Supplemental Appendix to
"A Deep Case for Shallow Integration"

John Sturm Becko — Princeton
Arnaud Costinot — MIT

A Proofs of Results in Section 5

A.1 Proof of Lemma 3

Proof. By contradiction, suppose that θ^* is not efficient. Then there exists $\hat{\theta} \in \Theta$ such that

$$W_i(\hat{\theta}) > W_i(\theta^*), \text{ for some } i, \tag{A.1}$$

$$W_j(\hat{\theta}) \geq W_j(\theta^*), \text{ for all } j \neq i. \tag{A.2}$$

Let $m^e(\hat{\theta}) \equiv \{m_i^e(\hat{\theta})\}$ and $d^e(\hat{\theta}) \equiv \{d_i^e(\hat{\theta})\}$ denote the vectors of extended net imports and domestic variables associated with the alternative policy $\hat{\theta}$. By definition of a competitive equilibrium, we know that for each country j, $d_j^e(\hat{\theta}) \in \mathcal{D}_j^e[m_j^e(\hat{\theta})]$. This implies

$$W_j(\hat{\theta}) = U_j[d_j^e(\hat{\theta}), z(\hat{\theta})] \leq \max_{d_j^e \in \mathcal{D}_j^e[m_j^e(\hat{\theta})]} U_j(d_j^e, z(\hat{\theta})) = V_j^e[m_j^e(\hat{\theta})] \text{ for all } j. \tag{A.3}$$

From (23), we also know that

$$W_j(\theta^*) = U_j[d_j^e(\theta^*), z(\theta^*)] = V_j^e[m_j^e(\theta^*)] \text{ for all } j. \tag{A.4}$$

Combining (A.1)-(A.2) with (A.3)-(A.4), we therefore get

$$V_i^e[m_i^e(\hat{\theta})] > V_i^e[m_i^e(\theta^*)], \text{ for some } i, \tag{A.5}$$

$$V_j^e[m_j^e(\hat{\theta})] \geq V_j^e[m_j^e(\theta^*)], \text{ for all } j \neq i. \tag{A.6}$$

To conclude, note that since $m^e(\hat{\theta}) \equiv \{m_i^e(\hat{\theta})\}$ is the vector of extended net imports that arises in a competitive equilibrium under $\hat{\theta}$, $m^e(\hat{\theta}) \in \mathcal{M}^e$. Thus, (A.5)-(A.6) contradict (22). $\square$

A.2 Proof of Lemma 4

Proof. ($\Leftarrow$) The argument is similar to the one used in Lemma 3. By contradiction, suppose that θ^N is not a Nash equilibrium. Then there exists a country i and $\hat{\theta}_i \in \Theta_i$ such that

$$W_i(\hat{\theta}_i, \theta^N_{-i}) > W_i(\theta^N). \tag{A.7}$$

Let $m^e(\hat{\theta}) \equiv \{m^e_i(\hat{\theta})\}$ and $d^e(\hat{\theta}) \equiv \{d^e_i(\hat{\theta})\}$ denote the extended net imports and domestic variables associated with the alternative policy $\hat{\theta} \equiv \{\hat{\theta}_i, \theta^N_{-i}\}$. By definition of a competitive equilibrium, we know that $d^e_i(\hat{\theta}) \in \mathcal{D}^e_i[m^e_i(\hat{\theta})]$. This implies

$$W_i(\hat{\theta}) = U_i[d^e_i(\hat{\theta}), z(\hat{\theta})] \leq \max_{d^e_i \in \mathcal{D}^e_i[m^e_i(\hat{\theta})]} U_i(d^e_i, z(\hat{\theta})) = V^e_i[m^e_i(\hat{\theta})]. \tag{A.8}$$

From (25), we also know that

$$W_i(\theta^N) = U_i[d^e_i(\theta^N), z(\theta^N)] = V^e_i[m^e_i(\theta^N)]. \tag{A.9}$$

Combining (A.7) with (A.8)-(A.9), we therefore get

$$V^e_i[m^e_i(\hat{\theta})] > V^e_i[m^e_i(\theta^N)]. \tag{A.10}$$

To conclude, note that since $m^e_i(\hat{\theta})$ is the vector of extended net imports that arises in a competitive equilibrium under $\hat{\theta}$, $m^e_i(\hat{\theta}) \in \mathcal{M}^e_i$ and $m^e_i(\hat{\theta}) \in \Omega_i(\theta^N_{-i})$. Thus, (A.10) contradicts (24).

($\Rightarrow$) By contradiction, suppose that there exists i such that $m^e_i(\theta^N)$ does not satisfy (24) or $d^e_i(\theta^N)$ does not satisfy (25). Since $m^e_i(\theta^N)$ and $d^e_i(\theta^N)$ arise in a competitive equilibrium under θ^N, we already know that $m^e_i(\theta^N) \in \mathcal{M}^e_i$, $m^e_i(\theta^N) \in \Omega_i(\theta^N_{-i})$, and $d^e_i(\theta^N) \in \mathcal{D}^e_i[m^e_i(\theta^N)]$. Hence for the previous condition to hold, there must exist $\hat{m}^e_i \in \mathcal{M}^e_i$ such that

$$V^e_i(\hat{m}^e_i) > V^e_i[m^e_i(\theta^N)], \tag{A.11}$$

$$\hat{m}^e_i \in \Omega_i(\theta^N_{-i}), \tag{A.12}$$

or $\hat{d}_i^e$ such that

$$U_i[\hat{d}_i^e, z(\theta^N)] > U_i[d_i^e(\theta^N), z(\theta^N)], \tag{A.13}$$

$$\hat{d}_i^e \in \mathcal{D}_i^e[m_i^e(\theta^N)]. \tag{A.14}$$

We consider each of these two cases in turn.

Suppose that (A.11)-(A.12) hold. In this case, we first show that there exists $\hat{\theta}_i = (\hat{t}_i, \hat{s}_i)$ such that $U_i[d_i^e(\hat{\theta}_i, \theta_{-i}^N), z(\hat{\theta}_i, \theta_{-i}^N)] = V_i^e(\hat{m}_i^e)$. Start from a solution $\tilde{d}_i^e(\hat{m}_i^e)$ to the Meade problem (21) conditional on $\hat{m}_i^e$. Since $\hat{m}_i^e \in \mathcal{M}_i^e$, we know that such a solution exists. We let $\hat{s}_i = \tilde{s}_i(\hat{m}_i^e)$ denote the associated domestic policies and let $\hat{p}_i = \tilde{p}_i(\hat{m}_i^e)$ denote the associated domestic prices. Since $\hat{m}_i^e \in \Omega_i(\theta_{-i}^N)$, we also know that there exists a vector of world prices $\hat{p}^w$ such that foreign equilibrium conditions hold. Given $\hat{p}_i$ and $\hat{p}^w$, set trade taxes $\hat{t}_i$ such that the non-arbitrage condition (7) holds, i.e.,

$$\hat{p}_{ig} = \hat{p}_g^w \times (1 + \hat{t}_{ig}).$$

Given Assumption 2, we know that such trade taxes are feasible. Since (A.12) holds, note that the combination of (i) the domestic variables $\tilde{d}_i^e(\hat{m}_i^e)$, which solve the Meade problem (21) conditional on $\hat{m}_i^e$ and (ii) the world price $\hat{p}^w$, foreign variables $\hat{d}_{-i}^e$, and net imports $\hat{m}_{-i}^e$ associated with the construction of the offer curve at $\hat{m}_i^e$ and θ_{-i}^N, and which therefore satisfy all foreign equilibrium conditions, forms a competitive equilibrium conditional on $(\hat{\theta}_i, \theta_{-i}^N)$. Since domestic variables solve the Meade problem (21) conditional on $\hat{m}_i^e$, the associated utility in country i is equal to $U_i[d_i^e(\hat{\theta}_i, \theta_{-i}^N), z(\hat{\theta}_i, \theta_{-i}^N)] = V_i^e(\hat{m}_i^e)$, as desired. Combining this equality with (A.11), we get

$$W_i(\hat{\theta}_i, \theta_{-i}^N) = U_i[d_i^e(\hat{\theta}_i, \theta_{-i}^N), z(\hat{\theta}_i, \theta_{-i}^N)] = V_i^e(\hat{m}_i^e) > V_i^e[m_i^e(\theta^N)] \geq W_i(\theta_i^N, \theta_{-i}^N),$$

where the final inequality follows from the same argument as in the proof of Lemma 3. This contradicts that θ_i^N is a best response to θ_{-i}^N.

