Speculative Growth and the AI "Bubble"

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Abstract

I develop a model in which temporary optimism about AI returns can induce investment that permanently raises productive capacity, even after a correction. AI capital supplies labor services, allowing firms to expand conventional capital and effective labor together while keeping the marginal product of capital constant during deployment. Income from AI services accrues to investors, whose propensity to save rises with wealth. Sufficient AI deployment capacity generates low- and high-capital steady states separated by a threshold. There is no rational equilibrium path from the low-capital steady state to the high-capital steady state. However, strong and persistent optimism can induce enough investment for the economy to converge to the high-capital steady state after a full correction. Later corrections can be larger without reversing accumulation. Investment demand puts upward pressure on the interest rate during the expansion. At the high-capital steady state, investors' greater wealth lowers the interest rate, wages are higher, and the labor share is lower. Investors bear the capital loss at the correction.

JEL Codes: E21, E22, E24, E44, O33, O41.

Keywords: AI, speculative growth, optimism, investment transition, multiple steady states, interest rates, labor share, wealth-in-utility, saving glut.

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1 Introduction

> "While some part of the investment which was going on in the world at large was doubtless ill-judged and unfruitful, there can, I think, be no doubt that the world was enormously enriched by the constructions of the quinquennium from 1925 to 1929; its wealth increased in these five years by as much as in any other ten or twenty years of its history." —Keynes (1931)

Temporary optimism about AI returns can induce an investment expansion that permanently raises productive capacity, even when asset prices subsequently fall. High valuations of AI-related firms are commonly assessed by asking whether they reflect fundamentals or a bubble. Their economic consequences also depend on the investment they induce: the capital installed while valuations are high changes production and the income available to investors after a correction. Evaluating an AI investment expansion therefore requires separating three questions: whether the technology is productive, whether peak valuations are sustained, and whether the increase in capital persists. I study an economy in which investment changes both productive capacity and investors' saving, so the expansion itself affects how much capital can be sustained after a correction.

AI capital performs tasks otherwise performed by workers. Firms allocate capital between conventional production and the provision of effective labor services. During deployment, the optimal allocation expands both uses together, keeping conventional capital per unit of effective labor constant. The marginal product of capital therefore remains constant over this range of accumulation. Before deployment begins and after deployment capacity is exhausted, effective labor is fixed and the marginal product of capital declines as capital accumulates. AI deployment capacity determines how much capital can accumulate without diminishing returns.

Income generated by AI services accrues to investors, whose propensity to save rises with wealth. During AI deployment, the marginal product of capital remains constant, while greater wealth lowers the steady-state interest rate. With sufficient AI deployment capacity, these production and saving responses generate low- and high-capital steady states separated by a middle steady state. The low-capital steady state describes the economy before a sustained expansion in AI deployment. At the high-capital steady state, deployment is complete, investors' wealth is greater, and the interest rate is lower.

There is no rational equilibrium path from the low-capital steady state to the high-capital steady state. The rational path from each capital stock is unique. The middle steady state is the capital threshold: below it, the path converges to the low-capital steady state; above it, the path converges to the high-capital steady state. The lower interest rate at the high-capital steady state reflects investors' greater wealth; at the low-capital steady state, the rational price implies replacement investment and the economy remains there.

I represent optimism by a belief wedge: investors overestimate the return on capital by a fixed amount, which they regard as permanent until an unanticipated correction. Optimistic investors value installed capital more highly. Firms respond to the higher valuation by investing more, with convex adjustment costs limiting the speed of accumulation. The belief wedge must be large enough for investment to exceed replacement throughout the

accumulation needed to cross the capital threshold. It must also persist long enough for that capital to be installed. The price adjusts immediately to a change in beliefs, whereas capital accumulates over time. Strong but short-lived optimism can end before the threshold is crossed; with persistent but insufficient optimism, capital converges to a steady state with no AI deployment.

A full correction eliminates the belief wedge. The subsequent equilibrium path depends on whether capital has crossed the threshold. The price falls at the correction. If capital remains below the threshold, the new price implies investment below replacement and the economy returns to the low-capital steady state. If the threshold has been crossed, the rational equilibrium path converges to the high-capital steady state. A fall in asset prices is consistent with either a temporary or a permanent increase in capital.

A partial correction reduces the belief wedge and can slow accumulation without reversing it. During deployment, the price rises along the optimistic path as capital accumulates, while the price at which investment just replaces depreciation remains constant. Later corrections can therefore be larger without reversing accumulation. The slower pace of accumulation can leave capital below the threshold at the full correction, in which case the economy returns to the low-capital steady state.

The investment expansion and the accumulation of investors' wealth affect the interest rate in opposite directions. Investment demand puts upward pressure on the rate during the expansion. At the high-capital steady state, investors are willing to hold their greater wealth at a lower interest rate, and the rate is below its initial level. The long-run effect of AI on interest rates therefore depends on who receives its income as well as on the investment it induces.

Wages are higher and the labor share is lower at the high-capital steady state. Conventional capital deepening raises wages before deployment and after capacity is exhausted; during deployment, wages remain constant because conventional capital and effective labor expand together. Income from AI-provided labor services accrues to investors, so the labor share falls during deployment even though the wage level rises over the full transition. Investors are wealthier at the high-capital steady state but bear the capital loss at the correction.

The model combines AI capital that substitutes for labor, wealth-in-utility preferences that generate non-homothetic saving (Straub, 2019), and convex adjustment costs that link the price of capital to investment. I characterize when temporary optimism permanently changes AI deployment and the aggregate capital stock through the interaction of production and saving. The analysis distinguishes the size and duration requirements for a transition, derives the effects of full and partial corrections, and relates these outcomes to interest rates and the distribution of income.

Related literature. The closest antecedent is Caballero et al. (2006), in which asset values and funding conditions affect investment, and capital accumulation affects asset values. I retain this interaction and introduce AI through the production function and optimism through a temporary belief wedge. AI capital supplies labor services, and the resulting income accrues to investors whose saving response lowers the long-run interest rate. Temporary optimism can then induce enough investment for the economy to converge to the high-capital steady state after a full correction.

The production structure is related to the automation literature's emphasis on capital performing tasks previously performed by workers. Acemoglu and Restrepo (2018) formalize automation and factor shares, while Moll, Rachel, and Restrepo (2022) study how automation affects returns to wealth and income and wealth inequality. Here, capital used to provide labor services expands together with conventional capital, producing a range over which the marginal product of capital is constant. Related work studies transitions to artificial general intelligence (Restrepo, 2025; Korinek and Suh, 2024) and the broader macroeconomics of transformative AI (Trammell and Korinek, 2023; Acemoglu, 2025; Aghion and Bunel, 2024; Jones and Tonetti, 2026; Brynjolfsson et al., 2025).

The saving mechanism draws on work connecting inequality, saving, and interest rates. Wealth-in-utility preferences represent the non-homothetic saving behavior in Straub (2019) and relate to Mian, Straub, and Sufi (2020) on the saving glut of the rich. AI deployment provides an endogenous source of income concentration: income from tasks performed by capital accrues to investors. Their greater wealth lowers the interest rate at which they are willing to hold it. Combining this saving response with the production structure generates the multiple steady states. The distribution of AI income therefore affects both the interest rate and the capital stock that can be sustained in the long run. Andrews and Farboodi (2025) report declines in long-maturity Treasury and TIPS yields around major AI model releases.

Pástor and Veronesi (2009) show how learning about a new technology can generate high prices that subsequently fall even when the technology is productive. Here, capital accumulation changes productive capacity and investors' saving, so investment induced by optimistic valuations can alter the steady state reached after a full correction. The relation between expectations and investment is supported by the evidence in Gennaioli, Ma, and Shleifer (2016). De Long and Shleifer (1991) provide a historical comparison by using closed-end fund premia to study overvaluation in the late-1920s market.

Martin and Ventura (2016) show how credit bubbles can expand investment by increasing the credit available to entrepreneurs. Caballero and Krishnamurthy (2006) show how bubbles can raise domestic investment by providing stores of value that reduce capital outflows, while exposing the economy to crashes and capital flow reversals. These papers relate speculative valuations to investment through financing conditions. I focus on whether the expansion in productive capacity induced by optimism persists after a full correction, through the effects of AI deployment on production and investors' saving.

The multiplicity of steady states also relates to work on takeoff and coordination failure (Rosenstein-Rodan, 1943; Murphy, Shleifer, and Vishny, 1989; Cooper and John, 1988; Azariadis and Drazen, 1990; Matsuyama, 1991). Coordination-failure models can have multiple rational equilibrium paths from the same initial condition. The model has multiple steady states and a unique rational equilibrium path from each capital stock. A transition from the low-capital steady state to the high-capital steady state requires an initial departure from rational expectations that induces investment until capital crosses the threshold.

Section 2 presents the technology, household saving, investment, and beliefs. Section 3 establishes the steady states and characterizes the rational equilibrium paths. Section 4 derives the size and duration requirements for speculative investment, studies full and

partial corrections, and examines interest rates, the post-correction yield curve, and distributional incidence. Section 5 concludes. The appendices provide the derivations, proofs, and numerical details.

2 Model

I consider a closed economy in continuous time with a single good used for consumption and investment. There are two types of households: workers, who supply labor and consume their wages, and investors, who own capital and derive utility from consumption and wealth. Competitive firms produce using conventional capital and labor services supplied by workers and AI capital. Investment is subject to convex adjustment costs.

2.1 Technology

AI capital supplies labor services that firms use together with conventional capital. Firms can therefore expand effective labor by installing capital, and investors receive the rental income from that capital. The allocation of capital between these two uses determines its marginal product and competitive rental rate.

Let L denote raw labor, K_c conventional capital, and N effective labor. With productivity $A > 0$, output is

$$Y = AK_c^{\alpha} N^{1-\alpha}, \qquad \alpha \in (0,1).$$

Total capital $K = K_c + K_\ell$ is allocated between conventional use and AI use. Each unit of AI capital K_ℓ supplies γ units of effective labor services, so $N = L + \gamma K_\ell$. AI deployment is bounded by $K_\ell \leq \bar{K}_\ell L$. The bound represents deployment capacity per worker, reflecting constraints such as data, power, and organizational capacity. This scaling makes the aggregate technology constant-returns in (K, L), so competitive payments to raw labor and total capital exhaust output. I normalize $L = 1$ after taking the factor derivatives.

At low capital stocks, firms allocate all capital to conventional production because its marginal product exceeds that in AI use. As capital accumulates, the marginal product of conventional capital falls relative to that of AI capital. Deployment begins when the two become equal. During deployment, the optimal allocation satisfies

$$\alpha A K_c^{\alpha-1} N^{1-\alpha} = (1-\alpha) A K_c^{\alpha} N^{-\alpha} \gamma.$$

This condition gives $N = bK_c$, with $b \equiv (1-\alpha)\gamma/\alpha$. Combining it with $K = K_c + K_\ell$ and $N = 1 + \gamma K_\ell$ gives

$$N = bK_c, \qquad K_\ell = \frac{bK - 1}{\gamma + b}, \tag{1}$$

which fixes $K_c/N = 1/b$ and hence the marginal product of capital (Appendix A). An increase in total capital is allocated partly to conventional capital and partly to AI capital. The additional AI services expand effective labor in proportion to conventional capital, so capital accumulation preserves the input ratio that determines both marginal products.

Deployment begins at $K_{\mathrm{AI}} = 1/b$, where $K_\ell = 0$, and reaches the capacity constraint at $K_{\mathrm{sat}} = [1 + (\gamma + b)\bar{K}_\ell]/b$. Using $b = (1-\alpha)\gamma/\alpha$,

$$K_{\mathrm{sat}} - K_{\mathrm{AI}} = \frac{(\gamma + b)\bar{K}_\ell}{b} = \frac{\bar{K}_\ell}{1-\alpha} > 0.$$

Thus the length of the deployment interval increases with capacity $\bar{K}_\ell$.

Let $d(K)$ denote the competitive rental rate of capital, equal to its marginal product, and let $w(K)$ be the wage per unit of labor supplied by workers. Because the optimized aggregate technology is constant-returns, the envelope theorem and Euler's theorem give

$$d(K) = \left.\frac{\partial Y(K,L)}{\partial K}\right|_{L=1}, \qquad w(K) = \left.\frac{\partial Y(K,L)}{\partial L}\right|_{L=1}, \qquad Y(K) = Kd(K) + w(K). \quad (2)$$

The rental rate is

$$d(K) = \begin{cases} \alpha A K^{\alpha-1}, & K < K_{\mathrm{AI}}, \\ d_{\mathrm{flat}} \equiv \alpha A b^{1-\alpha}, & K_{\mathrm{AI}} \le K < K_{\mathrm{sat}}, \\ \alpha A (K - \bar{K}_\ell)^{\alpha-1}(1 + \gamma\bar{K}_\ell)^{1-\alpha}, & K \ge K_{\mathrm{sat}}. \end{cases} \quad (3)$$

Before deployment, all capital is conventional and its marginal product declines with K. During deployment, the allocation in (1) keeps conventional capital per unit of effective labor constant, so the marginal product remains at d_{flat}. After the deployment constraint binds, effective labor is fixed at $1 + \gamma\bar{K}_\ell$; further accumulation increases conventional capital and its marginal product declines again. Deployment capacity thus determines the interval over which additional capital can be installed without diminishing returns.

The constant marginal product of capital during deployment extends to other constant-returns technologies whose interior allocation fixes the input ratio. In such a region, the marginal products depend on that ratio, so both remain constant as firms expand the two inputs together. Under Cobb–Douglas, $d_{\mathrm{flat}} = \gamma w_{\mathrm{flat}}$, where $w_{\mathrm{flat}} = (1-\alpha)Ab^{-\alpha}$: the rental income from a unit of AI capital equals the wage value of the labor services it supplies.

Workers receive $w = Y_L$, while investors receive capital income $Kd(K)$. Once the deployment constraint binds, workers' wages also include the value of the AI deployment capacity associated with their labor. The labor share is workers' wage income divided by output. Applying (2) region by region gives

$$s_L(K) = \frac{w(K)}{Y(K)} = \begin{cases} 1-\alpha, & K < K_{\mathrm{AI}}, \\ \dfrac{1}{1+\gamma K}, & K_{\mathrm{AI}} \le K < K_{\mathrm{sat}}, \\ (1-\alpha) - \dfrac{\alpha\bar{K}_\ell}{K - \bar{K}_\ell}, & K \ge K_{\mathrm{sat}}. \end{cases} \quad (4)$$

Before deployment, workers receive the Cobb–Douglas share $1-\alpha$. During deployment, output increases while the wage is constant, and the income from the additional AI-provided labor services accrues to investors. The labor share therefore falls. After saturation, further

conventional capital deepening raises the labor share toward $1-\alpha$, although it remains below its pre-deployment value. The wage level rises before deployment, remains constant during deployment, and rises again after saturation. A falling labor share during AI deployment is consistent with a higher wage at the high-capital steady state.

