

# The Safe-Debt Laffer Curve

Ricardo J. Caballero\*

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## Abstract

Government debt is often treated as the canonical safe asset, but its safety depends on more than sovereign solvency. This paper develops a theory in which safe government-debt claims are produced jointly by a solvent fiscal authority and a financial system that absorbs rollover risk. The government compensates that service through an excess yield on its debt; households receive the safe return net of the rollover-insurance premium. As the debt stock rises, rollover exposure rises relative to financial capacity, and the excess yield required to preserve safety increases. In steady state, this safety-production cost is financed through a lower primary deficit. At low debt levels, the direct wealth effect of additional insured debt dominates this fiscal adjustment. At high debt levels, the required fiscal adjustment dominates, so additional debt reduces aggregate demand and lowers the equilibrium safe rate. The mechanism generates a safe-debt Laffer curve for both aggregate demand and the safe interest rate. The rollover-insurance premium continues to rise with debt even where the equilibrium safe rate turns down. Financial capacity expands fiscal capacity (the peak of the safe-debt Laffer curve), higher debt strains financial capacity, and scarce rollover-insurance capacity tilts optimal debt management toward longer maturity despite the associated term-premium compensation. Preliminary diagnostic calculations place the United States close to the peak of the safe-debt Laffer curve.

**JEL Codes:** E21, E43, E62, H63, G12.

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# 1 Introduction

Government debt is often treated as the canonical safe asset of the modern financial system. It is the benchmark store of value, the collateral behind many private contracts, and the asset most naturally used by households to safely transfer resources over time. This paper starts from a distinction that is usually left implicit: the safety of public debt is a joint fiscal-financial property, produced by a solvent fiscal authority and by a financial system that absorbs the risk of rolling the debt over. In a non-Ricardian economy, such safe wealth supports aggregate demand and raises the equilibrium real safe rate. For major safe-asset issuers, this logic often supports a monotone presumption: conditional on public debt retaining its safe-asset status, more debt expands the supply of safe claims, strengthens demand, and puts upward pressure on the safe rate.

This paper challenges that presumption from within the safe-asset region. The mechanism is distinct from the familiar sovereign-risk channel in which high debt becomes contractionary because the sovereign claim itself is no longer safe. Here the claim remains safe, but preserving its safety requires financial-system capacity. The fiscal authority supplies the promise; the financial system supplies the market-making, warehousing, and rollover-insurance capacity that keeps outstanding claims safe when refinancing conditions deteriorate. As the debt stock rises, rollover pressure rises relative to that capacity, and the government must pay a larger excess yield to compensate the rollover-insurance service.

The central result is that debt can become contractionary even while households continue to hold safe government-debt claims. Additional insured debt has a direct positive wealth effect on households, but it also raises the fiscal cost of producing safety for the entire stock. When that cost absorbs more demand than the marginal unit of debt creates, the aggregate-demand effect of debt turns negative. In steady state, with taxes fixed in the benchmark, the government finances the safety-production cost by lowering government demand. The equilibrium safe rate then falls because a lower rate is required to raise the value of other assets (human wealth in the model) and restore goods-market clearing.

This mechanism generates a safe-debt Laffer curve. The curve is an aggregate-demand object: it describes how the contribution of insured public debt to goods demand first rises and then falls with the debt stock. The peak occurs when the marginal fiscal cost of the excess yield required to produce safety equals the direct perpetual-youth wealth effect of additional debt. On the increasing branch, more debt raises aggregate demand and the equilibrium safe rate. On the declining branch, more debt lowers aggregate demand and the equilibrium safe rate.

The analysis is steady-state: it asks how the economy behaves after a permanent change in debt has passed through the rollover structure. In the short run, a higher safety premium affects the primary deficit only as maturing debt is refinanced; in steady state, the higher premium is paid on the full rolled-over stock. This distinction is central to the safe-debt Laffer curve. An increase in the safety-production premium is a market-clearing price for insuring the rollover risk embedded in the stock of government debt. When the debt stock rises, rollover pressure rises, and the equilibrium premium eventually applies to the full insured stock. The marginal fiscal cost of safety therefore includes both the excess yield on the marginal unit and the higher premium paid on the rolled-over stock. This stock-wide rollover cost is what makes the marginal cost of safety production rise quickly once rollover pressure becomes severe.

The model has two blocks. The macro block explains why insured government-debt claims

affect aggregate demand. Households have finite horizons and demand claims that are safe across aggregate rollover states. Appendix B derives this demand from Epstein–Zin preferences (Epstein and Zin, 1989) with unit intertemporal elasticity and high aversion to aggregate payoff risk. In the infinite-risk-aversion limit, households fully insure their government-bond holdings against rollover-stress losses. The household problem then collapses to a perpetual-youth economy: consumption is a constant share of safe financial wealth plus human wealth, and insured public debt has a non-Ricardian wealth effect.

The financial block explains how insured claims are produced. The benchmark starts from the case in which rollover exposure is maximal: the full outstanding stock may have to be absorbed by investors when refinancing conditions deteriorate. This normalization isolates how a larger debt stock strains financial capacity even when the sovereign promise remains safe. The financial system contains a pool of potential rollover insurers with limited absorption capacity. Appendix G derives the safety premium from a global game in which insurers decide whether to participate after observing private signals about rollover conditions, following the threshold-selection logic of Morris and Shin (1998). The marginal participant’s zero-profit condition prices the rollover-insurance service. Financial capacity determines how much rollover exposure can be absorbed without stress; when debt rises relative to this capacity, the premium embedded in the government’s excess yield rises.

The positive-maturity version gives the same mechanism a price component. When Treasury claims have longer maturity, future rollover-premium conditions affect current bond prices before refinancing actually occurs. A higher required premium lowers bond prices, weakens effective financial capacity, and feeds back into the rollover premium through a balance-sheet multiplier. This multiplier separates the quantity effect of rollover exposure from the price effect generated by longer maturity. Maturity choice then applies this premium schedule to the tradeoff between rollover exposure and term-premium exposure.

The paper develops three results. The first is the safe-debt Laffer curve. The second is a financial-capacity comparative static: greater financial capacity expands fiscal capacity by lowering the marginal cost of producing safety and shifting the Laffer peak outward. The third is a positive-maturity extension that derives the balance-sheet multiplier linking rollover premia to bond prices and then studies maturity choice as an application.

The paper then takes the theory to a preliminary sign diagnostic for the United States. The empirical section asks where the United States lies relative to the aggregate-demand peak of the safe-debt Laffer curve. The diagnostic combines three quarterly objects around the large post-2020 increase in issuance: the Treasury debt coming due and therefore exposed to rollover, the dealer-parent balance-sheet capacity available to absorb that exposure, and the premium charged for bearing it. The premium has two components: a rollover or balance-sheet component inferred from Treasury-swap spreads and a duration component inferred from debt-weighted real term premia. Two empirical patterns drive the result: rollover exposure rises sharply relative to measured capacity, and the marginal premium schedule changes sign. Before 2020 the premium falls with supply, consistent with a strong convenience-yield cushion; after 2020 it rises with supply, consistent with strained safety-production capacity. Combining these estimates places the United States close to, but still on the upward side of, the aggregate-demand peak. The baseline estimate leaves a small positive marginal safe-rate gap of 42 basis points, meaning that additional debt still raises the aggregate-demand contribution of safe debt, but only slightly.

**Literature review.** The paper connects several literatures: safe-asset supply, fiscal policy in non-Ricardian economies, risk-centric macro-finance, Treasury-market intermediation, sovereign rollover risk, and debt maturity.

This paper is closely related to the safe-asset-shortage literature. Caballero, Farhi, and Gourinchas (2008, 2016, 2017, 2021) and Caballero and Farhi (2018) study the macroeconomic implications of scarce safe assets, including low safe rates, global imbalances, and recessionary pressure. Krishnamurthy and Vissing-Jorgensen (2012) document safety and liquidity premia on Treasury debt. Greenwood, Hanson, and Stein (2015) and D’Avernas and Vandeweyer (2024) study the supply and maturity structure of short safe claims. Maggiori (2017) and Farhi and Maggiori (2018) emphasize the international and intermediary-capacity dimensions of reserve-currency asset supply. Itskhoki and Mukhin (2026) provides a recent synthesis of the global-imbalances branch of this literature. Brunnermeier, Merkel, and Sannikov (2024) provide a recent incomplete-markets treatment of safe-asset service flows. The contribution here is to model safe government-debt claims as the output of a fiscal-financial production process with a debt-dependent marginal cost, which generates a safe-debt Laffer curve.

The paper also relates to the broader asset-shortage view. Caballero (2006, 2015) argue that low real rates, high asset valuations, bubbles, and macroeconomic fragility can arise when demand for stores of value exceeds the economy’s capacity to produce financial assets. Auclert, Malmberg, Rognlie, and Straub (2025) provide a recent asset-supply and asset-demand framework for studying U.S. wealth, real interest rates, and fiscal sustainability. The present paper focuses on the safe-asset branch of this broader logic: public debt is a store of value only insofar as the fiscal and financial systems can preserve its safety services.

The demand block builds on the non-Ricardian effects of government debt. Barro (1974) gives the Ricardian benchmark. Blanchard (1985) and Weil (1989) provide finite-horizon and overlapping-family environments in which government debt is private wealth. Mian, Straub, and Sufi (2021) study indebted demand, and Mian, Straub, and Sufi (2025) study fiscal deficits in a non-Ricardian environment. Relative to Mian, Straub, and Sufi, this paper adds the production side of safe debt. Households value insured public debt as safe wealth, but producing the insured claim requires financial-system capacity and fiscal resources. This additional safety-production margin is what generates the safe-debt Laffer curve. The declining branch is also related to classic work on fiscal adjustment and aggregate demand, including Giavazzi and Pagano (1990) and Alesina and Perotti (1997), but the mechanism here is a steady-state fiscal-financial one: the fiscal adjustment is required to finance the production of safe claims.

A separate empirical literature estimates the effect of public debt and fiscal positions on long-term government yields. Laubach (2009), Gruber and Kamin (2012), and Gamber and Seliski (2019) find positive effects of projected debt or fiscal positions on advanced-economy long rates; World Bank (2026) uses this literature in its discussion of how advanced-economy debt affects emerging-market and developing-economy borrowing costs. Recent work on fiscal news and Treasury repricing is also relevant. Gómez-Cram, Kung, and Lustig (2024) distinguish safe and risky government-debt regimes and use the COVID episode to connect Treasury valuations to fiscal news, term premia, inflation, and convenience-yield compression. Gómez-Cram, Kung, and Lustig (2025) use CBO cost estimates to identify fiscal news at high frequency and show that deficit-increasing proposals raise Treasury yields through expected inflation, term premia, and lower convenience yields. Jiang, Richmond, and Zhang (2025) document a supply-driven

decline in long-maturity Treasury convenience yields. The object here is complementary: the paper focuses on the marginal cost of producing safe Treasury claims and the resulting safe-rate Laffer condition.

The macro-finance channel relates to risk-centric models in which asset prices, financial conditions, and risk-bearing capacity shape aggregate demand. [Caballero and Simsek \(2020, 2021, 2023\)](#) develop models in which risk premia and asset prices affect demand and policy transmission, and [Caballero, Caravello, and Simsek \(2024\)](#) studies financial-conditions targeting. This paper studies the safe-asset services of public debt and treats the financial system's risk-bearing capacity as a macroeconomic state variable.

The Treasury-market intermediation literature gives institutional content to the financial-capacity block. [Kashyap, Stein, Wallen, and Younger \(2025\)](#) study a Treasury market with asset managers, hedge funds, and broker-dealers in which leveraged cash–futures basis positions and limited dealer balance-sheet capacity can generate market dysfunction as Treasury supply rises. Related work on the March 2020 episode and Treasury-market resilience includes [Duffie \(2020\)](#); [Duffie and Keane \(2023\)](#), [Schrimpf, Shin, and Sushko \(2020\)](#), [Kruttli, Monin, Petrsek, and Watugala \(2021\)](#), [He, Nagel, and Song \(2022\)](#), and [Du, Hébert, and Li \(2023\)](#). This literature provides empirical counterparts for the model's capacity variable and premium schedules; the contribution here is to embed that capacity margin in a macroeconomic safe-debt Laffer curve.

The production block relates to sovereign rollover risk and safe-asset determination. [Calvo \(1988\)](#) and [Cole and Kehoe \(2000\)](#) develop self-fulfilling rollover-crisis mechanisms. [Eaton and Gersovitz \(1981\)](#), [Arellano \(2008\)](#), and [Aguiar and Amador \(2014\)](#) study sovereign borrowing with endogenous spreads and repayment incentives. [He, Krishnamurthy, and Milbradt \(2019\)](#) is especially close because the safe-asset status of debt is endogenous and debt-size dependent. Their analysis centers on asset-market determination of safety. Here, the cost of preserving safety enters the government budget constraint, aggregate demand, and the equilibrium safe rate.

The maturity extension result relates to theories of government debt maturity and financial intermediation. [Angeletos \(2002\)](#), [Buera and Nicolini \(2004\)](#), and [Faraglia, Marcet, Oikonomou, and Scott \(2019\)](#) study the maturity structure of government debt as a tool for managing fiscal risk and hedging shocks. [Greenwood, Hanson, and Stein \(2015\)](#) emphasize comparative advantage in the supply of short safe debt. The extension below first derives how positive maturity turns rollover risk into a price component through a balance-sheet multiplier, and then uses that premium schedule to study the maturity tradeoff between rollover exposure and term-premium exposure.

The remainder of the paper is organized as follows. Section 2 introduces the perpetual-youth economy and the steady-state government budget constraint. Section 3 derives the cost of producing safe government-debt claims from rollover pressure. Section 4 combines the two blocks and derives the equilibrium safe-debt Laffer curve. Section 5 studies financial capacity and fiscal capacity. Section 6 derives the positive-maturity price component of rollover risk and then studies maturity choice. Section 7 constructs the empirical sign diagnostic. Section 8 concludes. The appendices collect the empirical construction, the household and insurance derivation, the rollover-premium formulas, the closed-form illustration, the balance-sheet amplification block, the term-premium extension, and the threshold-participation foundation for the pricing measure.

## 2 Environment

Time is continuous and output is normalized to one. The steady state is described by two key objects. The first is the stock of outstanding public debt, denoted by  $b$ . The second is financial-system capacity, denoted by  $N > 0$ , which summarizes the balance-sheet resources of the intermediaries and investors that absorb rollover risk in government-debt markets. Households receive the net safe return  $r$  on the insured government-debt claim.

Debt is refinanced gradually. Each unit of debt matures according to a Poisson process with hazard  $\delta > 0$ . The parameter  $\delta$  is the rollover intensity, so average maturity is  $1/\delta$ . The rollover share of the outstanding stock over a unit model period is

$$\mu(\delta) \equiv 1 - e^{-\delta}. \quad (1)$$

Thus the stock of debt exposed to rollover during the period is  $\mu(\delta)b$ . This exposure is increasing in  $\delta$  and converges to the outstanding debt stock as maturity becomes arbitrarily short. The benchmark in Section 3 takes this short-maturity limit,  $\delta \rightarrow \infty$ , so  $\mu(\delta) \rightarrow 1$ . Section 6 studies finite rollover intensity,  $\delta < \infty$ , and the positive-maturity price component of rollover risk.

### 2.1 Households

This subsection states the household block used in the main text. Appendix B derives it from preferences, portfolio choice, and rollover-insurance demand. The key output of that derivation is that households hold insured government-debt claims and consume a constant share of safe financial wealth plus human wealth.

Households are of the perpetual-youth type. Each household dies with Poisson intensity  $\lambda > 0$ , and newborn households replace dying households one-for-one. The parameter  $\lambda$  governs the strength of non-Ricardian behavior: because households may die before future taxes are levied, they internalize only part of future fiscal adjustments. Households discount utility at rate  $\rho$ . With logarithmic flow utility and actuarially fair annuity markets, the marginal propensity to consume out of total wealth is

$$A \equiv \rho + \lambda. \quad (2)$$

The term  $\rho$  is impatience, while  $\lambda$  is the mortality component that makes wealth more valuable for current consumption.

Taxes are fixed in the benchmark. With output normalized to one, the constant tax share is  $\tau$ , and the after-tax flow received by households is

$$\chi \equiv 1 - \tau. \quad (3)$$

Let  $h$  denote human wealth, the present value of the future disposable-income flow  $\chi$ . Aggregate consumption is

$$c = A(b + h), \quad (4)$$

where  $b$  is the safe government-debt claim held by households. The claim is safe because rollover risk is insured by the financial system and compensated through the yield spread described below. In steady state, human wealth is

$$h(r) = \frac{\chi}{r + \lambda}. \quad (5)$$

Appendix B derives (4)–(5) from the household problem. It starts from Epstein–Zin preferences (Epstein and Zin, 1989) with unit intertemporal elasticity and risk aversion  $\sigma$ , lets households choose government-bond holdings and rollover-insurance purchases, and studies the limiting problem as  $\sigma \rightarrow \infty$ . In that limiting problem, households choose full insurance: government bonds pay the government yield, rollover insurance absorbs the risky component, and the net return on insured debt is the safe rate  $r$ . The insurance premium is covered by the excess yield paid by the government.

