

# Advanced Economic Growth: Lecture 10, Appropriate Technologies and Barriers to Technology Adoption

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# Introduction

- A satisfactory theory of technology differences among countries must pay attention to barriers to technology adoption and to potential inefficiencies in the organization of production, leading to apparent technology differences across countries.
- Emphasis on whether or not technologies that are available from the world technology frontier are *appropriate* for the needs of less-developed countries.
- Inefficient technology adoption resulting from contracting problems among firms.

# Appropriate and Inappropriate Technologies and Productivity Differences

- Why does rapid diffusion of ideas not remove all, or at least most, cross-country technology differences?
- “Technology” differences and income gaps can remain substantial even with free flow of ideas because technologies of the world technology frontier may be *inappropriate* to the needs of specific countries.
- Technologies and skills consist of bundles of complementary attributes that vary across countries
- Three versions of this story. Appropriateness stemming from differences in:
  - 1 exogenous (e.g., geographic) conditions,
  - 2 capital intensity,
  - 3 skill intensity.

## Inappropriate Technologies. Example: Health Innovations

- Productivity in country  $j$  at time  $t$ ,  $A_j(t)$ , is a function of whether there are effective cures against certain diseases affecting their populations.
- Two different diseases, heart attack and malaria.
- $j = 1, \dots, J'$  are affected by malaria and not by heart attacks.
- $j = J' + 1, \dots, J$  are affected by heart attacks, not malaria.
- If the disease affecting country  $j$  has no cure,  $A_j(t) = \underline{A}$ .
- When a cure is introduced,  $A_j(t) = \bar{A}$ .
- A new cure against heart attacks is discovered and becomes freely available to all countries.
- Productivity in countries  $j = J' + 1, \dots, J$  increases from  $\underline{A}$  to  $\bar{A}$ , but productivity in countries  $j = 1, \dots, J'$  remains at  $\underline{A}$ .

# Inappropriate Technologies

- Technologies of the world frontier may be “inappropriate” to the needs of some of the countries (the  $J'$  countries).
- A technological advance that is freely available to all increases productivity in a subset of the countries and creates cross-country income differences.
- Could issues of the sort be important? Yes and no:
  - ▶ Over 90% of the world R&D is carried out in OECD economies; technologies should be optimized for the conditions in OECD countries.
  - ▶ But, other than the issue of disease, there are not many obvious *fixed* country characteristics that will create this type of “inappropriateness”.
  - ▶ The issue is much more likely to be important in the context of whether new technologies will function well at different *factor intensities*.

# Capital-Labor Ratios and Inappropriate Technologies I

- Atkinson and Stiglitz (1969): technological change shifts isoquants (increasing productivity) at a given capital-labor ratio.
- Technological changes are localized for specific capital-labor ratios:
  - ▶ e.g., discovery that favors firm that is using a type of tractor with a single worker can be used by any other firm employing the same tractor with a single worker, but not by firms using oxen or less (or even more) advanced tractors.
- Implications for cross-country income differences: technologies developed for high capital-intensive production processes in OECD countries may be of little use to labor-abundant less-developed economies (Basu and Weil, 1998).

# Capital-Labor Ratios and Inappropriate Technologies II

- Production technology for all countries in the world:

$$Y = A(k | k') K^{1-\alpha} L^\alpha,$$

- Output per worker:

$$y \equiv \frac{Y}{L} = A(k | k') k^{1-\alpha},$$

where  $k = K/L$ .

- $A(k | k')$  is the (total factor) productivity of technology designed to be used with  $k'$  when used instead with  $k$ .
- Suppose that

$$A(k | k') = A \min \left\{ 1, \left( \frac{k}{k'} \right)^\gamma \right\}$$

for some  $\gamma \in (0, 1)$ .

# Capital-Labor Ratios and Inappropriate Technologies III

- New technologies developed in richer economies, with greater  $k$ .
- Productivity in country with capital-labor ratio  $k < k'$  will be

$$y = A(k | k') k^{1-\alpha} = Ak^{1-\alpha+\gamma} (k')^{-\gamma}. \quad (1)$$

- (1) implies less-developed countries will be less productive even when producing with the same techniques.
- Moreover this productivity disadvantage will be larger when the gap between  $k$  and  $k'$  is greater.
- Might be important for understanding cross-country income differences:
  - ▶ With  $\alpha \approx 2/3$ , an economy with  $k' = 8k$  would only be twice as rich, when there is no issue of inappropriate technologies.
  - ▶ But if  $\gamma = 2/3$ , the difference would be eightfold.



# Endogenous Technological Change and Appropriate Technology I

- The evidence discussed before suggests differences in human capital may be particularly important in the adoption of technology.
- Moreover, the past 30 years have witnessed the introduction of skill-biased technologies.
- A mismatch between the skill requirements of frontier technologies and skills of workers in less-developed countries may be more important than differences in capital intensity.
- Model here emphasizes implications of this mismatch, uses ideas of directed technical change, and provides tractable multi-sector growth model (Acemoglu and Zilibotti, 2001).

# Endogenous Technological Change and Appropriate Technology II

- Two groups of countries, North and South.
- Two types of workers, skilled and unskilled.
- Two differences between North and South:
  - ① All R&D and new innovations take place in the North; the South copies. Because of lack of intellectual property rights in the South, the main market of new technologies will be Northern firms.
  - ② The North is more skill-abundant:

$$H^n / L^n > H^s / L^s,$$

- Many Northern and many Southern countries.
- No population growth.

# Endogenous Technological Change and Appropriate Technology III

- All countries have access to the same set of technologies; no issue of slow technology diffusion, all differences in productivity arise from mismatch between technology and skills.
- All economies admit a representative household with the standard preferences with  $n_j = 0$  for all countries.
- The final good in each country is produced as:

$$Y_j(t) = \exp \left[ \int_0^1 \ln y_j(i, t) di \right] \quad (2)$$

- Total output is spent on  $C_j(t)$ ,  $X_j(t)$ , and also in the North on  $Z_j(t)$ .