2.2 Households and the interest-rate schedule

Workers supply labor, hold no assets, and consume their wage.[1] Investors own the capital claim and can trade a riskless short-term bond. Its instantaneous interest rate is i_t. Investors' aggregate wealth is $W_t = q_t K_t$, where q_t is the price of installed capital in units of the consumption good (Tobin's q). The bond is in zero net supply and therefore does not add to aggregate wealth; its market-clearing interest rate is the rate at which investors are willing to reallocate consumption over time.

Let c_t denote investors' consumption. Investors have wealth-in-utility preferences,

$$\int_0^\infty e^{-\rho t}\Big[\log c_t + \theta W_t\Big]\, dt, \tag{5}$$

where $\rho > 0$ is the rate of time preference and $\theta > 0$ is the weight on wealth in instantaneous utility. The Euler equation (Appendix B) is

$$\frac{\dot{c}_t}{c_t} = i_t - \rho + \theta c_t. \tag{6}$$

Because wealth provides utility directly, saving yields a benefit in addition to the financial return. The term θc_t in (6) measures this benefit relative to the marginal utility of consumption. For a given consumption level and growth rate, investors are therefore willing to hold wealth at a lower interest rate than when $\theta = 0$. Rearranging (6) gives

$$i_t = \rho - \theta c_t + \frac{\dot{c}_t}{c_t}. \tag{7}$$

The bond is riskless, and investors perceive no risk in the capital claim. Given their beliefs, the no-arbitrage condition equates the perceived return on capital to i_t.

Setting $\dot{c} = 0$ in (6) gives $i = \rho - \theta c$. In a rational steady state the capital claim and the bond have the same return, and constant aggregate wealth requires $c = iW$. Solving these two equations for c and i gives

$$c^{ss}(W) = \frac{\rho W}{1+\theta W}, \qquad i^{ss}(W) = \frac{\rho}{1+\theta W}, \qquad \frac{di^{ss}}{dW} = -\frac{\rho\theta}{(1+\theta W)^2} < 0. \tag{8}$$

Consumption increases less than proportionately with wealth, so the consumption-wealth ratio falls. Investors are willing to hold a larger stock of wealth at a lower steady-state interest rate. This non-homothetic saving response connects the distribution of AI income to the interest rate: capital installed during deployment and the income from its labor

[1] Investors bear the entire capital loss under this ownership assumption. For a given capital path, the wage results in Section 4.5 are independent of asset ownership.

services belong to investors, whose propensity to save rises with wealth. These schedules describe steady-state household choices. During an investment expansion, investors' consumption also depends on the resources used for investment, and the interest rate includes the consumption-growth term in (7).

2.3 Investment, prices, and beliefs

Let I_t denote gross investment and $j_t = I_t/K_t$ the gross investment rate. Expenditure on new capital, including adjustment costs, is $K_t\Gamma(j_t)$ units of the consumption good, where

$$\Gamma(j) = \psi\left(e^{j/\psi} - 1\right), \qquad \psi > 0. \tag{9}$$

Adjustment costs per unit of existing capital are $\Gamma(j) - j$. The investment first-order condition is $q_t = \Gamma'(j_t) = e^{j_t/\psi}$. With depreciation rate $\delta > 0$,

$$j_t = \psi \log q_t, \qquad \frac{\dot{K}_t}{K_t} = \psi \log q_t - \delta. \tag{10}$$

The capital stock is continuous, while its price can jump when beliefs change. The gross investment rate is increasing in q_t and equals depreciation at $q_t = \bar{q} \equiv e^{\delta/\psi}$. The firm invests up to the point at which marginal investment expenditure, including adjustment costs, equals the price of installed capital. A higher price therefore increases investment, but convex adjustment costs limit how quickly capital accumulates. Net investment is positive when $q_t > \bar{q}$ and negative when $q_t < \bar{q}$.

The economy is closed, and workers consume their wages, so investors' saving funds all investment expenditure. Investors receive capital income $K_t d(K_t)$, pay for investment and adjustment costs, and consume the residual:

$$c_t = C(K_t, q_t) = K_t\left[d(K_t) - \psi(q_t - 1)\right]. \tag{11}$$

At a fixed capital stock, a higher q_t increases investment and the associated expenditure. Investors' consumption falls while wealth $W_t = q_t K_t$ rises. The decline in consumption represents the additional saving attracted by high perceived returns; in a broader setting, this saving could also come from other households or abroad.

The realized return on capital R_t^a includes the change in the value of the capital claim and the cash flow distributed to investors, both measured relative to $W_t = q_t K_t$:

$$R_t^a = \frac{\dot{q}_t}{q_t} + \psi \log q_t - \delta + \frac{d(K_t) - \psi(q_t - 1)}{q_t}, \qquad W_t = q_t K_t. \tag{12}$$

These definitions satisfy the aggregate wealth identity

$$\dot{W}_t = R_t^a W_t - c_t. \tag{13}$$

I represent optimism about future AI returns as an overestimate of the return on the aggregate capital claim. During the optimistic phase, this overestimate is a fixed amount

x, the *belief wedge*, with $0 < x < \rho$. The upper bound ensures a positive realized return in a steady state with this wedge. Investors perceive the return on capital as $R_t^a + x$, so the no-arbitrage condition given their beliefs is

$$i_t = R_t^a + x. \tag{14}$$

The resource constraint (11) and the wealth identity (13) use realized income and expenditure. Optimism changes the return investors perceive, the price of installed capital, and firms' investment. Investors' consumption and investment expenditure, including adjustment costs, continue to sum to capital income.

At each date, investors evaluate saving under the belief that the current wedge will remain fixed. They perceive the return on capital as i_t, while the realized return is $i_t - x$. For the same initial wealth and consumption path, forecast wealth is at least e^{xt} times market wealth. The transversality condition applies to forecast wealth and, for a fixed wedge $0 \leq x < \rho$, is equivalent to

$$\lim_{t\to\infty} e^{-(\rho-x)t} \frac{q_t K_t}{c_t} = 0. \tag{15}$$

An equilibrium path for a fixed wedge satisfies equations (6), (10)–(12), and (14), has positive capital, price, and consumption, and satisfies the transversality condition (15). This condition excludes solutions on which consumption tends to zero and is needed to determine the initial equilibrium price. Appendix B derives it from the household's optimization problem.

The economy follows the equilibrium path for the current wedge until an unanticipated correction at τ, then follows the equilibrium path for the revised wedge. At a full correction, the wedge falls to zero and the no-arbitrage condition becomes $i_t = R_t^a$. Rational beliefs correspond to $x = 0$.

3 Steady States and Rational Equilibrium Paths

The technology and saving behavior determine the steady-state capital stocks. I first derive these steady states and then characterize the rational equilibrium paths that converge to them. The existence of several steady states leaves open whether a rational equilibrium path from the low-capital steady state can converge to the high-capital steady state.

3.1 Multiple steady states

A rational steady state requires $\dot{K} = 0$ and $\dot{q} = 0$. By (10), the first condition gives $q_t = \bar{q}$, at which gross investment just replaces depreciation. All rational steady states therefore have the same price per unit of capital. Investors' wealth $\bar{q}K$ is greater at the high-capital steady state because the capital stock is larger. A permanently larger stock is thus consistent with a temporary increase in the price of capital during the transition.

At a rational steady state, the return on capital equals the interest rate. The household Euler equation determines the interest rate at which investors are willing to hold wealth $W = \bar{q}K$:

$$i^{ss}(\bar{q}K) = \frac{\rho}{1 + \theta\bar{q}K}.$$

Replacement uses $\Gamma(\delta) = \psi(\bar{q} - 1)$ units of output per unit of capital. The net cash flow available for investors' consumption is therefore $d(K) - \Gamma(\delta)$ per unit of capital, and the return on a claim priced at $\bar{q}$ is

$$\frac{d(K) - \Gamma(\delta)}{\bar{q}}.$$

Thus K is a steady state only if this return equals the interest rate:

$$\frac{d(K) - \Gamma(\delta)}{\bar{q}} = \frac{\rho}{1 + \theta\bar{q}K}.$$

Define the steady-state user cost of capital $\Phi(K)$ and the difference between the rental rate and the user cost, $G(K)$, by

$$\Phi(K) \equiv \Gamma(\delta) + \frac{\bar{q}\rho}{1 + \theta\bar{q}K}, \qquad G(K) \equiv d(K) - \Phi(K). \tag{16}$$

The user cost $\Phi(K)$ is replacement expenditure, including adjustment costs, plus the return required on capital valued at $\bar{q}$. At a steady state, the rental rate equals the user cost:

$$G(K) = 0, \qquad \text{equivalently} \qquad d(K) = \Phi(K).$$

At the common steady-state price $\bar{q}$, a larger K implies greater wealth for investors and a lower interest rate. Replacement expenditure per unit is unchanged, so the user cost $\Phi(K)$ decreases with K. During AI deployment, firms expand conventional capital and effective labor together, keeping the rental rate constant. The difference $G(K)$ therefore increases with K. This interaction between AI deployment and the effect of wealth on saving allows the rental rate to cross the user cost from below in the deployment region.

Figure 1 plots the rental rate $d(K)$ and the user cost $\Phi(K)$. The example has three intersections, one in each technology region.

If $\theta = 0$, the steady-state Euler equation gives $i = \rho$ at every level of wealth. The user cost is then constant at $\Phi(K) = \Gamma(\delta) + \bar{q}\rho$. Unless this level equals d_{flat}, the rental rate equals the user cost at exactly one capital stock. A user cost that decreases with capital is necessary for three isolated steady states. During deployment, the rental rate is constant while greater wealth lowers the steady-state interest rate and the user cost. Under the conditions below, the rental rate therefore crosses the user cost from below between the low- and high-capital steady states.

After saturation, the rental rate decreases with capital. If $d(K)$ falls below $\Phi(K)$, there is a high-capital steady state in addition to the low and middle steady states. The following proposition gives sufficient conditions.

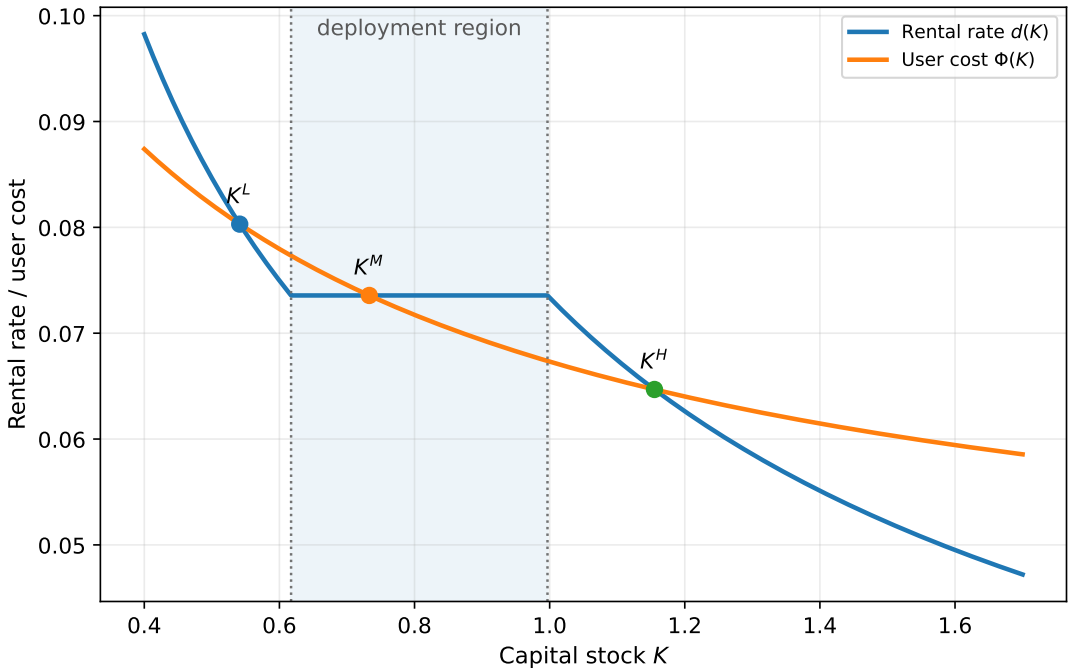

Figure 1: Rational steady states. The rental rate $d(K)$ equals the user cost $\Phi(K)$ below the deployment interval, within it, and above it. These intersections determine the low, middle, and high steady states.

Proposition 1 (Deployment capacity and multiple steady states)**.** *Suppose*

$$\Phi(K_{\text{AI}}) > d_{\text{flat}} > \Gamma(\delta). \tag{17}$$

Then there exists a deployment capacity $\bar{K}_\ell^* > 0$ *such that, for every* $\bar{K}_\ell > \bar{K}_\ell^*$*, the steady-state equation* $G(K) = 0$ *has at least one solution in* $(0, K_{\text{AI}})$*, a unique solution in* $(K_{\text{AI}}, K_{\text{sat}})$*, and at least one solution in* (K_{sat}, ∞)*. If, in addition,*

$$G'(K) < 0 \qquad \text{for } K \in (0, K_{\text{AI}}) \cup (K_{\text{sat}}, \infty), \tag{18}$$

then the low- and high-capital steady states are unique. At each, the rental rate crosses the user cost from above. The economy has exactly three steady states, denoted $K^L < K^M < K^H$*.*

The first inequality in (17) requires the rental rate to be below the user cost at the start of deployment. Since $d_{\text{flat}} = \gamma w_{\text{flat}}$, the second requires the rental income from AI-provided labor services to exceed replacement expenditure per unit of capital. As deployment capacity rises, $K_{\text{sat}} = [1 + (\gamma + b)\bar{K}_\ell]/b$ increases. Since $\Phi(K) \downarrow \Gamma(\delta)$ as $K \to \infty$, sufficiently large capacity gives $\Phi(K_{\text{sat}}) < d_{\text{flat}}$, and hence

$$\Phi(K_{\text{AI}}) > d_{\text{flat}} > \Phi(K_{\text{sat}}). \tag{19}$$

The middle steady state lies in the deployment region, where the rental rate is constant and the user cost decreases with capital. The deployment constraint must allow enough

accumulation for $\Phi(K)$ to fall below d_{flat} before diminishing returns resume. A wider deployment interval permits a larger decline in $\Phi(K)$; if capacity is exhausted before the two are equal, this mechanism does not generate the middle and high steady states. After saturation, the rental rate declines again. The slope condition (18) requires the rental rate to fall faster than the user cost before and after deployment, ensuring unique low and high steady states. It also implies that both are saddles, as established in Lemma 1. Appendix D proves the proposition.

