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On the Alignment of Consumer Surplus and Total Surplus under Competitive Price Discrimination†

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We study the role of information in Bertrand competition with differentiated goods and heterogeneous production costs. When producers know their costs and consumers know their values, consumer surplus and total surplus are aligned, in the sense that the information and equilibrium that maximize consumer surplus also maximize total surplus. Alignment may fail if consumers do not know their values: Partial information about values makes purchases less efficient but intensifies price competition. We illustrate this within a Hotelling duopoly framework. (*JEL* D11, D43, D82, D83)

Overview of Results.—An elementary observation about discriminatory pricing is that it can be extremely beneficial in terms of the total welfare of society: A monopolist who can perfectly price discriminate will charge a price equal to each consumer's willingness to pay, and a sale will take place whenever the consumer's value is above cost. The resulting outcome, while socially efficient, is dismal for the consumer, who obtains zero net value from their purchase.[1] For a long time, this was the only known mechanism by which discriminatory pricing could result in socially efficient outcomes. From that state of affairs, one might conclude that there is a fundamental trade-off between consumer surplus and total surplus, and that for noncompetitive markets to operate efficiently, the consumer must suffer.

Contrary to this conventional wisdom, Bergemann, Brooks, and Morris (2015b)—hereafter, BBM—showed that actually there are many ways in which discriminatory pricing might yield a socially efficient outcome. In fact, there are even ways of segmenting a market so that the resulting outcome is socially efficient, but the monopolist does not benefit from discriminatory pricing at all, and all the gains in surplus from segmentation go to the consumer. To put it concisely, consumer surplus and total surplus can be *aligned*; the segmentations that maximize consumer surplus

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[1] Throughout our exposition, we refer to a single representative consumer. All of our results can be interpreted as applying to a market consisting of a mass of nonatomic consumers. Our representative consumer's information and value should then be interpreted as the empirical distribution of information and values in the population.

also maximize total surplus. Moreover, consumer surplus and producer surplus are *opposed*; the segmentations that maximize consumer surplus also minimize producer surplus. Thus, these segmentations of the market would be optimal for any social planner whose objective is a weighted sum of maximizing total surplus, maximizing consumer surplus, and minimizing producer surplus. At a high level, these outcomes are achieved by pooling together high-value types of consumers with low-value types, in such a way that the monopolist is just barely willing to lower prices. The resulting outcome is efficient, but the high-value consumers reap all the benefits from lower prices.[2]

The result of BBM considers the welfare consequences of the producer's information. A closely related result was subsequently obtained by Roesler and Szentes (2017)—hereafter, RS—concerning the consumer's information. In their main model, they suppose that there is no segmentation of the market, but that the consumer may have imperfect information about their value. RS compute maximum consumer surplus across all models of consumer information, and they show that consumer surplus could be even higher than that obtained through market segmentation. RS also observe that their solution would remain optimal even if we also allow for market segmentation.[3] Moreover, consumer surplus is again maximized in an efficient outcome, thus extending alignment to the case where both consumer and producer information are allowed to vary. Kartik and Zhong (2023)—hereafter, KZ—show that the alignment result extends to monopoly seller settings, where the producer's cost and the consumer's value are correlated, and there are even more general forms of information, in which the producer may have more information than the consumer about the value.[4]

The present paper extends these analyses beyond the monopoly case to a setting in which there are a number of producers engaged in Bertrand competition. In the special case where the producers' goods are perfect substitutes for one another, and when all of the producers have the same cost of production, then, in equilibrium, price is competed down to cost, the outcome is socially efficient, and consumers obtain all of the gains from trade. But if the goods are differentiated and costs are heterogeneous, as we suppose, then, in general, the equilibrium outcome with no segmentation and the consumer knowing their value is neither efficient nor does it need to be especially good for the consumer. In an extreme case, it could be that the goods are not at all substitutable for one another, and we are effectively back to monopoly. In between, there is a rich plethora of possibilities in which both market

[2] In fact, BBM characterize the entire set of producer and consumer surplus pairs that can be attained with some form of market segmentation. In our view, the most striking aspect of this characterization is that consumer surplus is maximized at an efficient point, and therefore, total surplus and consumer surplus can be aligned. In this paper, we do not explore what can be achieved for other welfare objectives aside from those that correspond to the aforementioned point.

[3] Thus, in the case of monopoly, when it is possible to freely choose consumer's information, market segmentation is not needed to maximize consumer surplus. This result does not extend to the case of more than one producer, as we explain below.

[4] The positive results of RS and KZ rely on the assumption there is common knowledge of gains from trade. They also show that alignment may fail in their respective settings without this assumption. We discuss this further below.

segmentation and consumer information could play an important role in equilibrium and welfare.

Our primary focus is on whether the aforementioned results of BBM, RS, and KZ extend to oligopoly: are consumer surplus and total surplus aligned, and are consumer surplus and producer surplus opposed? And, more broadly, what are the limits of consumer welfare? Throughout our analysis, we hold fixed the joint distribution of producers' costs and the consumer's values for the different producers' goods. We first assume that the consumer knows their values and producers know their costs. We consider the effect of segmentation of the market, in that each producer observes a "signal" about the consumer's willingness to pay for their product, as well as possibly about the consumer's willingness to pay for other producers' products, other producers' cost of supplying the good, and other producers' signals. This signal represents any characteristics of the consumer or other producers on which the producer is able to condition prices. We refer to a specification of these signals for all producers as an information structure. Given the information structure, the producers play an equilibrium of the game in which producers simultaneously set prices based on their signals, and the consumer buys from whichever producer offers them the most surplus, with ties broken uniformly. For our main result, we restrict attention to strategy profiles in which producers set prices above their own costs. Theorem 1 shows that just as in the monopoly case, consumer surplus and total surplus can be aligned, and consumer surplus and producer surplus are opposed. Specifically, we construct an information structure and equilibrium that simultaneously maximizes consumer surplus and total surplus, and minimizes producer surplus. Thus, as in the monopoly case, we obtain a characterization of which welfare outcomes can be achieved for a social planner whose objective is any weighted sum of maximizing total surplus and consumer surplus, and minimizing producer surplus. Such objectives would correspond to valuing market efficiency as well as the redistribution of surplus from producers to consumers. The broad takeaway is that from any such policy perspective, discriminatory pricing and market segmentation under Bertrand oligopoly have the potential to facilitate extremely favorable outcomes.

Maximum consumer surplus is easy to describe. Recall that producers are assumed to price above costs. Thus, a worst case for each producer is that their competitors price as aggressively as possible, and set their prices equal to their respective costs. Irrespective of the particulars of their signals, a producer can always price optimally against this worst case, and guarantee themselves a lower bound on profit. We show that there is an information structure and equilibrium in which each producer's surplus is precisely this lower bound. The outcome is also efficient, and, hence, also maximizes consumer surplus. Note that if there were no segmentation at all, producers would generally all price above cost and producer surplus would be higher. Thus, the segmentation of the market serves both to induce producers to price more aggressively and drive down profits, and also to facilitate an efficient outcome without giving extra rents to producers.

A model of price setting by competing producers is a reverse (or procurement) auction. Our results on the reverse auction have immediate counterparts in standard auction settings. In particular, consider a standard single-unit, first-price auction with the twist that the auctioneer has a heterogeneous cost of delivery to the winning bidder (not necessarily known by bidders) and a bid wins if the net bid (bid minus delivery cost) exceeds other bidders' net bids. Now producers' costs are like bidders' values, and the auctioneer's delivery costs are like the consumer's heterogeneous values. Our assumptions that producers know their costs and the consumer knows their heterogeneous values correspond to assuming that bidders know their values and the auctioneer knows the heterogeneous delivery costs. The finding that total surplus and consumer surplus can be aligned immediately implies that in the standard auction setting, total surplus and revenue can be aligned. Similarly, our results imply that bidder surplus and revenue can be opposed.[5]

Our main result relies on the assumptions that the consumer knows the values for all of the goods and that producers know their costs and never price below cost. In the remainder of the paper, we consider what happens when these assumptions are dropped.

