MIT 14.662 Spring 2026: Lecture Slides 7 — Skills, Tasks, and Technologies (Part 1)

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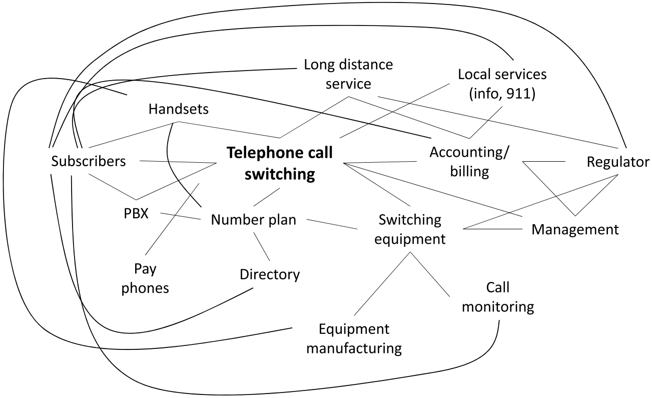
Interdependencies: Organizations, technologies, and workers

8528
Feigenbaum and Gross: *Organizational and Economic Obstacles to Automation*
Management Science, 2024, vol. 70, no. 12, pp. 8520–8540, © 2024 INFORMS

Figure 2. Integrating Automation into the AT&T Production System

(a)



(b)

Feigenbaum and Gross, *Management Science* '24

Interdependencies: Organizations, technologies, and workers

(b)

AT&T Corporate
–Develop + test equipment
–Equipment mfg. at scale
–Educate operating company managers on the tech
–Make data-driven recommendations for adoption
–Integrate w/ AT&T Long Lines, other markets

Regulators
–Telephone rate changes
–Public concerns

Central Offices
–Install equipment
–Re-wire exchange
–Integrate with manual
  –Auto-manual boards
  –Traditional operator (contingent labor)
–New approaches to:
  –Information services
  –Emergency services
  –Call monitoring
  –Caller assistance
–Personnel challenges:
  –Labor management
  –Transitional labor
  –New maintenance staff, training, processes
–New building design
–New cost accounting

User Behavior
–User acceptance of dial
–User training on dial
  –On-site training
  –Media campaigns
–Changes in organization (e.g., secretaries)
–Integration w/ PBX

User Technology
–New handsets, w/ dial
–New numbering plans
–New telephone directories
–Method for mapping alphanumeric IDs to a fully-numeric dial

Notes. (a) Example interdependencies in the AT&T system. (b) Major activities and changes required to adapt this system to mechanical switching.

Feigenbaum and Gross, *Management Science* '24

Agenda

The canonical model

Elegantly, powerfully operationalizes supply and demand for skills

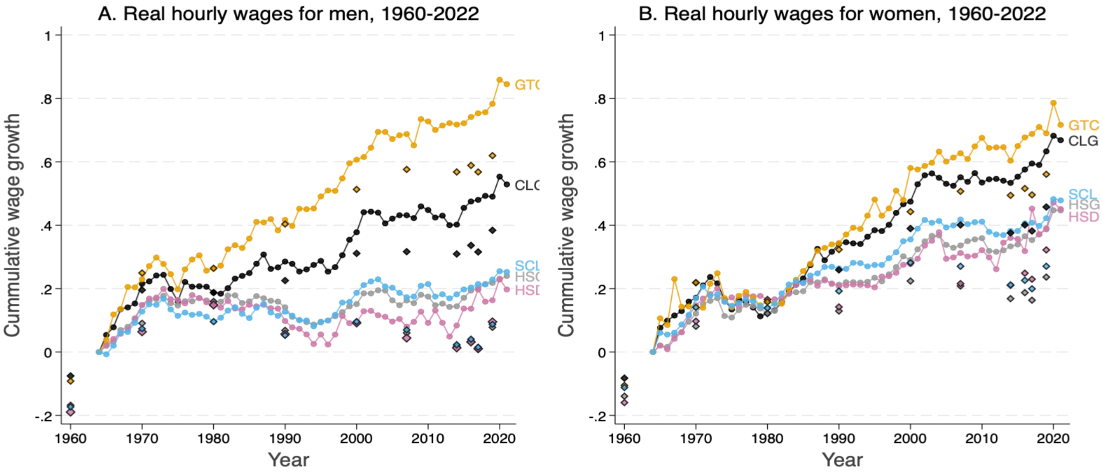
- A formalization of Tinbergen's "Education Race" analogy
- Two distinct skill groups that perform two different and imperfectly substitutable tasks

Model is a theoretical and empirical success

- Katz and Murphy '92
- Autor, Katz, Krueger '98
- Card and Lemieux '01
- Acemoglu, Autor and Lyle '04
- Goldin and Katz '08
- Carneiro and Lee '11
- Vogel, 25 (forthcoming)

But its limitations are also apparent

Declining real wages among non-college workers after 1980 – despite falling relative supply



Autor 2019, Acemoglu/Restrepo 2022, 2023

Wage inequality rises less than predicted by the canonical model

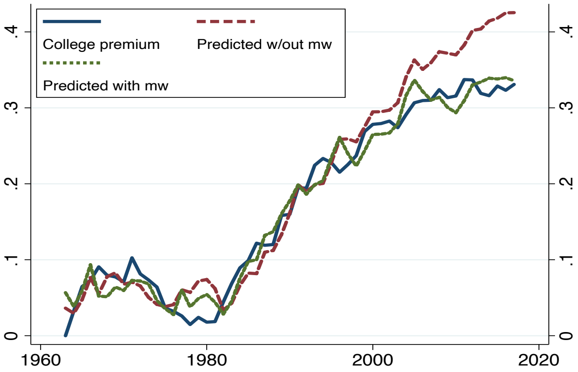
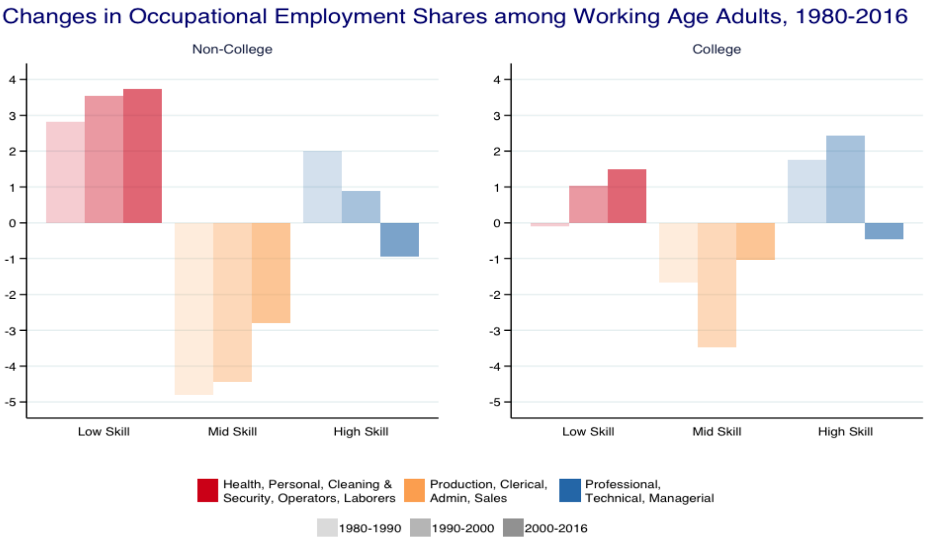


Figure A.1: Out-of-sample predictions of the (national) canonical model, estimated on the sample 1963-1987 including and not including the real minimum wage

Notes: The predicted national college premium from estimating equation (1) on the sample of years 1963-1987, both including and excluding the real minimum wage.