At the low-capital steady state K^L, AI deployment has not begun. At the high-capital steady state K^H, deployment capacity has been exhausted, investors' wealth is greater, and the interest rate is lower. Both production and saving have changed: AI services expand effective labor and capital income, while investors are willing to hold their greater wealth at a lower interest rate. These changes allow the larger capital stock to be maintained under rational expectations.

Lemma 2 establishes that the middle steady state K^M is the *capital threshold*: the rational equilibrium path converges to K^L below the threshold and to K^H above it. At K^M, the only equilibrium path is stationary.

3.2 Rational equilibrium paths

The initial price determines investment and subsequent accumulation. Convergence to a saddle steady state requires the capital-price pair to lie on its *saddle path*, the set of pairs whose equilibrium paths converge to that steady state. A rational transition from K^L to K^H requires the saddle path to K^H to be defined at K^L. The next two lemmas characterize local stability and the domain of each saddle path.

Capital accumulation satisfies

$$\dot{K}_t = K_t(\psi \log q_t - \delta),$$

so the $\dot{K} = 0$ locus is the horizontal line $q = \bar{q}$. Above that line investment exceeds depreciation and K_t rises; below it K_t falls.

Along a rational path, the return on the capital claim equals the interest rate, $R^a_t = i_t$; writing out both sides using (12) and (7),

$$\underbrace{\frac{\dot{q}_t}{q_t} + \psi \log q_t - \delta + \frac{d(K_t) - \psi(q_t - 1)}{q_t}}_{\text{realized return } R^a_t} = \underbrace{\rho - \theta c_t + \frac{\dot{c}_t}{c_t}}_{\text{interest rate } i_t}. \tag{20}$$

The resource constraint (11) determines consumption given K and q. Rational equilibrium paths satisfy (15) with $x = 0$.

Substituting $c_t = C(K_t, q_t)$ and $\dot{c}_t = C_K \dot{K}_t + C_q \dot{q}_t$ into (20) gives an equation linear in $\dot{q}_t$. Solving it gives the law of motion for q. The $\dot{q} = 0$ nullcline equates the realized return excluding changes in q to the interest rate from the Euler equation, including the consumption growth generated by changing K. Appendix C reports the law of motion and the nullcline.

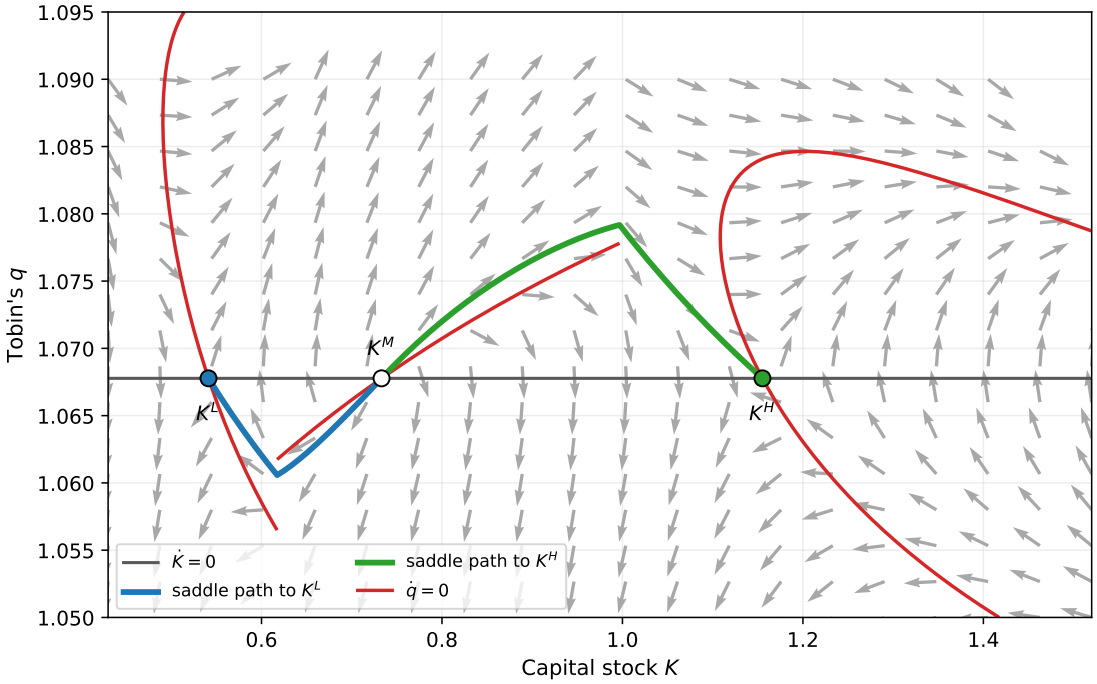

Figure 2: Rational phase diagram. The low- and high-capital steady states are saddles, and the middle steady state is an unstable node. Followed backward in time, both saddle paths approach the middle steady state. The horizontal line is $\dot{K} = 0$; the red curves are $\dot{q} = 0$. The breaks in the $\dot{q} = 0$ locus at K_{AI} and K_{sat} arise from the discontinuities in $d'(K)$.

At $q = \bar{q}$, the sign of $\dot{q}$ is opposite to that of the rental rate minus the user cost:

$$\operatorname{sign} \dot{q}\Big|_{q=\bar{q}} = -\operatorname{sign} G(K). \tag{21}$$

At $q = \bar{q}$, gross investment replaces depreciation. Where the rental rate exceeds the user cost, the no-arbitrage condition under rational beliefs requires an anticipated price decline; where the rental rate falls short, it requires an anticipated price increase. Equation (21) gives the sign of $\dot{q}$ at the given capital-price pair. Figure 2 shows the nullclines and direction field.

Lemma 1 (Local stability under rational beliefs)**.** *The sign of $\dot{q}$ at $q = \bar{q}$ is given by (21). At every steady state $(K^j, \bar{q})$ of the rational equilibrium system, the sign of the Jacobian determinant equals the sign of $G'(K^j)$. Consequently:*

(i) the middle steady state K^M is an unstable node. In the deployment region $d'(K^M) = 0$, which implies $G'(K^M) > 0$; the trace is positive and the eigenvalues are real there;

(ii) a low or high steady state K^j, $j \in \{L, H\}$, is a saddle whenever $G'(K^j) < 0$. In particular, both steady states are saddles under (18).

The Jacobian computation is in Appendix E.

The sign of $G'(K)$ determines local stability. Differentiating (16),

$$G'(K) = \underbrace{d'(K)}_{\leq 0} - \underbrace{\Phi'(K)}_{< 0}.$$

Diminishing returns lower $d(K)$, while investors' greater wealth lowers $\Phi(K)$. At the low and high steady states, the slope condition requires the rental rate to fall faster than the user cost. At the middle steady state, $d'(K^M) = 0$, so only the user cost falls. Hence $G'(K^M) > 0$, and the steady state is unstable.

The restrictions that give three steady states also determine the rational equilibrium path from each capital stock. Appendix F gives the proof.

Lemma 2 (Global rational equilibrium paths)**.** *Suppose (19) and (18) hold. There is exactly one rational equilibrium path from each $K_0 > 0$. It converges to K^L for $K_0 < K^M$, to K^H for $K_0 > K^M$, and is stationary at $K_0 = K^M$.*

On the saddle paths to the low- and high-capital steady states, the price is a function of capital defined on $(0, K^M)$ and (K^M, ∞), respectively, and consumption is positive. For $K^L < K < K^M$, the low saddle path has $q < \bar{q}$; for $K^M < K < K^H$, the high saddle path has $q > \bar{q}$. On these intervals, both saddle paths approach $(K^M, \bar{q})$ backward in time. Below K^L, the low saddle path has $q > \bar{q}$; above K^H, the high saddle path has $q < \bar{q}$. Capital converges monotonically along each saddle path.

For $K^L < K < K^M$, the rational equilibrium price is below $\bar{q}$, so gross investment is less than depreciation and capital declines toward K^L. For $K^M < K < K^H$, the price exceeds $\bar{q}$, investment exceeds depreciation, and capital rises toward K^H.

Proposition 2 (Equilibrium path from the low-capital steady state)**.** *Under (19) and (18), the unique rational equilibrium path with $K(0) = K^L$ is stationary at $(K^L, \bar{q})$. There is no rational equilibrium path from K^L to K^H.*

The proof is in Appendix G.

The rational equilibrium price at K^L is $\bar{q}$, and gross investment equals depreciation. A price above $\bar{q}$ would induce positive net investment. Along the resulting path under rational beliefs, investors' consumption would converge to zero and their transversality condition would fail. The high-capital steady state is a rational equilibrium at a different capital stock. A transition to it requires optimism to persist until capital exceeds K^M. A full correction after that point is followed by convergence to K^H. Section 4 characterizes the size and duration requirements for this investment expansion.

4 Speculative Investment and Corrections

Suppose investors in an economy initially at K^L unexpectedly adopt the belief wedge $x > 0$ at date zero. Its size and duration determine whether the economy converges to K^H after a full correction.

4.1 The required wedge

Under the no-arbitrage condition (14), the perceived return on capital equals the interest rate, while the realized return is lower by x. For a given income stream, investors therefore pay a higher price, which increases investment through (10). Relative to Figure 2, the wedge changes the law of motion for q and shifts the $\dot{q} = 0$ locus.

With a permanent wedge x, the steady-state Euler equation gives $R^a + x = \rho - \theta c$. The wealth identity remains $c = R^a W$, so the realized return is

$$R^a = \frac{\rho - x}{1 + \theta W},$$

while the interest rate is $i = R^a + x$. Substituting $R^a = [d(K) - \Gamma(\delta)]/\bar{q}$ and $W = \bar{q}K$ gives the steady-state condition

$$d(K) = \Gamma(\delta) + \frac{\bar{q}(\rho - x)}{1 + \theta \bar{q} K}. \tag{22}$$

Relative to Figure 1, the wedge lowers the user cost at each K by $\bar{q}x/(1 + \theta\bar{q}K)$. At a fixed wedge, investors' greater wealth lowers the steady-state interest rate and the realized return $i - x$, reducing the user cost.

In the deployment region, the rational middle steady state satisfies

$$K^M = \frac{1}{\theta \bar{q}} \left[\frac{\bar{q}\rho}{d_{\text{flat}} - \Gamma(\delta)} - 1 \right]. \tag{23}$$

Let $x^*(K)$ denote the required wedge: starting from $K \leq K^M$, capital crosses K^M under a constant wedge if and only if $x > x^*(K)$. This wedge is given by

$$x^*(K) = \theta \Big[d_{\text{flat}} - \Gamma(\delta) \Big] \Big(K^M - \max\{K, K_{\text{AI}}\} \Big) \qquad \text{for } K \leq K^M. \tag{24}$$

At the initial stock $K^L < K_{\text{AI}}$, the required wedge is

$$x^*(K^L) = \frac{1 + \theta \bar{q} K_{\text{AI}}}{\bar{q}} \Big[\Phi(K_{\text{AI}}) - d_{\text{flat}} \Big].$$

The required wedge is proportional to the difference between the user cost and the rental rate at the start of deployment. A larger difference means that the rental rate is further below the amount needed to cover replacement expenditure and the required return at $q = \bar{q}$. A larger wedge is therefore needed for net investment to remain positive at the start of AI deployment.

During deployment, $x^*(K)$ is the wedge for which the perceived middle steady state has capital stock K. Below K_{AI}, the required wedge is the value at which the perceived low and middle steady states merge at K_{AI}. Thus $x^*(K)$ is constant before deployment and decreases with K during deployment. The rental rate remains constant while the user cost decreases with K, reducing the optimism needed for further accumulation. The required wedge reaches zero at K^M, above which the rational equilibrium path converges to K^H.

Let $Q^H(K;x)$ denote the price on the saddle path of the perceived high steady state. When the wedge exceeds $x^*(K^L)$, the unique equilibrium path from each capital stock lies on this saddle path. Below that wedge, the perceived system has three steady states.

Lemma 3 (Perceived equilibrium paths)**.** *Suppose* (19) *and* (18) *hold.*

(i) For $x^*(K^L) < x < \rho$, there is a single steady state $K^H(x) > K_{\text{sat}}$ and a unique equilibrium path from every $K_0 > 0$. It converges to $K^H(x)$, with $Q^H(K;x) > \bar{q}$ below that stock and $Q^H(K;x) < \bar{q}$ above it.

(ii) For $0 \le x < x^*(K^L)$, there are three steady states

$$K^L(x) < K_{\text{AI}} < K^M(x) < K_{\text{sat}} < K^H(x).$$

The unique equilibrium path converges to $K^L(x)$ below $K^M(x)$, to $K^H(x)$ above it, and is stationary at $K^M(x)$. Capital converges monotonically along the saddle paths.

(iii) At $x = x^*(K^L)$, capital below K_{AI} converges to K_{AI}; capital above it converges to $K^H(x)$. At K_{AI}, the only equilibrium path is stationary. The equilibrium initial price is jointly continuous in (K_0, x) for $K_0 > 0$ and $0 \le x < \rho$, including the wedge at which the low and middle steady states merge.

With $0 < x < x^*(K^L)$, the equilibrium path from K^L converges to the perceived low steady state $K^L(x) < K_{\text{AI}}$. At the required wedge $x = x^*(K^L)$, capital converges to K_{AI}. A transition to the high-capital steady state therefore requires $x > x^*(K^L)$. Since $K^H(x) > K^H > K^M$ for $x > 0$, every wedge in $(x^*(K^L), \rho)$ implies investment above replacement throughout $[K^L, K^M]$, so capital crosses K^M if optimism persists long enough. Appendix F proves the lemma.

4.2 Correction timing and the duration requirement

Let the wedge remain at x until an unanticipated full correction at τ, when it falls to zero. Before the correction, the price is $Q^H(K;x)$, and capital evolves according to

$$q_t = Q^H(K_t;x), \qquad \dot{K}_t = K_t\Big[\psi \log Q^H(K_t;x) - \delta\Big], \qquad 0 \le t < \tau, \tag{25}$$

with consumption given by (11). The wedge is fixed throughout this phase; price and investment change as capital accumulates.

Take $x \in (x^*(K^L), \rho)$. Lemma 3(i) ensures that the perceived high saddle path is defined at the initial stock, with $Q^H(K; x) > \bar{q}$ from K^L through K^M. Capital therefore rises. The time needed to reach the capital threshold is

$$T(x) = \int_{K^L}^{K^M} \frac{dK}{K\Big[\psi \log Q^H(K; x) - \delta\Big]}. \tag{26}$$

Capital must be strictly above K^M at the full correction for the subsequent rational path to converge to K^H. This requires $\tau > T(x)$.