# Endogenous Technological Change and Appropriate Technology IV

- Technology for producing intermediate  $i$  in country  $j$  at time  $t$ :

$$y_j(i, t) = \frac{1}{1 - \beta} \left[ \int_0^{N_L(t)} x_{L,j}(i, \nu, t)^{1-\beta} d\nu \right] [(1 - i)l_j(i, t)]^\beta \quad (3) \\ + \frac{1}{1 - \beta} \left[ \int_0^{N_H(t)} x_{H,j}(i, \nu, t)^{1-\beta} d\nu \right] [i\omega h_j(i, t)]^\beta .$$

- where:

- ▶  $l_j(i, t)$  = unskilled workers working in intermediate  $i$  in country  $j$  at time  $t$ , and  $h_j(i, t)$  is defined similarly.
- ▶  $x_{L,j}(i, \nu)$  = machines of type  $\nu$  used with unskilled workers, and  $x_{H,j}(i, \nu)$  is defined similarly.
- ▶  $N_L(t)$  and  $N_H(t)$  = number of machine varieties available to be used with skilled and unskilled workers.

# Endogenous Technological Change and Appropriate Technology V

- Note:
  - ① Intermediates can be produced using two alternative technologies, one using skilled and the other using unskilled.
  - ② Pattern of *cross-industry comparative advantage*: skilled (unskilled) workers relatively more productive in higher (lower) indexed intermediates.
  - ③ Skilled workers have an *absolute advantage*, captured  $\omega > 1$ .
  - ④  $N_L(t)$  and  $N_H(t)$  not indexed by  $j$ : all technologies are available to all countries.
- Final good sectors and the labor markets are competitive.

# Endogenous Technological Change and Appropriate Technology VI

- A technology monopolist produces machines at marginal cost  $\psi$  and sets prices  $p_{L,j}^x(v, t)$  and  $p_{H,j}^x(v, t)$ .
- These prices do not depend on  $i$ , since machines are not sector-specific, but skill-specific.
- Profit maximization by the final good producers leads to the demands for machines:

$$x_{L,j}(i, v, t) = \left[ p_j(i, t) ((1-i)l_j(i, t))^\beta / p_{L,j}^x(v, t) \right]^{1/\beta},$$
$$x_{H,j}(i, v, t) = \left[ p_j(i, t) (i\omega h_j(i, t))^\beta / p_{L,j}^x(v, t) \right]^{1/\beta},$$

where  $p_j(i, t)$  = relative price of intermediate  $i$  in country  $j$  at time  $t$  in terms of the final good (the numeraire in each country).

# Endogenous Technological Change and Appropriate Technology VII

- In each Southern economy a “technology” firm adopts the new technology invented in the North (at no cost) and acts as the monopolist supplier of that machine for the producers in its own country.
- The marginal cost of producing machines for this firm is the same as the inventor in the North ( $\psi = 1 - \beta$ ).
- Symmetry between the North and the South: price and thus demand for machines will take the same form in all countries.
- Thus output in sector  $i$  in any country  $j$  is:

$$y_j(i, t) = \frac{1}{1 - \beta} p_j(i, t)^{(1-\beta)/\beta} [N_L(t) (1 - i) l_j(i, t) + N_H(t) i \omega h_j(i, t)] \quad (4)$$

- For each economy,  $N_L(t)$  and  $N_H(t)$  are the state variables.

# Endogenous Technological Change and Appropriate Technology: Threshold sector

**Proposition** In any country  $j$ , given the world technologies  $N_L(t)$  and  $N_H(t)$ , there will exist a threshold  $l_j(t) \in [0, 1]$  such that skilled workers will be employed only in sectors  $i > l_j(t)$ , that is, for all  $i < l_j(t)$ ,  $h_j(i, t) = 0$ , and for all  $i > l_j(t)$ ,  $l_j(i, t) = 0$ .

Moreover, prices and labor allocations across sectors will be such that: for all  $i < l_j(t)$ ,  $p_j(i, t) = P_{L,j}(t) (1 - i)^{-\beta}$  and  $l_j(i, t) = L_j / l_j(t)$ , while for all  $i > l_j(t)$ ,  $p_j(i, t) = P_{H,j}(t) i^{-\beta}$  and  $h_j(i, t) = H_j / (1 - l_j(t))$  where the positive numbers  $P_{L,j}(t)$  and  $P_{H,j}(t)$  can be interpreted as the price indices for labor-intensive and skill-intensive intermediates.



# Endogenous Technological Change and Appropriate Technology VIII

- The technology for the final goods sector in (2) implies:

$$\frac{P_{H,j}(t)}{P_{L,j}(t)} = \left( \frac{N_H(t) \omega H_j / (1 - I_j(t))}{N_L(t) L_j / I_j(t)} \right)^{-\beta}. \quad (5)$$

- The threshold sector  $I_j(t)$  in country  $j$  at time  $t$  is indifferent between using skilled and unskilled workers (and technologies) for production, thus

$$P_{L,j}(t) (1 - I_j(t))^{-\beta} = P_{H,j}(t) I_j(t)^{-\beta}$$

- Combining with (5):

$$\frac{P_{H,j}(t)}{P_{L,j}(t)} = \left( \frac{N_H(t) \omega H_j}{N_L(t) L_j} \right)^{-\beta/2}, \quad (6)$$

# Endogenous Technological Change and Appropriate Technology IX

- Equilibrium threshold  $l_j(t)$  is uniquely pinned down by

$$\frac{l_j(t)}{1 - l_j(t)} = \left( \frac{N_H(t) \omega H_j}{N_L(t) L_j} \right)^{-1/2}. \quad (7)$$

- Combining, total output in economy  $j$  is:

$$Y_j(t) = \exp(-\beta) \left[ (N_L(t) L_j)^{1/2} + (N_H(t) \omega H_j)^{1/2} \right]^2, \quad (8)$$

- And the skill premium:

$$\frac{w_{H,j}(t)}{w_{L,j}(t)} = \omega \left( \frac{N_H(t)}{N_L(t)} \right)^{1/2} \left( \frac{\omega H_j}{L_j} \right)^{-1/2} \quad (9)$$

- (8) shows the multi-sector model leads to allocation so that output is identical to that given a constant elasticity of substitution production function with elasticity of substitution equal to 2.

