Advanced Economic Growth: Lecture 3, Review of Endogenous Growth: Schumpeterian Models

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Introduction

- Most process innovations either increase the quality of an existing product or reduce the costs of production.
- *Competitive* aspect of innovations: a newly-invented superior computer often *replaces* existing vintages.
- Realm of Schumpeterian *creative destruction*.
- Schumpeterian growth raises important issues:
    1. Direct price competition between producers with different vintages of quality or different costs of producing
    2. Competition between incumbents and entrants: *business stealing effect.*

Preferences and Technology I

- Continuous time.
- Representative household with standard CRRA preferences.
- Constant population L; labor supplied inelastically.
- Resource constraint:

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{1}$$

- Normalize the measure of inputs to 1, and denote each machine line by $\nu \in [0, 1]$.

Preferences and Technology II

- Engine of economic growth: *quality improvement*.
- $q(\nu, t)$ =quality of machine line ν at time t.
- "Quality ladder" for each machine type:

$$q(\nu, t) = \lambda^{n(\nu,t)} q(\nu, 0) \text{ for all } \nu \text{ and } t, \tag{2}$$

  where:
  - $\lambda > 1$
  - $n(\nu, t)$ =innovations on this machine line between 0 and t.
- Production function of the final good:

$$Y(t) = \frac{1}{1-\beta} \left[\int_0^1 q(\nu, t) x(\nu, t \mid q)^{1-\beta} d\nu \right] L^\beta, \tag{3}$$

  where $x(\nu, t \mid q)$=quantity of machine of type ν quality q.

Preferences and Technology III

- Implicit assumption in (3): at any point in time only one quality of any machine is used.
- *Creative destruction*: when a higher-quality machine is invented it will replace ("destroy") the previous vintage of machines.

Technology for producing machines and innovation possibilities frontier I

- Cumulative R&D process.
- $Z(\nu, t)$ units of the final good for research on machine line ν, quality $q(\nu, t)$ generate a flow rate

$$\eta Z(\nu, t) / q(\nu, t)$$

  of innovation.
- Note one unit of R&D spending is proportionately less effective when applied to a more advanced machine.
- Free entry into research.
- The firm that makes an innovation has a perpetual patent.
- But other firms can undertake research based on the product invented by this firm.

Technology for producing machines and innovation possibilities frontier II

- Once a machine of quality $q(\nu, t)$ has been invented, any quantity can be produced at the marginal cost $\psi q(\nu, t)$.
- New entrants undertake the R&D and innovation:
  - The incumbent has weaker incentives to innovate, since it would be replacing its own machine, and thus destroying the profits that it is already making (*Arrow's replacement effect*).

Equilibrium

- Allocation: time paths of
  - consumption levels, aggregate spending on machines, and aggregate R&D expenditure $[C(t), X(t), Z(t)]_{t=0}^{\infty}$,
  - machine qualities $[q(\nu, t)]_{\nu \in [0,1], t=0}^{\infty}$,
  - prices and quantities of each machine and the net present discounted value of profits from that machine, $[p^x(\nu, t \mid q), x(\nu, t), V(\nu, t \mid q)]_{\nu \in [0,1], t=0}^{\infty}$, and
  - interest rates and wage rates, $[r(t), w(t)]_{t=0}^{\infty}$.

Equilibrium: Innovations Regimes

- Demand for machines similar to before:

$$x(\nu, t \mid q) = \left(\frac{q(\nu, t)}{p^x(\nu, t \mid q)} \right)^{1/\beta} L \quad \text{for all } \nu \in [0, 1] \text{ and all } t, \tag{4}$$

  where $p^x(\nu, t \mid q)$ refers to the price of machine type ν of quality $q(\nu, t)$ at time t.

- Two regimes:
  1. innovation is "drastic" and each firm can charge the unconstrained monopoly price,
  2. limit prices have to be used.