The non-Ricardian wealth effect is immediate from (4): holding the safe rate, and hence human wealth, fixed, an additional unit of insured public debt raises private demand by  $A$ .

## 2.2 Government budget constraint

The government levies the fixed tax share  $\tau$ , issues debt  $b$ , and pays the gross yield  $r^G(b, N)$  on that debt. Government demand is denoted by  $g$ . The gross yield paid by the government differs from the safe return received by households. Households hold the insured safe claim and receive the safe return  $r$ . The financial system absorbs the rollover risk embedded in the government bond and is compensated by the yield spread

$$s(b, N) \equiv r^G(b, N) - r. \quad (6)$$

The premium schedule  $s(b, N)$  is derived in Section 3. The yield spread closes the government budget: the government pays  $r + s(b, N)$ , while households receive the safe return  $r$ .

The steady-state government budget constraint is

$$\tau = g(b, N; r) + r^G(b, N)b. \quad (7)$$

Tax revenue  $\tau$  finances government demand  $g$  and interest payments on the debt. Throughout the paper, taxes are held fixed. Hence any increase in the fiscal cost of producing safe debt is absorbed by lower government demand; equivalently, because taxes are fixed, a decline in  $g$  is a decline in the primary deficit. Solving for government demand after using  $r^G(b, N) = r + s(b, N)$  gives

$$g(b, N; r) = \tau - rb - s(b, N)b. \quad (8)$$

For given taxes and a given safe rate, a larger debt stock leaves less room for government demand because the government must service the debt and pay the yield spread that produces safe government-debt claims.

It is useful to separate standard debt service from the safety-production component. Define

$$g^0(b; r) = \tau - rb, \quad S(b, N) = s(b, N)b. \quad (9)$$

The function  $g^0$  is government demand after standard safe-rate debt service but before the safety premium. The function  $S$  is the total fiscal cost of the yield spread. Section 3 derives the premium schedule and its marginal cost. With this notation,

$$g(b, N; r) = g^0(b; r) - S(b, N). \quad (10)$$

At a fixed safe rate, the marginal effect of debt on government demand is

$$g_b(b, N; r) = -r - S_b(b, N). \quad (11)$$

The first term is conventional debt-service arithmetic. The second term is the marginal fiscal cost of producing safe government-debt claims, derived in (27). The central result is that this safety-production component can turn debt expansion from an aggregate-demand force into an aggregate-demand drag once its marginal cost exceeds the direct wealth effect.

### 2.3 Equilibrium rate schedule

Using (5) in the consumption rule (4), and using the government budget rule (8), goods-market clearing is

$$A \left( b + \frac{\chi}{r + \lambda} \right) + \tau - rb - S(b, N) = 1. \quad (12)$$

The steady-state safe rate is the value of  $r$  that satisfies (12). Appendix B derives this condition from the household problem, the government budget constraint, and the resource constraint.

Implicit differentiation of (12) gives the slope of the safe-rate schedule. Let

$$F(b, N, r) \equiv A \left( b + \frac{\chi}{r + \lambda} \right) + \tau - rb - S(b, N) - 1. \quad (13)$$

Equilibrium requires  $F(b, N, r) = 0$ . Holding financial capacity fixed,

$$F_b(b, N, r) = A - r - S_b(b, N), \quad F_r(b, N, r) = -\frac{A\chi}{(r + \lambda)^2} - b. \quad (14)$$

The first derivative says that a marginal unit of debt raises private demand by  $A$ , lowers government demand through standard debt service by  $r$ , and lowers government demand through the marginal cost of safety production by  $S_b$ . The second derivative says that a higher safe rate reduces human wealth and raises the debt-service burden on the outstanding stock. Hence

$$r_b(b, N) = \frac{A - r - S_b(b, N)}{b + A\chi/(r + \lambda)^2}. \quad (15)$$

The denominator is positive, so the sign of the safe-rate response is

$$\text{sign } r_b(b, N) = \text{sign} [A - r - S_b(b, N)]. \quad (16)$$

This rate schedule reflects the same marginal aggregate-demand force as the safe-debt Laffer curve. At a fixed safe rate, additional debt raises private demand by  $A$ , lowers government demand by the standard debt-service cost  $r$ , and lowers government demand further by the marginal cost of safety production  $S_b(b, N)$ . If the net effect is positive, the safe rate must rise to reduce human wealth and restore goods-market clearing. If the net effect is negative, the safe rate must fall to raise human wealth and restore goods-market clearing.

### 3 Rollover Insurance and Safe Public Debt

This section answers the benchmark question: how does the debt stock strain financial capacity when rollover exposure is maximal? The benchmark takes the short-maturity limit of the Poisson rollover structure introduced in Section 2. In this limit,

$$\delta \rightarrow \infty, \quad \mu(\delta) = 1 - e^{-\delta} \rightarrow 1. \quad (17)$$

Thus the full outstanding stock is potentially exposed to rollover during the insurance episode. The short-maturity limit is useful because it isolates rollover-insurance production from term-premium compensation: as  $\delta \rightarrow \infty$ , average maturity collapses and the term-premium component vanishes. The premium  $s(b, N)$  is the benchmark rollover-insurance premium and  $S(b, N) = s(b, N)b$  is its fiscal cost. Section 6 studies finite rollover intensity, positive maturity, and the tradeoff between rollover exposure and term-premium compensation.

The participation state is denoted by  $\alpha$ . It is the fraction of potential absorption capacity that is active when rollover occurs. The state  $\alpha = 1$  corresponds to full participation by the potential rollover-insurance pool. Lower values of  $\alpha$  describe stress states in which fewer balance sheets are willing or able to absorb rollover exposure. The lower bound  $\underline{\alpha} \in (0, 1)$  is the inelastic investor base: this fraction of capacity is always present in the market. Thus  $\alpha \in [\underline{\alpha}, 1]$ .

Active absorption capacity in participation state  $\alpha$  is

$$B(\alpha, N) = \alpha N. \quad (18)$$

Financial-system capacity is measured in units of rollover exposure over the model period. If the active participation share is  $\alpha$ , the amount of rollover exposure the system can absorb without stress is  $\alpha N$ . Thus a larger  $N$  raises absorption capacity in every participation state, while a lower  $\alpha$  reduces the active share of that capacity.

In the benchmark limit  $\mu(\delta) = 1$ , the debt stock exposed to rollover is  $b$ . The participation threshold required to absorb this exposure is

$$q(b, N) = \frac{b}{N}. \quad (19)$$

This statistic is rollover exposure divided by full-participation absorption capacity. If  $q(b, N) = 0.3$ , for example, then 30 percent of the potential rollover-insurance pool is enough to absorb the rollover exposure. If  $q(b, N) \leq \underline{\alpha}$ , the inelastic investor base is sufficient, and no safety premium is required. If  $q(b, N) > \underline{\alpha}$ , some participation states have too little active capacity to absorb the rollover exposure. Those are the rollover-stress states that generate the safety premium.

Let  $\bar{q}(b, N) = \min\{q(b, N), 1\}$ . The rollover shortfall in participation state  $\alpha$  is

$$X(b, N, \alpha) = \max\{b - \alpha N, 0\}. \quad (20)$$

A shortfall  $X$  creates a support burden  $\Gamma(X)$ , where

$$\Gamma(0) = 0, \quad \Gamma'(X) > 0, \quad \Gamma''(X) \geq 0. \quad (21)$$

The support burden is the resource cost borne by the financial system when it preserves the safe-asset services of public debt in a stress state. The per-participating-unit assessment is

$$a(b, N, \alpha) = \frac{\Gamma(X(b, N, \alpha))}{\alpha}. \quad (22)$$

Thus a lower participation state raises the burden on each active balance-sheet unit.

The global game in Appendix G implies that the marginal rollover insurer prices participation states using a uniform measure over  $[\underline{\alpha}, 1]$ . In the diffuse-prior limit, the marginal participant's posterior makes each active participation share between the inelastic base and full participation equally likely, so the zero-profit premium averages support costs across precisely the states in which rollover stress is active. This measure is the competitive pricing distribution for the rollover-insurance service. The per-unit safety premium is therefore

$$s(b, N) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N)} \frac{\Gamma(b - \alpha N)}{\alpha} d\alpha, \quad (23)$$

with  $s(b, N) = 0$  when  $q(b, N) \leq \underline{\alpha}$ . In the interior stress region  $\underline{\alpha} < q(b, N) < 1$ , the moving-boundary term vanishes because the shortfall is zero at  $\alpha = q(b, N)$ . Differentiating gives

$$s_b(b, N) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \frac{\Gamma'(b - \alpha N)}{\alpha} d\alpha > 0, \quad (24)$$

while

$$s_N(b, N) = -\frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \Gamma'(b - \alpha N) d\alpha < 0. \quad (25)$$

Higher debt raises the safety premium because it increases rollover pressure. Higher financial capacity lowers the premium because it expands absorption capacity in every participation state. This monotone rollover-insurance premium response is the safety-production counterpart to the non-monotone equilibrium safe-rate response derived below. Appendix C derives these formulas in the general finite- $\delta$  case; equations (23)–(25) are the benchmark limit  $\mu(\delta) = 1$ . The appendix also derives the second derivative  $S_{bb}$  used below.

The total fiscal cost of producing safety is

$$S(b, N) = s(b, N)b. \quad (26)$$

The marginal cost is

$$S_b(b, N) = s(b, N) + bs_b(b, N). \quad (27)$$

This decomposition is the central financial mechanism. The first term,  $s(b, N)$ , is the safety-production cost of the marginal unit of debt. The second term,  $bs_b(b, N)$ , is the repricing of the outstanding stock. The premium is a common market-clearing insurance price for the entire stock, and the government pays it through an excess yield on all insured debt. Institutionally, this captures the idea that the required compensation for market-making, balance-sheet warehousing, and rollover support is set by aggregate rollover pressure. The marginal unit therefore makes the outstanding stock more expensive to support. Appendix G explains why the market-clearing premium applies to the whole insured stock.

The financial block delivers the input needed for the equilibrium analysis: the rollover premium  $s(b, N)$ , the fiscal cost  $S(b, N)$ , and the marginal cost  $S_b(b, N)$ . The next section combines these objects with the household block and the government budget constraint.

## 4 Equilibrium and the Safe-Debt Laffer Curve

Sections 2 and 3 describe the two building blocks of the model. The first block gives household demand for insured government-debt claims and the government budget constraint. The second block derives the rollover-insurance premium that enters the government yield. This section combines the two blocks and states the main result.

The key marginal object is the effect of an additional unit of debt on aggregate demand, evaluated at the prevailing safe rate. Household consumption is given by (4). At a given safe rate, human wealth in (5) is fixed, so an additional unit of insured public debt raises private demand by  $A$ . On the government side, equation (11) shows that the same increase in debt lowers government demand by standard debt service and by the marginal cost of safety production. Hence

$$\left. \frac{\partial(c + g)}{\partial b} \right|_{r, N} = A + g_b(b, N; r) = A - r - S_b(b, N). \quad (28)$$

The term  $A$  is the perpetual-youth wealth effect of an additional unit of insured public debt. The term  $r$  is standard steady-state debt service. The term  $S_b$  is the marginal fiscal cost of the yield spread required to produce safe government-debt claims. Using (27), this last term is  $s + bs_b$ : the premium on the marginal unit plus the repricing of the outstanding stock.

The safe-debt Laffer curve is the relationship between the debt stock and the aggregate-demand contribution of insured public debt. Debt is expansionary when

$$A - r > S_b(b, N), \quad (29)$$

and contractionary when

$$A - r < S_b(b, N). \quad (30)$$

The peak satisfies

$$A - r(b^L, N) = s(b^L, N) + b^L s_b(b^L, N), \quad (31)$$

where  $r(b^L, N)$  is the equilibrium safe rate at the peak. The left-hand side is the direct wealth effect net of standard debt service. The right-hand side is the marginal fiscal cost of producing safety.

**Proposition 1** (Safe-debt Laffer curve). *Let  $r(b, N)$  be the equilibrium safe rate that clears the goods market in (12), and define*

$$H(b, N) \equiv A - r(b, N) - S_b(b, N).$$

*Suppose the economy enters the stress region at  $\widehat{b}(N)$ , with  $S_b(\widehat{b}, N) = 0$ . Assume that  $S_b(b, N)$  is continuous and strictly increasing on the stress region, that  $S_{bb}(b, N) > 0$  at any interior zero of  $H$ , and that  $S_b(b, N) \rightarrow \infty$  as  $b \rightarrow \infty$ . If  $H(\widehat{b}, N) > 0$  and  $A - r(b, N)$  remains bounded on the stress region, then there is a unique Laffer peak  $b^L(N) > \widehat{b}(N)$  satisfying (31). Debt raises aggregate demand for  $b < b^L(N)$  and lowers aggregate demand for  $b > b^L(N)$ .*

The proposition states the main economic force. Below the peak, the wealth effect of insured debt exceeds standard debt service and the marginal cost of producing safety. Above the peak, the cost of maintaining the insured safe claim dominates the wealth effect. Existence follows because  $H(\widehat{b}, N) > 0$ , while  $S_b(b, N) \rightarrow \infty$  as  $b \rightarrow \infty$  and bounded  $A - r(b, N)$  imply that  $H(b, N) < 0$  for some finite  $b$  in the stress region. Continuity then gives a zero of  $H$  in the stress region.

Uniqueness follows from the same marginal condition that governs the safe-rate response. Since  $H(\widehat{b}, N) > 0$ , any zero lies in the interior of the stress region. At any such zero  $b^*$  of  $H$ , equation (15) gives  $r_b(b^*, N) = H(b^*, N)/[b^* + A\chi/(r + \lambda)^2] = 0$ . Hence

$$H_b(b^*, N) = -r_b(b^*, N) - S_{bb}(b^*, N) = -S_{bb}(b^*, N) < 0. \quad (32)$$

Every zero is a strict downcrossing. A second zero would require an intervening upcrossing, which (32) rules out. The zero is therefore unique. Equation (15) also shows that the equilibrium safe rate moves with the same marginal force: it rises with debt when debt is expansionary and falls with debt when debt is contractionary. At the Laffer peak,  $r_b(b^L, N) = 0$ .

**Corollary 1** (Coincident aggregate-demand and safe-rate peaks). *The aggregate-demand Laffer peak and the peak of the equilibrium safe-rate schedule coincide at  $b^L(N)$ . Hence the sign of the debt derivative of the equilibrium safe rate identifies the branch of the safe-debt Laffer curve.*

Figure 1 illustrates the mechanism in the closed-form case of Appendix D. The stress threshold  $\widehat{b}$  is the debt level at which rollover pressure first exceeds the inelastic investor base. The Laffer peak  $b^L$  occurs later, when the marginal cost of safety production has risen enough to offset the direct wealth effect net of standard debt service. The lower panel plots the equilibrium safe rate and shows the same turning point.

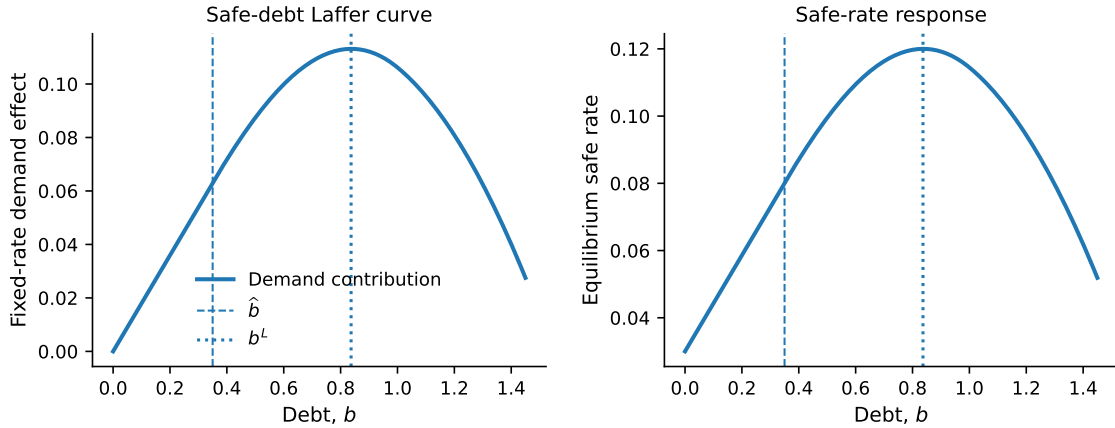


Figure 1: The safe-debt Laffer curve and the equilibrium safe rate. The figure uses the closed-form linear-support-cost case of Appendix D. The vertical dashed line marks the stress threshold  $\widehat{b}$ ; the dotted line marks the Laffer peak  $b^L$ . Above  $\widehat{b}$ , the marginal cost of safety production rises with debt. At  $b^L$ , the marginal fiscal adjustment equals the wealth effect net of standard debt service.