Now suppose that (A.13)-(A.14) hold. We can follow a similar strategy. In this case, we first show that there exists $\hat{\theta}_i$ such that $U_i[d_i^e(\hat{\theta}_i, \theta_{-i}^N), z(\theta^N)] = V_i^e[m_i^e(\theta^N)]$. Start from a solution $\tilde{d}_i^e[m_i^e(\theta^N)]$ to the Meade problem (21) conditional on $m_i^e(\theta^N)$. Since $m_i^e(\theta^N) \in \mathcal{M}_i^e$, we know that such a solution exists. Again, we let $\hat{s}_i = \tilde{s}_i[m_i^e(\theta^N)]$ denote the associated domestic policies and let $\hat{p}_i = \tilde{p}_i[m_i^e(\theta^N)]$ denote the associated domestic prices. Since $m_i^e(\theta^N) \in \Omega_i(\theta_{-i}^N)$, we also know that there exists a vector of world prices $\hat{p}^w$ such that foreign equilibrium conditions hold. Given $\hat{p}_i$ and $\hat{p}^w$, set trade taxes $\hat{t}_i$ such that the non-arbitrage condition (7) holds, i.e.,

$$\hat{p}_{ig} = \hat{p}_g^w \times (1 + \hat{t}_{ig}).$$

Given Assumption 2, we know again that such trade taxes are feasible. And since $m_i^e(\theta^N) \in \Omega_i(\theta_{-i}^N)$, the combination of (i) the domestic variables $\tilde{d}_i^e[m_i^e(\theta^N)]$, which solve the Meade problem (21) conditional on $m_i^e(\theta^N)$ and (ii) the world price $\hat{p}^w$, foreign variables $\hat{d}_{-i}^e$, and net imports $\hat{m}_{-i}^e$ associated with the construction of the offer curve at $m_i^e(\theta^N)$ and θ_{-i}^N, and which satisfy all foreign equilibrium conditions, forms a competitive equilibrium conditional on $(\hat{\theta}_i, \theta_{-i}^N)$. Since domestic variables solve the Meade problem (21) conditional on $m_i^e(\theta^N)$, the associated utility in country i is equal to $U_i[d_i^e(\hat{\theta}_i, \theta_{-i}^N), z(\theta^N)] = V_i^e[m_i^e(\theta^N)]$, as desired. Combining this equality with (A.13)-(A.14), we get

$$\begin{aligned} W_i(\hat{\theta}_i, \theta_{-i}^N) &= U_i[d_i^e(\hat{\theta}_i, \theta_{-i}^N), z(\theta^N)] = V_i^e[m_i^e(\theta^N)] \\ &\geq U_i[\hat{d}_i^e, z(\theta^N)] > U_i[d_i^e(\theta^N), z(\theta^N)] = W_i(\theta_i^N, \theta_{-i}^N), \end{aligned}$$

which again contradicts that θ_i^N is a best response to θ_{-i}^N. □

A.3 Proof of Theorem 2

Proof. (Existence) We start by defining a competitive equilibrium with lump-sum transfers of the "as-if" economy. It corresponds to $(\bar{p}^{w,e}, \bar{m}^e, \bar{D})$ such that:

$$\bar{m}_i^e \in \arg\max_{m_i^e \in \mathcal{M}_i^e} \quad V_i^e(m_i^e) \quad \text{s.t.} \quad \bar{p}^{w,e} \cdot m_i^e = \bar{D}_i, \text{ for all } i, \tag{A.15}$$

$$\sum_i \bar{m}_i^e = 0, \tag{A.16}$$

$$\bar{D}_i = \bar{p}^{w,z} \cdot \bar{m}_i^z, \tag{A.17}$$

where the equilibrium price vector $\bar{p}^{w,e} = (\bar{p}^w, \bar{p}^{w,z})$ comprises the price vector $\bar{p}^w$ for all goods as well as the price vector $\bar{p}^{w,z} \equiv \{\bar{p}_{ijn}^{w,z}\}$ for all externalities supplied by country i and demanded in country j. As discussed in Section 5.2, we assume such a competitive equilibrium exists.

For each country i, start from $\bar{m}_i^e = (\bar{m}_i, \bar{m}_i^z) \in \mathcal{M}_i^e$, with $\bar{z}$ the vector of externalities associated with $\bar{m}_i^z$. We note that the resource constraint for externalities embedded in (A.16) implies $\bar{z}$ is common across all countries. Then consider the solution to the Meade problem conditional on $\bar{m}_i^e$,

$$\bar{d}_i^e \in \arg\max_{d_i^e \in \mathcal{D}_i^e(\bar{m}_i^e)} \quad U_i(d_i^e, \bar{z}). \tag{A.18}$$

Since $\bar{m}_i^e \in \mathcal{M}_i^e$, such a solution exists. Let $\bar{p}_i$ and $\bar{s}_i$ denote the domestic prices and domestic policies associated with $\bar{d}_i^e$. Given domestic prices, set trade taxes $\bar{t}_i$ such that the non-arbitrage condition (7) holds between the domestic prices $\bar{p}_i$ and the world prices $\bar{p}^w$, i.e.,

$$\bar{p}_{ig} = \bar{p}_g^w \times (1 + \bar{t}_{ig}).$$

Assumption 2 guarantees that this is feasible. By construction, conditional on the policy vector $\theta^P \equiv \{\bar{t}_i, \bar{s}_i\}$, the profile $\{\bar{c}_h, \bar{y}_f, \bar{m}_i, \bar{z}, \bar{p}_i, \bar{p}^w, \bar{T}_i\}$ is a competitive equilibrium, with $\{\bar{c}_h, \bar{y}_f, \bar{T}_i\}$ the consumption, output, and transfers associated with $\{\bar{d}_i^e\}$. By construction, we also have: $m^e(\theta^P) = \bar{m}^e$, $d^e(\theta^P) = \bar{d}^e$, and $p^{w,e}(\theta^P) \equiv (p^w(\theta^P), \bar{p}^{w,z}) = \bar{p}^{w,e}$. Hence, (A.15)-(A.17) imply θ^P is an extended political optimum.

