

Financial Conditions Targeting in a Multi-Asset Open Economy

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Abstract

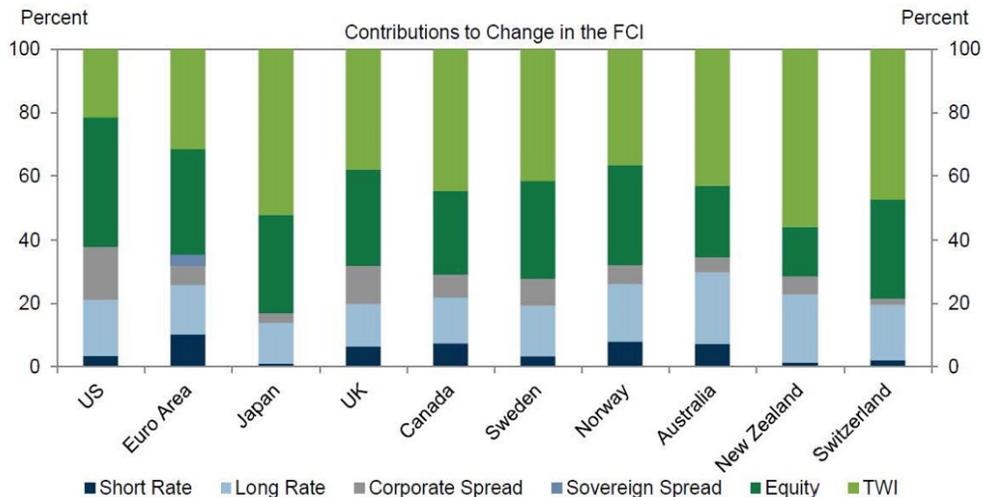
We analyze monetary policy responses to noisy financial conditions in an open economy where exchange rates and domestic asset prices affect aggregate demand. Noise traders operate in both markets, and specialized arbitrageurs have limited risk-bearing capacity. Monetary policy creates cross-market spillovers: by adjusting the interest rate to stabilize one market, the central bank influences volatility in the other. We show that targeting a financial conditions index (FCI)—a weighted average of exchange rates and domestic asset prices—delivers substantial macroeconomic benefits. FCI targeting commits the central bank to respond to unexpected movements in financial conditions beyond what discretionary monetary policy implies. These stronger responses improve diversification across markets: each market becomes more exposed to external shocks but less exposed to its own. This reduces volatility in both markets and activates the recruitment effect from Caballero et al. (2025b) in each market—lower variance induces arbitrageurs to trade against noise, further dampening volatility. Foreign exchange (FX) targeting can also be effective when the exchange rate is the primary source of noise, with benefits that increase as the economy becomes more open. In this case, FX targeting recruits arbitrageurs to stabilize the FX market, reducing volatility and dampening the macroeconomic impact of noise. However, FX targeting also raises volatility in non-targeted markets through anti-recruitment effects, limiting its effectiveness relative to FCI targeting, especially when domestic asset markets also matter for financial conditions and are comparably noisy.

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Source: Goldman Sachs Global Investment Research

Figure 1: Source: Goldman Sachs Global Investment Research (2017), “Our New G10 Financial Conditions Indices,” *Global Economics Analyst*, 20 April 2017. TWI denotes the trade-weighted exchange rate index.

1. Introduction

Financial conditions indices (FCIs) have become key inputs for monetary policy decisions worldwide. These indices aggregate the effects of various financial variables—borrowing rates, equity prices, and exchange rates—on economic activity by weighting each according to its estimated impact on GDP. Figure 1 shows that equity prices and exchange rates dominate FCI movements across developed economies, with the exchange rate’s role rising in more open economies. This dominance reflects their much higher volatility compared to interest rates (see Section 2.1). The finance literature has long documented that a substantial portion of this volatility cannot be explained by fundamentals, with a significant component driven by financial noise—supply-demand effects orthogonal to fundamentals that influence asset prices due to limits to arbitrage. This observation raises a fundamental question: how does monetary policy’s response to noisy financial conditions shape macroeconomic outcomes?

We addressed this question in Caballero et al. (2025b), for a closed economy, demonstrating that an FCI targeting policy framework can substantially improve macroeconomic stabilization. This improvement results from a “recruitment effect:” when arbitrageurs know that the central bank will keep FCI close to a pre-announced target, they join the central bank in offsetting the impact of noise on financial conditions and aggregate activity. In this paper, we develop a tractable *open-economy* model to revisit this question and evaluate the performance of financial conditions targeting versus exchange rate targeting.

The model builds on the observation, reflected in the construction of FCIs, that both the exchange rate and domestic market portfolio substantially influence aggregate demand and output.

The exchange rate affects the economy through expenditure switching—a depreciation makes domestic goods cheaper relative to foreign goods, boosting net exports. The domestic market portfolio affects the economy through wealth effects—higher stock prices increase household spending. In our model, this wealth effect stands in for a variety of channels by which domestic equity and bond prices affect economic activity in practice. Output depends on a financial conditions index that reflects these two effects—a weighted average of the exchange rate and the domestic market portfolio.

The model features two key departures from standard frameworks. First, noise traders’ demands generate deviations from fundamentals in both foreign exchange and domestic equity markets. Second, these markets feature arbitrageurs with segmented market access: FX specialists trade currencies, while domestic specialists trade the domestic market portfolio, with limited capacity for cross-market arbitrage. The impact of noise on asset prices depends on the arbitrageurs’ risk-bearing capacity. Each group of specialists has limited risk-bearing capacity, so in each market noise has an *endogenous* impact on asset prices and economic activity that grows with market volatility. Since monetary policy affects asset price volatility in both markets, it has the potential to either amplify or mitigate the impact of noise. Our focus is on how different policy regimes affect the impact of noise in each market and resulting macroeconomic fluctuations.

We start by analyzing the standard monetary policy in which the central bank focuses on stabilizing output gaps but also would like to adjust interest rates gradually. This conventional policy already creates cross-market spillover effects. When noise inflates equity prices or depreciates the exchange rate, financial conditions loosen and output rises above potential. The central bank responds by raising the interest rate. This rate increase simultaneously tightens both markets—dampening the initial shock in its source market but transmitting it to the other market. For instance, noise-driven currency depreciations trigger rate hikes that depress domestic asset prices; conversely, noise-driven domestic asset price booms trigger rate hikes that appreciate the currency. The central bank thus connects otherwise segmented markets. Importantly, these spillover effects are typically benign: a *diversification* effect dominates. While each market becomes more exposed to noise originating elsewhere, it becomes less exposed to its own noise. On net, this enables the arbitrageurs in different markets to share noise risks more effectively, reducing the average noise risk that each arbitrageur faces. However, in practice, this stabilizing effect is limited by policymakers’ preference for gradual interest rate adjustments. This gradualism means that noise shocks to exchange rate or domestic asset prices are only partially offset, even when these shocks push financial conditions away from levels needed to close output gaps.

We then establish that augmenting conventional monetary policy with a financial conditions target delivers substantial macroeconomic benefits. The recruitment effect identified in Caballero et al. (2025b) emerges here as well, enhanced by a diversification mechanism. First, FCI targeting effectively commits the central bank to respond to noise shocks more aggressively than it would

choose under discretion (which enhances the noise diversification described earlier). Second, because FCI targeting partially offsets the price impact of macroeconomic shocks in each market, it directly dampens those shocks and diversifies their incidence across markets. These effects reduce the volatility of both the exchange rate and the market price. This volatility reduction then activates the recruitment effect—lower variance encourages arbitrageurs to trade against noise, increasing market elasticity and further reducing the impact of noise and volatility in all markets. Consequently, we find that some degree of FCI targeting is always optimal—even after accounting for potential interest rate adjustment costs. In fact, in our calibrations, FCI targeting can also reduce interest rate volatility: since arbitrageurs absorb noise in the exchange rate and domestic asset prices more effectively, the central bank faces a smaller burden to adjust interest rates.

Narrower FX targeting can also deliver welfare gains when the exchange rate is the primary source of noise-driven fluctuations. The mechanism is the same: announcing a state-contingent exchange rate target recruits arbitrageurs to stabilize the FX market, reducing volatility and dampening the macroeconomic impact of noise. Importantly, this is not a call for fixed exchange rates—the target moves with fundamentals to keep overall financial conditions consistent with macroeconomic objectives. The benefits of FX targeting increase when the exchange rate accounts for a larger share of financial conditions. In the limiting case where the exchange rate equals the FCI, FX targeting coincides with FCI targeting, and we recover the results from Caballero et al. (2025b).

The main caveat is that FX targeting also raises volatility in non-targeted markets. By focusing policy responses on the exchange rate, FX targeting transmits FX market shocks to domestic asset markets and triggers anti-recruitment effects there—higher volatility discourages arbitrageurs from trading against noise, amplifying its impact. This limits the effectiveness of FX targeting relative to FCI targeting, which exploits diversification by spreading policy responses across all relevant markets. When equity market noise is substantial, these anti-recruitment effects can dominate the recruitment benefits in the FX market, making FX targeting counter-productive. FCI targeting, by contrast, remains effective regardless of where noise originates.

Related Literature. This paper builds upon our earlier work, Caballero et al. (2025b), which investigates the implications of financial noise and limits to arbitrage for business cycles and optimal monetary policy. That paper uses a closed economy model to show that financial noise can drive macroeconomic fluctuations and that an FCI targeting policy framework stabilizes noise-induced fluctuations. The present paper extends this mechanism to an *open economy* setup in which the FCI is driven by the exchange rate as well as domestic asset prices, and both markets feature financial noise and limits to arbitrage with limited cross-market arbitrage opportunities. We show that FCI targeting still works in this setting and is strengthened by a new *diversification* channel: responding to a broad FCI allows monetary policy to spread noise across markets, lowering aggregate volatility and amplifying recruitment in each market. We

then use this framework to contrast the stabilization properties of *multi-market* (FCI) targeting with those of *single-market* (FX-only) targeting. We show that the former exploits diversification across markets while the latter can be equally effective in economies where exchange rate noise dominates, though it is less robust when the composition of noise is uncertain.

A closely related line of research studies *exchange-rate stabilization under noisy flows*, including Jeanne and Rose (2002); Gabaix and Maggiori (2015); Itskhoki and Mukhin (2021b, 2023). These papers highlight mechanisms through which credible exchange rate stabilization can either deter noise-trader activity or crowd in informed traders, generating a “virtuous cycle.” Our contribution is to show that, once multiple noisy markets matter for aggregate demand, exchange rate stabilization can backfire by raising volatility and inducing “anti-recruitment” effects in domestic asset markets. This provides one rationale for the disappointing performance of the Monetary Conditions Index (MCI) frameworks adopted in Canada and New Zealand during the 1990s (see Section 5.4). Our theoretical analysis motivates the need for broader FCI-based approaches that internalize cross-market volatility dynamics.

In a related paper, Fontanier and Simsek (2025) analyze exchange rate stabilization for emerging markets, where domestic arbitrageurs might face binding risk limits and foreign investors are concerned with exchange rate risk. They show that stabilizing the exchange rate can increase foreign investment and relax domestic arbitrageurs’ constraints, enhancing monetary transmission—a result that contrasts with the conventional Mundellian trilemma. However, they also find that excessive exchange rate targeting can backfire by raising policy rate volatility and tightening arbitrageur risk limits—a version of the anti-recruitment effect we identify here.¹

In terms of the modeling ingredients, our paper is part of an emerging literature on New Keynesian models with risk and asset prices (e.g., Caballero and Farhi (2018); Caballero and Simsek (2020, 2021, 2023, 2024); Caballero et al. (2025a); Pflueger et al. (2020); Kekre et al. (2023); Kekre and Lenel (2022, 2024b,a, 2025); Beaudry et al. (2024); Adrian and Duarte (2018); Adrian et al. (2020, 2025)). The distinctive features of our model are that: (i) we allow for risk-driven fluctuations in *both* the exchange rate and the domestic market portfolio, and (ii) we focus on fluctuations driven by financial noise and limits to arbitrage. This connects our analysis with a large finance literature that emphasizes asset price fluctuations driven by noise and limits to arbitrage (see Black (1986); Shleifer and Summers (1990); De Long et al. (1990) for early contributions and Gromb and Vayanos (2010) for a review). In recent work, Gabaix and Maggiori (2015); Gourinchas et al. (2022); Greenwood et al. (2023) and Gabaix and Koijen (2021) apply these ideas to the exchange rate market and the equity market, respectively. This literature helps to motivate our modeling ingredients (see also Section 2.2).

¹More broadly, there is a large macroeconomics literature that examines optimal monetary policy in small open economies. In this literature, the conclusions about exchange rate targeting typically depend on assumptions about price-setting behavior—whether firms set prices in producer currency, local currency, or a dominant currency (see, e.g., Clarida et al. (2002); Gali and Monacelli (2005); Corsetti and Pesenti (2005); Engel (2011); Egorov and Mukhin (2023)). We abstract from these pricing frictions and instead focus on how financial noise and limits to arbitrage in multiple markets shape the trade-off between narrow exchange rate stabilization and broad financial conditions targeting.

	Policy	10y risk-free	Corp. spread	Equity	TWI (FX)
US	4.4	45.1	39.6	4.9	6.0
EA (+ sovereign 27.0)	17.3	34.3	11.4	2.1	7.8
JP	12.6	59.1	19.3	2.0	7.1
UK	13.7	38.2	33.6	3.7	10.9
CH	6.2	56.0	13.7	3.3	20.8
SE	6.4	46.2	27.9	2.3	17.2
NO	12.9	50.4	18.7	2.6	15.5
CA	12.2	46.6	27.7	2.5	11.0
AU	11.8	59.7	17.0	2.5	9.0
NZ	2.6	59.6	24.7	2.2	11.0

Table 1: **Weights in Goldman Sachs G10 FCIs (daily headline series)**. Euro Area also includes a sovereign-spread component (27%). Source: Goldman Sachs Global Investment Research (2017).

The paper proceeds as follows. Section 2 presents stylized facts about global financial conditions. Section 3 introduces the model and characterizes equilibrium under discretionary policy. Section 4 demonstrates how FCI targeting improves macroeconomic outcomes through cross-market diversification effects that recruit arbitrageurs to trade against noise. Section 5 analyzes FX targeting and shows when partial stabilization succeeds or fails. Section 6 concludes, and is followed by an appendix with omitted proofs and microfoundations.

2. Financial Conditions, Noise, and Propagation

This section documents three empirical regularities that motivate our theoretical framework. First, equity prices and exchange rates account for most FCI fluctuations in developed economies. Second, both exhibit substantial noise-driven volatility. Third, this financial noise propagates to the real economy.

2.1. Equity and FX Dominate Financial Conditions Fluctuations

To understand the empirical composition of open-economy financial conditions indices (FCIs), we start with the comprehensive indices constructed by Goldman Sachs for G10 economies (Hatzius et al. (2017)). These indices provide a harmonized measure of financial conditions across large and small advanced economies, updated daily. Each country’s FCI is a weighted average of five key variables: (i) a short-term policy rate, (ii) a ten-year sovereign yield, (iii) a corporate credit spread, (iv) an equity price variable scaled by a ten-year moving average of earnings, and (v) a broad trade-weighted exchange rate. For the euro area, a sovereign spread term is added to capture fragmentation risk.

The weights are calibrated to each variable’s estimated impact on real GDP growth one year ahead, as implied by the country-specific macro model. A 1 percentage point increase in the FCI is estimated to reduce GDP by approximately 1 percent after four quarters.

Table 1 shows the weights for each financial variable across countries. To interpret these

weights, consider the U.S.' FCI. A 220 basis point increase in the 10-year Treasury yield (weight 45 percent), a 250 basis point widening in corporate spreads (weight 40 percent), a 20 percent decline in equity prices (weight 5 percent), or a 17 percent currency appreciation (weight 6 percent) would each tighten the FCI by roughly 1 point and reduce GDP by approximately 1 percent over the subsequent year.

Equity has small weights (2-5 percent), reflecting positive but modest estimated wealth and investment effects (see, e.g., Chodorow-Reich et al. (2021) on wealth effects). Yet higher volatility makes equity a dominant source of FCI fluctuations, as Figure 1 illustrates. While a 1 percent equity change has modest impact, a 20 percent decline—roughly a *one* standard-deviation event in a given year—tightens the FCI by 1 percentage point and reduces GDP by approximately 1 percent. Achieving a comparable FCI impact from bond yields would require a 220 basis point increase in the 10-year Treasury yield in a year, roughly a *two* standard-deviation event. Since large equity swings occur far more frequently than equivalent bond yield movements, stocks contribute more to FCI variation than bonds despite their modest weights in Table 1.

Exchange rates receive larger formal weights than equity (6-21 percent), meaning a 1 percent exchange rate change has a larger estimated GDP impact than an equivalent equity price change. These estimates reflect expenditure switching effects, which the literature suggests can be sizeable. Combined with high volatility, exchange rates constitute another dominant source of short-term FCI fluctuations.

Table 1 reveals that, as expected, small open economies assign relatively high weights to exchange rates, whereas larger economies such as the United States load more heavily on equity prices and corporate spreads. Despite these differences in composition, equity prices and exchange rates combined account for over 50 percent of FCI variance across all the developed economies portrayed in Figure 1.

2.2. Equity and FX are Influenced by Financial Noise

A second empirical pillar is that both dominant components of open-economy FCIs—equity prices and exchange rates—exhibit substantial noise-driven fluctuations. By noise, we mean price movements driven by supply-demand shifts from a subset of market participants that are unrelated to fundamentals and persist because arbitrageurs with limited risk-bearing capacity cannot fully offset them. This framework builds on a large theoretical literature emphasizing noise and limits to arbitrage in asset pricing. Consistent with this foundation, an influential body of empirical research suggests that noisy flows account for a sizeable fraction of asset price variation in both equity and foreign exchange markets.

Equity Market Noise

Gabaix and Koijen (2021) document that the U.S. aggregate equity market has a low elasticity of approximately 0.2: a one-dollar flow into equities increases aggregate market valuation

by five dollars. This inelasticity reflects limited arbitrage capacity. When noise traders create buying or selling pressure, arbitrageurs’ risk-bearing constraints or mandates prevent them from fully absorbing the flow, which impacts prices. Using granular flow-of-funds data, they show that idiosyncratic flows from different investor sectors—particularly households and foreign investors—drive substantial equity price movements. These flow-driven shocks account for approximately one-third of quarterly equity return variance. Critically, the flows appear largely unrelated to fundamental news about cash flows or discount rates, suggesting they represent genuine noise rather than rational responses to unobserved information.²

Exchange Rate Noise

Exchange rates exhibit similar patterns of noise-driven fluctuations. Evans and Lyons (2002) show that daily order flow—signed transaction volume between dealers and customers—explains up to 60% of daily exchange rate returns, far exceeding the explanatory power of macro variables. Order flow appears largely unrelated to contemporaneous macro news, suggesting it reflects noise trading, hedging demands, or portfolio rebalancing rather than information about fundamentals (Love and Payne (2008)). Consistent with limited arbitrage capacity in FX markets, the price impacts of these flows are substantial. In recent work, Itskhoki and Mukhin (2021a) show that fundamental factors—even with complete information about monetary policy shocks, inflation expectations, and risk premia—leave a substantial share of exchange rate variance unexplained, with the residual appearing driven by financial shocks and noise.