The rate response has the same economic interpretation as the Laffer curve. When  $A - r - S_b > 0$ , debt creates excess demand at the old safe rate, and the safe rate rises to reduce human wealth

and restore goods-market clearing. When  $A - r - S_b < 0$ , debt creates deficient demand at the old safe rate, and the safe rate falls to raise human wealth and restore goods-market clearing. Debt can therefore become contractionary even while households continue to hold safe government-debt claims: producing those claims has become costly enough to absorb more demand than the marginal unit creates.

## 5 Financial Capacity and Fiscal Capacity

The Laffer peak depends on the marginal cost of producing safety. This section shows how financial capacity shifts that peak in the benchmark short-maturity limit.

Financial capacity lowers rollover pressure. Differentiating (19) gives

$$q_b(b, N) = \frac{1}{N} > 0, \quad q_N(b, N) = -\frac{1}{N}q(b, N) < 0. \quad (33)$$

A higher debt stock increases rollover exposure relative to capacity. A higher  $N$  expands absorption capacity and lowers the participation share required to absorb the same rollover exposure.

Appendix C shows that  $S_N < 0$  and  $S_{bN} < 0$ . Higher financial capacity lowers both the total fiscal cost of the safety premium and the marginal fiscal cost of producing safe government-debt claims. Since lower total safety cost raises aggregate demand at a given safe rate, goods-market clearing implies  $r_N > 0$ . The equilibrium effect on the Laffer peak follows by differentiating the peak condition jointly with goods-market clearing.

**Proposition 2** (Financial capacity and fiscal capacity). *Let  $b^L(N)$  denote an interior Laffer peak satisfying*

$$A - r(b^L, N) - S_b(b^L, N) = 0.$$

*Then*

$$\frac{db^L}{dN} = \frac{-r_N(b^L, N) - S_{bN}(b^L, N)}{S_{bb}(b^L, N)}, \quad r_N(b, N) = -\frac{S_N(b, N)}{b + A\chi/(r + \lambda)^2} > 0. \quad (34)$$

*If the capacity-dominance condition*

$$-S_{bN}(b^L, N) > r_N(b^L, N) \quad (35)$$

*holds, then  $db^L/dN > 0$ .*

The proposition separates two effects of financial capacity. The direct effect is the one emphasized by the mechanism: higher  $N$  lowers rollover pressure and reduces the marginal fiscal cost of safety production, captured by  $S_{bN} < 0$ . The equilibrium-rate effect works through goods-market clearing. Because higher  $N$  also lowers the level of the safety-production cost, it raises aggregate demand at a given safe rate and therefore raises the equilibrium safe rate. Condition (35) says that financial capacity expands fiscal capacity when the marginal-cost effect dominates this level effect. This is the relevant case in the benchmark figures and in the closed-form illustration.

Figure 2 illustrates this comparative static in the same closed-form case as Figure 1. A higher  $N$  shifts the stress threshold and the Laffer peak to the right. A lower  $N$  brings the stress region forward and makes the aggregate-demand reversal occur at a lower debt stock.

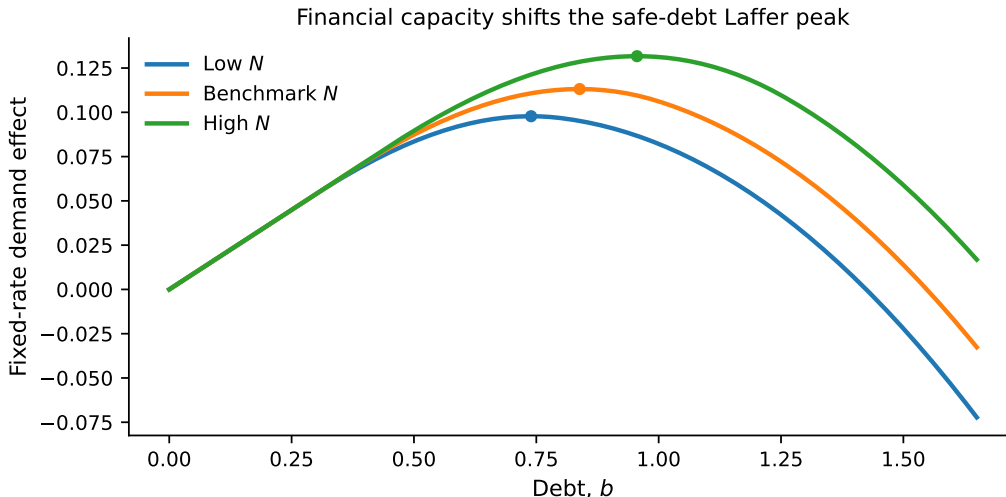


Figure 2: Financial capacity shifts the Laffer peak. The figure plots the aggregate-demand contribution of insured public debt for three levels of financial capacity in the closed-form case. Higher  $N$  lowers rollover pressure and shifts the peak outward.

The benchmark comparative static holds maturity at the short-maturity limit and therefore studies how debt strains financial capacity when the full stock is exposed to rollover. The next section studies finite rollover intensity and the positive-maturity price component of rollover risk.

## 6 Positive Maturity, Rollover-Risk Pricing, and Financial Capacity

The benchmark takes the short-maturity limit  $\delta \rightarrow \infty$ , in which the full stock is exposed to rollover and maturity collapses. This section restores positive maturity by considering a finite rollover intensity  $\delta < \infty$ . It uses three objects. The direct rollover component,  $s^{base}$ , is the premium implied by rollover exposure before price feedback. The duration component,  $s^X$ , is compensation for price risk on positive-maturity debt. The balance-sheet multiplier,  $M$ , converts the direct rollover component into the amplified rollover component  $s^R = Ms^{base}$ . The total premium is  $s^T = s^R + s^X$ .

### 6.1 Finite rollover intensity and direct rollover pricing

For a fixed rollover intensity  $\delta$ , the share of the outstanding stock exposed to rollover over the insurance episode is

$$\mu(\delta) = 1 - e^{-\delta}.$$

A higher  $\delta$  means shorter average maturity and larger rollover exposure  $\mu(\delta)b$ . A lower  $\delta$  means longer average maturity and less frequent rollover.

The direct rollover-insurance premium, before the price feedback derived in Appendix E, is

$$s^{base}(b, N, \delta) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N, \delta)} \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} d\alpha, \quad (36)$$

where

$$q(b, N, \delta) = \frac{\mu(\delta)b}{N}, \quad \bar{q}(b, N, \delta) = \min\{q(b, N, \delta), 1\}. \quad (37)$$

Thus finite rollover intensity scales the benchmark rollover-pressure variable by  $\mu(\delta)$ . For fixed  $\delta$ , higher debt raises  $q$  and higher financial capacity lowers  $q$ . Hence the direct rollover-insurance premium is increasing in public debt and decreasing in financial capacity on the stress region where the integrand is active.

## 6.2 The price component of rollover risk

Positive maturity turns rollover risk into a price object. A positive-maturity government bond is exposed to the current rollover-insurance premium and to the valuation consequences of future rollover-premium conditions. Let  $s^T(b, N, \delta)$  denote the total excess-yield premium for a claim with rollover intensity  $\delta$ , and define the associated effective discount-plus-hazard rate by

$$\rho^T(b, N, \delta) = r + s^T(b, N, \delta) + \delta. \quad (38)$$

The premium  $s^T$  is determined as a fixed point because the price component depends on duration, and duration depends on the total premium.

The term-premium component summarizes compensation for the standard price volatility of longer bonds. Let  $\sigma_p^2$  denote the variance of the persistent premium or rate shock that moves the price of the long-bond portfolio. Then

$$s^X(b, N, \delta) = \vartheta(N) \frac{\sigma_p^2}{[\rho^T(b, N, \delta)]^2}, \quad \vartheta_N(N) < 0. \quad (39)$$

A lower  $\delta$  lengthens duration and raises price sensitivity. The function  $\vartheta(N)$  captures the price of bearing term-premium risk; higher financial capacity lowers that price. Appendix F derives this form from the duration  $1/\rho^T$  of a Poisson-maturity bond.

The new rollover-risk price effect enters through the balance sheet. A higher required premium lowers the price of outstanding longer bonds. The mark-to-market loss weakens effective financial capacity and raises the rollover-insurance premium further. The balance-sheet multiplier  $M(b, N, \delta)$  is the fixed point of this loop: an initial increase in the direct rollover premium generates a price loss, the price loss reduces effective capacity, and lower capacity raises the premium again. To express this feedback, define the direct sensitivity of the term-premium component to financial capacity by

$$g_{dir}^X(b, N, \delta) \equiv - \left. \frac{\partial s^X(b, N, \delta)}{\partial N} \right|_{\rho^T} > 0, \quad (40)$$

where the derivative holds the effective discount-plus-hazard rate fixed and isolates the direct price-of-risk effect of financial capacity. Together with the direct rollover-premium sensitivity  $s_N^{base}(b, N, \delta) < 0$ , this gives

$$M(b, N, \delta) \equiv \frac{\rho^T(b, N, \delta) + g_{dir}^X(b, N, \delta)}{\underbrace{\rho^T(b, N, \delta) + s_N^{base}(b, N, \delta)}_{\text{balance-sheet amplification multiplier}}}. \quad (41)$$

The numerator captures the direct term-premium sensitivity to financial capacity. The denominator captures the balance-sheet feedback: a higher premium lowers bond prices, weakens intermediary net worth, and raises the premium further through the direct rollover-premium sensitivity  $s_N^{base}$ . Since  $s_N^{base} < 0$ , stronger capacity sensitivity reduces the denominator and amplifies  $M$ , subject to the admissibility condition below. The rollover component of the positive-maturity premium is therefore

$$s^R(b, N, \delta) = \frac{\rho^T(b, N, \delta) + g_{dir}^X(b, N, \delta)}{\rho^T(b, N, \delta) + s_N^{base}(b, N, \delta)} s^{base}(b, N, \delta). \quad (42)$$

The total positive-maturity premium is

$$s^T(b, N, \delta) = s^R(b, N, \delta) + s^X(b, N, \delta). \quad (43)$$

**Proposition 3** (Rollover intensity and the price multiplier). *Fix  $\delta < \infty$  and consider the active stress region  $\underline{\alpha} < q(b, N, \delta) < 1$ . The direct rollover-insurance component is increasing in rollover intensity:*

$$s_\delta^{base}(b, N, \delta) = \frac{\mu'(\delta)b}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N, \delta)} \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} d\alpha > 0. \quad (44)$$

Let  $M(b, N, \delta)$  be the amplification multiplier defined in (41). Under the admissibility condition

$$\rho^T(b, N, \delta) + s_N^{base}(b, N, \delta) > 0,$$

the multiplier satisfies  $M(b, N, \delta) > 1$ . Its response to rollover intensity is

$$M_\delta = \frac{(s_N^{base} - g_{dir}^X)\rho_\delta^T + (\rho^T + s_N^{base})g_{dir, \delta}^X - (\rho^T + g_{dir}^X)s_{N\delta}^{base}}{(\rho^T + s_N^{base})^2}. \quad (45)$$

Here  $M_\delta$  is the total derivative of the reduced-form multiplier evaluated along the fixed-point premium schedule. Thus  $\rho_\delta^T = 1 + s_\delta^T$  in (45); the equation is a derivative identity for the fixed-point object, not a closed-form sign formula.

The two components move through different margins. A higher  $\delta$  increases the direct rollover component by exposing a larger share of the debt stock to rollover. The multiplier captures the price feedback created by positive maturity. Equation (45) shows that  $M_\delta$  contains two opposing forces. The duration-compression force lowers the multiplier: raising  $\delta$  shortens maturity, raises the effective discount-plus-hazard rate, and reduces the direct term-premium

sensitivity. The rollover-capacity-sensitivity force raises the multiplier: raising  $\delta$  makes the direct rollover premium more sensitive to financial capacity, so  $s_N^{base}$  becomes more negative and the denominator of  $M$  falls.

The boundary cases discipline the sign. As  $\delta \rightarrow \infty$ , maturity collapses,  $\rho^T \rightarrow \infty$ , and the duration-price component vanishes:

$$s^X(b, N, \delta) \rightarrow 0, \quad g_{dir}^X(b, N, \delta) \rightarrow 0.$$

Moreover,  $\mu'(\delta) = e^{-\delta} \rightarrow 0$ , so the marginal effect of  $\delta$  on the rollover-capacity sensitivity also disappears. Hence

$$M(b, N, \delta) \rightarrow 1 \quad \text{as} \quad \delta \rightarrow \infty,$$

and  $M_\delta < 0$  for sufficiently high  $\delta$ . At the opposite boundary,  $\mu(0) = 0$ . If the economy is below the rollover-stress threshold at  $\delta = 0$ , then  $s^{base} = 0$  and  $s_N^{base} = 0$ , while the price component is maximal:

$$M(b, N, 0) = 1 + \frac{g_{dir}^X(b, N, 0)}{\rho^T(b, N, 0)} > 1.$$

In the no-stress neighborhood of  $\delta = 0$ , raising  $\delta$  compresses maturity and lowers  $g_{dir}^X/\rho^T$ , so  $M_\delta < 0$ .

The sign ambiguity is therefore an interior stress-region phenomenon. When duration compression dominates, increasing rollover intensity raises the direct rollover load  $s^{base}$  but reduces the price multiplier  $M$ . When the rollover-capacity-sensitivity effect dominates, both the direct load and the multiplier rise with  $\delta$ . This decomposition separates the quantity effect of rollover exposure from the price effect generated by positive maturity. The multiplier is the channel through which positive maturity converts rollover-risk compensation into a price component of the safety-production premium.

The total positive-maturity premium is

$$s^T(b, N, \delta) = M(b, N, \delta)s^{base}(b, N, \delta) + s^X(b, N, \delta).$$

Positive maturity adds a price component to the rollover-risk mechanism through  $M(b, N, \delta)$ : a higher required premium lowers bond prices, weakens effective financial capacity, and feeds back into the rollover premium.

### 6.3 Optimal maturity choice

Once the premium schedule is defined for each finite rollover intensity  $\delta$ , one can ask which rollover intensity minimizes the cost of producing safe debt. The maturity choice is

$$\delta^*(b, N) \in \arg \min_{\delta > 0} s^T(b, N, \delta). \quad (46)$$

Figure 3 illustrates the financial-capacity comparative static. The vertical axis reports optimal average maturity,  $1/\delta^*$ . As financial capacity rises, the government can sustain a higher rollover intensity and therefore chooses shorter average maturity. When financial capacity is scarce, rollover exposure is more costly and the optimum shifts toward longer maturity.

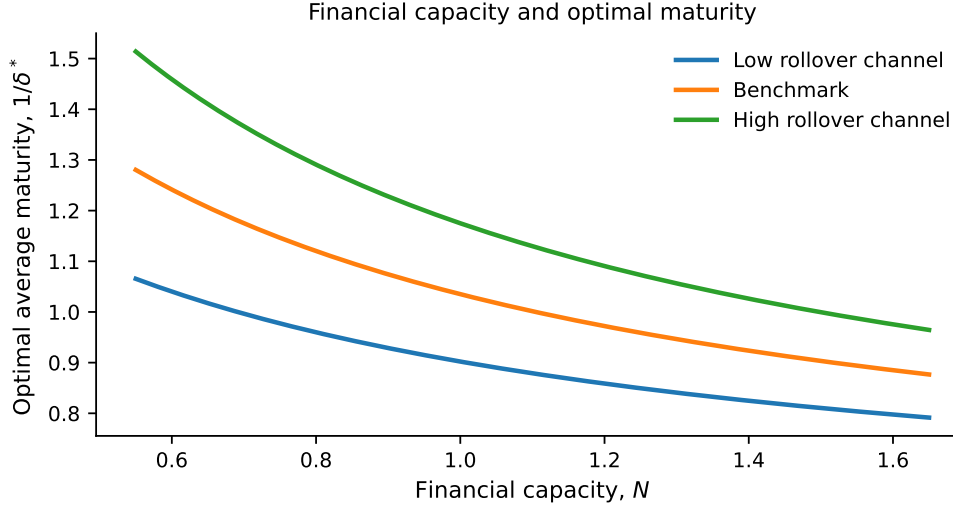


Figure 3: Financial capacity and optimal maturity. The figure plots the optimal average maturity  $1/\delta^*$  against financial capacity  $N$ . The downward slope means that stronger financial capacity permits a higher optimal rollover intensity, while scarce financial capacity tilts issuance toward longer maturity. The three curves vary the strength of the rollover channel and the term-premium compensation to show the robustness of the comparative static.

