

How Tariffs Affect Trade Deficits*

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We study the positive (not normative) effect of a permanent import tariff on trade deficits. We consider a two-period trade model with general preferences and technology. We first develop an aggregation result showing one can work with induced preferences over aggregate imports and exports. Our main results provide conditions under which tariffs are neutral as well as conditions under which they reduce deficits. A central theme is that the slope of the *static* Engel path for aggregate imports and exports is a key determinant of this *dynamic* question.

1 Introduction

How do import tariffs affect trade imbalances? Setting welfare consequences aside, can a permanent increase in tariffs reduce an ongoing trade deficit?

The answer depends on whom you ask. Politicians and the general public often assume that tariffs, by discouraging imports, will narrow the trade deficit. The great trade policy disaster of the 1930s is a case in point. As [Irwin \(2011\)](#) convincingly argues, trade wars over that period were not primarily driven by lobbying and other forms of redistributive politics, but rather by countries' desire to correct trade imbalances via a rise in trade protection. The "reciprocal tariffs" put forward on April 2, 2025, by the Trump administration seem to derive from a similar belief that an increase in US tariffs can lower trade deficits by choking off imports.

Economists are quick to point out that this is only part of the story. Everything else being equal, tariffs may reduce imports, but why would exports be unaffected? Trade economists may note that import tariffs are equivalent to export taxes, an expression of Lerner symmetry ([Costinot and Werning, 2019](#)). Macroeconomists may add that, following textbook analyses, trade imbalances are fundamentally shaped by national savings

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and investment decisions that are orthogonal to trade policy. Tariffs affect the extent and nature of intratemporal trade, but the trade balance issue is one of intertemporal trade (Clark Center Economic Expert Panel, 2025).¹

Intuitions aside, formal analyses of the impact of tariffs on trade imbalances are scarce. An early exception using an intertemporal approach is Razin and Svensson (1983). They pointed out that temporary tariffs do not satisfy Lerner symmetry and differentially affect the cost of living over time, which then affects borrowing and lending. Relatedly, if exogenous economic conditions are not stationary and, instead, vary over time, then even a permanent tariff may have a different incidence on the cost of living today versus tomorrow, with implications for trade imbalances.

The question we address in this paper is whether one might still expect a systematic effect of permanent tariffs on the trade deficit in a stationary scenario. Our main finding is that, perhaps surprisingly, there may indeed be such a systematic effect. Our analysis is based on the notion that even if economic primitives are unchanged between today and tomorrow, the fact that a country is currently running a trade deficit implies that aggregate consumption is different today and tomorrow. This opens the door, endogenously, for the non-neutrality of a permanent tariff on the trade deficit.

Section 2 considers a neoclassical trade model over two periods. We allow for an arbitrary number of goods, general preferences, and general technology. Imports may be used as final goods or as inputs into production. International prices are taken as given in our baseline model; we later extend the analysis. The government levies a uniform tariff on all imports in both periods and rebates the revenue back to households. We assume expectations are rational, so there is perfect foresight.

The starting point of our analysis is a new aggregation result. We show that in each period, one can summarize all the relevant implications of our general model for trade deficits into a preference relation over aggregate imports and aggregate exports only. This reduces the analysis of deficits to an “as-if” consumer problem with two goods, imports and exports, and two periods. The associated substitution and income effects then determine whether or not tariffs reduce trade deficits. This observation turns out to be key to simplifying our analysis and to generating novel insights.

Section 3 sets the stage for our main results. It provides sufficient conditions for two extreme scenarios: neutral tariffs and autarky-inducing tariffs. The trade balance is locally unaffected by tariffs in a stationary economy with preferences that are appropriately ho-

¹Link to the Clark Center Economic Expert Panel Poll: <https://www.kentclarkcenter.org/surveys/tariffs-reciprocal-and-retaliatory-2/>. Expressions of the conventional wisdom that tariffs are unlikely to affect trade deficits can also be found in Baldwin (2024) and Krugman (2024).

homothetic when every good is strictly imported or strictly exported, with no non-tradables. These assumptions are strong and, it turns out, always violated for large enough tariffs. Indeed, we prove that large enough tariffs drive the economy to zero aggregate imports, zero aggregate exports, or both, depending on the economy's net foreign asset position. When it is initially zero, the economy moves to autarky and tariffs fully close the deficit.

Our main contribution is a general characterization of how tariffs affect deficits through substitution and income effects. Section 4 isolates substitution effects by focusing on the impact of tariffs around free trade, where income effects are second order. To assess whether tariffs reduce trade deficits under these conditions, we highlight one key statistic: the slope of the *static* Engel path for aggregate imports and exports. If this slope is steeper in the first period, then tariffs increase the marginal cost of consumption disproportionately more in this period. This raises the domestic real interest rate, which creates incentives to save and reduces the deficit.

In the baseline with fixed international prices, no additional information is required. This then raises the question: Why would the slope of the Engel path vary? In a stationary environment, one might expect it to be the same across periods. We show that this need not be the case. Rather, differences in the slopes today versus tomorrow are dictated by the curvature of the Engel path, i.e. whether aggregate imports are a luxury good. If the Engel path is linear, then they are not, and tariff neutrality holds. If the Engel path is strictly convex, then they are, and tariffs reduce trade deficits. Interestingly, even when static preferences over individual goods are homothetic, the Engel path may be strictly convex. This is because the relevant preferences over aggregate imports and exports also capture technological considerations within our “as-if” aggregation construct. In the context of an endowment model with CES preferences, which nests the standard Armington model, we further show that the curvature of the Engel path depends on whether there is action at the extensive margin, i.e. shifting of goods between imported and non-traded, or between non-traded and exported. If there is none, then the Engel path is linear, as in our earlier neutrality result. In contrast, if goods do shift between these categories, their domestic prices endogenously change as aggregate consumption varies, and the Engel path becomes strictly convex.

The last part of Section 4 extends the analysis to endogenous international prices. In this case, the slope of the Engel path remains one of the sufficient statistics to sign the response of the domestic real interest rate to tariffs, along with the direct effects of tariffs on international prices. In the context of a Ricardian model, although the Engel path is linear in each period, slopes differ across periods because of the systematic relationship between aggregate consumption and a country's terms of trade. Specifically, when there

is home bias, an increase in aggregate consumption improves a country's terms of trade, which makes the slope of the Engel path steeper in periods of deficits. This creates another reason for tariffs to incentivize savings and reduce trade deficits.

Section 5 turns to how tariffs affect trade deficits through income effects. Away from free trade, tariffs create distortions that are no longer second order. This creates a new force through which tariffs tend to reduce trade deficits. When tariffs distort production, they make the economy poorer in both periods, but as a fraction of aggregate consumption, the income loss is smaller when aggregate consumption is high. If intertemporal preferences are homothetic, so that a wealth loss lowers aggregate consumption by the same proportion in both periods, we show that this differential income effect is also sufficient to cause a decline in the trade deficit.

Section 6 concludes with a series of quantitative examples. We illustrate the range of possible effects that might emerge under typical calibrations, both in the case of an endowment economy, with fixed international prices, and a Ricardian economy, with endogenous international prices. Our simulation results suggest that very different predictions about the impact of tariffs on deficits may arise from models that have been calibrated to the same levels of openness to trade, both intra- and intertemporally, as well as the same trade elasticity. In line with our analytical results, these different predictions go hand in hand with different slopes of the Engel paths, either because of extensive margin or terms-of-trade considerations. To bring additional credibility to simulations of the effects of tariffs on deficits, we discuss how to use world interest rate shocks to estimate the slope of the Engel path directly and use it as a new target in calibration.

Related Literature

In their influential work, Obstfeld and Rogoff (2000) emphasize that trade costs create a wedge between the real interest rates faced by borrowing and lending countries. In the context of an Armington economy, they show that the magnitude of this wedge is larger when trade imbalances are larger as well. Based on this observation, they argue that trade costs may keep trade imbalances in a modest range, thereby explaining the high correlation between domestic savings and investment and offering a solution to the Feldstein-Horioka puzzle.² Although Obstfeld and Rogoff (2000) do not formally study how changes in either trade costs or tariffs affect trade imbalances, the interest rate chan-

²A similar emphasis on the relationship between trade costs and trade imbalances can be found in Dornbusch (1983). In his paper, it is the existence of non-tradable goods that creates a wedge between domestic and world real interest rates whose magnitude varies with aggregate consumption.

nel that they highlight is at the heart of our analysis.³ More specifically, we demonstrate that the shape of the Engel paths encodes all the information that is necessary to evaluate how tariffs affect trade deficits via the interest rate channel.

Motivated by Obstfeld and Rogoff’s original work, [Eaton et al. \(2016\)](#) offer a quantitative exploration of the role of trade costs. At their preferred calibration, they find that very large changes in trade costs, going all the way to zero trade costs, can raise the US trade deficit to 20% of US GDP. Using a related model, [Reyes-Heroles \(2016\)](#) concludes that US trade deficits observed in the late 2000s could have been three times smaller absent his estimated changes in trade costs. Although we study tariffs, not iceberg trade costs, our analysis can be applied with some adjustments. In particular, it highlights the (implicit) role played by the shape of the Engel paths at different points in time, driven by terms-of-trade effects in the context of a Ricardian model.⁴

A related literature centered on income effects discusses the impact of terms-of-trade shocks, such as oil shocks, either temporary or permanent, on the current account. Classic references include [Harberger \(1950\)](#) and [Laursen and Metzler \(1950\)](#). [Obstfeld \(1982\)](#) and [Svensson and Razin \(1983\)](#) offer formal treatments of this issue. We show that if income effects derive from tariff distortions, they create another systematic reason why permanent tariffs can reduce deficits, even in a stationary environment.

Recent events have created a surge of interest in the macroeconomic implications of tariffs and their impact on trade deficits in particular. [Aguiar et al. \(2025\)](#), [Itskhoki and Mukhin \(2025\)](#), and [Bayas-Erazo and Lorenzoni \(2026\)](#) study how tariffs may affect net foreign asset positions and, in turn, the present discounted value of trade deficits.⁵ [Auclert et al. \(2025\)](#) consider the effects of a temporary tariff in a New Keynesian model. [Caliendo et al. \(2025\)](#) analyze tariffs and deficits in a Ricardian model with complete markets where countries may perfectly insure themselves against tariff fluctuations. [Ignatenko et al. \(2025\)](#) and [Rodríguez-Clare et al. \(2025\)](#) focus on the incidence of the “Liberation Day” tariffs in quantitative trade models with exogenous trade deficits. We propose instead a new aggregate approach to study the effect of a permanent and unanticipated tariff on current trade deficits.

³[Lorenzoni \(2019\)](#) offers an early discussion of the implications of [Obstfeld and Rogoff \(2000\)](#) for assessing the impact of tariffs on trade deficits.

⁴[Reyes-Heroles’](#) quantitative results also derive from the time-varying nature of trade costs, which are falling over the period he considers. Our analysis of permanent tariffs has nothing to say about this issue. Other quantitative analyses of the impact of changes in trade costs on trade imbalances include [Fitzgerald \(2012\)](#) and [Alessandria and Choi \(2021\)](#).

⁵[Bayas-Erazo and Lorenzoni \(2026\)](#) also study the impact of tariffs on deficits in an economy à la [Obstfeld and Rogoff \(2000\)](#). In their environment, terms of trade operate through their effects on the slope of the Engel path, as discussed in Section 4.3.

Our crucial aggregation result combines elements of the perspectives put forward by [Hicks \(1939\)](#) and [Meade \(1952\)](#). It emphasizes preferences over exports and imports, as in [Meade \(1952\)](#), and further creates aggregate composites of the two, as in [Hicks \(1939\)](#). This approach allows us to study trade deficits in a tractable way.⁶ Despite the fact that we keep preferences and technologies general, our analysis is no more complex than in a simple two-good economy. Through the lens of our aggregation result, richer economies with a continuum of goods and active extensive margins of trade implicitly induce non-homothetic preferences over aggregate exports and imports, which is what the impact of tariffs on deficits depends on.

2 A Neoclassical Model of Trade Deficits and Tariffs

To study the causal relationship between tariffs and trade deficits, we start from a rich static neoclassical trade model and extend it to two periods. A representative household with general preferences makes consumption choices. Production is handled by firms using a general technology, with any number of factors. Imports may be used as final goods or as inputs into production. We abstract from capital investment decisions. The government levies a uniform tariff τ on all imports in both periods, and rebates the revenue back to consumers. For now, we consider a small open economy that takes international prices as given and delay the discussion of terms-of-trade effects to subsequent sections.

2.1 Environment

Preferences. The representative household has utility

$$U(C_1, C_2),$$

where U is increasing and concave in aggregate consumption (C_1, C_2) . Some of our analysis applies without further restriction on U , but other results rely on U being homothetic, so that the marginal rate of substitution $U_1(C_1, C_2)/U_2(C_1, C_2)$ is a function of C_1/C_2 . This is a common benchmark assumption for intertemporal decision problems.

Aggregate consumption C_t is given by an aggregator

$$C_t = G_t(c_t),$$

⁶Different questions may, of course, call for different aggregate approaches. For instance, to study how factor prices respond to changes in trade costs, one may want to construct aggregate preferences over factor services instead, as in [Adao et al. \(2017\)](#).

where c_t is a finite or infinite dimensional vector representing all consumption goods, and G_t is assumed to be increasing and concave. In some cases it will be useful to specialize further and assume that G_t is homogeneous of degree one, so that preferences are homothetic within each period.

Technology. Technology is described by an aggregate production set Y_t for $t = 1, 2$. Given vectors of imports $m_t \geq 0$ and exports $x_t \geq 0$, the domestic consumption vector c_t is feasible if

$$(c_t, m_t, x_t) \in Y_t.$$

This formulation captures general production technologies, using any number of factors of production owned or hired by firms. Domestic firms import m_t as inputs, produce c_t for domestic consumption, and produce x_t for foreign consumption.⁷ The three vectors c_t , m_t , and x_t may, in principle, have different dimensions. As is standard in general equilibrium theory, factors can be incorporated into c_t as negative entries or implicitly folded into Y_t .

The setup allows for general trade costs, which may derive from the transportation services associated with international shipping as well as from the specific inputs required for domestic distribution and retail. For example, a non-tradable good i is one for which technology dictates that $x_{it} = m_{it} = 0$. Likewise, an economy with so-called iceberg trade costs is a special case where Y_t is given by requiring

$$y_i = c_i + (1 + \delta_{i,x})x_i - m_i / (1 + \delta_{i,m}), \text{ with } y \in \Omega_t, \quad (1)$$

where $\delta_{i,x} \geq 0$ and $\delta_{i,m} \geq 0$ denote exporting and importing costs, respectively, and Ω_t represents a domestic net-production set.⁸

International Prices and Intertemporal Trade Balance. All international prices are fixed. International prices consist of import prices p_{mt}^* , export prices p_{xt}^* , and the world interest rate R^* . Import and export prices may simply differ because imports and exports are dif-

⁷This implicitly assumes that all imports (and exports) are performed by firms, not directly by consumers. This is realistic and without loss of generality, as one can always introduce importing firms that produce final goods one-for-one from imports.

⁸In equation (1), both x_i and m_i measure units of good i available at the “foreign dock.” To sell x_i units abroad, domestic firms need to produce $(1 + \delta_{i,x})x_i$ units at home. Likewise, if they buy m_i units from abroad, they can only offer consumption of $m_i / (1 + \delta_{i,m})$ units at home. This is one among many possible conventions. For instance, we could have also chosen to measure goods at the “domestic dock,” in which case $y_i = c_i + x_i - m_i$, with iceberg trade costs $\delta_{i,x}$ and $\delta_{i,m}$ now folded into export and import prices. Note also that goods indexed by i can equivalently be interpreted as good–trading-partner pairs, so the analysis accommodates an arbitrary number of trading partners.

ferent goods—recall that we do not even require m_t and x_t to have the same dimension—or more generally because of foreign tariffs and transportation costs—which create a wedge between the price at which domestic firms can sell a good abroad and the price at which foreign firms would be willing to sell the same good to them.

Evaluated at international prices, the trade deficit in period t is

$$D_t = p_{mt}^* \cdot m_t - p_{xt}^* \cdot x_t.$$

The intertemporal trade balance condition is then

$$D_1 + \frac{1}{R^*} D_2 = NFA, \quad (2)$$

where NFA is an inherited net foreign asset position, which we take as given as well.⁹

Tariffs. It is clear that time-varying tariffs may affect the incentives to borrow and lend. Here, we focus on the more interesting situation where tariffs are permanent. That is, all imports are subject to the same ad-valorem tariff τ .¹⁰ Each period, tariff revenues are rebated to the representative household via a lump-sum transfer

$$T_t = \tau \times (p_{mt}^* \cdot m_t).$$

2.2 Definition of a Competitive Equilibrium

In a competitive equilibrium, households choose (c_1, c_2) subject to their budget constraint, taking as given domestic consumption prices p_{ct} , the world interest rate R^* , firm profits, and government transfers. Firms choose (c_t, m_t, x_t) to maximize profits, taking as given the tariff rate τ as well as domestic consumption prices p_{ct} , import prices p_{mt}^* , and export prices p_{xt}^* . Finally, market clearing in international markets requires the intertemporal trade balance condition (2). These conditions are standard and their formal description is relegated to Appendix A.

Note that a competitive equilibrium not only includes the import and export prices, p_{mt}^* and p_{xt}^* , but also a vector of domestic prices p_{ct} for all goods that enter the vector of consumption c_t . Domestic prices are not necessarily equal to international prices, both

⁹In general, even a small open economy may be subject to revaluation effects, since trade taxes can directly affect domestic prices and, in turn, the value of domestic assets and liabilities. We discuss such considerations in Costinot and Werning (2019).

