# Patent Races and Disclosure: Theory and Testable Implications

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**Abstract**

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This dissertation introduces strategic publications to a patent race and studies their effect on R&D. Publishing changes the state of the prior art, affecting the patentability of related innovation. The model is a multi-stage patent race model in which the competitors can strategically publish research results. Two identical firms compete over an innovation. The winner of the race is the first to achieve $n$ innovative steps above the prior art. Innovation is random and modeled as a Poisson process. If a firm chooses to publish, it increases the prior art.

The use of publications is first investigated in a model with a constant investment policy. In the Markov perfect equilibrium, firms publish when they are behind in the race and their rival is close to winning it. Publication sets the leader back and gives the follower a chance to catch up. I refer to this type of publications as defensive publications. Firms are more likely to publish the more patient they are and the higher their probability of success. With a joint decision on research intensity and publications, publication is a strategic substitute to investment.

The second chapter investigates whether the publications model is empirically refutable and determines if a particular data set is consistent with equilibrium behavior. Based on the insights derived from Afriat theorem, I derive nonparametric testable implications of the model and show that it is empirically refutable. The necessary data requirement would

have to include investment levels and publications along the equilibrium path. If we observe data that are consistent with the model, then it is possible to construct a cost function that rationalizes the data.

Chapter 3 studies the testable restrictions of a simple moral hazard model of mutual insurance. I show that the moral hazard model has minimal testable restrictions when only data associated with equilibrium choices are observed. When information on the probabilities of loss associated with the effort level that is not chosen in equilibrium is observed , then further restrictions can be derived. Under both assumptions on the data, if the data can be rationalized then I construct a rationalizing utility function.

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# Contents

Chapter 3

# Introduction

Intellectual property, particularly patents, are of vital and growing importance to the global economy. Although patent races have received considerable attention in the economic literature, the strategic use of publications in innovation races have not. This dissertation focuses on one important reason for public disclosure of research results: defensive publication. Firms can use publications to defend their own inventions from being patented by another company, to increase the probability of leapfrogging the leader and winning the race, and to preempt a competitor from obtaining a patent.

The main novelty of this work is in demonstrating the use of defensive publications in a dynamic R&D race. Importantly, and in contrast to most previous economic models of R&D races, a firm's progress is compared not only to its rival's progress but also to the state of the prior art. In the model, to be eligible for a patent a firm needs to be the first to accomplish a fixed number of steps above the prevailing prior art. Firms that publish can alter the state of the prior art so that more innovation steps are required. Publication sets back both competitors relative to their goal of advancing $n$ steps above the state of the prior art. Thus, the race considered in this paper is one in which there is no fixed finish line.

I present a model where two identical firms are engaged in a race for the same invention. The winner of the race enjoys a prize. The loser obtains no payoff from the innovation and incurs a loss of its R&D investment. Progress is measured in discrete steps. We assume that the number of successes each firm has and the state of the prior art are observed. There is uncertainty with respect to R&D which is modeled as a Poisson process. To emphasize the defensive purpose of research disclosure, publications are assumed to have no direct payoff, and firms do not possess market power over an innovation once it is made public.

Unlike a single stage race, in the multi-stage race there are situations in which one firm has a lead over its rival. This position of relative strength may bring about victory, but it may also be reversed. A multi-stage race allows us to study the behavior of leaders and followers and it allows for strategic interaction between the competitors during the race, in particular the strategic use of publications.

First consider publications in a simple model in which the instantaneous probability of success is fixed. Here, the only decision the firms face is whether or not to publish. The nature of the Markov perfect equilibrium can be best described as follows: a competitor who is behind in the race will publish research results in order to set back the leader when the leader is very close to obtaining a patent. Because publishing prolongs the race and delays the eventual reception of the prize, the follower will want to wait for as long as possible before publishing. By doing so the follower avoids unnecessary publications in the event he makes favorable progress in R&D. For the same reason, the follower will choose to publish the least possible to prevent the competitor from winning, and will do so only when he is far enough behind.

When the firms are patient and have no marginal cost of investment, it is simple to solve the dynamic game. The instantaneous probability of success does not enter the dynamic programming equations. Firms publish whenever their rival completed $n-1$ steps and the expected value in equilibrium depends only on the relative position of the competitors.

Assuming a small publication fee, this Markov perfect equilibrium is unique.

To investigate the effect of publications when the classical assumption that the instantaneous probability of success increases with investment holds, I consider a race with a convex cost function. The model becomes significantly more complex when firms decide on both publications and the intensity of research. Having the ability to respond to a change in their competitive positions with a change in investment, firms may avoid publications. Whether or not firms publish in equilibrium depends on the cost function, and the patentability requirement. When investment is costly, followers defensively publish. For a quadratic cost function, we numerically find equilibria varying parameter values showing that publications substitute effort. We find that the leader invests more than the follower and discuss the welfare affects of publications. When the patentability requirement is high leaders may also have an incentive to publish.

An important question relevant to any applied theoretical model is what are its empirical implications. In the second chapter of this dissertation I study the non parametric testable restrictions of Markov perfect equilibria in the defensive publications model. The analysis completes the theoretical results in showing that the model is refutable and providing (implicitly) its testable restrictions. This analysis serves as an example of how to apply insights from Afriat theorem to the investigation of the testability of Markov perfect equilibria in a strategic environment. I describe the data that might be observed in order to test the model. This data would consist of publications and investment levels along a realized equilibrium path. Then, I define dynamic consistency of the data which is a basic set of restrictions on data so that testing for equilibria would make sense. For a data set that is dynamically consistent I use the equilibrium conditions from chapter 1 and a set of Afriat-like inequalities (to guarantee the convexity of the cost function) to provide the non parametric testable restrictions. I show that the model is refutable by deriving an explicit restriction on the parameters of the model. The restriction is on investment levels that can be observed. There exist dynamically consistent data sets that do not satisfy this restriction. Thus, the model is testable. Chapter 2 also shows how to check for empirical existence of defensive publications equilibria in the absence a general existence result.

While chapter 2 shows that it is possible to test the defensive publications model using data from an observed equilibrium path, sometimes when variables off an equilibrium path cannot be observed, it might be impossible to refute a model. The freedom to choose values for the unobserved variables may allow us to rationalize any data set. That is, its always possible to choose some parametric function for which the data is consistent with equilibrium. Such a model would not be testable. An example of this difficulty is provided in Chapter 3 of this dissertation.

In the third chapter, a moral hazard model of mutual insurance is studied. In the model, there are infinitely many identical consumers. Wealth can take two possible values. An agent can choose a private action that affects his distribution of wealth. A choice of high effort increases the probability of the good state, but is more costly. Agents can insure by buying securities that pay a unit of consumption contingent on the realization of their random wealth. An insurance company that serves as an intermediary sells the assets. Due to the asymmetric information, an insurer cannot set prices that depend on the effort level unless he imposes incentive compatibility constraints. If the data only contain information related to agents' optimal effort choice, but no information regarding the off equilibrium effort level, then the testable implications are minimal. The only implications are the following weak accounting restrictions: a budget constraint, actuarially fair prices and consumption smoothing. However, if additional information is observed on the probabilities of loss associated with the effort level that is not chosen in equilibrium, then further restrictions can be derived. There are data sets where the budget constraint, fair prices and the consumption smoothing conditions are satisfied but the data cannot be rationalized by our simple moral hazard model. Under both assumptions on the data, if the data can be rationalized then I construct a rationalizing utility function. The results of this chapter suggest that the testable implications of moral hazard models may be minimal if only variables related to the equilibrium path are observed.

# Chapter 1

## 1 Defensive Publications in a Patent Race

### 1.1 Motivation and Evidence

The purpose of this chapter is to provide a formal model of a dynamic innovation race with strategic publications. The analysis takes into account the fact that innovation is evaluated against the state of the prior art. Publications increase the state of the prior art, thus making it more difficult for competitors to patent (or to make profit).

According to the patent law, the prior art is composed of patented innovations, inventions that are known and used by others and printed publications. Patent applicants provide prior art citations and the patent examiner can search for additional prior art that may have a bearing on patentability. To be entitled for patent protection, an invention must sufficiently improve upon the prior state of the art. Patent examiners are instructed to review and analyze patent applications in conjunction with the state of the prior art. They need to determine whether the claims define a useful, novel, nonobvious, and enabled invention. According to the Manual of Patent Examining Procedures: "By far the most frequent ground of rejection is on the ground of unpatentability in view of the prior art, that is, that the claimed subject matter is either not novel...or else it is obvious...".[1] By publishing research results, firms can change the state of the prior art, thus affecting the

[1] See section 706.02 in the Manual of Patent Examining Procedure (1993).

patentability of related inventions.

The patentability requirement is determined by the interpretations of the novel and nonobvious requirement (O'Donoghue 1998). According to Schotchmer and Green (1990) "The requirements of novelty and nonobviousness are hard to interpret. They are judicially determined standards, administered by the patent office and litigated in Federal Courts." The strength of the patentability requirement can serve as a tool of patent policy. The patententability requirement can be changed by a change in the standards for patentability. Issuing new guidelines can strengthen the patentablity requirement and make patenting more difficult. For example on the decision to make gene-patenting more difficult see Pollack (2000). In this paper, the patentability requirement is modeled as the need to advance a discrete number of innovation steps $n$ above the state of the prior art in order to get the prize. While the interpretation here is of a patentability requirement, we can alternatively consider an innovation race in which profits from the innovation can be made only with a sufficient improvement over publicly available technology.

There are various ways of securing defensive publications. Nearly any dated publication may count as a defensive publication. However, a clear publication that is easily available to patent examiners might provide a safer defense. Two places were companies may consider posting their defensive publications are on the web site IP.com and in the journal *Research Disclosure*, published by Kenneth Mason Publications Ltd. The journal explains the rationale for its existence as follows:

> "Research Disclosure is a defensive-type publication serving the scientific and

patent communities worldwide...The pages of the journal are available to companies who, due to the special nature of an invention, seek a low cost alternative, or supplement, to obtaining patents and require prompt publication whilst maintaining freedom for their own use of that invention."

The company IP.com, Inc. was formed with a web based service that allows defensive publication. "IP.com offers a defensive publication service that enables inventors to rapidly place inventions into the public domain." According to Milstein (2002) IP.com found interest mainly among midsize companies.

Some companies publish their own journal containing research disclosures. Xerox for instance publishes the *Xerox Disclosure Journal.*

> "Xerox, like several other large R&D corporations, publishes on the subject matter of inventions which it might use in the future but for which patent protection is not warranted. In this way, it preserves the right to use the invention against any third parties who may later come up with the invention and try to patent it"[2].

Baker, Lichtman and Mezzeti study the case of IBM's disclosures. They find that approximately one out of six patents issued to IBM (between 1/96 and 7/01) cite as prior art at least one article from the IBM Technical Disclosure Bulletin. This data may support the use of publications in an on going patent race.

[2]See http.xerox.com/research/xdj/.

This example was provided in a working paper by Baker, Lichtman and Mezzeti.

Interestingly, the US Patent and Trademark office offers innovators the choice of defensive publication. A statutory invention registration (SIR) has the defensive attributes of a patent but comes without the enforceable attributes[3]. Such publications must include a waiver of the applicant's right to receive a patent on the invention. A notice of the defensive publication appears in the *Official Gazette*.

I review some related literature in section 1.2. The model is presented in section 1.3. Section 1.4 analyzes the simple fixed instantaneous probability race. In section 1.5, I consider a race with joint investment and publication decisions.

## 1.2 Related Literature

Parchomovsky (2000) drew attention to the fact that patentability is a function of the state of the prior art which gives firms the power to affect the patentability of a rival's invention by altering the state of the prior art. Lichtman et al. (2000) challenge and expand the work of Parchomovsky. In contemporary independent work, Baker Lichtman and Mezzetti model disclosure strategies. In their model a patent is issued when the knowledge exceeds the prior art by some sufficient measure determined by the law. Their game has three period. In the first period, one firms conducts research and decides on disclosure, in the second period the second firm conducts research and in the third period the first firm conducts research again if a patent was not issued yet. Their work emphasis leader publications. Following

[3]Among the companies with SIR publications are Shell Oil company with at least 40, Exxon with at least 9 and Fuji and Xerox with at least 6 each. For details on SIR see 35 U.S.C. 157 in the Manual of Patent Examining Procedure (2000).

the model, Baker et al present a case study of IBM's disclosure.

In the standard dynamic models of a patent race such as Dasgupta and Stiglitz (1980) and Reinganum (1981), the race ends with a victory after a single success and the focus is on the investment decisions. Fudenberg et al. (1983) show that a multi-stage R&D process allows leapfrogging. Reinganum (1985) studies a market with a sequence of innovations in which a leader enjoys temporary monopoly power. In her paper, firms' R&D efforts influence the time of discovery. The literature on patent policy design often considers the cumulative nature of innovation. Green and Scotchmer (1995) study the roles of patent length and breadth when innovation occurs in two stages. Scotchmer and Green (1990) and O'Donoghue (1998) investigate the patentability (or novelty) requirement. According to their models, a strong patentability requirement can stimulate R&D spending. In this paper, we show how a strong patentability requirement may lower R&D investment when publications are considered.

The model in this paper is closely related to the one in Harris and Vickers (1987). In their model, each player is striving to reach a given finishing line before his rival. Every state is two-dimensional, one dimension for each player's current distance from the finish line. They focus on how the efforts of competitors in a race vary with the intensity of rivalry. In contrast to their work, the finish line in this paper is not fixed since the prior art can be changed by publications. A main result in their paper as well as in Grossman and Shapiro (1987) is that the leader invests more than the follower. We verify that this continues to hold when the cost of effort is quadratic.

## 1.3 The Model

### 1.3.1 Payoffs and States

Two identical firms, $a$ and $b$, are engaged in a patent race. The race is a multi-stage race; an innovation is composed of discrete steps. Time is continuous and the horizon is infinite. Innovation occurs according to a Poisson process. The date at which a new step will be discovered is random and depends on both firms' instantaneous probabilities of success (the hazard rates). The hazard rate $\lambda^i$ for each firm $i = a, b$ increases with its research effort. Firm $i$'s probability of discovering the next new step is $\frac{\lambda^i}{\lambda^i+\lambda^j}$, where $i \neq j = a, b$. Research has a flow cost $c(\lambda^i)$.

An innovation is evaluated against the prior state of the art which is a measure of the current knowledge in the market regarding the innovation. The prior art is normalized to zero at the beginning of the race. To obtain a patent, a firm needs to be the first to accomplish a fixed number of innovation steps, $n$, above the state of the prior art. Firms know how many steps have been achieved by each firm and the state of the prior art.[4] However, they cannot observe the content of their rival's research, unless the firm chose to disclose that information. A firm may choose to publish research results. Publication changes the state of the prior art. For simplicity, we assume (unless we note otherwise) that

[4] Common knowledge of the state of the prior art simplifies the analysis. We find this assumption reasonable for the following reasons: 1) previous patents are public, 2) publishing firms want to be sure the patent examiner has access to their ideas (see Milstein (2002)) and 3) innovating firms are likely to make efforts to be well informed about the state of the prior art since it can help their research and in order to avoid losses due to rejected applications and law suits.

publication is free. Each successful step that was published increases the prior state of the art by one step. A simultaneous publication of one step by both firms has the same effect as a publication by one of the firms, since it is assumed to add the same information to the state of the prior art.

The states of the world are described by a pair of state variables that measure the number of unpublished innovation steps of each firm above the prior state of the art. The state $\mathbf{s} = (s^a, s^b)$ denotes the situation in which firm $a$ had $s^a$ unpublished innovation steps above the state of the prior art and firm $b$ had $s^b$ innovation steps above the prior art. Let the state space be:

$$S = \left\{ (s^a, s^b) : s^i = 0, 1, 2, ..., n \right\}.$$

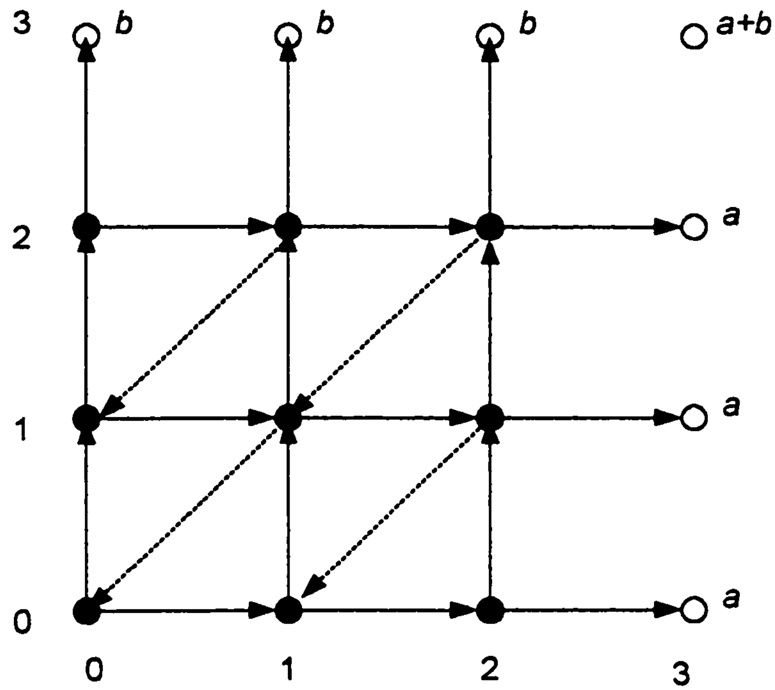
The number of successful innovation steps for each firm $s^i$ increases by one whenever firm $i$ experiences a success. If firm $i$ publishes the results of $k$ innovation steps, the state of the prior art increases by $k$ steps. Consequently, the number of steps above the prior art for each firm decreases by $k$ successes (or is equal to zero if a firm had less than $k$ successes). Figure 1a (on the next page) illustrates the states of the world and the feasible transition from each state. From each node, there is a transition to the right (upwards) if firm $a$ ($b$) was the first to advance a new step. A publication is illustrated with a transition diagonally down. Figure 1b illustrates an example of an equilibrium path.

The value of the patent is $v$ for the winning firm, and zero for the loser. The reward function at the terminal states is

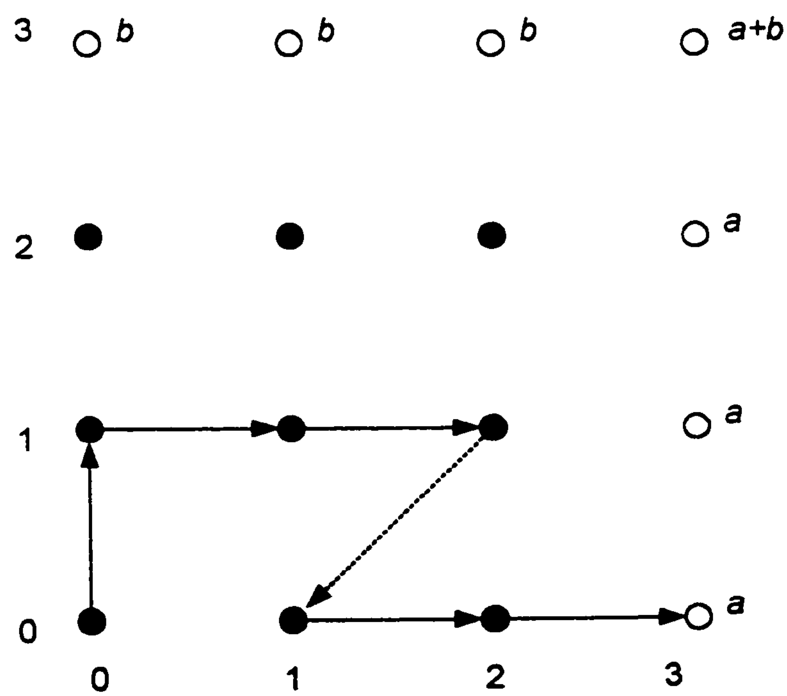
$$r^i(s^a, s^b) = \begin{cases} v \text{ if } s^j < s^i = n \\ \frac{v}{2} \text{ if } s^j = s^i = n \\ 0 \text{ if } s^i < s^j = n \end{cases}.$$

The tie-breaking rule can be interpreted as each firm obtaining the patent with probability $\frac{1}{2}$. The flow cost of research is $-c(\lambda)dt$. The payoff of the game at state s is simply the discounted expected value of rewards for firm $i$.

**Figure 1**




1a. Feasible transitions in a race with n=3




1b. A sample equilibrium path

### 1.3.2 Strategies and Equilibrium.

A strategy may depend at every point of time on the entire history of the game up to that point. Markov strategies are such that past play influences current play only through its effect on the state variables. In every state $\mathbf{s} = (s^a, s^b)$, $s^i < n$ firms make their publication choice, followed by a choice of effort when no publication occurred. Instead, if a firm published, there is a transition to a new state. In the publication stage, let the set of actions for firm $i$ be $A^i(\mathbf{s}) = \{0, 1..., s^i\}$. In the investment stage, let the set of actions be $R_+$, all possible hazard rates.

