Monetary-Fiscal Interactions: A Reappraisal*

George-Marios Angeletos
Northwestern University & NBER

Chen Lian
UC Berkeley & NBER

Christian K. Wolf
MIT & NBER

Dalton Rongxuan Zhang
Northwestern University

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Abstract

The possibility of fiscal dominance in the representative-agent New Keynesian model (RANK) hinges on the assumption that income is *perpetually* demand-determined: fiscal deficits can drive output and inflation within that model only insofar as they trigger infinitely lasting, self-sustained shifts in aggregate spending and income. Moving to heterogeneous-agent New Keynesian models (HANK) opens the door to a different pathway: classical non-Ricardian effects, due to finite horizons or liquidity constraints. A refinement motivated by the model's intended focus on short-run phenomena—requiring a return to flexible-price outcomes in finite time—arrests the infinite feedback loop between spending and income, leaving only the classical non-Ricardian mechanism, and makes sure that the study of monetary-fiscal interactions is not centered on hard-to-test assumptions regarding beliefs at infinity.

*E-mail: angeletos@northwestern.edu, chen_lian@berkeley.edu, ckwolf@mit.edu, and r.zhang@u.northwestern.edu.
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1 Introduction

The post-COVID experience has reignited interest in monetary-fiscal interactions. Do fiscal deficits cause inflationary pressures? And can the Fed arrest such pressures by hiking interest rates?

Following Leeper (1991), a large literature addresses these questions by studying how the equilibria of the representative-agent New Keynesian model (RANK) depend on two different policy regimes. In the conventional regime, the monetary authority is dominant in the sense that it can regulate output and inflation in the familiar manner (as described, e.g., in Woodford, 2003; Galí, 2008), and fiscal deficits are entirely irrelevant.[1] In the opposite regime, known as fiscal dominance and tightly connected to the Fiscal Theory of the Price Level (FTPL), these predictions flip: fiscal deficits become the main driver of household spending, output, and inflation, despite households being Ricardian; and interest rate hikes may actually amplify, rather than dampen, inflationary pressures.

In this paper, we revisit the theoretical foundations of this literature. We first argue that the New Keynesian model admits a continuum of equilibria *regardless* of monetary and fiscal policy; the reason is that income in that model can remain demand-determined forever. We next show that fiscal dominance (or "active" fiscal policy) leverages this property, and indeed hinges on it: in this regime, fiscal deficits are inflationary only insofar as they trigger infinitely lasting self-fulfilling booms in household spending. A refinement motivated by the idea that aggregate demand matters only in the short run—formally, requiring a return to flexible-price outcomes in finite time—rules out this mechanism and delivers monetary dominance even if the Taylor principle fails.

Getting fiscal policy to matter in spite of our refinement thus requires a primitive failure of Ricardian equivalence, due to finite horizons, liquidity frictions, or imperfect rationality, as in the HANK literature. In particular, our analysis makes clear that such environments not only help reconcile the study of monetary-fiscal interactions with microeconomic evidence, but also decouple it from inherently hard-to-test assumptions about beliefs at far-ahead horizons.

Standard setting, new lens. We consider the standard New Keynesian model: prices are sticky, output is demand-determined, taxes are non-distortionary, and households are Ricardian.[2] But instead of studying the model's competitive equilibria and the eigenvalues of its linearized representation, we recast the fully non-linear economy as a game among the households and study its Nash equilibria. At the heart of this representation is an elementary assumption regarding consumer rationality: house-

[1] In this regime, the monetary authority is "active" in the sense that it raises nominal interest rates more than one-to-one with inflation (equivalently, the Taylor principle holds), while the fiscal authority is "passive" in the sense that it adjusts taxes as needed to ensure intertemporal budget balance. The opposite assumptions define the fiscal-dominance regime.

[2] We say that households are "Ricardian" when they are rational, have infinite horizons, and face no liquidity constraints, just as in Barro (1974). Note that these assumptions hold in RANK regardless of the policy regime. Also, our terminology should not be confused with that used in Woodford (1995); there, the term "non-Ricardian" refers to an active fiscal authority rather than to a violation of the aforementioned assumptions about households.

holds understand the structure of the economy, including the dependence of aggregate outcomes on their joint behavior. Under this lens, the following model properties become evident:

1. *There exists a continuum of equilibria, corresponding to multiple self-fulfilling beliefs about permanent income.* This multiplicity is present globally, regardless of the monetary-fiscal policy mix,[3] and is distinct from that in Sargent and Wallace (1975). It instead derives from two properties: (i) output, and thus also income, is demand-determined in all periods; and (ii) the households' cumulative marginal propensity to consume (MPC) is one.[4] To see the logic, suppose that prices are completely rigid and the monetary authority pegs the interest rate to a value equal to the households' subjective discount rate. Optimal consumption then equals permanent income, just as in Friedman (1957). But since one's income is determined by others' spending, the following is also true: if all households raise their spending by one unit in all periods, a household's permanent income and optimal consumption increase by exactly the same amount. Perpetually higher spending thus sustains higher permanent income, which in turn sustains perpetually higher spending, one-for-one. More succinctly, the economy resembles a game where the average best response coincides with the 45-degree line: consumer optimality gives $c = y$, the assumption that income is demand-determined gives $y = c$, and the two properties combined yield a best response of the form $c = c$, which obviously has a continuum of fixed points.

2. *Unlike monetary policy, fiscal policy is payoff-irrelevant.* Holding aggregate spending fixed, monetary policy enters each household's payoff and best response by influencing the real interest rate it uses to discount future income and spending. By contrast, fiscal policy drops out of each household's payoff and best response for the exact same reason as in Barro (1974): rational households understand that, conditional on aggregate spending, any fiscal operation—the timing of taxes, helicopter drops of government bonds, explicit default, or implicit default via inflation—is immaterial for their lifetime budget.

An integral part of our contribution is to show that these properties are hidden behind traditional treatments of the New Keynesian model: they hold true regardless of the degree of nominal rigidity, the slope of the Phillips curve, the specification of monetary and fiscal policy, and the structure of government debt (real vs. nominal, short-term vs. long-term). They furthermore immediately imply that fiscal deficits, by virtue of their payoff irrelevance, can influence output and inflation in RANK if, and only if, they trigger self-fulfilling shifts in household spending—shifts that *must be* infinitely lasting, as otherwise the loop delivering equilibrium multiplicity would not close.

[3] This is a statement about *global* indeterminacy and does not contradict the familiar result on *local* determinacy under the Taylor principle. See Section 3.3 for a detailed discussion.

[4] Note that, in the New Keynesian framework, output is fully demand-determined even if prices are only partially rigid, because rationing is ruled out (Barro, 2025; Flynn et al., 2026).

Fiscal dominance under our prism. To illustrate how the above insights shed new light on the existing literature on monetary-fiscal interactions, consider first the following example: government debt is held in real, one-period bonds (as in Barro, 1974); and the fiscal authority is "active" in the particular sense that it pegs future taxes to, say, 30 percent of income, regardless of how big today's deficit is. Such a policy would have led to government default if income were exogenously fixed (again as in Barro, 1974); now, however, with income being demand-determined (unlike Barro, 1974), default can be averted by households coordinating on a self-fulfilling boom of a specific size—one such that the resulting expansion in the tax base finances today's deficit without any change in tax rates.

Consider next the more familiar FTPL scenario, where government debt is nominal and the fiscal authority pegs future tax revenues instead of future tax rates. Under our prism, this scenario is a mere translation of the above example: households now coordinate on whatever self-fulfilling boom is necessary for the resulting inflation and debt erosion to substitute for the missing tax revenue. Hence, while the literature has suggested that the link from fiscal deficits to output and inflation is fundamentally different once government debt is nominal, we find no support for this idea within the New Keynesian model. Turning finally to the claim that an active fiscal authority can induce a unique equilibrium, we question this claim on the following grounds: because Ricardian households understand, by virtue of their rationality, that fiscal policy is payoff-irrelevant, they also understand that they can coordinate on a level of spending different from that required to support an active fiscal rule, compelling the fiscal authority to adjust taxes or default.[5]

A refinement. If the New Keynesian model admits multiple equilibria regardless of the monetary-fiscal policy mix, then how can the analyst make concrete predictions about the economy's response to shocks, or about the effects of changes in monetary and fiscal policy design? We propose a simple principle. Because the New Keynesian model is intended to speak *only* to short-run phenomena, we refine the equilibrium concept by requiring that long-run beliefs are anchored in the following sense: households believe that, in the absence of future shocks, the economy returns to flexible-price, supply-determined outcomes at some arbitrarily large but finite horizon.

Under this refinement, the Keynesian feedback between income and spending can operate in the short run, but not in the long run—income is *a fortiori* supply-determined after a finite lag. As a result, the effective strategic complementarity in spending is now less than one, the infinite feedback loop upon which an active fiscal policy rests breaks down, and the monetary authority alone regulates output and inflation even if the Taylor principle fails (e.g., even under an interest rate peg). Within RANK, fiscal irrelevance and monetary dominance thus both emerge as natural and *necessary* implications of the requirement that the long run is supply-determined, in the sense made precise by our refinement.

[5] The contrast between the our take on equilibrium determinacy and the FTPL reflects a subtle difference in the solution concept (Nash vs. competitive equilibrium) and the treatment of rationality, discussed at length in our main analysis.

Monetary-fiscal interactions in HANK. The above results explain why, in our view, the study of monetary-fiscal interactions—at least within the New Keynesian framework—should be built on two foundations: (i) non-Ricardian behavior due to, e.g., finite horizons or liquidity constraints, delivering payoff relevance of fiscal policy; and (ii) robustness to our refinement, severing the short and long run. The *combination* of these properties guarantees that monetary and fiscal policies operate exclusively via short-run effects, quantifiable through evidence on marginal propensities to consume—and not via hard-to-test shifts in expectations of far-ahead income away from flexible-price outcomes.

We illustrate this point in an NK environment where government debt enters the household utility function to proxy for the behavior of non-Ricardian households in heterogeneous agent models. The resulting fundamental (payoff-relevant) effects of fiscal policy are similar to those of monetary policy: postponing taxes stimulates aggregate demand just like a real interest rate cut. But since an infinite feedback loop between income and spending is still possible, assumptions about the policy mix may operate not only via those fundamental effects, but also via self-fulfilling forces similar to those present in RANK. By design, our refinement shuts down the second channel (which is hard to test), thus isolating the first one (which instead is disciplined by micro evidence).

Practical takeaway. Combining all the above lessons, we arrive at our paper's main takeaway: within the bounds of the Keynesian framework, the study of monetary-fiscal interactions can be true to the idea that the long run is supply-determined, and quantifiable by standard micro and macro data, only in models with non-Ricardian consumers and only in equilibria that are robust to our refinement.

Related literature. Our paper makes a methodological contribution to the study of monetary-fiscal interactions. We start by arguing that the New Keynesian model's equilibrium indeterminacy is distinct from that of flexible-price models such as Sargent and Wallace (1975), being instead rooted in the assumption that income is perpetually demand-determined. Building on this insight, we then shed new light on how fiscal policy operates in two strands of the New Keynesian literature: one that assumes Ricardian households (e.g., see Davig and Leeper, 2011; Bianchi and Melosi, 2017; Smets and Wouters, 2024); and another that relaxes this assumption, whether by employing quantitative HANK models (e.g., Kaplan, Moll and Violante, 2018; Auclert, Rognlie and Straub, 2024; Eichenbaum, Guerreiro and Obradovic, 2026), or by developing simpler, analytically tractable settings (e.g., Aguiar, Amador and Arellano, 2023; Angeletos, Lian and Wolf, 2024a,b, 2026; Bilbiie, 2025; Kaplan, 2025; Rachel and Ravn, 2026). In both cases, robustness to our refinement reveals whether those works rely on infinite feedback loops and thus hard-to-test assumptions about beliefs of the very long run.[6]

Like all the works cited above, our analysis assumes that the monetary authority does not directly

[6]We have explicitly checked that Angeletos et al. (2024a,b, 2026) pass this test. We suspect the same is true for key contributions such as Kaplan et al. (2018) and Auclert et al. (2024), because of their focus on short-run mechanisms.

care about fiscal conditions, nor is it subject to any political pressure. Relaxing this assumption allows fiscal deficits to drive output and inflation via a different mechanism—the central bank *deliberately* lowering interest rates in order to aid the fiscal authority. This could provide a bridge between our analysis and a literature that models monetary-fiscal interactions as the solution to Ramsey problems with limited commitment (e.g., Bocola et al., 2026).

Although our game-theoretic approach brings to mind Bassetto (2002)'s contribution to the FTPL, there is a crucial difference: that paper studies a flexible-price economy, thus shutting down both the feedback loop between spending and income (the Keynesian cross) and the tight link between real economic activity and inflation (the Phillips curve). This explains why our insights do not apply to that paper's setting, and *vice versa*.[7] Our approach is instead more closely connected to Angeletos and Lian (2018), Angeletos and Huo (2021) and Farhi and Werning (2019). But whereas these papers focus on bounded rationality in settings without a fiscal sector, we shift the focus to monetary-fiscal interactions and highlight the fragility of FTPL-like equilibria to our refinement. Complementary in this regard is Angeletos and Lian (2023), which anchors expectations of aggregate spending at far-ahead horizons via small, appropriate noise (as in the global-games literature), as well as the earlier, familiar criticisms of the FTPL (Kocherlakota and Phelan, 1999; Buiter, 2002; Niepelt, 2004).

Last but not least, although our analysis agrees with Atkeson, Chari and Kehoe (2010), Cochrane (2011), Neumeyer and Nicolini (2025) and others on the pitfalls of the Taylor principle, it ultimately provides a new justification for RANK's conventional solution (as reviewed, e.g., in Galí, 2008): this solution is the only one consistent with the principle that aggregate demand matters only in the short run, in the sense made precise by our refinement. By contrast, the backward-looking or FTPL-based solutions advocated by Cochrane (2017, 2018) violate this principle, precisely because they hinge on the type of infinitely lasting self-sustained fluctuations highlighted above.

Outline. We begin in Section 2 with a simple two-period RANK model, allowing us to transparently explain the essence of our results on the game representation, equilibrium multiplicity, and the role of fiscal policy. The much more general infinite-horizon analogue follows in Section 3. We then turn to our refinement in Section 4, and finally develop the HANK extension in Section 5.

2 The two-period model

We begin with a stripped-down version of the New Keynesian model, featuring two periods, real government debt, and (for part of the analysis) fully rigid prices. This allows us to transparently commu-

[7] It is an open question whether a translation of Bassetto (2002) to the New Keynesian framework is possible, and how this could be reconciled with our insights. The same applies to Norman and Willems (2025), who explore a cooperative-game foundation of the FTPL within a flexible-price model.

nicate our main results; Section 3 will show that all insights extend to a general RANK environment.

2.1 Environment

There are two periods, indexed by $t \in \{0,1\}$, a continuum of households, indexed by $i \in [0,1]$, and a government. Each household is a combined consumer-worker-firm: it is the monopolist of a differentiated good, which is produced by the household's labor, and consumes a composite of all the goods in the economy. This assumption simplifies the exposition, reducing the private sector to a single set of agents. There is an exogenous unit of account, used to quote prices, but there is no money—the economy is "cashless," as is commonplace in the New Keynesian framework (e.g. Woodford, 2003; Galí, 2008), indeed even when fiscal dominance is assumed (e.g. Bianchi and Melosi, 2017; Cochrane, 2018; Smets and Wouters, 2024). Finally, we let households save and borrow using a real, risk-free, one-period bond, whose net supply equals government debt.[8]

Households. A household's lifetime utility is given by

$$\mathscr{U}_i = \sum_{t\in\{0,1\}} \beta^t \Big\{ U\big(C_{i,t}\big) - V\left(L_{i,t}\right) \Big\}, \tag{1}$$

where $C_{i,t}$ and $L_{i,t}$ denote consumption and labor in period t, $\beta \in (0,1)$ is the subjective discount factor, $U : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, strictly concave, continuously differentiable, and bounded, with $\lim_{C\to 0} U'(C) = \infty$ and $\lim_{C\to\infty} U'(C) = 0$, and $V : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing and convex. Consumption is the familiar constant elasticity of substitution (CES) composite:

$$C_{i,t} = \left(\int C_{i,j,t}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}},$$

where $C_{i,j,t}$ is i's consumption of good j and $\epsilon > 1$ is the elasticity of substitution.

The household's flow budgets in periods 0 and 1 are given by, respectively,

$$\int \frac{P_{j,0}}{P_0} C_{i,j,0}\, dj + R_0^{-1} D_{i,1} \leq D_0 + Y_{i,0} - T_0 \qquad \text{and} \qquad \int \frac{P_{j,1}}{P_1} C_{i,j,1}\, dj \leq D_{i,1} + Y_{i,1} - T_1,$$

where $P_{j,t}$ is the price of good j in date t, $P_t \equiv \left(\int P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index, $Y_{i,t}$ is the household's real income in period t, T_t is the (assumed common) tax bill in period t, $D_{i,0}$ is the amount of real government bonds inherited at the beginning of date 0, $D_{i,1}$ is the amount of the new bonds purchased at the end of date 0, and R_0 is the real interest rate. Without any loss of generality, we let $D_{i,0} = D_0$ for all i, i.e., we abstract from heterogeneity in initial wealth. Assuming that production is linear in labor and normalizing labor productivity to 1, we have that real income is $Y_{i,t} \equiv \frac{P_{i,t}}{P_t} L_{i,t}$ and

[8] At first glance, the assumption that government debt is real (as in Barro, 1974) may appear to disconnect our analysis from Leeper (1991), the FTPL, and the related literature on monetary-fiscal interactions, which all assign a prominent role to nominal debt; we will discuss later why that is not the case.

that $\frac{P_{i,t}}{P_t}$ can be interpreted as the real wage faced by household i. The CES demand structure furthermore as usual gives i's optimal demand for good j as $C_{i,j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} C_{i,t}$, and so total expenditure as $\int P_{j,t} C_{i,j,t} dj = P_t C_{i,t}$. We can thus rewrite the household's flow budget constraints as

$$C_{i,0} + R_0^{-1} D_{i,1} \le D_0 + Y_{i,0} - T_0 \qquad \text{and} \qquad C_{i,1} \le D_{i,1} + Y_{i,1} - T_1. \tag{2}$$

Combining these flow budgets yields the following lifetime budget:

$$C_{i,0} + R_0^{-1} C_{i,1} \le Z_0 + Y_{i,0} + R_0^{-1} Y_{i,1}, \tag{3}$$

where $Z_0 \equiv D_0 - T_0 - R_0^{-1} T_1$ measures initial private assets net of tax obligations.

To see next how the household's income is determined, note that the demand for the good it produces is $Q_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t$, where $C_t \equiv \int C_{i,t} di$ denotes aggregate consumption. Imposing market clearing ($L_{i,t} = Q_{i,t}$), we infer that i's labor supply and real income are pinned down by aggregate spending and by the relative price of its good:

$$L_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t \qquad \text{and} \qquad Y_{i,t} \equiv \frac{P_{i,t} L_{i,t}}{P_t} = \left(\frac{P_{i,t}}{P_t}\right)^{1-\epsilon} C_t. \tag{4}$$

Rigid prices. We begin by assuming that prices are fully rigid. Although this assumption is obviously restrictive and will be relaxed in Section 2.3 below (as well as in the more general, infinite-horizon framework of Section 3), it is useful for two reasons: it simplifies the exposition; and it helps make clear that the indeterminacy issues at the heart of our analysis are real, and as such distinct from the nominal indeterminacy familiar from Sargent and Wallace (1975). We thus fix $P_{i,t} = 1\ \forall i, t$, in which case condition (4) specializes to

$$L_{i,t} = Y_{i,t} = C_t. \tag{5}$$

This sharpens the dependence of individual labor and income on aggregate spending, and reduces the household problem to the choice of a consumption plan $\mathbf{C}_i \equiv (C_{i,0}, C_{i,1})$ that maximizes its lifetime utility (1) subject to its lifetime budget (3) and the market-clearing restriction (5).

Government. The government, consisting of unified monetary and fiscal authorities, sets interest rates and taxes. The real rate R_0 is set according to the rule

$$R_0 = \mathscr{R}(C_0), \tag{6}$$

for some continuous function $\mathscr{R} : \mathbb{R}_+ \to \mathbb{R}_{++}$. Under the assumption of fully rigid prices, R_0 is both the real and the nominal rate, so equation (6) can be interpreted as a Taylor-type feedback rule for the nominal rate. Later, we will consider both partially sticky prices and more general interest rate rules.

Turning to fiscal policy, taxes T_0 and T_1 are set according to the following rules:

$$T_0 = \mathscr{T}_0(D_0, C_0, C_1, R_0) \qquad \text{and} \qquad T_1 = \mathscr{T}_1(D_0, C_0, C_1, R_0), \tag{7}$$

where $\mathcal{T}_t : \mathbb{R}^4 \to \mathbb{R}$ for $t \in \{0,1\}$.[9] For our main analysis, we furthermore rule out government default as well as government consumption, thus delivering the following two flow budget constraints:

$$D_0 = T_0 + R_0^{-1} D_1 \qquad \text{and} \qquad D_1 = T_1, \tag{8}$$

where $D_1 \equiv \int D_{i,1} di = R_0 \int \left(D_0 + C_0 - T_0 - C_{i,0}\right) di$ is aggregate private saving (and so also government debt issued) in period 0. By combining the two flow constraints in (8), we immediately get the following equation, which is commonly known as the government's intertemporal budget constraint:

$$D_0 = T_0 + R_0^{-1} T_1. \tag{9}$$

Prior work has debated whether the infinite-horizon, nominal-bonds counterpart of equation (9) is a primitive constraint, or an implication of consumer optimality (and in particular of the representative agent's transversality condition). This distinction will prove immaterial under our lens; all we will leverage is that rational households understand the validity of equation (9) and of its generalizations.

Equilibria. Below we will characterize and compare two solution concepts: our game representation and its Nash equilibria; and the standard notion of a competitive equilibrium studied in prior work.[10]

2.2 Game representation and Nash equilibria

Consistent with the rational expectations hypothesis, we assume that households understand the economy's structure, including the government's behavior. More specifically, we endow them with direct knowledge of the market-clearing condition (5), the policy rules (6)-(7), and the flow budgets (8), and thus also of equation (9). Our economy then admits the following game representation.

Game Representation. *There is a continuum of players, corresponding to the households. Household i's action is the consumption plan $\mathbf{C}_i = (C_{i,0}, C_{i,1}) \in \mathbb{R}^2_{++}$. Letting $\mathbf{C} \equiv (C_0, C_1) \in \mathbb{R}^2_{++}$ be the corresponding average action, household i's payoff is given by*

$$\mathcal{U}(\mathbf{C}_i, \mathbf{C}) \equiv \sum_{t=0}^{1} \beta^t \left\{ U\left(C_{i,t}\right) - V\left(C_t\right) \right\} - \mathbb{I}\left[C_0 + (\mathcal{R}(C_0))^{-1} C_1 - C_{i,0} - (\mathcal{R}(C_0))^{-1} C_{i,1}\right] \tag{10}$$

where $\mathbb{I}[\bullet]$ is the penalty indicator of the household's lifetime budget set.[11]

[9] Notice that we express the tax rules in sequence form, i.e., as arbitrary functions of the initial debt and the entire path of aggregate output and interest rates—(C_0, C_1, R_0) here, and $\{C_t, R_t\}_{t=0}^{\infty}$ in our later infinite-horizon extension. This general specification nests the recursive tax rules that are popular in applied work since Leeper (1991), while also going materially beyond them: it allows, without strictly requiring, that tax policy be both backward- and forward-looking.

[10] We use the term competitive equilibrium to refer not only to perfectly competitive markets, but also to *monopolistically* competitive markets, thus accommodating price-setting behavior.

[11] Throughout, $\mathbb{I}$ denotes the penalty indicator of the relevant lifetime budget set: it equals zero when the household's plan has finite present value and satisfies its lifetime budget constraint, and it equals $+\infty$ otherwise. A best response is required to have finite penalty.

The logic of this game representation is straightforward. Because rational households understand that equation (9) must hold regardless of government policy and market behavior, they understand that their holdings of government bonds and their tax obligations necessarily net out to zero, and hence that their intertemporal budget constraint reduces to

$$C_{i,0} + R_0^{-1} C_{i,1} \leq Y_{i,0} + R_0^{-1} Y_{i,1}.$$

Next, because households understand that the real interest rate satisfies the policy rule (6), and that their income and labor supply are determined by aggregate spending according to (4) (along with $P_{i,t} = P_t = 1$, as prices are rigid), they in fact understand that their intertemporal budget constraint even further reduces to

$$C_{i,0} + (\mathscr{R}(C_0))^{-1} C_{i,1} \leq C_0 + (\mathscr{R}(C_0))^{-1} C_1.$$

In words, households understand that *both* their permanent income and the real interest rate are determined by the consumption of other households. Our game representation is then completed by incorporating the above constraint via a penalty indicator in the households' payoffs.[12] Fiscal policy is thus payoff-irrelevant, which directly implies that the set of Nash equilibria characterized below is invariant to the specification of the fiscal rules $\mathscr{T}_0(\bullet)$ and $\mathscr{T}_1(\bullet)$ and to the initial debt D_0.

With this representation in hand, an equilibrium can be defined in a standard fashion as a Nash equilibrium, i.e., as a fixed point of the households' best responses.

Definition 1. A Nash equilibrium (NE) is a collection of consumption plans $(\mathbf{C}_i)_{i\in[0,1]}$ such that $\mathbf{C}_i \in \arg\max_{\mathbf{C}_i \in \mathbb{R}^2_{++}} \mathscr{U}(\mathbf{C}_i, \mathbf{C})$ for all i, with $\mathbf{C} \equiv \int \mathbf{C}_i di$.

What are the economy's equilibria under this solution concept? And how do they compare to those defined and studied in the literature we are concerned with? We address the first question in the next theorem, and the second one in Section 2.4.

Theorem 1. *There exists a continuum of Nash equilibria and they have the following structure:*

1. $(\mathbf{C}_i)_{i\in[0,1]}$ *is a Nash equilibrium if and only if* $\mathbf{C}_i = \mathbf{C}$ *for all* i *and* $\mathbf{C}$ *solves the aggregate Euler equation*

$$U'(C_0) = \beta \mathscr{R}(C_0) U'(C_1). \tag{11}$$

2. *For any* $\xi \in \mathbb{R}_{++}$, *there is a Nash equilibrium in which the discounted present value of aggregate spending, namely* $C_0 + C_1/R_0$, *equals* ξ.

[12] This last step of including the budget constraint through a penalty indicator serves the following, non-essential, purpose. In a standard formulation of games, a player's choice set is *not* allowed to depend on the choices of other players. This creates a tension with any application in which an agent's feasibility set depends on endogenous outcomes. The tension is resolved here by recasting the infeasible as prohibitively costly.