- Assume drastic innovations regime: λ is sufficiently large

$$\lambda \geq \left(\frac{1}{1-\beta} \right)^{\frac{1-\beta}{\beta}}. \tag{5}$$

- Again normalize $\psi \equiv 1 - \beta$

Monopoly Profits

- Profit-maximizing monopoly:

$$p^x(\nu, t \mid q) = q(\nu, t). \tag{6}$$

- Combining with (4)

$$x(\nu, t \mid q) = L. \tag{7}$$

- Thus, flow profits of monopolist:

$$\pi(\nu, t \mid q) = \beta q(\nu, t) L.$$

Characterization of Equilibrium I

- Substituting (7) into (3):

$$Y(t) = \frac{1}{1-\beta} Q(t) L, \tag{8}$$

where

$$Q(t) = \int_0^1 q(\nu, t) d\nu \tag{9}$$

- Aggregate spending on machines:

$$X(t) = (1-\beta) Q(t) L. \tag{10}$$

- Equilibrium wage rate:

$$w(t) = \frac{\beta}{1-\beta} Q(t). \tag{11}$$

Characterization of Equilibrium II

- Value function for monopolist of variety ν of quality $q(\nu, t)$ at time t:

$$r(t)\,V(\nu, t \mid q) - \dot{V}(\nu, t \mid q) = \pi(\nu, t \mid q) - z(\nu, t \mid q)\,V(\nu, t \mid q), \tag{12}$$

  where:
  - $z(\nu, t \mid q)$=rate at which new innovations occur in sector ν at time t,
  - $\pi(\nu, t \mid q)$=flow of profits.
- Last term captures the essence of Schumpeterian growth:
  - when innovation occurs, the monopolist loses its monopoly position and is replaced by the producer of the higher-quality machine.
  - From then on, it receives zero profits, and thus has zero value.
  - Because of Arrow's replacement effect, an entrant undertakes the innovation, thus $z(\nu, t \mid q)$ is the flow rate at which the incumbent will be replaced.

Characterization of Equilibrium III

- Free entry:

$$\eta V(\nu, t \mid q) \leq \lambda^{-1} q(\nu, t) \tag{13}$$
$$\text{and } \eta V(\nu, t \mid q) = \lambda^{-1} q(\nu, t) \text{ if } Z(\nu, t \mid q) > 0.$$

- Note: Even though the $q(\nu, t)$'s are stochastic as long as the $Z(\nu, t \mid q)$'s, are nonstochastic, average quality $Q(t)$, and thus total output, $Y(t)$, and total spending on machines, $X(t)$, will be nonstochastic.
- Consumer maximization implies the Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \tag{14}$$

- Transversality condition:

$$\lim_{t \to \infty} \left[\exp\left(-\int_0^t r(s)\, ds \right) \int_0^1 V(\nu, t \mid q)\, d\nu \right] = 0 \tag{15}$$

for all q.

Definition of Equilibrium

- $V(\nu, t \mid q)$, is nonstochastic: either q is not the highest quality in this machine line and $V(\nu, t \mid q)$ is equal to 0, or it is given by (12).
- An equilibrium can then be represented as time paths of
  - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$ that satisfy (1), (10), (15),
  - $[Q(t)]_{t=0}^{\infty}$ and $[V(\nu, t \mid q)]_{\nu \in [0,1], t=0}^{\infty}$ consistent with (9), (12) and (13),
  - $[p^x(\nu, t \mid q), x(\nu, t)]_{\nu \in [0,1], t=0}^{\infty}$ given by (6) and (7), and
  - $[r(t), w(t)]_{t=0}^{\infty}$ that are consistent with (11) and (14)
- Balanced Growth Path defined similarly to before (constant growth of output, constant interest rate).

Balanced Growth Path I

- In BGP, consumption grows at the constant rate g_C^*, that must be the same rate as output growth, g^*.
- From (14), $r(t) = r^*$ for all t.
- If there is positive growth in BGP, there must be research at least in some sectors.
- Since profits and R&D costs are proportional to quality, whenever the free entry condition (13) holds as equality for one machine type, it will hold as equality for all of them.
- Thus,

$$V(\nu, t \mid q) = \frac{q(\nu, t)}{\lambda \eta}. \tag{16}$$

- Moreover, if it holds between t and $t + \Delta t$, $\dot{V}(\nu, t \mid q) = 0$, because the right-hand side of equation (16) is constant over time—$q(\nu, t)$ refers to the quality of the machine supplied by the incumbent, which does not change.

