

Should Monetary Policy Target Financial Stability?

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This version: July 22, 2022

Abstract

Monetary policy can promote financial stability and improve household welfare. We consider a macro model with a financial sector in which banks do not actively issue equity, output and growth depend on the aggregate level of bank equity, and equilibrium is inefficient. Monetary policy rules responding to the financial sector are ex-ante stabilizing because their effects on risk premia decrease the likelihood of crises and boost leverage during downturns. Stability gains from monetary policy increase welfare whenever macroprudential policy is poorly targeted. If macroprudential policy is sufficiently well-targeted to promote financial stability, then monetary policy should not target financial stability.

Keywords: Central bank mandate, Leaning against the wind, Fed Put, Macroprudential policy, Banks, Liquidity.

JEL classification: E44, E52, E58, G01, G12, G20, G21.

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We are grateful for feedback from Marco Del Negro, Fernando Duarte, Jesús Fernández-Villaverde, Simon Gilchrist, and Alejandro Van der Ghote, and from seminar participants at the Federal Reserve Bank of New York, the Darden School of Business at the University of Virginia, and the Office of Financial Research, Department of the U.S. Treasury. The errors are our own.

1 Introduction

Economists increasingly debate where financial stability fits into a central bank's mandate. Core to this debate is whether the financial sector creates inefficiencies with aggregate consequences that monetary policy can adequately address. Monetary policy should consider targeting financial conditions in addition to (or independent of) conventional targets only if the financial sector creates additional externalities.¹ In other words, to justify targeting financial stability in addition to inflation or output gaps, the financial sector must be more than a source of shocks to the rest of the economy; it must be an inefficient source of shocks. This raises several fundamental questions. How do monetary policy rules affect financial stability? Can monetary policy effectively correct financial-sector externalities? Do any benefits of targeting financial stability with monetary policy persist if macroprudential tools are available?

It is widely recognized that central bankers may pursue aggressive policies to stabilize the financial system during downturns—i.e., enacting a “Fed Put” to cut borrowing costs—which encourages excessive risk-taking and leverage in good times and may ex-ante increase the probability of financial crises. To counteract excessive risk-taking, some economists suggest that central banks should “lean against the wind” (“LAW”) in good times to mitigate overheating in the financial sector. Proponents argue that systematically raising the cost of intermediation in good times will decrease the probability of (extremely costly) financial crises.² Implicitly, LAW turns the standard dual mandate into a triple mandate of price stability, full employment, and financial stability. There are prevailing doubts that the benefits of LAW outweigh the costs, and LAW may exacerbate crises if the economy enters a crisis starting from a weaker position (Svensson, 2017). Opponents of LAW prescribe macroprudential tools as the appropriate instruments for addressing financial instability and thus support continued adherence of monetary policy to a dual mandate.

Our primary contribution to this debate is a quantitative model demonstrating that when macroprudential policy is poorly targeted, monetary policy should address financial stability by imple-

¹Targeting the output gap would automatically address output gaps caused by shocks to the financial sector; financial-sector shocks would just be demand or supply shocks for the central bank to respond to in the usual way.

²See BIS (2014, 2016), Borio (2014); Borio et al. (2018) and Juselius et al. (2017). See also Adrian and Liang (2016); Adrian and Duarte (2016); Adrian et al. (2019).

menting a Fed Put. The intuition is that both macroprudential policy and a Fed Put correct externalities due to financial frictions by recapitalizing the financial sector at a faster rate. However, if macroprudential policy is imperfect or improperly timed, then it fails to address financial-sector externalities. To the extent that macroprudential tools are inflexible and difficult to apply correctly, our analysis suggests that the central bank mandate should include financial stability.

We arrive at these conclusions using a continuous-time stochastic general equilibrium model in which financial frictions endogenously create inefficient instability and systemic risk, building on [Brunnermeier and Sannikov \(2014\)](#). Banks invest in productive capital, but due to frictions banks can issue only risk-free debt and not additional equity. As a result, banks invest more when they have more equity, and capital is allocated more efficiently when banks are well-capitalized. Limited equity issuance creates a distortion between the private and social values of bank equity, and so policies that improve financial stability can potentially increase household welfare. To this setting we add the model of monetary policy transmission from [Drechsler, Savov, and Schnabl \(2018\)](#) in which the nominal interest rate determines the liquidity premium on banks' investments. To capture constraints on monetary policy from inflation concerns, we also add inflation costs. We assess optimal monetary policy and explain the underlying mechanisms by investigating the impact of interest rate rules on the amplification of shocks, nonlinear dynamics, and systemic risk, which are central features of the framework in [Brunnermeier and Sannikov \(2014\)](#). We then evaluate whether macroprudential policy in the form of state-contingent leverage constraints changes the optimal monetary policy.

In our model, monetary policy can affect the return on banks' investments, the rate of bank equity growth, and banks' leverage decisions. Through these effects, monetary policy can change the frequency and duration of good and bad outcomes. We solve for the global dynamics of the economy and find that the impact of interest rate rules on bank profitability, and thus on bank equity growth, varies systematically with the state of the economy. How bank leverage varies over the financial cycle depends primarily on how monetary policy *changes* over the cycle, much more than on the overall level of rates.

Optimal monetary policy implements a Fed Put because it increases financial stability *ex-ante*

while minimizing inflation costs in good times. A Fed Put increases leverage *in bad times*, which improves aggregate outcomes and raises the expected rate at which banks recapitalize during crises. Surprisingly, even though banks expect accommodation during crises, bank leverage in good times does not sharply increase, so the probability of crises is *ex-ante lower* with a Fed Put. The key reason is that effects of monetary policy on returns, leverage, and stability are state-dependent.

Many economists have argued that macroprudential policy (MaP) should be the main tool for addressing financial crises (see [Svensson, 2017](#)). An important concern is whether monetary policy still improves financial stability and/or welfare when macroprudential policy has been applied. We find that if macroprudential policy is poorly targeted, then monetary policy should continue to address financial stability, and conversely, if macroprudential tools are well-targeted, then monetary policy should not address financial stability. When macroprudential policy does not sufficiently restrict leverage at the right times, monetary policy has scope to improve stability and welfare. If macroprudential tools properly target excessive risk taking across the financial cycle, then monetary policy is free to focus on conventional targets alone (e.g. inflation). However, our quantitative results suggest that for this condition to be met, MaP must be so effective that it almost completely prevents crises, which may be unrealistic.

Related Literature Methodologically, our paper follows the stochastic continuous-time macro literature, pioneered by [Brunnermeier and Sannikov \(2014, 2015, 2016a\)](#) and [He and Krishnamurthy \(2012, 2013, 2019\)](#), who analyze the nonlinear global dynamics of economies with financial frictions, building on seminal results from [Kiyotaki and Moore \(1997\)](#) and [Bernanke et al. \(1999\)](#). Within this literature, we combine the models of [Drechsler et al. \(2018\)](#) and [Phelan \(2016\)](#) to study how monetary policy affects global dynamics. [Drechsler et al. \(2018\)](#) develop an asset-pricing model in which banks hold liquidity to insure deposits against funding risks. By influencing the liquidity premium on deposits, monetary policy affects the risk premium component of the cost of capital. Risk-tolerant agents issue deposits to buy assets, and lower nominal rates raise leverage, resulting in lower risk premia as well as higher asset prices and volatility.³

³[Drechsler et al. \(2017\)](#) empirically confirm this transmission mechanism by examining how nominal rates affect the supply of retail bank deposits. [Bernanke and Kuttner \(2005\)](#) document empirically that monetary policy shocks decrease risk premia. [Kekre and Lenel \(2019\)](#) consider a New Keynesian model in which monetary policy shocks

We find that monetary policy has different effects on stability depending on the extent to which the returns on productive factors are affected. Hence, our mechanism is distinct from that in [Brunnermeier and Sannikov \(2012, 2016a\)](#), in which cutting interest rates mechanically increases bond prices, raising bank equity via direct recapitalization. In our model, cutting rates does not necessarily increase the price of capital per se (no direct recapitalization) and the timing of rate cuts matters. Simply cutting interest rates is not enough for indirect recapitalization to occur, but rates must be cut at the appropriate time.

It is helpful to contrast our results with several closely-related papers. [Svensson \(2017\)](#) assumes that a weaker economy in good times leads to a weaker economy in a crisis. This generally need not be the case in reality, and in our model a Fed Put leads to a *less* severe crisis because bank leverage increases in a crisis, a feature that is shared by [Caballero and Simsek \(2019, 2020\)](#). These latter papers study the role of monetary and macroprudential policies to mitigate the severity of demand-driven recessions. However, in these papers while the severity of a crisis is endogenous, the probability is exogenous, whereas in our model both the severity and likelihood of crises are endogenous. [Van der Gote \(2021\)](#) shows that in a New Keynesian version of the model in [Brunnermeier and Sannikov \(2014\)](#), it is optimal for monetary policy to implement a countercyclical employment gap. In that model, monetary policy affects banks' risk-taking incentives only indirectly through the impact of the nominal rate on price dispersion. In current work, [Van der Gote \(2019\)](#) includes money as a means of payments to examine the real effects of interest-rate corridor policies, which affect the cost of liquidity in financial markets.

There is a broad literature studying the relationship between monetary policy and financial stability. [Cieslak and Vissing-Jorgensen \(2020\)](#) find that the Federal funds rate responds to asset prices (the stock market); we consider responses to the condition of the financial sector (banks' equity levels). Several papers have cautioned against central banks intervening in financial markets ([Diamond and Rajan, 2012](#); [Farhi and Tirole, 2012](#)). We take maturity mismatch and correlated risks as given and then consider, given these features, how monetary policy affects stability. Because of the general equilibrium effects in our model, a Fed Put increases leverage when rates are low but not before and the increase in leverage in bad times is stabilizing. Moreover, a Put does not lower risk premia by redistributing wealth toward agents with greater propensities to invest in risky capital.

introduce a commitment or time-consistency problem because it is ex-ante stabilizing. [Bornstein and Lorenzoni \(2018\)](#) develop a simple model where passive monetary policy causes overborrowing due to an aggregate demand externality. Our results suggest that moral hazard may not be a severe concern when a Fed Put addresses resource misallocation.

Our paper's insight is also distinct from other research affirming the use of monetary policy to address financial stability. [Stein \(2012\)](#) provides a model in which monetary policy can enhance financial stability by reducing excessive short-term debt. [Gertler and Karadi \(2011\)](#), [Cúrdia and Woodford \(2011, 2016\)](#), [Christiano et al. \(2015\)](#), and [Caballero and Simsek \(2019\)](#) study the ability of monetary policy to address aggregate demand externalities in models with financial frictions, and show that prudential monetary policy can mitigate the severity of a demand-driven recession. In our economy, monetary policy corrects a pecuniary externality that produces dynamic resource misallocation affecting both the severity and the probability of crises. These results together show monetary policy can mitigate aggregate demand *and* aggregate supply issues caused by financial frictions. [Hansen \(2018\)](#) characterizes LQ-optimal monetary policy near an efficient deterministic steady state in a New Keynesian economy with a [Bernanke and Gertler \(1989\)](#) financial accelerator. Equilibrium in our model is inefficient with a stochastic steady state.

2 Model

The economy is populated by households and banks, and households own the banks. Capital can be used to produce a final consumption good, which can be re-invested to create new capital. Banks have an advantage at producing the consumption good. As a result, output and growth depend endogenously on capital ownership. The financial friction is costly equity issuance. Outcomes will depend on the level of equity in the banking sector. The model combines elements of the models in [Brunnermeier and Sannikov \(2014\)](#), [Phelan \(2016\)](#), and [Drechsler et al. \(2018\)](#).

The baseline model does not include macroprudential policy. Section [5](#) introduces macroprudential policy via leverage limits and considers the interaction between macroprudential and monetary policy rules. Proofs and additional results are given in Appendix [A](#).

2.1 Technology, Environment, and Markets

Time is continuous and infinite, and aggregate productivity shocks follow a Brownian motion.

Capital The effective capital quantity k_t evolves according to equation (1),

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta) dt + \sigma dW_t, \quad (1)$$

where $\Phi(\cdot)$ is a standard investment technology, ι is the rate of internal investment, σ is the volatility of capital growth, and W_t is an exogenous standard Brownian motion. For tractability, we choose the internal investment function $\Phi(\iota) = \delta + \frac{1}{\varepsilon} \log(\varepsilon \iota + 1 - \varepsilon \delta)$.⁴ In equilibrium, ι_t is the same for banks and households. To reduce notation, we directly use this result when writing the growth rate.

Capital can be used to produce the final consumption good. Banks produce consumption goods at the rate $a_b k_t$ while households produce at the rate $a_h k_t$, where $a_b > a_h$. Thus, it is statically efficient to allocate all capital to banks.⁵

Capital is traded in a perfectly competitive market at a real price Q_t . We postulate that the real capital price (the “asset price”) follows the process

$$\frac{dQ_t}{Q_t} = \mu_{Q,t} dt + \sigma_{Q,t} dW_t, \quad (2)$$

which will be determined endogenously in equilibrium. The return to owning capital includes dividends from production and the capital gains on the value of the capital. By Ito’s Lemma, the real rate of return to agent i using capital is

$$dr_{i,t} = \left(\frac{a_i - \iota_t}{Q_t} + \Phi(\iota_t) - \delta + \mu_{Q,t} + \sigma \sigma_{Q,t} \right) dt + (\sigma + \sigma_{Q,t}) dW_t.$$

⁴A second-order approximation of Φ around $\iota = \delta$ yields quadratic adjustment costs with parameter ε . Brunnermeier and Sannikov (2016b) recommend the log functional form because of its computational advantages. Following He and Krishnamurthy (2019), we deviate slightly from Brunnermeier and Sannikov (2016b)’s exact specification. First, we use a constant term of $1 - \varepsilon \delta$ rather than 1 inside the log so that adjustments costs are zero when $\iota = \delta$. Second, we add the δ to the front so that a second-order approximation yields quadratic adjustment costs.

⁵While this assumption is common in the literature, in reality, banks may misallocate capital during economic booms (e.g. NINJA mortgages, lax lending standards). Appendices D and E relax this assumption and demonstrate that our results are robust to a setting in which the possibility of bank misallocation makes it statically efficient for households to hold some capital.

The volatility of returns on investments is $\sigma + \sigma_{Q,t}$, which includes fundamental risk σ and endogenous price risk $\sigma_{Q,t}$.

Deposits and Monetary Policy There is a market for risk-free deposits in zero net-supply. The endogenous real risk-free interest rate implied by households' stochastic discount factor is $dr_{f,t}$. We assume $dr_{f,t}$ has paths of finite variation. The central bank sets the nominal interest rate i_t on deposits. (See Appendix B for details on implementation.) Inflation is determined according to a standard Fisher equation

$$i_t = dr_{f,t} + \pi_t.$$

We define policy functions for i_t later. We assume inflation is locally deterministic, i.e. $\frac{dp_t}{p_t} = \pi_t dt$.

Within the framework of our model, monetary policy is said to “target financial stability” when the nominal interest rate deviates from some baseline path. A straightforward baseline is a constant nominal rate, which never responds to the state of the financial sector, and therefore does not treat financial stability as a target. Another baseline would be a rule that implements a constant level of inflation regardless of the state of the financial sector.

2.2 Households, Banks, and Asset Pricing

There is a continuum of risk-averse households denoted by $h \in [0, 1]$ with initial wealth $w_{h,0}$ and a continuum of banks, denoted by $b \in [0, 1]$, with initial book value (“equity”) $n_{b,0}$. Each household owns a bank, so their wealth $w_{h,0}$ equals the sum of their “non-bank net worth” $n_{h,0}$ and the value of the shares in bank equity. Lifetime utility is given by

$$V_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \left(\log(c_{h,t}) - \frac{\lambda}{2} (\pi_t - \bar{\pi})^2 \right) dt \right],$$

where $c_{h,t}$ is flow consumption, r is the discount rate, and the last term captures flow utility costs from inflation. Quadratic losses centered around an inflation target of $\bar{\pi}$ is a standard way to represent welfare losses from output gaps in a New Keynesian model (see Section 4.4 of Galí (2015); see Appendix A of Nuño and Thomas (2016) for a formalization in continuous time).

Banks invest in capital and issue deposits. Banks are owned by households, who choose dividend payouts, the level of deposits, the level of liquid reserves, and the portfolio weight on capital used by banks. Because of un-modeled financial frictions, banks are subject to two constraints. First, equity issuance is infinitely costly (i.e., dividends must be positive). Second, the value of banks' assets minus liabilities $n_{b,t}$ cannot become negative (bankruptcy).⁶

Banks maximize the present value of dividends subject to their constraints. Dividends are discounted by households' stochastic discount factor ξ_t . Since households have log utility, the stochastic discount factor is $\xi_t = e^{-rt} c_{h,t}^{-1}$. Due to banks' productivity advantage, banks will borrow debt to hold capital. To avoid bankruptcy, they will never finance their portfolio with only debt.

The distinction between $n_{h,t}$ (not including bank holdings) and wealth held in bank equity is useful because banks control dividend payouts and cannot issue equity. This is precisely the role of financial frictions. Investment into bank equity is not controlled by households, but household wealth also includes their own capital holdings and bank deposits, which households do control. Non-bank net worth refers to these latter quantities.

Following [Drechsler et al. \(2018\)](#) we assume deposits are subject to funding shocks, requiring banks to self-insure by holding liquid assets. Since this behavior is not frictionless, borrowing with deposits imposes a “liquidity premium” of operations that is proportional to the nominal interest rate. See Appendix B for details. Our results do not rely on this specific microfoundation for how monetary policy affects liquidity premia, but only need a mechanism with two properties. First, the nominal interest rate creates a wedge in banks' first-order condition for their portfolio weight on capital but not in households' first-order condition. Second, the wedge increases with the level of the nominal interest rate. Other microfoundations would imply similar results.⁷

⁶[Acharya et al. \(2011\)](#) provide empirical support that bank equity is “sticky”: during the financial crisis, banks raised capital in the form of debt, not common equity. [Adrian and Shin \(2010, 2014\)](#) provide evidence that the predetermined balance sheet variable for banks is equity, not assets. Our results generalize so long as banks do not issue equity too frequently. Relatedly, [Gambacorta and Shin \(2016\)](#) provide evidence that bank capital matters for monetary policy transmission.

⁷Nominal interest rates may determine liquidity premia because banks have market power in the deposit market (see [Drechsler et al., 2017](#); [Brunnermeier and Koby, 2018](#)), because the central bank operates monetary policy through a corridor system and open market operations (see [Van der Ghote, 2019](#); [Bigio and Sannikov, 2021](#)), or because banks use short bonds as collateral to back inside money ([Lenel et al., 2019](#)). While the Federal Reserve has used a floor system since 2008, many central banks (e.g. the Riksbank) continue to operate corridor systems. Modeling monetary policy implementation as a corridor system still has empirical relevance.

Households' Problem Let $(x_{h,t}, x_{hd,t}, x_{e,t})$ be household's portfolio weight on capital, debt, and bank equity respectively. Formally, households solve the problem

$$\max_{x_{h,t}, x_{e,t} \geq 0, x_{hd,t}, c_{h,t}, l_t} \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \left(\log(c_{h,t}) - \frac{\lambda}{2} (\pi_t - \bar{\pi})^2 \right) dt \right],$$

subject to

$$\frac{dw_{h,t}}{w_{h,t}} = x_{hd,t} dr_{f,t} + x_{he,t} dr_{e,t} + x_{h,t} dr_t^h + \left(\mathcal{T}_{h,t} - \frac{c_{h,t}}{w_{h,t}} \right) dt, \quad (3)$$

$$w_{h,t}, x_{h,t}, x_{e,t} \geq 0, \quad (4)$$

$$x_{h,t} + x_{hd,t} + x_{e,t} = 1, \quad (5)$$

where $dr_{e,t}$ denotes the return on bank equity, $\mathcal{T}_{h,t}$ denotes all transfers received by households normalized by their net worth. Equation (3) characterizes the evolution of households' total wealth. Transfers include dividends from banks and any payouts from the government.

Proposition 5 in Appendix A characterizes households' optimal decisions. Since banks are shareholder-maximizing firms rather than separate agents, households internalize the value of future bank dividends in their consumption policy. Therefore, households consume a fraction r of their total wealth and hold positive quantities of capital if the expected excess returns equal the covariance of their consumption with returns.

Banks' Problem Let $(x_{b,t}, x_{bd,t})$ be the portfolio weights on capital and debt, respectively. Formally, banks solve the problem

$$\max_{x_{b,t} \geq 0, x_{bd,t}, d\zeta_t \geq 0, l_t} U_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty \frac{\xi_t}{\xi_\tau} d\zeta_t \right],$$

subject to two constraints on the evolution of their (book) equity

$$\frac{dn_{b,t}}{n_{b,t}} = x_{bd,t} dr_{f,t} + x_{b,t} dr_{b,t} + \left(\mathcal{T}_{b,t} - (x_{b,t} - 1) \gamma i_t \right) dt - \frac{d\zeta_t}{n_{b,t}} \quad (6)$$

$$n_{b,t}, x_{b,t}, d\zeta_t \geq 0, \quad x_{b,t} + x_{bd,t} = 1. \quad (7)$$

Banks pay interest on their deposits, earn a return from their capital holdings, pay the liquidity premium from holding reserves to back deposits, and pay dividends at rate $d\zeta_t$.⁸ They also receive transfers $\mathcal{T}_{b,t}$, which includes a wealth tax rate of T to ensure a nondegenerate stationary distribution as well as any payouts from the government.

By homogeneity and price-taking, we can write the maximized value of a bank as

$$\theta_\tau n_{b,\tau} \equiv \max_{x_{b,t} \geq 0, x_{bd,t}, d\zeta_t \geq 0, l_t} \mathbb{E}_\tau \left[\int_\tau^\infty \frac{\xi_t}{\xi_\tau} d\zeta_t \right], \quad (8)$$

where θ_t is the marginal value of equity. The value θ_t equals 1 plus the multiplier on the equity-issuance constraint and reflects the aggregate condition of the financial sector. Proposition 6 in Appendix A further characterizes banks' optimality conditions, and in particular describes the equilibrium diffusion for θ_t , which we write

$$\frac{d\theta_t}{\theta_t} = \mu_{\theta,t} dt + \sigma_{\theta,t} dW_t. \quad (9)$$

We summarize these optimality conditions here. Banks will not pay dividends when $\theta_t \geq 1$; θ_t can never be less than one because banks can always pay out the full value of equity, guaranteeing a value of at least $n_{b,t}$. Banks will hold capital only if the expected excess return equals the sum of banks' risk premium and the liquidity premium on deposits. Banks demand a risk premium that is higher than households' because although banks discount dividends using the household SDF, banks are also concerned about their solvency due to costly equity issuance.

Asset Pricing To characterize the equilibrium asset pricing conditions, we first define

$$HP_t \equiv \sigma_{ch,t}(\sigma + \sigma_{Q,t}), \quad SP_t \equiv -\sigma_{\theta,t}(\sigma + \sigma_{Q,t}), \quad LP_t \equiv \gamma l_t \quad (10)$$

⁸To minimize notation, we write banks' problem after substituting their first-order condition for the portfolio weight on reserves. Let $x_{M,t}$ be the weight on reserves. The microfoundation for monetary policy from Drechsler et al. (2018) implies $x_{M,t} = -\gamma x_{bd,t} = \gamma(x_{b,t} - 1)$. In general, this formulation of the bank problem is correct as long as the microfoundation implies that a bank scales its holdings of reserves in proportion to the level of its deposits.

as households' required risk premium, banks' solvency risk premium, and the liquidity premium, respectively. The term $\sigma_{ch,t}$ is the volatility of household consumption. The solvency risk premium SP_t represents the portion of banks' required risk premium (or instantaneous level of risk aversion) caused by costly equity issuance and solvency concerns.

Households choose capital so that

$$\mathbb{E}[dr_{h,t} - dr_{f,t}] \leq HP_t \quad (11)$$

where the inequality reflects that households may not always hold capital in equilibrium.

Banks' first-order condition for their portfolio weight on capital implies

$$\mathbb{E}[dr_{b,t} - dr_{f,t}] = HP_t + SP_t + LP_t. \quad (12)$$

Banks' risk premium on capital equals $HP_t + SP_t + LP_t$ because returns must compensate for the covariance of returns with household consumption (the usual SDF term), solvency concerns due to costly equity issuance, and the cost of managing liquidity using reserves. This condition binds with equality because we look for an equilibrium where the risk premium and liquidity premium are both non-negative, hence banks will almost surely hold capital.

Differencing (12) and (11) yields the equilibrium asset pricing condition:

$$\frac{a_b - a_h}{Q_t} \geq SP_t + LP_t. \quad (13)$$

Relative to households, banks earn extra returns of at least $SP_t + LP_t$ because they have higher productivity but also bear the risk of holding capital while facing an equity issuance constraint. Condition (13) binds with equality whenever households hold a positive quantity of capital.

2.3 Characterizing Equilibrium

A competitive equilibrium is characterized by the market price for the risky asset, together with portfolio allocations and consumption decisions such that given prices, agents optimize and mar-

kets clear. Due to equity issuance frictions, banks' decisions depend on their level of equity, and so equilibrium depends on banks' equity levels and monetary policy has scope to affect equilibrium.

We solve for the global equilibrium dynamics using the methods in [Brunnermeier and Sannikov \(2014\)](#). Define $N_{b,t} = \int n_{b,t} db$ as aggregate bank equity. Because capital grows geometrically and the bank problem is homogenous, the equilibrium state variable of interest is aggregate bank equity as a fraction of the total value of capital:

$$\eta_t = \frac{N_{b,t}}{Q_t K_t}.$$

Equilibrium consists of a law of motion for η_t , and allocations and prices as functions of η .⁹ The asset prices are $Q(\eta)$ and $\theta(\eta)$, and the allocation is the fraction of capital held by banks $\psi(\eta)$. We derive the evolution of η_t using Ito's Lemma and the equations for returns and budget constraints.

Lemma 1. *The equilibrium law of motion of η will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_{\eta,t} dt + \sigma_{\eta,t} dW_t + d\Xi_t, \quad (14)$$

where $d\Xi_t$ is an impulse variable creating a regulated diffusion. Furthermore,

$$\begin{aligned} \mu_{\eta,t} &= \frac{a_b - l_t}{Q_t} + \mathcal{T}_{b,t} + \left(\frac{\psi_t}{\eta_t} - 1 \right) ((\sigma_{ch,t} - \sigma_{\theta,t}) - (\sigma + \sigma_{Q,t}))(\sigma + \sigma_{Q,t}) \\ \sigma_{\eta,t} &= \left(\frac{\psi_t}{\eta_t} - 1 \right) (\sigma + \sigma_{Q,t}), \quad d\Xi_t = \frac{d\zeta_t}{N_{b,t}}, \end{aligned}$$

where $d\zeta_t = \int d\zeta_{b,t} db$.

We convert the equilibrium conditions into a system of differential equations ("ODE") in Q and θ . Given $Q(\eta)$, $\theta(\eta)$, and $\theta'(\eta)$, we can use equilibrium returns and allocations to derive $Q'(\eta)$ and $\theta''(\eta)$. We solve the ODE using appropriate boundary conditions (see [Appendix A](#)).

⁹Because we model banks as firms which maximize shareholder value, the representative household consumes at a rate $r(N_{h,t} + \theta_t N_{b,t})$, or, as a fraction of the total value of capital, $C_{h,t}/(Q_t K_t) = r((1 - \eta_t) + \theta_t \eta_t)$, where total wealth is $W_{h,t} = N_{h,t} + \theta_t N_{b,t}$ and $N_{h,t}$ is aggregate non-bank net worth. This consumption policy is different from those obtained in previous literature because of the additional $\theta_t \eta_t$. However, as long as θ is a function of η in equilibrium, the household consumption policy remains a function of η and therefore does not require a separate state variable to capture movements in non-bank net worth, e.g. $N_{h,t}/(Q_t K_t)$.

Proposition 1. *The equilibrium domain of the functions $Q(\eta)$, $\theta(\eta)$, and $\psi(\eta)$ is an interval $[0, \eta^*]$. The marginal price of bank value θ is a decreasing function, and the following boundary conditions hold: (i) $\theta(\eta^*) = 1$; (ii) $\theta'(\eta^*) = 0$; (iii) $Q(\eta^*) = \bar{Q}$; (iv) $Q'(\eta^*) = 0$; (v) $Q(0) = \underline{Q}$; (vi) $\lim_{\eta \rightarrow 0^+} \theta(\eta) = \infty$. Over $[0, \eta^*]$, $\theta_t \geq 1$ and $d\zeta_t = 0$, and $d\zeta_t > 0$ at η^* creating a regulated barrier for the process η_t . The quantity $\underline{Q}$ is the price of capital in an economy with no banks, and the quantity $\bar{Q}$ is the price of capital required by market-clearing for consumption at η^* .*

The system ranges between 0 and η^* , at which point banks pay dividends because the marginal value of debt outweighs the marginal value of additional equity. When interest rates do not vary too much, there exists $\bar{\eta} \in (0, \eta^*)$ such that $\psi(\eta) = 1$ for $\eta > \bar{\eta}$ and $\psi(\eta) < 1$ for $\eta \leq \bar{\eta}$. For high levels of η , banks can hold the entire capital stock. The evolution of η induces a stationary density (PDF) $f(\eta)$ with CDF $F(\eta)$; the density $f(\eta)$ solves a Kolmogorov-Forward equation.