**Definition 1** *A pure Markov strategy for player i is a function:*

$$f^i : S \rightarrow A^i(\mathbf{s}) \times R_+.$$

A Markov perfect equilibrium $(f^a, f^b)$ is a pair of Markov strategies that yield a Nash equilibrium in every proper subgame.

**Definition 2** *A Markov perfect equilibrium $(f^a, f^b)$ is a pair of Markov strategies such that at every subgame with initial state* $\mathbf{s}$ *and for every strategy g for that subgame,*

$$V^i(\mathbf{s}, f^i, f^j) \geq V^i(\mathbf{s}, g, f^j).$$

Denote firm $i's$ equilibrium value at state $(s^a, s^b)$, $V^i_{s^a,s^b}$. We restrict attention to symmetric Markov perfect equilibria. In a symmetric equilibrium,

$$V^a_{s,s'} = V^b_{s',s}.$$

For simplicity of notation, we sometimes omit the firms' index.

## 1.4 Defensive Publications

We begin this section with the general formulation of the dynamic programing equations making no assumptions on the interest rate $r$ and the cost function $c(\lambda)$. We then turn to the fixed hazard rate model.

### 1.4.1 Discrete Time Model

In this subsection I derive the continuous time dynamic programing equations as a limit of a discrete time version of the model.

Suppose time is discrete. Let the interval between two time periods be $\Delta$. Denote firm $i's$ probability of success in period $t$ and state $(s^i, s^j)$ in the absence of publications by $p^i_{s^i,s^j} = \Delta\lambda^i_{s^i,s^j}$. The probability that firm $i$ does not succeed is $1 - p^i_{s^i,s^j}$. At time $t + \Delta$ the new state is $(s^i + 1, s^j)$ if firm $i$ had a success and firm $j$ did not. The probability of this event is $p^i_{s^i,s^j} \times (1 - p^j_{s^i,s^j})$. If only firm $j$ had a success the state becomes $(s^i, s^j + 1)$ The probability of this event is $p^j_{s^i,s^j} \times (1 - p^i_{s^i,s^j})$. Both succeed to advance another state with probability $p^i_{s^i,s^j} \times p^j_{s^i,s^j}$. The state in this event is $(s^i + 1, s^j + 1)$. Finally, no firm advances with probability $(1 - p^i_{s^i,s^j}) \times (1 - p^j_{s^i,s^j})$ in which case the state remains $(s^i, s^j)$. Let the discount factor be $\frac{1}{1+r\Delta}$ . The discrete time cost function is $C(p^i_{s^i,s^j})$ and denote

$$c(\lambda^i_{s^i,s^j}) = \lim_{\Delta\to 0} \frac{C(p^i_{s^i,s^j})}{\Delta}.$$

The value to firm $i$ at state $(s^i, s^j)$ is the discounted expected value from all four states to which a transition is possible minus the cost of investment. The value at state $(s^i, s^j)$ is:

$$V^i_{s^i,s^j} = \frac{1}{1+r\Delta}[p^i_{s^i,s^j} p^j_{s^i,s^j} V^i_{s^i+1,s^j+1} + p^i_{s^i,s^j}(1 - p^j_{s^i,s^j})V^i_{s^i+1,s^j} + p^j_{s^i,s^j}(1 - p^i_{s^i,s^j})V^i_{s^i,s^j+1}$$
$$+(1 - p^i_{s^i,s^j})(1 - p^j_{s^i,s^j})V^i_{s^i,s^j}] - C(p^i_{s^i,s^j}).$$

We multiply the equation by $1 + r\Delta$ and rearrange to obtain:

$$0 = [p^i_{s^i,s^j} p^j_{s^i,s^j} V^i_{s^i+1,s^j+1} + p^i_{s^i,s^j}(1 - p^j_{s^i,s^j})V^i_{s^i+1,s^j} + p^j_{s^i,s^j}(1 - p^i_{s^i,s^j})V^i_{s^i,s^j+1}$$
$$+(p^i_{s^i,s^j} p^j_{s^i,s^j} - p^i_{s^i,s^j} - p^j_{s^i,s^j} - r\Delta)V^i_{s^i,s^j}] - (1 + r\Delta)\, C(p^i_{s^i,s^j}).$$

We divide this equation by $\Delta$ and recall that $p^i_{s^i,s^j} = \Delta\lambda^i_{s^i,s^j}$ :

$$0 = [\Delta\lambda^i_{s^i,s^j} \lambda^j_{s^i,s^j} V^i_{s^i+1,s^j+1} + \lambda^i_{s^i,s^j}(1 - \Delta\lambda^j_{s^i,s^j})V^i_{s^i+1,s^j} + \lambda^j_{s^i,s^j}(1 - \Delta\lambda^i_{s^i,s^j})V^i_{s^i,s^j+1}$$
$$+(\Delta\lambda^i_{s^i,s^j} \lambda^j_{s^i,s^j} - \lambda^i_{s^i,s^j} - \lambda^j_{s^i,s^j} - r)V^i_{s^i,s^j}] - (1 + r\Delta)\, \frac{C(\Delta\lambda^i_{s^i,s^j})}{\Delta}.$$

Take the limit as the time interval becomes infinitesimal, $\Delta \to 0$ and obtain:

$$0 = [\lambda^i_{s^i,s^j} V^i_{s^i+1,s^j} + \lambda^j_{s^i,s^j} V^i_{s^i,s^j+1} + (-\lambda^i_{s^i,s^j} - \lambda^j_{s^i,s^j} - r)V^i_{s^i,s^j}] - c(\lambda^i_{s^i,s^j}).$$

Rearrange to find the continuous time dynamic programming equation:

$$rV^i_{s^i,s^j} = [\lambda^i_{s^i,s^j}(V^i_{s^i+1,s^j} - V^i_{s^i,s^j}) + \lambda^j_{s^i,s^j}(V^i_{s^i,s^j+1} - V^i_{s^i,s^j}) - c(\lambda^i_{s^i,s^j}).$$

In continuous time, from a given state $(s^i, s^j)$ we may transition to one of the two states $(s^i + 1, s^j)$ or $(s^i, s^j + 1)$ may occur. the event where both competitors made progress at exactly the same instant has a zero probability. Thus a continuous time model simplifies the analysis and notation. It also eliminates phenomena that rely heavily on the unlikely event that both firms make progress at the same instant. Consider the following example in the discrete time model.

**Example 1** *Suppose the probability of success is fixed at* $p = 1$, *the number of steps required for the patent is* $n = 3$ *and the discount rate is* $r = 0$. *Starting from the* $(0,0)$ *state, firms will advance at the same time to state* $(1,1)$, *then* $(2,2)$ *and then end the game with a tie and split the prize each getting* $\frac{v}{2}$. *What does the Markov Perfect equilibrium strategy predict at the states* $(0,1)$ *and* $(1,2)$ *which are not on the equilibrium path? The follower expects to loose. If the follower's strategy is to publish at state* $(1,2)$ *then the leader would never get the prize unless she publishes at state* $(1,0)$. *If she does not publish in state* $(1,0)$, *the states would keep changing from* $(0,1)$ *to* $(1,2)$ *and back to* $(0,1)$. *If on the other hand the leader publishes at state* $(1,0)$ *then they reach the symmetric state that will let them enjoy the prize. Thus, we found an example with leader publications at* $(1,0)$. *In the continuous time model, the transition from state* $(1,0)$ *is to either one of the states* $(1,1)$ *or* $(2,0)$ *and there are no leader publications at* $(1,0)$.

### 1.4.2 Dynamic Programming

When each firm $i$ exerts effort that yields a hazard rate $\lambda^i_{s^i,s^j}$ the expected payoff $V^i_{s^i,s^j}$ satisfies the dynamic programming equation:

$$rV^i_{s^i,s^j} = \lambda^i_{s^i,s^j}(V^i_{s^i+1,s^j} - V^i_{s^i,s^j}) + \lambda^j_{s^i,s^j}(V^i_{s^i,s^j+1} - V^i_{s^i,s^j}) - c(\lambda^i_{s^i,s^j}).$$

That is, the rate of return equals the expected gains and losses minus the flow cost. Solving for the expected value for firm $i$ at $(s^i, s^j)$ we obtain:

$$V^i_{s^i,s^j} = \frac{\lambda^i_{s^i,s^j} V^i_{s^i+1,s^j} + \lambda^j_{s^i,s^j} V^i_{s^i,s^j+1} - c(\lambda^i_{s^i,s^j})}{r + \lambda^i_{s^i,s^j} + \lambda^j_{s^i,s^j}}.$$

In a symmetric equilibrium

$$\lambda^j_{s^i,s^j} = \lambda^i_{s^j,s^i}.$$

Insert $\lambda^j_{s^i,s^j}$ and for simplicity of notation drop the firm index to obtain:

$$V_{s^i,s^j} = \frac{\lambda_{s^i,s^j} V_{s^i+1,s^j} + \lambda_{s^j,s^i} V_{s^i,s^j+1} - c(\lambda_{s^i,s^j})}{r + \lambda_{s^i,s^j} + \lambda_{s^j,s^i}}. \tag{1}$$

When at least one of the firms achieved $n$ innovation steps, the patent is granted and the game ends. The values at such terminal nodes are given by:

$$V^i_{n,s^j} = v, \quad V^j_{n,s^j} = 0 \qquad \text{for } 0 \leqslant s^j \leqslant n-1. \tag{2}$$

If either firm publishes $k \leqslant \min(s^a, s^b)$ steps at state $(s^a, s^b)$, the values at that state will satisfy:

$$V^i_{s^a,s^b} = V^i_{s^a-k,s^b-k} \qquad \text{for } i = a, b. \tag{3}$$

Let us fix a strategy for firm $j$ and consider firm $i$'s best response at states in which firm $j$ does not publish. Firm $i$'s best response strategy solves[5]

$$\max_{A^i(s)\times R_+} \left[ \overbrace{V^i_{s^i-1,s^j-1}}^{\text{payoff if } i \text{ publishes}}, \overbrace{\frac{\lambda^i_{s^i,s^j} V_{s^i+1,s^j} + \lambda^j_{s^i,s^j} V_{s^i,s^j+1} - c(\lambda^i_{s^i,s^j})}{r + \lambda^i_{s^i,s^j} + \lambda^j_{s^i,s^j}}}^{\text{payoff is } i \text{ does not publish}} \right].$$

[5]We consider the value after publication of one step. Publication is possible at state $(s^i - 1, s^j - 1)$, therefore, the value at this state is at least as high as the value if the firm publishes $k > 1$ steps.

The use of publication sets back the race. Firms will need to achieve more innovation steps as a result of publications since inventions are evaluated against the prior state of the art and only unpublished steps count. Thus, publication may prolong the race. This may be costly to the firms, if it increases their research expenses. If the discount rate is positive, prolonging the race also reduces the expected discounted value of the patent. On the other hand, publication may be beneficial to a firm if it improves its chances of winning the race, and if the use of publications substitutes effort. In the presence of this trade-off between the potential cost and benefit of publications, we expect it to be used in some but not all races.

### 1.4.3 The Race With a Fixed Probability of Success

In this subsection we consider the cost function

$$c(\lambda) = \begin{cases} 0 \text{ for } \lambda \leqslant \overline{\lambda} \\ \infty \text{ for } \lambda > \overline{\lambda} \end{cases} .$$

With this cost function, the hazard rate is fixed and equal for both firms in all states. We focus on publication as the choice variable avoiding the extra complexity that an investment decision inflicts. This choice simplifies the model sufficiently to offer an analytical solution to the model.

Since the hazard rate is equal for both firms and constant, both firms have equal probabilities to be the first to advance a step. The expected discounted value at $(s^a, s^b)$ is given by

$$V^i_{s^a,s^b} = \gamma(V^i_{s^a+1,s^b} + V^i_{s^a,s^b+1}), \qquad \text{where } \gamma = \frac{\overline{\lambda}}{r + 2\overline{\lambda}}. \tag{4}$$

The first theorem shows us that for any patentability requirement of $n \geq 3$ innovation steps, there are parameter values for which publishing research results is in the best interest for firms engaged in a patent race. The incentive to publish arises for a follower. We need at least 3 steps to have a state in which a follower has something to publish. When $n = 1, 2$ no publication in all states is an equilibrium strategy.

**Theorem 1** *(Incentive to Publish)*

*For $n \geq 3$, a Markov perfect equilibrium must involve publications when $n - \frac{1-\gamma}{\gamma^2} > 0$, which holds when $r$ is small enough, when $\overline{\lambda}$ is high enough or for a large number of steps $n$.*

We recall that all proofs are provided in the Appendix. Intuitively, publications prolong the race but may improve a follower's probability of catching up with the leader. With a small interest rate or with a large hazard rate, the cost of prolonging the race is small and the benefit outweighs it. To prove the theorem, we show that if the no publication strategy is used by both firms, then the follower has an incentive to deviate and publish when the leader is one step away from obtaining the patent, thereby showing that no publication cannot be an equilibrium.

We now investigate the use of publications in equilibrium. In the presence of the trade-off, between the cost and benefit of publications, we expect a follower to wait with publications to the last moment (when the leader has attained $n-1$ steps). In this way the follower can use publication when there is risk that the leader might win the race after the next innovation step, but avoid unnecessary publications if he gets lucky. Further publication may be postponed until the next time the firm would lose, following an innovation step of its rival.

Patient firms can enjoy the benefit of publication (improving the probability of catching up) without worrying about prolonging the race. This suggests that the follower will always publish when the leader is a step away from winning the race. We refer to this strategy as the "publish as a last resort" strategy. When $r = 0$, the values of the game in every state are independent of $\overline{\lambda}$. This simplification, allows us to find an explicit form for the values of the game in every state under the proposed strategy. In proposition 1 we find these values.

**Proposition 1** *(Value Function)*

*When $r = 0$ and the firms choose the "publish as a last resort" strategy the values in every state are:*

$$V^i_{s^i,s^j} = \frac{n + s^i - s^j}{2n} v \qquad \text{for all} \quad 0 \leqslant s^i, s^j < n. \tag{5}$$

We use a mathematical induction to prove that without discounting, when the firms use the "publish as a last resort" strategy, each value $V_{s^a,s^b}$ equals the value $V_{s^a-1,s^b-1}$. Thus, the values depend only on the distance between the two competitors. The follower publishes whenever the leader is close to the patent. Hence, the leader can secure a patent only after the follower has gradually published all his successes.

In theorem 2 we present the results on equilibrium publishing behavior for patient and impatient firms. For patient firms, we show that "publish as a last resort" is a Markov perfect equilibrium. For impatient firms, theorem 2 shows that if in equilibrium the leader does not publish and the follower publishes at some state $(s, n-1)$, then the follower will also publish at all states in which the leader completed $n-1$ steps and the follower completed less than $s$ steps. Thus, when the leader is a step away from the patent, the follower is more likely to publish the further behind he is.

**Theorem 2** *(Equilibrium Publishing Behavior)*

1. *There is some* $\bar{s}$ *such that in equilibrium (if the leader does not publish) the follower publishes in states* $(s, n-1)$ *if* $0 < s < \bar{s}$ *and does not publish if* $s > \bar{s}$.

2. *When the discount rate is small enough, the strategy "publish as a last resort" is a Markov perfect equilibrium.*

To prove the first part of theorem 2, we show that if the follower's expected value of publishing at state $(s, n-1)$ is larger than his expected value if he does not publish, then this inequality will also hold in state $(s-1, n-1)$. Intuitively this monotonicity result holds true since the follower will avoid publication when his chances of leapfrogging ahead and winning the race are large enough. The closer the follower is to the leader, the higher these chances are.

To prove the second part we use the continuation values for each state from proposition 1, and verify that the equilibrium conditions are met. Finally, we argue that the result hold in a neighborhood of $r = 0$.

A further assumption is required to obtain uniqueness of the Markov perfect equilibrium for patient firms. The reason for multiple equilibria is that a simultaneous decision on publication implies that when one firm publishes in some state, the other firm is indifferent between publishing and not publishing at that state. Moreover, if the interest rate is $r = 0$, by proposition 1, $V_{s^a,s^b} = V_{s^a-1,s^b-1}$ for $0 \leqslant s^a, s^b < n$; therefore, there are additional payoff equivalent strategies with more publications. We can ensure uniqueness by assuming a small publications fee[6]. In the next proposition we show the uniqueness of equilibrium for a small discount rate under this assumption.

**Proposition 2** *(Uniqueness)*

*With a small enough discount rate and a small publication fee, publish as a last resort is the unique Markov perfect equilibrium.*

Publications may increase the follower's probability of catching up with the leader, but they prolong the race and delay the prize. For intermediate values of the interest rate, this trade-off may only be resolved with mixed strategies.[7] We solve for the mixed strategy equilibria in a race with $n = 3$ innovation steps.

### 1.4.4 An Example

In order to find the equilibrium strategy in a model with arbitrary discount rate, we restrict attention to a model where only $n = 3$ innovation steps above the prior state of the art are required in order to obtain a patent. In the Markov perfect equilibrium, a leader never has an incentive to publish. We expect to find that if publication occurs, it is by the follower at state $(1, 2)$.

[6] Posting a document on IP.com's web site costs only $155 and requires no lawyers. See Milstein (2002).

[7] Furthermore, with intermediate values of the interest rate and a large patentability requirement, there is a difficulty in ruling out leader publications. Given the behavior of the follower, the leader might have an incentive to narrow the distance between them and maintain a closer race.

When the discount rate is small, the benefit for the follower from publishing (increasing his probability of leapfrogging) outweighs the cost of prolonging the race. In this case there is a pure strategy equilibrium with publications. When the interest rate is large, the cost of prolonging the race offsets the benefit from publications and there is a pure strategy equilibrium with no publications. With intermediate interest rates, the trade-off between the cost and benefit of publications is resolved with mixed strategies. If no one publishes, the follower has an incentive to publish, but if the follower always publishes, the value at state $(1,2)$ is lowered so that a deviation to no publication in this state is profitable. For these parameter values we find a mixed strategy equilibrium in which the follower publishes at state $(1,2)$ with some probability $q$.

**Proposition 3** *(Equilibrium)*

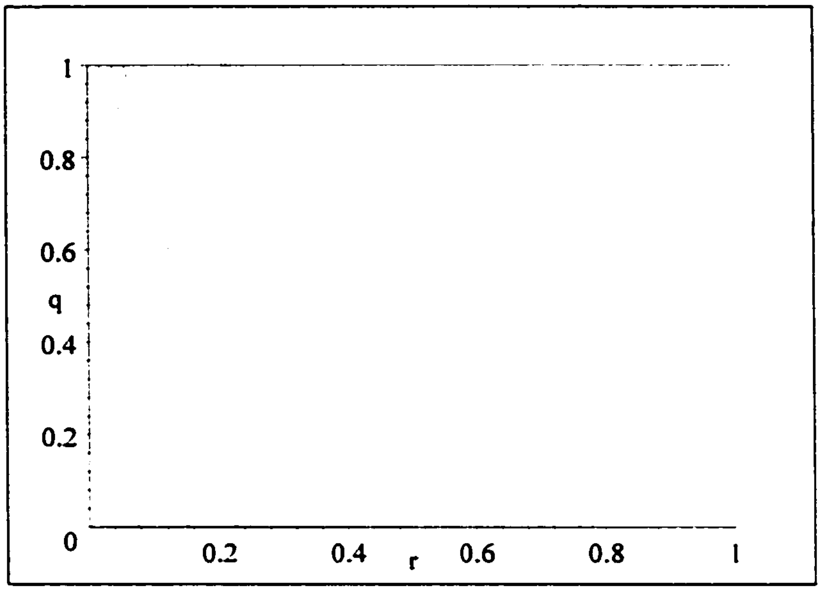
*For every $\overline{\lambda}$, there exist $0 < \underline{r}_{\overline{\lambda}} < \overline{r}_{\overline{\lambda}} < \infty$ such that the unique symmetric Markov perfect equilibrium strategy is: publish only as a follower in state $(1,2)$ with probability:*

$$q = \begin{cases} 1 & \text{for } 0 \leqslant r \leqslant \underline{r}_{\overline{\lambda}} \\ q_\gamma \in (0,1) & \text{for } \underline{r}_{\overline{\lambda}} < r < \overline{r}_{\overline{\lambda}} \\ 0 & \text{for } \overline{r}_{\overline{\lambda}} \leqslant r \leqslant \infty \end{cases},$$

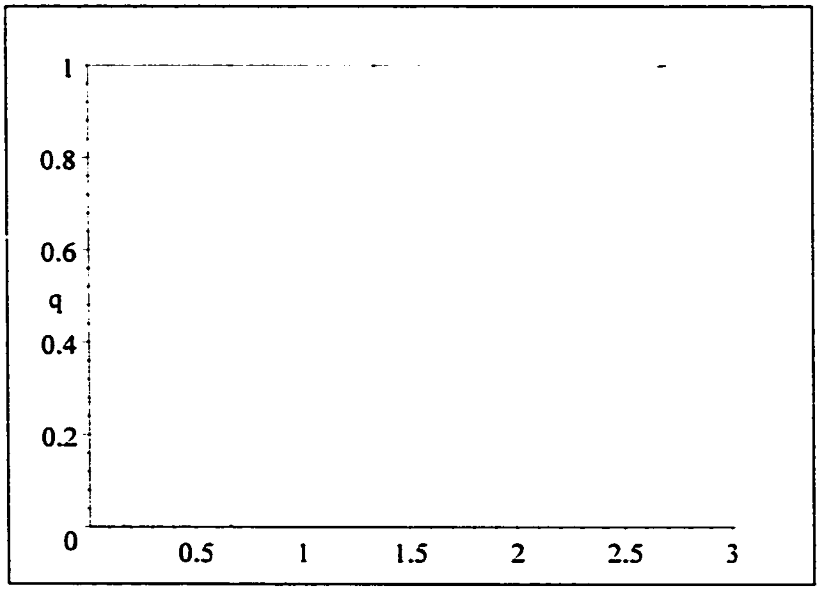
*where $q_\gamma = \frac{\gamma + 3\gamma^2 - 1}{\gamma(1 + 3\gamma^3 - 2\gamma)}$ and $\gamma = \frac{\overline{\lambda}}{r + 2\overline{\lambda}}$.*

Having found an explicit form for the probability of publication, we can derive comparative static results. We find that the lower the interest rate, the more likely the firms are to publish. With respect to the hazard rate, we find that the higher $\overline{\lambda}$, the larger the probability for follower publication. With a high instantaneous probability of success, the cost of publication caused by prolonging the race is relatively low since research advances rapidly. The results are presented in figures 2a, 2b. The probability that a follower publishes $q$ is plotted against the discount rate $r$ for three different values of $\overline{\lambda}$ and against the hazard rate $\overline{\lambda}$ for different values of $r$.