The proof of Theorem 1 is both straightforward and instructive. Taking the aggregate plan $\mathbf{C} = (C_0, C_1)$ as given, the individual plan $\mathbf{C}_i = (C_{i,0}, C_{i,1})$ is optimal—i.e., it is a best response—for household i if and only if it solves that household's Euler equation and intertemporal budget constraint:

$$U'(C_{i,0}) = \beta\mathscr{R}(C_0)U'(C_{i,1}), \tag{12}$$

$$C_{i,0} + [\mathscr{R}(C_0)]^{-1} C_{i,1} = C_0 + [\mathscr{R}(C_0)]^{-1} C_1. \tag{13}$$

Because households have identical and single-peaked payoffs, any Nash equilibrium is necessarily symmetric: $\mathbf{C}_i = \mathbf{C}$ for all i. But then equation (12) reduces to the aggregate Euler equation (11), so this equation is necessary for Nash equilibrium. To prove that this equation is also sufficient, pick any $\mathbf{C}$ that solves this equation and let $\mathbf{C}_i = \mathbf{C}$ for all i. Then, *both* (12) and (13) are satisfied for all i, which verifies that $\mathbf{C}$ is a Nash equilibrium and completes the proof of Part 1. The existence of a continuum of Nash equilibria is then immediate: just pick an arbitrary C_0 and solve (11) for C_1. The proof of Part 2 is completed by verifying that, by varying the solution of (11), we can match any value $\xi > 0$ for the discounted present value of $\mathbf{C}$. This last, more technical, step is relegated to the Appendix, while the economic essence is further discussed below. Finally, note that ξ can be arbitrarily large because we abstract from any physical upper bound on labor supply and output; if we instead add a physical upper bound on labor and thus output, there still exists a continuum of Nash equilibria with spending in both periods below that bound.

Strategic complementarity and equilibrium multiplicity. To illustrate the mechanism behind Theorem 1, it is instructive to consider the special case in which $\mathscr{R}(\bullet) = \beta^{-1}$, i.e., a monetary authority that pegs the interest rate at the households' subjective discount rate. In this case, optimal consumption is constant across time and is equated to the household's permanent income, just as in Friedman (1957):

$$C_{i,0} = C_{i,1} = \tfrac{1}{1+\beta} Y_{i,0} + \tfrac{\beta}{1+\beta} Y_{i,1}.$$

It follows that, if household i expects other households to spend ξ in *both* periods, then it also expects its permanent income to be ξ, and thus best-responds by spending ξ in *both* periods, closing the cycle. Away from this special case, the interest-rate rule $\mathscr{R}(\bullet)$ regulates the slope of aggregate spending over time, but still leaves its present discounted value, or the permanent income, indeterminate, and so the essence of the argument is unchanged.[13]

Zooming out, we see that equilibrium multiplicity is rooted in two model properties (which later will directly extend to the infinite-horizon context): first, that the households' *cumulative*, lifetime, marginal propensity to consume (MPC) is one; and second, that income is demand-determined in *all*

[13] Strictly speaking, an additional source of multiplicity is possible if $\mathscr{R}(\bullet)$ is sufficiently non-linear: the same level of permanent income could be consistent with different time profiles of spending. Since most of the literature (log-)linearizes, this possibility is not present there and is thus irrelevant for our main purposes. See Appendix C for details.

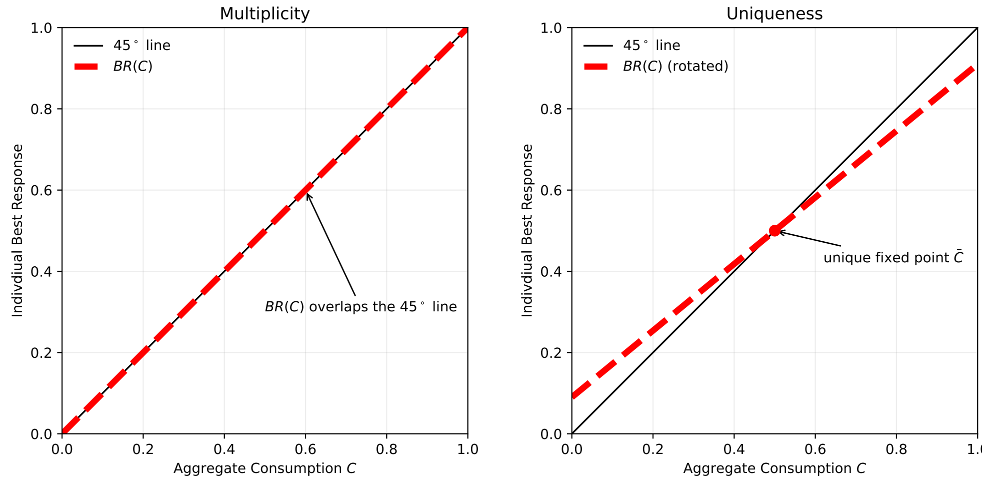

Figure 1: Schematic representation of individual best responses to aggregate consumption in our economy (left panel), and with weakened strategic complementarity (right panel).

periods. Together, these properties imply that the overall, intertemporal, degree of strategic complementarity in household spending is *exactly* one, delivering a continuum of equilibria.

We provide a visual illustration of this point in the left panel of Figure 1. The two properties stressed above—a lifetime MPC of 1, and demand-determined output—translate to a best-response mapping from aggregate to individual consumption (averaged across periods) that overlaps perfectly with the 45-degree line, delivering a continuum of equilibria. This illustration however also hints at the following idea: if we could *somehow* reduce the strategic complementarity between individual and aggregate spending, we could obtain uniqueness, as in the right panel of Figure 1. We will provide such a pathway in Section 4, via a refinement of far-ahead beliefs in the infinite-horizon case.

The aggregate Euler equation. An integral part of Theorem 1 is the *sufficiency* of the aggregate Euler equation (11). To understand this sufficiency, note that budget constraints bite at the individual level because households take their income as given. At the aggregate level, however, income is itself determined by aggregate spending (because output is fully demand-determined), so the representative agent's budget is automatically satisfied for *any* $\mathbf{C}$ that solves equation (11). In short, the sufficiency of the aggregate Euler equation is the mirror image of the multiplicity mechanism discussed above.

2.3 From rigid to sticky prices

In this section, we let prices be "sticky" instead of fully rigid, in the following sense: prices can be adjusted each period, but only at a cost (as in Rotemberg, 1982). This will introduce a micro-founded Phillips curve and will smooth-paste to the general RANK setting of Section 3, but will not change our lessons regarding the nature of the equilibrium multiplicity and the payoff irrelevance of fiscal policy.

Any predictions about inflation—and, later, about nominal debt erosion, too—will be purely ancillary.

Extended environment. Each household is still both a consumer and the monopolist of the differentiated good it produces. The only change is that it can now optimally set the price of this good, subject to an adjustment cost. Specifically, changing the period-t price from $P_{i,t-1}$ to $P_{i,t}$ requires $\kappa^{-1} g(P_{i,t-1}, P_{i,t})$ additional units of labor, where $g : \mathbb{R}^2_{++} \to \mathbb{R}_+$ attains its minimal value of zero only when $P_{i,t} = P_{i,t-1}$ and $\kappa \in (0,\infty)$ parameterizes the degree of price flexibility. Production remains linear in labor, so i's total labor and real income are, respectively,

$$L_{i,t} = \left(\tfrac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t + \kappa^{-1} g(P_{i,t-1}, P_{i,t}), \qquad Y_{i,t} = \left(\tfrac{P_{i,t}}{P_t}\right)^{1-\epsilon} C_t. \tag{14}$$

The first term in $L_{i,t}$ is the labor required to meet demand, and the second is the price-adjustment cost. Finally, all households are again endowed with the same initial real wealth D_0, inherit the same initial price $P_{i,-1} = 1$, and pay the same lump-sum taxes T_0 and T_1.

As in Rotemberg (1982), the adjustment cost $g(\bullet)$ micro-founds a Phillips curve, that is, a structural relation between real quantities and nominal prices, which will be denoted by $\mathscr{P}(\bullet)$ in Theorem 2 below. Unlike Rotemberg (1982), we wish this relation to be well-defined and single-valued globally, and not only locally in the neighborhood of zero inflation. Put simply, we wish to rule out "backward bending" Phillips curves (under which $\mathscr{P}$ is a correspondence rather than a function), in keeping with the related literature. To guarantee this, we assume that the labor disutility $V(\bullet)$ is linear and the adjustment cost $g(\bullet)$ satisfies two regularity conditions, stated as Assumptions A1-A2 in the proof of Theorem 2. All other assumptions on the model are unchanged.[14]

Revisiting the main lessons. We now verify that the lessons of the previous section extend for any degree of price stickiness, i.e., for any $\kappa \in (0,\infty)$. We start with the game representation.

Game Representation. *Household i's action is the vector $(\mathbf{C}_i, \mathbf{P}_i) \in \mathbb{R}^4_{++}$, describing how much it spends as a consumer and what prices it sets as a producer. Its payoff is given by*

$$\begin{aligned}\mathscr{U}(\mathbf{C}_i, \mathbf{P}_i; \mathbf{C}, \mathbf{P}) \quad \equiv \quad & \sum_{t=0}^{1} \beta^t \Big\{ U\big(C_{i,t}\big) - \mathscr{L}_t\big(\mathbf{P}_i, P_t, C_t\big) \Big\} \\ & - \mathbb{I}\big[\mathscr{Y}(P_{i,0}, P_0, C_0) + (\mathscr{R}(C_0))^{-1} \mathscr{Y}(P_{i,1}, P_1, C_1) - C_{i,0} - (\mathscr{R}(C_0))^{-1} C_{i,1}\big]\end{aligned}$$

where $C_t \equiv \int C_{i,t}\, di$ and $P_t \equiv \left(\int P_{i,t}^{1-\epsilon}\, di\right)^{\frac{1}{1-\epsilon}}$ capture the aggregate actions, $\mathscr{L}_t\big(\mathbf{P}_i, P_t, C_t\big) \equiv \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t + \kappa^{-1} g(P_{i,t-1}, P_{i,t})$ captures i's disutility of labor in period t, $\mathscr{Y}(P_{i,t}, P_t, C_t) \equiv \left(\frac{P_{i,t}}{P_t}\right)^{1-\epsilon} C_t$ captures i's real income in period t, and $\mathbb{I}[\cdot]$ is the same penalty indicator as before.

[14] In particular, we continue to assume (6), which can now be reinterpreted as the following Taylor rule: $I_0 = \Pi_1 \mathscr{R}(C_0)$, where I_0 is the nominal interest rate between periods 0 and 1 and $\Pi_1 \equiv P_1/P_0$ is the corresponding inflation rate. More general feedback rules from inflation and output to nominal rates are accommodated in Section 3.

This representation contains three natural modifications relative to its rigid-price counterpart. First, each household now chooses, not only a consumption path $\mathbf{C}_i = (C_{i,0}, C_{i,1})$, but also a price path $\mathbf{P}_i = (P_{i,0}, P_{i,1})$. Second, a household's income depends both on aggregate spending and on its relative price (essentially, its relative wage). And third, the price adjustment cost enters payoffs via the disutility of labor. Notwithstanding these modifications, the following two key properties carry over from the case with rigid prices: that fiscal policy is payoff irrelevant; and that individual income scales one-for-one with aggregate spending. We thus have the following extension of Theorem 1:

Theorem 2. *There exists a function $\mathscr{P}: \mathbb{R}^2_{++} \to \mathbb{R}^2_{++}$, which is invariant to fiscal policy, such that the path of aggregate spending $\mathbf{C} \equiv (C_0, C_1)$ and the path of the price level $\mathbf{P} \equiv (P_0, P_1)$ constitute a Nash equilibrium under sticky prices if and only if:*

1. *$\mathbf{C}$ is a Nash equilibrium under rigid prices.*
2. *$\mathbf{P} = \mathscr{P}(\mathbf{C})$.*

Equilibrium now makes predictions not only about the path of real spending and output, but also about the path of the price level. The price path is pinned down by the output path via a Phillips curve (i.e., the function $\mathscr{P}$ above), which itself is invariant to fiscal policy; and the output path is still characterized by Theorem 1. The scope and nature of the equilibrium multiplicity are thus *exactly* the same as in the case with rigid prices, and any implications about inflation are purely ancillary.

2.4 Competitive equilibria and active fiscal policy

Our preceding analysis agrees with Cochrane (2011, 2023) that interest-rate rules cannot deliver equilibrium determinacy, but differs from the FTPL literature in concluding that fiscal rules cannot do so either. This difference reflects a subtle but important difference in the solution concept. In this section, we thus compare our economy's Nash equilibria to its competitive counterparts; and in Section 2.5, we provide an economic interpretation of the differences between the two.

CE definition. We start by defining a competitive equilibrium (CE) for our setting.[15]

Definition 2. A collection of $\left((\mathbf{C}_i, \mathbf{P}_i)_{i\in[0,1]}, \mathbf{C}, \mathbf{P}, R_0, T_0, T_1\right)$ is a *competitive equilibrium* (CE) if:

1. For all i, the pair $(\mathbf{C}_i, \mathbf{P}_i)$ maximizes (1) over consumption plans $\mathbf{C}_i \in \mathbb{R}^2_{++}$ and price plans $\mathbf{P}_i \in \mathbb{R}^2_{++}$ subject to (3) and (14), and the corresponding aggregates are $C_t \equiv \int C_{i,t} di$ and $P_t \equiv \left(\int P_{i,t}^{1-\epsilon} di\right)^{1/(1-\epsilon)}$.
2. The policy rules (6) and (7) are satisfied.

[15] We continue with the sticky-price setting of Section 2.3. The earlier scenario with fully rigid prices is nested by fixing $P_{i,t} = 1$ in all the subsequent statements.

Since production is subsumed in the household problem, Condition 1 imposes not only optimal consumption-saving behavior but also optimal price-setting behavior, and embeds market clearing in equation (14). Condition 2, on the other hand, requires that the interest rate and taxes satisfy the corresponding policy rules. Finally, notice the following: echoing the FTPL literature, Definition 2 does *not* a priori impose equation (9), the government's intertemporal budget constraint; nevertheless, by Walras's law, this equation *has* to hold in every competitive equilibrium.[16]

CEs vs. NEs. We are now ready to map the set of competitive equilibria (CE), as defined above, to the set of Nash equilibria (NE), as previously defined and characterized.

Proposition 1. *The collection* $(\mathbf{C}_i, \mathbf{P}_i)_{i\in[0,1]}$ *is part of a competitive equilibrium (CE) if and only if it is a Nash equilibrium (NE) and, in addition, the aggregate consumption path solves*

$$D_0 = \mathscr{T}_0(D_0, C_0, C_1, \mathscr{R}(C_0)) + \mathscr{R}(C_0)^{-1}\mathscr{T}_1(D_0, C_0, C_1, \mathscr{R}(C_0)). \tag{15}$$

It follows that every CE is necessarily an NE, but the converse need not be true: to check if a given NE is also a CE, one must check whether (15)—which equates initial government debt to the present discounted value of taxes—is satisfied. Put differently, one can find the economy's CEs in two steps. First, find the economy's NEs. Next, keep any NE such that the associated consumption path (C_0, C_1) satisfies equation (15), and disregard any NE that fails this test.

Clearly, fiscal policy does *not* enter the first step, which we completed in Theorem 1, but looms large in the second step, because it enters equation (15). To characterize when this has bite, in the sense of excluding some NEs, we introduce the following classification.

Definition 3. Fiscal policy is *passive* if the tax rule $\mathscr{T}(\bullet)$ satisfies the following restriction for *every* $(D_0, C_0, C_1, R_0) \in \mathbb{R}^3_+ \times \mathbb{R}_{++}$:

$$D_0 = \mathscr{T}_0(D_0, C_0, C_1, R_0) + R_0^{-1}\mathscr{T}_1(D_0, C_0, C_1, R_0). \tag{16}$$

Fiscal policy is *active* otherwise.

In words, fiscal policy is passive if taxes adjust to ensure that the government's intertemporal budget constraint holds regardless of the initial debt burden, the path of aggregate spending, and the real interest rate—and it is active if the opposite is true. This definition and its upcoming infinite-horizon extension are identical in economic essence to that in Leeper (1991), though going beyond that paper's particular focus on bounded, log-linearized systems. Going back to Proposition 1, we now obtain a sharp characterization of how fiscal policy shapes CEs, in spite of its irrelevance for the set of NEs.

[16]To see this, note that household optimality requires that the lifetime budget constraint (3) is satisfied with equality for all i; aggregating this across i, we immediately get $Z_0 = 0$, or equivalently equation (9).

Theorem 3. *A CE always translates to an NE, but the converse depends on the fiscal rule:*

1. *When fiscal policy is passive, every NE translates to a CE.*
2. *For each NE, there exists an active fiscal rule such that this NE translates to the unique CE.*

As shown in Theorem 1, the economy admits multiple NEs, because permanent income is demand-determined, and the set of NEs is invariant to fiscal policy, because rational households understand that government bonds are not net wealth. When fiscal policy is passive, equation (16) is automatically satisfied in *all* NEs, so the sets of NEs and CEs coincide, and equilibrium is indeterminate under either solution concept. When instead fiscal policy is active, equation (16) may be satisfied for some NEs but not others, and so fiscal policy can now shape the set of CEs in spite of its irrelevance for the set of NEs. In fact, for any given NE, it is straightforward to design a fiscal rule $\mathscr{T}(\bullet)$ such that (16) is satisfied only for that NE; an active fiscal rule can thus select a particular NE as the unique CE.

An example. To illustrate how active fiscal policy works under the CE concept, consider again the case in which the interest rate is pegged to $R_0 = \beta^{-1}$ and recall that the set of NEs then corresponds to $C_0 = C_1 = \xi$ for arbitrary $\xi > 0$. Suppose further that $D_0 > 0$. Next, let the fiscal authority be "active" in that its tax rule does not depend directly on the initial debt level D_0: it sets lump-sum tax revenue equal to a fixed share of aggregate income. That is, let $\mathscr{T}_0(\bullet) = \tau_y C_0$ and $\mathscr{T}_1(\bullet) = \tau_y C_1$, for some fixed $\tau_y \in (0,1)$. Equation (16) then becomes

$$D_0 = \underbrace{\tau_y \xi}_{T_0} + \underbrace{\beta \tau_y \xi}_{R_0^{-1} T_1}. \tag{17}$$

Clearly, this equation pins down a unique ξ, namely $\xi = \xi^* \equiv \frac{D_0}{(1+\beta)\tau_y}$, so the corresponding NE emerges as the unique CE. Intuitively, households coordinate on whatever level of spending it takes to support the assumed fiscal rule in the competitive equilibrium. Furthermore, as we raise D_0 for given τ_y, we raise ξ^*: household spending increases with a "helicopter drop" of government bonds. Along this unique CE, Ricardian equivalence thus fails despite households being Ricardian.

Link to FTPL. We will build a precise bridge between the above example and the FTPL in Section 3.5, but the main idea is easy to convey. When government debt is nominal, its real value in period 0 is no longer historically predetermined. Instead, it depends on inflation, which in turn depends on aggregate spending via the Phillips curve. As a result, aggregate spending may enter *both* sides of the government's intertemporal budget constraint, or of equation (17) in our example. This does not change the essence of how active fiscal policy selects a particular NE as the unique CE, but it may change the particular NE that supports any given active fiscal policy. Intuitively, any given boom in spending translates to more inflation and more debt erosion when the Phillips curve is steeper, so the same taxes can then be supported with a smaller boom.

Perfectly flexible prices. This section has allowed an arbitrary degree of price flexibility, except for perfectly or infinitely flexible prices ("$\kappa = \infty$"). In this knife-edge case, two fundamental properties of the New Keynesian model cease to apply: output is no longer demand-determined, and inflation is no longer tied to output via a Phillips curve. This manifests as a discontinuity of the equilibrium set at $\kappa = \infty$ and underscores, once again, that the *real* indeterminacy at the heart of the New Keynesian model is different from the *nominal* indeterminacy emphasized in Sargent and Wallace (1975).[17]

2.5 Discussion

Our solution concept and its CE counterpart disagree on whether an active fiscal authority can induce a unique equilibrium. This disagreement reflects a subtle difference in the treatment of consumer rationality. To see what we mean, note that, although equation (9) must hold in any NE and CE alike, the definition of a CE does not embed household understanding of this equation—with the opposite being true for our solution concept. It is this additional account of consumer rationality that drives our conclusions. Furthermore, as shown in Appendix B.2, this logic holds regardless of whether households know equation (9) *a priori*, as part of the economy's exogenous structure, or can infer it from knowledge of others' optimality. This makes irrelevant the debate on whether this equation is a primitive constraint or an implication of consumer optimality—which, in the infinite-horizon case, translates into a debate on whether the government's no-Ponzi condition is exogenously imposed or derived from the representative agent's transversality condition.

Similarly, our analysis finds no support for the idea that the assumption of an active fiscal policy is more appealing when government debt is nominal than when it is real. Our characterization of the set of NEs and the logic of their mapping to CEs remain unchanged as we move from real to nominal debt, as we show in Section 3.5—moving to nominal debt just changes the size of the real boom that supports active fiscal policy. Put simply, because the New Keynesian model ties inflation to real spending via a Phillips curve, active fiscal policy *has* to operate via real spending, and this is equally true whether government debt is real or nominal.

[17] In Sargent and Wallace (1975), real quantities are fixed by endowments and the nominal price level is a free variable. In the New Keynesian model, this is reversed: prices are pinned down by quantities via the Phillips curve, but now quantities are themselves indeterminate. Theorem 2 has sharpened this point by showing that, in our two-period setting, the real indeterminacy problem under sticky prices is *exactly* the same as that under rigid prices. Section 3 will extend this lesson to the canonical, infinite-horizon, New Keynesian model. Finally, whether the discontinuity at $\kappa = \infty$ reflects a limitation of the New Keynesian framework or a limitation of flexible-price models is a foundational question outside the scope of our paper. Here we are simply stressing a fundamental difference between these frameworks and, by extension, between sticky- and flexible-price approaches to the study of monetary-fiscal interactions.

The plausibility of equilibrium selection via active fiscal policy then boils down to the following question: can the government *guarantee* that households will coordinate on the requisite level of spending, such as $C_0 = C_1 = \xi^*$ in our earlier example? We interpret our results as a negative answer to this question. By virtue of their rationality, Ricardian households should be able to understand that, conditional on the others' behavior, any fiscal operation—the design of the fiscal rule $\mathscr{T}(\bullet)$, helicopter drops of government bonds, or even default—is irrelevant for their lifetime budgets. Households may therefore disregard the fiscal rule, coordinate on a different level of spending (i.e., on $\xi \neq \xi^*$ in our example), and ultimately compel the government to adjust taxes, or else default. We corroborate this interpretation in Appendix B.1 by incorporating default. Any given active fiscal rule $\mathscr{T}(\bullet)$ then becomes consistent with a continuum of equilibria, along which lower levels of household spending translate to larger haircuts on government debt.

We finally emphasize that our insights about what it takes for fiscal deficits to drive output and inflation within RANK do not depend on whether equilibrium is defined in the NE or CE sense, and thus also not on how one resolves the determinacy debate. Since fiscal deficits have zero net wealth effects in equilibrium, it *has* to be that any deficit-driven boom is entirely self-sustained. Put differently, an infinite multiplier must be compensating for the zero direct effect.

Next steps. The remainder of the paper will achieve three purposes. First, we will show that all the lessons of this section extend seamlessly to a more general RANK setting, featuring infinite horizons, general Phillips curves, and nominal debt (Section 3). Second, we will propose a new and, in our assessment, more credible way of dealing with equilibrium multiplicity (Section 4); as our refinement will be designed to arrest self-fulfilling forces, it will rule out any fiscal influences on output and inflation, will guarantee monetary dominance, and will preclude any monetary-fiscal interactions in RANK. With this backdrop, we will finally turn to what we view as the more suitable framework for thinking about monetary-fiscal interactions: one featuring non-Ricardian consumer behavior along with robustness to our refinement (Section 5).

3 The infinite-horizon economy

We extend the analysis to an infinite-horizon economy—a setting that more accurately represents the textbook New Keynesian model (e.g. Galí, 2008), the modern, sticky-price version of the FTPL (e.g., Cochrane, 2017, 2018), and the related literature on monetary-fiscal interactions. Our analysis proceeds largely in parallel to that of Section 2: we describe the environment in Section 3.1, characterize Nash equilibria in Section 3.2, and translate those to competitive equilibria in Section 3.4. Differently from before, here we further add two separate subsections, Section 3.3 and Section 3.5, connecting our analysis, respectively, to existing treatments of local and global determinacy, and to the logic and canonical predictions of the FTPL.

3.1 Environment

Time is discrete and infinite, indexed by $t \in \{0, 1, ...\}$. There is a continuum of households, who consume, save, and supply labor; a continuum of monopolistic firms, which use the households' labor to produce differentiated intermediate goods; a competitive retailer, who uses those intermediate goods to produce the single final good, which in turn is consumed by the households; and a government, which sets interest rates and supplies the only asset in the economy. That asset is a risk-free real bond in our baseline specification and a nominal bond in the extension of Section 3.5. For clarity, we finally abstract from aggregate risk and study how perfect-foresight equilibria depend on the initial level of public debt, the timing of taxes, and the fiscal and monetary rules.[18]

The remainder of this section describes all model parts in detail. Zooming out, the key insight will be that the heart of the model continues to be the fact that output is demand-determined. This will give rise to basically the same feedback loop between lifetime spending and permanent income as that articulated in our two-period setting, now simply translated to the infinite-horizon case and also augmented with a more flexible Phillips curve.

Households. Household i has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [U(C_{i,t}) - V(L_{i,t})],$$

and its intertemporal budget constraint is

$$\sum_{t=0}^{\infty} Q_t C_{i,t} \leq D_0 + \sum_{t=0}^{\infty} Q_t \left(Y_{i,t} - T_t\right), \tag{18}$$

where $C_{i,t}$, $L_{i,t}$, and $Y_{i,t}$ are consumption, labor supply and income in period t, Q_t is defined by $Q_0 \equiv 1$ and $Q_t \equiv \prod_{s=0}^{t-1} R_s^{-1} \forall t \geq 1$, with R_s being the real interest rate between periods s and $s+1$, D_0 is the initial level of government debt (and so also private wealth), T_t are the uniform real taxes paid in period t, and finally the functions $U(\bullet)$, $V(\bullet)$ satisfy the same properties as in Section 2.[19] The household's total income is $Y_{i,t} = W_t L_{i,t} + X_t$, where W_t is the real wage and X_t are firm profits distributed uniformly as dividends. To simplify, we let labor supply be intermediated and allocated uniformly by labor unions,

[18]The monetary and fiscal disturbance sequences, introduced below and denoted by $\boldsymbol{\epsilon}^m$ and $\boldsymbol{\epsilon}^f$, are fully realized and observed by all agents at the beginning of period 0; there is no further uncertainty thereafter. By the well-known equivalence between linearized perfect-foresight transition paths and first-order perturbation solutions of stochastic economies, our conclusions also immediately extend to the question of whether equilibrium outcomes respond to fiscal shocks in a stochastic extension.

[19]Implicit in equation (18) is the requirement that the infinite sums converge. We will verify this condition in our upcoming analysis.

so that $L_{i,t} = L_t$ for all i, t. It then also follows that all households make the same income, $Y_{i,t} = Y_t$.

Firms. The final good is produced by the usual CES composite of a continuum of intermediate goods, with each intermediate good being supplied by a monopolistic firm. For our main analysis, we make the following reduced-form assumptions about the supply side of the economy. First, all firms operate the same technology, produce the same quantity, hire the same amount of labor, and set the same price. Second, this price adjusts mechanically according to a general Phillips curve:

$$\Pi_t = \mathscr{P}_t\left(\{Y_s\}_{s=0}^{\infty}\right) \ \forall t \geq 0, \tag{19}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate between $t-1$ and t (with P_{-1} normalized to 1), and $\mathscr{P}_t(\bullet)$ is an arbitrary, possibly time-varying, function. Finally, the amount of labor hired by the typical firm satisfies $L_t = \mathscr{L}_t(Y_t, \Pi_t)$, for some function $\mathscr{L}_t(\bullet)$. We furthermore continue to abstract, as in our two-period setting, from any physical upper bound on labor supply and output, as well as from backward bending Phillips curves (which would make $\mathscr{P}$ a correspondence instead of a function). Using boldface to denote sequences, we finally rewrite our Phillips curve more concisely as

$$\Pi = \mathscr{P}(\mathbf{Y}). \tag{20}$$

Before proceeding, we conclude our description of the firm and pricing block with further discussion of the role of our reduced-form assumptions, and of limiting flexible-price outcomes.