Balanced Growth Path II

- Since R&D for each machine type has the same productivity, constant in BGP:

$$z(\nu, t) = z(t) = z^*$$

- Then (12) implies

$$V(\nu, t \mid q) = \frac{\beta q(\nu, t) L}{r^* + z^*}. \tag{17}$$

- Note the *effective discount rate* is $r^* + z^*$.
- Combining this with (16):

$$r^* + z^* = \lambda \eta \beta L. \tag{18}$$

- From the fact that $g_C^* = g^*$ and (14), $g^* = (r^* - \rho) / \theta$, or

$$r^* = \theta g^* + \rho. \tag{19}$$

Balanced Growth Path III

- To solve for the BGP equilibrium, we need a final equation relating g^* to z^*. From (8)

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)}.$$

- Note that in an interval of time Δt, $z(t)\Delta t$ sectors experience one innovation, and this will increase their productivity by λ.
- The measure of sectors experiencing more than one innovation within this time interval is $o(\Delta t)$—i.e., it is second-order in Δt, so that

$$\text{as } \Delta t \to 0,\ o(\Delta t)/\Delta t \to 0.$$

- Therefore, we have

$$Q(t+\Delta t) = \lambda Q(t) z(t) \Delta t + (1 - z(t)\Delta t) Q(t) + o(\Delta t).$$

Balanced Growth Path IV

- Now subtracting $Q(t)$ from both sides, dividing by Δt and taking the limit as $\Delta t \to 0$, we obtain

$$\dot{Q}(t) = (\lambda - 1) z(t) Q(t).$$

- Therefore,

$$g^* = (\lambda - 1) z^*. \tag{20}$$

- Now combining (18)-(20), we obtain:

$$g^* = \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}}. \tag{21}$$

Summary of Balanced Growth Path

Proposition Consider the model of Schumpeterian growth described above. Suppose that

$$\lambda \eta \beta L > \rho > (1-\theta) \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}} . \qquad (22)$$

Then, there exists a unique balanced growth path in which average quality of machines, output and consumption grow at rate g^* given by (21). The rate of innovation is $g^* / (\lambda - 1)$.

- Important: *Scale effects* and implicit *knowledge spillovers* are present.
    - knowledge spillovers arise because innovation is *cumulative*.

Transitional Dynamics

Proposition In the model of Schumpeterian growth described above, starting with any average quality of machines $Q(0) > 0$, there are no transitional dynamics and the equilibrium path always involves constant growth at the rate g^* given by (21).

- Note only the average quality of machines, $Q(t)$, matters for the allocation of resources.
- Moreover, the incentives to undertake research are identical for two machine types ν and ν', with different quality levels $q(\nu, t)$ and $q(\nu', t)$

Pareto Optimality

- This equilibrium is typically Pareto suboptimal.
- But now distortions more complex than the expanding varieties model.
  - monopolists are not able to capture the entire social gain created by an innovation.
  - Business stealing effect.
- The equilibrium rate of innovation and growth can be too high or too low.

Social Planner's Problem I

- Quantities of machines used in the final good sector: no markup.

$$x^S(\nu, t \mid q) = \psi^{-1/\beta} L$$
$$= (1-\beta)^{-1/\beta} L.$$

- Substituting into (3):

$$Y^S(t) = (1-\beta)^{-1/\beta} Q^S(t) L,$$

Social Planner's Problem II

- Maximization problem of the social planner:

$$\max \int_0^\infty \frac{C^S(t)^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt$$

subject to

$$\dot{Q}^S(t) = \eta(\lambda - 1)(1-\beta)^{-1/\beta} \beta Q^S(t) L - \eta(\lambda - 1) C^S(t),$$

where $(1-\beta)^{-1/\beta} \beta Q^S(t) L$ is net output.

Social Planner's Problem III

- Current-value Hamiltonian:

$$\hat{H}\left(Q^S, C^S, \mu^S\right) = \frac{C^S(t)^{1-\theta} - 1}{1-\theta} + \mu^S(t)\left[\begin{array}{c} \eta(\lambda-1)(1-\beta)^{-1/\beta}\beta Q^S(t)L \\ -\eta(\lambda-1)C^S(t) \end{array}\right].$$

Social Planner's Problem IV

- Necessary conditions:

$$\begin{aligned}\hat{H}_C(\cdot) &= C^S(t)^{-\theta} - \mu^S(t)\,\eta(\lambda-1) \\ &= 0 \\ \hat{H}_Q(\cdot) &= \mu^S(t)\,\eta(\lambda-1)(1-\beta)^{-1/\beta}\beta L \\ &= \rho\mu^S(t) - \dot{\mu}^S(t)\end{aligned}$$

$$\lim_{t\to\infty}\left[\exp(-\rho t)\,\mu^S(t)\,Q^S(t)\right] = 0$$

- Combining:

$$\frac{\dot{C}^S(t)}{C^S(t)} = g^S \equiv \frac{1}{\theta}\left(\eta(\lambda-1)(1-\beta)^{-1/\beta}\beta L - \rho\right). \tag{23}$$

Summary of Social Planner's Problem

- Total output and average quality will also grow at the rate g^S.
- Comparing g^S to g^*, either could be greater.
  - When λ is very large, $g^S > g^*$. As $\lambda \to \infty$, $g^S / g^* \to (1-\beta)^{-1/\beta} > 1$.