## Figure 2




2a. Probability of publication as a function of r. From left to right $\lambda = .25, .5, .75$.




2b. Probability of publication as a function of the hazard rate $\lambda$. From left to right.

$r = .05, .1, .2.$

## 1.5 Publications and R&D Investment

In this section we combine the standard consideration of strategic R&D investments in an uncertain race with the new feature, publication. We show that for an open set of parameter values the Markov perfect equilibrium must involve publications. Publications may substitute effort when the marginal cost of investment is high.

Harris and Vickers (1987) acknowledged the apparent difficulty in obtaining analytical results in the no publications model. The consideration of strategic publications in addition to the investment decision adds a significant complexity to the model. While the dynamics of the patent race when no publication is allowed is such that each state is only visited once in the publication model, states may be revisited. In the no publication model, the values can be calculated from state $(n-1, n-1)$ backwards. In the publications model, one can no longer do so. We use the results from the simplified fixed hazard rate model to identify the incentive to publish in the race with investment.

Let the cost of research as a function of the hazard rate be:

$$c(\lambda) = \begin{cases} c_0 C(\lambda - \overline{\lambda}) \text{ for } \lambda \geq \overline{\lambda} \\ 0 \text{ for } \lambda < \overline{\lambda} \end{cases}, \tag{6}$$

where $c_0 > 0$ and $\overline{\lambda} \geq 0$. The function $C(.)$ is increasing and strictly convex. We interpret $\overline{\lambda}$ as a fixed hazard rate that each firm has due to its fixed investment. Investment increases the hazard rate, $\lambda$ above the fixed hazard rate $\overline{\lambda}$, and has decreasing returns. In equilibrium, if no publication occurs at state s, and if the optimal hazard rate exceeds $\overline{\lambda}$ then it satisfies the first order condition derived from equation (1)

$$V_{s^i+1,s^j} - V_{s^i,s^j} = c'(\lambda_{s^i,s^j}). \tag{7}$$

That is, in every state the marginal cost of effort equals its marginal benefit. When $\overline{\lambda} > 0$, as $c_0$ goes to infinity, the model approaches the fixed hazard rate model. This ensures the use of defensive publications on a large range of parameter values.

**Proposition 4** *(Incentive to Publish in Investment Race)*

*The Markov perfect equilibrium must involve publications when $n - \frac{1-\gamma}{\gamma^2} > 0$ for a sufficiently large $c_0$.*

Recall that by theorem 1, $n - \frac{1-\gamma}{\gamma^2} > 0$ where $\gamma = \frac{\overline{\lambda}}{r+2\overline{\lambda}}$ is the condition for an equilibrium with publications in the fixed hazard rate model. This condition holds true for example when there is not discounting. As $c_0 \to \infty$, the values of the game converge to the values in the fixed hazard rate model. Thus, the follower has an incentive to publish when the leader is close to the end of the race. Intuitively, since defensive publications substitute investment, they are more likely when the marginal cost is high. Moreover, since publications prolong the race, they are less costly when the firms are patient and when the success probability is high and its average cost low.

**Corollary 1** *When $n - \frac{1-\gamma}{\gamma^2} > 0$ for a sufficiently large $c_0$ publications prolong the race.*

When the probability of success is fixed and the race involves publications, the expected duration of the race must be longer than in a race with no publications. The number of successes above the state if the prior art achieved in this game is larger and the hazard rate fixed. In the investment race, high investments can expedite innovation. However, when the cost parameter increases, the duration of the race must be close to that in the fixed hazard rate model and therefore the race with publications takes longer.

### 1.5.1 An Example

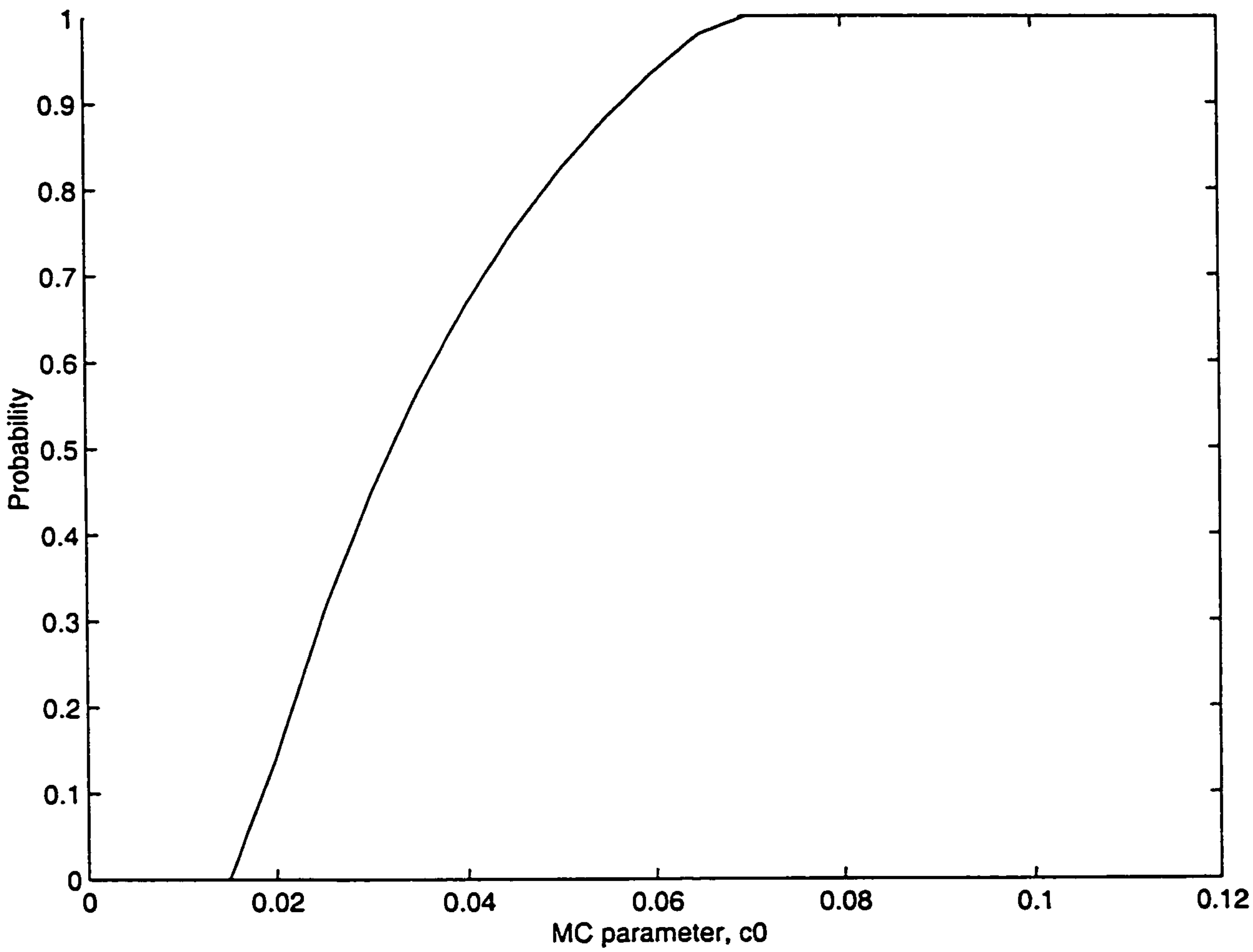
We consider the race with $n = 3$ requirement and patient firm. In this case, the marginal cost is linear, simplifying computation of equilibria. Varying parameter values, we find the equilibria and numerically compute the equilibrium levels of investments. The results are summarized in the following claims.

**Claim 1** *For every $\overline{\lambda} > 0$, there exist $0 < \underline{c}_{\overline{\lambda}} < \overline{c}_{\overline{\lambda}} < \infty$ such that the unique symmetric Markov perfect equilibrium strategy involves follower publication at state* $(1,2)$ *with probability*

$$q = \begin{cases} 0 & \text{for } 0 < c_0 \leqslant \underline{c}_{\overline{\lambda}} \\ q \in (0,1) & \text{for } \underline{c}_{\overline{\lambda}} < c_0 < \overline{c}_{\overline{\lambda}} \\ 1 & \text{for } \overline{c}_{\overline{\lambda}} \leqslant c_0 < \infty \end{cases} .$$

Figure 3 shows the equilibrium probability of publication at state (1,2) as a function of the cost parameter $c_0$. The probability or publication increases with the cost parameter $c_0$. Thus publications substitute investment.

Figure 3




The equilibrium probability of defensive publications at state *(1,2)* for different values of the cost parameter $c_0$. Other parameter values in this simulation where fixed at $v=1$, $\lambda=1$, $r=0$.

We now compare the findings of this model in a race with $n = 3$, to the results of the standard no publications races in Grossman and Shapiro (1987) and Harris and Vickers (1987). We find that when the follower publishes at state $(1,2)$ the leader makes a greater effort than the follower.

**Proposition 5** *(Leader Invests More than Follower).*

*If equilibrium involves publication at state* $(1,2)$, *then for* $s^i > s^j$, $\lambda^i_{s^i,s^j} > \lambda^j_{s^i,s^j}$.

Harris and Vickers (1987) find two additional properties of the equilibrium investment behavior that hold in the no publications race. We find that these properties hold in the publication race as well.

**Claim 2** *The ratio of the leader's efforts to those of the follower's decreases,* $(\frac{\lambda_{2,0}}{\lambda_{0,2}} > \frac{\lambda_{1,0}}{\lambda_{0,1}})$ *and the follower speeds up as the gap between the players narrows.*

In state $(2,0)$, in the publications model, if the leader is the first to complete the next step, she secures the patent. While if the follower advances first, not only the gap will narrow but the follower can also publish. Therefore, we expect both players to invest more compared to a no publications model[8].

**Claim 3** *1. The flows of effort* $\lambda_{0,2}$ *and* $\lambda_{2,0}$ *are higher in the model with publications.*

*2. In all other states,* $\lambda_s$ *is at least as high in the no publications model.*

While the three steps only involves defensive publications, in a race with a strong patentability requirement a leader may also have an incentive to publish. In the invest-

[8] I thank an anonymous referee for this suggestion.

ment race, if $\overline{\lambda} = 0$, and a small number of innovation steps is required, the no publication strategy is an equilibrium. The follower's ability to adjust investment eliminates his incentive to publish. However, when the patentability requirement is high enough ($n \geq 6$, according to numeric computations), the equilibrium must involve strategic publications. The distance from the end of the race has a complex cyclical behavior (see Harris and Vickers (1987)). In this case, a leader has an incentive to deviate from a no publication strategy and publish preemptively. Publications allows the leader to avoid states with fierce competition.

## 1.6 Welfare

Typical welfare issues that concern patent races are: the incentive to innovate; market power; and the duplication of innovation. Not surprisingly, the welfare analysis of this model is complex, and the effects are ambiguous. The patent race model does not include an analysis from which consumer surplus can be determined, the discovery process is random and when no discounting is assumed, an additional difficulty in defining the first best and measuring welfare is presented. It would be particularly interesting if welfare analysis can shed light on policy implications. The public authority can change the patentability requirement and affect the use of publications.

O'Donoghue (1998) suggests that a patentability requirement can stimulate R&D investment and enhance welfare. Scotchmer and Green (1990) compare a weak and a strong patentability requirement. A strong patentability requirement serves the social goal of protecting profits and a weak requirement serves the goal of disclosure through the patent system. The analysis in this dissertation suggests that increasing the patentability requirement may induce strategic information disclosure not accounted for in a model without strategic publications. An important distinction between disclosure in the patent system and publications is that disclosure through the patent system is done by a leader and facilitates rival R&D. Publications on the other hand make patenting more difficult and are not protected in the patent system. Moreover, publications may diminish the ex-ante value of the race. Thus, lowering the incentive to innovate.

Consider a race with a quadratic cost function . Normalize the prize to be $v = 1$ for a race with a patentability requirement $n = 3$. Let the fixed hazard rate component be $\overline{\lambda} = 1$ and the interest rate be $r = 0$. For this game, I numerically found the equilibrium hazard rates varying the value of the parameter $c_0$. The probability to transition from state $(s^i, s^j)$ to state $(s^i + 1, s^j)$ is the probability that firm $i$ is the first to progress an additional step i.e. $\frac{\lambda^i_{s^i,s^j}}{\lambda^i_{s^i,s^j}+\lambda^j_{s^i,s^j}}$.

To find the number of steps published in the game with defensive game we find the member of times state $(1,2)$ is visited. From state $(0,1)$ the probability that the game ends without visiting state $(1,2)$ (i.e. without publication) is:

$$p = 1 - \frac{\lambda^i_{1,0}}{\lambda_{1,0} + \lambda^j_{1,0}} \frac{\lambda^i_{2,0}}{\lambda_{2,0} + \lambda^j_{2,0}}.$$

With probability $(1-p)$, state $(1,2)$ is visited, one step is published and the state becomes $(0,1)$ again. The race is expected to end after $\frac{1}{p}$ such trials. Therefore, the number of published steps is expected to be

$$E(\#\text{ published steps}) = \frac{1}{p} - 1.$$

I compare the equilibrium in a race with publications to a race without publications. While a race with no publication ends with exactly $n = 3$ patented innovation steps above the state of the prior art, in the race with publication firms disclose up to three additional steps. The information disclosed in publications during the race is free of monopoly power. The expected number of steps above the prior state of the art increases with $c_0$. This information disclosure can be welfare enhancing when there are positive externalities to the information disclosed and when disclosure not through the patent system increases competition.

However, publications may have a welfare cost. The expected initial value $V_{00}$ of the race for each firm is lower in the publication race. Thus, publications diminish the incentive to enter the R&D race. In a race without discounting, the expected total cost of investment throughout the race can be inferred from the initial value

$$E(cost) = v - 2V_{00}.$$

Publication has the benefit of free information disclosure, at the cost of decreased incentive to invest and more duplicate research. Weighting the overall effect of publication on the social welfare depends on how we value these effects. The social benefit can be a weighted sum of the expected number of patented and published innovation steps. Since we do not discount, let us consider the time average of the net benefits as a measure of welfare:

$$W = \frac{n + aE(\#\text{ published steps}) - E(cost)}{E(\text{duration of the race})}.$$

If published and patented steps are equally weighed ($a = 1$) then the no publications race (with $n = 2$ or 3) does better. When the weight on disclosed steps is sufficiently large the race with publications gives rise to higher welfare.

## 1.7 Modifying the Model

In this section we discuss some possible extensions to the defensive publications model. This work intended to focus on defensive publications, that is publications by a follower that make it harder for the leader to win. However with some modification, the model allows analyzing additional incentives to publish. I consider three possible extensions in this section.

### 1.7.1 Reward for Publications

According to the model in this chapter there is no direct payoff to publications. Thus, the incentive to publish is derived by the possibility of increasing ones probability of winning the race. In some cases it may be reasonable to assume a positive payoff to publications. Publications can improve a firm's reputation, reward its scientists or allow the use of technology that would if patented by another be costly to use. Such payoff can be modeled as a positive reward $w$ for each publication. While the no publications dynamic programing equation (1) remains unchanged, the value function in a state with publication, equation (3) changes to

$$V_{s,s'} = V_{s-1,s'-1} + w.$$

**Example 2** *Consider a race with $n = 2$, patient firms, fixed hazard rates $\lambda^a = \lambda^b > 0$ and a reward for publications, $w > 0$. Firms have an incentive to publish at state $(1,1)$ since* *they can collect the reward for publications without decreasing their probability of winning the race. Payoffs in the game without publications are provided in table 1. Equilibrium payoffs in the game with publications are provided in table 2.*

*Table 1*

*Firm a's payoffs if firms do not publish*

| $s^b = 0$ | $s^b = 1$ | $s^b = 2$ | $s^b/s^a$ |
|---|---|---|---|
| $v$ | $v$ | $\frac{v}{2}$ | $s^a = 2$ |
| $\frac{3v}{4}$ | $\frac{v}{2}$ | 0 | $s^a = 1$ |
| $\frac{v}{2}$ | $\frac{v}{4}$ | 0 | $s^a = 0$ |

*Table 2*

*Firm a's equilibrium payoff. Firms' strategy is to publish at* $(1,1)$

| $s^b = 0$ | $s^b = 1$ | $s^b = 2$ | $s^b/s^a$ |
|---|---|---|---|
| $v$ | $v$ | $\frac{v}{2}$ | $s^a = 2$ |
| $\frac{3v}{4} + s$ | $\frac{v}{2} + 2s$ | 0 | $s^a = 1$ |
| $\frac{v}{2} + s$ | $\frac{v}{4} + s$ | 0 | $s^a = 0$ |

### 1.7.2 Asymmetric Race

The model analyzed in previous sections assumed that firms are symmetric in every way. Generalizing the model to asymmetric firms can provide additional incentive to publish by a strong firm that has less to loose from setting back the race.

**Example 3** *Consider a race with* $n = 2$, *patient firms and with fixed hazard rates* $\lambda^a > \lambda^b > 0$. *The strong firm, a has an incentive to publish at state (1,1) since with a larger* *distance from the finish line its advantage is more significant. Payoffs in the game without publications are provided in table 3. It is easy to verify that* $V_{00} > V_{11}$. *Equilibrium payoffs in the game with publications are provided in table 4.*

*Table 3*

*Firm a's payoffs in firms do not publish*

| $s^b = 0$ | $s^b = 1$ | $s^b = 2$ | $s^b/s^a$ |
|---|---|---|---|
| $v$ | $v$ | $\frac{v}{2}$ | $s^a = 2$ |
| $\frac{\lambda^a(\lambda^a+2\lambda^b)}{(\lambda^a+\lambda^b)^2}v$ | $\frac{\lambda^a}{\lambda^a+\lambda^b}v$ | 0 | $s^a = 1$ |
| $\frac{(\lambda^a)^2(\lambda^a+3\lambda^b)}{(\lambda^a+\lambda^b)^3}v$ | $\frac{(\lambda^a)^2}{(\lambda^a+\lambda^b)^2}v$ | 0 | $s^a = 0$ |

*Table 4*

*Firm a's equilibrium payoff. Strong firm's strategy is to publish at* $(1,1)$

| $s^b = 0$ | $s^b = 1$ | $s^b = 2$ | $s^b/s^a$ |
|---|---|---|---|
| $v$ | $v$ | $\frac{v}{2}$ | $s^a = 2$ |
| $\frac{(\lambda^a)^3+\lambda^a(\lambda^b)^2+(\lambda^a)^2\lambda^b}{[(\lambda^a)^2+(\lambda^b)^2](\lambda^a+\lambda^b)}v$ | $\frac{(\lambda^a)^2}{(\lambda^a)^2+(\lambda^b)^2}v$ | 0 | $s^a = 1$ |
| $\frac{(\lambda^a)^2}{(\lambda^a)^2+(\lambda^b)^2}v$ | $\frac{(\lambda^a)^3}{[(\lambda^a)^2+(\lambda^b)^2](\lambda^a+\lambda^b)}v$ | 0 | $s^a = 0$ |

### 1.7.3 Fixed Cost

When there is a fixed component to the instantaneous cost of investment

$$c(\lambda) = c_0 + C(\lambda),$$

a competitor would drop out of the race when its expected value of staying becomes negative. In such a model, a leader may have an incentive to publish if setting back the race would make the follower drop out.

# Chapter 2

## 2 Empirical Implications of Markov Perfect Equilibria: The Defensive Publications Model

### 2.1 Introduction

This chapter studies the non-parametric empirical implications of a patent race model with publications. In the previous chapter the model was presented and a symmetric Markov perfect equilibrium was derived. This chapter shows that the hypothesis that a data set is consistent with this equilibrium has nonvacuous testable restrictions. Moreover, I show how to construct a cost function that rationalizes data that satisfy the restrictions.

