omega(1-\rho_d)}\rho_d(d_t+\varepsilon_t) \\ \sum_{t=0}^{+\infty}\beta^t y_t &= -\frac{\sigma}{(1-\rho_d)(1-\beta\rho_d)} r_0 + \frac{(1-\beta\omega)(1-\omega)}{\omega(1-\rho_d)(1-\beta\rho_d)}\rho_d(d_0+\varepsilon_0). \end{aligned} \tag{90}$$

where we use (17) for the second equation. Putting them into (87) and using (3), we have, for $k \geq 0$,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\left(\frac{\beta}{1-\beta\rho_d}\right) r_0 + \frac{\sigma\left(\kappa\frac{D^{ss}}{Y^{ss}} + \tau_y\right)}{(1-\beta\rho_d)(1-\rho_d)} r_0 = \frac{\kappa\frac{D^{ss}}{Y^{ss}} + \tau_y}{(1-\beta\rho_d)(1-\rho_d)}\frac{(1-\beta\omega)(1-\omega)}{\omega}\rho_d(d_0+\varepsilon_0) \tag{91}$$

Now we turn to the RANK-FTPL economy sharing the same path of $\{r_t = r_t^{HANK}\}_{t=0}^{\infty}$. Similar to (11), the equilibrium path of $\{y_t^{FTPL}\}_{t=0}^{\infty}$ can be characterized by the familiar DIS equation, for $t \geq 0$,

$$y_t^{FTPL} = -\sigma r_t + y_{t+1}^{FTPL}. \tag{92}$$

Similar to (90) but without imposing $y_\infty^{FTPL} \equiv \lim_{t\to\infty} y_t^{FTPL} = 0$,

$$y_t^{FTPL} = -\frac{\sigma}{1-\rho_d} r_t + y_\infty^{FTPL} \tag{93}$$

$$\sum_{t=0}^{+\infty} \beta^t y_t^{FTPL} = -\frac{\sigma}{(1-\rho_d)(1-\beta\rho_d)} r_0 + \frac{1}{1-\beta} y_\infty^{FTPL}$$

Putting them into (88) and using (3), we have, for $k \geq 0$,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \left(\frac{\beta}{1-\beta\rho_d}\right) r_0 + \frac{\sigma\left(\kappa \frac{D^{ss}}{Y^{ss}} + \tau_y\right)}{(1-\beta\rho_d)(1-\rho_d)} r_0 = \frac{\kappa \frac{D^{ss}}{Y^{ss}} + \tau_y}{1-\beta} y_\infty^{\text{FTPL}}.$$

Compared with (91), we know that $y_\infty^{\text{FTPL}} = \frac{(1-\beta)(1-\beta\omega)(1-\omega)}{(1-\beta\rho_d)(1-\rho_d)\omega} \rho_d (d_0 + \varepsilon_0)$. From (69), we know that y_∞^{FTPL} has the same sign as ε_0.

From this point on, we will use the positive fiscal deficit shock $\varepsilon_0 > 0$ as an example, which means $y_\infty^{\text{FTPL}} > 0$; the proof with $\varepsilon_0 < 0$ is symmetric. With $\varepsilon_0 > 0$, from (17) and (69), we know that, in HANK, $\pi_t > 0$ and $y_t > 0$ because $\chi > 0$ and $\rho_d \in (0,1)$. When $\phi \in [0, \bar{\phi}), r_t \geq 0$ for all $t \geq 0$. From (92), $y_0^{FTPL} \leq y_1^{FTPL} \leq y_2^{FTPL} \leq \cdots$. From (3), we have

$$0 < \pi_0 = \pi_0^{FTPL} \leq \pi_1^{FTPL} \leq \pi_2^{FTPL} \leq \cdots.$$

We hence know that $\pi^{\dagger,FTPL} \leq 1-\beta < 1-\beta\rho_d = \pi^{\dagger,HANK}$. When $\phi \in \left(-\frac{1}{\sigma}, 0\right)$, $r_t = \rho_d^t r_0 < 0$ for all $t \geq 0$. From $y_\infty^{\text{FTPL}} > 0$ and (93), we know that $y_{t+1}^{FTPL} > \rho_d y_t^{FTPL} > 0$ for all $t \geq 0$. From (3), we know that $\pi_{t+1}^{FTPL} > \rho_d \pi_t^{FTPL} > 0$ for all $t \geq 0$. We hence know that $\pi^{\dagger,FTPL} < 1-\beta\rho_d = \pi^{\dagger,HANK}$.

