

Forward Guidance and Financial Conditions

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Abstract

Forward guidance has recently been contested, most visibly in the United States. We study its benefits and costs in a model where monetary policy operates through financial conditions. Noisy flows move those conditions and create output gaps because the policy rate adjusts gradually. Guidance reveals the central bank's view of future demand, reducing policy uncertainty and inducing arbitrageurs to absorb more financial noise. But guidance also moves financial conditions on announcement. On average, this movement raises output-gap variance. Optimal guidance balances these effects; it rises with financial noise, policy uncertainty, and the persistence of the central bank's view. We then compare interest rate and financial conditions guidance. A projected rate also reflects the central bank's view of the risk premium, so it conveys less guidance for a given communication effort. Financial conditions guidance can always attain the optimal intensity; rate guidance may not and requires more effort when it does. What a central bank communicates therefore determines how much it should say.

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1. Introduction

Monetary policy reaches households and firms through financial markets. Current spending responds to broad financial conditions—long-term yields, equity and house prices, credit spreads, and exchange rates—rather than to the overnight policy rate alone. Because these conditions are set in markets rather than by the central bank, how the central bank communicates with market participants is a key part of how policy works. Central banks have largely acted on that logic, moving over three decades toward greater transparency about their objectives, their outlook, and their expected policy path (Dincer and Eichengreen, 2014). The share of major central banks using forward guidance rose from about half in the early 2000s to nearly all by the late 2010s (Briggs et al., 2026). That direction is now contested, most visibly in the United States. Federal Reserve Chair Kevin Warsh has cut the post-meeting statement to less than half its former length and declined to submit his own projection to the Summary of Economic Projections. He has described forward guidance as ill suited to the current policy conjuncture, on the view that investors should respond to incoming data rather than to the central bank’s forward-looking commentary. This renewed debate raises two questions: how much forward guidance should a central bank provide, and what should it communicate?

We address these questions in a model in which financial conditions are central to policy transmission and are themselves buffeted by financial noise. We find that guidance has both a benefit and a cost, so that more of it is not always better, and that it is more valuable when markets are noisier and policy is more uncertain. We then ask what the central bank should talk about, and show that guidance about the interest rate is a systematically weaker instrument than guidance about financial conditions.

We motivate our model and arguments by presenting five empirical facts about the interactions between monetary policy and financial markets. First, monetary policy transmits to the real economy primarily through financial conditions—a broad set of asset prices and interest rates that shape aggregate demand. Second, financial conditions are driven predominantly by risky asset prices and therefore contain substantial noise. This noise induces output gaps that the central bank stabilizes only with delay, because it adjusts the policy rate gradually. Third, forward guidance moves financial conditions on announcement, even when the conditions the central bank wants today have not changed. Fourth, sophisticated financial market participants are opinionated, frequently holding beliefs that diverge from the central bank’s. They nonetheless want to know the central bank’s belief, because it sets policy, and guidance reduces their uncertainty about the policy path. Fifth, the neutral interest rate r^* moves with the risk premium in addition

to macroeconomic fundamentals, whereas FCI^* —the level of financial conditions that closes the expected output gap—reflects macroeconomic fundamentals alone. Hence, the central bank’s interest rate announcements are contaminated by its risk premium perceptions, whereas its FCI^* announcements are more closely linked to its macroeconomic perceptions.

In our model, financial conditions are represented by an aggregate asset price, which is the main channel from monetary policy to aggregate demand. This asset price is determined in financial markets that feature noise and limits to arbitrage. Noisy flows affect the asset price and induce output gaps, and the central bank (which we refer to as “the Fed”) adjusts the policy interest rate gradually, so it cannot fully offset these gaps with its policy rate alone. Risk-averse arbitrageurs trade against noise, but they require compensation for doing so. How much of the noise they absorb therefore depends on the variance they perceive.

Perceived variance has several sources, including financial noise and uncertainty about future policy. The component that guidance can reduce arises from disagreement about the Fed’s future view of aggregate demand. At each date the Fed has its own view of near-future demand, which need not align with the arbitrageurs’ view. The arbitrageurs disagree with the Fed but nonetheless want to learn its view because the Fed sets policy. They can infer the Fed’s current view from its current policy decision, but its future view remains uncertain. The main question is whether forward guidance—explicit communication about that future view—can reduce this uncertainty and recruit the arbitrageurs for policy objectives.

In this model, forward guidance has both a benefit and a cost. On the one hand, it resolves uncertainty about the Fed’s view of future aggregate demand. This lowers the asset-price variance arbitrageurs perceive and recruits them to trade more aggressively against noisy flows, mitigating noise-induced output gaps. On the other hand, guidance moves financial conditions on announcement. Because asset prices are forward-looking, news about future policy affects conditions today. The resulting movement need not match the conditions required for current stabilization, and gradual interest-rate adjustment offsets it only partially. The movement may occasionally offset an existing gap; on average, however, it raises output-gap variance.

How much to communicate. The optimal guidance intensity balances the benefit of recruiting arbitrageurs against the cost of the forward-guidance shock. It rises when markets are noisier, policy uncertainty is greater, or the Fed’s view is more persistent. Noise raises the variance arbitrageurs perceive directly. Policy uncertainty raises uncertainty

about the Fed’s future view, while persistence magnifies its effect on financial conditions. All three therefore increase the value of recruiting arbitrageurs.

What to communicate. In practice, central banks guide in terms of interest rates. Because policy transmits through financial conditions, we compare rate guidance with guidance about future FCI^* . For either projection, greater communication effort makes the announcement more precise. But a projected rate combines the Fed’s view of aggregate demand with its view of the risk premium. This contamination means that, for a given effort, rate guidance delivers less guidance intensity. The attenuation is stronger when markets are noisier, policy uncertainty is greater, or the Fed’s view is more persistent. The same conditions that make forward guidance valuable therefore make interest rates a less useful thing to talk about.

Financial conditions guidance can always attain the optimal guidance intensity. Rate guidance does so only when guidance is not particularly valuable and, even then, requires more effort. A central bank that speaks about interest rates must speak more to be understood equally well.¹

Related Literature. Our paper contributes to a theoretical literature on the benefits and potential costs of central bank communication. The literature has largely favored more communication. Because policy works through expectations of future short rates, conveying the central bank’s views reduces the market’s uncertainty about the policy path, so longer-term rates track the central bank’s intentions more closely and carry less extraneous noise (Blinder, 1998; Woodford, 2005; Blinder et al., 2008). The literature has also identified potential costs. Morris and Shin (2002) show that when agents care about coordinating with one another, they overweight a public signal relative to their private information, so a more precise announcement can move actions away from fundamentals. Amato et al. (2002) apply the argument to monetary policy: transparency helps coordinate expectations, but for that reason it also magnifies the central bank’s mistakes (see also Morris and Shin, 2008). Our environment sets that channel aside. The central bank and the market observe the same data, and what the market wants to learn is the central bank’s own view, about which the central bank is better informed by construction. No information aggregation problem arises. Our paper captures the benefit the advocates

¹An additional practical consideration, outside the model, is that an interest rate projection may be interpreted as a commitment because the central bank controls the policy rate. A projection of appropriate financial conditions is arguably less likely to be read as a promise about realized conditions, which the central bank influences but does not control. That limited control may make the conditional nature of financial conditions guidance clearer.

emphasize and isolates a different cost. Communication reduces the market’s uncertainty about future policy and dampens the pass-through of noise into financial conditions. But the announcement is itself a shock. Because asset prices are forward-looking, it moves financial conditions today, and gradual policy leaves part of that movement unoffset.²

Our paper is also related to the literature on forward guidance. That literature has mostly studied guidance as a commitment device for escaping the effective lower bound, following Eggertsson and Woodford (2003). We instead study guidance as a communication policy, which is its more common form in practice. Across major central banks, Briggs et al. (2026) count only four episodes of explicit commitment to a rate path, with the rest conveying a likely path without committing to it.³ We identify a trade-off between clarifying the central bank’s views for market participants and delivering a shock to current financial conditions. Sections 2.3 and 2.4 present evidence on both sides of this trade-off.

The model builds on our earlier work on the connections between financial markets and monetary policy (see Caballero and Farhi (2018); Caballero and Simsek (2020, 2021, 2023, 2024b,a, 2026); Caballero et al. (2024, 2025)). The most closely related paper is Caballero et al. (2024), where we develop a New Keynesian model with noise and limits to arbitrage that provides a rationale for Financial Conditions Targeting—a framework in which the central bank *commits* to stabilize future financial conditions around a target level to encourage arbitrageurs to trade against noisy flows. This policy improves monetary policy by recruiting the arbitrageurs to insulate the real economy from financial noise.

The present paper replaces commitment with information design. When arbitrageurs are uncertain about the central bank’s beliefs, communication alone can activate the same recruitment channel, but it also generates current policy news. The model characterizes the balance between these two effects.⁴

The theory is also related to our earlier work on the policy implications of disagreements between markets and the central bank (Caballero and Simsek, 2022). In that model communication is useful because it mitigates tantrums, in which the current policy rate

²See Geraats (2002, 2014); Van der Cruijsen and Eijffinger (2010); Blinder et al. (2024) for reviews. A related literature studies whether central banks should present a unified message or the diverse perspectives of policymakers; see, for example, Ehrmann and Fratzscher (2007); Vissing-Jorgensen (2019).

³On guidance as commitment versus communication, see Campbell et al. (2012); Woodford (2013); Bassetto (2019). Jimenez-Gimpel (2026) treats the degree of commitment as a choice. The central bank chooses how vague and how data dependent its announcement is, balancing the anchoring of market expectations against the flexibility to respond to later news.

⁴This paper is also part of a large literature on New Keynesian models with risk and asset prices (Kashyap and Stein, 2023; Pflueger et al., 2020; Pflueger and Rinaldi, 2022; Drechsler et al., 2018; Kekre and Lenel, 2022, 2025; Kekre et al., 2023; Beaudry et al., 2024; Adrian and Duarte, 2018; Adrian et al., 2020; Fontanier and Simsek, 2026).

does not fully reveal the central bank’s view and the market misreads it. It does not feature the cost we identify here, because the central bank adjusts its rate without constraint and can always offset an undesired effect of guidance on current conditions. The present paper differs in two ways. First, policy transmits through noisy financial conditions rather than the policy rate alone, which creates a new benefit of communication in the form of noise reduction. Second, the central bank adjusts its rate gradually, so it cannot fully offset the forward guidance shock. Together these generate the trade-off that the earlier model did not feature. We also show that the communication variable matters: financial conditions guidance conveys more information and weakly dominates rate guidance.

We organize the rest of the paper as follows. Section 2 presents the empirical facts about monetary policy and financial markets that motivate our analysis. Section 3 develops the dynamic model. Section 4 derives the forward guidance trade-off and the optimal guidance intensity. Section 5 compares guidance about the future policy rate with guidance about future financial conditions. Section 6 concludes. Appendix A presents the microfoundations of the model and Appendix B contains proofs.

2. Motivating facts about monetary policy and financial market interactions

“So, of course, monetary policy does, famously, work with long and variable lags. The way I think of it is, our policy decisions affect financial conditions immediately. In fact, financial conditions have usually been affected well before we actually announce our decisions. Then, changes in financial conditions begin to affect economic activity... within a few months.” (Chair Jerome Powell’s Press Conference, September 21, 2022)

We present five facts that motivate the analysis. Monetary policy transmits through financial conditions. Those conditions contain substantial noise that moves output, and policy adjusts gradually. Communication about future policy moves financial conditions today. Sophisticated market participants hold their own views but still want the central bank’s, and guidance reduces their uncertainty about policy. Finally, the interest rate a central bank projects reflects its view of the risk premium alongside its view of the economy. These facts synthesize insights from several strands of the literature.