2.3. Financial Noise Propagates to the Real Economy

A natural question is whether noise-driven fluctuations in equity prices and exchange rates affect aggregate demand and output in the same way as fundamental price changes. The construction of the Goldman Sachs FCIs suggests the answer is yes. These indices aggregate asset prices without distinguishing between noise-driven and fundamental movements, reflecting the assumption that households and firms respond to changes in wealth, borrowing costs, and international competitiveness regardless of whether the underlying asset price movements reflect fundamentals or noise.

In Caballero et al. (2025b), we empirically investigate this question for the U.S. equity market, building on Gabaix and Koijen (2021). We show that noisy flows influence not only stock prices but also financial conditions and macroeconomic activity. Specifically, we estimate a Bayesian VAR with an external instrument for financial noise shocks constructed from Flow of Funds data, following Gabaix and Koijen (2024). Our estimates show that a noise shock that increases equity demand loosens financial conditions on impact, as measured by the FCI. The resulting stimulus drives output above potential while raising inflation, with effects resembling

²Their identification strategy uses granular instrumental variables to isolate idiosyncratic sectoral flows, controlling for common factors and macroeconomic conditions.

those of a conventional aggregate demand shock. Monetary policy adjusts rates gradually in response, but this adjustment fails to fully neutralize the shock. Quantitatively, our estimates suggest that financial noise shocks account for up to 40% of FCI forecast variance on impact and contribute up to 25% of output gap forecast variance over a two-year horizon—a contribution comparable to the high end of estimates for monetary policy shocks in this literature. We refer readers to Caballero et al. (2025b) for detailed impulse responses and variance decompositions.

3. An Open Economy Model with Noise and Segmented Markets

In this section, we present an open economy macroeconomic model with the following key features: (i) financial conditions are influenced by both the exchange rate and equity markets; (ii) financial noise affects both the exchange rate and the equity market; (iii) in each market, specialized portfolio managers trade against noise with limited arbitrage capacity, (iv) monetary policy is gradualist, and cannot fully insulate financial conditions and output from financial noise. Within this framework, we demonstrate that: (a) noise shocks affect both asset prices and macroeconomic activity; (b) the impact of noise on asset prices and aggregate activity increases with the *endogenous* variance in the exchange rate and the equity market. These results suggest that monetary policy that targets financial variables can in principle improve outcomes by reducing variance. In subsequent sections, we use this framework to analyze alternative monetary policies that target financial conditions and the exchange rate.

3.1. Environment and equilibrium conditions

To simplify the exposition, we delegate the details and the microfoundations of the model to Appendix A.2. In the main text, we summarize the model ingredients and present the log-linearized equation system. We then solve the equation system for a benchmark case with discretionary monetary policy.

Model summary. The model is set in discrete time t . There are two currencies: domestic currency (peso) and foreign currency (dollars). The exchange rate is the price of dollars in pesos. There are two goods: domestic (domestically produced) tradeables and global tradeables. Domestic nominal prices are fully sticky in pesos. Global tradeables’ dollar price is constant. Hence, the exchange rate determines the relative price of global vs domestic tradeables.

There are three assets: a domestic market portfolio—which is a claim on the capital’s share of domestic output, a domestic safe asset—whose return is set by the central bank, and a foreign safe asset—whose return is exogenous.

Domestic output is determined by aggregate demand and can deviate from potential output. Aggregate demand comes from domestic households’ spending on domestic tradeables as well

as from exports. Domestic households' spending depends on the price of the market portfolio through a standard wealth effect. Exports (in pesos) depend on the exchange rate through a standard expenditure switching effect as well as on global output. Therefore, the model relates domestic output to a weighted-average of the market portfolio and the exchange rate—the model counterpart to the FCIs that we discussed in Section 2—with an adjustment for global output that influences domestic demand.

Market portfolio price and exchange rate are both determined endogenously in segmented markets that feature *financial noise* and *limits to arbitrage*. In particular, there is a “national market” where agents trade the market portfolio and the peso safe asset, and a “global market” where agents trade the dollar safe asset and the peso safe asset. Each market has (market-specific) noise traders as well as (market-specific) arbitrageurs that trade against noisy deviations in asset prices. The arbitrageurs in the national market maximize the expected log peso returns. Arbitrageurs in the currency market maximize the expected log dollar returns.

Log-linearized equilibrium conditions. These assumptions lead to the following log-linearized equilibrium conditions

$$y_t = \phi(e_t + y_t^g) + (1 - \phi)p_t \quad (1)$$

$$e_t = \rho + E_t[e_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma_e^2\right) + \mu_t^e \frac{\sigma_e^2}{\alpha} \quad (2)$$

$$p_t = \rho + (1 - \beta)E_t[y_{t+1}] + \beta E_t[p_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma_m^2\right) + \mu_t^p \frac{\sigma_m^2}{\alpha} \quad (3)$$

$$r_{t+1}^m = \rho + (1 - \beta)y_{t+1} + \beta p_{t+1} - p_t \quad (4)$$

$$\text{where } \sigma_e^2 \equiv \text{Var}_t(e_{t+1}) \quad \text{and} \quad \sigma_m^2 \equiv \text{Var}_t(r_{t+1}^m).$$

Here, $y_t = \log \frac{Y_t}{\bar{Y}}$, $y_t^g = \log \frac{Y_t^g}{\bar{Y}^g}$, $p_t = \log \frac{P_t}{\bar{P}}$, $e_t = \log \frac{E_t}{\bar{E}}$, denote the log deviations of output, global output, the asset price, and the exchange rate from their steady-state levels, $r_{t+1}^m = \log R_{t+1}^m$ denotes the log return on the market portfolio, $r_t^f = \log R_t^f$ denotes the log peso risk-free rate. We assume the dollar risk-free rate is equal to the domestic discount rate, $r^{f,d} = \rho = -\log \beta$. The variables μ_t^e and μ_t^p capture aggregate noisy demand for the dollar (vs peso) in the global market and for the market portfolio (vs the peso safe asset) in the national market, respectively. The parameter α captures the mass of arbitrageurs in both markets (assumed to be the same in both markets for simplicity). The derived parameter ϕ captures the impact of trade on domestic aggregate demand. The residual parameter $1 - \phi$ captures the impact of the market portfolio price on domestic aggregate demand through wealth effects.

Eq. (1) says that output is influenced by a weighted-average of the exchange rate and the price of the market portfolio, in view of expenditure switching and wealth effects. Output is also influenced by global output through demand for exports. After rearranging, we can write

this equation as

$$\begin{aligned}
y_t - y_t^* &= f_t - f_t^* \\
\text{where } f_t &= \phi e_t + (1 - \phi) p_t \\
f_t^* &= \phi e_t^* + (1 - \phi) y_t^*, \quad \text{with } e_t^* = y_t^* - y_t^g.
\end{aligned} \tag{5}$$

Here, f_t is the model counterpart (in price rather than yield space) to the financial conditions indices discussed in Section 2. We also define f_t^* (“FCI-star”) as the level of financial conditions that sets the output gap to zero at time t . Then, by definition, output deviations from potential are driven by FCI deviations from the corresponding FCI-star. One convenient representation of FCI-star is as a weighted average of the frictionless exchange rate (“estar”) and the frictionless market price (“pstar”). In this model, the former is driven by the relative potential output between the domestic economy and the global economy, and the latter is driven by the domestic potential output.

Eq. (2) describes the equilibrium exchange rate. This equation emerges from a risk-adjusted UIP condition in the global market. The exchange rate (dollar-peso rate) decreases (appreciates) with the peso interest rate r_t^f , increases (depreciates) with the dollar interest rate ρ , and increases (depreciates) with the future exchange rate with an adjustment for the exchange rate variance, $E_t[e_{t+1}] - \frac{1}{2}\sigma_e^2$.³ Crucially, the exchange rate is also influenced by the noisy demand for the dollar μ_t^e . The impact of noise is increasing in the exchange rate variance σ_e^2 and decreasing in the mass of arbitrageurs α . Since the exchange rate variance σ_e^2 is *endogenous*, this creates a key channel by which monetary policy affects the impact of noise on the exchange rate and the economy.

Eqs. (3) and (4) describe the equilibrium price and the return of the market portfolio. The return is a weighted-average of the future output (expected cash flows) and the future price (capital gains). Therefore, the price is increasing in the expected output and the expected price. The price is also decreasing in the peso interest rate r_t^f and a standard risk premium $\frac{1}{2}\sigma_m^2$.⁴ Crucially, the price is also influenced by the noisy demand for the market portfolio μ_t^p . Similar to the exchange rate, the impact of noise is increasing in the *endogenous* market return variance σ_m^2 and decreasing in the mass of arbitrageurs α .

³This is a Jensen’s adjustment that captures all else equal the expected dollar return from the peso safe asset is greater when the dollar-peso exchange rate is more volatile. Specifically, the expected dollar return from holding pesos is given by $r_t^f - E_t[\Delta e_{t+1}] + \frac{1}{2}\sigma_e^2$.

⁴This variance term combines a Jensen’s adjustment for the return of the market portfolio with a risk premium. The risk premium effect is qualitatively the dominant force. Specifically, the risk-adjusted return from the market portfolio is $E_t[r_{t+1}^m] + \frac{1}{2}\sigma_m^2 - \sigma_m^2 = E_t[r_{t+1}^m] - \frac{1}{2}\sigma_m^2$.

We assume that the exogenous shocks follow processes given by

$$\begin{aligned}
y_t^* &= y_{t-1}^* + z_t^p \quad \text{and} \quad e_t^* = y_t^* - y_t^g = e_{t-1}^* + z_t^e & (6) \\
\text{where } z_t^p &\sim N(0, \sigma_{z,p}^2), z_t^e \sim N(0, \sigma_{z,e}^2) \quad \text{and} \quad \text{corr}(z_t^p, z_t^e) = \rho_z \in (-1, 1) \\
\mu_t^p &\sim N(0, \sigma_{\mu,p}^2), \mu_t^e \sim N(0, \sigma_{\mu,e}^2) \quad \text{and} \quad \text{corr}(\mu_t^p, \mu_t^e) = \rho_\mu \in (-1, 1).
\end{aligned}$$

Domestic potential output (“ystar”) and global output both follow a random walk. Thus, the difference between domestic potential output and global output (“estar”) also follows a random walk. We denote the shocks to “ystar” and “estar” with z_t^p and z_t^e , since in equilibrium they drive the market price and the exchange rate, respectively. We refer to these shocks as “macroeconomic shocks” to differentiate them from noise shocks. Macroeconomic shocks follow a normal distribution with arbitrary (but less than perfect) correlation. The noise shocks in both markets also follow a normal distribution with arbitrary (but less than perfect) correlation. For simplicity, noise shocks and macroeconomic shocks are independent.

Given the shock processes in (6), the equilibrium is a solution to the system of equations (1–4), together with a monetary policy rule that sets the peso risk free rate r_t^f . We leave monetary policy unspecified for now since we will consider various rules.

Symmetric special case. For analytical tractability, we will also focus on a special case that makes the characterization of the exchange rate and the market price symmetric.

Assumption (S). Suppose $\phi = \frac{1}{2}$, variances satisfy $\sigma_{z,e}^2 = \sigma_{z,p}^2 \equiv \sigma_z^2, \sigma_{\mu,e}^2 = \sigma_{\mu,p}^2 \equiv \sigma_\mu^2$ with arbitrary correlations $\rho_z, \rho_\mu \in (-1, 1)$, and consider the limit $\beta \rightarrow 1$ (and thus, $\rho \rightarrow 0$).

In this case, the equilibrium system simplifies to the following:

$$y_t - y_t^* = f_t - f_t^*, \quad \text{where } f_t = \frac{e_t + p_t}{2}, f_t^* = \frac{e_t^* + y_t^*}{2} \quad (7)$$

$$e_t = E_t[e_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma_e^2\right) + \frac{\mu_t^e}{\alpha}\sigma_e^2 \quad (8)$$

$$p_t = E_t[p_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma_p^2\right) + \frac{\mu_t^p}{\alpha}\sigma_p^2 \quad (9)$$

$$\text{where } \sigma_e^2 \equiv \text{Var}_t(e_{t+1}) \quad \text{and} \quad \sigma_p^2 \equiv \text{Var}_t(p_{t+1}).$$

The exchange rate and the market price are equally important for output and they are subject to equally volatile exogenous macroeconomic shocks and noise shocks. Note also that we drop the market return from the notation since $\beta \rightarrow 1$ ensures the return variance is equal to the price variance. These assumptions simplify most of our analysis by allowing for a symmetric equilibrium in which the exchange rate and the market price have the same (endogenous) variance, $\sigma^2 = \sigma_e^2 = \sigma_p^2$. Later, when we consider exchange rate targeting, we drop this assumption since targeting the exchange rate breaks the asymmetry across the two markets even when the underlying parameters are symmetric.

Monetary policy objectives. We assume the central bank’s *true* loss function is given by

$$L_t = \tilde{y}_t^2 + \frac{1}{\theta} \left(r_t^f - E_{t-1} [r_t^f] \right)^2 + \tilde{\beta} E_t [L_{t+1}], \quad \text{where } \tilde{y}_t = y_t - y_t^*. \quad (10)$$

The central bank would like to reduce the output gap while also keeping the interest rate stable relative to the ex ante market expectations.⁵ The parameter $1/\theta > 0$ captures the importance of interest rate smoothing relative to the output gap objective. The parameter $\tilde{\beta}$ captures the central bank’s discount factor.⁶ We introduce an interest rate smoothing term into the objective function to match the central bank gradualism that we observe in practice. We will consider different *operational* rules for the central bank and evaluate their performance in terms of minimizing the loss function in (10).

3.2. Equilibrium with discretionary monetary policy

As a benchmark, suppose the central bank sets the policy rate in each period, r_t^f , without commitment, to minimize its actual loss function in (10). The optimality condition for this problem implies the central bank follows an interest rate rule reminiscent of the Taylor rule

$$r_t^f = E_{t-1} [r_t^f] + \theta \tilde{y}_t. \quad (11)$$

The central bank does not fully eliminate output gaps because it changes the interest rate gradually. The parameter θ captures the response of the interest rate to the output gap along the equilibrium path. The following result characterizes the rest of the equilibrium and establishes its comparative statics.

Proposition 1 (Equilibrium with Discretionary Monetary Policy). *Suppose the parameters satisfy Assumption (S) and $\alpha^2 > 4\sigma_\mu^2\sigma_z^2$. Suppose monetary policy minimizes its actual loss in*

⁵Central banks often describe stabilization objectives in terms of inflation deviations from target. Since in our model prices are fully sticky, we write the objective (and conduct the analysis) in terms of output gaps, which keeps the analysis and equilibrium characterization substantially simpler; moreover, in standard New Keynesian environments inflation is mapped to the output gap via a Phillips curve (up to expected inflation and a cost-push component). In Caballero et al. (2025b) we endogenize inflation and show that the recruitment effect and the associated policy recommendations emphasized here are unchanged; see Section 4.6.2 and Appendix A.5, which embed FCI targeting in an NKPC framework with an inflation–output stabilization trade-off.

⁶We allow $\tilde{\beta}$ to be different than the households’ discount factor β since we consider the limit as $\beta \rightarrow 1$. This discount factor does not play an important role for our results.

(10) *without commitment. Then, there is an equilibrium in which*

$$y_t - y_t^* = f_t - f_t^* = \frac{1}{1+\theta} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \quad (12)$$

$$r_t^f = -\frac{\sigma^2}{2} + \frac{\theta}{1+\theta} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \quad (13)$$

$$e_t = e_t^* + \left[\frac{1+\theta/2}{1+\theta} \mu_t^e - \frac{\theta/2}{1+\theta} \mu_t^p \right] \frac{\sigma^2}{\alpha} \quad (14)$$

$$p_t = y_t^* + \left[\frac{1+\theta/2}{1+\theta} \mu_t^p - \frac{\theta/2}{1+\theta} \mu_t^e \right] \frac{\sigma^2}{\alpha}. \quad (15)$$

Here, σ^2 denotes the conditional variance of both e_{t+1} and p_{t+1} (as well as r_{t+1}^m) and it corresponds to the smallest solution to the following quadratic:

$$\sigma^2 = \sigma_z^2 + \nu_\mu \left(\frac{\sigma^2}{\alpha} \right)^2, \quad \text{where } \nu_\mu \equiv \text{var} \left(\frac{1+\theta/2}{1+\theta} \mu_t^e - \frac{\theta/2}{1+\theta} \mu_t^p \right). \quad (16)$$

The common variance in both markets σ^2 is increasing in the macroeconomic variance σ_z^2 and the noise variance σ_μ^2 and it is decreasing in the responsiveness of monetary policy θ , that is, $\frac{d\nu_\mu}{d\theta} < 0$ and thus $\frac{d\sigma^2}{d\theta} < 0$.

We discuss the intuition for the result and relegate the proof to the appendix. Eqs. (12) and (13) show that the central bank responds to the *average* noise across the sub-markets $\frac{\mu_t^e + \mu_t^p}{2}$. Intuitively, if the central bank were not to respond, then noise in these markets would affect the FCI (under Assumption (S)) $f_t = \frac{e_t + p_t}{2}$ and thus also output. The central bank attempts to insulate FCI and output from financial noise. The same equations show that the central bank responds to the average noise only partially due to interest rate gradualism concerns.

Eqs. (14) and (15) show that noise affects the equilibrium asset prices, not only in the markets they originate but also in the other market, e.g., exchange rate noise affects the market price (and vice versa). To understand these equations, first consider the special case with extremely gradual monetary policy $\theta = 0$. In this case, the central bank does not respond to noise shocks and we have

$$\begin{aligned} e_t - e_t^* &= \mu_t^e \frac{\sigma^2}{\alpha} \\ p_t - y_t^* &= \mu_t^p \frac{\sigma^2}{\alpha}. \end{aligned}$$

Absent central bank response, noise has a relatively large but *localized* effect. Now consider the other extreme case with $\theta \rightarrow \infty$. In this case, the central bank responds to noise shocks

forcefully and fully insulates FCI from average noise, $r_t^f \sim \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}$. Thus, we have

$$\begin{aligned} e_t - e_t^* &= \left(\mu_t^e - \frac{\mu_t^e + \mu_t^p}{2} \right) \frac{\sigma^2}{\alpha} = \frac{\mu_t^e - \mu_t^p}{2} \frac{\sigma^2}{\alpha} \\ p_t - y_t^* &= \left(\mu_t^p - \frac{\mu_t^e + \mu_t^p}{2} \right) \frac{\sigma^2}{\alpha} = \frac{\mu_t^p - \mu_t^e}{2} \frac{\sigma^2}{\alpha}. \end{aligned}$$

In this case, noise has a smaller (though still non-zero) localized effect since the interest rate response to noise mitigates some of the impact. However, the same interest rate response *transfers* the financial noise in one market to other markets. For instance, a noise-driven exchange rate depreciation $\mu_t^e > 0$ induces the central bank to hike the interest rate, which reduces the market price.