We first analyze what happens if there is market segmentation, but the consumer has only partial information about their values for the goods, thus generalizing the model of RS to oligopoly. An immediate implication of our main result is that consumer surplus and total surplus are *interim aligned*, in the sense that if we hold fixed the consumer's information, then the market segmentation and producer strategies that maximize consumer surplus also maximize total surplus (Proposition 2). However, it may be that the associated outcome is still inefficient ex post simply because the consumer may not know which product generates the highest surplus or if that highest surplus is positive. If the goods are perfect substitutes and there is common knowledge of gains from trade (there is at least one producer whose cost is less than the value), then this issue is moot, and the efficient outcome is feasible regardless of the consumer's information. In this case, we show that consumer surplus and total surplus can be aligned (Theorem 2). However, if either of these assumptions fails, we give examples showing that consumer surplus could be maximized with consumer information that renders the ex post efficient outcome infeasible, and, hence, consumer surplus and total surplus are not aligned. This is the case for a Hotelling duopoly that we study in Section III.

More broadly, we wish to understand the structure of optimal information, even when consumer surplus and total surplus are not aligned. To that end, we completely characterize consumer optimal information in a canonical duopoly setting where the products are differentiated along a Hotelling line. Our main result here, Theorem 3, is a description of the optimal form of segmentation and consumer information and the resulting welfare. In particular, the optimal distribution of the consumer's interim values has a two-sided Pareto shape, and the market is divided into two segments, corresponding to which producer's good has the higher interim value. One can view this model as the natural generalization of the characterization of RS to a duopoly environment. The qualitative insight is that when goods

[5] In the discussion of the literature below, we reinterpret all results that were originally stated for standard first-price auctions, in particular, Bergemann, Brooks, and Morris (2015a) and (2017), within the current framework of price competition.

are heterogeneous, consumer surplus is maximized when the market learns which producer is interim efficient, but it is generally optimal to muddle the consumer's information about which good is ex post efficient, so as to make the goods more substitutable and intensify price competition.

In Supplemental Appendix C, we also ask what happens if producers do not know their own costs. In this case, producers may price below their cost (which they may not know), but we maintain the requirement that producers not set prices that they know are below their cost with probability one. Obviously, this makes no difference when there is no uncertainty about costs. However, we show in Theorem 5, that if values are homogeneous, there are two or more producers, and the support for costs is sufficiently large, then it is possible to attain the same welfare outcomes as if we dropped the assumption that producers price above cost altogether (as in Theorem 4): Consumer surplus is arbitrarily close to the efficient total surplus, and producer surplus is arbitrarily close to zero.

Related Literature.—We analyze a model of competitive price discrimination where producers with heterogeneous products and heterogeneous costs compete for one consumer with unit demand. Relative to the seminal model of oligopoly with product differentiation and uncertain willingness-to-pay of Perloff and Salop (1985), we also allow for uncertainty and private information regarding the production costs and consumer values.

In the special case where products are homogeneous with a commonly known value, our main result (Theorem 1) was proved in Theorem 3 of our working paper, Bergemann, Brooks, and Morris (2015a).[6] In the special case where production costs are commonly known and normalized to zero for all producers, our Theorem 1 was proved independently as Theorem 1 of Elliott et al. (2024). Thus, a contribution of this paper is to show that alignment is satisfied whether or not there is common knowledge of homogeneous values or common knowledge of homogeneous costs. Both these papers build on the third-price discrimination result of BBM. We can relate the results visually in Figure 1.

The case where the consumer does not know their value was studied by RS for the case of one producer. Consistent with our Theorem 2, RS showed alignment when there is common knowledge of gains from trade. The contribution of our Theorem 2 is to extend this result to multiple competing producers, when values are unknown but homogeneous.

		Value	
		Complete Information	Incomplete Information
Cost	Complete Information	Complete information bertrand	Elliott et al. (2024)
	Incomplete Information	Bergemann, Brooks, and Morris (2015a)	Theorem 1

Figure 1

[6]Theorem 3 of Bergemann, Brooks, and Morris (2015a) is unpublished and briefly discussed in Section IVD of the published version, Bergemann, Brooks, and Morris (2017).

We also consider what happens when the assumptions of Theorem 2 fail. Consistent with Theorem 2, RS show (in their Appendix) that alignment fails when there is no common knowledge of gains from trade. The Hotelling model is a leading example for the case of heterogeneous values. It corresponds to the special case of our general model, which follows Perloff and Salop (1985), when there are two producers whose costs are commonly known to be zero and whose goods' values to the consumer are perfectly negatively correlated. Armstrong and Zhou (2022) characterize the information structure of the consumer that maximizes consumer surplus, assuming the producers have no information about the consumer's values beyond the prior, restricting attention to pure strategy equilibria. By contrast, we consider the impact of information on both sides of the market. The two-sided nature of the information design is important in our work, and Theorem 1 would not hold if producers had no information about their competitors. In Section III, we show how additional information for producers leads to more consumer surplus and more efficient allocations than when producers have no information (i.e., the setting of Armstrong and Zhou 2022). The specific information that the producers receive in the optimal information structure is simply to learn whether or not they are the efficient producers.

Bergemann, Brooks, and Morris (2017) considered the case where producers do not know their costs, but values are homogeneous and common knowledge. Their Theorem 2 is closely related to our Theorem 5, as we discuss below.

Our focus in this paper is on maximizing consumer surplus across information structures and equilibria. By contrast, some of the papers described above, and others in the literature, characterize information structures and equilibria, maximizing producer surplus (see, e.g., Bergemann, Brooks, and Morris 2017, 2021; Armstrong and Vickers 2019, 2022; Elliott, Galeotti, Koh, and Li 2024).

The rest of this paper proceeds as follows. Section I presents the baseline model with known values and known costs. Section II contains our main results on the alignment of consumer surplus and total surplus, and the opposition of consumer surplus and producer surplus. Section III presents additional results concerning unknown values. Section IV concludes the paper. Appendix A contains omitted proofs from the main text. Supplemental Appendix B studies equilibrium welfare when producers may price above cost, and Supplemental Appendix C explores the case when producers have imperfect information about their own costs.

I. Model

There are producers $i = 1, \ldots, N$ and a single representative consumer.[7] The consumer demands a unit of a good that may be purchased from at most one producer. The consumer's value for producer i's good is v_i. The cost to producer i of supplying the good is c_i. The fundamental uncertainty about values and costs is described by a Borel probability measure, $\mu(dv, dc) \in \Delta\left(\mathbb{R}_+^{2N}\right)$. For analytical

[7] As mentioned in the introduction, all of our results have an equivalent interpretation where there is a mass of nonatomic consumers, and probability distributions are reinterpreted as the population distribution of types.

simplicity, we assume that values are bounded above by $\bar{v} < \infty$. We also assume that the support for costs is finite.

The producers simultaneously choose prices $p_1, \ldots, p_i, \ldots, p_N$. The consumer does not purchase if $v_i < p_i$ for all i. Otherwise, the consumer buys from a producer i that maximizes $v_i - p_i$, breaking ties uniformly. Thus, an implicit assumption of our model is that the consumer knows their values perfectly at the time they make a purchase. This assumption will be relaxed in Section III. We write $W(p,v)$ for the set of producers that the consumer is willing to purchase from and $q_i(v,p)$ for the likelihood that producer i makes a sale when the prices are $p = (p_1, \ldots, p_i, \ldots, p_N)$ and the values are $v = (v_1, \ldots, v_i, \ldots, v_N)$, that is,[8]

$$W(p,v) \equiv \{i \,|\, v_i - p_i = \max\{0, v_1 - p_1, \ldots, v_N - p_N\}\};$$

$$q_i(p,v) \equiv \begin{cases} \frac{1}{|W(p,v)|}, & \text{if } i \in W(p,v) \\ 0, & \text{otherwise.} \end{cases}$$

At the time of setting prices, each producer knows their cost and may have additional information about values and others' costs. This is described by an *information structure* (S, ϕ), where $S = \prod_i S_i$ is a product space of signal profiles, each S_i is a measurable space, and ϕ is a joint probability measure $\phi(ds, dv, dc)$ whose marginal on (v, c) is μ.

A *strategy* for producer i is a measurable function ρ_i that associates to each $(s_i, c_i) \in S_i \times \mathbb{R}_+$ a probability measure on $\{p_i \in \mathbb{R}_+ \,|\, p_i \geq c_i\}$. In other words, we assume that producers price weakly above cost. We identify a strategy profile $\rho = (\rho_1, \ldots, \rho_N)$ with the measurable function that maps each (s, c) into the product measure $\rho(dp \,|\, s, c) = \prod_i \rho_i(dp_i \,|\, s_i, c_i)$. In Supplemental Appendix B, we explore what would happen if we allow producers to price below cost.