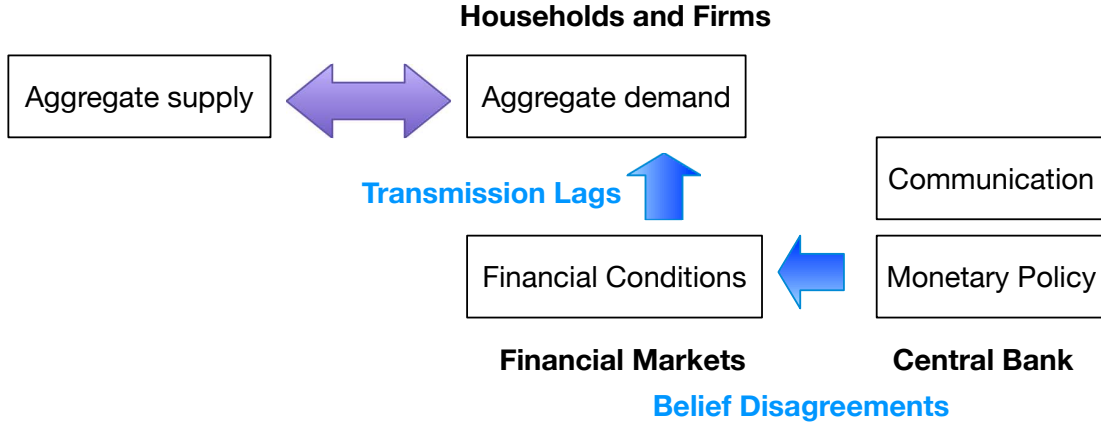


Figure 1: Monetary policy transmission through financial conditions.

2.1. Monetary policy transmits through financial conditions

Chair Powell’s quote illustrates that monetary policy influences aggregate demand indirectly—through financial markets—rather than operating directly in goods and services markets as fiscal policy does. When a central bank adjusts its policy rate or conveys policy news, the immediate impact is on asset prices, which then shape broader financial conditions and ultimately real economic activity. In fact, Keynes (1936) noted:

“...there are not many people who will alter their way of living because the rate of interest has fallen from 5% to 4% (...) Perhaps the most important influence (...) depends on the effect of these changes on the appreciation or depreciation in the prices of securities” (as quoted by Beaudry et al. (2024)).

Moreover, financial conditions affect the economy with significant lags. Chodorow-Reich et al. (2021) find that the labor-market effects of a stock-price shock peak four to eight quarters later, while Romer and Romer (2004) find similar lags for monetary-policy shocks. Together with the need to estimate unobservable variables such as potential output, these lags make monetary policy inherently belief dependent and create a central role for communication.

As shown in Figure 1, the central bank uses its tools to influence financial conditions, which in turn shape the behavior of households and firms. Policy moves financial conditions through the expected path of interest rates—the channel used in our model—and through risk premia.⁵ Conceptually, the term “financial conditions” in this figure

⁵See, among others, Bernanke and Kuttner (2005); Bauer et al. (2023); Boehm and Kroner (2024); Nagel and Xu (2024).

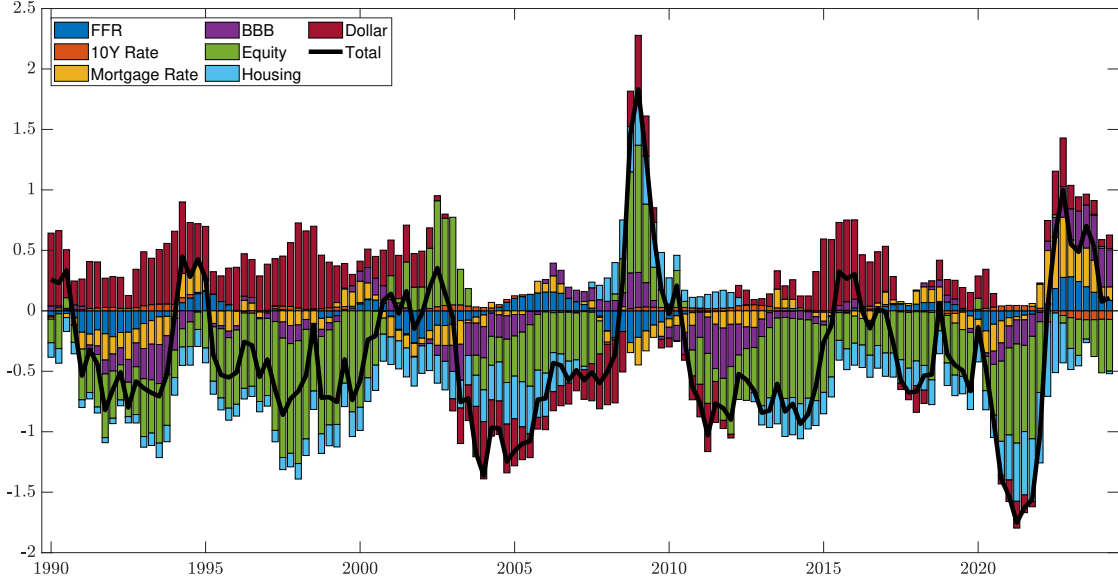


Figure 2: FCI-G index (with a three-year lookback) and its drivers over 1990Q1–2024Q2. Positive values imply recent financial conditions will reduce GDP growth in the next year. Each bar is the contribution of one financial variable, computed as a weighted sum of its changes over the previous twelve quarters, with weights given by the estimated effect of those changes on GDP growth over the next year. The bars sum to the index. Data are from Ajello et al. (2024).

represents a weighted average of aggregate asset prices—such as stock prices, borrowing rates, and exchange rates—that drive economic activity. Recently developed financial conditions indices (FCIs) measure this average. An FCI quantifies the impact of recent developments in financial markets on future GDP growth, and therefore measures the *market-based* transmission plotted in Figure 1. Several are well known, such as the Goldman Sachs FCI (see Hatzius and Stehn, 2018), the National FCI by the Chicago Fed, and the FCI-Growth index (FCI-G) by Ajello et al. (2023), which we use in our illustrations. The FCI-G is notable for relying on the FRB/US model and other large-scale models developed by the Federal Reserve, which incorporate the various channels by which financial markets affect economic activity—the borrowing and investment effects of various interest rates, the wealth effects of stock and house prices, and the expenditure switching effects of exchange rates.

2.2. Financial conditions contain noise that moves output, and policy adjusts gradually

Figure 2 plots the FCI-G index along with its drivers. Fluctuations in the index are driven primarily by risky asset prices—specifically, stock prices and exchange rates, and occasionally house prices—rather than by various yields. While these assets may not individually influence economic activity more strongly than interest rates, their heightened volatility generates larger swings in overall financial conditions, making them disproportionately important for monetary policy transmission.

This matters because risky asset prices feature “excess” volatility that is unrelated to expected cash flows, driven in part by time-varying sentiment (e.g., Shiller (2014)), time-varying risk premiums (e.g., Cochrane (2011)), or time-varying noise in inelastic markets (e.g., De Long et al. (1990); Gabaix and Koijen (2021)). Given that financial conditions greatly impact macroeconomic activity according to the Fed’s structural models, a natural question arises: does the “excess volatility” in asset prices also induce “excessive” macroeconomic fluctuations?

We have recently addressed this question in two studies. Caballero et al. (2024) builds upon Gabaix and Koijen (2021), who provide a measure of noisy flows into the aggregate stock market and show that this type of noise can induce substantial swings in stock prices. We show that these noisy flows also influence financial conditions and macroeconomic activity: a noisy flow shock into the stock market increases stock prices and loosens financial conditions, which in turn increases output gaps and inflationary pressures. The Fed eventually raises the policy interest rate and stabilizes the output gap, but with a substantial delay, reflecting the gradualism of policy rate adjustment. This interpretation is consistent with estimated policy rules that place a large weight on the previous period’s rate (Clarida et al., 2000; English et al., 2013), and with Coibion and Gorodnichenko (2012), who show that the persistence reflects deliberate smoothing rather than persistent shocks.

Caballero et al. (2025) introduces and empirically estimates FCI^* , the analogue of r^* , within a framework in which financial conditions along with demand shocks drive economic activity as in Figure 1. Conceptually, FCI^* is the level of financial conditions (measured with the FCI-G index) that closes the expected output gap. The estimated FCI^* mainly reflects *macroeconomic* forces, loosening in demand recessions and tightening in an inflationary boom, whereas the observed FCI partly reflects financial market shocks. Consequently, the observed FCI often differs from our estimated FCI^* . These FCI gaps are sizable, especially at the onset of major recessions such as the Global Financial Crisis,

when the observed FCI tightened sharply while FCI^* loosened.

Although the two studies use very different methodologies, they reach a similar conclusion: “excess volatility” in financial markets induces “excessive” macroeconomic fluctuations that are only partially stabilized by the Fed. As we argue later, appropriate communication with sophisticated market participants can enable the Fed to better smooth these fluctuations.

2.3. Forward guidance moves financial conditions today

Because asset prices depend on expected future policy, communication about the future changes financial conditions today (Gürkaynak et al., 2005; Nakamura and Steinsson, 2018). Swanson (2021) separates FOMC announcement surprises into changes in the current federal funds rate, forward guidance, and asset purchases. The guidance and purchase components move Treasury yields, corporate bond yields, equity prices, and exchange rates by amounts comparable to those of funds rate changes in normal times. Guidance is therefore an intervention in current financial conditions as much as an information release.

Nothing guarantees that this movement matches the financial conditions the central bank needs today. An announcement conveys the bank’s view of a future state, while its current stabilization problem depends on the current state. When the two differ, conveying the first displaces the second.

The 2013 taper episode illustrates the gap. On May 22, 2013, Chairman Bernanke told the Joint Economic Committee that the Fed could reduce the pace of its asset purchases in the next few meetings if the labor market continued to improve (Bernanke, 2013). The prospective tightening was premised on a recovery that had not yet occurred. The purchases themselves were unchanged, and the federal funds rate was at its lower bound. Long-term yields and mortgage rates nevertheless rose immediately and continued rising through the summer, and financial conditions tightened. The Fed did not want this tightening at the time, as revealed by what it did next: it left the pace of purchases unchanged in September, against widely held expectations of a reduction (Federal Open Market Committee, 2013). Anticipation had tightened conditions in an economy whose current state called for accommodation, and the instrument available for offsetting the movement was already at its bound.

2.4. Markets want the central bank’s view, and guidance reduces their uncertainty

Central bank communication is directed primarily toward sophisticated financial market participants. A defining feature of this audience is that its members are both *informed* and *opinionated* (see Caballero and Simsek (2022)). Market participants often hold strong, independent beliefs about the state of the economy and the appropriate monetary policy response—views that frequently diverge from those of the central bank. Forward interest rates, which largely reflect market expectations for the federal funds rate, often deviate from the Fed’s own median projections, and these divergences are a persistent feature of the policy landscape rather than a rare exception. They are correlated with differences in macroeconomic outlooks such as inflation expectations, suggesting that they stem in part from differing views about the economy (see Caballero and Simsek (2022)).

These differences do not reflect “different information” available to markets and the central bank—both are sophisticated with access to similar data. As Blinder (2007) notes, in monetary policy virtually all the data that matter are common knowledge, and Bauer and Swanson (2023) confirm that market participants do not consider the Fed to have superior information about the state of the economy. Rather, forecast differences mostly reflect different interpretations of the same data—what we refer to as belief disagreements.