The chapter serves as an example of how insights from revealed preference theory can be applied to derive testable implications of strategic games. The method has been successfully applied to general equilibrium models. However, two difficulties may arise when testing a game theoretical model. First, the literature on testing general equilibrium models normally assumes that several observations of economies sharing the same functional characteristics can be observed[9]. A game, such as a patent race between two firms, may have unique characteristics (cost function) so that in fact, we can only hope to obtain one observation of an equilibrium path. This difficulty can be overcome by using the stationary nature of the model. Firms' investments in different states provide multiple realizations of points on the

[9] An exeption is Kubler's (1999) work on financial markets.

graph of the cost function, so that data from a single equilibrium path can be rich enough to test the model. The second difficulty in testing equilibrium behavior, as well as models of asymmetric information is that it may not be possible to observe variables associated with the player's behavior off the equilibrium path. Chapter 3 demonstrates for a moral hazard model that without observing variables associated with off equilibrium behavior, the testable restrictions of the moral hazard model are minimal. For the defensive publications model however, this will not pose a problem. The variables on the equilibrium path suffice to refute the model.

In chapter 1 I solve for the equilibrium with a quadratic cost function and an innovation requirement $n = 3$ innovation steps above the state of the prior art. For a large range of parameter values, the symmetric Markov perfect equilibrium of the model is a publications equilibrium in which the follower with one step publishes when the leader has two steps and is therefore one step away from obtaining a patent.

We assume that the data consists of investment levels (costs) and publication behavior on an equilibrium path. Given this assumption about what data are observed, I show that the hypothesis that the model is consistent with the proposed equilibrium notion for the model has nonvacuous non-parametric testable restrictions. Note that while in the theoretical model it is assumed that firms observe their rivals progress, it is not required that this information be directly revealed to the economists. This is inferred from the observations on investment.

The equilibrium conditions of a publications equilibrium consists of the following types of restrictions: Dynamic programing conditions, first order conditions and conditions for optimal publications behavior. The dynamic conditions state the dependence of the optimal value at one state on the optimal values at the states to which it is possible to transition from the current state. The first order conditions guarantee optimal investment behavior in every state. The optimal publishing conditions guarantee that it is not possible to make a profitable deviation to a strategy with more or with less publications. The testable restrictions consist of the equilibrium conditions and conditions for the convexity of the rationalizing cost function. These conditions are based on the Afriat inequalities (Afriat 1967). See Varian (1984) for an exposition.

The complexity of the model allowed explicitly deriving equilibrium strategies only for a specific cost function, a quadratic cost function. The analysis in this chapter is non parametric so that if the data satisfy the testable restrictions of the proposed equilibrium, then it is possible to construct a rationalizing convex cost function, not necessary quadratic. The non-parametric analysis allows testing of the proposed equilibrium notion independent of the test of the specific functional form of the cost function.

The analysis is extended to a generalization of the model where the patentability requirement is $n$. The conditions for data to be consistent with equilibrium are derived for data containing observation of each firm's progress in addition to investments and publications. For the general model, a theoretical existence of a particular equilibrium notion was not established. The results from the nonparametric approach can be interpreted as an empirical existence theorem as suggested in Snyder (1998). Section 2.2 describes a useful partition of the states of the world. Section 2.3 describes data. Section 2.4 presents the set of equilibrium inequalities. Section 2.5 derives the non-parametric testable implications of the model.

## 2.2 Partition of States

The set of states in the publications model is $S = \{(i,j) : i,j = 0,1,...,n\}$. Let $\{i,j\}$ denote the pair of states $(i,j)$ and $(j,i)$ i.e. the event in which one firm had $i$ steps and the other $j$. If one of the firms achieves $n$ steps the patent is granted to that firm and the race ends. The set of terminal states is

$$S_T = \{(i,j) : Max\{i,j\} = n\}.$$

We denote the states in which firm $i$ chooses to publish in equilibrium by $S^i_{pub}$. In a symmetric equilibrium,

$$(s,s') \in S^a_{pub} \text{ if and only if } (s',s) \in S^b_{pub}.$$

The states in which at lease one of the firms publishes are denoted by $S_{pub}$, $S_{pub} = S^a_{pub} \cup S^b_{pub}$. Firms invest and don't publish in all other states. Denote the states with investment $S_{inv}$. For states with publications, the equilibrium strategy specifies investment levels that would have been chosen if there was no publications, We denote these two sets of off equilibrium states by $S^i_{dp}$. Each state in $S^i_{dp}$ corresponds to a state in $S^i_{pub}$. Let $S_{dp} = S^a_{dp} \cup S^b_{dp}$.

In the defensive publications equilibrium, the follower publishes when the leader is one step away from obtaining the patent. For a three step model the sets described above are:

$$S^a_{pub} = \{(1,2)\},\ S^b_{pub} = \{(2,1)\},\ S_{pub} = \{(1,2),(2,1)\},$$

$$S_{inv} = \{(0,0),(1,0),(0,1),(1,1),(2,0),(0,2)\} \text{ and}$$

$$S_T = \{(3,0),(3,1),(3,2),(0,3),(1,3),(2,3)\}.$$

## 2.3 Data Observation

Suppose we observe two firms' R&D investment levels and publications over time from the beginning of a patent race to its end, when a patent is granted. Let $t = t_0, t_1, t_2 ... t_m$ denote the points in time in which there was a change in investment levels or a publication. Let $T = t_{m+1}$ be the time in which the race ended with a patent. The sequence of per period investment and publication is denoted by: $\{(p^a(t_k), p^b(t_k)), (c^a(t_k), c^b(t_k))\}_{k=0..m}$ where $c^i(t_k) > 0$ is the flow investment level from time $t_k$ to time $t_{m+1}$ and $p^i(t_k) = 1$ if the firm published at time $t_k$ and zero otherwise. In the theoretical model, time is continuous and $c(\lambda)$ is the flow cost. For the purpose of data collecting the flow cost can be interpreted as per period investment levels. The points in time $t_k$ are defined from the collected data so that the per period investment is constant between time $t_k$ and time $t_{k+1}$.

The patent race begins at state $(0, 0)$. Once one of the firms had its first success, the state is $\{1, 0\}$. Theses states must be on the equilibrium path, and their corresponding investment levels can be observed. The investment levels corresponding optimal investment in the states with publications $S_{dp}$, is off the equilibrium path and cannot be observed. Possibly there are other states off the equilibrium path.

## 2.4 Equilibrium Conditions

The pure strategy symmetric Markov perfect equilibrium conditions are derived in the first chapter. The conditions include dynamic programing equations that define the relation between optimal values in different states, first order conditions guarantying optimal investment levels and optimal decisions over publications. Let $\Psi$ be the set of twice contentiously differentiable convex cost functions, $c(\lambda)$ satisfying $c(0) = c'(0) = 0$. For cost functions in $\Psi$, there is an interior solution in the investment stage so that a first order condition is satisfied[10]. Let $c(\lambda_{i,j}) = c_{i,j}$ and $\frac{dc(\lambda)}{d\lambda}|_{\lambda_{i,j}} = c'(\lambda_{i,j}) = mc_{i,j}$. We formally state the equilibrium conditions:

C1. The dynamic programing equations:

For all $s \in S_{inv}$,

$$rV_{s^i,s^j} = \lambda_{s^i,s^j}(V_{s^i+1,s^j} - V_{i,s^j}) + \lambda_{s^j,s^i}(V_{s^i,s^j+1} - V_{s^i,s^j}) - c_{s^i,s^j}. \tag{8}$$

For all $s \in S_{pub}$,

$$rV^{dp}_{s^i,s^j} = \lambda_{s^i,s^j}(V_{s^i+1,s^j} - V^{dp}_{s^i,s^j}) + \lambda_{s^j,s^i}(V_{s^i,s^j+1} - V^{dp}_{s^i,s^j}) - c_{s^i,s^j}$$

and

$$V_{s^i,s^j} = V_{s^i-1,s^j-1}.$$

C2. The first order conditions

For all $s \in S_{inv}$,

$$V_{s^i+1,s^j} - V_{s^i,s^j} = mc_{s^i,s^j}. \tag{9}$$

For all $s \in S_{pub}$,

$$V_{s^i+1,s^j} - V^{dp}_{s^i,s^j} = mc_{s^i,s^j}.$$

C3. Optimal publications behavior:

[10] An interior solution facilitates notation but is not necessary for deriving testable implications.

For all $s^i, s^j \in \{0, .., n-1\}$,

$$V_{s^i+1, s^j+1} \geq V_{s^i, s^j} \text{ and } V_{s^i+1,0} \geq V_{s^i,0}$$

and for all $s \in S^i_{dp}$,

$$V_s \geq V_s^{dp}.$$

C4. A convex cost function satisfies the inequality

$$c_s \geq c_{s'} + mc_{s'}(\lambda_s - \lambda_{s'}). \tag{10}$$

The analogue condition for a concave cost function is known as the Afriat inequality. A strictly convex cost function satisfies this condition with a strict inequality. Denote the strict condition C4'.

## 2.5 Testable Implications

### 2.5.1 Dynamic Consistency

A necessary requirement for data to be consistent with the publications model is for the data to follow the expected dynamics that the model predicts. According the patent race model, transitions from state to state happen if one firm succeed to advance a step or if one of the firms publishes. In the case one firm had a success, there is a transition to a state in which the successful firm has one additional step, and the unsuccessful firms remains with the same number of steps as before. Such transition is from state $(s^i, s^j)$ to state $(s^i + 1, s^j)$ or $(s^i, s^j + 1)$. In the case publication occurred, the transition is to a state in which each firm's number of successes above the prior art decreases by the number of published steps. When one step is published the transition is from state $(s^i, s^j)$ to state $(s^i - 1, s^j - 1)$.

To test the model, we would need to match between the observed investment levels and the equilibrium investment level. Let $C_{inv} = \{c_s : s \in S_{inv}\}$ be variables corresponding to equilibrium investment levels. If the progress of firms is observed, it is clear how to match the observed investments to $C_{inv}$. Even when the states cannot be observed, it may be possible to infer the state at particular period.

In special cases, such as the defensive publications equilibrium of the three step model, we can infer the states of the world without observing them. The first state as in all races is $(0,0)$. In the three steps model $s_{pub}$ is $\{1,2\}$. A patent can be secured only after a follower published all his successes and has no knowledge above the state of the prior art. The last states before a patent is issued must be $\{0,1\}$ and then $\{0,2\}$. If an additional state with equal investment for both firms is observed, it must be state $(1,1)$. Variables that cannot be observed or inferred are referred to as free variables. For a defensive publications equilibrium of a three steps model, from a given data set, we infer the investment levels at states on the equilibrium path as follows:

**Example 4** *When testing a defensive publications equilibrium in a three steps model, for a data set*

$$\{(p^a(t_k), p^b(t_k)), (c^a(t_k), c^b(t_k))\}_{k=0..m}$$

*let the investment levels for states* $s \in S_{inv}$ *be defined as follows:*

$$\begin{aligned} c_{0,0} &= c^a(t_0) \\ c_{2,0} &= c^w(t_m) \textit{ where } w \textit{ is the winner} \\ c_{0,2} &= c^l(t_m) \textit{ where } l \textit{ is the looser} \\ c_{1,0} &= c^w(t_{m-1})) \\ c_{0,1} &= c^l(t_{m-1}) \\ c_{1,1} &= c^a(t_i) \textit{ if } c^a(t_i) = c^b(t_i) \textit{ for some } i > 0 \end{aligned}$$

*The investment level* $c_{1,1}$ *remains a free variable if there is not such* $i$*.*

In definition 4 we formally define the expected dynamics of a symmetric Markov perfect equilibrium.

**Definition 3** *A data set*

$$\{(p^a(t_k), p^b(t_k), (c^a(t_k), c^b(t_k)))\}_{k=0..m}$$

*follows the expected dynamics of a symmetric Markov perfect equilibrium with publications at states* $S_{pub}$ *if there exist values for the free variables in* $C_{inv}$ *such that for all* $k$ *and for* $i \neq j \in \{a, b\}$

$$c^i(t_k) = c_{s^i,s^j} \textit{ and } c^j(t_k) = c_{s^j,s^i} \textit{ for some } c_{s^i,s^j} \in C_{inv}.$$

$$\textit{If } p^i(t_k) = p^j(t_k) = 0 \textit{ and } c^i(t_{k-1}) = c_{s^i,s^j} \textit{ then } c^i(t_k) = c_{s^i+1,s^j} \textit{ or } c^i(t_k) = c_{s^i,s^j+1}.$$

$$\textit{If } p^i(t_k) = 1 \textit{ and } c^i(t_k) = c_{s^i,s^j} \textit{ then } c^i(t_{k-1}) = c_{s^i+\ell-1,s^j+\ell} \textit{ or } c_{s^i+\ell,s^j+\ell-1} \textit{ for some } \ell > 0$$

$$\textit{and } (s^i+\ell, s^j+\ell) \in S_{pub}.$$

The first requirement for data to satisfy the expected dynamics is that all investment levels observed are in the set of equilibrium investments and that the pair of investments at states $\{s^i, s^j\}$ match. The second requirement concerns with the transition in states without publications. The third requirement concerns the transition in states with publications and the last requirement identifies the state with publication.

### 2.5.2 Rationalizing Data

We say that we can rationalize the data as a Markov Perfect equilibrium of the publications model with publications at states in a subset of states $S_{pub}$ if we can find a cost function such that the data is consistent with such an equilibrium in the model with this cost function.

**Definition 4** *A function* $c(\lambda)$ *rationalizes a data set*

$$\{(c^a(t_k), c^b(t_k)), (p^a(t_k), p^b(t_k))\}_{k=0..m}$$

*as a symmetric Markov perfect equilibrium with publications at* $S_{pub}$ *if such equilibrium for the model with cost function* $c(\lambda)$ *could generate an equilibrium path with the publications and investments given in the data.*

If a cost function in $\Psi$ rationalizes a data set, then the data is consistent with the dynamics of the defensive publications equilibrium. Moreover, the equilibrium conditions are met and the hazard rates and their associated costs must lie on a curve of a convex function.

**Proposition 6** *If a cost function $c(\lambda) \in \Psi$ rationalizes a data set as a symmetric Markov perfect equilibrium then the data satisfy dynamic consistency and there exist numbers*

$$\lambda_s, V_s, mc_s, c_s \in R_{++}$$

*for all $s \in S \cup S_{dp}$ such that conditions C1-C4 are met.*

**Proof.** Let $\lambda_s$ be the equilibrium investment levels, let $mc_s = c'(\lambda_s)$ and $c_s = c(\lambda_s)$. Let $V_s$ be the value at state $s$ when the equilibrium strategy is used and let $V_s^{dp}$ for $s \in S^{dp}$ be the values associated with the off equilibrium optimal investment in states with publications. Then, C4 must hold from convexity of the cost function and C1-C3 must hold since these are the equilibrium conditions for the model. ∎

Given a data set, we would like to know if it is consistent with a defensive publications equilibrium. Moreover, we want a method to construct a rationalizing function. The following proposition provides conditions for the data to be consistent with equilibrium. When the conditions are met. the proof shows how to construct a rationalizing cost function.

**Proposition 7** *There exist a smooth convex strictly increasing cost function that rationalizes the data if there exist*

$$\lambda_s, V_s, mc_s, c_s, V_{s'}^{dp} \in R_{++}$$

*for all $s \in S$ and $s' \in S_{dp}$ such that the data satisfies the expected dynamics and the conditions C1-C4' hold.*

To prove the proposition I use the following lemma from Chiappori and Rochet (1987). The notation was changed according to the notation in this chapter and I use a convex rather than concave function.

**Lemma 1** *If* $\{c_s,\ s \in S\}$ *and* $\{mc_s,\ s \in S\}$ *satisfy the condition* $c_s - c_{s'} > mc_{s'}(\lambda_s - \lambda_{s'})$ *for any* $s \neq s'$, *there exists a convex, strictly increasing, infinitely differentiable* $c : R \rightarrow R$, *such that:* $\forall s,\ c(\lambda_s) = c_s$ *and* $c'(\lambda_s) = mc_s$.

The idea behind the proof of this lemma is to use the Afriat construction of a piecewise linear convex function by taking the maximum of a collection of linear functions each satisfying the property $c(\lambda_s) = c_s$ and $c'(\lambda_s) = mc_s$. Then regularize the function using convolution. For a formal proof see Chiappori and Rochet (1987)

**Proof of proposition 7.** If there exist values for the unobserved such that the conditions C1-C4' hold, then by the lemma I construct a convex strictly increasing smooth function $c(\lambda)$ such that $c(\lambda_s) = c_s$ and $c'(\lambda_s) = mc_s$. For the model with this cost function the equilibrium conditions are satisfied so $c(\lambda)$ rationalizes the data. ∎

### 2.5.3 Refuting the Three Step Model

In this section I show that it is possible to test the hypothesis that a data set is consistent with a symmetric Markov perfect equilibrium with publications in the $n = 3$ patent race model. The model is consistent and refutable. Chapter 1 derives a symmetric Markov perfect stationary equilibrium in the $n = 3$ model with a quadratic cost function of the form $c(\lambda) = c_0(\lambda - \overline{\lambda})^2$. If the fixed hazard rate parameter $\overline{\lambda}$ or the cost parameter $c_0$ are sufficiently high the equilibrium is one in which the follower publishes at state $(1, 2)$ and there are no publications in other states. Let us refer to this equilibrium as a *defensive* *publications equilibrium.* We focus on testing the hypothesis that a data set is consistent with a defensive publications equilibrium.

Since the follower publishes if he had one success and the leader had two, prior to securing the patent, the leader must have two steps while the follower none. The states

$$\{(0,0),\{1,0\},\{2,0\}\}$$

must be visited on every equilibrium path. Therefore, the data observations would include investment levels for these states. The state $(1,1)$ may or may not realize. The optimal investment levels at states $\{1,2\}$ are off the equilibrium path and cannot be collected as data. The defensive publications equilibrium hypothesis is testable if there exist data sets that can refute it. If it is the case that for all data sets we can construct a rationalizing cost function, then the hypothesis cannot be refuted. We would like to show, that there exists a data set that is inconsistent with a defensive publications equilibrium. Clearly, any data set that is inconsistent with the dynamics of the equilibrium refutes the defensive publications equilibrium hypothesis. However, since this basic restriction is one that we might impose on the data collected in the process of preparing the data for testing the theoretical model, it would be interesting to refute the equilibrium even with data that satisfies dynamic consistency. In the next proposition I show that not every data set that follows the expected dynamics can be rationalized as a defensive publications equilibrium path.

**Proposition 8** *The defensive publications equilibrium hypothesis is refutable. Moreover, there exist dynamically consistent data sets that are inconsistent with a defensive publications equilibrium.*

**Proof.** Let us show that there exist observations consistent with the dynamics of the model that cannot be generated as defensive publications equilibrium of the model. Suppose we observe investment and publication data consistent with the dynamics of the model. Suppose $c^a(t_i) = c^b(t_i)$ and that $c^d(t_i) < c^d(t_{m-1})$ for some $i > 0$.Then I show that the data cannot be rationalized as a defensive publications equilibrium path. The symmetric state at $t_i$ must be $(1, 1)$. Thus $c_{1,1} < c_{0,1}$. If the data is on an equilibrium path then the dynamic programing equation (8) and the first order condition (9) must hold. Specializing (8) to $s = (1, 1)$ we get

$$rV_{1,1} = \lambda_{1,1}(V_{2,1} - V_{1,1}) + \lambda_{1,1}(V_{1,2} - V_{1,1}) - c_{1,1}.$$

Divide by $\lambda_{1,1} > 0$ to obtain

$$\frac{rV_{1,1}}{\lambda_{1,1}} = V_{2,1} - V_{1,1} + V_{1,2} - V_{1,1} - \frac{c_{1,1}}{\lambda_{1,1}}.$$

But $\frac{rV_{1,1}}{\lambda_{1,1}} + \frac{c_{1,1}}{\lambda_{1,1}} \geq 0$ therefore, $V_{21} - V_{11} \geq V_{11} - V_{12}$. With publication at state $(1, 2)$ $V_{12} = V_{01}$. Therefore, $V_{21} - V_{11} \geq V_{11} - V_{01}$ By (9) this implies that $mc_{11} \geq mc_{01}$. If there exists a strictly increasing and convex cost function that rationalizes the data then, $mc_{11} \geq mc_{01}$ implies that $c_{11} \geq c_{01}$. If the data satisfies the expected dynamics, this condition can be written as $c^d(t_i) \geq c^d(t_{m-1})$. Thus if we observe $c^d(t_i) < c^d(t_{m-1})$ then the data cannot be rationalized as a defensive publications equilibrium path. ■

A publication at state $(1,2)$ implies that $V_{12} = V_{01}$ even if the follower's strategy is to mix between publishing and not publishing at this state. The reason is that if the follower publishes with some positive probability, then he must be indifferent between publishing and not publishing at this state. Therefore, a data set in which we observe a publication at state $(1,2)$ for which $c_{11} < c_{01}$ refutes any Markov perfect equilibrium not just the pure defensive publications equilibrium. We summarize this result in the following corollary.