C.9 Proof of Proposition 7

Imposing $y_t = c_t$ (goods market clearing), $a_t = d_t$ (asset market clearing), and using the government's flow budget (4), we can write aggregate demand (1) recursively

$$y_t = -\sigma r_t + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega} \left(d_t - t_t + \beta \frac{D^{ss}}{Y^{ss}} r_t\right) + \mathbb{E}_t [y_{t+1}]. \tag{94}$$

Given (15), we know that, for $t \geq H$, (63) also holds under HANK. As a result, there exists a unique equilibrium. In this equilibrium, $y_t = \pi_t = 0$ for $t \geq H$. We find the equilibrium path of $\{y_t, \pi_t, d_t\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0 d_H \quad \text{with} \quad \chi_0 = 0. \tag{95}$$

Applying the fiscal and monetary rules (8) and (10) in (94), we know that, for $t \leq H-1$,

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)} (d_t + \varepsilon_t) + \frac{1}{1+\sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)} \mathbb{E}_t [y_{t+1}]. \tag{96}$$

As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}(d_t+\varepsilon_t) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)} + \frac{\frac{1}{\beta}\left(1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_{H-t}\right)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\chi_{H-t-1}, \tag{97}$$

Rearranging terms, we find the following recursive formula for the χs:

$$\chi_{H-t} = \frac{\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi_{H-t-1}}{\beta}\right)(1-\tau_d)}{1+\sigma\phi+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi_{H-t-1}}{\beta}\right)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)} \equiv g\left(\chi_{H-t-1}\right), \tag{98}$$

where $\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}>0$ and $1+\sigma\phi>0$ because $\phi\in(\underline{\phi},\bar{\phi})$ and

$$g'(\chi) = \frac{1-\tau_d}{\beta}\frac{1+\sigma\phi}{\left(1+\sigma\phi+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi}{\beta}\right)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2} \geq 0 \quad \forall\chi\geq 0.$$

We thus know that

$$\chi_k \in \left(0, \frac{1-\tau_d}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}\right) \quad \forall k\geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{99}$$

Now let's find the fixed point of g such that $g(\chi)=\chi$, where

$$\omega\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi^2+\left(\beta\omega(1+\sigma\phi)-\omega(1-\tau_d)+(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)\chi-(1-\beta\omega)(1-\omega)(1-\tau_d)=0.$$

We know that there is only one of such fix point such that $\chi>0$ because $-(1-\beta\omega)(1-\omega)(1-\tau_d)<0$ and $\omega\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)>0$. From (99), we have that $\lim_{k\to+\infty}\chi_k=\chi$. Moreover, because (96) also holds under the HANK equilibrium we characterized in Proposition 4, so the fixed point $\chi>0$ here corresponds to the χ in the equilibrium (17) in Proposition 4.

From (3), (4), and (5), we can construct the equilibrium path of $\{y_t,\pi_t,d_t\}_{t=0}^{H-1}$ based on $\{\chi_k\}_{k=0}^{H}$. In particular, for $t\leq H-1$,

$$\mathbb{E}_0[d_t] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_{H-j}\right)(d_0+\varepsilon_0), \tag{100}$$

where

$$\frac{1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi_k}{\beta} \to \frac{1-\tau_d-\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\chi}{\beta} = \rho_d \in (0,1),$$

where ρ_d is the one in (17) in Proposition 4. Together with $\lim_{k\to+\infty}\chi_k=\chi$, we know that, for any $T>0$, as $H\to\infty$, $\{y_t,\pi_t\}_{t=0}^{T}$ converges to its counterpart in Propositions 4, for all realizations of uncertainty.

C.10 Proof of Proposition 8

We first characterize the HANK equilibrium with $\omega < 1$, $\tau_y > 0$, $\tau_d \in [0,1)$, $\delta > 0$, and $\phi = 0$. Applying period-t expectation to (23) leads to the same $\mathbb{E}_t[d_{t+1}]$ as applying period-t expectation to (4). As a result, (64) in Proposition 3 for the $\delta = 0$ case characterizing the evolution from $\left(d_t + \varepsilon_t, y_t\right)'$ to $\left(\mathbb{E}_t[d_{t+1}], \mathbb{E}_t[y_{t+1}]\right)'$ is exactly the same under $\delta > 0$ case. This means that any equilibrium path of $\{d_t, y_t\}_{t=0}^{+\infty}$ still takes the form of

$$y_t = \chi(d_t + \varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \varepsilon_t),$$

where χ and ρ_d are uniquely given by the same (68) in the proof of Proposition 3 for the $\delta = 0$ case, continuous in $\left(\beta, \omega, \tau_y, \tau_d\right)$. As a result,

$$\pi^{\dagger,HANK} = 1 - \beta\rho_d > 1 - \beta. \tag{101}$$

The maturity of government debt $\delta > 0$, however, matters for the mapping from ε_t to $d_t - \mathbb{E}_{t-1}[d_t]$ in (69) and (70). From (3) and (57), we can find d_0 as a function of the deficit shock ε_0:

$$\begin{aligned}
d_0 &= -\frac{D^{ss}}{Y^{ss}}\pi_0^{\delta} = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{1-\beta\rho_d}\sum_{k=0}^{+\infty}(\beta\delta)^k\mathbb{E}_0[y_k] \\
&= -\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\frac{\kappa}{1-\beta\rho_d}\chi(d_0+\varepsilon_0) = -\frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}\varepsilon_0.
\end{aligned} \tag{102}$$