¹⁰This rules out, in particular, discriminatory tariffs that vary across origin countries (by the same general interpretation of goods as in footnote 8). We discuss how our main result extends to an environment with heterogeneous tariffs, including the case of discriminatory tariffs, in Section 4.1 and Appendix E.

because of the tariff τ and because of other trade costs. More importantly, the possibility that some goods are not traded in equilibrium implies that domestic prices may not be fully pinned down by a no-arbitrage condition. In the rest of this section, we develop an aggregate approach that, for the purposes of studying how a change in the tariff rate τ affects trade deficits, will allow us to set aside the endogenous determination of domestic prices.

2.3 Static Equilibrium Conditions: Meade Meets Hicks

Our approach focuses on the determination of aggregate imports and exports in a competitive equilibrium. We start here with a discussion of static equilibrium conditions. Dynamic equilibrium conditions will be discussed in Section 2.4. The formal derivation of both sets of conditions, leading to Proposition 1, can be found in Appendix B. Here, we offer a heuristic argument.

Conditional on the vector of imports and exports, domestic consumption and production choices in a competitive market are efficient. The tariff makes imports artificially more expensive than exports, so imports and exports will not be chosen efficiently overall. However, since the tariff is uniform across imports, it affects neither relative prices within imports nor relative prices within exports, and equilibrium choices are efficient within each of these categories. Thus, the competitive equilibrium is efficient conditional on aggregate imports and aggregate exports, even though it is inefficient in its choice of aggregate imports and aggregate exports.

Preferences Over Aggregate Imports and Exports. The previous reasoning suggests subsuming both intra-period preferences G_t and technology Y_t by defining

$$\mathcal{C}_t(M_t, X_t) \equiv \max_{(c_t, m_t, x_t) \in Y_t} G_t(c_t),$$

subject to

$$p_{mt}^* \cdot m_t = M_t,$$

$$p_{xt}^* \cdot x_t = X_t.$$

Here $M_t \geq 0$ and $X_t \geq 0$ are scalars representing the international value of aggregate imports and aggregate exports, respectively. The trade deficit, in turn, is simply

$$D_t = M_t - X_t.$$

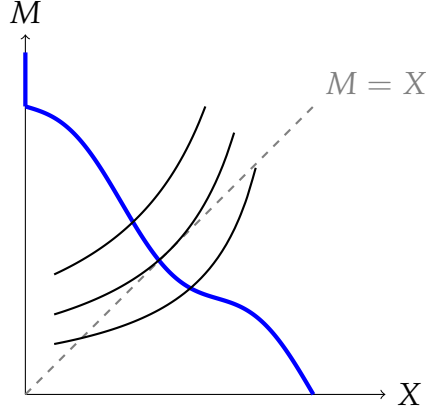


Figure 1: Preferences over Aggregate Imports and Exports

Notes: This figure plots three indifference curves (in solid black) associated with the utility $\mathcal{C}_t$ over (M, X) along with the Engel path for $\tau = 0$ (in solid blue) and the zero deficit condition $M = X$ (in dashed grey).

All considerations that shape international trade in a given period t , either coming from preferences (G_t), technology (Y_t), or international prices (p_{mt}^*, p_{xt}^*), are encoded in the induced utility function $\mathcal{C}_t$ over (M, X) . As we will argue, this is key to studying the impact of tariffs on trade deficits.¹¹ Figure 1 provides a graphical illustration of the aggregate preferences, with three indifference curves and an Engel path. The latter is defined as the locus of points (M, X) where $\mathcal{C}_{tM}/\mathcal{C}_{tX}$ is constant. It traces out how the optimal choices of M and X simultaneously vary with period t 's aggregate consumption or, equivalently, spending, holding fixed their relative price.¹² The slope of the static Engel path will play a key role below. Note that since exports reduce domestic consumption, aggregate exports X are a “bad.” Thus, indifference curves are upward-sloping and the Engel path downward-sloping.

Hicksian Demand for Aggregate Imports and Exports. We will make extensive use of the expenditure function associated with $\mathcal{C}_t$,

$$e_t(\tau, C) \equiv \min_{M, X \geq 0} \{(1 + \tau)M - X\}$$

$$s.t. : \mathcal{C}_t(M, X) \geq C.$$

¹¹Since the existence of the induced utility function $\mathcal{C}_t$ derives from the efficiency of the competitive equilibrium, we conjecture that our analysis extends, without further qualifications, to monopolistically competitive environments in which the equilibrium is also efficient, as in [Krugman \(1980\)](#) or [Melitz \(2003\)](#).

¹²[Mas-Colell et al. \(1995\)](#) call the map from spending to the vector (M, X) the Engel function and its image the wealth expansion path. The Engel path is thus the implicit representation of the two underlying Engel curves, which give M and X separately as functions of period t 's spending.

The static equilibrium conditions require imports and exports to solve this problem. We let $M_t(\tau, C)$ and $X_t(\tau, C)$ denote its solution. We can then define the deficit function

$$\mathcal{D}_t(\tau, C) \equiv M_t(\tau, C) - X_t(\tau, C).$$

Under free trade, the expenditure and deficit functions coincide: $e_t(0, C) = \mathcal{D}_t(0, C)$. But if tariffs are positive, the deficit function differs from the expenditure function, with $\mathcal{D}_t(\tau, C) = e_t(\tau, C) - \tau M_t(\tau, C)$, a reflection of the distortionary effect of the tariff.

Historical Note. The notion that preferences and domestic production can be combined, exploiting domestic efficiency, follows and extends a perspective introduced by [Meade \(1952\)](#) and further formalized by [Dixit and Norman \(1980\)](#). However, because of the questions they were studying, they worked with the vectors of net imports $m - x$. In contrast, we do not net out and keep imports and exports separate. We do so because the import tariff and other general trade costs may create different prices for the same good depending on whether it is imported or exported. In addition, we aggregate the import vector m to the scalar M and the export vector x to the scalar X . This reflects our interest in the impact of a uniform tariff τ that affects the price of (all) imports relative to (all) exports, which allows us to apply what [Deaton and Muellbauer \(1980\)](#) refer to as the Composite Commodity Theorem due to [Hicks \(1939\)](#). Thus, our analysis extends and combines elements of Meade and Hicks.

2.4 Dynamic Equilibrium Conditions

Using the expenditure function associated with the static equilibrium conditions, one can show that solving the household problem is equivalent to solving

$$\begin{aligned} & \max_{C_1, C_2} U(C_1, C_2) \\ & e_1(\tau, C_1) + \frac{1}{R^*} e_2(\tau, C_2) = NFA + T, \end{aligned}$$

where $T = \tau M_1(\tau, C_1) + \frac{1}{R^*} \tau M_2(\tau, C_2)$ is the present discounted value of tariff revenues, which is taken as given. Despite the fact that domestic prices p_{ct} enter the original budget constraint of the household, they can be conveniently omitted from the previous problem.

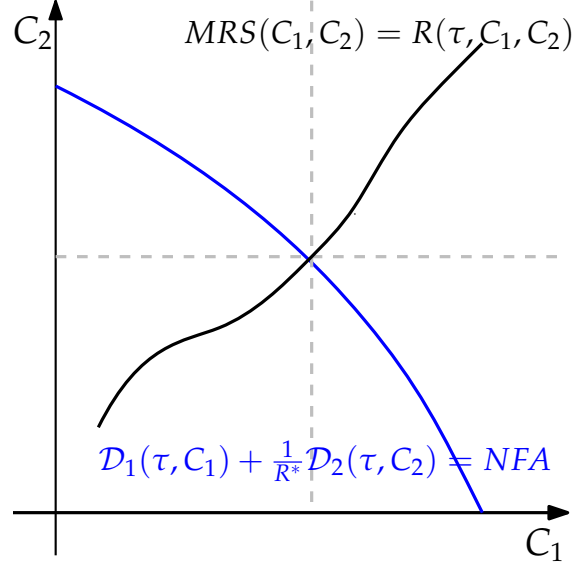


Figure 2: How Tariffs Affect Trade Deficits

Notes: This figure plots the Euler condition (3) (in solid black) and intertemporal trade balance condition (4) (in solid blue). The dashed-grey lines represent iso-deficit curves in each period $t = 1, 2$.

The dynamic equilibrium conditions are then

$$MRS(C_1, C_2) = R(\tau, C_1, C_2), \quad (3)$$

$$\mathcal{D}_1(\tau, C_1) + \frac{1}{R^*} \mathcal{D}_2(\tau, C_2) = NFA, \quad (4)$$

where $MRS(C_1, C_2)$ denotes the marginal rate of substitution between aggregate consumption in the two periods and $R(\tau, C_1, C_2)$ their relative marginal cost, which is also the domestic real interest rate,

$$\begin{aligned} MRS(C_1, C_2) &\equiv U_1(C_1, C_2) / U_2(C_1, C_2), \\ R(\tau, C_1, C_2) &\equiv R^* e_{1C}(\tau, C_1) / e_{2C}(\tau, C_2), \end{aligned}$$

with the convention $e_{tC} \equiv \partial e_t / \partial C$.

We are ready to describe the equilibrium trade deficit $D_t(\tau)$ in each period $t = 1, 2$ as a function of the tariff τ .

Proposition 1. *For a given tariff τ , the trade deficit in period t is equal to $D_t(\tau) = \mathcal{D}_t(\tau, C_t(\tau))$ with the equilibrium consumption levels $(C_1(\tau), C_2(\tau))$ a solution to (3) and (4).*

Proposition 1 encapsulates our aggregate approach. It shows how the preferences $\mathcal{C}_t$ over aggregate imports and exports—as well as its dual e_t , and the associated functions $\mathcal{D}_t$ and R —shape the causal relationship between tariffs and trade imbalances in a general neoclassical model. This relationship is illustrated in Figure 2, which plots both the Euler

condition (3) (in solid black) and trade balance condition (4) (in solid blue). Given a tariff τ , the equilibrium deficit $D_t(\tau)$ in period t then only depends on the value of aggregate consumption in this period, as captured by the dashed grey lines.

Whether or not tariffs reduce trade deficits boils down to a horse race between two potential effects. First, a change in τ may affect the domestic real interest rate, as captured by the partial derivative $\partial R/\partial \tau$. This is a substitution effect, which corresponds to a shift in the Euler condition. Second, a tariff, because it is distortionary, may also raise the level of the deficit, i.e. the transfer from the rest of the world required to achieve a given level of consumption, as captured by the partial derivatives $\partial D_t/\partial \tau$. This is an income effect, which corresponds to a shift in the trade balance condition.

3 Setting the Stage

In the rest of the paper, we use Proposition 1 to conduct comparative statics and study how trade deficits $D_t(\tau)$ vary with the tariff τ . We start with two extreme results that illustrate the range of effects that a permanent increase in tariffs may have on trade deficits, from neutral to closing them entirely. Formal proofs not given in the main text are in the appendix.

3.1 Neutrality Result

Our first comparative static result provides sufficient conditions under which a marginal tariff increase is neutral around free trade.

Proposition 2 (Neutrality). *Suppose static preferences are homothetic: G_t is homogeneous of degree one; the environment is stationary: $G_1 = G_2$, $Y_1 = Y_2$, $p_{m1}^* = p_{m2}^*$, and $p_{x1}^* = p_{x2}^*$; the equilibrium has each good i either strictly imported ($m_{it} > 0$) or strictly exported ($x_{it} > 0$) subject to iceberg trade costs (equation 1); and the sets of goods imported and exported are the same across periods, both at and around free trade. Then, starting from free trade ($\tau = 0$), tariffs do not affect trade deficits: $D'_t(\tau) = 0$.*

Proposition 2 resonates well with the broad intuition that unlike temporary tariffs, permanent tariffs may have no effect on a country's incentives to borrow and save. It should also be clear that even a permanent tariff may affect a country's incentives to borrow and save if primitives differ in periods $t = 1$ and $t = 2$. Proposition 2 therefore requires preferences, technology, trade costs, and prices to be stationary.¹³ Interestingly,

¹³Razin and Svensson (1983) show how neutrality breaks down when the same goods are exported and imported in both periods, but the environment is not stationary.

though, the proof of Proposition 2 also requires assumptions about the tradability of different goods. We will explain in detail why in the next section. But before doing so, we turn to another extreme scenario in which tariffs are far from neutral.

3.2 Autarky Result

Our next result provides conditions under which a large enough tariff leads to zero aggregate imports, zero aggregate exports, or both. In the special case where both imports and exports are zero, the economy is under autarky and tariffs fully close the deficit. This occurs when $NFA = 0$. Otherwise the economy runs persistent deficits, if $NFA > 0$, or persistent surpluses, if $NFA < 0$, as the next proposition demonstrates.

Proposition 3 (Autarky). *Suppose U and C_t are such that all aggregate commodities, C_t , M_t , and $-X_t$, are normal; the environment is stationary: $C_1 = C_2 = C$; and C has bounded derivatives. Then there exists a $\hat{\tau}$ such that for all $\tau \geq \hat{\tau}$: (i) $M_t = X_t = 0$ and $D_t = 0$, if $NFA = 0$; (ii) $X_t = 0$ and $D_t > 0$, if $NFA > 0$; and (iii) $M_t = 0$ and $D_t < 0$, if $NFA < 0$.*

We view the technical conditions imposed in Proposition 3 as very mild.¹⁴ The normality of all aggregate commodities is only required to establish that autarky is the unique competitive equilibrium. The stationarity of the environment could be dispensed with.

At this point, one may be tempted to interpret Propositions 2 and 3 as establishing that small permanent tariffs are neutral, whereas large enough tariffs are not. Next, we will show that this interpretation is incorrect and that there are systematic reasons why even a small permanent tariff may reduce the deficit.

4 Tariffs and Deficits I: Substitution Effects

As discussed in Section 2, tariffs have two types of effects: (i) a substitution effect, which we also refer to as the interest rate channel via $\partial R / \partial \tau$; and (ii) an income effect, which we also refer to as the distortion channel via $\partial D_t / \partial \tau$. For expositional purposes, we focus on these effects one at a time. Here, we study a small change in the tariff around free trade ($\tau = 0$). This implies that the income effect of the tariff is second order and that only the substitution effect is active. In the next section, we will study the opposite scenario.

¹⁴We assume the derivatives of C are finite, even at $M = 0$ or $X = 0$ for simplicity, to avoid the need for a limit $\tau \rightarrow \infty$ argument. Economically, this represents the realistic assumption of finite choke prices for supply and demand. Note, also, that G_t may still satisfy the Inada condition $G_{tc_i} \rightarrow \infty$ as $c_{ti} \rightarrow 0$, since $m_i = 0$ does not necessarily imply $c_i = 0$ if good i can be produced domestically.

4.1 A Sufficient Statistic: Slope of the Engel Path

Interest Rate Channel. Consider the domestic real interest rate,

$$R(\tau, C_1, C_2) = R^* \frac{e_{1C}(\tau, C_1)}{e_{2C}(\tau, C_2)}.$$

How does a change in the tariff τ affect the real interest rate, holding fixed aggregate consumption in the two periods? To answer this question, we can take logs and differentiate the previous expression,

$$\frac{\partial \ln R(\tau, C_1, C_2)}{\partial \ln(1 + \tau)} = \frac{\partial \ln e_{1C}(\tau, C_1)}{\partial \ln(1 + \tau)} - \frac{\partial \ln e_{2C}(\tau, C_2)}{\partial \ln(1 + \tau)}. \quad (5)$$

In any given period, the first and cross-derivatives of the expenditure function satisfy

$$\begin{aligned} e_C(\tau, C) &= (1 + \tau)M_C(\tau, C) - X_C(\tau, C) \geq 0 \\ e_\tau(\tau, C) &= M(\tau, C) \geq 0, \\ e_{C\tau}(\tau, C) &= M_C(\tau, C) \geq 0, \end{aligned}$$

where we have dropped the subscript t for notational convenience. It follows that

$$\frac{\partial \ln e_C(\tau, C)}{\partial \ln(1 + \tau)} = \frac{(1 + \tau)M_C(\tau, C)}{(1 + \tau)M_C(\tau, C) - X_C(\tau, C)}. \quad (6)$$

The right-hand side is a decreasing function of $M_C(\tau, C)/X_C(\tau, C)$, which is the slope of the Engel path pictured in Figure 1,

$$\frac{dM}{dX} = \frac{M_C(\tau, C)}{X_C(\tau, C)} \leq 0.$$

Equations (5) and (6) imply that an increase in the tariff shifts up the real interest rate, $\partial R/\partial \tau > 0$, if and only if the slope of the static Engel path, dM/dX , is steeper (larger in absolute value) at $t = 1$ than at $t = 2$.¹⁵

¹⁵In applied work, it is common to estimate household-level Engel *curves*. They describe how consumption of, or expenditure on, a given good varies with total expenditure (see, e.g., Deaton and Muellbauer, 1980). In the present context, we focus on country-level Engel curves for aggregate imports and exports, M and X . Their slopes are $M_C\tilde{C}_e$ and $X_C\tilde{C}_e$, with $\tilde{C}(\tau, e) \equiv \max_{M, X}\{C(M, X) | (1 + \tau)M - X = e\}$ the static indirect utility function associated with expenditure e . It follows that the slope of the Engel *path* is equal to the ratio of the slopes of the underlying Engel *curves*.