**Corollary 2** *The hypothesis that a data set is consistent with some symmetric Markov perfect equilibrium of the $n = 3$ model is refutable.*

### 2.5.4 Tests in the General $n$ Model

Consider the publications model with a patentability requirement of $n$ steps above the prior art. In this more general setting I did not find an equilibrium strategy for all parameter values. Nevertheless, with the non-parametric empirical approach, we can test whether a data set is consistent with some symmetric pure strategy Markov perfect equilibrium of the publications model. This result can be viewed as an empirical existence result. Suppose, in addition to investment and publication decisions we also observe the number of steps each firm had in every investment period. Suppose we would like to test if the model is consistent with a pure strategy symmetric MPE with firm $i$ publishing one step at each state in $S^i_{pub}$. There is a finite number of possible sets $S^a_{pub}$. For a give set $S^a_{pub}$ we can test whether the data is consistent with some pure strategy MPE path in which firms publish at states in the set $S^i_{pub}$ and don't publish otherwise.We might also be interested in checking if the data can be rationalized as equilibrium with some property of interest. For example, one with publications at $(s, n-1)$ only.

The $n$ steps model can also be refuted. There exist data consistent with the dynamics of the model that is inconsistent with equilibrium. The example in proposition 8 refutes the general $n$ model If the data shows publication at state $(2,1)$ and investments $c_{11} < c_{01}$ then it is inconsistent with the model for all $n$ and $r$.

# Chapter 3

## 3 Testable Implications of a Simple Moral Hazard Model of Mutual Insurance

### 3.1 Introduction

In this chapter, I derive the testable implications of a simple moral hazard model of mutual insurance in a general equilibrium economy. Moreover, I characterize the utility functions that rationalize a data set consistent with the moral hazard model. The model described in this chapter (suggested by Gottardi (1998)) is a simplified version of general equilibrium models with asymmetric information for example Prescott and Townsend (1984), Bennardo and Chiappori (1998).

Moral hazard refers to asymmetric information economies in which an agent can choose costly private actions that affect the probability of suffering a loss. These models are widely used in applied areas of microeconomics (see Stadler and Castrillo (1999), Chiappori (1998)). In most applications, the utility functions satisfy some parametric restrictions. This joint hypothesis does not allow an independent test of the moral hazard model. In this work, our analysis is nonparametric, hence eliminating the specification error implicit in assuming that utility functions are members of some parametric family. Afriat (1967) initiated the method of constructing non-parametric tests for utility maximization. Afriat's theorem shows the equivalence of the existence of a concave monotonic continuous utility function that rationalizes the data, an axiom of revealed preference (a condition on observed variables only), and the existence of a solution to a system of linear inequalities (with utilities and marginal utilities as unknowns). Varian (1982,1983) takes this approach to derive nonparametric tests of consumer behavior under different assumptions (separability, homotheticity, the expected utility model). Brown and Mazkin (1996) derive nonparametric testable restrictions on the equilibrium manifold in a pure trade economy. They introduce the use of the Tarski-Seidenberg theorem and quantifier elimination to derive testable restrictions of the model. Snyder (1999) follows Brown and Matzkin's method to examine the testable restrictions of Pareto optimality in a public goods economy

In this work, I assume that we observe a finite data set consisting of $n$ observations. Each observation includes the variables: prices, consumption, endowment and probabilities. These observed values are assumed to be the values for the infinity of identical agents in every observed economy. We look for restrictions that will enable us to test the hypothesis that the data is consistent with the model. By assumption, the infinity of agents are identical across observations in the sense that they have the same Bernoulli utility function and the same effort costs in terms of utility. However, observations may differ in the possible values wealth can take, in the probabilities, and therefore also in the optimal choices of effort and consumption. The data may consist of cross sectional observations, for example different states in the United States. The Bernoulli utility function is the same for agents in all states but the risk differs across states.

One problem that arises when seeking to test moral hazard models is that if agents choose high effort, we may only observe the economic consequences associated with this effort choice. The data may contain no information about what would have happened if the agent had chosen the low effort. For example when agents choose high effort we may learn from the data the probability of outcomes contingent on high effort, but not the probability distribution contingent on low effort. We first explore the case where the economist can only observe variables related to the effort that agents optimally choose. Then we turn to the case where the economist also knows the probability distribution that corresponds to the unchosen low effort level.

When the data consists of variables relating only to the optimal effort level that the agents actually chose, that is endowment consumption bundles and probabilities and insurance prices associated with agents' optimal choices of effort, the nonparametric testable restrictions on the data are very weak. When the optimal choice is high effort, the budget constraint, fair prices and consumption smoothing are the only testable restrictions that can be derived from the moral hazard model. Hence, it is difficult to reject the model. Given a data set that satisfies these restrictions I show existence of a strictly concave utility function (a CRRA utility) that rationalizes the data.

To show that a data set is consistent with the model given a particular concave utility function, we need to find values for the unobserved effort costs and the low effort probabilities so that all the conditions for an equilibrium in the model are satisfied for all the observations. In an equilibrium where agents choose high effort the incentive compatibility constraint binds. Equilibrium consumption is therefore the point of intersection of the binding incentive compatibility constraint and the budget constraint. The budget constraint is observed and the observed consumption point is assumed to lie on it. The incentive compatibility constraint on the other hand, depends on unobserved utility function effort costs and low effort probabilities. The data can be rationalized if the unobserved can be chosen to make the incentive compatibility curve pass exactly through the observed consumption bundle. When the probability associated with low effort is not observed, this free parameter can be used to move the incentive compatibility curve to make it intersect with the budget line exactly in the observed consumption bundle

To be a little more precise, given data, if we pick some concave Bernoulli utility function, we are left with the difference between effort costs and the low effort probability as free variables. In order to rationalize the data with the utility we picked, we need to choose effort costs and probabilities such that the consumption bundles will solve the incentive compatibility constraint with equality, and choosing high effort will be better for the agent than choosing low effort and fully insuring herself. The incentive compatibility constraint and the condition for optimal choice of effort level imply a lower bound and an upper bound respectively on the probability of the good state contingent on low effort in every observation. Combining these bounds gives a necessary and sufficient condition for a concave utility function to rationalize a given data set.

The risk neutral utility function can rationalize any data satisfying the budget constraint, fair prices and the smoothing condition. The intuition behind this is that a risk neutral agent cares only about her expected consumption. Therefore, when the cost of high effort is not too large, high effort is preferred since it allows the agent to enjoy a higher expected consumption. Finding effort costs and probabilities so that the data is consistent with the model when the utility is risk neutral is therefore always possible. The risk neutral utility function is a special case of the family of CRRA utility functions. It satisfies the necessary and sufficient condition that I derive for a utility function to rationalize the data. We show that if the coefficient of relative risk aversion is small enough (the agent is not too risk averse) then CRRA utility also satisfies these necessary and sufficient conditions so that the utility function rationalizes the data as high effort moral hazard equilibria.

Surprisingly, if in addition to the observations with consumption smoothing, we have observations with full insurance then we can still find a utility function and effort costs to rationalize the data with the moral hazard model. Hence, the additional observations do not imply additional testable restrictions. Consider a data set with n consumption smoothing observations that satisfy the budget constraint and $m$ observations with full insurance that also satisfy the budget constraint. We can interpret such data as generated by our model treating the $n$ consumption smoothing observations as high effort moral hazard equilibria and the $m$ full insurance observation as equilibria in which agents chose low effort and a full insurance contract. We show that any utility function that rationalizes the first n consumption smoothing observations also rationalizes the entire data set. To rationalize the data I first rationalize the high effort observations. Then for full insurance observations I choose the unobserved high effort probabilities in a way that will make the endowment bundle be the optimal choice of a high effort agent and this will always be worse than low effort and full insurance.

I also look for testable implications of the model when the economist has information about the probabilities associated with both levels of effort (or the corresponding prices) in addition to the consumption bundles and endowments. Even if we have data consisting of high effort observations, we may be able to observe the probability of a good state if low effort were chosen. This information can be determined from past periods in which high effort was either not available or too costly so that the agents employed low effort. Or, it may be that people know the consequence of low effort, for example it may be clear that with low effort a bad state always occurs. Or it can be that the insurers know the low effort probabilities from research or experiments and this information is inferred by the economist from prices in an insurance contract that is available but not purchased.

When the data contains information about the probabilities associated with both effort levels the model is shown to be testable in the sense that we derive a set of necessary and sufficient linear inequalities that the data must satisfy to be consistent with the moral hazard model. In this case the budget constraint, fair prices and consumption smoothing are no longer sufficient for the data to be consistent with the model, stronger restrictions on the observable variables must hold in order for the data to be rationalized by some concave utility function. In this case where both probabilities are observed, the Bernoulli utility function and effort costs are the only unobserved. Since these need to be the same for all observations, we lose the freedom (that we have if low effort probability is unobserved) to move each incentive compatibility constraint to pass through the different consumptions bundles. In some data sets, the different probabilities and consumptions in different observations may not allow for a choice of one common utility function that generates incentive compatibility constraints that pass through the corresponding consumption points. There are data sets inconsistent with the model hence it is testable.

When the data can be rationalized, the rationalizing concave utility function can be constructed as in Afriat (1967), but we can no longer guaranty that there is a CRRA utility function that rationalizes the data. When we can observe the probabilities associated with both effort levels, it is possible to find data sets that satisfy the budget constraint, fair prices and the smoothing condition but for which it is impossible to find effort costs such that the data satisfy the incentive compatibility constraint.

Section 3.2 describes the moral hazard model and gives some previously known results about the equilibrium of the model. In subsection 3.3.1 I derive the testable implications of the model when the data contain information related to the optimal effort choice, and I characterize the utility functions that rationalize a specific data set. In subsection 3.3.2 I consider data sets that contain information associated with both effort levels. We derive the linear equilibrium inequalities. We show that the system is nontrivial and that the restrictions are stronger than the ones derived earlier.

## 3.2 Model

In the model, there are infinitely many identical agents. There are two time periods $t = 0, 1$. There is one consumption good consumed at $t = 1$. Agents face risk, each agent's endowment (or wealth) is a random variable $W$ that is realized at $t = 1$. Wealth can take one of two possible values $W_G > W_B > 0$. The difference $W_G - W_B$ can be thought of as a possible loss. For example, the bad state might be one where an accident happens to the agent. The random endowments of all agents are independent and identically distributed (IID). Since we have identical agents and I focus on symmetric equilibria, I drop the agent index in order to keep notation simple.

An agent can effect the distribution of wealth (or her chance to suffer a loss) by her choice of effort level. There are two effort levels available for each agent, $e = H$ is the high effort and $e = L$ is the low effort. Agents choose effort level at time $t = 0$. High effort can be interpreted as being more careful. The effort is costly to the agents in terms of utility. The costs of high effort and low effort are denoted $V_H$ and $V_L$ respectively. High effort costs more, $V_H > V_L$. The probability distribution for $W$ depends on the agent's effort. Wealth $W$ takes the large (good) value $W_G$ with probability $\Pi_e$ ($e = L$ or $H$), and the small (bad) value $W_B$ with probability $1 - \Pi_e$. There is a higher probability for the good state if the agent chooses high effort, $1 > \Pi_H > \Pi_L > 0$.

This is a model of mutual insurance where an agent can insure by buying securities that pay a unit of consumption contingent on the realization of her random wealth, $W$. Assets are sold by an insurance company that serves as an intermediary. Every agent can trade with the insurance company only in her own securities. Insurance is assumed to be fair and the insurance market clears. Contracts are exclusive, the agents trade with one intermediary who observes their trades. Each agent's preferences are represented by a Von Neumann-Morgenstern expected utility function separable[11] in wealth and effort costs, $E[u(x)|e] - V_e$, where $u(x)$ is a concave increasing Bernoulli utility function. The expectation and the cost of effort $V_e$ depend on the effort level $e$.

We will now characterize equilibrium in the model. The results in this section are well

[11] Separability is a strong assumption. It is often made in moral hazard models. Bennardo and Chiappri (1998) show how relaxing separability changes the properties of the equilibrium of the model. Since the analysis in this paper assumes separability, rejection of the model does not necessarily imply its rejection with a non-separable utility function.

known from previous analysis (e.g. Bennardo and Chiappori (1998) and Gottardi (1998)), and included in this essay since our analysis is based on these results. Lemma 2 summarizes the characterization of equilibrium as a system of polynomial inequalities. This format will be most helpful for the study of testable implications in section 3.3.

First let us consider the symmetric information case where the effort, $e$ is observed.

Let $P_{Ge}$ be the price at $t = 0$ of the asset delivering one unit of good in state $G$, if the effort level is $e$. And $P_{Be}$ be the price at $t = 0$ of the asset delivering one unit of good in state $B$, if the effort level is $e$.

The agent's choice problem is:

$\max_{\{x,e\}} E[u(x)|e] - V_e$

Subject to:

$$X_G > 0, X_B > 0$$

Budget constraint:

$$P_{Ge}(X_G - W_G) + P_{Be}(X_B - W_B) = 0 \tag{11}$$

The market clearing condition is:

$$\Pi_e(X_G - W_G) + (1 - \Pi_e)(X_B - W_B) = 0 \tag{12}$$

This is the market clearing condition since by the law of large numbers, the fraction of individuals that get a good realization of $W$ is $\Pi_e$, when all agents choose effort level $e$. Note that each agent trades her own securities, that is, the assets an agent buys will deliver a unit of good contingent on the realization of this agent's random endowment and not on other agents. Agents cannot trade in securities contingent on someone else's realization of endowments. The market clearing condition is written in expectation however, since there are an infinite number of agents, markets exactly clear, meaning that the insurance company will have zero profit.

If the parameters of the model are such that:

$$u(E(W|H)) - V_H > u(E(W|L) - V_L,$$

then the competitive equilibrium of the complete information model is:

$$X_G^* = X_B^* = E(W|H),$$

$$e^* = H,$$

$$P_{Ge} = \Pi_e, \; P_{Be} = 1 - \Pi_e.$$

Hence, in the complete information case there is full insurance, a Pareto efficient outcome.

Now consider the asymmetric information case where the choice of effort is unobserved. The insurer needs to impose the incentive compatibility constraints in order to be able to condition prices on the unobserved effort. Agents can buy assets at prices $P_{Ge}$ and $P_{Be}$ only if the amounts of assets they demand satisfy the incentive compatibility constraint for effort level e. According to this model agents choose an effort level $H$ or $L$ and consumption levels $X_G$ and $X_B$ (or purchases of state contingent assets) to maximize expected utility subject to an additional constraint, the incentive compatibility constraint:

$$\Pi_e U(X_G) + (1 - \Pi_e) U(X_B) - V_e \geq \Pi_{e'} U(X_G) + (1 - \Pi_{e'}) U(X_B) - V_{e'} \; (e \neq e\prime) \qquad (13)$$

To solve this problem the agent can find the optimal consumption bundle contingent on her choosing effort level $H$, $(X_{GH}, X_{BH})$ and the optimal bundle contingent on her choosing effort level $L$, $(X_{GL}, X_{BL})$. Then she chooses the effort level that gives her highest utility, and the corresponding optimal bundle.

**Definition 5** *Given probabilities,* $1 \geq \Pi_H > \Pi_L \geq 0$*, endowments,* $W_G > W_B > 0$ *and costs of effort,* $V_H > V_L$*,*

$$\{(P_{GH}, P_{BH}), (P_{GL}, P_{BL}), (X_{GH}, X_{BH}), (X_{GL}, X_{BL})\} \in R^8_+$$

*is a high effort moral hazard equilibrium if:*

*1. For* $e = L, H$ *the pair* $(X_{Ge}, X_{Be})$ *solves the agent's choice problem, the maximization of his expected utility subject to non-negativity, the budget and the incentive compatibility constraint.*

*2. High effort is preferred to low effort:*

$$\Pi_H u(X_{GH}) + (1 - \Pi_H)u(X_{BH}) - V_H \geq \Pi_L u(X_{GL}) + (1 - \Pi_L)u(X_{BL}) - V_L \qquad (14)$$

*3. The market clearing condition*[12] *(12) for* $e = H, L$*.*

[12]The equilibrium concept here is one in which the market clearing condition is assumed to hold for the off equilibrium effort level as well as the equilibrium effort level. This means, as we will discuss later, that equilibrium prices are proportional to the probabilities for both effort levels. Considering equilibria in which the off equilibrium prices may not be fair requires further discussion on the behavior of insurance firms. We may replace the low effort market clearing condition with the assumption that there is a finite number of profit maximizing competitive insurance firms. Equilibrium profits must be zero that is, high effort prices are fair. Moreover, whenever there is a high effort equilibrium with low effort prices that are not fair, there

*Similarly, I define a low effort equilibrium by changing the direction of the inequality (14).*

Let us characterize the solution to the agent's problem for each effort level in the case

$$(P_{GH}, P_{BH}) \propto (\Pi_H, 1 - \Pi_H) \text{ and } (P_{GL}, P_{BL}) \propto (\Pi_L, 1 - \Pi_L)$$

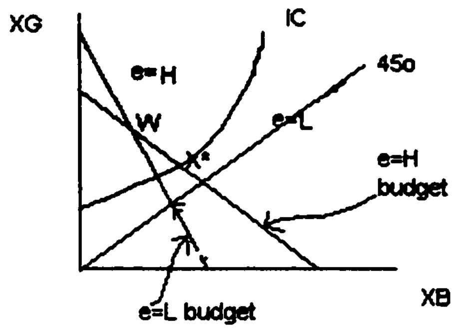
(prices proportional to the probabilities). For the low effort level, $L$, the solution to the agent's choice problem for $e = L$ is the full insurance point

$$X_{GL} = X_{BL} = \Pi_L W_G + (1 - \Pi_L) W_B$$

(Point F in figure 4). The reason is that this point is feasible for the agent and being risk averse she prefers it to any other point on her budget line. For the high effort level, we will see that the solution to the agent's choice problem is the point of intersection between the high effort budget line and the incentive compatibility constraint when solved with equality (point $X^*$ in figure 4). The pairs ($X_G$, $X_B$) in $R^2_+$ can be partitioned to a set for which the incentive compatibility constraint with $e = H$ holds (the shaded area), a set for which incentive compatibility with $e = L$ holds, and a set (curve) for which incentive compatibility holds with equality (see figure 4).

must also be a high effort equilibrium with fair low effort prices. If not, then at the fair prices agents prefer low effort to high effort but then an insurance firm can make a profitable deviation by offering insurance at low effort prices that are close to the fair prices yet give some positive profit. Such changes in the equilibrium concept will therefore not affect the results.

Figure 4




Consumption Possibility Set and the Optimal Choices

Shaded area- consumer prefers high effort. *W*- endowment. *X**- consumers optimal choice when she prefers high effort. F- full insurance, consumer's optimal consumption if she chooses low effort.

The budget lines are decreasing. The curve denoted in figure 4 with IC is the collection of points for which the incentive compatibility constraint holds with equality. On this curve, $X_G$ is implicitly defined as a function of $X_B$ by the incentive compatibility when solved with equality. This is an increasing function (not necessarily convex). All points on the $45^o$ line are such that low effort is preferred to high effort since when $X_G = X_B$ incentive compatibility reduces to $-V_H < -V_L$. If the set of feasible points in the $e = H$ problem is non-empty then there is a unique intersection of IC and the budget line. All points above IC are such that high effort is preferred. Since the agent has an increasing utility function, her optimal choice must be on the budget line. The point at which IC intersects the budget line second order stochastically dominates any other feasible point on the budget line. Therefore it is the optimal choice for the agent[13].

An equilibrium in this model exists. Let

$$(P_{GH}, P_{BH}) = (\Pi_H, 1 - \Pi_H)$$

and

$$(P_{GL}, P_{BL}) = (\Pi_L, 1 - \Pi_L).$$

If the point of intersection of IC and the budget line, $X^*$ is preferred to the full insurance low effort level point, F, then there is a high effort moral hazard equilibrium (agents all

[13]If the IC intersects the budget line on a point to the left of the endowments ($W$ is at the $e = L$ region) we may want to add $W$ to the feasible $e = H$ set since choosing high effort and no trade should always be feasible to the agent. In a case like this $W$ will be better to the agent than the best IC feasible choice for her if she chooses high effort. However it will be the case that in equilibrium the agent will prefer low effort to high effort. So $W$ will not be her equilibrium consumption choice anyway.

choose high effort and consume $X^*$). Otherwise, there is a low effort equilibrium with full insurance consumption. The equilibrium is constrained Pareto optimal (under incentive compatibility constraint).