Similarly, for $t \geq 1$, from (3) and (55),

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}\varepsilon_t. \tag{103}$$

Together with (3) and (17), we find the unique equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$. In particular, for all $t \geq 0$,

$$\pi_t - \mathbb{E}_{t-1}[\pi_t] = \pi_{\varepsilon,0}^{HANK}\cdot\varepsilon_t, \quad \pi_{\varepsilon,0}^{HANK} \equiv \frac{\frac{\kappa\chi}{1-\beta\rho_d}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}, \quad \text{and} \quad \pi_{\varepsilon,k}^{HANK} \equiv \frac{d\mathbb{E}_t[\pi_{t+k}]}{d\varepsilon_t} = \rho_d^k\pi_{\varepsilon,0}^{HANK}.$$

As a result,

$$\pi_{\varepsilon}^{\delta,HANK} = \sum_{k=0}^{\infty}(\beta\delta)^k\pi_{\varepsilon,k}^{HANK} = \frac{\frac{\kappa\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}}{\frac{\kappa\frac{D^{ss}}{Y^{ss}}\chi}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}+1}, \tag{104}$$

From the proof of Proposition 1, we know that ρ_d and χ are continuous and decreasing in $\tau_d \in [0,1)$. From 104, $\pi_{\varepsilon}^{\delta,HANK}$ is continuous and decreasing in $\tau_d \in [0,1)$.

Now we focus on the case of $\tau_d = 0$, focused in Proposition 8. In that case, from (23), we know that,

$1-\beta\rho_d = \tau_y \chi$. As a result,

$$\pi_\varepsilon^{\delta,HANK} = \sum_{k=0}^{\infty} (\beta\delta)^k \pi_{\varepsilon,k}^{HANK} = \frac{\frac{\kappa}{\tau_y(1-\beta\delta\rho_d)}}{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y(1-\beta\delta\rho_d)}+1} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\rho_d\right)} < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\right)}. \tag{105}$$

We now characterize the RANK-FTPL equilibrium with $\omega = 1$, $\tau_d = 0$, $\delta > 0$, and $\phi = 0$. (11) and (59) remain to hold no matter δ. As a result, as in Section 3 for the $\delta = 0$ case, any equilibrium must satisfy (60), with $\varrho = 1$. From (57), the government's intertemporal budget constraint (6) at $t = 0$, and (8), we have

$$\frac{D^{ss}}{Y^{ss}}\pi_0^{\delta,FTPL} + \tau_y \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0\left[y_k^{FTPL}\right] = \varepsilon_0.$$

Together with (3) and (60), we know that $\sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0\left[y_k^{FTPL}\right] = \frac{1-\beta\delta}{\kappa}\pi_0^{\delta,FTPL}$. As a result,

$$\pi_0^{\delta,FTPL} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\right)}\varepsilon_0.$$

Similarly, $\pi_t^{\delta,FTPL} - \mathbb{E}_{t-1}\left[\pi_t^{\delta,FTPL}\right] = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}(1-\beta\delta)}\varepsilon_t$. As a result,

$$\pi_\varepsilon^{\delta,FTPL} = \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1-\beta\delta\right)}.$$

Together with (105), we know that $\pi_\varepsilon^{\delta,HANK} < \pi_\varepsilon^{\delta,FTPL}$. Moreover, the distance between the two vanishing when $\tau_y \to 0$, $\kappa \to \infty$, or $\delta \to 0$. This proves Proposition 8.

Finally, from (60) (with $\varrho = 1$) and (101),

$$\pi^{\dagger,HANK} > \pi^{\dagger,FTPL} = 1-\beta.$$

C.11 Proof of Proposition 9

From the proof of Proposition 1, we know that ρ_d and χ are continuous and decreasing in $\tau_y \in (0,1)$. From (104), $\pi_\varepsilon^{\delta,HANK}$ are continuous and decreasing in $\tau_y \in (0,1)$. Also from the proof of Proposition 1, we know that ρ_d and χ are independent of $\frac{D^{ss}}{Y^{ss}}$, κ, and δ. As a result, $\pi_\varepsilon^{\delta,HANK}$ decreases with $\frac{D^{ss}}{Y^{ss}}$, increases with κ, and increases with δ.