Appealingly, treating the functions $\mathscr{P}_t(\bullet)$ and $\mathscr{L}_t(\bullet)$ as exogenous primitives allows our analysis to simultaneously be consistent with a range of different possible assumptions on microfoundations. First, our particular reduced-form structure readily nests a standard Rotemberg-style setting in which firms pay a (labor) cost to adjust prices; one could readily extend the setting of Section 2.3 to an infinite-horizon counterpart. While that microfoundation yields inflation as a function of output *and real interest rates*, we in (19) drop the latter as we will anyway later specify interest rates as a function of economic activity.[20] Second, relative to such standard microfoundations, our formulation allows the Phillips curve to be more flexible, featuring a dependence on both past and future output—as, for example, in the so-called hybrid NKPC. Similarly, our formulation can be consistent with nominal rigidities in either prices or wages, as all that matters for our purposes is that output is demand-determined. In particular, switching between price and wage rigidities naturally changes the cyclicality of the real wage and thus the *split* of total income between wages and dividends, but this split is inconsequential with a representative consumer. In summary, our assumptions on the supply side at the same time allow us to be appealingly general and yet materially simplify our up-

[20]The labor function $\mathscr{L}_t(\bullet)$ features output directly because of production, and inflation because of price adjustment costs. A similar relation would hold with a Calvo microfoundation, just with current and past inflation reducing the economy's effective total factor productivity by distorting relative prices and thereby the cross-sectional allocation of labor.

coming game representation by letting the households be the only explicit players, just as we did in the two-period, Phillips-curve setting of Section 2.3.

For future reference, we will also define the flexible-price output level corresponding to our economy. We define that output level as the solution to the equation

$$U'(Y^*) = \mu V'(Y^*),$$

for some $\mu \geq 1$ that captures the monopoly distortion (net of any corrective subsidy). The Phillips curve (19) is such that this output level is consistent with zero inflation, i.e., if $Y_t = Y^* \forall t$, then (19) returns $\Pi_t = 1 \forall t$. This flexible-price output level will play a key role in our connection to prior work in Section 3.3 and in our refinement in Section 4.

Government. Echoing our two-period analysis, we assume that the government sets the paths for real interest rates as well as taxes (or surpluses). The monetary policy rule is

$$\mathbf{R} = \mathscr{R}\left(\mathbf{Y}, \boldsymbol{\epsilon}^m\right), \tag{21}$$

where the variable $\boldsymbol{\epsilon}^m$ captures possible exogenous variation in monetary policy.[21] (21) is our generalized version of a Taylor-type rule for monetary policy, and can be interpreted in two interchangeable ways. Most literally, we can let the monetary authority directly set the real rate of return on government bonds. And less literally, we can introduce a nominal bond (in zero net supply here, or in positive net supply in Section 3.5 below) and let a monetary authority set the *nominal* rate on this bond according to $\mathbf{I} = \mathscr{I}(\Pi, \mathbf{Y}, \boldsymbol{\epsilon}^m)$, for some Taylor rule $\mathscr{I}(\bullet)$; together with the Phillips curve, this translates to $\mathbf{R} = \mathscr{R}(\mathbf{Y}, \boldsymbol{\epsilon}^m)$, for a unique $\mathscr{R}(\bullet)$ that is a composite function of $\mathscr{I}(\bullet)$ and $\mathscr{P}(\bullet)$, so it is now *as if* the real rate is set as a function of the state of the economy. Finally, we do not a priori impose the Taylor principle or any other restriction on $\mathscr{I}(\bullet)$ or $\mathscr{R}(\bullet)$.[22]

Next, the fiscal rule for taxes is

$$\mathbf{T} = \mathscr{T}\left(D_0, \mathbf{Y}, \mathbf{R}, \boldsymbol{\epsilon}^f\right), \tag{22}$$

where $\boldsymbol{\epsilon}^f$ now captures possible exogenous variation in fiscal policy. This rule is our generalized version of a Leeper rule for fiscal policy, herein stated in a sequence form (instead of a recursive form). For the time being, we impose no restriction on $\mathscr{T}$. In particular, we note that the flow constraint

$$D_{t+1} = R_t(D_t - T_t) \tag{23}$$

does not restrict the set of possible $\mathscr{T}$, simply because for any path of T_t one can construct the path

[21]Throughout the remainder of the paper we maintain $\epsilon_t^m > 0$ for all t.

[22]We only preclude fiscal variables such as the initial debt D_0 and the sequence of deficits from entering into $\mathscr{R}(\bullet)$. As discussed in Section 4.3, relaxing this assumption may capture a central bank that cares *directly* about fiscal conditions, or that yields to political pressure to accommodate fiscal deficits. This, however, is not the mechanism in Leeper (1991), Cochrane (2011, 2023), and all the literature we are concerned with.

of D_t using the above equation. A non-trivial restriction obtains if we add the familiar no-Ponzi condition, i.e., that $\lim_{t\to\infty}(\prod_{s=0}^{t-1} R_s^{-1})D_t = 0$. One can then arrive at the relation

$$D_0 = \mathrm{NPV}(\mathbf{T}, \mathbf{R}), \tag{24}$$

where the operator NPV($\bullet$) returns present discounted values, i.e., $\mathrm{NPV}(\mathbf{X}, \mathbf{R}) \equiv \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1} R_s^{-1}\right) X_t$. As noted earlier, the literature has debated whether equation (24) is a primitive constraint or an equilibrium restriction (Cochrane, 2005, 2023), and Appendix B.2 addresses this issue. Briefly, what matters is that rational households understand the validity of equation (24), not how they arrive at it.

Equilibria. The rest of this section characterizes the equilibria of our infinite-horizon economy, proceeding in the same fashion as in Section 2: we first study Nash equilibria; we then relate our solution concept to the competitive equilibrium counterpart used in the literature.

3.2 Game representation and Nash equilibria

At the heart of our solution concept is the same elementary assumption about rationality as in our two-period analysis: rational households understand that their bond holdings and tax obligations cancel, as well as that their employment and income are determined by aggregate demand. It follows that our infinite-horizon economy translates to the following game.

Game Representation. *There is a continuum of players, corresponding to the households. Household i's action is the consumption plan $\mathbf{C}_i = \{C_{i,t}\}_{t=0}^{\infty} \in \mathbb{R}_{++}^{\infty}$. Letting $\mathbf{C} = \{C_t\}_{t=0}^{\infty}$ be the corresponding average action (with $C_t \equiv \int C_{i,t} di$ for all t), household i's payoff is*

$$\mathscr{U}\left(\mathbf{C}_i, \mathbf{C}, \boldsymbol{\epsilon}^m\right) \equiv \sum_{t=0}^{\infty} \beta^t U\left(C_{i,t}\right) - \mathbb{I}\left[\mathrm{NPV}\left(\mathbf{C}_i, \mathscr{R}\left(\mathbf{C}, \boldsymbol{\epsilon}^m\right)\right), \mathrm{NPV}\left(\mathbf{C}, \mathscr{R}\left(\mathbf{C}, \boldsymbol{\epsilon}^m\right)\right)\right], \tag{25}$$

where $\mathbb{I}[\bullet,\bullet]$ is the penalty indicator defined in Footnote 11, adapted to the infinite horizon.

This game representation echoes that of Section 2.2: the first term on the right-hand side of (25) is the household's discounted utility from consumption; the two arguments of the penalty impose the household's lifetime budget constraint by comparing the present discounted values of spending and income. Throughout the infinite-horizon analysis, we normalize each household's payoff by omitting labor disutility, which, conditional on the aggregate consumption path $\mathbf{C}$, is invariant to each household's own action and therefore does not affect its best response. Leveraging this representation, and defining Nash equilibria as the fixed points of the corresponding best response (i.e., exactly as in Definition 1), we reach the following result, which extends Theorems 1 and 2 to the present context.

Theorem 4. *The economy's Nash equilibria have the following structure.*

1. $(\mathbf{C}_i)_{i\in[0,1]}$ *is a Nash equilibrium if and only if* $\mathbf{C}_i = \mathbf{C}$ *for all* i *and* $\mathbf{C}$ *satisfies the aggregate Euler equation*

$$U'(C_t) = \beta\mathscr{R}_t(\mathbf{C},\boldsymbol{\epsilon}^m)U'(C_{t+1})\ \forall t, \tag{26}$$

along with the restriction $\mathrm{NPV}\big(\mathbf{C},\mathscr{R}\big(\mathbf{C},\boldsymbol{\epsilon}^m\big)\big) < \infty$.

2. *Suppose that* $\mathscr{R}_t(\bullet)$ *is continuous and uniformly bounded away from* 0 *and* ∞ *for all* t*, and that* $\mathscr{R}_t(\bullet) = \beta^{-1}$ *for* $t \geq T$ *where* T *is arbitrarily high. Then, there is a continuum of Nash equilibria: for each* $\xi \in \mathbb{R}_{++}$*, there exists an equilibrium with* $\mathrm{NPV}(\mathbf{C},\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)) = \xi$.

The core insights of our two-period analysis thus extend seamlessly to the present, richer, infinite-horizon setting. First, the Euler equation is again not only necessary but also sufficient for characterizing aggregate equilibrium outcomes, subject to a minor qualification: for the household's problem to be well defined, permanent income must be finite, which yields $\mathrm{NPV}\big(\mathbf{C},\mathscr{R}\big(\mathbf{C},\boldsymbol{\epsilon}^m\big)\big) < \infty$ as an additional equilibrium condition.[23] The transversality condition is instead redundant, for the reason anticipated above: the intertemporal budget constraint is automatically satisfied for any path that solves the Euler equation and has a finite present value. Second, the equilibrium set is invariant to the fiscal rule $\mathscr{T}(\bullet)$, the initial debt level D_0, and the fiscal disturbance $\boldsymbol{\epsilon}^f$, simply because all of these objects are payoff-irrelevant. By contrast, the equilibrium set naturally depends on the monetary rule $\mathscr{R}(\bullet)$ and the monetary shock $\boldsymbol{\epsilon}^m$. It follows that fiscal variables can serve *only* as sunspots (i.e., coordination devices), whereas monetary policy can matter *both* as a fundamental (i.e., via payoffs) and as a sunspot. We will isolate the fundamental role of monetary policy in Section 4. Finally, the economy continues to admit multiple equilibria and for the same reason as before: income is demand-determined, and the lifetime MPC equals one, so the effective strategic complementarity is also one.

The logic is yet again easiest to see if the real rate is pegged at β^{-1} throughout. Pick any $C > 0$ and suppose that all households $j \neq i$ spend $C_t = C$ in all periods; then, household i's permanent income is $Y_i^{\mathrm{perm}} \equiv (1-\beta)\sum_{s=0}^{\infty}\beta^s Y_s = C$ and its best response is just to spend $C_{i,t} = Y_i^{\mathrm{perm}} = C$ in all periods, which closes the loop. To prove part 2 of Theorem 4, suppose now that the real rate is pegged at β^{-1} only after some finite date T. Households can coordinate on an arbitrary level of spending at $t \geq T$ by the above logic; but different anticipated paths of aggregate spending after T imply different levels of permanent income already before T, thereby supporting multiple equilibria throughout.

Finally, it is worth noting that the assumption in part 2 of Theorem 4 that $\mathscr{R}_t(\bullet) = \beta^{-1}$ for $t \geq T$ is a condition that facilitates the proof under general monetary rules and utility specifications, rather than an essential ingredient of the result. In Section 3.3, we specialize to the standard isoelastic utility

[23] With infinite horizons, this condition cannot be taken for granted. It is, however, satisfied for *all* solutions of the Euler equation if $\mathscr{R}(\bullet)$ is such that the real rate stabilizes *eventually* at β^{-1} regardless of $\mathbf{C}$, as assumed in part 2 of Theorem 4.

and Taylor-type rule used in the applied literature, and show that the same multiplicity conclusion obtains without this restriction; this specialization also allows us to connect our analysis to the applied literature on local and global determinacy.

Multiple equilibria vs. multiple steady states. The preceding argument for the case in which $\mathscr{R}_t(\bullet) = \beta^{-1}$ shows immediately that the New Keynesian model admits a continuum of steady states: because in this model output is *always* demand-determined, output can depart from the flexible-price benchmark not only in the short to medium run but also in the long run. We further elaborate on this point below, where we connect our analysis to conventional treatments of equilibrium determinacy, and we return to it again in Section 4, where we arrest multiplicity by appropriately anchoring household expectations to the unique *flexible-price* steady state.

3.3 Local vs global indeterminacy, and the Taylor principle

Theorem 4 established the multiplicity of Nash equilibria without reference to the Taylor principle. This conclusion contrasts with the role this principle plays in textbook treatments and a vast applied literature. We here explore the connection between the two, substantiating our claim that the income-spending complementarity is *the* source of multiplicity in the New Keynesian framework.

Adjusted environment. To speak as transparently as possible to standard practice in the prior literature, we specialize our environment in three ways. First, we let preferences take the familiar power form, namely $U(C) = \frac{C^{1-1/\sigma}-1}{1-1/\sigma}$, thus delivering the following Euler equation

$$C_t = (\beta R_t)^{-\sigma} C_{t+1},$$

where $\sigma > 0$ is the elasticity of intertemporal substitution. Second, we let the monetary authority set the nominal rate according to the following Taylor-type rule:

$$I_t = \beta^{-1} \Pi_{t+1}^{\psi} \epsilon_t^m,$$

where $\psi \in \mathbb{R}$ parameterizes the policy reaction to inflation and ϵ_t^m captures exogenous variation in policy, as before. And third, we assume that the Phillips curve takes the following special form:

$$\Pi_{t+1} = (Y_t / Y^*)^{\kappa},$$

where $Y^* > 0$ and $\kappa > 0$ represent, respectively, the flexible-price steady state and the slope of the Phillips curve. In this special case, the real interest rate rule becomes

$$R_t = \beta^{-1} (Y_t / Y^*)^{\phi} \epsilon_t^m,$$

with $\phi \equiv (\psi - 1)\kappa$. Zooming out, we note that we make these assumptions because they allow our model to mimic the textbook, three-equation version of the New Keynesian model, but without requiring a local approximation—our analysis remains global.

Revisiting equilibrium multiplicity. We now arrive at the following variant of Theorem 4.

Proposition 2. *Suppose that* $U(C) = \frac{C^{1-1/\sigma}-1}{1-1/\sigma}$ *and* $\mathscr{R}_t(\bullet) = \beta^{-1}(Y_t/Y^*)^{\phi}\epsilon_t^m$*, with* $\sigma \neq 1$*,* $\sigma\phi > -1$*, and* $\epsilon_t^m = 1$ *for all* $t \geq T$*, where* T *is finite but may be arbitrarily large.*[24] *There is a continuum of Nash equilibria, supporting a continuum of values for* $\mathrm{NPV}(\mathbf{C}, \mathscr{R}(\mathbf{C}))$.

Exactly as in Theorem 4 we see that there is equilibrium multiplicity—again in the strong sense of a continuum of equilibria supporting a continuum of values of $\mathrm{NPV}(\mathbf{C}, \mathscr{R}(\mathbf{C}))$—independently of the Taylor rule coefficient ϕ. In fact, generalizing Theorem 4, we here do not assume that $\mathscr{R}_t(\bullet) = \beta^{-1}$ for $t \geq T$ where T is arbitrarily high; the Taylor-type rule now is followed forever, and yet the global multiplicity conclusions survive. The remainder of the section comments further on the role of ϕ.

The role of ϕ. We consider three cases in turn: $\phi < 0$ (i.e., passive monetary policy), $\phi > 0$ (i.e., active monetary policy), and $\phi = 0$. In the first case, which the literature customarily associates with indeterminacy, the Euler condition admits a continuum of solutions, all of which converge to Y^*. And since all these solutions trivially deliver a finite permanent income, they all correspond to (Nash) equilibria. While thus all equilibria share the same steady state, they are supported by the very spending feedback mechanism we stressed before, in the formal sense that there is a one-to-one mapping between paths of spending and equilibrium values of permanent income (see Appendix C). In other words, the *entire* multiplicity in this familiar case is actually due to the feedback between lifetime spending and permanent income that we stress.[25]

The second case, an active monetary reaction ($\phi > 0$), is typically associated with *local* determinacy. Analogously, in our global analysis here, only one solution to the Euler condition converges to Y^*—the unique bounded, or locally determinate, equilibrium considered in applied work. The Euler equation nevertheless also admits a continuum of other solutions that do not converge to Y^* and yet deliver finite permanent income, and are thus proper equilibria. Among these "unbounded" equilibria, those featuring consumption diverging to infinity (when $\sigma < 1$) stretch our simplifying assumption that supply can be arbitrarily high; those converging to zero (when $\sigma > 1$), however, do not have this problem and translate to self-fulfilling liquidity traps once the zero lower bound on nominal rates is taken into account (Benhabib et al., 2001). Furthermore, all these equilibria become bounded

[24]The first two restrictions ($\sigma \neq 1$ and $\sigma\phi > -1$) rule out knife-edge cases; the last restriction, $\epsilon_t^m = 1$ for all $t \geq T$, merely sharpens the exposition by requiring the disturbance to return permanently to its no-shock value after a finite date.

[25]As already previewed in Footnote 13, outside the passive-policy case ($\phi < 0$) considered here, an additional type of multiplicity is possible: multiple paths of spending can be consistent with the same permanent income, due to non-monotone or non-linear feedbacks between real spending and real rates. While this possibility is allowed in Theorem 4, it is tangential to our purposes and irrelevant for understanding how fiscal dominance works in the existing literature. See Appendix C for a detailed discussion.

if the economy's horizon is finite (as in Section 2) or if the monetary authority switches to stabilizing the real rate at β^{-1} at long horizons (as in Theorem 4). It follows that the familiar distinction between local and global determinacy is in fact an artifact of auxiliary assumptions; ultimately, the feedback between lifetime spending and permanent income in a fully demand-determined economy remains the fundamental source of equilibrium multiplicity.[26]

The third case, $\phi = 0$, corresponds to a time-varying peg for the real rate, and is closest in spirit to our (eventual) assumptions on monetary policy in Theorem 4. In that case there is a continuum of bounded equilibria, each one centered around a different steady state, illustrating the mechanism common to all three cases: a continuum of equilibria is possible because, and only because, of the feedback between lifetime spending and permanent income.

Takeaway. The central takeaway of this section along with Section 3.2 is that, in the New Keynesian model, the intertemporal strategic complementarity in private spending is the common mechanism behind all of the following: a continuum of steady states; a continuum of equilibria under a time-varying peg for the real interest rate; and, under a more standard feedback rule for monetary policy, a continuum of equilibria that either converge asymptotically to the flexible-price steady state (when the Taylor principle fails) or diverge away from it (when the Taylor principle holds). Echoing our earlier two-period analysis, below we show how active fiscal policy leverages this multiplicity. In Section 4 we then introduce an equilibrium refinement that, instead, removes this multiplicity.

3.4 From Nash equilibria to competitive equilibria

We now turn to the mapping between Nash and competitive equilibria. Our analysis proceeds exactly in parallel to Section 2.4.

CE definition. Let $\mathscr{C}(D_0, \mathbf{Y}, \mathbf{T}, \mathbf{R})$ denote the households' optimal consumption path as a function of initial private wealth and of the paths of income, taxes, and real interest rates. We can then adapt our earlier definition of competitive equilibrium to the present setting.

Definition 4. A collection of $\left((\mathbf{C}_i)_{i\in[0,1]}, \mathbf{Y}, \Pi, \mathbf{R}, \mathbf{T}\right)$ is a *competitive equilibrium* (CE) in the infinite-horizon economy if:

1. Households optimize over consumption plans $\mathbf{C}_i \in \mathbb{R}^{\infty}_{++}$, with $\mathbf{C}_i = \mathscr{C}(D_0, \mathbf{Y}, \mathbf{T}, \mathbf{R})$ for all i.
2. The output market clears, $\mathbf{Y} = \mathbf{C} \equiv \int \mathbf{C}_i di$, and inflation satisfies the Phillips curve $\Pi = \mathscr{P}(\mathbf{Y})$.
3. The monetary and fiscal policy rules hold, $\mathbf{R} = \mathscr{R}(\mathbf{Y}, \boldsymbol{\epsilon}^m)$ and $\mathbf{T} = \mathscr{T}\left(D_0, \mathbf{Y}, \mathbf{R}, \boldsymbol{\epsilon}^f\right)$.

[26]See Appendix C for a more detailed analysis of these unbounded equilibria. In Section 4, we show that our refinement provides a real anchor and delivers a unique equilibrium without imposing the Taylor principle.

Condition 1 is household optimality, and so it embeds both the Euler equation and the transversality condition, i.e., the infinite-horizon counterpart of not leaving wealth unconsumed at the end of one's lifetime. Conditions 2 and 3, which impose output market-clearing, the Phillips curve, as well as the policy rules, are similarly direct analogues of our definitions in the two-period analysis. Like its two-period counterpart, Definition 4 does *not* presume equation (24), accommodating the FTPL view that this equation is an equilibrium implication instead of a primitive constraint.[27]

CEs vs. NEs. We next turn to the mapping between Nash equilibria and competitive equilibria.

Proposition 3. *A collection* $\left((\mathbf{C}_i)_{i\in[0,1]}, \mathbf{Y}, \Pi, \mathbf{R}, \mathbf{T}\right)$ *is a competitive equilibrium (CE) if and only if:*

1. $(\mathbf{C}_i)_{i\in[0,1]}$ *is a Nash equilibrium (NE).*
2. $\mathbf{Y} = \mathbf{C}$, $\Pi = \mathscr{P}(\mathbf{C})$, *and* $\mathbf{R} = \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)$.
3. $(\mathbf{Y}, \Pi, \mathbf{R}, \mathbf{T})$ *jointly satisfy* $\mathbf{T} = \mathscr{T}\left(D_0, \mathbf{Y}, \mathbf{R}, \boldsymbol{\epsilon}^f\right)$ *and equation* (24).

The set of CEs can thus be constructed in the same two steps as before: first, identify all NEs; then, keep those that satisfy equation (24) under the given fiscal rule $\mathscr{T}(\bullet)$, and discard the rest. As we have seen, the fiscal rule does not enter the characterization of the set of NEs, since rational households understand that government bonds are not net wealth. Fiscal rules can, however, play an equilibrium selection role: a given fiscal rule can be consistent with a CE—or equivalently, it can be supported by an NE—only if equation (24), the government's intertemporal solvency condition, holds. Depending on the fiscal rule, it is therefore possible that all, some, or none of the economy's NEs translate to CEs.

Active fiscal policy as equilibrium selection. To make further progress on the equilibrium selection role of fiscal policy, we yet again introduce the concepts of passive and active fiscal rules, now for our infinite-horizon setting.

Definition 5. In the infinite-horizon economy, fiscal policy is passive if $\mathscr{T}(\bullet)$ is such that equation (24) holds for all $(D_0, \mathbf{Y}, \mathbf{R}, \boldsymbol{\epsilon}^f)$. Otherwise, fiscal policy is active.

With infinite horizons, our definition of passive fiscal rules is more permissive than that in Leeper (1991) and better aligned with that in Cochrane (2005, 2023): the essential issue is not whether government debt stabilizes and remains bounded, but whether the government is solvent in the present value sense as in equation (24). A passive fiscal rule always satisfies this requirement, and thus in particular for every NE; an active rule may fail this requirement for some, or even all, NEs. We thus find that the two-period Theorem 3 on NEs and CEs under different fiscal rules extends verbatim:

[27] To see how equation (24) can be derived as an implication of any CE, take the household's intertemporal budget constraint and rewrite it as $D_0 \geq \mathrm{NPV}(\mathbf{C}_i - \mathbf{Y} + \mathbf{T}, \mathbf{R})$. Optimality requires that this constraint holds with equality for each household. Using this fact and aggregating across households yields $D_0 = \mathrm{NPV}(\mathbf{C} - \mathbf{Y} + \mathbf{T}, \mathbf{R})$. Together with market clearing ($\mathbf{C} = \mathbf{Y}$), this gives $D_0 = \mathrm{NPV}(\mathbf{T}, \mathbf{R})$, which is equation (24).

Theorem 5. *A CE always translates to an NE, but the converse depends on the fiscal rule:*

1. *When fiscal policy is passive, every NE translates to a CE.*
2. *For each NE, there exists an active fiscal rule $\mathscr{T}(\bullet)$ such that this NE translates to the unique CE.*

Under the CE concept, active fiscal rules thus play the same equilibrium selection role as in the two-period framework: this selection amounts to households coordinating on one out of many possible self-sustaining paths of aggregate spending and income. In Section 4 we will argue against the plausibility and robustness of this selection; first, however, we in the remainder of this section illustrate this selection under particular fiscal rules, making further clear the connections between our treatment, the FTPL, and the existing literature on monetary-fiscal interactions.

3.5 Revisiting fiscal dominance

We use two examples to illustrate the equilibria selected by active fiscal policy. This analysis allows us to complete the bridge between our discussion and the familiar predictions of the FTPL.

The "Fiscal Theory of Output". We begin with a stark example similar to that considered at the end of Section 2.4: government debt is real, real interest rates are held fixed, and inflation enters neither the government budget nor the determination of equilibrium output. At first glance, this example may appear disconnected from the FTPL and the related literature on monetary-fiscal interactions—literatures in which government debt is nominal, interest rates move, and inflation takes center stage. It will turn out, however, that the connection is in fact close, and the example is instructive.

Formally, we assume that the monetary authority pegs the real interest rate to $R_t = \beta^{-1}$ at all dates t, and that the fiscal authority adopts the following active fiscal rule:

$$T_t = -\epsilon_t^f + \tau_y Y_t, \quad \forall t, \tag{27}$$

where $\tau_y \in (0,1)$ is an exogenous scalar and $\boldsymbol{\epsilon}^f = \{\epsilon_t^f\}_{t=0}^{\infty}$ is an exogenous sequence of fiscal (transfer) shocks. In particular, τ_y can be interpreted as the steady-state rate of taxation, ϵ_0^f as an unexpected date-0 lump-sum transfer, and $\{\epsilon_t^f\}_{t=1}^{\infty}$ as date-0 news about future lump-sum transfers.[28] Given these assumptions, the government's intertemporal budget constraint (24) reduces to

$$D_0 + \tilde{\epsilon}_0^f = \tau_y \sum_{t=0}^{\infty} \beta^t Y_t, \tag{28}$$

[28] If the steady-state tax rate τ_y is distortionary, then the flexible-price steady-state output level must be adjusted accordingly, to the unique solution of $\mu V'(Y^*) = (1-\tau_y)U'(Y^*)$. A similar point applies if the fiscal shocks are distortionary: these shocks may then enter the Phillips curve; but unless the monetary authority conditions the real interest rate on them (either directly or by responding to inflationary pressures), these shocks will still not enter the determination of output. In any event, these possibilities are absent from the related literature and orthogonal to our purposes.

where $\tilde{\epsilon}_0^f \equiv \sum_{t=0}^{\infty} \beta^t \epsilon_t^f$ captures the deficit shock in present-value terms (with $D_0 + \tilde{\epsilon}_0^f > 0$). At the same time, since the real rate is pegged at β^{-1}, we know that the set of NEs corresponds to $Y_t = C_t = \xi$ for all t and for arbitrary $\xi > 0$. We conclude that (28) is satisfied in an NE if and only if ξ solves

$$D_0 + \tilde{\epsilon}_0^f = \tfrac{1}{1-\beta} \tau_y \xi. \tag{29}$$

A positive deficit shock (higher $\tilde{\epsilon}_0^f$) is thus equivalent to a helicopter drop of government bonds (higher D_0) and triggers a *permanent* increase in equilibrium output (higher ξ).