Intuitively, inequality (7) captures a higher reliance, at the margin, on imports in the first period. A tariff then raises the cost of consumption relatively more in the first period, creating a substitution effect away from C_1 towards C_2 , i.e. an incentive to save, which reduces the trade deficit. A naive intuition might have been that what matters is the direct incidence of the tariff on an aggregate consumption price index (CPI). Our result shows that this intuition, however, is generally lacking. First, the use of a CPI presumes homotheticity of preferences, but our result does not invoke such an assumption. Thus, our result allows for inferior, superior, or luxury goods. Second, in general, imports may be used in production as inputs, rather than directly consumed as final goods, in which case a CPI-based approach is doomed or requires more work to trace out the effect. Third and most surprisingly, according to our sufficient statistic, the consumption price vector p_{ct} does not directly enter the picture. Neither does the share of expenditure on non-traded goods. According to Proposition 4, only the relative expansion of imports to exports matters. In short, the static Engel path holds the key to the dynamic question of interest.¹⁶

4.2 Slope of the Engel Path: Extensive Margin Considerations

Non-linearity of the Engel path and non-neutrality of tariffs. Proposition 4 implies that a permanent tariff may reduce trade deficits even though preferences, technology, and, in turn, Meade preferences over imports and exports, C_t , are invariant over time. This can happen even though the Engel path is the same in the two periods. To see this, note that if the Engel path is strictly convex, then $C_1 > C_2$ further implies $|dM_1/dX_1| > |dM_2/dX_2|$, as described in Figure 4b. This is sufficient for a permanent tariff τ to have a non-neutral effect on deficits.

Corollary 1. *Consider a stationary economy with invariant Meade preferences, $C_1 = C_2 = C$, and a strictly convex Engel path for C . Suppose the economy runs a larger trade deficit in period 1 than in period 2: $D_1(\tau) > D_2(\tau)$. Then starting from free trade ($\tau = 0$), an increase in tariffs reduces the deficit in period 1: $D'_1(\tau) < 0$.*

Of course, since the intertemporal trade balance condition is unaffected, a reduction in the trade deficit in period 1 must be accompanied by a reduction in the trade surplus

¹⁶Appendix E shows how the same logic extends to an environment with heterogeneous tariffs. In this case, the effect of tariffs on deficits also depends on the slopes of the Engel path for different groups of imports, each subject to its own tariff change, as formally established in Proposition 4-H. Specifically, tariff changes reduce deficits in period 1 if, in addition to the Engel path for aggregate imports and exports being steeper in this period, import groups experiencing relatively larger tariff changes have relatively steeper slopes in this period as well. This captures the same general idea: a higher reliance, at the margin, on goods subject to larger tariff changes implies a disproportionate increase in the cost of consumption in period 1, leading to a substitution effect and an incentive to save.

in period 2, i.e. $D'_2(\tau) > 0$, as can be seen from Figure 3. In other words, under the conditions of Corollary 1, a small tariff reduces trade imbalances.¹⁷

Extensive margin considerations. In Corollary 1, the non-linearity of the Engel path induces differences in its slopes across periods and, in turn, the non-neutrality of tariffs. But this raises the question: Why would one expect Engel paths to be non-linear?

Non-homotheticity in the preferences over goods (represented by G_t) may lead to non-homotheticity in the preferences over aggregate imports and exports (represented by C_t). For instance, one might expect aggregate imports to be a luxury in many developing countries, whereas the converse is true in developed countries.¹⁸ More surprisingly, we will show that even if preferences over goods are homothetic, extensive margin considerations can give rise to non-homothetic preferences over aggregate imports and exports, i.e. non-linear Engel paths in (M, X) space.

For our next result, we focus on a standard environment in the international trade and macro literatures: preferences are CES; the supply of each good is fixed; and international trade is subject to iceberg trade costs. We do not restrict the number of goods, taste shifters, endowments, or international prices. So this environment nests the Armington model with one domestic good and one foreign good, as in Obstfeld and Rogoff (2000).¹⁹ The next proposition shows that whether or not the extensive margin of trade is active—in the sense that changes in aggregate consumption, from C_1 to C_2 , affect the sets of goods being exported, imported, or non-traded—is critical for the convexity of the Engel path.

Proposition 5 (Convex Engel Path). *In an endowment economy with CES preferences, the Engel path of C_t is convex for all $(M, X) > 0$ and strictly convex if the extensive margin of trade is active.*

The broad intuition is as follows. By definition, both imports and exports must be non-negative. When aggregate consumption C goes up, we move along an Engel path, increasing aggregate imports M and decreasing aggregate exports X . At the micro level,

¹⁷Like our subsequent results, Corollary 1 allows NFA to be non-zero, in which case the country may be running a deficit in both periods: $D_1(\tau) > D_2(\tau) > 0$. For the interested reader, Appendix F extends Corollary 1 to an infinite-horizon economy. In line with other theoretical results in international trade (e.g., Jones and Scheinkman, 1977 and Deardorff, 1980), going beyond “two-ness” weakens the restrictions imposed by theory. With more than two periods, these restrictions apply only to the periods with the largest and smallest deficits.

¹⁸Using a sample of 27 European countries and 13 large countries, Fajgelbaum and Khandelwal (2016) estimate that exports from richer countries have higher income elasticities, in line with the previous observation. In the case of the United States, though, Borusyak and Jaravel (2021) document that import shares are flat throughout the income distribution.

¹⁹Note, though, that even if foreigners have no endowment of the domestic good, we treat the domestic economy as a small open economy. We will introduce terms-of-trade considerations in Section 4.3.

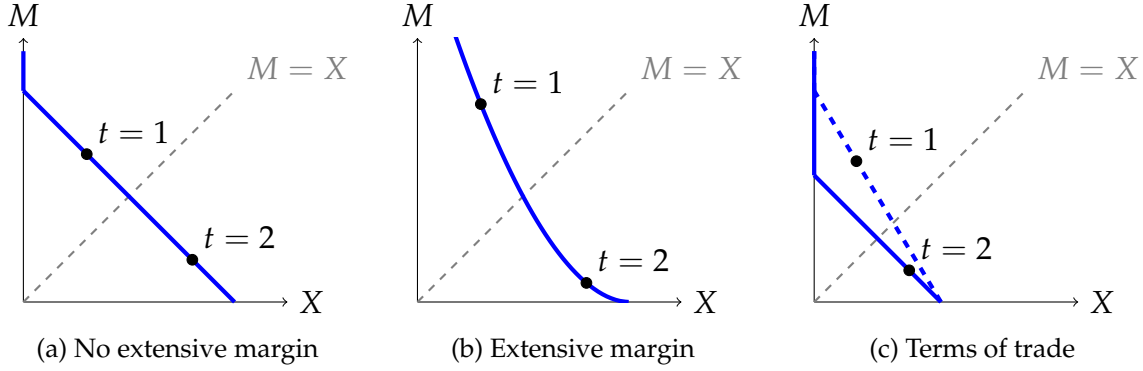


Figure 4: Slope of the Engel Path Across Periods

Notes: Each panel plots Engel paths (in blue) along with the equilibrium values of M and X at $t = 1$ and $t = 2$. Since the country runs a trade deficit at $t = 1$ and a surplus at $t = 2$, the $t = 1$ point lies above the 45-degree line and the $t = 2$ point below it, with aggregate consumption higher at $t = 1$. In both Figures 4a and 4b, the Engel path is identical across periods, as in Section 4.2. In Figure 4a the Engel path is linear, the slope is the same at $t = 1$ and $t = 2$, and a permanent tariff is neutral. In Figure 4b, the Engel path is strictly convex, so it is steeper at $t = 1$, and a permanent tariff reduces the deficit. In Figure 4c, the Engel path varies across periods (dashed at $t = 1$, solid at $t = 2$), as in Section 4.3. The path is linear in each period, but steeper at $t = 1$, so a permanent tariff also reduces the deficit.

the imports and exports of individual goods adjust. Due to the homotheticity of preferences, absent non-negativity constraints, these adjustments are linear, implying a linear Engel path.²⁰ However, this is no longer the case when non-negativity constraints bind. In particular, as consumption C rises, a greater number of goods that were not previously imported start being imported, going from $m_i = 0$ to $m_i > 0$, facilitating the adjustment along the aggregate import M margin. Likewise, as consumption C rises, the non-negativity constraint for exports becomes binding for a greater number of goods, going from $x_i > 0$ to $x_i = 0$, blunting the adjustment on the aggregate export X margin. Both extensive margin considerations tend to make Engel paths strictly convex. The dual of this observation, left implicit in our Meade representation, is the endogenous adjustment of domestic prices to aggregate consumption: as C increases, so do the prices of the goods that become non-traded or start being imported.

In the Armington model, with one domestic good and one foreign good, the extensive margin is inactive for all $M, X > 0$. As a result, Engel paths are linear, and if the environment is stationary, the slope of the Engel path is identical across periods, as described in

²⁰The role of CES preferences is to guarantee that quasi-homotheticity, and hence linearity of the Engel path, is preserved when non-negativity constraints bind for any given subset of goods. This makes CES the sharpest benchmark for isolating the role of extensive margin considerations: since preferences contribute no curvature on their own, any strict convexity is attributable to the extensive margin alone. Away from CES, the same general forces operate, with binding non-negativity constraints still pushing the Engel path toward greater local convexity, but now on top of whatever curvature preferences themselves generate.

Figure 4a. In contrast, if there is a continuum of domestic goods, the extensive margin is generically active, and Engel paths are smooth and convex. Even in a stationary environment, slopes differ across periods, as described in Figure 4b. Intermediate cases with a finite number of domestic goods give rise to Engel paths with finitely many kinks. Since strict convexity implies that tariffs reduce deficits, as discussed in Corollary 1, this highlights that the absence of extensive margin considerations in Proposition 2 was critical for neutrality to hold.²¹

4.3 Slope of the Engel Path: Terms-of-Trade Considerations

So far, we have assumed fixed international prices. This implicitly restricts how the variation in aggregate consumption between today and tomorrow may affect domestic prices and, as a result, shape the incidence of a tariff. In general, a change in C may also affect a country's terms of trade. This is at the heart of the transfer problem debated by Keynes and Ohlin and studied by Dornbusch et al. (1977). To conclude this section, we show how terms-of-trade considerations, in addition to extensive margin considerations, can affect the slope of the Engel path and, in turn, create a systematic relationship between a permanent tariff and the domestic real interest rate.

Interest rate channel with terms-of-trade effects. Consider a generalized version of the environment in Section 2 in which import prices $p_{mt}^*(\tau)$ and export prices $p_{xt}^*(\tau)$ may vary with τ . R^* is still fixed, without loss of generality.

Our aggregate approach remains valid in this case. Formally, we just need to keep track of the fact that $C_t(M, X; p_{mt}^*(\tau), p_{xt}^*(\tau))$ and the associated expenditure function $e_t(\tau, C; p_{mt}^*(\tau), p_{xt}^*(\tau))$ now also depend on τ through its impact on import and export prices. Accordingly, the response of the domestic real interest rate to a change in the tariff τ is given by

$$\frac{d \ln R}{d \ln(1 + \tau)} = \frac{d \ln e_{1C}}{d \ln(1 + \tau)} - \frac{d \ln e_{2C}}{d \ln(1 + \tau)}.$$

This is identical to (5), except for the fact that $d \ln e_{tC} / d \ln(1 + \tau)$ is now a total derivative,

$$\frac{d \ln e_{tC}}{d \ln(1 + \tau)} = \frac{\partial \ln e_{tC}}{\partial \ln(1 + \tau)} + \frac{\partial \ln e_{tC}}{\partial \ln p_{mt}^*} \cdot \frac{\partial \ln p_{mt}^*}{\partial \ln(1 + \tau)} + \frac{\partial \ln e_{tC}}{\partial \ln p_{xt}^*} \cdot \frac{\partial \ln p_{xt}^*}{\partial \ln(1 + \tau)}. \quad (8)$$

The partial effect of a tariff, $\partial \ln e_{tC} / \partial \ln(1 + \tau)$, is still given by equation (6). As before, it only depends on the slope of the Engel path M_C / X_C . This slope, however, may now

²¹For the interested reader, Appendix H offers a graphical way to relate our results to, and distinguish them from the original analysis of Obstfeld and Rogoff (2000).

vary with the endogenous international prices, since $C_t(M, X; p_{mt}^*(\tau), p_{xt}^*(\tau))$ depends on these.

The extra terms in (8) further capture the direct effect of tariffs on international prices, $\partial \ln p_{mt}^* / \partial \ln(1 + \tau)$ and $\partial \ln p_{xt}^* / \partial \ln(1 + \tau)$. Conveniently, their incidence on e_{tC} is itself a function of the slope of the Engel path M_C / X_C , as we discuss below. In sum, the slope of the Engel path, which is now endogenous to a country's terms of trade, continues to be a key determinant in this more general environment.

Ricardian economy. An illuminating example is the “small economy” limit of a Ricardian model à la [Eaton and Kortum \(2002\)](#). This limit obtains when the rest of the world is infinitely large, but its share of expenditure on domestic goods is infinitely small, as discussed in [Demidova et al. \(2024\)](#).²² In this environment, import prices and the world interest rate are fixed, but export prices are proportional to the domestic wage $w_t(\tau, C_t)$, which is an endogenous function of aggregate consumption C_t and the import tariff τ :

$$p_{i,xt}^* = \bar{p}_{i,xt}^* \times w_t(\tau, C_t). \quad (9)$$

Combining equations (6), (8), and (9), and dropping the subscript t for notational convenience, one can then show

$$\frac{d \ln e_C}{d \ln(1 + \tau)} = \frac{(1 + \tau)M_C(\tau, C; w(\tau, C)) - X_C(\tau, C; w(\tau, C))\epsilon_w(\tau, C)}{(1 + \tau)M_C(\tau, C; w(\tau, C)) - X_C(\tau, C; w(\tau, C))}, \quad (10)$$

where $\epsilon_w(\tau, C) \equiv \partial \ln w(\tau, C) / \partial \ln(1 + \tau)$ denotes the elasticity of the domestic wage with respect to the tariff and $M_C(\tau, C; w(\tau, C))$ and $X_C(\tau, C; w(\tau, C))$ now also depend on C through its impact on the wage $w(\tau, C)$, which enters the aggregate preferences $\mathcal{C}$.

For the purposes of signing the impact of the tariff on the interest rate, there are now two sufficient statistics. In addition to the slope of the Engel path, M_C / X_C , the new statistic ϵ_w is needed because export prices also respond to the tariff, $\partial \ln p_{xt}^* / \partial \ln(1 + \tau) \neq 0$. No further statistic is needed because the incidence of these export-price changes depends only on the marginal spending on aggregate imports and exports, which M_C / X_C already captures.

In this Ricardian economy, terms-of-trade effects create another reason for an increase in τ to reduce trade deficits via the interest rate channel, as we now demonstrate.

²²There are many possible ways to take a “small economy” limit. One could vary country size along with productivity, as in [Alvarez and Lucas \(2007\)](#), or along with trade costs, as in [Gali and Monacelli \(2005\)](#). Here we implicitly follow [Gali and Monacelli \(2005\)](#) and [Demidova et al. \(2024\)](#), an approach that also delivers a non-zero share of domestic expenditure on domestic goods. Details can be found in Appendix I.

Proposition 6 (Terms-of-Trade Effects). *Consider the small economy limit of a stationary Ricardian economy à la Eaton and Kortum (2002). Suppose the economy runs a larger trade deficit in period 1 than in period 2: $D_1(\tau) > D_2(\tau)$. Then an increase in τ raises the domestic real interest rate, $d \ln R / d \ln(1 + \tau) > 0$, increasing incentives to save in period 1.*

There are two distinct forces that shape the interest rate channel, each captured by one of the two sufficient statistics in (10). The first is general and extends beyond the small economy limit of a Ricardian economy. Because of home bias, an increase in C improves the country's terms of trade by raising w . This increase in the relative price of exports raises the marginal spending on imports relative to exports M_C/X_C , which makes the cost of living more sensitive to tariffs in periods of deficits. Graphically, this makes the Engel path steeper at $t = 1$ than at $t = 2$, as illustrated in Figure 4c. This is the counterpart to the mechanism at play in Proposition 5 via extensive margin considerations. The only difference is that here it is the adjustment in the price of tradables, rather than non-tradables, that induces variation in the slope of the Engel path across periods.²³

The second force relates to how the elasticity of tradable prices with respect to tariffs varies with the level of aggregate consumption C . In general, import and export prices may become more or less manipulable as C varies. In the small economy limit of a Ricardian economy, however, one can check that ϵ_w is also increasing in C . Thus an increase in τ must disproportionately raise export prices in periods of deficits, magnifying the direct effect of tariffs on the cost of living. In this limit, both forces reinforce each other and a tariff tends to increase incentives to save in periods of deficits.

Note that unlike our previous results, Proposition 6 does not restrict tariffs to be around zero. This reflects the fact that it focuses purely on the interest rate channel and leaves open the question of how income effects may affect the trade balance condition. Next we conclude our analysis with a discussion of income effects for large tariffs.

5 Tariffs and Deficits II: Income Effects

We return to the environment of Section 2, without terms-of-trade effects. But in contrast to our analysis in Section 4, we now study the case where the distortion channel is active,

²³It is worth emphasizing that the Engel path in any Ricardian economy is linear, exactly as in an Armington model. A Ricardian model à la Eaton and Kortum (2002), in particular, generates richer extensive margin adjustments to changes in trade costs, but not to changes in aggregate consumption, which is what matters for our results. Linearity of the Engel path reflects the deeper equivalence between the Ricardian economy and a fictitious endowment economy in which countries trade their own labor, as noted by Wilson (1980). As a result, there is only one non-negativity constraint that is relevant: the one associated with the labor content of a country's exports, bringing us back to the Armington case with one domestic good.

which creates income effects.

Distortion Channel. Formally, we no longer restrict the tariff τ to be around zero so that

$$\mathcal{D}_{t\tau}(\tau, C) = e_{t\tau}(\tau, C) - M_t(\tau, C) - \tau M_{t\tau}(\tau, C) = -\tau M_{t\tau}(\tau, C) > 0,$$

if tariffs distort aggregate imports, $M_{t\tau}(\tau, C) < 0$. This implies that the trade balance condition (4) (displayed in solid blue in Figure 2) may shift in response to a change in τ . Are there reasons to expect a systematic effect of a permanent tariff on deficits through this second channel? The answer again is yes, as we will demonstrate.