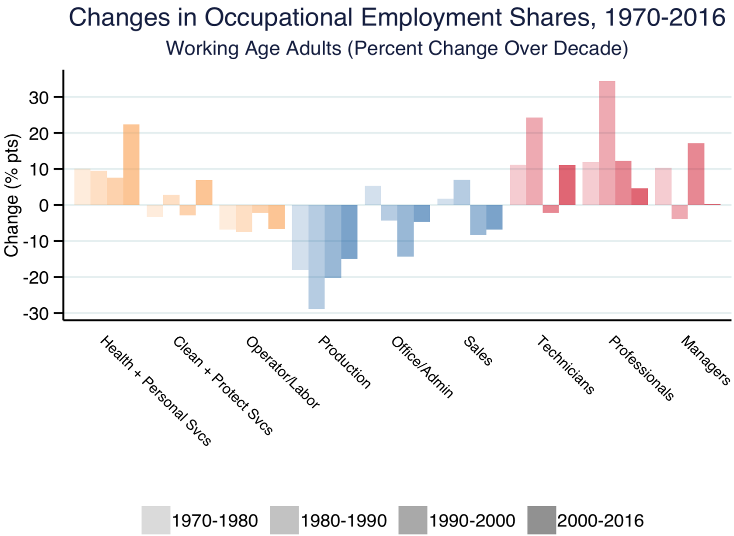
Vogel 2022

Changes in employment shares 1970 – 2016 by broad occupational category: Non-college and college workers



Autor 2019

Occupational polarization, 1970 – 2016: %Δ in employment by occupational category

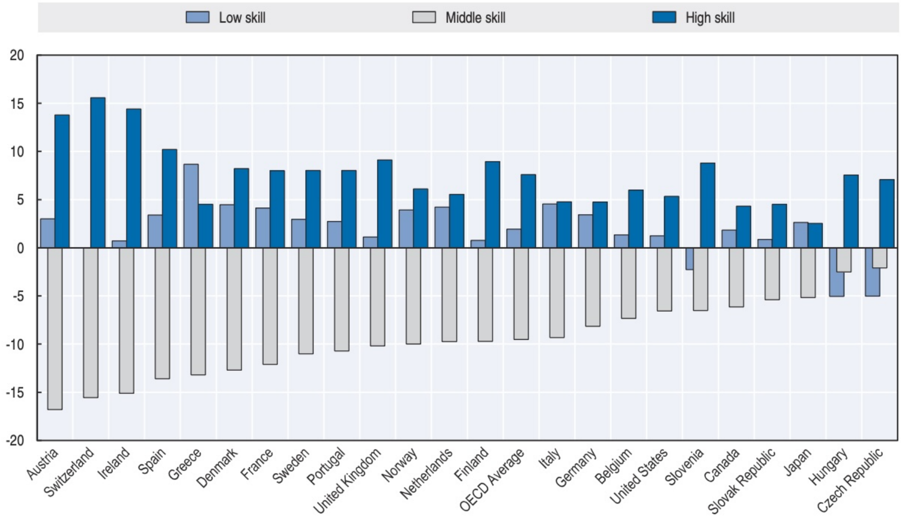


Autor 2019

Occupational polarization in 23 OECD countries, 1995-2015

Figure 3.A1.1. **Job polarisation by country**

Percentage point change in share of total employment, 1995 to 2015[a, b, c, d]



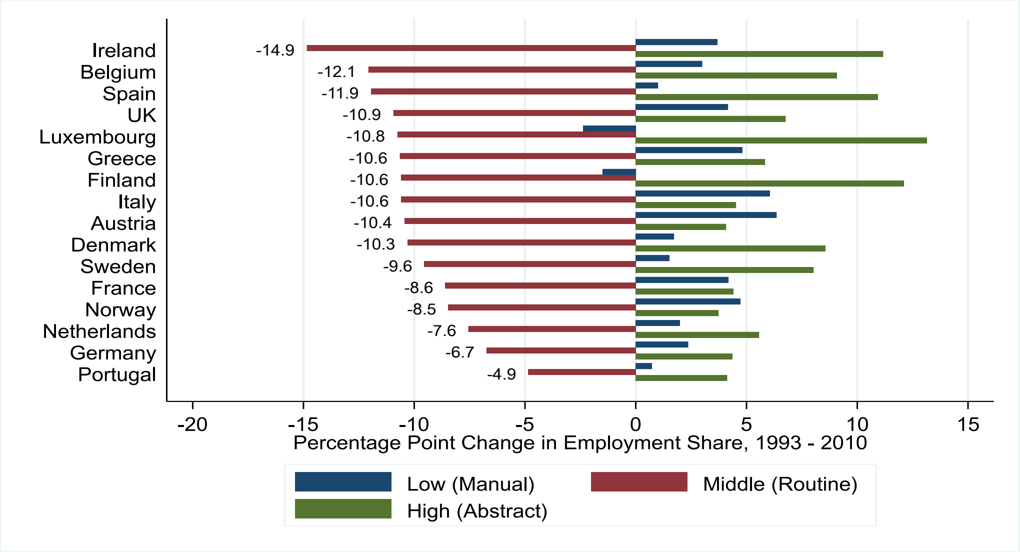
OECD 2017

Occupational polarization in 23 OECD countries: Table notes

Note: High-skill occupations include jobs classified under the ISCO-88 major groups 1, 2, and 3. That is, legislators, senior officials, and managers (group 1), professionals (group 2), and technicians and associate professionals (group 3). Middle-skill occupations include jobs classified under the ISCO-88 major groups 4, 7, and 8. That is, clerks (group 4), craft and related trades workers (group 7), and plant and machine operators and assemblers (group 8). Low-skill occupations include jobs classified under the ISCO-88 major groups 5 and 9. That is, service workers and shop and market sales workers (group 5), and elementary occupations (group 9). As agricultural, fishery and mining industries were not included in the analysis, those occupations within ISCO-88 group 6 (skill agricultural and fisheries workers) were likewise excluded. The above chart includes 15 of the 18 listed industries. The excluded industries are the following: Agriculture, hunting, forestry and fishing (1), Mining and quarrying (2), and Community, social and personal services (18). As a result of unavailable data for 1995, a different starting year was used for some countries. Norway, Slovenia, and Hungary used 1996; Finland, Sweden and the Czech Republic used 1997, while the Slovak Republic used 1998. The OECD average is a simple unweighted average of the selected OECD countries. Data for Japan over the period examined is reported under four different industry classifications and highly aggregate occupation groups.

OECD 2017

Occupational polarization in sixteen EU countries, 1993-2010



Goos, Manning and Salomons 2014

Labor's falling share of national income

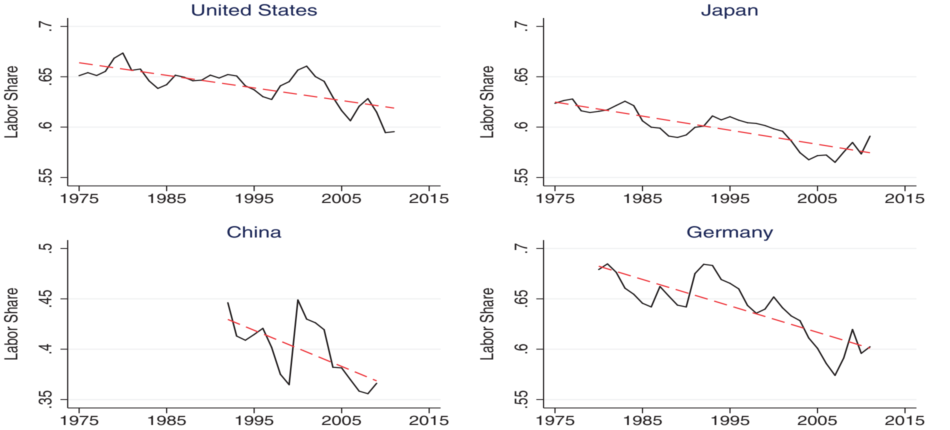


Figure II
Declining Labor Share for the Largest Countries

Karabarbounis and Neiman, 2014

Agenda

Task model: A model of skills, tasks and technologies

1. **Explicit distinction between skills and tasks**
   - Tasks—Unit of work activity that produces output
   - Skill—Worker's capability at performing various tasks
2. **Allow for comparative advantage among workers and machines in performing tasks**
   - Assignment of workers to tasks is endogenous (Roy, 1951)
3. **Allow for multiple sources of competing task 'supplies'**
   - Workers of different skill levels
   - Automation: Tasks subsumed by machines, AKA extensive margin technological Δ
   - Capital deepening: Intensive margin technological Δ
   - Trade in tasks also feasible (though won't develop that here)
4. **Fluid interplay between skills, tasks and technologies**
   - Technological advances can: displace workers from tasks; increase productivity; augment or reduce labor demand; affect labor's share of output

Task framework: Basic notions

1. **Production requires the completion of a range of tasks**
2. **Need not assume that task space is fixed/static**
   - Creation of new tasks will ultimately be important
3. **Tasks are complements**
   - Automating a subset does not make the remainder redundant
   - Extreme example: O-Ring Production Function (Kremer '93)