Importantly, the proposition further shows that the impact of noise on the equilibrium variables scales with the *endogenous* variance in individual markets σ^2 . This is because a greater variance makes arbitrageurs less willing to trade against noise, allowing it to have a greater impact. The endogenous variance σ^2 corresponds to the solution to the quadratic in (16). This quadratic captures an amplification mechanism: a greater variance allows noise to have a larger impact, which then increases variance, and so on. As expected, the endogenous variance is increasing in the exogenous drivers of variance. Less obviously, the variance is *decreasing* in the responsiveness of monetary policy θ . A stronger monetary policy response (higher θ) reduces the direct impact of noise in its own market while creating spillover effects on other markets. Due to *diversification*, the decline in the direct effect more than compensates for the spillover effect. Spreading noise impact across multiple markets, with the help of monetary policy, reduces the average variance in each market—as illustrated by the ν_μ term in (16).

In sum, discretionary monetary policy helps to spread noise-driven volatility across markets, and by doing so improves financial and macroeconomic stability. The key role played by endogenous volatility suggests that monetary policy can further improve stability by *targeting* financial variables, as we explore next. We consider two separate financial variables: a financial conditions index (FCI) and the foreign exchange rate (FX).

4. FCI targeting

In this section, we show that a framework where the central bank sets a (soft) financial conditions target for the upcoming period enhances the central bank’s ability to achieve its standard macroeconomic goals. The policy works by *recruiting* the arbitrageurs to trade more aggressively against noise and thus reducing the impact of noise for both the exchange rate and the price of the market portfolio. This in turn insulates the FCI from noise more effectively and improves monetary policy objectives.

We formalize FCI targeting by assuming the central bank solves the following *modified* prob-

lem:

$$L_t^{FCI} = \min_{r_t^f, \bar{f}_{t+1}} \tilde{y}_t^2 + \frac{1}{\theta} \left(r_t^f - E_{t-1} [r_t^f] \right)^2 + \frac{\psi}{\theta} (f_t - \bar{f}_t)^2 + \tilde{\beta} E_t [L_{t+1}^{FCI}]. \quad (17)$$

In each period, in addition to setting the interest rate, the central bank announces an FCI target for the next period, denoted by \bar{f}_{t+1} . Thus, in the current period, the central bank has an inherited FCI target \bar{f}_t . In addition to its usual output gap and interest rate smoothing objectives, the central bank aims to keep the FCI close to this pre-announced target. The parameter ψ captures the strength of FCI targeting relative to standard macroeconomic objectives.

It is important to emphasize that, while the central bank operationally minimizes (17), its actual objective is still given by (10): the central bank does not care about FCI stability directly. We assume the central bank can commit to adopting an operational FCI target and ask whether this type of commitment can improve its standard macroeconomic objectives.

4.1. Equilibrium with FCI targeting

We show that the optimal target announcement is next period's expected FCI-star in the, $\bar{f}_t = E_{t-1} [f_t^*]$. The optimality condition for problem (17) then implies that the central bank follows the modified rule

$$r_t^f = E_{t-1} [r_t^f] + \theta \tilde{y}_t + \psi (f_t - E_{t-1} [f_t^*]). \quad (18)$$

In addition to output gaps, the central bank responds to surprise changes in the FCI. The following result characterizes the rest of the equilibrium.

Proposition 2 (Equilibrium with FCI targeting). *Suppose the parameters satisfy Assumption (S) and $\alpha^2 > 4\sigma_\mu^2\sigma_z^2$. Suppose monetary policy minimizes the modified objective in (17). Then, there is an equilibrium in which the central bank optimally announces the expected FCI-star as the target $\bar{f}_t = E_{t-1} [f_t^*] = f_{t-1}^*$ and the equilibrium variables are given by*

$$y_t - y_t^* = f_t - f_t^* = -\frac{\psi}{1 + \theta + \psi} \frac{z_t^p + z_t^e}{2} + \frac{1}{1 + \theta + \psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \quad (19)$$

$$r_t^f = -\frac{\sigma^2}{2} + \frac{\psi}{1 + \theta + \psi} \frac{z_t^p + z_t^e}{2} + \frac{\theta + \psi}{1 + \theta + \psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \quad (20)$$

$$e_t = \underbrace{c_t^*}_{e_{t-1}^* + z_t^e} - \frac{\psi}{1 + \theta + \psi} \frac{z_t^e + z_t^p}{2} + \left[\frac{1 + (\theta + \psi)/2}{1 + \theta + \psi} \mu_t^e - \frac{(\theta + \psi)/2}{1 + \theta + \psi} \mu_t^p \right] \frac{\sigma^2}{\alpha} \quad (21)$$

$$p_t = \underbrace{y_t^*}_{y_{t-1}^* + z_t^p} - \frac{\psi}{1 + \theta + \psi} \frac{z_t^e + z_t^p}{2} + \left[\frac{1 + (\theta + \psi)/2}{1 + \theta + \psi} \mu_t^p - \frac{(\theta + \psi)/2}{1 + \theta + \psi} \mu_t^e \right] \frac{\sigma^2}{\alpha}. \quad (22)$$

Here, σ^2 denotes the conditional variance of both e_{t+1} and p_{t+1} and it corresponds to the

smallest solution to the following quadratic:

$$\sigma^2 = \nu_z + \nu_\mu \left(\frac{\sigma^2}{\alpha} \right)^2 \quad \text{where} \quad \begin{aligned} \nu_z &\equiv \text{var} \left(\frac{1+\theta+\psi/2}{1+\theta+\psi} z_t^e - \frac{\psi/2}{1+\theta+\psi} z_t^p \right) \\ \nu_\mu &\equiv \text{var} \left(\frac{1+(\theta+\psi)/2}{1+\theta+\psi} \mu_t^e - \frac{(\theta+\psi)/2}{1+\theta+\psi} \mu_t^p \right). \end{aligned} \quad (23)$$

Increasing the strength of FCI targeting ψ reduces the endogenous variance, $\frac{d\sigma^2}{d\psi} < 0$, as well as its components driven by macroeconomic shocks and noise shocks, $\frac{d\nu_z}{d\psi} < 0$, $\frac{d\nu_\mu}{d\psi} < 0$.

For intuition, recall from (7) that output gaps are driven by FCI gaps, $\tilde{y}_t = f_t - f_t^*$. Thus, the policy rule in (18) implies

$$\begin{aligned} r_t^f &= E_{t-1} [r_t^f] + (\theta + \psi) \tilde{y}_t + \psi (f_t^* - E_{t-1} [f_t^*]) \\ &= E_{t-1} [r_t^f] + (\theta + \psi) \tilde{y}_t + \psi \frac{z_t^e + z_t^p}{2}. \end{aligned} \quad (24)$$

Here, the second line uses (6) to describe the FCI surprises in terms of the underlying shocks (recall $f_t = \frac{e_t + p_t}{2}$). Comparing this rule with (11), implies that FCI targeting induces two differences. First, it makes the central bank *more responsive* to noise-driven output gaps than it would desire according to its preferences. Second, it induces the central bank to adjust the interest rate to partially offset macroeconomic shocks driving the exchange rate and market price, because these shocks move the FCI away from the central bank's preannounced target. The equilibrium characterized in (19 – 22) verifies these intuitions: the outcomes are as if the planner has a greater responsiveness parameter than indicated by its preferences ($\theta + \psi$), coupled with an additional objective to offset the impact of macroeconomic shocks. Consequently, noise shocks have a smaller impact on the FCI and the output gap but macroeconomic shocks also induce some output gaps.

Crucially, the result shows that FCI targeting *reduces* the endogenous variance σ^2 of both the exchange rate and the market price. This happens for two reasons. First, noise shocks have a smaller impact on both variables due to the diversification mechanism emphasized in the previous section: the stronger policy responsiveness to noise strengthens the diversification mechanism. Second, macroeconomic shocks have a smaller impact on both variables because their impact is partly offset by monetary policy.

4.2. Macro-stabilization effects of FCI targeting

While FCI targeting reduces financial variance, the policy ultimately values output gap stabilization and smooth interest rates. Does FCI targeting improve the central bank's standard macro-stabilization objectives? Our next result shows that in our model some FCI targeting is always optimal.

To state the result, we consider the central bank loss L_t from (10) and we define the expected

loss:

$$L_t^e \equiv E[L_t] = E \left[\sum_{h=0}^{\infty} \tilde{\beta}^h \left[\tilde{y}_t^2 + \frac{1}{\theta} \left(r_t^f - E_{t-1} [r_t^f] \right)^2 \right] \right]. \quad (25)$$

We evaluate the policy performance with L_t^e (rather than L_t) because this averages the current-period gaps across all shocks. To evaluate this expectation, observe that Eq. (19) implies that the output and interest rate gaps are given by:

$$\tilde{y}_t = -\frac{\psi}{1+\theta+\psi} \frac{z_t^p + z_t^e}{2} + \frac{1}{1+\theta+\psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2(\psi)}{\alpha} \quad (26)$$

$$r_t^f - E_{t-1} [r_t^f] = \frac{\psi}{1+\theta+\psi} \frac{z_t^p + z_t^e}{2} + \frac{\theta+\psi}{1+\theta+\psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2(\psi)}{\alpha}. \quad (27)$$

We can then calculate and decompose the central bank loss as follows

$$L^e(\psi) = L_z^e(\psi) + L_\mu^e(\psi), \quad \text{where,} \quad (28)$$

$$L_z^e(\psi) = \frac{1}{1-\tilde{\beta}} \sigma_z^2 \frac{1+\rho_z}{2} \left(1 + \frac{1}{\theta} \right) \left(\frac{\psi}{1+\theta+\psi} \right)^2$$

$$L_\mu^e(\psi) = \left(\frac{\sigma^2(\psi)}{\alpha} \right)^2 l_\mu(\psi)$$

$$\text{with } l_\mu(\psi) = \frac{1}{1-\tilde{\beta}} \sigma_\mu^2 \frac{1+\rho_\mu}{2} \left(\left(\frac{1}{1+\theta+\psi} \right)^2 + \frac{1}{\theta} \left(\frac{\theta+\psi}{1+\theta+\psi} \right)^2 \right).$$

Here, $L_z^e(\psi)$ and $L_\mu^e(\psi)$ capture the contributions of macroeconomic shocks and noise shocks to the central bank loss, respectively. The function, $l(\theta, \psi)$ captures (up to a scale) the noise-driven loss for a *given* endogenous variance and the function $\sigma^2(\psi)$ captures the effect through the endogenous variance. We next characterize how FCI targeting affects $L^e(\psi)$ and its components.

Proposition 3 (Macrostabilization Effects of FCI Targeting). *Consider the equilibrium in Proposition 2. FCI targeting reduces the expected central bank loss*

$$\frac{dL^e(\psi)}{d\psi} \Big|_{\psi=0} < 0.$$

Therefore, the loss-minimizing policy features some FCI targeting, $\psi^* = \arg \min_{\psi \geq 0} L^e(\psi) > 0$. FCI targeting raises the loss driven by macroeconomic shocks as well as the exogenous component of the loss driven by noise shocks, but both effects are second order

$$\frac{dL_z^e(\psi)}{d\psi} \geq 0, \quad \frac{dl_\mu(\theta, \psi)}{d\psi} \geq 0, \quad \text{with } \frac{dL_z^e(\psi)}{d\psi} \Big|_{\psi=0} = \frac{dl_\mu(\theta, \psi)}{d\psi} \Big|_{\psi=0} = 0.$$

FCI targeting has competing effects on output and interest rate gaps as illustrated by Eqs. (26 – 28). On the one hand, it creates a new source of gaps driven by macroeconomic shocks. Moreover, it also distorts the policy’s “ideal” discretionary response to noise shocks. If variance

σ^2 was exogenous, these forces would increase policy losses. On the other hand, since FCI targeting also reduces the endogenous variance σ^2 , it also reduces the impact of noise on both the output gap and interest rates and therefore mitigates the associated losses. The result shows that the loss-raising distortionary forces are second order—this is because the policy without FCI targeting is optimized, so small deviations are not very costly. In contrast, the loss-reducing variance reduction force is first order as we have seen in Proposition 2. Therefore, an appropriately calibrated FCI targeting always improves macroeconomic stabilization.

Conceptually, FCI targeting helps to address an important time inconsistency problem in monetary policy that we first identified in Caballero et al. (2025b). Since arbitrageurs' behavior depends on *future* financial variance, policies that commit to reducing *future* financial variance can improve *current* policy objectives by influencing the arbitrageurs' behavior. In particular, by reducing the financial variance in each sub-market, FCI targeting *recruits* the arbitrageurs to insulate FCI from financial noise and contribute to macroeconomic stability.

How does the FCI targeting affect the dimensions of the policy objective: output gap stabilization and interest rate smoothing? Eq. (26) implies that a small degree of FCI targeting always reduces the output gap losses

$$E[\tilde{y}_t^2] = \sigma_z^2 \frac{1 + \rho_z}{2} \left(\frac{\psi}{1 + \theta + \psi} \right)^2 + \sigma_\mu^2 \frac{1 + \rho_\mu}{2} \left(\frac{1}{1 + \theta + \psi} \right)^2 \left(\frac{\sigma^2(\psi)}{\alpha} \right)^2.$$

The additional gaps due to macroeconomic shocks are second order, but the gap reductions from noise shocks are first order. In fact, FCI targeting reduces the impact of noise shocks on the output gap through two channels: by reducing the variance and thus the impact of noise ($\sigma^2(\psi)$) and by increasing the responsiveness of policy to noise ($\frac{1}{1 + \theta + \psi}$).

In contrast, Eq. (27) shows that the impact of FCI targeting on interest rate variance (and associated losses) is more nuanced:

$$E\left[\left(r_t^f - E_{t-1}\left[r_t^f\right]\right)^2\right] = \sigma_z^2 \frac{1 + \rho_z}{2} \left(\frac{\psi}{1 + \theta + \psi} \right)^2 + \sigma_\mu^2 \frac{1 + \rho_\mu}{2} \left(\frac{\theta + \psi}{1 + \theta + \psi} \right)^2 \left(\frac{\sigma^2(\psi)}{\alpha} \right)^2.$$

While the new gaps through macroeconomic shocks are still second order, the policy *might* increase the interest rate gaps created by noise shocks. This is because it makes the interest rate respond to noise more aggressively ($\frac{\theta + \psi}{1 + \theta + \psi}$). On the other hand, the policy still exerts a stabilizing effect on interest rate gaps by reducing the variance ($\sigma^2(\psi)$). In our numerical simulations, we find that this second effect typically dominates because of the amplification mechanism captured by the quadratic in (16). Since the decline in variance recruits the arbitrageurs to trade against noise more aggressively, noise has a smaller impact on financial conditions and aggregate demand. This in turn reduces the burden on the central bank to adjust the interest rate in response to noise shocks. Consequently, FCI targeting not only reduces the central bank's total losses, it can actually mitigate both components of these losses—*inducing more stable output*

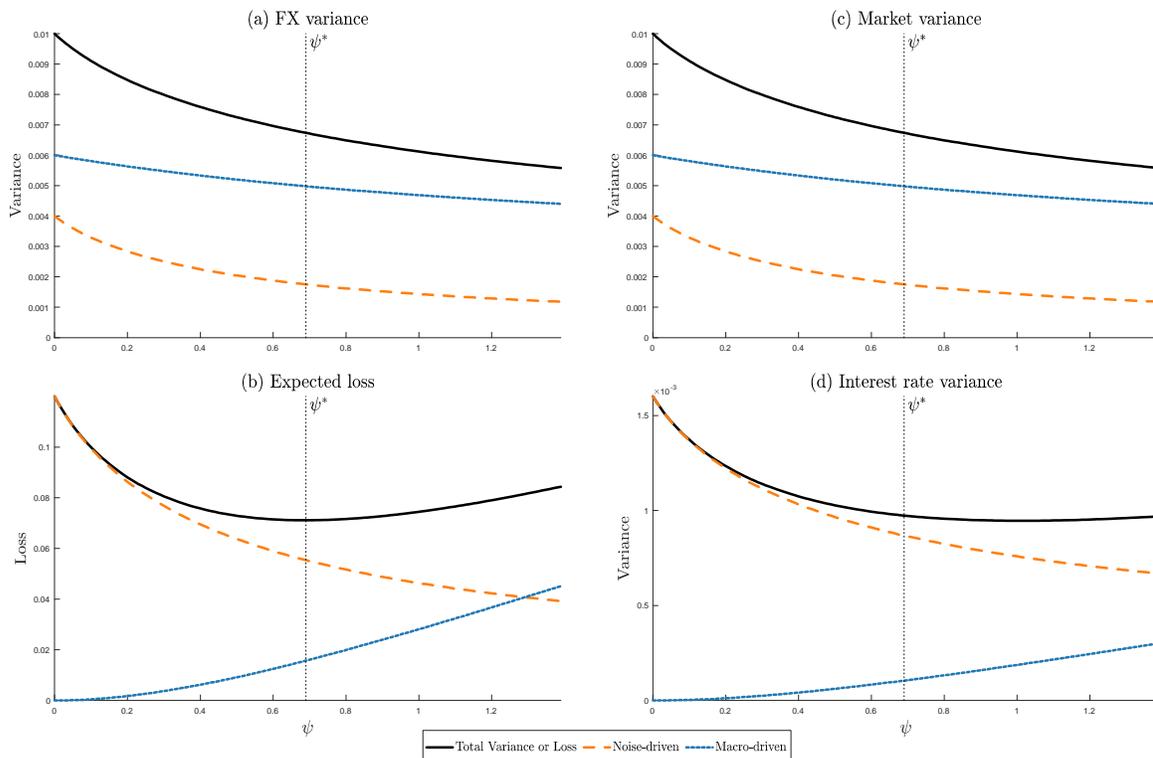


Figure 2: The top panels show the effect of FCI targeting ψ on the (identical) variance of the exchange rate and the market price. The bottom left panel shows the effect on the expected output gap loss. The bottom right panel shows the effect on the expected interest rate variance. In each panel, solid black line shows the total variance or loss, the dashed orange line shows the component induced by noise shocks, and the dotted blue line shows the component induced by macroeconomic shocks. We use the parameters in Appendix A.1.

along with smoother interest rates.

Numerical illustration. Figure 2 illustrates the effects of FCI targeting in a numerical example. The example is calibrated so that, with discretionary policy, a substantial fraction (40%) of the variance stems from noise shocks and the remaining from macroeconomic shocks. In addition, these shocks are uncorrelated across markets, so that cross-market diversification opportunities are relatively large (see Appendix A.1 for the parameters).