It is useful to divide the state space into three regions and label them. We define a *boom* as when banks hold the entire capital stock. The economy is in good times during a boom. We define *distress* as when banks are so constrained that they have to sell capital at fire-sale prices to households (i.e., $\psi < 100\%$, $\eta < \bar{\eta}$). We define a *crisis* as when households hold at least 50% of the capital stock ($\psi < 50\%$), indicating severe misallocation due to fire sales. Thus, the economy is in bad times when it is in distress or a crisis. The stability of the economy can be measured by either the long-run probability the economy is not in distress or the long-run probability the economy is not in crisis.

The fundamental problem caused by financial frictions is the inability of banks to always hold the entire capital stock. Households are worse-off when fire sales are more frequent. Our measures of stability directly quantify this problem. While alternative measures like asset price volatility can be more easily mapped to data, our paper is primarily theoretical, so we prefer the transparency provided by these definitions of stability.¹⁰ Higher stability will imply lower asset price volatility.

If the price function is twice-continuously differentiable, then the evolutions of the capital price

¹⁰He and Krishnamurthy (2019) use a similar measure of stability. They define the threshold for financial distress as the equity level at the 33rd percentile based on the stationary distribution, and they define a crisis as when a capital constraint binds for financial intermediaries. In our model, the equivalent notion of a binding capital constraint is precisely the sale of capital by banks because low bank equity restricts their ability to hold capital. However, because the output losses are relatively modest when households hold a small share of capital, we choose to reserve the label of “crisis” for when households hold at least 50% of the capital stock.

and marginal bank value (equations (2) and (9)) are functions of η

$$\frac{dQ_t}{Q_t} = \mu_Q(\eta_t)dt + \sigma_Q(\eta_t)dW_t, \quad \frac{d\theta_t}{\theta_t} = \mu_\theta(\eta_t)dt + \sigma_\theta(\eta_t)dW_t,$$

where the drift and variance terms are determined by the derivatives of $Q(\eta)$ and $\theta(\eta)$. For the remainder of the paper, we suppress the dependence on η_t and on time t for notational ease.

2.4 Welfare and Efficiency

The model aggregates up to a representative household with utility

$$V_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \left(\log(C_{h,t}) - \frac{\lambda}{2}(\pi_t - \bar{\pi})^2 \right) dt \right], \quad (15)$$

where in equilibrium aggregate household consumption is given by $C_{h,t} = (a_h + (\psi_t - 1)a_b - \iota)K_t$.

Because households have log utility, we can write their value function as

$$V = \frac{\log(W_h)}{r} + h(\eta),$$

where $h(\eta)$ is a wealth-independent term. Because total household wealth satisfies $W_h = Q(1 + (\theta - 1)\eta)K$,¹¹ the value function can be re-written into a function of η and K :

$$V(\eta, K) = \frac{\log(K)}{r} + H(\eta),$$

where $H(\eta) \equiv \log(Q(1 + (\theta - 1)\eta))/r + h(\eta)$. Since η and K are initial conditions, for our purposes, we may normalize $K = 1$ and focus on $V(\eta) \equiv V(\eta, 1)$.

Since positive nominal interest rates create liquidity costs, in the absence of financial frictions

¹¹Households' non-bank net worth equals the value of household capital holdings $(1 - \psi)QK$ plus the value of household deposits $\psi QK - N_b$. Adding bank equity holdings, total household wealth is $N_h + \theta N_b = QK + (\theta - 1)\eta QK > QK$ for $\eta < \eta^*$ since $\theta > 1$, where $N_{h,t}$ denotes aggregate non-bank net worth. In other words, total wealth in the economy exceeds the market value of capital. This is precisely the consequence of financial frictions: banks' value $((\theta - 1)\eta QK)$ exceeds the market value of their capital held as equity (ηQK) , due to intermediation and financial frictions. Without financial frictions, $\theta = 1$ always; financial frictions make possible $\theta > 1$, which creates a distortion between the market value of capital and the value of bank holdings. Variations in θ is precisely why policy can increase welfare with financial frictions.

and inflation costs, the Friedman Rule would be optimal in our model. Proposition 2 states this explicitly for the case when banks can freely issue equity and utility losses from inflation are zero.

Proposition 2. *If banks can freely issue equity (no financial frictions) and inflation costs are zero ($\lambda = 0$), then the generically optimal nominal interest rate is $i^* = 0$ (the Friedman Rule holds).*

The intuition is simple. The liquidity premium acts as a tax on banks' incentives to hold capital without creating any real benefit. Thus, when banks can finance themselves without frictions and inflation costs are not a concern, there is no need to impose liquidity costs on deposit funding. Technically, the optimal rate is not unique because the premium only needs to be sufficiently small to guarantee that banks hold the entire capital stock while the risk-free rate adjusts to absorb a small liquidity premium without changing allocations.¹² Proposition 7 in Appendix A characterizes the optimal rule when $\lambda > 0$. With positive inflation costs, the optimal rule trades off deviations from the inflation target for a higher allocation of capital to banks.

A constrained planner can achieve the first-best allocation so long as the planner has sufficient access to state-contingent taxes. A constrained planner chooses allocations, while respecting agents' constraints and allowing market clearing to determine prices, to affect prices and equilibrium dynamics, and equilibrium outcomes depend on the evolution of prices. A Constrained Social Planner chooses capital shares, consumption, and investment, $\{\psi_t, C_{h,t}, i_t\}$, to maximize welfare in equation (15) subject to constraints on output (1), returns (2), budget constraints and laws of motion for wealth (3)–(7), and bank and household portfolio choices (9)–(11).

The competitive equilibrium is constrained efficient if a social planner, who directly chooses allocations subject to agents' constraints but lets the goods and capital markets clear competitively, cannot improve households' welfare. Brunnermeier and Sannikov (2014) show in their setting that the first-best is achievable by introducing an asset whose value increases when η decreases, and thus when banks hold this asset then their wealth increases after losses from capital (perhaps the asset pays state-contingent taxes on households). Similarly, a constrained planner could achieve the first-best allocation by taxing household's capital holdings so that only banks hold capital

¹²However, only the Friedman rule is always optimal, regardless of the productivity loss when households use capital. The Friedman rule would also be the unique optimal rule if it was not optimal for banks to hold the entire capital stock, which is the case in the two-good extension in Appendix D.

and ensuring that the risk-free rate adjusts to recapitalize banks sufficiently after losses to avoid insolvency.¹³ Proposition 3 follows as a consequence.

Proposition 3. *In a constrained-efficient equilibrium with zero inflation costs, the Friedman rule is generically optimal.*

However, achieving the first-best in a constrained-efficient equilibrium requires a tremendous degree of state-contingent flexibility when setting taxes. Since one underlying friction in the model is that banks borrow only using non-contingent debt, providing the planner with such state-contingent flexibility when setting taxes seems inappropriate. A more appropriate constrained optimization problem is to consider the planner as a “regulator” who can choose leverage limits with some degree of state-contingency. With these policy restrictions, Proposition 4 asserts that, local to the endogenous barrier η^* , aggregate bank leverage exacerbates systemic risk and, under an additional condition, reduces social welfare.

Proposition 4. *Consider a competitive equilibrium in which (i) banks freely issue dividends without distortions; (ii) inflation costs are zero ($\lambda = 0$); (iii) monetary policy follows an exogenous function of η ; (iv) and banks hold the entire capital stock at the endogenous barrier η^* . The marginal social value of aggregate bank leverage at η^* is given by*

$$\mathcal{L}(\psi | \eta^*) = \underbrace{\frac{1+r}{r^2} \frac{a_b - a_h}{a_b + \frac{1-\varepsilon\delta}{\varepsilon}}}_{\text{Asset Price Level Channel}} + \underbrace{\frac{1}{r} (1 - \eta^*) \sigma^2 H''(\eta^*)}_{\text{Systemic Risk Channel}}. \quad (16)$$

First, the “Systemic Risk Channel” is always negative while the “Asset Price Level Channel” is always positive. Second, if

$$H''(\eta^*) < -\frac{1}{(1 - \eta^*) \sigma^2} \frac{1+r}{r} \frac{a_b - a_h}{a_b + \frac{1-\varepsilon\delta}{\varepsilon}}, \quad (17)$$

then the marginal social value of aggregate bank leverage is also negative.

¹³The model would therefore be similar to the equilibrium in Basak and Cuoco (1998): one agent would exclusively hold capital and the capital price would be constant, but the real interest rate would approach negative infinity as the banks’/experts’ wealth approached zero.

Proposition 4 decomposes the marginal social value of aggregate bank leverage into two channels, an “Asset Price Level Channel” and a “Systemic Risk Channel.” The “Asset Price Level Channel” can be interpreted as the static effect of lower leverage and the “Systemic Risk Channel” as the dynamic effect. Lower leverage near η^* reduces household welfare by depressing the asset price level (consumption market-clearing links the current allocation of capital to asset prices) but also improves welfare by decreasing σ_η , which quantifies systemic risk in our model. Higher systemic risk raises the probability of crises. When (17) holds, the dynamic benefits from lower systemic risk dominate the costs of lower asset prices today.

Farther away from η^* , banks still take excessive risk because the model with financial frictions features pecuniary externalities. Thus, the Friedman Rule need not hold when the regulator is constrained. Our quantitative results below confirm that a positive liquidity premium can increase welfare across the state space by decreasing banks’ risk-taking to improve stability.

3 Calibration

This section discusses the calibration for the model parameters and functional forms for monetary policy. Tables 1 and 2 report our calibration and the targets. Several parameters are disciplined by the existing empirical literature. The remaining parameters must be calibrated by matching empirical moments of the data. Our results are qualitatively robust to alternative calibrations.

Panel A of Table 1 lists the parameters drawn from the literature. A discount rate of $r = 2\%$ and an annualized depreciation rate of $\delta = 10\%$ are standard values in the literature (see He and Krishnamurthy, 2019). The liquidity premium coefficient γ is based on the microfoundation from Drechsler et al. (2018). In that paper, the coefficient is $\gamma = \kappa/m$, where $\kappa = 40.85\%$ is the funding shock size, and $m = 4$ is the liquidity services multiplier on reserves, implying $\gamma \approx 0.102$.¹⁴ A second-order approximation of the internal investment function’s implied adjustment costs yields

¹⁴To ensure banks always choose full insurance, we set the funding shock frequency to $\chi = 10\%$ following Drechsler et al. (2018), and the fire sale loss to $\rho = 21\%$, which corresponds to the loan haircut in the secondary market during the global financial crisis following Li (2021). Under these parameters, the maximum nominal interest rate is 7.5%. If $\rho = 15\%$ as in Drechsler et al. (2018), then the maximum rate falls to 5%. Under either calibration, our main analysis is not limited by the maximum rate.

Table 1: Model parameters. Panel A lists parameters calibrated according to the existing literature. Panel B lists parameters calibrated by matching empirical moments for U.S. data.

Description	Parameter	Value
<i>Panel A: Parameters Drawn from Existing Literature</i>		
Discount Rate	r	2%
Depreciation Rate	δ	10%
Liquidity Premium Coefficient	γ	10.2%
Investment Adjustment Cost	ε	3
Inflation Costs	λ	419.42
Inflation Target	$\bar{\pi}$	2%
<i>Panel B: Parameters Calibrated to Match Empirical Moments</i>		
Bank Productivity	a_b	0.127
Household Productivity	a_h	0.102
Fundamental Volatility	σ	1.5%
Tax on Banks	T	4.8%
Lift-Off from Put	η^{Put}	2.0%
Start LAW	η^{LAW}	6.0%
Put Interest Rate	i^{Put}	0%
LAW Interest Rate	i^{LAW}	4.1%

the standard quadratic specification with parameter ε . We calibrate $\varepsilon = 3$ to match the literature. The inflation cost λ is determined based on the New Keynesian microfoundation in Section 4.4 of [Galí \(2015\)](#), which provides empirical measures of mark-ups and the frequency of price changes; λ matters for welfare but does not affect equilibrium dynamics. The inflation target is 2% following the Federal Reserve and the European Central Bank.

Panel B of Table 1 lists the parameters we calibrate by jointly targeting empirical moments from U.S. data, which are listed in Table 2.

Our chosen moments for technology parameters are standard choices from the literature. Following [He and Krishnamurthy \(2019\)](#), we target a 3% long-run probability of crisis, which we defined as whenever $\psi < 50\%$. We match the average GDP growth volatility of 2.2% over the pe-

Table 2: Empirical moments. Averages of variables are computed by integrating the moment of interest with respect to the stationary density. Panel A lists targeted moments from U.S. data while Panel B lists untargeted moments. For the latter, some moments are not directly estimated from the data (e.g. inverse investment-price elasticity), but all the moments are typical calibration targets in the literature.

Empirical Moment	Data	Model
<i>Panel A: Targeted Moments</i>		
Prob. of Crisis ($\psi < 50\%$)	3%	3.15%
GDP Growth Volatility	2.2%	2.43%
Risk-Free Real Rate	1.85%	1.88%
Market Leverage	10	10.6
Normal Times	$\psi = 95\%$	95.0%
Crisis	$\psi = 50\%$	50.2%
Inflation Outside of ZLB	2%	1.98%
<i>Panel B: Untargeted Moments</i>		
Sharpe Ratio	48%	44.68%
Inverse Investment-Price Elasticity	0.25	0.31
Output Loss in Crisis	-8%	-7.61%
Investment-Capital Ratio	9%	10.52%

riod 1950-2019 and the average risk-free interest rate following [van Binsbergen et al. \(2022\)](#), who estimate risk-free interest rates unaffected by convenience yields on safe assets and find an average rate of 1.85% on a one-year bond. [Begenau and Landvoigt \(2021\)](#) calculate a market leverage ratio of 10 for banks over the period 1999-2019.

We calibrate the monetary policy rule as implementing the inflation target when unconstrained by the ZLB but responding to the financial sector by cutting rates when the economy is in distress or a crisis.¹⁵ For parsimony and tractability we consider the following piecewise linear class of

¹⁵While the calibration of the monetary policy rule is important for calibrating the remaining parameters of the model, the overall results are not so sensitive to the exact MP rule. A natural alternative is to suppose that the central bank pursues an inflation targeting rule that implements $\bar{\pi}$ for all η . The policy rule has important implications for stability (the main result of our paper), and thus for the average value of critical empirical moments. Nonetheless, the moments of the model are not so different when the monetary policy rule is inflation targeting instead of the Calibrated

policy rules for the nominal interest rate:

$$i(\eta) = \begin{cases} i^{Put} & \eta < \eta^{Put}, \\ i^{Put} + \frac{i^{LAW} - i^{Put}}{\eta^{LAW} - \eta^{Put}}(\eta - \eta^{Put}) & \eta^{Put} \leq \eta < \eta^{LAW}, \\ i^{LAW} & \eta \geq \eta^{LAW}. \end{cases} \quad (18)$$

This policy rule captures the idea of a Fed Put with LAW in a boom. For low η , the interest rate is lowered to the constant level i^{Put} , and for high η , the interest rate is raised to the constant level i^{LAW} . For intermediate η , the interest rate increases at a linear rate from i^{Put} to i^{LAW} . Where the Put and LAW regions begin and end are determined by η^{Put} and η^{LAW} . A higher η^{Put} is more accommodative in two senses. First, i^{Put} holds for a larger portion of the state space. Second, the interest rate decreases at a faster rate from i^{LAW} . A lower η^{LAW} has the opposite effects. Note that this class of rules includes constant nominal rates, which occur when $i^{LAW} = i^{Put}$.

We set the Put rate $i^{Put} = 0\%$ because of the ZLB. To reflect the Federal Reserve's monetary policy stance before they adopted average inflation targeting, we target an average inflation rate of 2% when the nominal rate is not constrained by the ZLB. It remains to calibrate the strikes η^{LAW} and η^{Put} as well as i^{LAW} . We suppose that rates begin to be cut when financial distress appears ($\psi < 100\%$) and that rates decrease to the zero lower bound (ZLB) during a crisis ($\psi < 50\%$). Thus, we choose η^{LAW} and η^{Put} so that the interest rates start to fall when $\psi = 95\%$ and hit zero when $\psi = 50\%$. For later reference, we denote this policy rule as “Calibration,” which is not the “optimal” rule (see below).

The model does a good job of matching the targeted empirical moments. In addition, the model does a fairly good job at matching several *untargeted* moments, which are listed in Panel B of Table 2. The reported moments are the average Sharpe ratio, output losses in a crisis, the inverse investment elasticity to the capital price, and the investment to capital ratio. The Sharpe ratio provides a sense of banks' risk-taking while output losses provide a sense of the severity of financial crises. Our average Sharpe ratio is close to He et al. (2017)'s empirical estimate of 48% as well as He and Krishnamurthy (2019)'s calibration, which yields a Sharpe of 45%. To Rule. See Table 4 in Appendix C for the comparison of the model moments when MP follows inflation targeting.

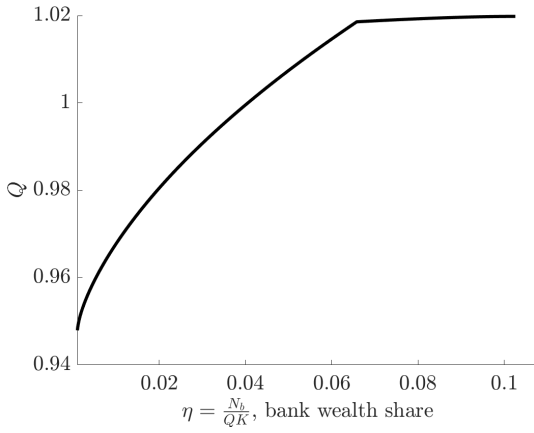
calculate output losses in a crisis, we simulate our model starting from a crisis (when $\psi = 50\%$) and compare total output over five years to total output if the economy had remained at the stochastic steady state. Our value of -7.6% is close to the empirical estimate of -8% from [Jordà et al. \(2013\)](#). The last two moments give a sense of the investment technology's quantitative properties. For the inverse elasticity of investment to the capital price, [Bernanke et al. \(1999\)](#) assert that a number between 0 and 0.50 is reasonable and select 0.25 for their calibration. [Jermann \(1998\)](#) obtains 0.23 while [Abel \(1980\)](#) estimates a number between 0.27 and 0.52. Finally, the investment to capital ratio is typical of values in the literature (e.g. [He and Krishnamurthy \(2019\)](#) obtain a number of 9%).

Figure 1 displays equilibrium prices, allocations, and dynamics for the calibrated model. When banks are well-capitalized, they own the entire capital stock, and asset prices are high. In this region, asset price volatility is nearly zero, and fundamental risk explains the majority of the volatility in η . As η decreases and banks become unwilling to hold the entire capital stock, fire sales depress asset prices, and volatility spikes. The kink in each plot occurs at $\bar{\eta}$, the threshold for fire sales. The plot for ψ looks like a scaled plot for Q , with ψ converging to 0 for $\eta = 0$. When η approaches zero, the economy is less volatile because banks are too small to have sizable aggregate impacts.

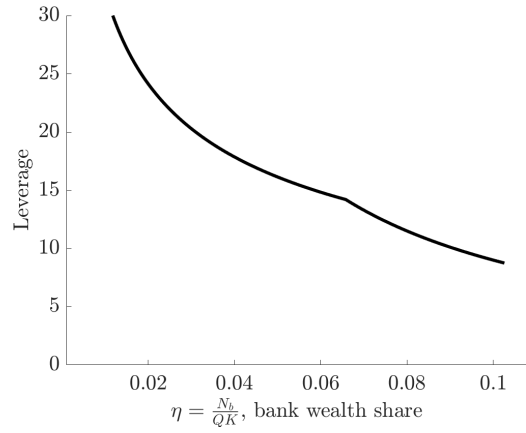
4 Quantitative Results: Stability and Optimal Monetary Policy

Our main quantitative results consider the effect of monetary policy on financial stability and household welfare in the calibrated model. It is outside the scope of this paper to solve the optimal interest rate path over the space of all policy functions. We can parsimoniously answer the questions of interest by focusing on two tractable spaces of policy rules. First, we consider the piecewise linear rules described in equation (18). Second, we consider rules that implement approximately constant inflation for all η , which we refer to as “inflation targeting.”

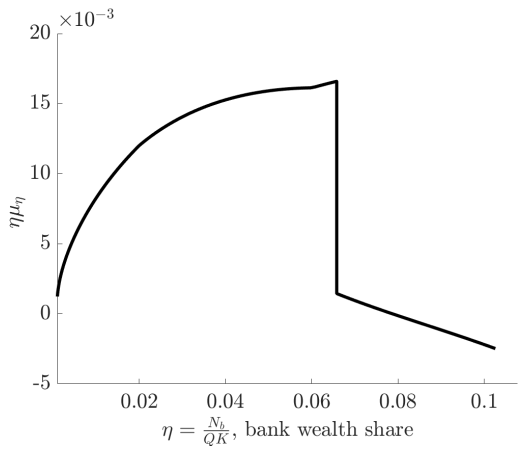
In this section, we omit macroprudential policy to focus on how monetary policy behaves in isolation. We find that monetary policy alone can achieve welfare-improving stability gains. Section 4.1 solves for the optimal policy within the class of rules we consider. Section 4.2 explains why optimal policy improves welfare by considering the positive effects of policy rules on stability.



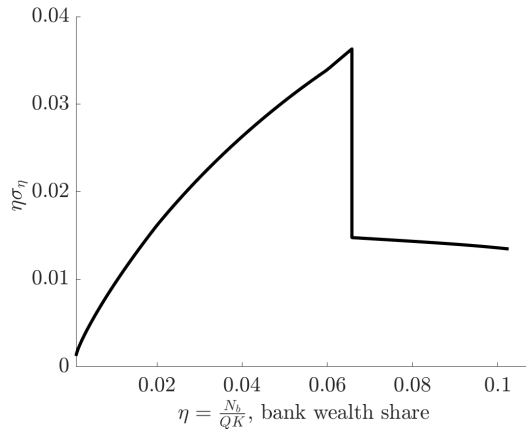
(a) Asset Price $Q(\eta)$



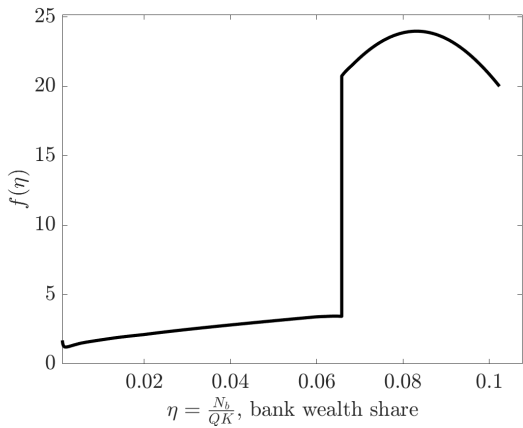
(b) Leverage $\psi(\eta)/\eta - 1$



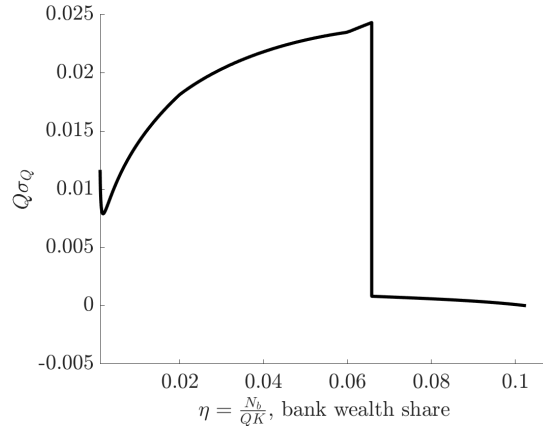
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$



(e) Stationary Density $f(\eta)$



(f) Endogenous price volatility $Q\sigma_Q$

Figure 1: Equilibrium prices, allocations, evolutions, and stationary density when the interest rate follows the Calibration policy rule. In each panel, the horizontal axis is the state variable η , which is the ratio of banks' book equity to the total value of capital.

In Section 5, we consider policy when macroprudential tools are available.

4.1 Welfare and Optimal Monetary Policy

The welfare for a given policy rule depends on the initial condition η . Thus, the preferred policy will generally vary with the initial condition. Accordingly, we take a “timeless” perspective on the initial condition η : the initial condition is not known, but we take expectations according to the stationary distribution. The stationary density $f(\eta)$ determines the ex-ante distribution of initial conditions (capitalization of the financial sector) and then computes $\mathbb{E}[V(\eta)]$ using the stationary distribution occurring in equilibrium.¹⁶

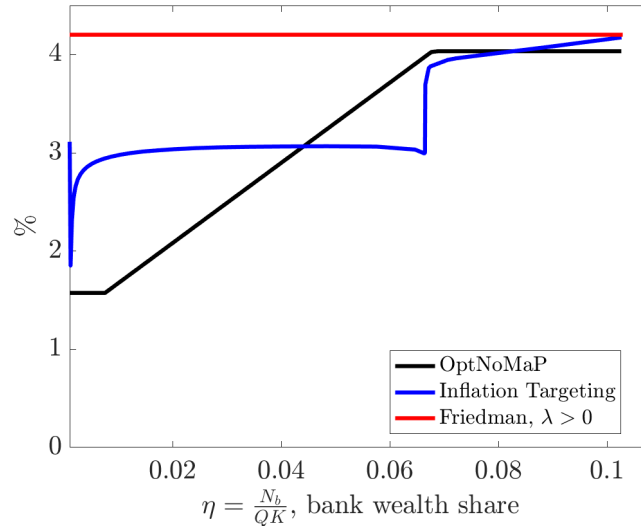


Figure 2: Optimal monetary policy rule and alternative rules without macroprudential regulation. The black line is the OptNoMaP rule. The blue line is the inflation targeting rule which implements approximately constant inflation of $\bar{\pi} = 2\%$. The red line is the optimal rule in the absence of financial frictions, i.e., the Friedman rule with inflation costs.

We define the “OptNoMaP” rule $(\tilde{i}^{Put}, \tilde{i}^{LAW}, \tilde{\eta}^{Put}, \tilde{\eta}^{LAW})$ as the welfare-maximizing (optimal)

¹⁶Because our class of monetary policy rules is parameterized by the four parameters i^{Put} , i^{LAW} , η^{Put} , and η^{LAW} , it is numerically feasible to search for the optimal choices of these parameters. We search over a compact parameter space to reduce computational time and because if i^{LAW} is too high, then equilibrium does not feature full-insurance. The parameter space is $(i^{Put}, i^{LAW}, \eta^{Put}, \eta^{LAW} - \eta^{Put}) \in [0, 0.04] \times [0, 0.075] \times [0, 0.05] \times [0, 0.10]$; the optimization algorithm is the interior-point method. To find a global optimum, we initialize the algorithm at many locations in the parameter space. Finally, we compare this result to the Inflation Targeting rule. When we introduce MaP, Inflation Targeting can dominate rules defined by (18).

policy rule in an economy without macroprudential policy, searching over the space of policy functions in (18). Given calibrated parameters, the optimal policy is given by

$$\tilde{i}^{Put} = 1.57\%, \quad \tilde{i}^{LAW} = 4.03\%, \quad \tilde{\eta}^{Put} = 0.75\%, \quad \tilde{\eta}^{LAW} = 6.78\%, \quad (\text{OptNoMaP})$$

which we plot in Figure 2 and compare to two important benchmarks. The constant red line plots the optimal nominal interest rate in an economy without financial frictions, i.e., the Friedman rule with $\lambda > 0$ (see Proposition 7); by definition this rule does not vary with η . Given our calibration, the Friedman rule with inflation costs is equivalent to inflation targeting in a stationary economy—in other words, the liquidity costs imposed by target inflation are sufficiently small that the liquidity premium gets absorbed in the risk-free rate without perturbing allocations. The state-contingent blue line is the inflation-targeting rule which implements approximately constant inflation of 2% for all η . Away from $\eta = 0$, we can approximate the inflation-targeting rule with the piecewise-linear rules in (18) (i.e., set $i^{Put} = 3\%$, $i^{LAW} = 4.03\%$ and $\eta^{Put} = \eta^{LAW} = 6.78\%$).

Comparing OptNoMaP to these two benchmarks reveals some important results. First, both OptNoMaP and inflation targeting imply rates that are strictly below the Friedman optimal in the absence of financial frictions. Hence, financial frictions lead to an optimal rule with *lower* rates, not higher rates to lean against the wind during booms.