Task model: Tributaries

1. **Origins**
   - Dornbusch, Fischer, Samuelson (1977): Continuum framework
   - Zeira (1998): Applied DFS to technology
   - Autor, Levy, Murnane (2003): Notion of routine and nonroutine tasks
   - Linn (2011): First empirical contribution on new work (new tasks)
   - Acemoglu and Autor (2011): Combines, ALM, and Acemoglu '98 directed tech Δ
   - Acemoglu and Restrepo (2018): Adds new tasks
   - Autor and Thompson (2025): Adds 'expertise'
2. **Definitive work on task displacement**
   - Acemoglu and Restrepo 2022 ECTA
   - Acemoglu and Restrepo forthcoming QJE
3. **Work on new task creation**
   - Acemoglu and Restrepo 2018 AER
   - Autor, Chin, Salomons, Seegmiller 2024 QJE and (forthcoming ARE)
4. **Recent work on Expertise framework**
   - Autor and Thompson 2025 JEEA
   - Hosseini and Lichtinger 2026 WP

Agenda for this and next lecture

1. **A simple task model**
   - Acemoglu and Restrepo (2018), "Artificial intelligence, automation, and work"
   - Automation vs. new task creation $\rightarrow$ productivity, labor share
   - This model has tasks and technologies but *not* skills
   - Hence no Roy selection – an important missing ingredient but still a useful starting point
2. **A full-blown model with worker heterogeneity**
   - Acemoglu-Autor (2011), "Skills, Task and Technologies"
   - Rich interplay among skill supplies, task assignments, wages
   - Integrates multiple skill groups, endogenous assignment of skills to tasks (Roy model)
3. **Expertise framework**
   - Will discuss in lecture on economics of AI

Agenda

1. **Motivation: the canonical model and its limitations**
2. **Motivation: the task model**
3. **A simplified task model with new tasks**
   - Capital augmenting technical change: Intensive margin Δ
   - Labor-augmenting technological change: Intensive margin Δ
   - The displacement effect—extensive margin technological change
   - New task creation: The opposite of automation
4. **Complete model of skills, tasks, and technologies**
   - Setup + three equilibrium conditions
   - Inequality implications: comparative statics
5. **Applications: Task displacement, productivity, and real and relative wages**
   - Real wage levels and technological change
   - Labor replacing technologies and inequality
   - Relationship to canonical model

A simple model of tasks and technologies

Goal of this first model is to fix key concepts — no distractions

1. Treat capital and labor symmetrically
2. Distinguish between intensive vs. extensive margin technological change
3. Allow for both displacement of 'old' tasks and creation of 'new' tasks
4. Consider how *different forms* of technological change affect
   - Productivity
   - Wage levels
   - Distribution of income among factors (labor v capital here)
   - Those forms are: (a) labor augmenting, (b) capital augmenting, (c) labor replacing, (d) labor 'instantiating'
5. Tie this back to canonical model that you know well

Aggregate output Y

- Produced by combining the services, $y(x)$, of a unit measure of tasks $x \in [N-1, N]$:

$$\ln Y = \int_{N-1}^{N} \ln y(x) dx,$$

- Tasks run between $N-1$ and N allows for changes in *range* of tasks
- Notice that this is a Cobb-Douglas structure with identical factor shares for services of each task
- It is the limiting case of the discrete Cobb-Douglas production function

$$Y = \prod_{n=1}^{N} y(n)^{1/N} \text{ as } N \to \infty$$

Simplified task framework: Task-level production function

Tasks produced by human labor, $\ell(x)$, <u>or</u> by machines, $m(x)$

- Tasks above I are **not technologically automated** and *must* be produced by labor:

$$y(x) = \begin{cases} \gamma_L(x)\ell(x) + \gamma_M(x)m(x) & \text{if } x \in [N-1, I] \\ \gamma_L(x)\ell(x) & \text{if } x \in (I, N]. \end{cases}$$

  - $\gamma_L(x)$ =productivity of labor in task x, increasing in x
  - $\gamma_M(x)$ =productivity of machines in automated tasks
- **Comparative advantage**: $\gamma_L(x)/\gamma_M(x)$ is increasing in x
- **Fixed (inelastic) supplies**: L workers and K units of capital (machines)

Aggregate output takes the form

$$Y = \Theta \left(\frac{K}{I - N + 1} \right)^{I-N+1} \left(\frac{L}{N - I} \right)^{N-I},$$

$$\Theta = \exp \left(\int_{N-1}^{I} \ln \gamma_M(x) dx + \int_{I}^{N} \ln \gamma_L(x) dx \right)$$

1. Why is this not a standard Cobb-Douglas production function?
   - This is a pure Cobb-Douglas with **non-constant shares**
2. What's going on with Θ?
   - $\Theta =$ Solow residual: *All* technological Δ generates Hicks-neutral TFP gain $\Delta\Theta$

Simplified task framework: Restrictions on comparative advantage of labor v. capital

Foundational assumption for what follows

$$\frac{\gamma_L(N)}{\gamma_M(N-1)} > \frac{W}{R} > \frac{\gamma_L(I)}{\gamma_M(I)} \tag{1}$$

- R is the capital rental rate
- Implies that tasks below I are produced with machines/offshoring

Assumption says that new tasks (rising N) raise output

1. Wage ratio not so high that new task creation lowers output
2. Not so low so that technologically automated tasks are still performed by labor

The demand for labor is given by

$$W = (N - I)\frac{Y}{L}$$

- This expression is equal to labor share of total output, $(N - I)$, times output Y divided by number of workers L
- The share of labor in national income is given by

$$s_L = \frac{WL}{Y} = N - I$$

- Note: this identity holds for *any* constant returns to production function

$$W = \frac{s_L Y}{L} \qquad R = \frac{s_K Y}{K}$$

- No rents: factors are paid their average products, which are also equal to their marginal products

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Simplified task framework: Four forces at play

1. **Capital-augmenting technological change**
   - Increases in the productivity of tasks that are already automated/offshored.
   - Corresponds to an increase in the $\gamma_M(x)$ function for tasks $x < I$
2. **Labor-augmenting technological advances**
   - Increases in the function $\gamma_L(x)$
   - This is the canonical factor-augmenting technical change
3. **Automation – tasks reallocated from labor to capital**
   - Expansion of the set of tasks that are technologically automated or trade-substituted, I
   - Not present in conventional models
4. **Creation of new tasks**
   - An increase in N
   - Mechanism due to Acemoglu-Restrepo '18 via Jeffrey Lin '11 *ReStat* (see also Autor, Chin, Salomons, Seegmiller '24)

Mechanism 1. Capital-augmenting technological change

Machines get better at what they do

- Consider an increase in the productivity of machines by $d \ln \gamma_M(x) = d \ln \gamma_M > 0$ for $x < I$, with no change in the extensive margin of automation, I
- Wage impact is

$$\frac{d \ln W}{d \ln \gamma_M} = \frac{d \ln Y/L}{d \ln \gamma_M} = (I - N + 1) \times d \ln \gamma_M > 0$$

This is a pure capital-labor complementarity

- Improved tractors make farm workers more efficient
- Better auto-assembly robots improve quality of welds (robots have been welding for years)
- New iPad—definitely doesn't do anything the old model didn't do

Agenda

Mechanism 2. Labor augmenting technological change

An increase in labor productivity, $d \ln \gamma_L(x) > 0$, with no Δ in extensive automation margin, I

- Wage impact is

$$\frac{d \ln W}{d \ln \gamma_L} = \frac{d \ln Y/L}{d \ln \gamma_L} = (N - I) \times d \ln \gamma_L > 0$$

- This is a a pure factor-augmenting technological change, as in the Katz-Murphy/Tinbergen model
- Could come from rising education or better management practices

Invariance of factor shares to labor and capital-augmenting technical change (Cobb-Douglas)

Factor-augmentation, automation and labor share

- Factor-augmenting technical Δ does not change labor share S_L in this model. *Why not?* *Cobb-Douglas structure implies that labor share is independent of the productivity of labor and capital*

- Even if underlying production f'n were CES with $\sigma > 1$, capital-augmenting tech Δ would have *indirect* (generally small) effect on S_L, automation would have *first-order* effect

Agenda

Mechanism 3. Extensive margin tech Δ — AKA, automation

Automation or trade/offshoring (an increase in I) generates a displacement effect

- From prior equation

$$\frac{d\ln W}{dI} = \underbrace{\frac{d\ln(N-I)}{dI}}_{\substack{\text{Displacement} \\ \text{effect} < 0}} + \underbrace{\frac{d\ln(Y/L)}{dI}}_{\substack{\text{Productivity} \\ \text{effect} > 0}}$$