What accounts for these stimulative effects of transfers? By virtue of rational expectations, households understand both that fiscal policy is payoff-irrelevant and that they can coordinate on multiple self-sustaining levels of aggregate spending and income, formalized here as multiple NEs and indexed by ξ. However, for such an outcome to support the assumed fiscal policy and translate to a CE, it must be that ξ solves equation (29). In words, of all of the possible self-sustaining levels of output, there is only one that generates the required fiscal revenue to substitute for missing taxes. We therefore term this model the "Fiscal Theory of Output" (FTY)—the only lever available to ensure fiscal balance is movements in output, so fiscal policy pins down the output level.

We finally note that, while here active fiscal policy selects a particular equilibrium level of output, this has immediate implications for inflation, through the Phillips curve $\mathscr{P}(\bullet)$. The Fiscal Theory of Output presented here is thus also a theory of the price level, though the implied mapping from fiscal deficits to inflation is different from that in the FTPL literature and does not hinge on debt being nominal. That more familiar mapping is what we turn to next.

The Fiscal Theory of the Price Level. We maintain our previous assumptions about monetary and fiscal policy, but now allow for a portion (or perhaps all) of government debt to be nominal. The real value of government debt is then given by $D_0 \equiv B_0^{nom}/P_0 + B_0^{real}$, where B_0^{nom} and B_0^{real} are the predetermined (as of $t = 0$) quantities of nominal and real one-period bonds, respectively, and P_0 is the initial price level. Rewriting in terms of inflation, we have

$$D_0 = \left(\nu \Pi_0^{-1} + 1 - \nu\right) \bar{D}_0,$$

where $\bar{D}_0 \equiv B_0^{nom}/P_{-1} + B_0^{real}$, $\nu \equiv B_0^{nom}/(P_{-1}\bar{D}_0)$, and $P_{-1} \equiv 1$ are all predetermined. This relation simply shows that, if government debt is at least partially nominal, then higher inflation has direct budgetary implications, reducing D_0 by eroding the real value of nominal bonds.

By the payoff irrelevance of fiscal policy, switching from real to (partially) nominal debt leaves the set of NEs entirely unchanged. What changes is only the mapping from NE to CE, that is, the answer to the following question: which is the particular NE that supports the given active fiscal policy? To see the logic as transparently as possible, we will specialize our general assumptions on the Phillips curve to the simple static special case $\mathscr{P}_t(\mathbf{Y}) = (Y_t/Y^*)^\kappa$, i.e., a static Phillips curve with slope $\kappa > 0$.

A given NE associated with an output level ξ thus now maps to inflation of $(\xi/Y^*)^\kappa$, and so for the intertemporal government budget constraint to be satisfied ξ must now solve

$$\left(\nu(\xi/Y^*)^{-\kappa}+1-\nu\right)\bar{D}_0+\tilde{\epsilon}_0^f=\tfrac{1}{1-\beta}\tau_y\xi. \tag{30}$$

In words, households coordinate their spending on the exact level ξ that balances the government budget through the combination of an expansion in the tax base and a reduction in the real value of public debt. Our FTY is nested at one extreme, with $\nu=0$, $\tau_y>0$, and the entirety of the adjustment taking place through the tax base. At the other extreme, with $\nu=1$, $\tau_y=0$, and $\tilde{\epsilon}_0^f<0$ (positive primary surpluses), we get a version of the FTPL in which the entirety of the adjustment takes place through debt erosion. In short, our FTY and the modern, sticky-price version of the FTPL are two sides of the same coin, translating the equilibrium selection logic of Section 3.4 into particular predictions for equilibrium output and inflation.

Variable rates and regime switches. Our two examples assume that the monetary authority pegs the real rate of interest at $R_t=\beta^{-1}$ regardless of the state of the economy, and that the fiscal authority follows the same active rule forever. The literature departs from these stark assumptions, allowing for endogenous movements in real rates and/or for stochastic switches in the policy regime. These modifications clearly matter for the exact properties of equilibrium paths; for example, with a strictly passive monetary rule (captured by $\phi<0$ in the log-linear context of Proposition 2), positive innovations in fiscal deficits are accompanied by lower real rates and thus induce *transitory* booms (rather than permanent booms, as in our above two examples). The essence, however, remains unchanged: fiscal policy is always payoff-irrelevant; any fiscally-led boom thus *has* to be self-sustaining; interest rates themselves would not have changed if households had instead coordinated on the NE that keeps spending and income unchanged; and the fiscal authority would have then been compelled to adjust taxes or default.[29]

4 Anchoring far-ahead beliefs

In the preceding two sections, we provided a novel perspective on equilibrium indeterminacy in the New Keynesian model, and its link to steady state multiplicity; identified the intertemporal feedback between spending and income as the reason behind the indeterminacy; and argued that neither an interest rate rule nor a fiscal rule can resolve the indeterminacy.

[29]That same logic also extends to other popular model alterations, such as long-term debt and partial fiscal dominance (e.g., Bianchi and Ilut, 2017; Smets and Wouters, 2024). As we move across the various cases, what changes is the size of output, inflation, and rate changes that finance a given fiscal shortfall; what does not change, however, is the economic essence—households coordinate on a self-sustaining boom that substitutes for the missing tax adjustment.

Informed by these results, we will now propose a refinement that addresses the indeterminacy problem. Our refinement centers on a simple principle: that the New Keynesian model was designed to analyze the role of aggregate demand in the short run, with long-run outcomes instead pinned down exclusively by aggregate supply. Accordingly, as our refinement, we will require that long-run expectations are anchored to flexible-price outcomes, in a sense made precise below. We then show that this seemingly mild requirement selects a unique equilibrium. Crucially, in that equilibrium, fiscal policy has no effect on output and inflation; instead, monetary policy alone regulates these outcomes, regardless of whether the Taylor principle holds or not.

4.1 The refinement

We consider the following refinement:

Refinement 1. *Households expect the economy to stabilize at the flexible-price outcomes in* finite time*: there exists an arbitrarily large but finite H such that $Y_t = Y^*$ and $R_t = R^* (\equiv \beta^{-1})$ for all $t \geq H$.*[30]

The motivation for this refinement was anticipated above. Following a long tradition in Keynesian thinking, the New Keynesian model seeks to speak to how aggregate demand matters for *short-run* fluctuations. However, the model also allows transitory demand shocks to drive *long-run* outcomes: as we have shown, one can construct equilibria along which the economy converges to steady states other than the unique flexible-price one and can even choose this steady state as a function of the transitory shocks. In our view, this is an unintended feature of the model, which we now circumvent via Refinement 1. Moving from our perfect-foresight set-up to analogous stochastic economies, this requirement simply means that long-run *expectations* are anchored to neoclassical outcomes, regardless of how policy is conducted or what shocks hit the economy in the short run.

It is important to note that our proposed refinement requires convergence to the flexible-price outcomes *in finite time*, not asymptotically,[31] and we argue that only this stronger requirement actually represents an adequate notion of anchored beliefs. In our two-period economy of Section 2, equilibrium outcomes in the short run (captured by period 0) are supported by expectations of different equilibrium outcomes in the long run (captured by period 1). In the infinite-horizon case, requiring only asymptotic convergence to the flexible-price steady state, as done in almost all applied work, means that expectations about the far-ahead future can still play a central role in supporting short-run outcomes. Our refinement breaks this link, appropriately anchoring long-run expectations.[32]

[30] Because expected and realized paths coincide under perfect foresight, the restriction on realized outcomes in Refinement 1 amounts to a restriction on the households' expectations of those outcomes.

[31] In stochastic economies with recurring shocks, the requirement applies after every shock realization: outcomes expected more than H periods ahead must coincide with their flexible-price counterparts.

[32] Put differently, our refinement makes sure that the theory's predictions do not hinge on subtle and hard-to-test as-

4.2 Unique equilibrium

We now show how our refinement arrests the infinite loop between aggregate spending and permanent income at the heart of equilibrium multiplicity in the New Keynesian model. We start by establishing this result in a simple benchmark; we next explain the economic logic behind it; and then finally connect it with familiar textbook treatments.

Exogenous real rate path. Consider a monetary authority that sets the real interest rate as a function only of time and of exogenous shocks $\boldsymbol{\epsilon}^m$, but for now not in response to endogenous macroeconomic outcomes. This scenario is nested in our earlier multiplicity result in Theorem 4 by letting $\mathscr{R}_t(\mathbf{Y}, \epsilon_t^m)$ be invariant with respect to $\mathbf{Y}$, and in Proposition 2 by setting $\phi = 0$. We previously showed that this scenario best encapsulates the real indeterminacy at the heart of the New Keynesian model, and so we now use that same special case to transparently explain the logic of our refinement.

Theorem 6. *Let the monetary authority peg the real interest rate to $\mathscr{R}_t(\mathbf{Y}, \epsilon_t^m) = \beta^{-1}\epsilon_t^m$, with arbitrary positive ϵ_t^m for $t < T$, arbitrary $T \geq 1$, and $\epsilon_t^m = 1$ for $t \geq T$.*[33] *Next, impose Refinement 1, for any $H \geq T$. Then, there exists a unique NE. In this equilibrium, $\mathbf{Y}$ and Π depend on $\boldsymbol{\epsilon}^m$ but are independent of $\boldsymbol{\epsilon}^f$.*

As discussed previously, without the refinement, this model economy would have featured a continuum of (Nash) equilibria, supported by self-fulfilling beliefs about perpetual spending and permanent income. By anchoring expectations of income at far-ahead horizons, our refinement eliminates the scope for such self-fulfilling beliefs and delivers a unique equilibrium. Along this equilibrium, fiscal policy plays no role precisely because it is payoff irrelevant—and monetary policy matters exclusively through its direct short-run effects via the path of real rates, and not by shifting expectations of real economic activity in the (very) long run.

The logic of the refinement. To transparently see the economic intuition for how the refinement works, it is useful to specialize even further to the case of outright fixed real rates, i.e., $R_t = \beta^{-1}\ \forall t$. In that case, as we already discussed in both Sections 2.2 and 3.2, household consumption is constant over time and given by date-0 permanent income:

$$C_{i,t} = (1-\beta)\sum_{k=0}^{\infty}\beta^k Y_k\ \forall t.$$

Replacing $Y_k = C_k$ gives the economy's best response relation as

$$C_{i,t} = (1-\beta)\sum_{k=0}^{\infty}\beta^k C_k \quad \forall t. \tag{31}$$

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sumptions about expectations of the very long run responding to short-run forces. Angeletos and Lian (2023) and Angeletos et al. (2024b) offer complementary perspectives, emphasizing the role of hard-to-test assumptions about social memory (in the first paper) and policy (in the second paper).

[33] The assumption that $\epsilon_t^m = 1$ for $t \geq T$, T arbitrarily large, captures that monetary policy shocks eventually die out.

For a constant aggregate path, $C_k = C$ for all k, the lifetime *cumulative* MPC is $(1-\beta)\sum_{k=0}^{\infty}\beta^k = 1$, so the economy resembles a one-shot game in which the best response is $C_i = a+bC$ with $a = 0$ and $b = 1$, exactly as discussed in Section 2.2. Thus, just as in the left panel of Figure 1, the best response overlaps with the 45-degree line, sustaining a continuum of equilibria. Refinement 1 then fundamentally alters this logic, with (31) becoming

$$C_{i,t} = (1-\beta)\sum_{k=0}^{H-1}\beta^k C_k + \beta^H Y^* \quad \forall t.$$

It is thus now *as if*, when $C_k = C$ for all $k < H$, the best response is $C_i = a + bC$ with $a = \beta^H Y^* > 0$ and $b = (1-\beta)\sum_{j=0}^{H-1}\beta^j = 1-\beta^H < 1$. That is, the economy instead resembles the one-shot game depicted in the right panel of Figure 1: the best response is (slightly) rotated away from the 45-degree line, guaranteeing a unique equilibrium.

This exposition illustrates how the multiplicity behind Theorem 4, and by extension the feasibility of active fiscal policy, are knife-edge. Put simply, any self-sustaining boom in spending and income must last *literally* forever, or else it cannot exist.

Translation to textbook treatments. In Section 3.3, we used Proposition 2 to translate the multiplicity result of Theorem 4 to a setting that mimicked the standard, log-linearized, three-equation version of the New Keynesian model found in textbook treatments such as Galí (2008). Here, we offer the same translation for the uniqueness result of Theorem 6. This makes clear that, although that theorem assumed a real interest rate peg in order to transparently show the function of our refinement, the logic directly extends to the type of feedback rules assumed in applied work.

Proposition 4. *Under the same assumptions as in Proposition 2, but imposing Refinement 1 for arbitrary $H \geq T$, there is a unique NE, regardless of ϕ. In this equilibrium, output (and thus also inflation) depends on monetary policy, but not on fiscal policy.*

Recall that, in Proposition 2, we showed that this textbook special case of our model admitted a continuum of (Nash) equilibria regardless of whether monetary policy is passive ($\phi < 0$) or active ($\phi > 0$), and we also explained that this pervasive multiplicity was itself possible because of the feedback between spending and permanent income. Proposition 4 now shows, exactly consistent with the intuition given above, that this multiplicity disappears as soon as we impose Refinement 1: there is now a unique equilibrium, independently of whether monetary policy is active or passive. Just as was the case for fixed real rates, once we anchor expectations of income at long horizons, we reduce the effective strategic complementarity in spending and thus remove equilibrium multiplicity.[34]

[34] Appendix B.3 verifies that the same result holds in the standard linearized three-equation New Keynesian model and under a *nominal* interest rate peg.

Compared to standard practice, our refinement therefore substitutes for *both* the Taylor principle and the boundedness requirement: there is a *globally* unique equilibrium surviving our refinement, and fiscal policy is irrelevant regardless of ϕ, the slope of the Taylor rule. By the same token, if we use a "King" rule (as in King, 2000; Cochrane, 2011) to separate the equilibrium-selection and stabilization roles of monetary policy, the former role becomes entirely redundant. This in turn explains why the equilibrium obtained in Proposition 4 can be readily replicated with a time-varying peg as in Theorem 6—which is to say, once again, that this theorem contains the essence of our refinement, similarly to how Theorem 4 contained the essence of the indeterminacy problem.

Zooming out, we conclude that, at least under our proposed lens, the textbook New Keynesian approach selects the appropriate equilibrium, as imposing the Taylor principle and excluding unbounded paths is equivalent to anchoring long-run expectations, in the sense formalized by our refinement. Put simply, the conventional solution of the New Keynesian model is the only one that is faithful to the model's guiding principle discussed above.

4.3 When can fiscal deficits drive output and inflation?

By ruling out self-sustaining variations in consumer spending and thus income, our refinement precludes fiscal influences on output and inflation. This raises the following question. Consider a researcher who wishes to study deficit-driven inflations and monetary-fiscal interactions within the New Keynesian framework, in a way that satisfies Refinement 1. How can they proceed?

There are at least two options. One path is to stay in RANK, reject active fiscal policy, but let the central bank deviate from its dual mandate, either because it succumbs to political pressure or because it cares about government debt dynamics for reasons outside the model. To illustrate, take Theorem 6, where the central bank alone regulates output, and assume that ϵ_t^m correlates with the fiscal outlook. This would capture a situation where the central bank *actively* and *deliberately* cuts interest rates in order to support the fiscal authority. Another path, which is directly supported by microeconomic evidence, is to let fiscal deficits influence household spending—and become payoff-relevant in the appropriate game representation—via a classical non-Ricardian channel. We spell out this route in the next section, building a bridge to HANK.

5 A bridge to HANK

We now depart from RANK by introducing a classical failure of Ricardian equivalence, consistent with empirical evidence on consumption-savings behavior and leveraged in the recent HANK literature. Fiscal policy then naturally becomes payoff-relevant in the economy's game representation, allowing it to continue to matter under our refinement, independently of monetary policy. We begin by first describing the extended model setting, before then characterizing equilibrium outcomes—with a focus on the role of fiscal policy—under our refinement.

5.1 Setting

We first describe how the environment is altered, before then turning to the new game representation.

Model changes. To capture non-Ricardian behavior, we will modify household utility as follows:

$$\mathscr{U}_i = \sum_{t=0}^{\infty} \beta^t \left(U(C_{i,t}, A_{i,t+1}) - V\left(L_{i,t}\right)\right), \tag{32}$$

where $C_{i,t}$ and $L_{i,t}$ are date-t consumption and labor supply, $A_{i,t+1}$ is household i's asset position, or liquidity, at the beginning of date $t+1$, and per-period preferences are given as $U(C,A) = u\left(C^{1-\chi}A^{\chi}\right)$, for some $\chi \in (0,1)$ and some $u : \mathbb{R}_{++} \to \mathbb{R}$ that is strictly increasing, strictly concave, continuously differentiable, bounded, with u' satisfying $\lim_{C\to 0} u'(C) = \infty$.[35] Throughout this section, we restrict household i's admissible plans to those satisfying $A_{i,t+1} > 0$ for all t.

Interpreted literally, with those adjusted preferences, households *directly* derive utility from holding assets (see, e.g., Michaillat and Saez, 2021; Mian et al., 2025). As is well-understood, however, such settings with direct preferences over wealth serve as tractable proxies for the precautionary savings motive that arises in standard incomplete-market models (see, e.g., Kaplan and Violante, 2018). In fact, by appropriately calibrating the utility flow from assets, this setting can mimic aggregate properties of HANK models even quantitatively (Auclert et al., 2024).[36]

Our focus continues to be on how assumptions on monetary and fiscal policy shape equilibrium outcomes. For monetary policy, we continue to assume that the monetary authority sets real interest rates according to a feedback rule of the form

$$\mathbf{R} = \mathscr{R}\left(\mathbf{Y}, \boldsymbol{\epsilon}^m\right).$$

For fiscal policy, given that household *wealth* directly appears in (32), we find it convenient to express fiscal policy not as a rule for taxes, but as a rule for government debt, i.e.,

$$\mathbf{D} = \mathscr{D}(\mathbf{Y}; D_0, \boldsymbol{\epsilon}^f).$$

Clearly this translation is without loss of generality: we can readily go back and forth between a rule $\mathscr{T}(\bullet)$ for taxes and a rule $\mathscr{D}(\bullet)$ for government debt issuances using the government's per-period

[35] We can readily accommodate unbounded $u(\bullet)$ under the same penalty-indicator convention. We could thus nest, for example, a specification like $U(C,A) = (1-\chi)\ln C + \chi \ln A$.

[36] Another popular proxy for HANK is overlapping generations of households, as explored in Farhi and Werning (2019), Aguiar et al. (2023), Angeletos et al. (2024a,b, 2026), and Rachel and Ravn (2026). For the present purposes, the bonds-in-the-utility proxy is better suited because it allows for a single set of players in the upcoming game representation.

budget constraint. Since government debt is the only asset in the economy, total household wealth holdings in equilibrium will equal total government debt; and since household preferences are directly over wealth holdings, the recasting of the fiscal rule to be in debt space will prove convenient. Throughout this section, we restrict fiscal policy to debt-issuance rules whose induced public debt path is strictly positive, i.e., $\mathscr{D}_{t+1}(\mathbf{Y}; D_0, \boldsymbol{\epsilon}^f) > 0$ for all t on the domain under consideration.

The adjusted game. Households as before are assumed to understand the structure of the economy, so household i's intertemporal budget constraint can still be expressed as

$$\text{NPV}\left(\mathbf{C}_i, \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)\right) \leq \text{NPV}\left(\mathbf{C}, \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)\right),$$

while its asset position in period t is given as

$$A_{i,t+1} = \mathscr{A}_{t+1}(\mathbf{C}_i, \mathbf{C}; D_0, \boldsymbol{\epsilon}^m, \boldsymbol{\epsilon}^f) = \sum_{k=0}^{t} \left(\prod_{s=k}^{t} \mathscr{R}_s(\mathbf{C}, \boldsymbol{\epsilon}^m) \right) (C_k - C_{i,k}) + \mathscr{D}_{t+1}(\mathbf{C}; D_0, \boldsymbol{\epsilon}^f). \tag{33}$$

Intuitively, since all households receive the same income, a household's asset position may exceed the corresponding average position, which here equals the total quantity of public debt, only to the extent that this household has spent less in the past than the average household.[37] Using these facts, we can once again map the economy to a game, with the households as the players and their consumption plans as the actions. What changes, however, is the payoff structure of that game.

Game Representation. *The economy's game representation is identical to that in Section 3, except that household i's payoff is now given by*

$$\begin{aligned}\mathscr{U}\left(\mathbf{C}_i, \mathbf{C}; D_0, \boldsymbol{\epsilon}^m, \boldsymbol{\epsilon}^f\right) \equiv & \sum_{t=0}^{\infty} \beta^t U\left(C_{i,t}, \mathscr{A}_{t+1}(\mathbf{C}_i, \mathbf{C}; D_0, \boldsymbol{\epsilon}^m, \boldsymbol{\epsilon}^f)\right) \\ & - \mathbb{I}\left[\text{NPV}\left(\mathbf{C}_i, \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)\right), \text{NPV}\left(\mathbf{C}, \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)\right)\right],\end{aligned}$$

with $\mathbb{I}[\cdot,\cdot]$ *defined as in Footnote 11.*

The fundamental difference relative to our earlier analysis is that, effectively by assumption, fiscal policy is now payoff-relevant: it enters the payoff $\mathscr{U}(\bullet)$ via $\mathscr{A}(\bullet)$, the household's stock of assets. In microfounded incomplete-markets models, this payoff relevance of fiscal policy arises because a larger amount of gross government debt facilitates self-insurance.[38]

[37] To obtain (33), take the per-period household budget, $A_{i,t+1} = R_t(A_{i,t} + Y_t - T_t - C_{i,t})$; iterate it backwards up to period 0; and finally replace $Y_t = C_t$ along with equation (23) and the policy rules. Finally, note that the asset mapping $\mathscr{A}(\bullet)$—and by extension the payoff function $\mathscr{U}(\bullet)$ in our upcoming game representation—depends on the two policy rules, but we suppress this dependence to ease the notation.

[38] It is not automatically the case that, in any incomplete-markets model, increases in government debt have to increase the ability to self-insure. For example, if households' borrowing capacity decreased one-to-one with their future tax obligations, then government debt and deficits would be neutral. In our reduced-form set-up this model variant would correspond to a preference not over total wealth, but instead *net* wealth, $Z_{i,t+1}$ (defined as gross assets minus the discounted present value of the household's tax obligations), returning us to payoff irrelevance of fiscal policy.

5.2 Equilibrium characterization

We now study how moving to a setting with payoff-relevant fiscal policy affects the main lessons of the previous sections. We proceed in two steps: we first show that this modification alone generally does *not* eliminate the multiplicity of equilibria, simply because it does not alter the key ingredients—lifetime propensities to consume of one, and demand-determined output. Once the model is augmented with our refinement, however, we again obtain uniqueness, but now in the resulting equilibrium *both* fiscal and monetary policy drive output and inflation.

As the extended HANK-like model of this section features a complicated fixed-point relation between government debt, aggregate spending, and real rates, we will not attempt to characterize equilibria for arbitrary fiscal and monetary rules. Instead, we focus on specific rules that suffice for the two goals stated above: to show that payoff-relevant fiscal policy does *not* by itself eliminate equilibrium multiplicity, and that our refinement restores uniqueness while allowing fiscal and monetary policy to drive output and inflation.

Multiplicity. We begin by extending the first part of Theorem 4, showing that the economy's equilibria correspond to any solution of the—now slightly more complicated—aggregate Euler equation.

Proposition 5. $(\mathbf{C}_i)_{i\in[0,1]}$ *is a Nash equilibrium if and only if* $\mathbf{C}_i = \mathbf{C}$ *for all* i *and* $\mathbf{C}$ *satisfies the aggregate Euler equation,*

$$U_C(C_t, D_{t+1}) = \beta R_t U_C(C_{t+1}, D_{t+2}) + R_t U_A(C_t, D_{t+1}) \; \forall t, \tag{34}$$

along with $\mathbf{R} = \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)$, $\mathbf{D} = \mathscr{D}(\mathbf{C}; D_0, \boldsymbol{\epsilon}^f)$, $D_{t+1} > 0$ *for all* t, *and* $\mathrm{NPV}\big(\mathbf{C}, \mathscr{R}\big(\mathbf{C}, \boldsymbol{\epsilon}^m\big)\big) < \infty$.

(34) is the aforementioned complicated fixed-point relation between government debt, aggregate spending, and real rates. We thus now further specialize our assumptions on monetary and fiscal policy to allow for a tractable analysis. To this end, we note first that our model economy admits, under flexible prices and with a fixed positive debt-to-output ratio of $\delta > 0$, a unique steady state, with output given as the unique solution to

$$U_C(Y^*, \delta Y^*) = \mu V'(Y^*),$$

where as before $\mu \geq 1$ captures the monopoly distortion, while the interest rate is given by

$$R^* = r(\delta) \equiv \frac{1}{\beta + U_A(Y^*, \delta Y^*)/U_C(Y^*, \delta Y^*)}.$$

This relation between R^* and δ is our model's analogue of the classic Aiyagari (1994) curve: the interest rate is depressed below β^{-1}, because of the value of holding assets—or, less literally, because of the precautionary motive. As the quantity of assets increases, this value diminishes and the interest rate approaches β^{-1}. In all of the sequel we will assume that δ is in fact sufficiently large so that $R^* > 1$.[39] With this flexible-price outcome as a reference point, we are now ready to state our specialized assumptions on monetary and fiscal policy. Specifically, we will assume that both real interest rates and government debt fluctuate around this flexible-price steady state, i.e., that

$$\mathscr{R}_t(\bullet) = r(\delta)\epsilon_t^m, \quad \mathscr{D}_{t+1}(\bullet) = \delta Y_t \epsilon_t^f, \tag{35}$$

where as before $\boldsymbol{\epsilon}^m, \boldsymbol{\epsilon}^f$ are sequences of monetary and fiscal shocks, respectively. For expositional convenience, we will furthermore assume that these shock sequences are moderate, in the sense that $\frac{\epsilon_t^m}{\epsilon_t^f} < \frac{(1-\chi)\delta}{r(\delta)\chi}$, and that the shocks take positive values.[40] We now arrive at the following variant of the second part of Theorem 4, showing that equilibrium multiplicity extends without change to this richer HANK-type environment.

Proposition 6. *Suppose that monetary and fiscal policy follow the rules* (35)*, with $\delta > \underline{\delta}$, $\epsilon_t^m = \epsilon_t^f = 1$ for $t \geq T$ and arbitrarily large T, and moderate shocks for $t < T$. Then the economy admits a continuum of Nash equilibria, indexed by a continuum of values of* $\mathrm{NPV}(\mathbf{C}, \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m))$.

The proposition shows that we indeed have equilibrium multiplicity of the exact same form as in our earlier analysis. To understand the result, begin with any date $t \geq T$, pick any $\xi > 0$, and guess an equilibrium of the form $Y_t = \xi$ and $D_{t+1} = \delta\xi$, for all such dates $t \geq T$. Because preferences are homothetic, the Euler equation is trivially satisfied from T onwards; and because $R^* > 1$, the corresponding permanent income is finite. This proves, in effect, that the economy admits a *continuum* of steady states under sticky prices, whereas it admitted a unique one under flexible prices, both exactly as in our earlier analysis. The origin of this multiplicity is also the same as in RANK—it is the feedback between perpetual spending and permanent income, with lifetime MPCs of one and output being fully demand-determined. The only novel twist here is that, in order to support multiple steady-state levels of spending and income with the *same* interest rate, the quantity of assets—which now directly enters household utility—must scale up in proportion with spending. Under our assumptions on policy, this is achieved by letting the fiscal authority target a given debt-to-GDP ratio δ, while the monetary authority targets a real rate $R = r(\delta)$. The argument is then completed exactly as in RANK: by varying the steady state at which the economy is expected to rest *after* T, we can support different equilibrium paths *before* T, no matter how far in the future T is. The precise equilibrium paths are then recovered directly through the (now more complicated) Euler equation.