**Lemma 2** *1. Given a strictly concave utility function*[14]*, effort costs $V_H > V_L$, endowments $W_G > W_B > 0$ and probabilities $\Pi_H > \Pi_L$,*

$$\{(X_{GH}, X_{BH}), (X_{GL}, X_{BL}), (P_{GH}, P_{BH}), (P_{GL}, P_{BL})\} \in R^8_+$$

*is a high effort moral hazard equilibrium if and only if the following conditions are satisfied:*

*C1.* $\Pi_H = \frac{P_{GH}}{P_{GH}+P_{BH}}$ *and* $\Pi_L = \frac{P_{GL}}{P_{GL}+P_{BL}}$

*C2.* $X_{GL} = X_{BL} = \Pi_L W_G + (1 - \Pi_L) W_B$

*C3.* $P_{GH}(X_{GH} - W_G) + P_{BH}(X_{BH} - W_B) = 0$

*C4.* $V_H - V_L = (\Pi_H - \Pi_L)[u(X_{GH}) - u(X_{BH})]$

*C5.* $\Pi_L u(X_G H) + (1 - \Pi_L) u(X_B H) - u(\Pi_L W_G + (1 - \Pi_L) W_B) > 0$

*2. These conditions imply consumption smoothing,*

$$W^i_G > X^i_{GH} > X^i_{BH} > W^i_B. \quad (15)$$

[14]If the utility function is weakly concave, C1-C5 are sufficient conditions. The conditions C1, C3, C5 are also necessary with a weak concave utility function. However, C2 and C4 are no longer necessary. The point of full insurance maximizes utility contingent on low effort also in the case of weakly concave utility function however, it may not be unique. Similarly, the point of intersection of the IC and budget solves maximization problem contingent on high effort but may not be the unique maximizer. For example, any point on the budget line left to this point will also be a possible equilibrium consumption bundle when the utility is linear.

The first condition means that prices are actuarially fair. The second condition states that agents fully insure if they choose low effort. The third condition is the budget constraint. The fourth condition is derived by rearranging the incentive compatibility constraint which holds with equality in equilibrium. The fifth condition derived from inequality (14) and C4, states that high effort is preferred to low effort. The proof is straightforward.

## 3.3 Testable Implications

We now derive the testable implications of the model. Assume that we observe a finite data set, index the observations $i = 1..n$. We look for restrictions on the observed data that will enable us to test the hypothesis that the data is consistent with the model. The data is consistent with the model (we can rationalize the data) when there exists a concave utility function and effort costs (common to all observations) such that the observed data can be explained as equilibria of the model.

### 3.3.1 Implications with Data Associated with the Optimal Action

In this section, we will consider data sets that only contain information about the actions taken by agents. Suppose we have a data set consisting of $n$ observations of consumption bundles, endowments, high effort probabilities and prices of the state contingent securities that agents face. Every economy from which we collect data has a large number of identical agents. There will be a fraction of agents who had a good realization of wealth and a fraction of agents who had a bad realization of wealth even though all agents chose the same effort level. Therefore, we are able to observe wealth and consumption in both good and bad states. We ask whether we can explain the data that we observe with the moral hazard model described in the previous section.

**Definition 6** *We say that a concave increasing Bernoulli utility function $u(x)$ rationalizes the data*

$$\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)\} \in R^6_+, i = 1 \ldots n,$$

*where $X^i_{GH} > X^i_{BH}$ and $W^i_G > W^i_B > 0$ as high effort moral hazard equilibria in the model if there exist probabilities of the good state (depending on the effort level $L$) $\Pi^i_L$ such that*

$$1 \geq \Pi^i_H > \Pi^i_L > 0$$

*and effort costs $V_H > V_L$ (independent of the $i$) so that*

$$\begin{aligned}\{(X^i_{GH}, X^i_{BH}), (X^i_{GL} &= X^i_{BL} = \Pi^i_L W^i_G + (1 - \Pi^i_L) W^i_B), \\ (P^i_{GH}, P^i_{BH}), (P^i_{GL} &= \Pi^i_L, P^i_{BL} = (1 - \Pi^i_L))\}\end{aligned}$$

*is a high effort moral hazard equilibrium for all $i$, for agents with utility function $u(x)$ and effort costs $V_H$,$V_L$. We say that we can rationalize a data set if there exists a concave increasing utility function that rationalizes the data.*

Theorem 3 provides the testable implications result given the assumptions we maid on the observed data.

**Theorem 3** *Given a data set $\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)\}$ $i = 1 \ldots n$ where $P^i_{GH} > 0$, $P^i_{BH} > 0$, $W^i_G > W^i_B \geq 0$, $X^i_{GH} > X^i_{BH} > 0$ the following conditions are equivalent:*

*I. There exists a strictly concave, increasing, smooth utility function (from the family of CRRA) that rationalizes the data.*

*II. The data satisfies for all $i = 1 \dots n$:*

*1. Budget constraint (11)*

*2. Fair prices:*

$$\Pi_H^i = \frac{P_{GH}^i}{P_{GH}^i + P_{BH}^i} \tag{16}$$

*3. Consumption smoothing (15)*

This theorem shows that the testable implications of the moral hazard model are minimal if we can only observe information related to the optimal effort choice. Any data set where prices are fair and where consumption bundles lie on the budget line and exhibit consumption smoothing can be rationalized with the model. Moreover, we can also rationalize it with a strictly concave smooth utility function. In fact, the proof shows that there is continuum of functions from the CRRA family with sufficiently low coefficients of relative risk aversion that rationalize the data. Note that each restriction in II depends on one observation only thus increasing a data set with more observations that satisfy these restrictions does not change the testability of the data at hand. However, it limits our choice of utility functions that can rationalize the data. Looking at the CRRA family, the more observations we have, the lower the coefficient of relative risk aversion must be in order to rationalize the data.

To prove this theorem I use two propositions. Proposition 9 gives a necessary and sufficient condition for a strictly concave utility function to rationalize the data. The result of proposition 9 is itself important, it allows us to verify for any given data set whether a particular utility function rationalizes the data. In proposition 10, for data sets where consumption bundles lie on the budget line and exhibit consumption smoothing, I show there exists a strictly concave CRRA utility function that rationalizes the data.

**Proposition 9** *Given a data set* $\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)\}$ *for* $i = 1 \ldots n$ *where* $P^i_{GH} > 0$, $P^i_{BH} > 0$, *fair prices (16), consumption smoothing (15), and the budget constraint (11) hold a necessary and sufficient condition for a strictly concave Bernoulli utility function to rationalize the data as high effort moral hazard equilibria is that for every* $i = 1 \ldots n$,

$$\Pi^i_H - \min_j \{ \frac{\Pi^j_H [u(X^j_{GH}) - u(X^j_{BH})]}{[u(X^i_{GH}) - u(X^i_{BH})]} \} \leq \Pi^{i^*}, \tag{17}$$

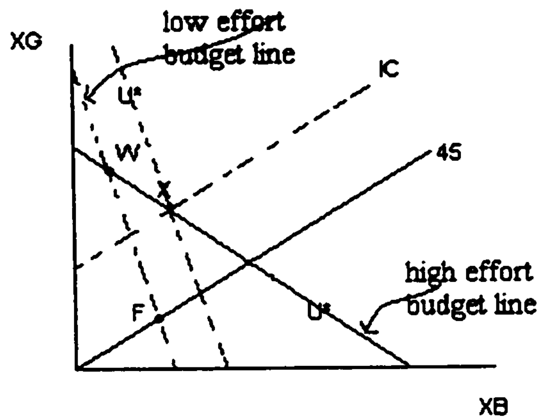
*where* $\Pi^{i^*}$ *is defined as the largest zero of the convex function:*

$$f^i(\Pi) = \Pi u(X^i_{GH}) + (1-\Pi) u(X^i_{BH}) - u[\Pi W^i_G + (1-\Pi) W^i_B] \tag{18}$$

*in the range* $[0, \Pi^i_H]$.

Before proving this proposition let us consider the case of a linear utility function $u(x) = x$. Each observation in the data set consists of prices proportional to probabilities, endowment, $W$ and consumption bundle, $X$ that lies on the budget line to the right of $W$ (see figure 5)

Figure 5




The Linear Case

Each observation includes the high effort budget line, consumption, *X*, and endowment, *W*. Low effort budget intersects high effort budget at *W*, IC has slope 1. The choice of low effort probabilities will determin the slope of the low effort budget line and the intersection of IC with the high effort budget line. Low effort and high effort indiference curves through *X* have the same utility level *U**.

The low effort budget must be steeper than the given high effort budget and pass through the endowment point $W$. The indifference curves are strait lines parallel to the budget lines. Incentive compatibility equality constraint in this case is a linear function,

$$V_H - V_L = (\Pi_H - \Pi_L)(X_{GH} - X_{BH}).$$

In the $(X_B, X_G)$ plain, IC is parallel to the 45° line. Effort levels and low effort probabilities are unobserved. All we need in order to rationalize the data as high effort moral hazard equilibria is to choose effort levels (the same for all observations) and low effort probabilities (or the slopes of the low effort budget lines), such that for every observation the IC line passes through the point $X$. If we can do so, we are guaranteed both that $X$ is the optimal consumption in case of high effort and that the agent prefers high effort to low effort. The later is true since by incentive compatibility the low effort and the high effort indifference curves at $X$ have the same utility value while the low effort indifference curve through $W$ (the utility if low effort is chosen) gives lower utility. I show later that we can always find effort costs and low effort probabilities such that the resulting IC line will pass through $X$.

With non linear utility function, in order to rationalize the data we need to worry not only about IC passing through the point $X$, (high effort equilibrium consumption) but also that $X$ will be preferred to F (low effort and full insurance). In order to rationalize the data we need to find effort levels and probabilities, $\Pi^i_L$ such that, for every observation both "high effort is preferred to low effort" and the incentive compatibility conditions are satisfied. We will see that the first condition implies an upper bound on $\Pi^i_L$. Intuitively if $\Pi^i_L$ is close to $\Pi^i_H$ then it is not worthwhile to pay the cost of high effort, and so low effort will be preferred. The second condition implies a lower bound on $\Pi^i_L$. We can rationalize the data if and only if for all $i$, the set of all possible $\Pi^i_L$ is not empty (namely the upper bound is larger than the lower bound). This is the condition stated in the proposition. I derive this condition formally in the proof which is provided in the appendix.

In lemma 3 I show that the risk neutral utility function satisfies the conditions in proposition 9 and therefore rationalizes the data.

**Lemma 3** *The risk neutral utility function, $u(x) = x$ rationalizes any data that satisfies the budget constraint and the smoothing condition.*

**Proof.** Proof. When $u(x) = x$,

$$f(\Pi^i_H) = \Pi^i_H X^i_{GH} + (1 - \Pi^i_H) X^i_{BH} - (\Pi^i_H W^i_G + (1 - \Pi^i_H) W^i_B) = 0$$

by the budget constraint. Therefore $\Pi^{i^*} = \Pi^i_H$ and the sufficient condition from proposition 9 is satisfied. ■

Now that we have established that the risk neutral utility function rationalizes the data, we will use this to find a strictly concave utility function. The idea is that if the risk neutral utility function rationalizes the data, we must be able to find some strictly concave utility "sufficiently close" to the risk neutral utility function that will also rationalize the data. To formalize "sufficiently close" let us take a parameterized family of utility functions from which the risk neutral utility function is obtained for some value of the parameter. In particular I choose to work with the widely used CRRA utility functions family.

Let us look at the family

$$u(\gamma, x) = \frac{x^{1-\gamma^2}}{1-\gamma^2}$$

of CRRA utility functions. With this representation $\gamma^2$ is the coefficient of relative risk aversion. This family is well defined for all $\gamma \in (-1, 1)$. When $\gamma = 0$, $u(0, x) = x$, the risk neutral utility. When $\gamma \neq 0$, $u(\gamma, x)$ is strictly concave and monotonically increasing. For this family of functions I show in lemma 4 in the appendix that $\Pi^*(\gamma)$ is continuous. Then I use the fact that the risk neutral utility rationalizes the data to find a continuum of functions with low enough coefficients of relative risk aversion that also rationalize the data. Intuitively, we need a low coefficient of relative risk aversion since the more risk averse the agent is the more likely she is to prefer low effort level together with a full insurance contract rather than the high effort partial insurance contract.

**Proposition 10** *For all data that satisfy the budget constraint, fair prices and the smoothing condition there exist strictly concave CRRA utility functions that rationalize the data.*

The proof of proposition 10 is provided in the appendix. So far, we have focused on data with consumption smoothing but not full insurance. We have rationalized the data as high effort moral hazard equilibria. Consider now a data set with $n$ consumption smoothing observations.

$$W_G^i > X_{GH}^i > X_{BH}^i > W_B^i$$

that satisfy the budget constraint and $m$ observations with full insurance $X_{GL}^j = X_{BL}^j$ that satisfy the budget constraint. Theorem 4 shows that any utility function and effort costs that rationalize the first $n$ observations will rationalize the $n+m$ observations. Thus, the additional observations add no further testable restrictions. While we explain the first $n$ observations as equilibria where high effort is the optimal choice, we may explain the additional $m$ full insurance observations as resulting form a choice of low effort.

**Theorem 4** *Given a data set*

$$\{(X_G^i, X_B^i), (W_G^i, W_B^i), (P_G^i, P_B^i), (\Pi^i)\} \text{ for } i = 1 \ldots n+m$$

*where*

$$P_G^i > 0, P_B^i \geq 0, \Pi^i = \frac{P_G^i}{P_G^i + P_B^i},$$

$$W_G^i > X_G^i > X_B^i > W_B^i > 0 \text{ for } i = 1...n$$

*and*

$$W_G^i > X_G^i = X_B^i > W_B^i > 0 \text{ for } i = (n+1)...m.$$

*that satisfies the budget constraint (11). Any strictly concave function $u(x)$ that rationalizes the first $n$ observations as high effort moral hazard equilibria rationalizes the given $n+m$ observations according to the model with the first $n$ observations being high effort equilibria and the last $m$ observations being low effort equilibria.*

The condition for $u(x)$ to rationalize the data is as in proposition 9 and the existence of such utility function follows from proposition 10. The proof is provided in the appendix.

I have assumed in the analysis above that we can observe every possible variable related with the equilibrium effort choice, that is consumption bundles, endowments, prices and the probabilities of good and bad states contingent on the equilibrium effort choice. What if we observe less? If we only observe either market prices or the probabilities and not both, we use the result that prices must be proportional to the probabilities in order to determine what the unobserved variable must be. The results of this section still hold, only fair prices are no longer a testable restriction. If neither the probabilities nor the prices are observed but endowments are different than consumption bundles, prices and probabilities can be inferred from the need to satisfy the budget constraint, and this restriction would no longer be a testable implication, leaving consumption smoothing as the only testable implication. If we do not observe endowments or consumption but observe the rest of the variables, we are left with fair prices as the only implication since given one of these variables we would be able to pick the other in a way that satisfies the budget constraint and the smoothing condition. The results in this section suggest that we cannot hope for substantial testable implications unless we observed variables related with the off equilibrium effort choice. I discuss the implications of the model under this assumption in the next section.

### 3.3.2 Implications with Data on Probabilities Associated Both Effort Levels

A set of assumptions that would yield more restrictive empirical implications consists of observing the probabilities of the states of the world (or asset prices) for both efforts (not only the optimal effort that agents choose). Even if the data consists of high effort observations, we may be able to observe the probability of a good state if low effort were to be used. It is possible to rationalize a data set when we can choose the unobserved utility function and effort costs in such a way that every incentive compatibility constraint, for every observation, passes exactly through the consumption point that lies on the budget line. Unlike the previous case, where the low effort probability was a free variable that could be used to move the incentive compatibility constraints to fit the different consumptions, in the case both probabilities are observed, we lose this freedom. The utility and effort costs need to be the same for all observations. It is no longer always possible to move the incentive compatibility constraint so that it will adjust to the different observations.

To find testable restrictions I follow the approach introduced in Brown and Matzkin (1996). The first step is to find a set of equilibrium inequalities, where the unknown values (utility levels and effort costs) are variables and the data are the coefficients, such that the data can be rationalized if and only if there is a solution to these inequalities. Then I appeal to the Tarski-Seidenberg theorem to prove that the equilibrium inequalities can be reduced to an equivalent finite family of polynomial inequalities in the coefficients of the system. This family contains all of the testable implications of the model. In general, the Tarski-Seidenberg algorithm does not terminate in polynomial time. We can prove that this system of inequalities in the data is nontrivial namely it is satisfied neither by all data sets nor by no data set. I show this by constructing an example where the data set is rationalized (there is a solution to the system) and an example of a data set that cannot be rationalized (there is no solution to the system). Further, we note that our family of inequalities is linear given a particular data set. Thus, checking for consistency of data with the model amounts to solving a finite system of linear inequalities. This can be done in polynomial time. Theorem 5 derives the system of linear inequalities and shows the testability of the model.

**Theorem 5** *Given a data set*

$$\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)(\Pi^i_L)\} \text{ for } i = 1 \ldots n$$

*where*

$$0 \leq \Pi^i_L < \Pi^i_H = \frac{P^i_{GH}}{P^i_{GH} + P^i_{BH}},$$

$$X^i_{GH} > X^i_{BH} \geq 0,$$

$$W^i_G > W^i_B > 0 \text{ and } P^i_{GH} > P^i_{BH} \geq 0.$$

*a. We can rationalize the data as high effort moral hazard equilibria with a strictly concave utility function if and only if there is a solution*

$$\{\{U^i_{se}\}\ ,\{M^i_{se} > 0\} \text{ for } i = 1...n \text{ and } V_H > V_L \geq 0\}$$

*to the following linear equilibrium inequalities: for all* $i, j = 1 \ldots n$

*C1-C3*

*C4'.* $V_H - V_L = (\Pi^i_H - \Pi^i_L)(U^i_{GH} - U^i_{BH})$

*C5'.* $\Pi^i_L U^i_{GH} + (1 - \Pi^i_L) U^i_{BH} - U^i_{GL} > 0$

*C6.* $U^i_{se} - U^j_{s'e'} < M^j_{s'e'}(X^i_{se} - X^j_{s'e'})$ *for all* $e, e' \in \{H, L\}$, $s, s' \in \{G, B\}$

*b. The model is testable as in Brown and Matzkin that is consistent (equilibrium inequalities sometimes have a solution) and refutable (equilibrium inequalities sometimes do not have a solution).*

Conditions C2 define the variables $X^i_{GL}, X^i_{BL}$. Conditions C1-C5' are as in lemma 2 only replacing utility functions with a variable representing the value of the utility. Condition

C6 assures that we have a concave function. This condition is a version of the Afriat inequalities. In this theorem we take the definition of high effort equilibrium to be one in which high effort is strictly preferred. Otherwise we need to take a weak inequality in C5' for the necessity statement. With a weakly concave utility function the conditions in part a of the lemma are sufficient. The proof of theorem 5 is provided in the appendix.

The testable implications that we can obtain when we have data on the probabilities (or prices) associated with both possible effort levels are more restrictive than those we had in the case where only information related to the optimal choice can be obtained.

# 4 Concluding Remarks

This dissertation introduces publication into a dynamic patent race. Publishing research results changes the state of the prior art, thus allowing firms to affect the patentability of their rival's inventions. Firms can use publication to set back a patent race when they lag behind.

When the instantaneous probability of success (the hazard rate) is fixed, the symmetric Markov perfect equilibrium strategy is for the follower to publish research results when the leader is close to obtaining a patent. Firms publish more the more patient they are and the larger their instantaneous probability of success. They publish when the patentability requirement is strong.

When firms have two variables of decision, the intensity of research and publications, the use of publications depends on the cost of investment and the patentability requirement. With a quadratic cost function, equilibrium involves publications if the patentability requirement $n$ is high. Firms use publications if the fixed hazard rate is large enough or with a high enough marginal cost parameter. For the race with $n = 3$ we show that the leader invests more than the follower. This is also the result in the standard no publication models (see Harris and Vickers (1987)).

Green and Scotchmer explain that the social goal of protecting profits is served by a strong novelty requirement while the goal of disclosure is served by a weak novelty requirement. O'Donoghue (1998) also suggests that a patentability requirement can stimulate R&D investment and enhance welfare. I showed that a strong patentability requirement also provides an opportunity for publications. While the disclosure of information is desirable, publications might have a negative effect on welfare. They could diminish the return from investment, thus lowering the incentive to innovate. Moreover, if races with publications encourage follower's investment, then publication may imply more duplication of research. Thus defensive publications present contradicting welfare effects.

Among the limitations of the model are the assumptions on the cost function and the information structure. The cost of investment could depend on the prior art and the firms' progress. Diminishing returns to R&D would suggest a higher cost, the larger the number of steps completed. Experience and additional information available would suggest a lower cost, the more steps completed. Thus, investment costs may conceivably go up or down with the number of steps completed. If research becomes more difficult, publication may become less attractive. While if it becomes easier to achieve the next step, publication may become more attractive.

Our assumption is that a firm observes the other firm's position in the race, as well as its own position and the state of the prior art. While it is reasonable that firms can obtain information about their rival's research progress, it may sometimes be more likely that information is asymmetric. A model with asymmetric information would be significantly more difficult to analyze since we should not expect to find stationary equilibrium behavior. A firm's belief about the current state of the world would depend not only on its success but also on time. The longer the time that passes, the more likely it is that a rival made progress. Therefore, it is likely that the decision to publish will depend on time.

Although some models with variables that are off equilibrium and cannot be observed may not be refutable or may have minimal testable implications like the simple moral hazard model we discussed in chapter 3, the analysis in chapter 2 shows that the defensive publications is testable. It can be refuted with data from an equilibrium path.