- The displacement effect implies that **wages—marginal product of labor—can decline**, despite the fact that output per worker rises
- Whether or not this occurs ultimately depends on the K/L ratio (see Autor and Kausik, 2025)
- **Wages necessarily grow by less than output per worker** $\rightarrow$ labor share falls

$$\frac{ds_L}{dI} = -1 < 0$$

Extensive margin technological change and capital supply

If capital supply is fixed, displacement effect on W magnified

- **With fixed supply of capital**
  - Automation at extensive margin increases the demand for capital
  - Raises the equilibrium rental rate, R
- **"Medium-run"**
  - Supply of machines expands as well (or more offshore supplies come online)
  - Capital accumulation bolsters the productivity effect by reducing the cost of machinery
- **With fully elastic supply of capital, R is effectively fixed**
  - Productivity effect necessarily dominates the displacement effect—all gains go to inelastically supplied factor
  - See "Robot arithmetic: New technologies and wages" by Caselli and Manning in *AER: Insights* 2019

Displacement also has a productivity effect

By reducing cost of a subset of tasks, automation/trade raises labor demand in remaining tasks

- Formally
$$\frac{d\ln(Y/L)}{dI} = \ln\left(\frac{W}{\gamma_L(I)}\right) - \ln\left(\frac{R}{\gamma_M(I)}\right) > 0$$
- What is this expression?
- $\ln\left[W/\gamma_L\left(I\right)\right] - \ln\left[R/\gamma_M\left(I\right)\right]$ is the **cost difference** btwn labor and capital in the marginal task I

Displacement also has a productivity effect

The overall impact on labor demand can be written as

$$\frac{d\ln W}{dI} = \underbrace{-\frac{1}{N-I}}_{\substack{\text{Displacement}\\ \text{effect} < 0}} \quad \underbrace{+\ln\left(\frac{W}{\gamma_L(I)}\right) - \ln\left(\frac{R}{\gamma_M(I)}\right)}_{\substack{\text{Productivity}\\ \text{effect} > 0}}$$

1. **Case 1: Productivity effect dominates displacement effect**: $\gamma_M(I)/R >> \gamma_L(I)/W$.
   Productivity jump big enough to overcome displacement effect

2. **Case 2: Displacement effect dominates productivity effect**: $\gamma_M(I)/R \approx \gamma_L(I)/W$.
   New technologies/trade are so-so

Aside: When is automation 'so-so'?

Whether automation is 'so-so' ultimately depends on K/L (see Autor and Kausik, 2025)

- Let $\alpha = s_K$ and $\lambda = 1 - \alpha$
- The value of α^* that maximizes wages is given by

$$\frac{1}{1-\alpha^*} + \ln\left(\frac{\alpha^*}{1-\alpha^*}\right) = \ln\frac{K}{L} + \ln\frac{\gamma_M(\alpha^*)}{\gamma_L(\alpha^*)}$$

- Wage-maximizing **capital share** is rising in K/L, and in marginal product of machines relative to labor
- Assume for simplicity that $\gamma_M(\alpha^*)/\gamma_L(\alpha^*)$ is constant:

$$\implies \frac{d\ln\alpha^*}{d\ln(K/L)} = (1-\alpha^*)^2$$

- Implication: *Labor share can be 'too low' (not enough automation to maximize wages) or 'too high' (additional automation lowers wages)*
- A variant of this expression holds for **every constant returns to production function**
- **Q**: Where did w, r go in this eq'n? **A:** Implied by the factor shares: $W = (1-\alpha)Y/L$, $R = \alpha Y/K$

Aside: When is automation 'so-so' (Autor-Kausik '25)?

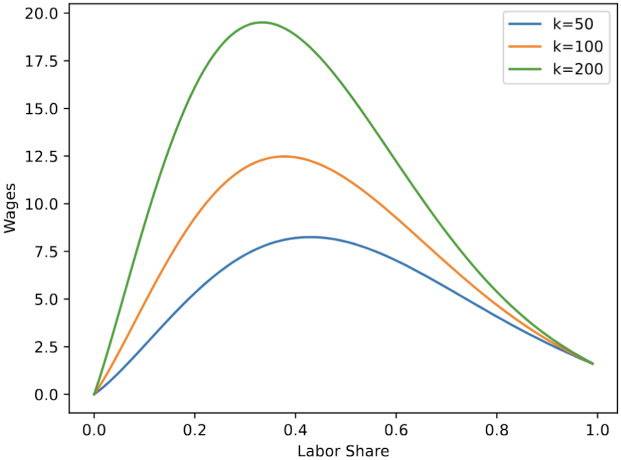


Figure 2: Wage vs Labor Share in the Cobb-Douglas Task Model.

The three curves show the relationship between the wage and the labor share for multiple values of the capital-to-labor ratio K/L in the Cobb-Douglas task model with factor productivity schedule $\gamma_M(\alpha)/\gamma_L(\alpha) = (1-\alpha)/(1+\alpha)$. Let $\lambda = 1-\alpha$ and $\lambda^* = 1-\alpha^*$, where α^* is given by equation (9). If $\lambda > \lambda^*$, greater automation (lower λ) will raise wages. Conversely, if $\lambda < \lambda^*$, reduced automation (higher λ) will raise wages.

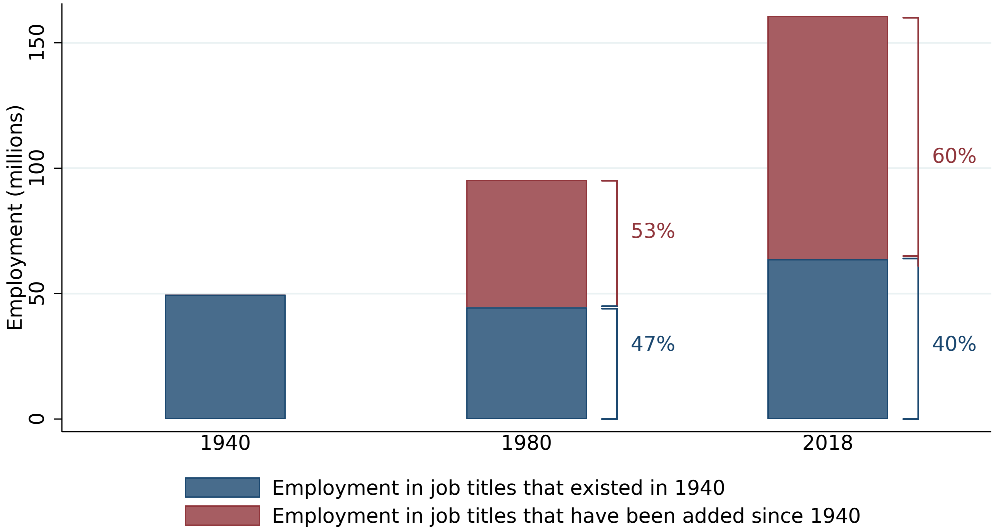
Agenda

Mechanism 4. New task creation

Creation of new, labor-using tasks maybe counterbalancing force

1. In 19th-century Britain, rapid expansion of new industries and jobs—engineers, machinists, repairmen, and managers (Landes, 1969, Chandler, 1977, and Mokyr, 1990)
2. In early 20th-century America, agricultural mechanization coincided with a large increase in employment in new industry and factory jobs (Olmstead and Rhode, 2001, Rasmussen, 1982)
3. From 1940 to 2018, new tasks and job titles explain large fraction of all employment growth (Autor, Chin, Salomons, Seegmiller, 2024)
4. In general, new tasks have in the last four decades tended to be more skill-intensive—which is both good and bad news. But this was not always so