Second, the optimal policy cuts rates in response to financial distress but nominal rates in booms are essentially the same as the inflation-targeting rule (the minor deviation reflects our restriction on policy rules). This provides support for a Fed Put but less for leaning against the wind relative to the inflation target.

Finally, even inflation targeting varies rates in response to η since fire sales decrease the risk-free rate. Nonetheless, the optimal rule cuts rates more gradually and ultimately more deeply. Hence, we find some support for the idea of the CB “keeping its powder dry” by cutting rates slowly rather than all at once. Cutting too quickly may incur unnecessary inflation costs without producing large gains in stability. If OptNoMaP can be said to lean against the wind at all, it is during light distress, not in the best of times. OptNoMaP imposes higher interest rates than the inflation targeting rule when the economy is in distress but not a crisis. While the optimal rule

employs a Put, the Put does not cut all the way to the ZLB because of the inflation costs from doing so. (For lower values of λ the optimal rule hits the ZLB; see Figure 6 in Appendix C.)

Table 3: Comparing welfare and stability under alternative policies to the optimal policy rule. Welfare is measured as $\mathbb{E}[V(\eta)]$. Welfare changes are reported in consumption equivalent percent differences from the optimal policy. Changes in probability of distress ($\psi < 100\%$) and probability of crisis ($\psi < 50\%$) are reported in percentage point differences from the optimal policy. A positive change in probability of distress (crisis) means that the probability of distress (crisis) is higher under the alternative policy than under OptNoMaP.

Alternative Rule	Δ Welfare	Δ Prob. Distress	Δ Prob. Crisis
Inflation Targeting	-0.59%	0.73	0.41
Constant $i = 1.57\% = \tilde{i}^{Put}$	-12.93%	-0.14	0.18
Constant $i = 4.03\% = \tilde{i}^{LAW}$	-1.45%	1.35	0.63
Constant $i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.74%	1.45	0.65
Calibration	-0.76%	-0.61	-0.35

Table 3 shows the welfare and stability losses from using alternative interest rate rules compared to OptNoMaP. Welfare losses are reported in consumption-equivalent percentages while stability losses are percentage point differences for the probabilities of distress and crises.¹⁷ Under OptNoMaP, the probability of a crisis or distress is slightly higher than under the Calibration, reflecting that the optimal policy employs a less aggressive Fed Put compared to the Calibration due to inflation costs (the crisis probability is 3.5% compared to 3.15% under Calibration, and the distress probability is 17.05% compared to 16.44% under Calibration).

We compare OptNoMaP to five alternatives. The first policy considers inflation targeting, which is the most natural benchmark. The OptNoMaP improves over inflation targeting by 0.59% consumption equivalent. Three policies consider constant rate rules with natural benchmarks. The second policy sets rates to 1.57%, which is the Put level of OptNoMaP. The third sets rates to 4.03%, which is the LAW level of OptNoMaP. The fourth sets rates to 4.2%, which is the optimal rate in the absence of financial frictions (Friedman). With one exception, these four policies all

¹⁷Figure 11 in the appendix plots the consumption-equivalent losses for each η . OptNoMaP leads to uniformly higher welfare across the state space, with greater gains when the initial condition is a low value of η .

lead to uniformly worse stability than the optimal rate. Keeping rates low at 1.57% produces a lower probability of distress compared to OptNoMaP but at the cost of a higher probability of crises. As discussed, Calibration leads to more stability but lower welfare.

The welfare losses from deviating from OptNoMaP are significant, with consumption losses of at least 0.5%. Inflation costs must be extremely high for the stability benefits of OptNoMaP to be outweighed by the losses from not targeting inflation. OptNoMaP outperforms inflation targeting even if we set inflation costs to 5 times the calibrated value. Even without inflation costs OptNoMaP leads to higher welfare than the other policies precisely because of the stability benefits (fewer crises). With $\lambda = 0$, the welfare losses from the alternative policies above are, in order, -0.74%, -0.36%, -1.20%, and -1.29%. Holding rates low at 1.57% no longer imposes such significant inflation losses but still decreases welfare because stability suffers (crisis probability increases). With $\lambda = 0$, inflation targeting does comparably worse because inflation costs are no longer a concern, and therefore the stability gains from OptNoMaP matter more.¹⁸

4.2 Stability, Returns, and Leverage

Equilibrium under a Fed Put generally resembles a combination of the characteristics of the equilibria under constant rate policies, with constant rates defined by the values i^{Put} and i^{LAW} (see also Figure 10 in Appendix C). Despite this behavior, the OptNoMaP policy *improves* stability ex-ante, in contrast to the worry that a Fed Put would be destabilizing. Stability under OptNoMaP, measured by the probabilities of crisis or distress, is generally better than under constant rule policies. Perhaps surprisingly, the $i = 4.2\%$ rule (unconditional LAW) results in the least stable economy, increasing the probabilities of both crisis and distress. Leaning against the wind in this way is counterproductive. Similarly, while keeping rates low everywhere decreases the probability of distress, it actually increases the probability of a crisis. Instead, the policy of high rates in a boom followed by low rates in a crisis improves stability *ex ante*.

These results contrast with those obtained by Drechsler et al. (2018). While the effects of

¹⁸The optimal policy changes as λ changes. Figure 6 in Appendix C plots the optimal policies for lower λ . The Put becomes more aggressive for lower inflation costs, and if costs are sufficiently low, the optimal policy also pursues an aggressive degree of LAW in booms.

monetary policy on asset prices and volatility in our model are similar to theirs, the impact of a Fed Put on stability and returns is different. Higher rates in the endowment economy of [Drechsler et al. \(2018\)](#) produce a less disperse and thus “stabler” stationary density. Although they do not report the stationary density for their model’s extension to a production economy with investment and output gaps, we presume that their results do not change. In contrast, OptNoMaP and constant low rates in our model yield greater stability than the constant high rate policy. The key reason is that financial frictions affect the output gap in our model whereas in [Drechsler et al. \(2018\)](#) the output gap does not depend on the health of bankers’ balance sheets.

For intuition about why the OptNoMaP policy promotes stability, we point to two mechanisms. First, a low nominal rate during crises and a high rate during booms produce better returns, which promote the expected rate at which banks recapitalize. All else equal, a faster rate of equity recovery reduces the duration of time for which η is low. Second, leverage does not increase substantially during booms. Moral hazard is a common concern in response to promising interest rate cuts in crises. If banks know that they will receive accommodation when their balance sheets are distressed, then they may take greater risk during booms and ex-ante destabilize the economy. However, in our model, when banks have healthy balance sheets, capital is efficiently allocated, so banks cannot take more leverage during booms. Instead, higher demand for capital completely passes through to its price. In the following two sections, we discuss these mechanisms in detail.

Returns A salient reason why a Fed Put improves stability is the state-dependent impact on returns. Combining banks’ asset-pricing equation with Lemma 1, the drift of η can be written as

$$\mu_{\eta} = \underbrace{\left(\frac{\psi}{\eta} - 1\right)(HP + SP)}_{\text{Effect of Risk Premium on Drift}} + \frac{a_b - l}{Q} + \mathcal{J}_b - \left(\frac{\psi}{\eta} - 1\right)(\sigma + \sigma_Q)^2.$$

From this expression, a higher household risk premium HP or bank solvency risk premium SP rebuilds bank equity at a faster rate, but a higher liquidity premium LP does not increase μ_{η} . Hence, the liquidity premium is indeed an opportunity cost: banks have to earn but it does not actually help them rebuild their equity.

When the economy is in distress or a crisis, the liquidity premium behaves like a substitute for the risk premium (see Figure 7 in Appendix C). When η is low, an increase in the nominal interest rate leads banks' risk premium to fall as if the two premia are substitutes. Asset pricing condition (13) illustrates why the liquidity premium substitutes for the risk premium when the economy is in distress or a crisis. Recall that if households hold positive quantities of capital, then

$$\frac{a_b - a_h}{Q} = SP + LP$$

must hold. Quantitatively, a higher LP barely changes the capital price Q , so the LHS of this equation is essentially constant. The reason is that households are marginal pricers of capital during times of distress or crisis. Since households are willing to sell capital due to their lower productivity, the price of Q does not respond strongly to the nominal interest rate. For the equality to hold, SP must decrease instead.

During a boom, however, the liquidity premium becomes a substitute for the risk-free interest rate instead of the risk premium (see Figure 8). In this case, the asset pricing condition (13) is slack, so expected returns now only need to satisfy (12):

$$\mathbb{E}[dr_{b,t} - dr_{f,t}] = HP + SP + LP.$$

The risk-free rate during a boom can (and does) move to absorb most of the effects of a higher nominal rate, whereas during times of distress or crisis, changes in LP have to be matched by changes in SP and/or Q . The lower liquidity premium corresponds to a higher risk-free rate. This result is consistent with the evidence in Gambacorta and Shin (2016) showing that bank capital matters for monetary policy transmission. The intuition is the same as in Drechsler et al. (2018): a higher liquidity premium implies a greater precautionary savings motive for banks and thus households because households own banks.

Together, these state-dependent impacts on returns imply that the OptNoMaP policy accelerates the expected recovery of bank equity by implementing an aggressive Put during crises and targeting inflation during booms. In a crisis, the risk premium is higher when i^{Put} is low because the liquidity

premium substitutes for the risk premium. In a boom, the nominal rate is not constrained by stability concerns because the liquidity premium substitutes for the risk-free rate rather than the risk premium. The first feature ensures faster recoveries from severe downturns while the second minimizes inflation costs.

Monetary policy improves financial stability because its effects on equilibrium depend on where the economy is located in the financial cycle: the differential impact on leverage and returns is determined by whether or not banks hold the entire stock of capital. A carefully targeted policy of high rates during booms, and only during booms, promotes stability. Raising interest rates has a detrimental effect on stability if higher rates increase bank funding costs during crises, precisely when rebuilding bank equity is most valuable. However, when banks are well-capitalized and hold the entire capital stock, raising rates does not reduce stability. Therefore, to stabilize the financial sector while minimizing inflation costs, i^{Put} should be low, and i^{LAW} should be high. Appendix C.5 explores further dimensions of monetary policy's state dependency and the effectiveness of LAW.

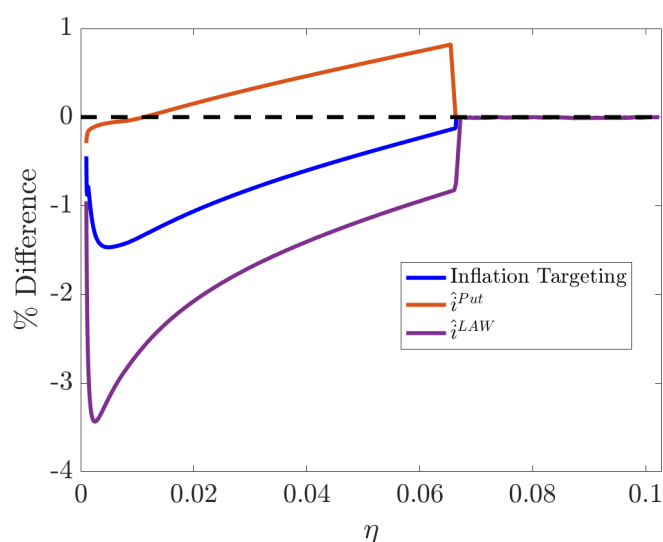


Figure 3: Percent differences in leverage relative to the OptNoMaP policy. Leverage in a crisis is higher under OptNoMap than under inflation targeting or constant rates. The blue, red, and purple lines correspond to inflation targeting, a constant rate at i^{Put} , and a constant rate at i^{LAW} . Note that the i^{Put} and i^{LAW} levels are from the OptNoMaP policy.

Leverage We define leverage in our model as the ratio of bank debt to *book* equity, i.e. $\frac{\psi QK - N_b}{N_b} = \frac{\psi}{\eta} - 1$. Figure 3 plots the percent difference in leverage under OptNoMaP compared to leverage under the alternative rules (a negative number means leverage is lower under the alternative than under OptNoMaP). Policy rules affect leverage differently in good and bad times. During booms, all policy rules generate the same leverage. Explicitly, since $\psi = 100\%$ in a boom, leverage must be $1/\eta - 1$ in equilibrium. Changes in the nominal interest rate (and changes in banks' demand for capital) affect the capital price in equilibrium but not bank leverage.¹⁹

When the economy is in distress or a crisis, however, leverage is actually higher under OptNoMaP compared to inflation targeting and substantially higher than a rule with high constant rates at $\tilde{i}^{LAW}$. In contrast, constant low rates at $\tilde{i}^{Put}$ generally leads to higher leverage except for the deepest crisis. This is why the probability of distress is lower with constant low rates at $\tilde{i}^{Put}$ compared to OptNoMaP. Surprisingly, for the lowest η , leverage under OptNoMaP is also higher than a rule with low constant rates set to $\tilde{i}^{Put}$, even though interest rates for those η are the same in both cases; general equilibrium effects across the state space actually support higher leverage under OptNoMaP in the deepest crisis.

Monetary policy affects leverage across the state space in this way because households are marginal pricers of capital during times of distress or crisis but only banks are marginal pricers in a boom. All else equal, low rates during times of distress or crisis incentivize banks to demand more capital. Banks' higher demand for capital is met by higher supply because households are less productive and are therefore willing to sell capital. In equilibrium, banks increase their leverage. In contrast, during booms banks hold the entire stock of capital, so interest rates affect asset prices instead of allocations (leverage).

¹⁹In Appendix D, we consider a two-good model where households specialized in the production of one good. In equilibrium they always hold some capital, hence banks during booms can actually increase their leverage. However, leverage in the two-good model still behaves like leverage in this one-good model. In a crisis, low rates boost leverage, but in a boom the prospect of low rates in the future does not significantly boost bank leverage. The reason is the same: interest rates during booms primarily affect asset prices, not allocations. Intuitively, households are not willing to sell during booms because their capital holdings are efficiently used. While low rates in the future incentivize higher leverage by banks, no agent is willing to sell, so prices change rather than allocations.

Discussion of Moral Hazard Many economists worry that the expectation of low rates during crises will trigger more risk-taking during booms and conclude that a Fed Put will decrease stability. This is not the case in our model, which is one reason why OptNoMaP produces an ex-ante more stable economy. A Fed Put does not lead to higher leverage in a boom, and high leverage during distress or crisis boosts stability because it allows banks to earn greater excess returns and rebuild equity quickly. This is precisely why OptNoMaP cuts rates by more than the inflation targeting rule.

There are two issues to consider with regards to moral hazard in our model. First is whether banks allocations can exceed the socially optimal level during booms. Second is whether banks can take actions that are undesirable at the individual level. Banks may take more leverage in a boom because they expect rate cuts when the economy is in distress or a crisis, and because of financial frictions leverage can be excessive. While aggregate leverage could be excessive, we have not modeled any reason why banks may take actions that households would not find desirable at the individual level.

On the first issue, a primary concern among policymakers is that the expectation of a Fed Put will lead banks to hold more assets than efficient during booms. But in our baseline model banks rightly hold the entire capital stock in a boom and cannot hold more capital. Increased demand for capital increases the capital *price* but does not affect bank *holdings*. We address this issue with two extensions.

Appendix D considers a modified model with two goods in which banks take excessive risk during booms, and it is no longer optimal for banks to hold all the capital stock. In this setting, it is now possible for a Put to produce a higher allocation of capital to banks during booms, not just a higher price. We show that our main normative and positive results continue to hold in this environment: a Fed Put with higher rates during booms is welfare-optimal and stabilizes the economy by increasing leverage during times of distress or crisis only (not creating excessive leverage in a boom), which improves the stationary distribution of the economy.

On the second issue, in reality, bank managers can be “dishonest” and misbehave. It is common to assume that bankers can “steal” capital to divert for their own consumption. Di Tella (2019) stud-

ies this form of dishonest moral hazard within a continuous-time macro-finance model and finds that bankers face a greater incentive to steal when prices are higher because they can earn more per unit of capital. In our setting, this externality would be worsened by accommodative monetary policy. Placing a similar mechanism into our model may mitigate the capacity of monetary policy to address financial stability. Banks also function as monitors for depositors (see [Diamond \(1984\)](#)), but screening risky investments is costly. Stable financial conditions may decrease screening effort, causing banks to systematically choose poorer quality projects. [Adrian and Shin \(2014\)](#) motivate a value-at-risk (VaR) constraint as the outcome of a moral hazard problem between depositors and the bank. Accommodative monetary policy may loosen banks' VaR constraints and produce even more leverage during booms, which could undo the stabilizing effects of monetary policy.

Appendix [E](#) considers a modified model in which banks' productivity exogenously depends on the aggregate state η . We choose a function such that bank productivity falls when η is too high so that output is the highest when the share of capital allocated to banks is between approximately 75% and 87%. This specification captures the idea that banks may misallocate capital during a boom (i.e. when $\psi = 100\%$). Despite the possibility of bank misallocation, the optimal policy rule implements a more aggressive Fed Put. Intuitively, even if capital can be misallocated by banks, as long as it is still optimal to avoid crises, the optimal policy rule is accommodative during downturns. The optimal policy rule also increases the probability of crisis, which reflects the fact that output is highest when $\psi < 100\%$.

More careful considerations of additional moral hazard problems are unlikely to reverse our results. Within our model, the effects of monetary policy depend on whether or not households are marginal pricers of capital and whether marginal pricers efficiently utilize capital. Timing rate changes around the marginal pricer's identity avoids creating a serious moral hazard problem.

5 Interaction of Monetary and Macroprudential Policies

We now investigate the extent to which monetary policy still improves financial stability and/or welfare when macroprudential policy (MaP) is also applied. A Fed Put promotes both financial stability and welfare in the absence of MaP. These results extend so long as macroprudential policy

is insufficient to address financial stability on its own. Consistent with our theoretical results (Propositions 2 and 3), with sufficiently well-targeted MaP, optimal monetary policy does not target financial stability. However, the empirical plausibility of this case seems quantitatively unlikely.

5.1 Positive Consequences of Macroprudential Policy

We model macroprudential policy as a leverage constraint that depends on η . For some function $L(\eta)$, the share of capital allocated to banks must satisfy

$$\frac{\psi}{\eta} - 1 \leq L(\eta).$$

Since all variables in the model depend on η in equilibrium, this constraint can also be written as a function of other quantities like the marginal value of bank equity rather than η directly. Leverage constraints change some of the equilibrium conditions used to derive Lemma 1 (see Appendix A).

Banks choose socially suboptimal levels of leverage only for certain ranges of η , so macroprudential policy must limit leverage at the correct locations in the state space. Because bank leverage is countercyclical, leverage constraints that are simply defined are likely to bind when the economy is already in a downturn. However, this is precisely when output is the worst and the need to rebuild equity is the greatest.

Macroprudential policy performs best if it primarily restricts leverage during a boom and allows more leverage during a crisis. One such rule ties leverage to the marginal value of bank equity:

$$L(\eta) = \alpha\theta. \tag{19}$$

This constraint is close to the incentive-compatibility constraint on leverage used in [Van der Ghote \(2021\)](#), and so we call (19) the “ICC” macroprudential policy. The coefficient α parameterizes how stringent the leverage constraint is. This rule encourages countercyclical leverage (θ decreases with η). When θ is low (during a boom) bank leverage cannot be too high, but if bank equity is low, then the constraint relaxes, allowing banks to borrow more.

The ICC constraint moves leverage in the desired direction and improves stability. Figure

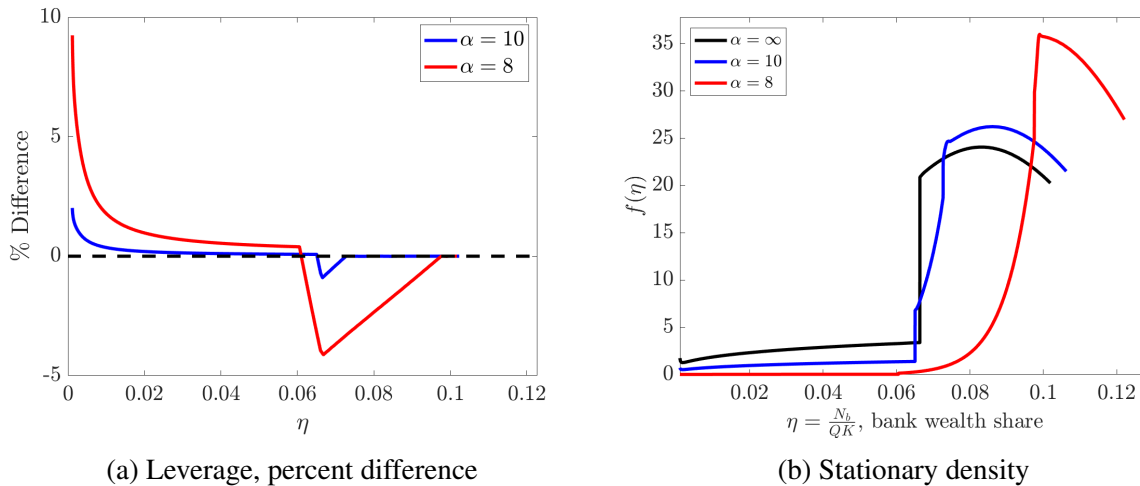


Figure 4: Leverage and stability when the ICC MaP rule is applied and MP follows the OptNoMaP rule. The left panel plots the difference in leverage between the OptNoMaP policy with no MaP and the OptNoMaP policy with a binding ICC rule. The blue line corresponds to $\alpha = 10$, and the red line corresponds to $\alpha = 8$. In the right panel, the black line is the OptNoMaP rule with no ICC.

4 plots leverage and the stationary density when the ICC policy is applied and monetary policy follows the Calibration rule. Figure 4(a) plots the percent differences in leverage compared to equilibrium with no ICC policy (i.e., $\alpha = \infty$). The ICC policy leads to lower leverage in a boom and higher leverage in a crisis.²⁰ Because the ICC policy boosts leverage when the economy is in distress or a crisis but dampens leverage in a boom, the economy is overall more stable, as illustrated in Figure 4(b): the mass of $f(\eta)$ is shifted toward higher states, and the endogenous boundary η^* increases as well. The drift of bank equity increases everywhere and systemic risk generally falls when the ICC policy is applied. Although the ICC policy increases the probability of distress, the higher drift and lower volatility prevent the economy from slipping into crisis.

This positive result informs why well-targeted MaP may allow MP to focus on inflation targeting. The previous analysis found that OptNoMaP outperformed inflation targeting precisely because OptNoMaP implemented a more stable economy (lower probability of crises). If MaP can stabilize the economy, then there may be less need for MP to play that role.

²⁰More precisely, the ICC policy causes lower leverage in the region of the state space that would be classified as a boom if the Calibration rule *without* an ICC policy was used.

5.2 Optimal Monetary Policy with Macroprudential Policy

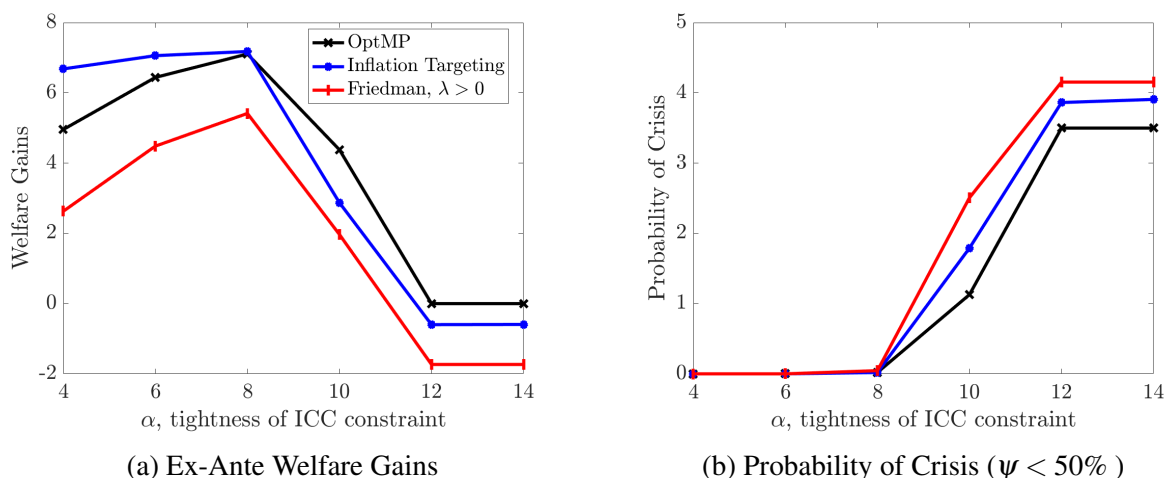


Figure 5: Welfare and stability varying the tightness of MaP (α). Monetary policy is set to OptMP (black), inflation targeting (blue), and Friedman $\lambda > 0$ (red). The OptMP and inflation targeting rules change with α . Welfare gains are calculated in consumption equivalent percent differences from OptNoMaP without MaP. Panel (b) shows the probability of crisis.

The presence of the ICC policy changes the welfare consequences of monetary policy rules. For sufficiently tight ICC, welfare is now maximized when monetary policy pursues inflation targeting. Figure 5 plots welfare and stability (crisis probability) while varying the tightness of the ICC policy α for three rules: (i) the optimal piecewise linear policy (i.e. policies in the class of (18)), which we call OptMP, (ii) inflation targeting, and (iii) the Friedman $\lambda > 0$ rule. Note that the OptMP and inflation targeting rules change with α . For high α , the OptNoMaP rule is the OptMP rule since the leverage constraint is slack. Welfare with the ICC policy is not monotonic in α , regardless of the monetary policy rule, reflecting the tradeoff between stability and output: tighter MaP improves stability, but at the cost of lower flow output and growth. Welfare is maximized for an interior value of the ICC.

More importantly, for sufficiently tight α , inflation targeting yields higher welfare than OptMP (see $\alpha < 8$ in Figure 5(a)). Thus, with sufficient MaP, inflation targeting is the optimal monetary rule, given the space of policy rules we consider. The ICC erases the need for active monetary policy because it restricts leverage during the appropriate phase of the financial cycle—the boom.

Asset price volatility is the highest when the economy enters distress and banks start to sell capital to households, hence the economy's pecuniary externality is the largest at that time (see [Van der Ghote, 2021](#)). The reason asset price volatility spikes when distress begins is that a boom sustains excessively high asset prices. The higher prices are before distress begins, the more they must fall once banks need to sell capital. Binding leverage constraints during a boom mitigate the pecuniary externality by reducing the severity of these initial fire sales. Lower leverage implies lower asset prices, so prices do not fall as much when banks need to sell capital.

Restricting leverage during a boom also causes a shallow recession. Indeed, a consequence of tight ICC constraints is that the probability of distress ($\psi < 1$) increases as α . However, this is precisely the point of having the constraint bind to prevent crises. By accepting some output losses earlier in the financial cycle, macroprudential policy lowers the probability of a severe recession and further reduces the severity of fire sales.

Although these results seem to suggest that macroprudential policy can address financial stability on its own, they require that macroprudential policy is unrealistically effective. Notice that the range of MaP over which inflation targeting is optimal implements an economy so stable that crises occur with virtually zero probability. In other words, for inflation targeting to dominate a policy rule which actively targets the financial sector, macroprudential policy needs to completely prevent crises. A more robust interpretation of our results is that macroprudential policy is unlikely to be sufficiently powerful that it obviates the need for monetary policy to target financial stability.

Discussion So far we have considered macroprudential policies which are well-targeted in the sense that they reduce leverage when it exacerbates the economy's pecuniary externality and permit leverage when it helps recapitalize the financial sector. These policies in theory can be sufficiently well-targeted that they obviate the need for monetary policy to target financial stability. Another dimension we have not explored is the effect of macroprudential policy when it binds in the wrong places and is thus poorly targeted.

If macroprudential policy is poorly targeted, then our results from Section 4 continue to hold. An obvious case of poorly-targeted leverage constraints are constraints which bind during a crisis. In these cases, the OptNoMaP rule outperforms constant rate rules or inflation targeting, even

though we are not reoptimizing the monetary policy rule as the tightness of the leverage constraint changes. Appendix C.6 discusses these results in detail.

Our results regarding the interaction of macroprudential and monetary policies can be interpreted in light of the so-called [Tinbergen \(1952\)](#) principle, that a policymaker with n independent objectives needs n policy tools. Our model is dynamic with state-contingent effects from policy (and externalities). To address financial and price stability together, policy makers do not only need 2 policy tools—they must also have sufficiently *flexible* tools. Addressing financial stability therefore requires appropriately designed macroprudential limits. In reality, macroprudential tools may be restricted as a result of regulation or challenges to implementation, whereas monetary policy rules can be extremely nimble, responding immediately to financial stability concerns if the central bank so chooses. If macroprudential tools are inflexible and cannot target sources of financial instability at the correct time, then scope remains for monetary policy to address financial stability (see also [Caballero and Simsek, 2019](#)). Conversely, monetary policy is free to focus on conventional targets (e.g. a stable price level) as its sole objectives only if macroprudential tools are sufficient to mitigate financial crises.