The top panels show that FCI targeting substantially reduces the exchange rate and market variances (which are identical). At the optimal targeting level, $\psi = \psi^*$ (vertical line), total variance in both markets declines substantially. This decline is mainly accounted for by the component of variance due to noise shocks.

The bottom left panel shows that appropriately calibrated FCI targeting reduces the policy loss. Small doses of FCI targeting reduce the total output loss and its component driven by noise shocks, while having only second-order effects on its component driven by macroeconomic shocks. As targeting intensity increases, the policy benefits gradually decline. The optimal level,

$\psi^* \simeq 0.7$, corresponds to the central bank putting about a third of the weight on FCI gaps as on output gaps (we set $\theta = 2$). This mild degree of FCI targeting is sufficient to reduce the total loss by about 40%.

The bottom panel shows that in this example FCI targeting also reduces the interest rate variance (and associated losses). With small doses of FCI targeting, interest rate variance declines because the large reduction in the financial variance (and the associated decline in the noise impact) dominates the modest increase in the interest rate responsiveness to macroeconomic and noise shocks. In this range, FCI targeting recruits the arbitrageurs to absorb noise more effectively, which reduces the burden on the central bank to respond to noise shocks. As targeting intensity increases, the interest rate variance eventually begins to increase due to diminishing returns in noise reduction and recruitment. Nonetheless, the interest rate variance with the optimal targeting, $\psi = \psi^*$, is still substantially lower than the variance with discretion.

5. FX targeting

The FCI targeting framework developed in Section 4 requires the central bank to track and respond to a broad set of financial variables. A natural question is whether and when much of the gain from FCI targeting can be achieved by focusing on a narrower set of assets. This question is particularly relevant for economies where the exchange rate—rather than the domestic stock market—is the dominant driver of financial conditions. In this section, we analyze FX targeting and identify conditions under which it can be effective.

We find two main results. First, we show that FX targeting can substantially improve welfare when two conditions hold jointly: (i) the exchange rate accounts for most of the variation in financial conditions (high ϕ), and (ii) the exchange rate is the primary source of noise-driven fluctuations. In economies satisfying both conditions, FX targeting can achieve welfare gains approaching those of full FCI targeting. Second, we characterize the limitations of this substitution. FX targeting forgoes the diversification mechanism that makes FCI targeting robust to the dominant source of noise. As a result, in some instances it may amplify rather than dampen macroeconomic volatility through anti-recruitment effects in non-targeted markets.

We formalize FX targeting by assuming the central bank solves the following version of problem (17)

$$L_t^{FX} = \min_{r_t^f, \bar{e}_{t+1}} \tilde{y}_t^2 + \frac{1}{\theta} \left(r_t^f - E_{t-1} [r_t^f] \right)^2 + \frac{\psi}{\theta} (e_t - \bar{e}_t)^2 + \tilde{\beta} E_t [L_{t+1}^{FX}]. \quad (29)$$

In each period, instead of announcing an FCI target, the central bank announces an exchange rate target. Thus, the central bank has an inherited exchange rate target \bar{e}_t that it would like to achieve in addition to its usual output gap and interest rate smoothing objectives.

In this case, Assumption (S) does not simplify the analysis as much as before because once the central bank targets the exchange rate, it breaks the symmetry across the exchange rate and

the market portfolio even when the underlying parameters are symmetric. Therefore, we drop this assumption and consider the more general system in (1 – 4). To keep the analysis tractable, however, we make the simplifying assumption that shocks to the exchange rate and the market portfolio are uncorrelated ($\rho_z = \rho_\mu = 0$).

5.1. Equilibrium with FX targeting

The optimal target announcement is the expected exchange rate in the next period, $\bar{e}_t = E_{t-1}[e_t]$ (this is also equal to expected “estar” up to a constant term, similar to before). Therefore, the optimality condition for problem (29) implies the policy rule

$$r_t^f = E_{t-1} \left[r_t^f \right] + \theta \tilde{y}_t + \psi (e_t - E_{t-1} [e_t]). \quad (30)$$

Here, the central bank responds to surprise deviations of the *exchange rate* relative to its expectation, rather than to deviations of the FCI (cf. (18)). The following result characterizes the rest of the equilibrium.

Proposition 4 (Equilibrium with FX targeting). *Consider the more general model without Assumption (S) but with uncorrelated shocks $\rho_z = \rho_\mu = 0$. Suppose the parameters are such that the system in (A.42 – A.43) in the appendix has a positive solution. Suppose monetary policy minimizes the modified objective in (29). Then, there is an equilibrium in which the central bank optimally announces the expected exchange rate as the target $\bar{e}_t = E_{t-1}[e_t]$ and the equilibrium variables are given by*

$$\underbrace{y_t - y_t^*}_{f_t - f_t^*} = -\frac{\psi}{1 + \theta + \psi} z_t^e + \frac{\phi - (1 - \phi)\psi}{1 + \theta + \psi} \mu_t^e \frac{\sigma_e^2}{\alpha} + \frac{(1 - \phi)(1 + \psi)}{1 + \theta + \psi} \mu_t^p \frac{\sigma_m^2}{\alpha} \quad (31)$$

$$r_t^f + \frac{\sigma_e^2}{2} = \rho + \frac{\psi}{1 + \theta + \psi} z_t^e + \frac{\phi\theta + \psi}{1 + \theta + \psi} \mu_t^e \frac{\sigma_e^2}{\alpha} + \frac{(1 - \phi)\theta}{1 + \theta + \psi} \mu_t^p \frac{\sigma_m^2}{\alpha} \quad (32)$$

$$e_t - \frac{\kappa^0}{\phi} = \underbrace{e_t^*}_{e_{t-1}^* + z_t^e} - \frac{\psi}{1 + \theta + \psi} z_t^e + \frac{1 + \theta(1 - \phi)}{1 + \theta + \psi} \mu_t^e \frac{\sigma_e^2}{\alpha} - \frac{(1 - \phi)\theta}{1 + \theta + \psi} \mu_t^p \frac{\sigma_m^2}{\alpha} \quad (33)$$

$$p_t + \frac{\kappa^0}{1 - \phi} = \underbrace{y_t^*}_{y_{t-1}^* + z_t^p} - \frac{\psi}{1 + \theta + \psi} z_t^e + \frac{1 + \phi\theta + \psi}{1 + \theta + \psi} \mu_t^p \frac{\sigma_m^2}{\alpha} - \frac{\phi\theta + \psi}{1 + \theta + \psi} \mu_t^e \frac{\sigma_e^2}{\alpha} \quad (34)$$

$$E_t \left[r_{t+1}^m - \frac{\psi}{1 + \theta + \psi} z_t^e \right] = \frac{z_t^p -}{1 + \theta + \psi} + \left[\begin{pmatrix} \frac{\phi - (1 - \phi)\psi}{1 + \theta + \psi} \\ -\beta\phi \end{pmatrix} \mu_t^e \frac{\sigma_e^2}{\alpha} + \begin{pmatrix} \frac{(1 - \phi)(1 + \psi)}{1 + \theta + \psi} \\ +\beta\phi \end{pmatrix} \mu_t^p \frac{\sigma_m^2}{\alpha} \right]. \quad (35)$$

where $\kappa^0 = \frac{\sigma_m^2 - \sigma_e^2}{1 - \beta} \frac{1 - \phi}{2}$. The terms σ_e^2 and σ_m^2 denote the conditional variances of e_{t+1} and r_{t+1}^m , respectively. They correspond to the stable solution to the following quadratic system (A.42 – A.43) in the appendix.

With FX targeting, the equilibrium is considerably more complicated than before due to the

lack of symmetry across the two markets. To unpack the results, consider the special case with Assumption (S). Note that the interest rate response to shocks is given by:

$$\begin{aligned} r_t^f &\sim \frac{\psi}{1+\theta+\psi} z_t^e + \frac{\theta/2+\psi}{1+\theta+\psi} \mu_t^e \frac{\sigma_e^2}{\alpha} + \frac{\theta/2}{1+\theta+\psi} \mu_t^p \frac{\sigma_m^2}{\alpha} \\ e_t &\sim z_t^e - \frac{\psi}{1+\theta+\psi} z_t^e + \frac{1+\theta/2}{1+\theta+\psi} \mu_t^e \frac{\sigma_e^2}{\alpha} - \frac{\theta/2}{1+\theta+\psi} \mu_t^p \frac{\sigma_m^2}{\alpha} \\ p_t &\sim z_t^p - \frac{\psi}{1+\theta+\psi} z_t^e + \frac{1+\theta/2+\psi}{1+\theta+\psi} \mu_t^p \frac{\sigma_m^2}{\alpha} - \frac{\theta/2+\psi}{1+\theta+\psi} \mu_t^e \frac{\sigma_e^2}{\alpha}. \end{aligned}$$

These expressions illustrate that increasing the degree of FX targeting, ψ , has three main effects on policy: (i) it makes the policy partially offset the price impact of *macroeconomic shocks to the exchange rate* (z_t^e), (ii) it makes the policy respond to *exchange rate noise* more aggressively ($\frac{\theta/2+\psi}{1+\theta+\psi}$ is increasing in ψ), and (iii) it makes the policy respond to the domestic market noise less aggressively ($\frac{\theta/2}{1+\theta+\psi}$ is decreasing in ψ).

Comparing these expressions with their counterparts with FCI targeting from Proposition 2 is also illustrative. In particular, recall that the interest rate with FCI targeting is given by [cf. (20)]

$$r_t^{f,FCI} \sim \frac{\psi}{1+\theta+\psi} \frac{z_t^p + z_t^e}{2} + \frac{(\theta+\psi)/2}{1+\theta+\psi} (\mu_t^e + \mu_t^p) \frac{\sigma^2}{\alpha}.$$

While FCI targeting dampens both macroeconomic and noise shocks equally, FX targeting dampens the exchange rate fundamentals and noise shocks more than before, and the market price noise shock less than before ($\frac{\theta/2+\psi}{1+\theta+\psi} > \frac{(\theta+\psi)/2}{1+\theta+\psi} > \frac{\theta/2}{1+\theta+\psi}$).

These expressions imply that, compared to an FCI targeting policy with similar strength, FX targeting exerts a greater stabilizing force on the exchange rate and a smaller stabilizing force on the market price. In particular, FX targeting mitigates the impact of all shocks on e_t for given variances σ_e^2 and σ_m^2 . In contrast, FX targeting tends to raise the impact of shocks on the market price: it always raises the impact of both noise shocks on p_t , and it also raises the impact of macroeconomic shocks on p_t —unless z_t^e and z_t^p are highly correlated.

These comparisons take the variances σ_e^2 and σ_m^2 as given. As before, these variances are endogenous to the policy regime. The result also characterizes the variances and shows that they solve a quadratic system. This system is more complicated than before so we resort to numerical solutions to investigate its properties.

5.2. Numerical illustration of variance effects of FX targeting

Effects of FX-Targeting on Volatility and Welfare. Figure 3 illustrates the effects of FX targeting in the same example that we analyzed in Figure 2. Specifically, we assume $\sigma_{z,e}^2 = \sigma_{z,p}^2 = \sigma_z^2$ and $\sigma_{\mu,e}^2 = \sigma_{\mu,p}^2 = \sigma_\mu^2$ along with $\rho_z = \rho_\mu = 0$ and set the remaining parameters the same as before. In particular, Assumption (S) holds so the exchange rate and the market price

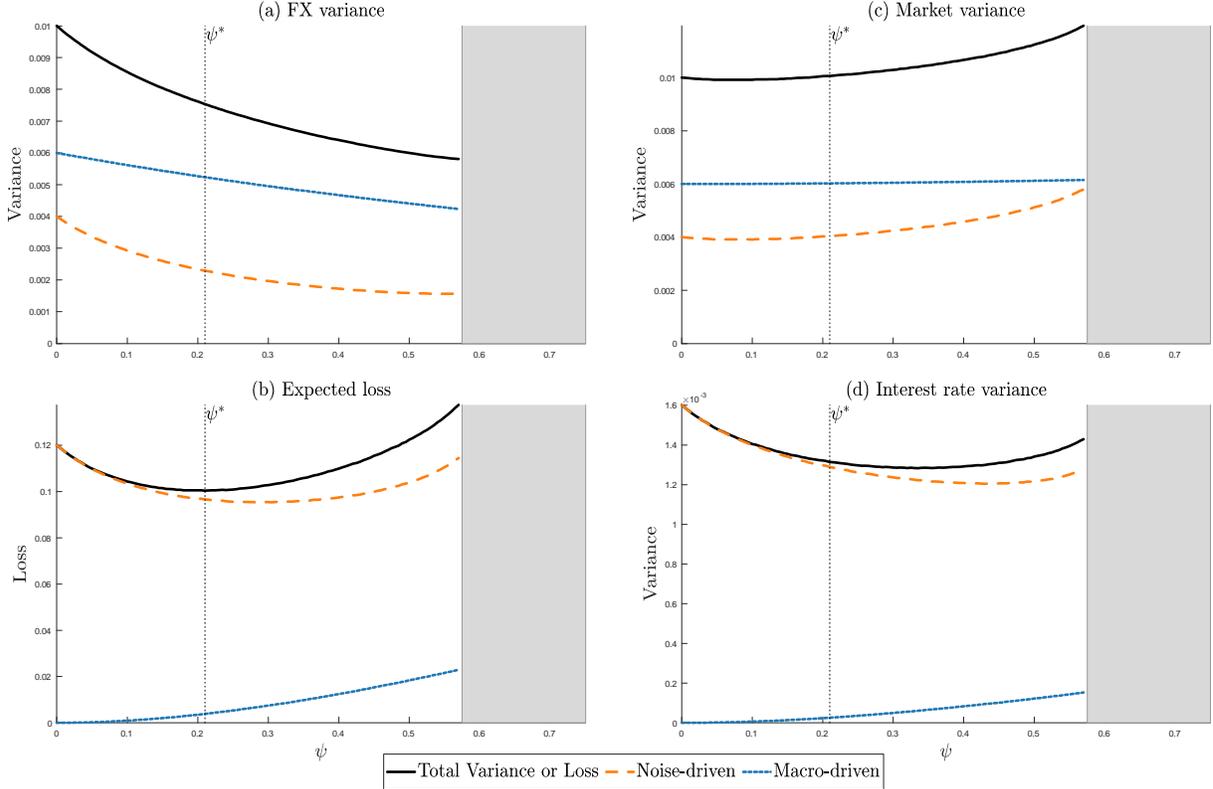


Figure 3: The top panels show the effect of FX targeting ψ on the variance of the exchange rate and the market price, respectively. The bottom left panel shows the effect on the expected output gap loss. The bottom right panel shows the effect on the expected interest rate variance. In each panel, solid black line shows the total variance or loss, the dashed orange line shows the component induced by noise shocks, and the dotted blue line shows the component induced by macroeconomic shocks. The shaded gray areas correspond to regions where a stable equilibrium with finite variance does not exist. We use the parameters in Appendix A.1.

are equally important for output, $\phi = 1/2$ (see Appendix A.1 for the full parameters).

The top left panel shows the effects of FX targeting on exchange rate variance. As the central bank increases ψ , total FX variance declines monotonically. A stronger policy reaction dampens the sensitivity of the exchange rate to both macro and noise shocks. The decomposition shows that most of this reduction early on comes from the noise component (orange dashed line), while the macroeconomic variance (blue dotted line) falls more gradually. This is the recruitment effect in action within the FX market.

The top right panel shows the effect on the market return variance. Market variance initially declines slightly, but eventually rises sharply, with contributions from both macroeconomic and noise components. This reflects a key difference from FCI targeting: while FX targeting recruits arbitrageurs in the FX market, it raises volatility in the equity market by transmitting exchange rate shocks there. This higher variance creates an anti-recruitment effect—arbitrageurs in the equity market become less willing to trade against noise, which raises the variance induced by

noise shocks. These negative feedback effects can become severe: for sufficiently aggressive FX targeting, no stable equilibrium with finite variance exists (gray shaded area).

The bottom left panel shows the impact on the expected output gap loss. In this symmetric benchmark, FX targeting reduces expected output gap loss by about 16% at the optimal level $\psi^* \simeq 0.2$. This is smaller than the 40% reduction from FCI targeting (Figure 2), reflecting the fact that FX targeting foregoes diversification gains when both markets are equally important and equally noisy.

Beyond the optimal level, total loss rises as anti-recruitment costs in the equity market begin to outweigh recruitment benefits in the FX market. As we show next, FX targeting performs considerably better when noise is concentrated in the FX market.

The last panel shows a similar pattern for the interest rate variance. After an initial decline due to the positive recruitment effect in the FX market, interest rate volatility starts to increase with ψ . This reflects the growing effort required by monetary policy to defend the FX target and to offset the greater market volatility induced by an ever increasing anti-recruitment effect.

In sum, the symmetric benchmark shows that FX targeting successfully reduces FX-market noise through recruitment, but cross-market spillovers limit the overall gains when both markets are equally noisy. This motivates the key question: under what conditions can FX targeting achieve larger welfare gains? We turn to this next.

5.3. The Case for FX Targeting: Noise Concentration and Openness

The symmetric benchmark shows that FX targeting faces an inherent tradeoff: recruitment gains in the FX market versus anti-recruitment costs in the equity market. This tradeoff determines when FX targeting improves welfare. In particular, the performance of FX targeting depends critically on two factors: the composition of noise across markets ($\sigma_{\mu,e}$ relative to $\sigma_{\mu,p}$) and the degree of openness, which determines the exchange rate's weight in financial conditions (ϕ).

Figure 4 investigates how performance depends on the composition of noise. The left panel shows that when FX noise is larger than equity noise ($\sigma_{\mu,e} > \sigma_{\mu,p}$, green line), FX targeting delivers substantial welfare gains—the optimal degree of stabilization is considerably higher than in the symmetric case, and the policy efficiently filters out the main source of noise-driven fluctuations. The anti-recruitment effects in equity markets remain, but they are less costly when equity noise is smaller. Conversely, when equity noise dominates ($\sigma_{\mu,p} > \sigma_{\mu,e}$, red line), FX targeting performs poorly: recruiting arbitrageurs in the less noisy market while triggering anti-recruitment in the noisier one.

The right panel decomposes these effects. The macro component (solid lines) rises smoothly with ψ and is similar across scenarios. The key difference lies in the noise component (dashed lines). When FX noise dominates, targeting FX reduces noise-driven losses substantially. When equity noise dominates, anti-recruitment amplifies equity market noise, overwhelming any FX gains.