(Efficiency) As in Theorem 1, the argument follows the proof of the First Welfare Theorem. By contradiction, suppose that $m^e(\theta^P)$ does not satisfy condition (22). Then there exists $\tilde{m}^e \in \mathcal{M}^e$ such that:

$$\begin{aligned}
&V_i^e(\tilde{m}_i^e) > V_i^e[m_i^e(\theta^P)] \text{ for some } i,\\
&V_j^e(\tilde{m}_j^e) \geq V_j^e[m_j^e(\theta^P)] \text{ for all } j \neq i,\\
&\sum_j \tilde{m}_j^e = 0.
\end{aligned}$$

Since $m^e(\theta^P)$ satisfies (26) and $\{V_j^e\}$ are locally nonsatiated, this further implies

$$\begin{aligned}
&p^{w,e}(\theta^P) \cdot \tilde{m}_i^e > \bar{D}_i(\theta^P),\\
&p^{w,e}(\theta^P) \cdot \tilde{m}_j^e \geq \bar{D}_j(\theta^P) \text{ for all } j \neq i.
\end{aligned}$$

Adding up the previous inequalities, we obtain

$$p^{w,e}(\theta^P) \cdot (\sum_j \tilde{m}_j^e) > \sum_j \bar{D}_j(\theta^P) = 0.$$

This contradicts $\sum_j \tilde{m}_j^e = 0$. Hence $m^e(\theta^P)$ satisfies condition (22). Since $d^e(\theta^P)$

satisfies (27), Lemma 3 implies that θ^P is efficient.

(Implementability) The argument follows the same strategy as in the proof of Lemma 4. By contradiction, suppose that θ^P is not a Nash equilibrium of the policy game under extended market-access commitment around θ^P. Then there exists a country i and $\hat{\theta}_i \in \Theta_i$ such that

$$W_i(\hat{\theta}_i, \theta^P_{-i}) > W_i(\theta^P), \tag{A.19}$$

$$p^{w,e}(\theta^P) \cdot m^e_i(\hat{\theta}_i, \theta^P_{-i}) = p^{w,e}(\theta^P) \cdot m^e_i(\theta^P). \tag{A.20}$$

Let $m^e(\hat{\theta}) \equiv \{m^e_i(\hat{\theta})\}$ and $d^e(\hat{\theta}) \equiv \{d^e_i(\hat{\theta})\}$ denote the extended net imports and domestic variables associated with the alternative policy $\hat{\theta} \equiv \{\hat{\theta}_i, \theta^P_{-i}\}$. By definition of a competitive equilibrium, we know that $d^e_i(\hat{\theta}) \in \mathcal{D}^e_i[m^e_i(\hat{\theta})]$. This implies

$$W_i(\hat{\theta}) = U_i[d^e_i(\hat{\theta}), z(\hat{\theta})] \leq \max_{d^e_i \in \mathcal{D}^e_i[m^e_i(\hat{\theta})]} U_i(d^e_i, z(\hat{\theta})) = V^e_i[m^e_i(\hat{\theta})]. \tag{A.21}$$

From (27), we also know that

$$W_i(\theta^P) = U_i[d^e_i(\theta^P), z(\theta^P)] = V^e_i[m^e_i(\theta^P)]. \tag{A.22}$$

Combining (A.19) with (A.21)-(A.22), we therefore get

$$V^e_i[m^e_i(\hat{\theta})] > V^e_i[m^e_i(\theta^P)]. \tag{A.23}$$

To conclude, note that $m^e_i(\hat{\theta}) \in \mathcal{M}^e_i$, since $m^e_i(\hat{\theta})$ is the vector of extended net imports that arises in the competitive equilibrium under $\hat{\theta}$, and $p^{w,e}(\theta^P) \cdot m^e_i(\hat{\theta}) = \bar{D}_i(\theta^P)$, by (A.20) since $p^{w,e}(\theta^P) \cdot m^e_i(\theta^P) = \bar{D}_i(\theta^P)$. Thus, (A.23) contradicts (26). $\square$

B Proofs of Results in Section 6: Imperfect Competition

B.1 A General-Equilibrium Model with Imperfect Competition

In each country i, there are two types of firms: perfectly competitive firms $f \in \mathcal{F}^P_i$, and imperfectly competitive firms, $f \in \mathcal{F}^I_i$, which we simply refer to as imperfect competitors. When all firms are perfectly competitive, i.e., $\mathcal{F}_i = \mathcal{F}^P_i$, the environment reduces to the baseline model from Section 3.[1]

Imperfect competitors' technology. Each imperfect competitor f sells a differentiated good $g = f$, both at home and abroad. We let $q_{fj} \geq 0$ denote the quantity produced by imperfect competitor f for destination j. We assume f's production set is separable across destination markets, $\mathcal{Y}_f(s_i) = \prod_j \mathcal{Y}_{fj}(s_i)$, with $\mathcal{Y}_{fj}(s_i)$ the sub-production set for destination j, and that each imperfect competitor only purchases inputs from perfectly competitive firms. We let $q^P_{fj} \geq 0$ denote the vector of inputs purchased by firm f to sell in j and $y_{fj} = (-q^P_{fj}, q_{fj})$ denote its associated vector of net output. Across all destinations, firm f's net output is $y_f = \{y_{fj}\}$.

Resource constraints. In each country i, the difference between total demand and total supply is equal to imports from the rest of the world. For any good g produced by perfectly competitive firms,

$$\sum_{h \in \mathcal{H}_i} c_{hg} + \sum_{f \in \mathcal{F}^I_i} \sum_j q^P_{fjg} = \sum_{f \in \mathcal{F}^P_i} y_{fg} + m^P_{ig}, \tag{B.1}$$

where m^P_{ig} denotes country i's net imports of good g. For any differentiated good produced by an imperfectly competitive firm f,

$$q_{fi} = \sum_{h \in \mathcal{H}_i} c_{hf} - \sum_{f' \in \mathcal{F}^P_i} y_{f'f}, \tag{B.2}$$

with $y_{f'f} \leq 0$ the input from firm f purchased by a competitive firm f'.

Across countries, the sum of total imports is zero. We denote each country i's net import vector by $m_i = (m^P_i, m^I_i)$, where $m^P_i = \{m^P_{ig}\}$ and where $m^I_i = \{m^I_{ifj}\}$ is i's net imports of imperfectly competitive goods, with $m^I_{ifj} = -q_{fj}$ for all $f \in \mathcal{F}^I_i$ and $j \neq i$, $m^I_{ifi} = q_{fi}$ if $f \in \mathcal{F}^I_j$ and $j \neq i$, and $m^I_{ifj} = 0$ otherwise. International market clearing is, then,

$$\sum_i m_i = 0. \tag{B.3}$$

Policies, prices, and budget constraints. In addition to the trade taxes and domestic policies considered in our baseline model, we assume that governments may

[1] For ease of notation, we omit externalities from our analysis in this section. The same results hold in the presence of local externalities.

impose trade taxes on the exports and imports of imperfect competitors as well as subsidies on the profits received by households who own them. We let $\Theta_i \equiv \mathcal{T}_i \times \mathcal{S}_i \times \mathcal{T}_i^\pi$ denote the set of feasible policies in country i, with $\mathcal{T}_i^\pi$ the set of feasible profit subsidies.

An imperfect competitor f exporting from i to $j \neq i$ may be subject to an export subsidy t_{fj}^X, imposed by country i, and to an import tariff t_{fj}^M, imposed by country j. For each $f \in \mathcal{F}_i^I$, trade taxes create wedges between local and world prices,

$$p_{fj}^X = p_{fj}^{w,I}(1 + t_{fj}^X), \tag{B.4}$$

$$p_{fj}^M = p_{fj}^{w,I}(1 + t_{fj}^M), \tag{B.5}$$

where p_{fj}^X is the price received by firm f when exporting to country j and p_{fj}^M the price paid by households in j. For intra-national trade, we use the convention $p_{fi}^M = p_{fi}^X$. The non-arbitrage condition for goods produced by perfectly competitive firms is unchanged and given by

$$p_{ig}^P = p_g^{w,P}(1 + t_{ig}^P), \tag{B.6}$$

where p_{ig}^P, $p_g^{w,P}$, and t_{ig}^P are i's local price, the world price, and i's trade taxes on good g, respectively.