Proposition In the model of Schumpeterian growth described above, the decentralized equilibrium is generally Pareto suboptimal, and may have a higher or lower rate of innovation and growth than the Pareto optimal allocation.

Policies I

- Creative destruction implies a natural *conflict of interest*, and certain types of policies may have a constituency.
- Suppose there is a tax τ imposed on R&D spending.
- This has no effect on the profits of existing monopolists, and only influences their net present discounted value via replacement.
- Since taxes on R&D will discourage R&D, there will be replacement at a slower rate, i.e., z^* will fall.
- This increases the steady-state value of all monopolists given by (17):

$$V(q) = \frac{\beta q L}{r^*(\tau) + z^*(\tau)},$$

- The free entry condition becomes

$$V(q) = \frac{(1+\tau)}{\lambda \eta} q.$$

Policies II

- $V(q)$ is clearly increasing in the tax rate on R&D, τ.
- Combining the previous two equations, we see that in response to a positive rate of taxation, $r^*(\tau) + z^*(\tau)$ must adjust downward.
- Intuitively, when the costs of R&D are raised because of tax policy, the value of a successful innovation, $V(q)$, must increase to satisfy the free entry condition. This can only happen through a decline the effective discount rate $r^*(\tau) + z^*(\tau)$.
- A lower effective discount rate, in turn, is achieved by a decline in the equilibrium growth rate of the economy:

$$g^*(\tau) = \frac{(1+\tau)^{-1}\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}}.$$

- This growth rate is strictly decreasing in τ, but incumbent monopolists would be in favor of increasing τ.

Outline

- Two major differences with previous model:
  1. Only one sector experiencing quality improvements rather than a continuum of machine types.
  2. The innovation possibilities frontier uses a scarce factor, labor.

Aghion-Howitt Model I

- Consumer side as before, but risk neutral consumers, so:

$$r^* = \rho$$

- Population constant at L; individuals supply labor inelastically.
- Aggregate production function of final good:

$$Y(t) = \frac{1}{1-\beta} x(t \mid q)^{1-\beta} (q(t) L_E(t))^{\beta}, \tag{24}$$

- Market clearing requires:

$$L_E(t) + L_R(t) \leq L.$$

- where $L_E(t)$ is labor used in production, $L_R(t)$ in the R&D sector.
- Once invented, a machine of quality $q(t)$ can be produced at the constant marginal cost ψ in terms of final goods.

Aghion-Howitt Model II

- Normalize $\psi \equiv 1 - \beta$.
- Innovation possibilities frontier: each worker employed in the R&D sector generates a flow rate η of a new machine.
- When the current machine used in production has quality $q(t)$, the new machine has quality $\lambda q(t)$.
- Assume that the monopolist can charge the unconstrained monopoly price.
- Then, the demand for the leading-edge machine of quality q is

$$x(t \mid q) = p^x(t)^{-1/\beta} q(t) L_E(t),$$

- Suppose that the monopoly price for highest quality machine is:

$$p^x(t \mid q) = \frac{\psi}{1-\beta} = 1.$$

- Why is this a "supposition"?

Aghion-Howitt Model III

- Thus demand for the machine of quality q at time t is:

$$x(t \mid q) = q(t) L_E(t),$$

- Monopoly profits:

$$\pi(t \mid q) = \beta q(t) L_E(t).$$

- Aggregate output:

$$Y(t \mid q) = \frac{1}{1-\beta} q(t) L_E(t),$$

- Equilibrium wage:

$$w(t \mid q) = \frac{\beta}{1-\beta} q(t).$$

- Focus on a "steady-state equilibrium" with constant flow rate of innovation z^*.