# 5 Appendix.

## 5.1 Proofs for chapter 1

**Proof of Theorem 1.** Consider the system of equations (1)-(2). It defines the value function when each firm's strategy is never to publish. The no publication strategy is an equilibrium when there is no state in which a firm can benefit from publishing. That is, if and only if the solution to this system satisfies the following inequalities:

$$V^i_{s^i,s^j} \geq V^i_{s^i-1,s^j-1} \text{ and } V^i_{s^i,0} \geq V^i_{s^i-1,0} \text{ for all } 1 \leqslant s^i, s^j \leqslant n-1.$$

However, I show that under some conditions on the parameters $n, r$ and $\overline{\lambda}$, the inequality

$$V^i_{s^i,n-1} < V^i_{s^i-1,n-2} \tag{19}$$

holds. This inequality implies that the follower would be better off by publishing at state $(s^i, n-1)$. Let us write the inequality (19) as a function of the parameters only. From (4) we know that $V^i_{s^i,n-1} = \gamma V^i_{s^i+1,n-1}$. Using recursion we obtain from (4) that:

$$V^i_{s^i,n-1} = \gamma^{n-s^i-1} V^i_{n-1,n-1} = \gamma^{n-s^i} v. \tag{20}$$

Substitute (20) in the left hand side of (19) and substitute (4) in the right hand side to obtain:

$$\gamma^{n-s^i} v < \gamma(V^i_{s^i,n-2} + V^i_{s^i-1,n-1}).$$

Substitute (20) again and rearrange to get:

$$\gamma^{n-s^i} v - \gamma^{n-s^i+2} v < \gamma V^i_{s^i,n-2}.$$

Substituting (4) in the right hand side and rearranging we have:

$$\gamma^{n-s^i}v - 2\gamma^{n-s^i+2}v < \gamma^2 V^i_{s^i+1,n-2}.$$

Continue this step iteratively to obtain:

$$\gamma^{n-s^i}v - (n - s^i - 1)\gamma^{n-s^i+2}v < \gamma^{n-s^i-1}V^i_{n-2,n-2}.$$

Substitute (4) in the right hand side:

$$\gamma^{n-s^i}v - (n - s^i - 1)\gamma^{n-s^i+2}v < \gamma^{n-s^i}[\gamma(1+\gamma)v + \gamma^2 v].$$

This inequality holds if and only if

$$s^i < n + 1 - \frac{1-\gamma}{\gamma^2}.$$

We found an *explicit* function for $s^*$, the cutoff point between having an incentive to publish or not at states $(s^i, n-1)$ when the strategies used are never to publish.

$$s^* = n + 1 - \frac{1-\gamma}{\gamma^2}. \tag{21}$$

When $s^* > 1$, if the firms never publish, then the follower at state $(1, n-1)$ prefers publishing; thus, publication must arise in equilibrium. As a function of the parameters, $s^*(n,\gamma)$ is increasing in $n$ and $\gamma$. If $n \geq 3$ with large enough $\gamma$, in equilibrium, the firms must publish in some states. Since $\gamma$ is increasing in the hazard rate $\overline{\lambda}$, and decreasing in the rate $r$ publications occur with large $\overline{\lambda}$ and with small $r$. ∎

**Proof of Proposition 1.** I show this in 3 steps.

**Step 1:** I show that

$$V_{s,n-1} = \dots = V_{s-k,n-1-k} \dots = V_{0,n-1-s}.$$

Since there is no discounting and no marginal cost of investment the payoffs in every state sum to the value of the patent $v$. The game and the strategies are symmetric. Therefore, whenever both firms have the same number of steps they share the patent value equally. Thus, $V_{s,s} = \frac{v}{2}$. Since the follower publishes at state $(s, n-1)$, according to equation (3), $V_{s,n-1} = V_{s-1,n-2}$. At state $(n-3, n-2)$ the firms do not publish. Therefore the value is:

$$V_{n-3,n-2} = \frac{1}{2}(V_{n-2,n-2} + V_{n-3,n-1}) = \frac{1}{2}(V_{n-3,n-3} + V_{n-4,n-2}) = V_{n-4,n-3}.$$

Next I show by induction that for all $(s, n-2), 0 < s < n-2$, the value equals the value at $(s-1, n-3)$. To do this I show that if $V_{s,n-2} = V_{s-1,n-3}$, then $V_{s-1,n-2} = V_{s-2,n-3}$. But,

$$V_{s-1,n-2} = \frac{1}{2}(V_{s,n-2} + V_{s-1,n-1}) = \frac{1}{2}(V_{s-1,n-3} + V_{s-2,n-2}) = V_{s-2,n-3}.$$

Therefore, $V_{s-1,n-2} = V_{s-2,n-3}$. It follows by induction that $V_{s,n-2} = V_{s-1,n-3}$ for all $s < n-2$. Furthermore, for all $s^b$ and $0 < s^a < s^b$, the value $V_{s^a,s^b}$ equals the value $V_{s^a-1,s^b-1}$. We just proved this result for $s^b = n-2$. Using two nested backwards inductions I now prove that the equality $V_{s^a,s^b} = V_{s^a-1,s^b-1}$ holds for all $s^b$ and $s^a$, $0 < s^a < s^b$. Suppose the result is true for $s^b = s$ and all greater number of steps, then we can show it is true for $s^b = s-1$. First the result is true for $s^b = s-1$ and $s^a = s-2$ since:

$$V_{s-2,s-1} = \frac{1}{2}(V_{s-1,s-1} + V_{s-2,s}) = \frac{1}{2}(V_{s-2,s-2} + V_{s-3,s-1}) = V_{s-3,s-2}.$$

If the result is true for $s^b = s-1$ and some $s^a < s-1$, then it is also true for $s^b = s-1$ and $s^a - 1$ since:

$$V_{s^a-1,s-1} = \frac{1}{2}(V_{s^a,s-1} + V_{s^a-1,s}) = \frac{1}{2}(V_{s^a-1,s-2} + V_{s^a-2,s-1}) = V_{s^a-2,s-2}.$$

by the induction assumptions. Therefore, for $s^b = s-1$ and for all $s^a < s^b$, $V_{s^a,s^b} = V_{s^a-1,s^b-1}$. Thus the induction is complete and $V_{s^a,s^b} = V_{s^a-1,s^b-1}$ for all $s^a < s^b$. Because $V_{s^b,s^a} = v - V_{s^a,s^b}$ the equality holds for all $(s^a, s^b)$.

**Step 2**: I show that for all $s$,

$$V_{0,s} = \frac{n-s}{2n}v \text{ and } V_{s,0} = \frac{n+s}{2n}v.$$

The values of the game are defined by the system of equations:

$V_{s,n} = 0,\ V_{n,s} = v$ for all $s < n-1$.

$V_{s,n-1} = V_{s-1,n-2}$ for all $0 < s < n-1$.

$V_{s^a,s^b} = \frac{1}{2}(V_{s^a+1,s^b} + V_{s^a,s^b+1})$ for all other $s^a, s^b$.

Using step 1 we reduce the system to:

$$V_{0,s} = \frac{1}{2}(V_{0,s-1} + V_{0,s+1}) \text{ for all } 0 < s < n,$$

where $V_{0,n} = 0$ and $V_{0,0} = \frac{1}{2}v$. Given these values we can find $V_{s,0} = v - V_{0,s}$, since there is no discounting and the patent value at every state is split between the firms. This is a system with $n-1$ variables $V_{0,s}$ for all $0 < s < n$ and $n-1$ equations. It is easy to verify that the unique solution to this system is $V_{0,s} = \frac{n-s}{2n}v$.

**Step 3**: I show that for all $s$,

$$V_{s^a,s^b} = \frac{n+s^a-s^b}{2n}v \text{ for all } 0 \leqslant s^a, s^b < n.$$

For $s^a \geq s^b$, by step 1, $V_{s^a,s^b} = V_{s^a-s^b,0}$ and from step 2, $V_{s^a-s^b,0} = \frac{n+s^a-s^b}{2n}v$. For $s^a < s^b$, by step 1, $V_{s^a,s^b} = V_{0,s^b-s^a}$ and from step 2, $V_{0,s^b-s^a} = \frac{n+s^a-s^b}{2n}v$. ∎

**Proof of Theorem 2.** 1. I show that if in equilibrium the follower, firm $i$, publishes at $(s, n-1)$, for $s > 1$, with some positive probability, then the follower also publishes at

$(s-1, n-1)$ with probability 1. That is, we need to show that if the firm publishes at $(s, n-1)$, then

$$\gamma V^i_{s,n-1} < V^i_{s-2,n-2}, \tag{22}$$

where the left hand side is the value for the follower, $i$, at state $(s-1, n-1)$ if the firm does not publish and the right hand side is the value if the firm publishes. Since the firm publishes at $(s, n-1)$, the value at this state is the same as the value at $(s-1, n-2)$, that is.

$$V^i_{s,n-1} = V^i_{s-1,n-2}.$$

So (22) holds if and only if

$$\gamma V^i_{s-1,n-2} < V^i_{s-2,n-2}.$$

As long as the leader does not publish at $(s-2, n-2)$, the value of $V^i_{s-2,n-2}$ is at least as large as the value that can be obtained in this state with no publication:

$$\gamma(V^i_{s-1,n-2} + V^i_{s-2,n-1}) \leqslant V^i_{s-2,n-2}.$$

Therefore (22) holds if:

$$\gamma V^i_{s-1,n-2} < \gamma(V^i_{s-1,n-2} + V^i_{s-2,n-1}).$$

But this is equivalent to

$$0 < \gamma V^i_{s-2,n-1},$$

which holds true since at state $(s-2, n-1)$ with some positive probability firm $i$ wins the race and gets a positive payoff.

2. I verify that the proposed strategy is an equilibrium. It suffices to show that the values $V_{s^a,s^b} = \frac{n+s^a-s^b}{2n}$ satisfy the following inequalities:

$$V_{s^a,s^b} \geq V_{s^a-1,s^b-1} \text{ for all } 0 < s^a, s^b < n. \tag{23}$$

$$V_{s,0} \geq V_{s-1,0} \text{ for all } 0 < s < n. \tag{24}$$

$$V_{s,n-1} \geq \gamma V_{s+1,n-1} \text{ for all } s < n-1. \tag{25}$$

The inequalities (23) and (24) ensure that a deviation to publication is not profitable and (25) ensures that a deviation to no publication at the states with publication is not profitable. It is easy to verify that the values (given in proposition 1) satisfy these inequalities.

I now claim that the results are true for $r$ small enough. The values of the game given the proposed strategy are calculated as a solution to an $(n-1) \times (n-1)$ system of linear independent equations. The values are therefore continuous in the parameter values. Since the solution of the system when $r = 0$ satisfies the inequalities (24) and (25) as strict inequalities, these inequalities will also hold for $r > 0$ sufficiently small. It can be verified that the solution to the system with small enough discount rate and cost also satisfies (23). Hence the strategy is a Markov perfect equilibrium. ■

**Proof of Proposition 2.** Let $\varepsilon$ be a small publication fee. I first show that in equilibrium, the leader strictly prefers not to publish at states $(n-1, s)$ and $(s, 0)$. And that the follower strictly prefers to publish at state $(s, n-1)$. For $\varepsilon = 0$, at state $(n-1, 0)$, the value for the leader if she does not publish is

$$\frac{1}{2}(v + V_{n-1,1}) \geq \frac{1}{2}(v + V_{n-2,0}) > V_{n-2,0}.$$

By induction, if $V_{s,0} > V_{s-1,0}$, then $V_{s-1,0} > V_{s-2,0}$ since the value for the leader at $(s-1,0)$ if she does not publish is $\frac{1}{2}(V_{s,0} + V_{s-1,1}) > \frac{1}{2}(V_{s-1,0} + V_{s-2,0}) \geq V_{s-2,0}$. Therefore,

$$V_{s,0} > V_{s-1,0}.$$

This inequality is strict and would also hold with a small publication fee. The leader at state $(n-1,s)$ will not publish either. If the follower publishes, then the leader is better off avoiding the publication fee. If the follower does not publish, then the leader's payoff if she does not publish is

$$\frac{1}{2}(v + V_{n-1,s+1}) > \frac{1}{2}(V_{n-k,s-k} + V_{n-1-k,s+1-k}),$$

which is the payoff if she publishes and $\varepsilon = 0$. Again this strict inequality would also hold with a small publication fee.

The follower at state $(s, n-1)$ will necessarily have an incentive to publish. Consider $\varepsilon = 0$. First I show this for $s = n-2$. The follower will publish in state $(n-2, n-1)$ if

$$\frac{1}{2}V_{n-1,n-1} < V_{n-3,n-2}. \tag{26}$$

The value[15] in state $(n-3, n-2)$ is given by (4), substituting it we obtain the condition:

$$\frac{1}{2}V_{n-1,n-1} < \frac{1}{2}(V_{n-2,n-2} + V_{n-3,n-1}).$$

When $r = 0$, $V_{n-1,n-1} = V_{n-2,n-2} = \frac{v}{2}$. Therefore, the inequality (26) becomes:

$$0 < \frac{1}{2}V_{n-3,n-1},$$

[15] If $n-3=0$ we have shown no publications. If $n-3>0$ the no publications value is at least as high as the publications value. Therefore, if a firm publishes at this state then both firms must be indifferent about the publication so the value would equal the no publications value given by (4).

which is true because the right hand side is positive. It now follows that the follower will publish at all other states $(s, n-1)$ (by the proof of theorem 2 since the leader does not publish at $(s, n-2)$). This strong incentive to publish would remain with a small enough publication fee.

We have shown that in equilibrium, the leader strictly prefers not to publish at states $(n-1, s)$ and $(s, 0)$, and that the follower strictly prefers to publish at state $(s, n-1)$. When $\varepsilon = 0$, any pair of Markov strategies for which these properties hold, result in the values given in (5) for the "publish as a last resort" strategy. Since these values satisfy both (3) and (4) at every state $(s^a, s^b)$ for $0 < s^a, s^b < n-1$. With a small publication fee, the value at state $(s^a, s^b)$ would have an expected benefit as in (5) minus the expected publication expenses. If there exists an equilibrium other than the "publish as a last resort", firms publish more than with "publish as a last resort". Therefore, at some state for some firm, the expected expenses would be higher than with "publish as a last resort". A deviation from its strategy to "publish as a last resort" would yield the same benefit and a lower expenditure on publication and therefore at this state it would be profitable. Contradiction. Equilibrium must be unique when there is a small publication fee. Finally, uniqueness holds in a neighborhood of $r = 0$. As $r \to 0$, $\gamma = \frac{\lambda}{r+2\lambda} \to \frac{1}{2}$. And we note that the inequalities involving $\gamma = \frac{1}{2}$ in the proof are strict and remain true as $\gamma \to \frac{1}{2}$. ■

**Proof of Proposition 3.** This result is obtained by solving the system of equations for the values of the game with no publications (1)-(2), the system for the case of publication at state $(1, 2)$ and the system for the case the firms publish at this state with a probability $q$. I check the equilibrium conditions (23)-(25) to find the parameter values for which each of these cases is an equilibrium. The equations depend on $\gamma$ and not on $\overline{\lambda}$ and $r$ independently. The solution obtained is:

$$q = \begin{cases} 0 & \text{for } \gamma \leqslant -\frac{1}{6}+\frac{1}{6}\sqrt{13} \\ \frac{\gamma+3\gamma^2-1}{\gamma(1+3\gamma^3-2\gamma)} & \text{for } -\frac{1}{6}+\frac{1}{6}\sqrt{13} < \gamma < \frac{1}{6}\sqrt{(30-6\sqrt{13})} \\ 1 & \text{for } \frac{1}{6}\sqrt{(30-6\sqrt{13})} \leqslant \gamma \end{cases}.$$

The parameter $\gamma = \frac{\overline{\lambda}}{r+2\overline{\lambda}}$, is monotone is $\overline{\lambda}$ and in $r$. Writing these conditions as a function of $r$ instead of $\gamma$ gives the following cutoff points:

$$\overline{r_{\overline{\lambda}}} = 2\overline{\lambda}\frac{4-\sqrt{13}}{-1+\sqrt{13}} \text{ and } \underline{r_{\overline{\lambda}}} = 2\overline{\lambda}\frac{3-\sqrt{(30-6\sqrt{13})}}{\sqrt{(30-6\sqrt{13})}}.$$

■

**Proof of Proposition 4.** The cost of increasing the hazard rate, $\overline{\lambda}$ above the fixed hazard rate, $\overline{\lambda}$ is convex. The difference between any two values at two states is bounded by the patent value, $v$. By the first order condition (7) the marginal cost of investment is therefore bounded by $v$. When $c_0 \to \infty$, it must be that the optimal hazard rate at each state approaches $\overline{\lambda}$, $\lambda_{s,s'}(c_0) \underset{c_0\to\infty}{\longrightarrow} \overline{\lambda}$. Otherwise, the marginal cost cannot be bounded, since $C'(\lambda) > 0$ for all $\lambda > 0$. Moreover, the cost function at the optimal equilibrium investments must approach zero as $c_0 \to \infty$. Otherwise, for high enough $c_0$ the firm can do better by choosing a corner solution $\lambda = \overline{\lambda}$ for which $c(\overline{\lambda}) = 0$. Since equilibrium hazard rates approach the fixed hazard rate $\overline{\lambda}$ and the cost approaches zero, the fixed probability race values are the limit of the equilibrium values as the cost parameter $c_0 \to \infty$.

For a quadratic cost function we can prove the above by directly solving for the optimal hazard rate from the first order condition (7). Substitute the optimal effort from equation

(7) into (1), multiply by $2c_0(\lambda_{s^i,s^j} + \lambda_{s^j,s^i})$ and rearrange to get:

$$-\frac{1}{2}(V_{s^i+1,s^j}-V_{s^i,s^j})^2+(V_{s^j+1,s^i}-V_{s^j,s^i})(V_{s^i,s^j}-V_{s^i,s^j+1})-2c_0\overline{\lambda}(V_{s^i+1,s^j}+V_{s^i,s^j+1}-2V_{s^i,s^j}) = 0. \tag{27}$$

The difference between any two continuation values is bounded by $-v$ and $v$. Thus when $c_0\overline{\lambda} \to \infty$, it must be the case that

$$V_{s^i+1,s^j} + V_{s^i,s^j+1} - 2V_{s^i,s^j} \to 0,$$

yielding the same system as in the non discounted fixed hazard rate race, for which we showed that firms publish. ■

**Proof of Proposition 5.** For all states $(s^i, s^j)$

$$V_{s^i+1,s^j} \geq V_{s^i,s^j}. \tag{28}$$

From the evolution of states in the game with publications it must be that:

$$v > V_{2,0} + V_{0,2} > V_{1,0} + V_{0,1} > 2V_{1,1}. \tag{29}$$

I rearrange equation (27) and obtain:

$$(V_{s^j+1,s^i}-V_{s^j,s^i})(V_{s^i,s^j}-V_{s^i,s^j+1})+2c_0\overline{\lambda}(V_{s^i,s^j}-V_{s^i,s^j+1}) = \frac{1}{2}(V_{s^i+1,s^j}-V_{s^i,s^j})^2+2c_0\overline{\lambda}(V_{s^i+1,s^j}-V_{s^i,s^j}). \tag{30}$$

From the first order condition

$$\lambda_{1,0} > \lambda_{0,1} \iff V_{2,0} - V_{1,0} > V_{1,1} - V_{0,1}$$

Assume by contradiction that $(V_{1,1} - V_{0,1}) \geq (V_{2,0} - V_{1,0})$ . By equation (30) specialized to states $(1,0)$ and $(0,1)$, if $(V_{1,0} - V_{1,1}) > (V_{0,1} - V_{0,2})$ then $(V_{1,1} - V_{0,1}) < (V_{2,0} - V_{1,0})$

which is a contradiction. If on the other hand $(V_{0,1} - V_{0,2}) \geq (V_{1,0} - V_{1,1})$ then

$$(V_{1,1} - V_{0,1}) \geq (V_{2,0} - V_{1,0}) \geq (V_{0,1} - V_{0,2}) \geq (V_{1,0} - V_{1,1}),$$

where the middle inequality holds by (29). It follows that $2V_{1,1} \geq V_{1,0} + V_{0,1}$ which is a contradiction to (29). I conclude that $(V_{1,1} - V_{0,1}) < (V_{2,0} - V_{1,0})$ and the leader at $(1,0)$ invests more than the follower. A similar argument shows that the leader at $(2,0)$ invests more than the follower. ■

## 5.2 Proofs for Chapter 3

**Proof of Proposition 9.** $\Longrightarrow$ (Sufficient) Suppose the data and the concave utility function $u(x)$ satisfy the conditions in the theorem we would like to show that $u(x)$ rationalizes the data as high effort moral hazard equilibria. By lemma 2 and definition 7, it suffices to show that there exist probabilities of the good state, $\Pi^i_L$ such that $1 \geq \Pi^i_H > \Pi^i_L \geq 0$ and there exist effort costs $V_H > V_L$ such that for all $i$ conditions C1-C5 hold.

