MIT 14.662 Spring 2026 Lecture Slides 4 — Discrimination and Misallocation, ‘A Roy-Fréchet’ Approach

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Agenda

What are the costs of discrimination?

1. **Transactional harms to targets of discrimination**: Harm to individuals who are discriminated against at the moment of discrimination — denied interview/job, received low wage offer (the focus of most economic literature)
2. **Pervasive harms to targets of discrimination**: Discrimination may directly affect skills acquisition/investment, productivity, or realized performance of targets of discrimination
3. **Societal externalities**: Discrimination may cause social maladies, violence, despair, drug abuse, crime, and attendant costs (incarceration, victimization, etc.)
4. **'Systemic discrimination:'** Discrimination in one set of decisions indirectly contributes to disparities in non-discriminatory domains (Bohren, Hull, and Imas *QJE* 2025)
5. **Market misallocation**: If discrimination prevents people from realizing their comparative advantage, there's potentially a large social opportunity cost. How large?
    - This is inherently a **Roy** question
    - Misallocation means people are not realizing their comparative advantage

Occupational dissimilarity and convergence

Occupational differences

- In 1960: 94% of doctors and lawyers were white men
- By 2010: Only 62% were white men
- Sandra Day O'Connor — 3rd in class at Stanford Law, could get job only as legal secretary (1952)
- Similar patterns across highly-skilled occupations

Central question

If innate talent is unlikely to have changed differently across groups, what explains this convergence? And what are its aggregate consequences?

Gender occupational segregation: 1971– 2009

$S_t \equiv \frac{1}{2} \times \sum_i |m_{it} - f_{it}| \in [0, 100]$, where m_{it}, f_{it} are share of all M or F emp in occ i in year t

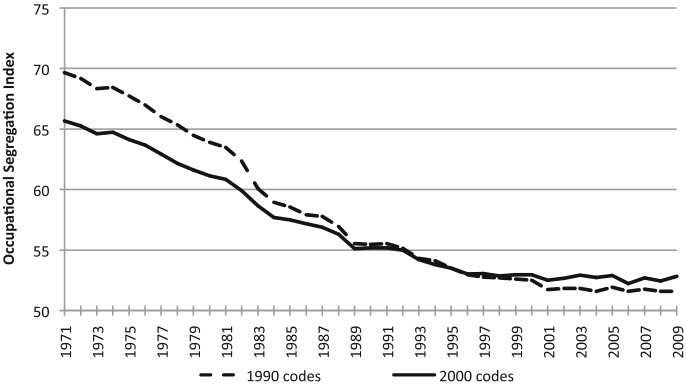
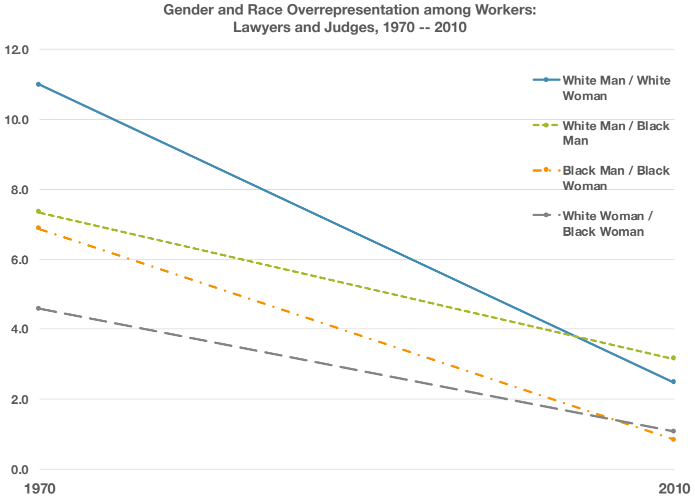


Fig. 1 Trends in occupational segregation using gender-specific CPS crosswalk (March CPS data) Estimates for the years 2000–2002 use actual (noncrosswalked) data from the BLS dual-coded data set

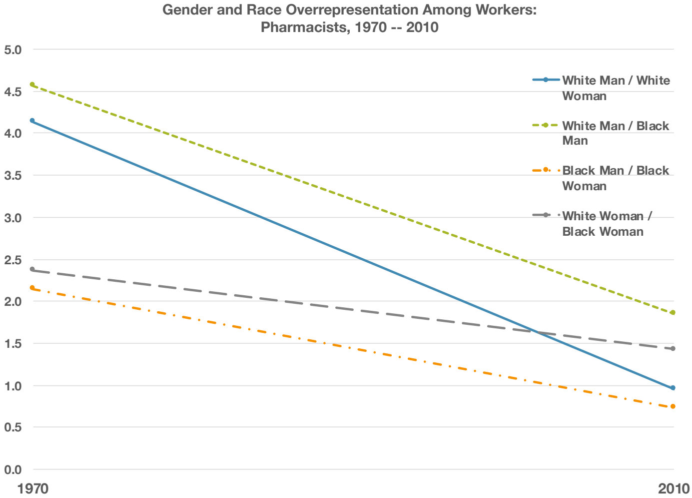
Blau, Brummond, Liu '13

Gender and race over/under-representation, 1970 v. 2010: Lawyers and judges



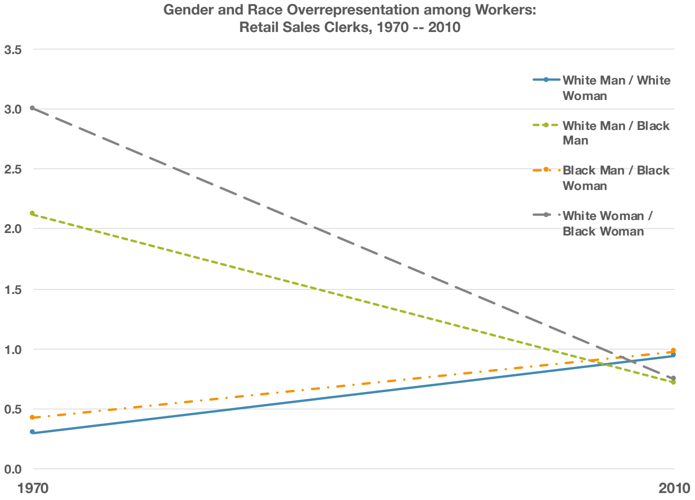
Bayers-Winston, Foaud, and Wen '15

Gender and Race over/under-representation, 1970 v. 2010: Pharmacists



Bayers-Winston, Foaud, and Wen ’15

Gender and race over/under-representation, 1970 v. 2010: Retail sales clerks



Bayers-Winston, Foaud, and Wen '15

General equilibrium implications of discrimination
Hsieh, Hurst, Jones & Klenow 2019 Ecta

Why are Blacks and women underrepresented in high paid occupations?