The contribution of new work to employment growth, 1940 – 2018



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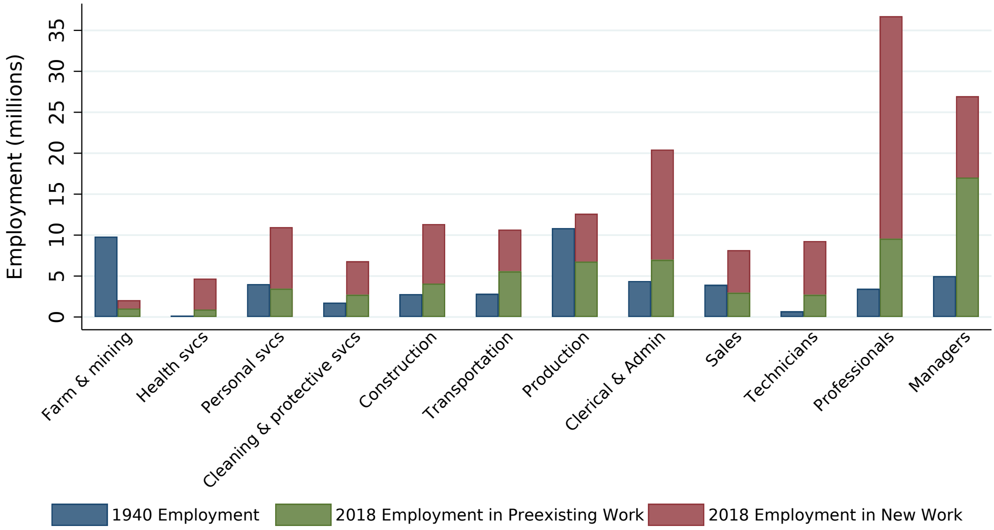
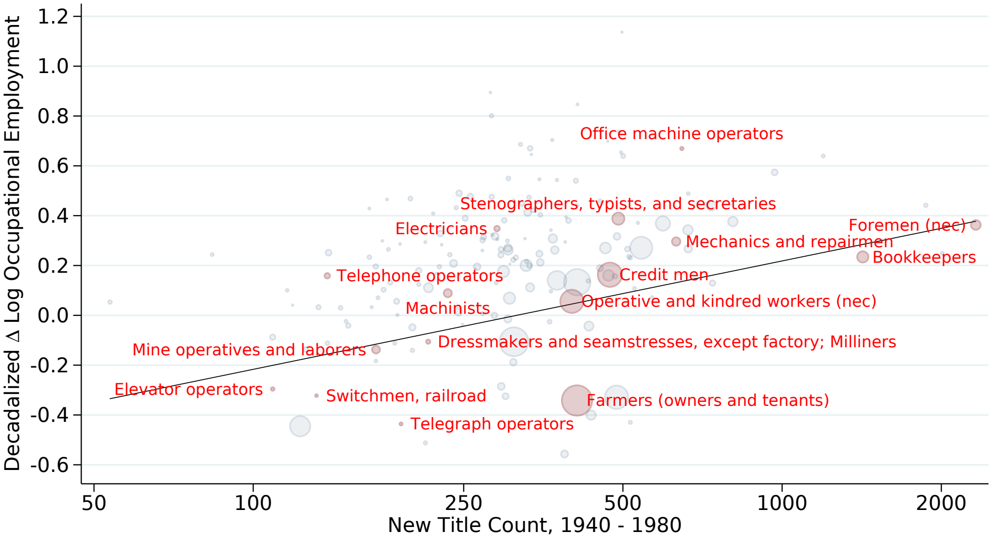
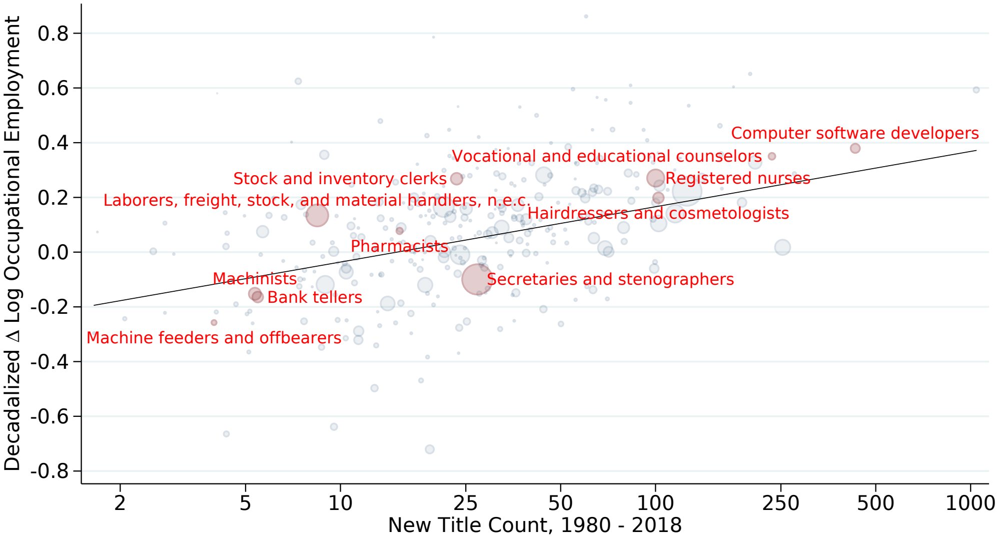


New titles flows vs. occupational employment growth, 1940 – 1980



New titles flows vs. occupational employment growth, 1980 – 2018



New tasks and the demand for labor

- **An increase in N—the creation of new tasks—raises productivity**

$$\frac{d\ln Y/L}{dN} = \ln\left(\frac{R}{\gamma_M(N-1)}\right) - \ln\left(\frac{W}{\gamma_L(N)}\right) > 0$$

  which is positive from Assumption 1

- Besides its effect on productivity, new tasks also increase labor demand and equilibrium wages by creating a *reinstatement effect*:

$$\frac{d\ln W}{dN} = \underbrace{\ln\left(\frac{R}{\gamma_M(N-1)}\right) - \ln\left(\frac{W}{\gamma_L(N)}\right)}_{\substack{\text{Productivity}\\ \text{effect} > 0}} + \underbrace{\frac{1}{N-I}}_{\substack{\text{Reinstatement}\\ \text{effect} > 0}}$$

The "race" between new tasks and automation

Creation of new tasks generates additional labor demand, raise share of labor in national income

- Total wage effect equals

$$d \ln W = \left[\ln \left(\frac{R}{\gamma_M(N-1)} \right) - \ln \left(\frac{W}{\gamma_L(N)} \right) \right] dN + \left[\ln \left(\frac{W}{\gamma_L(I)} \right) - \ln \left(\frac{R}{\gamma_M(I)} \right) \right] dI + \frac{1}{N-I}(dN - dI),$$

  and also for the labor share, we get

$$ds_L = dN - dI.$$

- Labor share stable, wages increase 1:1 w/productivity **iff** new tasks, N, appear at same rate as automation, I
- But keep in mind that labor share and wages can move in opposite directions

The endogenous evolution of new tasks

Some reasons why new tasks, N, may keep up with automation

- Rapid automation may endogenously generate incentives for firms to introduce new labor-intensive tasks (Acemoglu and Restrepo, 2018)
- Some automation technology platforms, especially AI, may eliminate old tasks faster than they generate new ones
- It is possible that we are heading to a future with a lower range of tasks done by human labor, $N - I$
  - If so, this implies a falling labor share, but not necessarily falling wages
  - Still, the political economy implications are concerning

Summary: Four forces at play

1. **Automation at the intensive margin – deepening of automation**
   - Increases in the productivity of tasks that are already automated/offshored.
   - Corresponds to an increase in the $\gamma_M(x)$ function for tasks $x < \boldsymbol{I}$
2. **Labor-augmenting technological advances**
   - Increases in the function $\gamma_L(x)$
   - This is the canonical factor-augmenting effect
3. **Automation at the extensive margin – task displacement**
   - Expansion of the set of tasks that are technologically automated or trade-substituted, $\boldsymbol{I}$
   - Not present in conventional models
4. **Creation of new tasks – labor 'instantiation'**
   - This is novel but *must be* important

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2. **Motivation: the task model**
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   - The displacement effect—extensive margin technological change
   - New task creation: The opposite of automation
4. **Complete model of skills, tasks, and technologies**
   - Setup + three equilibrium conditions
   - Inequality implications: comparative statics
5. **Applications: Task displacement, productivity, and real and relative wages**
   - Real wage levels and technological change
   - Labor replacing technologies and inequality
   - Relationship to canonical model

The full model of skills, tasks and technologies: What we're now adding

1. **Adding skill groups (L, M, H): Roy makes a triumphant return**
2. **Explicit distinction between skills and tasks**
   - Tasks—Unit of work activity that produces output
   - Skill—Worker's capability at performing various tasks
3. **Allow for comparative advantage among workers and machines in performing tasks**
   - Assignment of workers to tasks is endogenous (Roy, 1951)
4. **No restrictions on where/how automation enters**
   - Technological advances can: displace workers from tasks; increase productivity; augment or reduce labor demand; affect labor's share of output

Agenda

Tasks into goods. Same Cobb-Douglas infrastructure

- Static environment with a unique final good, Y
- Y produced with continuum of *tasks* on the unit interval, $[0, 1]$
- Cobb-Douglas technology describing the aggregation of tasks into the final good:

$$\ln Y = \int_0^1 \ln y(i) di,$$

where $y(i)$ is the "service" or production level of task i

- Cobb-Douglas structure is inessential. CES would be fine
- Price of the final good, Y, is numeraire

Three types of labor: High, Medium and Low

- Fixed, inelastic supply of the three types. Supplies are L, M and H
- Later introduce capital or technology

Each task on continuum has production function

$$y(i) = A_L \alpha_L(i)\, l(i) + A_M \alpha_M(i)\, m(i) + A_H \alpha_H(i)\, h(i) + A_K \alpha_K(i)\, k(i),$$

- A terms are factor-augmenting technologies
- $\alpha_L(i)$, $\alpha_M(i)$ and $\alpha_H(i)$ are *task productivity schedules*
- For example, $A_L \alpha_L(i)$ is the productivity of low skill workers in task i, and $l(i)$ is the number of low skill workers allocated task i

Q: How are factors combined in task level production function?