The denominator in (26) is net investment. A higher price along the path raises net investment and reduces $T(x)$. The duration $T(x)$ is strictly decreasing in the wedge and diverges as the wedge approaches $x^*(K^L)$ from above, as established in Proposition 3.

Figure 3 plots the required-wedge schedule, the duration requirement, and capital paths with early and late corrections. The two paths have the same wedge and differ only in the correction date.

Proposition 3 (Feasibility and duration)**.** *Under (19) and (18), for $x \in (x^*(K^L), \rho)$:*

(i) The perceived high-capital saddle path is defined at the initial low stock, and capital reaches K^M in finite time while the wedge remains constant.

(ii) For any such x, the economy converges to the high-capital steady state if $\tau > T(x)$ and to the low-capital steady state if $\tau < T(x)$, where $T(x)$ is given by (26). If $\tau = T(x)$, the post-correction path is stationary at the middle steady state.

(iii) The duration $T(x)$ is strictly decreasing on $(x^*(K^L), \rho)$, and $T(x) \to \infty$ as $x \downarrow x^*(K^L)$.

The size requirement ensures that investment exceeds replacement throughout $[K^L, K^M]$. The duration requirement determines whether capital has crossed K^M by the full correction. Prices can adjust immediately when investors become optimistic, but convex adjustment costs make capital accumulate gradually. Even with a wedge above the required level, a full correction can occur before enough capital has been installed. A larger wedge raises the price and speeds accumulation along the optimistic path, reducing the time needed to cross the threshold. Appendix G contains the proofs.

4.3 Corrections and interest rates

Corollary 1 (Price after a full correction)**.** *Along any pre-correction path governed by (25), if $\tau > T(x)$, the price jumps at the correction to $Q^H(K_\tau; 0)$ on the rational high saddle path, and the economy converges to K^H. If $\tau < T(x)$, the price jumps to the value on the rational low saddle path at K_τ, and the economy returns to K^L.*

A full correction lowers the price. Its effect on subsequent investment depends on whether capital has crossed K^M. After an early full correction, $K_\tau < K^M$, and the rational price lies on the low saddle path with $q < \bar{q}$. Gross investment falls below depreciation,

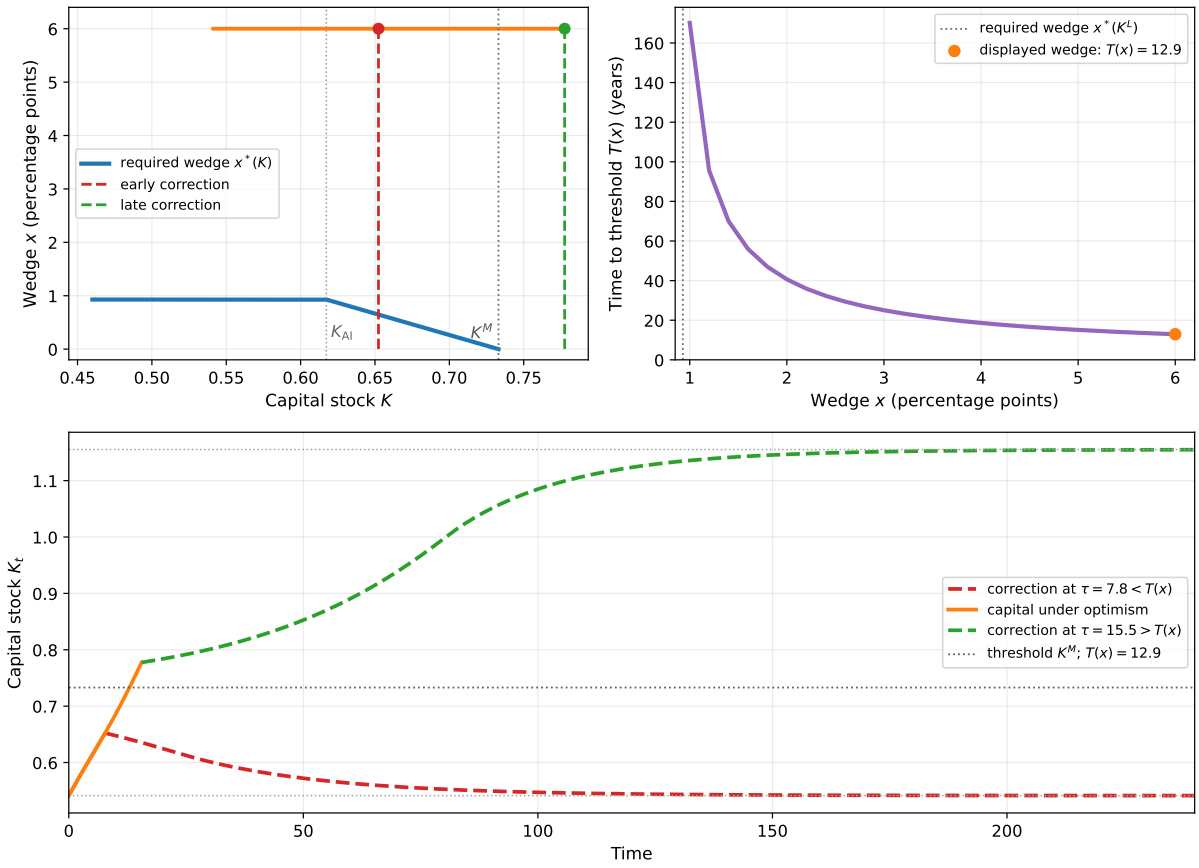

Figure 3: Required optimism and correction timing. The upper-left panel shows $x^*(K)$ and a common wedge ending at two dates. The upper-right panel plots the time $T(x)$ to reach the capital threshold. In the lower panel, capital converges to K^L after the early correction and to K^H after the late correction. The upper-left and lower panels use $x = 0.06$; all panels use the illustrative parameters in Appendix I.

so capital gradually returns to K^L. After a late correction with $K_\tau \in (K^M, K^H)$, the price falls to the rational high saddle path but remains above $\bar{q}$. Net investment remains positive and capital converges to K^H. A fall in asset prices is consistent with either a temporary or a permanent increase in capital.

Interest rates during the transition. Optimism raises the price of installed capital and firms' investment demand. The additional investment requires more saving, putting upward pressure on the interest rate. As investors accumulate wealth, they are willing to hold it at a lower steady-state interest rate.

For a fixed belief wedge x, the steady-state household schedules are

$$c_x^{ss}(W) = \frac{(\rho - x)W}{1 + \theta W}, \qquad i_x^{ss}(W) = \frac{\rho + x\theta W}{1 + \theta W}. \tag{27}$$

The first expression follows from the steady-state Euler equation and wealth identity; the second uses the no-arbitrage condition $i = R^a + x$. They give consumption and the interest rate consistent with constant household wealth at a given W and belief wedge. At a given wealth level, a larger belief wedge raises the benchmark interest rate. I compare investors' consumption along the transition with steady-state consumption at the same wealth and belief wedge. Since $i_x^{ss}(W) + \theta c_x^{ss}(W) = \rho$, the Euler equation gives

$$i_t = i_x^{ss}(W_t) + \theta\Big[c_x^{ss}(W_t) - c_t\Big] + \frac{\dot{c}_t}{c_t}. \tag{28}$$

To ensure that consumption remains below $c_x^{ss}(W)$, suppose $d(K) + \psi + Kd'(K) > 0$ along the path as capital increases toward the perceived high-capital steady state. The derivative of total capital income $Kd(K)$ is $d(K) + Kd'(K)$, so the condition limits any decline in capital income to less than ψ per additional unit of capital. It holds automatically before and during deployment, when capital income rises. Consumption then satisfies

$$c_t < c_x^{ss}(W_t) = \left(1 - \frac{x}{\rho}\right) c^{ss}(W_t). \tag{29}$$

Appendix H proves this bound and gives a sufficient parameter restriction for the condition to hold after saturation. The consumption-compression term $\theta[c_x^{ss}(W_t) - c_t]$ in (28) is strictly positive during accumulation and vanishes at the steady state for the fixed wedge. It measures the upward contribution of investment demand to the interest rate. The remaining growth term accounts for the expected change in consumption along the equilibrium path.

The steady-state interest rate $i_x^{ss}(W)$ decreases with wealth for $x < \rho$. As AI deployment increases productive capacity and investors accumulate the associated wealth, the interest rate at which they are willing to hold that wealth falls. Following a full correction and convergence to a steady state, consumption growth and the consumption-compression term vanish. The high-capital steady state has $i^H = \rho/(1 + \theta\bar{q}K^H) < i^L$. During the transition, the positive consumption-compression term and the decline in the steady-state interest rate have opposite effects on i_t. The level of i_t relative to i^L, and its total response to optimism, depend on wealth, consumption, and consumption growth.

The yield curve after a correction. After a full correction near the capital threshold, yields at sufficiently long maturities exceed the short rate if accumulation reverses and lie below it if accumulation continues. Along the rational equilibrium path, a zero-coupon bond of remaining maturity $h > 0$ has continuously compounded yield $y_\tau(h) = h^{-1} \int_0^h i_{\tau+s}\, ds$. Hence

$$\lim_{h\to\infty} y_\tau(h) = \begin{cases} i^L, & K_\tau < K^M, \\ i^H, & K_\tau > K^M. \end{cases} \tag{30}$$

As capital at the correction approaches K^M from either side, the short rate immediately after the correction approaches i^M, with $i^H < i^M < i^L$. The two limiting yields differ because investors' wealth is higher, and the steady-state interest rate lower, at the high-capital steady state. A correction just below the threshold is followed by a return toward

the low-capital steady state and its higher interest rate, so yields at sufficiently long maturities exceed the current short rate. Just above the threshold, further accumulation leads to the lower rate at K^H, so yields at sufficiently long maturities lie below the current short rate.

As capital at the correction approaches the threshold, the subsequent path remains close to the middle steady state for longer. At each fixed maturity, the yield and the short rate both converge to i^M, so their spread approaches zero (Appendix G).

Figures 4 and 5 illustrate a transition with a late full correction. The interest rate is above its initial level during the optimistic phase and converges to a lower level after the correction.

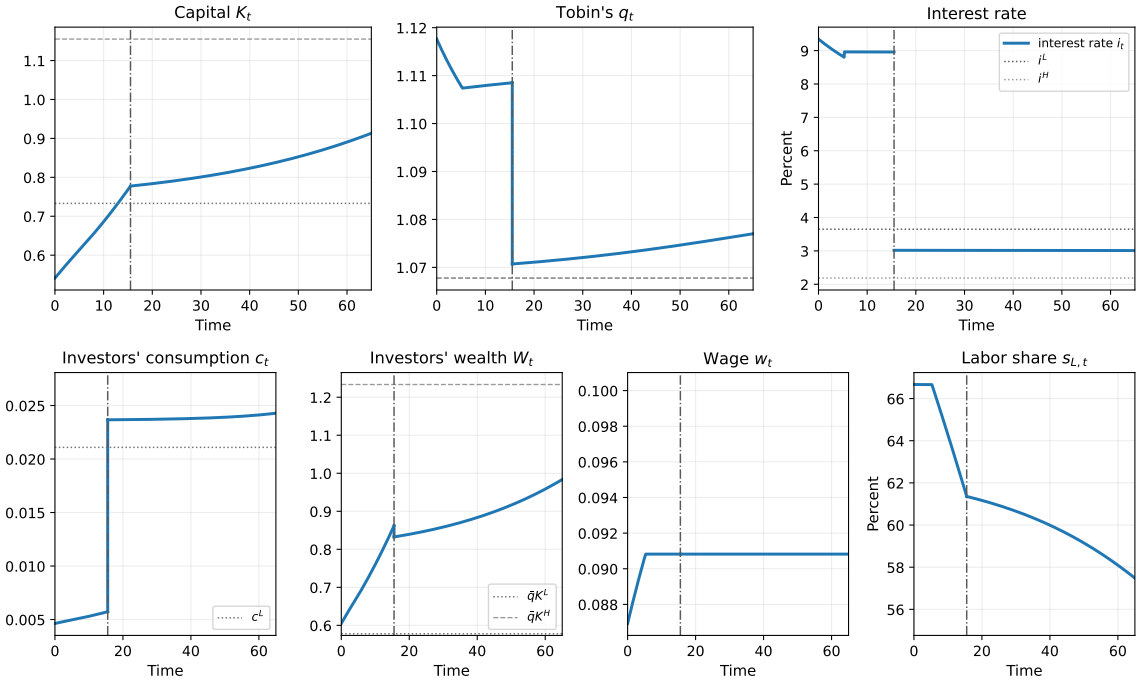

Figure 4: Transition with a late correction. Capital, price, the interest rate, investors' consumption and wealth, the wage, and the labor share for $\tau > T(x)$. Before the correction, investors' consumption is below its initial level and increasing, so the interest rate exceeds its initial level. Investors' wealth falls at the correction and converges to a level above its initial value. The vertical line marks the correction date. The figure uses $x = 0.06$ and the illustrative parameters in Appendix I.

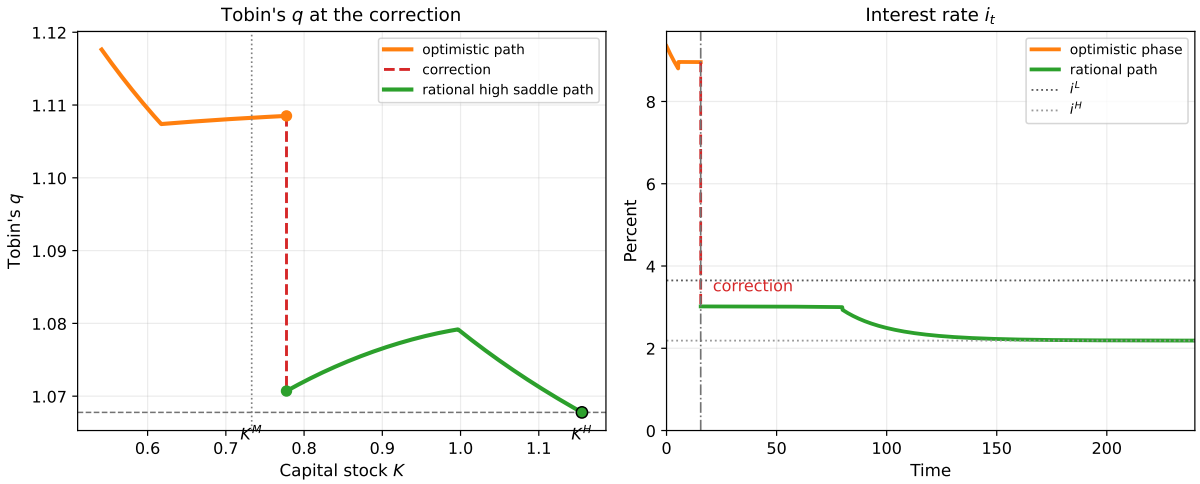

Figure 5: Price and interest rates after a late correction. At the correction, $K > K^M$ and the price falls to $Q^H(K;0)$. Capital then converges to K^H. The interest rate converges to a value below its initial level. The figure uses $x = 0.06$ and the illustrative parameters in Appendix I.