C.12 Proof of Proposition 10

Let $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\phi = 0$, and $\mu \in (0,1)$. We work with the flow government budget (23) allowing $\delta \in [0,1)$, nesting the short-term debt case in (4). Imposing $y_t = c_t$ (goods market clearing) and $a_t = d_t$

(asset market clearing) in (26),

$$
\begin{aligned}
y_t &= (1-\beta\omega)\,d_t + (\mu + (1-\mu)(1-\beta\omega))\left((y_t - t_t) + \frac{(1-\mu)(1-\beta\omega)}{\mu + (1-\mu)(1-\beta\omega)}\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k (y_{t+k} - t_{t+k})\right]\right) \\
&= \frac{1-\beta\omega}{(1-\mu)\,\beta\omega} d_t - \frac{\mu + (1-\mu)(1-\beta\omega)}{(1-\mu)\,\beta\omega} t_t + \frac{1-\beta\omega}{\beta\omega}\mathbb{E}_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k (y_{t+k} - t_{t+k})\right] \\
&= \frac{1-\beta\omega}{(1-\mu)\,\beta\omega} d_t - \frac{\mu + (1-\mu)(1-\beta\omega)}{(1-\mu)\,\beta\omega} t_t + \mathbb{E}_t[y_{t+1}] + \mathbb{E}_t\left[-\frac{1-\beta\omega}{1-\mu} d_{t+1} + \frac{\mu}{1-\mu} t_{t+1}\right] \\
&= \frac{1-\beta\omega}{(1-\mu)\,\beta\omega} d_t - \frac{\mu + (1-\mu)(1-\beta\omega)}{(1-\mu)\,\beta\omega} t_t + \mathbb{E}_t[y_{t+1}] + \mathbb{E}_t\left[-\frac{1-\beta\omega}{\beta(1-\mu)}(d_t - t_t) + \frac{\mu}{1-\mu} t_{t+1}\right] \\
&= \frac{(1-\beta\omega)(1-\omega)}{\beta\omega(1-\mu)} d_t - \left(\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right) t_t + \mathbb{E}_t[y_{t+1}] + \frac{\mu}{1-\mu}\mathbb{E}_t[t_{t+1}].
\end{aligned}
$$

Applying the fiscal rule (8), we have, for all $t \geq 0$,

$$
y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega(1-\mu)}}{1 + \left(\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)\tau_y} d_t + \frac{1 + \frac{\mu}{1-\mu}\tau_y}{1 + \left(\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)\tau_y}\mathbb{E}_t[y_{t+1}] + \frac{\left(\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)}{1 + \left(\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\right)\tau_y}\varepsilon_t. \tag{106}
$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (23) and rearranging (106), we have, for all $t \geq 0$,

$$
\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega(1-\mu)}}{1+\frac{\mu}{1-\mu}\tau_y} & \left(1 + \frac{\frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}\tau_y}{1+\frac{\mu}{1-\mu}\tau_y}\right) \end{pmatrix}\begin{pmatrix} d_t \\ y_t \end{pmatrix} + \begin{pmatrix} \frac{1}{\beta} \\ -\frac{\frac{\mu}{1-\mu} + \frac{(1-\omega)(1-\beta\omega)}{\beta\omega(1-\mu)}}{1+\frac{\mu}{1-\mu}\tau_y} \end{pmatrix}\varepsilon_t.
$$

The two eigenvalues of the system ($\lambda_1 > \lambda_2$) are given by the solutions of

$$
f(\lambda) \equiv \lambda^2 - \lambda\left(\frac{1}{\beta} + 1 + \frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)}\right) + \frac{1}{\beta} = 0.
$$

Because $f(0) > 0$ and $f(1) < 0$, we know that $\lambda_1 > 1 > \lambda_2 > 0$. Moreover,

$$
\begin{aligned}
\lambda_2 &= \frac{\frac{1}{\beta} + 1 + \frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)} - \sqrt{\left(\frac{1}{\beta} + 1 + \frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)}\right)^2 - \frac{4}{\beta}}}{2} \\
&= \frac{\frac{2}{\beta}}{\left(\frac{1}{\beta} + 1 + \frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)} + \sqrt{\left(\frac{1}{\beta} + 1 + \frac{(1-\omega)(1-\beta\omega)\tau_y}{\beta\omega(1-(1-\tau_y)\mu)}\right)^2 - \frac{4}{\beta}}\right)},
\end{aligned}
$$

which decreases in $\mu \in [0,1)$.

Similar to Proposition 3, there is a unique equilibrium where

$$
y_t = \chi_d d_t + \chi_\varepsilon \varepsilon_t \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d d_t + \rho_\varepsilon \varepsilon_t, \tag{107}
$$

where

$$
\chi_d = \frac{1-\beta\rho_d}{\tau_y} > 0, \quad \rho_d = \lambda_2, \quad \text{and} \quad \chi_\varepsilon = \frac{1-\beta\rho_\varepsilon}{\tau_y} > \chi_d. \tag{108}
$$

Because λ_2 decreases in $\mu \in [0,1)$ and the baseline HANK case in Proposition 8 corresponds to $\mu = 0$. We know that $\rho_d < \rho_d^{HANK}$.