Given an information structure (S, ϕ) and strategy profile ρ, the resulting ex ante expected surplus for producer i, consumer surplus, and total surplus are, respectively,

$$PS_i(S, \phi, \rho) \equiv \int_{s,v,c,p} (p_i - c_i)\, q_i(v,p) \rho(dp \,|\, s, c) \phi(ds, dv, dc);$$

$$CS(S, \phi, \rho) \equiv \sum_{i=1}^{N} \int_{s,v,c,p} (v_i - p_i)\, q_i(v,p) \rho(dp \,|\, s, c) \phi(ds, dv, dc);$$

$$TS(S, \phi, \rho) \equiv \sum_{i=1}^{N} \int_{s,v,c,p} (v_i - c_i)\, q_i(v,p) \rho(dp \,|\, s, c) \phi(ds, dv, dc).$$

[8] The particulars of the tie-breaking rule do not matter for our results, since in the equilibria we construct, ties occur with zero probability, and in the event that a tie occurs after a deviation, the deviator's surplus is nonpositive. We have modeled tie breaking in simplest and most natural manner possible.

Ex ante expected producer surplus is $PS(S,\phi,\rho) \equiv \sum_i PS_i(S,\phi,\rho)$. Note that $PS + CS = TS$.

The strategy profile ρ is a (*Bayes Nash*) *equilibrium* if $PS_i(S,\phi,\rho) \geq PS_i(S,\phi,\rho_i',\rho_{-i})$ for every i and strategy ρ_i'. Note that in any information structure and strategy profile, total surplus is bounded above by the efficient total surplus $\overline{TS}$:

$$TS(S,\phi,\rho) \leq \overline{TS} \equiv \int_{v,c} \max\{0, v_1 - c_1, \ldots, v_N - c_N\} \mu(dv,dc).$$

We say that consumer surplus and total surplus can be *aligned* if there exists an information structure and equilibrium (S,ϕ,ρ) that simultaneously maximizes both welfare criteria, across all information structures and equilibria. Consumer surplus and producer surplus are *opposed* if there is an information structure and equilibrium (S,ϕ,ρ) that simultaneously maximizes consumer surplus and minimizes producer surplus. The primary objective of our analysis is to characterize when consumer surplus and total surplus can be aligned. A secondary objective is to understand when consumer surplus and producer surplus are opposed.

II. The Alignment of Consumer Surplus and Total Surplus

We now present our main results for the model just described. First, we define a lower bound on producer surplus in any information structure and equilibrium. Then we construct an information structure and equilibrium in which this lower bound is attained and the outcome is socially efficient.

A. *Main Result*

To that end, we now describe a lower bound on producer surplus given by

$$(1) \qquad \underline{PS}_i \equiv \sup_{f:\mathbb{R}_+\to\mathbb{R}_+} \int_{v,c} [f(c_i) - c_i] q_i(v, f(c_i), c_{-i}) \mu(dv,dc).$$

This is the highest producer surplus that producer i can obtain if the other producers are pricing at cost, and producer i chooses a best response $f: \mathbb{R}_+ \to \mathbb{R}_+$ that conditions on their own cost. Let $\underline{PS} \equiv \sum_i \underline{PS}_i$. As the following result shows, $\underline{PS}_i$ is a lower bound on producer i's profit in any equilibrium under any information structure.

PROPOSITION 1 (Lower Bound for Producer Surplus): *For any* (S,ϕ) *and equilibrium* ρ, $PS_i(S,\phi,\rho) \geq \underline{PS}_i$.

PROOF:

Observe that $q_i(p,v)$ is nondecreasing in p_{-i}, and since $p_{-i} \geq c_{-i}$, we have that $q_i(p,v) \geq q_i(p_i,c_{-i},v)$. Let f be a function that attains a value of $\underline{PS}_i - \varepsilon$ for some

$\varepsilon > 0$, and let ρ_i' be a strategy that for every (s_i, c_i) puts probability one on $f(c_i)$. Since ρ is an equilibrium, we have

$$\begin{aligned} PS_i(S,\phi,\rho) &\geq \int_{s,v,c,p} \big[f(c_i) - c_i\big] q_i\big(v, f(c_i), p_{-i}\big) \rho\big(dp \,|\, s,c\big) \phi\big(ds,dv,dc\big) \\ &\geq \int_{s,v,c,p} \big[f(c_i) - c_i\big] q_i\big(v, f(c_i), c_{-i}\big) \rho\big(dp \,|\, s,c\big) \phi\big(ds,dv,dc\big) \\ &= \int_{v,c,p} \big[f(c_i) - c_i\big] q_i\big(v, f(c_i), c_{-i}\big) \mu\big(dv,dc\big) \\ &\geq \underline{PS}_i - \varepsilon. \end{aligned}$$

Since ε was arbitrary, the result follows. ∎

What the proof effectively shows is that each producer always has the option to ignore their signal and just price as a function of their own cost, and best respond as if other producers were pricing at cost. The resulting worst-case payoff is then a lower bound on what a producer can achieve when a producer has more information available and others' prices are weakly greater than costs. We now present our main result.

THEOREM 1 (Alignment): *Consumer surplus and total surplus can be aligned. Consumer surplus and producer surplus are opposed. Moreover, there is an information structure and an equilibrium in which each producer's surplus is* $\underline{PS}_i$, *total surplus is* $\overline{TS}$, *and consumer surplus is* $\overline{TS} - \sum_i \underline{PS}_i$.

The formal proof of Theorem 1 is in the Appendix. Here, we will motivate and sketch the construction of the information structure and equilibrium that simultaneously maximize consumer surplus, maximize total surplus, and minimize producer surplus.[9]

Competitive Pricing under No Information and Complete Information.—To motivate the construction, let us consider two natural benchmarks for the producers' information. If producers had no information beyond knowing their own costs (so $|S_i| = 1$ for each i), then the equilibrium would generally be in mixed strategies, and producers would sometimes price strictly above cost. The outcome would therefore be inefficient because consumers would not buy when their values are close to but above cost.

At the opposite extreme, suppose producers have complete information, meaning that there is common knowledge of (v, c) (e.g., $S_i = \mathbb{R}_+^N \times \mathbb{R}_+^{N-1}$, with typical element $(\tilde{v}^i, \tilde{c}_{-i}^i)$, and the joint distribution ϕ is such that with probability

[9] We note that the proof of Theorem 1 shows an even stronger result: In any information structure and equilibrium that maximizes consumer surplus, total surplus is necessarily maximized and producer surplus is necessarily minimized. However, there are information structures and equilibria that maximize total surplus but do not maximize consumer surplus or minimize producer surplus. We thank a referee for a stimulating discussion of this issue.

one, $(\tilde{v}^i, \tilde{c}^i_{-i}) = (v, c_{-i})$ for each i). All equilibria have the following properties (Blume 2003; Kartik 2011): The *efficient surplus* is

$$TS(v,c) \equiv \max\{0, v_1 - c_1, \ldots, v_N - c_N\}.$$

In equilibrium, if $TS(v,c) = 0$, then producers can set any prices above cost, and the consumer does not purchase. If $TS(v,c) > 0$, then some producer i which maximizes the efficient surplus $TS(v,c)$ prices at the consumer's *residual willingness to pay* r_i:

$$r_i \equiv v_i - TS(v_{-i}, c_{-i}), \tag{2}$$

where

$$TS(v_{-i}, c_{-i}) \equiv \max\{0, v_1 - c_1, \ldots, v_{i-1} - c_{i-1}, v_{i+1} - c_{i+1}, \ldots, v_N - c_N\}.$$

Some of the *runner-up* producers that are efficient among $-i$ play mixed strategies that induce the efficient producer to price at r_i.

The outcome under complete information is socially efficient, and all except for the efficient producer are pricing (nearly) at cost, and in that sense competition is more aggressive than under no information. At the same time, the efficient producer receives their entire marginal contribution to total surplus, since

$$p_i - c_i = r_i - c_i = v_i - TS(v_{-i}, c_{-i}) - c_i = TS(v,c) - TS(v_{-i}, c_{-i}).$$

Hence, producers still retain quite a bit of monopoly power.