The mechanics of how MaP and MP affect welfare and stability are somewhat different. Monetary policy increases welfare primarily by increasing leverage when the economy is in distress or a crisis so that flow outcomes improve and banks rebuild equity more quickly. In contrast, the ICC policy improves outcomes *on average* by forcing banks to sell capital sooner, creating a small recession so that the likelihood of a severe recession falls. When leverage limits bind, intermediation falls and flow outcomes suffer, but binding leverage limits increase banks' investment returns and so banks may rebuild equity faster. When MaP is well-targeted, the general equilibrium consequences support higher leverage in Crisis times, mirroring the effects of decreasing interest rates following a Fed Put. These differences between how MaP and MP affect welfare and stability are precisely why well-targeted leverage constraints obviate the need for monetary policy to target financial conditions, allowing monetary policy to simply target inflation.²¹

²¹Leverage constraints are not the only instrument in the macroprudential policy toolkit. Appendix A.3.3 models equity injections, which amounts to changing the boundary conditions for the equilibrium ODE. We find that equity injections can also substitute for active monetary policy. If these injections are sufficiently aggressive so that banks are able to always hold the entire capital stock, then monetary policy becomes unnecessary for stabilization.

6 Conclusion

Central bank mandates may need to be expanded to include financial stability whenever macroprudential policy is imperfect. We provide a macroeconomic model with a financial sector in which monetary policy endogenously determines the stability of the economy and therefore determines the probability and severity of crises. Monetary policy influences outcomes by affecting risk and liquidity premia. Policies that combine high nominal rates in good times with accommodative rates during financial crises can substantially improve stability and household welfare. The consequences of monetary policy for financial stability are state-dependent, so the stability benefits of monetary policy depend critically on the timing.

Cutting rates during financial crises (and only then) yield the greatest stability gains. When macroprudential policies are poorly targeted, monetary policy should target financial stability, but when macroprudential policies are sufficiently well-targeted, then monetary policy should not target financial stability. Our quantitative results suggest, however, that macroprudential policy is unlikely to be sufficiently well-targeted. Unless macroprudential policy can fully eliminate the possibility of crisis, there remains scope for monetary policy to actively target financial stability.

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Appendices for Online Publication

A Proofs and Additional Derivations and Analytical Results

A.1 Proofs and Propositions

Household Problem

Define $\omega_t \equiv n_{h,t}/w_{h,t}$, which is the share of a household’s total wealth that is held in their non-bank net worth (i.e., excluding bank holdings), let $\sigma_{e,t}$ denote the volatility of bank equity returns, and let $\sigma_{U,t}$ denote the volatility of $U_t \equiv \theta_t N_{b,t}$.

Proposition 5. Let $\sigma_{ch,t}$ denote the volatility of a household's consumption. Then households choose $x_{h,t}$ and $c_{h,t}$ such that (i) $\mathbb{E}[dr_{h,t} - dr_{f,t}] \leq \sigma_{ch,t}(\sigma + \sigma_{Q,t})$, and the inequality binds if and only if $x_{h,t} > 0$; (ii) $c_{h,t} = rw_{h,t}$; (iii) $\sigma_{ch,t} = \omega_t \sigma_{nh,t} + (1 - \omega_t) \sigma_{U,t}$; (iv) $\mathbb{E}[dr_{e,t} - dr_{f,t}] = \sigma_{ch,t} \sigma_{e,t}$.

Proof of Proposition 5, households' optimality decisions. Conjecture that households have a twice-differentiable value function

$$V_t = \frac{\log(w_{h,t})}{r} + h_t,$$

where h_t is independent of the level of household wealth. By Ito's lemma, households' HJB is

$$\begin{aligned} \log(w_{h,t}) + rh_t &= \max_{x_{h,t} \geq 0, c_{h,t}, l_t} \log(c_{h,t}) - \frac{\lambda}{2}(\pi_t - \bar{\pi})^2 + \frac{\mu_{wh,t} w_{h,t}}{rw_{h,t}} - \frac{(\sigma_{wh,t} w_{h,t})^2}{2rw_{h,t}^2} \\ &= \max_{x_{h,t} \geq 0, c_{h,t}, l_t} \log(c_{h,t}) - \frac{\lambda}{2}(\pi_t - \bar{\pi})^2 + \frac{1}{r} \mu_{wh,t} - \frac{1}{2r} \sigma_{wh,t}^2. \end{aligned}$$

Further, because $\sigma_{wh,t} = x_{h,t}(\sigma + \sigma_{Q,t}) + x_{e,t} \sigma_{e,t}$,

$$\frac{\partial \sigma_{wh,t}}{\partial x_{h,t}} = (\sigma + \sigma_{Q,t}), \quad \frac{\partial \sigma_{wh,t}}{\partial x_{e,t}} = \sigma_{e,t}.$$

Finally, households' law of motion for their net worth can also be written as

$$\frac{dw_{h,t}}{w_{h,t}} = dr_{f,t} + x_{h,t}(dr_t^h - dr_{f,t}) + x_{e,t}(dr_{e,t} - dr_{f,t}) + \left(\mathcal{I}_{h,t} - \frac{c_{h,t}}{w_{h,t}} \right) dt.$$

Given these observations and (3), the first-order conditions for portfolio weights are

$$\begin{aligned}
(x_{h,t}) : \quad 0 &= \frac{1}{r}(\mathbb{E}[dr_{h,t} - dr_{f,t}]) - \frac{1}{r}\sigma_{wh,t} \frac{\partial \sigma_{wh,t}}{\partial x_{h,t}} \\
&= (\mathbb{E}[dr_{h,t} - dr_{f,t}] - \sigma_{wh,t}(\sigma + \sigma_{Q,t})) \\
\Rightarrow \mathbb{E}[dr_{h,t} - dr_{f,t}] &= \sigma_{wh,t}(\sigma + \sigma_{Q,t}) \\
(x_{e,t}) : \quad 0 &= \frac{1}{r}(\mathbb{E}[dr_{e,t} - dr_{f,t}]) - \frac{1}{r}\sigma_{wh,t} \frac{\partial \sigma_{wh,t}}{\partial x_{e,t}} \\
&= \mathbb{E}[dr_{e,t} - dr_{f,t}] - \sigma_{wh,t}\sigma_{e,t} \\
\Rightarrow \mathbb{E}[dr_{e,t} - dr_{f,t}] &= \sigma_{wh,t}\sigma_{e,t},
\end{aligned}$$

and the conditions for consumption and internal investment are

$$\begin{aligned}
(c_{h,t}) : \quad 0 &= \frac{1}{c_{h,t}} - \frac{1}{rw_{h,t}}, \\
\Rightarrow c_{h,t} &= rw_{h,t}, \\
(l_t) : \quad \Phi'(l_t) &= \frac{1}{Q_t}.
\end{aligned}$$

Since consumption is proportional to wealth,

$$\sigma_{ch,t} = x_{h,t}(\sigma + \sigma_{Q,t}) + x_{e,t}\sigma_{e,t}. \quad (20)$$

Hence the first-order conditions for $x_{h,t}$ and $x_{e,t}$ equate expected excess returns with the covariance of household consumption and returns.

We can reframe the problem as if each household held ownership in a bank directly (e.g., a household gets split between a “consumer” and a “banker” as is a common modeling technique in the literature). This allows us to define an implied wealth share ω_t , which would correspond to the fraction of household wealth held in bank holdings, taking into account market clearing. We first solve this version of the household problem. After, we verify that the implied price of bank equity shares coincides with the price required by the previous version of the household problem.

Household wealth can be decomposed as $n_{h,t} + \theta_t n_{b,t}$, where $n_{h,t}$ is non-bank net worth, $n_{b,t}$ are the dollars of bank equity held by households, and θ_t converts the book value of bank equity to market value. In equilibrium, households must hold in total $U_t = \theta_t N_{b,t}$ in bank equity, and we therefore can write $\sigma_{ch,t} = \omega_t \sigma_{nh,t} + (1 - \omega_t) \sigma_{U,t}$ in equilibrium. We now write

$$V_t = \frac{\log(w_{h,t})}{r} + h_t = \frac{\log(n_{h,t} + \theta_t n_{b,t})}{r} + h_t.$$

With a mild abuse of notation, households' law of motion for their non-bank net worth can be written as

$$\frac{dn_{h,t}}{n_{h,t}} = dr_{f,t} + x_{h,t}(dr_t^h - dr_{f,t}) + \left(\mathcal{T}_{h,t} - \frac{c_{h,t}}{n_{h,t}} \right) dt,$$

where transfers are re-scaled by non-bank net worth and the portfolio weight $x_{h,t}$ is over non-bank net worth, not total wealth. Then $w_{h,t}$ follows the process

$$\begin{aligned} \frac{dw_{h,t}}{w_{h,t}} &= \{ \omega_t \mu_{nh,t} + (1 - \omega_t)(\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t}) \} dt \\ &\quad + \{ \omega_t \sigma_{nh,t} + (1 - \omega_t)(\sigma_{\theta,t} + \sigma_{nb,t}) \} dW_t. \end{aligned} \tag{21}$$

Further, because $\sigma_{nh,t} = x_{h,t}(\sigma + \sigma_{Q,t})$,

$$\frac{\partial \sigma_{wh,t}}{\partial x_{h,t}} = \omega_t \frac{\partial \sigma_{n,t}}{\partial x_{h,t}} = \omega_t (\sigma + \sigma_{Q,t}).$$

Given these observations and (21), the first-order conditions are

$$\begin{aligned}
(x_{h,t}) : \quad 0 &= \frac{\omega_t}{r} (\mathbb{E}[dr_{h,t} - dr_{f,t}]) - \frac{1}{r} \sigma_{wh,t} \frac{\partial \sigma_{wh,t}}{\partial x_{h,t}} \\
&= \omega_t (\mathbb{E}[dr_{h,t} - dr_{f,t}] - \sigma_{wh,t} \omega_t (\sigma + \sigma_{Q,t})) \\
\Rightarrow \mathbb{E}[dr_{h,t} - dr_{f,t}] &= \sigma_{wh,t} (\sigma + \sigma_{Q,t}) \\
(c_{h,t}) : \quad 0 &= \frac{1}{c_{h,t}} - \frac{1}{r} \omega_t \frac{1}{n_{h,t}} = \frac{1}{c_{h,t}} - \frac{1}{rw_{h,t}} \\
\Rightarrow c_{h,t} &= rw_{h,t} \\
(\iota_t) : \quad \Phi'(\iota_t) &= \frac{1}{Q_t}
\end{aligned}$$

which are the same conditions derived above when we allowed households to optimize over bank equity holdings (i.e., the ω_t term falls out). Since consumption is proportional to wealth,

$$\sigma_{ch,t} = \omega_t \sigma_{nh,t} + (1 - \omega_t) (\sigma_{\theta,t} + \sigma_{nb,t}), \quad (22)$$

In other words, the drift and volatility terms in (21) define the equilibrium returns on bank equity. Since all banks earn identical returns (there is only an aggregate shock), households will trade equity to hold identical ω_t . We can consider ω_t to be the actual fraction of wealth held directly in banks, or we can consider this the equivalent holdings that households achieve when trading equity shares.

Finally, it remains to price traded bank equity shares and show that the prices coincide in either version of the household's problem. As was just shown, ω_t is defined to determine the volatility of a household wealth as a function of η and the price of traded shares does not affect the other returns in equilibrium. As with the pricing of the risk-free asset (see Lemma 2 below), there is a subtlety that arises for the price of shares as a result of the household transfers due to seignorage and taxes on banks. This is not a central feature of our model and simply reflects how lump-sum transfers are modeled. For example, if the government were to rebate transfers to households through bank equity shares, then shares could be priced without a discount; similarly if the lump-sum rebates were traded as a risk-free claim, the additional term would go away.

To start, the return to bank equity includes dividends paid $d\zeta_t$ and capital gains. The aggregate value of bank dividends, discounted by the household stochastic discount factor, is $U_t = \theta_t N_{b,t}$. From Proposition 6, banks pay dividends only when $\theta_t = 1$. Since the dividend payout decreases bank wealth by exactly the amount paid, this means that the value $U_t = \theta_t N_{b,t}$ does not change when dividends are paid (there is an identical offsetting term in the capital gain). By the proof of Proposition 6 and the fact that consumption is proportional to wealth, banks' HJB implies that

$$r + \mu_{ch,t} - \sigma_{ch,t}^2 = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{\theta,t}.$$

From Lemma 2, the risk-free rate is given by $dr_{f,t} = \left(r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathcal{T}_{h,t}\right) dt$. Note that $\mathcal{T}_{h,t}$ is transfers scaled by a household's *non-bank* net worth rather than its total wealth. It follows that

$$\mathbb{E}[dr_{f,t} + \mathcal{T}_{h,t}] = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}(\sigma_{nb,t} + \sigma_{\theta,t}),$$

and so

$$\mathbb{E}[\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - dr_{f,t} - \mathcal{T}_{h,t}] = \sigma_{ch,t}(\sigma_{\theta,t} + \sigma_{nb,t}). \quad (23)$$

Note that for the case when $\mathcal{T}_{h,t} = 0$, then the market price of traded bank shares equals $\theta_t N_{b,t}$ so that $\mathbb{E}[\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - dr_{f,t}] = \sigma_{ch,t}(\sigma_{\theta,t} + \sigma_{nb,t})$, where $\mathbb{E}[dr_{e,t}] = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t}$ and $\sigma_{e,t} = \sigma_{\theta,t} + \sigma_{nb,t}$. The expected return on equity includes expected changes in the book equity $\mu_{nb,t}$ and expected changes in the marginal valuation $\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}$, where the last term is the Ito interaction term. We need not include the dividend yield $d\zeta_t$ directly for the reasons just given. In addition, the volatility of returns is given by $\sigma_{e,t} = \sigma_{\theta,t} + \sigma_{nb,t}$. From households' optimization over bank equity, $\mathbb{E}[dr_{e,t} - dr_{f,t}] = \sigma_{ch,t}\sigma_{e,t}$.

There is an additional subtlety when $\mathcal{T}_{h,t} > 0$, in which case bank shares trade at a constant premium relative to $\theta_t N_{b,t}$. We have $\sigma_{e,t} = \sigma_{\theta,t} + \sigma_{nb,t}$ and from (23)

$$\mathbb{E}[\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \mathcal{T}_{h,t} - dr_{f,t}] = \sigma_{ch,t}\sigma_{e,t}.$$

and therefore household optimization requires

$$\mathbb{E}[dr_{e,t}] = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \mathcal{T}_{h,t},$$

i.e., returns are low because the risk-free rate is depressed by $\mathcal{T}_{h,t}$ relative to $r + \mu_{ch,t} - \sigma_{ch,t}^2$, which arises from the household discount factor (used by banks to discount dividends). This may at first glance be a surprising result, but it actually reflects the underlying discounting that occurs. Banks discount future dividends using households' stochastic discount factor (see Proposition 6), which implies a discount of $r + \mu_{ch,t} - \sigma_{ch,t}^2$. The market price of dividends gets discounted according to the risk-free rate, which is depressed by the $\mathcal{T}_{h,t} > 0$ term since the transfers are received risklessly from banks, implying a higher discount value and therefore lower return on bank equity, which explains the distinction. $\square$

Risk-free rate

Lemma 2. *The equilibrium risk-free rate is*

$$dr_{f,t} = (r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathcal{T}_{h,t}) dt.$$

Proof. To prove this claim, we employ the stochastic maximum principle (see Brunnermeier and Sannikov (2016b)). The Hamiltonian is

$$\mathcal{H}_t = e^{-rt} \log(c_{h,t}) + \xi_t \mu_{wh,t} w_{h,t} - \zeta_t \xi_t \sigma_{wh,t} w_{h,t}$$

where the drifts and volatilities are

$$\begin{aligned} \mu_{wh,t} w_{h,t} &= w_{h,t} \omega_t (dr_{f,t} + \mathcal{T}_{h,t} + x_{h,t} (\mathbb{E}[dr_{h,t} - dr_{f,t}])) - c_{h,t} \\ &\quad + w_{h,t} (1 - \omega_t) (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t}) \\ \sigma_{wh,t} w_{h,t} &= w_{h,t} \omega_t \sigma_{nh,t} + w_{h,t} (1 - \omega_t) (\sigma_{\theta,t} + \sigma_{nb,t}) \\ \sigma_{nh,t} &= x_{h,t} (\sigma + \sigma_Q). \end{aligned}$$

The FOCs with respect to $c_{h,t}$ and $x_{h,t}$ are

$$\begin{aligned} 0 &= e^{-rt} \frac{1}{c_{h,t}} - \xi_t \\ 0 &= \xi_t w_{h,t} \omega_t (\mathbb{E}[dr_{h,t} - dr_{f,t}]) - \zeta_t \xi_t w_{h,t} \omega_t (\sigma + \sigma_{Q,t}) \\ &= \xi_t w_{h,t} \omega_t (x_{h,t} (\mathbb{E}[dr_{h,t} - dr_{f,t}]) - \zeta_t \sigma_{nh,t}). \end{aligned}$$

The FOC w.r.t. $c_{h,t}$ implies that $\xi_t = e^{-rt}/c_{h,t}$, hence by Ito's lemma

$$\frac{d\xi_t}{\xi_t} = (-r - \mu_{ch,t} + \sigma_{ch,t}^2) dt - \sigma_{ch,t} dW_t.$$

By the stochastic maximum principle,

$$\begin{aligned} \mu_{\xi,t} \xi_t &= -\frac{\partial}{\partial w_{h,t}} \mathcal{H}_t \\ &= -\xi_t \omega_t ((dr_{f,t} + \mathcal{T}_{h,t} + x_{h,t} (\mathbb{E}[dr_{h,t} - dr_{f,t}])) - \zeta_t \sigma_{nh,t}) \\ &\quad - \xi_t (1 - \omega_t) (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t} + \zeta_t (\sigma_{\theta,t} + \sigma_{nb,t})) \\ \zeta_{\xi,t} &= -\sigma_{ch,t} \end{aligned}$$

By the proof of Proposition 6, banks' HJB implies that

$$r + \mu_{ch,t} - \sigma_{ch,t}^2 = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t} - \sigma_{ch,t} \sigma_{nb,t} - \sigma_{ch,t} \sigma_{\theta,t}.$$

Using the above equation, the FOC on $x_{h,t}$, and the implications of the stochastic maximum principle for $\mu_{\xi,t}$ and $\zeta_{\xi,t}$, we obtain

$$\mu_{\xi,t} = -\omega_t (\mathbb{E}[dr_{f,t}] + \mathcal{T}_{h,t}) - (1 - \omega_t) (r + \mu_{ch,t} - \sigma_{ch,t}^2)$$

Substitute in for $\mu_{\xi,t}$ to acquire

$$\begin{aligned} -(r + \mu_{ch,t} - \sigma_{ch,t}^2) &= -\omega_t(\mathbb{E}[dr_{f,t}] + \mathcal{J}_{h,t}) - (1 - \omega_t)(r + \mu_{ch,t} - \sigma_{ch,t}^2) \\ 0 &= -\omega_t(dr_{f,t} + \mathcal{J}_{h,t}) + \omega_t(r + \mu_{ch,t} - \sigma_{ch,t}^2) \\ \mathbb{E}[dr_{f,t}] &= r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathcal{J}_{h,t}, \end{aligned}$$

and hence $dr_{f,t} = (r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathcal{J}_{h,t})dt$. □

Bank Problem

Proposition 6. Assume θ_t and $c_{h,t}$ follow the finite diffusions

$$\frac{d\theta_t}{\theta_t} = \mu_{\theta,t} dt + \sigma_{\theta,t} dW_t, \quad (24)$$

$$\frac{dc_{h,t}}{c_{h,t}} = \mu_{ch,t} dt + \sigma_{ch,t} dW_t, \quad (25)$$

with $\sigma_{\theta,t} \leq 0$. Then $\theta_t n_{b,t}$ represents the maximal future expected payoff that a bank with book value $n_{b,t}$ can attain, and $\{x_{b,t}, d\zeta_t, \iota_t\}$ is optimal if and only if (i) $\theta_t \geq 1 \forall t$, (ii) $d\zeta_t > 0$ only when $\theta_t = 1$, and $d\zeta_t = 0$ otherwise, (iii) $\mu_{\theta,t} = T_t/(1 - \eta_t) - \gamma \iota_t + \sigma_{ch,t} \sigma_{\theta,t}$, (iv) $\mathbb{E}[dr_t^b - dr_{f,t}] - \gamma \iota_t \leq (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t})$, with strict equality when $x_{b,t} > 0$, (v) $\Phi'(\iota_t) = 1/Q_t$, (vi) The transversality condition $\mathbb{E}[\xi_t \theta_t n_{b,t}] \rightarrow 0$ holds under $\{x_{b,t}, d\zeta_t\}$.

Proof of Proposition 6, banks' optimality decisions. Homogeneity and price-taking imply that banks' value function takes the form $U_t = \theta_t n_{b,t}$, where θ_t is the marginal value of banks' equity. Further, households' discount factor is $\xi_t = e^{-rt} c_{h,t}^{-1}$ because households have log utility. Therefore, the HJB can be written as

$$r\theta_t n_{b,t} c_{h,t}^{-1} = \max_{x_{b,t} \geq 0, d\zeta_t, \iota_t} d\zeta_t c_{h,t}^{-1} + \mathbb{E}[d(\theta_t n_{b,t} c_{h,t}^{-1})], \quad (26)$$

subject to the constraints (6) and (7).

Conjecture that θ_t and $c_{h,t}$ both follow diffusions. By Ito's product rule,

$$\frac{d(\theta_t n_{b,t})}{\theta_t n_{b,t}} = (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t}) dt + (\sigma_{\theta,t} + \sigma_{nb,t}) dW_t.$$

By Ito's product and quotient rules,

$$\begin{aligned} & (\theta_t n_{b,t} c_{h,t}^{-1}) \mathbb{E} \left[\frac{d(\theta_t n_{b,t} c_{h,t}^{-1})}{\theta_t n_{b,t} c_{h,t}^{-1}} \right] \\ &= \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t} + \sigma_{ch,t}^2 - \mu_{ch,t} - \sigma_{ch,t} (\sigma_{\theta,t} + \sigma_{nb,t}). \end{aligned}$$

After dropping the differential dt , equation (26) becomes

$$\begin{aligned} r \theta_t n_{b,t} c_{h,t}^{-1} = & \max_{x_{b,t} \geq 0, \iota_t, d\zeta_{b,t} \geq 0} c_{h,t}^{-1} d\zeta_{b,t} + \theta_t n_{b,t} c_{h,t}^{-1} (\mu_{\theta,t} + \mu_{nb,t} - \mu_{ch,t} + \sigma_{ch,t}^2 \\ & + \sigma_{\theta,t} \sigma_{nb,t} - \sigma_{ch,t} \sigma_{nb,t} - \sigma_{ch,t} \sigma_{\theta,t}). \end{aligned}$$

Dividing through by $\theta_t n_{b,t} c_{h,t}^{-1}$, substituting wealth terms, and plugging $x_{bd,t} = 1 - x_{b,t}$ obtains

$$\begin{aligned} r = & \max \frac{d\zeta_{b,t}}{\theta_t n_{b,t}} + \mu_{\theta,t} + dr_{f,t} + x_{b,t} (\mathbb{E}[dr_{b,t} - dr_{f,t}]) - \gamma x_{b,t} \dot{\iota}_t + \gamma \dot{\iota}_t + \mathcal{T}_{b,t} - \frac{d\zeta_{b,t}}{n_{b,t}} \\ & - \mu_{ch,t} + \sigma_{ch,t}^2 + x_{b,t} \sigma_{\theta,t} (\sigma + \sigma_{Q,t}) - x_{b,t} \sigma_{ch,t} (\sigma + \sigma_{Q,t}) - \sigma_{ch,t} \sigma_{\theta,t} \end{aligned}$$

The first-order conditions imply

$$d\zeta_{b,t} > 0 \text{ if } \theta_t \leq 1$$

$$d\zeta_{b,t} = 0 \text{ if } \theta_t > 1$$

$$\mathbb{E}[dr_{b,t} - dr_{f,t}] = (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) + \gamma \dot{\iota}_t$$

$$\Phi'(\iota_t) = \frac{1}{Q_t}.$$

The FOC on capital holdings can be re-written as

$$\mathbb{E}[dr_{b,t} - dr_{f,t}] = (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) + \gamma i_t.$$

The FOC on the dividends process implies $\mu_{nb,t} = \mu_{nb,t} - d\zeta_{b,t}/n_{b,t}$, hence we may put $\mu_{nb,t}$ back into the HJB and re-arrange to obtain the expression mentioned in the proof of Lemma 2.

$$\begin{aligned} r &= \max_{x_{b,t} \geq 0, t_t} \mu_{\theta,t} + \mu_{nb,t} - \mu_{ch,t} + \sigma_{ch,t}^2 \\ &\quad + \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{\theta,t} \\ r + \mu_{ch,t}^2 - \sigma_{ch,t}^2 &= \max_{x_{b,t} \geq 0, t_t} \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}(\sigma_{nb,t} + \sigma_{\theta,t}). \end{aligned}$$

Plug the remaining FOCs into the HJB to acquire

$$r = \mu_{\theta,t} + \mathbb{E}[dr_{f,t}] + \gamma i_t + \mathcal{T}_{b,t} - \mu_{ch,t} + \sigma_{ch,t}^2 - \sigma_{ch,t}\sigma_{\theta,t}$$

By Lemma 2,

$$0 = \mu_{\theta,t} - \mathcal{T}_{h,t} + \gamma i_t + \mathcal{T}_{b,t} - \sigma_{ch,t}\sigma_{\theta,t}.$$

In the interior of the state space, $\mathcal{T}_{h,t}$ includes seigniorage payments and the wealth tax on banks. The value of seigniorage payments $\mathcal{S}_t$ is the quantity of total debt multiplied by the liquidity premium (see Drechsler et al. (2018)):

$$\mathcal{S}_t \equiv \eta_t Q_t K_t (x_{b,t} - 1) \gamma i_t = (\psi_t - \eta_t) Q_t K_t \gamma i_t \quad (27)$$

The wealth tax per dollar of book equity is a constant T . In the aggregate

$$\mathcal{T}_{h,t} = \frac{TN_{b,t} + (1 - \eta_t)\mathcal{S}_t}{N_{h,t}} = \frac{\eta_t}{1 - \eta_t} T + (\psi_t - \eta_t) \gamma i_t, \quad (28)$$

$$\mathcal{T}_{b,t} = \frac{-TN_{b,t} + \eta_t \mathcal{S}_t}{N_{b,t}} = -T + (\psi_t - \eta_t) \gamma i_t. \quad (29)$$

Therefore, the drift of θ is

$$\begin{aligned}
 \mu_{\theta,t} &= \mathcal{T}_{h,t} - \mathcal{T}_{b,t} - \gamma i_t + \sigma_{ch,t} \sigma_{\theta,t} \\
 &= \frac{\eta_t}{1 - \eta_t} T + T - \gamma i_t + \sigma_{ch,t} \sigma_{\theta,t} \\
 &= \frac{\eta_t + 1 - \eta_t}{1 - \eta_t} T - \gamma i_t + \sigma_{ch,t} \sigma_{\theta,t} \\
 &= \frac{1}{1 - \eta_t} T - \gamma i_t + \sigma_{ch,t} \sigma_{\theta,t}.
 \end{aligned}$$

□

Evolution of η

Proof of Lemma 1, law of motion for η_t . After substituting banks' asset pricing condition and using $x_{b,t} = \psi_t / \eta_t$, the process for banks' aggregate equity $N_{b,t}$ is

$$\begin{aligned}
 \frac{dN_{b,t}}{N_{b,t}} &= \left(\mathbb{E}[dr_{f,t}] + \mathcal{T}_{b,t} - \gamma \left(\frac{\psi_t}{\eta_t} - 1 \right) i_t + \frac{\psi_t}{\eta_t} (\sigma_{ch,t} - \sigma_{\theta,t}) (\sigma + \sigma_{Q,t}) + \frac{\psi_t}{\eta_t} \gamma i_t \right) dt - \frac{d\zeta_{b,t}}{N_{b,t}} \\
 &\quad + \frac{\psi_t}{\eta_t} (\sigma + \sigma_{Q,t}) dW_t \\
 &= \left(\mathbb{E}[dr_{f,t}] + \mathcal{T}_{b,t} + \gamma i_t + \frac{\psi_t}{\eta_t} (\sigma_{ch,t} - \sigma_{\theta,t}) (\sigma + \sigma_{Q,t}) \right) dt - \frac{d\zeta_{b,t}}{N_{b,t}} + \frac{\psi_t}{\eta_t} (\sigma + \sigma_{Q,t}) dW_t.
 \end{aligned}$$

The law of motion for the value of the aggregate capital stock is

$$\frac{d(Q_t K_t)}{Q_t K_t} = (\mu_{Q,t} + \Phi(\iota_t) - \delta + \sigma \sigma_{Q,t}) dt + (\sigma + \sigma_{Q,t}) dW_t.$$

Because banks' asset pricing condition always holds in equilibrium, we may write

$$\mu_{Q,t} = \mathbb{E}[dr_{f,t}] + (\sigma_{ch,t} - \sigma_{\theta,t}) (\sigma + \sigma_{Q,t}) + \gamma i_t - \frac{a_b - \iota_t}{Q_t} - (\Phi(\iota_t) - \delta) - \sigma \sigma_{Q,t}.$$

Substituting into the law of motion for the value of capital

$$\begin{aligned}\frac{d(Q_t K_t)}{Q_t K_t} &= \left(\mathbb{E}[dr_{f,t}] + (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) + \gamma i_t - \frac{a_b - l_t}{Q_t} \right) dt \\ &\quad - ((\Phi(l_t) - \delta) - \sigma \sigma_{Q,t} + \Phi(l_t) - \delta + \sigma \sigma_{Q,t}) dt + (\sigma + \sigma_{Q,t}) dW_t \\ &= \left(\mathbb{E}[dr_{f,t}] + (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) + \gamma i_t - \frac{a_b - l_t}{Q_t} \right) dt + (\sigma + \sigma_{Q,t}) dW_t.\end{aligned}$$

By Ito's quotient rule,

$$\frac{d(1/(Q_t K_t))}{1/(Q_t K_t)} = \left((\sigma + \sigma_{Q,t})^2 - \mathbb{E}[dr_{f,t}] - (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) - \gamma i_t + \frac{a_b - l_t}{Q_t} \right) dt - (\sigma + \sigma_{Q,t}) dW_t.$$

Using Ito's product rule,

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \left(\mathbb{E}[dr_{f,t}] + \mathcal{T}_{b,t} + \gamma i_t + \frac{\psi_t}{\eta_t}(\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) \right) dt - \frac{d\zeta_{b,t}}{N_{b,t}} \\ &\quad + \left((\sigma + \sigma_{Q,t})^2 - \mathbb{E}[dr_{f,t}] - (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) - \gamma i_t + \frac{a_b - l_t}{Q_t} \right) dt \\ &\quad - \frac{\psi_t}{\eta_t}(\sigma + \sigma_{Q,t})^2 dt + \left(\frac{\psi_t}{\eta_t} - 1 \right) (\sigma + \sigma_{Q,t}) dW_t.\end{aligned}$$

The drift simplifies to

$$\mu_{\eta,t} = \left(\frac{\psi_t}{\eta_t} - 1 \right) ((\sigma_{ch,t} - \sigma_{\theta,t}) - (\sigma + \sigma_{Q,t}))(\sigma + \sigma_{Q,t}) + \frac{a_b - l_t}{Q_t} + \mathcal{T}_{b,t} - \frac{d\zeta_{b,t}}{N_{b,t}}.$$

Define $d\Xi_t \equiv d\zeta_{b,t}/N_{b,t}$ as a control creating an upper reflecting barrier. The drift and volatility of $d\eta_t/\eta_t$ now match (14). $\square$

For the remainder of the proofs, time subscripts are suppressed unless required for clarity.