# Endogenous Technological Change and Appropriate Technology X

- More generally: by changing the pattern of comparative advantage of skilled and unskilled workers in different sectors, one can obtain aggregate production functions of any elasticities of substitution.
- The type of technologies,  $N_L(t)$  and  $N_H(t)$ , will impact economies with different factor proportions differently.
- For example, consider the case  $H^s = 0$ . Then an increase in  $N_H(t)$  will increase productivity in the North, but will have no effect in the South.
- In general: an increase in  $N_H(t)$  relative to  $N_L(t)$  will benefit the skill-abundant North more than the skill-scarce South.
- Conversely, an increase in  $N_L(t)$  will tend to benefit Southern economies relatively more.

# Endogenous Technological Change and Appropriate Technology X

- Since new technologies are developed in the North and there are no intellectual property rights for Northern R&D in the South, new technologies will be developed—*designed*—for the North.
- Suppose the simplest version of the directed technical change model (with the lab equipment specification) and:

$$\dot{N}_L(t) = \eta Z_L(t) \text{ and } \dot{N}_H(t) = \eta Z_H(t), \quad (10)$$

where  $\eta_L$  and  $\eta_H$  have been set equal to each other.

# Summary of Endogenous Technological Change and Appropriate Technology

**Proposition** The unique steady-state equilibrium involves:

$$\frac{P_H^n}{P_L^n} = \left( \frac{\omega H^n}{L^n} \right)^{-\beta}$$
$$\frac{N_H^*}{N_L^*} = \frac{\omega H^n}{L^n}. \quad (11)$$

Moreover, in the North the threshold sector satisfies

$$\frac{1 - I^{n*}}{I^{n*}} = \frac{\omega H^n}{L^n}$$

and the skill premium is

$$\frac{w_H^{n*}}{w_L^{n*}} = \omega.$$

This steady-state equilibrium is globally saddle path stable.

# Directed Technical Change I

- To understand the implications of directed technical change for equilibrium relative technologies  $N_L$  and  $N_H$ , define:

- ▶ *Net output* in country  $j$ :

$$NY_j \equiv Y_j - X_j,$$

- ▶ *Income per capita* and *income per effective unit of labor* in different countries:

$$y_j \equiv \frac{Y_j}{L_j + H_j} \text{ and } y_j^{\text{eff}} \equiv \frac{Y_j}{L_j + \omega H_j}.$$

- All of these quantities are functions of labor supplies and of relative technologies,  $N_H/N_L$ .

# Directed Technical Change II

**Proposition** Consider the above-described model. Then:

The steady-state equilibrium technology ratio  $N_H^*/N_L^*$  is such that, given a constant level of for given  $N_H + N_L$ , it achieves the unique maximum of net output in the North,  $NY^n$ , as a function of relative technology  $N_H/N_L$ .

At the steady-state equilibrium technology ratio  $N_H^*/N_L^*$ , we have  $y_n > y_s$  and  $y_n^{eff} > y_s^{eff}$ .

## Directed Technical Change III

- 1 The steady-state equilibrium technology is appropriate for the needs of the North; research firms are targeting Northern markets. Moreover, since there is a unique maximum of  $NY_n$  (given  $N_H + N_L$ ),  $NY_s$  will *not* be maximized by  $N_H^*/N_L^*$ .
- 2 Technologies are inappropriate for the needs of the South. Hence, income per capita and income per effective units of labor in the North will be higher than in the South.
- 3 The process of directed technical change, combined with import of frontier technologies to less-developed economies, creates an advantage for the more advanced economies and acts as a force towards greater cross-country inequality.
- 4 This source of cross-country income differences can be quite substantial in practice (Acemoglu and Zilibotti, 2001)



# Contracting Institutions and Technology Adoption

- An important determinant of differences in technology and technology adoption are institutional differences.
- Differences in the ability to write contracts between firms and their suppliers (or firms and their workers) may have first-order effect on technology adoption.
- Emphasize the other side of the issue of technology adoption, i.e., how the conditions in the adopting country affect the use of these technologies by firms.

# Description of the Environment I

- Simplified version of Acemoglu, Antras and Helpman (2007)
- Static world and focus on a single country.
- Continuum of final goods  $q(z)$ , with  $z \in [0, M]$ .
- All consumers have identical preferences,

$$u = \left( \int_0^M q(v)^\beta dv \right)^{1/\beta} - \psi e, \quad 0 < \beta < 1, \quad (12)$$

where  $e$  is total effort, and  $\psi$  its cost in terms of real consumption.

- $\beta \in (0, 1)$  determines elasticity of demand and implies that the elasticity of substitution between final goods,  $1 / (1 - \beta) > 1$ .

## Description of the Environment II

- Preferences imply the demand function ( $p(v)$  is the price of good  $v$ ,  $A$  is the aggregate spending level):

$$q(v) = \left[ \frac{p(v)}{p^I} \right]^{-1/(1-\beta)} \frac{A}{p^I},$$

for each  $v \in [0, M]$ , where

$$p^I \equiv \left[ \int_0^M p(v)^{-\beta/(1-\beta)} dv \right]^{-(1-\beta)/\beta}$$

is the ideal price index, taken as the numeraire, i.e.,  $p^I = 1$ .