From (3) and (57), we can find π_0^δ as a function of the deficit shock ε_0 :

$$\begin{aligned}
\pi_0^\delta &= \pi_0 + \sum_{k=0}^{+\infty} (\beta\delta)^{k+1} \mathbb{E}_0 [\pi_{k+1}] \\
&= -\frac{1}{1-\beta\delta\rho_d} \frac{\kappa\chi_d}{1-\beta\rho_d} \frac{D^{ss}}{Y^{ss}} \pi_0^\delta + \kappa \left(\chi_\varepsilon + \frac{\chi_d \beta\rho_\varepsilon}{1-\beta\rho_d}\right) \varepsilon_0 + \frac{\beta\delta}{1-\beta\delta\rho_d} \frac{\kappa\chi_d\rho_\varepsilon}{1-\beta\rho_d} \varepsilon_0 \\
&= \frac{\kappa\chi_\varepsilon + \frac{\kappa\chi_d\beta\rho_\varepsilon}{1-\beta\rho_d}\left(1 + \frac{\delta}{1-\beta\delta\rho_d}\right)}{1 + \frac{1}{1-\beta\delta\rho_d} \frac{\kappa\chi_d}{1-\beta\rho_d} \frac{D^{ss}}{Y^{ss}}} \varepsilon_0 \\
&= \frac{\frac{\kappa}{\tau_y} \frac{1}{1-\beta\delta\rho_d} \left(1 + \beta\delta\left(\rho_\varepsilon - \rho_d\right)\right)}{1 + \frac{1}{1-\beta\delta\rho_d} \frac{\kappa}{\tau_y} \frac{D^{ss}}{Y^{ss}}} \varepsilon_0.
\end{aligned}$$

As a result,

$$\pi_\varepsilon^\delta = \frac{1 + \beta\delta\left(\rho_\varepsilon - \rho_d\right)}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1 - \beta\delta\rho_d\right)}. \tag{109}$$

When $\delta = 0$, together with Proposition 1,

$$\pi_{\varepsilon,0} = \pi_\varepsilon^\delta = \frac{\kappa}{\tau_y + \frac{D^{ss}}{Y^{ss}}\kappa} = \pi_\varepsilon^{HANK} = \pi_\varepsilon^{FTPL}.$$

When $\delta > 0$, from (108), we know that $\rho_\varepsilon < \rho_d$. Moreover,

$$\pi_\varepsilon^\delta < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1 - \beta\delta\rho_d\right)} < \frac{1}{\frac{D^{ss}}{Y^{ss}} + \frac{\tau_y}{\kappa}\left(1 - \beta\delta\rho_d^{HANK}\right)}.$$

Together with (105), we know that

$$\pi_\varepsilon^\delta < \pi_\varepsilon^{\delta,HANK} < \pi_\varepsilon^{\delta,FTPL}.$$

C.13 Proof of Proposition 11

We first derive some properties under the hybrid NKPC (28) shared by both HANK and RANK-FTPL. From the hybrid NKPC (28), for all $t \geq 0$,

$$(1-\xi)\,\mathbb{E}_t [\pi_{t+1}] - \frac{1}{\beta}\pi_t + \xi\pi_{t-1} = -\frac{\kappa}{\beta} y_t. \tag{110}$$

Consider two roots of

$$(1-\xi)\,\lambda^2 - \frac{1}{\beta}\lambda + \xi = 0,$$

given by

$$\Lambda_1 = \frac{1-\sqrt{1-4\beta^2\xi(1-\xi)}}{2\beta(1-\xi)} = \frac{2\xi\beta}{1+\sqrt{1-4\beta^2\xi(1-\xi)}} \le \frac{2\xi\beta}{1+|2\xi-1|} < 1,$$
$$\Lambda_2 = \frac{2\xi\beta}{1-\sqrt{1-4\beta^2\xi(1-\xi)}} = \frac{1+\sqrt{1-4\beta^2\xi(1-\xi)}}{2\beta(1-\xi)} > \frac{1+|1-2\xi|}{2\beta(1-\xi)} > \frac{1}{\beta} > 1.$$

We can rewrite (110) as

$$\pi_t - \Lambda_1\pi_{t-1} = \Lambda_2^{-1}\left(\frac{\kappa}{\beta(1-\xi)}y_t + \mathbb{E}_t[\pi_{t+1}] - \Lambda_1\pi_t\right).$$

Iterating forward and use $\pi_{-1}=0$, we have

$$\pi_0 = \frac{\kappa}{\beta(1-\xi)}\sum_{k=0}^{+\infty}\Lambda_2^{-k-1}\mathbb{E}_0[y_k] \quad \text{and} \quad \pi_t - \Lambda_1\pi_{t-1} = \frac{\kappa}{\beta(1-\xi)}\sum_{k=0}^{+\infty}\Lambda_2^{-k-1}\mathbb{E}_t[y_{t+k}]. \tag{111}$$