Importantly, these disagreements do not imply that markets ignore central bank communications. On the contrary, markets follow them closely precisely because the central bank sets policy, and policy decisions are influenced by the central bank’s beliefs. Communication does not persuade markets of the Fed’s views, but lets them infer the likely direction of policy. Disagreements therefore increase the need for communication. When the central bank’s views closely mirror market views, disagreement adds little uncertainty about the policy path; when they diverge, it adds considerably more. Consistent with this, Swanson (2006) finds that as the Federal Reserve became more transparent, private forecasters predicted the funds rate more accurately, were less surprised by announcements, and were more certain of their own forecasts. See also Ehrmann et al. (2012) for cross-country evidence that greater transparency lowers dispersion among professional forecasters. Briggs et al. (2026) find the same across major central banks, with the introduction of forward guidance among the reforms that lowered year-ahead rate volatility most.

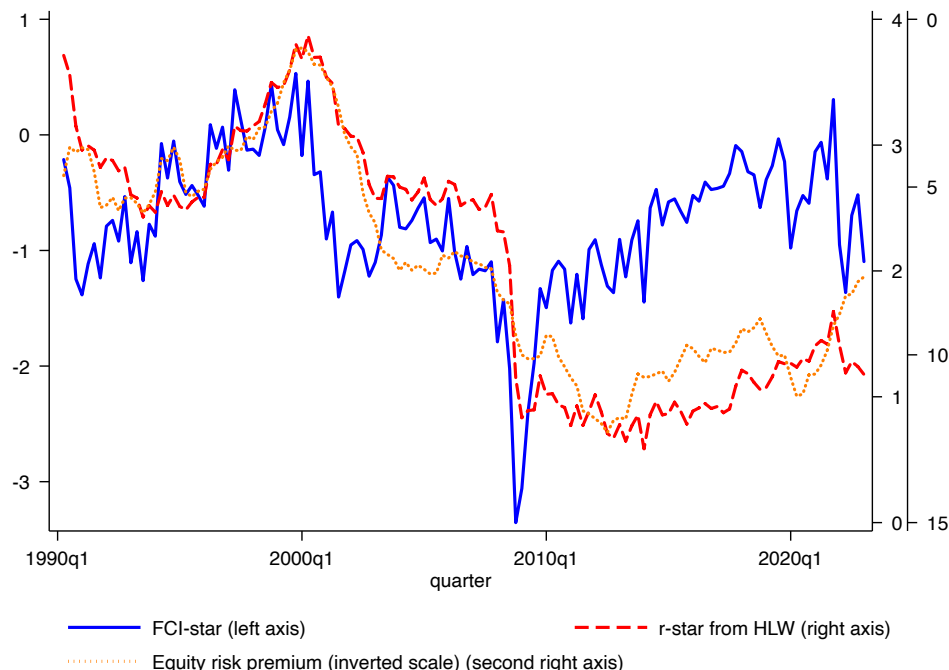


Figure 3: r^* as estimated in Holston et al. (2023) alongside the estimate of FCI^* in Caballero et al. (2025). The equity risk premium measure is from Duarte and Rosa (2015) and is available through 2023Q1.

2.5. Interest rate views partly reflect risk premium views

The preceding facts say that communication is potentially valuable but not what it should be about. In practice central banks announce expected policy rates, and those expectations depend on their view of the neutral rate r^* , the policy rate that closes the expected output gap. It therefore matters what r^* itself responds to.

The neutral rate depends partly on “non-fundamental” fluctuations in risky asset prices driven by risk premia, sentiment, or noise. An asset price bust that is not matched by a change in aggregate productivity reduces aggregate demand, which forces the central bank to cut the policy rate to stabilize aggregate demand, and conversely for an asset price boom. In Caballero et al. (2025) we show that the corresponding financial conditions object— FCI^* —is by construction insulated from asset price fluctuations and depends only on macroeconomic fundamentals. We also estimate FCI^* in U.S. data.

Figure 3 illustrates the contrast. The Holston–Laubach–Williams r^* is strongly correlated with the equity risk premium, whereas FCI^* is not once macroeconomic conditions are held fixed. The figure does suggest some comovement between FCI^* and the risk premium, but this reflects periods in which financial booms and busts correlate with the

business cycle, such as the late 1990s and early 2000s. In multivariate regressions controlling for output gaps, the equity risk premium has no independent relationship with FCI^* , while r^* remains strongly correlated with it. These observations suggest that a central bank projecting an interest rate path is projecting an object that embeds its own view of the risk premium alongside its view of the economy.

The autumn of 2023 illustrates the point. Long-term Treasury yields rose by roughly a percentage point between August and October, and the FOMC held the funds rate at its September and November meetings. Logan (2023) explained what the increase implied for policy: if long-term rates remain elevated “because of higher term premiums, there may be less need to raise the fed funds rate,” whereas to the extent that economic strength lay behind the increase, “the FOMC may need to do more.” Chair Powell made the same distinction (Powell, 2023). Projecting a rate path at that moment therefore required a view on how much of the increase was a premium movement, and on how long it would last. Projecting FCI^* would have required no view about the risk premium, only about the economy.

3. A Model of Forward Guidance and Financial Conditions

In this section we develop a dynamic model consistent with the empirical facts in Section 2. The model has four types of agents: households, noise traders, arbitrageurs (denoted by the subscript A) and the central bank (the Fed, denoted by the subscript F). Households transmit financial conditions into aggregate spending decisions; noise traders submit noisy financial flows that move financial conditions; the arbitrageurs absorb these flows subject to limited aggregate risk-bearing capacity and partly stabilize financial conditions; and the Fed sets the interest rate to steer financial conditions and close the output gap (see Figure 1). Importantly, the arbitrageurs and the Fed may hold different beliefs about aggregate demand, which creates a role for communication. As discussed in Section 2.4, these disagreements reflect different interpretations of broadly common data. The Fed’s internal view of those data is not observed until policy or communication reveals it.

Throughout, we present the model in the limit $\beta \rightarrow 1$, where β denotes the households’ discount factor. Appendix A derives the general case and shows that nothing substantive depends on this limit: it only removes constants that cancel from every equilibrium condition. We also relegate microfoundations to Appendix A and proofs to Appendix B.

Real economy. Potential output is a positive constant, $y^* > 0$. For simplicity, we assume prices are fully sticky, so that output is determined by aggregate demand and can depart from potential. This can be relaxed by allowing partially flexible prices and a standard Phillips curve without changing anything substantive (see Caballero et al. (2024)). Letting p_t denote the FCI, with a higher value corresponding to looser financial conditions, the *output–financial conditions* relation is

$$y_t = p_t + \delta_t. \quad (1)$$

Looser financial conditions raise spending and output through a consumption wealth effect, and δ_t is an *aggregate demand shifter* that captures other factors affecting spending, such as consumer sentiment, fiscal policy, or discount rate shocks. Note that this is the opposite of the convention used by standard FCIs, where an increase indicates tighter conditions. We use the price convention because it makes the model’s equations easier to read. The policy rate does not enter separately: it affects output only through the FCI, consistent with the evidence in Sections 2.1 and 2.5. The true FCI-star that ensures output is at potential is

$$p_t^* = y^* - \delta_t. \quad (2)$$

Financial markets. The equilibrium FCI is given by

$$p_t = \mathbb{E}_t^A[p_{t+1}] - (i_t + \tfrac{1}{2}Q) + Q\mu_t, \quad (3)$$

where $Q \equiv \text{Var}_t^A(p_{t+1})$ is the *perceived FCI variance*, the variance of next period’s FCI under the arbitrageurs’ beliefs.

This equation emerges in a setup in which arbitrageurs with standard preferences interact with noise traders. The noise traders submit exogenous flows, denoted by μ_t , realized at the beginning of the period and publicly observed—by the Fed as well as the arbitrageurs. We measure these flows in price-impact units, so σ_μ^2 captures both the size of the raw flows and the arbitrageurs’ risk-bearing capacity. Appendix A derives the mapping. The flows follow

$$\mu_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\mu^2). \quad (4)$$

The arbitrageurs maximize expected log utility, resulting in standard mean-variance optimization. In equilibrium they absorb the noise, but they require a risk premium to do so. Hence the impact of noise on the FCI is proportional to Q . As the perceived FCI variance rises, they require a greater price movement to absorb a given noise-trader flow.

Combined with (1), this implies that financial noise affects output in proportion to the perceived variance. Communication affects this variance and therefore the pass-through of financial noise. The remaining terms are familiar: the FCI depends on the price expected next period, and inversely on the rate at which that price is discounted, $i_t + \frac{1}{2}Q$ —the policy rate plus the average risk premium the arbitrageurs require. Equivalently, μ_t can be read as the negative of risk-premium pressure: positive noise buying compresses the premium, while a positive risk-premium shock corresponds to $\mu_t < 0$.

Monetary policy. We assume the Fed sets the policy interest rate before observing the current demand shock. This assumption makes monetary policy depend on the Fed’s beliefs about aggregate demand. Let $\delta_t^F \equiv \mathbb{E}_t^F[\delta_t]$ denote the Fed’s current demand view. Its perceived FCI-star and output gap are

$$p_t^{*,F} = y^* - \delta_t^F, \quad \mathbb{E}_t^F[y_t] - y^* = p_t - p_t^{*,F}. \quad (5)$$

By (1), the realized output gap decomposes as

$$y_t - y^* = (p_t - p_t^{*,F}) + (\delta_t - \delta_t^F). \quad (6)$$

The first term is the gap the Fed perceives. The second is its forecast error about current demand, which neither the policy rate nor communication can affect.

The Fed dislikes output gaps and movements in the policy rate, and minimizes the loss

$$\mathcal{L} \equiv \mathbb{E}^F \left[(y_t - y^*)^2 + \frac{1}{\theta} (i_t - i^*)^2 \right], \quad \theta > 0, \quad (7)$$

where $\mathbb{E}^F$ is the unconditional expectation under the Fed’s belief. It sets the policy rate under discretion, period by period, taking the arbitrageurs’ expectation of p_{t+1} and the realized noise as given. By (6) its forecast error about current demand is orthogonal to its date- t information, so that term contributes to $\mathcal{L}$ but cannot be influenced by i_t . Since $\partial p_t / \partial i_t = -1$ by (3), the first-order condition gives

$$i_t = i^* + \theta (p_t - p_t^{*,F}). \quad (8)$$

The Fed adjusts the policy rate only partially toward the gap it perceives. A Fed that cares less about rate movements has a larger θ and closes that gap faster. Here $i^* = -Q/2$ is the steady-state natural rate, chosen to ensure the unconditional expected output gap is zero. We call i^* the natural rate and reserve *neutral rate* for the time-varying rate that

closes the Fed’s perceived output gap, introduced in Section 5.

We view the Fed’s aversion to rate movements as capturing in reduced form the tendency of central banks to adjust interest rates gradually. Gradualism implies that the Fed cannot fully control financial conditions with the interest rate alone, so it needs to communicate with the fast-moving arbitrageurs to recruit them for its policy objectives.

The equilibrium pair (i_t, p_t) is determined jointly at the beginning of the period and is publicly observed. We next describe the Fed’s beliefs that drive policy according to (8).

Fed’s beliefs. The Fed’s view of aggregate demand δ_t^F evolves as⁶

$$\delta_{t+1}^F = \rho \delta_t^F + \xi_{t+1}^F + \omega_{t+1}^F, \quad 0 \leq \rho < 1. \quad (9)$$

Here, ξ_{t+1}^F and ω_{t+1}^F denote two innovations to the date- $(t+1)$ view that are orthogonal to each other and to the date- t view, with distributions

$$\xi_{t+1}^F \stackrel{\text{iid}}{\sim} N(0, v), \quad \omega_{t+1}^F \stackrel{\text{iid}}{\sim} N(0, v_\omega), \quad v > 0, \quad v_\omega \geq 0. \quad (10)$$

The two innovation terms differ in when the Fed learns them. The *advance news* ξ_{t+1}^F is already known at date t , so the Fed’s date- t projection is

$$\bar{\delta}_{t+1|t}^F \equiv \mathbb{E}_t^F[\delta_{t+1}^F] = \rho \delta_t^F + \xi_{t+1}^F. \quad (11)$$

The *revision* ω_{t+1}^F is the change in the Fed’s view that arises at date $t+1$. It is not known at date t , so no date- t announcement can convey it. Appendix A derives (9) from a signal-extraction problem under the Fed’s subjective model.