Q: All sources of supply are perfect substitutes (logical but can be relaxed)

Role of comparative advantage

- All tasks i can be performed by low, medium or high skill workers

$$y(i) = A_L \alpha_L(i)\, l(i) + A_M \alpha_M(i)\, m(i) + A_H \alpha_H(i)\, h(i) + A_K \alpha_K(i)\, k(i)$$

$$\Rightarrow \frac{\partial y(i)}{\partial l(i)} = A_L \alpha_L(i), \; \frac{\partial y(i)}{\partial m(i)} = A_M \alpha_M(i), \; \frac{\partial y(i)}{\partial h(i)} = A_H \alpha_H(i), \; \frac{\partial y(i)}{\partial k(i)} = A_K \alpha_K(i)$$

But comparative advantage differs among skill groups $\{\alpha_L(i), \alpha_M(i), \alpha_H(i)\}$

- **Assumption:** $\alpha_L(i)/\alpha_M(i)$ *and* $\alpha_M(i)/\alpha_H(i)$ *are continuously differentiable and strictly decreasing*
  - Comparative advantage of H relative to M, and M relative to L, monotonically rising in task index
  - Higher indices correspond to "more complex" tasks
  - No assumption made about *absolute* advantage (i.e., the level of $\alpha_J(i)$ vs. $\alpha_K(i)$)
- **Key observation:** Which skill group performs which tasks in equilibrium depends on both relative productivity $(A_J \alpha_J(i) \gtrless A_K \alpha_K(i))$ and relative wages $(W_J \gtrless W_K)$

I_L, I_H are equilibrium task thresholds: In any equilibrium there exist I_L and I_H such that

- $0 < I_L < I_H < 1$ and for any $i < I_L$, $m(i) = h(i) = 0$
- For any $i \in (I_L, I_H)$, $l(i) = h(i) = 0$
- For any $i > I_H$, $l(i) = m(i) = 0$

Allocation of tasks to skill groups determined by I_H, I_L

- Tasks $i > I_H$ will be performed by high skill workers (Abstract)
- Tasks $i < I_L$ will be performed by low skill workers (Manual)
- Middle tasks $I_L \leq i \leq I_H$ will be performed by medium skill workers (Routine)

Boundaries of these sets are endogenous

- Given skill supplies, firms (equivalently workers) decide which skills perform which tasks $\rightarrow$ *Substitution of skills across tasks*

Solution concept: Three equilibrium conditions

1. **Law of one price for skills**
2. **Equal division of labor among tasks within a skill group**
3. **No arbitrage across skill groups**

1. Law of one price for skills

- Let $p(i)$ denote the price of services of task i.
- In equilibrium, all tasks employing L workers must pay them the same wage, W_L, and same for M and H

$$W_L = A_L p(i) \alpha_L(i) \text{ for any } i < I_L.$$

$$W_M = A_M p(i) \alpha_M(i) \text{ for any } I_L < i < I_H.$$

$$W_H = A_H p(i) \alpha_H(i) \text{ for any } i > I_H.$$

where $p(i)$ is the equilibrium price of task i services (from upper-level production function)

- *Why?* Workers are identical within skill groups, can costlessly move across tasks within their skill group

1. **Law of one price for skills**

- In equilibrium all tasks employing L workers must pay them the same wage, W_L, and similarly for H, L:

$$W_L = A_L p(i) \alpha_L(i) \text{ for any } i < I_L.$$

where $p(i)$ is the equilibrium price of task i services (from upper-level production function)

- Observe that wage paid a worker doing a task assigned to L will be proportional to W_L
- **This has an important implication**
  - $p(i)\alpha_L(i) = p(i')\alpha_L(i') \equiv P_L$ for any $i, i' < I_L$
  - $p(i)\alpha_M(i) = p(i')\alpha_M(i') \equiv P_M$ for any $I_H > i, i' > I_L$
  - $p(i)\alpha_H(i) = p(i')\alpha_H(i') \equiv P_H$ for any $i, i' > I_H$

2. Equal division of labor among tasks within a skill group

- The Cobb-Douglas technology implies

$$p(i)y(i) = p(i')y(i')$$

- Substituting $y(i) = A_L \alpha_L(i) l(i)$ for any $i < I_L$:

$$p(i) A_L \alpha_L(i) l(i) = p(i') A_L \alpha_L(i') l(i')$$

- From law of one price (as shown above): $p(i)\alpha_L(i) = p(i')\alpha_L(i')$ for all $i, i' < I_L$
- These terms cancel, leaving

$$\Rightarrow l(i) = l(i') \text{ for any } i, i' < I_L$$

$\Longrightarrow$ All tasks performed exclusively by workers of a skill group must use identical amounts of labor

Three equilibrium conditions: Equal division of labor

2. Equal division of labor among tasks within a skill group

$$l(i) = l(i')$$

- Implication

$$l(i) = \frac{L}{I_L} \text{ for any } i < I_L,$$
$$m(i) = \frac{M}{I_H - I_L} \text{ for any } I_H > i > I_L,$$
$$h(i) = \frac{H}{1 - I_H} \text{ for any } i > I_H.$$

- Any two tasks performed exclusively by workers of a skill group must use identical amounts of labor
- That amount is group's labor supply divided by the fraction of tasks performed by group
- Otherwise, workers of the same skill level would earn different wages in different tasks

3. No arbitrage between tasks

- Recall that $P_J \equiv p(i)\alpha_J(i)$ is constant within each skill group (law of one price)
- Wages equal marginal products:

$$W_L = P_L A_L = A_L p\,(i)\,\alpha_L\,(i) \text{ for } i < I_L$$

$$W_M = P_M A_M = A_M p\,(i)\,\alpha_M\,(i) \text{ for } I_L < i < I_H$$

$$W_H = P_H A_H = A_H p\,(i)\,\alpha_H\,(i) \text{ for } i > I_H$$

Three equilibrium conditions: No arbitrage between skill groups

3. No arbitrage across skill groups

- Since each task earns equal revenue (Cobb-Douglas), the threshold task I_H must yield the same output whether performed by H or M workers (and similarly for I_L):

$$A_H \alpha_H (I_H) \times \frac{H}{(1 - I_H)} = A_M \alpha_M (I_H) \times \frac{M}{(I_H - I_L)}$$

and

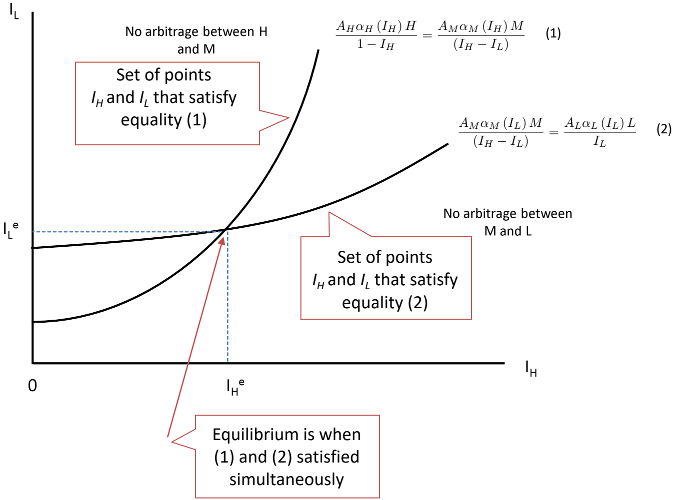
$$A_M \alpha_M (I_L) \times \frac{M}{(I_H - I_L)} = A_L \alpha_L (I_L) \times \frac{L}{I_L}$$