Consider a small change in tariff from τ to $\tau + d\tau$. Let $(\delta C_t)_{D_t=cst}$ denote the change in aggregate consumption that would be necessary to hold the trade deficit in period t fixed at its original level, $D_t = \mathcal{D}_t(\tau, C_t)$, under the new tariff, $\tau + d\tau$. It is given by

$$\mathcal{D}_C(\tau, C_t(\tau)) \times (\delta C_t)_{D_t=cst} + \mathcal{D}_\tau(\tau, C_t(\tau)) \times d\tau = 0.$$

By definition, an increase in tariffs from τ to $\tau + d\tau$ reduces the deficit in period t if and only if the change in aggregate consumption δC_t observed along the equilibrium is less than $(\delta C_t)_{D_t=cst}$. Note that the sum of the trade deficits across the two periods must remain equal to the country's NFA, which is fixed. So, trade deficits cannot move in the same direction in both periods. It follows that a necessary and sufficient condition for the trade deficit to go down in period 1 is

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} < \frac{\delta C_1}{\delta C_2}.$$

No Interest Rate Channel. To isolate the role of the distortion channel, we go back to the same economy as in Proposition 2. Static preferences are homothetic; the environment is stationary; and there are no extensive margin considerations. This guarantees that

$$\partial R(\tau, C_1, C_2) / \partial \tau = 0.$$

In this case, the Euler condition (3) (displayed in solid black in Figure 2) is fixed. So, we can rearrange the condition for the trade deficit to go down in period 1 as

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} < \frac{(\delta C_1)_{MRS=R}}{(\delta C_2)_{MRS=R}}, \quad (11)$$

where $\delta C_t = (\delta C_t)_{MRS=R}$ denotes the equilibrium change in C_t obtained from moving along the *dynamic* Engel path in (C_1, C_2) . This is the situation represented in Fig-

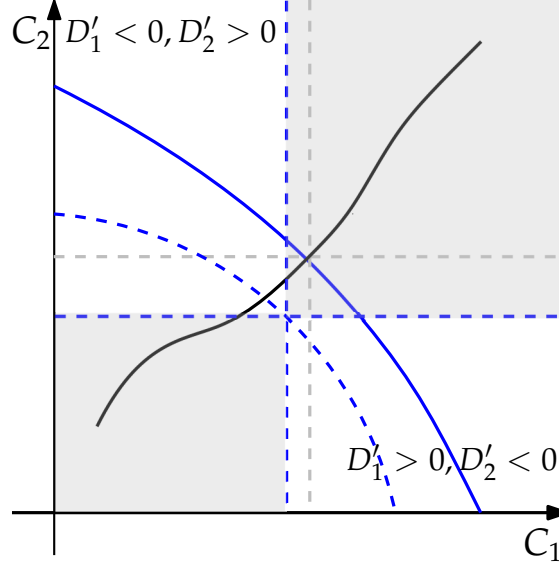


Figure 5: Income Effects: The Distortion Channel

Notes: This figure illustrates how a shift in the deficit functions, $\partial D_t / \partial \tau > 0$, causes a shift in the iso-deficit curves (in dashed blue) as well as the trade balance condition (4) (also in dashed blue) and, in turn, affects the equilibrium size of the trade deficits. In this example, an increase in τ lowers the deficit in period 1.

ure 5 where the intersection between the Euler condition (3) (in solid black) and the new trade balance condition (4) (in dashed blue) lies above the intersection of the new iso-deficit curves (also in dashed blue). Intuitively, what matters for trade deficits is not only the difference between the magnitude of the income shocks in the two periods, reflected in $(\delta C_1)_{D_1=cst} / (\delta C_2)_{D_2=cst}$, but also how this difference relates to the difference in the propensity to consume out of wealth in the two periods, as reflected in $(\delta C_1)_{MRS=R} / (\delta C_2)_{MRS=R}$.

Condition (11) is related to previous work by Svensson and Razin (1983). They derive a version of this condition in the context of an endowment economy subject to a permanent terms-of-trade shock. They then show that non-homotheticity in intertemporal preferences, which creates differences in the propensity to consume across periods, implies the non-neutrality of this permanent income shock on the deficit. In contrast, the next proposition shows that even when intertemporal preferences are homothetic, a permanent tariff shock can have a non-neutral effect on the deficit.

Proposition 7 (Distortions). *Consider the same economy as in Proposition 2, but with intertemporal preferences that are homothetic: U is homogeneous of degree one. Suppose that the economy runs a larger trade deficit in period 1 than in period 2: $D_1(\tau) > D_2(\tau)$. Then away from free trade ($\tau > 0$), an increase in τ lowers the deficit in period 1: $D'_1(\tau) \leq 0$, with strict inequality if and only if tariffs distort production.*

The logic is the following. Since intertemporal preferences are homothetic and there is no interest rate channel, dynamic Engel paths are linear and $C_1(\tau)/C_2(\tau)$ is fixed. It follows that $(\delta C_1)_{MRS=R}/(\delta C_2)_{MRS=R}$ must be equal to $C_1(\tau)/C_2(\tau)$ as well. Given condition (11), the question then is whether $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst}$ is lower than this ratio. Proposition 7 states that it must be.

By distorting aggregate imports, $M_{t\tau}(\tau, C) < 0$, an increase in τ makes the economy poorer: $\mathcal{D}_{t\tau}(\tau, C) > 0$, which implies that a larger deficit is necessary to sustain the same level of aggregate consumption. In the case of an endowment economy, the import distortion is equal to the consumption distortion and is therefore proportional to C . This implies $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst} = C_1(\tau)/C_2(\tau)$ and, in turn, the neutrality of tariffs, as in Proposition 2, even away from $\tau = 0$. In the general case, however, the import distortion also includes a production distortion, which is the same in the two periods, by stationarity. As formally shown in Appendix J, $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst} = [C_1(\tau) + a]/[C_2(\tau) + a]$, with $a > 0$ if and only if production is distorted, in which case tariffs reduce the deficit in period 1, since $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst} < C_1(\tau)/C_2(\tau)$.²⁴

Two departures from Svensson and Razin (1983) are key to establishing Proposition 7. First, we allow for a general neoclassical economy rather than an endowment economy. Second, we endogenize income fluctuations across periods through the distortionary effects of tariffs rather than take income shocks as given. Here, increasing a permanent tariff makes the economy poorer in both periods. But it “succeeds” in reducing the trade deficit in period 1 because, as a fraction of aggregate consumption, it induces a disproportionately smaller income loss in this period.²⁵ If one relaxes the assumption that intertemporal preferences are homothetic, the mechanism emphasized by Svensson and Razin (1983) reappears alongside ours, in line with (11). The two mechanisms reinforce each other if the propensity to consume out of wealth is higher in the deficit period, and work against each other otherwise.

²⁴The absence of a production distortion can also occur, at least locally, in an economy with production. The Ricardian model is an important example. In the absence of terms-of-trade effects, the pattern of specialization in a Ricardian model is independent of the level of the tariff as long as aggregate exports are non-zero. This is true even if there is a continuum of goods, as in Dornbusch et al. (1977). The existence of terms-of-trade effects is therefore critical for the non-neutrality of tariffs in Proposition 6.

²⁵A related but distinct question is whether imposing a tariff would also decrease deficits in period 1 as a share of domestic value added in this period. The answer depends on the prices used to evaluate domestic value added as well as the sign of the net foreign asset position. Focusing on $NFA = 0$, if international prices are used to evaluate both trade deficits and domestic value added, then a permanent tariff leaves the deficit-to-value-added ratio unchanged. If instead international prices are used to evaluate deficits and domestic prices to evaluate domestic value added, in line with national accounts practices, then a permanent tariff decreases the deficit-to-value-added ratio as well. Appendix K establishes both results.

6 Quantitative Examples

We have provided formal propositions about the impact of a permanent tariff on trade deficits when either the interest rate channel or the distortion channel is active. In either of these two polar cases, we have highlighted reasons why an increase in τ may reduce trade deficits, either because its impact on the cost of living systematically varies with C , as in Section 4, or because its impact on income does, as in Section 5. There is little more that can be said analytically when both channels are active and interact.²⁶ To conclude our analysis, we therefore turn to simulations. The goal is not to offer a definitive answer about what we expect the impact of tariffs on trade deficits to be in practice, but rather to explore the range of possible answers that might emerge under typical calibrations.

Parametric assumptions. In line with our previous analysis, we consider endowment economies, as in Section 4.2, as well as the “small economy” limit of a Ricardian economy, as in Section 4.3. So the interest rate channel may either derive from extensive margin considerations or from terms-of-trade effects affecting the slope of the Engel path. The environment is stationary and the world interest rate is normalized to unity: $R^* = 1$. Intertemporal preferences are CES with elasticity of substitution μ and static preferences are CES with elasticity of substitution σ . All goods enter preferences symmetrically. Iceberg trade costs are uniform: $\delta_{i,x} = \delta_{i,m} = \delta$.

In the case of endowment economies, there is a fixed supply $y_i = [1 + e^{(i-0.5)/\gamma}]^{-1}$ of each good $i \in [0, 1]$. The parameter γ captures the importance of extensive margin considerations. As γ goes to zero, the model converges to the Armington model with one domestic good (with positive endowment) and one foreign good (with zero endowment). International prices are fixed and we normalize them so that $p_m^* = p_x^* = 1$. In the case of the Ricardian economy, there is an exogenous endowment of labor L and a distribution of productivity that gives rise to a gravity equation with trade elasticity $\varepsilon > 0$. The domestic wage w is endogenously determined by the domestic labor market clearing condition, with δ determining the relative importance of domestic and foreign demand and ε determining the importance of terms-of-trade considerations. The expressions for aggregate imports and exports as well as the labor market clearing condition can be found in equations (I.1)-(I.3) of Appendix I.

²⁶Our working paper, Costinot and Werning (2025), offers one such result in the special case of an endowment economy in which both U and G_t are CES, with identical elasticity of substitution.

Calibration. We set the intertemporal elasticity of substitution to $\mu = 1$. We consider two endowment economies, one with $\gamma = 0.1$ and another with $\gamma = 0.005$, close to Armington. For each version, we choose σ to target a trade elasticity $\varepsilon = 5$. In the case of the Ricardian economy, we directly set $\varepsilon = 5$. Finally, in all three economies, we set the discount factor β and the iceberg trade cost δ to match a share of imports to domestic value added equal to 15% and a ratio of deficit to domestic value added equal to 6% in the free trade equilibrium ($\tau = 0$).²⁷ We set NFA to zero.

Simulation results. Figure 6a illustrates the relationship between tariffs and deficits in these three economies: endowment with (almost) no extensive margin ($\gamma = 0.005$) and no terms-of-trade effects; endowment with an extensive margin around free trade ($\gamma = 0.1$) and no terms-of-trade effects; and Ricardian with terms-of-trade effects. In all three cases, we see that an increase in τ lowers the trade deficit, in line with our analytical results. The extent to which it does, however, varies significantly across models, despite the fact that all models are consistent with the same levels of openness to trade, both intra- and intertemporally, as well as the same trade elasticity.

Figure 6b explains why. As emphasized in our earlier analysis, how the slope of the static Engel path varies across periods is a key determinant of the structural relationship between tariffs and deficits. Matching the previous moments, however, imposes little discipline on the value of this statistic across the three models. In the Ricardian economy, the trade elasticity is important because it shapes terms-of-trade effects, which then drive the variation in the slope of the Engel path. But in the endowment economies, the trade elasticity is irrelevant: it is the parameter γ that controls extensive margin considerations and drives this variation.²⁸ Different variations in the slopes across periods, in turn, lead to different predictions. The contrast is particularly stark around free trade, where the economy with neither terms-of-trade effects nor extensive margin adjustments is very close to neutrality.

To bring additional credibility to simulations of the effects of tariffs on deficits, our theoretical analysis calls for estimating the slope of the Engel path directly and using it as a new target in calibration. To estimate this key statistic, one could proceed as follows. For a given tariff τ and a given level of aggregate consumption C , consider small exogenous shocks to the world interest rate ΔR^* . Everything else being equal, each shock ΔR^* causes a change in expenditure and, in turn, in aggregate consumption ΔC , with

²⁷Following national accounts practices, we use international prices to evaluate imports and deficits and domestic prices to evaluate domestic value added.

²⁸Similar differences across these three models can be observed for alternative values of the trade elasticity. Appendix L reports the counterparts of Figure 6 for $\varepsilon = 2$ and 8.

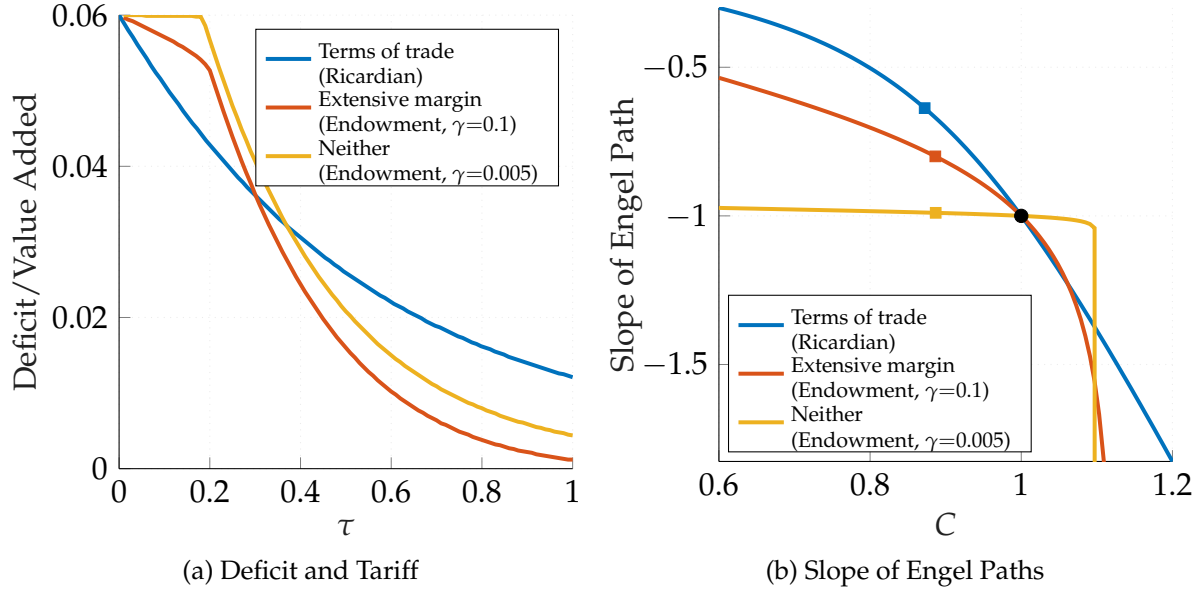


Figure 6: Quantitative Examples

Notes: Figure 6a plots the deficit D_1 , expressed as a share of domestic value added, as a function of the tariff τ , in three economies: an endowment economy with (almost) no extensive margin adjustments ($\gamma = 0.005$) and no terms-of-trade effects (in yellow); an endowment economy with extensive margin adjustments ($\gamma = 0.1$) and no terms-of-trade effects (in red); and a Ricardian economy with terms-of-trade effects (in blue). All three economies are calibrated to match a share of imports to domestic value added equal to 15%, a ratio of deficit to domestic value added equal to 6%, and a trade elasticity $\varepsilon = 5$ in the free trade equilibrium ($\tau = 0$). Figure 6b plots the slopes of the Engel paths M_C/X_C for the same three economies. All economies are normalized so that $C_1 = 1$ and the slope of the Engel path is -1 at $C = 1$. The black dot represents the common slope at C_1 ; squares represent the slopes at C_2 in each economy.

associated changes in imports and exports, $\Delta M = M_C \Delta C$ and $\Delta X = X_C \Delta C$. Using this exogenous variation in world interest rates, one can trace out a country's Engel path and estimate its slope locally as the ratio of ΔM to ΔX , i.e. the ratio of the slopes of its Engel curves for imports and exports. We are not aware of empirical work along these lines, but we hope that our analysis can provide further motivation.

7 Concluding Remarks

We have used a flexible trade model to study the effect of tariffs on the trade balance and to isolate the relevant sufficient statistics. We have shown that the response to this dynamic question is controlled by the static Engel path for aggregate imports and exports.

Except in special cases, economic theory predicts that tariffs do affect trade imbalances and are likely to reduce them. Our results provide a step forward in understanding the underlying mechanism and determining the magnitudes. One central idea is that even if economic conditions are unchanged between today and tomorrow, the fact that a country

is currently running a trade deficit implies that aggregate consumption will, in general, be different today and tomorrow. As a result, a permanent tariff can affect the domestic real interest rate and, in turn, the incentives to borrow and save. This can occur both through extensive margin and terms-of-trade considerations.

We conclude with a caveat and a reminder. First, intertemporal substitution requires that households and firms anticipate the endogenous effects of policy on future prices and income. Our analysis relied on the benchmark assumption of rational expectations. Alternatives with more backward-looking specifications of expectations would likely mitigate the effects of tariffs on deficits. Second, we started this paper by noting that our focus was a positive rather than a normative one. We are interested here in whether tariffs can affect trade deficits, not whether tariffs should be used to affect them. The answer to the second question is well known. Regardless of whether or not tariffs affect deficits, a country may gain from unilaterally imposing tariffs, if it has the ability to affect international prices, and it may lose from foreign retaliation, if the rest of the world does as well.

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A Definition of a Competitive Equilibrium

This appendix lays out the full set of equilibrium conditions.

The representative household faces a domestic budget constraint

$$p_{c1} \cdot c_1 + \frac{1}{R^*} p_{c2} \cdot c_2 = NFA + \Pi_1 + T_1 + \frac{1}{R^*} (\Pi_2 + T_2),$$

where p_{ct} are the domestic prices of consumption goods in period t ; NFA is a fixed net foreign asset position; Π_t are total profits from firms in the same period; and T_t is the transfer from the government.