Producer Pricing under Partial Information.—We can do even better for the consumer by applying the ideas from third-degree price discrimination, as analyzed by BBM. In a setting with a single producer, Theorem 1 of BBM says that there exists a signal and associated optimal pricing strategy for the monopolist with the property that producer surplus is the same as if the monopolist has no information, but the induced outcome is socially efficient. Since it is impossible for the producer to obtain less than the no-information payoff (they can always ignore the information they receive), consumer surplus attains an upper bound, and must therefore be maximized. Roughly speaking, this is achieved by pooling a relatively large proportion of low-value types with some higher-value types of the consumer in such a way that the monopolist is just barely willing to drop the price, and the higher value consumer types reap all the gains in total surplus.

To apply the above segmentation logic to the setting with many producers, first fix the identity i and cost c_i of an efficient producer. As we have already observed, if producers $-i$ price at cost, then there is an induced *residual* (*willingness to pay*), denoted r_i, which essentially plays the same role as the value v_i in the monopoly case. There is an associated lower bound on profit, which is achieved by setting a price $p_i^*(c_i)$, which is the best response when other producers price at cost, producer i has no additional information beyond their own cost, and all ties are broken in favor of producer i. This last assumption is problematic, but continuing with it for the moment, we may then invoke the result of BBM to conclude that there is a signal for producer i about r_i such that they would still be willing to price at $p_i^*(c_i)$ (and therefore do not benefit from the information). Moreover, producer i is also willing to set a price equal to the lowest value of the residual willingness to pay r_i that is in support of the posterior distribution. The resulting outcome is socially efficient, and, hence, the bounds on surplus in Theorem 1 are achieved. Moreover, since producer i sets a price $p_i \leq r_i$ with probability one, we have that the consumer's willingness to pay for the good of producer $j \neq i$ is at most

$$v_j - (v_i - p_i) \leq v_j - (v_i - r_i) = v_j - TS(v_{-i}, c_{-i}) \leq v_j - (v_j - c_j) = c_j,$$

so that producer j can only make a sale by pricing weakly below their cost. Hence, the inefficient producers have no profitable deviation either, and we are done.

The only problem with this argument is the presumption that ties are broken in favor of the efficient producer, whereas, in fact, they are broken uniformly. In the formal proof, we finesse this issue using the same kind of mixing as in the complete information case (Blume 2003).

To summarize the information structure: (i) it publicly reveals the identity of the efficient producer; (ii) it generates a signal for the efficient producer i about r_i, using the construction of BBM, so that under the premise that $p_{-i} = c_{-i}$, producer i would get the payoff $\underline{PS}_i$, but they also make a sale whenever it is efficient to do so; and (iii) there is an additional signal for any producer j that might tie with the efficient producer i that tells them how to randomize to break ties in favor of the efficient producer, while still inducing the efficient producer to price as needed for (ii). The associated strategies are such that the efficient producer i sets a price equal to the lowest possible value of r_i, conditional on their information, and the inefficient producers either price at cost or randomize as per case (iii). The resulting outcome is efficient and producers are held down to their lower bound surplus, and, hence, consumer surplus is maximized.

B. *An Example with Binary Costs and Binary Values*

We now illustrate how this construction works with a simple example.

EXAMPLE 1: *There are two producers that offer differentiated products with uncertain cost $c_i \in \{0,1\}$ and value $v_i \in \{1,4\}$. Each profile of costs and values (v_1, v_2, c_1, c_2) is equally likely. We now apply the construction underlying Theorem 1 to obtain the consumer-surplus maximizing information structure. The residual willingness to pay r_i, as defined earlier in (2), for producer i is given in the following table:*

		r_i		
$(v_i, c_i) \backslash (v_j, c_j)$	$(1,1)$	$(1,0)$	$(4,1)$	$(4,0)$
$(1,1)$	0	0	−2	−3
$(1,0)$	1	0	−2	−3
$(4,1)$	4	3	0	0
$(4,0)$	4	3	1	0

Note that both producers are efficient on the diagonal, and producer i is efficient only in the region including and below the diagonal. Moreover, producer i's cost is less than the residual on and below the diagonal, which is precisely when they are the efficient producer.

We now construct the consumer-surplus maximizing information structure and pricing policy. First, producers learn whether or not they have the same profile. If they do, they price at cost, and the consumer receives all the surplus. Otherwise, they learn who is the efficient producer. The inefficient producer j will randomize over an interval $\left(c_j, c_j + \varepsilon\right)$ just so as to make it suboptimal to set any price not in the support of r_i. If the efficient producer only knew their own cost, then the conditional likelihoods of each residual are given by

		r_i		
c_i	Prob	1	3	4
1	$1/3$	0	$1/2$	$1/2$
0	$2/3$	$1/2$	$1/4$	$1/4$

Given $c_i = 1$, the optimal price is $p_i = 3$, and the resulting outcome is efficient. In this case, there is no need to give the efficient producer any additional information. However, given $c_i = 0$, the optimal price is $p_i = 3$, which would lead to an inefficient allocation. To amend this, we give the efficient producer an additional signal when $c_i = 0$, following the construction of BBM. In particular, we "divide" the distribution of r_i into two segments according to the following table:

		r_i		
Segment	Prob	1	3	4
$p_i = 1$	$3/4$	$2/3$	$1/6$	$1/6$
$p_i = 3$	$1/4$	0	$1/2$	$1/2$
Total	1	$1/2$	$1/4$	$1/4$

It is easily verified that the given prices are optimal on their respective segments and would result in the efficient producer always making a sale. Moreover, the unconditional optimal price of 3 is also optimal in both segments, so producer surplus has not changed.

III. Market Segmentation and Unknown Values

We now consider what happens if the consumer may have only partial information about their value for the products. We first observe that the logic of alignment of Theorem 1 goes through, holding fixed the consumer's information. This immediately delivers Proposition 2 on interim alignment of consumer surplus and total surplus, and interim opposition of consumer surplus and producer surplus. Theorem 2 then establishes that our main result about alignment holds under the ex ante notion of efficiency, under the hypotheses that there is common knowledge of gains from trade and when the goods are homogeneous. We then show how misalignment can occur where the goods are not homogeneous. We provide a complete analysis of the optimal information structure in the Hotelling model of competition in Theorem 3. In this canonical model of horizontal differentiation, the values of the consumer are heterogeneous, as they depend on the location of the consumer.

A. *Interim Alignment*

We model partial information of the consumer by generalizing our definition of an information structure. We say that a distribution $\mu'(dv', dc)$ is a *value garbling* of μ if there is a probability transition kernel $\eta : \mathbb{R}_+^2 \to \Delta(\mathbb{R}_+)$ such that

$$\mu(dv, dc) = \int_{v'} \mu'(dv', dc) \eta(dv \,|\, v', c)$$

and

$$\int_{v} v\eta(dv \,|\, v', c) = v'.$$

In other words, the distribution $\mu(dv, dc)$ is obtained from $\mu'(dv', dc)$ by adding noise to v' that has mean zero conditional on (v', c). This noise represents the consumer's residual uncertainty about the value. An *unknown values information structure* is an information structure as defined in Section I, except that we only require that the marginal distribution of the joint distribution (of the information structure) ϕ on (v, c) is a value garbling of μ. (We previously required that this marginal of ϕ is exactly μ.)

This definition of an unknown values information structure builds in a nontrivial restriction. Namely the producers only have information about the consumer's interim expected value, and not directly about the ex post value of the consumer. Without this assumption, it could be that producers know more about the true value than does the consumer. And if producers can price based on such information, then the consumer might end up with a nontrivial inference problem about their true value, given the prices they observe. Our assumption that the consumer knows everything the producers know about v shuts down this signaling channel.[10]

We say that consumer surplus and total surplus are *interim aligned* if holding fixed the marginal on (v, c), there is an information structure and equilibrium that simultaneously maximizes both consumer surplus and total surplus. Similarly, we say that consumer surplus and producer surplus are *interim opposed* if holding fixed the marginal on (v, c), there is an information structure and equilibrium that simultaneously maximizes consumer surplus and minimizes producer surplus. In particular, let us define interim analogues of the bounds from Theorem 1:

$$\overline{TS}(\mu') \equiv \int_{v,c} \max\{0, v_1 - c_1, \ldots, v_N - c_N\} \mu'(dv, dc),$$

$$\underline{PS}_i(\mu') \equiv \sup_{f: \mathbb{R}_+ \to \mathbb{R}_+} \int_{v,c} [f(c_i) - c_i] q_i(f(c_i), c_{-i}, v) \mu'(dv, dc) \;;$$

[10] For a discussion of what might happen with such signaling through prices in the monopoly context, see Kartik and Zhong (2023).