At date t , the Fed also forms an advance forecast μ_{t+1}^F of next period’s noise pressure. Conditional on public date- t information, we assume this forecast is orthogonal to the Fed’s advance demand news ξ_{t+1}^F .

Arbitrageurs’ beliefs and agree-to-disagree. The arbitrageurs and the Fed agree on the joint distribution of the Fed’s views $(\delta_t^F, \xi_{t+1}^F, \omega_{t+1}^F, \mu_{t+1}^F)$. They disagree about how these views relate to fundamentals: the Fed interprets its views as informative about demand and financial noise, while the arbitrageurs do not. In particular, under the arbitrageurs’ beliefs, ω_{t+1}^F is Fed-specific belief noise, unrelated to fundamentals and to innovations in their own demand assessment. In Caballero and Simsek (2022), by contrast,

⁶We place the disagreement on aggregate demand; Caballero and Simsek (2022) obtain similar results when it concerns potential output instead.

the market expects future information to move the Fed's assessment toward its own. That learning dampens the response of expected policy to current disagreement and causes the two sides' rate forecasts to diverge across horizons. Here we isolate uncertainty about the Fed's future view, which is sufficient for the communication trade-off. The arbitrageurs still value information about the Fed's views because those views determine policy.

The arbitrageurs do not need to be told the Fed's current view. In the equilibrium we construct, the policy decision reveals δ_t^F exactly, because the rate responds to the gap the Fed perceives. What policy leaves unrevealed is the Fed's view about *future* demand. Together with the fact that the later revision ω_{t+1}^F is not yet known at date t , this leaves the advance news ξ_{t+1}^F as the only object forward guidance can convey.

Until Section 5 we remain agnostic about the communication technology. We assume only that (i) the arbitrageurs update by Bayes' rule, and (ii) the Fed sends an informative signal that reduces their uncertainty about its demand views. Formally, after hearing the date- t signal, their posterior over the advance news is

$$\xi_{t+1}^F \stackrel{A}{\sim} N(\hat{\xi}_{t+1}^F, \hat{v}), \quad 0 \leq \hat{v} \leq v. \quad (12)$$

For example, if the Fed announces the advance news with independent noise,

$$\tilde{\xi}_{t+1}^F = \xi_{t+1}^F + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \text{ independent}, \quad (13)$$

then Bayes' rule gives

$$\hat{\xi}_{t+1}^F = \frac{v}{v + \sigma_\epsilon^2} \tilde{\xi}_{t+1}^F, \quad \hat{v} = \frac{v \sigma_\epsilon^2}{v + \sigma_\epsilon^2}. \quad (14)$$

A more precise signal moves the posterior mean more and leaves less residual variance. The following lemma shows this is a general property of Bayesian updating.

Lemma 1 (Resolved and residual variance). *Ex ante, $\hat{\xi}_{t+1}^F \sim N(0, v - \hat{v})$, so that*

$$\underbrace{v}_{\text{prior}} = \underbrace{\text{Var}^F(\hat{\xi}_{t+1}^F)}_{\text{resolved by the announcement}} + \underbrace{\hat{v}}_{\text{left unresolved}}. \quad (15)$$

The Fed and the arbitrageurs assign the same distribution to $\hat{\xi}_{t+1}^F$.

Communication divides the variance of the advance news v into a resolved and a residual component. A more informative announcement leaves a smaller residual $\hat{v}$ and moves the posterior mean more. The second part holds because the posterior mean is a function of the announcement alone. The two parties therefore agree on how far the Fed's

stated view will move, while disagreeing about whether those movements track demand.

Since communication divides the variance in this way, we index it by the share resolved. Let $\lambda \in [0, 1]$ denote the *guidance intensity*, defined by

$$\hat{v} = (1 - \lambda)v, \quad \text{so that} \quad \text{Var}^F(\hat{\xi}_{t+1}^F) = \lambda v. \quad (16)$$

Here, $\lambda = 0$ corresponds to silence and $\lambda = 1$ to full disclosure of the advance news ξ_{t+1}^F . For intermediate cases, consider the example (13). The coefficient in (14) is exactly λ , so $\hat{\xi}_{t+1}^F = \lambda \tilde{\xi}_{t+1}^F$ and $\lambda/(1 - \lambda)$ is the announcement's signal-to-noise ratio. We return to this mapping between signals and guidance intensity in Section 5.

Timing and equilibrium. At date t , current financial noise μ_t is public. The Fed observes its advance news ξ_{t+1}^F and its premium forecast μ_{t+1}^F , then sends its announcement. The market updates its belief about ξ_{t+1}^F , and (i_t, p_t) are determined jointly. At date $t + 1$, ω_{t+1}^F is realized and the Fed's current view is revised.

Definition 1 (Stationary equilibrium). *Fix a communication structure as described above. A stationary equilibrium consists of a constant $Q > 0$, policy and FCI processes (i_t, p_t) , and a Bayesian posterior $(\hat{\xi}_{t+1}^F, \hat{v})$ such that the policy rule (8), the pricing condition (3), and the communication structure hold, and*

$$Q = \text{Var}_t^A(p_{t+1}). \quad (17)$$

4. How Much to Communicate: The Costs and Benefits of Forward Guidance

We now characterize the equilibrium and use it to study the forward guidance trade-off. Guidance reduces the arbitrageurs' uncertainty about future policy and recruits them to mitigate the impact of noise. It also delivers a forward guidance shock that moves current financial conditions and opens a new source of gaps. We then derive the optimal guidance intensity and its comparative statics.

4.1. Equilibrium and the forward guidance trade-off

We first verify that the current policy-rate–FCI pair reveals the Fed's current demand estimate, because the Fed's policy rule reacts to its perceived current target. Formally,

solving the policy rule for the Fed's perceived FCI-star,

$$p_t^{*,F} = p_t - \frac{i_t - i^*}{\theta}. \quad (18)$$

Combining with $p_t^{*,F} = y^* - \delta_t^F$,

$$\delta_t^F = y^* - p_t + \frac{i_t - i^*}{\theta}. \quad (19)$$

The equilibrium pair reveals δ_t^F exactly, so by (11) the predictable component $\rho\delta_t^F$ of the next projection is public as well. A signal about the projected view $\bar{\delta}_{t+1|t}^F$ therefore delivers the same posterior as a signal about ξ_{t+1}^F .

Our next result characterizes the equilibrium. For compactness, write $B \equiv 1 + \theta - \rho$.

Proposition 1 (Equilibrium with forward guidance). *Fix a guidance intensity λ , so that the arbitrageurs' posterior is given by (12) with $\hat{v} = (1 - \lambda)v$. Suppose $\sigma_\mu^2 > 0$ and the quadratic below has two positive solutions. Then there is a stable equilibrium given by*

$$p_t - p_t^{*,F} = \underbrace{\frac{1 - \rho}{B} \delta_t^F}_{\text{current Fed view}} + \underbrace{\frac{Q(\lambda)}{1 + \theta} \mu_t}_{\text{financial noise}} - \underbrace{\frac{\theta}{(1 + \theta)B} \hat{\xi}_{t+1}^F}_{\text{forward guidance shock}}, \quad (20)$$

$$i_t = i^* + \theta(p_t - p_t^{*,F}), \quad (21)$$

where $Q(\lambda) = \text{Var}_t^A(p_{t+1})$ is the smaller solution to⁷

$$Q(\lambda) = \underbrace{\left(\frac{\theta}{B}\right)^2 [v_\omega + (1 - \lambda)v]}_{\text{unresolved Fed view}} + \underbrace{\left(\frac{\theta}{(1 + \theta)B}\right)^2 \lambda v}_{\text{next period's forward guidance shock}} + \underbrace{\left(\frac{Q(\lambda)}{1 + \theta}\right)^2 \sigma_\mu^2}_{\text{financial noise}}. \quad (22)$$

Raising the guidance intensity λ lowers $Q(\lambda)$ and the noise pass-through into the FCI gap, $Q(\lambda)/(1 + \theta)$. It also raises $\text{Var}^F(\hat{\xi}_{t+1}^F) = \lambda v$, and with it the variance of the forward guidance shock component of the gap, $(\theta/[(1 + \theta)B])^2 \text{Var}^F(\hat{\xi}_{t+1}^F)$.

Equations (20) and (21) characterize the equilibrium. The FCI is centered on the Fed's perceived FCI-star but departs from it for three reasons. First, the Fed does not fully stabilize the demand shocks according to its view, because it adjusts gradually, $\theta < \infty$. Second, gradualism likewise leaves financial noise to open FCI gaps. The magnitude of both effects declines with the Fed's reaction speed θ . Beyond gradualism, the impact of noise also depends on the variance the arbitrageurs perceive.

⁷The larger solution corresponds to an unstable equilibrium. See the proof in Appendix B.

Third, and importantly, forward guidance that shifts the market's posterior about the *future* also opens FCI gaps in the *present*. Suppose the Fed announces that it expects strong demand next period, $\hat{\xi}_{t+1}^F > 0$. The Fed will then want tighter conditions at $t + 1$. Prices are forward-looking, so financial conditions tighten immediately. The Fed's FCI-star for date t has not moved, however, because the news concerns date $t + 1$. The movement may occasionally narrow an existing gap. On average, however, it raises gap variance. This is the *forward guidance shock*. The size of this effect depends on the Fed's reaction speed in two opposing ways, through the θ in the numerator of its loading and the θ in the denominator. A perfectly gradual Fed, $\theta = 0$, would not move future conditions with its views and so would not move current conditions either. A perfectly fast Fed, $\theta = \infty$, would offset the current gap as soon as the announcement opened it. The forward guidance shock is therefore largest at intermediate degrees of gradualism.

Equation (22) shows that the variance the arbitrageurs perceive is itself endogenous to the guidance intensity. Next period's FCI inherits the same three loadings, so the conditional uncertainty the arbitrageurs face at date t has the same three sources: the unresolved component of the Fed's date- $(t + 1)$ view, with variance $(1 - \lambda)v + v_\omega$, next period's guidance announcement, with variance λv by Lemma 1, and next period's noise.⁸ Since the impact of noise depends on the perceived variance, $Q(\lambda)$ solves a fixed point. The stable equilibrium corresponds to the smaller root.

The fixed point implies that on net guidance lowers the variance the arbitrageurs perceive. While guidance does not remove the variance in the Fed's advance news, it relocates it in a way that reduces its impact on financial conditions. Raising λ moves an amount v from the first source to the second, one for one. The two do not enter with the same weight, however. The unresolved view enters $Q(\lambda)$ with loading $(\theta/B)^2$, and the announcement with the same loading divided by $(1 + \theta)^2$. The difference between the two loadings is the Fed's response. An announcement moves financial conditions away from a target that has not moved, and the Fed offsets part of that movement by adjusting the policy rate. Relocation therefore lowers $Q(\lambda)$, and the lower perceived variance recruits the arbitrageurs to absorb noise more aggressively.

Forward guidance therefore involves a trade-off. By resolving the Fed's views it recruits the arbitrageurs, which lowers the gaps that financial noise opens. By delivering a forward guidance shock it opens a gap in (20) today. Communication therefore lowers gaps through the recruitment channel and raises them through the forward guidance shock.