- Thus each final good producer will face a demand function

$$q = Ap^{-1/(1-\beta)}$$

# Description of the Environment III

- Revenue function for the firm:

$$R = A^{1-\beta} q^\beta. \quad (13)$$

- Production depends on the technology choice of the firm, denoted by  $N \in \mathbb{R}_+$ .
- More advanced technologies involve a greater range of intermediate goods (inputs), supplied by different suppliers.
- The transactions between the producer and the suppliers will necessitate contracting relationships.
- For each  $j \in [0, N]$ , let  $X(j)$  be the quantity of intermediate input  $j$ .

## Description of the Environment IV

- Production function of representative firm (CES):

$$q = N^{\kappa+1-1/\alpha} \left[ \int_0^N X(j)^\alpha dj \right]^{1/\alpha}, \quad (14)$$

where  $0 < \alpha < 1$ , so that the elasticity of substitution between inputs,  $\varepsilon \equiv 1/(1 - \alpha) > 1$ .

- Assume  $\kappa > 0$ .
- Without  $N^{\kappa+1-1/\alpha}$ , total output would be  $q = N^{1/\alpha} X$ , and both the elasticity of substitution between inputs and the elasticity of output to changes in technology,  $N$ , would be governed by  $\alpha$ .
- With  $N^{\kappa+1-1/\alpha}$  in front of the integral, we separate the elasticities.

# Description of the Environment V

- A large number of profit-maximizing suppliers can produce the intermediate goods.
- Each supplier has the same outside option  $w_0 > 0$ , given.
- Each intermediate input needs to be produced by a different supplier with whom the firm needs to contract.
- A supplier of an intermediate input  $j$  needs to undertake relationship-specific investments in a unit measure of (symmetric) activities  $x(i, j)$ .
- The marginal cost of investment for each activity is  $\psi$  as in (12).

## Description of the Environment VI

- Production function of intermediate inputs (Cobb-Douglas and symmetric):

$$X(j) = \exp \left[ \int_0^1 \ln x(i, j) di \right], \quad (15)$$

- A subset of the investments are nonverifiable and thus noncontractible.
- Adopting a technology  $N$  involves costs  $\Gamma(N)$ , where (to get interior solutions):
  - 1 For all  $N > 0$ ,  $\Gamma(N)$  is twice continuously differentiable, with  $\Gamma'(N) > 0$  and  $\Gamma''(N) > 0$ .
  - 2 For all  $N > 0$ ,  $N\Gamma''(N) / [\Gamma'(N) + w_0] > [\beta(\kappa + 1) - 1] / (1 - \beta)$ .
- Relationship between producer and suppliers requires contracts to ensure that suppliers deliver.
- Two payments to supplier  $j$ :
  - 1  $\tau(j) \in \mathbb{R}$  before the investment levels  $x(i, j)$  take place,
  - 2  $s(j)$  after the investments.

## Description of the Environment VII

- Payoff to supplier  $j$  (taking account of her outside option):

$$\pi_x(j) = \max \left\{ \tau(j) + s(j) - \int_0^1 \psi_x(i, j) di, w_0 \right\}. \quad (16)$$

- Payoff to the firm:

$$\pi = R - \int_0^N [\tau(j) + s(j)] dj - \Gamma(N), \quad (17)$$

where  $R$  is revenue.

- Substituting (14) and (15) into (13):

$$R = A^{1-\beta} N^{\beta(\kappa+1-1/\alpha)} \left[ \int_0^N \left( \exp \left( \int_0^1 \ln x(i, j) di \right) \right)^\alpha dj \right]^{\beta/\alpha}. \quad (18)$$



# Equilibrium under Complete Contracts I

- The firm has full control over all investments and pays each supplier her outside option.
- Corresponds to the case in which markets are complete, and intermediates of different qualities can be bought and sold in a quasi-competitive fashion.
- Game:
  - ▶ firm chooses a technology level  $N$  and makes a contract offer  $\left[ \{x(i, j)\}_{i \in [0,1]}, \{s(j), \tau(j)\} \right]$  for every input  $j \in [0, N]$ .
  - ▶ If supplier accepts contract for input  $j$ , she is obliged to supply  $\{x(i, j)\}_{i \in [0,1]}$  in exchange for  $\{s(j), \tau(j)\}$ .
- *Subgame perfect equilibrium*: strategy combination for firm and suppliers such that suppliers maximize (16) and the firm maximizes (17).

## Equilibrium under Complete Contracts II

- Equilibrium can be alternatively represented as a solution to:

$$\max_{N, \{x(i,j)\}_{i,j}, \{s(j), \tau(j)\}_j} R - \int_0^N [\tau(j) + s(j)] dj - \Gamma(N) \quad (19)$$

subject to (18) and the suppliers' *participation constraint*,

$$s(j) + \tau(j) - \psi \int_0^1 x(i,j) di \geq w_0 \text{ for all } j \in [0, N]. \quad (20)$$

- Since the firm has no reason to provide rents to the suppliers, it chooses payments  $s(j)$  and  $\tau(j)$  that satisfy (20) with equality.
- Moreover, with complete contracts,  $\tau(j)$  and  $s(j)$  are perfect substitutes, so only  $s(j) + \tau(j)$  matters.

## Equilibrium under Complete Contracts III

- Since the firm's objective (19) is (jointly) concave in  $x(i, j)$  and these investments are all equally costly, the firm chooses the same  $x$  for all activities in all intermediate inputs.
- Substituting for (20) in (19):

$$\max_{N, x} A^{1-\beta} N^{\beta(\kappa+1)} x^\beta - \psi N x - \Gamma(N) - w_0 N. \quad (21)$$

- First-order conditions:

$$(N^*)^{\frac{\beta(\kappa+1)-1}{1-\beta}} A \kappa \beta^{1/(1-\beta)} \psi^{-\beta/(1-\beta)} = \Gamma'(N^*) + w_0, \quad (22)$$

$$x^* = \frac{\Gamma'(N^*) + w_0}{\kappa \psi}. \quad (23)$$

- Equations (22) and (23) can be solved recursively.