[39] The existence of such a threshold $\underline{\delta}$ follows from the continuity of $r(\delta)$ in δ and the fact that $\lim_{\delta\to\infty} r(\delta) = \beta^{-1} > 1$. Also note that, if R^* were less than 1, permanent income would be infinite and the household's problem would not admit a solution—unless we were to add a borrowing constraint, as is commonplace in HANK models.

[40] This assumption guarantees interior solutions for all our subsequent arguments. Without this assumption, if monetary policy were sufficiently contractionary or fiscal policy sufficiently tight, desired saving could become so strong that the interior Euler equation no longer applies and the equilibrium instead features a corner solution for consumption. Such cases can be accommodated by modifying the Euler equation, but we set them aside for clarity.

Returning to the transmission of fiscal and monetary policy, we see that both instruments—and in particular *transitory* fluctuations in either—can once again operate by affecting the steady state in which the economy is expected to rest after T. Exactly as in RANK, both therefore transmit through their effects on long-run beliefs, and hence on consumer spending in the long run. Our refinement, to which we turn next, again rules out this mechanism.

Fiscal policy under the refinement. We invoke the same refinement as in Section 4.[41]

Proposition 7. *Under the same assumptions as in Proposition 6, but now imposing Refinement 1 for arbitrary $H \geq T$, there exists a unique Nash equilibrium. In this equilibrium, output, and thus inflation, depends on fiscal policy if and only if $\chi > 0$.*

The refinement selects a unique equilibrium in the present HANK-like setting for the same reason as it did in RANK: by anchoring expectations of income at far-ahead horizons and thereby moderating strategic complementarity. Along this unique equilibrium, both policies operate *exclusively* because of their direct, short-run effects on household payoffs. Fiscal policy thus now matters even under our refinement, and through the same direct, short-run payoff channel as monetary policy. To see the logic most transparently, suppose that preferences take the special form $U(C,A) = (1-\chi)u(C)+\chi u(A)$, where $u(\cdot) \equiv \log(\cdot)$ and $\chi \in (0,1)$. The Euler equation (34) then, under our assumptions on policy, simplifies as follows:

$$u'(C_t) = \beta R^* \epsilon_t^m \left[1 - \frac{R^* \chi \epsilon_t^m}{(1-\chi)\delta \epsilon_t^f}\right]^{-1} u'(C_{t+1}). \tag{36}$$

Starting with $C_t = Y^*$ for $t \geq H$ and then solving equation (36) backward, we immediately get a unique equilibrium path for output as a function of $\boldsymbol{\epsilon}^m$ and $\boldsymbol{\epsilon}^f$. Two features of the resulting equilibrium path are noteworthy. First, fiscal deficits are now expansionary via their effects on household payoffs. For example, if the government increases the initial fiscal deficit, then output on impact jumps, simply because greater wealth holdings encourage spending. Second, fiscal and monetary policy are interchangeable: lower rates and higher deficits both act like a negative discount-rate shock, frontloading consumer spending in partial equilibrium, and thus leading to a boom in general equilibrium (for an earlier discussion of such equivalence in HANK-type models, see Wolf, 2025).

Relation to the literature. Zooming out, the analysis in this and the preceding sections makes clear how studies of monetary-fiscal interactions in HANK-type settings fundamentally differ from their earlier RANK counterparts. Key applied lessons of that recent literature—e.g., the output and inflation

[41] As discussed in Section 4, the spirit of the refinement is that the economy eventually returns to flexible-price outcomes. With a direct preference of households over government debt, those flexible-price outcomes are not invariant to policy: the steady-state real interest rate is now $r(\delta)$ rather than β^{-1}. We accordingly read Refinement 1 with $R^* = r(\delta)$ throughout this section.

effects of fiscal transfers, or how fiscal policy shapes monetary policy transmission—are driven by what we have termed the direct, short-run payoff effects of fiscal policy, and not by hard-to-test effects on household beliefs about the distant future. We have explicitly verified this claim only for the results of Angeletos et al. (2024a,b, 2026), but we expect it to extend to the results of Aguiar et al. (2023), Auclert et al. (2025), Kaplan et al. (2018), and Rachel and Ravn (2026), among many others.

6 Conclusion

We revisited the theoretical foundations of a literature that studies monetary-fiscal interactions in RANK. By recasting this familiar environment as a game among rational, Ricardian households, we isolated three important properties: that fiscal policy is payoff-irrelevant; that equilibrium multiplicity is rooted in output being perpetually demand-determined; and that, by its very nature, multiplicity is present regardless of the policy mix. Leveraging these observations, we questioned the claim that an active fiscal authority can regulate output and inflation in RANK and we showed that, whenever fiscal deficits trigger fluctuations in output and inflation in that model, these fluctuations are both purely self-sustained and infinitely lasting. We finally argued that this mechanism is at odds with the New Keynesian model's intended purpose—to speak to cyclical, short-run fluctuations against a long run that is supply-determined. Formalizing that premise as a belief refinement, namely requiring that households expect a return to flexible-price outcomes at some finite horizon, guaranteed both fiscal irrelevance and monetary dominance.

The practical takeaway was that, for fiscal policy to matter in spite of our refinement, Ricardian equivalence must fail at a more primitive level. HANK models naturally pass this test: once households are non-Ricardian, fiscal deficits become expansionary through a quantifiable, short-run demand channel, which translates to payoff relevance of fiscal policy in the appropriate game representation. Nonetheless, HANK models preserve the assumption of perpetually demand-determined outcomes, thus also preserving RANK's scope for infinitely lasting, self-fulfilling fluctuations. HANK equilibria that are robust to our refinement therefore deliver what we view as the right foundations for studying monetary-fiscal interactions—foundations in which policy effects are disciplined by microeconomic evidence and not driven by hard-to-test assumptions about far-ahead beliefs.

By highlighting how output is perpetually demand-determined in the New Keynesian framework, our analysis touched upon one of the most subtle properties of that framework, one deserving of further scrutiny. Our refinement ruled out equilibria that hinge upon this property, but it did not address its source. In our view, there is significant value in thinking more carefully about this and other related foundational issues, such as the possibility of rationing and the determination of long-run inflation expectations. Doing so need not overturn this paper's conclusions, but will advance our understanding of the possible pathways from fiscal deficits to output and inflation.[42]

Finally, we wish to emphasize that, although we questioned the theoretical foundations of the existing, RANK-based literature on monetary-fiscal interactions and the related literature on the FTPL, this does not imply that we challenged those literatures' real-world motivation or practical takeaways, such as the idea that unfunded fiscal deficits can trigger inflationary pressure (the main thesis of Cochrane, 2023) or the complementary idea that a dovish monetary reaction may intensify fiscally led inflations (Bianchi and Melosi, 2017; Smets and Wouters, 2024). Instead, our point is to offer guidance on the theoretical frameworks that are best suited for studying these important policy issues. In this spirit, our companion work in Angeletos et al. (2024b) shows how HANK models help reconcile the aforementioned ideas, as well as the specific FTPL-like predictions emphasized in Barro and Bianchi (2024), with the requirements laid out above: consumption behavior being consistent with microeconomic evidence; and output gaps closing in finite time.

References

Aguiar, Mark, Manuel Amador, and Cristina Arellano, "Pareto improving fiscal and monetary policies: Samuelson in the New Keynesian model," *NBER Working Paper*, 2023.

Aiyagari, S. Rao, "Uninsured idiosyncratic risk and aggregate saving," *Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

Angeletos, George-Marios and Chen Lian, "Forward guidance without common knowledge," *American Economic Review*, 2018, *108* (9), 2477–2512.

_ **and** _ , "Determinacy without the Taylor principle," *Journal of Political Economy*, 2023, *131* (8), 2125–2164.

_ **and Zhen Huo**, "Myopia and anchoring," *American Economic Review*, 2021, *111* (4), 1166–1200.

[42]For instance, Flynn et al. (2026) show that an extension of the New Keynesian model featuring rationing is isomorphic—in terms of aggregate dynamics—to the textbook New Keynesian model, subject to letting aggregate productivity be a function of aggregate spending. This model can in turn be nested in the analysis of Section 3, just with an appropriate specification of the Phillips curve (i.e., with aggregate spending driving inflation not only via the output gap but also via aggregate productivity). This suggests that our lessons are robust to the accommodation of rationing. Turning to long-run inflation, note that, within our framework and the basic New Keynesian model alike (see, e.g., the derivation in Hazell et al., 2022), expectations of long-run inflation are tied to expectations of long-run output gaps. The literature has sometimes addressed this feature by allowing prices to be automatically indexed to an exogenous, time-varying reference point interpreted as the inflation trend (see, e.g., Del Negro and Eusepi, 2011). Although this approach can be useful for quantitative analysis, the determination of long-run inflation expectations and the closure of the New Keynesian model at far-ahead horizons remain open questions.

— , **Chen Lian, and Christian Wolf**, "Can deficits finance themselves?," *Econometrica*, 2024, *92* (5), 1351–1390.

— , — , **and** — , "Deficits and inflation: HANK meets FTPL," *NBER Working Paper*, 2024.

— , — , **and** — , "Fiscal inaction as monetary support," *NBER Working Paper*, 2026.

Atkeson, Andrew, Varadarajan Chari, and Patrick Kehoe, "Sophisticated monetary policies," *Quarterly Journal of Economics*, 2010, *125* (1), 47–89.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub, "The intertemporal Keynesian cross," *Journal of Political Economy*, 2024, *132* (12), 4068–4121.

— , — , **and** — , "Fiscal and monetary policy with heterogeneous agents," *Annual Review of Economics*, 2025, *17*, 539–562.

Barro, Robert, "Are government bonds net wealth?," *Journal of Political Economy*, 1974, *82* (6), 1095–1117.

— , "The old Keynesian model," *NBER Working Paper*, 2025.

— **and Francesco Bianchi**, "Fiscal influences on inflation in OECD countries, 2020-2023," *NBER Working Paper*, 2024.

Bassetto, Marco, "A game-theoretic view of the fiscal theory of the price level," *Econometrica*, 2002, *70* (6), 2167–2195.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe, "The perils of Taylor rules," *Journal of Economic Theory*, 2001, *96* (1–2), 40–69.

Bianchi, Francesco and Cosmin Ilut, "Monetary/fiscal policy mix and agent's beliefs," *Review of Economic Dynamics*, 2017, *26*, 113–139.

— **and Leonardo Melosi**, "Escaping the Great Recession," *American Economic Review*, 2017, *107* (4), 1030–1058.

Bilbiie, Florin, "Monetary policy and heterogeneity: An analytical framework," *Review of Economic Studies*, 2025, *92* (4), 2398–2436.

Bocola, Luigi, Gaston Chaumont, Alessandro Dovis, and Rishabh Kirpalani, "Accounting for credibility: Monetary-fiscal interactions and the credibility of the central bank mandate," *Working Paper*, 2026.

Buiter, Willem, "The fiscal theory of the price level: A critique," *Economic Journal*, 2002, *112* (481), 459–480.

Cochrane, John, "Money as stock," *Journal of Monetary Economics*, 2005, *52* (3), 501–528.

—, "Determinacy and identification with Taylor rules," *Journal of Political Economy*, 2011, *119* (3), 565–615.

—, "The New-Keynesian liquidity trap," *Journal of Monetary Economics*, 2017, *92*, 47–63.

—, "Michelson-Morley, Fisher, and Occam: The radical implications of stable quiet inflation at the zero bound," *NBER Macroeconomics Annual*, 2018, *32* (1), 113–226.

—, *The fiscal theory of the price level*, Princeton University Press, 2023.

Davig, Troy and Eric Leeper, "Monetary-fiscal policy interactions and fiscal stimulus," *European Economic Review*, 2011, *55* (2), 211–227.

Eichenbaum, Martin, Joao Guerreiro, and Jana Obradovic, "Ricardian non-equivalence," *NBER Working Paper*, 2026.

Farhi, Emmanuel and Iván Werning, "Monetary policy, bounded rationality, and incomplete markets," *American Economic Review*, 2019, *109* (11), 3887–3928.

Flynn, Joel, George Nikolakoudis, and Karthik Sastry, "Pricing and production without the invisible hand," *Working Paper*, 2026.

Friedman, Milton, *A theory of the consumption function*, Princeton University Press, 1957.

Galí, Jordi, *Monetary policy, inflation, and the business cycle: An introduction to the New Keynesian framework*, Princeton University Press, 2008.

Hazell, Jonathon, Juan Herreno, Emi Nakamura, and Jón Steinsson, "The slope of the Phillips curve: Evidence from US states," *Quarterly Journal of Economics*, 2022, *137* (3), 1299–1344.

Kaplan, Greg, "Implications of fiscal-monetary interaction from HANK models," *NBER Working Paper*, 2025.

— **and Giovanni Violante**, "Microeconomic heterogeneity and macroeconomic shocks," *Journal of Economic Perspectives*, 2018, *32* (3), 167–194.

___ , **Benjamin Moll, and Giovanni Violante**, "Monetary policy according to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

King, Robert, "The new IS-LM model: Language, logic, and limits," *Federal Reserve Bank of Richmond Economic Quarterly*, 2000, *86* (3), 45–104.

Kocherlakota, Narayana and Christopher Phelan, "Explaining the fiscal theory of the price level," *Federal Reserve Bank of Minneapolis Quarterly Review*, 1999, *23* (4), 14–23.

Leeper, Eric, "Equilibria under "active" and "passive" monetary and fiscal policies," *Journal of Monetary Economics*, 1991, *27* (1), 129–147.

Mian, Atif, Ludwig Straub, and Amir Sufi, "A Goldilocks theory of fiscal deficits," *American Economic Review*, 2025, *115* (12), 4253–4291.

Michaillat, Pascal and Emmanuel Saez, "Resolving New Keynesian anomalies with wealth in the utility function," *Review of Economics and Statistics*, 2021, *103* (2), 197–215.

Negro, Marco Del and Stefano Eusepi, "Fitting Observed Inflation Expectations," *Journal of Economic Dynamics and Control*, 2011, *35* (12), 2105–2131.

Neumeyer, Pablo and Juan Pablo Nicolini, "The incredible Taylor principle: A comment," *Journal of Political Economy*, 2025, *133* (4), 1382–1399.

Niepelt, Dirk, "The fiscal myth of the price level," *Quarterly Journal of Economics*, 2004, *119* (1), 277–300.

Norman, Thomas and Tim Willems, "A game-theoretic foundation for the fiscal theory of the price level," *Working Paper*, 2025.

Rachel, Łukasz and Morten Ravn, "Brothers in arms: Monetary-fiscal interactions without Ricardian equivalence," *Working Paper*, 2026.

Rotemberg, Julio, "Sticky prices in the United States," *Journal of Political Economy*, 1982, *90* (6), 1187–1211.

Sargent, Thomas and Neil Wallace, ""Rational" expectations, the optimal monetary instrument, and the optimal money supply rule," *Journal of Political Economy*, 1975, *83* (2), 241–254.

Smets, Frank and Raf Wouters, "Fiscal backing, inflation and US business cycles," *Working Paper*, 2024.

Wolf, Christian, "Interest rate cuts versus stimulus payments: An equivalence result," *Journal of Political Economy*, 2025, *133* (4), 1235–1275.

Woodford, Michael, "Price-level determinacy without control of a monetary aggregate," in "Carnegie-Rochester Conference Series on Public Policy," Vol. 43 1995, pp. 1–46.

_ , *Interest and prices*, Princeton University Press, 2003.

Online Appendices for

Monetary-Fiscal Interactions: A Reappraisal

This online appendix contains supplemental material for the article "Monetary-Fiscal Interactions: A Reappraisal." We provide: (i) proofs for all results; (ii) details for several model extensions; and (iii) supplementary discussion on the nature of equilibrium multiplicity in the New Keynesian model.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."–"C." refer to the main article.

A Proofs

A.1 Proof of Theorem 1

The main arguments—necessity and sufficiency of the Euler equation, and existence of a continuum of Nash equilibria—were proved in the main text. Here, we complete the last step, namely we prove the claim that we can support an arbitrary value $\xi > 0$ for the discounted present value of spending. Define the NPV function

$$\text{NPV}(C_0) \equiv C_0 + \mathscr{R}(C_0)^{-1} C_1(C_0),$$

where $C_1(C_0)$ denotes the solution of the Euler equation for C_1 as a function of C_0. Continuity of $\mathscr{R}(\bullet)$ and $U'(\bullet)$ implies continuity of $C_1(\bullet)$ and $\text{NPV}(\bullet)$. First, $\text{NPV}(C_0) \geq C_0$, so

$$\lim_{C_0 \to \infty} \text{NPV}(C_0) = \infty.$$

Second, using the Euler equation,

$$\mathscr{R}(C_0)^{-1} C_1(C_0) = \beta \frac{C_1(C_0) U'(C_1(C_0))}{U'(C_0)}.$$

Given $U(\bullet)$ is concave and bounded,

$$U'(C) \leq \frac{U(C) - U(C/2)}{C/2} \leq \frac{2 \sup_{x>0} U(x)}{C},$$

and therefore $C\, U'(C) \leq K$ for some $K < \infty$. It follows that

$$0 \leq \mathscr{R}(C_0)^{-1} C_1(C_0) \leq \frac{\beta K}{U'(C_0)} \to 0 \qquad \text{as } C_0 \downarrow 0,$$

where the last step uses the Inada condition at zero. Hence

$$\lim_{C_0 \downarrow 0} \text{NPV}(C_0) = 0.$$

By continuity, NPV takes every value in $(0, \infty)$, so an equilibrium with $\text{NPV}(C_0) = \xi$ exists for all $\xi > 0$.

A.2 Proof of Theorem 2

We first state the household problem under sticky prices, then establish symmetry of any Nash equilibrium, show that the symmetric consumption allocations coincide with those under rigid prices, and finally construct the price mapping $\mathscr{P}$. Under Rotemberg pricing, household i takes the aggregate plan $(\mathbf{C}, \mathbf{P})$ as given and chooses $(\mathbf{C}_i, \mathbf{P}_i) \in \mathbb{R}^4_{++}$ to maximize

$$\sum_{t=0}^{1} \beta^t \Big[U(C_{i,t}) - \left(\tfrac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t - \kappa^{-1} g(P_{i,t-1}, P_{i,t}) \Big] \tag{A.1}$$

subject to the intertemporal budget constraint

$$C_{i,0} + R_0^{-1} C_{i,1} \le \left(\tfrac{P_{i,0}}{P_0}\right)^{1-\epsilon} C_0 + R_0^{-1} \left(\tfrac{P_{i,1}}{P_1}\right)^{1-\epsilon} C_1, \tag{A.2}$$

where $R_0 = \mathscr{R}(C_0)$, $P_{i,-1} = 1$ for all i, and $V(L) = L$. The adjustment cost $g : \mathbb{R}^2_{++} \to \mathbb{R}_+$ satisfies the following two assumptions:

A1. *(Optimal pricing uniqueness.)* The adjustment cost g is such that, for any aggregate output and prices, the household's pricing first-order conditions are necessary and sufficient for optimality and admit a unique solution.[43]

A2. *(Phillips curve regularity.)* For each period, the relevant marginal cost of raising prices is strictly increasing in the price being raised, and has range $\mathbb{R}$.[44]

All households share the same preferences, the same κ, the same initial price $P_{i,-1} = 1$, and the same initial wealth D_0. It follows that every household faces the identical optimization problem (A.1)–(A.2). By Assumption **A1**, this problem has a unique solution given any aggregate path, so that all best responses coincide and any NE is symmetric.

Given a consumption path (C_0, C_1), the symmetric pricing first-order conditions are obtained by differentiating (A.1) with respect to $P_{i,t}$, multiplying by $P_{i,t}$, using the consumption Euler equation, and imposing symmetry $P_{i,t} = P_t$. Define the static markup gap $\Lambda(C) \equiv C[\epsilon - (\epsilon - 1)\, U'(C)]$. The period-1 condition gives

$$\kappa^{-1} P_1\, g_2(P_0, P_1) = \Lambda(C_1), \tag{A.3}$$

and the period-0 condition gives

$$\kappa^{-1} P_0\, g_2(1, P_0) + \beta \kappa^{-1} P_0\, g_1(P_0, P_1) = \Lambda(C_0), \tag{A.4}$$

[43] A sufficient condition is that, when the household's problem is reformulated as choosing the quantity produced $y_{i,t}$ in each period, the composed adjustment cost $g\big(P_{i,t-1}(y_{i,t-1}), P_{i,t}(y_{i,t})\big)$ is jointly convex in $(y_{i,t-1}, y_{i,t})$. Consequently, a sufficient primitive condition is that the matrix $M(P, P') \equiv \begin{pmatrix} g_{11} + \frac{\epsilon+1}{P} g_1 & g_{12} \\ g_{12} & g_{22} + \frac{\epsilon+1}{P'} g_2 \end{pmatrix}$ is positive semidefinite for all $(P, P') \in \mathbb{R}^2_{++}$.

[44] A sufficient condition is that, for each fixed $P > 0$, the marginal adjustment cost $P' g_2(P, P')$ is strictly increasing in P' with range $\mathbb{R}$; and for each $z \in \mathbb{R}$, let $p(P; z)$ be the unique solution to $p(P; z)\, g_2\big(P, p(P; z)\big) = z$. Then $P\, g_2(1, P) + \beta P\, g_1\big(P, p(P; z)\big)$ is strictly increasing in P with range $\mathbb{R}$.

where g_1 and g_2 denote the partial derivatives of g with respect to its first and second arguments, respectively. This is a system of two equations in two unknowns (P_0, P_1), with (C_0, C_1) entering through Λ. Under Assumption **A2**, the marginal adjustment cost $P_1 g_2(P_0, P_1)$ is strictly increasing in P_1 with range $\mathbb{R}$ for each fixed P_0. It follows that, for any $P_0 > 0$, equation (A.3) has a unique solution

$$P_1 = p\big(P_0; \kappa\Lambda(C_1)\big).$$

Substituting this into (A.4) yields one equation in P_0, whose left-hand side is strictly increasing and has range $\mathbb{R}$, again by Assumption **A2**. The price mapping $\mathscr{P}(C_0, C_1) = (P_0^*, P_1^*)$ is therefore well-defined.

In a symmetric NE, $P_{i,t} = P_t$ for all i and household i's real income is $Y_{i,t} = C_t$, exactly the same as under rigid prices. The budget constraint (A.2) therefore reduces to

$$C_{i,0} + R_0^{-1} C_{i,1} \leq C_0 + R_0^{-1} C_1,$$

and the consumption Euler equation is $U'(C_0) = \beta\mathscr{R}(C_0)\,U'(C_1)$, identical to the rigid-price condition of Theorem 1. To verify sufficiency, fix any $\mathbf{C}$ satisfying this Euler equation, let $C_{i,t} = C_t$ and $P_{i,t} = P_t$ for all i, and note that both the consumption and pricing first-order conditions are satisfied (the pricing condition is the NKPC derived above). By strict concavity of U and Assumption **A1**, these conditions are jointly sufficient for the household's optimality. Hence (C_0, C_1) is a NE under sticky prices if and only if it is a NE under rigid prices.

Remark. Two aspects of our environment deserve brief comment. First, we specify a relatively general functional form for the adjustment cost, only requiring that the household problem and the Phillips curve are well-behaved. However, the standard quadratic adjustment cost specification, $g(P, P') = \frac{1}{2}(P'/P - 1)^2$, does not satisfy Assumption **A1** globally: it introduces non-concavity in the household's pricing problem, opening the door to the possibility that the best response is discontinuous and, hence, that a pure-strategy equilibrium may fail to exist. Within the neighborhood of the flexible-price steady state, however, this problem may be ruled out. Second, the linear labor disutility $V(L) = L$ ensures that the price mapping $\mathscr{P}$ is a function. With non-linear V, the symmetric pricing conditions could in principle admit multiple solutions for a given consumption path, so that $\mathscr{P}$ becomes a correspondence. However, this feature, known as the backward bending Phillips curve, is orthogonal to the central message of the paper: part 2 of Theorem 2 simply becomes $(P_0, P_1) \in \mathscr{P}(C_0, C_1)$.

A.3 Proof of Proposition 1

To maximize clarity, we start with the case of fully rigid prices. Recall that in this case the household's best response in the game is the same as the optimal consumption function, denoted $\mathbf{C}_i = \mathscr{C}(D_0, \mathbf{C}, \mathbf{T}, \mathscr{R}(\mathbf{C}))$, with boldface denoting paths, evaluated at $D_0 = T_0 = T_1 = 0$. Any Nash equilibrium therefore satisfies $\mathbf{C}_i = \mathscr{C}(0,\mathbf{C},0,\mathscr{R}(\mathbf{C}))$ for all i. Using equation (15), the household's problem is the same in an NE and in a CE; that is, $\mathscr{C}(0,\mathbf{C},0,\mathscr{R}(\mathbf{C})) = \mathscr{C}(D_0,\mathbf{C},\mathbf{T},\mathscr{R}(\mathbf{C}))$. Finally, any aggregate consumption path generated by a Nash equilibrium and satisfying equation (15) constitutes a competitive equilibrium by definition. Turning now to the case with sticky prices, the argument goes through verbatim, modulo the reinterpretation of the best response in the game as the pair of the optimal consumption and pricing functions.

A.4 Proof of Theorem 3

Part 1. From the proof of Proposition 1, every Nash equilibrium translates into a competitive equilibrium provided equation (15) holds. Since passive fiscal policy, by definition, guarantees that (16) holds for *every* $(D_0, C_0, C_1, R_0) \in \mathbb{R}^3_+ \times \mathbb{R}_{++}$, any NE is necessarily a CE.

Part 2. Starting from any Nash equilibrium with aggregate consumption path $\mathbf{C}^*$, define a fiscal policy by

$$\mathscr{T}_0(D_0,\mathbf{C},R_0) = \begin{cases} D_0, & \mathbf{C} = \mathbf{C}^*, \\ 0, & \mathbf{C} \neq \mathbf{C}^* \text{ and } D_0 \neq 0, \\ 1, & \mathbf{C} \neq \mathbf{C}^* \text{ and } D_0 = 0, \end{cases} \qquad \mathscr{T}_1(D_0,\mathbf{C},R_0) = 0.$$

By construction, only $\mathbf{C}^*$ constitutes a CE: any candidate equilibrium with $\mathbf{C} \neq \mathbf{C}^*$ fails equation (15). This also shows that the fiscal policy is active.