Firm profits in period t are equal to

$$\Pi_t = p_{ct} \cdot c_t + p_{xt}^* \cdot x_t - (1 + \tau) p_{mt}^* \cdot m_t.$$

Given an import tariff τ , international prices $\{p_{mt}^*, p_{xt}^*\}_{t=1,2}$ and R^* , as well as net foreign asset position NFA , a competitive equilibrium is $\{(c_t, m_t, x_t), p_{ct}\}_{t=1,2}$ such that

1. consumption (c_1, c_2) maximizes utility subject to the budget constraint taking p_{ct} , Π_t , and T_t as given;
2. each period, $(c_t, m_t, x_t) \in Y_t$ maximizes profits Π_t taking p_{ct} , p_{mt}^* , p_{xt}^* and τ as given;
3. transfers satisfy

$$T_t = \tau \times (p_{mt}^* \cdot m_t).$$

B Proof of Proposition 1

We draw on the full equilibrium definition from Appendix A. We proceed in two steps. First, we establish that for any given C_t , the equilibrium in period t can be found using the aggregates and C_t , as argued in Section 2.3. Second, we establish that we can find the aggregates C_1 and C_2 by focusing on the intertemporal problem in Section 2.4. To save notation, we fix τ , p_{mt}^* and p_{xt}^* and drop these variables from the arguments of all functions.

Static equilibrium conditions. For a given price p_c , the firm problem in period t is

$$\begin{aligned} \tilde{e}_t(p_c) &\equiv \min_{c, m, x} (1 + \tau) p_{mt}^* \cdot m - p_{xt}^* \cdot x - p_c \cdot c \\ \text{s.t. } & (c, m, x) \in Y_t. \end{aligned}$$

The dual of the household problem can be written as

$$\begin{aligned}\hat{e}_t(p_c, C) &\equiv \min_c p_c \cdot c \\ \text{s.t. } &G_t(c) = C.\end{aligned}$$

For a given C_t , a static equilibrium at t corresponds to (p_{ct}, c_t, m_t, x_t) that solve both the firm and household problems. A static equilibrium thus also solves

$$\begin{aligned}e_t(C) &\equiv \min_{c, m, x} (1 + \tau) p_{mt}^* \cdot m - p_{xt}^* \cdot x \\ \text{s.t. } &(c, m, x) \in Y_t, \\ &G_t(c) = C.\end{aligned}$$

At the equilibrium price p_{ct} , note that

$$e_t(C_t) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C_t).$$

The minimization problem $e_t(C)$ is identical to minimizing $(1 + \tau)M - X$ subject to $\mathcal{C}_t(M, X) = C$. We can then find m, x from the maximization problem defining the aggregate preferences $\mathcal{C}_t$. This establishes that for any given C_t the equilibrium in period t can be found using the aggregates and $\mathcal{C}_t$.

Dynamic equilibrium conditions. Since preferences are separable over time, we can use two-stage budgeting to express the intertemporal household problem as

$$\begin{aligned}\max_{C_1, C_2} &U(C_1, C_2) \\ \hat{e}_1(p_{c1}, C_1) + \frac{1}{R^*} \hat{e}_2(p_{c2}, C_2) &= NFA + T,\end{aligned}$$

where $\hat{e}_t(p_c, C)$ denotes the static expenditure function of the household and the transfer $T = T_1 + \frac{1}{R^*} T_2$ is taken as given. This gives the first-order condition

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = R^* \frac{\hat{e}_{1C}(p_{c1}, C_1)}{\hat{e}_{2C}(p_{c2}, C_2)}.$$

To establish that this matches the first-order condition (3) in Section 2.4, it is sufficient to show that

$$e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t).$$

We already argued above that for $C = C_t$,

$$e_t(C_t) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C_t).$$

Now we want to show that for $C \neq C_t$,

$$e_t(C) \geq \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C). \quad (\text{B.1})$$

This then implies that $e_t(\cdot)$ is an upper envelope for $\hat{e}_t(p_{ct}, \cdot)$ and, in turn, that

$$e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t),$$

as desired.

To establish (B.1) we first rewrite

$$\begin{aligned} e_t(C) &= \min_{c_d, c_s, m, x} (1 + \tau) p_{mt}^* \cdot m - p_{xt}^* \cdot x \\ \text{s.t. } &:(c_s, m, x) \in Y_t, \\ &G_t(c_d) = C, \\ &c_d = c_s. \end{aligned}$$

Then for any p_{ct} , we note that

$$\begin{aligned} e_t(C) &\geq \min_{c_d, c_s, m, x} (1 + \tau) p_{mt}^* \cdot m - p_{xt}^* \cdot x + p_{ct} \cdot (c_d - c_s) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C) \\ \text{s.t. } &:(c_s, m, x) \in Y_t, \\ &G_t(c_d) = C, \end{aligned}$$

where the inequality derives from the fact that the right-hand side is a relaxed version of the original expenditure problem and the equality derives from the fact that c_d and c_s only appear in the household's and firm's problems, respectively. This completes the proof.

C Proof of Proposition 2

Let us first demonstrate that under the stated assumptions, the derivative of the expenditure function $e_{tC}(\tau, C_t)$ is independent of C_t . Start from the firm problem, as described in the proof of Proposition 1. Since the environment is stationary and trade is subject to

iceberg trade costs, we have

$$\begin{aligned}\tilde{e}_t(p_{ct}) &\equiv \min_{c,m,x,y} (1+\tau)p_m^* \cdot m - p_x^* \cdot x - p_{ct} \cdot c \\ \text{s.t. } &y \in Y, \\ &y_i = c_i + (1 + \delta_{i,x})x_i - \frac{m_i}{(1 + \delta_{i,m})},\end{aligned}$$

where p_{ct} denotes the equilibrium price vector for consumption in period t . Since the equilibrium has each good i either strictly imported ($m_{it} > 0$) or strictly exported ($x_{it} > 0$) subject to iceberg trade costs ($\delta_{i,m}, \delta_{i,x} \geq 0$), the first-order conditions of the firm problem require $p_{c1} = p_{c2} = p_c(\tau)$, with

$$p_{i,c}(\tau) = \begin{cases} (1+\tau)(1+\delta_{i,m})p_{i,m}^* & \text{if } m_{i1}, m_{i2} > 0, \\ p_{i,x}^*/(1+\delta_{i,x}) & \text{if } x_{i1}, x_{i2} > 0. \end{cases}$$

Next, consider the dual of the household problem, as also described in the proof of Proposition 1. In this stationary environment, it can be written as

$$\begin{aligned}\hat{e}_t(p_{ct}, C_t) &= \min_c p_c(\tau) \cdot c \\ \text{s.t. } &G_t(c) = C_t.\end{aligned}$$

Since G_t is homogeneous of degree one, we therefore have

$$\hat{e}_t(p_{ct}, C_t) = \beta(\tau)C_t.$$

In the proof of Proposition 1, we have already shown that $e_{tC}(\tau, C_t) = e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t)$. We therefore obtain, as desired,

$$e_{tC}(\tau, C_t) = \beta(\tau). \tag{C.1}$$

To complete the argument, note that we can rearrange the equilibrium conditions (3) and (4) using (C.1) as

$$\begin{aligned}MRS(C_1, C_2) &= R^*, \\ \mathcal{D}_1(\tau, C_1) + \frac{1}{R^*}\mathcal{D}_2(\tau, C_2) &= NFA.\end{aligned}$$

So the Euler condition is independent of τ . Around $\tau = 0$, we have

$$\mathcal{D}_{t\tau}(\tau, C_t) = -\tau M_{t\tau}(\tau, C_t) = 0.$$

So the trade balance condition is also independent of τ . By the Implicit Function Theorem, we therefore have the neutrality of (C_1, C_2) with respect to the tariff τ . Since $D_t(\tau) = \mathcal{D}_t(\tau, C_t(\tau))$, we get $D'_t(\tau) = 0$.

D Proof of Proposition 3

We first formally state our normality assumptions for $\mathcal{C}$ and U . We assume that M and $-X$ are normal goods.

Assumption (Normality of M and $-X$). *The utility function $\mathcal{C}(M, X)$ is concave in $(M, -X)$ and has decreasing interior Engel paths:*

$$-\frac{\mathcal{C}_M(M, X)}{\mathcal{C}_X(M, X)} = 1 + \tau$$

defines a downward sloping relation between M and X for $M, X > 0$.

We also introduce the required normality assumptions for aggregate consumption over time:

Assumption (Normality of C_1 and C_2). *The utility function U is concave and has increasing Engel paths, so that for any $R > 0$ the condition*

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = R$$

defines a strictly upward sloping relation between C_1 and C_2 for $C_1, C_2 > 0$.

Let $\tilde{C}(\tau, e)$ denote the static indirect utility function, i.e., the aggregate consumption level associated with expenditure e given a tariff τ ,

$$\tilde{C}(\tau, e) = \max_{M, X \geq 0} \mathcal{C}(M, X) \quad (1 + \tau)M - X = e,$$

and let $V(\tau, e_1, e_2)$ denote the intertemporal indirect utility function,

$$V(\tau, e_1, e_2) \equiv U(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2)).$$

The next lemma shows that normality for (C_1, C_2) in U implies normality for (e_1, e_2) in V .

Lemma. If C_1 and C_2 are normal and $\tilde{C}(\tau, e)$ is concave in e then the solution to

$$\begin{aligned} \max_{e_1, e_2} & V(\tau, e_1, e_2) \\ \text{s.t.} & e_1 + \frac{1}{R^*} e_2 = T \end{aligned}$$

has both e_1 and e_2 strictly increasing in T .

We are now ready to derive our autarky result.

Proof of autarky result ($NFA = 0$). Choose τ large enough so that

$$\frac{C_M(0, 0)}{1 + \tau} + C_X(0, 0) \leq 0. \quad (\text{D.1})$$

Since M and $-X$ are normal, the static optimum as a function of expenditure e ,

$$\tilde{C}(\tau, e) = \max_{M, X \geq 0} C(M, X) \quad (1 + \tau)M - X = e$$

is obtained at the corners: $X = 0$ for $e \geq 0$ or $M = 0$ for $e \leq 0$. Thus,

$$\tilde{C}(\tau, e) = \begin{cases} C(\frac{e}{1+\tau}, 0) & e \geq 0, \\ C(0, -e) & e \leq 0. \end{cases}$$

Furthermore, one can verify that $\tilde{C}(\tau, e)$ is a concave function of e . We denote the right derivative by $\tilde{C}_{e+}(\tau, e)$ and the left derivative by $\tilde{C}_{e-}(\tau, e)$. For $e \neq 0$ they coincide, but with $e = 0$ we have $\tilde{C}_{e+}(\tau, e) = \frac{1}{1+\tau} C_M(\frac{e}{1+\tau}, 0) > 0$ and $\tilde{C}_{e-}(\tau, e) = -C_X(0, -e) > 0$. Note that $\tilde{C}_{e-}(\tau, 0) = -C_X(0, 0) \geq \frac{1}{1+\tau} C_M(0, 0) = \tilde{C}_{e+}(\tau, 0)$ so there is a concave kink at $e = 0$.

Households solve the intertemporal problem

$$\begin{aligned} \max_{e_1, e_2} & U(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2)) \\ \text{s.t.} & e_1 + \frac{1}{R^*} e_2 = T \end{aligned}$$

taking T as given. In equilibrium

$$T = \left(M_1 + \frac{1}{R^*} M_2 \right) \tau.$$

For any value of T the problem is strictly convex so the optimum is unique.

The first order conditions are

$$U_1(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2))\tilde{C}_{e+}(\tau, e_1) - U_2(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2))R^*\tilde{C}_{e-}(\tau, e_2) \leq 0,$$

$$-U_1(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2))\tilde{C}_{e-}(\tau, e_1) + U_2(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2))R^*\tilde{C}_{e+}(\tau, e_2) \leq 0.$$

The first inequality ensures that it is not optimal to increase e_1 ; the second condition ensures it is not optimal to lower e_1 . Note that when $e_1 \neq 0$ we have $\tilde{C}_{e-}(\tau, e_1) = \tilde{C}_{e+}(\tau, e_1)$ so the two conditions are equivalent to

$$U_1(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2))\tilde{C}_e(\tau, e_1) - U_2(\tilde{C}(\tau, e_1), \tilde{C}(\tau, e_2))R^*\tilde{C}_e(\tau, e_2) = 0.$$

We first verify that $e_1 = e_2 = 0$ is an equilibrium with $T = 0$.

$$U_1(\tilde{C}(\tau, 0), \tilde{C}(\tau, 0))\tilde{C}_{e+}(\tau, 0) - U_2(\tilde{C}(\tau, 0), \tilde{C}(\tau, 0))R^*\tilde{C}_{e-}(\tau, 0) \leq 0,$$

$$-U_1(\tilde{C}(\tau, 0), \tilde{C}(\tau, 0))\tilde{C}_{e-}(\tau, 0) + U_2(\tilde{C}(\tau, 0), \tilde{C}(\tau, 0))R^*\tilde{C}_{e+}(\tau, 0) \leq 0.$$

Using that

$$\tilde{C}_{e+}(\tau, 0) = \frac{1}{1+\tau}\mathcal{C}_M(0, 0),$$

$$\tilde{C}_{e-}(\tau, 0) = -\mathcal{C}_X(0, 0),$$

this is equivalent to

$$\frac{1}{1+\tau}\mathcal{C}_M(0, 0) + \beta R^*\mathcal{C}_X(0, 0) \leq 0, \tag{D.2}$$

$$\beta R^*\frac{1}{1+\tau}\mathcal{C}_M(0, 0) + \mathcal{C}_X(0, 0) \leq 0, \tag{D.3}$$

where we have defined

$$\beta \equiv \frac{U_2(\tilde{C}(\tau, 0), \tilde{C}(\tau, 0))}{U_1(\tilde{C}(\tau, 0), \tilde{C}(\tau, 0))}.$$

There are two cases to consider. If $\beta R^* \leq 1$ let $\hat{\tau}$ denote the lowest value of τ for which condition (D.2) is satisfied. Then for any $\tau \geq \hat{\tau}$ conditions (D.1)-(D.3) are verified. If $\beta R^* > 1$ let $\hat{\tau}$ denote the lowest value of τ for which condition (D.3) is satisfied. Then for any $\tau \geq \hat{\tau}$ conditions (D.1)-(D.3) are verified. We conclude that $e_1 = e_2 = 0$ is a solution to the household's intertemporal problem for $T = 0$. It follows that $M_1 = X_1 = 0$ and $M_2 = X_2 = 0$ is an equilibrium.

To establish that this is the unique equilibrium, we need to rule out $T > 0$. We proceed by contradiction. Suppose $T > 0$. Since C_1 and C_2 are normal, the previous lemma implies

$e_1 \geq 0$ and $e_2 \geq 0$. But then

$$T = e_1 + \frac{1}{R^*}e_2 = (1 + \tau)M_1 + \frac{1}{R^*}(1 + \tau)M_2 > \tau M_1 + \frac{1}{R^*}\tau M_2 = T.$$

This concludes our proof for $NFA = 0$.

Proof of autarky result ($NFA \neq 0$). The argument is similar. The only difference is that for τ large the economy will not converge to $M_1 = X_1 = 0$ and $M_2 = X_2 = 0$, but instead to whatever level of aggregate imports or exports is consistent with the level of NFA .

Formally, if $NFA > 0$, define (M_1^a, M_2^a) that solves

$$\begin{aligned} \max_{M_1, M_2} & U(\mathcal{C}(M_1, 0), \mathcal{C}(M_2, 0)) \\ \text{s.t.} & : M_1 + \frac{1}{R^*}M_2 = NFA. \end{aligned}$$

One can then check the first-order conditions (D.1)-(D.3) around $M_1 = M_1^a$, $X_1 = 0$, $M_2 = M_2^a$, and $X_2 = 0$ to establish that for τ large enough this is the unique equilibrium.

Likewise, if $NFA < 0$, define (X_1^a, X_2^a) that solves

$$\begin{aligned} \max_{X_1, X_2} & U(\mathcal{C}(0, X_1), \mathcal{C}(0, X_2)) \\ \text{s.t.} & : -X_1 - \frac{1}{R^*}X_2 = NFA. \end{aligned}$$

The same approach as before shows that for τ large enough, the unique equilibrium is $M_1 = 0$, $X_1 = X_1^a$, $M_2 = 0$, and $X_2 = X_2^a$.

E Extension to Heterogeneous Tariffs

In this appendix, we extend Proposition 4 to the case of heterogeneous tariffs.

Environment with Heterogeneous Tariffs. Consider the same general environment as in Section 2.1, except that tariffs are potentially heterogeneous across groups. Formally, there is a partition of imported goods into groups $g = 1, \dots, G$, such that all goods i in group g are subject to the same ad-valorem tariff τ_g . Each period, tariff revenues are rebated to the representative household via a lump-sum transfer

$$T_t = \sum_g \tau_g M_{gt},$$

where $M_{gt} = \sum_{i \in g} p_{it}^* m_{it}$ denotes the value of group g 's imports. Our baseline analysis corresponds to the case of a single group. We are interested in the effects of an arbitrary increase in tariffs $d \ln(1 + \tau_g) \geq 0$, with strict inequality for some g . Without loss of generality, we order groups so that $0 \leq d \ln(1 + \tau_1) \leq \dots \leq d \ln(1 + \tau_G)$.