**Proposition 4** (Maturity choice and financial capacity). *Let  $\delta^*(b, N)$  be an interior solution of the maturity problem. Suppose the second-order condition*

$$s_{\delta\delta}^T(b, N, \delta^*) > 0$$

*holds. Write the rollover component as  $s^R = Ms^{base}$ . Its full maturity-capacity cross-partial is*

$$s_{\delta N}^R = Ms_{\delta N}^{base} + M_N s_{\delta}^{base} + M_{\delta} s_N^{base} + M_{\delta N} s^{base}. \quad (47)$$

*If the full total-premium cross-partial satisfies*

$$s_{\delta N}^T(b, N, \delta^*) = s_{\delta N}^R(b, N, \delta^*) + s_{\delta N}^X(b, N, \delta^*) < 0, \quad (48)$$

*then*

$$\frac{d\delta^*(b, N)}{dN} > 0. \quad (49)$$

*Thus higher financial capacity permits a higher optimal rollover intensity; equivalently, lower financial capacity lengthens maturity.*

At an interior optimum,

$$s_{\delta}^T(b, N, \delta^*) = 0. \quad (50)$$

This first-order condition balances two margins. Raising  $\delta$  shortens maturity and increases rollover pressure through  $\mu(\delta)$ , which raises the amplified rollover component  $s^R$ . The same

increase in  $\delta$  shortens duration and lowers term-premium compensation, which lowers  $s^X$ . Differentiating the first-order condition with respect to  $N$  gives

$$\frac{d\delta^*}{dN} = -\frac{s_{\delta N}^T(b, N, \delta^*)}{s_{\delta\delta}^T(b, N, \delta^*)}. \quad (51)$$

Appendix F derives the full cross-partial  $s_{\delta N}^T$ . Equation (48) requires the entire rollover-capacity effect, including the response of the balance-sheet multiplier, to dominate the term-premium price-of-risk channel. Together with the second-order condition for a local minimum, this proves Proposition 4.

The economic force is that financial capacity changes the marginal cost of short maturity. The direct rollover-capacity term is negative: higher capacity lowers the stress cost of additional rollover exposure. The proposition requires that force, including the induced response of the balance-sheet multiplier, to dominate the opposing term-premium component. Under that condition, a high- $N$  government can choose a higher rollover intensity, while scarce rollover-insurance capacity lengthens maturity.

A transparent sufficient case isolates the direct rollover-capacity channel by holding the amplification multiplier fixed in the  $(\delta, N)$  comparison. Then  $M_N = M_\delta = M_{\delta N} = 0$ , so

$$s_{\delta N}^R = M s_{\delta N}^{base} < 0. \quad (52)$$

In that benchmark, the condition in Proposition 4 reduces to the checkable comparison

$$-M s_{\delta N}^{base} > s_{\delta N}^X. \quad (53)$$

The full amplified economy is governed by (48); the benchmark only shows how the direct rollover-capacity force generates the sign when multiplier variation does not overturn it.

Maturity policy changes the cost of producing safety and therefore shifts fiscal capacity. Debt can still force government demand down even if the current premium level is zero, because the Laffer condition is governed by the marginal fiscal cost of safety production, now evaluated at the chosen maturity:

$$A - r = s^T(b, N, \delta^*) + b \frac{\partial s^T(b, N, \delta^*)}{\partial b}. \quad (54)$$

Maturity management therefore moves the Laffer peak by changing the marginal cost of safety production. Scarce rollover-insurance capacity lengthens maturity and reduces rollover pressure, but the aggregate-demand peak remains governed by the same fiscal-financial margin: once safety production is sufficiently costly, higher debt requires a larger steady-state fiscal adjustment.

## 7 A Quantitative Sign Diagnostic for the Safe-Debt Laffer Curve

This section uses quarterly data on Treasury rollover exposure, dealer-parent intermediation capacity, and Treasury premium components around the large increase in issuance after 2020 to

construct a quantitative sign diagnostic for the safe-debt Laffer curve. The empirical question is where the United States lies relative to the aggregate-demand peak of the curve.

The model’s Laffer condition is governed by the marginal cost of producing safe government debt. In the notation of the theory, this object is

$$S_b(b, N) \equiv s(b, N) + b s_b(b, N), \quad \Omega(b, N) \equiv A - r - S_b(b, N). \quad (55)$$

Additional debt raises the aggregate-demand contribution of public debt—and, equivalently, the equilibrium safe rate—when  $\Omega > 0$ , and lowers both when  $\Omega < 0$  (Proposition 1 and Corollary 1). At current U.S. values, the baseline estimate gives  $\Omega = +42$  basis points, so the marginal aggregate-demand contribution of additional safe debt remains positive, but small. The empirical exercise constructs the data counterpart of this sign condition.

## 7.1 Rollover exposure and premium components

Rollover exposure is measured relative to Treasury-intermediation capacity:

$$Q_t = \frac{\mu_t b_t}{N_t}, \quad (56)$$

where  $b_t$  is privately held marketable Treasury debt,  $\mu_t$  is the share of that debt maturing within one year, and  $N_t$  is measured Treasury-intermediation capacity. The numerator  $\mu_t b_t$  is therefore privately held marketable Treasury debt maturing within one year.

The headline capacity measure is dealer-parent holdable Treasury capacity. “Dealer-parent” refers to the parent banking organizations that own the dealers active in Treasury intermediation; “holdable” means the amount of Treasury inventory those organizations could hold, given regulatory leverage capacity, plus the Treasury inventory they already hold. The measure is

$$N_t^{\text{hold}} = \sum_i \left( \frac{T1_{i,t}}{m} - LE_{i,t} \right) + \sum_i UST_{i,t}, \quad m = 6\%. \quad (57)$$

Here  $T1$  is Tier-1 capital,  $LE$  is total leverage exposure, and  $UST$  is Treasury securities already held by dealer-parent institutions. The first term measures unused balance-sheet space under a Supplementary Leverage Ratio (SLR)-type leverage constraint: with minimum leverage ratio  $m$ , total leverage exposure cannot exceed  $T1/m$ . The second term adds Treasuries already held, so the measure captures both unused room to absorb Treasuries and Treasury inventory already warehoused by dealer-parent institutions. The measure is used to construct the headline rollover-stress series  $Q_t$ .

Total safety-production premia are measured as the sum of two components,

$$s_t^T = s_t^R + s_t^D. \quad (58)$$

This decomposition mirrors the maturity section’s separation between the rollover component and the term-premium component in (43). Both empirical components are aggregated across maturity buckets using outstanding Treasury debt weights. The first component,  $s_t^R$ , is the rollover or balance-sheet component. It is measured using a Treasury-swap spread, with the

sign convention chosen so that a higher  $s_t^R$  means Treasuries are cheaper relative to matched swaps. This convention captures the balance-sheet cost of warehousing cash Treasuries rather than holding an unfunded synthetic position. It has no duration-risk component by construction.

The second component,  $s_t^D$ , is the duration component. It is measured as the debt-weighted real term premium on the maturity structure of outstanding Treasuries. This empirical duration component corresponds to the model's term-premium component  $s^X$ . The decomposition therefore assigns balance-sheet capacity costs to  $s_t^R$  and duration compensation to  $s_t^D$ , so the separate component regressions do not mechanically mix duration compensation into the rollover component.

## 7.2 Diagnostic slopes

The sign diagnostic requires the marginal response of total premia to rollover stress. The exercise uses quarterly data from 2015Q1 through 2026Q1, giving 45 observations. For each premium component, the following levels specification constructs point slopes before and after 2020Q2:

$$s_t = a + bQ_t + cD_t + d(Q_t D_t) + eQE_t + fQT_t + \varepsilon_t, \quad D_t = \mathbf{1}\{t \geq 2020Q2\}, \quad (59)$$

for  $s_t \in \{s_t^R, s_t^D\}$ . The pre-2020 slope is  $b$ ; the post-2020 slope is  $b + d$ . The variables  $QE_t$  and  $QT_t$  are Federal Reserve balance-sheet regime indicators constructed from SOMA Treasury holdings:  $QE_t = 1$  in quarters when those holdings expand,  $QT_t = 1$  in quarters when they contract, and both are zero in quarters with no material change. Their role is to absorb average level differences in premia across balance-sheet expansion and runoff regimes.

The break date is placed a priori at 2020Q2, when rollover exposure rose sharply relative to measured intermediation capacity, separating a low-stress regime from a high-stress regime. The specification is designed to summarize low-frequency comovement between rollover stress and Treasury premia.

Table 1 reports the diagnostic slopes, and the result is a sign reversal in the duration component. Before 2020, higher Treasury supply went together with lower real term premia—the convenience-yield regime in which scarce safe assets are bid up. After 2020 the sign reverses: the duration premium rises with rollover stress, and the post-2020 marginal premium is positive and economically large. The rollover component, by contrast, slopes upward in both regimes. The reversal is the empirical signature of the model's threshold mechanism: once rollover exposure is large relative to capacity, additional debt raises the marginal cost of producing safe government claims. The post-2020 total slope is

$$s_Q^T = s_Q^R + s_Q^D \approx 40.3 + 56.6 = 96.9 \text{ basis points per unit of } Q. \quad (60)$$

Table 1: Diagnostic premium slopes

	$s_Q^R$	$s_Q^D$	$s_Q^T = s_Q^R + s_Q^D$
Pre-2020	+39.2	-191.1	-151.9
Post-2020	+40.3	+56.6	+96.9

Notes: Entries are point slopes in basis points per unit of  $Q$ . The post-2020 slope is the slope relevant for the current sign diagnostic. The pre-2020 slope comes from a short low-stress sample dominated by convenience-yield forces.

### 7.3 The sign diagnostic

The empirical counterpart of the model's marginal safety-production cost is

$$S_Q \equiv s^T + Qs_Q^T. \quad (61)$$

The first term is the premium level. The second term is the inframarginal repricing term: a marginal increase in rollover stress changes the premium applied to the outstanding stock. In the model, the stock-repricing term maps into the  $Q$ -derivative through  $Q = \mu b/N$ : for a marginal debt change at a fixed maturity share and capacity measure,  $dQ/db = \mu/N = Q/b$ , so  $bs_b$  corresponds to  $Qs_Q^T$ . The empirical slope  $s_Q^T$  is a time-series point slope, not a separately identified structural partial derivative. It is used as a debt-pressure proxy for the model derivative because the post-2020 rise in  $Q_t$  is driven mainly by the increase in short-maturity privately held Treasury exposure, while the dealer-parent capacity measure is relatively flat by comparison. The diagnostic is invariant to a constant rescaling of measured capacity: if  $N_t$  is rescaled by a constant,  $Q_t$  and the estimated slope  $s_Q^T$  move in offsetting directions, leaving  $Q_t s_Q^T$  unchanged. Capacity therefore enters through the time-series shape of  $Q_t$ , rather than through the units in which  $N_t$  is measured. The corresponding sign diagnostic is

$$\Omega \equiv A - r - S_Q. \quad (62)$$

The economy is on the increasing branch when  $\Omega > 0$  and on the declining branch when  $\Omega < 0$ .

At the end of the sample, the one-year privately held maturing Treasury stock divided by dealer-parent holdable Treasury capacity gives

$$Q = 2.59.$$

The premium level is measured from the same end-of-sample premium components used in the slope exercise: the rollover component is about  $s^R = 29$  basis points and the duration component is about  $s^D = 8$  basis points. Hence

$$s^T = s^R + s^D \approx 29 + 8 = 37 \text{ basis points.}$$

Using the post-2020 marginal slope  $s_Q^T = 96.9$  basis points per unit of  $Q$ , the marginal safety-production cost is

$$S_Q = s^T + Qs_Q^T \approx 37 + 2.59 \times 96.9 \approx 288 \text{ basis points.} \quad (63)$$

The macro safe-rate gap  $A - r$  is the direct aggregate-demand value of an additional unit of safe wealth net of standard debt service. Numerically, the baseline  $A - r = 330$  basis points corresponds to an annual marginal propensity to consume out of safe wealth of roughly 4.3 percent, net of a real safe rate near 1 percent. The calculation uses this value as the baseline macro safe-rate gap;  $\Omega$  depends on the difference between this gap and  $S_Q$ , so the sensitivity table can be read as showing how close the calculation is to the peak for alternative gaps between  $A - r$  and the measured marginal safety-production cost. With  $A - r = 330$  basis points,

$$\Omega = A - r - S_Q \approx 330 - 288 = 42 \text{ basis points.} \quad (64)$$

The headline calculation leaves  $\Omega$  positive but small. Of the 288 basis points of marginal safety-production cost, the premium level accounts for about 37; the remaining 251 come from the inframarginal term  $Qs_Q^T$ . This inframarginal term is a steady-state rollover cost: after the debt stock has passed through the maturity structure, the higher safety-production premium induced by marginal debt applies to the full rolled-over stock. The diagnostic is large not because premia are high today, but because a marginal unit of debt eventually raises the safety cost of the entire stock. Figure 4 plots the resulting  $\Omega_t$  path: well above zero in the low-stress regime, it falls as the marginal premium turns positive and reaches +42 basis points at the current debt level.

## 7.4 Sensitivity and interpretation

The most informative sensitivity is to the marginal premium slope. Holding  $Q = 2.59$ ,  $s^T = 37$  basis points, and  $A - r = 330$  basis points fixed, the diagnostic is

$$\Omega = 330 - (37 + 2.59s_Q^T).$$

Table 2 reports the implied values.

Table 2: Sensitivity to the marginal premium slope

$s_Q^T$	$S_Q = s^T + Qs_Q^T$	$\Omega = A - r - S_Q$
75.0	231	99
90.0	270	60
96.9	288	42
110.0	322	8
113.1	330	0
125.0	361	-31
150.0	426	-96

Notes: Entries are basis points. The table holds  $Q = 2.59$ ,  $s^T = 37$  basis points, and  $A - r = 330$  basis points fixed. The headline post-2020 slope is  $s_Q^T = 96.9$ .

The headline diagnostic leaves the economy just inside the increasing branch. The sensitivity table shows how narrow the margin is: a marginal slope near 113 basis points per unit of  $Q$ ,

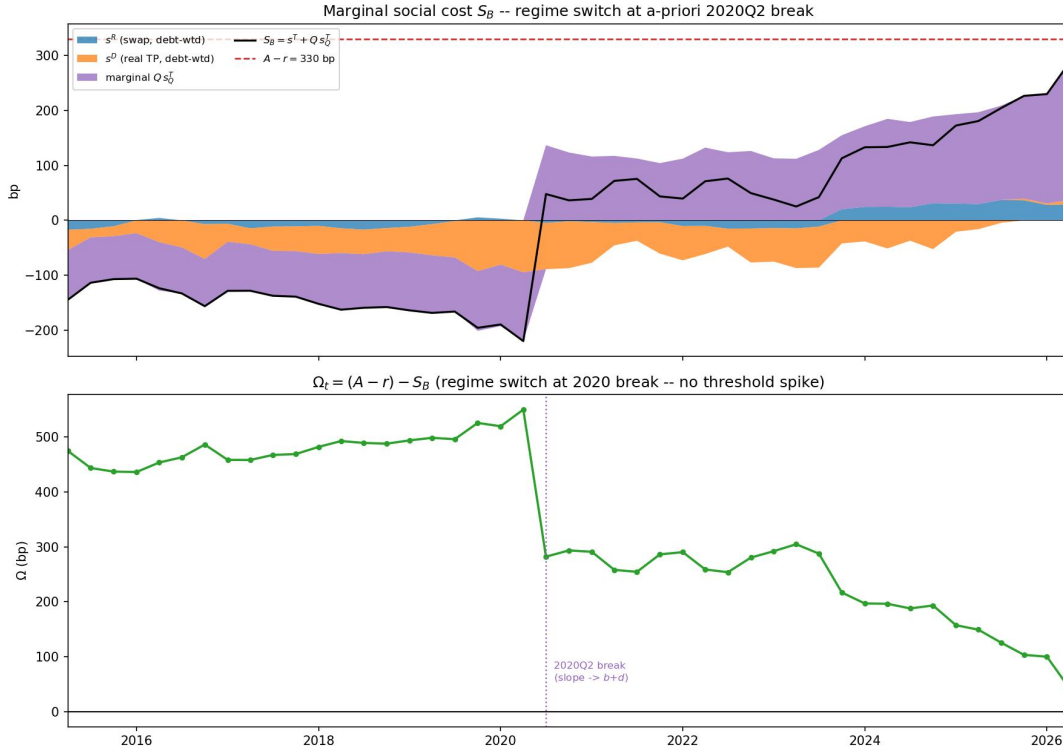


Figure 4: Construction of the empirical sign diagnostic. The upper panel decomposes the marginal safety-production cost  $S_Q = s^T + Qs_Q^T$  into the rollover component  $s^R$ , the duration component  $s^D$ , and the inframarginal repricing term  $Qs_Q^T$ . The dashed horizontal line is the baseline macro safe-rate gap,  $A - r = 330$  basis points. The lower panel plots  $\Omega_t = A - r - S_Q$ , the empirical counterpart of  $H(b, N)$  in Proposition 1, along the realized debt path. The vertical line marks the a-priori 2020Q2 break used in (59). The diagnostic remains above zero but reaches +42 basis points at the current debt level.

against the headline value of 97, brings  $\Omega$  to zero. The signed measurement forces in the calculation point in the same direction.