A.5 Proof of Theorem 4

Part 1. To prove the result, first consider an aggregate path $\mathbf{C}$ with infinite present value. By Footnote 11, every household plan with finite penalty has finite present value and can be improved by a finite increase in $C_{i,0}$ without triggering the penalty. Strict monotonicity of U therefore implies that no best response exists at such a profile. Hence any Nash equilibrium must have $\mathrm{NPV}\big(\mathbf{C},\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)\big) < \infty$. Fix any aggregate path with this property. By strict concavity, the household's problem admits at most one solution. Any solution satisfies

$$U'(C_{i,t}) = \beta \mathscr{R}_t\big(\mathbf{C},\boldsymbol{\epsilon}^m\big) U'(C_{i,t+1}) \qquad \forall t,$$

together with $\mathrm{NPV}\big(\mathbf{C}_i,\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)\big) = \mathrm{NPV}\big(\mathbf{C},\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)\big)$. Conversely, because the objective is concave and the lifetime budget constraint is linear, any plan satisfying these conditions is the unique solution. Whenever it exists, write household i's best response as $\mathbf{C}_i = \mathscr{F}(\mathbf{C},\boldsymbol{\epsilon}^m)$. Note that,

$$\mathscr{F}(\mathbf{C},\boldsymbol{\epsilon}^m) = \mathscr{C}\big(0,\mathbf{C},0,\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)\big),$$

that is, conditional on $\mathbf{C}$, households take as given (i) the income and interest-rate paths pinned down by aggregate spending and (ii) that government bonds are not net wealth (as if debt and taxes were zero). Hence, given $\mathbf{C}$, all households face the same optimization problem and therefore have the same best response whenever it exists; it follows that any Nash equilibrium must be symmetric, i.e. $\mathbf{C}_i = \mathbf{C}$ for all i. The individual Euler condition then reduces to the aggregate Euler equation

$$U'(C_t) = \beta \mathscr{R}_t(\mathbf{C},\boldsymbol{\epsilon}^m)\, U'(C_{t+1}) \qquad \forall t.$$

Together with the necessity of $\text{NPV}(\mathbf{C},\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)) < \infty$, this proves necessity. For sufficiency, take any $\mathbf{C}$ satisfying the aggregate Euler equation and $\text{NPV}(\mathbf{C},\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)) < \infty$. Then $\mathbf{C}$ satisfies the household's Euler equation under $\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)$ and exactly exhausts the present-value resources, so the unique best response is $\mathscr{F}(\mathbf{C},\boldsymbol{\epsilon}^m) = \mathbf{C}$. Hence $\mathbf{C}_i = \mathbf{C}$ for all i is a Nash equilibrium.

Part 2. Fix any $\xi > 0$. By Part 1, it suffices to construct a path $\mathbf{C}$ satisfying the aggregate Euler equation at every date with $\text{NPV}(\mathbf{C},\mathscr{R}(\mathbf{C},\boldsymbol{\epsilon}^m)) = \xi$. To avoid branch-selection issues, we construct the equilibrium path and the target present value jointly in one finite-dimensional fixed-point problem.

By assumption, there exists a finite date $T > 1$ such that $\mathscr{R}_t(\cdot) = \beta^{-1}$ for all $t \geq T$. For any constant tail $C_t = x_T$ with $t \geq T$, the Euler equation is automatically satisfied. So it remains to determine the $T+1$ unknowns $x = (x_0,\dots,x_T)$, where $x_0,\dots,x_{T-1}$ are pre-T consumption and x_T is the tail level. Using boundedness of $\mathscr{R}_t \in [\underline{r},\bar{r}]$, define for each $z > 0$ the backward consumption envelopes $\underline{B}_T(z) = \bar{B}_T(z) = z$ and, for $t = T-1,\dots,0$,

$$\underline{B}_t(z) := \big(U'\big)^{-1}\big(\beta\bar{r}\, U'\big(\underline{B}_{t+1}(z)\big)\big), \qquad \bar{B}_t(z) := \big(U'\big)^{-1}\big(\beta\underline{r}\, U'\big(\bar{B}_{t+1}(z)\big)\big).$$

By induction, $0 < \underline{B}_t(z) \leq \bar{B}_t(z)$ for all t and $z > 0$, and each envelope is continuous and increasing in z. Define the associated present-value envelopes

$$\underline{V}(z) := \sum_{t=0}^{T-1} \bar{r}^{-t}\, \underline{B}_t(z) + \bar{r}^{-T}\frac{z}{1-\beta}, \qquad \overline{V}(z) := \sum_{t=0}^{T-1} \underline{r}^{-t}\, \bar{B}_t(z) + \underline{r}^{-T}\frac{z}{1-\beta}.$$

Under the Inada condition, $\bar{B}_t(z) \to 0$ as $z \to 0$ for each t, hence $\overline{V}(z) \to 0$ as $z \to 0$. Moreover, $\underline{V}(z) \geq \bar{r}^{-T} z/(1-\beta) \to \infty$ as $z \to \infty$. Therefore we can choose $0 < z_- < z_+$ such that

$$\overline{V}(z_-) < \xi < \underline{V}(z_+). \tag{A.5}$$

Now define the compact convex set $K := \prod_{t=0}^{T-1} [\underline{B}_t(z_-), \bar{B}_t(z_+)] \times [z_-, z_+]$. For any $x \in K$, let $\hat{C}(x)$ denote the path with $\hat{C}_t = x_t$ for $t \leq T-1$ and $\hat{C}_t = x_T$ for $t \geq T$, and define

$$\mathscr{N}(x) := \sum_{t=0}^{T-1}\left(\prod_{s=0}^{t-1} \mathscr{R}_s^{-1}(\hat{C}(x),\boldsymbol{\epsilon}^m)\right) x_t \;+\; \left(\prod_{s=0}^{T-1} \mathscr{R}_s^{-1}(\hat{C}(x),\boldsymbol{\epsilon}^m)\right)\frac{x_T}{1-\beta},$$

which equals $\mathrm{NPV}(\hat{C}(x), \mathscr{R}(\hat{C}(x), \boldsymbol{\epsilon}^m))$ and is continuous. Define $F: K \to K$ by

$$F(x) := \Big(\Gamma_0(x), \ldots, \Gamma_{T-1}(x),\ \max\big(z_-,\ \min(z_+,\ x_T + \xi - \mathscr{N}(x))\big)\Big),$$

where $\Gamma_t(x) := (U')^{-1}\big(\beta \mathscr{R}_t(\hat{C}(x), \boldsymbol{\epsilon}^m)\, U'(x_{t+1})\big)$ for $t = 0, \ldots, T-1$. The first T coordinates impose the Euler equations; the last coordinate adjusts the tail level upward when permanent income falls short of ξ and downward when it exceeds ξ, truncated to remain in $[z_-, z_+]$. By the envelope bounds, the first T coordinates map into the required intervals; by truncation, the last coordinate stays in $[z_-, z_+]$. Since $\mathscr{R}_t(\cdot)$, $U'(\cdot)$, and $(U')^{-1}$ are continuous, F is a continuous self-map of K. By Brouwer's fixed-point theorem, there exists $x^* \in K$ with $F(x^*) = x^*$.

We claim $x_T^* \in (z_-, z_+)$. Suppose $x_T^* = z_-$. Since x^* is a fixed point, backward iteration using $\mathscr{R}_t \geq \underline{r}$ yields $x_t^* \leq \bar{B}_t(z_-)$ for all $t \leq T-1$. Combined with $\mathscr{R}_s^{-1} \leq \underline{r}^{-1}$, this gives $\mathscr{N}(x^*) \leq \overline{V}(z_-) < \xi$ by (A.5). Hence $x_T^* + \xi - \mathscr{N}(x^*) > z_-$, so truncation cannot return z_-, a contradiction. Similarly, if $x_T^* = z_+$, then $x_t^* \geq \underline{B}_t(z_+)$ for all t and $\mathscr{N}(x^*) \geq \underline{V}(z_+) > \xi$, so $x_T^* + \xi - \mathscr{N}(x^*) < z_+$, again a contradiction. Therefore $x_T^* \in (z_-, z_+)$, the truncation is inactive, and the last-coordinate fixed-point condition gives $x_T^* = x_T^* + \xi - \mathscr{N}(x^*)$, hence $\mathscr{N}(x^*) = \xi$.

Setting $\mathbf{C} := \hat{C}(x^*)$, the Euler equation holds at every date (by the fixed-point conditions for $t < T$, and by $\mathscr{R}_t = \beta^{-1}$ with constant consumption for $t \geq T$), and $\mathrm{NPV}(\mathbf{C}, \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)) = \xi$. By Part 1, $\mathbf{C}$ is a Nash equilibrium. Since $\xi > 0$ was arbitrary, the proof is complete.

A.6 Proof of Proposition 2

Work with normalized variables $c_t \equiv C_t / Y^*$ and $v_0 \equiv \mathrm{NPV}(C, R(C, \epsilon^m))/Y^*$, where

$$v_0 = \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=0}^{t-1} c_s^{-\phi} (\epsilon_s^m)^{-1} \right) c_t$$

denotes the discounted present value of normalized income (whenever the sum converges). Let T denote the finite date after which $\epsilon_t^m = 1$. Fix an arbitrary $c_0 > 0$. Under the Taylor rule and CRRA preferences, the aggregate Euler equation $U'(C_t) = \beta R_t U'(C_{t+1})$ becomes

$$c_t^{-1/\sigma} = c_t^{\phi} \epsilon_t^m c_{t+1}^{-1/\sigma}.$$

Let $\alpha \equiv 1 + \sigma\phi$. Since $\sigma\phi > -1$, we have $\alpha > 0$. Rearranging the Euler equation gives

$$c_{t+1} = c_t^{\alpha} (\epsilon_t^m)^{\sigma}. \tag{A.6}$$

Iterating forward from c_0 gives:

$$c_t = c_0^{\alpha^t} \prod_{s=0}^{t-1} (\epsilon_s^m)^{\sigma \alpha^{t-1-s}}. \tag{A.7}$$

Define $\mu_t \equiv \prod_{s=0}^{t-1} (\epsilon_s^m)^{\sigma\alpha^{t-1-s}}$, so that $c_t = c_0^{\alpha^t}\mu_t$. Since $\epsilon_t^m = 1$ for $t \geq T$, no new shocks enter the recursion (A.6) after T, and for $t \geq T$ the path evolves as $c_{t+1} = c_t^{\alpha}$, i.e., $c_t = c_T^{\alpha^{t-T}}$ with $c_T = c_0^{\alpha^T}\mu_T$.

We now compute the exponent of c_0 in each term of v_0. Using (A.7), the product $\prod_{s=0}^{t-1} c_s^{-\phi}$ contributes $c_0^{-\phi\sum_{s=0}^{t-1}\alpha^s}$ to the t-th term. Combined with $c_t = c_0^{\alpha^t}\mu_t$, the total exponent of c_0 in the t-th term is

$$\alpha^t - \phi\sum_{s=0}^{t-1}\alpha^s = \alpha^t - \phi\,\frac{\alpha^t - 1}{\alpha - 1}.$$

Substituting $\phi = (\alpha-1)/\sigma$ (from $\alpha = 1 + \sigma\phi$):

$$\alpha^t - \frac{(\alpha-1)/\sigma \cdot (\alpha^t - 1)}{\alpha - 1} = \alpha^t - \frac{\alpha^t - 1}{\sigma} = \frac{(\sigma-1)\alpha^t + 1}{\sigma} \equiv e_t.$$

It follows that each term of the series can be written as $\beta^t\omega_t c_0^{e_t}$, where

$$e_t \equiv \frac{\sigma-1}{\sigma}\alpha^t + \frac{1}{\sigma},$$

and $\omega_t > 0$ collects all terms depending only on the (fixed) shock sequence $\{\epsilon_s^m\}$ and not on c_0.[45] Note that $e_0 = 1$. When $\phi < 0$ (Case 2 below), ω_t converges to a positive constant as $t \to \infty$ (since $\epsilon_t^m = 1$ and $c_t \to 1$ for $t \geq T$, while $c_0^{e_t} \to c_0^{1/\sigma}$), ensuring in particular that $\sum_{t=0}^{\infty}\beta^t\omega_t < \infty$. We now verify the two parts of the proposition. We proceed case by case.

Case 1: $\phi = 0$. Then $\alpha = 1$ and $c_t = c_0\mu_t$ for all t, with $\mu_t = \mu_T$ for all $t \geq T$. In particular $e_t = 1$ for all t, so each term of the series equals $\beta^t\omega_t c_0$. Since ω_t is bounded (it equals a fixed positive constant for $t \geq T$) and $\beta < 1$, the series converges for all $c_0 > 0$:

$$v_0 = c_0\sum_{t=0}^{\infty}\beta^t\omega_t < \infty.$$

By varying $c_0 \in (0,\infty)$, one can span any value of $v_0 \in (0,\infty)$.

Case 2: $\phi < 0$ *(with* $\sigma\phi > -1$*).* Then $\alpha \in (0,1)$, so $\alpha^t \to 0$ as $t \to \infty$ and $e_t \to 1/\sigma > 0$. We claim $e_t > 0$ for all t: if $\sigma > 1$, then $(\sigma-1)/\sigma > 0$, so $e_t > 1/\sigma > 0$; if $\sigma < 1$, then $(\sigma-1)\alpha^t \geq \sigma - 1 > -1$ (since $\alpha^t \leq 1$), giving $e_t \geq \sigma/\sigma = 1 > 0$. In either case, e_t is bounded between $\min(1, 1/\sigma)$ and $\max(1, 1/\sigma)$. It follows that, for all $c_0 > 0$,

$$\beta^t\omega_t c_0^{e_t} \leq \beta^t\omega_t \max\big(c_0^{1/\sigma}, c_0\big),$$

so the series defining v_0 converges. Moreover, since $e_t > 0$ for all t, each term $\beta^t\omega_t c_0^{e_t}$ is strictly increasing in c_0. Therefore v_0 is strictly increasing in c_0, with $v_0 \to 0$ as $c_0 \to 0$ (since $e_t > 0$ for all t, each term vanishes) and $v_0 \to \infty$ as $c_0 \to \infty$ (since the $t = 0$ term is $\omega_0 c_0 = c_0$). By continuity of v_0 in c_0 and the intermediate value theorem, v_0 spans $(0,\infty)$, so one can support a continuum of Nash equilibria, one for each $c_0 \in \mathbb{R}_{++}$.

[45] Explicitly, $\omega_t = \mu_t\prod_{s=0}^{t-1}\mu_s^{-\phi}(\epsilon_s^m)^{-1}$. Since $\epsilon_s^m > 0$ for all s and $\mu_t > 0$ by construction, we have $\omega_t > 0$.

Case 3: $\phi > 0$. Then $\alpha > 1$, so $\alpha^t \to \infty$ and $e_t \to +\infty$ if $\sigma > 1$ while $e_t \to -\infty$ if $\sigma < 1$. Define

$$\bar{c} \equiv \mu_T^{-1/\alpha^T},$$

i.e., the unique $c_0 > 0$ such that

$$c_T = c_0^{\alpha^T} \mu_T = 1.$$

As a special example, when $\epsilon_t^m = 1$ for all t, $\mu_T = 1$ and $\bar{c} = 1$. Since the first T terms of v_0 form a finite sum and are therefore always finite, convergence of v_0 is governed entirely by the tail $t \geq T$. In this tail, $\epsilon_t^m = 1$ and the path evolves as $c_t = c_T^{\alpha^{t-T}}$, so the t-th tail term is proportional to $\beta^t c_T^{\bar{e}_{t-T}}$, where $\bar{e}_j \equiv \frac{\sigma-1}{\sigma}\alpha^j + \frac{1}{\sigma}$, with a proportionality constant that is positive, finite, and independent of t.

Sub-case $\sigma > 1$: Here $\bar{e}_j \geq 1 > 0$ for all j (since $(\sigma-1)/\sigma > 0$ and $\alpha^j \geq 1$), with $\bar{e}_j \to +\infty$. If $c_T > 1$ (i.e., $c_0 > \bar{c}$), then $c_T^{\bar{e}_j} \to +\infty$ and so $v_0 = +\infty$; the Euler path fails to identify a NE. If $c_T \leq 1$ (i.e., $c_0 \leq \bar{c}$), then $c_T^{\bar{e}_j} \leq 1$ for all j (since $\bar{e}_j > 0$ and $c_T \leq 1$), giving $v_0 < \infty$. The Euler path thus identifies a NE if and only if $c_0 \leq \bar{c}$.

Sub-case $\sigma < 1$: Here $\bar{e}_0 = 1$ but $\bar{e}_j \to -\infty$ (since $(\sigma-1)/\sigma < 0$). If $c_T < 1$ (i.e., $c_0 < \bar{c}$), then $c_T^{\bar{e}_j} \to +\infty$ (a number less than 1 raised to an increasingly negative power) and so $v_0 = +\infty$. If $c_T \geq 1$ (i.e., $c_0 \geq \bar{c}$), then $\bar{e}_j \leq 1$ for all j (since $\bar{e}_0 = 1$ and $\bar{e}_j$ is decreasing when $\sigma < 1$), so $c_T^{\bar{e}_j} \leq c_T^0 = 1$ when $\bar{e}_j \leq 0$ and $c_T^{\bar{e}_j} \leq c_T$ when $0 < \bar{e}_j \leq 1$; in either case $c_T^{\bar{e}_j} \leq \max(1, c_T)$, giving $v_0 < \infty$. The Euler path thus identifies a NE if and only if $c_0 \geq \bar{c}$.
Sub-case $\sigma = 1$: Then $e_t \equiv 1$ for all t, and $v_0 = c_0 \sum \beta^t \omega_t < \infty$ for all $c_0 > 0$.

In every sub-case, v_0 is continuous in c_0 on the admissible domain, and takes on a continuum of values.

A.7 Proof of Proposition 3

From CE to conditions 1–3. Suppose $\big((\mathbf{C}_i)_{i\in[0,1]}, \mathbf{Y}, \mathbf{\Pi}, \mathbf{R}, \mathbf{T}\big)$ is a CE according to Definition 4.
By condition 1 of Definition 4, household i optimizes. In particular, each household satisfies its Euler equation and transversality condition. The transversality condition, combined with the budget constraint holding with equality, implies that

$$\text{NPV}(\mathbf{C}_i, \mathbf{R}) = D_0 + \text{NPV}(\mathbf{Y} - \mathbf{T}, \mathbf{R}).$$

Aggregating across households and using $\mathbf{C} = \mathbf{Y}$ (market clearing from condition 2 of Definition 4), we obtain

$$\text{NPV}(\mathbf{Y}, \mathbf{R}) = D_0 + \text{NPV}(\mathbf{Y} - \mathbf{T}, \mathbf{R}),$$

which simplifies to $D_0 = \text{NPV}(\mathbf{T}, \mathbf{R})$, establishing the government budget constraint. By conditions 2 and 3 of Definition 4, the Phillips curve and policy rules are satisfied, giving conditions 2 and 3.

To verify condition 1, note that in a CE, each household solves its problem with initial wealth D_0 and taxes $\mathbf{T}$. Since the government budget constraint holds, the household's intertemporal budget constraint becomes

$$\text{NPV}(\mathbf{C}_i, \mathbf{R}) \leq D_0 + \text{NPV}(\mathbf{Y} - \mathbf{T}, \mathbf{R}) = \text{NPV}(\mathbf{Y}, \mathbf{R}).$$

That is, conditional on $\mathbf{C}$ (and hence $\mathbf{Y} = \mathbf{C}$ and $\mathbf{R}$), each household's feasibility set and optimal choice coincide exactly with those in the game representation where D_0 and $\mathbf{T}$ are absent (i.e., where the budget constraint is $\text{NPV}(\mathbf{C}_i, \mathbf{R}) \leq \text{NPV}(\mathbf{C}, \mathbf{R})$). Each household's consumption plan is therefore a best response in the game, confirming that $(\mathbf{C}_i)_{i \in [0,1]}$ is a NE.

From conditions 1–3 to CE. Conversely, suppose conditions 1–3 hold. Since $(\mathbf{C}_i)_{i \in [0,1]}$ is a NE, by Theorem 4, $\mathbf{C}_i = \mathbf{C}$ for all i and $\mathbf{C}$ satisfies the aggregate Euler equation along with $\text{NPV}(\mathbf{C}, \mathscr{R}(\mathbf{C}, \epsilon^m)) < \infty$. By condition 3, the government budget constraint holds, so the household's intertemporal budget constraint is equivalent to $\text{NPV}(\mathbf{C}_i, \mathbf{R}) \leq \text{NPV}(\mathbf{C}, \mathbf{R})$, which is exactly the constraint in the game. Each household's Euler equation and transversality condition are satisfied (the latter following from the NE requiring finite permanent income and the budget holding with equality at the optimum). Hence condition 1 of Definition 4 is satisfied. Conditions 2 and 3 of Definition 4 follow directly from conditions 2 and 3 of the proposition. Therefore the collection is a CE.

A.8 Proof of Theorem 5

Similarly to the two-period case, this result is a direct implication of Theorem 4 and Definition 5. If fiscal policy is passive, equation (24), the government's intertemporal budget constraint, is trivially satisfied for every NE, so all NEs translate to CEs, proving the first part. If instead fiscal policy is active, then equation (24) might be satisfied for some but not all NEs. What is more, by designing the fiscal rule appropriately, one can trivially select a specific NE as the unique CE. For instance, it suffices to set

$$\mathscr{T}_0(D_0, \mathbf{C}, \mathbf{R}, \boldsymbol{\epsilon}^f) = \begin{cases} D_0, & \mathbf{C} = \mathbf{C}^*, \\ 0, & \mathbf{C} \neq \mathbf{C}^* \text{ and } D_0 \neq 0, \\ 1, & \mathbf{C} \neq \mathbf{C}^* \text{ and } D_0 = 0, \end{cases} \qquad \mathscr{T}_t(D_0, \mathbf{C}, \mathbf{R}, \boldsymbol{\epsilon}^f) = 0, \quad t > 0.$$

We offer more standard examples of active fiscal policies in Section 3.5.

A.9 Proof of Theorem 6

By Theorem 4, the set of NE corresponds to all consumption paths $\mathbf{C}$ satisfying the aggregate Euler equation

$$U'(C_t) = \beta R_t U'(C_{t+1}) \quad \forall\, t \geq 0,$$

together with $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$. Substituting $R_t = \beta^{-1}\epsilon_t^m$, the Euler equation simplifies to

$$U'(C_t) = \epsilon_t^m U'(C_{t+1}) \quad \forall\, t \geq 0. \tag{A.8}$$

Unique path. By Refinement 1, $C_t = Y^*$ for all $t \geq H$. We now solve (A.8) backwards from this terminal condition. At $t = H-1$, equation (A.8) gives

$$U'(C_{H-1}) = \epsilon_{H-1}^m U'(Y^*).$$

Since U' is continuous, strictly decreasing, with range $(0, \infty)$ (by the Inada conditions and strict concavity of U), it is globally invertible. Hence

$$C_{H-1} = (U')^{-1}\big(\epsilon_{H-1}^m U'(Y^*)\big),$$

which is uniquely determined. Proceeding inductively: given C_{t+1}, the equation $U'(C_t) = \epsilon_t^m U'(C_{t+1})$ uniquely determines $C_t = (U')^{-1}\big(\epsilon_t^m U'(C_{t+1})\big)$. Iterating from $t = H-1$ down to $t = 0$, we obtain a unique path $\{C_t\}_{t=0}^{H-1}$.

Finite permanent income. For $t \geq T$, $R_t = \beta^{-1}$ and $C_t = Y^*$, so the tail of the NPV series is $Q_T \sum_{j=0}^{\infty} \beta^j Y^* = \frac{Q_T Y^*}{1-\beta} < \infty$, where $Q_T \equiv \prod_{s=0}^{T-1} R_s^{-1} < \infty$ by uniform boundedness of $\mathscr{R}_s(\bullet)$ away from zero on $\{0, \ldots, T-1\}$. The finitely many terms for $t < T$ are each finite (since $C_t > 0$ and $R_t > 0$). Hence $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$, confirming that the unique solution to the Euler equation is indeed a NE.

Since neither the Euler equation (A.8) nor the terminal condition $C_H = Y^*$ involves the fiscal shock $\boldsymbol{\epsilon}^f$, the initial debt D_0, or the fiscal rule $\mathscr{T}(\bullet)$, the unique equilibrium path is entirely determined by $\boldsymbol{\epsilon}^m$. Inflation is then given by $\boldsymbol{\Pi} = \mathscr{P}(\mathbf{C})$, which likewise depends only on ϵ^m.

A.10 Proof of Proposition 4

Under the assumed CRRA preferences, $U'(C) = C^{-1/\sigma}$, and the aggregate Euler equation $U'(C_t) = \beta R_t U'(C_{t+1})$ becomes

$$C_t^{-1/\sigma} = (C_t / Y^*)^{\phi} \epsilon_t^m C_{t+1}^{-1/\sigma}.$$

Working in log-deviations $y_t \equiv \log(C_t/Y^*)$ (so that $y_t = 0$ corresponds to $C_t = Y^*$), and using $C_t^{-1/\sigma} = (Y^*)^{-1/\sigma} e^{-y_t/\sigma}$, we abuse notation and let ϵ_t^m now be the log of monetary policy shocks. The Euler equation now becomes

$$-\frac{1}{\sigma} y_t = \phi y_t + \epsilon_t^m - \frac{1}{\sigma} y_{t+1},$$

which rearranges to

$$\left(1 + \sigma\phi\right) y_t = y_{t+1} - \sigma \epsilon_t^m. \tag{A.9}$$

Let $\alpha \equiv 1 + \sigma\phi$. By assumption, $\alpha \neq 0$. Refinement 1 imposes $y_t = 0$ for all $t \geq H$. In particular, $y_H = 0$. Furthermore, since $\epsilon_t^m = 0$ for $t \geq T$ and $H \geq T$, we have $y_t = 0$ for all $t \geq T$ (this can be verified by backward induction from H: if $y_{t+1} = 0$ and $\epsilon_t^m = 0$, then $\alpha y_t = 0$, so $y_t = 0$). For $t \leq T - 1$, equation (A.9) gives

$$y_t = \delta y_{t+1} - \sigma\delta \epsilon_t^m,$$

where $\delta \equiv 1/\alpha = 1/(1 + \sigma\phi)$. Since $\alpha \neq 0$, δ is well-defined and finite. Starting from $y_T = 0$ and iterating backwards gives:

$$\begin{aligned}
y_{T-1} &= -\sigma\delta \epsilon_{T-1}^m, \\
y_{T-2} &= \delta y_{T-1} - \sigma\delta \epsilon_{T-2}^m = -\sigma\delta \epsilon_{T-2}^m - \sigma\delta^2 \epsilon_{T-1}^m, \\
&\vdots \\
y_t &= -\sigma\delta \sum_{k=0}^{T-1-t} \delta^k \epsilon_{t+k}^m.
\end{aligned}$$

More compactly,

$$y_t = -\frac{\sigma}{1 + \sigma\phi} \sum_{k=0}^{T-1-t} \left(\frac{1}{1 + \sigma\phi}\right)^k \epsilon_{t+k}^m, \qquad t = 0, 1, \ldots, T-1. \tag{A.10}$$

Each y_t is uniquely determined. This establishes uniqueness of the equilibrium output path. For $t \geq T$, $C_t = Y^*$ and $R_t = \beta^{-1}$, so the tail of permanent income is finite. For $t < T$, $C_t = Y^* e^{y_t}$ is finite and positive (since y_t is a finite linear combination of the ϵ_t^m's). Hence $\text{NPV}(\mathbf{C}, \mathbf{R}) < \infty$, confirming that the unique solution is a valid NE.

A.11 Proof of Proposition 5

The proof follows the same logic as Theorem 4 part 1, adapted to the modified payoff structure of Section 5. Fix an aggregate path $\mathbf{C}$, and let $\mathbf{R} = \mathscr{R}(\mathbf{C}, \epsilon^m)$ and $\mathbf{D} = \mathscr{D}(\mathbf{C}; D_0, \boldsymbol{\epsilon}^f)$, with $D_{t+1} > 0$ for all t. Given $\mathbf{C}$, household i's payoff depends on its own consumption plan through both current consumption $C_{i,t}$

and the induced asset position $A_{i,t+1}$. By equation (33), household i's asset position is

$$A_{i,t+1} = \sum_{k=0}^{t} \left(\prod_{s=k}^{t} R_s \right) (C_k - C_{i,k}) + D_{t+1}.$$

Because the map $\mathbf{C}_i \mapsto (C_{i,t}, A_{i,t+1})$ is affine and U is strictly concave, the household's objective is strictly concave in $\mathbf{C}_i$, so its problem admits at most one solution. Since all households face the same problem, they have the same best response whenever it exists; hence any NE is symmetric, with $\mathbf{C}_i = \mathbf{C}$ for all i. At any equilibrium, the common plan must have finite penalty and therefore finite present value by Footnote 11; symmetry then implies $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$.