Our first result on the unknown values model is the following:

PROPOSITION 2 (Interim Alignment): *Consumer surplus and total surplus are interim aligned, and consumer surplus and producer surplus are interim opposed. In particular, if there is an optimal information structure such that the marginal on* (v,c) *is* μ', *then there is a consumer surplus maximizing information structure and equilibrium in which each producer's surplus is* $\underline{PS}_i(\mu')$, *total surplus is* $\overline{TS}(\mu')$, *and consumer surplus is* $\overline{TS}(\mu') - \sum_i \underline{PS}_i(\mu')$.

PROOF:

Applying Theorem 1 to the case where the prior is μ', we conclude that holding fixed μ', there is an information structure and equilibrium that simultaneously maximizes consumer surplus, maximizes total surplus, and minimizes producer surplus, and attains the welfare outcome in the statement of the proposition. The result then follows immediately. ∎

B. *Homogenous Values*

We now give conditions under which consumer surplus and total surplus can be aligned, even when there are unknown values. We say that values are *homogeneous* if $v_1 = \ldots = v_N$ μ-almost surely, meaning that the goods are perfect substitutes from the consumer's perspective. We say that there is *common knowledge of gains from trade* if $\max_i v_i - c_i \geq 0$ μ-almost surely.

THEOREM 2 (Alignment with Unknown Values): *Suppose that values are unknown and homogeneous, and there is common knowledge of gains from trade. Then consumer surplus and total surplus can be aligned, and consumer surplus and producer surplus are opposed. In particular, if consumer surplus is maximized when the marginal on* (v,c) *is* μ', *then there is a consumer surplus maximizing information structure and equilibrium in which each producer's surplus is* $\underline{PS}_i(\mu')$, *total surplus is* $\overline{TS}(\mu') = \overline{TS}$, *and consumer surplus is* $\overline{TS} - \sum_i \underline{PS}_i(\mu')$.

Theorem 2 shows that when values are unknown and goods are homogeneous, consumer surplus and total surplus can be aligned. However, the theorem does not provide a detailed characterization of the optimal interim value distribution for the consumer. For the special case of one producer, RS characterize the consumer surplus maximizing information: The consumer's interim expected value has a truncated Pareto distribution, so that the producer is willing to price at the bottom of the support, and the parameters of that distribution minimize the price subject to the constraint that the interim value distribution is a mean-preserving contraction of the prior.

Beyond the monopoly case, we are not aware of a general characterization of the consumer-surplus maximizing information. In the working paper version, Bergemann, Brooks, and Morris (2023), we fully solve the following example with two producers.

EXAMPLE 2 (Duopoly with Unknown and Homogenous Values): *A consumer's ex post value is in the interval* $[0,1]$ *and has distribution F. Producer has cost* $c_1 = 0$, *and producer 2 has cost* $c_2 \in [0,1]$.

Note that this example satisfies the hypotheses of Theorem 2, so that consumer surplus and total surplus can be aligned, and consumer surplus will be maximized at an outcome that is ex post efficient. It is straightforward to see that producer 2 will price at cost, so that the consumer's willingness to pay for producer 1's good is the minimum of their interim value v and producer 2's cost, c_2. Even in this simple case, the optimal information of the consumer departs significantly from the solution of RS. The reason is that what matters for producer 1 is the interim residual willingness to pay $\min\{v, c_2\}$, and the mean-preserving constraint on v imposes only weak restrictions on the distribution $\min\{v, c_2\}$.

As suggested by Theorem 2, even when values are homogenous, alignment may fail when there is no common knowledge of gains from trade. In fact, RS and KZ have noted the simplest example of this when there is a single producer, μ puts probability one on a particular cost c_1, and there is positive probability that $v_1 < c_1$. While the baseline model of RS assumes common knowledge of gains from trade, their Appendix contains an extension to the case where the consumer's value is less than the producer's cost with positive probability, and they find that the information that maximizes consumer surplus can result in inefficient trade. KZ similarly finds that the outcome can be inefficient in the monopoly setting when cost and value are correlated, and there is no common knowledge from gains from trade. We can illustrate the inefficiency with the following simple example.

EXAMPLE 3 (Monopoly without Common Knowledge of Gains from Trade): *The monopolist's cost is* $c_1 = 1$ *and* $v_1 \in \{0,4\}$, *with both values equally likely. For trade to be efficient, the consumer must learn their value exactly. But in that case, the optimal price is* $p_1 = 4$, *so that consumer surplus is zero. On the other hand, if the consumer knew nothing, then the expected value would be* 2, *and this would be the optimal price. Finally, if there is small probability* ε *such that the consumer learns their value, and otherwise learns nothing, then* 2 *is still the optimal price, and the consumer earns positive surplus when they learn that their value is* 4, *which occurs with probability* $\varepsilon/2$, *though the outcome is inefficient.*

C. *Heterogeneous Values and the Hotelling Model*

We now consider what happens with heterogeneous values, focusing on the Hotelling duopoly model. Producers $i = 1,2$ have zero cost $c_i = 0$. Values are symmetrically and perfectly negatively correlated, with $v_1 + v_2 = \bar{v}$. Recall that r_i is the residual willingness to pay for the product of producer i:

$$r_i = v_i - v_j \in [-\bar{v}, \bar{v}].$$

We denote by F the distribution of $r_i \in [-\bar{v}, \bar{v}]$. By the assumed symmetry of the producers, r_1 and r_2 have the same distribution, and thus we drop the subscript i on r for the remainder of this section.

This model can be viewed as a generalization of RS to more than one producer. It was also recently studied by Armstrong and Zhou (2022), who only considered variations in the consumer information, but fixed the information of the producers to be the prior. In contrast, and like RS, we allow for the consumer and the producers' information to vary and to shape welfare. Our main result is a complete characterization of the information and equilibrium that maximizes consumer surplus. In RS, market segmentation plays no role, and maximum consumer surplus can be achieved without any market segmentation. As we will see, with more than one producer, nontrivial market segmentation plays a key role in pinning down maximum consumer surplus.

Now, to see why consumer surplus and total surplus may not be aligned, we can consider the following simple binary example.

EXAMPLE 4 (Hotelling with Binary Values): *The value profiles* $(v_1, v_2) \in \{(0,1),(1,0)\}$ *are equally likely, so that* r *is equally likely to be* ± 1. *For the outcome to be efficient, the consumer would have to learn which producer gives them the higher value. In that case, each producer knows that the consumer's residual for their good is equally likely to be 0 and 1, so the optimal price is* $p_i = 1$, *and consumer surplus is zero. On the other hand, if the consumer has no information about the value, then their expected value is* $1/2$ *for both producers. The producers will compete the price down to cost, so* $p_1 = p_2 = 0$. *Consumer surplus is equal to* $1/2$, *which is also the total surplus. The first takeaway is that uncertainty about the consumer's ex post value can induce more competition, thereby raising consumer surplus but lowering efficiency.*[11]

We now characterize the distribution of interim residual willingness to pay that maximizes consumer surplus in the Hotelling model. Given parameters $0 < b < B$, we define the cumulative distribution function:

$$G_b^B(r) \equiv \begin{cases} 0, & \text{if } r \leq -B \\ -\frac{b}{2r}, & \text{if } -B < r \leq -b \\ 1/2, & \text{if } -b < r \leq b \\ 1 - \frac{b}{2r}, & \text{if } b < r \leq B \\ 1, & \text{if } r > B; \end{cases}$$

Thus, G_b^B is a two-sided truncated Pareto distribution with bounds $\pm b$ and $\pm B$. Each segment has a mass point at $|B|$, and the distribution G_b^B is constant between $[-b, b]$.

[11] In Bergemann, Brooks, and Morris (2023), we also provide a complete solution to the Hotelling model in which r is uniformly distributed on $[-1, 1]$.

THEOREM 3 (Consumer Surplus Maximizing Information Structure in Hotelling Model): *In the Hotelling model, there exists bounds b and B such that the interim residual distribution G_b^B maximizes consumer surplus. Under the consumer surplus maximizing information and equilibrium, producers learn which of them is the interim efficient producer. The inefficient producer prices at cost and the efficient producer sets a price $p = b$.*

The proof in the Appendix closely follows that of Lemma 1 of Roesler and Szentes (2017) separately for each efficient producer and then joins the solution across the segments. The problem of computing the consumer surplus maximizing information structure is thus reduced to optimization over all parameters (b, B), subject to the constraint that F is a mean-preserving spread of G_b^B.