⁸These components are mutually orthogonal under the arbitrageurs' measure.

4.2. Optimal guidance intensity

We next characterize the optimal guidance intensity. The Fed chooses λ ex ante to minimize $\mathcal{L}$. By (8) the rate deviation is θ times the perceived gap, so (6) gives

$$\mathcal{L}(\lambda) = (1 + \theta)L(\lambda) + \text{Var}^F(\delta_t - \delta_t^F), \quad L(\lambda) \equiv \mathbb{E}^F \left[\left(p_t - p_t^{*,F} \right)^2 \right]. \quad (23)$$

The second term is the variance of the Fed's forecast error about current demand, which is independent of λ . The Fed's problem is therefore to minimize L .

Proposition 2 (Optimal guidance intensity). *Suppose $\theta > 0$, $v > 0$, $v_\omega \geq 0$, $\sigma_\mu^2 > 0$, and the smaller solution $Q(\lambda)$ of (22) exists for every $\lambda \in [0, 1]$. Then L is strictly convex and the loss-minimizing guidance intensity λ^* is unique. Silence is optimal ($\lambda^* = 0$) when $2\sigma_\mu^2 Q(0) \leq 1$, and full disclosure is optimal ($\lambda^* = 1$) when $2\sigma_\mu^2 Q(1) \geq 1$. Otherwise the optimum is interior, $\lambda^* \in (0, 1)$, and is characterized by*

$$Q(\lambda^*) = \frac{1}{2\sigma_\mu^2}, \quad \text{equivalently} \quad s(\lambda^*) = \frac{1}{2(1 + \theta)^2}, \quad (24)$$

where $s(\lambda) \equiv \sigma_\mu^2 Q(\lambda)/(1 + \theta)^2$ is the share of the perceived FCI variance attributable to financial noise. Moreover λ^* is weakly increasing in the financial noise variance σ_μ^2 , in the revision variance v_ω , and in the persistence of the Fed's view ρ , strictly so in the interior.

A proof sketch clarifies the intuition. Write out the loss. Let $\sigma_{\delta^F}^2 = (v + v_\omega)/(1 - \rho^2)$ denote the stationary variance of the Fed's demand view. Using (20), orthogonality, and (16),

$$L(\lambda) = \underbrace{\left(\frac{1 - \rho}{B} \right)^2 \sigma_{\delta^F}^2}_{\text{current Fed view}} + \underbrace{\left(\frac{Q(\lambda)}{1 + \theta} \right)^2 \sigma_\mu^2}_{\text{financial noise}} + \underbrace{\left(\frac{\theta}{(1 + \theta)B} \right)^2 \lambda v}_{\text{forward guidance shock}}. \quad (25)$$

Only the last two terms depend on communication. Differentiating,

$$L'(\lambda) = \underbrace{\frac{2\sigma_\mu^2 Q(\lambda) Q'(\lambda)}{(1 + \theta)^2}}_{\text{recruitment}} + \underbrace{\left(\frac{\theta}{(1 + \theta)B} \right)^2 v}_{\text{forward guidance shock}}. \quad (26)$$

The second term is strictly positive and is the marginal cost of the forward guidance shock. The first term is weakly negative and is the recruitment benefit. Proposition 2 follows by differentiating the fixed point in Proposition 1 and substituting $Q'(\lambda)$ into this first-order condition. Appendix B gives the steps.

The second term in (26) shows that the cost of the forward guidance shock does not depend on how much the Fed has already said. Each unit of resolved variance reaches the current FCI with the same loading. However, the first term shows the benefit does depend on it. Noise passes through in proportion to the perceived variance, so as guidance lowers Q it also lowers the gain from lowering Q further. More guidance is therefore worthwhile only while the benefit still exceeds the cost, and the two are equal when Q has fallen to $1/(2\sigma_\mu^2)$. The second condition in (24) states optimality in terms of the noise share of the perceived variance. Guidance mitigates the impact of noise and thereby lowers its share, so the optimum obtains when the share has fallen to a threshold set by how responsive policy is.

That noise-share characterization also explains the comparative statics with respect to financial noise. More financial noise raises $s(\lambda)$ at every λ and therefore raises λ^* . Intuitively, greater financial noise makes recruiting arbitrageurs more valuable.

To see the remaining comparative statics, consider the characterization $Q(\lambda^*) = 1/(2\sigma_\mu^2)$. More irreducible policy uncertainty v_ω raises $Q(\lambda)$ at every λ , and so also raises the optimal intensity. Intuitively, greater policy uncertainty increases the noise impact and thus it also makes recruiting arbitrageurs more valuable.

Greater persistence ρ raises the optimal intensity for a different reason. It does not change how uncertain the Fed's view is, but it changes how much that view matters for financial conditions. A more persistent view means a given innovation moves the Fed's target for longer, and forward-looking prices capitalize the whole path, so the FCI responds more to a given innovation. This is the loading θ/B in (22), which rises with ρ because $B = 1 + \theta - \rho$. The same uncertainty about the Fed's view therefore translates into a larger $Q(\lambda)$, and resolving it becomes more valuable.

5. What to Communicate: Financial Conditions or Interest Rates

Section 4 took the informational content of the announcement as given. The Fed must also choose what to talk about. In practice, the Fed provides forward guidance in terms of interest rates, but our model suggests that guidance in terms of financial conditions can also be valuable, given the central role they play in the analysis. This section compares two objects the Fed could project: the financial conditions it is aiming for, and the interest rate that would implement them. We show that for a given communication effort the second delivers a strictly lower guidance intensity, because the implementing rate also

reflects the risk premium the Fed expects to offset. Rate guidance therefore caps the intensity the Fed can attain, and the cap is more likely to bind in the environments where guidance is most valuable.

5.1. Rate guidance communicates less than FCI-star guidance

The Fed can communicate either of two projections. The first variable is the Fed’s projected FCI-star,

$$p_{t+1|t}^{*,F} = y^* - \bar{\delta}_{t+1|t}^F. \quad (27)$$

The second is its projected *neutral interest rate*: the date- t projected policy rate $\bar{i}_{t+1|t}$ that would set the Fed’s expected FCI equal to its projected FCI-star. This projection depends on the Fed’s advance forecast μ_{t+1}^F of next period’s noise pressure. Formally, $\mu_{t+1}^F \equiv \mathbb{E}_t^F[\mu_{t+1}]$. We assume $\text{Var}^F(\mu_{t+1}) = \sigma_\mu^2$, so the law of total variance implies $\sigma_{\mu^F}^2 \equiv \text{Var}^F(\mu_{t+1}^F) \in (0, \sigma_\mu^2]$. The arbitrageurs regard this forecast as uninformative about realized future noise. Evaluating the pricing condition (3) at $t + 1$ under that rate and imposing $\mathbb{E}_t^F[p_{t+1}] = p_{t+1|t}^{*,F}$ gives

$$\bar{i}_{t+1|t} = i^* + \ell \bar{\delta}_{t+1|t}^F + Q(\lambda) \mu_{t+1}^F, \quad \ell \equiv \frac{(1 + \theta)(1 - \rho)}{1 + \theta - \rho} \in (0, 1]. \quad (28)$$

Here ℓ captures the amount the projected rate loads onto the Fed’s projected demand view. A projection of the policy rate itself, $i_{t+1|t}^F \equiv \mathbb{E}_t^F[i_{t+1}]$, is an affine transformation of (28),

$$i_{t+1|t}^F = \frac{1}{1 + \theta} i^* + \frac{\theta}{1 + \theta} \bar{i}_{t+1|t}. \quad (29)$$

So the analysis covers dot-plot-style policy-rate projections.

The two projections differ in what they contain. The target (27) is a function of the Fed’s demand view alone. The rate (28) is not: implementing a level of financial conditions requires offsetting the risk premium the Fed expects to prevail, so μ_{t+1}^F enters alongside $\bar{\delta}_{t+1|t}^F$. This is the model counterpart of the contrast in Section 2.5, where the neutral rate moves with the risk premium while FCI-star does not. Recall also that by (11), the Fed’s demand projection consists of a public component and advance news:

$$\bar{\delta}_{t+1|t}^F = \underbrace{\rho \delta_t^F}_{\text{public at date } t} + \underbrace{\xi_{t+1}^F}_{\text{news}}. \quad (30)$$

Hence, both projections contain news about the Fed’s future demand, but the interest rate projection also contains news about the Fed’s expected risk premium.

The Fed announces a noisy version of either projection:

$$\tilde{p}_{t+1|t}^{*,F} = p_{t+1|t}^{*,F} + \epsilon_t \quad \text{or} \quad \tilde{i}_{t+1|t} = \bar{i}_{t+1|t} + \epsilon_t, \quad (31)$$

with $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ independent. By (28) and (30), the news in the first announcement is ξ_{t+1}^F , with variance v , and the news in the second is $\ell \xi_{t+1}^F + Q(\lambda) \mu_{t+1}^F$. For either announcement we assume the share of that news the market learns is a constant $\gamma \in [0, 1]$,

$$\gamma = \frac{v}{v + \sigma_\epsilon^2} \quad \text{under FCI-star guidance,} \quad (32)$$

$$\gamma = \frac{\text{Var}^F(\ell \xi_{t+1}^F + Q(\lambda) \mu_{t+1}^F)}{\text{Var}^F(\ell \xi_{t+1}^F + Q(\lambda) \mu_{t+1}^F) + \sigma_\epsilon^2} \quad \text{under rate guidance.} \quad (33)$$

Thus, $\gamma/(1 - \gamma)$ is the announcement's signal-to-noise ratio. Announcing the projection exactly is $\sigma_\epsilon^2 = 0$ and $\gamma = 1$, an announcement carrying as much noise as news is $\gamma = 1/2$, and silence is $\gamma = 0$.

We call γ *communication effort* because the Fed can raise it by making its projection more precise or easier to interpret. It is scale independent and measured in information rather than words. We assume the same γ applies to both interest rate and financial conditions projections so that we can compare communication formats for a given effort.

Communication effort γ is related to the guidance intensity λ of Section 4, but the two coincide only under FCI-star guidance. There, $p_{t+1|t}^{*,F} = y^* - \rho \delta_t^F - \xi_{t+1}^F$ and the first two terms are public at date t , so observing $\tilde{p}_{t+1|t}^{*,F}$ is observing ξ_{t+1}^F with independent noise. That is exactly the announcement (13), and hence

$$\lambda_F(\gamma) = \gamma. \quad (34)$$

Under rate guidance the two differ. Writing the announcement out,

$$\tilde{i}_{t+1|t} = i^* + \ell \rho \delta_t^F + \ell \xi_{t+1}^F + Q(\lambda) \mu_{t+1}^F + \epsilon_t, \quad (35)$$

where the first two terms are public at date t . Subtracting them and dividing by ℓ shows that the announcement is informationally equivalent to

$$\xi_{t+1}^F + \tilde{\epsilon}_t, \quad \tilde{\epsilon}_t \equiv \frac{Q(\lambda) \mu_{t+1}^F + \epsilon_t}{\ell}, \quad \text{Var}^F(\tilde{\epsilon}_t) = \frac{\sigma_{\mu^F}^2 Q(\lambda)^2 + \sigma_\epsilon^2}{\ell^2}, \quad (36)$$

where $\sigma_{\mu^F}^2$ is the variance of the Fed's premium forecast defined above. Recall that $\ell \leq 1$ is the loading of the projected rate on the Fed's view. This has the form (13), but with a

larger noise variance, $\text{Var}^F(\tilde{\epsilon}_t) > \sigma_\epsilon^2$: the Fed's premium forecast enters the announcement as noise from the market's point of view. In particular, the guidance intensity is now given by

$$\lambda_R = \frac{v}{v + \text{Var}^F(\tilde{\epsilon}_t)}. \quad (37)$$

The rate projection therefore contains additional noise from the market's perspective. Comparing the two formats at a common communication effort also requires accounting for the amount of news each projection contains. A direct mapping between effort and intensity provides that comparison. Combining (37) with (36) and (33) gives⁹

$$\lambda_R = \gamma \Phi(\lambda_R), \quad \Phi(\lambda) \equiv \frac{\ell^2 v}{\ell^2 v + \sigma_{\mu^F}^2 Q(\lambda)^2} \in (0, 1), \quad (38)$$

where we have used the fact that under rate guidance the prevailing intensity is λ_R itself. We refer to $\Phi(\lambda)$ as the *attenuation coefficient*. It captures the extent to which the interest rate signal tracks the Fed's view of fundamentals rather than its perceived risk premium response. A lower Φ means the signal is more contaminated by the Fed's risk premium perceptions.