1. Labor market discrimination
2. Barriers to skill acquisition
3. Preferences/tastes
4. Differences in comparative advantage (i.e., talent)

And what are the implications of partial race and gender convergence in occupational stature?

General equilibrium implications of discrimination

Clearly, a model is needed to analyze this question

- What are the fundamental distortions (discrimination, human capital)?
- What are the underlying differences in talent and tastes?
- How can we empirically distinguish misallocation due to distortions from optimal allocations due to tastes or comparative advantage?
- How do we quantify consequences?

General equilibrium implications of discrimination

- Hsieh and Klenow. 2009. "Misallocation and Manufacturing TFP in China and India" *QJE*
- Cavalcanti and Tavares. 2016. "The Output Cost of Gender Discrimination: A Model-based Macroeconomics Estimate" *The Economic Journal*
- Hsieh, Hurst, Jones, Klenow. 2019. "The Allocation of Talent and U.S. Economic Growth" *ECTA*
- Aizer, Boone, Lleras-Muney, Vogel. 2020. "Discrimination and Racial Disparities in Labor Market Outcomes: Evidence from WWII" NBER WP
- Hurst, Rubinstein, Shimizu. 2024. "Task Based Discrimination" *AER*. Focusing on *contact tasks*
- Ashraf, Bandiera, Minni, Quintas-Martnez. 2024. "Gender Gaps Across the Spectrum of Development: Local Talent and Firm Productivity" NBER WP
- Autor, Nybom, Markussen, Restrepo, Røed, Salomons. 2026. "In Heels and Backwards: The Economic Costs of Women Doing More for Less" (slides)
- Many other recent contributions...

Agenda

Model of occupational choice in general equilibrium

Goal is to identify and quantify forces causing individuals to choose occupations where they *don't* have comparative advantage:

1. **Labor market discrimination** (τ^w_{ig})
   - Wedge between wages and marginal products
2. **Human capital barriers** (τ^h_{ig})
   - Increased costs of accumulating occupation-specific human capital
   - Historical restrictions on college admission, school quality differences
3. **Preferences/social norms** (z_{ig})
   - Group-specific utility from working in occupation i
   - Includes home sector preferences, fertility timing
   - These are not necessarily "misallocation" if preferences are taken as given

Note that i denotes occupations, g demographic groups (white men, white women, black men), $\tau's$ are 'taxes'—really, they're wedges since they are dissipative (no one collects the revenue).

Key identifying assumption

The distribution of **innate talent** of women and black men—relative to white men—is **constant over time**

Note: this assumption is testable via AFQT scores (NLSY 1979, 1997) — women score similarly to men.

Implication: Changes in occupational distribution must be driven by

- Changes in labor market frictions (τ^w)
- Changes in human capital frictions (τ^h)
- Changes in occupational preferences ($\tilde{z}$)

Cannot identify levels, only changes relative to a benchmark group (white men)

Key modeling innovation/trick: A "Roy-Fréchet model"

"Roy-Fréchet model": Skill follows an extreme value distribution. Formally:

- If x is distributed Fréchet
$$\Pr[X > x] = 1 - e^{-x^{-\alpha}}$$
where $\alpha \in (0, \infty)$ is the shape parameter with lower values meaning more dispersion
- Workers draw x_i
- Say workers select into sector if $wx_i > \kappa$, where w is the wage and x_i is their individual talent draw
- This selection rule will produce a conditional mean of sectoral wages, δ, that does not vary with w
$$E[wx \mid wx > \kappa] = \delta$$
- If wage w rises, $E[x \mid wx > \kappa]$ falls by an exactly offsetting amount, so that $E[wx \mid wx > \kappa]$ is constant
- *Why?* Fréchet has Pareto-like tails where $E[x \mid x > c] \propto c$. So if threshold is $c = \kappa/w$, then $w \cdot E[x \mid x > \kappa/w] = \text{const}$

Illustrating the Roy-Fréchet property

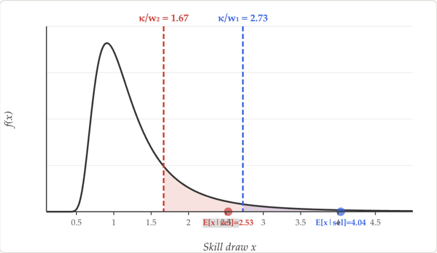
Roy-Fréchet Selection: Why Conditional Wages Don't Depend on w

Workers select into a sector when $wx_i > \kappa$, i.e. when their skill draw $x_i > \kappa/w$. Because the Fréchet distribution has Pareto-like tails, the conditional mean $E[x \mid x > c]$ is proportional to c. So when w rises, the lower threshold draws in less-skilled workers — but the wage premium exactly offsets this dilution.

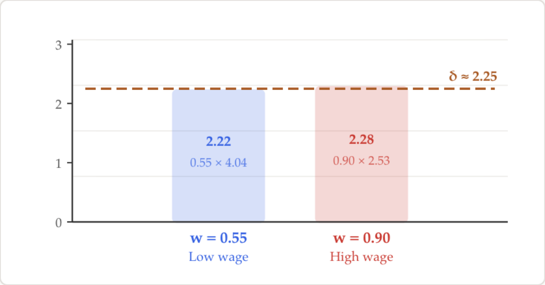
Low wage (w_1) = 0.55 High wage (w_2) = 0.90

$\alpha = 3$, $\kappa = 1.5$

Fréchet density of skill x with selection thresholds



Conditional mean earnings $E[wx \mid \text{selected}]$



Structure

Environment

- Continuum of workers in M discrete sectors (including home)
- Workers indexed by occupation i, group g (race/gender), cohort c
- Each worker has heterogeneous abilities ϵ_i *or* preferences μ_i
- Not clear that M makes another appearance in the paper