Preferences Over Import Groups and Aggregate Exports. To extend Proposition 4, one can proceed as we did in Section 2.3. First, define the induced utility function,

$$\mathcal{C}_t(\{M_{gt}\}, X_t) \equiv \max_{(c_t, m_t, x_t) \in Y_t} G_t(c_t),$$

subject to

$$\begin{aligned} \sum_{i \in g} p_{it}^* m_{it} &= M_{gt}, \\ p_{xt}^* \cdot x_t &= X_t. \end{aligned}$$

The associated expenditure function is

$$\begin{aligned} e_t(\boldsymbol{\tau}, C) &\equiv \min_{\{M_g\}, X \geq 0} \left\{ \sum_g (1 + \tau_g) M_g - X \right\} \\ \text{s.t. } &\mathcal{C}_t(\{M_g\}, X) \geq C, \end{aligned}$$

with $\boldsymbol{\tau} \equiv \{\tau_g\}$ the vector of tariffs, and the associated deficit function is

$$\mathcal{D}_t(\boldsymbol{\tau}, C) \equiv \sum_g M_{gt}(\boldsymbol{\tau}, C) - X_t(\boldsymbol{\tau}, C),$$

with $\{M_{gt}(\boldsymbol{\tau}, C)\}$ and $X_t(\boldsymbol{\tau}, C)$ the Hicksian demand functions that solve the expenditure minimization problem. In line with our baseline analysis, we assume that all import groups are normal goods: $M_{gtC} > 0$ for all g . The equilibrium deficit in period t is $D_t(\boldsymbol{\tau}) = \mathcal{D}_t(\boldsymbol{\tau}, C_t(\boldsymbol{\tau}))$.

The Interest Rate Channel. Next consider the domestic real interest rate,

$$R(\boldsymbol{\tau}, C_1, C_2) = R^* \frac{e_{1C}(\boldsymbol{\tau}, C_1)}{e_{2C}(\boldsymbol{\tau}, C_2)}.$$

For an arbitrary change in the vector of tariffs $\{d \ln(1 + \tau_g)\}$, the same argument as in Section 4.1 implies

$$d \ln R(\boldsymbol{\tau}, C_1, C_2) = \sum_g \frac{\partial \ln e_{1C}(\boldsymbol{\tau}, C_1)}{\partial \ln(1 + \tau_g)} d \ln(1 + \tau_g) - \sum_g \frac{\partial \ln e_{2C}(\boldsymbol{\tau}, C_2)}{\partial \ln(1 + \tau_g)} d \ln(1 + \tau_g)$$

as well as

$$\frac{\partial \ln e_C(\boldsymbol{\tau}, C)}{\partial \ln(1 + \tau_g)} = \frac{(1 + \tau_g)M_{gC}(\boldsymbol{\tau}, C)}{M_C(\boldsymbol{\tau}, C) - X_C(\boldsymbol{\tau}, C)}, \quad (\text{E.1})$$

where we have dropped the subscript t for notational convenience and where $M_C(\boldsymbol{\tau}, C) \equiv \sum_g (1 + \tau_g)M_{gC}(\boldsymbol{\tau}, C)$.

Slopes of the Engel Path as Sufficient Statistics. As in our baseline analysis with uniform tariff changes, we refer to

$$\frac{dM}{dX} = \frac{M_C(\boldsymbol{\tau}, C)}{X_C(\boldsymbol{\tau}, C)} \leq 0$$

as the slope of the Engel path for aggregate imports and exports. In addition, we refer to

$$\frac{dM_g}{dM_k} = \frac{M_{gC}(\boldsymbol{\tau}, C)}{M_{kC}(\boldsymbol{\tau}, C)} \geq 0$$

as the slope of the Engel path for imports of groups g and k .

Using the previous notation, Proposition 4 extends as follows.

Proposition 4-H (Sufficient statistics with heterogeneous tariffs). *Starting from free trade ($\boldsymbol{\tau} = 0$), an increase in tariffs $0 \leq d \ln(1 + \tau_1) \leq \dots \leq d \ln(1 + \tau_G)$, with at least one strict inequality, reduces the deficit in period 1,*

$$dD_1 = \sum_g \frac{\partial D_1(\boldsymbol{\tau})}{\partial \ln(1 + \tau_g)} d \ln(1 + \tau_g) < 0,$$

if the Engel paths satisfy

$$\left| \frac{dM_1}{dX_1} \right| > \left| \frac{dM_2}{dX_2} \right| \text{ and } \frac{dM_{g1}}{dM_{k1}} \geq \frac{dM_{g2}}{dM_{k2}} \text{ for all } g > k. \quad (\text{E.2})$$

Conversely, the same increase in tariffs raises the deficit in period 1,

$$dD_1 = \sum_g \frac{\partial D_1(\boldsymbol{\tau})}{\partial \ln(1 + \tau_g)} d \ln(1 + \tau_g) > 0,$$

if the Engel paths satisfy

$$\left| \frac{dM_1}{dX_1} \right| < \left| \frac{dM_2}{dX_2} \right| \text{ and } \frac{dM_{g1}}{dM_{k1}} \leq \frac{dM_{g2}}{dM_{k2}} \text{ for all } g > k. \quad (\text{E.3})$$

Proof. Consider the case $dD_1 < 0$. By the same logic as in Section 4.1, in order to establish

that $dD_1 < 0$, it is sufficient to show that

$$\begin{aligned} d \ln e_{1C} &= \sum_g \frac{\partial \ln e_{1C}(\boldsymbol{\tau}, C_1)}{\partial \ln(1 + \tau_g)} d \ln(1 + \tau_g) \\ &> \sum_g \frac{\partial \ln e_{2C}(\boldsymbol{\tau}, C_2)}{\partial \ln(1 + \tau_g)} d \ln(1 + \tau_g) = d \ln e_{2C}. \end{aligned} \quad (\text{E.4})$$

In each period t , (E.1) implies

$$d \ln e_{tC} = \frac{M_{tC}(\boldsymbol{\tau}, C_t)}{M_{tC}(\boldsymbol{\tau}, C_t) - X_{tC}(\boldsymbol{\tau}, C_t)} \left[\sum_g \frac{(1 + \tau_g) M_{gtC}(\boldsymbol{\tau}, C_t)}{M_{tC}(\boldsymbol{\tau}, C_t)} d \ln(1 + \tau_g) \right]. \quad (\text{E.5})$$

By (E.2), we know that

$$\frac{M_{1C}(\boldsymbol{\tau}, C_1)}{M_{1C}(\boldsymbol{\tau}, C_1) - X_{1C}(\boldsymbol{\tau}, C_1)} > \frac{M_{2C}(\boldsymbol{\tau}, C_2)}{M_{2C}(\boldsymbol{\tau}, C_2) - X_{2C}(\boldsymbol{\tau}, C_2)}. \quad (\text{E.6})$$

By definition of $M_{1C}(\boldsymbol{\tau}, C_1)$, we also know that

$$\begin{aligned} \frac{(1 + \tau_g) M_{g1C}(\boldsymbol{\tau}, C_1)}{M_{1C}(\boldsymbol{\tau}, C_1)} &\in [0, 1], \\ \sum_g \frac{(1 + \tau_g) M_{g1C}(\boldsymbol{\tau}, C_1)}{M_{1C}(\boldsymbol{\tau}, C_1)} &= 1. \end{aligned}$$

Thus (E.2) further implies

$$\left\{ \frac{(1 + \tau_g) M_{g1C}(\boldsymbol{\tau}, C_1)}{M_{1C}(\boldsymbol{\tau}, C_1)} \right\} \succeq_{\text{MLRP}} \left\{ \frac{(1 + \tau_g) M_{g2C}(\boldsymbol{\tau}, C_2)}{M_{2C}(\boldsymbol{\tau}, C_2)} \right\}.$$

Since stochastic dominance in terms of MLRP implies first-order stochastic dominance and $d \ln(1 + \tau_g)$ is nondecreasing in g , we therefore have

$$\sum_g \frac{(1 + \tau_g) M_{g1C}(\boldsymbol{\tau}, C_1)}{M_{1C}(\boldsymbol{\tau}, C_1)} d \ln(1 + \tau_g) \geq \sum_g \frac{(1 + \tau_g) M_{g2C}(\boldsymbol{\tau}, C_2)}{M_{2C}(\boldsymbol{\tau}, C_2)} d \ln(1 + \tau_g). \quad (\text{E.7})$$

Combining (E.5), (E.6), and (E.7) and using the fact that $M_{g2C} > 0$ for all g , we obtain (E.4), as desired. The case $dD_1 > 0$ can be dealt with in a similar manner. $\square$

F Extension to Infinite Horizon

In this appendix, we extend Corollary 1 to an infinite-horizon economy.

Infinite-horizon Economy. Consider the same general environment as in Section 2, except that there is an infinite number of periods indexed by t and that intertemporal preferences are CES,

$$U(\{C_t\}) = \sum_{t=1}^{\infty} \beta^{t-1} \frac{(C_t)^{1-1/\mu}}{1-1/\mu}.$$

We can then express the household problem as

$$\begin{aligned} \max_{\{C_t\}} \sum_{t=1}^{\infty} \beta^{t-1} \frac{(C_t)^{1-1/\mu}}{1-1/\mu} \\ \sum_{t=1}^{\infty} \frac{1}{(R^*)^{t-1}} e_t(\tau, C_t) = NFA + T, \end{aligned}$$

where the lump-sum transfer $T = \sum_{t=1}^{\infty} \frac{1}{(R^*)^{t-1}} \tau M_t(\tau, C_t)$ is taken as given.

Dynamic Equilibrium Conditions. Suppose, in line with the assumptions in Corollary 1, that the economy is stationary. The dynamic equilibrium conditions are then

$$\begin{aligned} \left(\frac{C_t}{C_s} \right)^{-1/\mu} &= \beta^{s-t} R^* \frac{e_C(\tau, C_t)}{e_C(\tau, C_s)} \equiv \beta^{s-t} R(\tau, C_t, C_s), \\ \sum_{t=1}^{\infty} \frac{1}{(R^*)^{t-1}} \mathcal{D}(\tau, C_t) &= NFA. \end{aligned} \tag{F.1}$$

Corollary 1 Revisited. Suppose t_0 is the period with the largest deficit: $D_{t_0}(\tau) > D_t(\tau)$ for all $t \neq t_0$, which implies $C_{t_0}(\tau) > C_t(\tau)$ for all $t \neq t_0$.

Let $\hat{C}_t(\tau, C_{t_0})$ denote the solution C to

$$\left(\frac{C_{t_0}}{C} \right)^{-1/\mu} = \beta^{t-t_0} R(\tau, C_{t_0}, C).$$

By the same argument as in Section 4.1, if the Engel path for $\mathcal{C}$ is strictly convex, $C_{t_0}(\tau) > C_t(\tau)$ implies

$$\frac{\partial \ln R(\tau, C_{t_0}, C_t)}{\partial \tau} > 0.$$

From the second-order condition of the household problem, we must therefore have

$$\frac{\partial \ln \hat{C}_t(\tau, C_{t_0})}{\partial \tau} > 0 \text{ and } \frac{\partial \hat{C}_t(\tau, C_{t_0})}{\partial C_{t_0}} > 0.$$

Now substituting $\hat{C}_t(\tau, C_{t_0})$ in the intertemporal budget constraint and differentiating

with respect to τ , we get

$$\sum_{t=1}^{\infty} \frac{1}{(R^*)^{t-1}} \mathcal{D}_{C_t} \left[\frac{\partial \hat{C}_t(\tau, C_{t_0})}{\partial \tau} + \frac{\partial \hat{C}_t(\tau, C_{t_0})}{\partial C_{t_0}} \frac{dC_{t_0}}{d\tau} \right] = 0.$$

It follows that $dC_{t_0}/d\tau < 0$ and, in turn, that $D'_{t_0}(\tau) = \mathcal{D}_{C_{t_0}} \times (dC_{t_0}/d\tau) < 0$.

Given the two previous observations, the following extension of Corollary 1 holds in an infinite-horizon economy.

Corollary 1-I (Infinite horizon). *Consider a stationary economy with CES intertemporal preferences and invariant Meade preferences, $C_t = C$ for all t , and a strictly convex Engel path for C . Suppose the economy runs its largest trade deficit in period t_0 : $D_{t_0}(\tau) > D_t(\tau)$ for all $t \neq t_0$. Then starting from free trade ($\tau = 0$), an increase in tariffs reduces the deficit in period t_0 : $D'_{t_0}(\tau) < 0$.*

A symmetric argument applied to the period t_1 with smallest deficit, i.e., $D_{t_1}(\tau) < D_t(\tau)$ for all $t \neq t_1$, implies $D'_{t_1}(\tau) > 0$.

G Proof of Proposition 5

Consider an endowment economy. For notational convenience, we drop the subscript t . The representative household has CES preferences,

$$G(c) = u^{-1} \left(\int \theta_i u(c_i) di \right),$$

with $u(c) = c^{1-\sigma}/(1-\sigma)$ and $\theta_i > 0$. The domestic output of each good i is fixed and equal to $\omega_i \geq 0$. Imports and exports of good i are subject to iceberg trade costs $\delta_{i,m}$ and $\delta_{i,x}$. Define the “domestic dock” prices,

$$\begin{aligned} p_{i,m} &= p_i^* (1 + \delta_{i,m}), \\ p_{i,x} &= p_i^* / (1 + \delta_{i,x}), \end{aligned}$$

with p_i^* the exogenous price of good i at the “foreign dock”. We let F denote the cdf taken over the exogenous characteristics $(\omega, \theta, p_m, p_x)$ of individual goods.

In an endowment economy with CES preferences, the induced utility function over aggregate imports and exports is given by

$$\mathcal{C}(M, X) = u^{-1} \left(\max_{m,x} \int \theta u \left(\omega + \frac{m(\omega, \theta, p_m, p_x)}{p_m} - \frac{x(\omega, \theta, p_m, p_x)}{p_x} \right) dF \right) \quad (\text{G.1})$$

subject to

$$\begin{aligned}\int x(\omega, \theta, p_m, p_x) dF &= X, \\ \int m(\omega, \theta, p_m, p_x) dF &= M.\end{aligned}$$

To establish the convexity of the Engel paths of $\mathcal{C}(M, X)$, we first need to characterize the solution to the previous maximization problem.

Since $p_x \leq p_m$ we can ignore the possibility that both $m(\omega, \theta, p_m, p_x) > 0$ and $x(\omega, \theta, p_m, p_x) > 0$. The associated first-order conditions are then

$$\begin{aligned}\frac{\theta}{p_m} u' \left(\omega + \frac{m(\omega, \theta, p_m, p_x)}{p_m} \right) &\leq \mu_m, \\ \frac{\theta}{p_x} u' \left(\omega - \frac{x(\omega, \theta, p_m, p_x)}{p_x} \right) &\geq \mu_x,\end{aligned}$$

with the usual complementary slackness, where μ_m and μ_x denote the Lagrange multipliers associated with the aggregate import and export constraints, respectively.

Together with $u(c) = c^{1-\sigma}/(1-\sigma)$, the two previous conditions imply

$$\begin{aligned}m(\omega, \theta, p_m, p_x) &= \max \left\{ 0, \mu_m^{-1/\sigma} \left(\frac{p_m}{\theta} \right)^{-1/\sigma} - \omega \right\}, \\ x(\omega, \theta, p_m, p_x) &= \max \left\{ 0, \omega - \mu_x^{-1/\sigma} \left(\frac{p_x}{\theta} \right)^{-1/\sigma} \right\}.\end{aligned}$$

For $k = m, x$, define

$$\begin{aligned}\tilde{\mu}_k &\equiv \mu_k^{-1/\sigma}, \\ \tilde{z}_k &\equiv (p_k)^{1-1/\sigma} \theta^{1/\sigma}, \\ \tilde{\omega}_k &\equiv p_k \omega.\end{aligned}$$

Using the previous notation, and summing across all goods, we can then express aggregate imports and exports as a function of the (transformed) Lagrange multipliers,

$$\hat{M}(\tilde{\mu}_m) = \int_{\tilde{\omega}_m / \tilde{z}_m \leq \tilde{\mu}_m} (\tilde{z}_m \tilde{\mu}_m - \tilde{\omega}_m) dF, \quad (\text{G.2})$$

$$\hat{X}(\tilde{\mu}_x) = \int_{\tilde{\omega}_x / \tilde{z}_x \geq \tilde{\mu}_x} (\tilde{\omega}_x - \tilde{z}_x \tilde{\mu}_x) dF. \quad (\text{G.3})$$

Note that

$$\hat{M}'(\tilde{\mu}_m) = \int_{\tilde{\omega}_m / \tilde{z}_m \leq \tilde{\mu}_m} \tilde{z}_m dF \equiv \bar{z}_m(\tilde{\mu}_m) \geq 0, \quad (\text{G.4})$$

$$\hat{X}'(\tilde{\mu}_x) = - \int_{\tilde{\omega}_x / \tilde{z}_x \geq \tilde{\mu}_x} \tilde{z}_x dF \equiv \bar{z}_x(\tilde{\mu}_x) \leq 0, \quad (\text{G.5})$$

with strict inequalities if $\hat{M}(\tilde{\mu}_m) > 0$ and $\hat{X}(\tilde{\mu}_x) > 0$, since this requires a positive mass of goods with $\tilde{z}_m \tilde{\mu}_m - \tilde{\omega}_m > 0$ and $\tilde{\omega}_x - \tilde{z}_x \tilde{\mu}_x > 0$, respectively. Equations (G.4) and (G.5) further imply that $\hat{M}'$ and $\hat{X}'$ are increasing and strictly so if there is an active extensive margin.