We adopt the following vector notation for prices and taxes: the vector of export prices and subsidies of imperfect competitors selling from i are $p_i^X \equiv \{p_{fj}^X\}_{f\in\mathcal{F}_i^I, j\neq i}$, and $t_i^X \equiv \{t_{fj}^X\}_{f\in\mathcal{F}_i^I, j\neq i}$; the vector of import prices and tariffs of imperfect competitors selling to i are $p_i^M = \{p_{fi}^M\}_{f\in\mathcal{F}_{-i}^I}$ and $t_i^M = \{t_{fi}^M\}_{f\in\mathcal{F}_{-i}^I}$; and the vector of domestic prices and trade taxes in i for goods produced by perfectly competitive firms are $p_i^P = \{p_{ig}^P\}$ and $t_i^P = \{t_{ig}^P\}$. Finally, the world price vector is

$$p^w = (p^{w,P}, p^{w,I}),$$

where $p^{w,I} = \{p_{fj}^{w,I}\}$ is the world price vector for goods produced by imperfect competitors, with the convention that $p_{fi}^{w,I} = p_{fi}^M = p_{fi}^X$ for $f \in \mathcal{F}_i^I$.

Each imperfect competitor $f \in \mathcal{F}_i^I$ may also be subject to a profit subsidy t_f^π, imposed by the country i in which it is located. This creates a wedge between the firm's profits and the capital payments $\hat{\pi}_f$ received by the households who own them,

$$\hat{\pi}_f = (1 + t_f^\pi)\pi_f. \tag{B.7}$$

The total capital payments received by a household $h \in \mathcal{H}_i$ are then equal to

$$\pi_h = \sum_{f \in \mathcal{F}_i^P} \omega_{hf} \pi_f + \sum_{f \in \mathcal{F}_i^I} \omega_{hf} \hat{\pi}_f. \tag{B.8}$$

Household and government budget constraints are as in the baseline model, except that governments now also pay for profit subsidies. We let $t_i^\pi = \{t_f^\pi\}_{f \in \mathcal{F}_i^I}$ denote government i's profit subsidies and note that, conditional on household budget constraints in country i, the government budget constraint is still equivalent to country i's trade balance.

Decentralized equilibrium. The problems of households and competitive firms are the same as in our baseline model,

$$c_h \in \arg\max_c \; u_h(c) \quad \text{s.t.} \quad x_h(c, p_i^P, p_i^M, s_i) = \pi_h + T_i, \text{ for all } h \in \mathcal{H}_i, \tag{B.9}$$

$$y_f \in \arg\max_y \; \pi_f(y, p_i^P, p_i^M, s_i) \quad \text{s.t.} \quad y \in \mathcal{Y}_f(s_i), \text{ for all } f \in \mathcal{F}_i^P. \tag{B.10}$$

Each imperfect competitor f in country i is imperfectly competitive with respect to the output of its differentiated good, but otherwise takes prices as given. Input cost minimization by firm $f \in \mathcal{F}_i^I$ producing q_{fj} units in j requires

$$q_{fj}^P \in \arg\min_{q^P} \; p_i^P \cdot q^P \quad \text{s.t.} \quad (-q^P, q_{fj}) \in \mathcal{Y}_{fj}(s_i). \tag{B.11}$$

We let $C_{fj}(q_{fj}, p_i^P, s_i)$ denote the associated cost function for each destination j.

Compared to perfectly competitive firms, imperfect competitors internalize how (a subset of) prices and quantities are jointly determined, as in Costinot and Werning (2019). Formally, each firm $f \in \mathcal{F}_i^I$ has a strategy set $\Sigma_f(s_i) = \prod_j \Sigma_{fj}(s_i)$, which is separable across markets. We let σ_{fj} denote firm f's strategy for market j. For each destination j, once all imperfect competitors have submitted their strategies $\sigma_j \equiv \{\sigma_{fj}\}$, their output in that country $q_j \equiv \{q_{fj}\}$ as well as their prices $p_j^M \equiv \{p_{fj}^M\}$ are determined by an "auctioneer" according to

$$q_j = \tilde{q}_j(\sigma_j, p_j^P, s_j, T_j, \{\pi_h\}_{h \in \mathcal{H}_j}) \quad \text{and} \quad p_j^M = \tilde{p}_j^M(\sigma_j, p_j^P, s_j, T_j, \{\pi_h\}_{h \in \mathcal{H}_j}), \tag{B.12}$$

subject to the constraints that households in j maximize their utility subject to budget constraints, perfectly competitive firms maximize their profits, and market clearing conditions hold for the goods sold by imperfect competitors in j, i.e., there exist $\{c_h\}_{h\in\mathcal{H}_j}$ and $\{y_f\}_{f\in\mathcal{F}_j^P}$ satisfying (B.2), (B.9) and (B.10). Given q_j, we assume uniqueness of $\{c_h\}_{h\in\mathcal{H}_j}$, $\{y_f\}_{f\in\mathcal{F}_j^P}$, and p_j^M that solve (B.2), (B.9) and (B.10); and given p_j^M, we assume uniqueness of $\{c_h\}_{h\in\mathcal{H}_j}$, $\{y_f\}_{f\in\mathcal{F}_j^P}$, and q_j that solve this system.

The problem of an imperfect competitor f from i selling in market j is then

$$\begin{aligned}\max_{\sigma_{fj}\in\Sigma_{fj}(s_i)} & \pi_{fj}(\sigma_{fj},\sigma_{-fj},p_i^P,p_j^P,s_i,s_j,T_i,T_j,\{\pi_h\}_{h\in\mathcal{H}_j}) \\ \equiv & \frac{1+t_{fj}^X}{1+t_{fj}^M}\tilde{p}_{fj}^M(\sigma_{fj},\sigma_{-fj},p_j^P,s_j,T_j,\{\pi_h\}_{h\in\mathcal{H}_j})\tilde{q}_{fj}(\sigma_{fj},\sigma_{-fj},p_j^P,s_j,T_j,\{\pi_h\}_{h\in\mathcal{H}_j}) \\ & -C_{fj}\left[\tilde{q}_{fj}(\sigma_{fj},\sigma_{-fj},p_j^P,s_j,T_j,\{\pi_h\}_{h\in\mathcal{H}_j}),p_i^P,s_i\right],\end{aligned} \tag{B.13}$$

where σ_{-fj} is the strategy profile of other imperfect competitors. We assume that (B.13) is strictly concave so its solution can be characterized by first-order conditions. Cournot competition corresponds to the special case where $\sigma_{fj}=q_{fj}$ and $\tilde{q}_{fj}(\sigma_{fj},\sigma_{-fj},p_j^P,s_j,T_j,\{\pi_h\}_{h\in\mathcal{H}_j})=q_{fj}$, whereas Bertrand competition corresponds to the special case where $\sigma_{fj}=p_{fj}^M$ and $\tilde{p}_{fj}^M(\sigma_{fj},\sigma_{-fj},p_j^P,s_j,T_j,\{\pi_h\}_{h\in\mathcal{H}_j})=p_{fj}^M$. In what follows, we restrict attention to these two polar cases.[2]

The rest of the equilibrium conditions are standard. They consist of the good market clearing conditions (B.1), (B.2), and (B.3); the non-arbitrage conditions (B.4)-(B.6); and the government budget constraints.

B.2 The Case for Market-Access Commitments

Assumption on available policy instruments. Our main result extends Theorem 1 to environments with imperfect competition under a stronger version of Assumption 2 that ensures each country has control over not only its local prices but also the post-tax profits of its imperfect competitors.