Timing

1. **Pre-period**: draw talent in each occ, choose occ, invest in human capital
2. **Working life**: three periods — "young," "middle," "old"

Key modeling choice

- Workers heterogeneous in *either* talent or preferences, not both
- Parameter δ: share sorting on preferences ($1 - \delta$ sort on ability)
- Baseline: $\delta = 0$ (sorting on ability only)

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Lifetime utility function

Lifetime utility for worker from group g, cohort c, choosing occupation i:

$$\log U = \beta \left[\sum_{t=c}^{c+2} \log C(c,t) \right] + \log[1 - s(c)] + \log z_{ig}(c) + \log \mu$$

Components:

- $C(c,t)$: consumption of cohort c at time t
- s: time allocated to human capital acquisition (pre-period)
- z_{ig}: common utility benefit for group g in occupation i
- μ: idiosyncratic utility benefit (individual-specific)
- β: trade-off between lifetime consumption and schooling time (discount rate)
- Note that there is no person-level subscript: i is occupation

Normalization: time endowment in pre-period $= 1$, so $1 - s(c)$ is leisure.

Human capital production

Human capital production function:

$$h_{ig}(c,t) = \bar{h}_{ig} \cdot \gamma(t-c) \cdot s_i(c)^{\phi_i} \cdot e_{ig}(c)^{\eta}$$

Parameters:

- $\bar{h}_{ig}$: permanent differences in human capital endowments (group-occupation specific)
- $\gamma(t-c)$: return to experience (function of age $= t - c$)
- ϕ_i: occupation-specific return to *time* investments
- η: elasticity of human capital w.r.t. *goods* expenditures e

Awkward modeling feature/bug: Schooling is purchased with both time s and goods e

- Both costs are needed to rationalize the results
- Foregone goods consumption is affected by market distortions — e_{ig} responds to frictions
- Foregone time needed to explain average wage differences across occupations—more human-capital time investment in more 'skill-sensitive' occupations (higher ϕ_i), higher wages
- But time investment not affected by market distortions

Budget constraint and frictions

Consumption equals "after-tax" earnings net of education:

$$C(c,t) = \left[1 - \tau_{ig}^{w}(t)\right] w_i(t)\epsilon h_{ig}(c,t) - e_{ig}(c,t)\left[1 + \tau_{ig}^{h}(c)\right]$$

Two types of frictions

1. **Labor market discrimination** τ_{ig}^{w}:
   - "Tax" on individual earnings
   - Affects all cohorts of group g in occupation i at time t
   - This shows up as time effects
2. **Human capital barriers** τ_{ig}^{h}:
   - Increases cost of education spending e
   - Fixed for a cohort-group after pre-period decision
   - This shows up as cohort effects

Identification strategy: use life-cycle wage dynamics to separate τ^w from τ^h.

Optimal human capital investments

First-order conditions yield optimal schooling choices:

$$s_i^* = \frac{1}{1 + \frac{1-\eta}{3\beta\phi_i}}$$

$$e_{ig}^* = \left(\frac{\eta(1-\tau_{ig}^w) w_i \bar{\gamma} \bar{h}_{ig} s_i^{\phi_i} \epsilon}{1+\tau_{ig}^h} \right)^{\frac{1}{1-\eta}}$$

where $\bar{\gamma} \equiv 1 + \gamma(1) + \gamma(2)$ sums experience terms over life-cycle.

Key observations:

- Time s_i^* depends *only* on ϕ_i — not on w_i, $\bar{h}_{ig}$, τ_{ig}^h, τ_{ig}^w
  - These factors affect wage gains and opportunity cost equally
- Goods e_{ig}^* *does* respond to frictions with elasticity $\frac{1}{1-\eta}$

Indirect utility

Substituting optimal choices, indirect utility becomes:

$$U_{ig}^{*} = \mu_i \left[\bar{\gamma}\tilde{w}_{ig}\epsilon_i\right]^{\frac{3\beta}{1-\eta}}$$

where the **composite wage** is:

$$\tilde{w}_{ig} \equiv w_i s_i^{\phi_i}(1-s_i)^{\frac{1-\eta}{3\beta}} \cdot \frac{\bar{h}_{ig}\tilde{z}_{ig}}{\tau_{ig}}$$

Composite friction:

$$\tau_{ig} \equiv \frac{(1+\tau_{ig}^{h})^{\eta}}{1-\tau_{ig}^{w}} \geq 1$$

Transformed preferences:

$$\tilde{z}_{ig} \equiv z_{ig}^{\frac{1-\eta}{3\beta}}$$

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Fréchet distribution of talent—this is crucial (for better and worse)

Distributional assumption (following McFadden 1974, Eaton-Kortum 2002):
Each person draws talent ϵ_i in each occupation from a **multivariate Fréchet**:

$$F_g(\epsilon_1, \ldots, \epsilon_M) = \exp\left[-\sum_{i=1}^{M} \epsilon_i^{-\theta}\right]$$

Parameter θ:

- Governs dispersion of skills
- Higher $\theta \Rightarrow$ *smaller* dispersion (less comparative advantage)
- Mean parameter normalized to 1 (isomorphic to $\bar{h}_{ig}$)

For preference draws: shape parameter $= \frac{\theta(1-\eta)}{3\beta}$

- This expression makes labor supply elasticity identical for *both* types of heterogeneity—talent and preference

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Occupational choice

Proposition 1 (occupational choice)

The fraction of people from cohort c and group g who choose occupation i:

$$p_{ig}(c) = \frac{\tilde{w}_{ig}(c)^{\theta}}{\sum_{s=1}^{M} \tilde{w}_{sg}(c)^{\theta}}$$

where $\tilde{w}_{ig}(c) \equiv w_i(c) s_i(c)^{\phi_i(c)} [1 - s_i(c)]^{\frac{1-\eta}{3\beta}} \cdot \frac{\bar{h}_{ig} \tilde{z}_{ig}(c)}{\tau_{ig}(c)}$

Interpretation:

- This is a standard share equation (you should recognize these even if sleep deprived and hungover)
- Occupational sorting depends on comparative advantage
- High w_i attracts more workers of all groups
- Group differences driven by $\tilde{z}$, $\bar{h}_{ig}$, τ^w, τ^h
- Low p_{ig} when: group dislikes occupation ($\tilde{z}_{ig}$ low), low ability ($\bar{h}_{ig}$ low), high discrimination (τ^w_{ig} high), or high HC barriers (τ^h_{ig} high)

Average worker quality

Proposition 2 (average quality of workers of group g in occ i)

For cohort c of group g at time t, geometric average of worker quality:

$$e^{\mathbb{E}[\log(h_{ig}(c,t)\epsilon_{ig}(c))]} = \bar{\Gamma} s_i(c)^{\phi_i(t)} \gamma(t-c) \left(\frac{\eta s_i(c)^{\phi_i(c)} \bar{\gamma} \bar{h}_{ig} w_i(c)[1-\tau^w_{ig}(c)]}{1+\tau^h_{ig}(c)} \right)^{\frac{\eta}{1-\eta}} \left(\frac{1}{p_{ig}(c)} \right)^{\frac{1-\delta}{\theta(1-\eta)}}$$

Quality selection effect (when $\delta = 0$, sorting on **ability**):

- Average quality *inversely related* to occupational share $p_{ig}(c)$. This is a **crucial implication**
- If women discriminated against as lawyers in 1960 $\Rightarrow$ only most talented chose it
- As barriers decline $\Rightarrow$ less talented enter $\Rightarrow$ average quality falls

When $\delta = 1$ (preferences only): quality selection effect absent, it's all taste.

Average worker quality

The parameter δ denotes the share of the population with idiosyncratic preferences (so $1-\delta$ is the share of workers with idiosyncratic ability) and $\bar{\Gamma}$ is a constant.[7] By varying δ, we can explore the robustness of our results to sorting that occurs completely on talent ($\delta = 0$), sorting that occurs completely on preferences ($\delta = 1$), or sorting that occurs on both margins. When all individuals possess heterogeneous abilities ($\delta = 0$), average quality is inversely related to the share of the group working in the occupation $p_{ig}(c)$. This captures the selection effect. For example, the model predicts that if the labor market discriminated against female lawyers in 1960, only the most talented female lawyers would have chosen to work in this occupation. And if the barriers faced by female lawyers declined after 1960, less talented female lawyers would move into the legal profession and thus lower the average quality of female lawyers. Conversely, in 1960, the average quality of white male lawyers would have been lower in the presence of labor market discrimination against women and black men. At the other extreme, when $\delta = 1$ (all workers sort on preferences), this selection effect is absent.

Occupational wages

Proposition 3 (occupational wages)

Geometric average earnings in occupation i by cohort c at date t of group g:

$$\overline{\text{wage}}_{ig}(c,t) \equiv (1-\tau^w_{ig}(t))w_i(t)e^{\mathbb{E}[\log(h_{ig}(c,t)\epsilon_{ig})]}$$

$$= \bar{\Gamma}\bar{\eta}\left[p_{ig}(c)^{\delta} m_g(c)\right]^{\frac{1}{\theta(1-\eta)}} \tilde{z}_{ig}(c)^{-\frac{1}{1-\eta}} [1-s_i(c)]^{-\frac{1}{3\beta}}$$
$$\times \frac{1-\tau^w_{ig}(t)}{1-\tau^w_{ig}(c)} \cdot \frac{w_i(t)}{w_i(c)} \cdot \frac{\gamma(t-c)}{\bar{\gamma}} \cdot \frac{s_i(c)^{\phi_i(t)}}{s_i(c)^{\phi_i(c)}}$$

where $m_g(c) \equiv \sum_{i=1}^{M} \tilde{w}_{ig}(c)^{\theta}$ and $\bar{\eta} \equiv \eta^{\eta/(1-\eta)}$.

Wages higher in occs where

- Schooling is esp. productive (high ϕ and hence high s)
- Workers have strong common disutility from being in the occ $\tilde{z}_{ig}$ is small

Key 'insight': wage gaps and selection

Key result for young workers ($t = c$) when $\delta = 0$: Average earnings diffs across occs only due to—

1. Differences in s_i (schooling productivity)
2. Differences in $\tilde{z}_{ig}$ (occupational preferences)

Paradoxically, wages are <u>not</u> higher where:

- Less labor market discrimination
- Lower human capital frictions
- Higher wage per efficiency unit w_i
- Higher group talent $\bar{h}_{ig}$

Why? *Composition effect exactly offsets direct effect* when talent is Fréchet.

- This is entirely functional form dependent – not a general result
- "Roy-Fréchet model" — Roy like, but not quite due to this knife-edge property

$\Rightarrow$ **General point stands:** Wage gap is *not* a robust measure of frictions/discrimination

Almost no cross-sectional relationship between occupational wage gaps and occupational under-representation: White women vs. white men in 1980

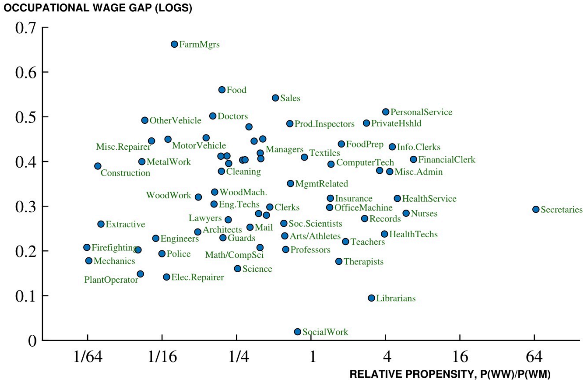

FIGURE 8.—Wage gaps versus propensities across occupations for white women in 1980. Note: The figure shows the relationship between the (log) occupational earnings gap for white women compared to white men (both in the young cohort) and the relative propensity to work in the occupation for the two groups, $p_{i,\mathrm{ww}}/p_{i,\mathrm{wm}}$ in 1980. A simple regression line through the scatter plot in the top panel yields a slope coefficient of 0.01 with a standard error of 0.01.