To show that the Engel path of $\mathcal{C}(M, X)$ is convex for all $(M, X) > 0$, start from

$$M(\tau, C) = \hat{M}(\tilde{\mu}_m(\tau, C)),$$

$$X(\tau, C) = \hat{X}(\tilde{\mu}_x(\tau, C)),$$

with the (transformed) Lagrange multipliers $\tilde{\mu}_m(\tau, C)$ and $\tilde{\mu}_x(\tau, C)$ such that $M = \hat{M}(\tilde{\mu}_m(\tau, C))$ and $X = \hat{X}(\tilde{\mu}_x(\tau, C))$ solve

$$-\frac{\mathcal{C}_M(M, X)}{\mathcal{C}_X(M, X)} = 1 + \tau, \quad (\text{G.6})$$

$$\mathcal{C}(M, X) = C. \quad (\text{G.7})$$

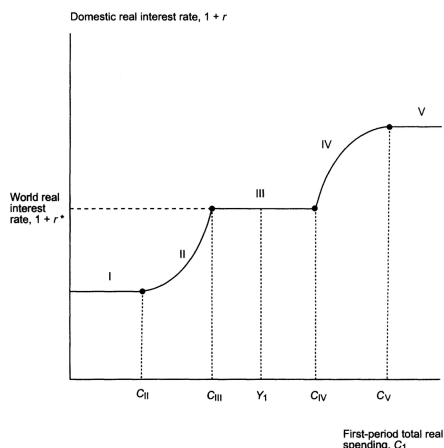
By the Envelope Theorem, applied to (G.1), the first-order condition (G.6) requires $\mu_m(\tau, C) = \mu_x(\tau, C)(1 + \tau)$ so that $\tilde{\mu}_m(\tau, C) = \tilde{\mu}_x(\tau, C)(1 + \tau)^{-1/\sigma}$. The slope of the Engel path is therefore equal to

$$\frac{M_C(\tau, C)}{X_C(\tau, C)} = \frac{(1 + \tau)^{-1/\sigma} \hat{M}'(\tilde{\mu}_x(\tau, C)(1 + \tau)^{-1/\sigma})}{\hat{X}'(\tilde{\mu}_x(\tau, C))}, \quad (\text{G.8})$$

with $\tilde{\mu}_x(\tau, C)$ an increasing function of C , since differentiating (G.7) implies

$$\tilde{\mu}_{x,C}(\tau, C) = \frac{1}{\mathcal{C}_M(1 + \tau)^{-1/\sigma} \hat{M}'(\tilde{\mu}_x(\tau, C)(1 + \tau)^{-1/\sigma}) + \mathcal{C}_X \hat{X}'(\tilde{\mu}_x(\tau, C))} > 0, \quad (\text{G.9})$$

where the inequality follows from $\mathcal{C}_M > 0$, $\mathcal{C}_X < 0$, $\hat{M}' > 0$, and $\hat{X}' < 0$. Combining (G.8) and (G.9) with the monotonicity of $\hat{M}'$ and $\hat{X}'$, we get that $|\frac{M_C(\tau, C)}{X_C(\tau, C)}|$ is increasing in C and strictly so if there is an active extensive margin. It follows that $\frac{dM}{dX}$ is increasing in X , since $X_C = \hat{X}' \tilde{\mu}_{x,C} < 0$, i.e., the Engel path is convex for all $(M, X) > 0$, strictly so with an active extensive margin.



(a) Domestic Real Interest Rate

	H t=1	F t=1	H t=2	F t=2
I	X	M	M	M
II	X	M	NT	M
III	X	M	X	M
IV	NT	M	X	M
V	M	M	X	M

(b) Pattern of Exports and Imports

Figure H.1: Original Analysis from **Obstfeld and Rogoff (2000)**

Notes: Figure H.1a is the original Figure 1 from OR, which describes the domestic real interest rate ($1+r$ in their notation, $R(\tau, C_1, C_2)$ in ours) as a function of C_1 . Figure H.1b describes whether each good (H or F) is exported (X), imported (M) or non-traded (NT) in a given period t .

H Relationship to **Obstfeld and Rogoff (2000)**

Obstfeld and Rogoff (2000) (OR) consider an Armington model with two goods, H and F , and trade costs. They analyze the relationship between the domestic real interest rate and aggregate consumption in period 1. Figure 1 of their original paper offers a graphical representation. We reproduce it here in Figure H.1a. OR emphasize five different regions, depending on whether the goods are exported (X), imported (M), or non-traded (NT). Figure H.1b describes the pattern of trade associated with each region at $t = 1$ and 2.

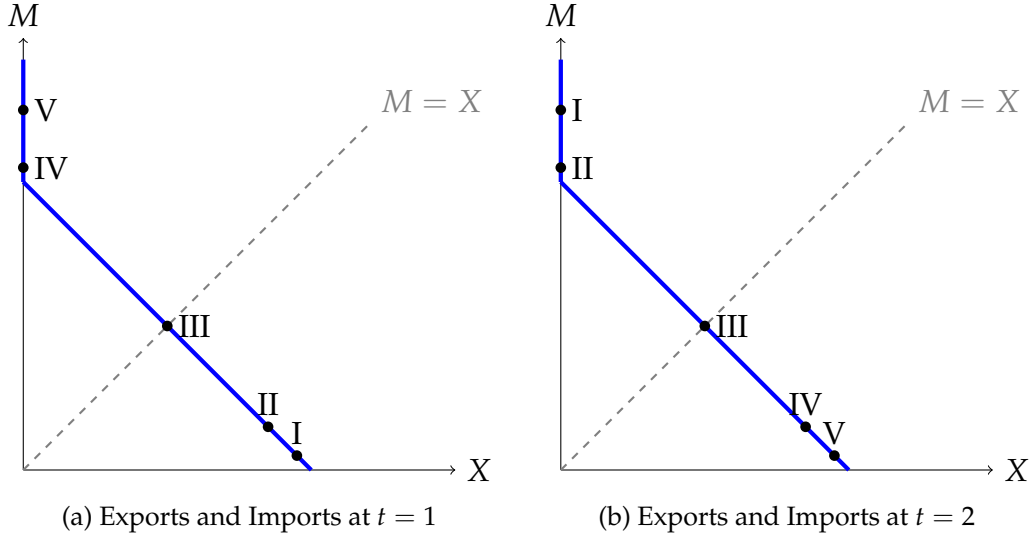


Figure H.2: Engel Path in **Obstfeld and Rogoff (2000)**

Notes: Each panel plots the Engel path in the Armington model considered by OR. In Figure H.2a, the five dots correspond to the pattern of exports and imports at $t = 1$ in the five regions of OR's original analysis. In Figure H.2b, the five dots correspond to the same five regions at $t = 2$.

In order to relate OR's original analysis to ours, Figure H.2 plots the Engel path for aggregate imports and exports in their Armington model. It is linear, as in Figure 4a. For each period $t = 1$ or 2 , we report the pattern of aggregate imports and exports that correspond to each of the 5 regions in OR's analysis. Figure H.2 illustrates the distinction between our sufficient statistic, the slope of the Engel path, and the domestic real interest rate emphasized in OR. From Figure H.1a, we see that the interest rate varies between regions I and II as well as between regions IV and V. This reflects variation in the price of non-tradables across these regions. From Figure H.2a, however, we see that the slope of the Engel path is the same in regions I and II and the same in regions IV and V. This slope, rather than the price of non-tradables, is what is relevant for the comparative static exercise that we are interested in, and that OR abstract from, namely the impact of changes in tariffs on the trade deficits.

I Proof of Proposition 6

We first derive equation (10). We then present the assumptions imposed in the small economy limit of a Ricardian economy à la **Eaton and Kortum (2002)**. Finally, we use equation (10) and the previous assumptions to establish Proposition 6.

Derivation of Equation (10). We drop t for notational convenience. Define

$$\mathcal{C}(M, X; w) \equiv \max_{(c, m, x) \in Y} G(c),$$

subject to

$$\begin{aligned} p_m^* \cdot m &= M, \\ w(\bar{p}_x^* \cdot x) &= X. \end{aligned}$$

The associated expenditure function is

$$\begin{aligned} e(\tau, C; w) &\equiv \min_{M, X \geq 0} \{(1 + \tau)M - X\} \\ \text{s.t. : } &\mathcal{C}(M, X; w) \geq C. \end{aligned}$$

We let $M(\tau, C; w)$ and $X(\tau, C; w)$ denote the solution.

Note that $\mathcal{C}(M, X; w) = \mathcal{C}(M, X/w; 1)$. Thus we can change variable, $\tilde{X} = X/w$, to rearrange the expenditure function as

$$\begin{aligned} e(\tau, C; w) &= \min_{M, \tilde{X} \geq 0} \{(1 + \tau)M - w\tilde{X}\} \\ \text{s.t. : } &\mathcal{C}(M, \tilde{X}; 1) \geq C. \end{aligned}$$

Its derivative and cross-derivative then satisfy

$$\begin{aligned} e_w(\tau, C; w) &= -\tilde{X}(\tau, C; w) = -X(\tau, C; w)/w \leq 0, \\ e_{Cw}(\tau, C; w) &= -\tilde{X}_C(\tau, C; w) = -X_C(\tau, C; w)/w \leq 0. \end{aligned}$$

This leads to

$$\frac{\partial \ln e_C}{\partial \ln w} = \frac{w e_{Cw}(\tau, C; w)}{e_C(\tau, C; w)} = -\frac{X_C(\tau, C; w)}{(1 + \tau)M_C(\tau, C; w) - X_C(\tau, C; w)}.$$

From equation (6), we know that

$$\frac{\partial \ln e_C}{\partial \ln(1 + \tau)} = \frac{(1 + \tau)M_C(\tau, C; w)}{(1 + \tau)M_C(\tau, C; w) - X_C(\tau, C; w)}.$$

Combining the two previous expressions and using the definition of ϵ_w , we finally get

$$\begin{aligned} \frac{d \ln e_C}{d \ln(1 + \tau)} &= \frac{\partial \ln e_C}{\partial \ln(1 + \tau)} + \frac{\partial \ln e_C}{\partial \ln w} \frac{\partial \ln w}{\partial \ln(1 + \tau)} \\ &= \frac{(1 + \tau)M_C(\tau, C; w)}{(1 + \tau)M_C(\tau, C; w) - X_C(\tau, C; w)} - \frac{\epsilon_w X_C(\tau, C; w)}{(1 + \tau)M_C(\tau, C; w) - X_C(\tau, C; w)}, \end{aligned}$$

which can be rearranged into (10).

Assumptions for a Small Ricardian Economy à la Eaton and Kortum (2002). There is a fixed endowment of labor L_t . Preferences are CES and the distribution of labor productivity is Fréchet, with shape parameter $\varepsilon > 0$. Under these assumptions, aggregate imports and exports satisfy

$$(1 + \tau)M_t = \frac{[(1 + \delta_t)(1 + \tau)]^{-\varepsilon}}{(w_t)^{-\varepsilon} + [(1 + \delta_t)(1 + \tau)]^{-\varepsilon}} \times P_t(\tau, w_t)C_t, \quad (\text{I.1})$$

$$X_t = w_t L_t - \frac{(w_t)^{-\varepsilon}}{(w_t)^{-\varepsilon} + [(1 + \delta_t)(1 + \tau)]^{-\varepsilon}} \times P_t(\tau, w_t)C_t, \quad (\text{I.2})$$

with the price index such that

$$P_t(\tau, w_t) \equiv [(w_t)^{-\varepsilon} + [(1 + \delta_t)(1 + \tau)]^{-\varepsilon}]^{-\frac{1}{\varepsilon}}.$$

Each period, the domestic wage w_t equalizes the global demand for domestic labor with its supply. In the limit when the rest of the world is infinitely large, but its share of expenditure on domestic goods is infinitely small, foreign spending on domestic labor is $[(1 + \delta_t)w_t]^{-\varepsilon}$, up to normalization of wages in the rest of the world. Thus this labor market clearing condition takes the form,

$$\frac{(w_t)^{-\varepsilon} \times P_t(\tau, w_t)C_t}{(w_t)^{-\varepsilon} + [(1 + \delta_t)(1 + \tau)]^{-\varepsilon}} + [(1 + \delta_t)w_t]^{-\varepsilon} = w_t L_t. \quad (\text{I.3})$$

The equilibrium wage function $w_t(\tau, C_t)$ is the solution to (I.3).

Proof of Proposition 6. If the environment is stationary, the expenditure function is the same in the two periods: $e_t(\tau, C; w) = e(\tau, C; w)$. So is the equilibrium wage function: $w_t(\tau, C) = w(\tau, C)$. Accordingly, we can write the domestic real interest rate

$$R(\tau, C_1, C_2) = R^* \frac{e_C(\tau, C_1; w(\tau, C_1))}{e_C(\tau, C_2; w(\tau, C_2))}.$$

It follows that

$$\frac{d \ln R}{d \ln(1 + \tau)} > 0 \text{ if and only if } \frac{d \ln e_C(\tau, C_1; w(\tau, C_1))}{d \ln(1 + \tau)} > \frac{d \ln e_C(\tau, C_2; w(\tau, C_2))}{d \ln(1 + \tau)}.$$

To establish Proposition 6, it is therefore sufficient to show that $d \ln e_C(\tau, C; w(\tau, C)) / d \ln(1 + \tau)$ is strictly increasing in C . We do so in three steps.

First, we establish that $\epsilon_w(\tau, C) \equiv \partial \ln w(\tau, C) / \partial \ln(1 + \tau) \in (0, 1)$ and is strictly in-

creasing in C . Second, we show that $X_C(\tau, C; w(\tau, C))/M_C(\tau, C; w(\tau, C))$ is strictly increasing in C as well. Third, we combine these two results with equation (10) to establish the monotonicity of $d \ln e_C(\tau, C; w(\tau, C))/d \ln(1 + \tau)$.

Step 1: $\partial \ln w(\tau, C)/\partial \ln(1 + \tau) \in (0, 1)$ *and is strictly increasing in C .* Define

$$\mathcal{L}(w) \equiv \left[\frac{w}{[(w)^{-\varepsilon} + (1 + \delta)^{-\varepsilon}]^{-\frac{1}{\varepsilon}}} \right]^{-(1+\varepsilon)},$$

$$\mathcal{L}^*(w) \equiv (1 + \delta)^{-\varepsilon} w^{-(1+\varepsilon)}.$$

Next, differentiate (I.3) with respect to τ . This implies

$$\frac{\partial \ln w(\tau, C)}{\partial \ln(1 + \tau)} = \frac{\alpha(\tau, C) \epsilon_L(w(\tau, C)/(1 + \tau))}{\alpha(\tau, C) \epsilon_L(w(\tau, C)/(1 + \tau)) + (1 - \alpha(\tau, C)) \epsilon_{L^*}(w(\tau, C))}, \quad (\text{I.4})$$

with

$$\alpha(\tau, C) \equiv \frac{\mathcal{L}(w(\tau, C)/(1 + \tau))C}{\mathcal{L}(w(\tau, C)/(1 + \tau))C + \mathcal{L}^*(w(\tau, C))},$$

$$\epsilon_L(w) \equiv -\frac{d \ln \mathcal{L}(w)}{d \ln w},$$

$$\epsilon_{L^*}(w) \equiv -\frac{d \ln \mathcal{L}^*(w)}{d \ln w}.$$

Note that

$$\epsilon_L(w) = (1 + \varepsilon) \times \left[1 - \frac{w^{-\varepsilon}}{w^{-\varepsilon} + (1 + \delta)^{-\varepsilon}} \right] > 0, \quad (\text{I.5})$$

$$\epsilon_{L^*}(w) = 1 + \varepsilon > \epsilon_L(w). \quad (\text{I.6})$$

From (I.4)-(I.6), it follows that $\partial \ln w(\tau, C)/\partial \ln(1 + \tau) \in (0, 1)$.

Let us now show that $\partial \ln w(\tau, C)/\partial \ln(1 + \tau)$ is strictly increasing in C . Define

$$\omega(\alpha, \epsilon_L, \epsilon_{L^*}) \equiv \frac{\alpha \epsilon_L}{\alpha \epsilon_L + (1 - \alpha) \epsilon_{L^*}}.$$

It satisfies $\omega_\alpha > 0$, $\omega_{\epsilon_L^*} < 0$, and $\omega_{\epsilon_L} > 0$. Second, note that

$$\begin{aligned}\frac{d\alpha}{dC} &= \frac{\mathcal{L}\mathcal{L}^*}{(\mathcal{L}C + \mathcal{L}^*)^2} + \alpha(1 - \alpha)(\epsilon_{L^*} - \epsilon_L)(w_C/w) > 0, \\ \frac{d\epsilon_L}{dC} &= \frac{\epsilon(1 + \epsilon)}{(1 + \tau)} \left[\frac{(w/(1 + \tau))^{-\epsilon-1}(1 + \delta)^{-\epsilon}}{((w/(1 + \tau))^{-\epsilon} + (1 + \delta)^{-\epsilon})^2} \right] w_C > 0, \\ \frac{d\epsilon_{L^*}}{dC} &= 0,\end{aligned}$$

where we have used the fact that $\epsilon_{L^*} > \epsilon_L$, $\epsilon > 0$, and $w_C > 0$, since $w(\tau, C)$ is the solution to (I.3). Totally differentiating (I.4) with respect to C and using the previous observations, we get $\partial \ln w(\tau, C) / \partial \ln(1 + \tau)$ strictly increasing in C .

Step 2: $X_C(\tau, C; w(\tau, C)) / M_C(\tau, C; w(\tau, C)) < 0$ is strictly increasing in C . From (I.1) and (I.2), we know that

$$\begin{aligned}(1 + \tau)M_C(\tau, C; w) &= \frac{[(1 + \delta)(1 + \tau)]^{-\epsilon}}{(w)^{-\epsilon} + [(1 + \delta)(1 + \tau)]^{-\epsilon}} \times P(\tau, w), \\ X_C(\tau, C; w) &= -\frac{(w)^{-\epsilon}}{(w)^{-\epsilon} + [(1 + \delta)(1 + \tau)]^{-\epsilon}} \times P(\tau, w).\end{aligned}$$

This implies

$$\frac{X_C(\tau, C; w(\tau, C))}{(1 + \tau)M_C(\tau, C; w(\tau, C))} = -\frac{(w(\tau, C))^{-\epsilon}}{[(1 + \delta)(1 + \tau)]^{-\epsilon}} < 0.$$

Since $w_C > 0$ and $\epsilon > 0$, we get that $X_C(\tau, C; w(\tau, C)) / M_C(\tau, C; w(\tau, C))$ is strictly increasing in C .