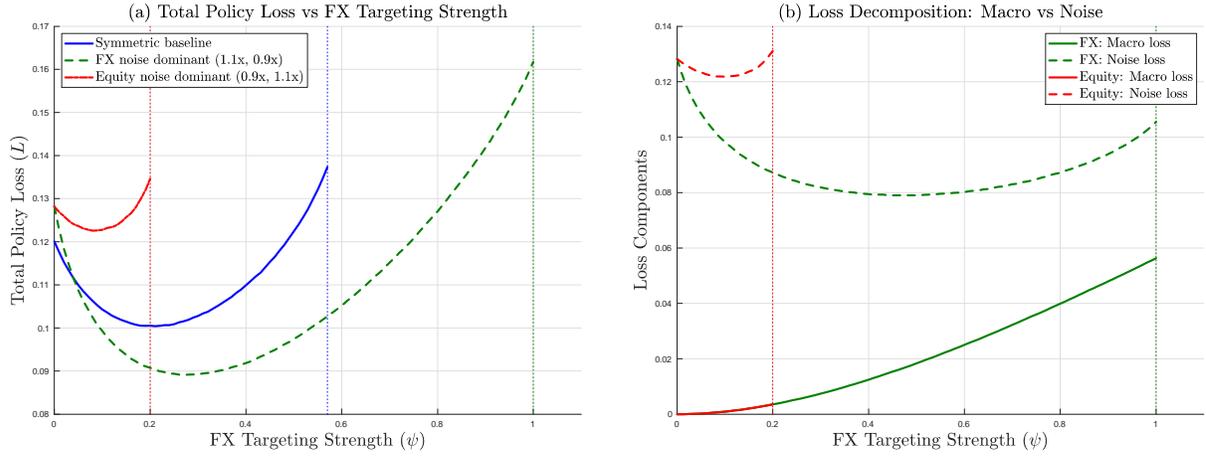


Figure 4: Panel (a) shows the total policy loss as a function of FX targeting strength ψ for three different noise variance scenarios. The solid blue line represents the symmetric case where all noise variances are the same $\sigma_{\mu,e}^2 = \sigma_{\mu,p}^2 = \sigma_{\mu}^2$. The dashed green line shows the case where FX market noise is dominant ($\sigma_{\mu,e}^2 = 1.1\sigma_{\mu}^2$ and $\sigma_{\mu,p}^2 = 0.9\sigma_{\mu}^2$). The dash-dotted red line shows the case where equity market noise is dominant ($\sigma_{\mu,e}^2 = 0.9\sigma_{\mu}^2$ and $\sigma_{\mu,p}^2 = 1.1\sigma_{\mu}^2$). Macroeconomic shock variances remain the same across all scenarios, $\sigma_{z,e}^2 = \sigma_{z,p}^2 = \sigma_z^2$. Panel (b) decomposes the policy loss into macro-driven components (solid lines) and noise-driven components (dashed lines) for the two asymmetric scenarios, with green representing the FX noise dominant case and red representing the equity noise dominant case. Vertical lines mark the threshold beyond which a stable equilibrium with finite variance does not exist. See Appendix A.1 for the remaining parameters.

The benefits of FX targeting also increase with openness. As ϕ increases—reflecting greater openness—the exchange rate accounts for a larger share of financial conditions fluctuations. Therefore, recruitment gains in the FX market translate more directly into macroeconomic stabilization. In the limiting case where $\phi = 1$, the exchange rate is the only component of financial conditions, FX targeting coincides with FCI targeting, and we recover the single-asset results from Caballero et al. (2025b).

Discussion. Our analysis shows that FX targeting can deliver welfare gains when the exchange rate is the primary source of noise, but it also highlights limitations relative to FCI targeting. By concentrating policy responses on a single market, FX targeting forgoes the diversification benefits of responding to broad financial conditions and can trigger anti-recruitment effects in domestic asset markets.

Our model further suggests that more flexible approaches can mitigate these costs while preserving the benefits of focusing on a dominant asset. The loss-minimizing exchange rate target adjusts to keep financial conditions consistent with macroeconomic objectives—it is not an absolute level to be defended regardless of circumstances. Therefore, a more flexible approach can: (i) frame targets as deviations from time-varying fundamental benchmarks (e^* or FCI^*), rather than fixed levels; (ii) monitor other market conditions even if not formally targeted; and (iii) broaden the targeting framework if the composition of noise shifts toward domestic asset markets.

5.4. Exchange Rate Stabilization in Practice: The MCI Experience

The flexible principles outlined above stand in contrast to the Monetary Conditions Index (MCI) frameworks implemented in Canada and New Zealand during the 1990s (see, e.g., Freedman (1995); Batini and Turnbull (2002); Drew and Hunt (1998); Coletti et al. (1999)). These experiments illustrate what can go wrong when narrow financial conditions targeting is applied mechanically.

Canada pioneered the approach in the early 1990s, constructing an MCI as a weighted average of the short-term interest rate and the trade-weighted exchange rate. A 1-percentage-point rise in the short rate was treated as equivalent to roughly a 3 percent currency appreciation, based on estimated impacts on aggregate demand. The Bank of Canada initially used the MCI as an operational target, adjusting policy rates to stabilize the index around a desired level.

In practice, this strategy proved problematic. When disturbances originated outside the MCI basket—particularly in equity or commodity markets—attempting to stabilize the index led to excessive interest-rate volatility without corresponding macroeconomic gains. The fixed weights were unstable across different conditions, and the framework could not easily distinguish between fundamental and noise-driven exchange-rate movements. By the late 1990s, the Bank of Canada abandoned the MCI as an operational target, acknowledging that “chasing the index” created more volatility than it resolved. New Zealand’s experience was similar: the Reserve Bank

adopted an MCI framework in 1996 but soon faced the same difficulties, especially during the Asian Financial Crisis when capital-flow reversals caused large exchange-rate swings unrelated to domestic conditions.

Viewed through the lens of our model, the MCI experience illustrates two limitations of narrow targeting. First, by focusing exclusively on the exchange rate and interest rates, MCI frameworks may have destabilized non-targeted markets such as equities—potentially raising volatility there and amplifying the impact of noise through anti-recruitment effects. Second, constructing the appropriate target proved difficult: distinguishing fundamental from noise-driven exchange rate movements requires assessing “ e^* ,” which depends on variables central banks found hard to track in real time. In our framework, broader FCI targeting would mitigate both problems. Recruitment benefits are shared more evenly across markets, reducing the risk of destabilizing any single market. And while FCI^* is also nontrivial to construct, it depends primarily on macroeconomic variables—supply and demand shocks—that are central to standard monetary policy analysis (see Caballero et al. (2025a)).

6. Final Remarks

This paper analyzes monetary policy responses to financial noise in open economies where both exchange rates and domestic asset prices affect aggregate demand. We develop a model in which targeting a financial conditions index—a weighted average of both markets—delivers substantial macroeconomic benefits. In our model, FCI targeting exploits diversification across segmented markets: by responding to noise and macroeconomic shocks in both markets, policy spreads volatility more effectively than discretionary policy alone. This reduces variance in each market, activating a recruitment effect where lower volatility encourages arbitrageurs to trade against noise, further dampening the impact of noise on financial conditions and output.

In our model, FX targeting can also deliver welfare gains when the exchange rate is the primary source of noise. In this case, FX targeting recruits arbitrageurs to stabilize the FX market, reducing volatility and dampening the macroeconomic impact of noise. The benefits increase when the exchange rate accounts for a larger share of financial conditions; in the limiting case where the exchange rate equals the FCI, FX targeting coincides with FCI targeting. Importantly, FX targeting, as we analyze it, is not a call for fixed exchange rates—the target moves with fundamentals to keep overall financial conditions aligned with macroeconomic objectives.

However, FX targeting also raises volatility in non-targeted markets through anti-recruitment effects. By concentrating policy responses on the exchange rate, FX targeting transmits FX shocks to domestic asset markets, discouraging arbitrageurs there from trading against noise. This limits the effectiveness of FX targeting relative to FCI targeting, which exploits diversification across markets. When equity market noise is substantial, these costs can outweigh the benefits of FX stabilization. The disappointing experience with Monetary Conditions Index frameworks in Canada and New Zealand during the 1990s illustrates these challenges: by focus-

ing narrowly on the exchange rate and interest rates, MCI frameworks may have destabilized non-targeted markets, while struggling to distinguish fundamental from noise-driven exchange rate movements.

Importantly, our analysis focuses on implementing exchange rate and financial conditions targets through interest rate policy, not through direct market interventions. Foreign exchange interventions may yield additional benefits beyond those we analyze. As emphasized by Gabaix and Maggiori (2015), direct interventions can reduce the noise flows that FX arbitrageurs must absorb. In our framework, such interventions would potentially generate additional stabilization gains: by suppressing noise at its source in the FX market, interventions would reduce volatility there and—through the spillover channels we identify—also stabilize domestic asset markets, activating recruitment effects across all components of financial conditions. Singapore’s framework provides an illustration of this mechanism: by managing the exchange rate directly, it contains FX-driven volatility at the source, which in turn contributes to a more stable interest-rate environment (see Khor et al. (2004); Parrado (2004); Monetary Authority of Singapore (2022)). We leave a formal analysis of direct FX interventions with cross-market spillover effects for future work.

More broadly, direct FX interventions paired with flexible interest rate policy can approximate some of the diversification benefits of FCI targeting by addressing different markets with different instruments. When FX interventions stabilize the exchange rate around a target that moves with fundamentals, interest rate policy is freed to respond more effectively to shocks in domestic asset markets. This instrument separation mitigates the central limitation of interest-rate-based FX targeting—that stabilizing the exchange rate through interest rates transmits shocks to domestic asset markets, triggering anti-recruitment effects there. The key lesson remains: robust stabilization in environments with multiple noisy markets is achieved by policy frameworks that exploit, rather than suppress, diversification across markets.

Finally, while this paper focuses on the benefits of commitment to financial conditions targets, communication alone can also generate recruitment effects. In Caballero and Simsek (2025), we show that when markets and the central bank disagree about near-term macroeconomic scenarios, communicating the central bank’s intended path for financial conditions—an “FCI-plot”—reduces ambiguity about the policy stance and narrows the range of outcomes arbitrageurs must consider. This clarity recruits arbitrageurs to trade against noise even absent formal commitment. Some central banks already recognize this logic. Norges Bank’s monetary policy strategy explicitly states that “with confidence that the central bank will stabilise inflation, the exchange rate will therefore contribute to stabilising inflation when unexpected changes in inflation occur”—acknowledging that credible policy recruits FX market participants to act as stabilizing forces (Norges Bank, 2024). Our framework suggests that extending this insight to explicit communication about financial conditions could amplify recruitment effects.

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Appendices

A. Theory Appendix

This appendix contains details related to the theoretical model. Section A.1 describes the parameters we use in the numerical examples that illustrate our findings. Section A.2 provides the microfoundations for the model. Section A.3 contains the proofs omitted from the main text.

A.1. Details of the numerical exercise

We next describe the parameters we use in the numerical examples that we use to illustrate the results in Sections 4 and 5.

Parameters for Section 4. We start by describing the parameters we use for Figure 2.

For policy objectives, we set $\theta = 2$ so that the policy changes the interest rate by 2 percentage points in response to a 1 percent change in the output gap.

To calibrate the mass of arbitrageurs (elastic funds) α , we observe that the price impact of a unit change in asset demand (as a fraction of supply) is given by

$$\mathcal{I} \equiv \frac{dp_t}{d\mu_t} = \frac{\sigma^2}{\alpha}.$$

We then set α to target a price impact equal to one, $\mathcal{I} = \frac{\sigma^2}{\alpha} = 1$. This is a conservative calibration since empirical analyses find that for aggregate risky assets this price impact is typically larger than one, e.g., Gabaix and Koijen (2021) argue that for the aggregate stock market it could be as large as 5.

To set the variance parameters, we set $\rho_z = \rho_\mu = 0$ so that cross-market diversification opportunities are relatively large. Eq. (16) then becomes

$$\begin{aligned} \sigma^2 &= \sigma_z^2 + \sigma_{noise}^2 \\ \text{where } \sigma_{noise} &= \left[\left(\frac{\theta/2}{1+\theta} \right)^2 + \left(\frac{1+\theta/2}{1+\theta} \right)^2 \right] \sigma_\mu^2 \left(\frac{\sigma^2}{\alpha} \right)^2. \end{aligned}$$

We ensure that with the discretionary policy 40% of the variance is driven by noise shocks and the remaining is driven by fundamental shocks, $\sigma_{noise}^2 = 40\% \times \sigma^2$ and $\sigma_z^2 = 60\% \times \sigma^2$. This ensures that noise is a sizeable source of volatility while still allowing for a stable equilibrium.⁷ We also target a standard deviation of 10%, which implies total variance $\sigma^2 = 0.01$. In this calibration, total variance does not change the optimal degree of FCI targeting because it simply scales all other variances.

Finally, we consider a quarterly calibration and set the planner's discount rate to 1%, $\tilde{\beta} = 0.99$. The

⁷The highest noise share we can set without triggering destabilizing dynamics is $\sigma_{noise}^2 = 50\% \times \sigma^2$.

full set of parameters are given by

$$\begin{aligned}
\theta &= 2 \\
\sigma^2 &= 0.01 \\
\sigma_z^2 &= 0.006 \\
\sigma_{noise}^2 &= 0.004 \text{ and } \sigma_\mu^2 = 0.0072 \\
\rho_\mu &= \rho_z = 0 \\
\alpha &= 0.01 \\
\tilde{\beta} &= 0.99.
\end{aligned} \tag{A.1}$$

Parameters for Section 5. For Figure 3, we assume symmetric shocks and use the same parameters as before. In particular, we set

$$\sigma_{z,e}^2 = \sigma_{z,p}^2 = \sigma_z^2 \text{ and } \sigma_{\mu,e}^2 = \sigma_{\mu,p}^2 = \sigma_\mu^2.$$

We then assume σ_z^2, σ_μ^2 and other parameters are given by (A.1).

For Figure 4, we adjust the variance parameters $\sigma_{\mu,e}^2, \sigma_{\mu,p}^2$ across different scenarios as described in the figure, while keeping the other parameters unchanged.

A.2. Microfoundations of the model

In this section, we provide the microfoundations of the model that we summarize in Section 3.1. We describe the ingredients and derive the log-linearized equilibrium conditions (1 – 4) that we use in the main text. The setup adjusts the closed-economy model of Caballero et al. (2025b) to introduce the exchange rate and exports driven by an expenditure switching effects. We allow for noise and limits to arbitrage for both the domestic market portfolio and exchange rate. We focus on the determination of these prices and their impact on aggregate demand, and we make simplifying assumptions to keep the rest of the model tractable.

The economy is set in discrete time $t \in \{0, 1, \dots\}$. There are two currencies: domestic currency (peso) and foreign currency (dollar). We let E_t denote the exchange rate—the price of a dollar in pesos.

There are two goods: domestic tradeables and a world basket of tradeables. The price of the world basket in dollars is normalized to one. The price of domestic tradeables is sticky in domestic currency and normalized to one. Its price in dollars is $1/E_t$.

A.2.1. Supply side

The domestic economy produces only domestic tradeables. There are two factors, capital and labor, that produce output according to, $Y_t = A_t K^\nu L^{1-\nu}$. Capital supply is fixed and normalized to one, $K = 1$. Potential labor supply is also constant and given by L^* (determined by preferences and technology as described in the appendix). Therefore, potential output is $Y_t^* = A_t (L^*)^{1-\nu}$. Potential output evolves according to

$$\log Y_t^* = \log Y_{t-1}^* + z_t^p. \tag{A.2}$$

Because domestic firms' prices are sticky, actual output, Y_t , and labor supply, L_t , are endogenous and determined by aggregate demand for domestic tradeables. For simplicity, we assume prices are fully

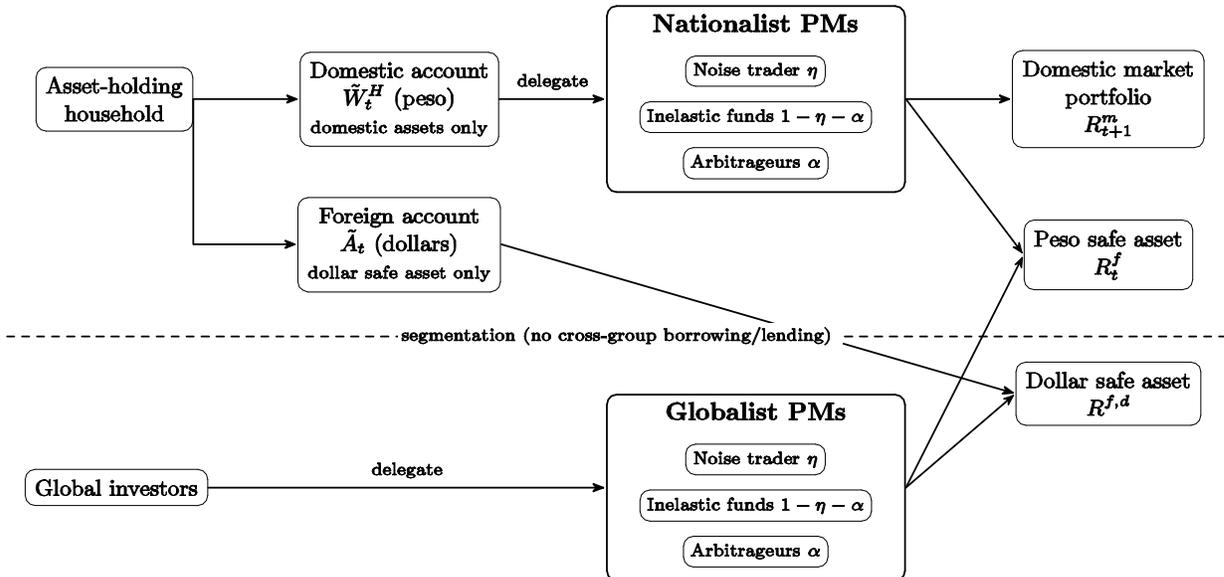


Figure 5: Conceptual diagram households' mental accounts (domestic vs. foreign) and the segmented portfolio management structure.

sticky and consider inflation in an extension. We also abstract away from the monopoly distortions of the New-Keynesian model and assume capital and labor receive their respective shares of output given by νY_t and $(1 - \nu) Y_t$.

Aggregate demand for domestic tradeables has two components:

$$Y_t = X_t + D_t. \quad (\text{A.3})$$

Here, X_t denotes foreign demand (exports in domestic currency) and D_t denotes domestic demand.

A.2.2. Demand side and financial markets

On the demand side, there are two types of domestic households: asset-holding households (households, for short) and hand-to-mouth agents. Our focus is on households who keep separate accounts: foreign and domestic (peso). They invest their foreign account in dollars and they delegate the investment of their domestic account to nationalist portfolio managers that choose between the domestic market portfolio and the peso risk-free asset. In equilibrium, nationalist portfolio managers determine the price of the market portfolio. There are also global investors who delegate their investment to globalist portfolio managers that choose between the dollar risk-free asset and the peso risk-free asset. In equilibrium, globalist portfolio managers determine the exchange rate. Figure 5 provides a schematic description of these key players and their portfolio managers. We next describe these ingredients in more detail.

Financial assets. There are three assets: domestic market portfolio, peso risk-free asset, and dollar risk-free asset.