The first force is convexity. The theory predicts a convex premium schedule under capacity pressure, while the linear specification averages the slope over post-2020 observations. If the current observation lies on the steeper part of the schedule, the local  $s_Q^T$  exceeds the regression average. The second force is QE attenuation: asset purchases absorb duration and collateral pressure, compressing the premium response to rollover stress. The third force is convenience compression: Treasury convenience services reduce measured Treasury yields and therefore compress the observed rollover component. The fourth force is level measurement: the diagnostic uses the observed premium level  $s^T$ , while a fitted high-stress premium level would raise  $S_Q$ . Table 2 varies only the slope while holding the premium level fixed.

Each force raises the marginal cost relative to the linear diagnostic. Under these signed adjustments, the headline  $\Omega$  of 42 basis points is an upper-bound reading of the remaining safe-rate gap. On the most generous reading, the economy sits close to the peak of the safe-debt

Laffer curve; once the convexity of the premium schedule, the attenuation from asset purchases, and the compression from convenience demand are taken into account, the diagnostic moves toward, and plausibly onto, the declining branch.

## 8 Conclusion

The paper's central result is a safe-debt Laffer curve: beyond a point, additional safe public debt lowers rather than raises the aggregate-demand contribution of public debt and the equilibrium safe rate. The mechanism is fiscal-financial. The safety quality of public debt is endogenous: safe-asset production depends jointly on sovereign solvency, debt-service arithmetic, and the financial system's capacity to roll over government debt and preserve its safety. When that capacity is abundant, additional debt expands the stock of safe claims and raises aggregate demand and the equilibrium safe rate. When it is scarce, the fiscal cost of producing safety can dominate the wealth effect: the added claims can remain safe, while their marginal contribution to aggregate demand and the safe rate turns negative. Preliminary diagnostic calculations using Treasury rollover exposure, dealer-parent intermediation capacity, and Treasury premium components place the United States close to the peak of this curve.

This perspective links the paper's three policy margins. Maturity policy changes rollover exposure and the price feedback embedded in longer claims; financial capacity changes the cost of absorbing that exposure; and the fiscal rule determines how the cost of safety production is financed in steady state. The balance-sheet multiplier links these margins by feeding valuation losses back into rollover premia. Together, they determine the aggregate-demand support that the economy obtains from additional safe public debt.

The aggregate-demand nature of the Laffer curve gives the result a direct welfare interpretation in environments where the safe rate cannot fully adjust. In the New Keynesian safe-asset environment studied by [Caballero and Farhi \(2018\)](#), with a potentially binding lower bound, additional debt issuance on the declining branch reduces the aggregate-demand contribution of public debt even though the added claims remain safe. When the lower bound binds, monetary policy cannot fully offset this decline, and the economy can enter a safety trap: gross safe debt remains abundant while its marginal demand contribution is negative. The logic also parallels [Spence \(1975\)](#)'s analysis of monopoly quality choice and regulation.<sup>1</sup> In Spence's setting, the monopolist's decision is governed by the marginal buyer, while the social value of changing quality depends on inframarginal buyers; in the present model, the analogous wedge is the stock-wide rollover-cost term  $bs_b(b, N)$ . This margin connects the mechanism to the aggregate-demand externalities in [Farhi and Werning \(2016\)](#) and [Korinek and Simsek \(2016\)](#).

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<sup>1</sup>I owe this connection to Matteo Maggiori.

## A Empirical Construction

This appendix records the measurement details behind Section 7. The body uses the empirical exercise as a quantitative sign diagnostic for the model’s marginal safety-production cost. The appendix therefore emphasizes construction, mechanical reproducibility, and the interpretation of the slopes used in the diagnostic.

### A.1 Capacity and rollover exposure

The headline capacity measure is dealer-parent holdable Treasury capacity,

$$N_t^{\text{hold}} = \sum_i \left( \frac{T1_{i,t}}{m} - LE_{i,t} \right) + \sum_i UST_{i,t}, \quad m = 6\%.$$

Here  $T1$  is Tier-1 capital,  $LE$  is total leverage exposure,  $UST$  is Treasuries already held by dealer-parent institutions, and SLR denotes the Supplementary Leverage Ratio. The first term measures the unused balance-sheet amount consistent with an SLR-type leverage constraint at the maintained ratio  $m = 6\%$ . The second term adds Treasuries already warehoused by the same dealer-parent institutions. The measure is deliberately broad: it treats all measured leverage slack as potentially usable for Treasury intermediation. This makes the denominator generous and therefore makes the headline rollover-stress statistic conservative.

The rollover-stress statistic is

$$Q_t = \frac{\mu_t b_t}{N_t}.$$

The headline construction uses a one-year rollover horizon. Thus  $\mu_t b_t$  is the dollar amount of privately held marketable Treasury debt maturing within one year, net of Federal Reserve SOMA holdings. The current value used in Section 7,  $Q = 2.59$ , is the end-of-sample value of this one-year privately held rollover exposure divided by  $N_t^{\text{hold}}$ .

Because the diagnostic  $S_Q = s^T + Qs_Q^T$  is invariant to constant rescalings of  $N_t$ , alternative denominators are informative through the time-series shape of  $Q_t$ , not through the level of the denominator. A constant rescaling  $N_t \rightarrow N_t/k$  implies  $Q_t \rightarrow kQ_t$ , while a slope measured per unit of  $Q_t$  rescales by  $1/k$ , leaving  $Qs_Q^T$  unchanged. The substantive issue is therefore whether an alternative capacity concept changes the time-series movements of  $Q_t$ , especially around the post-2020 issuance episode.

Figure 5 plots the capacity measure, rollover stress, and premium components. The capacity series is much flatter than the post-2020 increase in privately held short-maturity exposure. This is the empirical reason the time-series slope with respect to  $Q_t$  is used in the body as a debt-pressure proxy for the theoretical marginal premium response.

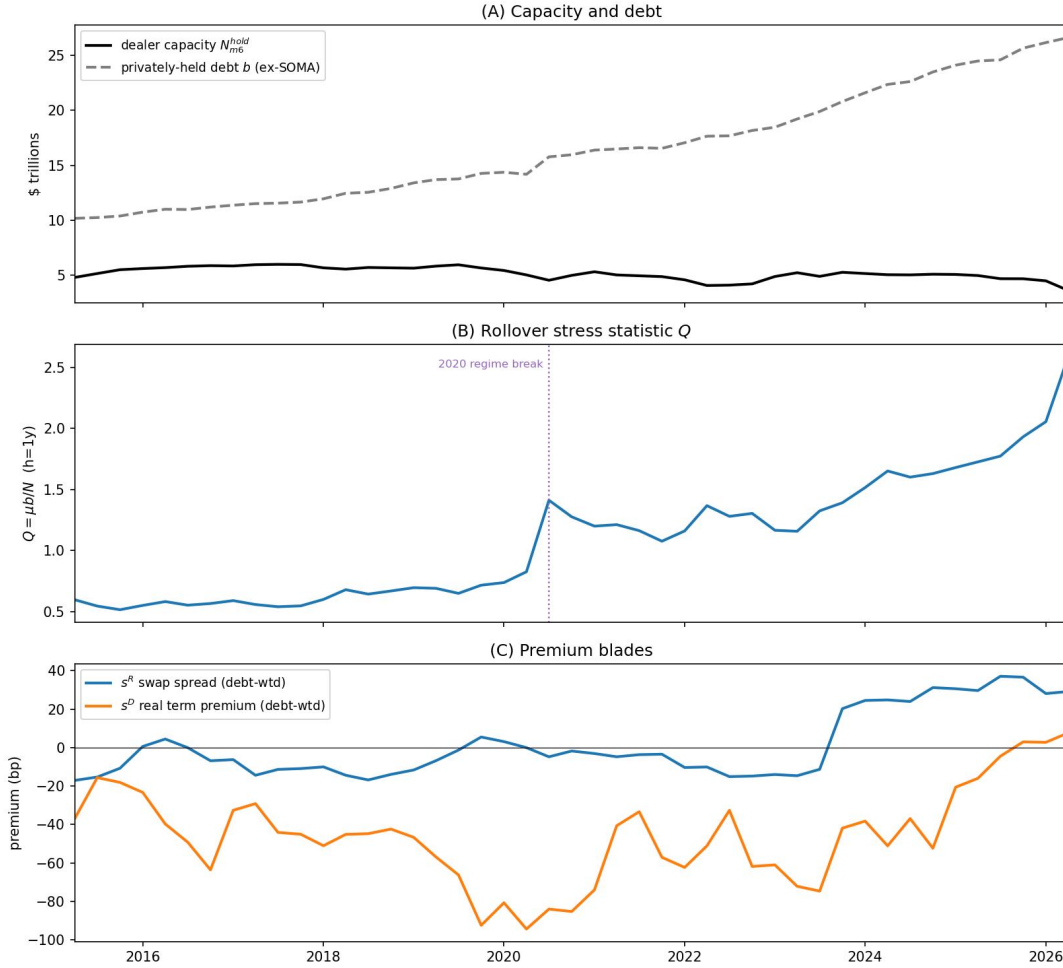


Figure 5: Capacity, rollover exposure, and premium components. Panel A reports privately held marketable Treasury debt net of SOMA holdings and the dealer-parent capacity measure  $N_t^{\text{hold}}$ . Panel B reports the resulting rollover-stress statistic  $Q_t = \mu_t b_t / N_t$ . Panel C reports the rollover component  $s^R$ , measured from Treasury-swap spreads, and the duration component  $s^D$ , measured from debt-weighted real term premia. The vertical line marks the 2020Q2 break used in the diagnostic regression.

## A.2 Premium components

The total premium used in the diagnostic is

$$s_t^T = s_t^R + s_t^D.$$

The rollover component  $s^R$  is measured using the Treasury-swap spread, defined as the Treasury par yield minus the maturity-matched par swap rate and aggregated across maturity buckets using outstanding Treasury debt weights. The sign convention is chosen so that a higher  $s^R$  means cash Treasuries are cheaper relative to swaps, consistent with a higher balance-sheet cost of warehousing Treasuries. This object is a cash-versus-swap balance-sheet wedge. It does not

include duration-risk compensation by construction. At the current observation, this component is about 29 basis points.

The Treasury-swap spread replaces the cash–futures basis as the headline quarterly proxy. Both objects compare cash Treasury exposure with synthetic exposure and therefore load on balance-sheet costs. The cash–futures basis is useful at high frequency during stress episodes, but at quarterly frequency it is affected by cheapest-to-deliver effects, delivery options, financing conditions, repo specialness, and price asynchrony. The swap spread is a more stable quarterly wedge for the stock of Treasury balance-sheet pressure.

The duration component  $s^D$  is measured as a debt-weighted real term premium on the maturity structure of outstanding Treasuries. The nominal term premium is interpolated across maturities and adjusted for an inflation-risk-premium component to match the real object in the model. Thus  $s^R$  captures the balance-sheet or capacity wedge, while  $s^D$  captures duration compensation on the same outstanding-debt maturity distribution. At the current observation,  $s^D$  is about 8 basis points, so the observed total premium level used in the headline calculation is  $s^T = s^R + s^D \approx 37$  basis points.

### A.3 Regression specification and slope construction

The body uses quarterly data from 2015Q1 through 2026Q1, giving 45 observations. For each component  $s_t \in \{s_t^R, s_t^D\}$ , the diagnostic regression is

$$s_t = a + bQ_t + cD_t + d(Q_t D_t) + eQE_t + fQT_t + \varepsilon_t, \quad D_t = \mathbf{1}\{t \geq 2020Q2\}.$$

The pre-2020 point slope is  $b$ . The post-2020 point slope, used in the current diagnostic, is  $b + d$ . The break date is fixed a priori at 2020Q2, the start of the high-issuance regime used in the body. The policy indicators are defined mechanically from Federal Reserve Treasury holdings:  $QE_t$  denotes quarters in which SOMA Treasury holdings are expanding, and  $QT_t$  denotes quarters in which SOMA Treasury holdings are running off. These indicators absorb level shifts associated with Federal Reserve balance-sheet regimes; they are not used as slope interactions.

Table 3 reports the coefficient values underlying the slopes in Table 1. The body uses only the point slopes for the sign diagnostic. The Newey–West  $t$ -statistics in the appendix are included for transparency about the fitted regression, not to turn Section 7 into a complete econometric evidence section.

Table 3: Diagnostic regression details, 2015Q1–2026Q1

	$s^R$	$s^D$
$Q$	39.2 (1.18)	-191.1 (-4.48)
$D$	-17.8 (-0.46)	-194.8 (-6.16)
$Q \times D$	1.1 (0.03)	247.7 (5.65)
$QE$	-5.6 (-0.59)	-7.0 (-0.78)
$QT$	-1.1 (-0.19)	-1.8 (-0.26)
Constant	-31.0 (-1.64)	71.0 (2.75)
Pre-2020 slope	39.2 (1.18)	-191.1 (-4.48)
Post-2020 slope	40.3 (2.94)	56.6 (6.60)
$R^2$	0.66	0.61

Notes: Entries are coefficients in basis points; Newey–West  $t$ -statistics with four quarterly lags are in parentheses. The diagnostic uses the post-2020 point slopes 40.3 and 56.6, so  $s_Q^T = 96.9$  basis points per unit of  $Q$ .

## A.4 Persistence and diagnostic interpretation

The baseline specification is in quarterly levels. This choice matches the mechanism: rollover stress, capacity pressure, and Treasury premia are slow-moving objects, and the diagnostic requires a low-frequency premium response to the level of  $Q_t$ , not a high-frequency response to quarter-to-quarter changes. Differencing would remove much of the variation that is economically relevant for the safe-debt Laffer condition.

The levels slope should nevertheless be interpreted as a diagnostic slope rather than as a fully identified structural partial derivative. The empirical regression follows the realized path of  $Q_t$ , while the theory’s derivative holds maturity and capacity fixed when mapping  $bs_b$  into  $Qs_Q^T$ . The mapping is informative in this sample because the large post-2020 variation in  $Q_t$  comes primarily from privately held short-maturity Treasury exposure, while measured dealer-parent capacity is relatively flat. A full empirical strategy would need separate variation in debt, maturity, and capacity. Section 7 uses the observed slope as a quantitative sign diagnostic.

## A.5 Robustness logic and alternative proxies

Two robustness issues matter most for the diagnostic. First, alternative capacity denominators affect the calculation through the time-series shape of  $Q_t$ . Pure level changes in measured capacity are absorbed by the scale invariance of  $Qs_Q^T$ . Second, the headline dealer-parent capacity measure is broad. A narrower capacity concept raises measured stress if it changes the path of  $Q_t$ , especially after 2020, and therefore raises the marginal safety-production cost in the diagnostic.

The cash–futures basis provides a high-frequency stress proxy for the same balance-sheet channel, but it is not the headline quarterly proxy for the stock of Treasury balance-sheet pressure. A high-frequency capacity proxy can be constructed from dealer-parent equity values. This proxy is useful for event studies, because equity values move within the quarter and therefore capture capacity contractions that are invisible in quarterly regulatory filings. The paper uses the

regulatory capacity measure for the headline calculation because the safe-debt Laffer diagnostic is a low-frequency stock calculation.

## A.6 Data sources

Table 4: Data sources for the empirical diagnostic

Series	Source	Frequency
Marketable debt and maturing share by maturity	Monthly Statement of the Public Debt and Treasury Fiscal Data	Monthly
Federal Reserve Treasury holdings	Federal Reserve SOMA holdings by CUSIP	Weekly
Tier-1 capital, leverage exposure, and Treasuries held by dealer parents	FR Y-9C regulatory filings	Quarterly
Treasury par yields and maturity-matched par swap rates	Treasury and swap-curve data	Daily
Nominal term premia	Term-premium estimates by maturity	Daily
Inflation-risk-premium adjustment	Inflation-risk-premium estimates	Monthly
Dealer-parent equity values for high-frequency capacity proxy	Dealer-parent equity-market data	Daily

## B Households, Insurance, and Perpetual-Youth Demand

This appendix derives the household side of the model. The derivation proceeds in three steps. First, households choose consumption, government-bond holdings, and rollover insurance under Epstein–Zin preferences with finite aversion to aggregate payoff risk. Second, the first-order conditions for bonds and insurance show how the insurance decision prices rollover losses. Third, the limiting problem as risk aversion to aggregate payoff states becomes infinite is solved at full insurance. In that limit the insured government-debt claim has the same payoff as the safe annuitized asset, the government yield contains the insurance premium, and the household problem collapses to the perpetual-youth block used in the main text.