4.4 Partial corrections and investment

A partial correction can leave net investment positive while slowing accumulation enough that capital remains below K^M at the subsequent full correction.

Suppose the wedge falls unexpectedly from $x_0 \in (x^*(K^L), \rho)$ to $x_1 \in [0, x_0)$. Investors treat the remaining wedge as permanent until another unanticipated correction. A full correction is the case $x_1 = 0$.

By Lemma 3, the sign of post-correction net investment is determined by the remaining wedge relative to $x^*(K)$. For $K_{\text{AI}} < K < K^M$, investment exceeds replacement if $x_1 > x^*(K)$ and falls below it if $x_1 < x^*(K)$. When $x_1 = x^*(K)$, the economy remains at the perceived middle steady state. Once $K > K^M$, the equilibrium path converges to K^H with $x_1 = 0$. During deployment below K^M, positive net investment requires a positive remaining wedge; as capital approaches K^M, the required wedge falls to zero.

I express correction size as a proportional price decline to compare corrections at different capital stocks. Write the pre-correction price as $Q_0(K) = Q^H(K; x_0)$, the post-correction equilibrium price as q^+, and the proportional price decline as $D = 1 - q^+/Q_0(K)$. By (10), capital growth immediately after the correction is

$$\frac{\dot{K}^+}{K} = \psi \log Q_0(K) - \delta + \psi \log(1 - D). \tag{31}$$

Net investment is zero at $q = \bar{q}$. Define the *correction boundary* as the proportional price decline that reduces the price to $\bar{q}$:

$$D^*(K) = 1 - \frac{\bar{q}}{Q_0(K)}. \tag{32}$$

Proposition 4 (Partial corrections and investment)**.** *Under* (19) *and* (18)*, suppose* $d(K) + \psi + Kd'(K) > 0$ *along the saddle path as capital increases toward the perceived high steady state. For* $K \in (K_{\text{AI}}, K^M)$*:*

(i) A partial correction $D < D^*(K)$ *lowers investment, with net investment remaining positive. A correction* $D > D^*(K)$ *implies negative net investment along the path toward the perceived low steady state. When* $D = D^*(K)$*, net investment is zero and the economy remains at the perceived middle steady state.*

(ii) The correction boundary is strictly increasing:

$$\frac{dD^*(K)}{dK} = \frac{\bar{q}\, Q_0'(K)}{Q_0(K)^2} > 0.$$

There exist two stocks $K_{\text{early}} < K_{\text{late}}$ *and equilibrium corrections* $D_{\text{early}} < D_{\text{late}}$ *for which net investment is negative after the smaller early correction and positive after the larger late correction.*

During deployment, the rental rate is constant and the user cost decreases with capital. The price rises along the high saddle path during accumulation, as shown in Appendix H. Because $\bar{q}$ is fixed, a larger proportional price decline is needed to reduce net investment to zero.[2]

The equilibrium corrections in Figure 6 satisfy $D^*(K_{\text{early}}) < D_{\text{early}} < D_{\text{late}} < D^*(K_{\text{late}})$. The later price decline is larger in both proportional and absolute terms. Net investment is negative after the early correction and positive after the late correction.

For $x_1 > x^*(K)$, the time from the partial correction to the capital threshold is

$$\mathcal{T}(K, x_1) = \int_K^{K^M} \frac{dk}{k[\psi \log Q^H(k; x_1) - \delta]}. \tag{33}$$

A lower x_1 reduces the price along the optimistic path and increases $\mathcal{T}(K, x_1)$. As $x_1 \downarrow x^*(K)$, the post-correction point approaches the perceived middle steady state and $\mathcal{T}(K, x_1) \to \infty$. Near that state, the price is only slightly above $\bar{q}$, so gross investment barely exceeds depreciation. Capital therefore remains close to its level at the correction for a long time. The divergence is logarithmic under the nondegenerate node conditions in Appendix H. For any fixed remaining duration of optimism, a sufficiently large correction below $D^*(K)$ leaves capital below K^M at the full correction. Net investment remains positive until the full correction, but the subsequent rational path returns to K^L.

[2]The result concerns the correction boundary $D^*(K)$. The price decline caused by a fixed reduction in the wedge need not increase with K.

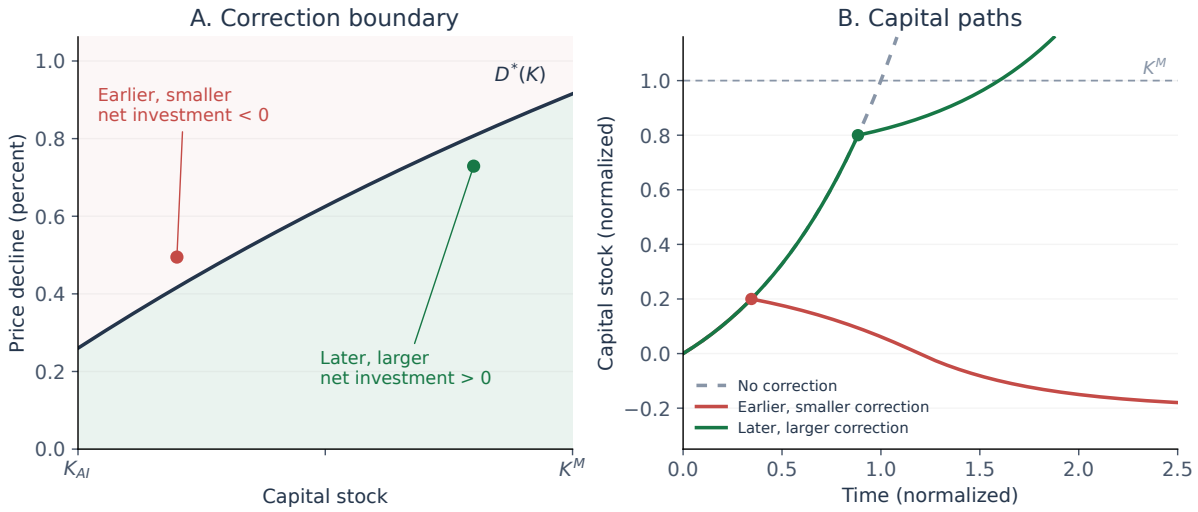

Figure 6: Partial corrections and net investment. The early correction exceeds $D^*(K)$, making net investment negative. The larger late correction is below $D^*(K)$, leaving net investment positive. The left panel reports price declines in percent. In the right panel, capital is normalized as $(K - K_{\text{AI}})/(K^M - K_{\text{AI}})$, and time is divided by the deployment duration without a correction. Both post-correction capital-price pairs lie on equilibrium saddle paths. The figure uses the illustrative parameters in Appendix I and $x_0 = 0.012$.

4.5 Distributional incidence

At the high-capital steady state, wages are higher and the labor share is lower. Investors are wealthier there but bear the capital loss at the correction. Their steady-state wealth is $W = \bar{q}K$, so the increase from K^L to K^H raises wealth even though the price per unit of capital is the same in both steady states.

For workers, the marginal product of labor in (2) gives

$$w(K) = \begin{cases} (1-\alpha)AK^{\alpha}, & K < K_{\text{AI}}, \\ w_{\text{flat}}, & K_{\text{AI}} \le K < K_{\text{sat}}, \\ Y(K)\left[(1-\alpha) - \dfrac{\alpha \bar{K}_{\ell}}{K - \bar{K}_{\ell}}\right], & K \ge K_{\text{sat}}. \end{cases} \tag{34}$$

The wage increases with capital before deployment. During deployment, $N = bK_c$ keeps conventional capital per unit of effective labor constant, so the wage is flat. After saturation, an additional unit of raw labor supplies labor directly and expands AI deployment capacity. Its competitive wage includes both effects, giving the bracketed term in (34). The resulting wage is increasing in K, and the pieces meet at the two technology boundaries.

By (4), the labor share is constant before deployment, falls during deployment, and increases afterward while remaining below $1-\alpha$. Since $K^L < K_{\text{AI}} < K_{\text{sat}} < K^H$, the wage is higher and the labor share is lower at the high-capital steady state.

Capital rises during the optimistic phase. After an early full correction, it converges to K^L from above; after a late correction, it converges to K^H. Since the stock never falls

below K^L on these paths and the wage is weakly increasing in capital, wages never fall below their initial level.

Corollary 2 (Wages and the labor share)**.** *In the technology of Section 2, the wage $w(K)$ is weakly increasing in K and strictly higher at the high-capital steady state. Along the paths with early and late corrections satisfying Proposition 3 and Corollary 1, the wage therefore does not fall below its initial level $w(K^L)$. At the high-capital steady state, the wage is higher and the labor share is lower than at the low-capital steady state.*

Appendix G gives the proof.

Investors fund investment and its adjustment costs and own the capital claim. Before a full correction, the realized return is below the perceived return by x, as in (14). The correction causes a capital loss as the price falls and reduces their wealth $W = qK$. Workers bear no capital loss, and their wage is unchanged at the correction because the capital stock is continuous.

After an early correction, investors' wealth and consumption return to their initial levels. After a late correction, wealth converges to $\bar{q}K^H$, and consumption converges to a higher level because the steady-state consumption rule in (8) is increasing in wealth. Investors bear a capital loss in both cases. A welfare comparison must account for consumption forgone to fund investment, the subsequent consumption and wealth paths, and the treatment of mistaken beliefs.

5 Conclusion

Temporary optimism about AI returns can induce investment that permanently increases productive capacity. During deployment, firms expand conventional capital and AI-provided labor services together, keeping the marginal product of capital constant. Income from those services accrues to investors. As their wealth increases, they are willing to hold it at a lower interest rate, allowing the larger capital stock to be sustained under rational expectations. The rational equilibrium path from the low-capital steady state remains there. Convergence to the high-capital steady state requires optimism large enough to raise investment above replacement and persistent enough for capital to cross the threshold.

Investment demand puts upward pressure on the interest rate during the expansion. At the high-capital steady state, investors' greater wealth lowers the interest rate, wages are higher, and the labor share is lower. Investors bear the capital loss at the correction.

The outcome of a correction depends on capital relative to the threshold and on the remaining belief wedge. A full correction before capital crosses the threshold returns the economy to the low-capital steady state; a full correction afterward is followed by convergence to the high-capital steady state. Partial corrections reduce investment. During deployment, the price along the optimistic path rises, so later corrections can be larger without reversing accumulation. Slower accumulation can leave capital below the threshold at the full correction, so the subsequent rational path returns to the low-capital steady state. A fall in AI valuations is consistent with either a temporary or a permanent increase in productive capacity.

Correction timing is exogenous in the model. If unusually rapid increases in AI valuations or investment made a correction more likely, a partial correction could slow accumulation while delaying a subsequent full correction; its effect on the transition would then depend on whether the additional time compensates for the slower pace of investment.

A Technology Details

Before normalizing raw labor, firms choose the split $K = K_c + K_\ell$, with $0 \le K_\ell \le \bar{K}_\ell L$, to maximize $Y = AK_c^\alpha N^{1-\alpha}$ with $N = L + \gamma K_\ell$. If (K, L, K_ℓ) is feasible, then scaling all three by the same factor remains feasible and scales output by that factor. The maximized technology is therefore constant-returns in (K, L).

The interior first-order condition equates the marginal products of capital in its two uses,

$$\alpha A K_c^{\alpha-1} N^{1-\alpha} = (1-\alpha) A K_c^\alpha N^{-\alpha} \gamma,$$

which gives $N = bK_c$ with $b = (1-\alpha)\gamma/\alpha$. Setting $L = 1$ gives (1). By the envelope theorem, the rental rate of capital is the marginal product in the use selected by the firm. It equals $\alpha A b^{1-\alpha}$ throughout the interior region. Below $K_{\text{AI}} = 1/b$, the constraint $K_\ell \ge 0$ binds and all capital is conventional. Above $K_{\text{sat}} = [1 + (\gamma + b)\bar{K}_\ell]/b$, the capacity constraint binds. Substituting the relevant allocation into the envelope derivative gives the three pieces of (3).

Before deployment, the marginal product of labor is the standard Cobb–Douglas wage. In the interior region, the optimized technology is linear in K and L, with derivatives d_{flat} and w_{flat}. After saturation,

$$Y(K, L) = A(K - \bar{K}_\ell L)^\alpha \Big[L(1 + \gamma \bar{K}_\ell)\Big]^{1-\alpha}.$$

Differentiating with respect to L and then setting $L = 1$ gives the last line of (34) and, after division by output, the last line of (4). Since the maximized technology is constant-returns, Euler's theorem gives $Y = Kd + w$, which proves (2).

B Wealth-in-Utility and the Consumption Rule

At a planning date, investors evaluate consumption and wealth along the path they anticipate. Let $\widetilde{W}_t$ denote forecast wealth, initialized at market wealth $\widetilde{W}_0 = W_0$. With a fixed belief wedge, the perceived budget constraint is

$$\dot{\widetilde{W}}_t = i_t \widetilde{W}_t - c_t, \tag{35}$$

because the perceived capital return equals the interest rate. Investors evaluate the utility in (5) using forecast wealth. The physical resource constraint and the accounting identity for $W = qK$ continue to use realized income and expenditure.

Investors can allocate wealth between the capital claim and a bond paying i_t. Let λ_t be the current-value costate, the marginal value of wealth in the household optimization problem. The consumption first-order condition is $1/c_t = \lambda_t$, and the costate equation is

$$\dot{\lambda}_t = (\rho - i_t)\lambda_t - \theta.$$

Using $\lambda_t = 1/c_t$ gives

$$\frac{\dot{c}_t}{c_t} = i_t - \rho + \theta c_t,$$

which is (6). Because the bond is in zero aggregate supply, investors' aggregate wealth equals the value of the capital claim, $W = qK$. The interest rate enters the Euler equation through the household's marginal saving decision. In a rational steady state, the no-arbitrage condition gives $i = R^a$, while constant aggregate wealth in (13) gives $c = iW$. Combining these conditions with the steady-state Euler equation yields $c^{ss}(W) = \rho W/(1+\theta W)$ and $i^{ss}(W) = \rho/(1+\theta W)$, with $di^{ss}/dW < 0$ as in (8).