Domestic market portfolio is a claim on the output share of domestic capital (νY_t). We denote its

ex-dividend price with P_t . Its endogenous return is then given by

$$R_{t+1}^m = \frac{\nu Y_{t+1} + P_{t+1}}{P_t}. \quad (\text{A.4})$$

Peso risk-free asset is in zero net supply (by definition of the market portfolio). The peso interest rate R_t^f is set by the central bank as we describe subsequently.

Dollar risk-free asset is in elastic supply. The dollar interest is determined in global financial markets and is constant for simplicity. To simplify the expressions, we assume the dollar interest rate is the inverse of the domestic discount factor, $R^{f,d} = 1/\beta$.

Foreign demand. Global consumers choose over individual-country tradable goods with a CES aggregator with elasticity of substitution. For simplicity, we take this elasticity to be one. We also assume global consumers' consumption is proportional to global output, denoted Y_t^g . Their consumption of domestic tradeables can then be written as:

$$X_t = \Gamma Y_t^g (1/E_t)^{-1} = \Gamma E_t Y_t^g. \quad (\text{A.5})$$

Here, Γ is a constant parameter. We assume global output follows a similar process as in (A.2)

$$\log Y_t^g = \log Y_{t-1}^g + z_t^g. \quad (\text{A.6})$$

Domestic demand. Hand-to-mouth agents: They receive the labor's share of output and spend all of their income. They consume both domestic tradeables and global tradeables according to preferences

$$\left(1 - \zeta^{HM}\right) \log C_t^{HM} + \zeta^{HM} \log I_t^{HM}.$$

Here, we denote the global tradeable spending with I_t^{HM} (imports). Hand-to-mouth agents spend a fixed fraction ζ^{HM} of their income on imports, which implies

$$E_t I_t^{HM} = \zeta^{HM} (1 - \nu) Y_t = \chi Y_t \text{ where } \chi = \zeta^{HM} (1 - \nu).$$

Here, the derived parameter χ captures hand-to-mouth spending on imports as a fraction of output. They spend the remaining fraction $1 - \zeta^{HM} = 1 - \frac{\chi}{1-\nu}$ of their income on the domestic tradable good. Therefore, their peso consumption of domestic good is:

$$C_t^{HM} = (1 - \nu - \chi) Y_t.$$

These agents help simplify expressions by ensuring that other households receive no labor income along the equilibrium path.

Asset-holding households (Households): These households hold financial assets but receive no labor income. They consume both domestic tradeables and foreign tradeables according to preferences

$$\sum_{h=0}^{\infty} \beta^h \left(\left(1 - \zeta^H\right) \log C_{t+h}^H + \zeta^H \log I_{t+h}^H \right). \quad (\text{A.7})$$

We assume these households are subject to a behavioral friction: they separate their domestic and foreign consumption into different “mental accounts.” Specifically, they hold two separate accounts to manage their domestic and foreign spending: a domestic account and a foreign account. The “domestic account” invests only in domestic assets and the foreign account invests only in foreign assets (the dollar safe asset). We let the beginning-of-period peso value of these accounts (inclusive of dividends and interest income) with W_t^H and $A_t E_t$ respectively (so A_t denotes the foreign account value in dollars). We let the end-of-period peso values with \tilde{W}_t^H and $\tilde{A}_t E_t$ (see Figure 5).

Given the two different accounts, households choose domestic consumption and imports according to the following rules

$$\begin{aligned} C_t^H &= (1 - \beta) W_t^H \\ I_t^H &= (1 - \beta) A_t. \end{aligned} \tag{A.8}$$

Intuitively, they allocate domestic assets to the domestic good and foreign assets to the foreign good. Within each category, they choose their consumption with a constant MPC out of wealth.⁸

The consumption rules in (A.8) provide analytical tractability by ensuring that foreign assets A_t do not affect the demand for domestic goods. In particular, combining the spending by households and hand-to-mouth agents, the demand for domestic goods by domestic agents is given by⁹

$$D_t = (1 - \nu - \chi) Y_t + (1 - \beta) W_t^H. \tag{A.9}$$

In practice, we expect the dependence of D_t on foreign assets A_t to be relatively small except for countries that have large net foreign asset or debt positions.

Given the households’ total consumption, their end-of-period peso wealth is given by

$$\tilde{W}_t^H + E_t \tilde{A}_t = \beta W_t^H + \beta E_t A_t.$$

Recall that \tilde{W}_t^H is the part of wealth invested in domestic assets. Since these households are the only agents that invest in the domestic assets, in equilibrium their domestic wealth is equal to the market portfolio

$$\begin{aligned} \tilde{W}_t^H &= P_t \\ E_t \tilde{A}_t &= \beta W_t^H + \beta E_t A_t - P_t = \beta E_t A_t + \beta W_t^H - P_t. \end{aligned} \tag{A.10}$$

Intuitively, in every period households “rebalance” their domestic and foreign portfolios to ensure their domestic wealth is equal to the domestic market portfolio. If their desired savings out of the domestic account exceeds the value of the market portfolio, $\beta W_t^H > P_t$ they move the difference to the foreign account (and vice versa).

⁸Observe that the preferences in (A.7) imply the rational rule that describes the *total* spending as constant MPC out of *total* assets, $C_t^H + I_t^H E_t = (1 - \beta) (W_t^H + A_t E_t)$. The households are boundedly rational in the sense that they follow a version of the rational rule but they apply it *within* each account.

⁹When the country runs net exports $NX_t > 0$ and its assets grow over time $A_{t+1} > A_t$, in the future they spend more on imports but not on the domestic good. This ensures we do not need to keep track of A_t as a state variable for D_t for our main results.

Investments and market segmentation. Given the different purposes of the domestic and foreign accounts, households also make different portfolio choices in each account (see Figure 5).

Foreign account. Since the foreign account will be spent on imports only, they simply invest this in the dollar safe asset, which implies

$$A_{t+1} = \tilde{A}_t R^{f,d}. \quad (\text{A.11})$$

Domestic account and the national PMs. In contrast, since the domestic account will be spent on domestic goods, households allocate this wealth between the market portfolio and the peso safe asset. They delegate this choice to national PMs. As we will describe subsequently, there are three types of national PMs. Their *total* assets under management in *pesos* evolves according to

$$W_{t+1}^H = \tilde{W}_t^H \left((1 - \bar{\omega}_t^m) R_t^f + \bar{\omega}_t^m R_{t+1}^m \right). \quad (\text{A.12})$$

Here, $\bar{\omega}_t^m$ denotes the aggregate portfolio weight on the market portfolio aggregated over all nationalist PMs. Recall also that in equilibrium we have $\tilde{W}_t^H = P_t$ (see (A.10)).

Global PMs. These portfolio managers invest on behalf of global investors that consume only the world basket of tradeables. They are restricted to choose between the dollar risk-free asset and the peso risk-free asset. Their assets under management *in dollars*, which corresponds to global investors wealth, evolves according to

$$W_{t+1}^{G,d} = \left(W_t^{G,d} - C_t^{G,d} \right) \left((1 - \bar{\omega}_t^d) \frac{R_t^f E_t}{E_{t+1}} + \bar{\omega}_t^d R^{f,d} \right). \quad (\text{A.13})$$

Here, $C_t^{G,d}$ denotes the dollar consumption of global investors (which does not play an important role) and $\frac{R_t^f E_t}{E_{t+1}}$ captures the dollar return from the peso risk-free asset.

Market segmentation and asset market clearing. We assume national PMs and global PMs collectively cannot borrow and lend across groups although they can borrow and lend within groups (see Figure 5). Therefore, we impose the following asset market clearing conditions

$$\bar{\omega}_t^m = \bar{\omega}_t^d = 1. \quad (\text{A.14})$$

National PMs are the only investors that can hold the market portfolio. In equilibrium, all of their wealth is invested back in the market portfolio (see also (A.10)). Likewise, global PMs invest their wealth back into dollars. Nonetheless, these investors' portfolio choice decisions determine the price of the domestic market portfolio P_t and the exchange rate E_t , respectively. In equilibrium, P_t and E_t adjusts to ensure that the national and global PMs find it optimal to choose $\bar{\omega}_t^m = 1$ and $\bar{\omega}_t^d = 1$, respectively.

Goods market clearing. Note that Eq. (A.12) and (A.4) along with $\tilde{W}_t^H = P_t$ and $\bar{\omega}_t^m = 1$ implies

$$W_{t+1}^H = P_t \frac{\nu Y_{t+1} + P_{t+1}}{P_t} = \nu Y_{t+1} + P_{t+1}.$$

In equilibrium, the beginning-of-period domestic wealth corresponds to the capital income plus the continuation value of the market portfolio. Substituting this into (A.9) and using (A.3), we obtain the goods market clearing condition

$$Y_t = \Gamma E_t Y_t^g + (1 - \nu - \chi) Y_t + (1 - \beta) (\nu Y_t + P_t).$$

After rearranging terms, we obtain the output asset price relation

$$Y_t = \frac{1}{\beta\nu + \chi} (\Gamma E_t Y_t^g + (1 - \beta)P_t). \quad (\text{A.15})$$

Here, $\frac{1}{\beta\nu + \chi}$ denotes a Keynesian multiplier. The equation captures two types of transmission from financial markets to the real economy: a depreciation of the exchange rate, which increases exports, and an increase in the price of the market portfolio, which increases demand for domestic goods.

We also derive the condition that determines the evolution of external assets. To this end, first observe that the goods market clearing condition implies

$$\beta\nu Y_t = (1 - \beta)P_t + \Gamma E_t Y_t^g - \chi Y_t.$$

Next we combine this with Eq. (A.10) to obtain

$$\begin{aligned} E_t \tilde{A}_t &= \beta E_t A_t + \beta (\nu Y_t + P_t) - P_t \\ &= E_t A_t + \underbrace{\Gamma E_t Y_t^g - \chi Y_t - (1 - \beta) E_t A_t}_{NX_t}. \end{aligned}$$

After substituting $A_{t+1} = \tilde{A}_t R^{f,d} = \tilde{A}_t / \beta$ (see (A.11)) and simplifying, we obtain the capital account dynamics

$$\beta A_{t+1} = \beta A_t + \Gamma Y_t^g - \frac{\chi Y_t}{E_t}. \quad (\text{A.16})$$

In equilibrium, positive net exports increase the external asset position and raise future imports due to households' spending rule out of external assets. In particular, all else equal, a depreciation of the currency (an increase in E_t) raises the dollar value of net exports and raise foreign assets, and vice versa for an appreciation of the currency.

Benchmark without uncertainty. To facilitate the analysis, we next characterize a benchmark steady-state equilibrium and log linearize (A.4) and (A.15) around this steady state.

Suppose there are no domestic or global shocks, $z_t = 0$ and $z_t^g = 0$, so potential output is constant $Y^* = \bar{Y}$, global output is constant, $Y_t^g \equiv \bar{Y}^g$, and exports depend only on the exchange rate $X_t = \Gamma \bar{Y}^g E_t$ (see (A.2) and (A.6)). Suppose also there is no financial noise or other source of return uncertainty and monetary policy keeps output at potential, $Y_t = \bar{Y}$. Then, we conjecture a steady state in which external assets remain constant at their initial level, $A_t = A_0 = \bar{A}$ and all asset returns are equal to one another

$$R_{t+1}^m = R_t^f = R^{f,d} = 1/\beta.$$

Combining this conjecture with (A.4) and (A.15), along with $Y_t = \bar{Y}$, we obtain

$$\begin{aligned} P_t &= \bar{P} = \frac{\nu \bar{Y}}{R^{f,d} - 1} = \frac{\beta \nu \bar{Y}}{1 - \beta} \\ X_t &= \bar{X} = \chi \bar{Y} \quad \text{and} \quad E_t = \bar{E} = \frac{\chi \bar{Y}}{\Gamma \bar{Y}^g}. \end{aligned} \quad (\text{A.17})$$

Substituting this into (A.16), we also verify that A_t remains constant at $A_0 = \bar{A}$.

Absent uncertainty, the price of the market portfolio is determined by a standard Gordon formula

with the discount rate determined by the risk-free rate. The exchange rate then adjusts to ensure exports are at the level required by the goods market clearing condition.

Log-linearized equilibrium conditions. To analyze the equilibrium with financial noise, we log-linearize Eq. (A.15) around the steady state $Y_t = \bar{Y}$ to obtain the approximate output-FCI relation

$$\begin{aligned} y_t &= f_t = (1 - \phi)p_t + \phi(e_t + y_t^g) \\ \text{where } f_t &= (1 - \phi)p_t + \phi e_t \text{ and } \phi = \frac{\chi}{\chi + \beta\nu}. \end{aligned} \quad (\text{A.18})$$

Here, $y_t = \log \frac{Y_t}{\bar{Y}}$, $y_t^g = \log \frac{Y_t^g}{\bar{Y}^g}$, $p_t = \log \frac{P_t}{\bar{P}}$, $e_t = \log \frac{E_t}{\bar{E}}$, denote the log deviations of domestic output, global output, the domestic market portfolio, and the exchange rate. The derived parameter ϕ captures the effect of a depreciation on output after incorporating the Keynesian multiplier effects. The variable f_t is the model counterpart to an open-economy financial conditions index: it describes how the domestic market portfolio and the exchange rate affect aggregate demand and output.

We further log-linearize (A.4) to obtain the approximate return relation

$$r_{t+1}^m = \rho + (1 - \beta)y_{t+1} + \beta p_{t+1} - p_t. \quad (\text{A.19})$$

Here, $r_{t+1}^m = \log R_{t+1}^m$ denotes the log return and $\rho \equiv \log 1/\beta$ denotes the steady state log return. The weight $1 - \beta = \frac{\nu\bar{Y}}{\nu\bar{Y} + \bar{P}}$ reflects the steady state dividend-to-price ratio in this economy. This is the Campbell-Shiller approximation in this model.

Finally, we define $y_t^* = \log (Y_t^*/\bar{Y})$ to denote the log deviation of potential output from its steady-state level and observe that Eq. (A.2) implies

$$y_t^* = y_{t-1}^* + z_t^p. \quad (\text{A.20})$$

Likewise, given $y_t^g = \log \frac{Y_t^g}{\bar{Y}^g}$, observe that Eq. (A.6) implies

$$y_t^g = y_{t-1}^g + z_t^g. \quad (\text{A.21})$$

Once we define $e_t^* = y_t^* - y_t^g$, these equations also imply

$$e_t^* = e_{t-1}^* + z_t^e \text{ where } z_t^e = z_t - z_t^g.$$

This verifies the macroeconomic shock processes described in (6).

Financial noise and limits to arbitrage. We next introduce the main ingredients of the model: financial noise and arbitrageurs with risk limits. We introduce these ingredients *within* both Globalist and Nationalist PMs. While Globalist and Nationalist groups in the aggregate cannot trade with one another, subgroups of investors within each group can trade with one another. This keeps the model tractable while introducing limited arbitrage pricing for both the exchange rate and the price of the market portfolio.

Specifically, within each group of PMs, there is a mass η of “noise traders,” a mass $(1 - \eta - \alpha)$ of “inelastic funds” with portfolio weights equal to the group average, and a mass α of “arbitrageurs” who choose their portfolio weight to maximize expected log assets-under-management and trade against the

noise (see Figure 5). We next describe these investors' positions and derive the equations that determine the log asset prices e_t and p_t , respectively.

Global Portfolio Managers and the UIP. *Inelastic funds.* Their portfolio weight on the dollar risk-free asset (relative to peso) is equal to the group average, $\omega_t^{i,d} = \bar{\omega}_t^d = 1$.

Noise traders. Their portfolio weight is given by $\omega_t^{n,d} = 1 + \frac{\mu_t^e}{\eta}$; in particular, their weight deviates from the group average by an amount $\frac{\mu_t^e}{\eta}$. This results in an excess demand for dollars relative to pesos equal to μ_t^e —measured in terms of globalists' average portfolio weight. We refer to μ_t^e as the exchange rate noise.

Arbitrageurs (Arbs). They choose their portfolio weight to maximize the log of the assets under management in (A.13). This implies the optimization problem

$$\max_{1-\omega_t^{a,d}} E_t \left[\log \left(R^{f,d} + (1 - \omega_t^{a,d}) \left(\frac{R_t^f E_t}{E_{t+1}} - R^{f,d} \right) \right) \right].$$

The first order condition is

$$E_t \left[\frac{R_t^f \frac{E_t}{E_{t+1}} - R_t^{f,d}}{R^{f,d} + (1 - \omega_t^{a,d}) \left(\frac{R_t^f E_t}{E_{t+1}} - R^{f,d} \right)} \right] = 0.$$

Assuming the exchange rate and the portfolio return approximately follow a jointly lognormal distribution, we obtain the optimality condition

$$(r_t^f - r^{f,d}) - E_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 = (1 - \omega_t^{a,d}) \sigma_e^2.$$

Here, $E_t [\Delta e_{t+1}] = E_t [e_{t+1}] - e_t$ is the expected exchange rate depreciation and $\sigma_e^2 \equiv \text{Var}_t(e_{t+1})$ is the conditional volatility of the log exchange rate. The left-side captures the expected excess return of the peso safe asset relative to the dollar asset, measured in dollars. The equation says that Arbs tilt their portfolio toward the peso safe asset, $1 - \omega_t^{a,d} > 0$, only if the peso safe asset generates a sufficiently large excess return.

Market clearing . Collectively, Globalist PMs hold no peso safe assets so their portfolio weights satisfy

$$\alpha \omega_t^{a,d} + \eta (1 + \mu_t^e / \eta) + (1 - \alpha - \eta) = \bar{\omega}_t^d = 1 \implies \omega_t^{a,d} = 1 - \frac{\mu_t^e}{\alpha}.$$

When noise traders' demand for dollar safe assets rises, the Arbs take the other side of these positions in equilibrium.

Combining this with the Arbs' optimality condition, we obtain the modified UIP condition:

$$(r_t^f - r^{f,d}) - E_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 = \frac{\mu_t^e}{\alpha} \sigma_e^2.$$

Rearranging this condition, and using our assumption that $R^{f,d} = 1/\beta$, which implies $r^{f,d} = \rho = \log 1/\beta$,

we also obtain an expression for the exchange rate

$$\begin{aligned} e_t &= \rho - r_t^f + E_t[e_{t+1}] - \frac{1}{2}\sigma_e^2 + \frac{\mu_t^e}{\alpha}\sigma_e^2 \\ &= \rho + E_t[e_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma_e^2\right) + \frac{\mu_t^e}{\alpha}\sigma_e^2. \end{aligned} \quad (\text{A.22})$$

Intuitively, when noise traders demand more dollar bonds, the peso depreciates to entice arbitrageurs to absorb the excess supply of peso bonds. Importantly, the price impact of noise on the exchange rate depends on *the volatility of the exchange rate*. This is because the Arbs' required excess return to absorb noise rises with volatility.

National Portfolio Managers and the Price of the Market Portfolio. *Inelastic funds.* Their portfolio weight on the market portfolio (relative to the safe asset) is equal to the group average, $\omega_t^{i,m} = \bar{\omega}_t^m = 1$.