### B.1 Preferences, states, and controls

Time is continuous. A household alive at date  $t$  dies with Poisson hazard  $\lambda$ . Death is independent of aggregate payoff states. Actuarially fair annuity markets rebate the wealth of deceased households to survivors, so surviving households earn the mortality return  $\lambda$  on financial wealth. The subjective discount rate is  $\rho$ , and we define

$$A \equiv \rho + \lambda. \quad (65)$$

The household has Epstein–Zin preferences (Epstein and Zin, 1989) with unit intertemporal elasticity of substitution and risk aversion  $\sigma$  over aggregate payoff states. Let  $\omega \in \Omega_t$  denote the aggregate payoff state at date  $t$ . The state  $\omega$  indexes rollover conditions for government debt and therefore the losses on uninsured government-debt positions. In the stationary environment, the continuation value can be written as

$$V_t = \int_t^\infty e^{-A(u-t)} \log C_u^\sigma du, \quad (66)$$

where  $C_u^\sigma$  is the certainty-equivalent consumption aggregator across aggregate payoff states at date  $u$ :

$$C_u^\sigma = \left( \mathbb{E}_u [c_u(\omega)^{1-\sigma}] \right)^{1/(1-\sigma)}. \quad (67)$$

The logarithm over time gives unit intertemporal elasticity. The parameter  $\sigma$  governs aversion to aggregate payoff risk within a date.

The household state variable is financial wealth  $a_i$ . At each date the household chooses state-contingent consumption  $c_i(\omega)$ , government-bond holdings  $b_i$ , and units of rollover insurance  $m_i$  written on those holdings:

$$\begin{aligned} c_i(\omega) &\geq 0, && \text{state-contingent consumption,} \\ b_i &\geq 0, && \text{government-bond holdings,} \\ m_i &\in [0, b_i], && \text{insured units of government-bond holdings.} \end{aligned}$$

The uninsured exposure is

$$u_i \equiv b_i - m_i \geq 0. \quad (68)$$

The portfolio also contains the residual safe annuitized asset  $a_i - b_i$ , which earns the safe rate  $r$ . The government bond pays yield  $r^G$ . Rollover insurance costs  $\pi$  per insured unit. An uninsured unit of government debt suffers a rollover-stress loss  $\ell(\omega) \geq 0$  in payoff state  $\omega$ .

The government yield  $r^G$  and the insurance price  $\pi$  are linked by the insured-limit equilibrium condition derived below.

## B.2 Budget constraint and finite- $\sigma$ problem

Disposable private income is fixed and denoted by

$$\chi \equiv 1 - \tau. \quad (69)$$

Given  $(a_i, b_i, m_i)$ , the household's state-contingent wealth law is

$$\dot{a}_i(\omega) = (r + \lambda)(a_i - b_i) + (r^G + \lambda)b_i - \pi m_i - \ell(\omega)(b_i - m_i) + \chi - c_i(\omega). \quad (70)$$

The first term is the return on residual safe wealth. The second term is the government yield plus the mortality return on government-bond wealth. The third term is the insurance premium. The fourth term is the rollover loss on the uninsured position.

It is useful to rewrite the same constraint in terms of insured and uninsured government-debt positions. Since  $b_i = m_i + u_i$ , equation (70) becomes

$$\dot{a}_i(\omega) = (r + \lambda)a_i + \chi - c_i(\omega) + (r^G - r - \pi) m_i + (r^G - r - \ell(\omega)) u_i. \quad (71)$$

This expression separates two portfolio payoffs. An insured unit of government debt pays the excess payoff  $r^G - r - \pi$  relative to the safe asset. An uninsured unit pays the state-contingent excess payoff  $r^G - r - \ell(\omega)$ . The insurance decision is the choice of how much of the government-bond position is converted from the uninsured payoff into the insured payoff.

For finite  $\sigma$ , the household solves

$$\begin{aligned} V(a_i) = & \sup_{\{c_i(\omega), m_i, u_i\}} \int_t^\infty e^{-A(u-t)} \log \left[ \left( \mathbb{E}_u c_i(u, \omega)^{1-\sigma} \right)^{1/(1-\sigma)} \right] du & (72) \\ \text{s.t. } & \dot{a}_i(u, \omega) = (r + \lambda)a_i(u) + \chi - c_i(u, \omega) \\ & + (r^G - r - \pi) m_i(u) + (r^G - r - \ell_u(\omega)) u_i(u), \\ & m_i(u) \geq 0, \quad u_i(u) \geq 0, \quad b_i(u) = m_i(u) + u_i(u), \end{aligned}$$

plus the standard no-Ponzi condition. This formulation is equivalent to choosing  $(b_i, m_i)$  with  $0 \leq m_i \leq b_i$ , but it makes the two relevant margins explicit: the insured position  $m_i$  and the uninsured rollover-risk position  $u_i$ .

## B.3 Finite- $\sigma$ first-order conditions

The date- $u$  certainty equivalent determines the state weights used to value state-contingent portfolio payoffs. Differentiating (67) gives

$$\frac{\partial C_u^\sigma}{\partial c_i(\omega)} = \left( \mathbb{E} c_i(\omega)^{1-\sigma} \right)^{\sigma/(1-\sigma)} c_i(\omega)^{-\sigma}. \quad (73)$$

The common positive term is the same across states, so the normalized within-date state-price weight is

$$\mathcal{M}_i^\sigma(\omega) \equiv \frac{c_i(\omega)^{-\sigma}}{\mathbb{E}[c_i(\omega)^{-\sigma}]}.$$
 (74)

Because the intertemporal aggregator is logarithmic in  $C_u^\sigma$ , the within-date certainty-equivalent weights price the marginal state-contingent payoffs generated by the bond and insurance choices at that date.

Consider first the uninsured rollover-risk position  $u_i$ . A marginal increase in  $u_i$  pays  $r^G - r - \ell(\omega)$  relative to the safe asset. The Kuhn–Tucker condition is

$$\mathbb{E} [\mathcal{M}_i^\sigma(\omega)(r^G - r - \ell(\omega))] = 0, \quad \text{if } u_i > 0, \quad (75)$$

$$\mathbb{E} [\mathcal{M}_i^\sigma(\omega)(r^G - r - \ell(\omega))] \leq 0, \quad \text{if } u_i = 0. \quad (76)$$

Thus the government yield must compensate households for the risk-adjusted rollover loss when they voluntarily hold uninsured exposure.

Now consider the insured position  $m_i$ . A marginal increase in  $m_i$  pays  $r^G - r - \pi$  relative to the safe asset in every state. The Kuhn–Tucker condition is

$$r^G - r - \pi = 0, \quad \text{if } m_i > 0, \quad (77)$$

$$r^G - r - \pi \leq 0, \quad \text{if } m_i = 0. \quad (78)$$

This is the no-arbitrage condition for an insured government-debt claim. When insured claims are held in positive amount, their net return equals the safe return.

The insurance purchase condition can also be written holding total bond holdings  $b_i$  fixed. Increasing  $m_i$  by one unit reduces  $u_i$  by one unit. The state-contingent marginal payoff is

$$\ell(\omega) - \pi. \quad (79)$$

The finite- $\sigma$  Kuhn–Tucker conditions for insurance, conditional on  $b_i$ , are therefore

$$\mathbb{E} [\mathcal{M}_i^\sigma(\omega)(\ell(\omega) - \pi)] = 0, \quad \text{if } 0 < m_i < b_i, \quad (80)$$

$$\mathbb{E} [\mathcal{M}_i^\sigma(\omega)(\ell(\omega) - \pi)] \leq 0, \quad \text{if } m_i = 0, \quad (81)$$

$$\mathbb{E} [\mathcal{M}_i^\sigma(\omega)(\ell(\omega) - \pi)] \geq 0, \quad \text{if } m_i = b_i. \quad (82)$$

For finite  $\sigma$ , the household compares the insurance premium with the risk-adjusted value of the rollover loss. The household may leave some exposure uninsured when the premium saving in good states compensates for the loss in bad states under the state-price weights  $\mathcal{M}_i^\sigma$ .

## B.4 The limit as risk aversion becomes infinite

Let

$$\ell^{\max} \equiv \text{ess sup}_{\omega \in \Omega} \ell(\omega) \quad (83)$$

be the worst-state rollover loss. In the equilibrium studied in the main text, the insurance premium  $\pi$  is the competitive average support cost across rollover states. With a nondegenerate distribution of rollover losses, this average is strictly below the worst-state loss:

$$\pi < \ell^{\max}. \quad (84)$$

As  $\sigma \rightarrow \infty$ , the certainty equivalent satisfies

$$\lim_{\sigma \rightarrow \infty} C_u^\sigma = \operatorname{ess\,inf}_{\omega \in \Omega_u} c_u(\omega), \quad (85)$$

and the normalized state-price weights concentrate on states that minimize consumption:

$$\mathcal{M}_i^\sigma(\omega) \implies \mathcal{M}_i^\infty(\omega), \quad \operatorname{supp} \mathcal{M}_i^\infty \subseteq \arg \min_{\omega \in \Omega_u} c_i(\omega). \quad (86)$$

Fix any allocation with positive uninsured exposure,  $u_i > 0$ . Then consumption resources are decreasing in  $\ell(\omega)$ , so the worst consumption state is a state with loss  $\ell^{\max}$ . The one-sided gain from increasing insurance and reducing uninsured exposure, evaluated at this fixed allocation, is therefore

$$\lim_{\sigma \rightarrow \infty} \mathbb{E} [\mathcal{M}_i^\sigma(\omega)(\ell(\omega) - \pi)] = \ell^{\max} - \pi > 0. \quad (87)$$

No allocation with  $u_i > 0$  solves the limiting max-min problem. The limiting allocation is fully insured:

$$m_i = b_i, \quad u_i = 0. \quad (88)$$

In the insured limit, the positive aggregate supply of government debt is held as insured government-debt claims. Equation (77) then implies

$$r^G = r + \pi. \quad (89)$$

A fully insured unit of government debt earns the government yield and pays the insurance premium, so its net return is

$$r^G - \pi = r. \quad (90)$$

The government yield spread  $r^G - r$  is the compensation for the rollover-insurance service embedded in the safe claim. In aggregate, the spread finances the premium flow on the insured debt stock.

Substituting  $u_i = 0$  and  $r^G - r - \pi = 0$  into (71) removes the state-contingent payoff and gives

$$\dot{a}_i = (r + \lambda)a_i + \chi - c_i. \quad (91)$$

In the insured limit, consumption is state-independent with respect to rollover losses. Therefore  $C_u^\sigma = c_u$  for every  $\sigma$ , including the infinite-risk-aversion limit, and the household problem reduces to the standard time-separable log perpetual-youth problem.

## B.5 Human wealth and consumption in the insured limit

Human wealth is the present discounted value of fixed disposable income while alive:

$$h_t = \int_t^\infty \exp \left[ - \int_t^u (r_v + \lambda) dv \right] \chi du. \quad (92)$$

Differentiating (92) gives the human-wealth law of motion

$$\dot{h}_t = (r_t + \lambda)h_t - \chi. \quad (93)$$

In a steady state with constant safe rate  $r$ ,

$$h(r) = \frac{\chi}{r + \lambda}. \quad (94)$$

With taxes fixed in the household problem, human wealth is determined by disposable income and the equilibrium safe rate. Government spending and debt affect human wealth through their effect on that rate.

Let total wealth be

$$w_i = a_i + h. \quad (95)$$

Combining (91) and (93) gives

$$\dot{w}_i = (r + \lambda)w_i - c_i. \quad (96)$$

The Euler equation for the log perpetual-youth problem is

$$\frac{\dot{c}_i}{c_i} = r - \rho. \quad (97)$$

Integrating the Euler equation from date  $t$  forward gives

$$c_i(u) = c_i(t)e^{(r-\rho)(u-t)}. \quad (98)$$

The no-Ponzi condition applied to (96) implies the intertemporal budget constraint

$$w_i(t) = \int_t^\infty e^{-(r+\lambda)(u-t)} c_i(u) du. \quad (99)$$

Substituting (98) into (99) yields

$$w_i(t) = c_i(t) \int_t^\infty e^{-(\rho+\lambda)(u-t)} du = \frac{c_i(t)}{\rho + \lambda}. \quad (100)$$

Therefore

$$c_i(t) = (\rho + \lambda)w_i(t) = Aw_i(t). \quad (101)$$

Aggregating over the unit-mass population and using the fact that fully insured public debt is the aggregate safe financial asset held by households yields

$$c = A(b + h), \quad (102)$$

which is equation (4). Public debt is net wealth because finite horizons imply partial capitalization of taxes paid by future households. Holding the safe rate fixed, human wealth is fixed, and the direct marginal effect of insured public debt on private demand is

$$\left. \frac{\partial c}{\partial b} \right|_r = A. \quad (103)$$

## B.6 Goods-market clearing and the safe rate

In the body of the paper, the insurance premium per unit of debt is denoted by  $s(b, N)$ . This is the aggregate counterpart of  $\pi$  in the household problem. The government yield is  $r + s(b, N)$ , and the government budget constraint with fixed taxes is

$$\tau = g + (r + s(b, N))b. \quad (104)$$

Hence

$$g = \tau - rb - S(b, N), \quad S(b, N) = s(b, N)b. \quad (105)$$

Substituting (94) into (102) and imposing goods-market clearing gives

$$A \left( b + \frac{\chi}{r + \lambda} \right) + \tau - rb - S(b, N) = 1. \quad (106)$$

This is equation (12). It defines the steady-state safe rate implicitly.

Differentiating (106) with respect to  $b$ , holding  $N$  fixed, gives

$$A - A \frac{\chi}{(r + \lambda)^2} \frac{dr}{db} - r - b \frac{dr}{db} - S_b(b, N) = 0. \quad (107)$$

Collecting the terms in  $dr/db$  yields

$$\left[ b + \frac{A\chi}{(r + \lambda)^2} \right] \frac{dr}{db} = A - r - S_b(b, N). \quad (108)$$

Therefore

$$\frac{dr}{db} = \frac{A - r - S_b(b, N)}{b + A\chi/(r + \lambda)^2}. \quad (109)$$

The denominator is positive. Hence the sign of the safe-rate response is the sign of the fixed-rate aggregate-demand effect,

$$\text{sign} \left( \frac{dr}{db} \right) = \text{sign} (A - r - S_b(b, N)). \quad (110)$$

This is the rate result used in the main text.

## C Rollover Premium and Marginal Safety Cost

This appendix derives the safety premium and the marginal safety-production cost in the general finite- $\delta$  environment. The benchmark formulas in Section 3 are obtained by taking  $\mu(\delta) = 1$ . The derivation is included to make clear which objects are primitive and which are equilibrium prices. In this appendix,  $\delta$  is held fixed when differentiating with respect to  $b$  and  $N$  unless the derivative explicitly contains  $\delta$ .

The primitive state variables are the public-debt stock  $b$ , financial capacity  $N$ , and the participation state  $\alpha$ . The rollover intensity is  $\delta$ , so the rollover exposure over the insurance

episode is  $\mu(\delta)b$ . Active financial capacity in participation state  $\alpha$  is  $\alpha N$ . The shortfall is therefore

$$X(b, N, \alpha) = \max\{\mu(\delta)b - \alpha N, 0\}. \quad (111)$$

The support burden is  $\Gamma(X)$ . Active participants share the burden, so the per-participant assessment is

$$a(b, N, \alpha) = \frac{\Gamma(X(b, N, \alpha))}{\alpha}. \quad (112)$$

The global-game foundation in Appendix G implies that the marginal rollover insurer prices the assessment under a uniform distribution over  $\alpha \in [\underline{\alpha}, 1]$ . Hence

$$s(b, N) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^1 a(b, N, \alpha) d\alpha. \quad (113)$$

Since  $a(b, N, \alpha) = 0$  whenever  $\alpha \geq q(b, N)$ , where

$$q(b, N) = \frac{\mu(\delta)b}{N}, \quad (114)$$

this becomes

$$s(b, N) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N)} \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} d\alpha, \quad \bar{q}(b, N) = \min\{q(b, N), 1\}. \quad (115)$$

When  $q(b, N) \leq \underline{\alpha}$ , the inelastic base absorbs the rollover exposure in every state, the integral is empty, and  $s(b, N) = 0$ . The interesting region is the interior stress region,

$$\underline{\alpha} < q(b, N) < 1. \quad (116)$$

In this region, the upper integration limit is  $q(b, N)$ , and the shortfall is zero at the boundary:

$$\mu(\delta)b - q(b, N)N = 0. \quad (117)$$

Because  $\Gamma(0) = 0$ , the moving-boundary term in Leibniz differentiation is zero. Differentiating (115) with respect to  $b$  gives

$$\begin{aligned} s_b(b, N) &= \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \frac{\Gamma'(\mu(\delta)b - \alpha N)\mu(\delta)}{\alpha} d\alpha \\ &= \frac{\mu(\delta)}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} d\alpha > 0. \end{aligned} \quad (118)$$

The sign is positive because more debt raises the rollover shortfall in every stress state.

Differentiating (115) with respect to  $N$  gives

$$\begin{aligned} s_N(b, N) &= \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \frac{\Gamma'(\mu(\delta)b - \alpha N)(-\alpha)}{\alpha} d\alpha \\ &= -\frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \Gamma'(\mu(\delta)b - \alpha N) d\alpha < 0. \end{aligned} \quad (119)$$

The sign is negative because a stronger financial system raises absorption capacity in every participation state.