Equilibrium ODE for Q and θ

Proof of Proposition 1, equilibrium ODE. To derive Lemma 1, we first conjectured that θ and Q are diffusions. We now verify these conjectures by applying Ito's lemma to write θ and Q as

functions of η . By Ito's lemma

$$\begin{aligned}\frac{dQ}{Q} &= \left(\frac{Q'}{Q} \mu_\eta \eta + \frac{1}{2} \frac{Q''}{Q} (\sigma_\eta \eta)^2 \right) dt + \frac{Q'}{Q} \sigma_\eta \eta dW_t, \\ \frac{d\theta}{\theta} &= \left(\frac{\theta'}{\theta} \mu_\eta \eta + \frac{1}{2} \frac{\theta''}{\theta} (\sigma_\eta \eta)^2 \right) dt + \frac{\theta'}{\theta} \sigma_\eta \eta dW_t.\end{aligned}$$

The boundary conditions for θ at η^* are straightforward. Banks' first-order condition for $d\zeta$ implies $\theta(\eta^*) = 1$. Since $d\Xi$ creates a reflecting barrier at η^* , a smooth-pasting condition applies at η^* . No arbitrage requires that

$$Q_t K_t = \mathbb{E}_t \left[\int_t^\infty \frac{\xi_\tau}{\xi_t} \text{Div}_\tau d\tau \right] = C_{h,t} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \frac{\text{Div}_s}{C_{h,\tau}} d\tau \right]$$

where Div_t are dividends from capital. Define $G_t \equiv Q_t K_t / C_{h,t}$. Then the previous equation simplifies to

$$G_t = \mathbb{E}_t \left[\int_t^\infty e^{-r(\tau-t)} \frac{\text{Div}_s}{C_{h,\tau}} d\tau \right],$$

which implies the equation

$$rG_t = \frac{\text{Div}_t}{C_{h,t}} + \mathbb{E}[dG_t].$$

Drop time subscripts to reduce clutter and plug in for household consumption.

$$G \equiv \frac{QK}{C_h} = \frac{QK}{rQ(1 + (\theta - 1)\eta)K} = \frac{1}{r(1 + (\theta - 1)\eta)}.$$

Smooth pasting requires

$$0 = \frac{\partial}{\partial \eta} G(\eta^*) = -G \frac{\theta'(\eta^*)\eta^* + \theta(\eta^*) - 1}{1 + (\theta(\eta^*) - 1)\eta^*}.$$

It follows that

$$1 = \theta'(\eta^*)\eta^* + \theta(\eta^*).$$

Since $\theta(\eta^*) = 1$, $\theta'(\eta^*) = 0$.

Market-clearing conditions pin down the behavior of Q . Aggregate household consumption is

$$r(N_h + \theta N_b) = r(1 - \eta + \theta\eta)QK.$$

Market-clearing for consumption requires

$$r(N_h + \theta N_b) + \iota K = (a_h + (a_b - a_h)\psi)K. \quad (30)$$

Using the identity $N_h = QK - N_b$, dividing both sides by K , and simplifying obtains

$$rQ(1 - \eta + \theta\eta) + \iota = a_h + (a_b - a_h)\psi. \quad (31)$$

When $\eta = 0$, households own all capital. The price of capital must satisfy

$$\underline{Q} = \frac{a_h - \iota(\underline{Q})}{r}, \quad (32)$$

where $\iota(\underline{Q})$ make explicit the dependence of ι on Q due to Tobin's Q . When $\eta = \eta^*$, $\theta(\eta^*) = 1$ implies that aggregate household consumption is

$$r(1 - \eta^* + \theta(\eta^*)\eta^*)Q = r(1 - \eta^* + \eta^*) = rQ.$$

Banks will also own all capital at η^* , hence

$$\overline{Q} = \frac{a_b - \iota(\overline{Q})}{r}. \quad (33)$$

To obtain the boundary condition for $Q'(\eta^*)$, notice that ψ will equal 1 over some interval $[\overline{\eta}, \eta^*]$,

where $\bar{\eta} < \eta^*$. Then the derivative of the market-clearing condition for capital with respect to η is

$$\begin{aligned} Q'(\eta^*)(1 - \eta^* + \theta(\eta^*)\eta^*) + Q(\eta^*)(\theta'(\eta^*)\eta^* + \theta(\eta^*) - 1) &= -\iota'(Q(\eta^*))Q'(\eta^*) \\ Q'(\eta^*)(1 - \eta^* + \eta^*) + Q(\eta^*)(0 + 1 - 1) &= -\iota'(Q(\eta^*))Q'(\eta^*) \\ Q'(\eta^*)(1 + \iota'(Q(\eta^*))) &= 0. \end{aligned}$$

The properties of the investment technology Φ imply $\iota'(Q(\eta^*)) \neq -1$, hence

$$Q'(\eta^*) = 0. \quad (34)$$

Finally, θ satisfies the boundary condition

$$\lim_{\eta \rightarrow 0^+} \theta(\eta) = \infty \quad (35)$$

because if a bank has capital when η is identically zero, then that bank can acquire arbitrarily high returns. In particular, $\eta = 0$ is an absorbing equilibrium. If η reaches 0, then Q will remain forever at $\underline{Q}$. Under the assumption that interest rates are sufficiently small near $\eta = 0$, a bank with an infinitesimal quantity of capital at $\eta = 0$ faces the excess returns

$$\frac{a_b - \iota}{\underline{Q}} + \Phi(\iota) - \delta - \mathbb{E}[dr_f] - LP > \frac{a_h - \iota}{\underline{Q}} + \Phi(\iota) - \delta - \mathbb{E}[dr_f] = \sigma_{ch}\sigma = \sigma^2.$$

Leverage does not reduce a bank's excess returns because the bank is infinitesimal. Therefore, the bank can borrow as much as it wants and obtain arbitrarily high returns, resulting in an infinite value function.

The boundary conditions for θ require it to be decreasing for at least a positive-measure subset of $[0, \eta^*]$. When the tax rate on bank equity T is sufficiently high, the equilibrium ODE for θ will satisfy $\theta'(\cdot) < 0$ and $\theta''(\cdot) > 0$ over $[0, \eta^*]$. $\square$

Equilibrium with Leverage Constraints

Solving equilibrium with leverage constraints. Suppose the government specifies an exogenous leverage constraint policy $L(\eta)$ and requires bank leverage to satisfy

$$\frac{\psi}{\eta} - 1 \leq L(\eta). \quad (36)$$

The leverage constraint applies to individual banks. Because the model admits a representative agent for banks, we directly specify the leverage constraint on the representative bank to economize on exposition. If leverage constraints bind, then banks' asset pricing condition (12) may not bind, which changes the law of motion for η . However, if banks are leverage constrained, then it must be the case that $\psi < 1$, hence households' asset pricing condition (11) must bind with equality. This condition implies

$$\frac{-l}{Q} + \Phi(l) - \delta + \mu_Q + \sigma\sigma_Q - \mathbb{E}[dr_f] = \sigma_{ch}(\sigma + \sigma_Q) - \frac{a_h}{Q},$$

so

$$\begin{aligned} \mathbb{E}[dr_b] - \mathbb{E}[dr_f] &= \frac{a_b}{Q} - \frac{l}{Q} + \Phi(l) - \delta + \mu_Q + \sigma\sigma_Q - dr_f \\ &= \frac{a_b - a_h}{Q} + \sigma_{ch}(\sigma + \sigma_Q) \end{aligned}$$

Banks' aggregate equity evolves according to

$$\begin{aligned} \frac{dN_{b,t}}{N_{b,t}} &= \left(\mathbb{E}[dr_{f,t}] + \mathcal{T}_{b,t} - \gamma \left(\frac{\psi_t}{\eta_t} - 1 \right) i_t + \frac{\psi_t}{\eta_t} \left(\frac{a_b - a_h}{Q_t} + \sigma_{ch,t}(\sigma + \sigma_{Q,t}) \right) \right) dt \\ &\quad - \frac{d\zeta_{b,t}}{N_{b,t}} + \frac{\psi_t}{\eta_t} (\sigma + \sigma_{Q,t}) dW_t. \end{aligned}$$

The aggregate wealth process $Q_t K_t$ also evolves differently when leverage constraints bind.

The law of motion is

$$\frac{d(Q_t K_t)}{Q_t K_t} = (\mu_{Q,t} + \Phi(l_t) - \delta + \sigma \sigma_{Q,t}) dt + (\sigma + \sigma_{Q,t}) dW_t,$$

and by Ito's quotient rule,

$$\frac{d(1/(Q_t K_t))}{1/(Q_t K_t)} = ((\sigma + \sigma_{Q,t})^2 - \mu_{Q,t} - (\Phi(l_t) - \delta) - \sigma \sigma_{Q,t}) dt - (\sigma + \sigma_{Q,t}) dW_t.$$

Households' asset pricing condition implies

$$-\mu_{Q,t} - (\Phi(l_t) - \delta - \sigma \sigma_{Q,t}) = \frac{a_h - l_t}{Q} - \mathbb{E}[dr_{f,t}] - \sigma_{ch,t}(\sigma + \sigma_Q),$$

hence

$$\frac{d(1/(Q_t K_t))}{1/(Q_t K_t)} = \left((\sigma + \sigma_{Q,t})^2 + \frac{a_h - l_t}{Q_t} - \mathbb{E}[dr_{f,t}] - \sigma_{ch,t}(\sigma + \sigma_{Q,t}) \right) dt - (\sigma + \sigma_{Q,t}) dW_t.$$

By Ito's product rule,

$$\begin{aligned} \frac{d\eta_t}{\eta_t} &= \left(\mathbb{E}[dr_{f,t}] + \mathcal{T}_{b,t} - \gamma \left(\frac{\psi_t}{\eta_t} - 1 \right) i_t + \frac{\psi_t}{\eta_t} \left(\frac{a_b - a_h}{Q_t} + \sigma_{ch,t}(\sigma + \sigma_Q) \right) \right) dt \\ &\quad - \frac{d\zeta_{b,t}}{N_{b,t}} + \frac{\psi_t}{\eta_t} (\sigma + \sigma_{Q,t}) dW_t \\ &\quad - \frac{\psi_t}{\eta_t} (\sigma + \sigma_{Q,t})^2 dt + \left((\sigma + \sigma_{Q,t})^2 + \frac{a_h - l_t}{Q_t} - dr_{f,t} - \sigma_{ch,t}(\sigma + \sigma_{Q,t}) \right) dt - (\sigma + \sigma_{Q,t}) dW_t \\ &= \left(\frac{\psi_t}{\eta_t} \frac{a_b}{Q_t} - \left(\frac{\psi_t}{\eta_t} - 1 \right) \frac{a_h}{Q_t} - \frac{l_t}{Q} + \mathcal{T}_{b,t} - \gamma \left(\frac{\psi_t}{\eta_t} - 1 \right) i_t \right) dt - \frac{d\zeta_{b,t}}{N_{b,t}} \\ &\quad + \left(\left(\frac{\psi_t}{\eta_t} - 1 \right) \sigma_{ch,t}(\sigma + \sigma_{Q,t}) - \left(\frac{\psi_t}{\eta_t} - 1 \right) (\sigma + \sigma_{Q,t})^2 \right) dt + \left(\frac{\psi_t}{\eta_t} - 1 \right) (\sigma + \sigma_{Q,t}) dW_t \\ &= \left(\frac{\psi_t}{\eta_t} \frac{a_b}{Q_t} - \left(\frac{\psi_t}{\eta_t} - 1 \right) \frac{a_h}{Q_t} - \frac{l_t}{Q} + \mathcal{T}_{b,t} - \gamma \left(\frac{\psi_t}{\eta_t} - 1 \right) i_t \right) dt - \frac{d\zeta_{b,t}}{N_{b,t}} \\ &\quad + \left(\frac{\psi_t}{\eta_t} - 1 \right) (\sigma_{ch,t} - (\sigma + \sigma_{Q,t})) (\sigma + \sigma_{Q,t}) dt + \left(\frac{\psi_t}{\eta_t} - 1 \right) (\sigma + \sigma_{Q,t}) dW_t. \end{aligned}$$

Thus, μ_η and σ_η when leverage constraints bind are given by

$$\mu_\eta = \frac{\psi}{\eta} \frac{a_b}{Q} - \left(\frac{\psi}{\eta} - 1 \right) \frac{a_h}{Q} - \frac{l}{Q} - \gamma \left(\frac{\psi}{\eta} - 1 \right) i + \mathcal{T}_b, \quad (37)$$

$$-d\Xi + \left(\frac{\psi}{\eta} - 1 \right) (\sigma_{ch} - (\sigma + \sigma_Q))(\sigma + \sigma_Q) \quad (38)$$

$$\sigma_\eta = \left(\frac{\psi}{\eta} - 1 \right) (\sigma + \sigma_Q). \quad (39)$$

Banks' Bellman equation also changes because they cannot obtain first-order optimality for their portfolio choice. Prior to substitution of the first-order condition for $x_{b,t}$, their Bellman equation is

$$0 = \mu_{\theta,t} - \mathcal{T}_{h,t} + \mathcal{T}_{b,t} - \sigma_{ch,t} \sigma_{\theta,t} + x_{b,t} (\mathbb{E}[dr_{b,t} - dr_{f,t}] + (\sigma_{\theta,t} - \sigma_{ch,t})(\sigma + \sigma_{Q,t}) - \gamma i_t) + \gamma i_t.$$

Using households' asset pricing condition, banks' Bellman equation becomes

$$\begin{aligned} & \mathcal{T}_{h,t} - \mathcal{T}_{b,t} - \gamma i_t + \sigma_{ch,t} \sigma_{\theta,t} \\ &= \mu_{\theta,t} + x_{b,t} \left(\frac{a_b - a_h}{Q_t} + \sigma_{ch,t}(\sigma + \sigma_{Q,t}) + (\sigma_{\theta,t} - \sigma_{ch,t})(\sigma + \sigma_{Q,t}) - \gamma i_t \right) \\ &= \mu_{\theta,t} + x_{b,t} \left(\frac{a_b - a_h}{Q_t} + \sigma_{\theta,t}(\sigma + \sigma_{Q,t}) - \gamma i_t \right). \end{aligned}$$

Re-arranging yields the new equation for μ_θ :

$$\mu_\theta = \mathcal{T}_h - \mathcal{T}_b - \gamma i + \sigma_{ch} \sigma_\theta - \frac{\psi}{\eta} \left(\frac{a_b - a_h}{Q} - (-\sigma_\theta(\sigma + \sigma_Q) + \gamma i) \right). \quad (40)$$

To summarize, when leverage constraints bind, μ_η , σ_η , and μ_θ are given by the equations derived above. The remaining quantities can be obtained using the same procedures as when banks are not constrained. Furthermore, when leverage constraints do not bind, μ_η , σ_η , and μ_θ are given by the exact same equations as before. Despite this fact, the global solution will change because the behavior of θ and Q in the regions of η for which the leverage constraint binds will be different, which affects their behavior elsewhere. $\square$

Welfare ODE/HJB

Derivation of the ODE for Welfare. To compute welfare, we solve the HJB for the representative household. Because households have log utility from consumption, the representative household's value function takes the form

$$V(W_h, \eta) = \frac{\log(W_h)}{r} + h(\eta).$$

Plugging in for household wealth, we obtain

$$V(K, \eta) = \frac{\log(QK(1 + (\theta - 1)\eta))}{r} + h(\eta) = \frac{\log(K)}{r} + \frac{\log(Q(1 + (\theta - 1)\eta))}{r} + h(\eta).$$

Define $H(\eta) = \log(Q(1 + (\theta - 1)\eta))/r + h(\eta)$. Then the value function simplifies to

$$V(K, \eta) = \frac{\log(K)}{r} + H(\eta).$$

The HJB is

$$\begin{aligned} r \left(\frac{\log(K)}{r} + H(\eta) \right) &= \log(C_h) - \frac{\lambda}{2}(\pi - \bar{\pi})^2 + E[dV] \\ \log(K) + rH &= \log(rQK(1 + (\theta - 1)\eta)) - \frac{\lambda}{2}(\pi - \bar{\pi})^2 + \mathbb{E}[dV] \\ rH &= \log(rQ(1 + (\theta - 1)\eta)) - \frac{\lambda}{2}(\pi - \bar{\pi})^2 + \mathbb{E}[dV]. \end{aligned}$$

It remains to characterize dV . By Ito's lemma, for any Ito process dX ,

$$d\log(X) = \frac{dX}{X} - \frac{(dX)^2}{2X^2}.$$

It follows that

$$\begin{aligned}\mathbb{E}[dV] &= \frac{1}{r}\mathbb{E}[d\log(K)] + \mathbb{E}[dH] \\ &= \frac{1}{r}\left(\Phi(Q) - \delta - \frac{1}{2}\sigma^2\right) + H'\mu_\eta\eta + \frac{1}{2}H''\sigma_\eta^2\eta^2.\end{aligned}$$

Plugging this quantity into the HJB yields a second-order differential equation for H :

$$\begin{aligned}rH &= \log(rQ(1 + (\theta - 1)\eta)) - \frac{\lambda}{2}(\pi - \bar{\pi})^2 + \frac{1}{r}\left(\Phi(Q) - \delta - \frac{1}{2}\sigma^2\right) \\ &\quad + H'\mu_\eta\eta + \frac{1}{2}H''\sigma_\eta^2\eta^2.\end{aligned}$$

This can be re-arranged into the explicit differential equation

$$H'' = \frac{2}{(\sigma_\eta\eta)^2}(rH - H'\mu_\eta\eta - \mathcal{J}),$$

where $\mathcal{J}$ is the inhomogeneous part of the differential equation

$$\mathcal{J} = \log(rQ(1 + (\theta - 1)\eta)) - \frac{\lambda}{2}(\pi - \bar{\pi})^2 + \frac{1}{r}\left(\Phi(Q) - \delta - \frac{1}{2}\sigma^2\right).$$

This differential equation is linear in H . There are multiple methods to solve linear differential equations; we opt for upwind finite differences because this approach simplifies the handling of boundary conditions. Because a reflecting barrier exists at η^* , we impose a reflecting boundary condition at $\eta = \eta^*$. Since the reflection of η at the barrier costs nothing to the household, the smooth-pasting condition is $V'(\eta^*) = H'(\eta^*) = 0$. We also impose a reflecting boundary condition at $\eta = 0$ so that $H'(0) = 0$ because the drift μ_η is strictly positive for η sufficiently close to zero, and the volatility σ_η vanishes as η approaches zero. $\square$

Friedman Rule/First Best

Proof of Proposition 2, Friedman Rule. When banks can freely issue equity, then equilibrium is stationary with constant asset price Q^* , constant capital allocation, and no endogenous risk or

instability, implying banks do not have a required risk premium. By Lemma 2, (28), and the fact that $\eta = 0$,

$$r_f = r + \Phi(\iota(Q^*)) - \delta - \sigma^2 - \psi LP.$$

Market-clearing for consumption requires

$$Q(\psi) = \frac{a_h + (a_b - a_h)\psi - \iota(Q(\psi))}{r}.$$

Note that we parameterize Q by ψ because it will be notationally convenient when deriving the optimal rate in the presence of inflation costs. The value of ψ is pinned down as follows:

$$\psi = \begin{cases} 1 & \text{if } a_b - a_h > LP \cdot Q(1) \\ \frac{rQ(\psi) + \iota(Q(\psi)) - a_h}{a_b - a_h} & \text{if } a_b - a_h = LP \cdot Q(\psi) \\ 0 & \text{if } a_b - a_h < LP \cdot Q(0). \end{cases}$$

In the absence of inflation costs, it is optimal for banks to hold all capital. Therefore, the generically optimal policy rule is the Friedman rule in the sense that, as long as $a_h \leq a_b$, $i = 0\%$ implements the first-best equilibrium.

□

Constrained Planner/Second Best

Proof of Proposition 3. One way to prove the proposition is to introduce an aggregate insurance asset with a cash flow process that is financed by taxes on household. We sketch the proof here, as the argument is essentially the same as in Brunnermeier and Sannikov (2014).

Assume that the planner controls the cash flow process on the insurance asset. Then the planner may endow the asset with any risk profile. By carefully choosing the risk profile, the planner can guarantee that the risk exposure of bank equity does not generate endogenous volatility ($\sigma_{\eta,t} = 0$). When endogenous volatility is zero, the economy becomes effectively static. Since inflation costs

are zero, it is optimal to adopt the Friedman rule because it guarantees that banks hold the entire capital stock and because banks are more productive than households. $\square$

Friedman Rule/First Best with Inflation Costs

Proposition 7. *In the first-best equilibrium with positive inflation costs, the optimal monetary policy rule can be characterized as follows. When inflation costs are sufficiently small and the productivity loss $a_b - a_h$ is sufficiently large, the optimal rule is given by*

$$i^* = \frac{r + \Phi(\iota(Q(1))) - \delta - \sigma^2 + \bar{\pi}}{1 + \gamma}, \quad Q(1) = \frac{a_b - \iota(Q(1))}{r}.$$

When inflation costs and/or the productivity loss are moderate, then the optimal rule is

$$i^* = \arg \max \log(rQ(\psi)) - \frac{\lambda}{2}(i - r - \Phi(\iota(Q(\psi))) + \delta + \sigma^2 + \psi\gamma i - \bar{\pi})^2 + \frac{1}{r}\Phi(\iota(Q(\psi)))$$

where $Q(\psi)$ and ψ satisfy

$$Q(\psi) = \frac{a_h + (a_b - a_h)\psi - \iota(Q(\psi))}{r}, \quad Q(\psi)\gamma i^* = a_b - a_h.$$

For large λ , the optimal rule is given by

$$\underline{i} = r + \Phi(\iota(Q(0))) - \delta - \sigma^2 + \bar{\pi}, \quad Q(0) = \frac{a_b - \iota(Q(0))}{r}.$$

Proof. With free equity issuance, $\theta = 1$. Then the household HJB is

$$\begin{aligned} rH &= \log(rQ(1 + (\theta - 1)\eta)) - \frac{\lambda}{2}(i - r_f - \bar{\pi})^2 + \frac{1}{r} \left(\Phi(\iota(Q)) - \delta - \frac{1}{2}\sigma^2 \right) + \mathbb{E}[dH] \\ &= \log(rQ) - \frac{\lambda}{2}(i - r_f - \bar{\pi})^2 + \frac{1}{r} \left(\Phi(\iota(Q)) - \delta - \frac{1}{2}\sigma^2 \right) + \mathbb{E}[dH]. \end{aligned}$$

Recognize that

$$H(\eta) = \frac{\log(Q(1 + (\theta - 1)\eta))}{r} + h(\eta) = \frac{\log(Q)}{r} + h(\eta).$$

The quantities $h(\eta)$ and Q are constant, so $\mathbb{E}[dH] = 0$. Further, $\eta = 0$, hence the risk-free rate is given by

$$r_f = r + \Phi(\iota(Q)) - \delta - \sigma^2 - \psi\gamma i.$$

To maximize household welfare, it suffices to solve

$$\max_i \log(rQ) - \frac{\lambda}{2}(i - r - \Phi(\iota(Q)) + \delta + \sigma^2 + \psi\gamma i - \bar{\pi})^2 + \frac{1}{r}\Phi(\iota(Q))$$

To determine the optimal interest rate, we need to consider three regions. For sufficiently small liquidity premia, Q does not depend on i . For intermediate liquidity premia, banks do not hold the entire capital stock. The level of Q adjusts until $(a_b - a_h) = Q \cdot LP$. For sufficiently large liquidity premia, households hold the entire capital stock, and Q again does not depend on i .

Consider the first region. Assuming it is optimal to be in this region, the optimal nominal rate is

$$\bar{i} = \frac{r + \Phi(\iota(Q(1))) + \delta - \sigma^2 + \bar{\pi}}{1 + \gamma}.$$

This rate is optimal as long as

$$\bar{i} \leq \frac{a_b - a_h}{\gamma Q(1)}. \quad (41)$$

If (41) does not hold, then we are in the second region. In this case, the price of capital and the

allocation of capital satisfy

$$Q(\psi) = \frac{a_h + (a_b - a_h)\psi - \iota(Q(\psi))}{r}$$

$$Q(\psi)\gamma i = a_b - a_h.$$

If the required ψ falls below zero, then we are in the third region, in which case the optimal rate must be

$$\underline{i} = r + \Phi(\iota(Q(0))) + \delta - \sigma^2 + \bar{\pi}.$$

□

Functional Form for Investment

We choose the functional form for Φ of

$$\Phi = \delta + \frac{1}{\varepsilon} \log(\varepsilon \iota + 1 - \varepsilon \delta)$$

because a second order Taylor expansion of the implied adjustment costs coincides exactly with quadratic adjustment costs. To see this, consider the required investment ι to obtain I units of new capital.

$$I = \delta + \frac{1}{\varepsilon} \log(\varepsilon \iota + 1 - \varepsilon \delta)$$

$$\exp(\varepsilon(I - \delta)) = \varepsilon \iota + 1 - \varepsilon \delta$$

$$\iota(I) = \frac{\exp(\varepsilon(I - \delta)) - (1 - \varepsilon \delta)}{\varepsilon}.$$

Take a second-order Taylor approximation around $I = \delta$.

$$\iota(I) \approx \frac{\exp(0) - 1 + \varepsilon \delta}{\varepsilon} + \frac{\exp(0)\varepsilon}{\varepsilon} (I - \delta) + \frac{1}{2} \frac{\exp(0)\varepsilon^2}{\varepsilon} (I - \delta)^2 = \delta + (I - \delta) + \frac{\varepsilon}{2} (I - \delta)^2.$$

He and Krishnamurthy (2019) specify quadratic adjustment costs as

$$I + \frac{\chi}{2}(I - \delta)^2 = \delta + (I - \delta) + \frac{\chi}{2}(I - \delta)^2.$$

Thus, their choice of χ is precisely our ε , up to a second order approximation.