Almost no longitudinal relationship between Δ occ wage gaps and Δ occ representation: White women vs. white men in 1980

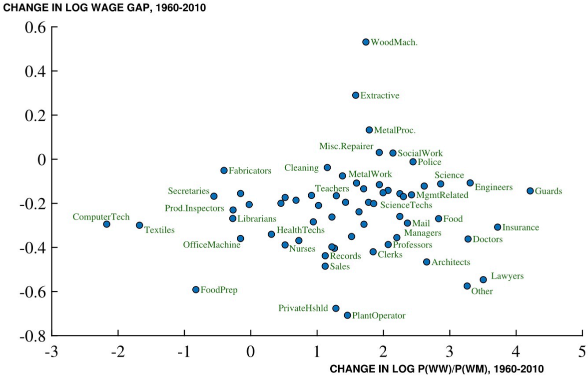

FIGURE 9.—Changes in wage gaps versus propensity gaps, young white women 1960–2010. Note: The figure shows the relationship between the change in (log) occupational earnings gaps for white women compared to white men (both in the young cohort) and the change in the relative propensity to work in the occupation for the two groups, $p_{i,\text{ww}}/p_{i,\text{wm}}$, between 1960 and 2010. A simple regression line through the scatter plot in the top panel yields a slope coefficient of 0.06 with a standard error of 0.05.

Relative occupational propensities (shares)

Proposition 4 (relative propensities)

The fraction of group g in occupation i relative to white men:

$$\frac{p_{ig}(c)}{p_{i,wm}(c)} = \left(\frac{\tau_{ig}(c)}{\tau_{i,wm}(c)}\right)^{-\frac{\theta}{1-\delta}} \left(\frac{\bar{h}_{ig}}{\bar{h}_{i,wm}}\right)^{\frac{\theta}{1-\delta}} \left(\frac{\overline{\text{wage}}_{ig}(c,c)}{\overline{\text{wage}}_{i,wm}(c,c)}\right)^{-\frac{\theta(1-\eta)}{1-\delta}}$$

Key result for identification:

- Relative propensity depends on: relative frictions, relative talent, wage gap
- Wage gap controls for effect of preferences on occupational choice
- "Residual" occupational choice driven only by τ
- Preferences $\tilde{z}_{ig}$ don't appear directly — they influence wage gaps
- Misallocation depend on dispersion of occupational wedges τ
- If wedges are same in every occ i, wedges affect LFP only

Relative labor force participation

Proposition 5 (relative labor force participation)

Let $\text{LFP}_g \equiv 1 - p_{\text{home},g}$. The share in home sector relative to white men:

$$\frac{1-\text{LFP}_g(c)}{1-\text{LFP}_{wm}(c)} = \frac{m_{wm}(c)}{m_g(c)}$$

where

$$\frac{m_{wm}(c)}{m_g(c)} = \left(\frac{\overline{\text{wage}}_{ig}(c,c)}{\overline{\text{wage}}_{i,wm}(c,c)}\right)^{-\theta(1-\eta)} \left(\frac{\tilde{z}_{ig}(c)}{\tilde{z}_{i,wm}(c)}\right)^{-\theta} \left(\frac{p_{ig}(c)}{p_{i,wm}(c)}\right)^{\delta} \quad \forall i \in \text{market}$$

Normalization: $\tilde{z} = 1$, $\tau^w = 0$, $\tau^h = 0$ for home sector.

Implications:

- Participation in market (relative to home) is affected by pay wedges, human capital wedges, and taste diffs
- The first two wedges (pay, human capital) can be conditioned out with wages
- LFP gap can identify $\tilde{z}$ in market sectors after conditioning on wages

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Aggregate production function

Final output from workers in M occupations:

$$Y = \left[\sum_{i=1}^{M} (A_i \cdot H_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where:

- $H_i \equiv \sum_g \sum_c q_g(c) p_{ig}(c) \mathbb{E}[h_{ig}(c) \epsilon_{ig}(c)]$: total efficiency units in occupation i
- A_i: exogenous productivity of occupation i
- σ: elasticity of substitution across occupations

This is a standard CES aggregator

Baseline: $\sigma = 3$

Note: when $\sigma \to \infty$ (perfect substitutes), $w_i = A_i$ is pinned down by technology.

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Simplified model for intuition

1. Two groups: men (undistorted) and women
2. Perfect substitutes $(\sigma \to \infty) \Rightarrow w_i = A_i$
3. No schooling $(\phi_i = 0)$, $\bar{h}_i = 1$, one-period lives
4. Selection on ability only $(\delta = 0)$

- Aggregate output is equal to sum of wages paid to men and women (gross of discrimination)

$$Y = q_m \cdot \bar{W}_m + q_f \cdot \frac{\bar{W}_f}{1 - \bar{\tau}^w},$$

- Q: What is $1 - \tau^w$ doing in the denominator? A: Need to inflate women's wage-bill to get real output—since they are paid less than MRPL
- No distortion of men's allocation

$$\bar{W} = \left(\sum_i^N A_i^\theta \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}}$$

- For women, $\bar{\tau}^w \equiv \sum_{i=1}^N \frac{p_{iw} \tau_i^w}{1 - \tau_i^w}$ is the earnings-weighted average of the discrimination wedge

$$\bar{W}_f = \left(\sum_i^N \frac{A_i^\theta \left(1 - \tau_i^w\right) \tilde{z}_i}{\left(1 + \tau_i^h\right) \eta} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}}$$

Simplified model for intuition

1. Two groups: men (undistorted) and women
2. Perfect substitutes ($\sigma \to \infty$) $\Rightarrow w_i = A_i$
3. No schooling ($\phi_i = 0$), $\bar{h}_i = 1$, one-period lives
4. Selection on ability only ($\delta = 0$)

Aggregate output:

$$Y = \underbrace{q_m \cdot \left(\sum_{i=1}^{M} A_i^{\theta} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}}}_{\text{output from men}} + \underbrace{\frac{q_w}{1 - \bar{\tau}^w} \cdot \left(\sum_{i=1}^{M} \left(\frac{A_i (1 - \tau_i^w) \tilde{z}_i}{(1 + \tau_i^h)^{\eta}} \right)^{\theta} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}}}_{\text{output from women}}$$

- Where $\bar{\tau}^w$ is earnings-weighted average labor friction and q_m, q_f are the quantities of men and women