Step 3: $d \ln e_C(\tau, C; w(\tau, C)) / d \ln(1 + \tau)$ is strictly increasing in C . Write (10) as

$$\frac{d \ln e_C(\tau, C; w(\tau, C))}{d \ln(1 + \tau)} = \frac{1 - g(C)h(C)}{1 - h(C)},$$

with

$$\begin{aligned}g(C) &\equiv \frac{\partial \ln w(\tau, C)}{\partial \ln(1 + \tau)}, \\ h(C) &\equiv \frac{X_C(\tau, C; w(\tau, C))}{(1 + \tau)M_C(\tau, C; w(\tau, C))}.\end{aligned}$$

Taking log and totally differentiating with respect to C , we then get

$$\frac{d \ln}{d \ln C} \left[\frac{d \ln e_C(\tau, C; w(\tau, C))}{d \ln(1 + \tau)} \right] = -\frac{g'(C)h(C)}{1 - g(C)h(C)} + h'(C) \times \left[\frac{1 - g(C)}{(1 - h(C))(1 - g(C)h(C))} \right].$$

Step 1 implies $g(C) \in (0, 1)$ and $g'(C) > 0$. Step 2 implies $h(C) < 0$ and $h'(C) > 0$. It follows that $d \ln e_C(\tau, C; w(\tau, C)) / d \ln(1 + \tau)$ is strictly increasing in C .

J Proof of Proposition 7

Let us first compute aggregate imports and exports as a function of aggregate consumption C_t and the permanent tariff τ . Since the economy satisfies the same assumptions as in Proposition 2, we can follow the same steps as our previous proof to show that the value function of the firm problem is

$$\tilde{e}_t(p_{ct}) = \min_{y \in Y} -(p_c(\tau) \cdot y) \equiv \alpha(\tau),$$

with

$$p_{i,c}(\tau) = \begin{cases} (1 + \tau)(1 + \delta_{i,m})p_{i,m}^* & \text{if } m_{i1}, m_{i2} > 0, \\ p_{i,x}^* / (1 + \delta_{i,x}) & \text{if } x_{i1}, x_{i2} > 0. \end{cases}$$

For future reference, we let $y(\tau)$ denote the associated solution to the firm problem. Likewise, since the economy satisfies the same assumptions as in Proposition 2, the value function of the household problem is

$$\hat{e}(p_{ct}, C_t) = \min_c \{p_c(\tau) \cdot c | G_t(c) = C_t\} \equiv \beta(\tau)C_t.$$

We let $c(\tau)$ denote the solution of the previous problem for $C_t = 1$. The values of aggregate imports and exports can then be expressed as

$$M(\tau, C_t) = \sum_{i \in \mathcal{M}} p_{i,m}^* (1 + \delta_{i,m}) [c_i(\tau)C_t - y_i(\tau)] \equiv -\alpha_M(\tau) + \beta_M(\tau)C_t, \quad (\text{J.1})$$

$$X(\tau, C_t) = \sum_{i \in \mathcal{X}} p_{i,x}^* [y_i(\tau) - c_i(\tau)C_t] / (1 + \delta_{i,x}) \equiv \alpha_X(\tau) - \beta_X(\tau)C_t, \quad (\text{J.2})$$

where $\mathcal{M}$ and $\mathcal{X}$ denote the set of imported and exported goods, respectively,

$$\alpha_M(\tau) \equiv \sum_{i \in \mathcal{M}} p_{i,m}^* (1 + \delta_{i,m}) y_i(\tau), \quad (\text{J.3})$$

$$\alpha_X(\tau) \equiv \sum_{i \in \mathcal{X}} p_{i,x}^* y_i(\tau) / (1 + \delta_{i,x}), \quad (\text{J.4})$$

$$\beta_M(\tau) \equiv \sum_{i \in \mathcal{M}} p_{i,m}^* (1 + \delta_{i,m}) c_i(\tau), \quad (\text{J.5})$$

$$\beta_X(\tau) \equiv \sum_{i \in \mathcal{X}} p_{i,x}^* c_i(\tau) / (1 + \delta_{i,x}). \quad (\text{J.6})$$

For future reference, note that $\alpha'_M(\tau) \geq 0$ and $\beta'_M(\tau) \leq 0$. For output choices, this follows from the fact that the value of output produced among $\mathcal{M}$ and $\mathcal{X}$ in the import and export sectors, $\alpha_M(\tau)$ and $\alpha_X(\tau)$, must solve

$$\begin{aligned} \max_{\alpha_M, \alpha_X} (1 + \tau)\alpha_M + \alpha_X \\ F(\alpha_M, \alpha_X) \leq 0, \end{aligned} \tag{J.7}$$

with $F(\alpha_M, \alpha_X)$ the aggregate production possibility frontier between the import and export sectors,

$$\begin{aligned} F(\alpha_M, \alpha_X) &\equiv \min_y H(y) \\ \sum_{i \in \mathcal{M}} p_{i,m}^* (1 + \delta_{i,m}) y_i &= \alpha_M, \\ \sum_{i \in \mathcal{X}} p_{i,x}^* y_i / (1 + \delta_{i,x}) &= \alpha_X, \end{aligned}$$

and $H(y)$ the production possibility frontier associated with the domestic net-production set Ω , i.e. $y \in \Omega$ if and only if $H(y) \leq 0$. The argument for consumption choices is similar.

Next, let us compute $(\delta C_t)_{D_t=cst}$ in this environment. Recall that it is defined by

$$\mathcal{D}_C(\tau, C_t(\tau)) \times (\delta C_t)_{D_t=cst} + \mathcal{D}_\tau(\tau, C_t(\tau)) \times d\tau = 0.$$

Using the fact that

$$\begin{aligned} \mathcal{D}_\tau(\tau, C) &= -\tau M_\tau(\tau, C) = -\tau[\beta'_M(\tau)C - \alpha'_M(\tau)], \\ \mathcal{D}_C(\tau, C) &= M_C(\tau, C) - X_C(\tau, C) = \beta_M(\tau) + \beta_X(\tau), \end{aligned}$$

we therefore get

$$(\delta C_t)_{D_t=cst} = \frac{\tau[\beta'_M(\tau)C_t(\tau) - \alpha'_M(\tau)]d\tau}{\beta_M(\tau) + \beta_X(\tau)}$$

and, in turn,

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} = \frac{\beta'_M(\tau)C_1(\tau) - \alpha'_M(\tau)}{\beta'_M(\tau)C_2(\tau) - \alpha'_M(\tau)}.$$

Since the economy runs a larger trade deficit in period 1, we know that $C_1(\tau) > C_2(\tau)$. It follows that

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} \leq \frac{C_1(\tau)}{C_2(\tau)}, \tag{J.8}$$

with strict inequality if and only if $\alpha'_M(\tau) > 0$.

Finally, recall that since the economy satisfies the same assumptions as in Proposition

2, we have

$$\partial R(\tau, C_1, C_2) / \partial \tau = 0.$$

In addition, intertemporal preferences are homothetic. Hence the Euler condition (3) implies that the ratio of consumption between the two periods must remain constant,

$$\frac{C_1(\tau + d\tau)}{C_2(\tau + d\tau)} = \frac{C_1(\tau)}{C_2(\tau)},$$

and, in turn, that

$$\frac{(\delta C_1)_{MRS=R}}{(\delta C_2)_{MRS=R}} = \frac{C_1(\tau + d\tau) - C_1(\tau)}{C_2(\tau + d\tau) - C_2(\tau)} = \frac{C_1(\tau)}{C_2(\tau)}. \quad (\text{J.9})$$

Combining (J.8) and (J.9), we get

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} \leq \frac{(\delta C_1)_{MRS=R}}{(\delta C_2)_{MRS=R}},$$

with strict inequality if and only if $\alpha'_M(\tau) > 0$. Following the same argument as in Section 5, the trade deficit must go down in period 1, $D'_1(\tau) \leq 0$, with strict inequality if and only if $\alpha'_M(\tau) > 0$.

K Change in Deficit as a Share of Domestic Value Added

In this Appendix, we impose the exact same assumptions as in Proposition 7, but study the effect of a permanent tariff τ on the ratio of trade deficit to domestic value added in period 1.

We start by computing the deficit in period 1, $D_1(\tau) = \mathcal{D}(\tau, C_1(\tau))$, in closed-form. As already argued in Appendix J, since intertemporal preferences are homothetic, we must have

$$C_2(\tau) = \gamma C_1(\tau),$$

with $\gamma \in (0, 1)$ under the assumption that the economy runs a larger deficit in period 1. From equations (J.1) and (J.2), we also know that

$$\mathcal{D}(\tau, C) = -[\alpha_M(\tau) + \alpha_X(\tau)] + [\beta_M(\tau) + \beta_X(\tau)] C. \quad (\text{K.1})$$

Combining the two previous expressions with the intertemporal budget constraint,

$$\mathcal{D}(\tau, C_1) + \frac{1}{R^*} \mathcal{D}(\tau, C_2) = NFA,$$

we can solve for aggregate consumption in period 1,

$$C_1(\tau) = \frac{R^* \times NFA + (R^* + 1) [\alpha_M(\tau) + \alpha_X(\tau)]}{(R^* + \gamma) [\beta_M(\tau) + \beta_X(\tau)]}.$$

Substituting into (K.1), we obtain after rearrangements,

$$D_1(\tau) = \mathcal{D}(\tau, C_1(\tau)) = \frac{R^* \times NFA}{R^* + \gamma} + [\alpha_M(\tau) + \alpha_X(\tau)] \times \left(\frac{1 - \gamma}{R^* + \gamma} \right).$$

Next, note that equations (J.3) and (J.4) imply that domestic value added evaluated at international prices is equal to

$$VA_{\text{international}}(\tau) = \alpha_M(\tau) + \alpha_X(\tau).$$

If $NFA = 0$, the two previous equations imply

$$\frac{D_1(\tau)}{VA_{\text{international}}} = \frac{1 - \gamma}{R^* + \gamma}.$$

So the ratio of deficit to domestic value added, measured at international prices, is constant, as stated in footnote 25.

Finally, consider the case in which domestic value added is evaluated at domestic prices,

$$VA_{\text{domestic}}(\tau) = (1 + \tau)\alpha_M(\tau) + \alpha_X(\tau).$$

If $NFA = 0$, we therefore have

$$\frac{D_1(\tau)}{VA_{\text{domestic}}} = \frac{\alpha_M(\tau) + \alpha_X(\tau)}{(1 + \tau)\alpha_M(\tau) + \alpha_X(\tau)} \times \left(\frac{1 - \gamma}{R^* + \gamma} \right). \quad (\text{K.2})$$

Since $\alpha_M(\tau)$ and $\alpha_X(\tau)$ solve (J.7), a standard Envelope argument implies

$$\frac{d[(1 + \tau)\alpha_M(\tau) + \alpha_X(\tau)]}{d\tau} = \alpha_M(\tau) \geq 0, \quad (\text{K.3})$$

and, in turn,

$$\frac{d[\alpha_M(\tau) + \alpha_X(\tau)]}{d\tau} = -\tau\alpha'_M(\tau) \leq 0, \quad (\text{K.4})$$

since $\alpha'_M(\tau) \geq 0$, as noted in Appendix J. (K.2)-(K.4), along with $\gamma \in (0, 1)$, imply that the ratio of deficit to domestic value added, measured at domestic prices, is decreasing in τ , as stated in footnote 25.

L Alternative Calibrations

In this Appendix, we report the counterparts of Figure 6 for alternative calibrations of the trade elasticity, namely for $\varepsilon = 2$ and $\varepsilon = 8$.

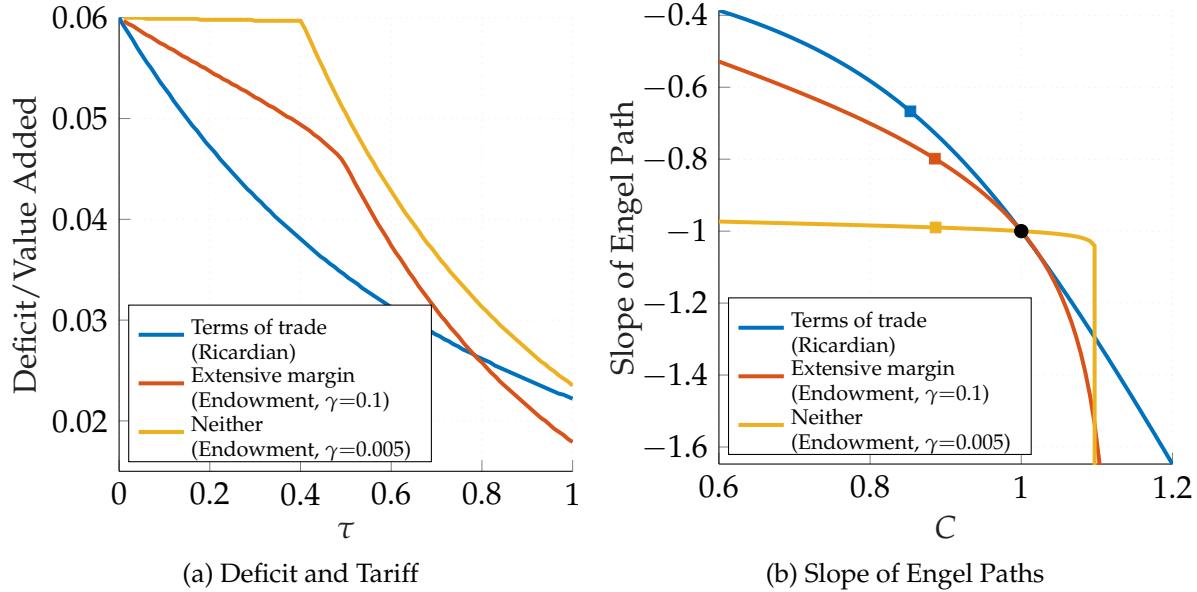


Figure L.1: Quantitative Examples ($\varepsilon = 2$)

Notes: Figure L.1a plots the deficit D_1 , expressed as a share of domestic value added, as a function of the tariff τ , in three economies: an endowment economy with (almost) no extensive margin adjustments ($\gamma = 0.005$) and no terms-of-trade effects (in yellow); an endowment economy with extensive margin adjustments ($\gamma = 0.1$) and no terms-of-trade effects (in red); and a Ricardian economy with terms-of-trade effects (in blue). All three economies are calibrated to match a share of imports to domestic value added equal to 15%, a ratio of deficit to domestic value added equal to 6%, and a trade elasticity $\varepsilon = 2$ in the free trade equilibrium ($\tau = 0$). Figure L.1b plots the slopes of the Engel paths M_C/X_C for the same three economies. All economies are normalized so that $C_1 = 1$ and the slope of the Engel path is -1 at $C = 1$. The black dot represents the common slope at C_1 ; squares represent the slopes at C_2 in each economy.

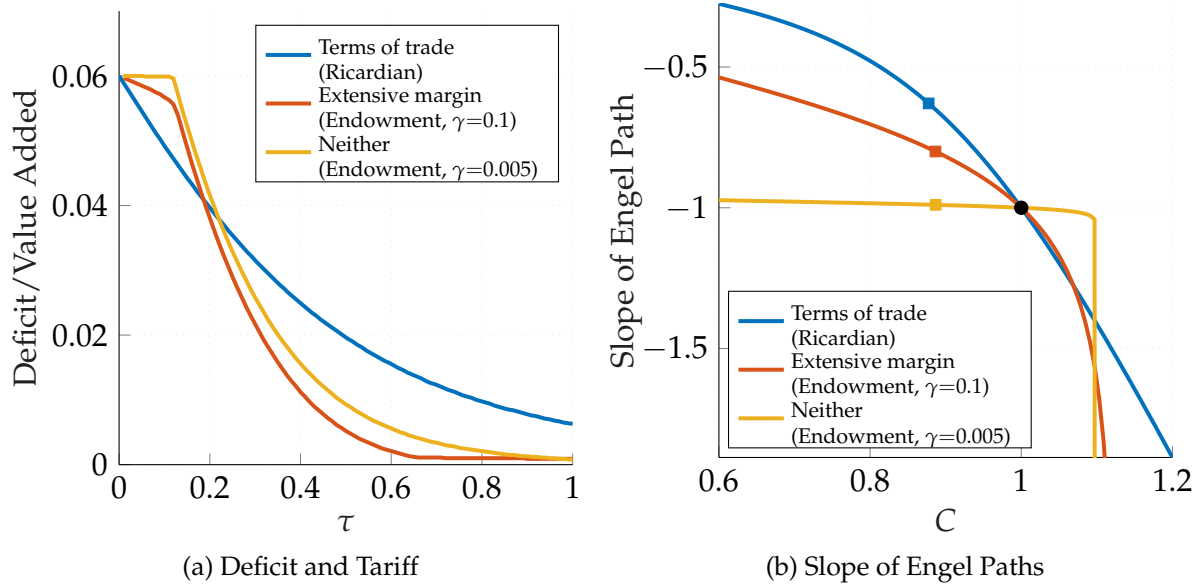


Figure L.2: Quantitative Examples ($\varepsilon = 8$)

Notes: Figure L.2a plots the deficit D_1 , expressed as a share of domestic value added, as a function of the tariff τ , in three economies: an endowment economy with (almost) no extensive margin adjustments ($\gamma = 0.005$) and no terms-of-trade effects (in yellow); an endowment economy with extensive margin adjustments ($\gamma = 0.1$) and no terms-of-trade effects (in red); and a Ricardian economy with terms-of-trade effects (in blue). All three economies are calibrated to match a share of imports to domestic value added equal to 15%, a ratio of deficit to domestic value added equal to 6%, and a trade elasticity $\varepsilon = 8$ in the free trade equilibrium ($\tau = 0$). Figure L.2b plots the slopes of the Engel paths M_C/X_C for the same three economies. All economies are normalized so that $C_1 = 1$ and the slope of the Engel path is -1 at $C = 1$. The black dot represents the common slope at C_1 ; squares represent the slopes at C_2 in each economy.