- Since $P_L A_L l(i) = P_L A_L l(i')$ for all $i < I_L$, and similarly for M and H, we have:

$$P_H A_H \times \frac{H}{(1 - I_H)} = P_M A_M \times \frac{M}{(I_H - I_L)}$$

$$P_M A_M \times \frac{M}{(I_H - I_L)} = P_L A_L \times \frac{L}{(I_L)}$$

No arbitrage across skill groups: Curves depict no arbitrage conditions between H v. M and M v. L as thresholds I_H and I_L are adjusted (note: curves not independent)



Relative supply & demand for skills across tasks: Supply curves static, demand curves not independent

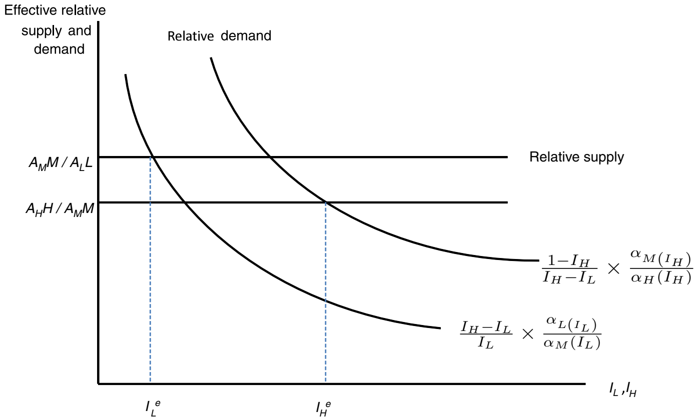


Figure 23 ***Equilibrium allocation of skills to tasks.***

Three equilibrium conditions

3. No arbitrage across skill groups

$$P_H A_H H / (1 - I_H) = P_M A_M M / (I_H - I_L)$$
$$P_M A_M M / (I_H - I_L) = P_L A_L L / (I_L)$$

- **Substituting the no arbitrage conditions into law of one price conditions, yields eq'm wage ratios:**

$$W_H = P_H A_H,\ W_M = P_M A_M,\ W_L = P_L A_L$$

$$W_H H / (1 - I_H) = W_M M / (I_H - I_L)$$
$$W_M M / (I_H - I_L) = W_L L / (I_L)$$

$$\Rightarrow \frac{W_H}{W_M} = \left(\frac{1 - I_H}{I_H - I_L}\right)\frac{M}{H},\ \frac{W_M}{W_L} = \left(\frac{I_H - I_L}{I_L}\right)\frac{L}{M},\ \frac{W_H}{W_L} = \left(\frac{1 - I_H}{I_L}\right)\frac{L}{H}$$

Full task model: Equlibrium for task assignments, division of labor, and wage ratios

- These three conditions [law of one price, equal shares, no arbitrage] imply that relative wages are solely a function of labor supplies and task thresholds

$$W_J = W_J\left[I_H, I_L | H, M, L, A_H, A_M, A_L, \alpha_H(\cdot), \alpha_M(\cdot), \alpha_L(\cdot)\right] \text{ for } J \in [H, M, L]:$$

$$\begin{aligned} \frac{W_H}{W_M} &= \left(\frac{1-I_H}{I_H - I_L}\right)\left(\frac{H}{M}\right)^{-1}, \\ \frac{W_M}{W_L} &= \left(\frac{I_H - I_L}{I_L}\right)\left(\frac{M}{L}\right)^{-1} \end{aligned}$$

- So, labor supplies L, M, H plus compare adv. $\alpha(L)$, $\alpha(M)$, $\alpha(H)$ determine task allocation, I_L and I_H, and hence wages
- *It's that simple!*

Equations for wage levels makes this clearer

- Output is

$$Y = B \times \underbrace{L^{I_L} M^{I_H - I_L} H^{1-I_H}}_{\text{Cobb-Douglas labor aggregate}}$$

where

$$B = \underbrace{\exp\left(\int_0^{I_L} \ln A_L \alpha_L(i)\, di + \int_{I_L}^{I_H} \ln A_M \alpha_M(i)\, di + \int_{I_H}^{1} \ln A_H \alpha_H(i)\, di\right)}_{\text{TFP (Solow residual)}}$$

- Wages equal

$$\begin{aligned} W_L = \frac{\partial Y}{\partial L} &= B \times I_L \times L^{I_L - 1} M^{I_H - I_L} H^{1-I_H} \\ &= \frac{I_L}{L} \times \left(B L^{I_L} M^{I_H - I_L} H^{1-I_H}\right) \\ &= I_L \times \frac{Y}{L} \end{aligned}$$

and similarly, $W_M = Y\left(I_H - I_L\right)/M$ and $W_H = Y\left(1 - I_H\right)/H$.

Agenda

1. **A rise in A_H (SBTC)**
2. **A rise in high-skilled labor supply H**
3. **Analogous comparative statics for rise in A_L or L**
4. **What about the effect of a rise in A_M or M on W_H/W_L?**

The effect of $\Delta's$ in technology and skill supplies on the division of labor

- **How do thresholds change w/increase in supply of H labor or H-augmenting technical change A_H?**

  1. $H's$ own task share?

  $$\frac{dI_H}{d\ln A_H} = \frac{dI_H}{d\ln H} < 0$$

  2. $L's$ task share?

  $$\frac{dI_L}{d\ln A_H} = \frac{dI_L}{d\ln H} < 0$$

  3. $M's$ task share?

  $$\frac{d\left(I_H - I_L\right)}{d\ln A_H} = \frac{d\left(I_H - I_L\right)}{d\ln H} < 0$$

- **Analogously for $d\ln L$ or $d\ln A_L$**

$$\frac{dI_H}{d\ln A_L} = \frac{dI_H}{d\ln L} > 0, \ \frac{dI_L}{d\ln A_L} = \frac{dI_L}{d\ln L} > 0$$

$$\text{and } \frac{d\left(I_H - I_L\right)}{d\ln A_L} = \frac{d\left(I_H - I_L\right)}{d\ln L} < 0$$

The effect of changes in skill supplies on wage inequality

- **Impact of an increase in the supply of labor on own relative wages**

  1. **An increase in H supply?**

$$\frac{d\ln\left(W_H/W_L\right)}{d\ln H}\gtrless 0,\ \frac{d\ln\left(W_H/W_M\right)}{d\ln H}\gtrless 0?$$

$$\frac{d\ln\left(W_H/W_L\right)}{d\ln H}< 0,\ \frac{d\ln\left(W_H/W_M\right)}{d\ln H}< 0$$

  2. **An increase in M supply?**

$$\frac{d\ln\left(W_H/W_M\right)}{d\ln M}> 0,\ \frac{d\ln\left(W_M/W_L\right)}{d\ln M}< 0$$

  3. **An increase in L supply?**

$$\frac{d\ln\left(W_M/W_L\right)}{d\ln L}> 0,\ \frac{d\ln\left(W_H/W_L\right)}{d\ln L}> 0$$

- **What about?**

$$\frac{d\ln\left(W_H/W_L\right)}{d\ln M}\ \text{and}\ \frac{d\ln\left(W_H/W_L\right)}{d\ln A_M}$$

  - **This one is subtle...**

How ΔA_M and ΔM affect W_H/W_L (subtle)

What happens when either M or A_M rises?

$$\frac{d\ln(W_H/W_L)}{d\ln M} \text{ and } \frac{d\ln(W_H/W_L)}{d\ln A_M}$$

- Depends critically on this term

$$\beta_H(I) \equiv \ln\alpha_M(I) - \ln\alpha_H(I), \beta_L(I) \equiv \ln\alpha_L(I) - \ln\alpha_M(I)$$

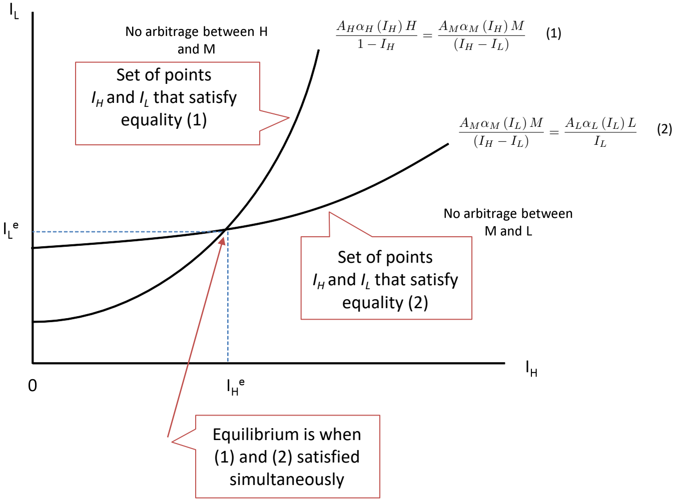
- β are comp. advantage of L versus H workers in M tasks
- $\beta_L'(I_L)\,I_L = \partial\beta_L/\partial I_L$ and $\beta_H'(I_H)(1-I_H) = \partial\beta_H/\partial I_H$
- If $\beta_L'(I_L)$ low relative to $\beta_H'(I_H)$, high skill workers have *strong comparative advantage* for tasks above I_H

Hence, rise in M displaces L workers more than H iff

$$\frac{d\ln(W_H/W_L)}{d\ln M} > 0 \text{ iff } \left|\beta_L'(I_L)\,I_L\right| < \left|\beta_H'(I_H)(1-I_H)\right|$$

Implicitly this occurs because I_L falls more than I_H rises

Revisiting that no arbitrage figure. (Non-independent curves depict no arbitrage conditions between H v. M and M v. L as thresholds I_H and I_L are adjusted)



Agenda

Agenda

Can a technological advance lower wages?