The investors' transversality condition. The transversality condition is

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t \widetilde{W}_t = \lim_{t\to\infty} e^{-\rho t}\frac{\widetilde{W}_t}{c_t} = 0. \tag{36}$$

Market wealth obeys $\dot{W} = (i-x)W - c$. Subtracting this identity from (35), with equal initial wealth, gives $\widetilde{W}_t \geq W_t > 0$. Moreover,

$$\frac{d}{dt}\log\frac{\widetilde{W}_t}{W_t} = x + c_t\left(\frac{1}{W_t} - \frac{1}{\widetilde{W}_t}\right) \geq x.$$

Hence $\widetilde{W}_t \geq e^{xt}W_t$, and (36) implies (15). Appendix F.1 proves the converse from the equilibrium equations, positivity, and (15).

Applying the transversality condition for rational beliefs directly to market wealth imposes no additional restriction when $x > 0$. In fact,

$$\frac{d}{dt}\left(e^{-\rho t}\frac{W_t}{c_t}\right) = -xe^{-\rho t}\frac{W_t}{c_t} - e^{-\rho t}(1+\theta W_t),$$

so that condition holds on every solution with positive consumption at all dates. Investors' transversality condition uses forecast wealth under the belief that the current wedge is permanent. After a correction, investors evaluate their plans using the new wedge.

C Equilibrium Dynamics

The resource constraint. Gross investment is $I = K\psi\log q$, and total investment expenditure, including adjustment costs, is $K\Gamma(I/K) = K\psi(q-1)$. Investors receive capital income $Kd(K)$, pay this expenditure, and consume the difference. Hence

$$C(K,q) = K\Big[d(K) - \psi(q-1)\Big], \qquad C_K = d(K) - \psi(q-1) + Kd'(K), \qquad C_q = -\psi K.$$

These expressions imply (11). Substituting (12) into $R^aW - c$, and using $W = qK$ and (10), gives $\dot{q}K + q\dot{K} = \dot{W}$.

The law of motion for q. Substituting $c = C(K,q)$, $\dot{c} = C_K\dot{K} + C_q\dot{q}$, and $\dot{K} = K(\psi\log q - \delta)$ into the Euler equation and the no-arbitrage condition $i = R^a + x$ gives

$$\dot{q} = \frac{\rho - \theta C + \dfrac{C_K}{C}K(\psi\log q - \delta) - (\psi\log q - \delta) - \dfrac{C}{qK} - x}{\dfrac{1}{q} - \dfrac{C_q}{C}}. \tag{37}$$

The denominator is strictly positive whenever $C > 0$, because $C_q = -\psi K < 0$. The rational system sets $x = 0$. Its $\dot{q} = 0$ locus satisfies

$$\psi \log q - \delta + \frac{C(K,q)}{qK} = \rho - \theta C(K,q) + \frac{C_K(K,q)}{C(K,q)} K(\psi \log q - \delta). \tag{38}$$

The left side is the realized return excluding changes in q; the right side is the interest rate implied by the Euler equation, including the consumption growth generated by changes in K. At $\dot{q} = 0$, capital can still accumulate and change consumption, which is why the consumption-growth term remains in the nullcline condition. Equation (37) together with (10) defines the vector field $F^x(K,q)$; F^0 is the rational benchmark shown in Figure 2.

The perceived system. A fixed belief wedge enters (37) through the term $-x$. The high-capital saddle path of this vector field is $Q^H(K;x)$. When $x > x^*(K^L)$, Lemma 3 establishes that the unique equilibrium path follows this saddle path before a full correction. The post-correction price is the value on the rational saddle path at the capital stock reached.

D Proof of Proposition 1

Recall that

$$G(K) = d(K) - \Phi(K), \qquad \Phi(K) = \Gamma(\delta) + \frac{\bar{q}\rho}{1 + \theta \bar{q} K}.$$

Since $\Phi(K) \downarrow \Gamma(\delta)$ as $K \to \infty$, the condition $d_{\text{flat}} > \Gamma(\delta)$ implies that there exists $\bar{K}_\ell^* > 0$ such that, for every $\bar{K}_\ell > \bar{K}_\ell^*$,

$$\Phi(K_{\text{sat}}) < d_{\text{flat}}.$$

Together with $\Phi(K_{\text{AI}}) > d_{\text{flat}}$, this gives the inequalities in (19). After saturation, $d'(K) = (\alpha - 1)d(K)/(K - \bar{K}_\ell) < 0$. The slope condition (18) is still needed because $G'(K) = d'(K) - \Phi'(K)$, and greater wealth lowers the user cost.

The existence of the three roots follows from continuity. As $K \to 0^+$, $d(K) \to \infty$ while $\Phi(K)$ is bounded, so $G(K) > 0$. At K_{AI}, the left inequality in (19) gives $G(K_{\text{AI}}) < 0$, so a low root exists in $(0, K_{\text{AI}})$. On the flat region, $d(K) = d_{\text{flat}}$ and Φ is strictly decreasing, so G is strictly increasing. The right inequality in (19) gives $G(K_{\text{sat}}) > 0$, so the middle steady state exists and is unique. Beyond saturation, $d(K) \to 0$ while $\Phi(K) \to \Gamma(\delta) > 0$, so $G(K) \to -\Gamma(\delta) < 0$; with $G(K_{\text{sat}}) > 0$, a high root exists. If (18) holds, G is strictly decreasing in both outer regions, so each outer root is unique. □

E Local Stability: Proof of Lemma 1

Write the rational vector field as $F^0 = (f_1, f_2)$, with

$$f_1(K,q) = K(\psi \log q - \delta)$$

and $f_2(K, q)$ equal to the right-hand side of (37) at $x = 0$. Its denominator is positive when consumption is positive. On the horizontal nullcline $q = \bar{q}$, capital growth vanishes and $C(K, \bar{q}) = K[d(K) - \Gamma(\delta)]$. The numerator of f_2 then becomes

$$\rho - \theta C(K, \bar{q}) - \frac{C(K, \bar{q})}{\bar{q}K} = -\frac{1 + \theta\bar{q}K}{\bar{q}}\, G(K).$$

This proves the sign identity (21): the price rises where the rental rate of capital falls short of its user cost and falls where it exceeds that cost.

At a steady state, $f_{1,K} = 0$ and $f_{1,q} = K\psi/\bar{q} > 0$. Because the numerator of f_2 is zero at the root, differentiating it with respect to K gives

$$f_{2,K}(K^j, \bar{q}) = -\frac{1 + \theta\bar{q}K^j}{\bar{q}} \frac{G'(K^j)}{\dfrac{1}{\bar{q}} - \dfrac{C_q}{C}}.$$

It follows that

$$\operatorname{sign} \det DF^0(K^j, \bar{q}) = \operatorname{sign} G'(K^j).$$

At a steady state with $G'(K^j) < 0$, the determinant is negative, so the steady state is a saddle.

At the middle steady state, $d' = 0$. In that region $C = K[d_{\text{flat}} - \psi(q-1)]$, so $C_K K/C = 1$ and the two terms involving capital growth cancel in the numerator of (37). Differentiating the remaining numerator with respect to q, and then evaluating at $(K^M, \bar{q})$, gives

$$\theta\psi K^M + \frac{\psi}{\bar{q}} + \frac{d_{\text{flat}} - \Gamma(\delta)}{\bar{q}^2} > 0.$$

The denominator of (37) is positive, so the trace is positive. Since $G'(K^M) = -\Phi'(K^M) > 0$, the determinant is also positive. Direct substitution of the flat-region expressions into the characteristic discriminant reduces it to a perfect square divided by a positive denominator. The eigenvalues are therefore real and positive: K^M is an unstable node. This proves Lemma 1. For a fixed wedge $0 \leq x < \rho$, the same calculation applies with $\rho - x$ in place of ρ. The saddle paths for the numerical example are shown in Figure 2. □

F Global Equilibrium Paths

This appendix proves Lemmas 2 and 3. Throughout, maintain (19) and (18) and hold the wedge fixed along each path. I first use transversality to establish compactness of equilibrium paths for $0 \leq x < \rho$, then construct the rational saddle paths.

Under rational beliefs, the signs of $\dot{q}$ on $q = \bar{q}$ keep the high saddle path above that line on (K^M, K^H) and the low saddle path below it on (K^L, K^M) as they are followed backward. A consumption bound for the high saddle path and an invariant region for the low saddle path establish that both approach $(K^M, \bar{q})$. These connections exclude periodic or other nonconvergent equilibrium paths. The arguments extend to positive wedges, with separate treatment of the case in which the low and middle steady states merge.

F.1 Transversality and compactness

For the proof, define

$$S(K) = d(K) + \psi, \quad B(K,q) = S(K) - \psi q,$$
$$g(q) = \psi \log(q/\bar{q}), \qquad M = \rho - x - \theta K B - B/q.$$

Thus $c = KB$, $W = qK$, and positive consumption requires $K > 0$ and $0 < q < S(K)/\psi$. Away from the technology boundaries, the equilibrium equations are

$$\dot{K} = Kg, \qquad \dot{q} = \frac{qB}{S}\left(M + \frac{Kd'}{B}g\right). \tag{39}$$

The wealth–consumption ratio $Z = W/c = q/B$ satisfies

$$\dot{Z} = (\rho - x)Z - 1 - \theta W. \tag{40}$$

Solving this equation with (15) gives

$$Z_t = \int_t^\infty e^{-(\rho-x)(s-t)}(1 + \theta W_s)\, ds \geq \frac{1}{\rho - x}. \tag{41}$$

The transformation is invertible:

$$q = \frac{S(K)Z}{1 + \psi Z}, \qquad c = \frac{KS(K)}{1 + \psi Z}. \tag{42}$$

In (K, Z) coordinates the field is

$$\begin{aligned} \dot{K} &= K\left[\psi \log\left(\frac{S(K)Z}{1+\psi Z}\right) - \delta\right], \\ \dot{Z} &= (\rho - x)Z - 1 - \theta\frac{KS(K)Z}{1+\psi Z}. \end{aligned} \tag{43}$$

It is continuous and locally Lipschitz on $(0,\infty)^2$, including the technology boundaries, and depends continuously on x.

Compactness. Every equilibrium path has its forward orbit in a compact subset of the positive-consumption region. Let $K_\delta > K_{\text{sat}}$ solve $d(K_\delta) = \Gamma(\delta)$. Positive consumption requires $q < 1 + d(K)/\psi$. Hence $K > K_\delta$ implies $q < \bar{q}$ and $\dot{K} < 0$, so

$$K_t \leq \overline{K} \equiv \max\{K_0, K_\delta\}.$$

Also $W = qK < KS(K)/\psi$. Since $Kd(K) \to 0$ as $K \downarrow 0$, wealth has a uniform upper bound $\overline{W}$ for $0 < K \leq \overline{K}$. Equation (41) gives

$$\frac{1}{\rho - x} \leq Z_t \leq \frac{1 + \theta\overline{W}}{\rho - x}. \tag{44}$$

The lower bound implies $q_t \geq S(K_t)/(\psi + \rho - x)$. Since $d(K) \to \infty$ as $K \downarrow 0$, this price exceeds $\bar{q}$ at sufficiently small stocks, where capital must increase. Capital is bounded away from zero. Equations (42) and (44) then bound price and consumption away from zero and price away from the zero-consumption boundary. These bounds are uniform when K_0 ranges over a compact subset of $(0, \infty)$ and x over a compact subset of $[0, \rho)$.

The bounds also establish the converse of the transversality implication in Appendix B. Along a solution of the equilibrium equations satisfying positivity and (15), consumption has a positive lower bound $\underline{c}$. Forecast wealth remains positive because $\widetilde{W} \geq W$. By the Euler equation,

$$\frac{d}{dt}\left(e^{-\rho t}\frac{\widetilde{W}_t}{c_t}\right) = -\theta c_t e^{-\rho t}\frac{\widetilde{W}_t}{c_t} - e^{-\rho t}.$$

Thus $0 \leq e^{-\rho t}\widetilde{W}_t/c_t \leq (W_0/c_0)e^{-\theta \underline{c} t} \to 0$. Given the equilibrium equations and positivity, condition (15) is therefore equivalent to the transversality condition on forecast wealth.

F.2 The rational high-capital saddle path

Set $x = 0$. Proposition 1 gives

$$0 < K^L < K_{\text{AI}} < K^M < K_{\text{sat}} < K^H.$$

The function G is positive on $(0, K^L)$ and (K^M, K^H), and negative on (K^L, K^M) and (K^H, ∞). The low- and high-capital steady states are saddles and the middle steady state is a source.

Set

$$\bar{B} = d_{\text{flat}} - \Gamma(\delta) > 0, \qquad c^M = K^M \bar{B}.$$

At the middle steady state,

$$\rho - \theta c^M = \bar{B}/\bar{q}, \qquad \psi + \rho - \theta c^M = (d_{\text{flat}} + \psi)/\bar{q}. \tag{45}$$

For $K^M < K \leq K^H$, the sign of G and the increasing steady-state consumption schedule imply

$$K[d(K) - \Gamma(\delta)] \geq \frac{\rho \bar{q} K}{1 + \theta \bar{q} K} > c^M. \tag{46}$$

Differentiating $c = KB$ along the rational path gives

$$\frac{\dot{c}}{c} = \left(1 + \frac{K d'}{S}\right) g + \psi - \frac{\psi q}{S}(\psi + \rho - \theta c). \tag{47}$$

On the boundary $c = c^M$ with $K^M < K < K^H$, (46) implies $q > \bar{q}$. Since $d' \leq 0$ and $S(K) \leq d_{\text{flat}} + \psi$,

$$\frac{\dot{c}}{c} \leq \psi\left[\log(q/\bar{q}) + 1 - q/\bar{q}\right] < 0. \tag{48}$$

The inequality holds on each side of the saturation boundary.

The local saddle path to the left of K^H has $q > \bar{q}$ and, close to the saddle, $c > c^M$. Follow it backward. At $q = \bar{q}$, $G > 0$ implies $\dot{q} < 0$, so backward flow points into $q > \bar{q}$. At $c = c^M$, (48) makes backward flow point into $c > c^M$. The saddle path remains in

$$K^M < K < K^H, \qquad q > \bar{q}, \qquad c > c^M.$$

Capital decreases backward. This region closes to $(K^M, \bar{q})$ at $K = K^M$; the only steady states in its closure are its endpoints. Its other boundaries cannot terminate the orbit. Monotonicity of K therefore gives the backward limit $(K^M, \bar{q})$. The saddle path is defined on (K^M, K^H), with a unique price at each capital stock.