It should be noted that the consumer surplus maximizing parameters (b, B) are generally distinct from those that minimize producer surplus, and hence consumer surplus and producer surplus are not opposed in the Hotelling model.

We now derive the parameters b and B that maximize consumer surplus for Example 4. Recall that under complete information, r is equally likely to be ± 1. Thus, the mean-preserving contraction constraints are automatically satisfied by G_b^B as long as $B \leq 1$. Given an interim value distribution G, total surplus can be written as the sum over the expectation of the unconditional value and the residual value of the representative producer, thus

$$\overline{TS}(G) \equiv \int_{v=0}^{\bar{v}} v\mu(dv) + \int_{r=0}^{\bar{v}} rG(dr). \tag{3}$$

Hence,

$$\overline{TS}(G_b^B) = \frac{1}{2} + \int_{r=b}^{1} rG_b^B(dr) = \frac{1}{2}\left[1 + b + b(\ln B - \ln b)\right].$$

Consumer surplus is

$$\overline{TS}(G_b^B) - b = \frac{1}{2}\left[1 - b + b(\ln B - \ln b)\right].$$

The optimal information structure sets $B^* = 1$ and $b^* = 1/e^2 \approx 0.07$, and the maximized consumer surplus is $1/2(1 + e^{-2}) \approx 0.57$. Note that the total surplus is $1/2(1 + 3e^{-2}) \approx 0.70$, whereas the efficient surplus is 1.

Armstrong and Zhou (2022) characterize the consumer optimal information structure for a Hotelling duopoly under the additional constraints that producers receive no information and the producers use pure strategies.[12] They find that the optimal distribution of interim residual values has a censored Pareto shape with different parameter values. In order to avoid the natural separation into two local monopolies, the distribution of interim residual value must have extra mass around zero. For our binary example, Armstrong and Zhou (2022) find that consumer surplus is maximized when both producers price at $b^* \approx 0.05$. The resulting total surplus is

[12] Armstrong and Zhou (2022) also consider maximum producer surplus, whereas our focus is on consumer surplus.

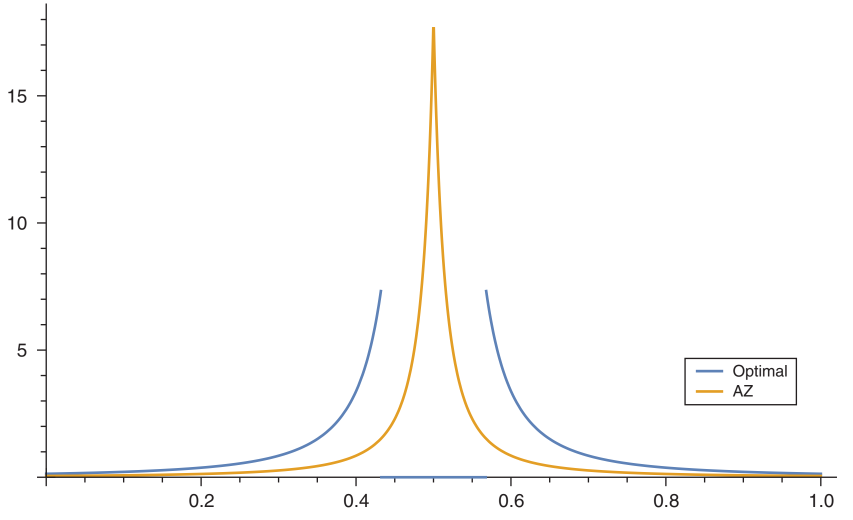

Figure 2. Probability Density of Consumer Surplus Maximizing Interim Values

$\approx$ 0.57, and the resulting consumer surplus is $\approx$ 0.52. Thus, total surplus, producer surplus, and consumer surplus are all lower when the producers are constrained to have no information, relative to the case studied in the present paper. Figure 2 compares the optimal distribution of r in our model and in Armstrong and Zhou (2022).

The logic underlying Theorem 3 readily generalizes to a considerably larger class of models. First, it is not essential that values are perfectly negatively correlated. Suppose that the values are distributed according to $\mu(v_1, v_2)$, with both v_1 and v_2 being nonnegative. By Proposition 2, it is still the case that in the consumer-surplus maximizing information structure, the producers learn which of them is efficient, and the residual willingness to pay for the efficient producer i's good is $r_i = v_i - v_j$. Thus, only information about the residual is strategically relevant to the producers, and the variation in levels of values is only important insofar as it contributes to the total surplus. Indeed, the efficient surplus can be more generally written as

$$\overline{TS}(\mu') = \int_{(v_1,v_2)} \left[\frac{v_1 + v_2}{2} + \frac{|v_1 - v_2|}{2}\right] \mu'(dv_1, dv_2).$$

In addition, while we assumed that the distribution of residuals was symmetric, this was not essential to our argument. The construction of the mean-preserving contraction in the proof of Theorem 3 was done separately, conditional on the identity of the efficient producer. In fact, the argument could even be applied with more than two producers: All that matters is the consumer's interim expectation of their residual r_i for the efficient producer i's good, assuming the other producers price at cost, and it is without loss to consider distributions of r_i that have the censored Pareto shape. The mean-preserving spread constraints do take a more complicated form when there are more than two producers.

D. *Unknown Costs*

Our focus in this section has been on the case where consumers may not perfectly know their values. Alternatively, we may also consider what happens if producers do not perfectly know their own costs. In Supplemental Appendix C, we investigate this possibility in detail. Of course, if there is no uncertainty about costs, then the preceding results would apply. However, if there is sufficient uncertainty about costs, then it is generally possible to support more extreme outcomes. In particular, Theorem 5 gives conditions under which there are efficient equilibria in which producer surplus is (nearly) zero. A key issue is that when producers are uncertain about costs, we can no longer restrict attention to strategies in which producers price above cost (which may be infeasible), and the milder requirement that producers not set prices that they are certain are above cost has very little bite. The consumer surplus maximizing equilibria then closely resemble the equilibria in complete information Bertrand in which producers price above cost (Blume 2003). We also consider examples where there is an intermediate amount of uncertainty about costs, in which case there are still nontrivial restrictions on the welfare outcomes that can arise in equilibrium.

IV. Conclusion

The purpose of this paper has been to investigate the role of information and competition in determining welfare in models of price competition with differentiated products. In the monopoly setting, BBM showed that consumer surplus and total surplus can be aligned, and consumer surplus and producer surplus are opposed. Our main result dramatically extends this finding to the oligopoly setting: It is possible for information to simultaneously maximize consumer surplus and total surplus, while the producers are no better off than if they had no information *and if their competitors priced as aggressively as possible*. A takeaway is that there is no inherent conflict between consumer surplus and total surplus. We have considered whether this finding extends when the consumer may have partial information about their values and when producers have partial information about their costs. In both cases, consumer surplus and total surplus may or may not be aligned, depending on what additional assumptions we make about the distribution of values and costs. For settings with unknown values and/or unknown costs, we have stopped short of a complete and general characterization of the information that maximizes consumer surplus. More broadly, even with known values and known costs, we have focused on characterizing maximum consumer surplus and total surplus. It remains an open question what is the whole set of welfare outcomes that are achievable with information and competition, even when values and costs are known.

Appendix A Appendix: Omitted Proofs

A1. *Proof of Theorem 1*

The information structure we construct has the form

$$S_i = \{0\} \cup \left(\{1,\ldots,N\} \times \mathbb{R} \times \Delta(\mathbb{R}) \times \{0,1\}\right).$$

Thus, each producer either gets a signal 0 or a signal that is a tuple $s_i = (k_i, \tilde{c}_i, x_i, l_i)$. Moreover, the first three components of the signal are public, meaning that with probability one, $k_1 = \ldots = k_N$, $\tilde{c}_1 = \ldots = \tilde{c}_N$, and $x_1 = \ldots = x_N$, and hence, we will drop the subscript and just write $(k, \tilde{c}, x)$.

First, the producers' signals are all 0 with likelihood $\left(1 - \sum_{k>0} q_k(v,c)\right)\mu(dv, dc)$. (Recall that $1 - \sum_k q_k(v,c)$ is either zero or one, and it is one if and only if production is inefficient.)