We now characterize the HANK equilibrium with $\omega<1$, $\tau_d=0$, $\delta=0$, $\phi=0$, and $\xi\in(0,1)$. Note that the evolution from $(d_t+\varepsilon_t, y_t)'$ to $(\mathbb{E}_t[d_{t+1}], \mathbb{E}_t[y_{t+1}])'$ is exactly the same as (64) in Proposition 3 for the $\xi=0$ case characterizing. This means that any equilibrium path of $\{d_t, y_t\}_{t=0}^{+\infty}$ still takes the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\varepsilon_t),$$

where χ and ρ_d are uniquely given by the same (68) in Proposition 3 for the $\xi=0$ case. The hybrid NKPC with $\xi>0$, however, matters for the mapping from ε_t to $d_t-\mathbb{E}_{t-1}[d_t]$ in (69) and (70). From (111) and the fact that $\mathbb{E}_t[y_{t+k}] = \rho_d^k y_t$, we have

$$\pi_0 = \frac{\kappa}{\beta(1-\xi)}\sum_{k=0}^{+\infty}\Lambda_2^{-k-1}\mathbb{E}_0[y_k] = \frac{\kappa}{\beta(1-\xi)}\frac{1}{\Lambda_2-\rho_d}y_0$$

From (5) and (3) , we can find d_0 as a function of the deficit shock ε_0 :

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{\beta(1-\xi)}\frac{1}{\Lambda_2-\rho_d}y_0 = -\frac{D^{ss}}{Y^{ss}}\frac{\kappa}{\beta(1-\xi)}\frac{\chi}{\Lambda_2-\rho_d}(d_0+\varepsilon_0) \tag{112}$$

As a result,

$$d_0 = -\frac{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}+1}\varepsilon_0 \quad \text{and} \quad \pi_0 = \frac{\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}+1}\varepsilon_0.$$

As a result,

$$\pi_{\varepsilon,0}^{HANK} = \frac{\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa\chi}{\beta(1-\xi)(\Lambda_2-\rho_d)}+1}.$$

When $\tau_d=0$, from (4), we know that, $\chi = \frac{1-\beta\rho_d}{\tau_y}$. As a result,

$$\pi_{\varepsilon,0}^{HANK} = \frac{\frac{\kappa(1-\beta\rho_d)}{\beta\tau_y(1-\xi)(\Lambda_2-\rho_d)}}{\frac{D^{ss}}{Y^{ss}}\frac{\kappa(1-\beta\rho_d)}{\beta\tau_y(1-\xi)(\Lambda_2-\rho_d)}+1}. \tag{113}$$

We now turn to the RANK-FTPL equilibrium with $\omega = 1$, $\tau_d = 0$, $\delta = 0$, $\phi = 0$, and $\xi \in (0,1)$. (11) and (59) remain to hold no matter ξ. As a result, as in Section 3 for the $\xi = 0$ case, any equilibrium in which $\{y_t^{FTPL}\}_{t=0}^{\infty}$ is bounded must satisfy (60), with $\varrho = 1$. In particular, $\mathbb{E}_t\left[y_{t+k}^{FTPL}\right] = y_t^{FTPL}$ for all $t, k \geq 0$. Following similar step as above (simply replace ρ_d with $\varrho = 1$), we have

$$\pi_0^{FTPL} = \frac{\kappa}{\beta(1-\xi)} \frac{1}{\Lambda_2 - 1} y_0^{FTPL}.$$

From (5), the government's intertemporal budget constraint (6) at $t = 0$, and (8), we have

$$\frac{D^{ss}}{Y^{ss}} \pi_0^{FTPL} + \tau_y \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0\left[y_k^{FTPL}\right] = \varepsilon_0.$$

Together, we have

$$\pi_0^{FTPL} = \frac{\frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)}}{\frac{D^{ss}}{Y^{ss}} \frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)} + 1} \varepsilon_0 \quad \text{and} \quad \pi_{\varepsilon,0}^{FTPL} = \frac{\frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)}}{\frac{D^{ss}}{Y^{ss}} \frac{\kappa(1-\beta)}{\beta\tau_y(1-\xi)(\Lambda_2-1)} + 1}.$$

Because $\Lambda_2 > \frac{1}{\beta}$ and $\rho_d \in (0,1)$,

$$\frac{1 - \beta\rho_d}{\Lambda_2 - \rho_d} > \frac{1-\beta}{\Lambda_2 - 1}.$$

Together with (113), we know that

$$\pi_{\varepsilon,0}^{HANK} > \pi_{\varepsilon,0}^{FTPL},$$

with the distance between the two vanishing when $\tau_y \to 0$ or $\kappa \to \infty$.

C.14 Proof of Proposition 12

In this proof, objects without superscripts (such as $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ and (ρ_d, χ)) capture relevant objects in the HANK economy characterized in Proposition 4. Objects with the superscript FTPL (such as $\{\pi_t^{FTPL}, d_t^{FTPL}, y_t^{FTPL}\}_{t=0}^{+\infty}$ and $(\rho_d^{FTPL}, \chi^{FTPL})$) capture the corresponding objects in the RANK-FTPL economy which shares the same path of (expected) real interest rates as the HANK economy.