Aghion-Howitt Model IV

- Even with constant z, consumption and output growth will not be constant because of the stochastic nature of innovation.
- A constant number (and thus fraction) of workers, L_R^*, must be working in research. Since $r^* = \rho$, this implies that the steady-state value of a monopolist is:

$$V(q) = \frac{\beta q (L - L_R^*)}{\rho + z^*},$$

- Free entry:

$$w(q) = \eta V(\lambda q).$$

- In addition, given the R&D technology, we must have

$$z^* = \eta L_R^*$$

Aghion-Howitt Model V

Combining the last four equations:

$$\frac{\lambda (1-\beta) \eta (L - L_R^*)}{\rho + \eta L_R^*} = 1,$$

- Which uniquely determines the steady-state number of workers in research as

$$L_R^* = \frac{\lambda (1-\beta) \eta L - \rho}{\eta + \lambda (1-\beta) \eta}, \tag{25}$$

  as long as this expression is positive.
- Since there is only one sector undergoing technological change and this sector experiences growth only at finite intervals, the growth rate of the economy will have an *uneven* nature.

Summary of Aghion-Howitt Model

Proposition Consider the one-sector Schumpeterian growth model presented in this section and suppose that

$$0 < \lambda (1 - \beta) \eta L - \rho < \frac{1 + \lambda (1 - \beta) \rho}{\ln \lambda}. \qquad (26)$$

Then there exists a unique steady-state equilibrium in which L_R^* workers work in the research sector, where L_R^* is given in equation (25). The economy has an average growth rate of $g^* = \eta L_R^* \ln \lambda$. Equilibrium growth is "uneven," in the sense that the economy has constant output for a while and then grows by a discrete amount when an innovation takes place.

Uneven Growth I

- The uneven pattern of economic growth in the previous model is driven by the discrete nature of innovations in continuous time.
- Another source of uneven growth more closely related to creative destruction is that future growth reduces the value of current innovations, because it causes more rapid replacement.
- We focus on an equilibrium path with endogenous growth cycles.
- Now assume that L_R workers in research leads to innovation at the rate

$$\eta(L_R) L_R,$$

where $\eta(\cdot)$ is a strictly decreasing function, representing an externality in the research process.

Uneven Growth II

- Free entry condition:

$$\eta\left(L_R\left(q\right)\right)V\left(\lambda q\right) = w\left(q\right)$$

- Look for an equilibrium with the cyclical property that the rate of innovation differs in an odd-numbered innovation versus an even-numbered innovation.
- Possible when all agents in the economy expect there to be such an equilibrium (i.e., it is a "self-fulfilling" equilibrium).
- Denote the number of workers in R&D for odd and even-numbered innovations by L_R^1 and L_R^2.
- Then, in any equilibrium with such cyclical pattern:

$$V^2\left(\lambda q\right) = \frac{\beta q\left(L - L_R^2\right)}{\rho + \eta\left(L_R^2\right)L_R^2} \text{ and } V^1\left(\lambda q\right) = \frac{\beta q\left(L - L_R^1\right)}{\rho + \eta\left(L_R^1\right)L_R^1}. \tag{27}$$

Uneven Growth III

- And the free entry conditions is:

$$\eta\left(L_R^1\right) V^2(\lambda q) = w(q) \text{ and } \eta\left(L_R^2\right) V^1(\lambda q) = w(q),$$

- Therefore, equilibrium conditions:

$$\begin{aligned}
\eta\left(L_R^1\right) \frac{\lambda(1-\beta) q\left(L - L_R^2\right)}{\rho + \eta\left(L_R^2\right) L_R^2} &= 1 \text{ and} \qquad (28)\\
\eta\left(L_R^2\right) \frac{\lambda(1-\beta) q\left(L - L_R^1\right)}{\rho + \eta\left(L_R^1\right) L_R^1} &= 1.
\end{aligned}$$

- These two equations can have solutions L_R^1 and $L_R^2 \neq L_R^1$, which would correspond to the possibility of a two-period endogenous cycle.

Labor Market Implications

- So far creative destruction only destroyed the monopoly rents of incumbent producers. In more realistic settings, it may:
  - Dislocate previously employed workers.
  - Destroy firm-specific skills: workers (and firms) may be less willing to make specific human capital and other investments.

Conclusions

- Forward-looking incentives driving growth in this Schumpeterian model as well.
- But the "industrial organization" of growth is richer than in the basic Romer or Grossman-Helpman model.
- Most important ideas:
  - *Creative destruction*
  - *Business stealing effect and conflict of interest*
  - *Growth and planning horizons of incumbents related*
  - *Possible uneven growth*