Noise traders. Their portfolio weight is given by $\omega_t^{n,m} = 1 + \frac{\mu_t^p}{\eta}$; in particular, their weight deviates from the group average by an amount $\frac{\mu_t^p}{\eta}$. This results in an excess demand for the market portfolio of μ_t^p —measured in terms of nationalists' average portfolio weight. We refer to μ_t^p as the market noise.

Arbitrageurs (Arbs). They choose their portfolio weight to maximize the log of the assets under management (A.12). This implies the optimization problem

$$\max_{\omega_t^{a,m}} E_t \left[\log \left(R_t^f + \omega_t^{a,m} \left(R_{t+1}^m - R_t^f \right) \right) \right].$$

The first order condition is given by

$$E_t \left[\frac{R_{t+1}^m - R_t^f}{R_t^f + \omega_t^{a,m} \left(R_{t+1}^m - R_t^f \right)} \right] = 0.$$

Assuming the market portfolio and the portfolio return approximately follow a jointly lognormal distribution, we obtain the optimality condition

$$E_t [r_{t+1}^m] - r_t^f + \frac{1}{2}\sigma_m^2 = \omega_t^{a,m}\sigma_m^2,$$

where $\sigma_m^2 = \text{Var}_t(r_{t+1}^m)$ is the conditional volatility of the market portfolio return.

Market clearing . Collectively, Nationalist PMs hold no peso safe assets so their portfolio weights satisfy

$$\alpha\omega_t^{a,m} + \eta \left(1 + \frac{\mu_t^p}{\eta}\right) + (1 - \alpha - \eta) = \bar{\omega}_t^m = 1 \implies \omega_t^{a,m} = 1 - \frac{\mu_t^p}{\alpha}.$$

When noise traders' demand for the market portfolio rises, the Arbs take the other side of these positions in equilibrium.

Combining this with the Arbs' optimality condition, we obtain the equilibrium condition

$$E_t [r_{t+1}^m] = r_t^f + \frac{1}{2}\sigma_m^2 - \frac{\mu_t^p}{\alpha}\sigma_m^2.$$

In equilibrium, the excess return on the market portfolio relative to the safe asset—the risk premium—depends on the variance σ_m^2 as well as on the noisy demand for the market portfolio. Substituting the expression for the return from (A.19), we further obtain

$$p_t = \rho + (1 - \beta) E_t [y_{t+1}] + \beta E_t [p_{t+1}] - \left(r_t^f + \frac{1}{2} \sigma_m^2 \right) + \frac{\mu_t^p}{\alpha} \sigma_m^2. \quad (\text{A.23})$$

Here, recall that y_t and p_t denote log deviations from the no-uncertainty steady state. Intuitively, when noise trader demand for the market portfolio increases, so does the price of the market portfolio. Importantly, the price impact of noise on the market portfolio depends on the *market return volatility* σ_m^2 . This is because the excess return the Arbs require to absorb noise rises with volatility.

This completes the derivation of the log-linearized equilibrium conditions that we use in the main text. In particular, note that Eqs. (A.18), (A.22), (A.23), (A.19) correspond to Eqs. (1–4) in the main text. We solve these equations with the shock processes described in (6).

A.3. Omitted proofs

This section contains the proofs omitted from the main text.

Proof of Proposition 1. We first prove that the policy optimally follows the rule (11). The central bank solves

$$L_t = \min_{r_t^f} (y_t - y_t^*)^2 + \frac{1}{\theta} \left(r_t^f - E_{t-1} [r_t^f] \right)^2 + \tilde{\beta} E_t [L_{t+1}].$$

The first order condition is given by

$$r_t^f = E_{t-1} [r_t^f] - \frac{\partial y_t}{\partial r_t^f} \theta (y_t - y_t^*).$$

Recall that $y_t - y_t^* = f_t - f_t^*$ where $f_t = \frac{e_t + p_t}{2}$. Eqs. (8) and (9) both imply $\frac{\partial p_t}{\partial r_t^f} = \frac{\partial e_t}{\partial r_t^f} = -1$. This in turn implies $\frac{\partial y_t}{\partial r_t^f} = -1$. Substituting this into the first order condition establishes that monetary policy follows the rule (11)

$$r_t^f = E_{t-1} [r_t^f] + \theta (y_t - y_t^*).$$

The equilibrium corresponds to allocations that satisfy this rule along with the equations in (7–9) given the shocks in (6).

We next conjecture (and verify) an equilibrium in which the variance is the same in the two markets, $\sigma_e^2 = \sigma_p^2 = \sigma^2$, and the expected output and asset prices are centered around their corresponding ideal levels

$$E_t [y_{t+1}] = E_t [p_{t+1}] = E_t [y_{t+1}^*] = y_t^* \text{ and } E_t [e_{t+1}] = E_t [e_{t+1}^*] = e_t^*. \quad (\text{A.24})$$

To characterize these equilibria, first observe that Eqs. (8) and (9) along with the assumption that e_t^* and y_t^* follow random walks imply

$$e_t - e_t^* = E_t [e_{t+1} - e_{t+1}^*] - \left(r_t^f + \frac{1}{2} \sigma^2 \right) + \frac{\mu_t^e}{\alpha} \sigma^2 \quad (\text{A.25})$$

$$p_t - y_t^* = E_t [p_{t+1} - y_{t+1}^*] - \left(r_t^f + \frac{1}{2} \sigma^2 \right) + \frac{\mu_t^p}{\alpha} \sigma^2. \quad (\text{A.26})$$

Aggregating and using (7), we obtain

$$\begin{aligned} f_t - f_t^* &= E_t [f_{t+1} - f_{t+1}^*] - \left(r_t^f + \frac{1}{2} \sigma^2 \right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \\ \implies \tilde{y}_t &= E_t [\tilde{y}_{t+1}] - \left(r_t^f + \frac{1}{2} \sigma^2 \right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}. \end{aligned} \quad (\text{A.27})$$

Substituting $E_t [\tilde{y}_{t+1}] = 0$ from (A.24), we further obtain

$$\tilde{y}_t = - \left(r_t^f + \frac{1}{2} \sigma^2 \right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}. \quad (\text{A.28})$$

Taking the ex-ante expectation in period $t - 1$, we further obtain

$$E_{t-1} [r_t^f] = -\frac{1}{2} \sigma^2.$$

Substituting this back into (A.28), we obtain

$$\tilde{y}_t = - \left(r_t^f - E_{t-1} [r_t^f] \right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}.$$

Combining this with (11), we solve

$$\begin{aligned} \tilde{y}_t &= \frac{1}{1+\theta} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \\ r_t^f &= -\frac{1}{2} \sigma^2 + \frac{\theta}{1+\theta} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}. \end{aligned}$$

This establishes Eqs. (12) and (13).

Next consider the exchange rate and the price of the market portfolio. Substituting r_t^f into (A.25) and using (A.24), we obtain

$$e_t = e_t^* - \frac{\theta}{1+\theta} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} + \mu_t^e \frac{\sigma^2}{\alpha}.$$

After rearranging terms, this proves Eq. (14). Likewise, substituting r_t^f into (A.26) and using (A.24), we obtain

$$p_t = y_t^* - \frac{\theta}{1+\theta} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} + \frac{\mu_t^p}{\alpha} \sigma^2.$$

This proves Eq. (15).

Next note that these expressions also verify the conjecture in (A.24). It remains to characterize the endogenous variances σ_e^2 and σ_p^2 . Taking the conditional variance of either e_t or p_t , and using the observation that shocks are i.i.d., the variance solves the quadratic

$$\begin{aligned} \sigma^2 &= \sigma_z^2 + \nu_\mu \left(\frac{\sigma^2}{\alpha} \right)^2 \\ \text{where } \nu_\mu &= \text{var} \left(\left(1 - \frac{\theta/2}{1+\theta} \right) \mu_t^e - \frac{\theta/2}{1+\theta} \mu_t^p \right) \end{aligned}$$

In particular, the variance is a root of a quadratic, $P(\sigma^2) = 0$, given by

$$\begin{aligned}
P(x) &= \frac{\nu_\mu}{\alpha^2} x^2 - x + \sigma_z^2 & (A.29) \\
\text{where } \nu_\mu &= f\left(\frac{\theta/2}{1+\theta}|\rho\right) \sigma_\mu^2 \\
\text{with } f(x|\rho) &= (1-x)^2 + x^2 - 2(1-x)x\rho.
\end{aligned}$$

As long as the parameters satisfy $\alpha^2 > 4\sigma_\mu^2\sigma_z^2$, this polynomial has two positive roots for any θ and ρ_μ . The larger root is unstable in the sense that small changes in volatility induce further changes in volatility that move the equilibrium away from this point. The smaller root corresponds to a stable equilibrium. This verifies that, for either the exchange rate e_t or the market price p_t , the equilibrium conditional variance is the smaller solution to the fixed point equation in (16). This in turn verifies that these variances are the same, $\sigma_e^2 \equiv \sigma_p^2 \equiv \sigma^2$, completing the characterization of equilibrium.

It remains to characterize the comparative statics of the equilibrium variance. Observe that $P(x)$ in (A.29) corresponds to an upward-sloping parabola with two positive roots. Observe also that increasing σ_z^2 or σ_μ^2 increases $P(x)$ for each x and therefore lifts the parabola upward. Therefore, increasing σ_z^2 or σ_μ^2 increases the smaller root (while reducing the larger root). Since the equilibrium variance σ^2 corresponds to the smaller root, increasing σ_z^2 or σ_μ^2 increases σ^2 . For the comparative statics with respect to θ , observe that the function $f(x)$ in (A.29) satisfies

$$f'(x|\rho_\mu) = (4x - 2)(1 + \rho_\mu) < 0 \text{ for each } x < 1/2 \text{ given } \rho_\mu > -1.$$

This in turn implies that $f\left(\frac{\theta/2}{1+\theta}|\rho_\mu\right)$ is decreasing in θ for each $\theta \geq 0$. This reduces the smaller root of $P(x)$, proving that increasing θ reduces σ^2 . \square

Proof of Proposition 2. We first assume that in each period $t - 1$ the central bank announces the target $\bar{f}_t = E_{t-1}[f_t^*] = f_{t-1}^*$ and show the allocations satisfy (19 – 23). We then show that it is optimal to announce $\bar{f}_t = E_{t-1}[f_t^*] = f_{t-1}^*$.

To characterize the allocations, we start by proving that the policy optimally follows the rules (18) and (24). The central bank solves

$$L_t = \min_{r_t^f} (y_t - y_t^*)^2 + \frac{1}{\theta} \left(r_t^f - E_{t-1} \left[r_t^f \right] \right)^2 + \frac{\psi}{\theta} (f_t - E_{t-1} [f_t^*])^2 + \tilde{\beta} E_t [L_{t+1}].$$

The first order condition is given by

$$r_t^f = E_{t-1} \left[r_t^f \right] - \frac{\partial y_t}{\partial r_t^f} \theta (y_t - y_t^*) - \frac{\partial f_t}{\partial r_t^f} \psi (f_t - E_{t-1} [f_t^*]).$$

Recall that $y_t - y_t^* = f_t - f_t^*$. Eqs. (8) and (9) both imply $\frac{\partial p_t}{\partial r_t^f} = \frac{\partial e_t}{\partial r_t^f} = -1$. This in turn implies $\frac{\partial f_t}{\partial r_t^f} = \frac{\partial y_t}{\partial r_t^f} = -1$. Substituting this into the first order condition establishes that monetary policy follows the rule in (18)

$$\begin{aligned}
r_t^f &= E_{t-1} \left[r_t^f \right] + \theta (y_t - y_t^*) + \psi (f_t - E_{t-1} [f_t^*]) \\
&= E_{t-1} \left[r_t^f \right] + (\theta + \psi) (y_t - y_t^*) + \psi (f_t^* - E_{t-1} [f_t^*]).
\end{aligned}$$

Here, the second line substitutes $y_t - y_t^* = f_t - f_t^*$. After observing $f_t^* = \frac{e_t^* + y_t^*}{2}$ and using (6), we further obtain (24)

$$r_t^f = E_{t-1} \left[r_t^f \right] + (\theta + \psi) (y_t - y_t^*) + \psi \frac{z_t^e + z_t^p}{2}.$$

The remaining steps closely follow the proof of Proposition 1. We construct equilibria in which the variance is the same in the two markets, $\sigma_e^2 = \sigma_p^2 = \sigma^2$, and the expectations satisfy (A.24). Then, the output gap still satisfies the recursive equation (A.27). Using (A.24), this further implies

$$y_t - y_t^* = - \left(r_t^f + \frac{1}{2} \sigma^2 \right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}.$$

Taking the ex-ante expectation in period $t - 1$, we further obtain $E_{t-1} \left[r_t^f \right] = -\frac{1}{2} \sigma^2$. Substituting this back into the previous equation, we obtain

$$y_t - y_t^* = - \left(r_t^f - E_{t-1} \left[r_t^f \right] \right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}.$$

Combining this with (24), we solve

$$\begin{aligned} y_t - y_t^* &= - \frac{\psi}{1 + \theta + \psi} \frac{z_t^e + z_t^p}{2} + \frac{1}{1 + \theta + \psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} \\ r_t^f &= - \frac{1}{2} \sigma^2 + \frac{\psi}{1 + \theta + \psi} \frac{z_t^e + z_t^p}{2} + \frac{\theta + \psi}{1 + \theta + \psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}. \end{aligned}$$

This establishes Eqs. (19) and (20).

Next consider the exchange rate and the price of the market portfolio. Note that Eq. (A.25) still holds. Substituting r_t^f and Eq. (A.24) into this, we obtain

$$e_t - e_t^* = - \frac{\psi}{1 + \theta + \psi} \frac{z_t^e + z_t^p}{2} - \frac{\theta + \psi}{1 + \theta + \psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} + \frac{\mu_t^e}{\alpha} \sigma_e^2.$$

After rearranging terms, this proves Eq. (21). Likewise, Eq. (A.26) still holds. Substituting r_t^f and Eq. (A.24) into this, we obtain

$$p_t - y_t^* = - \frac{\psi}{1 + \theta + \psi} \frac{z_t^e + z_t^p}{2} - \frac{\theta + \psi}{1 + \theta + \psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} + \frac{\mu_t^p}{\alpha} \sigma_p^2.$$

This proves Eq. (22).

Next note that these expressions verify the conjecture in (A.24). It remains to characterize the endogenous variances σ_e^2 and σ_p^2 . Taking the conditional variance of either e_t or p_t , and using the observation that shocks are independent, the variance solves the quadratic quadratic in (23)

$$\sigma^2 = \nu_z + \nu_\mu \left(\frac{\sigma^2}{\alpha} \right)^2 \quad \text{where} \quad \begin{aligned} \nu_z &= \text{var} \left(\frac{1 + \theta + \psi/2}{1 + \theta + \psi} z_t^e - \frac{\psi/2}{1 + \theta + \psi} z_t^p \right) \\ \nu_\mu &= \text{var} \left(\frac{1 + (\theta + \psi)/2}{1 + \theta + \psi} \mu_t^e - \frac{(\theta + \psi)/2}{1 + \theta + \psi} \mu_t^p \right). \end{aligned}$$

This corresponds to a zero of the following polynomial

$$\begin{aligned}
P(x|\psi) &= \frac{\nu_\mu}{\alpha^2}x^2 - x + \nu_z & (A.30) \\
\text{where } \nu_z &= f\left(\frac{\psi/2}{1+\theta+\psi}|\rho_z\right)\sigma_z^2 \\
\nu_\mu &= f\left(\frac{(\theta+\psi)/2}{1+\theta+\psi}|\rho_\mu\right)\sigma_\mu^2 \\
\text{with } f(x|\rho) &= (1-x)^2 + x^2 - 2(1-x)x\rho.
\end{aligned}$$

As long as the parameters satisfy $\alpha^2 > 4\sigma_\mu^2\sigma_z^2$, this polynomial has two positive roots for any $\theta, \psi, \rho_\mu, \rho_z$. As before, the solution corresponds to the smaller root. This verifies that the variance of e_t and p_t are the same and completes the characterization of equilibrium allocations.

We next verify that the optimal target announcement in each period is $\bar{f}_t = E_{t-1}[f_t^*] = f_{t-1}^*$. Suppose the equilibrium allocations are given by (19–23) for all periods t and consider a deviation in period $t-1$ where the planner sets an arbitrary target \bar{f}_t . We characterize the resulting allocations in period t given \bar{f}_t and show that the central bank finds it optimal to set $\bar{f}_t = f_{t-1}^*$.

Given the inherited target \bar{f}_t , in period t the optimality conditions still imply the following version of the policy rule (24)

$$\begin{aligned}
r_t^f &= E_{t-1}\left[r_t^f\right] + \theta(y_t - y_t^*) + \psi(f_t - \bar{f}_t) \\
&= -\frac{1}{2}\sigma^2 + (\theta + \psi)(y_t - y_t^*) + \psi(f_t^* - \bar{f}_t).
\end{aligned}$$

Here, the second line substitutes $E_{t-1}\left[r_t^f\right] = -\frac{1}{2}\sigma^2$. Next note that from time t onward, Eqs. (A.27) and (A.24) still hold at time t . Combining these observations, we obtain

$$y_t - y_t^* = -\left(r_t^f + \frac{1}{2}\sigma^2\right) + \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha}.$$

Substituting the interest rate from the policy rule, this further implies

$$y_t - y_t^* = \frac{1}{1+\theta+\psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} - \frac{\psi}{1+\theta+\psi} (f_t^* - \bar{f}_t).$$

Substituting this along with the interest rate into the central bank objective, we calculate the central bank's operational loss as

$$\begin{aligned}
L_t^{FCI}(\bar{f}_t) &= \left(\frac{1}{1+\theta+\psi} \frac{\mu_t^e + \mu_t^p}{2} \frac{\sigma^2}{\alpha} - \frac{\psi}{1+\theta+\psi} (f_t^* - \bar{f}_t)\right)^2 \\
&\quad + \frac{1}{\theta} \left((\theta + \psi)(y_t - y_t^*) + \psi(f_t^* - \bar{f}_t)\right)^2 + \tilde{\beta} E_t[L_{t+1}^{FCI}]. & (A.31) \\
&\quad + \frac{\psi}{\theta} \left(\underbrace{y_t - y_t^* + f_t^*}_{f_t} - \bar{f}_t\right)^2
\end{aligned}$$

At the ex-ante period $t-1$, the central bank announces \bar{f}_t that minimizes $E_{t-1}[L_t^{FCI}(\bar{f}_t)]$. Since $E_{t-1}[y_t - y_t^*] = 0$ and $E_{t-1}\left[\frac{\mu_t^e + \mu_t^p}{2}\right] = 0$, it is easy to check that Eq. (A.31) implies that the solution

satisfies

$$E_{t-1} [f_t^* - \bar{f}_t] = 0 \implies \bar{f}_t = E_{t-1} [f_t^*] = f_{t-1}^*.$$

The optimal FCI target is equal to the expected FCI-star in the next period. Deviating from this announcement creates an average bias that raises gaps and policy losses.