Now we describe the signals when production is efficient. We first construct the joint distribution of (k,v,c) to be $q_k(v,c)\mu(dv,dc)$ for $k > 0$. In other words, k is the identity of the producer that the consumer would choose to purchase from if all producers priced at cost, with ties broken uniformly. We define, for all i,

$$r_i(v, c_{-i}) \equiv \min_{j \neq i} v_i - v_j + c_j.$$

This is the "residual" willingness to pay of the consumer for producer i's good when other producers price at cost. We can then define a measure $\zeta^i(dr_i, dv, dc)$, according to

$$\zeta^i(X) \equiv \int_{\{(v,c) | (r_i(v,c_{-i}),v,c) \in X\}} q_i(v,c)\mu(dv,dc).$$

This measure can then be disintegrated as $\zeta^i(dr_i, dv, dc) = \eta^i(dc_i)\nu^i(dr_i | c_i)\gamma^i(dv, dc_{-i} | r_i, c_i)$.

Claim: For every i and c_i, there is a solution to

$$\max_{p_i} (p_i - c_i) \int_{r_i} \mathbf{1}_{\{r_i \geq p_i\}} \nu^i(dr_i | c_i),$$

which we denote by $p_i^*(c_i)$. This follows from the fact that the integral is simply the upper cumulative distribution of the random variable r_i, which is upper semi-continuous, and the domain of p_i can, without loss, be restricted to $[c_i, \bar{v}]$ (since $q_i(v, p_i, p_{-i}) = 0$ when $p_i > \bar{v}$, ν^i almost surely).

Claim: For every i,

$$\underline{PS}_i = \int_{c_i} \eta^i(dc_i)\left[p_i^*(c_i) - c_i\right] \int_{r_i \geq p_i^*(c_i)} \nu^i(dr_i | c_i).$$

To prove the claim, observe that in (1) it is without loss to restrict attention to f such that $f(c_i) \geq c_i$ for all i, since otherwise the contribution to the right-hand side is necessarily nonpositive. Among such functions, let f be one for which the right-hand side of (1) is at least $\underline{PS}_i - \varepsilon$. Note that if $q_i(v,c) = 0$ (meaning that producer i is *not* an efficient producer), then $q_i(v, f(c_i), c_{-i}) = 0$ as well. Thus, the contribution to the right-hand side of the event where producer i is not efficient is zero. Moreover, if $q_i(c,v) \in (0,1)$, meaning that there is more than one efficient producer, then the contribution must be zero as well. The reason is that if $f(c_i) = c_i$, then the contribution is zero because producer i is pricing at cost, and if

$f(c_i) > c_i$, then $q_i(f(c_i), c_{-i}, v) = 0$, because the consumer would not want to buy from producer i at a price strictly higher than c_i. Thus, the contribution to the right-hand side is strictly positive only if producer i is the *unique* efficient producer, and hence,

$$\int_{v,c} [f(c_i) - c_i] q_i(v, f(c_i), c_{-i}) \mu(dv, dc)$$

$$= \int_{v,c} [f(c_i) - c_i] q_i(v, f(c_i), c_{-i}) q_k(c, v) \mu(dv, dc)$$

$$\leq \int_{v,c} [f(c_i) - c_i] \mathbf{1}_{\{r_i(v, c_{-i}) \geq f(c_i)\}} q_k(v, c) \mu(dv, dc)$$

$$\leq \int_{v,c} [p_i^*(c_i) - c_i] \mathbf{1}_{\{r_i(v, c_{-i}) \geq p_i^*(c_i)\}} q_k(v, c) \mu(dv, dc).$$

In the first inequality, we used the fact that if $q_i(f_i(c_i), c_{-i}, v) > 0$, then $r_i(v, c_{-i}) \geq f(c_i)$ (otherwise the consumer would not be willing to purchase from producer i with positive probability). To complete the proof of the claim, it only remains to show that there exist fs for which the gap is arbitrarily small. Let $f(c_i) = p^*(c_i) - \varepsilon$. Then, $r_i(v, c_{-i}) \geq p_i^*(c_i)$ implies that $q_i(v, f(c_i), c_{-i}) = 1$, so

$$\int_{v,c} [f(c_i) - c_i] q_i(v, f(c_i), c_{-i}) q_k(c, v) \mu(dv, dc)$$

$$\geq \int_{v,c} [f(c_i) - c_i] \mathbf{1}_{\{r_i(v, c_{-i}) \geq p_i^*(c_i)\}} q_k(v, c) \mu(dv, dc)$$

$$\geq \int_{v,c} [p_i^*(c_i) - c_i] \mathbf{1}_{\{r_i(v, c_{-i}) \geq p_i^*(c_i)\}} q_k(v, c) \mu(dv, dc) - \varepsilon$$

$$= \int_{c_i} [p_i^*(c_i) - c_i] \eta^i(dc_i) \int_{\{r_i \geq p_i^*(c_i)\}} \nu^i(dr_i | c_i) - \varepsilon,$$

as desired.

Now, we invoke Theorem 1B of BBM, which says that for every c_i, there exists a *uniform profit preserving segmentation*, which we write as $\sigma_i(\cdot | c_i) \in \Delta\Delta(\mathbb{R})$and $\sigma_i(dx | c_i)$, where x is itself a probability measure on the reals, with the properties that for every x in the support of $\sigma_i(\cdot | c_i)$ and p_i in the support of x,

$$(p_i - c_i) x([p_i, \bar{v}]) = \min \operatorname{supp} x,$$

$$p_i^*(c_i) \in \operatorname{supp} x,$$

and

$$\int_x x(dr_i) \sigma_i(dx | c_i) = \nu^i(dr_i | c_i).$$

Now, we define a measure over (k, c_k, x, v, c) according to

$$\phi(k, c_k, dx, dv, dx) = \eta^k(c_k)\, \sigma_k(dx \,|\, c_k) \int_{r_k} x(dr_k)\, \gamma^k(dv, dc_{-k} \,|\, c_k, r_k).$$

Finally, we describe the private component of the signal, l_i. The purpose of this component is to "alert" producers if they need to randomize, in order to break ties in favor of the efficient producer. If the realized segment x does not have a mass point at $\underline{r} = \min \operatorname{supp} x$, or if there is a mass point at $\underline{r}$ but $\underline{r} = c_k$, then we simply set $l_j = 0$ for each j. On the other hand, if there is a mass point at $\underline{r}$, then we set $l_j = 1$ for any producer j with $\underline{r} = v_i - v_j + c_j$, and $l_j = 0$ otherwise. This completes the construction of the information structure.

We now describe the strategies. First, at the signal (k, x, l_i), let $\underline{r} = \min \operatorname{supp} x$. If $i = k$, then $\rho_i(\underline{r} \,|\, k, x, l_i) = 1$. In other words, the efficient producer sets a price equal to the lowest residual willingness to pay in the segment x. If $k \neq i$ and $l_i = 0$, then $\rho_i(c_i \,|\, k, x, 0) = 1$. Finally, if $l_i = 1$, then producer i randomizes on an interval just above c_i according to a distribution that we now define. Since $l_i = 1$, there is a mass point at $\underline{r}$. Since the efficient producer is indifferent between different prices in the support, it must be that there is a gap in the support. (If not, then the efficient producer would not be willing to set a price just above $\underline{r}$, which would entail a discrete drop in demand from the consumer with residual willingness to pay $\underline{r}$.) Let $\hat{r} = \min\{r \in \operatorname{supp} x \,|\, r > \underline{r}\}$ be the second lowest residual willingness to pay. Then a producer with $l_i = 1$ randomizes according to the distribution

$$\rho\big([c_i, c_i + \varepsilon] \,|\, k, x, 1\big) = \begin{cases} 0, & \text{if } \varepsilon < 0 \\ 1 - \frac{\underline{r} - c_k}{\underline{r} - c_k + \varepsilon}, & \text{if } 0 \leq \varepsilon < \dfrac{(\hat{r} - \underline{r})}{2} \\ 1, & \text{if } \varepsilon > \dfrac{(\hat{r} - v_1 = 3\underline{r})}{2}. \end{cases}$$

Note that if $l_i = 1$, then $\underline{r} > c_k$, so that the distribution is nondegenerate.