## Equilibrium under Complete Contracts IV

- Restrictions on  $\Gamma$  above ensure (22) has a unique solution for  $N^*$ , and with (23) a unique solution for  $x^*$ .
- When all investment levels are identical and equal to  $x$ , output is:

$$q = N^{\kappa+1}x.$$

- Since a total of  $NX = Nx$  inputs are used, a measure of productivity is output divided by total input:

$$P = N^{\kappa}$$

- In the case of complete contracts this is increasing in the level of technology:

$$P^* = (N^*)^{\kappa}, \quad (24)$$

# Summary of Equilibrium under Complete Contracts

**Proposition** Consider the above described model, take  $A$  as given and suppose that there are complete contracts. Then there exists a unique equilibrium with technology and investment levels  $N^* > 0$  and  $x^* > 0$  given by (22) and (23). Furthermore, this equilibrium satisfies:

$$\frac{\partial N^*}{\partial A} > 0, \quad \frac{\partial x^*}{\partial A} \geq 0, \quad \frac{\partial N^*}{\partial \alpha} = \frac{\partial x^*}{\partial \alpha} = 0.$$

- The size of the market,  $A$ , is exogenous from the viewpoint of firm and has positive effect on investments and productivity, because it makes both suppliers' and the producer's investments more productive.
- Technology and thus productivity do not depend on the elasticity of substitution between intermediate inputs,  $1/(1 - \alpha)$ .

# Equilibrium under Incomplete Contracts I

- There exists a  $\mu \in [0, 1]$  such that, for every intermediate input  $j$ , investments in activities  $0 \leq i \leq \mu$  are observable and verifiable and therefore contractible, while investments in activities  $\mu < i \leq 1$  are not contractible.
- Contract stipulates investment levels  $x(i, j)$  for the  $\mu$  contractible activities, but not for the remaining  $1 - \mu$  noncontractible activities.
- Suppliers choose their investments in noncontractible activities in anticipation of the ex post distribution of revenue, and may decide to withhold their services in these activities.
- Economies with weak contracting institutions: low  $\mu$ .
- Ex post distribution of revenues in noncontractible activities: multilateral bargaining between the firm and its suppliers (*Shapley value* solution).

# Equilibrium under Incomplete Contracts II

- Timing of events:

- ▶ Firm adopts technology  $N$  and offers contract  $\left[ \{x_c(i, j)\}_{i=0}^{\mu}, \tau(j) \right]$  for every intermediate input  $j \in [0, N]$ , where  $x_c(i, j)$  is an investment level in a contractible activity and  $\tau(j)$  is an upfront payment to supplier  $j$  (can be positive or negative).
- ▶ Potential suppliers decide whether to apply for the contracts. Then the firm chooses  $N$  suppliers, one for each intermediate input  $j$ .
- ▶ All suppliers  $j \in [0, N]$  simultaneously choose investment levels  $x(i, j)$  for all  $i \in [0, 1]$ . In the contractible activities  $i \in [0, \mu]$  the suppliers will invest  $x(i, j) = x_c(i, j)$ .
- ▶ The suppliers and the firm bargain over the division of revenue; suppliers can withhold their services in noncontractible activities.
- ▶ Output is produced and sold, and the revenue  $R$  is distributed according to the bargaining agreement.

## Equilibrium under Incomplete Contracts III

- Look for *symmetric subgame perfect equilibrium* (SSPE) of this game, where bargaining outcomes in all subgames are determined by Shapley values.
- SSPE (denoted by  $\{\tilde{N}, \tilde{x}_c, \tilde{x}_n\}$ ) can be described by a tuple  $\{\tilde{N}, \tilde{x}_c, \tilde{x}_n, \tilde{\tau}\}$  in which:
  - ▶  $\tilde{N}$  =level of technology,
  - ▶  $\tilde{x}_c$  =investment in contractible activities,  $\tilde{x}_n$ =investment in noncontractible activities, and
  - ▶  $\tilde{\tau}$  =upfront payment to every supplier.
- That is, for every  $j \in [0, \tilde{N}]$  the upfront payment is  $\tau(j) = \tilde{\tau}$ , and the investment levels are  $x(i, j) = \tilde{x}_c$  for  $i \in [0, \mu]$  and  $x(i, j) = \tilde{x}_n$  for  $i \in (\mu, 1]$ .
- Use backward induction.



## Equilibrium under Incomplete Contracts IV

- Penultimate stage of the game, with  $N$  as the level of technology,  $x_c$  as the level of investment in contractible activities.
- Suppose each supplier other than  $j$  has chosen a  $x_n(-j)$  (all the same, symmetric equilibrium), while the investment level in every noncontractible activity by supplier  $j$  is  $x_n(j)$ .
- Given these investments, the suppliers and the firm will engage in multilateral bargaining.
- Denote the return to supplier  $j$  resulting from this bargaining by  $\bar{s}_x [N, x_c, x_n(-j), x_n(j)]$ .
- Optimal investment by supplier  $j$ :  $x_n(j)$  must be chosen to maximize  $\bar{s}_x [N, x_c, x_n(-j), x_n(j)]$  minus the cost of investment in noncontractible activities,  $(1 - \mu) \psi x_n(j)$ .

# Equilibrium under Incomplete Contracts V

- In a symmetric equilibrium,  $x_n(j) = x_n(-j)$ ; i.e.,  $x_n$  is a fixed-point given by:

$$x_n \in \arg \max_{x_n(j)} \bar{s}_x [N, x_c, x_n, x_n(j)] - (1 - \mu) \psi x_n(j). \quad (25)$$

- (25) is an “incentive compatibility constraint” with symmetry.
- There will be a unique maximizer (“ $\in$ ” can be replaced with “ $=$ ”).