Two channels for τ^w effects

With log-normal distributions, output from women:

$$\ln Y_w = \ln q_w + \ln \left(\sum_{i=1}^{M} A_i^{\theta} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}} + \underbrace{\frac{\eta}{1-\eta} \cdot \ln(1-\bar{\tau}^w)}_{\text{mean effect}} - \underbrace{\frac{1}{2} \cdot \frac{\theta - 1}{1-\eta} \cdot \text{Var}[\ln(1-\tau_i^w)]}_{\text{dispersion effect}}$$

Channel 1: mean of τ^w

- Changes return to human capital investment
- Magnitude depends on η (HC elasticity)
- This is an investment distortion

Channel 2: dispersion of τ^w

- Affects allocation of female labor across occupations
- Reduced dispersion $\Rightarrow$ better allocation $\Rightarrow$ higher output
- This is an allocation distortion
- If dispersion of τ^w were 0, but $\tau^w > 0$, and even if human capital were not important, there would still be a distortion.
- *Q: What would this distortion be?*
- *A: Reduced female LFP*

Why inference's about misallocation are not overly sensitive to θ, the (inverse of the) variance of talent draws

Intuition from simplified model:

- Effect of unequal barriers seems increasing in θ.
- But inference about variance of τ^w from data *also* depends on θ:

$$\mathrm{Var}[\ln(1-\tau^w)] = \frac{1}{\theta^2} \cdot \mathrm{Var}\left[\ln \frac{p_{ig}}{p_{i,wm}}\right]$$

Net effect: elasticity of Y_w w.r.t. variance of observed propensities:

$$\frac{1}{2} \cdot \frac{\theta - 1}{(1-\eta)\theta^2}$$

Higher θ: larger effect of misallocation, but *smaller* implied misallocation given data.

$\Rightarrow$ Results moderately robust to θ when recalibrating to match data.

Selection on preferences ($\delta = 1$)

When individuals sort only on preferences:

$$\ln Y_w = \ln q_w + \ln \frac{\sum_{i=1}^{M} A_i^{\theta+1}}{\sum_{j=1}^{M} A_j^{\theta}} + \theta \cdot \text{Cov}[\ln A_i, \ln(1 - \tau_i^w)]$$

Key difference:

- Output increases in *covariance* of A_i and $1 - \tau_i^w$
- Output falls when groups are under-represented in high-A_i occupations
- This simply says that if barriers are higher in higher productivity occupations, that's bad
- Even though workers not selecting on individual ability (constant when $\delta = 1$), they still select on average occupational wages
- Dispersion of τ alone doesn't affect output (with log-normal)

Empirical relevance:

- Women and black men moving into high-A_i (high wage) occupations over time
- This generates growth even without selection on ability

Agenda

1. Discrimination through the lens of the Roy model
2. A model of discrimination and misallocation
3. Intuition: Simplified model
4. **Data and inference**
5. Main results
6. Conclusion

Data sources

Primary data:

- 1960, 1970, ..., 2000 decennial Censuses
- 2010–2012 American Community Surveys (pooled)

Sample restrictions:

- Four groups: white men, white women, black men, black women
- Ages 25–54 (post-schooling, pre-retirement)
- Exclude unemployed and active military

Synthetic cohorts:

- Young (25–34), Middle (35–44), Old (45–54)
- Example: Young in 1960 $\rightarrow$ Middle in 1970 $\rightarrow$ Old in 1980
- 8 cohorts observed; 4 at all three life-cycle points

Occupations: 66 market occupations $+$ home sector $=$ 67 total

Standard deviation of relative occupational shares has fallen over time

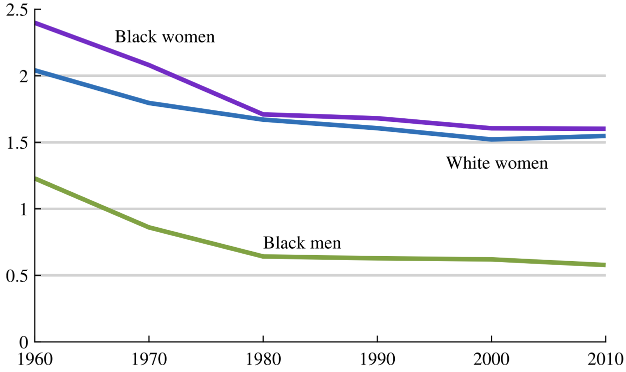

FIGURE 1.—Standard deviation of relative occupational shares. Note: The figure shows the standard deviation of $\ln(\frac{p_{ig}}{p_{\text{wm}}})$ across occupations for each group weighting each occupation by the share of earnings in that occupation. Specifically, we show the data for young white women (middle line), young black men (bottom line), and young black women (top line) relative to young white men.

Parameter estimation: θ and η

Estimating $\theta(1-\eta)$:

- Wages within occupation-group follow Fréchet with shape $\theta(1-\eta)$
- MLE on residuals from: $\log w$ = occupation $\times$ group $\times$ age dummies
- Result: $\theta(1-\eta) \approx 1.36$ (average across years)

Estimating η:

- η = elasticity of HC w.r.t. education spending = share of output on HC
- Education spending / GDP $\approx 6.6\%$; Labor share $\approx 64.1\%$
- $\eta = 0.066/0.641 = 0.103$
- $\theta(1-\eta) = 1.36 \rightarrow \theta(1-0.103) = 1.36 \rightarrow \theta = 1.52$

Alternative estimate of θ from labor supply elasticity:

- Extensive margin elasticity $= \theta(1 - \text{LFP}_g)$
- Meta-analysis: elasticity ≈ 0.26 for men; LFP $\approx 90\%$
- Implies $\theta \approx 2.57$

Baseline: $\theta = 2$, $\eta = 0.103$ (compromise)

Identification strategy: overview

Parameter	Definition	Empirical target
$A_i(t)$	Technology by occupation	Occupations of young white men
$\phi_i(t)$	Time elasticity of HC	Education by occ., young WM
$\tau^h_{ig}(c)$	Human capital barriers	Occupations of young, by group
$\tau^w_{ig}(t)$	Labor market barriers	Life-cycle wage growth, by group
$\tilde{z}_{ig}(c)$	Occupational preferences	Wages by occ. for young
$\gamma(1), \gamma(2)$	Experience terms	Age-earnings profile of WM

Key normalizations:

- $\tau^w_{i,wm} = \tau^h_{i,wm} = 0$ (white men undistorted)
- $\bar{h}_{ig} / \bar{h}_{i,wm} = 1$ (equal innate talent)
- $\tilde{z}_{\text{home},g} = 1$, $\tau^w_{\text{home}} = \tau^h_{\text{home}} = 0$ (home undistorted)

Inference: composite frictions vs. preferences

From Proposition 1: differences in occupational choice driven by $\tilde{z}/\tau$ ratio.