This contamination is endogenous, because noise has a greater impact on financial conditions when the arbitrageurs perceive greater variance. A higher $Q(\lambda)$ means the implementing rate must offset a larger expected premium, so the announcement carries proportionally more premium information. This in turn raises the market's policy uncertainty and $Q(\lambda)$. Hence (38) is a fixed point in the guidance intensity. The following result establishes existence and describes the comparative statics of the attenuation.

Proposition 3 (Rate guidance attenuates communication). *Suppose $\sigma_{\mu^F}^2 > 0$ and $\Phi'(\lambda) < 1$ for every $\lambda \in [0, 1]$. Then for each $\gamma \in [0, 1]$ the fixed point (38) has a unique solution $\lambda_R(\gamma)$, which is strictly increasing in γ with range $[0, \bar{\lambda}_R]$, where $\bar{\lambda}_R \equiv \lambda_R(1) < 1$. Moreover, for every $\gamma \in (0, 1]$, rate guidance results in a smaller guidance intensity than FCI-star guidance,*

$$\lambda_R(\gamma) < \lambda_F(\gamma) = \gamma. \quad (39)$$

Equivalently, delivering a given guidance intensity λ requires effort λ under FCI-star guidance and $\lambda/\Phi(\lambda) > \lambda$ under rate guidance. Finally, for every $\gamma \in (0, 1]$ the delivered intensity $\lambda_R(\gamma)$ is strictly decreasing in the financial noise variance σ_μ^2 , in the revision variance v_ω , and in the persistence of the Fed's view ρ .

⁹Write $V \equiv \ell^2 v + \sigma_{\mu^F}^2 Q(\lambda)^2$ for the variance of the news the announcement carries. Multiplying the numerator and denominator of (37) by ℓ^2 gives $\lambda_R = \ell^2 v / (V + \sigma_\epsilon^2)$, while (33) reads $\gamma = V / (V + \sigma_\epsilon^2)$. The two share a denominator, so dividing one by the other leaves $\lambda_R / \gamma = \ell^2 v / V = \Phi(\lambda)$.

All three comparative statics work by lowering the attenuation coefficient $\Phi(\lambda)$ in (38). More financial noise and greater revision variance raise the arbitrageurs' perceived variance $Q(\lambda)$. The rate that implements a given level of financial conditions must then offset a larger expected premium, so the announcement carries proportionally more premium information and less news about the target.

Greater persistence has this same force, since it also raises $Q(\lambda)$, but it has an additional one. Persistence lowers the loading of the projected rate on the Fed's demand view

$$\ell = \frac{(1 + \theta)(1 - \rho)}{1 + \theta - \rho}.$$

A persistent movement in the target is partly delivered by expected future conditions, so implementing it requires a smaller adjustment of the rate. In contrast, the exposure to the premium forecast is undiminished. So when demand shocks are more persistent, the target's share of the announcement news is smaller also through this channel.

5.2. Optimal communication

We next analyze the optimal communication within each class. Under either format the Fed chooses an effort $\gamma \in [0, 1]$, which delivers guidance intensity $\lambda_F(\gamma) = \gamma$ or $\lambda_R(\gamma)$. Say that a format *attains* an intensity λ if some effort delivers it. By Proposition 3, FCI-star guidance attains every $\lambda \in [0, 1]$, whereas rate guidance attains only $\lambda \in [0, \bar{\lambda}_R]$ with $\bar{\lambda}_R < 1$. The Fed chooses the attainable intensity that minimizes the loss L of Section 4.

Proposition 4 (Optimal communication). *Maintain Propositions 2 and 3. Under FCI-star guidance the Fed attains the unconstrained optimum λ^* of Proposition 2. Under rate guidance it attains $\min\{\lambda^*, \bar{\lambda}_R\}$, since L is strictly convex and λ^* may lie above the ceiling. FCI-star guidance therefore weakly dominates rate guidance, and strictly so if and only if $\lambda^* > \bar{\lambda}_R$. When instead $0 < \lambda^* \leq \bar{\lambda}_R$, both formats attain λ^* , but rate guidance requires effort $\lambda^*/\Phi(\lambda^*) > \lambda^*$ against λ^* under FCI-star guidance.*

The two formats coincide when the intensity the Fed wants is within reach of a rate projection, $\lambda^* \leq \bar{\lambda}_R$, and differ otherwise. Since σ_μ^2 , v_ω and ρ all raise λ^* by Proposition 2 and lower $\bar{\lambda}_R$ by Proposition 3, they make strict dominance more likely on both counts. Noisier markets, more uncertain policy, and more persistent demand views therefore make FCI-star guidance more valuable.

The effort comparison sharpens this. For any positive intensity attainable under both formats, rate guidance requires more effort. A central bank that speaks about interest rates must speak more to be understood equally well.

6. Conclusion

Guidance in our model has two effects. By revealing the central bank’s view of future demand, it lowers the variance arbitrageurs perceive and recruits them against noisy flows. But guidance also moves financial conditions on announcement. On average, this movement raises output-gap variance. The optimal intensity balances these effects and rises with financial noise, policy uncertainty, and the persistence of the central bank’s view.

The communication variable determines how closely guidance can approach that optimum. A projected rate bundles the central bank’s demand view with its risk-premium forecast, so it delivers less guidance intensity for a given effort and may not reach the optimum. Financial conditions guidance does not face this ceiling.

The model does not settle the current debate over how much central banks should say, but it locates the disagreement. The case against guidance is that announcements move asset prices before the events they describe. The case for guidance is that uncertainty about policy leaves financial conditions more exposed to non-fundamental flows. Which force dominates is an empirical question about how much noise financial conditions carry and how much policy uncertainty markets face.

What a central bank should communicate has a more concrete answer. Interest rate guidance is contaminated by the central bank’s view of the risk premium, so a central bank that speaks in terms of interest rates must say more to convey the same guidance and might not be able to convey the optimal amount at all. Financial conditions guidance faces neither limit. What a central bank talks about therefore determines how much it has to say.

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A. Microfoundations of the model

This appendix provides the microfoundations for the two behavioral relations used in Section 3: the output–financial conditions relation (1) and the asset pricing relation (3). The real side of the economy follows the baseline model in Caballero and Simsek (2023). The financial market side follows Caballero et al. (2024) and allows for noise shocks.

The economy is set in discrete time $t \in \{0, 1, \dots\}$. There are four agent types: asset-holding households (households), hand-to-mouth agents, portfolio managers, and the central bank. The hand-to-mouth agents primarily serve to decouple labor supply decisions from household consumption behavior. The asset-holding households are the main drivers of aggregate demand through their consumption and savings choices. The portfolio managers act on behalf of these households by making portfolio allocation decisions that determine asset prices. The central bank conducts monetary policy.

A.1. Supply side

Hand-to-mouth agents provide all of the labor supply and spend all of their income (they do not save). Their problem is

$$\begin{aligned} \max_{N_t} \log C_t^{HM} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}, \\ \mathcal{P}_t C_t^{HM} = \mathcal{W}_t N_t + T_t, \end{aligned} \tag{A.1}$$

where φ denotes the inverse Frisch elasticity of labor supply, $\mathcal{P}_t$ the nominal price of the final good, $\mathcal{W}_t$ the nominal wage, and T_t lump-sum transfers from the government. The optimality condition implies a standard labor supply equation, $\mathcal{W}_t/\mathcal{P}_t = \chi N_t^\varphi C_t^{HM}$.

The rest of the supply side is the standard New Keynesian block. A competitive final goods producer combines intermediate goods with the CES technology $Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\varepsilon/(\varepsilon-1)}$ where $Y_t(\nu) = Z_t N_t(\nu)^{1-\alpha}$, where α governs decreasing returns to labor and $\varepsilon > 1$ is the elasticity of substitution that determines firm markups. Goods and labor market clearing require $Y_t = C_t^H + C_t^{HM}$ and $\int_0^1 N_t(\nu) d\nu = N_t$.

To simplify the distribution of output, the government transfers $T_t = (1 - \alpha) \mathcal{P}_t Y_t - \mathcal{W}_t N_t$ to hand-to-mouth agents, financed by a lump-sum tax on firms' profits. This leaves after-tax profits equal to $\alpha \mathcal{P}_t Y_t$ and implies $C_t^{HM} = (1 - \alpha) Y_t$. Substituting this into goods market clearing gives

$$Y_t = \frac{C_t^H}{\alpha}. \tag{A.2}$$

Hand-to-mouth agents create a Keynesian multiplier effect, but output is ultimately determined by the asset-holding *households'* spending, C_t^H .

Potential output and sticky prices. Consider the flexible-price allocation. The intermediate firm sets an optimal markup over marginal cost, which yields a labor demand equation $\mathcal{W}_t/\mathcal{P}_t = \frac{\varepsilon-1}{\varepsilon} (1-\alpha) Z_t N_t^{-\alpha}$. Combining this with labor supply and $C_t^{HM} = (1-\alpha) Y_t$ delivers potential labor N^* as the solution to $\chi(N^*)^{1+\varphi} = \frac{\varepsilon-1}{\varepsilon}$, and potential output $Y^* = Z(N^*)^{1-\alpha}$. In the main text we work in logs and take Z constant, so that potential output is the constant $y^* \equiv \log Y^*$.

With fully sticky prices, intermediate good firms have a preset nominal price $\mathcal{P}_t(\nu) = \mathcal{P}^*$, so the final good price is also fixed. Since each firm operates with a markup, for small aggregate demand shocks it optimally chooses to meet the demand for its goods. Therefore output is determined by aggregate demand according to (A.2). Allowing partially flexible prices adds a standard New Keynesian Phillips curve without affecting the mechanisms studied here. See Caballero et al. (2024).

A.2. Demand side and financial markets

Financial assets. There are two assets. The market portfolio is a claim on firms' profits αY_t . Let P_t denote its ex-dividend price. Its gross return is

$$R_{t+1} = \frac{\alpha Y_{t+1} + P_{t+1}}{P_t}. \quad (\text{A.3})$$

There is also a risk-free asset in zero net supply, whose gross return $R_t^f = \exp(i_t)$ is set by the central bank.

Households' consumption-savings decisions. Households have preferences $\mathbb{E}_t [\sum_{h=0}^{\infty} \beta^h \log C_{t+h}^H]$ and the budget constraint

$$\begin{aligned} W_{t+1} + C_{t+1}^H &= W_t \left((1-\omega_t) R_t^f + \omega_t R_{t+1} \right) = D_{t+1} + K_{t+1}, \\ \text{where } D_{t+1} &= W_t \left[(1-\omega_t) (R_t^f - 1) + \omega_t \frac{\alpha Y_{t+1}}{P_t} \right] \\ \text{and } K_{t+1} &= W_t \left[1 - \omega_t + \omega_t \frac{P_{t+1}}{P_t} \right]. \end{aligned} \quad (\text{A.4})$$

Here W_t is end-of-period wealth and ω_t the market portfolio weight. The last two lines decompose the portfolio payoff at date $t+1$ into interest and dividend income (D_{t+1}) and a residual capital component (K_{t+1}).