## Equilibrium under Incomplete Contracts VI

- In a symmetric equilibrium with technology  $N$ , investment in contractible activities  $x_c$  and with investment in noncontractible activities  $x_n$ , revenue of the firm is:

$$R = A^{1-\beta} \left( N^{\kappa+1} x_c^\mu x_n^{1-\mu} \right)^\beta.$$

- Let  $s_x(N, x_c, x_n) = \bar{s}_x(N, x_c, x_n, x_n)$ , then the Shapley value of the firm is obtained as a residual:

$$s_q(N, x_c, x_n) = A^{1-\beta} \left( N^{\kappa+1} x_c^\mu x_n^{1-\mu} \right)^\beta - N s_x(N, x_c, x_n).$$

- Now consider the stage in which the firm chooses  $N$  suppliers from a pool of applicants.

## Equilibrium under Incomplete Contracts VII

- If suppliers expect to receive less than  $w_0$  the pool of applicants is empty.
- For production to take place, the final-good producer has to offer a contract that satisfies the participation constraint of suppliers under incomplete contracts, i.e.,

$$\bar{s}_x(N, x_c, x_n, x_n) + \tau \geq \mu \psi x_c + (1 - \mu) \psi x_n + w_0 \quad (26)$$

for  $x_n$  that satisfies (25).

- i.e., given  $N$  and  $(x_c, \tau)$ , each supplier  $j \in [0, N]$  should expect her Shapley value plus the upfront payment to cover the cost of investment in contractible and noncontractible activities and the value of her outside option.

## Equilibrium under Incomplete Contracts VIII

- Maximization problem of the firm:

$$\max_{N, x_c, x_n, \tau} s_q(N, x_c, x_n) - N\tau - \Gamma(N)$$

subject to (25) and (26).

- With no restrictions on  $\tau$ , the participation constraint (26) will be satisfied with equality; otherwise the firm could reduce  $\tau$  without violating (26) and increase its profits.
- Solving for  $\tau$  from (26) and substituting into the firm's objective:

$$\max_{N, x_c, x_n} s_q(N, x_c, x_n) + N[\bar{s}_x(N, x_c, x_n, x_n) - \mu\psi x_c - (1 - \mu)\psi] - \Gamma(N) - w_0 N, \quad (27)$$

subject to (25).

# Equilibrium under Incomplete Contracts IX

- The SSPE  $\{\tilde{N}, \tilde{x}_c, \tilde{x}_n\}$  solves this problem, and the corresponding upfront payment satisfies

$$\tilde{\tau} = \mu\psi\tilde{x}_c + (1 - \mu)\psi\tilde{x}_n + w_0 - \bar{s}_x(\tilde{N}, \tilde{x}_c, \tilde{x}_n, \tilde{x}_n). \quad (28)$$

- The key issue now is that the payments from the firm to its suppliers will be determined ex post through bargaining rather than through contractual arrangements.

# Shapley Value under Incomplete Contracts

**Proposition** Suppose that supplier  $j$  invests  $x_n(j)$  in her noncontractible activities, all the other suppliers invest  $x_n(-j)$  in their noncontractible activities, every supplier invests  $x_c$  in her contractible activities, and the level of technology is  $N$ . Then the Shapley value of supplier  $j$  is

$$\bar{s}_x[\cdot] = (1 - \gamma) A^{1-\beta} \times \left[ \frac{x_n(j)}{x_n(-j)} \right]^{(1-\mu)\alpha} x_c^{\beta\mu} x_n(-j)^{\beta(1-\mu)} N^{\beta(\kappa+1)-1}, \quad (29)$$

where

$$\gamma \equiv \frac{\alpha}{\alpha + \beta}. \quad (30)$$

# Features of Shapley Value under Incomplete Contracts I

- $\gamma \equiv \alpha / (\alpha + \beta)$ , the bargaining power of the firm, is increasing in  $\alpha$  and decreasing in  $\beta$ .
  - ▶ Higher elasticity of substitution between intermediate inputs (higher  $\alpha$ ): every supplier is less essential.
  - ▶ Higher elasticity of demand for the final good (higher  $\beta$ ): marginal contribution of the firm to the coalition's payoff falls.
- In equilibrium  $x_n(j) = x_n(-j) = x_n$ , and so

$$\begin{aligned} s_x(N, x_c, x_n) &= \bar{s}_x(N, x_c, x_n, x_n) & (31) \\ &= (1 - \gamma) A^{1-\beta} x_c^{\beta\mu} x_n^{\beta(1-\mu)} N^{\beta(\kappa+1)-1} \\ &= (1 - \gamma) \frac{R}{N}, \end{aligned}$$

where  $R = A^{1-\beta} x_c^{\beta\mu} x_n^{\beta(1-\mu)} N^{\beta(\kappa+1)}$  is the total revenue of the firm.



## Features of Shapley Value under Incomplete Contracts II

- Thus, the joint Shapley value of the suppliers,  $Ns_x(N, x_c, x_n)$ , equals the fraction  $1 - \gamma$  of the revenue, and the firm receives the remaining fraction  $\gamma$ , i.e.,

$$\begin{aligned} s_q(N, x_c, x_n) &= \gamma A^{1-\beta} x_c^{\beta\mu} x_n^{\beta(1-\mu)} N^{\beta(\kappa+1)} \\ &= \gamma R. \end{aligned} \quad (32)$$

- When  $\alpha$  is smaller,  $\bar{s}_x[N, x_c, x_n(-j), x_n(j)]$  is more concave with respect to  $x_n(j)$ : greater complementarity between the intermediate inputs implies that a given change in the relative employment of two inputs has a larger impact on their relative marginal products.
- $\beta$  affects the concavity of revenue in output (see (13)), but has no effect on the concavity of  $\bar{s}_x$ , because with a continuum of suppliers, a single supplier has an infinitesimal effect on output.