It remains to establish the comparative statics for the variance. Consider the polynomial $P(x|\psi)$ in (A.30). Observe that $f'(x|\rho) < 0$ for each $x < 1/2$ as long as $\rho > -1$. Thus, increasing ψ strictly reduces both $f\left(\frac{(\theta+\psi)/2}{1+\theta+\psi}|\rho_\mu\right)$ and $f\left(\frac{\psi/2}{1+\theta+\psi}|\rho_z\right)$. This in turn reduces $P(x|\psi)$ for each x . Since σ^2 is the smaller root of this polynomial, increasing ψ reduces σ^2 . This completes the proof. \square

Proof of Proposition 3. Differentiating the expressions in (28), we obtain

$$\begin{aligned} \frac{dL_z^e(\psi)}{d\psi} &= \frac{1}{1-\tilde{\beta}} \sigma_z^2 \frac{1+\rho_z}{2} \left(1 + \frac{1}{\theta}\right) 2 \frac{\psi(1+\theta)}{(1+\theta+\psi)^3}. \\ \frac{dl_\mu(\theta, \psi)}{d\psi} &= \frac{1}{1-\tilde{\beta}} \sigma_\mu^2 \frac{1+\rho_\mu}{2} \frac{2}{(1+\theta+\psi)^3} \frac{\psi}{\theta} \end{aligned}$$

These expressions show that $\frac{dL_z^e(\psi)}{d\psi} \geq 0$, $\frac{dl_\mu(\theta, \psi)}{d\psi} \geq 0$ with $\frac{dL_z^e(\psi)}{d\psi}\Big|_{\psi=0} = \frac{dl_\mu(\theta, \psi)}{d\psi}\Big|_{\psi=0} = 0$. Proposition 2 shows $\frac{d\sigma^2(\psi)}{d\psi} < 0$. Combining these observations, we obtain

$$\frac{dL^e}{d\psi}\Big|_{\psi=0} = \underbrace{\frac{dL_z^e(\psi)}{d\psi}}_{=0} + \underbrace{\left(\frac{\sigma^2}{\alpha}\right)^2 \frac{dl_\mu(\theta, \psi)}{d\psi}}_{>0} + \underbrace{\frac{d(\sigma^2(\psi)/\alpha)^2}{d\psi}}_{<0} \underbrace{l_\mu(\psi)}_{>0} < 0.$$

This further implies $\psi^* = \arg \min_{\psi \geq 0} L^e(\psi) > 0$, completing the proof. \square

Proof of Proposition 4. As before, we take the optimal target as given $\bar{e}_t = E_{t-1}[e_t]$ and solve for the resulting equilibrium. We then verify that announcing this target is optimal.

In this case, we conjecture (and verify) that the exchange rate and the asset price satisfy

$$e_t = \frac{\kappa^0}{\phi} + e_t^* + \kappa_e^{z,e} z_t^e + \kappa_e^{z,p} z_t^p + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha} \quad (\text{A.32})$$

$$p_t = -\frac{\kappa^0}{1-\phi} + y_t^* + \kappa_p^{z,e} z_t^e + \kappa_p^{z,p} z_t^p + \kappa_p^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_p^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha}. \quad (\text{A.33})$$

Here, $\kappa^0, \kappa_e^{z,e}, \kappa_e^{\mu,e}, \kappa_e^{\mu,p}, \kappa_p^{z,e}, \kappa_p^{z,p}, \kappa_p^{\mu,e}, \kappa_p^{\mu,p}$ are coefficients to be determined. We impose some structure on these coefficients by: (i) by conjecturing that $\kappa_e^{z,e} = \kappa_p^{z,e} \equiv \kappa^{z,e}$ and $\kappa_e^{z,p} = \kappa_p^{z,p} \equiv \kappa^{z,p}$ —since macroeconomic shocks will make asset prices deviate from their natural levels only because of a common policy response, (ii) by choosing the intercepts to ensure $E_{t-1}[y_t] = y_t^*$. In particular, since $f_t = \phi e_t + (1-\phi)p_t$ and $y_t - y_t^* = f_t - f_t^*$, these conjectures also imply that the output gap satisfies

$$y_t - y_t^* = \begin{bmatrix} (\phi \kappa_e^{z,e} + (1-\phi) \kappa_p^{z,e}) z_t^e \\ + (\phi \kappa_e^{z,p} + (1-\phi) \kappa_p^{z,p}) z_t^p \end{bmatrix} + \begin{bmatrix} (\phi \kappa_e^{\mu,e} + (1-\phi) \kappa_p^{\mu,e}) \mu_t^e \frac{\sigma_e^2}{\alpha} \\ + (\phi \kappa_e^{\mu,p} + (1-\phi) \kappa_p^{\mu,p}) \mu_t^p \frac{\sigma_m^2}{\alpha} \end{bmatrix}. \quad (\text{A.34})$$

Output is still centered around its potential on average but it can deviate due to noise shocks as well as a policy response to macroeconomic shocks.

We next solve for the intercept coefficient κ^0 . To this end, observe that the exchange rate and the

market price still satisfy the following versions of (A.25) and (A.26)

$$e_t - e_t^* = \rho + E_t [e_{t+1} - e_{t+1}^*] - \left(r_t^f + \frac{1}{2} \sigma_e^2 \right) + \frac{\mu_t^e}{\alpha} \sigma_e^2 \quad (\text{A.35})$$

$$p_t - y_t^* = \rho + \left(\begin{array}{c} (1 - \beta) E_t [y_{t+1} - y_{t+1}^*] \\ + \beta E_t [p_{t+1} - y_{t+1}^*] \end{array} \right) - \left(r_t^f + \frac{1}{2} \sigma_m^2 \right) + \frac{\mu_t^p}{\alpha} \sigma_m^2. \quad (\text{A.36})$$

Taking the ex-ante expectation, and using the conjectures, we obtain

$$\begin{aligned} E_{t-1} [e_t - e_t^*] &= \rho + \frac{\kappa^0}{\phi} - E_{t-1} [r_t^f] - \frac{1}{2} \sigma_e^2 \\ E_{t-1} [p_t - y_t^*] &= \rho - \frac{\beta \kappa^0}{1 - \phi} - E_{t-1} [r_t^f] - \frac{1}{2} \sigma_m^2. \end{aligned}$$

Using the conjectures once more, we have $E_{t-1} [e_t - e_t^*] = \frac{\kappa^0}{\phi}$ and $E_{t-1} [p_t - y_t^*] = \frac{-\kappa^0}{1 - \phi}$. Substituting, we solve

$$E_{t-1} [r_t^f] = \rho - \frac{1}{2} \sigma_e^2 \quad (\text{A.37})$$

$$\kappa^0 = \frac{\sigma_m^2 - \sigma_e^2}{1 - \beta} \frac{1 - \phi}{2} \quad (\text{A.38})$$

In this model, when the two variances differ the interest rate responds only to the variance of *the exchange rate*. This creates a non-zero intercept for both the asset price and the exchange rate that results from the differences in their respective variances and the interest rate response.

We next solve for the remaining coefficients. To this end, observe that Eqs. (A.35) and (A.36) together with $f_t = \phi e_t + (1 - \phi) p_t$ imply

$$f_t - f_t^* = \left(\begin{array}{c} E_t [f_{t+1} - f_{t+1}^*] \\ + (1 - \phi) (1 - \beta) E_t \left[\begin{array}{c} y_{t+1} - y_{t+1}^* \\ (p_{t+1} - y_{t+1}^*) \end{array} \right] \end{array} \right) - \left(\begin{array}{c} r_t^f - \rho + \\ \frac{1}{2} (\phi \sigma_e^2 + (1 - \phi) \sigma_m^2) \end{array} \right) + \bar{M}_t$$

$$\text{where } \bar{M}_t = \phi \mu_t^e \sigma_e^2 / \alpha + (1 - \phi) \mu_t^p \sigma_m^2 / \alpha.$$

After substituting $E_t [y_{t+1} - y_{t+1}^*] = 0$ and $E_t [p_{t+1} - y_{t+1}^*] = \frac{-\kappa^0}{1 - \phi} = -\frac{\sigma_m^2 - \sigma_e^2}{1 - \beta} \frac{1}{2}$, and using $y_t - y_t^* = f_t - f_t^*$, we obtain

$$\tilde{y}_t = (\sigma_m^2 - \sigma_e^2) \frac{1 - \phi}{2} + E_t [\tilde{y}_{t+1}] - \left(r_t^f - \rho + \frac{1}{2} (\phi \sigma_e^2 + (1 - \phi) \sigma_m^2) \right) + \bar{M}_t.$$

Simplifying, we obtain the following version of (A.27)

$$\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \left(r_t^f - \rho + \frac{1}{2} \sigma_e^2 \right) + \bar{M}_t \quad (\text{A.39})$$

$$\text{where } \bar{M}_t = \phi \mu_t^e \sigma_e^2 / \alpha + (1 - \phi) \mu_t^p \sigma_m^2 / \alpha.$$

Substituting our conjecture that $E_t [\tilde{y}_{t+1}] = 0$, and using (A.37), this further implies

$$\tilde{y}_t = - \left(r_t^f - E_{t-1} [r_t^f] \right) + \bar{M}_t. \quad (\text{A.40})$$

Next note that the similar steps as in the proof of Proposition 2 imply that given the target announcement $\bar{e}_t = E_{t-1}[e_t]$, the policy rule is given by (30)

$$r_t^f = E_{t-1} \left[r_t^f \right] + \theta \tilde{y}_t + \psi (e_t - E_{t-1}[e_t]).$$

After substituting our conjecture, this rule implies

$$r_t^f - E_{t-1} \left[r_t^f \right] = \theta \tilde{y}_t + \psi \left((1 + \kappa^{z,e}) z_t^e + \kappa^{z,p} z_t^p + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha} \right).$$

Here, we have used $e_t^* - E_{t-1}[e_t^*] = z_t^e$. Combining this with with (A.40), we solve

$$\begin{aligned} \tilde{y}_t &= \frac{1}{1+\theta} \bar{M}_t - \frac{\psi}{1+\theta} \left((1 + \kappa^{z,e}) z_t^e + \kappa^{z,p} z_t^p + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha} \right) \\ r_t^f - E_{t-1} \left[r_t^f \right] &= \frac{\theta}{1+\theta} \bar{M}_t + \frac{\psi}{1+\theta} \left((1 + \kappa^{z,e}) z_t^e + \kappa^{z,p} z_t^p + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha} \right). \end{aligned} \quad (\text{A.41})$$

Next we substitute (A.41) into the equation for the exchange rate (A.35), together with (A.37), and use $E_t[e_{t+1} - e_{t+1}^*] = \frac{\kappa^0}{\phi}$ to obtain

$$\begin{bmatrix} e_t - e_t^* \\ -\frac{\kappa^0}{\phi} \end{bmatrix} = -\frac{\theta}{1+\theta} \bar{M}_t - \frac{\psi}{1+\theta} \begin{pmatrix} (1 + \kappa^{z,e}) z_t^e + \kappa^{z,p} z_t^p \\ + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha} \end{pmatrix} + \frac{\mu_t^e}{\alpha} \sigma_e^2.$$

Observe that Eq. (A.32) implies the left hand side is given by

$$\kappa^{z,e} z_t^e + \kappa^{z,p} z_t^p + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha}.$$

Substituting $\bar{M}_t = \phi \mu_t^e \sigma_e^2 / \alpha + (1 - \phi) \mu_t^p \sigma_m^2 / \alpha$ and matching coefficients, we solve

$$\begin{aligned} \kappa^{z,e} &= \frac{-\psi}{1+\theta+\psi} \\ \kappa^{z,p} &= 0 \\ \kappa_e^{\mu,e} &= 1 - \phi \frac{\theta}{1+\theta} - \frac{1}{1+\theta} \psi \kappa_e^{\mu,e} \implies \kappa_e^{\mu,e} = \frac{1+\theta(1-\phi)}{1+\theta+\psi} \\ \kappa_e^{\mu,p} &= -(1-\phi) \frac{\theta}{1+\theta} - \frac{1}{1+\theta} \psi \kappa_e^{\mu,p} \implies \kappa_e^{\mu,p} = -\frac{(1-\phi)\theta}{1+\theta+\psi} \end{aligned}$$

This proves (33).

Similarly, we substitute (A.41) into the equation for the market price (A.35) to obtain

$$\begin{bmatrix} p_t - y_t^* \\ +\frac{\kappa^0}{1-\phi} \end{bmatrix} = -\frac{\theta}{1+\theta} \bar{M}_t - \frac{\psi}{1+\theta} \begin{pmatrix} (1 + \kappa^{z,e}) z_t^e + \kappa^{z,p} z_t^p \\ + \kappa_e^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_e^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha} \end{pmatrix} + \frac{\mu_t^p}{\alpha} \sigma_m^2.$$

Observe that Eq. (A.33) implies the left side is given by

$$\kappa^{z,e} z_t^e + \kappa^{z,p} z_t^p + \kappa_p^{\mu,e} \mu_t^e \frac{\sigma_e^2}{\alpha} + \kappa_p^{\mu,p} \mu_t^p \frac{\sigma_m^2}{\alpha}.$$

After substituting $\bar{M}_t = \phi \mu_t^e \sigma_e^2 / \alpha + (1 - \phi) \mu_t^p \sigma_m^2 / \alpha$ and matching coefficients, we verify that $\kappa^{z,e}$ and

$\kappa^{z,p}$ are the same as in the exchange rate expression and we solve

$$\begin{aligned}\kappa_p^{\mu,e} &= -\phi \frac{\theta}{1+\theta} - \frac{1}{1+\theta} \psi \underbrace{\frac{1+\theta(1-\phi)}{1+\theta+\psi}}_{\kappa_e^{\mu,e}} \implies \kappa_p^{\mu,e} = -\frac{\phi\theta + \psi}{1+\theta+\psi} \\ \kappa_p^{\mu,p} &= 1 - (1-\phi) \frac{\theta}{1+\theta} + \frac{1}{1+\theta} \psi \underbrace{\frac{(1-\phi)\theta}{1+\theta+\psi}}_{-\kappa_e^{\mu,p}} \implies \kappa_p^{\mu,p} = \frac{1+\phi\theta + \psi}{1+\theta+\psi}\end{aligned}$$

This proves (34).

Substituting the solution for the exchange rate into (A.41), we further solve for the interest rate

$$r_t^f - \underbrace{E_{t-1} [r_t^f]}_{\rho - \frac{1}{2} \sigma_e^2} = \frac{\psi}{1+\theta+\psi} z_t^e + \frac{\phi\theta + \psi}{1+\theta+\psi} \mu_t^e \frac{\sigma_e^2}{\alpha} + \frac{(1-\phi)\theta}{1+\theta+\psi} \mu_t^p \frac{\sigma_m^2}{\alpha}.$$

This proves (32).

Substituting the solution for the exchange rate and output into (A.34), we also solve for the FCI and output

$$y_t - y_t^* = -\frac{\psi}{1+\theta+\psi} z_t^e + \frac{\phi - (1-\phi)\psi}{1+\theta+\psi} \mu_t^e \frac{\sigma_e^2}{\alpha} + \frac{(1-\phi)(1+\psi)}{1+\theta+\psi} \mu_t^p \frac{\sigma_m^2}{\alpha}.$$

This proves (31).

Finally, using (4) and the solutions, we find the drivers of the market portfolio

$$r_{t+1}^m - E_t [r_{t+1}^m] = -\frac{z_{t+1}^p}{1+\theta+\psi} z_{t+1}^e + \left[\left(\frac{\phi - (1-\phi)\psi}{1+\theta+\psi} \right) \mu_{t+1}^e \frac{\sigma_e^2}{\alpha} + \left(\frac{(1-\phi)(1+\psi)}{1+\theta+\psi} + \beta\phi \right) \mu_{t+1}^p \frac{\sigma_m^2}{\alpha} \right].$$

This proves (35).

We next characterize the variances σ_e^2 and σ_m^2 . Taking the variance of the exchange rate, using the assumption that shocks are uncorrelated across the two markets ($\rho_z = \rho_\mu = 0$), we obtain

$$\sigma_e^2 = \left(\frac{1+\theta}{1+\theta+\psi} \right)^2 \sigma_{z,e}^2 + \left(\frac{1+\theta(1-\phi)}{1+\theta+\psi} \right)^2 \sigma_{\mu,e}^2 \left(\frac{\sigma_e^2}{\alpha} \right)^2 + \left(\frac{(1-\phi)\theta}{1+\theta+\psi} \right)^2 \sigma_{\mu,p}^2 \left(\frac{\sigma_m^2}{\alpha} \right)^2 \quad (\text{A.42})$$

$$\sigma_m^2 = \left(\left[\begin{array}{c} \sigma_{z,p}^2 + \\ \left(\frac{\psi}{1+\theta+\psi} \right)^2 \sigma_{z,e}^2 \end{array} \right] \right) + \left(\frac{\phi - (1-\phi)\psi}{1+\theta+\psi} \right)^2 \sigma_{\mu,e}^2 \left(\frac{\sigma_e^2}{\alpha} \right)^2 + \left(\frac{(1-\phi)(1+\psi)}{1+\theta+\psi} + \beta\phi \right)^2 \sigma_{\mu,p}^2 \left(\frac{\sigma_m^2}{\alpha} \right)^2 \quad (\text{A.43})$$

In particular, the variances correspond to a zero of a two dimensional quadratic function, $P(\sigma_e^2, \sigma_p^2 | \psi) = 0$, given by

$$P(x, y | \psi) = \left[\begin{array}{c} \frac{x^2}{\alpha^2} \left(\frac{1+\theta(1-\phi)}{1+\theta+\psi} \right)^2 \sigma_{\mu,e}^2 - x + \left(\frac{1+\theta}{1+\theta+\psi} \right)^2 \sigma_{z,e}^2 + \left(\frac{(1-\phi)\theta}{1+\theta+\psi} \right)^2 \sigma_{\mu,p}^2 \frac{y^2}{\alpha^2} \\ \frac{y^2}{\alpha^2} \left(\frac{(1-\phi)(1+\psi)}{1+\theta+\psi} + \beta\phi \right)^2 \sigma_{\mu,p}^2 - y + \left[\begin{array}{c} \sigma_{z,p}^2 + \\ \left(\frac{\psi}{1+\theta+\psi} \right)^2 \sigma_{z,e}^2 \end{array} \right] + \left(\frac{\phi - (1-\phi)\psi}{1+\theta+\psi} \right)^2 \sigma_{\mu,e}^2 \frac{x^2}{\alpha^2} \end{array} \right].$$

Let $\{x_i, y_i\}_i$ denote the zeros of this equation system. We assume the parameters are such that all of these zeros are strictly positive. It can also be seen that the zeros are strictly ranked. The solution

corresponds to the smallest zero.

It remains to show that it is optimal to announce the expected exchange rate as the target, $\bar{e}_t = E_{t-1}[e_t]$. This follows from a similar argument as in the proof of Proposition 2, completing the characterization. \square