The total fiscal cost of producing safety is the per-unit excess yield times the insured stock:

$$S(b, N) = bs(b, N). \quad (120)$$

Its first derivative is

$$S_b(b, N) = s(b, N) + bs_b(b, N). \quad (121)$$

This is the key marginal-cost formula. The first term is the excess yield on the marginal unit. The second term is the increase in the premium on all outstanding insured debt. The second term appears because the premium is a stock-wide market-clearing price for rollover insurance.

The second derivative is

$$S_{bb}(b, N) = 2s_b(b, N) + bs_{bb}(b, N). \quad (122)$$

To compute  $s_{bb}$ , differentiate (118). There are two pieces. The interior derivative is

$$\frac{\partial}{\partial b} \left[ \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} \right] = \frac{\mu(\delta)\Gamma''(\mu(\delta)b - \alpha N)}{\alpha}. \quad (123)$$

The moving-upper-limit term evaluates the integrand in (118) at  $\alpha = q(b, N)$ . Since the shortfall is zero at that boundary, this term is

$$q_b(b, N) \frac{\Gamma'(0)}{q(b, N)}. \quad (124)$$

Therefore

$$s_{bb}(b, N) = \frac{\mu(\delta)}{1 - \underline{\alpha}} \left[ \mu(\delta) \int_{\underline{\alpha}}^{q(b, N)} \frac{\Gamma''(\mu(\delta)b - \alpha N)}{\alpha} d\alpha + q_b(b, N) \frac{\Gamma'(0)}{q(b, N)} \right] \geq 0. \quad (125)$$

The first term is the curvature of the support burden inside the stress region. The second term is the boundary effect from expanding the set of stress states. Since  $s_b > 0$  in the stress region, (122) implies

$$S_{bb}(b, N) > 0. \quad (126)$$

Thus the marginal cost of producing safe debt rises once rollover stress is active.

The same premium block also delivers the financial-capacity cross-partial used in the main text. Since

$$S_{bN}(b, N) = s_N(b, N) + bs_{bN}(b, N), \quad (127)$$

we first differentiate (118) with respect to  $N$ . There are again two terms. The interior derivative is

$$\frac{\partial}{\partial N} \left[ \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} \right] = -\Gamma''(\mu(\delta)b - \alpha N). \quad (128)$$

The moving-upper-limit term evaluates the integrand in (118) at  $\alpha = q(b, N)$  and multiplies it by  $q_N(b, N)$ . At the boundary, the shortfall is zero, so this term is

$$\frac{\mu(\delta)}{1 - \underline{\alpha}} q_N(b, N) \frac{\Gamma'(0)}{q(b, N)}. \quad (129)$$

Because  $q(b, N) = \mu(\delta)b/N$ ,

$$q_N(b, N) = -\frac{1}{N}q(b, N). \quad (130)$$

Combining these terms gives

$$\begin{aligned} s_{bN}(b, N) &= -\frac{\mu(\delta)}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \Gamma''(\mu(\delta)b - \alpha N) d\alpha \\ &\quad - \frac{\mu(\delta)}{(1 - \underline{\alpha})N} \Gamma'(0) < 0. \end{aligned} \quad (131)$$

The first term is negative because higher financial capacity lowers the marginal shortfall created by debt in every active stress state. The second term is the boundary effect: higher capacity shrinks the set of participation states in which stress is active. Since  $s_N < 0$  from (119) and  $s_{bN} < 0$  from (131), (127) implies

$$S_{bN}(b, N) < 0. \quad (132)$$

Thus higher financial capacity lowers the marginal fiscal cost of producing safe government-debt claims.

The same differentiation also gives the effect of the rollover intensity. Let  $\mu_\delta(\delta) \equiv e^{-\delta}$ . In the interior stress region,

$$s_\delta(b, N, \delta) = \frac{\mu_\delta(\delta)b}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N, \delta)} \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} d\alpha > 0. \quad (133)$$

A higher rollover intensity raises rollover exposure  $\mu(\delta)b$  and therefore raises the premium. Differentiating  $s_b$  with respect to  $\delta$  gives

$$\begin{aligned} s_{b\delta}(b, N, \delta) &= \frac{\mu_\delta(\delta)}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N, \delta)} \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} d\alpha \\ &\quad + \frac{\mu(\delta)\mu_\delta(\delta)b}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N, \delta)} \frac{\Gamma''(\mu(\delta)b - \alpha N)}{\alpha} d\alpha + \frac{\mu_\delta(\delta)\Gamma'(0)}{1 - \underline{\alpha}}. \end{aligned} \quad (134)$$

The first two terms are interior effects: a higher  $\delta$  raises the level of the shortfall and the marginal effect of debt on that shortfall. The last term is the moving-boundary effect; it appears because a higher  $\delta$  expands the set of participation states in which rollover stress is active. Since  $S_b = s + bs_b$ ,

$$S_{b\delta}(b, N, \delta) = s_\delta(b, N, \delta) + bs_{b\delta}(b, N, \delta) > 0. \quad (135)$$

This derivative characterizes how finite rollover intensity shifts marginal safety-production costs in the general case used by Section 6.

## D Closed-Form Illustration

This appendix solves the premium block in closed form under a linear support burden. The closed form draws the figures and illustrates the general comparative statics derived in Appendix C.

Assume

$$B(\alpha, N) = \alpha N, \quad \Gamma(X) = \ell X, \quad \ell > 0. \quad (136)$$

The stress threshold is the debt level at which the inelastic base just absorbs rollover exposure. The benchmark figures impose  $\mu(\delta) = 1$ , so  $\widehat{b}(N) = \underline{\alpha}N$ . The formulas below are written for the more general finite- $\delta$  case and collapse to the benchmark by setting  $\mu(\delta) = 1$ :

$$\widehat{b}(N) = \frac{\underline{\alpha}N}{\mu(\delta)}. \quad (137)$$

For  $b \leq \widehat{b}(N)$ , no stress state is reached and  $s(b, N) = 0$ . For

$$\widehat{b}(N) < b < \frac{N}{\mu(\delta)}, \quad (138)$$

we have  $\underline{\alpha} < q(b, N) < 1$ . Substituting the linear burden into (115) gives

$$\begin{aligned} s(b, N) &= \frac{\ell}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N)} \frac{\mu(\delta)b - \alpha N}{\alpha} d\alpha \\ &= \frac{\ell}{1 - \underline{\alpha}} \left[ \mu(\delta)b \int_{\underline{\alpha}}^{q(b, N)} \frac{1}{\alpha} d\alpha - N \int_{\underline{\alpha}}^{q(b, N)} 1 d\alpha \right] \\ &= \frac{\ell}{1 - \underline{\alpha}} \left[ \mu(\delta)b \log \left( \frac{\mu(\delta)b}{\underline{\alpha}N} \right) - \mu(\delta)b + \underline{\alpha}N \right]. \end{aligned} \quad (139)$$

The derivative with respect to debt is obtained term by term:

$$\begin{aligned} s_b(b, N) &= \frac{\ell}{1 - \underline{\alpha}} \left[ \mu(\delta) \log \left( \frac{\mu(\delta)b}{\underline{\alpha}N} \right) + \mu(\delta) - \mu(\delta) \right] \\ &= \frac{\ell\mu(\delta)}{1 - \underline{\alpha}} \log \left( \frac{\mu(\delta)b}{\underline{\alpha}N} \right) > 0. \end{aligned} \quad (140)$$

The middle  $+\mu(\delta)$  comes from differentiating  $\mu(\delta)b \log(\mu(\delta)b/(\underline{\alpha}N))$ , and it cancels the derivative of  $-\mu(\delta)b$ . The second derivative is

$$s_{bb}(b, N) = \frac{\ell\mu(\delta)}{(1 - \underline{\alpha})b} > 0. \quad (141)$$

The derivative with respect to financial capacity is

$$s_N(b, N) = -\frac{\ell}{1 - \underline{\alpha}} [q(b, N) - \underline{\alpha}] < 0. \quad (142)$$

The marginal safety-production cost is

$$\begin{aligned}
S_b(b, N) &= s(b, N) + bs_b(b, N) \\
&= \frac{\ell}{1 - \underline{\alpha}} \left[ \mu(\delta)b \log \left( \frac{\mu(\delta)b}{\underline{\alpha}N} \right) - \mu(\delta)b + \underline{\alpha}N \right] + \frac{\ell}{1 - \underline{\alpha}} \left[ \mu(\delta)b \log \left( \frac{\mu(\delta)b}{\underline{\alpha}N} \right) \right] \\
&= \frac{\ell}{1 - \underline{\alpha}} \left[ 2\mu(\delta)b \log \left( \frac{\mu(\delta)b}{\underline{\alpha}N} \right) - \mu(\delta)b + \underline{\alpha}N \right]. \tag{143}
\end{aligned}$$

This expression isolates the repricing force: the log term appears twice, once from the average premium on the stock and once from the effect of debt on the premium itself. When rollover exposure exceeds the full inelastic-support capacity,  $b \geq N/\mu(\delta)$ , the upper integration limit in (115) is one. The corresponding marginal cost is

$$S_b(b, N) = \frac{\ell}{1 - \underline{\alpha}} \left[ 2\mu(\delta)b \log \left( \frac{1}{\underline{\alpha}} \right) - N(1 - \underline{\alpha}) \right], \tag{144}$$

which grows without bound as  $b \rightarrow \infty$ .

In the interior stress region, the cross-partial is

$$S_{bN}(b, N) = \frac{\ell}{1 - \underline{\alpha}} \left[ -\frac{2\mu(\delta)b}{N} + \underline{\alpha} \right] < 0. \tag{145}$$

The sign follows because  $\mu(\delta)b > \underline{\alpha}N$ . In the saturated region  $b \geq N/\mu(\delta)$ , (144) gives  $S_{bN}(b, N) = -\ell < 0$ . Hence higher financial capacity lowers the marginal cost of producing safety throughout the stress region.

If the Laffer peak lies in the interior stress region, it solves

$$A - r = \frac{\ell}{1 - \underline{\alpha}} \left[ 2\mu(\delta)b^L \log \left( \frac{\mu(\delta)b^L}{\underline{\alpha}N} \right) - \mu(\delta)b^L + \underline{\alpha}N \right]. \tag{146}$$

If the peak lies in the saturated region, the right-hand side is instead given by (144) evaluated at  $b = b^L$ . The left-hand side is the direct wealth effect net of standard debt service. The right-hand side is the marginal cost of safety production. A larger  $N$  or larger inelastic base  $\underline{\alpha}$  lowers the right-hand side for a given  $b$  and pushes the peak outward. A larger unit support cost  $\ell$  or rollover intensity  $\delta$  raises the right-hand side through the rollover share  $\mu(\delta)$  and pulls the peak inward.

## E Balance-Sheet Amplification

This appendix derives the local amplification formula discussed in Section 6. The state variables are  $b$ ,  $N$ , and the rollover intensity  $\delta$ . The object being priced is the premium required to compensate the financial system for producing safety. The key loop is that a stress assessment weakens capacity, lower capacity raises future premia, and higher future premia reduce the value of outstanding long debt, which further weakens capacity.

The baseline rollover assessment premium is

$$s^{base}(b, N, \delta) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N, \delta)} \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} d\alpha, \tag{147}$$

where  $\bar{q}(b, N, \delta) = \min\{\mu(\delta)b/N, 1\}$ . Its sensitivity to financial capacity is  $s_N^{base}(b, N, \delta) < 0$ . Thus a capacity loss  $dN < 0$  raises the baseline premium by approximately  $-s_N^{base}(-dN)$ .

A stress assessment lowers future financial capacity. If  $a$  is the state-contingent assessment, the local law of motion is

$$dN^+ = -\kappa_N a, \quad \kappa_N > 0, \quad (148)$$

where  $\kappa_N$  converts assessment losses into units of financial capacity. In the price-loss loop below,  $N$  is measured in units of mark-to-market long-bond capacity, so a unit valuation loss is a unit capacity loss. This is a units convention for the local representation; equivalently, any conversion coefficient for that valuation channel is absorbed into the local sensitivity  $|s_N^T|$ .

## E.1 Price sensitivity of long debt

Let  $P$  denote the price of a Poisson-maturity long bond. The bond pays a unit flow while alive and matures with rollover intensity  $\delta$ . If the safe discount rate is  $r$  and the required total premium is  $s^T$ , the effective discount-plus-hazard rate is

$$\rho^T(b, N, \delta) = r + s^T(b, N, \delta) + \delta. \quad (149)$$

The price is the present value of the unit flow:

$$P = \int_0^\infty e^{-\rho^T u} du = \frac{1}{\rho^T}. \quad (150)$$

A persistent increase  $ds^T$  in the premium raises  $\rho^T$  one-for-one. Therefore

$$d \log P = -\frac{1}{\rho^T} ds^T. \quad (151)$$

The local log-price sensitivity to a persistent premium increase is  $1/\rho^T$ . The persistence qualifier matters: the valuation formula applies to the component of the stress-induced premium increase that investors expect to persist over the effective life of the long bond.

## E.2 The feedback loop

Let  $s^R$  denote the rollover component after balance-sheet feedback. Suppose a unit loss of capacity occurs. The direct effect is to raise the baseline rollover premium by  $-s_N^{base}$ . If long bonds are outstanding, this premium increase lowers their price. From (151), the valuation loss per unit of persistent premium increase is proportional to  $1/\rho^T$ . This valuation loss further erodes capacity and triggers another premium increase.

Let  $m$  denote the local amplification generated by this price-loss loop. If the total premium sensitivity to capacity is  $|s_N^T|$ , then a unit capacity loss raises the total premium by  $|s_N^T|$ . A persistent premium increase of this size lowers the mark-to-market value of the long-bond position by  $|s_N^T|/\rho^T$ , using (151). Since  $N$  is measured in mark-to-market capacity units, this valuation loss is a further capacity loss. The derivative  $|s_N^T|$  is expressed per unit of long-bond

exposure relative to capacity, so the position size is embedded in the local capacity derivative. The price-loss channel therefore has local size

$$m = \frac{|s_N^T|}{\rho^T}. \quad (152)$$

The amplification term  $m$  is the fixed-point amplification generated by the local interaction between capacity pressure, the term-premium component, and the bond-price response. It is not a one-round feedback ratio to be summed geometrically. The self-reference enters through the  $(1 + m)$  term in the capacity derivative of the premium schedule. Thus the converged rollover component is multiplied by  $1 + m$ ,

$$s^R = (1 + m)s^{base}. \quad (153)$$

Holding  $m$  fixed in the local derivative gives

$$-s_N^R = (1 + m)(-s_N^{base}). \quad (154)$$

The first term  $-s_N^{base}$  is the direct effect of capacity on the rollover premium. The term  $m(-s_N^{base})$  is the additional effect generated by the price-loss feedback on outstanding long debt. The fixed point for  $m$  below uses the total sensitivity  $|s_N^T|$ , so the feedback is already incorporated in the converged multiplier  $1 + m$ .

Let  $s^X$  denote term-premium compensation and define its direct sensitivity to financial capacity, holding  $\rho^T$  fixed, by

$$g_{dir}^X(b, N, \delta) \equiv - \left. \frac{\partial s^X(b, N, \delta)}{\partial N} \right|_{\rho^T} > 0. \quad (155)$$

Appendix F derives this object from the term-premium component. The total sensitivity of the required premium to financial capacity is therefore

$$|s_N^T| = (1 + m)(-s_N^{base}) + g_{dir}^X. \quad (156)$$

Combining (156) and (152) gives the fixed point

$$m = \frac{(1 + m)(-s_N^{base}) + g_{dir}^X}{\rho^T}. \quad (157)$$

Solving for  $m$  yields

$$m = \frac{-s_N^{base} + g_{dir}^X}{\rho^T + s_N^{base}}, \quad \rho^T + s_N^{base} > 0. \quad (158)$$

The regularity condition  $\rho^T + s_N^{base} > 0$  keeps the feedback finite. This is the same local admissibility condition used in Proposition 3: the feedback from capacity to prices amplifies the base premium without making the local fixed point explosive. Substituting (158) into (153) gives

$$s^R(b, N, \delta) = \frac{\rho^T(b, N, \delta) + g_{dir}^X(b, N, \delta)}{\rho^T(b, N, \delta) + s_N^{base}(b, N, \delta)} s^{base}(b, N, \delta). \quad (159)$$

The numerator captures the direct term-premium sensitivity to financial capacity. The denominator captures the balance-sheet feedback: a higher premium lowers bond prices, weakens intermediary net worth, and raises the premium further through the direct rollover-premium sensitivity  $s_N^{base}$ . The formula is a local linearization around a steady state that shows how balance-sheet fragility magnifies the cost of safety production.

## F Term-Premium Compensation and Maturity Choice

This appendix gives the details behind the maturity tradeoff in Section 6. The government chooses the rollover intensity  $\delta$ , equivalently the frequency with which debt must be rolled over. A higher  $\delta$  means shorter average maturity, faster rollover, and lower duration. A lower  $\delta$  means longer average maturity, slower rollover, and higher duration.