- **Reminder: wage levels in task model**

$$Y = B \times \underbrace{L^{I_L} M^{I_H - I_L} H^{1-I_H}}_{\text{Cobb-Douglas labor aggregate}}$$

where

$$B = \underbrace{\exp\left(\int_0^{I_L} \ln A_L \alpha_L(i)\, di + \int_{I_L}^{I_H} \ln A_M \alpha_M(i)\, di + \int_{I_H}^{1} \ln A_H \alpha_H(i)\, di\right)}_{\text{TFP}}$$

- Wages equal

$$\begin{aligned} W_L = \frac{\partial Y}{\partial L} &= B \times I_L \times L^{I_L - 1} M^{I_H - I_L} H^{1-I_H} \\ &= \frac{I_L}{L} \times \left(B L^{I_L} M^{I_H - I_L} H^{1-I_H}\right) \\ &= I_L \times \frac{Y}{L} \end{aligned}$$

and similarly, $W_M = Y(I_H - I_L)/M$ and $W_H = Y(1 - I_H)/H$.

Can a technological advance lower wages?

Consider a task-replacing tech Δ, embodied in K,that displaces workers from some M tasks

- Assume initially that there are some tasks performed by K that have crowded out some previously M-using tasks

$$Y = \underbrace{\exp\left(\int_0^{I_L} \ln A_L \alpha_L(i)\, di + \int_{I_L}^{I_K} \ln A_K \alpha_K(i)\, di + \int_{I_K}^{I_H} \ln A_M \alpha_M(i)\, di + \int_{I_H}^{1} \ln A_H \alpha_H(i)\, di\right)}_{\equiv B_K}$$

$$\times\, L^{I_L} K^{I_K - I_L} M^{I_H - I_K} H^{1 - I_H}$$

- How does W_M respond to a further encroachment of K into $M's$ task space?

$$W_M = \frac{\partial Y}{\partial M} = (I_H - I_K) \times \frac{Y}{M}$$

$$\frac{\partial \ln W_M}{\partial I_K} = \underbrace{\frac{-1}{I_H - I_K}}_{<0} + \underbrace{\frac{\partial \ln Y/M}{\partial I_K}}_{\geq 0}$$

Can a technological advance lower wages?

- **How does W_M respond to a further encroachment of K into $M's$ task space?**

$$W_M = \frac{\partial Y}{\partial M} = (I_H - I_K) \times \frac{Y}{M}$$

$$\frac{\partial \ln W_M}{\partial I_K} = \underbrace{\frac{-1}{I_H - I_K}}_{\substack{<0 \\ \textbf{Displacement Effect}}} + \underbrace{\frac{\partial \ln Y/M}{\partial I_K}}_{\substack{\geq 0 \\ \textbf{Productivity Effect}}}$$

- **Two offsetting forces at work**
  1. Task displacement – Technology displaces workers from a subset of tasks, lowers labor share
  2. Productivity effect — Technology increases output, thereby raises wages (you can think of this as q-complementarity)
- **Net effect — Depends on which dominates**

Agenda

Example: Routine Task-Replacing technology

- **Capital that out-competes M in a subset of tasks i' in the interval $I_L < i' < I_H$**
- **Own wage effects**
  - Immediately lowers relative wage of M by narrowing set of M tasks
- **Cross-price effects on W_L and W_H?**
  - Again depend on $\left|\beta_L'(I_L)\, I_L\right| \gtreqless \left|\beta_H'(I_H)(1 - I_H)\right|$
  - If M workers better suited to L than H tasks, then W_H/W_L rises

Routine task replacing technology

Focal case

- Task replacing technology concentrated in middle-skill/routine tasks
- Strong comparative advantage of H relative to L at respective margins with M

Leads to wage and employment 'polarization'

1. Wages:
   - Middle wages fall relative to top and bottom.
   - Top rises relative to bottom
2. Employment:
   - Middle-skill/routine tasks mechanized
   - Declining labor input in Routine tasks
   - Given comparative advantage, middle-skill workers move disproportionately downward in task distribution.

Agenda

A special case demonstrating CES equivalence: Two skills, H and L, carefully chosen $\alpha's$

- **Carefully chosen schedule of comparative advantage**

$$\alpha_L(i) = 1 - i \quad \text{and} \quad \alpha_H(i) = i$$

- **There will be single task threshold I that satisfies**

$$\frac{1-I}{I} = \left(\frac{A_H H}{A_L L}\right)^{1/2}$$

- **The relative price of tasks performed by H and L workers is**

$$\frac{P_H}{P_L} = \left(\frac{A_H H}{A_L L}\right)^{1/2}$$

- **The skill premium is therefore**

$$\frac{W_H}{W_L} = \left(\frac{A_H}{A_L}\right)^{1/2} \left(\frac{H}{L}\right)^{-1/2}$$

- Thus, the model is isomorphic to the canonical model with an elasticity of substitution of 2
- One can do this for any $\sigma \geq 1$ by choosing α's appropriately

Offshoring works identically to capital that competes for tasks

- In this sense, model is akin to Grossman and Rossi-Hansberg (2008)
- But the comparative advantage setup here is much more general

Three further extensions

1. **Endogenous choice of skills (going extra-Roy!)**
   - Workers can have a bundle of l, m, and h skills
   - When comparative advantage of one skill sufficiently eroded, may switch skills
   - Example: Former manager, now driving delivery truck
2. **Endogenous technical change (assumes task continuum is static)**
   - Endogenous tech change favoring *skills* is well understood from Acemoglu (1998, 2007)
   - Also consider endogenous technical change *favoring tasks* in this model
3. **Creation of new tasks (exogenous or endogenous) — as above**
   - Addition of new labor-using tasks: Task continuum not static. See Lin '11 *ReStat* for ingenious evidence
   - Acemolgu-Restrepo '18 *AER* — theoretical development + high level evidence (see also A-R '19 *JEP*)
   - Autor-Chin-Salomons-Seegmiller '24 — detailed evidence on new task creation + mechanisms

Full task model: Summary

Model's inputs

1. Explicit distinction between *skills* and *tasks*
2. *Comparative advantage* among workers in different tasks
3. Multiple sources of competing task 'supplies'

What the model delivers

- A natural concept of occupations (bundles of tasks)
- An endogenous mapping from skill to tasks via comparative advantage
- Technical change (offshoring) that can raise and *lower* wages
- Migration of skills across tasks as technology changes
- Polarization of wages and employment as *one possible outcome*

Some implications

1. **Task displacement**: Automation (or trade) can directly substitute for labor
2. **Comparative advantage**: Can forecast which tasks will be displaced by understanding comparative advantage of workers, machines, foreign suppliers, etc.
3. **Complementarity**: Automation (or trade) should boost *productivity and wages* in tasks not displaced: workers/tasks that are not not substituted should be *complemented*
4. **'Ripple effects'**: Displacement can reduce wages of skill groups that are substitutes with those displacement (Acemoglu-Restrepo *Ecta* '22)
5. **New task creation**: May 'reinstate' labor by creating new labor-using tasks (see Acemoglu-Restrepo '18, Autor-Chin-Salomons-Seegmiller '24)
6. **Speed of adjustment:** Gains typically diffuse, possibly slow-moving—demand effects, income effects, capital deepening
7. **Welfare:** Technological change or trade/outsourcing Pareto-improving *only* in restrictive special cases