# SSPE Under Incomplete Contracts I

- Derive the incentive compatibility constraint using (25) and (29):

$$x_n = \arg \max_{x_n(j)} (1 - \gamma) A^{1-\beta} \left[ \frac{x_n(j)}{x_n} \right]^{(1-\mu)\alpha} x_c^{\beta\mu} x_n^{\beta(1-\mu)} N^{\beta(\kappa+1)-1} - \psi (1 - \gamma)$$

- Two differences with producer's first-best:
  - The term  $(1 - \gamma)$  implies that supplier is not the full residual claimant of the return from her investment in noncontractible activities and thus underinvests in these activities.
  - Multilateral bargaining distorts the perceived concavity of the private return relative to the social return. Using the first-order condition of this problem and solving for the fixed point by substituting  $x_n(j) = x_n$  yields a unique  $x_n$ :

$$x_n = \bar{x}_n(N, x_c) \equiv \left[ \alpha (1 - \gamma) \psi^{-1} x_c^{\beta\mu} A^{1-\beta} N^{\beta(\kappa+1)-1} \right]^{1/[1-\beta(1-\mu)]} . \quad (33)$$

## SSPE Under Incomplete Contracts II

- (33) implies investments in noncontractible activities are increasing in  $\alpha$  ( $\alpha(1 - \gamma) = \alpha\beta / (\alpha + \beta)$  is increasing in  $\alpha$ ).
- This is the outcome of two opposing forces (the second dominates):
  - 1 The share of the suppliers,  $(1 - \gamma)$ , is decreasing in  $\alpha$ , because greater substitution between the intermediate inputs reduces the suppliers' ex post bargaining power.
  - 2 But higher  $\alpha$  also reduces the concavity of  $\bar{s}_x(\cdot)$  in  $x_n$ , increasing the marginal reward from investing further in noncontractible activities.
- Note also contractible and noncontractible activities are complements; in particular  $\bar{x}_n(N, x_c)$  is increasing in  $x_c$ .

# SSPE Under Incomplete Contracts III

- The effect of  $N$  on  $x_n$  is ambiguous: investment declines with  $N$  when  $\beta(\kappa + 1) < 1$  and increases with  $N$  when  $\beta(\kappa + 1) > 1$ .
- Again, two opposite effects on a supplier's incentives to invest. A greater number of inputs:
  - 1 Increases the marginal product of investment due to the “love for variety” embodied in the technology
  - 2 The bargaining share of a supplier,  $(1 - \gamma) / N$ , declines with  $N$ .
- For large values of  $\kappa$  effect 1 dominates, while for small values of  $\kappa$  2 dominates.

## SSPE Under Incomplete Contracts IV

- Using (31), (32) and (33):

$$\begin{aligned} \max_{N, x_c} & A^{1-\beta} \left[ x_c^\mu \bar{x}_n (N, x_c)^{1-\mu} \right]^\beta N^{\beta(\kappa+1)} - \psi N \mu x_c \quad (34) \\ & - \psi N (1 - \mu) \bar{x}_n (N, x_c) - \Gamma(N) - w_0 N \end{aligned}$$

where  $\bar{x}_n(N, x_c)$  is defined in (33).

- Substituting (33) and differentiating yield a unique solution  $(\tilde{N}, \tilde{x}_c)$ :

$$\begin{aligned} & \tilde{N}^{\frac{\beta(\kappa+1)-1}{1-\beta}} A \kappa \beta^{\frac{1}{1-\beta}} \psi^{-\frac{\beta}{1-\beta}} \times \quad (35) \\ & \left[ \frac{1 - \alpha (1 - \gamma) (1 - \mu)}{1 - \beta (1 - \mu)} \right]^{\frac{1-\beta(1-\mu)}{1-\beta}} [\beta^{-1} \alpha (1 - \gamma)]^{\frac{\beta(1-\mu)}{1-\beta}} \\ & = \Gamma'(\tilde{N}) + w_0, \end{aligned}$$

$$\tilde{x}_c = \frac{\Gamma'(\tilde{N}) + w_0}{\kappa \psi}. \quad (36)$$

## SSPE Under Incomplete Contracts V

- Again these conditions determine the equilibrium recursively. First, (35) gives  $\tilde{N}$ , and then given  $\tilde{N}$ , (36) yields  $\tilde{x}_c$ .
- Moreover, using (33), (35), and (36) gives the level of investment in noncontractible activities as

$$\tilde{x}_n = \frac{\alpha(1-\gamma)[1-\beta(1-\mu)]}{\beta[1-\alpha(1-\gamma)(1-\mu)]} \left( \frac{\Gamma'(\tilde{N}) + w_0}{\kappa\psi} \right). \quad (37)$$

- For a given  $N$ ,  $\tilde{x}_c$  is identical to  $x^*$  (compare (23) to (36)).
- Thus differences in investments in contractible activities between these economic environments only result from differences in technology adoption ( $N$ ).
- $\tilde{N}$  and  $N^*$  differ only because of the two bracketed terms on the left-hand side of (35).
- These represent the distortions created by bargaining between the firm and its suppliers.

## SSPE Under Incomplete Contracts VI

- Intuitively, technology adoption is distorted because incomplete contracts reduce investments in noncontractible activities below the level of investment in contractible activities and this “underinvestment” reduces the profitability of technologies with high  $N$ .
- As  $\mu \rightarrow 1$  (and contractual imperfections disappear), both of these bracketed terms on the left-hand side of (35) go to 1 and  $(\tilde{N}, \tilde{x}_c) \rightarrow (N^*, x^*)$ .
- Comparative static results are facilitated by the block-recursive structure of the equilibrium; any change in  $A$ ,  $\mu$  or  $\alpha$  that increases the left-hand side of (35) also increase  $\tilde{N}$ , and the effect on  $\tilde{x}_c$  and  $\tilde{x}_n$  can then be obtained from (36) and (37).