### F.1 Rollover component

The rollover component uses the same pricing formula as the benchmark, now treating  $\delta$  as the government's choice:

$$s^{base}(b, N, \delta) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N, \delta)} \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} d\alpha. \quad (160)$$

Since rollover exposure is  $\mu(\delta)b$ , the derivative  $s_\delta^{base}$  is positive in the stress region. Let  $\mu_\delta(\delta) \equiv e^{-\delta}$ . Differentiating gives

$$s_\delta^{base}(b, N, \delta) = \frac{\mu_\delta(\delta)b}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N, \delta)} \frac{\Gamma'(\mu(\delta)b - \alpha N)}{\alpha} d\alpha > 0. \quad (161)$$

The cross-partial with financial capacity is negative. Differentiating (161) with respect to  $N$  gives an interior term and a moving-boundary term:

$$\begin{aligned} s_{\delta N}^{base}(b, N, \delta) &= -\frac{\mu_\delta(\delta)b}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{q(b, N, \delta)} \Gamma''(\mu(\delta)b - \alpha N) d\alpha \\ &\quad - \frac{\mu_\delta(\delta)b}{(1 - \underline{\alpha})N} \Gamma'(0) < 0. \end{aligned} \quad (162)$$

The first term says that higher capacity lowers the marginal shortfall generated by shorter maturity in every stress state. The second term is the boundary effect: higher capacity shrinks the set of states in which a marginal increase in  $\delta$  creates rollover stress.

### F.2 Term-premium component

For term risk, consider a Poisson-maturity bond with effective discount-plus-hazard rate

$$\rho^T(b, N, \delta) = r + s^T(b, N, \delta) + \delta. \quad (163)$$

As shown in Appendix E, the price of this bond is locally proportional to  $1/\rho^T$ , so its duration is

$$\mathcal{D}\Pi\nabla = \frac{1}{\rho^T}. \quad (164)$$

If the variance of the persistent premium or rate shock is  $\sigma_P^2$ , local price variance is proportional to  $\mathcal{D}\Pi\nabla^2\sigma_P^2$ . If the market price of bearing term risk is  $\vartheta(N)$ , the term-premium component is

$$s^X(b, N, \delta) = \vartheta(N) \frac{\sigma_P^2}{[\rho^T(b, N, \delta)]^2}, \quad (165)$$

with

$$\vartheta(N) = \bar{\vartheta}N^{-\psi}, \quad \bar{\vartheta} > 0, \quad \psi > 0. \quad (166)$$

Higher financial capacity lowers the market price of bearing term risk. Holding  $\rho^T$  fixed, the direct sensitivity is

$$g_{dir}^X(b, N, \delta) = - \left. \frac{\partial s^X}{\partial N} \right|_{\rho^T} = \frac{\psi}{N} s^X(b, N, \delta). \quad (167)$$

Holding  $s^T$  fixed inside  $\rho^T$ , the direct cross-partial of the term-premium term is

$$s_{\delta N}^{X,dir}(b, N, \delta) = \frac{2\psi}{N} \frac{\vartheta(N)\sigma_P^2}{(\rho^T)^3} > 0. \quad (168)$$

This term has the opposite sign from (162): higher capacity lowers the price of term risk, which reduces the term-premium saving from shortening maturity.

### F.3 Fixed point and maturity choice

The total premium solves the fixed point

$$s^T(b, N, \delta) = \frac{\rho^T + g_{dir}^X}{\rho^T + s_N^{base}} s^{base}(b, N, \delta) + \vartheta(N) \frac{\sigma_P^2}{(\rho^T)^2}, \quad \rho^T = r + s^T + \delta. \quad (169)$$

The first term is rollover insurance with balance-sheet amplification. The second term is compensation for term risk. We maintain the local contraction condition

$$\sup_{s^T} \left| \frac{\partial}{\partial s^T} \left[ \frac{\rho^T + g_{dir}^X}{\rho^T + s_N^{base}} s^{base} + \vartheta(N) \frac{\sigma_P^2}{(\rho^T)^2} \right] \right| < 1, \quad (170)$$

which ensures that (169) defines a unique local premium schedule for each  $(b, N, \delta)$ . The government chooses  $\delta$  to minimize that well-defined schedule:

$$\delta^*(b, N) \in \arg \min_{\delta > 0} s^T(b, N, \delta). \quad (171)$$

At an interior optimum,

$$s_{\delta}^T(b, N, \delta^*) = 0, \quad s_{\delta\delta}^T(b, N, \delta^*) > 0. \quad (172)$$

The first-order condition equates the marginal rollover cost of shorter maturity with the marginal term-premium saving from shorter maturity.

The comparative static follows by differentiating the first-order condition:

$$s_{\delta\delta}^T(b, N, \delta^*) \frac{d\delta^*}{dN} + s_{\delta N}^T(b, N, \delta^*) = 0. \quad (173)$$

Thus

$$\frac{d\delta^*}{dN} = -\frac{s_{\delta N}^T}{s_{\delta\delta}^T}. \quad (174)$$

The sign of  $s_{\delta N}^T$  decomposes into the rollover-capacity term and the term-premium-price term:

$$s_{\delta N}^T = s_{\delta N}^R + s_{\delta N}^X. \quad (175)$$

Because  $s^R = Ms^{base}$ , the rollover part of the cross-partial is

$$s_{\delta N}^R = Ms_{\delta N}^{base} + M_N s_{\delta}^{base} + M_{\delta} s_N^{base} + M_{\delta N} s^{base}. \quad (176)$$

The first term is the direct rollover-capacity force. It is negative because  $M > 0$  and  $s_{\delta N}^{base} < 0$ , as shown in (162). The remaining terms capture how the balance-sheet multiplier itself changes with maturity and capacity. They are part of the full cross-partial.

The direct term-premium cross-partial in (168) is positive. The total maturity-capacity cross-partial is therefore

$$s_{\delta N}^T = Ms_{\delta N}^{base} + M_N s_{\delta}^{base} + M_{\delta} s_N^{base} + M_{\delta N} s^{base} + s_{\delta N}^X. \quad (177)$$

The maturity comparative static holds when this full expression is negative:

$$Ms_{\delta N}^{base} + M_N s_{\delta}^{base} + M_{\delta} s_N^{base} + M_{\delta N} s^{base} + s_{\delta N}^X < 0. \quad (178)$$

Equivalently,

$$-\left(Ms_{\delta N}^{base} + M_N s_{\delta}^{base} + M_{\delta} s_N^{base} + M_{\delta N} s^{base}\right) > s_{\delta N}^X. \quad (179)$$

Under this condition,

$$s_{\delta N}^T < 0. \quad (180)$$

A checkable benchmark fixes the amplification multiplier in the  $(\delta, N)$  comparison. In that case  $M_N = M_{\delta} = M_{\delta N} = 0$ , and (176) becomes

$$s_{\delta N}^R = Ms_{\delta N}^{base}. \quad (181)$$

Since  $M > 0$  and  $s_{\delta N}^{base} < 0$ , this rollover-capacity term is negative. The full condition (178) then reduces to

$$-Ms_{\delta N}^{base} > s_{\delta N}^X. \quad (182)$$

This benchmark is not used to drop the multiplier-derivative terms in the full model. It records the primitive comparison that obtains when the direct rollover-capacity channel is isolated; outside this benchmark, the full cross-partial in (177) is the relevant object.

Together with the second-order condition, implicit differentiation gives

$$\frac{d\delta^*}{dN} = -\frac{s_{\delta N}^T}{s_{\delta\delta}^T} > 0. \quad (183)$$

Equivalently, lower rollover-insurance capacity lowers  $\delta^*$ , which means longer maturity. Scarce rollover-insurance capacity therefore induces the government to term out the debt, trading off reduced rollover pressure against higher term-premium compensation.

## G A Global-Game Foundation for Rollover Insurance Pricing

This appendix derives the pricing measure used in Section 3 from an explicit rollover-insurance game. The object to be derived is the premium

$$s(b, N) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N)} \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} d\alpha, \quad \bar{q}(b, N) = \min\{q(b, N), 1\}, \quad (184)$$

where

$$q(b, N) = \frac{\mu(\delta)b}{N}. \quad (185)$$

The derivation has three steps. First, potential rollover insurers choose whether to participate after observing noisy private signals about the participation environment. Second, conditional on participation, they share the support burden required to preserve the safety of government debt in rollover-stress states. Third, in the diffuse-prior global-game limit, the participation state relevant for pricing is uniform on  $[\underline{\alpha}, 1]$ . Equation (184) is then the zero-profit premium for the marginal rollover insurer.

### G.1 Rollover need, financial capacity, and the support burden

The government has a stock of debt  $b$ . The rollover share over the insurance episode is  $\mu(\delta) = 1 - e^{-\delta}$ , so rollover need is

$$R(b, \delta) = \mu(\delta)b. \quad (186)$$

There is a continuum of potential rollover insurers. A fraction  $\underline{\alpha} \in (0, 1)$  is inelastic and always participates. The remaining elastic insurers have total mass one; their participation share is denoted  $\tilde{\alpha} \in [0, 1]$ . Total active participation is

$$\alpha = \underline{\alpha} + (1 - \underline{\alpha})\tilde{\alpha} \in [\underline{\alpha}, 1]. \quad (187)$$

If total participation is  $\alpha$ , active absorption capacity is

$$B(\alpha, N) = \alpha N. \quad (188)$$

The state variable  $N$  is aggregate financial-system capacity. It converts active participation into balance-sheet absorption capacity.

A rollover-stress shortfall occurs when rollover need exceeds active absorption capacity. The shortfall is

$$X(b, N, \alpha) = \max\{\mu(\delta)b - \alpha N, 0\}. \quad (189)$$

Equivalently, stress occurs when  $\alpha < q(b, N)$ , where  $q(b, N)$  is defined in (185). The support burden required to preserve the safety of government debt is

$$\Gamma(X), \quad \Gamma(0) = 0, \quad \Gamma'(X) > 0, \quad \Gamma''(X) \geq 0. \quad (190)$$

This burden is the real rollover-risk object in the paper. If active capacity is too low, the insurance pool must absorb a shortfall to keep insured government debt safe. The burden is the resource cost of covering the rollover gap in the stress state.

The burden is shared by the active rollover insurers. Per unit of active participation, the assessment is

$$a(b, N, \alpha) = \frac{\Gamma(X(b, N, \alpha))}{\alpha} = \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} \mathbf{1}_{\{\alpha < q(b, N)\}}. \quad (191)$$

The division by  $\alpha$  is important. In low-participation states, fewer balance sheets share the support burden, so the per-participant assessment is larger.

## G.2 The participation game

There is an aggregate fundamental  $\theta \in \mathbb{R}$  that shifts the willingness or ability of elastic insurers to participate. A larger  $\theta$  is a worse participation environment. Elastic insurer  $i$  observes the private signal

$$x_i = \theta + \eta \varepsilon_i, \quad (192)$$

where  $\eta > 0$  is signal noise and  $\varepsilon_i$  has continuous distribution  $F$  with strictly positive density  $f$ . Conditional on  $\theta$ , the idiosyncratic signal noises are independent across insurers, so the law of large numbers pins down aggregate participation.

Each elastic insurer chooses a participation decision

$$e_i \in \{0, 1\}, \quad (193)$$

where  $e_i = 1$  means that the insurer enters the rollover-insurance pool. A participating insurer receives the premium  $s$  per unit of insured exposure and pays the assessment  $a(b, N, \alpha)$  if a support burden is realized. A non-participant receives zero. Thus the payoff of insurer  $i$ , per unit of insured exposure, is

$$\Pi_i(e_i, \alpha; b, N) = e_i [s - a(b, N, \alpha)]. \quad (194)$$

The state variables in the participation problem are  $(b, N, \theta, x_i)$ . The decision variable is  $e_i$ . The aggregate object taken into account by the insurer is the equilibrium participation rate  $\alpha$ , because  $\alpha$  determines the assessment that active insurers must pay.

A symmetric threshold strategy is a cutoff  $x^*$  such that

$$e_i(x_i) = 1 \iff x_i \leq x^*. \quad (195)$$

The direction of the inequality follows from the convention that higher  $\theta$  is a worse participation environment. A lower signal makes the insurer more optimistic about aggregate participation and therefore about the assessment it will face.

Under the threshold strategy (195), the elastic participation share is

$$\tilde{\alpha}(\theta; x^*) = F\left(\frac{x^* - \theta}{\eta}\right), \quad (196)$$

and total participation is

$$\alpha(\theta; x^*) = \underline{\alpha} + (1 - \underline{\alpha})F\left(\frac{x^* - \theta}{\eta}\right). \quad (197)$$

This equation is the aggregate law of motion of participation inside the global game.

### G.3 Finite-noise equilibrium condition

Given a candidate threshold  $x^*$ , the expected payoff from participating after observing signal  $x_i$  is

$$V(x_i; x^*, s, b, N) = s - \mathbb{E}\left[a(b, N, \alpha(\theta; x^*)) \mid x_i\right]. \quad (198)$$

The threshold  $x^*$  is an equilibrium threshold if the marginal insurer is indifferent:

$$V(x^*; x^*, s, b, N) = 0. \quad (199)$$

Equivalently,

$$s = \mathbb{E}\left[a(b, N, \alpha(\theta; x^*)) \mid x_i = x^*\right]. \quad (200)$$

Equation (200) is the finite-noise pricing condition. It says that the premium equals the expected assessment faced by the marginal participant.

The threshold structure is supported by the monotonicity of the continuation value. Since  $\alpha(\theta; x^*)$  is decreasing in  $\theta$ , and since the assessment  $a(b, N, \alpha)$  is weakly decreasing in  $\alpha$  over the stress region, a higher signal makes high- $\theta$ , low-participation states more likely and lowers the payoff from participation. Thus the participation payoff satisfies a single-crossing property in  $x_i$ . The marginal signal  $x^*$  separates insurers who enter from insurers who stay out.

### G.4 Diffuse-prior limit and the uniform participation measure

It remains to compute the distribution of  $\alpha$  that appears in the marginal insurer's pricing equation. Let the prior density over  $\theta$  be  $p(\theta)$ . Conditional on observing  $x_i = x^*$ , the posterior density of  $\theta$  is proportional to

$$f\left(\frac{x^* - \theta}{\eta}\right)p(\theta). \quad (201)$$

In the diffuse-prior limit,  $p(\theta)$  is locally constant over the region where the likelihood has mass. Hence the posterior density is proportional to

$$f\left(\frac{x^* - \theta}{\eta}\right). \quad (202)$$

Define

$$z = F\left(\frac{x^* - \theta}{\eta}\right). \quad (203)$$

Then  $z \in [0, 1]$  and

$$\frac{dz}{d\theta} = -\frac{1}{\eta} f\left(\frac{x^* - \theta}{\eta}\right). \quad (204)$$

The likelihood term in (202) is therefore exactly offset by the Jacobian in the change of variables from  $\theta$  to  $z$ . Consequently, conditional on the marginal signal  $x_i = x^*$ , the random variable  $z$  is uniform on  $[0, 1]$ .

Since total participation is

$$\alpha = \underline{\alpha} + (1 - \underline{\alpha})z, \quad (205)$$

participation  $\alpha$  is uniform on  $[\underline{\alpha}, 1]$  from the perspective of the marginal participant. This distribution is independent of the threshold  $x^*$ . Therefore the premium can be evaluated without solving explicitly for the equilibrium threshold. This is the pricing measure used in the main text.

Substituting the uniform distribution of  $\alpha$  into the finite-noise pricing condition (200) gives

$$s(b, N) = \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^1 \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} \mathbf{1}_{\{\alpha < q(b, N)\}} d\alpha \quad (206)$$

$$= \frac{1}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{\bar{q}(b, N)} \frac{\Gamma(\mu(\delta)b - \alpha N)}{\alpha} d\alpha, \quad (207)$$

where  $\bar{q}(b, N) = \min\{q(b, N), 1\}$ . This is the premium formula in equation (23).

## G.5 Interpretation

The global game gives the rollover premium a precise foundation. Participation is endogenous, and rollover risk is real because low-participation states generate an actual rollover shortfall. The support burden  $\Gamma(X)$  is the cost of keeping insured government debt safe when active absorption capacity is insufficient. The premium  $s(b, N)$  is the zero-profit compensation required by the marginal rollover insurer for bearing that state-contingent assessment. In equilibrium this compensation is embedded in the excess yield paid by the government and nets out of the safe return received by households on insured debt.

The uniform measure on  $[\underline{\alpha}, 1]$  is the limiting belief of the marginal participant in a threshold participation game with diffuse priors. From that perspective, every participation share between the inelastic base  $\underline{\alpha}$  and full participation is equally likely. The safety premium therefore averages the per-participant assessment over precisely the states in which rollover support is needed.

This foundation also explains why the premium is a stock-wide price. The participation state and the shortfall are aggregate objects determined by total debt  $b$ , financial capacity  $N$ , and the active participation share  $\alpha$ . The premium that clears the rollover-insurance market applies to the insured stock and is financed by the excess yield on government debt. When  $b$  rises, the market-clearing assessment changes for the whole stock, which is why the marginal fiscal cost of safety production contains both the direct premium term and the repricing term  $bs_b$ .

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