Necessity. In any symmetric NE, $C_{i,t} = C_t$ for all i, t, and so $A_{i,t+1} = D_{t+1}$. The household's problem is

$$\max_{\{C_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, A_{i,t+1}) \quad \text{s.t.} \quad A_{i,t+1} = R_t(A_{i,t} + C_t - T_t - C_{i,t}).$$

Forming the Lagrangian with multipliers $\beta^t \lambda_t$ on the budget constraint gives:

$$U_C(C_{i,t}, A_{i,t+1}) = \lambda_t R_t, \qquad U_A(C_{i,t}, A_{i,t+1}) + \beta \lambda_{t+1} R_{t+1} = \lambda_t.$$

From the first equation, $\lambda_t = U_C(C_{i,t}, A_{i,t+1})/R_t$. Substituting into the second, multiplying through by R_t, and evaluating at $C_{i,t} = C_t$, $A_{i,t+1} = D_{t+1}$ gives:

$$U_C(C_t, D_{t+1}) = \beta R_t U_C(C_{t+1}, D_{t+2}) + R_t U_A(C_t, D_{t+1}). \tag{A.11}$$

Together with $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$, this proves necessity.

Sufficiency. Suppose $\mathbf{C}$ satisfies (A.11) and $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$. Set $\mathbf{C}_i = \mathbf{C}$ for all i. Then each household's Euler equation is satisfied and its budget constraint holds with equality (since $\mathrm{NPV}(\mathbf{C}_i, \mathbf{R}) = \mathrm{NPV}(\mathbf{C}, \mathbf{R})$). It remains to show that no admissible plan improves on $\mathbf{C}_i = \mathbf{C}$. The objective is concave in $\mathbf{C}_i$, as noted above. Raising $C_{i,t}$ by one unit yields $\beta^t U_C(C_t, D_{t+1})$ directly but, by (33), lowers the household's asset holdings from date $t+1$ onward. Hence the derivative of the lifetime objective at the candidate is

$$\Lambda_t \;\equiv\; \beta^t U_C(C_t, D_{t+1}) - \underbrace{\sum_{s \geq t} \beta^s U_A(C_s, D_{s+1}) \prod_{k=t}^{s} R_k}_{\Xi_t}.$$

Here Ξ_t is the marginal value of the resulting reduction in future asset holdings. Iterating (A.11) forward expresses $\beta^t U_C(C_t, D_{t+1})$ as the corresponding finite partial sum plus a strictly positive remainder. Since $U_A > 0$, these partial sums are increasing and bounded above by $\beta^t U_C(C_t, D_{t+1})$; hence Ξ_t converges and $\Lambda_t \geq 0$. Separating the $s = t$ term from Ξ_t and using (A.11) gives $\Lambda_t = R_t \Lambda_{t+1}$, and therefore $\Lambda_t = \lambda Q_t$, where $\lambda \equiv \Lambda_0 \geq 0$ and $Q_t \equiv \prod_{s=0}^{t-1} R_s^{-1}$.

It suffices to consider an admissible deviation $\mathbf{C}'_i$ that does not trigger the penalty. Let $\Delta C_t \equiv C'_{i,t} - C_t$ and $\Delta A_{t+1} \equiv A'_{i,t+1} - D_{t+1}$. Equation (33) gives

$$\Delta A_{t+1} = -\sum_{k=0}^{t} \left(\prod_{s=k}^{t} R_s \right) \Delta C_k.$$

Moreover,

$$\sum_{t=0}^{\infty} Q_t |\Delta C_t| \le \mathrm{NPV}(\mathbf{C}'_i, \mathbf{R}) + \mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty,$$

while (A.11) implies

$$0 \le \Xi_t \le \beta^t U_C(C_t, D_{t+1}) \le U_C(C_0, D_1) Q_t.$$

Together with the asset mapping above, these bounds imply

$$\sum_{t=0}^{\infty} \beta^t U_A(C_t, D_{t+1}) |\Delta A_{t+1}| \le \sum_{t=0}^{\infty} \Xi_t |\Delta C_t| < \infty.$$

Thus the relevant series are absolutely convergent, and their order of summation can be changed. Concavity then gives

$$\begin{aligned}
\mathscr{U}(\mathbf{C}'_i) - \mathscr{U}(\mathbf{C}) &\le \sum_{t=0}^{\infty} \beta^t \left[U_C(C_t, D_{t+1}) \Delta C_t + U_A(C_t, D_{t+1}) \Delta A_{t+1} \right] \\
&= \sum_{t=0}^{\infty} \left[\beta^t U_C(C_t, D_{t+1}) - \sum_{s \ge t} \beta^s U_A(C_s, D_{s+1}) \prod_{k=t}^{s} R_k \right] \Delta C_t \\
&= \sum_{t=0}^{\infty} \Lambda_t \Delta C_t = \lambda \left[\mathrm{NPV}(\mathbf{C}'_i, \mathbf{R}) - \mathrm{NPV}(\mathbf{C}, \mathbf{R}) \right] \le 0.
\end{aligned}$$

Thus $\mathbf{C}_i = \mathbf{C}$ is a global optimum. No separate transversality condition is needed: λ is precisely the multiplier on the household's lifetime budget constraint. Hence $\mathbf{C}_i = \mathbf{C}$ for all i constitutes a NE.

A.12 Proof of Proposition 6

Consider $t \ge T$, where $\epsilon_t^m = \epsilon_t^f = 1$. Fix any $\xi > 0$ and set $C_t = \xi$ and $D_{t+1} = \delta\xi$ for all $t \ge T$. Under $U(C, A) = u(C^{1-\chi} A^{\chi})$, the Euler equation at any such t reads

$$U_C(\xi, \delta\xi) = \beta\, r(\delta)\, U_C(\xi, \delta\xi) + r(\delta)\, U_A(\xi, \delta\xi).$$

Note that the ratio $U_A(C, \delta C)/U_C(C, \delta C)$ is independent of C:

$$\frac{U_A(\xi, \delta\xi)}{U_C(\xi, \delta\xi)} = \frac{\chi}{(1-\chi)\delta} \equiv \rho(\delta).$$

The Euler equation reduces to $1 = \beta\, r(\delta) + r(\delta)\, \rho(\delta)$, or equivalently $r(\delta) = 1/(\beta + \rho(\delta))$, which is precisely how $r(\delta)$ is defined. Hence the Euler equation is satisfied at $C_t = \xi$ for *any* $\xi > 0$. Next, $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$ because $R_t = r(\delta) > 1$ for $t \ge T$ (guaranteed by $\delta > \underline{\delta}$), so the tail of the series converges geometrically. Moreover, since $R_t = r(\delta)\epsilon_t^m$ does not depend on aggregate consumption, the discount factors $Q_t \equiv \prod_{s=0}^{t-1} R_s^{-1}$ are independent of ξ, and the tail contribution to permanent income equals $K_T\,\xi$, where $K_T \equiv \sum_{t\geq T} Q_t > 0$.

For $t < T$: Given any $\xi > 0$ as the terminal condition $C_T = \xi$, we need to show existence of a solution to the Euler equation for $t = 0,\dots,T-1$. At each date $t < T$, the Euler equation under $D_{t+1} = \delta C_t \epsilon_t^f$ reads

$$U_C(C_t, \delta C_t \epsilon_t^f) = \beta\, r(\delta)\epsilon_t^m\, U_C(C_{t+1}, D_{t+2}) + r(\delta)\epsilon_t^m\, U_A(C_t, \delta C_t \epsilon_t^f).$$

Under $U(C,A) = u(C^{1-\chi}A^{\chi})$ with $A = \delta C_t \epsilon_t^f$, both U_C and U_A are proportional to $u'\big((\delta\epsilon_t^f)^{\chi} C_t\big)$:

$$U_C(C_t, \delta C_t \epsilon_t^f) = (1-\chi)(\delta\epsilon_t^f)^{\chi}\, u'\big((\delta\epsilon_t^f)^{\chi} C_t\big),$$

$$U_A(C_t, \delta C_t \epsilon_t^f) = \chi\,(\delta\epsilon_t^f)^{\chi-1}\, u'\big((\delta\epsilon_t^f)^{\chi} C_t\big).$$

The Euler equation therefore becomes

$$\underbrace{\big[(1-\chi)\delta\epsilon_t^f - r(\delta)\epsilon_t^m\,\chi\big]}_{B_t}\,(\delta\epsilon_t^f)^{\chi-1}\, u'\big((\delta\epsilon_t^f)^{\chi} C_t\big) = \beta\, r(\delta)\epsilon_t^m\, U_C(C_{t+1}, D_{t+2}). \tag{A.12}$$

The right-hand side is a known positive constant (given C_{t+1}). The coefficient $B_t \equiv (1-\chi)\delta\epsilon_t^f - r(\delta)\epsilon_t^m \chi > 0$ by assumption. Since u' is strictly decreasing, the left-hand side is a strictly decreasing function of C_t, ranging from $+\infty$ as $C_t \to 0$ to 0 as $C_t \to \infty$. By the intermediate value theorem, there is a unique $C_t > 0$ solving the equation. Thus, for each terminal condition $C_T = \xi$, backward iteration yields a unique path.

The pre-T contribution $\sum_{t=0}^{T-1} Q_t C_t(\xi)$ is continuous in ξ, since the backward iteration of (A.12) is continuous by strict monotonicity of its left-hand side in C_t. Therefore $\text{NPV}(\mathbf{C},\mathbf{R})$ is continuous in ξ and bounded below by $K_T\,\xi$, hence unbounded as $\xi \to \infty$. By the intermediate value theorem, $\text{NPV}(\mathbf{C},\mathbf{R})$ takes a continuum of values as ξ ranges over $(0,\infty)$.

A.13 Proof of Proposition 7

Refinement 1, read with $R^* = r(\delta)$ as discussed in Footnote 41, requires $Y_t = Y^*$ and $R_t = r(\delta)$ for all $t \geq H$, for some arbitrarily large but finite $H \geq T$. This pins down the terminal condition $C_t = Y^*$ for all $t \geq H$. Combined with the fiscal rule $D_{t+1} = \delta Y_t \epsilon_t^f$ and $\epsilon_t^f = 1$ for $t \geq T$ (recall $H \geq T$), it follows that $D_{t+1} = \delta Y^*$ for all $t \geq H$. For $t < H$, $D_{t+1} = \delta C_t \epsilon_t^f$ remains pinned down by the fiscal rule jointly with C_t at each step of the backward iteration below. We solve the Euler equation backwards from this terminal condition. Under $U(C,A) = u(C^{1-\chi}A^{\chi})$ with $D_{t+1} = \delta C_t \epsilon_t^f$, the Euler equation at date t reads

$$U_C(C_t, \delta C_t \epsilon_t^f) = \beta\, r(\delta)\epsilon_t^m\, U_C(C_{t+1}, D_{t+2}) + r(\delta)\epsilon_t^m\, U_A(C_t, \delta C_t \epsilon_t^f). \tag{A.13}$$

Using the explicit forms of the marginal utilities under $U(C,A) = u(C^{1-\chi}A^{\chi})$ with $A = \delta C_t \epsilon_t^f$:

$$U_C(C_t, \delta C_t \epsilon_t^f) = (1-\chi)(\delta\epsilon_t^f)^{\chi}\, u'\big((\delta\epsilon_t^f)^{\chi} C_t\big),$$
$$U_A(C_t, \delta C_t \epsilon_t^f) = \chi\,(\delta\epsilon_t^f)^{\chi-1}\, u'\big((\delta\epsilon_t^f)^{\chi} C_t\big),$$

equation (A.13) becomes

$$\underbrace{\big[(1-\chi)\delta\epsilon_t^f - r(\delta)\epsilon_t^m \chi\big]}_{B_t}(\delta\epsilon_t^f)^{\chi-1}\, u'\big((\delta\epsilon_t^f)^{\chi} C_t\big) = \beta\, r(\delta)\epsilon_t^m\, U_C(C_{t+1}, D_{t+2}). \tag{A.14}$$

The right-hand side is a known positive constant at each backward step: C_{t+1} has been pinned down at the previous step (or, when $t+1 = H$, by the terminal condition), and $D_{t+2} = \delta C_{t+1}\epsilon_{t+1}^f$ then follows from the fiscal rule. For the left-hand side, $B_t > 0$ for all t by the moderate-shock assumption. At each date t, the LHS of (A.14) is a strictly decreasing function of C_t (since $B_t > 0$, $(\delta\epsilon_t^f)^{\chi-1} > 0$, and u' is strictly decreasing), ranging from $+\infty$ as $C_t \to 0$ (by the Inada condition on u) to 0 as $C_t \to \infty$. Since the RHS is a fixed positive constant, the intermediate value theorem guarantees a unique $C_t > 0$ solving (A.14). Starting from $C_H = Y^*$ and iterating backwards, each step uniquely determines C_t given C_{t+1}. The unique equilibrium path $\{C_t\}_{t=0}^{H-1}$ follows by induction. Finally, $\text{NPV}(\mathbf{C}, \mathbf{R}) < \infty$: the tail converges geometrically since $C_t = Y^*$ and $R_t = r(\delta) > 1$ for $t \geq H$, and there are only finitely many earlier terms.

For $\chi > 0$, output depends on fiscal policy. To see this, fix $t < T$ and hold all shocks other than ϵ_t^f fixed. As ϵ_t^f approaches from above the lower boundary

$$\underline{\epsilon}_t^f \equiv \frac{r(\delta)\chi\epsilon_t^m}{(1-\chi)\delta}$$

of the moderate-shock region, B_t in (A.14) converges to zero. By the backward recursion, C_{t+1} and D_{t+2} are unchanged, so the right-hand side of (A.14) remains fixed and strictly positive. If C_t were independent of ϵ_t^f, then, since $\underline{\epsilon}_t^f > 0$ and $C_t > 0$, the remaining factors on the left-hand side would converge to finite positive values, forcing the left-hand side to zero, a contradiction. Hence C_t depends on ϵ_t^f.

Along the unique equilibrium, the Euler equation (A.13) features $U_A(C_t, D_{t+1})$ with $D_{t+1} = \delta C_t \epsilon_t^f$. Since $U_A \neq 0$, the fiscal shock ϵ_t^f enters the Euler equation through the marginal value of asset holdings, providing a direct, payoff-relevant channel through which fiscal policy can affect output. This contrasts with the case of RANK ($\chi = 0$), under which the Euler equation reduces to $U'(C_t) = \beta R_t U'(C_{t+1})$, which is independent of fiscal variables. Fiscal deficits therefore cannot influence the unique equilibrium under the refinement.

B Supplementary materials

B.1 Default

In the main text, we highlighted two competing interpretations of what the uniqueness of a CE under active fiscal policy may mean. The first one, encapsulated in the concept of fiscal dominance, is that the fiscal authority can guarantee that the private sector behaves according to this particular equilibrium. Our preferred interpretation instead emphasizes that, even if we rule out non-equilibrium behavior (now in the sense of NE), private agents could still coordinate on a different equilibrium, forcing the government either to adjust taxes or to default. To further substantiate this interpretation, and to close the conceptual gap between the two different equilibrium concepts, we will now explicitly accommodate government default.

In particular, we return to our general infinite-horizon model and modify it in the following way: the government may apply a haircut on the initial real debt, that is, instead of repaying D_0 it may repay $(1-h)D_0$, for some $h \in [0,1]$. This does not affect the game representation and the characterization of Nash equilibria: because default is a zero net transfer, household payoffs are invariant to it, just as they were invariant to implicit default via inflation in the FTPL example of Section 3.5. The set of NE thus remains exactly as characterized in Theorem 4.

What changes, though, is the Walrasian representation of the economy and, with it, the mapping between NE and CE. In this extended setting, the household's intertemporal budget constraint becomes

$$\sum_{t=0}^{\infty} Q_t C_{i,t} \leq (1-h) D_0 + \sum_{t=0}^{\infty} Q_t \left(Y_{i,t} - T_t\right),$$

where $Q_t \equiv \prod_{s=0}^{t-1} R_s^{-1}$ as before. Accordingly, the definition of a competitive equilibrium adjusts as follows.

Definition B.1. *In the setting with default, a collection* $\left((\mathbf{C}_i)_{i\in[0,1]}, \mathbf{Y}, \mathbf{\Pi}, \mathbf{R}, \mathbf{T}, h\right)$ *is a competitive equilibrium (CE) if:*

1. *Households optimize,* $\mathbf{C}_i = \mathscr{C}((1-h)D_0, \mathbf{Y}, \mathbf{T}, \mathbf{R})$ *for all* i.
2. *The output market clears,* $\mathbf{Y} = \mathbf{C} \equiv \int \mathbf{C}_i \, di$, *and inflation satisfies the Phillips curve* $\Pi = \mathscr{P}(\mathbf{Y})$.
3. *The monetary and fiscal policy rules hold,* $\mathbf{R} = \mathscr{R}(\mathbf{Y}, \boldsymbol{\epsilon}^m)$ *and* $\mathbf{T} = \mathscr{T}\left((1-h)D_0, \mathbf{Y}, \mathbf{R}, \boldsymbol{\epsilon}^f\right)$.

Compared to Definition 4, the only changes are the inclusion of h in the collection of objects that constitute a CE and the replacement of D_0 with $(1-h)D_0$ in the optimal consumption function and the tax rule. While this modification is trivial, the value of incorporating an option to default becomes clear in the next result.

Proposition B.1. *Consider the Fiscal Theory of Output example: let $\mathcal{R}(\cdot) = \beta^{-1}$ and $T_t = -\epsilon_t^f + \tau_y Y_t$ for all t. There is still a single CE without default, but there is also a continuum of CEs with default ($h \geq 0$). In particular, letting ξ^* be the unique solution to*

$$D_0 + \tilde{\epsilon}_0^f = \frac{1}{1-\beta} \tau_y \xi^*, \tag{B.1}$$

where $\tilde{\epsilon}_0^f \equiv \sum_{t=0}^{\infty} \beta^t \epsilon_t^f$, assuming $D_0 > 0$ and $D_0 + \tilde{\epsilon}_0^f > 0$, and letting

$$\underline{\xi} \equiv \max\left\{0,\ \xi^* - \frac{(1-\beta)\, D_0}{\tau_y}\right\},$$

the following is true: for any strictly positive $\xi \in [\underline{\xi}, \xi^*]$, the NE in which consumers coordinate on $C_t = \xi$ for all t translates to a CE in which the haircut is

$$h = \frac{\tau_y}{(1-\beta)\, D_0} \left(\xi^* - \xi\right) \in [0,1].$$

Proof. The set of NEs is unchanged by the introduction of default, since default is payoff-irrelevant: it is a transfer from bondholders to the government that nets out in the household's lifetime budget under the maintained assumption that households understand the government's intertemporal budget constraint. By Theorem 4, the NEs of this economy correspond to $C_t = \xi$ for all t and for arbitrary $\xi > 0$ (since $R_t = \beta^{-1}$ is pegged). Without default ($h = 0$), the government's intertemporal budget constraint requires ξ to solve (B.1), yielding the unique CE at $\xi = \xi^*$, exactly as in Section 3.5. Now allow default ($h > 0$). For a given NE at level ξ, the government's intertemporal budget constraint under the modified CE definition becomes

$$(1-h)\, D_0 + \tilde{\epsilon}_0^f = \frac{1}{1-\beta} \tau_y \xi.$$

Subtracting this from (B.1) gives:

$$h\, D_0 = \frac{\tau_y}{1-\beta} \left(\xi^* - \xi\right),$$

which gives

$$h = \frac{\tau_y}{(1-\beta)\, D_0} \left(\xi^* - \xi\right).$$

For any strictly positive $\xi \in [\underline{\xi}, \xi^*]$, this delivers $h \in [0,1]$: the upper bound $\xi = \xi^*$ gives $h = 0$, and the lower bound $\xi = \underline{\xi}$ gives $h = \min\{1, \tau_y \xi^* / ((1-\beta) D_0)\}$, which equals 1 whenever $\tilde{\epsilon}_0^f > 0$. In each case, $h \in [0,1]$, the tax rule is satisfied, and the household's budget constraint under $(1-h) D_0$ is consistent with the NE at level ξ. Hence each such (ξ, h) pair constitutes a valid CE. □

The economic content of this result is the following. The ability of an active fiscal policy to select a specific NE, and thus to pin down a unique CE, here reflects the joint assumption that neither tax adjustment nor default is available. Once the option to default is restored, the tight link between the fiscal rule and a particular level of output breaks down: if households coordinate on a level of spending $\xi < \xi^*$, the resulting tax revenue falls short of the amount needed to service the initial debt D_0 under the assumed fiscal rule; the government, unable or unwilling to adjust taxes, is compelled to default on a fraction of its obligations, and this default precisely makes up the budgetary shortfall. The haircut h is proportional to the gap $\xi^* - \xi$ between the intended and the realized level of output.

Put simply, our earlier argument that consumers could coordinate on a different equilibrium than the one selected by active fiscal policy, and thereby force the government to revise its policy or to default, is now formalized: a NE in which consumers coordinate on $C_t = \xi$ for strictly positive $\xi \in [\underline{\xi}, \xi^*)$ and the government nonetheless insists on not adjusting taxes triggers a haircut h proportional to $\xi^* - \xi$. The logic extends beyond this example: an active fiscal rule need not guarantee that households will coordinate on a particular equilibrium, and the self-fulfilling forces emphasized throughout this paper can no longer be obscured by the use of the CE concept in place of our NE concept.

Relation to the literature. In this extension, we added default while maintaining the original definition of active fiscal policy. One could, in principle, adjust this definition to include a commitment not to default under any circumstances, and argue that an active policy redefined in this way can still select a unique CE. This would amount to arguing that, out of the many NEs, there is one that happens to be consistent with both no tax adjustment and no default. We would still argue that rational Ricardian consumers could ignore fiscal policy and coordinate on a different NE, underscoring the limitations of relying on active fiscal policy for equilibrium selection.

Bassetto (2002) also incorporates default, but does so in a model with flexible prices: as we have emphasized throughout, the relevant equilibrium determinacy and selection issues are fundamentally different in the New Keynesian framework, so we see an important gap between that paper and ours. Similarly, a core thesis of that paper and Cochrane (2005, 2023), that nominal debt is unlike real debt, does not apply in our framework: in the eyes of rational Ricardian households, nominal debt erosion is equivalent to a haircut on real debt, and neither one has wealth effects. Households may therefore choose not to condition their spending on fiscal policy, unless they expect their permanent income to vary with it via the self-fulfilling mechanism articulated in this paper.

B.2 On the meaning of the government's intertemporal budget constraint

The main text assumes that households have direct, a priori, knowledge of equation (9), commonly known as the government's intertemporal budget constraint. This amounts to treating this equation—or its infinite-horizon counterpart (24)—as a primitive, an interpretation that potentially differs from the one preferred in the FTPL literature (e.g., Cochrane, 2005, 2023). We now show that the constraint can instead be inferred as an equilibrium implication. In particular, even if equation (9) or its infinite-horizon counterpart is not imposed a priori, rational households can infer its validity from the optimality of other households and, in the infinite-horizon case, the household no-Ponzi condition. The resulting equilibrium consumption profiles are precisely the Nash equilibria characterized in the main text that also satisfy the government's intertemporal budget constraint. This clarifies the debate over whether that constraint is a primitive or an implication of consumer optimality, and reinforces the key role of rationality in our result: households can infer in equilibrium (i) that fiscal operations only reshuffle assets and corresponding obligations, with no effect on their net wealth, and (ii) that multiple spending paths can be sustained because their income is determined by the spending of others.

The two-period case. To maximize transparency, we develop this point in the two-period setting of Section 2.1; once the two-period argument is completed, its translation to the infinite-horizon case will follow the same logic. Thus consider the setting of Section 2.1, and in particular the case with rigid prices, but now make the following modification: give the government the option to issue debt maturing after the second period and give households the option to buy such debt.[46] As shown momentarily, such debt is not traded in equilibrium. This modification nevertheless allows us to capture the following FTPL narratives: that equation (9) is not a primitive constraint, and that government bonds can have wealth effects *off* equilibrium even though they end up having no wealth effects *in* equilibrium.

Consider first how this modification affects our game representation. The players are still the households, but their action space is expanded to include the option to purchase bonds maturing after the world ends and their payoffs are adjusted to exclude prior knowledge of equation (9). That is, household i chooses the vector

$$\mathbf{S}_i = (C_{i,0}, C_{i,1}, \widetilde{D}_i) \in \mathbb{R}^2_{++} \times \mathbb{R}_+,$$

where $\widetilde{D}_i$ is the date-0 value of the household's terminal-debt purchase. Individual payoffs in turn are given by

$$\begin{aligned}\mathscr{U}\left(\mathbf{C}_i, \widetilde{D}_i, \mathbf{C}; \mathscr{T}(\bullet), D_0\right) \equiv \sum_{t=0}^{1} \beta^t \Big\{U\big(C_{i,t}\big) - V\big(C_t\big)\Big\} \\ - \mathbb{I}\left[C_0 + R_0^{-1}C_1 + \left(D_0 - T_0 - R_0^{-1}T_1\right) - C_{i,0} - R_0^{-1}C_{i,1} - \widetilde{D}_i\right].\end{aligned} \tag{B.2}$$

Here the penalty indicator $\mathbb{I}$ is defined in Footnote 11 and, to ease notation, we let $R_0 = \mathscr{R}(C_0)$ and

[46] Formally, at date 1 the government posts a price $q > 0$ for debt maturing at date 2 and stands ready to supply the quantity demanded. Household i chooses $D_{i,2} \geq 0$, and bond-market clearing requires $D_2 = \int D_{i,2}\, di$. We define $\widetilde{D}_i \equiv R_0^{-1} q D_{i,2}$ as the date-0 value of the household's purchase. The government's date-1 flow budget is $D_1 = T_1 + qD_2$. Date-2 wealth does not enter household utility.

$T_t = \mathscr{T}_t(D_0, C_0, C_1, R_0)$. Compared with the payoff structure in the main text, there are exactly two changes. First, we have introduced $\widetilde{D}_i$ as an additional choice variable. Second, we have not used equation (9) to substitute out fiscal policy, because we have not assumed prior knowledge of this equation.

As a result, fiscal policy is no longer payoff-irrelevant off equilibrium: if we fix the actions of other households and vary $\mathscr{T}$, we also vary the lifetime budget and the payoff of player i. Concretely, consider a reduction in taxes for given D_0, or a "helicopter drop" of government bonds for given taxes. Then, holding the actions of others fixed, household i's lifetime budget expands. This is the direct analogue of the corresponding CE property and of the FTPL narrative that government bonds have wealth effects off equilibrium.