For each country i, let $\mathcal{T}_i^P$ denote the set of feasible trade taxes $\{t_{ig}^P\}$ on goods g produced by competitive firms, with P the number of goods that they produce; let $\mathcal{T}_i^M$ denote the set of feasible import tariffs $\{t_{fi}^M\}$ on foreign imperfect competitors $f\in\mathcal{F}_{-i}^I$, with M_i the number of foreign imperfect competitors; and let $\mathcal{T}_i^X$ denote the set of feasible export subsidies $\{t_{fj}^X\}$ on domestic imperfect competitors $f\in\mathcal{F}_i^I$ and foreign destinations $j\neq i$, with X_i the number of domestic imperfect competitor-destination pairs. Recall that $\mathcal{T}_i^\pi$ denotes the set of feasible profit subsidies $\{t_f^\pi\}$ on

[2]Monopolistic competition à la Dixit-Stiglitz obtains as a limit case with many firms.

domestic imperfect competitors $f \in \mathcal{F}_i^I$, with F_i the number of country i's imperfect competitors.

Assumption 2-I (Governments Have Access to a Complete Set of Trade Taxes and Profit Subsidies on Imperfect Competitors)**.** *In each country i, the set of feasible trade taxes is $\mathcal{T}_i = \mathcal{T}_i^P \times \mathcal{T}_i^M \times \mathcal{T}_i^X$ with $\mathcal{T}_i^P = \mathbb{R}^P$, $\mathcal{T}_i^M = \mathbb{R}^{M_i}$, and $\mathcal{T}_i^X = \mathbb{R}^{X_i}$, and the set of feasible profit subsidies is $\mathcal{T}_i^\pi = \mathbb{R}^{F_i}$.*

Domestic preferences over net imports. The first step towards extending Theorem 1 is to define Meade utilities with imperfect competition. We let

$$V_i^I(m_i) \equiv \max_{d_i \in \mathcal{D}_i^I(m_i)} \; U_i(d_i), \tag{B.14}$$

where $\mathcal{D}_i^I(m_i)$ consists of the domestic profiles

$$d_i = (\{c_h\}_{h\in\mathcal{H}_i}, \{y_f\}_{f\in\mathcal{F}_i^P \cup \mathcal{F}_i^I}, \{\sigma_{fi}\}_{f\in\mathcal{F}_i^I}, p_i^P, p_i^M, \{\pi_h\}_{h\in\mathcal{H}_i}, \{\hat{\pi}_f\}_{f\in\mathcal{F}_i^I}, s_i, T_i)$$

that satisfy the following equilibrium conditions in country i: (i) consumption solves the utility-maximization problem (B.9) for all $h \in \mathcal{H}_i$ with profit income given by (B.8), (ii) production solves the profit-maximization problem (B.10) for all $f \in \mathcal{F}_i^P$, (iii) imperfect competitors' inputs solve the cost-minimization problem (B.11) for all $f \in \mathcal{F}_i^I$ and j, (iv) imperfect competitors' domestic strategies σ_{fi} solve (B.13), with $j = i$, for all $f \in \mathcal{F}_i^I$, (v) quantities and prices of goods imported from imperfect competitors satisfy (B.12) with $j = i$, (vi) country i's consumption, production and imports satisfy (B.1) and (B.2), and (vii) country i's domestic policies $s_i \in \mathcal{S}_i$.[3] Note that the Meade problem only requires the optimality of imperfect competitors' behavior when they sell in their own domestic markets. We define $\mathcal{M}_i^I$ as the set of net imports m_i for which $\mathcal{D}_i^I(m_i)$ is non-empty. We assume that V_i^I is locally nonsatiated within $\mathcal{M}_i^I$ and that there exists a competitive equilibrium of the corresponding "as-if" exchange economy.

[3]Compared to the case of perfect competition, conditions (iv) and (v) depend on the strategies σ_{fi} of all imperfect competitors, not just those located in i. This does not, however, create complications in defining the Meade utility. Under Cournot competition, the strategies of imperfect competitors located abroad, $\sigma_{fi} = q_{fi}$ for $f \in \mathcal{F}_j^I$ with $j \neq i$, are determined by country i's vector of net imports m_i. Under Bertrand competition, these strategies, $\sigma_{fi} = p_{fi}^M$ for $f \in \mathcal{F}_j^I$ with $j \neq i$, are determined by country i's domestic profile d_i.

Political optimum. Having defined Meade utilities with imperfect competition, we can define a political optimum following the same approach as in Section 4.

Definition 4-I. *A global policy vector θ^P is a political optimum if net imports, $m(\theta^P) \equiv \{m_i(\theta^P)\}$, domestic variables, $d(\theta^P) \equiv \{d_i(\theta^P)\}$, and world prices, $p^w(\theta^P)$, satisfy for all i:*

$$m_i(\theta^P) \in \arg\max_{m_i \in \mathcal{M}_i^I} \quad V_i^I(m_i) \quad s.t. \quad p^w(\theta^P) \cdot m_i = 0, \tag{B.15}$$

$$d_i(\theta^P) \in \arg\max_{d_i \in \mathcal{D}_i^I[m_i(\theta^P)]} \quad U_i(d_i). \tag{B.16}$$

Market-access commitments work. We can now state a version of Theorem 1 extended to our environment with imperfect competition.

Theorem 1-I. *Given Assumption 2-I, there exists a political optimum θ^P that is both efficient and a Nash equilibrium of the policy game under market-access commitment around θ^P.*

Proof. Efficiency and implementability both follow from the same argument used for Theorem 1 in Section 4. The argument for the existence of a political optimum is new. We begin, as in the proof of Theorem 1, by recalling we have assumed the existence of an "as-if" competitive equilibrium with $\{m_i\}$ and p^w satisfying

$$m_i \in \arg\max_{\hat{m}_i \in \mathcal{M}_i^I} \; V_i^I(\hat{m}_i) \quad \text{s.t.} \quad p^w \cdot \hat{m}_i = 0,$$
$$0 = \sum_i m_i.$$

We then extend these variables to a full equilibrium profile. For each country i, we select a domestic profile

$$d_i = \{\{c_h\}_{h \in \mathcal{H}_i}, \{y_f\}_{f \in \mathcal{F}_i^P \cup \mathcal{F}_i^I}, \{\sigma_{fi}\}_{f \in \mathcal{F}_i^I}, p_i^P, p_i^M, \{\pi_h\}_{h \in \mathcal{H}_i}, \{\hat{\pi}_f\}_{f \in \mathcal{F}_i^I}, s_i, T_i\}$$

that solves i's Meade problem (B.14) at m_i, and, using Assumption 2-I, we set import tariffs t_i^M on imperfectly competitive goods and trade taxes t_i^P on perfectly competitive goods to satisfy (B.5) and (B.6), respectively. At this point, we have determined all variables except exporting strategies σ_{fj} for all $f \in \mathcal{F}_i^I$ and $j \neq i$, as well as the associated export prices p_i^X, export subsidies t_i^X, and profit subsidies t_i^π. And our construction has ensured that the profile constructed so far satisfies all equilibrium conditions except (B.4), (B.7), and (B.13) when $i \neq j$.

Next, for each country i, we select exporting strategies σ_{fj} and export subsidies t^X_{fj} to ensure that for all imperfect competitors $f \in \mathcal{F}^I_i$ and $j \neq i$, (B.13) holds. This is possible by Assumption 2-I and Lemma 5-I, stated and proved separately below. Finally, we set export prices p^X_{fj} to satisfy (B.4) and profit subsidies t^π_f to satisfy (B.7).