Households take their portfolio allocation as given (it is delegated to the managers) and follow the consumption rule

$$C_t^H = (1-\beta) (D_t + K_t \exp(\delta_t)). \quad (\text{A.5})$$

When $\delta_t = 0$ this is the optimal rule under log preferences. When $\delta_t > 0$ (resp. < 0), households spend more (resp. less) than the optimal rule prescribes. This is the model's aggregate demand shifter.¹⁰

Portfolio managers and market clearing. Households delegate their portfolio choice to managers, who do not consume themselves. Each household invests with a continuum of managers randomly sampled from all managers, which ensures there is no portfolio heterogeneity across households.

In each period, a fraction η of managers are “noise traders” with portfolio weight $\omega_t^N = 1 + \frac{1}{\eta}\mu_t$. That is, they deviate from the optimal portfolio benchmark by $\frac{1}{\eta}\mu_t$. The remaining mass $1 - \eta$ are “arbitrageurs” who choose portfolio weights to maximize expected log assets under management, after observing the risk-free rate and the current noise μ_t :

$$\max_{\omega_t^A} \mathbb{E}_t^A \left[\log \left(W_t \left(R_t^f + \omega_t^A \left(R_{t+1} - R_t^f \right) \right) \right) \right].$$

We allow the arbitrageurs to hold their own beliefs. If returns are log-normally distributed, the approximate optimality condition is

$$\omega_t^A \sigma_{t,r_{t+1}}^A = \frac{\mathbb{E}_t^A [r_{t+1}] + \frac{1}{2} \left(\sigma_{t,r_{t+1}}^A \right)^2 - i_t}{\sigma_{t,r_{t+1}}^A}, \quad (\text{A.6})$$

so the arbitrageurs' demand for risk equals their perceived equilibrium Sharpe ratio. Financial markets clear when households in the aggregate hold the market portfolio, both before and after the portfolio allocation:

$$W_t = P_t \quad \text{and} \quad \omega_t = (1 - \eta) \omega_t^A + \eta \left(1 + \frac{\mu_t}{\eta} \right) = 1. \quad (\text{A.7})$$

Market clearing therefore gives

$$\omega_t^A = 1 - \frac{\mu_t}{1 - \eta} = 1 - \kappa \mu_t, \quad \kappa \equiv (1 - \eta)^{-1}. \quad (\text{A.8})$$

In equilibrium the arbitrageurs adjust their portfolio weight to absorb aggregate noise. A smaller arbitrageur sector raises κ , the price pressure associated with a given aggregate noise-trader flow. The main text measures noise in these price-impact units, writing μ_t for κ times the raw flow, so that σ_μ^2 subsumes both the size of the flow and arbitrageur capacity.

Remark 1 (Interpreting noise shocks). *In practice, noisy flows emerge from various sources. Retail investors may engage in sentiment-driven trading based on excessive optimism or pes-*

¹⁰Demand shocks could alternatively be modeled as shocks to the discount factor β in a fully optimizing framework. The specification in (A.5) keeps the analysis tractable and permits a flexible process for δ_t .

simism about assets. Institutional investors often face client inflows and outflows that require them to expand or reduce positions regardless of market conditions. Portfolio rebalancing can generate mechanical trading needs unrelated to asset fundamentals. The model remains agnostic about the source of these flows. Its key assumption is that they are less sensitive to return variance than the arbitrageurs' positions. This captures the standard limits-to-arbitrage mechanism in which risk constrains arbitrageurs' capacity to absorb mispricing (Shleifer and Vishny, 1997).

Output–financial conditions relation. Combining (A.4) and (A.7) gives $D_t = \alpha Y_t$ and $K_t = P_t$: dividends equal the firms' share of output, and capital equals the ex-dividend value of the market portfolio. Substituting into the consumption rule (A.5) yields $C_t^H = (1 - \beta)(\alpha Y_t + P_t \exp(\delta_t))$. Substituting (A.2) then gives

$$\begin{aligned} Y_t &= (1 - \beta) \frac{1}{\alpha\beta} P_t \exp(\delta_t) \\ \implies y_t &= m + p_t + \delta_t, \quad \text{where } y_t \equiv \log Y_t, \ p_t \equiv \log P_t, \ m \equiv \log \left(\frac{1 - \beta}{\alpha\beta} \right). \end{aligned} \quad (\text{A.9})$$

Output depends on aggregate wealth P_t , the MPC out of wealth $1 - \beta$, the demand shifter δ_t , and the Keynesian multiplier $1/(\alpha\beta)$. The derived constant m is suppressed in the main text. See the discussion of the limit below.

Financial market equilibrium condition. Substituting $\omega_t^A = 1 - \kappa\mu_t$ into (A.6) gives

$$\mathbb{E}_t^A [r_{t+1}] = i_t + \frac{1}{2} \left(\sigma_{t,r_{t+1}}^A \right)^2 - \kappa \left(\sigma_{t,r_{t+1}}^A \right)^2 \mu_t. \quad (\text{A.10})$$

The expected return depends on the risk premium, the perceived return variance, and the noise. The impact of noise increases in perceived variance and inverse arbitrageur capacity.

Campbell–Shiller approximation and the pricing relation. From (A.3), $r_{t+1} = \log(1 + X_{t+1}) + p_{t+1} - p_t$, where $X_t = \alpha Y_t / P_t$ is the dividend price ratio. Evaluating (A.9) at $\delta_t = 0$ and $Y = Y^*$ gives the steady-state ratio $X^* = \frac{1-\beta}{\beta}$. Log-linearizing $\log(1 + X_{t+1})$ around X^* and collecting constants yields

$$r_{t+1} = \varrho - (1 - \beta) m + (1 - \beta) y_{t+1} + \beta p_{t+1} - p_t = \varrho + p_{t+1} - p_t + (1 - \beta) \delta_{t+1},$$

where $\varrho \equiv -\log \beta$ and the second equality substitutes (A.9). Substituting this into (A.10) gives the present discounted value relation

$$p_t = \varrho + \mathbb{E}_t^A [p_{t+1}] + (1 - \beta) \mathbb{E}_t^A [\delta_{t+1}] - \left(i_t + \frac{1}{2} \left(\sigma_{t,r_{t+1}}^A \right)^2 \right) + \left(\sigma_{t,r_{t+1}}^A \right)^2 \kappa \mu_t.$$

The limit $\beta \rightarrow 1$. The main text presents this system in the limit $\beta \rightarrow 1$, taken as a limit of economies with $\beta < 1$ so that the households' problem remains well defined. Three simplifications follow, none of them substantive.

First, $\varrho \equiv -\log \beta \rightarrow 0$, so the discount-rate constant drops out of the pricing relation.

Second, the cash-flow term vanishes under bounded conditional second moments. Its contribution to the conditional mean is multiplied by $1 - \beta$. Its variance and covariance contributions are

$$(1 - \beta)^2 \text{Var}_t^A(\delta_{t+1}) + 2(1 - \beta) \text{Cov}_t^A(p_{t+1}, \delta_{t+1}),$$

and therefore also vanish. Hence return variance converges to FCI variance, $(\sigma_{t,r_{t+1}}^A)^2 = \text{Var}_t^A(p_{t+1}) \equiv Q$, and we drop the return from the notation.

Third, the constants m and ϱ are level normalizations. The constant m fixes the origin of the FCI scale and ϱ the level of the policy rate, and both cancel from every equilibrium condition: policy and pricing depend on the FCI only through the gap $p_t - p_t^{*,F}$, from which m cancels identically, while ϱ shifts the pricing intercept and the policy intercept by the same amount. Setting them to zero is therefore without loss of generality, and would be so even away from the limit.

Together these deliver (1) and (3) in the main text.

A.3. The Fed's view process

Equation (9) is the innovations representation of a standard Gaussian filter under the Fed's subjective model. Let

$$\delta_{t+1} = \rho \delta_t + u_{t+1}, \quad s_{t+1}^F = u_{t+1} + e_{t+1}^s, \quad d_{t+1}^F = \delta_{t+1} + e_{t+1}^d, \quad (\text{A.11})$$

with independent Gaussian innovations. The Fed observes the advance signal s_{t+1}^F at date t and a second signal d_{t+1}^F at date $t+1$. Its advance news is $\xi_{t+1}^F = \mathbb{E}_t^F[u_{t+1} \mid s_{t+1}^F]$, and the subsequent update is

$$\omega_{t+1}^F = K (d_{t+1}^F - \mathbb{E}_t^F[d_{t+1}^F]), \quad (\text{A.12})$$

where K is the steady-state Kalman gain. Gaussian projection makes the two orthogonal, delivering (10).

This construction establishes that the Fed's advance news and subsequent revision are orthogonal innovations under its own model. Under the arbitrageurs' beliefs, the revision is instead Fed-specific belief noise, unrelated to fundamentals and to innovations in their own demand assessment. At date $t+1$, the equilibrium pair (i_{t+1}, p_{t+1}) reveals δ_{t+1}^F exactly by (19). The arbitrageurs therefore learn the Fed's realized view from policy without treating the revision as information about demand.

B. Proofs

B.1. Notation used in this appendix

A remark on the two measures. The perceived FCI variance $Q(\lambda) = \text{Var}_t^A(p_{t+1})$ is taken under the arbitrageurs' beliefs, since it is what governs their risk-bearing, while the loss $\mathcal{L}$ is taken under the Fed's. The pass-through coefficient $Q(\lambda)/(1 + \theta)$ in (20) belongs to the equilibrium law of motion, so its ranking is measure-free, and the two parties agree on the unconditional variances that enter $\mathcal{L}$ even though they disagree about the relation between the Fed's view and realized demand.

Under the rate guidance of Section 5, the posterior residual for ξ_{t+1}^F loads on μ_{t+1}^F . The latter remains orthogonal to realized μ_{t+1} under the arbitrageurs' measure because they regard the Fed's advance noise signal as uninformative.

The proofs use the following shorthand, which does not appear in the main text:

$$A \equiv (1 + \theta)^2, \quad H_\xi \equiv \frac{A\theta^2 v}{B^2}, \quad H_\omega \equiv \frac{A\theta^2 v_\omega}{B^2}, \quad (\text{B.1})$$

$$x_\xi \equiv 4\sigma_\mu^2 H_\xi, \quad x_\omega \equiv 4\sigma_\mu^2 H_\omega. \quad (\text{B.2})$$

The next-period FCI implied by (20) is

$$p_{t+1} = y^* - \frac{\theta}{B} \delta_{t+1}^F - \frac{\theta}{(1 + \theta)B} \hat{\xi}_{t+2}^F + \frac{Q}{1 + \theta} \mu_{t+1}, \quad (\text{B.3})$$

whose date- t conditional variance under the arbitrageurs' measure gives (22). In this notation the fixed point (22) reads $Q(\lambda) = A^{-1}[\sigma_\mu^2 Q(\lambda)^2 + D(\lambda)]$ with

$$D(\lambda) \equiv H_\omega + H_\xi [1 - (1 - A^{-1}) \lambda], \quad Q(D) = \frac{A - \sqrt{A^2 - 4\sigma_\mu^2 D}}{2\sigma_\mu^2}, \quad (\text{B.4})$$

and the stable solution exists on $[0, 1]$ if and only if

$$A^2 > 4\sigma_\mu^2 (H_\xi + H_\omega), \quad (\text{B.5})$$

which is the existence hypothesis of Proposition 2.