We first characterize the HANK equilibrium with $\omega < 1$, $\tau_y > 0$, $\tau_d = 0$, $\delta > 0$, and $\phi \in (\underline{\phi}, \bar{\phi})$. Applying period-$t$ expectation to (23) leads to

$$\mathbb{E}_t[d_{t+1}] = \frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t,$$

similar to applying period-t expectation to (4). As a result, (76) in Proposition 4 for the $\delta = 0$ case characterizing the evolution from $(d_t + \varepsilon_t, y_t)'$ to $(\mathbb{E}_t[d_{t+1}], \mathbb{E}_t[y_{t+1}])'$ is exactly the same under $\delta > 0$ case. Moreover, when $\tau_d = 0$, from (80), we know that $\lambda_1 > 1$. Similar to the proof of Proposition 3, any equilibrium path of $\{d_t, y_t\}_{t=0}^{+\infty}$ takes the form of

$$y_t = \chi(d_t + \varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \varepsilon_t), \tag{114}$$

where χ and ρ_d are uniquely given by the same (84) in Proposition 4. The maturity of government debt $\delta > 0$, however, matters for the mapping from ε_t to $d_t - \mathbb{E}_{t-1}[d_t]$ in (69) and (70). In particular, from (3) and (57), we can find d_0 as a function of the deficit shock ε_0 :

$$\begin{aligned} d_0 &= -\frac{D^{ss}}{Y^{ss}}\left(\pi_0^\delta + r_0^\delta\right) = -\frac{D^{ss}}{Y^{ss}}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\sum_{t=0}^{+\infty}(\beta\delta)^t\mathbb{E}_0[y_t] \\ &= -\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi(d_0+\varepsilon_0) = -\frac{\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi}{\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi + 1}\varepsilon_0. \end{aligned} \tag{115}$$

Now consider any realization of the initial fiscal shock ε_0, abstract from any future shocks. When $\tau_d = 0$, from (57), the government's intertemporal budget constraint (6) at $t = 0$, and (8), we have

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\sum_{t=0}^{+\infty}\left(\beta^{t+1} - (\beta\delta)^{t+1}\right)r_t = \frac{D^{ss}}{Y^{ss}}\pi_0^\delta + \tau_y\sum_{t=0}^{+\infty}\beta^t y_t = \frac{D^{ss}}{Y^{ss}}\pi_0^\delta + \frac{\tau_y}{\kappa}\pi_0, \tag{116}$$

where we use (3) for the second equality. Together with (114) and (115), we know that

$$\pi_0^\delta = \frac{\kappa}{\tau_y(1-\beta\delta\rho_d) + \frac{D^{ss}}{Y^{ss}}\kappa}\left(\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\sum_{t=0}^{+\infty}\left(\beta^{t+1} - (\beta\delta)^{t+1}\right)r_t\right) = \frac{\frac{\kappa}{(1-\beta\delta\rho_d)(1-\beta\rho_d)}\chi}{\frac{\frac{D^{ss}}{Y^{ss}}}{1-\beta\delta\rho_d}\left(\frac{\kappa}{1-\beta\rho_d} + \beta\delta\phi\right)\chi + 1}\varepsilon_0. \tag{117}$$

From the government budget (23) and the recursive AD (75), for $t \geq 0$,

$$y_t = -\sigma r_t + \frac{(1-\beta\omega)(1-\omega)}{\omega}\rho_d(d_t + \varepsilon_t) + y_{t+1},$$

where $\varepsilon_t = 0$ for all $t \neq 0$. Same as (89), we still have

$$r_t = \rho_d^t r_0 = \phi\rho_d^t y_0. \tag{118}$$

Because $\rho_d \in (0,1)$ so $\lim_{t\to\infty} y_t = 0$ in the HANK equilibrium, we have, for $t \geq 0$,

$$\begin{aligned} y_t &= -\frac{\sigma}{1-\rho_d}r_t + \frac{(1-\beta\omega)(1-\omega)}{\omega(1-\rho_d)}\rho_d(d_t+\varepsilon_t) \\ \sum_{t=0}^{+\infty}\beta^t y_t &= -\frac{\sigma}{(1-\rho_d)(1-\beta\rho_d)}r_0 + \frac{(1-\beta\omega)(1-\omega)}{\omega(1-\rho_d)(1-\beta\rho_d)}\rho_d(d_0+\varepsilon_0). \end{aligned} \tag{119}$$

where we use (17) for the second equation. Putting them into (116) and using (3), we have, for $k \geq 0$,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}}\left(\frac{\beta}{1-\beta\rho_d} - \frac{\beta\delta}{1-\beta\delta\rho_d}\right)r_0 + \frac{\sigma\left(\kappa\frac{D^{ss}}{Y^{ss}}\frac{1}{(1-\beta\rho_d\delta)} + \tau_y\right)}{(1-\beta\rho_d)(1-\rho_d)}r_0 = \frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{1}{(1-\beta\rho_d\delta)} + \tau_y}{(1-\beta\rho_d)(1-\rho_d)}\frac{(1-\beta\omega)(1-\omega)}{\omega}\rho_d(d_0+\varepsilon_0). \tag{120}$$

Together with (114), (115), and (117), we know that π_0, π_0^δ, $d_0 + \varepsilon_0$, and y_0 have the same sign as ε_0. For example, with $\varepsilon_0 > 0$,we have $\pi_0 > 0$, $\pi_0^\delta > 0$, $d_0 + \varepsilon_0 > 0$, and $y_0 > 0$.