Now let us verify that these strategies are an equilibrium. We first verify this for the efficient producer. Suppose that producer i is efficient and the realized segment is x. Producer i is setting a price $\underline{r} = \min \operatorname{supp} x$, which induces a profit of $\underline{r} - c_i$. If $\underline{r} = c_i$, then it must be that there is a tie for efficient producer, because $c_i = \underline{r} = v_i - v_j + c_j$ for some $j \neq i$. Moreover, that producer j is pricing at cost (because $l_j = 0$ for all j in this case), and the only way for the efficient producer to make a sale is with a price $p_i \leq c_i$, that would induce nonpositive profit. Thus, there is no profitable deviation. We now consider what happens if $\underline{r} > c_i$. If there is no mass point on $\underline{r}$, then ties occur with zero probability at $\underline{r}$, and if there is a mass point on $\underline{r}$, then any producer j with $\underline{r} = v_i - v_j + c_j$ received a signal $l_j = 1$, and hence, they are randomizing on the interval $\big[c_j, c_j + (\hat{r} - \underline{r})/2\big]$, where $\hat{r}$ is the second lowest element of the support of x. This induces a residual demand curve, where the probability of making a sale from a price $p_i \in \big[\underline{r}, (\underline{r} + \hat{r})/2\big]$ is $\big((\underline{r} - c_i)/(p_i - c_i)\big)^L$, where $L = \sum_i l_i \geq 1$. Setting any other price that is not in $\operatorname{supp} x \cup \big[\underline{r}, (\hat{r} + \underline{r})/2\big]$ is clearly dominated. From the properties of a uniform profit preserving segmentation, if ties were broken in favor of the efficient producer, then setting any price in the support of x must induce the same profit. Since we break ties uniformly, such prices induce a weakly lower profit than a price of $\underline{r}$. Finally, setting a price $p_i \in \left[\underline{r}, \left(\underline{r} + \hat{r}\right)/2\right]$ induces an interim expected producer surplus of

$$(p_i - c_i)\left(\frac{\underline{r} - c_i}{p_i - c_i}\right)^L \leq (p_i - c_i)\frac{\underline{r} - c_i}{p_i - c_i} = \underline{r} - c_i,$$

as desired.

Next, for any inefficient producer j,

$$p_i \leq r_i = \min_{k \neq i} v_i - v_k + c_k \leq v_i - v_j + c_j.$$

So, for producer j to make a sale, they would have to set a price weakly below cost, and hence, they cannot make positive profit. Thus, the proposed strategies are a best response.

Finally, we verify that the welfare outcome is the one described in the theorem. By the properties of a uniform profit-preserving segmentation, the efficient producer i is indifferent to pricing at $p_i^*(c_i)$ for any signal realization x. Thus, they are indifferent to *always* pricing at $p_i^*(c_i)$, so that their resulting payoff is $\underline{PS}_i$. But an efficient producer always makes a sale, so that total surplus is $\overline{TS}$. This completes the proof. ∎

A2. *Proof of Theorem 2*

Because of homogeneous values, we have that for all (v', c) in the support of μ',

$$v_i' = \int_v v_i \eta(dv \mid v', c) = \int_v v_j \eta(dv \mid v', c) = v_j',$$

so that μ' satisfies homogeneous values as well. Moreover, under common knowledge of gains from trade, for all (v', c) in the support of μ', we have

$$\begin{aligned}
\max_i(v_i' - c_i) &= v_1' - \min_i c_i \\
&= \int_v v_1 \eta(dv \mid v', c') - \min_i c_i \\
&= \int_v \left(v_1 - \min_i c_i\right)\eta(dv \mid v', c') \\
&= \int_v \max_i(v_i - c_i)\eta(dv \mid v', c') \\
&\geq 0.
\end{aligned}$$

Hence, μ' also satisfies common knowledge of gains from trade. Thus,

$$\begin{aligned}
\overline{TS}(\mu') &= \int_{v',c} \max\{0\} \cup \{v_1' - c_1, \ldots, v_N' - c_N\}\mu'(dv', dc) \\
&= \int_{v',c}\left(v_1' - \min_i c_i\right)\mu'(dv', dc) \\
&= \int_{v,c}\left(v_1 - \min_i c_i\right)\mu(dv, dc) \\
&= \overline{TS}.
\end{aligned}$$

It then follows immediately from Proposition 2 that consumer surplus and total surplus can be aligned.

Now suppose that there is another information structure and equilibrium in which $PS < \sum_i \underline{PS}_i(\mu')$. Let μ'' be the marginal on (v, c) associated with this information structure. By the argument in the preceding paragraph, $\overline{TS}(\mu'') = \overline{TS}$. By Proposition 2, $PS \geq \sum_i \underline{PS}_i(\mu'')$, and also there is an information structure and equilibrium in which the outcome is efficient and producer surplus is precisely $\underline{PS}_i(\mu'')$. In this outcome, consumer surplus is therefore $\overline{TS} - \sum_i \underline{PS}_i(\mu'') \geq \overline{TS} - PS > \overline{TS} - \sum_i \underline{PS}_i(\mu')$, which contradicts the hypothesis that μ' corresponds to a consumer surplus maximizing information structure. Thus, it must be that $\sum_i \underline{PS}_i(\mu')$ is also minimum producer surplus, and consumer surplus and producer surplus are opposed. ■

A3. *Proof of Theorem 3*

The distribution $G \in \Delta[-\bar{v}, \bar{v}]$ has to form a mean-preserving contraction of the underlying distribution F, thus,

$$\int_{x=r}^{\infty} [G(x) - F(x)] dx \geq 0, \ \forall r.$$

By the symmetry of the problem, it is without loss to consider symmetric interim residual value distributions that satisfy $G(-r) = 1 - G(r)$ for $r \geq 0$.

Recall the expression 3 for total surplus. The first integral is a constant and independent of the choice of the optimal information structure with interim residual value distribution G. The sum of the producers' surplus is the sum of the revenue across the efficient producers:

$$\underline{PS}(G) = \max_{p \geq 0} \{pG^-(-p)\} + \max_{p \geq 0} \{p[1 - G^-(p)]\},$$

where G^- denotes the limit from the left, and optimal consumer surplus is $\overline{TS}(G) - \underline{PS}(G)$. Now, if the sum of the producers' surplus is b (and thus jointly the producers sell with probability one), then conditional on being the efficient producer, a producer's surplus must be at most b. This is equivalent to the interim distribution G satisfying

$$r\frac{1 - G^-(r)}{1/2} \leq b, \ \forall r \geq 0,$$

$$-r\frac{G^-(r)}{1/2} \leq b, \ \forall r \leq 0;$$

in which case the above constraints are equivalent to

$$G^-(r) \geq 1 - \frac{b}{2r}, \ \forall r \geq 0,$$

$$G^-(r) \leq -\frac{b}{2r}, \ \forall r \leq 0.$$

So, we can focus on choosing $G(r)$, subject to the aforementioned pricing constraints and mean-preserving contraction constraints.

Now, suppose that there is an interim value distribution G for which producer surplus is b. We claim that there is a B such that G_b^B is a symmetric mean-preserving contraction of G. To prove the claim, first note that conditional on $r \geq 0$, the distribution G first-order stochastically dominates $G_b^{\bar{v}}$, and G is first-order stochastically dominated by G_b^b. Hence, conditional on $r \geq 0$, the expectation under G is between the expectations under $G_b^{\bar{v}}$ and G_b^b. Because the expectation under G_b^B is continuous in B, by the intermediate value theorem, there is a $B \in [b, \bar{v}]$ such that the expectation of r conditional on $r \geq 0$ is the same under G and G_b^B, and in particular,

$$\int_{x=0}^{\bar{v}} \left[G_b^B(x) - G(x)\right] dx = 0.$$

Since $G(r) \geq G_b^B(r)$ for $r < B$ and $G(r) \leq G_b^B(r)$ for all $r \geq B$, we conclude that for all $r \geq 0$,

$$\int_{x=r}^{\infty} \left[G_b^B(x) - G(x)\right] dx \geq 0.$$

By symmetry, we conclude that G_b^B is a mean-preserving contraction of G, and hence, is also a mean-preserving contraction of F.

Note that $\underline{PS}\left(G_b^B\right) = b$, so the lower bound on producer surplus has not changed. Moreover, because G_b^B is separately a mean-preserving contraction of G on either side of zero, we have not changed the expectation of $|r|$, and hence, total surplus has not changed as well. Thus, it is without loss to optimize consumer surplus over distributions of the form G_b^B that are mean-preserving contractions of F. ∎

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