By construction, this profile is an equilibrium under the chosen policies $\theta^P = \{\theta^P_i \equiv (s_i, t^P_i, t^M_i, t^X_i, t^\pi_i)\}$. Since we have assumed there is a unique equilibrium associated with any policy profile, we have $m_i = m_i(\theta^P)$ and $d_i = d_i(\theta^P)$ for all i. By construction, these quantities satisfy (B.15) and (B.16). Thus, θ^P is a political optimum. $\square$

The extension of Theorem 1 to imperfect competition uses the following lemma.

Lemma 5-I. *Fix any two countries i and $j \neq i$, net import vectors $m_i \in \mathcal{M}^I_i$ and $m_j \in \mathcal{M}^I_j$, domestic profiles $d_i \in \mathcal{D}^I_i(m_i)$ and $d_j \in \mathcal{D}^I_j(m_j)$, and tariffs t^M_{fj} imposed by j on imperfect competitors $f \in \mathcal{F}^I_i$ that export from i. For each exporter $f \in \mathcal{F}^I_i$, consider the exporting strategy σ_{fj} implied by j's net imports m_j and domestic profile d_j, either $\sigma_{fj} = q_{fj}$ under Cournot competition or $\sigma_{fj} = p^M_{fj}$ under Bertrand competition. There exists an export subsidy t^X_{fj} imposed by country i such that σ_{fj} solves (B.13) given t^M_{fj}, with all other variables evaluated at their values implied by m_j, d_j, and d_i.*

Proof. First, suppose that all imperfect competitors compete à la Cournot in destination j. To assess whether the prescribed strategy $\sigma_{fj} = q_{fj}$ is optimal for each exporter $f \in \mathcal{F}^I_i$, we note that its first-order condition is

$$\tilde{p}^M_{fj}(q_{fj}, q_{-fj}, p^P_j, s_j, T_j, \{\pi_h\}_{h\in\mathcal{H}_j}) = \frac{1}{1-\eta_{fj}(q_{fj})} \frac{1+t^M_{fj}}{1+t^X_{fj}} \frac{\partial C_{fj}(q_{fj}, p^P_i, s_i)}{\partial q_{fj}} \qquad \text{(B.17)}$$

where $\eta_{fj}(q_{fj})$ is the inverse elasticity of the residual demand faced by f in destination j,

$$\eta_{fj}(q_{fj}) \equiv -\frac{\partial \log \tilde{p}^M_{fj}(q_{fj}, q_{-fj}, p^P_j, s_j, T_j, \{\pi_h\}_{h\in\mathcal{H}_j})}{\partial \log q_{fj}}.$$

Setting t^X_{fj} to satisfy (B.17) ensures (B.13) holds.

Second, suppose that all imperfect competitors compete à la Bertrand in destination j. To assess whether the prescribed strategy $\sigma_{fj} = p^M_{fj}$ is optimal for each exporter $f \in \mathcal{F}^I_i$, we note that its first-order condition is

$$p^M_{fj} = \frac{\rho_{fj}(p^M_{fj})}{\rho_{fj}(p^M_{fj}) - 1} \frac{1+t^M_{fj}}{1+t^X_{fj}} \frac{\partial C_{fj}(q_{fj}, p^P_i, s_i)}{\partial q_{fj}}, \qquad \text{(B.18)}$$

where $\rho_{fj}(p_{fj}^M)$ is the elasticity of demand faced by f in destination j,

$$\rho_{fj}(p_{fj}^M) \equiv -\frac{\partial \log \tilde{q}_{fj}(p_{fj}^M, p_{-fj}^M, p_j^P, s_j, T_j, \{\pi_h\}_{h\in\mathcal{H}_j})}{\partial \log p_{fj}^M}.$$

So, t_{fj}^X can be set so that (B.18) and hence (B.13) hold, i.e., each exporter finds it optimal to set any desired price p_{fj}^M. $\square$

C Proofs of Results in Section 6: Bargaining

C.1 A General-Equilibrium Model with Bargaining

In each country i, there are three types of firms: perfectly competitive firms $f \in \mathcal{F}_i^P$, suppliers of differentiated inputs, $f \in \mathcal{F}_i^S$, and buyers of differentiated inputs, $f \in \mathcal{F}_i^B$. Each input supplier from country i serving a foreign destination $j \neq i$ is matched with a separate buyer in this destination.[4] Within each bilateral relationship, bargaining determines input prices between suppliers and their buyers. When all firms are perfectly competitive, i.e., $\mathcal{F}_i = \mathcal{F}_i^P$, the environment again reduces to the baseline model from Section 3. In line with the extension to imperfect competition developed in Supplemental Appendix B, we assume that in addition to the trade taxes and domestic policies considered in our baseline model, governments may impose trade taxes on the exports and imports of differentiated inputs, t_{fj}^X and t_{fj}^M, as well as subsidies on the profits received by households who own suppliers and buyers of these inputs, t_f^π.[5]

Suppliers of differentiated inputs. In each origin country i, suppliers of differentiated inputs have the same technology as imperfect competitors in Supplemental Appendix B. Each supplier f buys a vector of inputs $q_{fj}^P \geq 0$ from perfectly competitive firms in country i in order to produce and deliver $q_{fj} \geq 0$ units of its own differentiated good to destination j. As before, we let $y_{fj} = (-q_{fj}^P, q_{fj})$ and $y_f = \{y_{fj}\}$. For all suppliers $f \in \mathcal{F}_i^S$ and all destinations $j \neq i$, input cost minimization requires

$$q_{fj}^P \in \arg\min_{q^P} \ p_i^P \cdot q^P \quad \text{s.t.} \quad (-q^P, q_{fj}) \in \mathcal{Y}_{fj}(s_i). \tag{C.1}$$

[4] Following Antràs and Staiger (2012a,b), we assume domestic transactions are competitive; all trade in differentiated inputs occurs across borders. This is innocuous for our results.

[5] As in Supplemental Appendix B, we omit externalities from our analysis for ease of notation.

We let $C_{fj}(q_{fj}, p_i^P, s_i)$ denote the associated cost function for each destination j, which also depends on the prices p_i^P charged by country i's perfectly competitive firms and the domestic policies s_i. We assume $C_{fj}(q_{fj}, p_i^P, s_i)$ is weakly convex in q_{fj}.

Buyers of differentiated inputs. In each destination country j, the buyer b matched with a supplier f can combine its q_{fj} units of the differentiated input with the inputs of perfectly competitive firms in order to produce goods that are locally sold on competitive markets. For all buyers $b \in \mathcal{F}_j^B$, revenue maximization implies

$$y_b^P \in \arg\max_{y^P} \; p_j^P \cdot y^P \quad \text{s.t.} \quad (y^P, -q_{fj}) \in \mathcal{Y}_b(s_j), \tag{C.2}$$

where $y_b^P \equiv \{y_{bg}^P\}$ denotes b's vector of goods sold on competitive markets in country j, with the convention that $y_{bg}^P \leq 0$ if good g is used as an input by b. We let $y_b = (y_b^P, -q_{fj})$ denote the buyer's net output vector. We let $R_b(q_{fj}, p_j^P, s_j)$ denote the associated revenue function of buyer b, conditional on its purchases q_{fj} of differentiated input from f as well as country j's domestic prices and policies, p_j^P and s_j, respectively. We assume $R_b(q_{fj}, p_j^P, s_j)$ is strictly concave in q_{fj}.