Another approach to calibrate ε is targeting the average elasticity of the price to investment rate ι . Tobin's Q implies

$$\begin{aligned} Q &= \varepsilon \iota + 1 - \varepsilon \delta \\ \log(Q) &= \log(\varepsilon \iota + 1 - \varepsilon \delta) \\ \frac{d \log(Q)}{d \log(\iota)} &= \frac{1}{\varepsilon \exp(\log(\iota)) + 1 - \varepsilon \delta} \varepsilon \exp(\log(\iota)) = \frac{\varepsilon \iota}{\varepsilon \iota + 1 - \varepsilon \delta}. \end{aligned}$$

We now derive the implications of the functional form choice for equilibrium. The first derivative of the investment function is

$$\Phi'(\iota) = \frac{1}{\varepsilon} \frac{1}{\varepsilon \iota + 1 - \varepsilon \delta} \varepsilon = \frac{1}{\varepsilon \iota + 1 - \varepsilon \delta}.$$

By Tobin's Q,

$$\iota(Q) = \frac{Q - (1 - \varepsilon \delta)}{\varepsilon}. \quad (42)$$

Plug (42) into (31) and re-arrange to acquire

$$Q = \frac{a_h + (a_b - a_h)\psi + \frac{1 - \varepsilon \delta}{\varepsilon}}{r(1 - \eta + \theta \eta) + \frac{1}{\varepsilon}}. \quad (43)$$

Constrained Regulator Problem

Proof of Proposition 4. Before beginning the proof, we explain the naming of the two channels. The first term in (16) is named the “Asset Price Level Channel” because it would vanish if we ignored the effect of lower bank leverage on the asset price level. The second term in (16) is named

the “Systemic Risk Channel” because it is proportional to the derivative of σ_η , which represents systemic risk, to bank leverage. Although the asset price level influences the magnitude of systemic risk in general, at the barrier η^* the derivative of σ_η with respect to ψ is the same whether or not we account for the effect of leverage on $Q(\eta^*)$.

Recall that households’ value function takes the form

$$V(K, \eta) = \frac{1}{r} \log(QK(1 + (\theta - 1)\eta)) + h(\eta) = \frac{\log(K)}{r} + H(\eta).$$

Define the marginal social value of aggregate bank leverage as

$$\mathcal{L}(\psi | \eta) \equiv \frac{\partial}{\partial \psi} V(K, \eta).$$

We subsume K as an argument because it is irrelevant to our analysis.

To make the proof easier to follow, we note which objects have zero derivatives with respect to ψ . Since K and η are state variables, they have a zero derivative with respect to ψ . Because θ does not directly depend on the local value of ψ and is instead determined by the global behavior of ψ , the derivative of θ with respect to ψ at a specific η is also zero. In contrast, the consumption market-clearing condition (see Appendix A.2.1 for the derivation)

$$Q = \frac{(a_b - a_h)\psi + a_h + \frac{1-\varepsilon\delta}{\varepsilon}}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}$$

implies a nonzero derivative of Q to ψ given some η . Finally, at $\eta = \eta^*$, any equilibrium in which banks freely issue dividends without distortions (e.g. taxes or subsidies) must satisfy the following boundary conditions:

- $\partial V(K, \eta^*) / \partial \eta = Q'(\eta^*) = \theta'(\eta^*) = 0$,
- $\theta(\eta^*) = 1$.

As argued previously, these conditions imply $H'(\eta^*) = 0$. Since these boundary conditions must hold regardless of ψ , the partial derivatives of $Q'(\eta^*)$ and $H'(\eta^*)$ with respect to ψ are also zero.

Given these observations, the marginal social value of aggregate bank leverage satisfies

$$\mathcal{L}(\psi | \eta) = \frac{\partial}{\partial \psi} H.$$

To sign $\partial H(\eta^*)/\partial \psi$, we turn to the HJB for the value function, which simplifies to

$$rH(\eta) = \log(rQ(1 + (\theta - 1)\eta)) + \mathbb{E}[dV].$$

By Ito's lemma, the HJB becomes

$$rH(\eta) = \log(rQ) + \log(1 + (\theta - 1)\eta) + \frac{1}{r} \left(\Phi(Q) - \delta - \frac{1}{2}\sigma^2 \right) + H'(\eta)\mu_\eta\eta + \frac{1}{2}H''(\eta)(\sigma_\eta\eta)^2.$$

Set $\eta = \eta^*$. Since $V'(\eta^*) = 0$ is a boundary condition and implies $H'(\eta^*) = 0$ and since $H''(\eta)$ depends only on the global behavior of ψ , the derivative of the previous expression with respect to ψ is

$$\begin{aligned} r \frac{\partial}{\partial \psi} H(\eta^*) &= \frac{1}{Q(\eta^*)} \frac{\partial}{\partial \psi} Q(\eta^*) + \frac{1}{1 + (\theta(\eta^*) - 1)\eta^*} \frac{\partial}{\partial \psi} (1 + (\theta(\eta^*) - 1)\eta^*) \\ &\quad + \frac{1}{r} \Phi'(Q(\eta^*)) \frac{\partial}{\partial \psi} Q(\eta^*) + H''(\eta^*)(\sigma_{\eta^*}\eta^*) \frac{\partial}{\partial \psi} [\sigma_{\eta^*}\eta^*]. \end{aligned}$$

The notation σ_{η^*} denotes the fact that this quantity is evaluated at η^* . In this analysis, because we differentiate with respect ψ , we implicitly treat ψ as an additional state variable and evaluate $\psi(\eta^*)$ *after* the differentiation.

Because Tobin's Q requires $\Phi'(Q) = 1/Q$, $\psi(\eta^*) = 1$, and $\theta(\eta^*) = 1$, the derivative simplifies to

$$r \frac{\partial}{\partial \psi} H(\eta^*) = \frac{1+r}{r} \left(\frac{1}{Q(\eta^*)} \frac{\partial}{\partial \psi} Q(\eta^*) \right) + H''(\eta^*)(\sigma_{\eta^*}\eta^*) \frac{\partial}{\partial \psi} [\sigma_{\eta^*}\eta^*].$$

Market-clearing for consumption implies

$$Q(\eta^*) = \frac{a_b + \frac{1-\varepsilon\delta}{\varepsilon}}{r + \frac{1}{\varepsilon}}$$

$$\frac{\partial}{\partial \psi} Q = \frac{a_b - a_h}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}} \Rightarrow \frac{\partial}{\partial \psi} Q(\eta^*) = \frac{a_b - a_h}{r + \frac{1}{\varepsilon}}.$$

Recall that

$$\begin{aligned}\sigma_\eta \eta &= (\psi - \eta)(\sigma + \sigma_Q) \\ &= (\psi - \eta) \left(\sigma + \frac{Q'(\eta)(\psi - \eta)}{Q(\eta) - Q'(\eta)(\psi - \eta)} \sigma \right) \\ &= (\psi - \eta) \left(\frac{(Q(\eta) - Q'(\eta)(\psi - \eta)) + Q'(\eta)(\psi - \eta)}{Q(\eta) - Q'(\eta)(\psi - \eta)} \right) \sigma \\ &= \frac{Q(\eta)(\psi - \eta)}{Q(\eta) - Q'(\eta)(\psi - \eta)} \sigma.\end{aligned}$$

Set $\eta = \eta^*$ and differentiate with respect to ψ to acquire

$$\begin{aligned}&\frac{\partial}{\partial \psi} [\sigma_{\eta^*} \eta^*] \\ &= \sigma \left(\frac{(\psi - \eta^*) \frac{\partial}{\partial \psi} Q(\eta^*) + Q(\eta^*)}{Q(\eta^*) - Q'(\eta^*)(\psi - \eta^*)} - \frac{Q(\eta^*)(\psi - \eta^*)}{(Q(\eta^*) - Q'(\eta^*)(\psi - \eta^*))^2} \left(\frac{\partial}{\partial \psi} Q(\eta^*) - Q'(\eta^*) \right) \right) \\ &= \sigma \left(1 + \frac{\psi - \eta^*}{Q(\eta^*)} \frac{\partial}{\partial \psi} Q(\eta^*) - \frac{\psi - \eta^*}{Q(\eta^*)} \frac{\partial}{\partial \psi} Q(\eta^*) \right) \\ &= \sigma.\end{aligned}$$

Thus, the derivative of $\sigma_\eta \eta$ with respect to ψ does not depend on $\partial Q / \partial \psi$, and we obtain

$$\begin{aligned}r \frac{\partial}{\partial \psi} H(\eta^*) &= \frac{1+r}{r} \frac{r + \frac{1}{\varepsilon}}{a_b + \frac{1-\varepsilon\delta}{\varepsilon}} \frac{a_b - a_h}{r + \frac{1}{\varepsilon}} + H''(\eta^*)(1 - \eta^*)\sigma^2 \\ &= \frac{1+r}{r} \frac{a_b - a_h}{a_b + \frac{1-\varepsilon\delta}{\varepsilon}} + H''(\eta^*)(1 - \eta^*)\sigma^2.\end{aligned}$$

Dividing both sides by r yields (16).

We now prove that the Systemic Risk Channel is unambiguously negative. Remember that

$$rH = \frac{(\sigma_\eta \eta)^2}{2} H'' + H' \eta \mu_\eta + \mathcal{J},$$

$$\mathcal{J} = \log(rQ(1 + (\theta - 1)\eta)) + \frac{1}{r} \left\{ \Phi(Q) - \delta - \frac{1}{2} \sigma^2 \right\}$$

At η^* , the second-order ODE in H becomes

$$rH(\eta^*) = \frac{(\sigma_{\eta^*} \eta^*)^2}{2} H''(\eta^*) + \log(rQ(\eta^*)) + \frac{1}{r} \left(\Phi(Q(\eta^*)) - \delta - \frac{1}{2} \sigma^2 \right)$$

$$H(\eta^*) = \frac{1}{r} \frac{(\sigma_{\eta^*} \eta^*)^2}{2} H''(\eta^*) + \left(\frac{1}{r} \log(rQ(\eta^*)) + \frac{1}{r^2} \left(\Phi(Q(\eta^*)) - \delta - \frac{1}{2} \sigma^2 \right) \right)$$

Notice that the second parenthetical term is the H in the household value function in the first-best equilibrium. In that equilibrium, the economy is static except for capital growth.

$$\log(Q^{(fb)}) + rh^{(fb)} = \log(rQ^{(fb)}) + \frac{1}{r} \left(\Phi(Q^{(fb)}) - \delta - \frac{1}{2} \sigma^2 \right)$$

$$h^{(fb)} = \frac{1}{r} \log(r) + \frac{1}{r^2} \left(\Phi(Q^{(fb)}) - \delta - \frac{1}{2} \sigma^2 \right)$$

$$H^{(fb)} = \frac{\log(Q^{(fb)})}{r} + h^{(fb)} = \frac{1}{r} \log(rQ^{(fb)}) + \frac{1}{r^2} \left(\Phi(Q^{(fb)}) - \delta - \frac{1}{2} \sigma^2 \right).$$

Because households have log utility, consumption market-clearing and our assumption that $\psi(\eta^*) = 1$ imply that $Q(\eta^*) = Q^{(fb)}$. Since $V^{(fb)}(K) = \log(K)/r + H^{(fb)}$, $V(K, \eta) = \log(K)/r + H(\eta)$, and since household welfare must be strictly lower in our model's competitive equilibrium than in the first best, it must be the case that $H^{(fb)} > H(\eta^*)$, hence

$$0 > \frac{(\sigma_{\eta^*} \eta^*)^2}{2} H''(\eta^*) = \frac{((1 - \eta^*)\sigma)^2}{2} H''(\eta^*)$$

This inequality establishes that the systemic risk channel is negative.

For the second claim, the inequality (17) immediately follows from re-arranging (16). $\square$

A.2 Numerical Algorithm

In this section, we show that equilibrium can be characterized as a system of differential equations for θ and Q . To handle the endogeneity of the state space boundary η^* , we reformulate the boundary value problem as a least-squares problem. For this section, we suppress time subscripts unless required for clarity.

A.2.1 No Leverage Constraints

From Proposition 6, the drift of θ is $\mu_\theta = T/(1 - \eta) - \gamma i + \sigma_{ch}\sigma_\theta$. Since θ decreases with η , a smaller drift of θ implies that θ , in expectation, moves toward larger values and bad times at a slower rate, reflecting the partial equilibrium intuition that higher interest rates should disincentivize excessive risk-taking. By Ito's lemma, we also have that

$$\mu_{\theta,t} = \frac{\theta'}{\theta} \eta_t \mu_{\eta,t} + \frac{1}{2} \frac{\theta''}{\theta} \eta_t^2 \sigma_{\eta,t}^2.$$

Setting these two equations equal to each other yields a second-order ODE in η_t for θ .

We obtain a differential equation for Q using the asset pricing condition (13). When $\psi = 1$, (13) is slack, and Q' can be obtained by directly differentiating (43) with respect to η :

$$\begin{aligned} Q &= \frac{a_h + (a_b - a_h)\psi + \frac{1-\varepsilon\delta}{\varepsilon}}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}} \\ \Rightarrow Q' &= -\frac{Q}{r(1 - \eta + \theta\eta) + \frac{1}{\varepsilon}} r(-1 + \theta'\eta + \theta) \\ &= \frac{rQ(\theta'\eta + (\theta - 1))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}. \end{aligned}$$

When $\psi < 1$, (13) binds with equality and can be re-arranged to

$$\sigma_Q = \sqrt{\frac{\frac{a_b - a_h}{Q} - LP}{-\frac{\theta'}{\theta}(\psi - \eta)}} - \sigma. \quad (44)$$

Recall that

$$\sigma_\eta = \left(\frac{\psi}{\eta} - 1 \right) (\sigma + \sigma_Q).$$

By Ito's lemma,

$$\sigma_Q = \frac{Q'}{Q} \sigma_\eta \eta,$$

hence

$$Q' = \frac{Q \sigma_Q}{\sigma_\eta \eta}.$$

Therefore, equilibrium is characterized by a solution to the system of differential equations

$$Q' = \begin{cases} \frac{Q \sigma_Q}{\sigma_\eta \eta} & \text{if } \psi < 1, \\ -\frac{rQ(\theta' \eta + (\theta - 1))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}} & \text{if } \psi = 1, \end{cases} \quad (45)$$

$$\theta'' = \frac{2}{(\sigma_\eta \eta)^2} (\theta \mu_\theta - \theta' \mu_\eta \eta), \quad (46)$$

where $\psi \leq 1$ and can be calculated from Q using (43).

A.2.2 Leverage Constraints

To implement leverage constraints numerically, during integration of θ and Q , we add a check for whether the leverage constraint $L(\eta)$ is violated by the ψ consistent with the proposed θ and Q according to market-clearing for consumption. If ψ exceeds the leverage constraint, then we set $\psi = \eta(L(\eta) + 1)$ and calculate the derivative Q' by directly differentiating the market-clearing

condition for consumption instead of using banks' asset pricing condition.

$$Q = \frac{a_h + (a_b - a_h)\eta(L(\eta) + 1) + \frac{1-\varepsilon\delta}{\varepsilon}}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}$$

$$\Rightarrow Q' = \frac{(a_b - a_h)(L(\eta) + 1 + \eta L'(\eta))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}} - \frac{rQ(\theta'\eta + (\theta - 1))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}.$$

Given sufficiently small step sizes, continuity of the numerical solutions guarantee that the approximations to θ and Q obey the leverage constraint.

A specific leverage constraint of interest is the incentive compatible leverage constraint used by [Van der Ghote \(2021\)](#). We assume each household delegate management of their bank to another agent, and banks cannot accept deposits from their owning household. This delegation is subject to a limited enforcement friction. The agent managing a bank can divert a portion $1/\alpha$ of their assets immediately after raising deposits from households. Should an agent do so, then that agent's bank immediately shuts down but they keep the assets they stole. As a result, a bank's portfolio choice has the following constraint:

$$\frac{1}{\alpha}x_b n_b \leq \theta n_b$$

The left-hand side is the value of the assets a bank manager could steal, and the right-hand side is the bank's value if the bank manager continues to operate the bank. After simplification, this constraint becomes

$$x_b \leq \alpha \theta.$$

This inequality asserts that bank leverage cannot exceed a multiple of the marginal value of a bank's equity. Otherwise, the bank manager would strictly benefit by diverting assets and closing down the bank.

For our model, this form of the constraint causes numerical problems. We instead consider the

following variant:

$$x_b \leq \alpha\theta + 1,$$

which implies

$$L \leq \alpha\theta.$$

A.2.3 Least-Squares Problem

The boundary conditions for Q at $\eta = 0$ and η^* can be calculated directly from the model's parameters. The boundary condition for θ at η^* is also known. Since $\lim_{\eta \rightarrow 0^+} \theta(\eta) = \infty$, it must be the case that $\lim_{\eta \rightarrow 0^+} \theta'(\eta) = -\infty$. Due to the singularity at $\eta = 0$, we solve over a grid slightly perturbed away from 0 to some $\underline{\eta}$, so we have to guess $\theta(\underline{\eta})$ and $\theta'(\underline{\eta})$. In practice, the choice of $\theta'(\underline{\eta})$ does not affect global dynamics as long as it is a sufficiently negative number. We fix $\theta'(\underline{\eta})$ at some guess and treat $\theta(\underline{\eta})$ as a control in a least-squares problem. Given $\theta(\underline{\eta})$ and $\theta'(\underline{\eta})$, we can integrate the equilibrium system of differential equations as if it is an initial-value problem with the boundary conditions at η^* as terminating events:

1. $\theta(\eta) < 1$,
2. $\theta'(\eta) > 0$,
3. $Q(\eta) > \bar{Q}$.

The first and third terminating events imply each other when we have found a solution, but in practice, omitting the third boundary condition makes the numerical algorithm perform better.

Our residual function is just a weighted sum of the squared errors in the boundary conditions:

$$F(\theta(\underline{\eta})) = w^T \begin{pmatrix} \theta(\eta^*) - 1 \\ \theta'(\eta^*) \\ Q - \bar{Q} \end{pmatrix} w,$$

where w is chosen to renormalize the magnitude of the errors to be more similar. With this residual function and an initial guess for $\theta(\underline{\eta})$, we can use any standard algorithm for least-squares minimization, such as the interior-point method or sequential quadratic programming, both of which are available through MATLAB's `fmincon` function. To reduce computational time, we start with a bisection search to find an interval in which $\theta(\underline{\eta})$ should belong. If the minimizing $\theta(\underline{\eta})$ implies a residual function sufficiently close to zero, then a solution has been found.

To summarize:

1. Select $\underline{\eta}$ close to zero, set $\theta'(\underline{\eta})$ to a very negative number (e.g. -5×10^4), and guess $\theta(\underline{\eta})$.
2. Integrate the differential equations for θ and Q until one of the terminating events occurs:
 - $\theta(\eta) < 1$,
 - $\theta'(\eta) > 0$.
3. Calculate the residuals at the terminating η .
4. Use a least-squares solver to find a minimizing $\theta(\underline{\eta})$. If the residual are sufficiently close to zero, then a solution has been found.

A.3 Additional Derivations

A.3.1 Consumption-Equivalent Welfare

To calculate welfare changes in consumption equivalents, define welfare under a baseline scenario as

$$V^{(0)} = \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\log(C_{h,t}^{(0)}) - \frac{\lambda}{2} (\pi_t^{(0)})^2 \right) dt \right],$$

where $C_{h,t}^{(0)}$ is the equilibrium consumption policy. Define $V^{(1)}$ similarly, and let it denote household welfare under an alternative scenario.

Suppose we want to compute how much consumption must change in order for households to obtain welfare of $V^{(1)}$ while facing the same prices and allocations (except for consumption) in the

baseline scenario. Then the consumption-equivalent increase in welfare is the quantity ϕ^+ , which satisfies

$$\begin{aligned}
 V^{(1)} &= \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\log((1 + \phi^+)C_{h,t}^{(0)}) - \frac{\lambda}{2}(\pi_t^{(0)})^2 \right) dt \right] \\
 &= \mathbb{E}_0 \left[\int_0^\infty e^{-rt} (\log(1 + \phi^+) + \log(C_{h,t}^{(0)})) - \frac{\lambda}{2}(\pi_t^{(0)})^2 dt \right] \\
 &= \log(1 + \phi^+) \int_0^\infty e^{-rt} dt + V^{(0)} \\
 &= -\frac{\log(1 + \phi^+)}{r} e^{-rt} \Big|_0^\infty + V^{(0)} \\
 &= \frac{\log(1 + \phi^+)}{r} + V^{(0)},
 \end{aligned}$$

Re-arranging yields

$$\phi^+ = \exp(r(V^{(1)} - V^{(0)})) - 1.$$

If we want to include differences in the convenience yield and inflation costs as part of the welfare calculation, then

$$\phi^+ = \exp(r(V^{(1)} - V^{(0)})) - 1.$$

Similarly, if household welfare decreases under the alternative scenario and ϕ^- is the consumption-equivalent *decrease*, then

$$\phi^- = 1 - \exp(r(V^{(1)} - V^{(0)})).$$

Thus, the consumption-equivalent difference in welfare is

$$\phi = |\exp(r(V^{(1)} - V^{(0)})) - 1|. \quad (47)$$

By linearity of expectations, we can also use (48) to compute the consumption-equivalent *ex ante*

welfare change.

It is also useful to consider how much consumption must change if we neglect the difference in inflation costs. Toward this end, define

$$V_c = \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \log(C_{h,t}) dt \right],$$

$$V_\pi = \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \frac{\lambda}{2} \pi_t^2 dt \right],$$

and let ϕ_c denote the consumption-equivalent difference when ignoring differences in inflation costs. Then

$$V^{(1)} = \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\log((1 + \phi_c) C_{h,t}^{(0)}) - \frac{\lambda}{2} (\pi_t^{(1)})^2 \right) dt \right]$$

$$V^{(1)} + V_\pi^{(1)} = \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\log(1 + \phi_c) + \log(C_{h,t}^{(0)}) - \frac{\lambda}{2} (\pi_t^{(0)})^2 \right) dt \right] + \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \frac{\lambda}{2} (\pi_t^{(0)})^2 dt \right]$$

$$V_c^{(1)} = \log(1 + \phi_c) \int_0^\infty e^{-rt} dt + V^{(0)} + V_\pi^{(0)}$$

$$= \frac{\log(1 + \phi_c)}{r} + V_c^{(0)}.$$

Therefore, by the same reasoning as before,

$$\phi_c = |\exp(r(V_c^{(1)} - V_c^{(0)})) - 1|. \quad (48)$$

To compute V_π , note that it satisfies

$$rV_\pi(\eta) = \frac{\lambda}{2} \pi(\eta)^2 + \mathbb{E}[dV_\pi] = \frac{\lambda}{2} \pi(\eta)^2 + V_\pi'(\eta) \mu_\eta \eta + \frac{1}{2} V_\pi''(\sigma_\eta \eta)^2. \quad (49)$$

We use finite differences to solve this second-order differential equation.

A.3.2 Stationary Distribution

If a stationary distribution exists, then the stationary probability density function $g(\eta)$ solves a Kolmogorov forward equation. Following [Brunnermeier and Sannikov \(2014\)](#), we simplify the

Kolmogorov forward equation to a first-order ODE, which can be solved as an initial value problem and renormalized to satisfy $\int_0^{\eta^*} g(\eta) d\eta = 1$.

A.3.3 Equity Injections

Suppose the government injects equity into the financial sector by establishing a reflecting barrier at $\underline{\eta}$. To fund equity injections, the government forcibly redistributes wealth from households to banks. Following [Brunnermeier and Sannikov \(2014\)](#), we choose the boundary condition

$$\theta'(\underline{\eta})\underline{\eta} + \theta(\underline{\eta}) = 0.$$

To obtain this condition, assume that per dollar of wealth banks receive, agents holding a piece of capital pay a tax τ_{TR} given by

$$\tau_{TR} = \frac{1}{r(1 + (\theta - 1)\eta)^2}.$$

These taxes are then redistributed back to banks and households. Because of this tax, the smooth-pasting condition for Q at $\underline{\eta}$ is

$$\tau_{TR} = \frac{\partial}{\partial \eta} G(\underline{\eta}) = -\frac{\theta'(\underline{\eta})\underline{\eta} + \theta(\underline{\eta}) - 1}{r(1 + (\theta(\underline{\eta}) - 1)\underline{\eta})^2},$$

where G was previously defined as QK/C_h . This condition simplifies to

$$0 = \theta'(\underline{\eta})\underline{\eta} + \theta(\underline{\eta}).$$

Smooth-pasting for banks requires

$$\begin{aligned} 0 &= \frac{\partial}{\partial \eta} \theta \underline{\eta} QK = (Q'(\underline{\eta})\theta(\underline{\eta})\underline{\eta} + Q(\underline{\eta})(\theta'(\underline{\eta})\underline{\eta} + \theta(\underline{\eta})))K \\ &\Rightarrow Q'(\underline{\eta}) = 0. \end{aligned}$$

We can still reformulate the boundary value problem as a least-squares problem. The differ-

ential equations can also still be explicitly integrated. Given a guess for $\theta(\underline{\eta})$, we can calculate $\theta'(\underline{\eta})$, so we have the right initial values for the θ ODE. For Q , we have the first derivative rather than $Q(0)$. Because $Q'(\underline{\eta}) = 0$, $\sigma_Q(\underline{\eta}) = 0$. The volatility of θ becomes

$$\sigma_\theta(\underline{\eta}) = \frac{\theta'(\underline{\eta})}{\theta(\underline{\eta})}(\psi - \eta)(\sigma + \sigma_Q(\underline{\eta})) = \frac{\theta'(\underline{\eta})}{\theta(\underline{\eta})}(\psi - \eta)\sigma.$$

Therefore, the value of $Q(\underline{\eta})$ consistent with $Q'(\underline{\eta}) = 0$ is

$$Q(\underline{\eta}) = \frac{a_b - a_h}{-\sigma_\theta(\underline{\eta})\sigma + \gamma i(\underline{\eta})}.$$

Market-clearing for consumption gives

$$Q \left(r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon} \right) = (a_b - a_h)\psi + a_h + \frac{1}{\varepsilon}.$$

Since $\theta(\underline{\eta})$ and $\underline{\eta}$ are taken as given, these two equations yield two equations for the two unknown $Q(\underline{\eta})$ and $\psi(\underline{\eta})$. By solving these two equations, we obtain the initial values $(\theta(\underline{\eta}), \theta'(\underline{\eta}), Q(\underline{\eta}))$.

B A Model of Monetary Policy Transmission

We microfound the relationship between the nominal interest rate and the liquidity premium on bank deposits using the model of monetary policy transmission from [Drechsler et al. \(2018\)](#).

B.1 Monetary Policy: Nominal Rates, Inflation, and Liquidity Premia

Following [Drechsler et al. \(2018\)](#), monetary policy determines the opportunity cost of holding liquid assets—namely, central bank reserves—rather than capital (i.e., determines the liquidity premium). To capture the money multiplier of reserves, each dollar of reserves yields $m > 1$ effective liquid assets. The central bank can create and withdraw reserves by exchanging government bonds through open market operations (see [Drechsler et al. \(2018\)](#) for implementation details).

Let M_t be the total dollar value of reserves in the economy, and let s_t be the value of a dollar in

consumption units. Letting reserves be the numeraire, s_t becomes the inverse price level, and the real value of liquid assets held by banks scaled by aggregate wealth is

$$S_t = \frac{s_t(m-1)M_t}{Q_t k_t}.$$

The remaining liquidity is given by the value of government bonds held by the central bank, $s_t M_t$.

We assume inflation is locally deterministic, i.e.

$$-\frac{ds_t}{s_t} = \frac{dp_t}{p_t} = \pi_t dt.$$

Liquidity Premia The liquidity premium on reserves is the opportunity cost of holding liquid assets. Because reserves pay no interest, their return is equal to their capital gain, so the liquidity premium that deposits earn is

$$dr_{f,t} - \frac{ds_t}{s_t} = dr_{f,t} + \pi_t = i_t,$$

which is precisely the nominal interest rate.

The government earns seigniorage from the liquid assets held by banks. To close the model, we assume the seigniorage is distributed to agents according to their wealth, following [Drechsler et al. \(2018\)](#), and the government maintains zero net worth.

B.2 Banks' Problem

Following [Drechsler et al. \(2018\)](#) we assume deposits are subject to funding shocks, which are modeled as a Poisson process J_t with constant intensity χ . J_t is an aggregate shock, and when J_t realizes, banks must immediately redeem a fraction $\frac{\kappa}{1+\kappa}$ of their deposits, where $\kappa > 0$. Only a fraction $1 - \rho \in (0, 1)$ of capital's value can be recovered quickly enough to absorb a funding shock. Banks may self-insure by holding liquid assets, which can be liquidated without causing fire sales when a funding shock realizes.