Proof of Proposition 1. Fix a guidance intensity λ and write $\hat{v} = (1 - \lambda)v$ for the residual advance-news variance, so that $\text{Var}^F(\hat{\xi}_{t+1}^F) = v - \hat{v}$. Conjecture an affine FCI gap

$$p_t - p_t^{*,F} = a\delta_t^F + b\hat{\xi}_{t+1}^F + d\mu_t.$$

Then

$$p_{t+1} - y^* = (a - 1)\delta_{t+1}^F + b\hat{\xi}_{t+2}^F + d\mu_{t+1}.$$

Using $\mathbb{E}_t^A[\delta_{t+1}^F] = \rho\delta_t^F + \hat{\xi}_{t+1}^F$ and the centering of future innovations and financial noise, the pricing condition and policy rule imply

$$(1 + \theta)(p_t - p_t^{*,F}) = \{1 + \rho(a - 1)\}\delta_t^F + (a - 1)\hat{\xi}_{t+1}^F + Q\mu_t.$$

Matching coefficients gives

$$a = \frac{1 - \rho}{B}, \quad b = -\frac{\theta}{(1 + \theta)B}, \quad d = \frac{Q}{1 + \theta},$$

which proves (20) and (B.3).

For variance consistency, separate the residual component of the innovation from its date- t posterior mean in (B.3):

$$p_{t+1} = \text{known date-}t \text{ terms} - \frac{\theta}{B} \left(\xi_{t+1}^F - \hat{\xi}_{t+1}^F + \omega_{t+1}^F \right) - \frac{\theta}{(1 + \theta)B} \hat{\xi}_{t+2}^F + \frac{Q}{1 + \theta} \mu_{t+1}. \quad (\text{B.6})$$

The three displayed components have variances $\hat{v} + v_\omega$, $v - \hat{v}$, and σ_μ^2 , respectively, and are orthogonal under the arbitrageurs' measure. Therefore

$$Q = \frac{\theta^2}{B^2}(v_\omega + \hat{v}) + \frac{\theta^2}{AB^2}(v - \hat{v}) + \frac{\sigma_\mu^2 Q^2}{A},$$

which is (22).

Writing the fixed point as $\sigma_\mu^2 Q^2 - AQ + D(\hat{v}) = 0$, the constant term satisfies

$$D(\hat{v}) = \frac{A\theta^2}{B^2}(v_\omega + \hat{v}) + \frac{\theta^2}{B^2}(v - \hat{v}), \quad D'(\hat{v}) = \frac{\theta^2(A - 1)}{B^2} > 0.$$

The smaller root is

$$G(\hat{v}) = \frac{A - \sqrt{A^2 - 4\sigma_\mu^2 D(\hat{v})}}{2\sigma_\mu^2}, \quad \frac{dG}{dD} = \frac{1}{\sqrt{A^2 - 4\sigma_\mu^2 D}} > 0.$$

It is strictly increasing in $\hat{v}$, so reducing residual uncertainty lowers Q and the noise pass-through coefficient. Since $\hat{v} = (1 - \lambda)v$, this is the statement that $Q(\lambda)$ is decreasing in the guidance intensity.

It remains to justify the selection of the smaller root. The right side of (22) maps the variance the arbitrageurs perceive into the variance the resulting price process delivers, and its derivative is $2\sigma_\mu^2 Q/A$. At the smaller root this slope is below one. A small increase in the

perceived variance raises the compensation the arbitrageurs demand and hence the noise pass-through, but it returns a smaller increase in realized variance than the one that started it, so the perturbation dies out and the fixed point is locally stable. At the larger root the slope exceeds one and the same perturbation returns amplified: higher perceived variance raises pass-through by enough to raise realized variance by more, and the loop feeds on itself. That equilibrium is not locally stable, and we discard it. The selected root is equivalently the one satisfying $A - 2\sigma_\mu^2 Q > 0$, the condition that recurs in the denominators below. $\square$

Proof of Proposition 2. Writing (22) in the notation of Appendix B yields (B.4). Implicit differentiation of

$$\sigma_\mu^2 Q(\lambda)^2 - A Q(\lambda) + D(\lambda) = 0$$

gives

$$Q'(\lambda) = -\frac{H_\xi(1 - A^{-1})}{A - 2\sigma_\mu^2 Q(\lambda)}.$$

The denominator is positive at the smaller root. Hence $Q'(\lambda) < 0$.

The three components of the FCI gap in (20) are orthogonal. The current-view component contributes $\left(\frac{1-\rho}{B}\right)^2 \sigma_{\delta_F}^2$, financial noise contributes $\sigma_\mu^2 Q(\lambda)^2/A$, and communication news contributes $\theta^2 \lambda v/(AB^2)$. This gives (25) and (26). Substituting $Q'(\lambda)$ into the recruitment term of (26) and using $H_\xi = A\theta^2 v/B^2$,

$$\frac{2\sigma_\mu^2 Q(\lambda)}{A} Q'(\lambda) = -\frac{\theta^2 v}{AB^2} \cdot \frac{2\sigma_\mu^2 Q(\lambda)(A-1)}{A - 2\sigma_\mu^2 Q(\lambda)},$$

so that, factoring out the cost term $\theta^2 v/(AB^2)$,

$$\begin{aligned} L'(\lambda) &= \frac{\theta^2 v}{AB^2} \left[1 - \frac{2\sigma_\mu^2 Q(\lambda)(A-1)}{A - 2\sigma_\mu^2 Q(\lambda)} \right] \\ &= \frac{\theta^2 v}{AB^2} \cdot \frac{[A - 2\sigma_\mu^2 Q(\lambda)] - 2\sigma_\mu^2 Q(\lambda)(A-1)}{A - 2\sigma_\mu^2 Q(\lambda)} \\ &= \frac{\theta^2 v}{AB^2} \cdot \frac{A [1 - 2\sigma_\mu^2 Q(\lambda)]}{A - 2\sigma_\mu^2 Q(\lambda)}, \end{aligned}$$

where the two terms in $2\sigma_\mu^2 Q(\lambda)$ carrying no factor of A cancel in the numerator. Cancelling A leaves

$$L'(\lambda) = \frac{\theta^2 v}{B^2} \frac{1 - 2\sigma_\mu^2 Q(\lambda)}{A - 2\sigma_\mu^2 Q(\lambda)}. \quad (\text{B.7})$$

Since $\sigma_\mu^2 > 0$ and the smaller root has $A - 2\sigma_\mu^2 Q(\lambda) > 0$, equation (B.7) shows that $L'(\lambda)$ has the sign of $1 - 2\sigma_\mu^2 Q(\lambda)$. To establish strict convexity, define $f(Q) = (1 - 2\sigma_\mu^2 Q)/(A - 2\sigma_\mu^2 Q)$.

Then

$$f'(Q) = -\frac{2\sigma_\mu^2(A-1)}{(A-2\sigma_\mu^2Q)^2} < 0.$$

Since $Q'(\lambda) < 0$, the derivative in (B.7) is strictly increasing in λ .

At any endpoint, $2\sigma_\mu^2Q \lesseqgtr 1$ if and only if $4\sigma_\mu^2D \lesseqgtr 2A-1$. At silence, $4\sigma_\mu^2D(0) = x_\xi + x_\omega$. At full disclosure, $4\sigma_\mu^2D(1) = x_\omega + x_\xi/A$. Strict convexity therefore gives the three cases of Proposition 2. In the interior case, $Q(\lambda^*) = 1/(2\sigma_\mu^2)$ and hence

$$D(\lambda^*) = \frac{2A-1}{4\sigma_\mu^2}.$$

Solving (B.4) for λ^* and differentiating with respect to v_ω gives

$$\frac{\partial \lambda^*}{\partial v_\omega} = \frac{(1+\theta)^2}{[(1+\theta)^2-1]v} > 0. \quad (\text{B.8})$$

Finally, consider the comparative statics. Raising v_ω raises λ^* by (B.8), strictly in the interior. Raising ρ raises $D(\lambda)$ and hence $Q(\lambda)$ at every λ . Raising σ_μ^2 also raises $Q(\lambda)$ at every λ , since implicit differentiation of $\sigma_\mu^2Q^2 - AQ + D = 0$ gives $\partial Q/\partial \sigma_\mu^2 = Q^2/(A-2\sigma_\mu^2Q) > 0$, and it additionally lowers the target $1/(2\sigma_\mu^2)$. By (B.7) and strict convexity, each therefore raises λ^* weakly, and strictly whenever the optimum is interior both before and after the change. $\square$

Proof of Proposition 3. Fix $\gamma \in (0, 1]$ and let $h_\gamma(\lambda) = \gamma\Phi(\lambda) - \lambda$. Because $Q'(\lambda) < 0$, Φ is weakly increasing, and $0 \leq \gamma\Phi'(\lambda) \leq \Phi'(\lambda) < 1$, so h_γ is strictly decreasing. Since $h_\gamma(0) = \gamma\Phi(0) > 0$ and $h_\gamma(1) = \gamma\Phi(1) - 1 < 0$, where the second inequality uses $\sigma_{\mu_F}^2 > 0$ and $Q(1) > 0$, there is a unique $\lambda_R(\gamma) \in (0, 1)$ solving (38). At $\gamma = 0$ the unique solution is $\lambda_R(0) = 0$.

Monotonicity in γ follows from h_γ being increasing in γ at fixed λ together with h_γ strictly decreasing in λ . Continuity of λ_R then gives the range $[0, \bar{\lambda}_R]$ with $\bar{\lambda}_R = \lambda_R(1)$, and $\bar{\lambda}_R < 1$ because $\Phi(1) < 1$.

For the attenuation statement, (38) and $\sigma_{\mu_F}^2 > 0$ give $\Phi(\lambda) < 1$ for every λ . Hence $\lambda_R(\gamma) = \gamma\Phi(\lambda_R(\gamma)) < \gamma$ for $\gamma > 0$, which is (39). Inverting (38), the effort required to deliver λ in rate space is $\gamma = \lambda/\Phi(\lambda)$, against $\gamma = \lambda$ in FCI-star space by (34).

At any fixed λ , each of σ_μ^2 , v_ω and ρ raises $Q(\lambda)$, and ρ additionally lowers ℓ . Both effects lower $\Phi(\lambda)$ in (38) pointwise. A pointwise fall in Φ lowers h_γ at every λ , so its unique zero falls. Hence $\lambda_R(\gamma)$ is strictly decreasing in each of the three, and taking $\gamma = 1$ gives the same for $\bar{\lambda}_R$. $\square$

Proof of Proposition 4. Proposition 3 gives the feasible sets

$$\Lambda_F = [0, 1], \quad \Lambda_R = [0, \bar{\lambda}_R], \quad \bar{\lambda}_R < 1.$$

By Proposition 2, L is strictly convex with unique minimizer λ^* . Since $\lambda^* \in \Lambda_F$, FCI-star guidance attains the unconstrained optimum. Strict convexity implies that L is strictly decreasing below λ^* and strictly increasing above it, so the minimizer of L on Λ_R is $\min\{\lambda^*, \bar{\lambda}_R\}$. FCI-star guidance therefore weakly dominates rate guidance, and strictly so if and only if $\lambda^* > \bar{\lambda}_R$, in which case $L(\bar{\lambda}_R) > L(\lambda^*)$.

If instead $0 < \lambda^* \leq \bar{\lambda}_R$, both formats attain λ^* . By Proposition 3 the effort required is λ^* under FCI-star guidance and $\lambda^*/\Phi(\lambda^*)$ under rate guidance, and $\Phi(\lambda^*) < 1$ gives $\lambda^*/\Phi(\lambda^*) > \lambda^*$. $\square$