Hold-up problem between suppliers and buyers. Following Antràs and Staiger (2012a), we assume that within each buyer-supplier pair, Nash bargaining takes place after suppliers have produced their differentiated inputs. For any supplier firm $f \in \mathcal{F}_i^S$ and any destination $j \neq i$, differentiated inputs have zero value outside the bilateral relationship and the buyer receives a fraction $\beta_{fj} \in [0, 1)$ of the ex-post surplus. Anticipating this hold-up, the supplier f chooses the quantity q_{fj} of its input for destination j in order to maximize its share of the ex-post surplus net of production costs,

$$q_{fj} \in \arg\max_q \; (1 - \beta_{fj}) \left[R_b(q, p_j^P, s_j) - (p_{fj}^M - p_{fj}^X) q \right] - C_{fj}(q, p_i^P, s_i), \tag{C.3}$$

where the difference between the buyer's cost of the input, $p_{fj}^M q$, and the supplier's revenue, $p_{fj}^X q$, measures trade taxes paid by the buyer-supplier pair.[6] In turn, the

[6] By taking p_{fj}^M and p_{fj}^X as given in (C.3), we assume that the supplier ignores that its choice of quantity q may also affect trade taxes paid through changes in prices at fixed ad-valorem rates. This is consistent with Antràs and Staiger (2012a) who assume specific trade taxes and thus abstract from such considerations. Taking into account these considerations does not affect our results.

price received by the supplier is equal to its per unit payment after bargaining,

$$p_{fj}^X = \frac{(1-\beta_{fj})\left[R_b(q_{fj}, p_j^P, s_j) - (p_{fj}^M - p_{fj}^X)q_{fj}\right]}{q_{fj}}. \tag{C.4}$$

Resource constraints. In each country i, the difference between total demand and total supply is equal to imports from the rest of the world. For any good g sold on competitive markets, the good market clearing condition is

$$\sum_{h\in\mathcal{H}_i} c_{hg} + \sum_{f\in\mathcal{F}_i^S}\sum_{j\neq i} q_{fjg}^P = \sum_{f\in\mathcal{F}_i^B} y_{fg}^P + \sum_{f\in\mathcal{F}_i^P} y_{fg} + m_{ig}^P. \tag{C.5}$$

Across countries, the sum of total imports is zero,

$$\sum_i m_i = 0, \tag{C.6}$$

where country i's net import vector $m_i = (m_i^P, m_i^S)$ comprises its net imports of goods sold on competitive markets $m_i^P = \{m_{ig}^P\}$ and its net imports of differentiated inputs $m_i^S = \{m_{ifj}^S\}$, with $m_{ifj}^S = -q_{fj}$ for all $f \in \mathcal{F}_i^S$ and $j \neq i$, $m_{ifi}^S = q_{fi}$ if $f \in \mathcal{F}_j^S$ and $j \neq i$, and $m_{ifj}^S = 0$ otherwise.

Decentralized equilibrium. The problems of households and competitive firms are the same as before,

$$c_h \in \arg\max\nolimits_c \; u_h(c) \quad \text{s.t.} \quad x_h(c, p_i^P, s_i) = \pi_h + T_i, \text{ for all } h \in \mathcal{H}_i, \tag{C.7}$$
$$y_f \in \arg\max\nolimits_y \; \pi_f(y, p_i^P, s_i) \quad \text{s.t.} \quad y \in \mathcal{Y}_f(s_i), \text{ for all } f \in \mathcal{F}_i^P, \tag{C.8}$$

except that total capital payments received by a household $h \in \mathcal{H}_i$ are now equal to

$$\pi_h = \sum_{f\in\mathcal{F}_i^P} \omega_{hf}\pi_f + \sum_{f\in\mathcal{F}_i^S\cup\mathcal{F}_i^B} \omega_{hf}\hat{\pi}_f, \tag{C.9}$$
$$\hat{\pi}_f = (1+t_f^\pi)\pi_f. \tag{C.10}$$

Suppliers of differentiated inputs and their buyers minimize cost and maximize revenue as in (C.1) and (C.2), respectively. Bilateral bargaining determines the quantity and price of differentiated inputs, as described by (C.3) and (C.4). Finally, the market clearing conditions (C.5) and (C.6) hold, along with the non-arbitrage conditions for differentiated inputs,

$$p^X_{fj} = p^{w,S}_{fj}(1 + t^X_{fj}), \tag{C.11}$$
$$p^M_{fj} = p^{w,S}_{fj}(1 + t^M_{fj}), \tag{C.12}$$

for all $f \in \mathcal{F}^S_i$ and $j \neq i$, the non-arbitrage condition for goods sold on competitive markets,

$$p^P_{ig} = p^{w,P}_g(1 + t^P_{ig}), \tag{C.13}$$

and the government budget constraints.

In line with the notation of Supplemental Appendix B, we let $p^X_i \equiv \{p^X_{fj}\}_{f\in\mathcal{F}^S_i,j\neq i}$, and $t^X_i \equiv \{t^X_{fj}\}_{f\in\mathcal{F}^S_i,j\neq i}$ denote country i's vectors of export prices and export subsidies for differentiated inputs, $p^M_i = \{p^M_{fi}\}_{f\in\mathcal{F}^S_{-i}}$ and $t^M_i = \{t^M_{fi}\}_{f\in\mathcal{F}^S_{-i}}$ denote its vector of import prices and tariffs for differentiated inputs, $p^P_i = \{p^P_{ig}\}$ and $t^P_i = \{t^P_{ig}\}$ denote its vector of domestic prices and trade taxes for goods sold on competitive markets, and $p^w = (p^{w,P}, p^{w,S})$ denote the world price vector, with $p^{w,S} = \{p^{w,S}_{fj}\}$ the world price vector for differentiated inputs.

C.2 The Case for Market-Access Commitments

Assumption on available policy instruments. We impose the same assumption on the set of available policy instruments as in Supplemental Appendix B, so that each country has control over not only its local prices but also the post-tax profits of suppliers and buyers of differentiated goods.

For each country i, let $\mathcal{T}^P_i$ denote the set of feasible trade taxes $\{t^P_{ig}\}$ on goods g sold in competitive markets, with P the number of these goods; let $\mathcal{T}^M_i$ denote the set of feasible import tariffs $\{t^M_{fi}\}$ on foreign input suppliers $f \in \mathcal{F}^S_{-i}$, with M_i the number of foreign input suppliers; and let $\mathcal{T}^X_i$ denote the set of feasible export subsidies $\{t^X_{fj}\}$ on domestic input suppliers $f \in \mathcal{F}^S_i$ and foreign destinations $j \neq i$, with X_i the number of domestic input supplier-destination pairs. Recall that $\mathcal{T}^\pi_i$ denotes the set of feasible profit subsidies $\{t^\pi_f\}$ on domestic suppliers and buyers $f \in \mathcal{F}^S_i \cup \mathcal{F}^B_i$, with F_i the number of country i's input suppliers and buyers.

Assumption 2-B (Governments Have Access to a Complete Set of Trade Taxes and Profit Subsidies on Suppliers and Buyers of Differentiated Inputs)**.** *In each country i,* *the set of feasible trade taxes is* $\mathcal{T}_i = \mathcal{T}_i^P \times \mathcal{T}_i^M \times \mathcal{T}_i^X$ *with* $\mathcal{T}_i^P = \mathbb{R}^P$, $\mathcal{T}_i^M = \mathbb{R}^{M_i}$, *and* $\mathcal{T}_i^X = \mathbb{R}^{X_i}$, *and the set of feasible profit subsidies is* $\mathcal{T}_i^\pi = \mathbb{R}^{F_i}$.