From Proposition 3 ($\delta = 0$, young):

- Wage gaps across occupations $\propto \tilde{z}^{-\frac{1}{1-\eta}}$
- If women poorly compensated as lawyers vs. secretaries $\Rightarrow$ higher utility from secretaries

$\Rightarrow$ **Infer $\tilde{z}$ from occupational wage gaps**

From Proposition 4:

$$\frac{p_{ig}(c)}{p_{i,wm}(c)} = \tau_{ig}(c)^{-\theta} \cdot \left(\frac{\overline{\text{wage}}_{ig}(c,c)}{\overline{\text{wage}}_{i,wm}(c,c)} \right)^{-\theta(1-\eta)}$$

$\Rightarrow$ **Infer τ from occupational shares, conditioning on wage gap**

Estimated mean composite occupational frictions

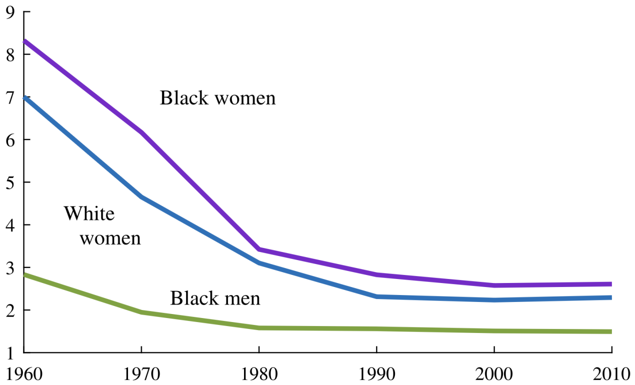

FIGURE 2.—Mean of composite occupational frictions. Note: Figure shows earnings-weighted mean of τ for each group.

Variance of occupational frictions, occupational preferences

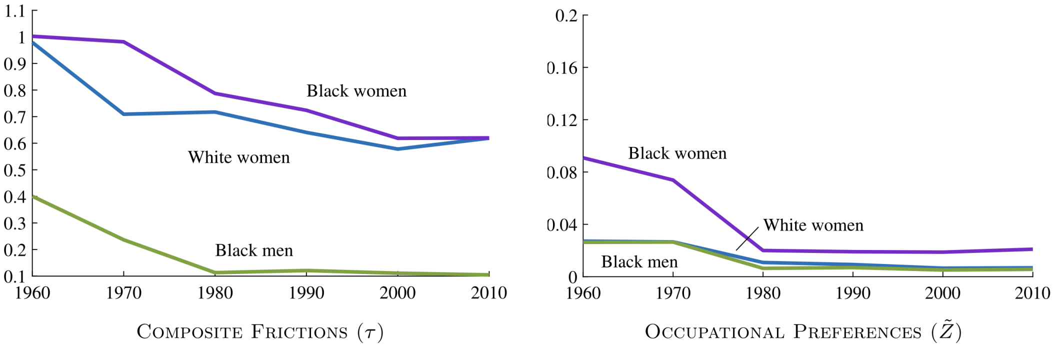

FIGURE 3.—Variance of composite frictions and occupational preferences. Note: Figure shows the earnings-weighted variance of $\ln \tau$ (left panel) and $\ln \tilde{z}$ (right panel).

Estimated barriers to white women, 1960–2010

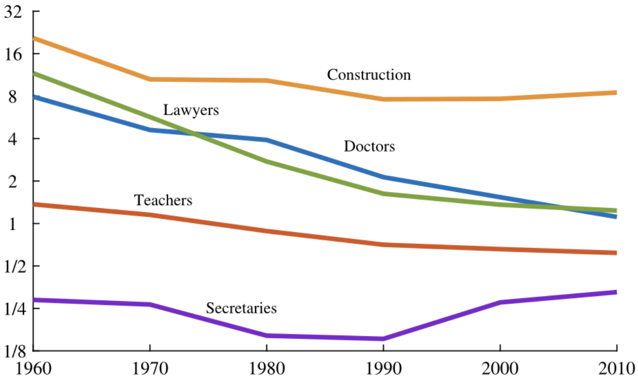

FIGURE 4.—Occupational barriers (τ_{ig}) for white women. Note: Author's calculations based on equation (7) using Census data and imposing $\theta = 2$ and $\eta = 0.103$.

Separating τ^h from τ^w: life-cycle variation

Key assumption:

- Human capital decisions made *before* entering labor market
- τ^h affects cohort throughout life $\Rightarrow$ cohort effect
- τ^w affects all cohorts at a given time $\Rightarrow$ time effect

From Proposition 3:

$$\frac{\text{gap}_{ig}(c,t)}{\text{gap}_{ig}(c,c)} \propto \frac{1-\tau^w_{ig}(t)}{1-\tau^w_{ig}(c)}$$

Identification:

- Change in wage gap over cohort's life-cycle $\Rightarrow$ change in τ^w over time
- Intercept shifts between cohorts (controlling for slope) $\Rightarrow$ change in τ^h

Life-cycle wage gaps: visual evidence

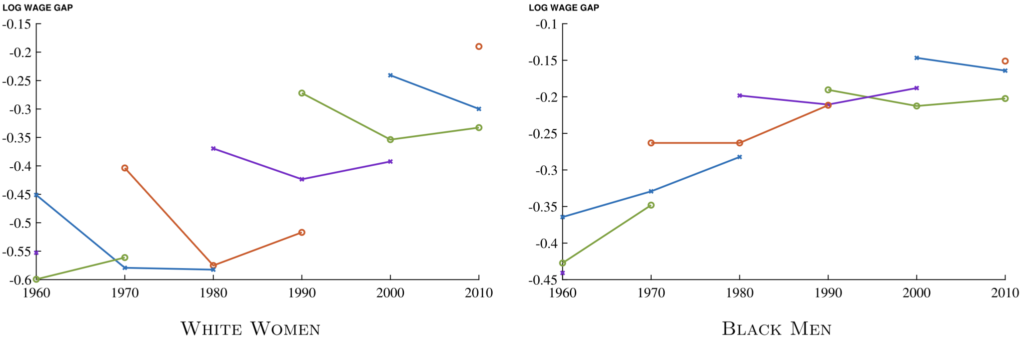

FIGURE 5.—Wage gaps relative to white men by time and cohort. Note: Log wage gaps are shown for the life-cycle of each cohort by connected line segments for young, middle-aged, and old periods.