The local saddle path to the right of K^H has $q < \bar{q}$. Where $q = \bar{q}$ lies in the positive-consumption region, $G < 0$ makes backward flow point into $q < \bar{q}$. Once $d(K) \leq \Gamma(\delta)$, positive consumption itself requires $q < \bar{q}$. Capital increases backward. The saddle path cannot terminate at a finite stock: bounded stocks on a backward interval give a uniform wealth bound on the subsequent path, and (41) keeps price and consumption in the interior. A finite limiting stock would have to be another steady state above K^H, which does not exist. The saddle path is defined for every $K > K^H$. Convergence to the interior high steady state guarantees transversality on both segments.

F.3 The rational low-capital saddle path

The low saddle path can be constructed without additional parameter restrictions. Differentiating M along (39) yields

$$\dot{M} = a_M M - b_M, \qquad a_M = \frac{B}{q} + \frac{\theta K \psi q B}{S} > 0, \qquad b_M = \theta K B g \left(1 + \frac{K d'}{S}\right). \tag{49}$$

This identity holds for every fixed wedge. On the interval (K^L, K^M),

$$S + K d' = \begin{cases} \psi + \alpha d > 0, & K < K_{\mathrm{AI}}, \\ \psi + d > 0, & K > K_{\mathrm{AI}}. \end{cases}$$

Define $\Xi = M - \theta K q g$ and consider the region

$$K^L < K < K^M, \qquad \theta K q g < M < 0. \tag{50}$$

The inequalities imply $g < 0$, so $q < \bar{q}$. Consumption is positive because $d(K) \geq d_{\mathrm{flat}} > \Gamma(\delta)$ on this interval.

Both boundaries make the region invariant backward in time. At $M = 0$, (49) gives $\dot{M} = -b_M > 0$. At $\Xi = 0$, direct differentiation gives

$$\dot{\Xi} = -\theta K q \left[\left(1 + \frac{K d' + \theta K q B}{S}\right) g^2 + \frac{K d'}{q} g\right] < 0. \tag{51}$$

The first coefficient is positive, while $d' \leq 0$ and $g < 0$ make the second term nonnegative. The directional inequalities hold on both sides of the technology boundary.

The local saddle path to the right of K^L enters this region. Let $\lambda_- < 0$ be the stable eigenvalue at the low saddle. Along that saddle path, linearizing (49) gives

$$\frac{M}{\theta K q g} \longrightarrow \frac{(B/q)(1 + Kd'/S)}{a_M - \lambda_-} \in (0, 1),$$

where the quantities on the right are evaluated at $(K^L, \bar{q})$. Here $0 < 1 + Kd'/S < 1$ and $a_M > B/q$. Since $g < 0$ on the right saddle path, the ratio establishes $\theta K q g < M < 0$ locally.

The region closes to $q = \bar{q}$ at both K^L and K^M. To see this, for $q < \bar{q}$,

$$M_q = \theta K \psi + S/q^2 > 0, \qquad \Xi_q = S/q^2 - \theta K g > 0.$$

At either root, $M = \Xi = 0$ when $q = \bar{q}$, so no lower price satisfies $\Xi > 0$. At an interior stock, $G < 0$ gives $M(K, \bar{q}) > 0$. The zero of M lies below $\bar{q}$, and the zero of Ξ lies below it; these two curves bound the region in (50). They stay away from $q = 0$, where M and Ξ tend to minus infinity, and consumption has a positive lower bound. Following the saddle path backward therefore gives increasing capital until its limit at $(K^M, \bar{q})$. The saddle path is defined on (K^L, K^M), with capital increasing monotonically backward in time.

The saddle path to the left of K^L has $q > \bar{q}$. Since $G > 0$ below K^L, backward flow at $q = \bar{q}$ points into $q > \bar{q}$, and capital decreases backward. The bounds from (41) prevent termination at an interior positive stock. There is no further steady state below K^L, so the saddle path is defined for every $0 < K < K^L$.

F.4 Rational uniqueness

The saddle paths and the middle stationary path establish existence. Every point on a saddle path converges to an interior steady state and satisfies transversality.

For uniqueness, use the continuous locally Lipschitz field (43). A periodic orbit has index $+1$, equal to the sum of the indices of the steady states it encloses. The middle source has index $+1$ and each saddle has index -1. A periodic orbit would have to enclose the source and neither saddle. But the construction of the high saddle path gives a trajectory from the source to the high saddle. It could not cross a periodic orbit, so an orbit enclosing the source would also enclose the high saddle. This contradicts the index requirement.

Compactness leaves a steady state or a collection of steady states and connecting trajectories as the possible omega-limit set.[3] The latter possibility is also excluded. The source has a neighborhood with strictly outward flow and cannot belong to the omega-limit set of another orbit. Every trajectory entering either saddle lies on a saddle path already constructed. Backward, these paths approach the source or leave every compact subset of the domain toward $K = 0$ or $K = \infty$. None can be a connection in a compact omega-limit set that excludes the source. Every equilibrium path therefore converges to one steady state.

[3]The Poincaré–Bendixson theorem for locally Lipschitz planar fields is discussed in Hanche-Olsen (2017, Chapter 6); Deconinck (2009, Section 6.4) states the finite-equilibrium limit-set classification. The index formula is presented in Pernarowski (n.d.).

A path converging to a saddle lies on its saddle path. The two saddle paths express the price as a function of capital on disjoint intervals, $(0, K^M)$ and (K^M, ∞). Only the stationary path converges to the source. This proves Lemma 2.

F.5 Equilibrium paths with positive wedges

For a fixed wedge, define the rental rate minus the user cost as

$$G_x(K) = d(K) - \Gamma(\delta) - \frac{\bar{q}(\rho - x)}{1 + \theta\bar{q}K} = G(K) + \frac{\bar{q}x}{1 + \theta\bar{q}K}. \tag{52}$$

On either outer region,

$$G'_x(K) = G'(K) - \frac{\theta\bar{q}^2 x}{(1 + \theta\bar{q}K)^2} < 0.$$

During deployment, $G'_x > 0$ for $x < \rho$. Also $G_x(K_{\text{sat}}) > 0$, and $G_x(K_{\text{AI}})$ is negative, zero, or positive according as x is below, equal to, or above $x^*(K^L)$. The required wedge satisfies $0 < x^*(K^L) < \rho$, since

$$\rho - x^*(K^L) = \frac{1 + \theta\bar{q}K_{\text{AI}}}{\bar{q}}\Big[d_{\text{flat}} - \Gamma(\delta)\Big] > 0.$$

Wedges below $x^*(K^L)$. For $0 \le x < x^*(K^L)$, the perceived system satisfies (19) and (18) with $\rho - x$ in place of ρ. The compactness, saddle-path, and uniqueness arguments apply with that substitution. They give the steady states and equilibrium paths in Lemma 3(ii).

Wedges above $x^*(K^L)$. For $x^*(K^L) < x < \rho$, $G_x > 0$ at every stock below its unique root $K^H(x) > K_{\text{sat}}$, and $G_x < 0$ above it. This steady state is a saddle. To its left, the saddle path has $q > \bar{q}$; backward flow at $q = \bar{q}$ points into this region. Capital decreases backward, and the bounds in (41) prevent termination at a positive stock. There is no lower steady state, so the saddle path is defined on $(0, K^H(x))$. The segment to the right is defined for all $K > K^H(x)$ by the same argument used for the rational high saddle.

Compactness and the horizontal-nullcline signs give uniqueness directly. Below $K^H(x)$, a path at or below $\bar{q}$ enters $q < \bar{q}$ and has declining capital, with no lower steady state to approach. Above $K^H(x)$, a path at or above $\bar{q}$ has increasing capital, with no higher steady state to approach. Neither can remain in a compact interior set. A nonstationary equilibrium path must therefore have $q > \bar{q}$ below $K^H(x)$ or $q < \bar{q}$ above it, and cannot cross the steady-state stock. Its capital converges monotonically to $K^H(x)$, so it belongs to the saddle path. This proves part (i).

The case $x = x^*(K^L)$. At $x = x^*(K^L)$, the low and middle steady states coincide at K_{AI}. The other steady state is $K^H(x)$, and $G_x > 0$ below it except at K_{AI}. The consumption bound for the high saddle path remains valid with $\rho - x$ replacing ρ, K_{AI} replacing K^M,

and $c = K_{\text{AI}}[d_{\text{flat}} - \Gamma(\delta)]$ at the lower endpoint. It gives a high saddle path defined for $K > K_{\text{AI}}$ whose left endpoint is $(K_{\text{AI}}, \bar{q})$.

Extending the pre-deployment technology smoothly across the boundary gives a saddle at $(K_{\text{AI}}, \bar{q})$, because the left derivative of G_x is negative. The saddle path on $K < K_{\text{AI}}$ has $q > \bar{q}$. Backward flow keeps it there, and compactness of the forward path prevents an endpoint at a positive stock. The saddle path is defined for all $0 < K < K_{\text{AI}}$. Extending the deployment technology to the left instead gives a source: its two eigenvalues are positive by the calculation in Appendix E. A nonstationary path cannot approach $(K_{\text{AI}}, \bar{q})$ while remaining to its right.

These facts also establish uniqueness. A path with $q \leq \bar{q}$ below the high steady state, other than the stationary path at K_{AI}, enters $q < \bar{q}$ and has decreasing capital. Compactness would require convergence to a lower steady state. The only candidate is K_{AI}, approached from its right, which the source calculation excludes. A path with $q \geq \bar{q}$ above the high steady state has increasing capital and no higher steady state to approach. The remaining equilibrium paths have monotone capital and converge to one of the two steady states. Convergence to the high saddle requires its saddle path, which is undefined at or below K_{AI}. Convergence to K_{AI} must occur from the left on the unique saddle path. At $K = K_{\text{AI}}$, only the stationary path is an equilibrium path.

F.6 Continuity of the equilibrium price

Let $Q(K; x)$ denote the unique equilibrium initial price. Consider $(K_n, x_n) \to (K_0, x_0)$, where $K_0 > 0$ and $0 \leq x_0 < \rho$. The bounds in Section F.1 place all the corresponding forward paths in a common compact subset of the positive-consumption region. Every subsequence of initial prices has a convergent further subsequence. In (K, Z) coordinates, continuous dependence on initial data and parameters gives convergence of the associated paths on every bounded time interval.[4]

The limiting path stays in the same compact interior set and has bounded Z. Since $\rho - x_0 > 0$, it satisfies (15) and is an equilibrium path. Uniqueness at (K_0, x_0), including the case $x_0 = x^*(K^L)$ proved above, implies that its initial price is $Q(K_0; x_0)$. Every convergent subsequence has this limit, proving joint continuity of Q. This completes Lemma 3. □

F.7 Prices as a function of the wedge

At a common (K, q) away from $q = \bar{q}$,

$$\frac{\partial}{\partial x}\left(\frac{dq}{dK}\right) = -\frac{1}{\dot{K}[1/q - C_q/C]}.$$

The derivative is negative on saddle paths with increasing capital and positive on saddle paths with decreasing capital. The low and high steady-state capital stocks increase with x, because $\partial_x G_x > 0$ and $G_x' < 0$ at those roots.

[4] See Hanche-Olsen (2017, Chapter 1) for continuous dependence of locally Lipschitz flows on initial data and parameters.

Consider $x_2 > x_1$. At $K^H(x_1)$, the x_2 high saddle path lies above $\bar{q}$. On the common growing region, a crossing of the two saddle paths could only take their price difference from positive to negative as K increases. This would contradict their ordering at $K^H(x_1)$. Hence $Q^H(K; x_2) > Q^H(K; x_1)$ throughout their common growth region.

For two low saddle paths with decreasing capital, $K^L(x_2) > K^L(x_1)$. At $K^L(x_2)$, the x_2 price equals $\bar{q}$, while the x_1 price is below it. On the common declining region, a crossing could only take the difference from negative to positive as K increases, so the initial positive ordering is preserved. The price on a low saddle path with declining capital is therefore also strictly increasing in the wedge.

G Proofs for Sections 3–4

Closed-form required wedge. In the flat deployment region, a permanent belief wedge changes the steady-state condition to

$$d_{\text{flat}} = \Gamma(\delta) + \frac{\bar{q}(\rho - x)}{1 + \theta \bar{q} K}.$$

Solving for the perceived middle steady state gives

$$K^M(x) = \frac{1}{\theta \bar{q}} \left[\frac{\bar{q}(\rho - x)}{d_{\text{flat}} - \Gamma(\delta)} - 1 \right] = K^M - \frac{x}{\theta [d_{\text{flat}} - \Gamma(\delta)]}.$$

For $K \in [K_{\text{AI}}, K^M]$, the wedge that moves this steady state to K solves $K = K^M(x)$:

$$x^*(K) = \theta [d_{\text{flat}} - \Gamma(\delta)](K^M - K).$$

For $K < K_{\text{AI}}$, the low and middle steady states merge at the start of deployment, so the required wedge remains $x^*(K_{\text{AI}})$. Combining the two cases yields (24). Lemma 3(i) establishes a unique equilibrium path from K^L along the saddle path to $K^H(x)$, with positive net investment through K^M.

Proposition 2. By Lemma 2, the unique rational equilibrium path from K^L lies on the low saddle path. At that stock the saddle path contains the stationary point $(K^L, \bar{q})$. The path remains there and cannot converge to K^H. □

Proposition 3(i). Let $x \in (x^*(K^L), \rho)$. By Lemma 3(i), $Q^H(K; x)$ is continuous and strictly above $\bar{q}$ on the compact interval $[K^L, K^M]$, and consumption is positive along the path. Hence $\psi \log Q^H(K; x) - \delta$ is bounded below by a positive number. Capital rises from K^L to K^M in finite time. □

Proposition 3(ii). Part (i) makes the denominator in (26) positive and bounded away from zero, so $0 < T(x) < \infty$. The solution reaches K^M exactly at $T(x)$. Since capital is continuous at the correction, $\tau < T(x)$ implies $K_\tau < K^M$, while $\tau > T(x)$ implies $K_\tau > K^M$. By Lemma 2, the rational path converges to K^L in the first case and to K^H in the second. When $\tau = T(x)$, the post-correction state is the middle steady state. □

Proposition 3(iii). Section F.7 establishes $Q^H(K; x_2) > Q^H(K; x_1)$ on the common growth region when $x_2 > x_1$. For $x_2 > x_1 > x^*(K^L)$, this region contains $[K^L, K^M]$. The denominator in (26) is therefore strictly larger at the higher wedge, making $T(x)$ strictly decreasing.