# Comparative Statics Under Incomplete Contracts I

**Proposition** Consider the above described model with incomplete contracts and suppose that the restrictions on  $\Gamma$  hold. Then there exists a unique SSPE under incomplete contracts,  $\{\tilde{N}, \tilde{x}_c, \tilde{x}_n\}$ , characterized by (35), (36) and (37). Furthermore,  $\{\tilde{N}, \tilde{x}_c, \tilde{x}_n\}$  satisfies  $\tilde{N}, \tilde{x}_c, \tilde{x}_n > 0$ ,

$$\tilde{x}_n < \tilde{x}_c,$$

$$\begin{aligned} \frac{\partial \tilde{N}}{\partial A} &> 0, \quad \frac{\partial \tilde{x}_c}{\partial A} \geq 0, \quad \frac{\partial \tilde{x}_n}{\partial A} \geq 0, \\ \frac{\partial \tilde{N}}{\partial \mu} &> 0, \quad \frac{\partial \tilde{x}_c}{\partial \mu} \geq 0, \quad \frac{\partial (\tilde{x}_n / \tilde{x}_c)}{\partial \mu} > 0, \\ \frac{\partial \tilde{N}}{\partial \alpha} &> 0, \quad \frac{\partial \tilde{x}_c}{\partial \alpha} \geq 0, \quad \frac{\partial (\tilde{x}_n / \tilde{x}_c)}{\partial \alpha} > 0. \end{aligned}$$



## Comparative Statics Under Incomplete Contracts II

- Thus suppliers invest less in noncontractible activities than in contractible activities. In particular:

$$\frac{\tilde{x}_n}{\tilde{x}_c} = \frac{\alpha (1 - \gamma) [1 - \beta (1 - \mu)]}{\beta [1 - \alpha (1 - \gamma) (1 - \mu)]} < 1, \quad (38)$$

which follows from (36) and (37) and from the fact that  $\alpha (1 - \gamma) = \alpha\beta / (\alpha + \beta) < \beta$  (recall (30)).

- Intuition:
  - ▶ Producer firm is the full residual claimant of the return to  $\tilde{x}_c$  and dictates these investments in the contract.
  - ▶ But investments in  $\tilde{x}_n$  are decided by suppliers, who are not full residual claimants of the returns (recall (31)) and thus underinvest in these activities.

# Comparative Statics Under Incomplete Contracts III

- Level of technology and investments in both activities are increasing in:
  - ▶ the size of the market,
  - ▶ the fraction of contractible activities (quality of contracting institutions),
  - ▶ the elasticity of substitution between intermediate inputs.
- Intuition:
  - ▶ Greater  $A$  makes production more profitable and thus increases investments and equilibrium technology.
  - ▶ Higher  $\mu$  imply a greater fraction of activities receive  $\tilde{x}_c$  rather than  $\tilde{x}_n < \tilde{x}_c$ . This makes the choice of higher  $N$  more profitable, and higher  $N$  increases profitability of further investments in  $\tilde{x}_c$  and  $\tilde{x}_n$ .
  - ▶ Higher  $\mu$  also closes the (proportional) gap between  $\tilde{x}_c$  and  $\tilde{x}_n$ : with high  $\mu$  the marginal return to investment in noncontractible activities is also higher.
  - ▶ Higher  $\alpha$ , i.e., lower complementarity between intermediate inputs, reduces the share of each supplier but also makes  $\bar{s}_x(\cdot)$  less concave, but the latter effect dominates.

# Complete vs. Incomplete Contracts I

- Key implication: contractual frictions (here, the incomplete contracts equilibrium) lead to underinvestment in quality, discourage technology adoption and reduce productivity.
- Note that productivity under incomplete contracts is  $\tilde{P} = \tilde{N}^\kappa$ , while productivity on the complete contracts,  $P^*$ , is given in (24).

**Proposition** Let  $\{\tilde{N}, \tilde{x}_c, \tilde{x}_n\}$  be the unique SSPE with incomplete contracts and let  $\{N^*, x^*\}$  be the unique equilibrium with complete contracts. Then

$$\tilde{N} < N^* \text{ and } \tilde{x}_n < \tilde{x}_c < x^*.$$

# Complete vs. Incomplete Contracts II

- Since incomplete contracts lead to the choice of less advanced (lower  $N$ ) technologies, they also reduce productivity and investments in contractible and noncontractible activities.
- Acemoglu, Antras and Helpman (2007) also show that the technology and income differences resulting from relatively modest differences in contracting institutions can be quite large.
- Therefore, the link between contracting institutions and technology adoption provides us with a theoretical mechanism that might generate significant technology differences across countries.

# Conclusions I

- ① Once we allow a relatively rapid diffusion of technologies, does there remain any reason for technology or productivity differences across countries (beyond differences in physical and human capital)? Yes: “appropriateness” of technologies and barriers to technological change.
- ② There are reasons to suspect that technology-skill mismatch may be more important, because of the organization of the world technology market. Two features are important:
  - ① The majority of frontier technologies are developed in a few rich countries.
  - ② The lack of effective intellectual property rights enforcement implies that technology firms in rich countries target the needs of their own domestic market.
- ③ Thus new technologies will be “too skill-biased” and this source of inappropriateness of technologies can create a large endogenous technology and income gap among nations.

## Conclusions II

- ④ Productivity differences also stem because production is organized differently around the world: a key reason for is institutions and policies in place in different parts of the world.
- ⑤ What types of contracts firms can write with their suppliers, can have an important effect on their technology adoption decisions and thus on cross-country differences on productivity.
- ⑥ But contracting institutions are only one of many potential organizational differences across countries that might impact upon equilibrium productivity.