We now feed the equilibrium path of the (expected) real rate obtained in the HANK equilibrium $\{r_t = r_t^{HANK}\}_{t=0}^\infty$ into the RANK-FTPL economy in which $\omega = 1$, fiscal policy follows the same rule as in our HANK economy (with $\tau_d = 0$), monetary policy follows the passive rule $r_t^{FTPL} = r_t$, and shares the same maturity of the HANK economy (with $\delta > 0$). Similar to (116), we have

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \left(\beta^{t+1} - (\beta\delta)^{t+1}\right) r_t = \frac{D^{ss}}{Y^{ss}} \pi_0^{\delta,FTPL} + \tau_y \sum_{t=0}^{+\infty} \beta^t y_t^{FTPL} = \frac{D^{ss}}{Y^{ss}} \pi_0^{\delta,FTPL} + \frac{\tau_y}{\kappa} \pi_0^{FTPL}, \tag{121}$$

where we use (3) for the second equality. Similar to (11), the equilibrium path of $\{y_t^{FTPL}\}_{t=0}^{\infty}$ can be characterized by the familiar DIS equation, for $t \geq 0$,

$$y_t^{FTPL} = -\sigma r_t + y_{t+1}^{FTPL}. \tag{122}$$

Similar to (119) but without imposing $y_\infty^{FTPL} \equiv \lim_{t\to\infty} y_t^{FTPL} = 0$,

$$y_t^{FTPL} = -\frac{\sigma}{1-\rho_d} r_t + y_\infty^{FTPL} \tag{123}$$

$$\sum_{t=0}^{+\infty} \beta^t y_t^{FTPL} = -\frac{\sigma}{(1-\rho_d)(1-\beta\rho_d)} r_0 + \frac{1}{1-\beta} y_\infty^{FTPL}$$

Putting them into (121) and using (3), we have,

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \left(\frac{\beta}{1-\beta\rho_d} - \frac{\beta\delta}{1-\beta\delta\rho_d} \right) r_0 + \frac{\sigma\left(\kappa \frac{D^{ss}}{Y^{ss}} \frac{1}{(1-\beta\rho_d\delta)} + \tau_y\right)}{(1-\beta\rho_d)(1-\rho_d)} r_0 = \frac{\kappa \frac{D^{ss}}{Y^{ss}} \frac{1}{1-\beta\delta} + \tau_y}{1-\beta} y_\infty^{\text{FTPL}}. \tag{124}$$

Compared with (120), we know that y_∞^{FTPL} has the same sign as $d_0 + \varepsilon_0$ and ε_0. From this point on, we will use the positive fiscal deficit shock $\varepsilon_0 > 0$ as an example, which means $y_\infty^{\text{FTPL}} > 0$. The proof with $\varepsilon_0 < 0$ is symmetric. With $\varepsilon_0 > 0$, from (114), we know that, in HANK, $\pi_t > 0$ and $y_t > 0$ because $\chi > 0$, $\rho_d \in (0,1)$, and $d_0 + \varepsilon_0 > 0$.

When $\phi \in [0, \bar{\phi}), r_t \geq 0$ for all $t \geq 0$. From (122), $y_0^{FTPL} \leq y_1^{FTPL} \leq y_2^{FTPL} \leq \cdots$. From (3), we have

$$\pi_0^{FTPL} \leq \pi_1^{FTPL} \leq \pi_2^{FTPL} \leq \cdots.$$

We hence know that $\pi_0^{FTPL} \leq (1-\beta\delta)\pi_0^{\delta,FTPL} < (1-\beta\delta\rho_d)\pi_0^{\delta,FTPL}$.

When $\phi \in \left(-\frac{1}{\sigma}, 0\right)$, $r_t = \rho_d^t r_0 < 0$ for all $t \geq 0$. From $y_\infty^{\text{FTPL}} > 0$ and (123), we know that $y_{t+1}^{FTPL} > \rho_d y_t^{FTPL} > 0$ for all $t \geq 0$. From (3), we know that $\pi_{t+1}^{FTPL} > \rho_d \pi_t^{FTPL} > 0$ for all $t \geq 0$. We hence know that $\pi_0^{FTPL} < (1-\beta\delta\rho_d)\pi_0^{\delta,FTPL}$.

Together with (121), we know that

$$\pi_0^{\delta,FTPL} > \frac{\kappa}{\tau_y(1-\beta\delta\rho_d) + \frac{D^{ss}}{Y^{ss}}\kappa} \left(\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{t=0}^{+\infty} \left(\beta^{t+1} - (\beta\delta)^{t+1}\right) r_t \right).$$

Together with (117), we have $\pi_\varepsilon^{\delta,FTPL} > \pi_\varepsilon^{\delta,HANK} > 0$.