As $x \downarrow x^*(K^L)$, joint continuity of the equilibrium price and continuous dependence of the flow give convergence on each bounded time interval to the equilibrium path from K^L with wedge $x^*(K^L)$. That path converges to $K_{\text{AI}} < K^M$ and never reaches K^M. A bounded sequence of hitting times would give a finite hitting time on the limiting path, a contradiction. Thus $T(x) \to \infty$. □

Corollary 1. At the full correction, the wedge falls from x to zero and the price adjusts to the rational equilibrium path from K_τ. If $\tau > T(x)$, then $K_\tau > K^M$, so the price lies on the rational high saddle path and capital converges to K^H. If $\tau < T(x)$, then $K_\tau \in (K^L, K^M)$, so the price lies on the rational low saddle path and capital converges to K^L. □

The post-correction yield curve. No arbitrage gives the price $\exp[-\int_0^h i_{\tau+s}\, ds]$ for a unit payoff at $\tau + h$, hence the yield formula. Corollary 1 and the Euler equation imply $i_{\tau+s} \to i^L$ below K^M and $i_{\tau+s} \to i^H$ above it. Time averaging gives (30).

As $K_\tau \to K^M$ along either rational saddle path, the price approaches $\bar{q}$. Continuity of the short rate then gives $i_{\tau+} \to i^M$. Since $i^H < i^M < i^L$ by (8), nearby stocks have $i^H < i_{\tau+} < i^L$; the long-yield limits then give the stated spread signs for all sufficiently large h.

On each fixed finite horizon, continuous dependence on initial conditions gives uniform convergence to the stationary middle path as $K_\tau \to K^M$. Thus $y_\tau(h) \to i^M$ and the spread vanishes at every fixed maturity. □

Corollary 2. The first two pieces of (34) are increasing and flat. In the saturation region, differentiating the last piece yields a positive derivative because $\alpha \in (0, 1)$ and $K > \bar{K}_\ell$. The pieces coincide at both technology boundaries, so $w(K)$ is weakly increasing everywhere and $w(K^H) > w(K^L)$. On the perceived path capital rises. After an early correction, the rational equilibrium path lies on the low saddle path with $q < \bar{q}$ (Lemma 2), so capital converges monotonically to K^L from above. Capital remains at or above K^L after either correction, so the wage remains at or above its initial value. □

H Partial Corrections and Investment

Lemma 3 determines the equilibrium path for any remaining wedge $0 \le x_1 < x_0 < \rho$. For $x_1 < x^*(K^L)$, capital converges to the low steady state below $K^M(x_1)$ and to the high steady state above it; at $K^M(x_1)$, the equilibrium path is stationary. For $x_1 > x^*(K^L)$, every equilibrium path converges to the high steady state. Capital increases toward the limiting steady state from below and decreases toward it from above. The equilibrium

price is continuous in capital and the wedge, including at $x_1 = x^*(K^L)$, where the low and middle steady states merge.

The equilibrium correction boundary. Let $\bar{B} = d_{\text{flat}} - \Gamma(\delta) > 0$. Equation (22) gives

$$K^M(x) = K^M - \frac{x}{\theta \bar{B}}.$$

At $K \in (K_{\text{AI}}, K^M)$, the wedge that places the middle steady state at K is $x^*(K)$. By Lemma 3, the post-correction price is above $\bar{q}$ when $x_1 > x^*(K)$, below $\bar{q}$ when $x_1 < x^*(K)$, and equal to $\bar{q}$ at the source. As the remaining wedge approaches the required wedge from either side, the post-correction price converges to $\bar{q}$. Substituting into $D = 1 - q^+/Q_0(K)$ proves (32) and part (i) of Proposition 4. Thus $D^*(K)$ is the supremum of corrections consistent with positive net investment. At $D = D^*(K)$, net investment is zero and capital remains constant.

The price during deployment. Fix x and define

$$B = d(K) - \psi(q-1), \qquad S = d(K) + \psi, \qquad g = \psi \log q - \delta,$$

and $M = \rho - x - \theta K B - B/q$. Consumption is KB, and the vector field (37) becomes

$$\dot{K} = Kg, \qquad \dot{q} = \frac{qB}{S}\left(M + \frac{Kd'(K)}{B}g\right). \tag{53}$$

Since

$$M_K = -\theta(B + Kd') - \frac{d'}{q}, \qquad M_q = \theta K \psi + \frac{S}{q^2},$$

the time derivative of M satisfies

$$\dot{M} = a_M M - b_M, \qquad a_M = \frac{B}{q} + \frac{\theta K \psi q B}{S} > 0, \qquad b_M = \theta K B g\left(1 + \frac{Kd'}{S}\right). \tag{54}$$

Along a growing high saddle path, $B > 0$ and $g > 0$. If $S + Kd' > 0$, then $b_M > 0$. At the limiting high steady state, $M = 0$ and a_M converges to a positive constant. Solving (54) with this bounded terminal condition yields

$$M_t = \int_t^\infty \exp\left[-\int_t^s a_{M,u}\, du\right] b_{M,s}\, ds > 0. \tag{55}$$

The identity applies piecewise across the technology kinks, where M is continuous. Wherever $d' = 0$, equations (53) and (55) imply $\dot{q} > 0$. Since $\dot{K} > 0$, the saddle path has $\partial_K Q^H(K; x) > 0$. In particular, $Q_0'(K) > 0$ during deployment, which proves the derivative in part (ii) of Proposition 4.

With $\theta = 0$, the forcing term b_M vanishes, the bounded terminal solution has $M = 0$, and q is constant wherever the rental rate is constant along an equilibrium path with

increasing capital. The strict increase in the correction boundary therefore depends on the effect of wealth on saving.

The regularity condition is automatic before and during deployment. After saturation,

$$S + Kd' = \psi + d(K)\frac{\alpha K - \bar{K}_\ell}{K - \bar{K}_\ell}.$$

A sufficient restriction in primitives is

$$\gamma \bar{K}_\ell (1 - 2\alpha) \leq \alpha^2. \tag{56}$$

It is equivalent to $K_{\text{sat}} \geq \bar{K}_\ell/\alpha$, which makes $d(K) + Kd'(K)$ nonnegative after saturation. The illustrative parameters satisfy this restriction. It is stronger than needed for $S + Kd' > 0$, because the positive term ψ can offset a negative derivative of capital income.

Consumption compression and interest rates. By the definition of M_t, $M_t = \rho - x - (\theta + 1/W_t)c_t$. Hence

$$c_x^{ss}(W_t) - c_t = \frac{W_t}{1 + \theta W_t} M_t > 0, \tag{57}$$

where the inequality follows from (55). This proves (29). Substituting (57) into (28) gives the equivalent expression

$$i_t = \frac{\rho + x\theta W_t}{1 + \theta W_t} + \frac{\theta W_t}{1 + \theta W_t} M_t + \frac{\dot{c}_t}{c_t}.$$

The consumption-compression term is positive along the high saddle path when capital is increasing and converges to zero with M_t at the steady state for the fixed wedge. Moreover, $\partial_W i_x^{ss}(W) = -\theta(\rho - x)/(1 + \theta W)^2 < 0$, while $i_0^{ss}(W) = i^{ss}(W)$ recovers the rational steady-state comparison.

A necessary condition for (29) is

$$d(K^H(x)) + \psi + K^H(x) d'(K^H(x)) \geq 0.$$

If this expression is negative, $b_{M,t} < 0$ for all sufficiently large t along the growing high saddle path. The equalities in (55) and (57) hold without the regularity restriction, so $M_t < 0$ and $c_t > c_x^{ss}(W_t)$ near the steady state.

Price as a function of the wedge. Section F.7 proves that the price is strictly increasing in the wedge on the common portions of the high saddle paths with increasing capital and of the low saddle paths with decreasing capital. At the wedge for which $K = K^M(x)$, the equilibrium price is $\bar{q}$. By continuity in x (Lemma 3), the prices on the low and high saddle paths converge to this value. Price declines sufficiently close to $D^*(K)$ on either side can therefore be generated by remaining wedges on the corresponding side of $x^*(K)$.

For $K_{\text{AI}} < K_{\text{early}} < K_{\text{late}} < K^M$, choose a sufficiently small $\varepsilon > 0$ and set

$$D_{\text{early}} = D^*(K_{\text{early}}) + \varepsilon, \qquad D_{\text{late}} = D^*(K_{\text{late}}) - \varepsilon.$$

Taking $2\varepsilon < D^*(K_{\text{late}}) - D^*(K_{\text{early}})$ gives $D_{\text{early}} < D_{\text{late}}$, with the early post-correction price on the declining low saddle path and the late post-correction price on the growing high saddle path. Both are attainable by the preceding continuity argument. Since $Q_0(K_{\text{early}}) < Q_0(K_{\text{late}})$, the late absolute price decline is also larger. This proves the remaining statement of Proposition 4.

Delay near the correction boundary. Fix $K_\circ \in (K_{\text{AI}}, K^M)$ and let $\mu = x_1 - x^*(K_\circ) \downarrow 0$. The post-correction source is

$$K^M(x_1) = K_\circ - \frac{\mu}{\theta \bar{B}}.$$

At $x_1 = x^*(K_\circ)$, the eigenvalues at the middle steady state are

$$\lambda_1 = \frac{\bar{B}}{\bar{q}}, \qquad \lambda_2 = \frac{\theta K_\circ \psi \bar{q} \bar{B}}{d_{\text{flat}} + \psi}.$$

Both are positive. Assume they are distinct, and let $\lambda_\circ$ be the eigenvalue associated with the eigenvector tangent to the saddle path; generically it is the smaller one. The local expansions are

$$\dot{K} = \lambda_\circ (K - K^M(x_1)) + \text{higher-order terms},$$
$$q - \bar{q} = \frac{\lambda_\circ \bar{q}}{K_\circ \psi}(K - K^M(x_1)) + \text{higher-order terms}.$$

The time to exit a fixed small neighborhood of the source is $\lambda_\circ^{-1} \log(1/\mu) + O(1)$, while the time from that neighborhood to K^M remains bounded. Because $q^+ - \bar{q} = Q_0(K_\circ)[D^*(K_\circ) - D]$ is proportional to μ to first order,

$$\mathcal{T}(K_\circ, x_1) = \frac{1}{\lambda_\circ} \log\left(\frac{1}{D^*(K_\circ) - D}\right) + O(1) \longrightarrow \infty. \tag{58}$$

This proves the duration claim near the correction boundary in Section 4.4.

Other convex adjustment costs. The arguments for the correction boundary and price monotonicity extend to other investment expenditure functions Γ that are smooth and strictly convex. Set $j(q) = (\Gamma')^{-1}(q)$, $B = d - \Gamma(j)$, $h = q/\Gamma''(j)$, $S = B + qh$, and $g = j - \delta$. Equations (53)–(55) hold with $a_M = B/q + \theta K h q B/S$ and the regularity condition $S + Kd' > 0$. The steady-state price is $\bar{q} = \Gamma'(\delta)$, so $D^*(K) = 1 - \bar{q}/Q_0(K)$ remains the correction boundary.

I Illustrative Numerical Example

The figures use the following parameters:

$$\alpha = \tfrac{1}{3}, \quad A = 0.16, \quad \rho = 0.089, \quad \delta = 0.040,$$
$$\theta = 2.49, \quad \gamma = 0.81, \quad \bar{K}_\ell = 0.253, \quad \psi = 0.61.$$

These values imply $\bar{q} = e^{\delta/\psi} = 1.0678$, $b = (1-\alpha)\gamma/\alpha$, $K_{\text{AI}} = 1/b = 0.6173$, and $K_{\text{sat}} = K_{\text{AI}} + \bar{K}_\ell/(1-\alpha) = 0.9968$. The rate of time preference is ρ; the steady-state interest rate is $i^{ss} = \rho/(1+\theta W)$ by (8), which is below ρ at all three steady states. The inequalities in (19) hold, $G' < 0$ on both outer regions, and the three steady states are $K^L = 0.5413$, $K^M = 0.7330$, and $K^H = 1.1551$. The associated steady-state interest rates are $i^L = 3.65$ percent, $i^M = 3.02$ percent, and $i^H = 2.19$ percent. The required wedge is $x^*(K^L) = 0.0093$.

The slope condition can also be verified analytically. At $\alpha = 1/3$, write $t = \theta\bar{q}$. Before deployment, the ratio $(-\Phi')/(-d')$ is proportional to $K^{5/3}/(1+tK)^2$, which increases until $K = 5/t$. Since $tK_{\text{AI}} < 5$, its maximum over that region is at K_{AI}, where the ratio is 0.455872. After saturation, the ratio is proportional to $(K - \bar{K}_\ell)^{5/3}/(1+tK)^2$, whose maximum is at $K = 5/t + 6\bar{K}_\ell$, where the ratio is approximately 0.420771. Both are below one, giving $G' < 0$ throughout the outer regions.

Lemmas 2 and 3 establish the rational and optimistic equilibrium paths at these parameters. Numerical integration supplies the paths plotted in the figures.

The parameters are illustrative, rather than calibrated. Tobin's q is the price per unit of installed capital. Rate parameters are annual, so transition times are in years.

Before the late correction, investors' consumption lies between 0.0046 and 0.0057, below its value $c^L = 0.0211$ at the low-capital steady state, and it is nondecreasing. The Euler equation (7) then implies an interest rate above its initial level throughout the optimistic phase in this illustration. The interest rate ranges from 8.80 to 9.35 percent. The realized return in (14) is six percentage points lower; the correction causes the discrete capital loss described in Section 4.5.

The full-correction figures use $x = 0.06$, about 6.5 times the required wedge $x^*(K^L)$. At this wedge, the time needed to reach K^M is $T(x) = 12.93$. For smaller wedges, the corresponding times are $T(0.01) = 170.13$, $T(0.02) = 40.65$, $T(0.03) = 25.04$, $T(0.04) = 18.62$, and $T(0.05) = 15.12$. The required duration rises without bound as the wedge approaches $x^*(K^L)$ from above. The early correction is set at $\tau = 0.60T(x) = 7.76$, when $K_\tau = 0.6523 < K^M$; the late correction is set at $\tau = 1.20T(x) = 15.52$, when $K_\tau = 0.7775 > K^M$. The two correction dates illustrate convergence to different rational steady states. At the late correction, the price falls from 1.1085 to 1.0707. The post-correction price lies on the rational high saddle path.

Figure 6 uses $x_0 = 0.012$ and corrections at 20 and 80 percent of the interval from K_{AI} to K^M. The remaining wedges imply price declines on opposite sides of the correction boundary, with the larger decline occurring later.

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