Despite this change in the game representation and the resulting payoff relevance of fiscal policy, any Nash equilibrium of the modified game satisfies the government's intertemporal budget constraint. Conditional on this constraint, households effectively play the same consumption game as that studied in the main text. Formally:

Proposition B.2. *Consider the game described above, in which households do not have a priori knowledge of equation* (9)*. A profile* $\big((\mathbf{C}_i, \widetilde{D}_i)\big)_{i\in[0,1]}$ *is a Nash equilibrium of this game if and only if* $\widetilde{D}_i = 0$ *for all* i*,* $(\mathbf{C}_i)_{i\in[0,1]}$ *is a Nash equilibrium of the game studied in Section 2.2, and*

$$D_0 = T_0 + R_0^{-1} T_1.$$

Proof. Fix household i. Household i knows that all other households $j \neq i$ are themselves rational. At any Nash equilibrium, each such household chooses a plan with finite penalty. If $\widetilde{D}_j > 0$, household j can instead choose $(C_{j,0} + \widetilde{D}_j, C_{j,1}, 0)$, preserving feasibility and strictly raising utility. Hence $\widetilde{D}_j = 0$ and, because $q > 0$, $D_{j,2} = 0$ for every $j \neq i$. A single household has measure zero, so bond-market clearing gives $D_2 = 0$. The government's date-1 flow budget then gives $D_1 = T_1$; combining this with its unchanged date-0 flow budget yields $D_0 = T_0 + R_0^{-1} T_1$. Thus, although households were not given a priori knowledge of this equation, each can infer it from the optimality of other households. The same deviation argument gives $\widetilde{D}_i = 0$ for every i. Conditional on the government constraint, the modified household problem coincides with the problem in Section 2.2, so $(\mathbf{C}_i)_{i\in[0,1]}$ is a Nash equilibrium of the original game.

Conversely, suppose that $\widetilde{D}_i = 0$ for every i, that $(\mathbf{C}_i)_{i\in[0,1]}$ is a Nash equilibrium of the original game, and that $D_0 = T_0 + R_0^{-1} T_1$. Any deviation $(\mathbf{C}'_i, \widetilde{D}'_i)$ with finite penalty in the modified game satisfies

$$C'_{i,0} + R_0^{-1} C'_{i,1} + \widetilde{D}'_i \leq C_0 + R_0^{-1} C_1.$$

Because $\widetilde{D}'_i \geq 0$, $\mathbf{C}'_i$ has finite penalty in the original game as well. Terminal debt provides no utility, and the original-game best-response property therefore implies that this deviation cannot raise household i's payoff. A deviation with an infinite penalty cannot improve upon the proposed plan. Thus the proposed profile is also a Nash equilibrium of the modified game. $\square$

The infinite-horizon case. The translation to the infinite-horizon case follows the same logic. Consider the economy of Section 3, but do not impose the government's no-Ponzi condition as a primitive and expand the households' action space along the same lines as above. In this setting, household j's intertemporal budget can be written as

$$\mathrm{NPV}(\mathbf{C}_j, \mathbf{R}) + \widetilde{D}_j = D_0 + \mathrm{NPV}(\mathbf{Y}_j - \mathbf{T}, \mathbf{R}),$$

where

$$\widetilde{D}_j \equiv \lim_{T\to\infty} Q_{T+1} A_{j,T+1}$$

is the date-0 value of wealth carried indefinitely, with $Q_0 = 1$ and $Q_t = \prod_{s=0}^{t-1} R_s^{-1}$ for $t \geq 1$. The standard household no-Ponzi condition requires $\widetilde{D}_j \geq 0$: a household cannot finance consumption by rolling over debt forever.[47]

Optimality implies that household j chooses $\widetilde{D}_j = 0$. Otherwise, it could consume part of this wealth at a finite date, continue to satisfy its no-Ponzi condition, and strictly increase utility. Knowing that all households $j \neq i$ are rational and satisfy the household no-Ponzi condition, household i can thus infer that

$$\mathrm{NPV}(\mathbf{C}, \mathbf{R}) = D_0 + \mathrm{NPV}(\mathbf{Y} - \mathbf{T}, \mathbf{R}).$$

Household i furthermore understands that $\mathbf{Y} = \mathbf{C}$, $\mathbf{R} = \mathscr{R}(\mathbf{C}, \boldsymbol{\epsilon}^m)$, and $\mathbf{T} = \mathscr{T}(D_0, \mathbf{C}, \mathbf{R}, \boldsymbol{\epsilon}^f)$. It can therefore conclude that, at any equilibrium,

$$D_0 = \mathrm{NPV}(\mathbf{T}, \mathbf{R}).$$

Put simply, because other households are rational and satisfy the standard household no-Ponzi condition, they will neither finance consumption by borrowing forever nor leave positive wealth unconsumed forever. The government therefore cannot place Ponzi debt with them in equilibrium, and the value of the initial government bonds they hold must cancel with the present value of their tax

[47] For every finite T, iterating the household's flow budget gives

$$\sum_{t=0}^{T} Q_t C_{j,t} + Q_{T+1} A_{j,T+1} = D_0 + \sum_{t=0}^{T} Q_t (Y_{j,t} - T_t).$$

We assume that the relevant present-value sums converge and may be aggregated across households. Taking limits gives the displayed intertemporal budget.

obligations. This conclusion follows from household optimality together with the household no-Ponzi condition; it does not require imposing the government's no-Ponzi condition as a primitive. Conditional on the government's intertemporal budget constraint, households effectively play the same game as that in Section 3.

Discussion. The intuition that emerges from the analysis in this appendix, together with that in Appendix B.1 above, is simple and powerful. Rational households can infer that other rational households will never pay for debt that matures after the world ends (or, under the household no-Ponzi condition, support a government-run Ponzi scheme in the infinite-horizon case). As a result, households understand that, if the present value of taxes falls short of the outstanding government debt under any given fiscal rule, the government will either have to adjust taxes or default—and, in either case, their lifetime budgets will remain unchanged, provided that other households keep spending the same amount. At the same time, households understand that, regardless of fiscal and monetary policy, their permanent income increases one-for-one with the present value of aggregate spending. All in all, households therefore understand that they can coordinate on multiple self-sustaining paths for aggregate spending and income. Provided that taxes adjust or default occurs as in Appendix B.1 when necessary, fiscal policy is payoff-irrelevant as a fundamental but may still serve as a sunspot (coordination device).

B.3 Refinement under alternative monetary policy specifications

This section verifies that the uniqueness result of Proposition 4 extends to the standard linearized three-equation New Keynesian model and to the case of a nominal interest rate peg.

The linearized three-equation model. Consider the standard system in log-deviations from the zero-inflation steady state ($y_t \equiv \log(Y_t/Y^*)$, $\pi_t \equiv \log \Pi_t$):

$$\begin{aligned} &\text{(IS)} && y_t = y_{t+1} - \sigma r_t, \\ &\text{(NKPC)} && \pi_t = \kappa y_t + \beta \pi_{t+1}, \\ &\text{(Taylor rule)} && i_t = \phi \pi_t + \varepsilon_t^m, \end{aligned}$$

where $r_t \equiv i_t - \pi_{t+1}$ is the real rate, $\sigma > 0$ is the elasticity of intertemporal substitution, $\kappa > 0$ is the slope of the Phillips curve, $\phi \in \mathbb{R}$ is the Taylor-rule coefficient, and $\varepsilon_t^m = 0$ for $t \geq T$. Substituting the Taylor rule into the real rate gives $r_t = \phi \pi_t - \pi_{t+1} + \varepsilon_t^m$.

Proposition B.3. *Under Refinement 1, imposed for any $H \geq T$, the three-equation system admits a unique equilibrium for generic ϕ[48]. In this equilibrium, $\{y_t\}$ and $\{\pi_t\}$ depend on $\boldsymbol{\varepsilon}^m$ but not on fiscal*

[48]Specifically for $\phi \notin \left\{-\frac{1}{\sigma\kappa}, \frac{1}{\beta}\right\}$.

policy.

Proof. Refinement 1 imposes $y_t = r_t = 0$ for all $t \geq H$. Since $\varepsilon_t^m = 0$ on this tail, the NKPC and the monetary rule imply

$$\pi_t = \beta \pi_{t+1}, \qquad \pi_{t+1} = \phi \pi_t.$$

Hence $(1-\beta\phi)\pi_t = 0$, so $\pi_t = 0$ for all $t \geq H$ provided $\phi \neq 1/\beta$. Combining (IS) with $r_t = \phi\pi_t - \pi_{t+1} + \varepsilon_t^m$, the system reduces to a backward recursion: given (y_{t+1}, π_{t+1}),

$$\begin{aligned} y_t &= y_{t+1} + \sigma\pi_{t+1} - \sigma\phi\pi_t - \sigma\varepsilon_t^m, \\ \pi_t &= \kappa y_t + \beta\pi_{t+1}. \end{aligned}$$

At each date $t < H$, these two equations jointly determine (y_t, π_t) from $(y_{t+1}, \pi_{t+1}, \varepsilon_t^m)$, provided $1 + \sigma\kappa\phi \neq 0$. Starting from $(y_H, \pi_H) = (0,0)$ and iterating for $t = H-1, H-2, \ldots, 0$, there is a unique solution. The two excluded values correspond to distinct knife-edge cases: at $\phi = 1/\beta$, the terminal restrictions on output and the real rate do not uniquely pin down inflation, while at $\phi = -1/(\sigma\kappa)$ the backward recursion is singular. Since neither (IS), (NKPC), nor the Taylor rule involves fiscal variables, the equilibrium is independent of fiscal policy. $\square$

The contrast with standard practice is instructive. Without the refinement, the three-equation system is solved forward, and the Blanchard–Kahn conditions require both eigenvalues of the companion matrix to lie outside the unit circle. For nonnegative Taylor-rule coefficients, this is equivalent to the Taylor principle ($\phi > 1$). The refinement replaces the boundedness condition at infinity with a terminal condition at finite date H, converting the problem to a backward recursion that admits a unique solution for all non-knife-edge values of ϕ.

Nominal interest rate peg. Now let the monetary authority peg the nominal rate: $i_t = \varepsilon_t^m$, corresponding to $\phi = 0$. The real rate is $r_t = \varepsilon_t^m - \pi_{t+1}$, and the backward recursion becomes

$$\begin{aligned} y_t &= y_{t+1} + \sigma\pi_{t+1} - \sigma\varepsilon_t^m, \\ \pi_t &= \kappa y_t + \beta\pi_{t+1}, \end{aligned}$$

with $(y_H, \pi_H) = (0,0)$. This is the $\phi = 0$ case of the system above.

C Dissecting equilibrium multiplicity

This section discusses the two distinct sources of equilibrium multiplicity in the non-linear New Keynesian model and the sense in which our refinement eliminates both. We first identify the two sources and illustrate the second one, then make them concrete in the setting of Proposition 2, establishing that the second source produces at most two equilibria per NPV. We then show that one of these two equilibria is unstable. We finally show that, in the setting of Proposition 2, our refinement eliminates both sources of multiplicity.

As established in Theorems 1 and 4, the New Keynesian model admits a continuum of Nash equilibria regardless of the stance of monetary or fiscal policy. Multiple equilibria exist for two conceptually distinct reasons.

Source 1: The New Keynesian economy is demand-determined. Because the New Keynesian economy is fully demand-determined and the cumulative lifetime MPC equals one, the economy's intertemporal strategic complementarity is exactly one. Different levels of normalized permanent income $\xi \equiv \text{NPV}(\mathbf{C}, \mathbf{R})/Y^*$ therefore correspond to different equilibria, generating a continuum parameterized by ξ. This is the mechanism at the heart of fiscal dominance: an active fiscal rule selects a particular ξ by requiring that the government's intertemporal budget constraint holds.

Source 2: Non-linear feedback from the monetary policy rule. Even holding normalized permanent income ξ fixed, there may be multiple output paths $\mathbf{C}$ satisfying the Euler equation and delivering $\text{NPV}(\mathbf{C}, \mathscr{R}(\mathbf{C}))/Y^* = \xi$. This occurs because the interest rate depends on the level of output, creating a nonlinear feedback: for example, if the monetary policy rule is active, higher output raises the interest rate, which discounts future output more heavily despite the faster output growth. The permanent income is potentially unchanged even as the composition of spending shifts. This source is generically discrete (at most two equilibria per ξ in the setting of Proposition 2, as we show below).

Figure C.1 illustrates the second, type-2 source of multiplicity directly. The horizontal line fixes ξ; because $v_0(c_0)$ is non-monotone, two distinct initial spending levels, c_0^A and c_0^B, satisfy $v_0(c_0) = \xi$ and therefore launch different equilibrium paths with the same permanent income.

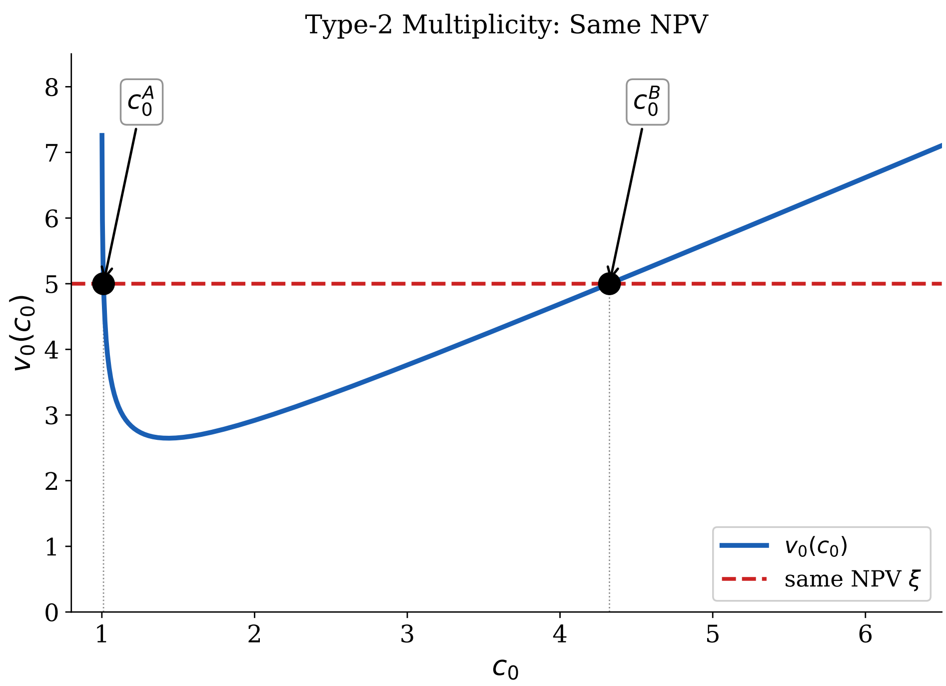

Figure C.1: Type-2 multiplicity: multiple equilibrium paths yielding the same NPV ($\sigma = 0.5$, $\phi = 2.5$, $\beta = 0.96$; no shocks).

Revisiting the standard practice. Consider the setting of Proposition 2: $U(C) = \frac{1}{1-1/\sigma} C^{1-1/\sigma}$ and $R_t = \beta^{-1}(Y_t/Y^*)^{\phi}\epsilon_t^m$, with $\sigma \neq 1$, $\sigma\phi > -1$, and $\epsilon_t^m = 1$ for t large enough. Following the proof of Proposition 2, iterating the Euler equation forward gives $c_t = c_0^{\alpha^t}\mu_t$, where $c_t \equiv C_t/Y^*$, $\alpha \equiv 1+\sigma\phi$, and $\mu_t > 0$ collects the shocks. The normalized permanent income is

$$v_0(c_0) = \sum_{t=0}^{\infty} \beta^t \omega_t c_0^{e_t}, \qquad e_t \equiv \frac{(\sigma-1)\alpha^t + 1}{\sigma},$$

where $\omega_t > 0$ depends only on the shock sequence and $e_0 = 1$. The two sources of multiplicity can be read directly off v_0: Source 1 corresponds to v_0 taking a continuum of values; Source 2 corresponds to v_0 being non-injective. Whether v_0 is monotone depends on the signs of e_t:

- $\phi \leq 0$ (passive monetary policy): $\alpha \in (0,1]$ and $e_t > 0$ for all t, so v_0 is strictly increasing. Each ξ corresponds to a unique equilibrium. (Only Source 1)
- $\phi > 0$, $\sigma \geq 1$: $\alpha > 1$ and $e_t \geq 1$ for all t. Again v_0 is strictly increasing. (Only Source 1)
- $\phi > 0$ (active monetary policy), $\sigma < 1$: $\alpha > 1$ but $e_t \to -\infty$. The function v_0 may be non-monotone (and is then U-shaped), so two distinct c_0 values may deliver the same ξ. (Source 1 and potentially Source 2)

When $\phi > 0$, only one Euler solution converges to Y^* (the bounded equilibrium). The remaining admissible solutions are "unbounded" but deliver finite NPV. Most differ from the bounded equilibrium in their permanent income (Source 1). When $\sigma < 1$ and v_0 is U-shaped, some may share the same permanent income but follow different trajectories (Source 2), because its two branches can deliver the same ξ.

Proposition C.1. *Assume the hypotheses of Proposition 2, and let D denote the admissible set of initial conditions characterized in its proof:*

$$D = \begin{cases} (0,\infty), & \phi \le 0, \\ (0,\bar{c}], & \phi > 0,\ \sigma > 1, \\ [\bar{c},\infty), & \phi > 0,\ \sigma < 1, \end{cases} \qquad \bar{c} \equiv \mu_T^{-1/\alpha^T}, \quad \alpha \equiv 1 + \sigma\phi.$$

Then:

(i) If $\phi \le 0$, or if $\phi > 0$ and $\sigma > 1$, the map v_0 is strictly increasing on D. Hence every attainable $\xi \in v_0(D)$ is delivered by exactly one equilibrium.

(ii) If $\phi > 0$ and $\sigma < 1$, then every attainable $\xi \in v_0(D)$ is delivered by at most two equilibria.

Proof. For part (i), note that in both cases all exponents e_t are strictly positive, while all coefficients $\beta^t\omega_t$ are strictly positive. Hence each term

$$\beta^t \omega_t c_0^{e_t}$$

is strictly increasing in c_0, and therefore so is

$$v_0(c_0) = \sum_{t=0}^{\infty} \beta^t \omega_t c_0^{e_t}$$

on the admissible domain D. This gives injectivity. For part (ii), set

$$a_t \equiv \beta^t \omega_t > 0, \qquad g(x) \equiv v_0(e^x) = \sum_{t=0}^{\infty} a_t e^{e_t x},$$

defined on $I \equiv \log D$. By Proposition 2, the series is finite on D, hence g is well defined on I. We claim that g is strictly convex. Let $x \neq y$ and $\lambda \in (0,1)$. Then, term by term,

$$a_t e^{e_t(\lambda x + (1-\lambda)y)} \le \lambda a_t e^{e_t x} + (1-\lambda) a_t e^{e_t y}.$$

Summing over t gives

$$g(\lambda x + (1-\lambda)y) \le \lambda g(x) + (1-\lambda) g(y).$$

The inequality is strict because the $t = 0$ term is

$$a_0 e^{e_0 x} = e^x,$$

and e^x is strictly convex. Hence g is strictly convex on I. A strictly convex function intersects any horizontal line at at most two points. Therefore, for any attainable ξ, the equation

$$g(x) = \xi$$

has at most two solutions in I. Since $x = \log c_0$ is one-to-one, the equation

$$v_0(c_0) = \xi$$

has at most two solutions in D. $\square$

When Source 2 is active, a natural question is whether both equilibria are equally robust. To answer this, fix $\xi > 0$ and consider the following fixed NPV best response exercise, in which each household solves,

$$\mathscr{U}\left(\mathbf{C}_i, \mathbf{C}, \boldsymbol{\epsilon}^m\right) \equiv \sum_{t=0}^{\infty} \beta^t U\big(C_{i,t}\big), \qquad \text{s.t.} \qquad \frac{\text{NPV}\big(\mathbf{C}_i, \mathscr{R}(\mathbf{C})\big)}{Y^*} = \xi.$$

Constraints are incorporated with the same approach as in the main text. We study stability under iteration of the scalar map $\text{BR}(c_0;\xi)$ derived below, fixing ξ. At each iteration, the aggregate consumption path is reconstructed as $C_t = Y^* c_0^{\alpha^t} \mu_t$ from the updated c_0. From the proof of Proposition 2, the aggregate interest-rate path $R_t = \beta^{-1}\big(c_0^{\alpha^t}\mu_t\big)^{\phi}\epsilon_t^m$ depends only on the aggregate c_0 and on the shock sequence, not on any individual household's choice. The household's individual Euler equation $U'(C_{i,t}) = \beta R_t U'(C_{i,t+1})$ therefore gives

$$\frac{C_{i,t+1}}{C_{i,t}} = (\beta R_t)^{\sigma},$$

so that, iterating forward from $C_{i,0} = c_{i,0} Y^*$:

$$c_{i,t} = c_{i,0} \prod_{s=0}^{t-1} (\beta R_s)^{\sigma}.$$

The product depends only on the aggregate path, not on the household's own choice. The household's NPV is therefore linear in its initial choice $c_{i,0}$:

$$\frac{\text{NPV}(\mathbf{C}_i, \mathbf{R})}{Y^*} = c_{i,0} \cdot \Omega(c_0), \qquad \Omega(c_0) \equiv \frac{v_0(c_0)}{c_0} = \sum_{t=0}^{\infty} \beta^t \omega_t c_0^{e_t - 1},$$

where $\omega_t > 0$ collects the shocks, exactly as in Proposition 2. Here $\Omega(c_0)$ is the present value of a unit of initial spending, evaluated at the aggregate interest-rate path. Setting normalized permanent income equal to ξ and solving for the household's optimal $c_{i,0}$ yields the best response:

$$\text{BR}(c_0;\xi) = \frac{\xi}{\Omega(c_0)}. \tag{C.1}$$

A Nash equilibrium at normalized permanent income ξ is a fixed point $c_0 = \text{BR}(c_0;\xi)$, equivalently $v_0(c_0) = \xi$.

Proposition C.2. *In the setting of Proposition 2 with $\phi > 0$ and $\sigma < 1$, suppose ξ admits two equilibria $c_0^A < c_0^B$. Under this scalar iteration:*

Equilibrium A is unstable (one-sided if $c_0^A = \bar{c}$): $BR(c_0;\xi) > c_0$ for every $c_0 \in (c_0^A, c_0^B)$.

Equilibrium B (on the ascending branch of v_0) is stable: $|BR'(c_0^B)| < 1$.

Proof. Let

$$\Omega(c_0) \equiv \frac{v_0(c_0)}{c_0} = \sum_{t=0}^{\infty} \beta^t \omega_t c_0^{e_t - 1}.$$

Then

$$\mathrm{BR}(c_0;\xi) = \frac{\xi}{\Omega(c_0)}.$$

Since $g(x) = v_0(e^x)$ is strictly convex and, by hypothesis, $v_0(c_0^A) = v_0(c_0^B) = \xi$ with $c_0^A < c_0^B$, it follows that

$$v_0(c_0) < \xi \qquad \text{for every } c_0 \in (c_0^A, c_0^B).$$

Using $\Omega(c_0) = v_0(c_0)/c_0$, we therefore have

$$\mathrm{BR}(c_0;\xi) = \frac{\xi c_0}{v_0(c_0)} > c_0 \qquad \text{for every } c_0 \in (c_0^A, c_0^B).$$

Since $\mathrm{BR}(\cdot;\xi)$ is continuous and has no fixed point in (c_0^A, c_0^B), scalar iterations starting just to the right of c_0^A move monotonically away from it until they leave any sufficiently small neighborhood. Thus c_0^A is unstable relative to the admissible domain.

The right equilibrium satisfies $c_0^B > c_0^A \geq \bar{c}$ and is therefore an interior point. Strict convexity gives $v_0'(c_0^B) > 0$. Since $v_0(c_0^B) = \xi = c_0^B \Omega(c_0^B)$, one has

$$\mathrm{BR}'(c_0^B) = 1 - \frac{v_0'(c_0^B)}{\Omega(c_0^B)} < 1.$$

Moreover,

$$v_0'(c_0) - \Omega(c_0) = \sum_{t=0}^{\infty} \beta^t \omega_t (e_t - 1) c_0^{e_t - 1}.$$

The $t = 0$ term vanishes because $e_0 = 1$, and for $t \geq 1$ one has $e_t - 1 < 0$ and $\omega_t > 0$, so every remaining term is negative. Therefore

$$0 < v_0'(c_0^B) < \Omega(c_0^B),$$

which implies

$$0 < \mathrm{BR}'(c_0^B) < 1.$$

Hence c_0^B is locally stable. □

The economics is transparent: at either equilibrium $c_0 \in \{c_0^A, c_0^B\}$, $\Omega(c_0) = \xi / c_0$ is the average permanent income per unit of initial spending. At the lower equilibrium, an admissible increase in aggregate c_0 reduces permanent income, so households respond by raising initial spending still further to meet their target. At the upper equilibrium, higher c_0 raises permanent income, dampening the feedback. The stable equilibrium is the one on which the natural comparative statics obtain.

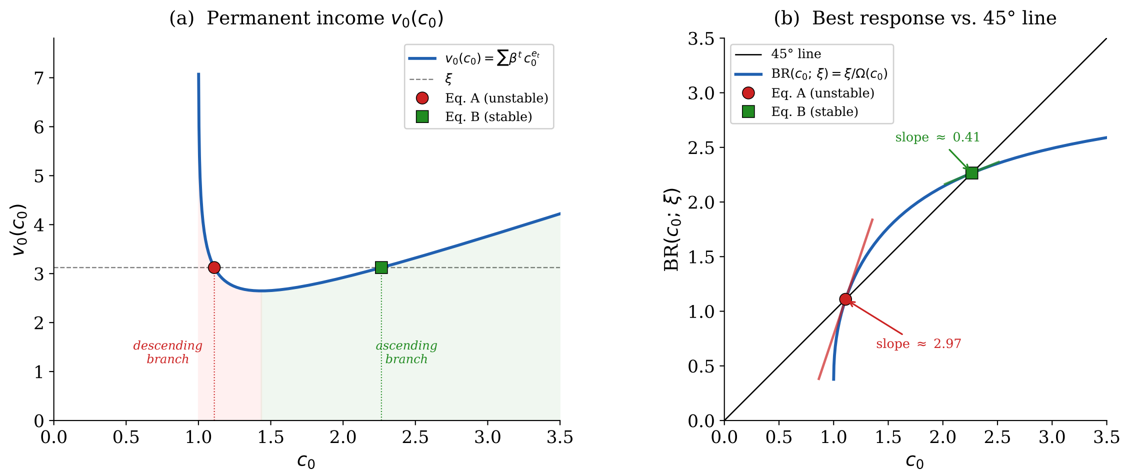

Figure C.2: Source 2 multiplicity and stability ($\sigma = 0.5$, $\phi = 2.5$, $\beta = 0.96$; no shocks).

We now show that Refinement 1 eliminates both sources simultaneously, by converting the forward recursion (which requires a global NPV condition to pin down c_0) into a backward recursion from a terminal condition (which uniquely determines each C_t given C_{t+1}).

Proposition C.3. *Make the same assumptions as in Proposition 2, with $\epsilon_t^m = 0$ for $t \geq T$ (in log-deviations). Impose Refinement 1 for arbitrary $H \geq T$. Then there is a unique equilibrium, regardless of ϕ.*

Proof. In log-deviations $y_t \equiv \log(C_t / Y^*)$, the Euler equation is $(1 + \sigma\phi)\, y_t = y_{t+1} - \sigma \epsilon_t^m$. Let $\delta \equiv 1/(1 + \sigma\phi)$. The refinement imposes $y_t = 0$ for $t \geq H$. Backward induction gives $y_t = 0$ for $t \geq T$, and for $t \leq T - 1$:

$$y_t = -\sigma\delta \sum_{k=0}^{T-1-t} \delta^k \epsilon_{t+k}^m.$$

Each y_t is uniquely determined. Moreover, $\mathrm{NPV}(\mathbf{C}, \mathbf{R}) < \infty$: for $t \geq T$, $C_t = Y^*$ and $R_t = \beta^{-1} > 1$, so the tail converges geometrically, while the earlier sum has finitely many finite terms. Thus the path defines a valid NE. □