Pattern for white women:

- Large *intercept* shifts between cohorts $\Rightarrow$ declining τ^h
- Small changes in *slopes* $\Rightarrow$ modest role for τ^w

Pattern for black men:

- Steepening slopes imply falling labor market frictions

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Estimated gains to reduced misallocation

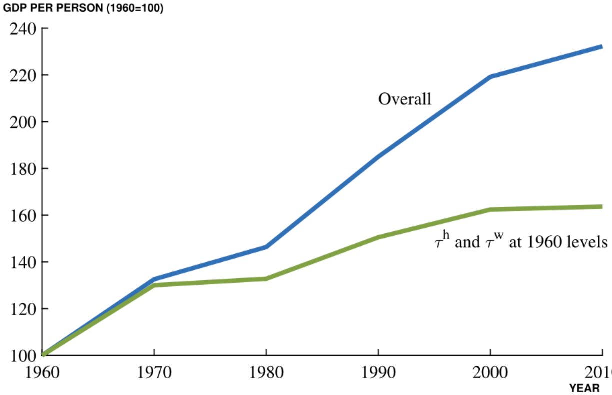

FIGURE 7.—GDP per person, data and model counterfactual. Note: The graph shows the cumulative growth in GDP per person (market), in the data (overall), and in the model with no changes in τ's as in Table V.

Main result: share of growth explained

	Share of growth accounted for by			
	τ^h and τ^w	τ^h, τ^w, $\tilde{z}$	τ^h only	τ^w only
Market GDP per person	41.5%	40.8%	36.0%	7.7%
Market earnings per person	38.4%	37.5%	18.9%	26.0%
Labor force participation	90.4%	112.7%	24.9%	56.2%
Market GDP per worker	24.0%	15.0%	40.0%	−9.8%
Home + market GDP per person	32.7%	32.1%	30.6%	4.4%

Key findings:

- Declining frictions explain ∼40% of market GDP/person growth
- Human capital barriers (τ^h) more important than labor market discrimination (τ^w)
- Preferences ($\tilde{z}$) explain little additional growth

Decomposition: τ^h vs. τ^w

Human capital barriers dominate:

- Declining τ^h: 36% of growth in market GDP/person
- Declining τ^w: 8% of growth

Why the difference?

- τ^h affects human capital investment decisions
- Effect stays with cohort throughout life
- Evidence: large intercept shifts in life-cycle wage profiles (especially for women)

Consistency with Figure 5 patterns:

- White women: mostly intercept shifts $\Rightarrow$ τ^h dominant
- Black men: both intercept shifts and slope changes $\Rightarrow$ both τ^h and τ^w matter

Why don't preferences explain growth?

Hypothesis: maybe women just didn't *like* certain occupations in 1960?

Model prediction ($\delta = 0$):

- If women disliked occupation $\Rightarrow$ would need *higher* wages to compensate
- Should see higher wages in occupations where women under-represented

Data show NO such pattern:

- Gender wage gap no lower in skilled occupations
- Gap didn't fall faster in skilled occupations as women's share rose
- Near-zero correlation between propensities and wage gaps (Figure 8)

$\Rightarrow$ Changing preferences reallocated women across occupations, but didn't generate growth because they weren't being *compensated* in disliked occupations.

Contribution by group

1960–2010	τ^h and τ^w	τ^h only	τ^w only
All groups	41.5%	36.0%	7.7%
White women	33.8%	29.8%	6.1%
Black men	1.2%	0.7%	0.6%
Black women	3.7%	3.2%	0.9%

Table: Share of growth in market GDP per person

White women drive most of the gains:

- Larger population share
- Larger initial barriers (especially in high-skilled occupations)

Remaining gains

Question: how much additional growth from eliminating remaining frictions?

Answer: if τ's reduced to zero in 2010, GDP would be **9.9% higher**.

Interpretation:

- Still some group differences in occupational choice and wages in 2010
- But most large gains already realized (1970–2000)
- Reason for pessimism about future growth from this channel

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Summary of main findings

1. **Quantitative importance:** 20–40% of U.S. GDP/person growth (1960–2010) from improved allocation of talent
2. **Human capital barriers dominate:** declining τ^h explains 36% of growth; declining τ^w explains 8%
3. **Preferences matter little:** changing $\tilde{z}$ contributes minimally to growth
4. **Conceptual insight:** occupational shares more robustly identify frictions than wage gaps (due to selection)
5. **Results robust to:** selection on preferences ($\delta > 0$), alternative θ, η, σ, and talent assumptions

Conclusions

Policy 'implications':

- Pre-market interventions (education access) may be more effective than anti-discrimination enforcement
- Most gains already realized; remaining frictions worth $\sim$10% GDP

Limitations and extensions:

- Assumed no correlation between absolute and comparative advantage
- Fréchet distribution aids tractability but limits flexibility
- No direct measure of ability, quality of match or human capital—but assumptions on these parameters drive the entire exercise

A Roy approach to discrimination's costs

1. **Transactional harms to targets of discrimination**: Harm to individuals who are discriminated against at the moment of discrimination — denied interview/job, received low wage offer (the focus of most economic literature)
2. **Pervasive harms to targets of discrimination**: Discrimination may directly affect skills, productivity, or realized performance of targets of discrimination
3. **Societal externalities**: Discrimination may cause social maladies, violence, despair, drug abuse, crime, and attendant costs (incarceration, victimization, etc.)
4. **Market misallocation**: If discrimination prevents people from realizing their comparative advantage, there's potentially a large social opportunity cost. How large?

- **There's much more to do on this agenda**