Let $(x_{b,t}, x_{M,t}, x_{bd,t})$ be portfolio weights on capital, reserves, and debt. Since the portfolio

weights sum to one, we rewrite $x_{bd,t} = 1 - x_{b,t} - x_{M,t}$. Formally, banks solve the problem

$$\max_{\{x_{b,t}, x_{M,t}\}} U_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty \frac{\xi_t}{\xi_\tau} d\zeta_t \right],$$

subject to

$$\begin{aligned} \frac{dn_{b,t}}{n_{b,t}} = & dr_{f,t} + (\mathcal{T}_{b,t} - x_{M,t}i_t) dt + x_{b,t}(dr_t^b - dr_{f,t}dt) - \frac{d\zeta_{b,t}}{n_{b,t}} \\ & - \frac{\rho}{1-\rho} \max \left\{ \frac{\kappa}{1+\kappa} (x_{b,t} + mx_{M,t} - 1) - mx_{M,t}, 0 \right\} dJ_t, \end{aligned} \quad (50)$$

$$n_{b,t}, x_{b,t}, x_{M,t} d\zeta_t \geq 0. \quad (51)$$

Banks earn the deposit rate, pay the liquidity premium on reserves, receive exogenous transfers, earn the risk premium on capital holdings, and pay dividends at rate $d\zeta_t$. The second line of (50) reflects the exposure of banks to funding shocks given their portfolio. Note that, unlike [Drechsler et al. \(2018\)](#), we abstract from government bonds and only include central bank reserves. (For the equivalent net worth equation, see (A7) in the appendix of [Drechsler et al. \(2018\)](#) and note that $x_{M,t}$ in our paper is w_M in their paper. Since we do not have government reserves, $w_G \equiv 0$ so that $w_L = mw_M$.)

Recall that in Appendix A, we solve banks' portfolio choice problem by conjecturing θ_t , $c_{h,t}$, and Q_t follow diffusion processes. Given these conjectures, we can calculate first-order conditions and derive the law of motion for the state variable η_t . Once we compute $d\eta_t$, we can apply Ito's lemma to verify our conjectures, namely that θ_t , $c_{h,t}$, and Q_t are diffusion processes in equilibrium.

The microfoundation of monetary policy transmission from [Drechsler et al. \(2018\)](#) can be easily placed within this solution strategy. Below, we derive a sufficient condition for the full-insurance equilibrium, assuming our conjectures for θ_t and $c_{h,t}$ hold. If this sufficient condition holds, then we can rewrite the bank's portfolio choice problem as the problem stated in the main text (Section 2.2). Thus, the proofs in Appendix A still hold with this microfoundation.

Lemma 3. Suppose θ_t and $c_{h,t}$ are diffusion processes

$$\begin{aligned}\frac{d\theta_t}{\theta_t} &= \mu_{\theta,t} dt + \sigma_{\theta,t} dW_t \\ \frac{dc_{h,t}}{c_{h,t}} &= \mu_{ch,t} dt + \sigma_{ch,t} dW_t.\end{aligned}$$

If χ and ρ are such that, for all i_t ,

$$\chi \frac{\rho}{1-\rho} \frac{\kappa}{1+\kappa} \geq LP_t, \quad (52)$$

where $LP_t \equiv \frac{\kappa}{m} i_t$ is the liquidity premium at interest rate i_t , then banks fully self-insure, and their liquidity demand is given by

$$mx_{M,t} = \max \{ \kappa(x_{b,t} - 1), 0 \}.$$

Proof Lemma 3. Banks' value function is $\theta_t n_{b,t}$. Banks also discount using the stochastic discount factor $e^{-rt} c_{h,t}^{-1}$. Under the conjectured law of motion for θ_t , Ito's product rule implies

$$\begin{aligned}\frac{d(\theta_t n_{b,t})}{\theta_t n_{b,t}} &= (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t} \sigma_{nb,t}) dt + (\sigma_{\theta,t} + \sigma_{nb,t}) dW_t \\ &\quad - \frac{\rho}{1-\rho} \max \left\{ \frac{\kappa}{1+\kappa} (x_{b,t} + mx_{M,t} - 1) - mx_{M,t}, 0 \right\} dJ_t.\end{aligned}$$

After suppressing the controls, dropping the differential dt , and using the laws of motion for $\theta_t n_{b,t}$ and $c_{h,t}$, equation (26) simplifies to

$$\begin{aligned}r\theta_t n_{b,t} c_{h,t}^{-1} &= \max c_{h,t}^{-1} d\zeta_t + \theta_t n_{b,t} c_{h,t}^{-1} (\mu_{\theta,t} + \mu_{nb,t} - \mu_{ch,t} + \sigma_{ch,t}^2 + \sigma_{\theta,t} \sigma_{nb,t} - \sigma_{ch,t} \sigma_{nb,t} - \sigma_{ch,t} \sigma_{\theta,t}) \\ &\quad - \chi \frac{\rho}{1-\rho} \max \left\{ \frac{\kappa}{1+\kappa} (x_{b,t} + mx_{M,t} - 1) - mx_{M,t}, 0 \right\}.\end{aligned}$$

Note that banks take θ_t and $c_{h,t}$ as given because the marginal value of bank equity θ_t will not change due to the decisions of an infinitesimal bank, and $c_{h,t}$ is not controlled by banks. Therefore,

the first-order condition for $x_{M,t}$ is

$$\begin{aligned} 0 &\leq -i_t - \chi \frac{\rho}{1-\rho} \left(\frac{\kappa}{1+\kappa} m - m \right) \\ i_t &\leq m \chi \frac{\rho}{1-\rho} \frac{1}{1+\kappa} \\ \frac{\kappa}{m} i_t &\leq \chi \frac{\rho}{1-\rho} \frac{\kappa}{1+\kappa} \end{aligned}$$

By our hypothesis, the FOC for $x_{M,t}$ is always satisfied. Since i_t in equilibrium depends on η_t , a bounded variable, there always exist χ, ρ sufficiently large to ensure that (52) always holds.

Finally, to ensure that the max function returns zero, $x_{M,t}$ must satisfy

$$0 = \frac{\kappa}{1+\kappa} (x_{b,t} - 1) - \frac{1}{1+\kappa} m x_{M,t},$$

which implies the desired demand function. □

Lemma 3 implies that if θ_t and $c_{h,t}$ are diffusions, then equation (52) is a sufficient condition for full insurance. If banks are fully insured against funding shocks, then their equity never jumps in equilibrium. If their equity never jumps in equilibrium, then the marginal value of bank equity will never jump. Since $c_{h,t} = r(n_{h,t} + \theta n_{b,t})$, household consumption will also never jump. Therefore, if parameters are chosen such that (52) holds, then an equilibrium in which θ_t and $c_{h,t}$ are diffusions is consistent with a bank's optimal portfolio choice.

In a full-insurance equilibrium, we may zero out the jump term in (50). Because banks have a productive advantage, they will always be levered, so we may substitute $m x_{M,t} = \kappa (x_{b,t} - 1)$ into the Bellman equation. Hence, the nominal interest rates i_t determine the liquidity premium LP_t in equilibrium, where $\gamma = \frac{\kappa}{m}$ in the main text.

C Additional Numerical Results on Welfare and Stability

C.1 Calibration

Table 4 compares the model moments when monetary policy follows inflation targeting to the Calibration. While the calibration of the monetary policy rule is important for calibrating the rest of the model, the overall calibration results are not so sensitive to the exact MP rule. A natural alternative is to suppose that the central bank pursues an inflation targeting rule that implements $\bar{\pi}$ for all η . The alternative monetary policy rule has important implications for stability (the main result of our paper), and thus for the average value of critical empirical moments. Nonetheless, the moments generated by the model are not so different when the monetary policy rule is inflation targeting instead of the Calibrated Rule. Note that the inflation targeting rule is only approximate.

Table 4: Empirical moments when monetary policy follows the inflation targeting rule. The alternative monetary policy rule has important implications for stability, and thus for the average value of critical empirical moments. Nonetheless, the moments generated by the model are not so different when the monetary policy rule is inflation targeting instead of the Calibrated Rule.

Empirical Moment	Target	Calibrated MP Rule	Inflation Targeting Rule
Prob. of Crisis ($\psi < 50\%$)	3%	3.15%	3.86%
GDP Growth Volatility	2.2%	2.43%	2.48%
Risk-Free Real Rate	1.85%	1.88%	1.87%
Market Leverage	10	10.6	10.48
Non-ZLB Inflation	2%	1.98%	2.00%
Sharpe Ratio	48%	44.68%	44.49%
Output Loss in Crisis	-8%	-7.61%	-8.01%

C.2 Varying Inflation Costs λ

OptNoMap from the main text was computed for a given level of inflation costs, namely the calibrated λ . In the main text we compared welfare under OptNoMap to the other policies when λ varied. But the optimal policy changes when λ changes. For lower inflation costs, the central bank

has more flexibility to deviate from inflation targeting without welfare losses. In this section we consider how optimal policy changes for lower inflation costs.

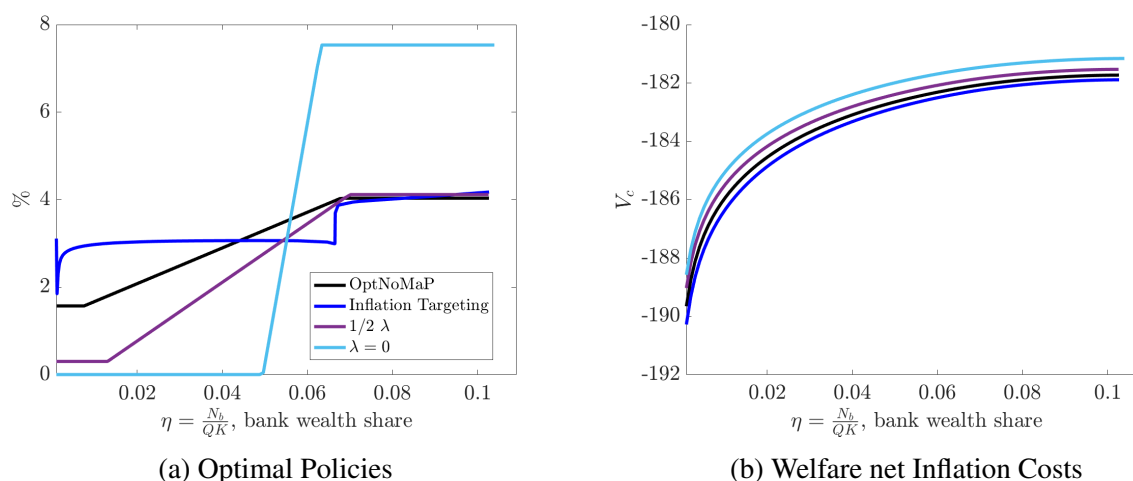


Figure 6: Optimal monetary policy and welfare for lower inflation costs. We plot the OptNoMaP rule under the calibrated λ (black), the inflation targeting rule (blue), the optimal MP rule when λ is half the calibrated value (purple), and the optimal MP rule when $\lambda = 0$ (teal).

Figure 6 considers the optimal policy rule setting λ to half the calibrated value and to zero. Figure 6(a) plots the policy rules, comparing to OptNoMaP and inflation targeting. Figure 6(b) plots the welfare net of inflation costs under each rule, i.e., welfare from consumption alone, denoted V_c . As is clear, V_c is highest for the rules that are optimal for lower λ . In these cases, the more aggressive Put/LAW combination leads to a more stable economy, but the larger fluctuations in interest rates away from inflation targeting would lead to higher costs from inflation. In the extreme case without inflation costs at all, the optimal rule is a very aggressive combination of a Put to the ZLB and a very high degree of LAW during a boom.

The reason why more aggressive LAW becomes optimal is the redistribution of seigniorage. Because banks receive some seigniorage and seigniorage increases in the nominal interest rate, the drift of η increases with the nominal interest rate, all else equal. A higher drift promotes a more stable stationary distribution. This effect is quantitatively small and is dwarfed by the destabilizing effects of high nominal rates when the economy is in distress or a crisis. As a result, even with relatively mild inflation costs, the optimal i^{LAW} is fairly low since the costs from inflation deviations

are far larger than the stability gains.

Redistributing some seigniorage to banks can be interpreted as partially undoing the wealth effects of the tax that the nominal interest imposes on banks. Higher nominal rates effectively raise the cost of funding for banks, so they behave like a tax on risk-taking, and seigniorage revenue is the collected tax revenue from banks. These taxes not only affect incentives to hold capital but also reduce the expected growth in wealth. By redistributing seigniorage according to agents' wealth, the government rebates some taxes back to banks but not all of the taxes. Thus, bank equity does not grow as slowly as it would if they received no seigniorage, but the wealth effects are not fully undone.

C.3 Excess Returns and Liquidity Premia

In this section we examine how i^{Put} and i^{LAW} affect returns during downturns. First, Figure 7 shows the effect of increasing i^{Put} by 3 percentage points above the optimal level on excess returns. Figure 7(a) displays the excess returns while Figure 7(b) decomposes the returns into the risk premium and liquidity premium components, as labeled above.

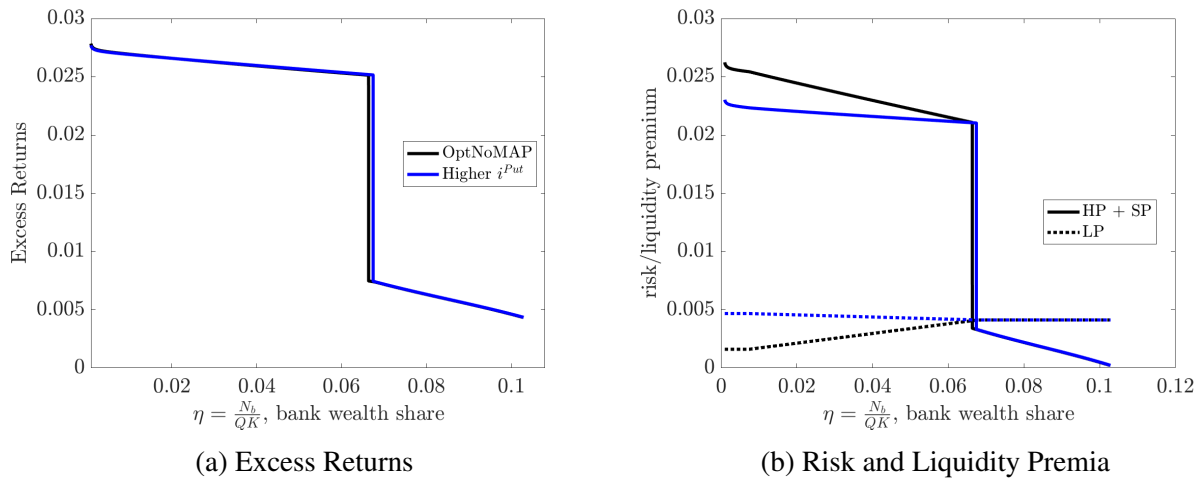


Figure 7: Equilibrium returns varying i^{Put} . In black, we plot the OptNoMaP rule while in blue, we plot the same rule but with i^{Put} increased by 3 percentage points. In panel (b), the solid lines are bank risk premia ($HP + SP$), and the dashed lines are liquidity premia.

When the economy is in distress, the liquidity premium behaves like a substitute for the risk

premium. For η close to zero, the risk premium in Figure 7(b) is higher under OptNoMaP than the higher i^{Put} rule even though the level of excess returns in Figure 7(a) is the same between the two policies. The difference arises because the liquidity premium is larger under the higher i^{Put} rule. Thus, when η is low, an increase in the nominal interest rate leads banks' risk premium to fall as if the two premia are substitutes. As η increases and the nominal interest rate begins to rise, the risk premium become the same across the two policies because the two policies have the same liquidity premium for high η .

Equilibrium asset pricing condition (13) illustrates why the liquidity premium substitutes for the risk premium during times of distress or crisis. Recall that if households hold positive quantities of capital, then

$$\frac{a_b - a_h}{Q} = SP + LP$$

must hold. Quantitatively, a higher LP barely changes the capital price Q , so the LHS of this equation is essentially constant. The intuition is that households are marginal pricers of capital in the distress region. Since the liquidity premium affects banks' incentives but not households', the price of Q does not respond strongly to the nominal interest rate. For the equality to hold, SP must decrease instead.

Now we examine how i^{LAW} affects returns during a boom. Figure 8 shows the effect of decreasing i^{LAW} by 3 percentage points below the optimal level on excess returns, the risk premium, the liquidity premium, and the risk-free interest rate. To make the effect clearer, we also set $\eta^{Put} = \tilde{\eta}^{LAW}$ so that decreasing i^{LAW} does not also lower rates in the distress region. During a boom, the liquidity premium becomes a substitute for the risk-free interest rate instead of the risk premium. Excess returns in Figure 8(a) are clearly different when η is high, but the risk premium in Figure 8(b) is unchanged. These two features directly contrast with Figure 7. The reason is that the asset pricing condition (13) is slack, so expected returns now only need to satisfy (12):

$$\frac{a_b - l}{Q} + \Phi(l) - \delta + \mu_Q + \sigma\sigma_Q - dr_f = \sigma_{ch}(\sigma + \sigma_Q) + SP + LP.$$

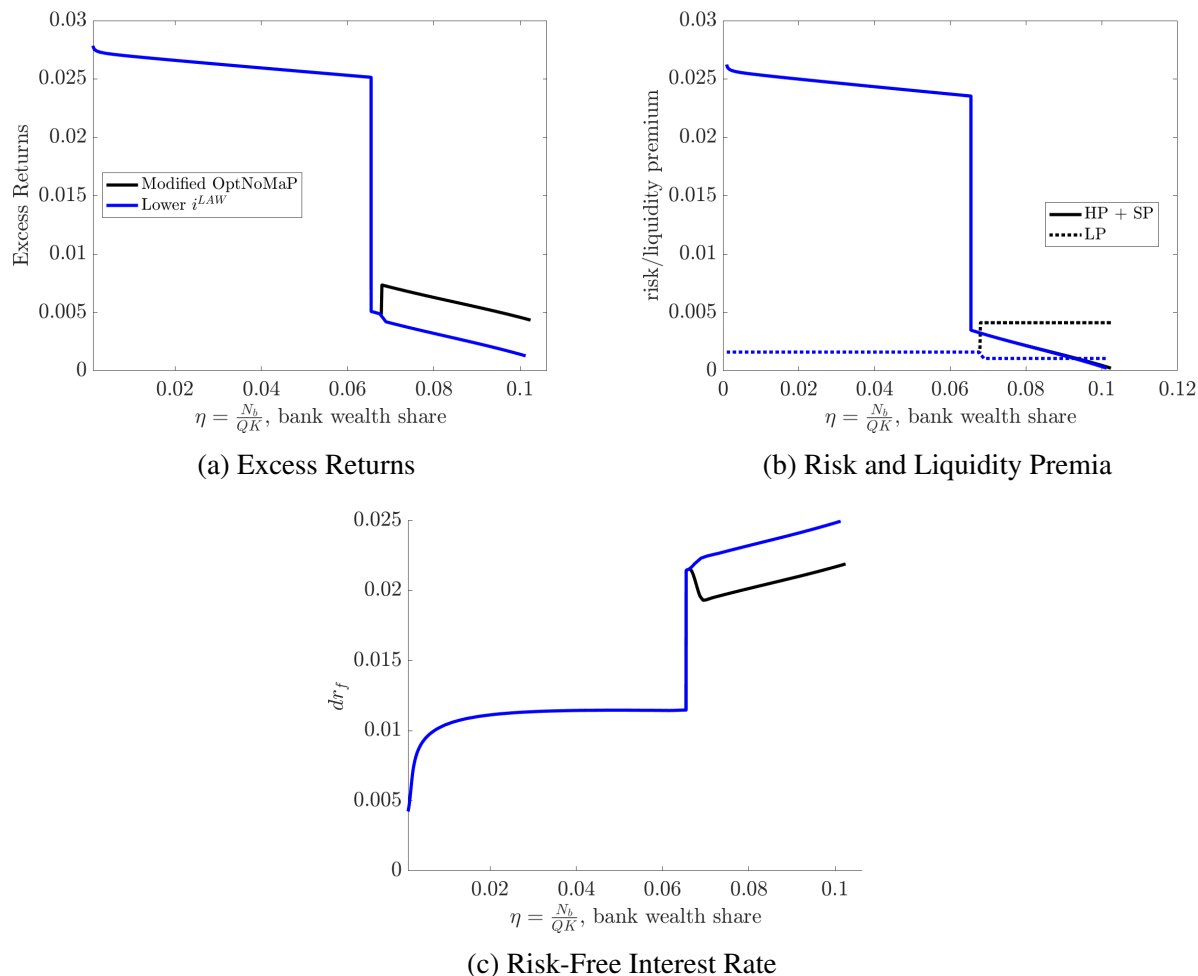


Figure 8: Equilibrium returns varying i^{LAW} . In black, we plot the OptNoMaP rule but with $\eta^{Put} = \tilde{\eta}^{LAW}$ while in blue we plot the same rule with i^{LAW} also lowered by 3 percentage points. In panel (b), the solid lines are bank risk premia ($HP + SP$), and the dashed lines are liquidity premia.

The risk-free interest rate in good times can move to absorb most of the effects of a higher nominal rate, whereas when the economy is in distress or a crisis, changes in LP have to be matched by changes in SP and/or Q . Figure 8(c) confirms that the lower liquidity premium corresponds to a higher risk-free interest rate.

In addition to the level of rates (i^{Put} and i^{LAW}), the locations of the strikes (η^{Put} and η^{LAW}) have state-dependent effects on returns. The locations determine whether rate cutting occurs when only banks are marginal pricers of capital or not. Cutting η^{LAW} , for example, can reduce the risk premium that banks earn because a lower η^{LAW} means that interest rates rise more quickly even

when households are still the marginal pricers of capital. Figure 9 confirms this intuition in an experiment where η^{LAW} is cut by 4 percentage points.

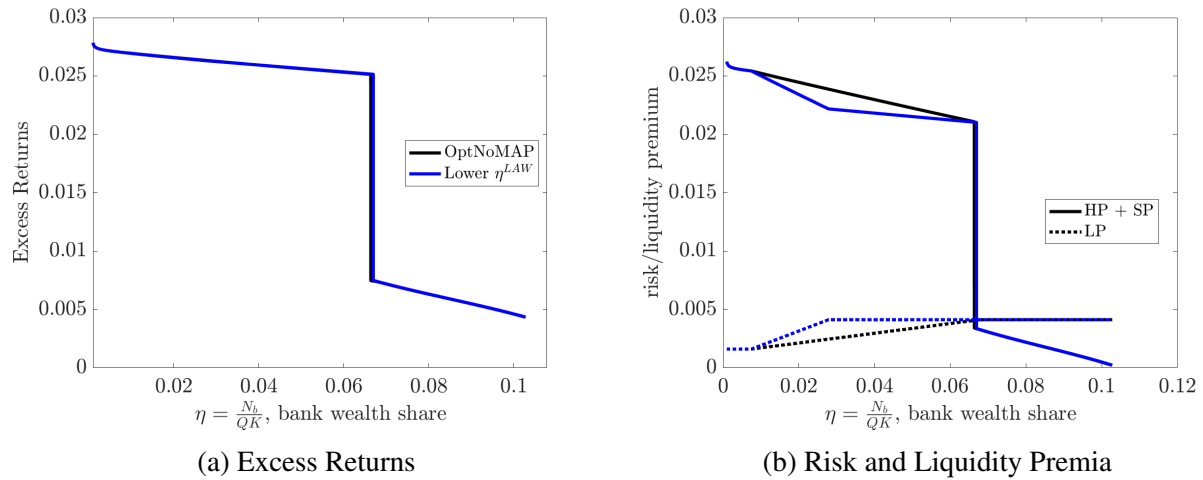


Figure 9: Equilibrium returns varying η^{LAW} . In black, we plot the OptNoMaP rule while in blue, we plot the same rule but with η^{LAW} lowered by 4 percentage points. In panel (b), the solid lines are bank risk premia ($HP + SP$), and the dashed lines are liquidity premia.

C.4 OptNoMaP and Alternative Policies

OptNoMaP vs. Constant Rate Rules Figure 10 plots key equilibrium variables under the OptNoMaP rule and constant interest rate rules set to the $\tilde{i}^{Put}$ and $\tilde{i}^{LAW}$ levels of the OptNoMaP rule. Interest rates in the OptNoMaP policy vary between the $\tilde{i}^{Put}$ and $\tilde{i}^{LAW}$ levels, hence the behavior according to OptNoMaP also generally varies between these two levels.

The volatilities of the asset price Q and bank equity η are higher with low constant interest rates, consistent with the standard intuition that low rates may be destabilizing by increasing volatility. Lower constant rates also increase the drift of bank equity (μ_η is higher), which is consistent with the intuition that low rates allow banks to finance themselves more cheaply and to build up equity. Under OptNoMaP, the drift μ_η , volatilities, and capital allocations largely fall in between the values under the constant rate rules.

Therefore, the OptNoMaP policy is more stable than the constant rate rules because it combines the benefits of both policies without their downsides. The high rate policy is undesirable because

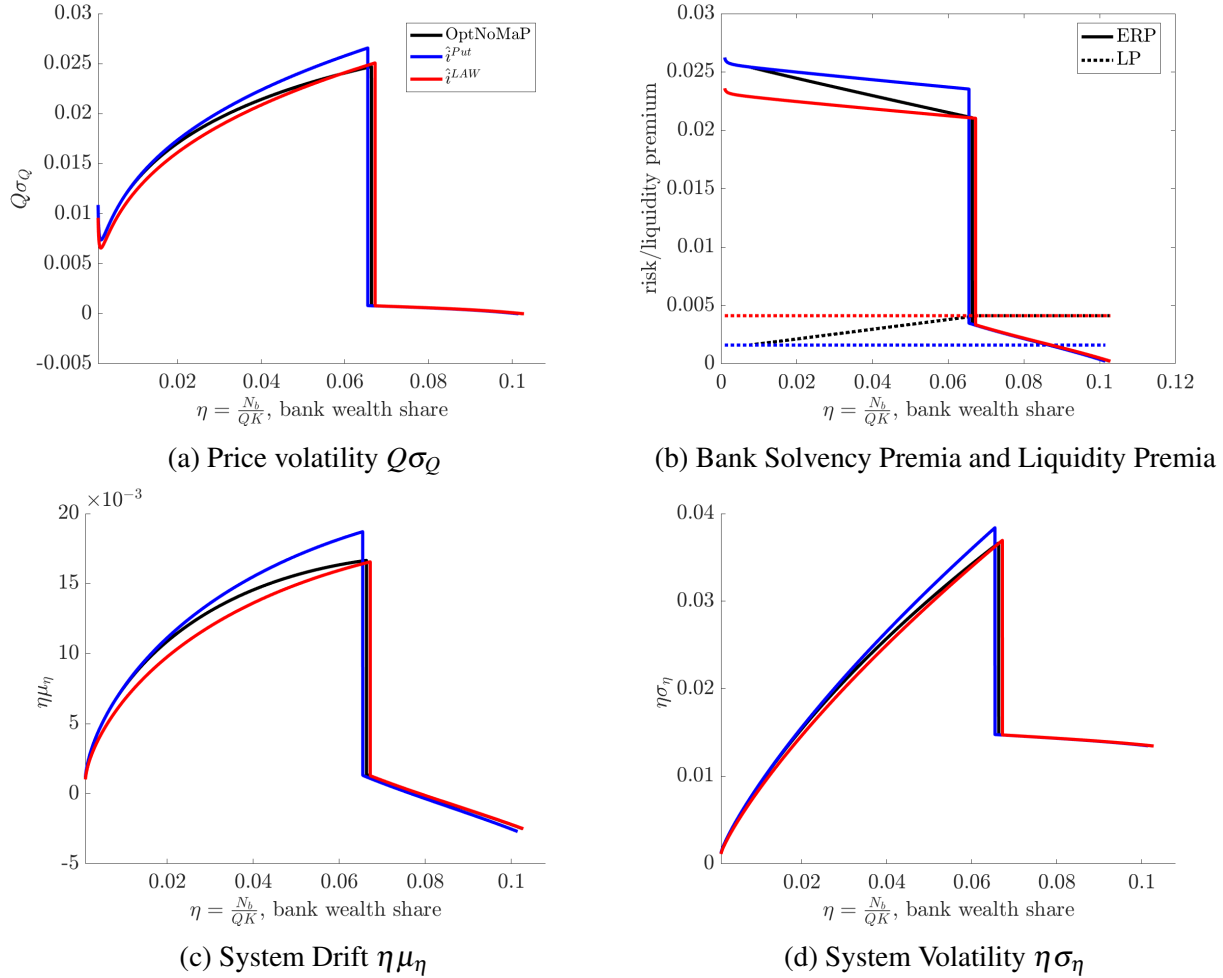


Figure 10: Equilibrium prices, return premia, and evolutions with constant rates and OptNoMaP. The OptNoMaP rule is in black while in blue we plot a constant rate of i^{Put} and in red a constant rate of i^{LAW} . In panel (b), the solid lines are bank risk premia ($HP + SP$), and the dashed lines are liquidity premia.

it reduces leverage in a crisis—precisely when it is needed. The low rate policy is undesirable because it causes volatilities to rise too much. Combining a Fed Put with high rates during a boom avoids these downsides while still obtaining the positive effects of these policies.