Condition C3 is given in the theorem. By the second condition

$$\Pi^i_H = \frac{P^i_{GH}}{P^i_{GH} + P^i_{BH}}.$$

Define $\Pi^i_L = P^i_{GL}$, and define

$$X^i_{GL} = X^i_{BL} = \Pi^i_L W^i_G + (1 - \Pi^i_L) W^i_B,$$

for $\Pi^i_L$ yet to be determined. This gives us conditions C1 and C2. Let $V_L = 0$. It is left to show that we can find probabilities of the good state for low effort, $\Pi^i_L$ such that

$$1 \geq \Pi^i_H > \Pi^i_L \geq 0$$

and high effort cost $V_H > 0$ such that for all $i$ conditions C4 (incentive compatibility satisfied with equality) and C5 (effort $H$ preferred to effort $L$) hold. Let

$$V_H = \min_j \{\Pi^j_H [u(X^j_{GH}) - u(X^j_{BH})]\},$$

$V_H > 0$. Let

$$\Pi^i_L = \Pi^i_H - \frac{V_H - V_L}{u(X^i_{GH}) - u(X^i_{BH})}$$

for all $i$. First I show that the low effort probabilities are well defined, that is $\Pi^i_L \in [0, \Pi^1_H)$.

Clearly,

$$\frac{\min_j\{\Pi^j_H[u(X^j_{GH}) - u(X^j_{BH})]\}}{u(X^i_{GH}) - u(X^i_{BH})} > 0.$$

Therefore $\Pi^i_L < \Pi^i_H$.

Since the left hand side is a minimum over all $j$,

$$\frac{\min_j\{\Pi^j_H[u(X^j_{GH}) - u(X^j_{BH})]\}}{u(X^i_{GH}) - u(X^i_{BH})} \leq \frac{\Pi^i_H[u(X^i_{GH}) - u(X^i_{BH})]}{[u(X^i_{GH}) - u(X^i_{BH})]} = \Pi^i_H.$$

Therefore $\Pi^i_L \geq 0$. Hence, the probabilities are well defined. Rearranging the definitions of $\Pi^i_L$ we see that C4, the incentive compatibility condition

$$V_H - V_L = (\Pi^i_H - \Pi^i_L)[u(X^i_{GH}) - u(X^i_{BH})]$$

is true for all $i$. It is left to verify that condition C5 (high effort is preferred to low effort) is satisfied for every observation. The condition is: for all $i$,

$$f^i(\Pi^i_L) \equiv \Pi^i_L u(X^i_{GH}) + (1 - \Pi^i_L)u(X^i_{BH}) - u(\Pi^i_L W^i_G + (1 - \Pi^i_L)W^i_B) \geq 0.$$

I defined $\Pi^{i*}$ to be the largest zero of the function $f^i(\Pi)$ in the range $[0, \Pi^i_H]$. Note that $f^i(\Pi)$ is a continuous convex function that is positive at $\Pi = 0$ (from monotonicity of $u(x)$ and the assumption $W^i_B < X^i_{BH}$). Also $f^i(\Pi)$ is non-positive at $\Pi = \Pi^i_H$ (from concavity of $u(x)$ and the budget constraint). Therefore $f^i(\Pi)$ obtains a zero in $(0, \Pi^i_H]$. If $f^i(\Pi^i_H) = 0$ then $\Pi^{i*} = \Pi^i_H$ and if $f^i(\Pi^i_H) < 0$ then by convexity the function $f^i(\Pi)$ has a unique zero in the segment, $\Pi^{i*}$. In the range $[0, \Pi^{i*})$, $f^i(\Pi^i_L) > 0$, high effort is preferred to low effort. In the range $(\Pi^{i*}, \Pi^i_H]$ $f^i(\Pi^i_L) < 0$, low effort is preferred. We only need to check $\Pi^i_L \leq \Pi^{i*}$.

But

$$\Pi^i_L = \Pi^i_H - \frac{V_H - V_L}{u(X^i_{GH}) - u(X^i_{BH})} = \Pi^i_H - \frac{\min_j\{\Pi^j_H[u(X^j_{GH}) - u(X^j_{BH})]\}}{u(X^i_{GH}) - (X^i_{BH})} \leq \Pi^{i^*}.$$

This holds for all $i$ by the assumption of the theorem. Hence, when

$$\Pi^i_H - \min_j\{\frac{\Pi^j_H[u(X^j_{GH}) - u(X^j_{BH})]}{[u(X^i_{GH}) - (X^i_{BH})]}\} \leq \Pi^{i^*} \tag{31}$$

for every $i = 1 \ldots n$ we can find probabilities and effort costs such that $u(x)$ rationalizes the data as high effort moral hazard equilibria.

$\Longleftarrow$ (necessary) Suppose a strictly concave utility function, $u(x)$ rationalizes the data as high effort moral hazard equilibria. By the definitions and lemma 2 we know that there exist probabilities of the good state $\Pi^i_L$ such that $1 \geq \Pi^i_H > \Pi^i_L \geq 0$ and effort costs $V_H > V_L \geq 0$ such that together with the data these satisfy, for all $i$, the conditions C1-C5. I need to show that for every $i$ condition (17) holds.

Condition C4, the incentive compatibility condition, can also be written as

$$\Pi^i_L = \Pi^i_H - \frac{V_H - V_L}{u(X^i_{GH}) - u(X^i_{BH})}.$$

The probability being non negative gives us the inequality

$$\Pi^i_H - \frac{V_H - V_L}{u(X^i_{GH}) - u(X^i_{BH})} \geq 0$$

for every $i$. Substituting the incentive compatibility for observation 1:

$$V_H - V_L = (\Pi^1_H - \Pi^1_L)[u(X^1_{GH}) - (X^1_{BH})].$$

These non negativity conditions become: for every $i$,

$$\Pi^i_H - \frac{(\Pi^1_H - \Pi^1_L)[u(X^1_{GH}) - (X^1_{BH})]}{u(X^i_{GH}) - u(X^i_{BH})} \geq 0.$$

Or for every $i$,

$$(\Pi_H^1 - \Pi_L^1) \leq \frac{\Pi_H^i[u(X_{GH}^i) - u(X_{BH}^i)]}{u(X_{GH}^1) - u(X_{BH}^1)}.$$

or

$$(\Pi_H^1 - \Pi_L^1) \leq \min_j \frac{\Pi_H^j[u(X_{GH}^j) - u(X_{BH}^j)]}{u(X_{GH}^1) - u(X_{BH}^1)}.$$

I use this inequality to obtain that for every $i$

$$\Pi_L^i = \Pi_H^i - \frac{V_H - V_L}{u(X_{GH}^i) - u(X_{BH}^i)} =$$

$$\Pi_H^i - (\Pi_H^1 - \Pi_L^1)\frac{u(X_{GH}^1) - u(X_{BH}^1)}{u(X_{GH}^i) - u(X_{BH}^i)} \geq$$

$$\Pi_H^i - \min_j\{\frac{\Pi_H^j[u(X_{GH}^j) - u(X_{BH}^j)]}{u(X_{GH}^i) - u(X_{BH}^i)}\}.$$

I will now show that $\Pi_L^i \leq \Pi^{i^*}$ and by this obtain the required inequality. Condition C5 tells us that high effort is preferred to low effort.

$$f^i(\Pi_L^i) \equiv \Pi_L^i u(X_{GH}^i) + (1 - \Pi_L^i)u(X_{BH}^i) - u(\Pi_L^i W_G^i + (1 - \Pi_L^i)W_B^i) \geq 0.$$

I defined $\Pi^{i^*}$ as the largest zero of the function $f^i(\Pi)$ in the range $[0, \Pi_H^i]$. In the range $[0, \Pi^{i^*}]$, $f^i(\Pi_H^i) \geq 0$, high effort is preferred to low effort. In the range $(\Pi^{i^*}, \Pi_H^i]$, $f(\Pi_H^i) < 0$, low effort is preferred. Since high effort is preferred to low effort $\Pi_L^i \leq \Pi^{i^*}$. Combining this with the previous result we see that when the data is rationalized as high effort moral hazard equilibria,

$$\Pi_H^i - \min_j\{\frac{\Pi_H^j[u(X_{GH}^j) - u(X_{BH}^j)]}{u(X_{GH}^i) - u(X_{BH}^i)}\} \leq \Pi^{i^*}.$$

for every $i = 1 \ldots n$. ■

**Lemma 4** *The function* $\Pi^{i^*}(\gamma)$ *is a continuous*

**Proof of lemma 4.** The function $\Pi^{i^*}(\gamma)$is defined as the zero of the function:

$$f^i(\gamma,\Pi) = \Pi u(\gamma, X^i_{GH}) + (1-\Pi)u(\gamma, X^i_{BH}) - u(\gamma, \Pi W^i_G + (1-\Pi)W^i_B) = 0,$$

in $[0, \Pi^i_H]$. For all $\gamma$ in $(-1,1)$, $f^i(\gamma,0) > 0$ , $f^i(\gamma,\Pi^i_H) \leq 0$ and $f^i(\gamma,\Pi^i)$ is continuous. Therefore we know that there exists a zero of the function in $(0,\Pi^i_H]$. The slope of $f^i(\gamma,\Pi)$ for a given $\gamma$ is:

$$f^i_\Pi(\gamma,\Pi) = \frac{(X^i_{GH})^{1-\gamma^2} - (X^i_{BH})^{1-\gamma^2}}{1-\gamma^2} - [\Pi W^i_G + (1-\Pi)W^i_B]^{-\gamma^2} \times (W^i_G - W^i_B).$$

$$f^i_\Pi(\gamma,\Pi) = X^i_{GH} - X^i_{BH} - (W^i_G - W^i_B) < 0$$

for all $\Pi$ by the smoothing condition. In particular[16] $f^i_\Pi(0,2) < 0$. The function $f^i_\Pi(0,\Pi)$ is a continuous function of $\Pi$ in $(-1,1)$ and therefore there exists an open neighborhood of $\gamma = 0$, $B_\varepsilon(0) \subset (-1,1)$ such that for all $\gamma$ in this neighborhood $f^i_\Pi(\gamma,2) < 0$. The function $f^i(\gamma,\Pi)$ is a convex function of $\Pi$ so $f^i_\Pi(\gamma,\Pi)$ is increasing in $\Pi$ and therefore $f^i_\Pi(\gamma,\Pi) < f^i_\Pi(\gamma,2) < 0$ for all $\gamma \in B_\varepsilon(0)$ and $\Pi \in (0,2)$. We conclude that for all $\gamma \in B_\varepsilon(0)$ and $\Pi \in (0,2)$, $f^i(\gamma,\Pi)$ is a strictly decreasing function of $\Pi$ and has a unique zero (that lies in the segment $(0,\Pi^i_H]$). The function $f^i(\gamma,\Pi^i)$ is a $C^1$ function on the open neighborhood $B_\varepsilon(0) \times (0,2)$ of $(0,\Pi^i_H)$. We know from the budget constraint that $f^i(\gamma,\Pi^i_H) = 0$. Differentiating the function $f^i(\gamma,\Pi)$ with respect to $\Pi$ and evaluating at $(0,\Pi^i_H)$ we obtain that

$$f^i_\Pi(0,\Pi^i_H) = X^i_{GH} - X^i_{BH} - (W^i_G - W^i_B) \neq 0$$

[16]The number 2 was arbitrarily chosen, to create an open neighborhood that contains the non-open sets of interest $(0,\Pi^i_H]$.

from the consumption smoothing condition. Therefore, we can apply the implicit function theorem to find that there exist neighborhoods $N_\gamma$ and $N_\Pi$ of zero and $\Pi^i_H$, $N_\gamma \times N_\Pi \subset B_\epsilon(0) \times (0,2)$ such that $f^i(\gamma, \Pi) = 0$ has a unique solution in $\Pi$ for all $(\gamma, \Pi^i) \in N_\gamma \times N_\Pi$. Furthermore the function $\phi : N_\gamma \to N_\Pi$ that uniquely defines this solution, is itself a $C^1$ function, in particular it is continuous. As we saw earlier, for all $\gamma \in B_\epsilon(0)$ and $\Pi \in (0,2)$, $f^i(\gamma, \Pi^i)$ has a unique zero that lies in the segment $(0, \Pi^i_H]$ we called it $\Pi^{i^*}(\gamma)$. So it must be then, that $\Pi^{i^*}(\gamma) \equiv \phi(\gamma)$ on $N_\gamma$. We have proved that $\Pi^{i^*}(\gamma)$ is a continuous function of $\gamma$. ∎

**Proof of Proposition 10.** Let us consider at the family of CRRA utility functions, $u(\gamma, x) = \frac{x^{1-\gamma^2}}{1-\gamma^2}$. The necessary and sufficient condition for $u(\gamma, x)$ to rationalize the data is

$$\Pi^i_H - \min_j \{ \frac{\Pi^j_H [u(\gamma, X^j_{GH}) - u(\gamma, X^j_{BH})]}{[u(\gamma, X^i_{GH}) - u(\gamma, X^i_{BH})]} \} - \Pi^{i^*}(\gamma) \leq 0.$$

As we obtained in proposition 10 this condition holds with a strict inequality for the linear utility function, that is for $\gamma = 0$. The function $u(\gamma, x)$ is a continuous function of $\gamma$, $u(\gamma, X^i_{GH}) \neq u(\gamma, X^i_{BH})$ and $\Pi^{i^*}(\gamma)$ is continuous on an open set containing zero (by lemma 4). Therefore, the left-hand side of the inequality is a continuous function of $\gamma$. Hence there is a neighborhood of $\gamma = 0$, $B_\epsilon(0)$ for which the inequality holds. For all $\gamma$ in this neighborhood, $u(\gamma, x)$ is a strictly concave utility function that rationalizes the data. ∎

**Proof.** I$\Longrightarrow$II. If the data is rationalized as high effort moral hazard equilibria, then by lemma 2 the budget constraint and the fair prices condition and consumption smoothing must hold for all observations.

II$\Longrightarrow$I. Follows immediately from proposition 10. ∎

**Proof of Theorem 4.** Let $u(x)$ be a strictly concave utility that rationalizes the first $n$ observations as high effort moral hazard equilibria with probabilities $\Pi_L^i$, $i = 1..n$ associated with low effort and effort costs $V_H > V_L$. In order for this utility function and effort levels to rationalize the additional $m$ full insurance observations there need to exist $(X_{GH}^i, X_{BH}^i)$, and $1 \geq \Pi_H^i > \Pi_L^i$ for $i = (n+1)...m$ such that low effort is preferred to high effort. That is

$$\Pi_H^i u(X_{GH}^i) + (1 - \Pi_H^i)u(X_{BH}^i) - V_H \leq u(\Pi_L^i W_G^i + (1 - \Pi_L^i)W_B^i) - V_L,$$

where $(X_{GH}^i, X_{BH}^i)$ is the best consumption choice of the agent if she were to choose high effort.

For each $i = (n+1)...m$ we consider two possible cases.

**Case 1:** Suppose

$$\frac{V_H - V_L}{u(W_G^i) - u(W_B^i)} < 1 - \Pi_L^i.$$

Let

$$\Pi_H^i = \Pi_L^i + \frac{V_H - V_L}{u(W_G^i) - u(W_B^i)}.$$

Then $1 > \Pi_H^i > \Pi_L^i$. Let

$$(X_{GH}^i, X_{BH}^i) = (W_G^i, W_B^i).$$

It is now the case that both the budget constraint for high effort,

$$\Pi_{GL}^i(W_G^i - W_G^i) + (1 - \Pi_{GL}^i)(W_B^i - W_B^i) = 0,$$

and the incentive compatibility constraint:

$$V_H - V_L = (\Pi_H^i - \Pi_L^i)[u(W_G^i) - u(W_B^i)]$$

are satisfied. Hence, $(W^i_{GH}, W^i_{BH})$ is the best choice in the case where high effort is chosen. It is also the case that low effort is strictly better than high effort since given the incentive compatibility constraint, this is equivalent to

$$\Pi^i_L u(W^i_{GH}) + (1 - \Pi^i_L) u(W^i_{BH}) - u(\Pi^i_L W^i_G + (1 - \Pi^i_L) W^i_B) < 0,$$

which is true by strict concavity of $u(x)$. Notice that both the condition that the probability is less than 1 and that low effort is preferred are satisfied with strict inequality when $(X^i_{GH}, X^i_{BH}) = (W^i_G, W^i_B)$. Therefore we can take a very small perturbation of the endowments

$$(X^i_{GH}, X^i_{BH}) = (W^i_G - \varepsilon, W^i_B + \delta),$$

so that the budget constraint will still hold with equality, the probability $\Pi^i_{GL}$ will still be well defined, and low effort will still be preferred. The perturbed endowments satisfy the smoothing condition.

**Case 2:** Suppose

$$\frac{V_H - V_L}{u(W^i_G) - u(W^i_B)} \geq 1 - \Pi^i_L.$$

In this case, the cost of high effort is so high that for any probability, $\Pi^i_H$, low effort is preferred to high effort. Take some $1 > \Pi^i_H > \Pi^i_L$. Given the assumption on effort costs the incentive compatibility constraint does not hold for the endowment bundle or for any consumption-smoothing bundle on the budget line. To see this we rearrange the incentive compatibility constraint to get:

$$(\Pi^i_H - \Pi^i_L)[u(X^i_{GH}) - u(X^i_{BH})] \geq V_H - V_L.$$

But from the assumption in case 1and by $1 \geq \Pi^i_H$

$$V_H - V_L > (1 - \Pi^i_L)[u(W^i_G) - u(W^i_B)] \geq (\Pi^i_H - \Pi^i_L)[u(W^i_G) - u(W^i_B)] \geq$$

$$(\Pi^i_H - \Pi^i_L)[u(X^i_{GH}) - u(X^i_{BH})],$$

by consumption smoothing. Therefore incentive compatibility cannot hold.

Since none of the consumption smoothing bundles satisfy incentive compatibility, if the agent chooses high effort only bundles on the budget line to the left of the endowment may be feasible (bundles with higher variance: $X^i_{BH} < W^i_B$). But these bundles give lower expected utility then the endowment bundle when high is chosen (by second order stochastic dominance). Low effort and no trade give higher utility than high effort and no trade as we saw above. Low effort and full insurance is strictly better than low effort and no trade by concavity. Therefore low effort and full insurance is strictly preferred to the best an agent can do if she chooses high effort. Thus in this case, low effort is preferred to high effort. We conclude that the observation at hand is consistent with a low effort equilibrium given the utility and effort costs that rationalize the first $n$ observations of the data. ■

**Proof of Theorem 5.** a. Suppose we can rationalize the data with a strictly concave increasing differentiable utility function $u(x)$. Then let $U^i_{se} = u(X^i_{se})$ and $M^i_{se} = u'(X^i_{se})$. The first five conditions are implied by the model (see lemma 2). The sixth condition follows from the strict concavity of $u$.

If there is a solution to the system, then by condition C6 and the construction in Afriat's theorem, there is a concave utility function such that $U^i_{se} = u(X^i_{se})$ and $M^i_{se} = u'(X^i_{se})$.

By Chiappori and Rochet (1987) we can take a strictly concave smooth function by using convolution and making a small perturbation. Note that conditions C1-C3 are conditions on the observed variables only. Condition C4' is an equality condition that defines the difference between effort costs. Conditions C5 and C6 are strict inequalities and so will hold after a small enough perturbation of the utility levels. The data satisfies all the conditions for high effort moral hazard equilibrium given the strictly concave utility constructed and therefore this utility function rationalizes the data.

b. Having a system of equilibrium inequalities I now show that there are possible data sets for which there is no solution to the system that is, there are data sets that cannot be rationalized by any concave utility function.

For the condition C4' to be satisfied for all $i$ it must be that for any $i, j$:

$$(\Pi^i_H - \Pi^i_L)(U^i_{GH} - U^i_{BH}) = (\Pi^j_H - \Pi^j_L)(U^j_{GH} - U_{BH}).$$

Or

$$\frac{\Pi^i_H - \Pi^i_L}{\Pi^j_H - \Pi^j_L} = \frac{U^j_{GH} - U^j_{BH}}{U^i_{GH} - U^i_{BH}}.$$

For any data with $X^j_{GH} > X^i_{GH} > X^i_{BH} > X^j_{BH}$ it must be that $U^j_{GH} > U^i_{GH} > U^i_{BH} > U^j_{BH}$. Therefore,

$$\frac{U^j_{GH} - U^j_{BH}}{U^i_{GH} - U^i_{BH}} > 1.$$

But it is possible to have the probabilities satisfy

$$\frac{\Pi^i_H - \Pi^i_L}{\Pi^j_H - \Pi^j_L} < 1.$$

Hence, the condition

$$\frac{\Pi_H^i - \Pi_L^i}{\Pi_H^j - \Pi_L^j} = \frac{U_{GH}^j - U_{BH}^j}{U_{GH}^i - U_{BH}^i}$$

cannot hold. For such a data set, there is no solution to the equilibrium inequalities. Note that we can find an example as described here while still satisfying fair prices, the budget and smoothing conditions so the restrictions in this theorem are more restrictive than those in theorem 3. It is clearly the case that the system of equilibrium inequalities sometimes has a solution. To find data for which the system has a solution we can simply take any data of consumption wealth and high effort prices that satisfy the smoothing and the budget conditions, and use the result in propositions 9 or 10 to find probabilities associated with the low effort. ■

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