Domestic preferences over net imports. Next, we define Meade utilities with bargaining. We let

$$V_i^B(m_i) \equiv \max_{d_i \in \mathcal{D}_i^B(m_i)} U_i(d_i), \tag{C.14}$$

where $\mathcal{D}_i^B(m_i)$ consists of the domestic profiles

$$d_i = (\{c_h\}_{h\in\mathcal{H}_i}, \{y_f\}_{f\in\mathcal{F}_i^P\cup\mathcal{F}_i^S\cup\mathcal{F}_i^B}, p_i^P, \{\pi_h\}_{h\in\mathcal{H}_i}, \{\hat{\pi}_f\}_{f\in\mathcal{F}_i^S\cup\mathcal{F}_i^B}, s_i, T_i)$$

that satisfy the following equilibrium conditions in country i: (i) consumption solves the utility-maximization problem (C.7) for all $h \in \mathcal{H}_i$ with profit income given by (C.9), (ii) production solves the profit-maximization problem (C.8) for all $f \in \mathcal{F}_i^P$, (iii) input suppliers solve the cost-minimization problem (C.1) for all $f \in \mathcal{F}_i^S$ and $j \neq i$, (iv) input buyers solve the revenue-maximization problem (C.2) for all $f \in \mathcal{F}_i^B$, (v) country i's consumption, production, and imports satisfy (C.5) and the net output of its input suppliers and buyers are consistent with its exports and imports of differentiated inputs m_i^S, and (vi) country i's domestic policies $s_i \in \mathcal{S}_i$. We define $\mathcal{M}_i^B$ as the set of net imports m_i for which $\mathcal{D}_i^B(m_i)$ is non-empty. We assume that V_i^B is locally nonsatiated within $\mathcal{M}_i^B$ and that there exists a competitive equilibrium of the corresponding "as-if" exchange economy.

Political optimum. Having defined Meade utilities with bargaining, we can define a political optimum as follows.

Definition 4-B. *A global policy vector* θ^P *is a political optimum if net imports,* $m(\theta^P) \equiv \{m_i(\theta^P)\}$, *domestic variables,* $d(\theta^P) \equiv \{d_i(\theta^P)\}$, *and world prices,* $p^w(\theta^P)$, *satisfy for all* i:

$$m_i(\theta^P) \in \arg\max_{m_i\in\mathcal{M}_i^B} \quad V_i^B(m_i) \quad s.t. \quad p^w(\theta^P)\cdot m_i = 0, \tag{C.15}$$

$$d_i(\theta^P) \in \arg\max_{d_i\in\mathcal{D}_i^B[m_i(\theta^P)]} \quad U_i(d_i). \tag{C.16}$$

Market-access commitments work. We are ready to state a version of Theorem 1 extended to our environment with bargaining.

Theorem 1-B. *Given Assumption 2-B, there exists a political optimum θ^P that is both efficient and a Nash equilibrium of the policy game under market-access commitment around θ^P.*

Proof. Efficiency and implementability again follow from the same argument used for Theorem 1. As in Supplemental Appendix B, only the argument for the existence of a political optimum is new.

We begin, as in the proof of Theorem 1, by recalling we have assumed the existence of an "as-if" competitive equilibrium with $\{m_i\}$ and p^w satisfying

$$\begin{aligned} m_i &\in \arg\max_{\hat{m}_i \in \mathcal{M}_i^B} \; V_i^B(\hat{m}_i) \quad \text{s.t.} \quad p^w \cdot \hat{m}_i = 0, \\ 0 &= \sum_i m_i. \end{aligned}$$

We then extend these variables to a full equilibrium profile. First, for each country i, we select a domestic profile

$$d_i = (\{c_h\}_{h \in \mathcal{H}_i}, \{y_f\}_{f \in \mathcal{F}_i^P \cup \mathcal{F}_i^S \cup \mathcal{F}_i^B}, p_i^P, \{\pi_h\}_{h \in \mathcal{H}_i}, \{\hat{\pi}_f\}_{f \in \mathcal{F}_i^S \cup \mathcal{F}_i^B}, s_i, T_i)$$

that solves i's Meade problem (C.14) at m_i, and, using Assumption 2-B, we set trade taxes t_i^P on perfectly competitive goods to satisfy (C.13). At this point, we have determined all variables except the export prices p_i^X and import prices p_i^M of differentiated inputs, the associated export subsidies t_i^X and import tariffs t_i^M, and profit subsidies t_i^π on suppliers and buyers of differentiated inputs. And our construction has ensured that the profile constructed so far satisfies all equilibrium conditions except (C.3), (C.4), (C.11), (C.12), and (C.10).

Next, for each bargaining supplier-buyer pair $f \in \mathcal{F}_i^S$ and $b \in \mathcal{F}_j^B$, with $j \neq i$, we select export and import prices p_{fj}^X and p_{fj}^M that ensure both (C.3) and (C.4) hold. This is possible by Lemma 5-B, stated and proved separately below. Finally, we set export subsidies t_{fj}^X to satisfy (C.11), import tariffs t_{fj}^M to satisfy (C.12), and profit subsidies t_f^π to satisfy (C.10).

By construction, this profile is an equilibrium under the chosen policies $\theta^P = \{\theta_i^P \equiv (s_i, t_i^P, t_i^M, t_i^X, t_i^\pi)\}$. Since we have assumed there is a unique equilibrium associated with any policy profile, we have $m_i = m_i(\theta^P)$ and $d_i = d_i(\theta^P)$ for all i. By construction, these quantities satisfy (C.15) and (C.16). Thus, θ^P is a political optimum. $\square$

The extension of Theorem 1 to bargaining uses the following lemma.

Lemma 5-B. *Fix any bargaining supplier-buyer pair* $f \in \mathcal{F}_i^S$ *and* $b \in \mathcal{F}_j^B$*, with* $j \neq i$*, net import vectors* $m_i \in \mathcal{M}_i^B$ *and* $m_j \in \mathcal{M}_j^B$*, and domestic profiles* $d_i \in \mathcal{D}_i^B(m_i)$ *and* $d_j \in \mathcal{D}_j^B(m_j)$*. There exist an export price* p_{fj}^X *and an import price* p_{fj}^M *at which (C.3) and (C.4) hold, with all other variables evaluated at their values implied by* m_j*,* d_j*, and* d_i*.*

Proof. Since $R_b(q_{fj}, p_j^P, s_j)$ is strictly concave in q_{fj} and $C_{fj}(q_{fj}, p_i^P, s_i)$ is weakly convex, the quantity chosen by the supplier f solves (C.3) if and only if the associated first-order condition holds, i.e.,

$$(1-\beta_{fj})\left[\frac{\partial R_b(q_{fj}, p_j^P, s_j)}{\partial q} - (p_{fj}^M - p_{fj}^X)\right] - \frac{\partial C_{fj}(q_{fj}, p_i^P, s_i)}{\partial q} = 0. \tag{C.17}$$

To construct p_{fj}^X and p_{fj}^M at which (C.3) and (C.4) hold, it is therefore sufficient to find p_{fj}^X and p_{fj}^M that solve (C.4) and (C.17). Such a solution is given by

$$p_{fj}^X = (1-\beta_{fj})\left[\frac{R_b(q_{fj}, p_j^P, s_j)}{q_{fj}} - \frac{\partial R_b(q_{fj}, p_j^P, s_j)}{\partial q}\right] + \frac{\partial C_{fj}(q_{fj}, p_i^P, s_i)}{\partial q},$$
$$p_{fj}^M = (1-\beta_{fj})\left[\frac{R_b(q_{fj}, p_j^P, s_j)}{q_{fj}}\right] + \beta_{fj}\left[\frac{\partial R_b(q_{fj}, p_j^P, s_j)}{\partial q} - \frac{1}{(1-\beta_{fj})}\frac{\partial C_{fj}(q_{fj}, p_i^P, s_i)}{\partial q}\right].$$

□