The key reason that OptNoMaP and constant low rates in our model yield greater stability than the constant high rate policy is that output gaps in our model are associated with financial distress, and movements in the nominal rate influence output gaps through their effect on the banking sector's risk-taking. This association produces state-dependent effects on returns, volatility, and

leverage that generate greater stability if monetary policy enacts a Fed Put when the economy is in a crisis and high rates when the economy is in a boom. In contrast, the output gap in Drechsler et al. (2018) depends only on deviations of the nominal rate from a benchmark rule and is independent of financial distress. Since financial frictions only affect investment and not the output gap, financial frictions cause fewer inefficiencies in the real economy, so accommodative monetary policy generates smaller benefits.

To be explicit, the output gap in our model is $o = (\psi - 1)(a_b - a_h)$, which implies the market-clearing condition $C = (a_b - \iota + o)K$, where C is aggregate consumption. Output gaps appear in our model when households hold capital ($\psi < 1$) because they are less productive than banks. Households hold capital only if bank equity is low. Thus, output gaps directly depend on the financial sector's capitalization. The nominal interest rate indirectly affects output gaps by controlling the liquidity premium banks face when borrowing with deposits. In contrast, while the market-clearing condition for consumption in Drechsler et al. (2018) is the same as ours, the definition of o is specified such that o turns negative (positive) when the nominal rate is shocked above (below) a benchmark level. The output gap in their model reflects New Keynesian effects of monetary policy shocks on the output gap and does not closely depend on the wealth share of banks.

Welfare Comparisons Figure 11 plots the consumption-equivalent losses given initial condition η for inflation targeting and Friedman compared to OptNoMaP.

Welfare with ICC MaP Table 5 provides more information about how the ICC MaP policy affects the consequences of different MP rules.

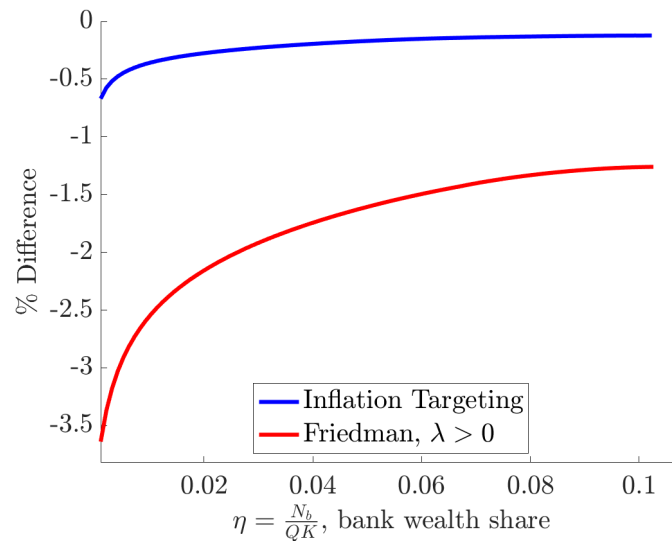


Figure 11: Welfare losses (consumption-equivalent), comparing alternative policies to OptNoMaP. We plot the welfare losses under inflation targeting in blue and the losses under the Friedman rule for $\lambda > 0$ in red.

Table 5: Welfare and stability comparing alternative interest rate rules to the optimal policy rule with ICC. Welfare changes are reported in consumption equivalent percent differences from the optimal policy. Changes in probability of distress and probability of crisis are reported in percentage point differences from the optimal policy. A positive change in probability of distress (crisis) means that the probability of distress (crisis) is higher under the alternative policy than under the optimal policy rule with ICC.

Alternative Rule	Δ Welfare	Δ Prob. Distress	Δ Prob. Crisis
<i>Panel A: MaP Constraint $\alpha = \infty$ (none)</i>			
Inflation Targeting	-0.59	0.73	0.41
$i = \tilde{i}^{Put}$	-12.93	-0.14	0.18
$i = \tilde{i}^{LAW}$	-1.45	1.35	0.63
$i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.74	1.45	0.65
<i>Panel B: MaP Constraint $\alpha = 8$</i>			
Inflation Targeting	-0.39	1.55	0.01
$i = \tilde{i}^{Put}$	-21.68	1.88	-0.01
$i = \tilde{i}^{LAW}$	-1.21	3.35	0.02
$i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.58	3.30	0.03
<i>Panel C: MaP Constraint $\alpha = 6$</i>			
Inflation Targeting	0.52	4.03	0.00
$i = \tilde{i}^{Put}$	-27.46	-5.64	0.00
$i = \tilde{i}^{LAW}$	-1.90	5.27	0.00
$i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.76	4.59	0.00

C.5 State-Contingent Easing and Leaning Against the Wind

When considering policies with constant rates, we have seen that constant low interest rates lead to a more stable distribution but higher volatility. A state-dependent policy can achieve both stability objectives, decreasing the probability of crises and decreasing volatility. High interest rates (at times) can generate low volatility, and low interest rates (at other times) can generate a favorable stationary distribution. Thus, leaning against the wind must be state-contingent, not “unconditional” as some economists consider. The global behavior of leverage depends on the state-dependent behavior of interest rates much more than on the overall level.

We now consider timing considerations regarding when to lean against the wind and when to ease. To cleanly isolate the i^{Put} and i^{LAW} components of the policy rule, we consider discontinuous piecewise rules where $\eta^{Put} = \eta^{LAW}$. Since we focus on stability rather than welfare in this section, we will also ignore the role of inflation costs. For this reason, in the subsequent analysis we shall interpret any case where $i^{LAW} > i^{Put}$ as leaning against the wind.

We solve the model with $i^{LAW} \in [0, 5\%]$ and $i^{Put} \in [0, 3\%]$. Figure 12 plots the long-run probability of distress for $\eta^{Put} = 3\%$ and 7% . In the first case, rates are adjusted in the distress region in which households are pricing capital. This also means that the i^{LAW} rate is held well into a crisis, and so a higher i^{LAW} will lead to higher interest rates in times of distress and crisis, which is precisely when we want low interest rates. In the second case, interest rates adjust before distress occurs, when banks are the marginal pricers of capital, and so variations in i^{LAW} only affect rates during a boom.

Two results emerge clearly. First, a more aggressive Fed Put (lower i^{Put}) is always more stabilizing regardless of the level of i^{LAW} . Second, the position of the policy strike matters. When η^{Put} is outside the crisis region, then a higher i^{LAW} is more stabilizing (Figure 12(b))—LAW in a boom is an effective policy for improving stability. However, if η^{Put} is so low that rates are high for most of a crisis, a higher i^{LAW} leads to a less stable economy, as is the case when $\eta^{Put} = 3\%$ (Figure 12(a)).

This evidence suggests that LAW in a boom can be an effective policy toward improving financial stability. Increasing interest rates has a detrimental effect on stability when higher rates

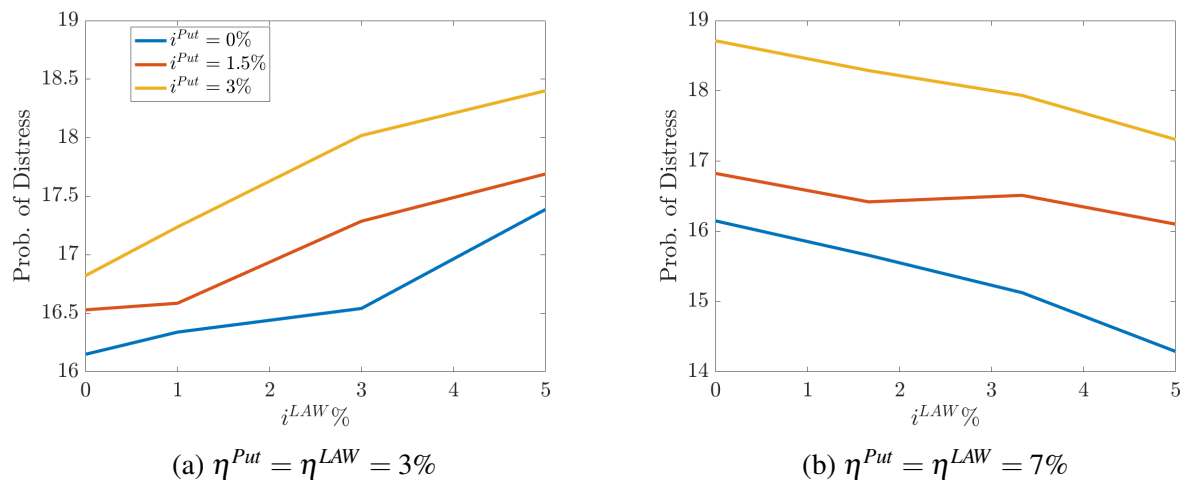


Figure 12: Stability with Leaning against the Wind together with a Fed Put, varying (i^{LAW}, i^{Put}) for a given strike $\eta^{Put} = \eta^{LAW}$. The policy rule used in these figures are discontinuous piecewise rules which jump at η^{Put} . The blue, red, and yellow lines correspond to a policy of i^{Put} equaling 0%, 1.5%, and 3%, respectively. The horizontal axis varies the i^{LAW} policy between 0% and 5%. The left panel sets strike to 3% while the right panel sets it to 7%.

increase bank funding costs during crises, precisely when increasing bank equity is most valuable, and precisely when changing funding costs has minimal effect on banks' investment returns. A carefully targeted policy of leaning against the wind in a boom, and only in a boom, can improve financial stability. While endogenous instability, represented either by σ_Q or σ_η , is highest for moderate values of η , excessive risk taking in terms of the effect on stability is highest for high η . Indeed, the ICC macroprudential policy considered in the main text improves welfare and stability precisely because it dampens excessive risk taking for high η .

C.6 Poorly Targeted Macroprudential Policy

Some research (e.g. Phelan, 2016) has found that binding leverage constraints in bad times can improve stability and welfare. Figure 13 illustrates such policies as linearly decreasing leverage constraints, which bind only when η is low.

Leverage constraints during a crisis turn out to be poorly targeted and thus fail to restrict leverage when it is most socially suboptimal. Table 6 shows the consumption equivalent welfare change and stability changes between OptNoMaP and alternative interest rate rules. The OptNoMaP rule

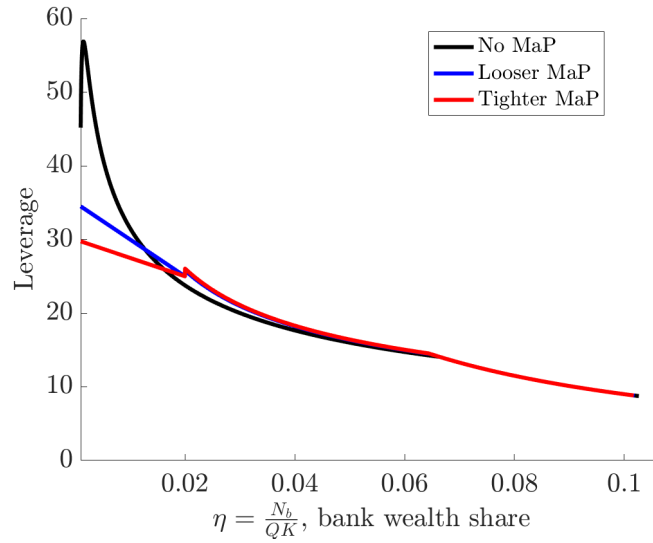


Figure 13: Linearly decreasing leverage constraints that bind during a crisis. The black line corresponds to no constraint on leverage. The blue line corresponds to a linearly decreasing constraint from $L = 35$ to $L = 25$. The red line corresponds to a linearly decreasing constraint from $L = 30$ to $L = 25$. In all cases, the nominal interest rate follows OptNoMaP.

outperforms inflation targeting and constant rate rules, even as the macroprudential constraint tightens. This result contrasts with the ICC rule, where a sufficiently tight α causes inflation targeting to outperform OptNoMaP.

Table 6: This table lists welfare and stability under the leverage constraint policies which bind during a crisis. The black, blue, and red lines in Figure 13 correspond to the “No MaP”, “Looser”, and “Tighter” panels comprising the table. The rows consider alternative rate rules. Welfare is reported first in consumption equivalent percent differences from the OptNoMaP monetary policy rule (but with leverage constraints potentially binding), and probabilities are reported second in percentage point differences. A positive change in probability of distress (crisis) means that the probability of distress (crisis) is higher under the alternative policy than under OptNoMaP.

Alternative Rule	Δ Welfare	Δ Prob. Distress	Δ Prob. Crisis
<i>Panel A: No MaP Constraint</i>			
Inflation Targeting	-0.59	0.73	0.41
$i = \tilde{i}^{Put}$	-12.93	-0.14	0.18
$i = \tilde{i}^{LAW}$	-1.45	1.35	0.63
$i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.74	1.45	0.65
<i>Panel B: Looser MaP Constraint</i>			
Inflation Targeting	-0.33	-0.57	-0.10
$i = \tilde{i}^{Put}$	-12.71	-0.35	0.17
$i = \tilde{i}^{LAW}$	-1.72	1.20	0.54
$i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.87	0.82	0.51
<i>Panel C: Tighter MaP Constraint</i>			
Inflation Targeting	-0.39	0.56	0.31
$i = \tilde{i}^{Put}$	-12.58	-0.33	0.01
$i = \tilde{i}^{LAW}$	-1.35	0.82	0.55
$i = 4.2\% = \text{Friedman}, \lambda > 0$	-1.68	1.59	0.81

D A Model with Two Goods

We now consider a model with two goods. In the baseline one-good model, the optimal allocation of capital for high η is for banks to hold all of the capital stock, which means that banks can never hold *too much* capital (more than socially optimal). We now consider a model in which the optimal allocation is for banks to hold a fraction of the capital stock and for high η banks will

hold more than socially efficient. Proposition 8 proves explicitly that banks will hold too much capital for high η when interest rates are low. The main results of our baseline model continue to go through—namely, that the combination of a Put and LAW can improve stability and welfare.

D.1 Setup

We modify the setup as follows. Capital can be used to produce two intermediate goods. Banks have an advantage for producing one good (the “bank-dependent sector”). As a result, output and growth depend endogenously on capital ownership.

One factor of production, capital, can be used to produce two types of intermediate goods at unit rate. The effective capital quantity y_t evolves according to equation (53),

$$\frac{dy_t}{y_t} = g_y^{ij} dt + \sigma dW_t, \quad (53)$$

where dW_t is an exogenous standard Brownian motion and g_y^{ij} depends on who manages capital and what it is used to produce. The values of g_y^{ij} for agent i and good j (Table 7) imply that banks are comparatively better at managing good-1 production (good-1 are bank-dependent investments). We define the parameter restriction on g_B more clearly later in this section.

Table 7: Expected capital productivity growth rates by agent and good produced.

g_y^{ij}	Good 1	Good 2
Households	$g - \ell$	g
Banks	g_B	g_B

Denote by Y_t the stock of effective capital at time t , which is also the flow production of goods at time t . The consumption good is produced using goods 1 and 2 according to

$$C_t = Y_{1t}^{\frac{1}{2}} Y_{2t}^{\frac{1}{2}},$$

where C_t is the quantity of the consumption good, Y_{jt} is the quantity of good j (equivalently the quantity of capital used to produce good j). Standard static optimization implies that the

equilibrium prices of intermediates are given by

$$p_{1t} = \frac{1}{2} \left(\frac{Y_{2t}}{Y_{1t}} \right)^{\frac{1}{2}} p_t, \quad p_{2t} = \frac{1}{2} \left(\frac{Y_{1t}}{Y_{2t}} \right)^{\frac{1}{2}} p_t,$$

where p_t is the price of consumption. Let $v_t = \frac{Y_{1t}}{Y_t}$ be the fraction of capital cultivating good 1. Then the real prices of intermediate goods ($P_{jt} = p_{jt}/p_t$) are

$$P_{1t} = \frac{1}{2} \left(\frac{1-v_t}{v_t} \right)^{\frac{1}{2}}, \quad P_{2t} = \frac{1}{2} \left(\frac{v_t}{1-v_t} \right)^{\frac{1}{2}}. \quad (54)$$

By Ito's Lemma, the rate of return to agent i using capital to produce good j is

$$dr_t^{ij} = \left(\frac{P_{jt}}{Q_t} + g_y^{ij} + \mu_{Q,t} + \sigma \sigma_{Q,t} \right) dt + (\sigma + \sigma_{Q,t}) dW_t,$$

where g_y^{ij} is appropriately defined for agent i .

For technical reasons, we can no longer solve the model using the same methods when households have log utility. Accordingly, we now suppose that households have linear utility. Lifetime utility is given by

$$V_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \left(c_{h,t} + \phi_L \delta_{h,t} - \frac{\lambda}{2} \pi_t^2 \right) dt \right], \quad (55)$$

where $c_{h,t}$ is household flow consumption, r is the discount rate, $\delta_{h,t}$ are bank deposits, $\phi_L > 0$ is the convenience yield preference parameter, and inflation costs are quadratic around zero. Households may consume positive and negative amounts, though in equilibrium their consumption will always be positive. It follows that households require an expected real return of r on any real investment and a real return of $r_L = r - \phi_L$ on deposits due to the convenience yield.

D.2 Equilibrium Asset Pricing and Excessive Risk Taking

Households will always produce good 2 and sometimes produce good 1. Because households require a return of r on real investments, it follows that household returns satisfy

$$\frac{P_{2t}}{Q_t} + g + \mu_{Q,t} + \sigma \sigma_{Q,t} = r, \quad (56)$$

$$\frac{P_{1t}}{Q_t} + g - \ell + \mu_{Q,t} + \sigma \sigma_{Q,t} \leq r, \quad (57)$$

where the inequality reflects that households may not always produce good 1 in equilibrium. Because households are risk neutral, there is no household risk premium, hence the bank risk premium coincides with the bank solvency risk premium. Since we look for an equilibrium where the risk premium and liquidity premium are both non-negative, banks will never produce good 2. From the banks' investment in good 1 we have

$$\frac{P_{1t}}{Q_t} + g + \mu_{Q,t} + \sigma \sigma_{Q,t} = r + SP_t + LP_t. \quad (58)$$

When households do not produce good 1 (banks and households specialize), we obtain a market-clearing condition for capital allocations by taking the difference between the equations (56) and (58):

$$SP_t + LP_t = \frac{P_{1t} - P_{2t}}{Q_t}. \quad (59)$$

With specialization, changes in the liquidity premium may affect the difference in returns between goods 1 and 2. In contrast, when households produce good 1, we have $\frac{P_{1t} - P_{2t}}{Q_t} = \ell$, which implies

$$SP_t + LP_t = \ell, \quad (60)$$

i.e., the sum of banks' investment premia equals the household return disadvantage. The differences between equations (59) and (60) provide a crucial insight. When banks are the marginal investors in the intermediation sector (specialization), a decrease in interest rates might, all else equal, decrease the relative return between sectors 1 and 2 ($P_1 - P_2$ decreases). When households

are the marginal investor in the intermediation sector, a decrease in interest rates must increase banks' equilibrium risk-premium, which will occur through higher leverage.

The model with financial frictions features pecuniary externalities so that banks generally take excessive risk. A positive liquidity premium can improve welfare by decreasing banks' risk-taking.

Proposition 8. *With costly equity issuance, the optimal interest rate at the stochastic steady-state η^* is positive. In other words, when interest rates are zero, the marginal social value of bank leverage is negative implying that banks hold too much capital at the steady state. A positive liquidity premium local to η^* corrects the pecuniary externality from financial frictions.*

Proof of Proposition 8. The welfare function $V(\eta)$ solves the following differential equation

$$rV(\eta) = z(\eta) + V'(\eta)\eta\mu^\eta + V(\eta)g_Y(\eta) + \frac{1}{2}V''(\eta)(\eta\sigma^\eta)^2 + V'(\eta)\eta\sigma^\eta\sigma, \quad (61)$$

where $z(\eta_t) = v_t^{1/2}(1 - v_t)^{1/2} + \phi_L(\psi_t - \eta_t)Q_t$ is output plus convenience yield utility and $g_Y(\eta_t) = g(1 - \psi_t) + \psi_t(g - \phi_L) - (v_t - \psi_t)\ell$ is aggregate productivity growth. Taking the derivative of the right-hand side of equation (61) with respect to ψ_t yields

$$\mathcal{L}(\psi_t) \equiv \frac{\partial z(\psi, v)}{\partial \psi} + V'(\eta)\eta \frac{\partial \mu^\eta}{\partial \psi} + V \frac{\partial g_Y}{\partial \psi} + (V''(\eta)\eta\sigma^\eta(\psi, v, \eta) + V'\eta\sigma) \frac{\partial \sigma^\eta}{\partial \psi}. \quad (62)$$

It suffices to show that if interest rates are zero then $\mathcal{L}(\psi_t) < 0$ near η^* . Near η^* , $\psi(\eta) = v(\eta)$, so that

$$\frac{\partial z(\psi, v)}{\partial \psi} = P_1 - P_2 + \phi_L q_t,$$

and $\frac{\partial g_Y}{\partial \psi} = -\phi_L$. By smooth-pasting, $V'(\eta^*) = 0$. Plugging in terms, we have

$$\mathcal{L}(\psi_t) = P_1 - P_2 + V''(\eta^*)\eta\sigma^\eta(\psi, v, \eta) \frac{\partial \sigma^\eta}{\partial \psi} - \phi_L(V(\eta^*) - Q(\eta^*)).$$

Rearranging equation (61) yields $V(\eta^*) = \frac{z(\eta^*)}{r - g_Y} + \frac{1}{2(r - g_Y)}V''(\eta^*)(\sigma^\eta(\psi, \eta))^2$. The first term is the present discounted value if the system did not move from η^* . Since welfare is strictly less

than $\frac{z(\eta^*)}{r-g_Y}$, $V''(\eta^*) < 0$. $V(\eta) \geq Q(\eta)$ because $\theta_t \geq 1$. When $v_t = \psi_t$, $P_1 - P_2 = -\sigma_t^\theta(\sigma + \sigma^\mathcal{Q})Q_t$ when liquidity premia are zero. Furthermore $\frac{\partial \sigma^\eta}{\partial \psi} > 0$. In competitive equilibrium $\theta'(\eta^*) = 0$, so that $\sigma_t^\theta = 0$, which implies $P_1 = P_2$ when the liquidity premium is zero. Hence $\mathcal{L}(\psi_t) < 0$. Since the marginal social value of ψ_t is negative at η^* if the liquidity premium is zero, welfare in competitive equilibrium would improve if leverage near η^* decreased on the margin, which occurs with a positive liquidity premium. $\square$

D.3 Calibrated Results

We solve the model numerically using the calibrated parameters from [Phelan \(2016\)](#) and [Drechsler et al. \(2018\)](#). The two most important variables are volatility σ and the monetary policy transmission value γ . The volatility $\sigma = 2\%$ corresponds to the volatility of TFP and also the typical volatility of bank assets (the two-good model requires a lower value for a calibration of volatility). We maintain the value of $\gamma = 10.2\%$. Remaining parameters are $r = 4\%$, $g = 2\%$, $\ell = 1\%$, and $\phi_L = 2\%$. See [Phelan \(2016\)](#) for additional calibration details. In the subsequent discussion, our definitions of a boom, distress, and crisis from the main text no longer apply. Since the results are similar to our baseline model's, we opt for a simple partition of the state space into “bad times” ($v < \psi$) versus “good times” ($v = \psi$).

Normative Results We first calculate welfare-maximizing monetary policy and then examine positive results regarding stability and leverage. Before turning to state-dependent policy in our model, we first consider two benchmarks for interest rates. First, we calculate the optimal interest rate i^{FB} in an economy without financial frictions but with inflation costs. The Friedman Rule is no longer optimal, so the optimal interest rate i^{FB} balances the liquidity cost from positive interest rates with the standard New Keynesian costs of deviating from i^* . For these parameters, $i^{FB} = 1.47\%$. As a second benchmark, we restrict monetary policy to a constant nominal rate $\tilde{i}$ and maximize expected welfare under the timeless initial condition. For these parameters, welfare is maximized near $\tilde{i} = 1.62\%$.

We now solve for optimal monetary policy functions choosing η^{LAW} , η^{Put} , i^{LAW} , and i^{Put} to

maximize expected welfare using the timeless perspective. For this class of functions, the best policy appears to be to set policy roughly as follows: $\eta^{Put} = 1\%$, $\eta^{LAW} = 3.5\%$, $i^{Put} = 1.4\%$, and $i^{LAW} = 1.7\%$. Compared to the first-best benchmark, the optimal policy leans against the wind by about 20bps in good times, and accommodates by about 10bps in bad times (comparing to the constant-rates benchmark the numbers are 10bps and 20bps). It is optimal to begin cutting rates before a crisis, but not too early. Thus, the central bank should keep its powder dry for a while.

Table 8 displays the results with $\lambda = 288, 432$, and 648 . The optimal policy strikes are roughly the same ($\eta^{Put} = 1\%$ and $\eta^{LAW} = 3.5\%$) except that with the lower inflation costs welfare is slightly higher for $\eta^{LAW} = 3.25\%$ so that it is optimal to hold powder dry for a little longer before cutting. The optimal constant interest rate $\tilde{i}$ is higher than the first-best i^{FB} (the gap is decreasing in the inflation cost). When considering state-dependent policies, it is always optimal to pursue a Fed Put and to lean against the wind. The extent of the rate drop is decreasing in the inflation cost, and for the baseline inflation cost is on the order of a single rate change (25bps).

Table 8: Optimal Interest Rate Policies with Inflation Costs

λ	288	432	648
i^{FB}	1.19%	1.47%	1.65%
$\tilde{i}$	1.43%	1.62%	1.75%
i^{LAW}	1.47%	1.70%	1.78%
i^{Put}	1.13%	1.40%	1.66%
Rate Drop	34bps	30bps	12bps

For λ even lower than 288, the optimal policy has a lower Put strike η^{Put} and also cuts to a much lower i^{Put} . Comparing the extent of a rate drop is somewhat harder to do in this case since two dimensions are moving: how deep to cut, and over what range. Simply subtracting i^{Put} from i^{LAW} doesn't tell the full story of how much rates are responding since policy takes longer to reach i^{Put} . If we consider $\lambda = 216$ and fix $\eta^{Put} = 1\%$ and $\eta^{LAW} = 3.25\%$ (the optimal values for $\lambda = 288$), then the optimal rate cut is roughly 62bps, indicating that two and a half rate cuts could be justified to support the financial sector. Lower values of λ may indeed be justified depending

on the microfoundation for nominal rigidities. Nuño and Thomas (2016) consider a continuous-time New Keynesian model with Rotemberg pricing and argue for a much lower calibration of inflation costs. Hence, we continue to find evidence in favor of leaning against the wind in good times and pursuing a Fed Put in crises. Nominal rigidities motivate keeping interest rates near a baseline target, but financial instability motivates a degree of state-dependent policy in response to conditions in the financial sector. Even with large inflation costs, a rate cut to support the financial sector can be justified.

Positive Results We now compare “Fed Put” (cutting rates in bad times) and “Fed Call” (raising rates in bad times) to examine the positive implications. Indeed, the positive implications of the baseline, one-good model persist. We consider the following interest rate rules: constant rates held at 0 and 4%, a Fed Put that sets $i^{LAW} = 4\%$ and $i^{Put} = 0\%$, and a Fed Call (the reverse of usual) with $i^{LAW} = 0\%$ and $i^{Put} = 4\%$, both with strikes near $\eta = 2\%$ (during crises). The behavior of stability, leverage, and returns, are comparable to what is in the baseline model.

Figure 14 plots leverage, returns (the Sharpe Ratio), and stability (stationary distribution), under these four policies. Consistent with our benchmark model, leverage under constant rate MP is nearly identical across equilibria. In contrast, the consequence of state-dependent MP is to affect leverage during crises. A Fed Put props up leverage in bad times, while a Fed Call depresses leverage in bad times. Leverage in good times is virtually the same across policies. Hence, even when banks can overaccumulate capital in good times, the consequence of a Fed Put is primarily to affect leverage in bad times, and in the desirable way. In addition, the Sharpe ratio is benefited globally by the Fed Put, with banks earning higher risk-adjusted returns everywhere, and especially when MP cuts rates. The Sharpe Ratio under the Fed Put is everywhere higher even than the policy that holds rates constant at 4%. The Fed Put economy is the most stable, with the least mass of the distribution for low η and the highest mass for high η . Hence, the positive results that a Fed Put can improve stability and welfare continues to hold in an economy where banks can overaccumulate capital in good times.

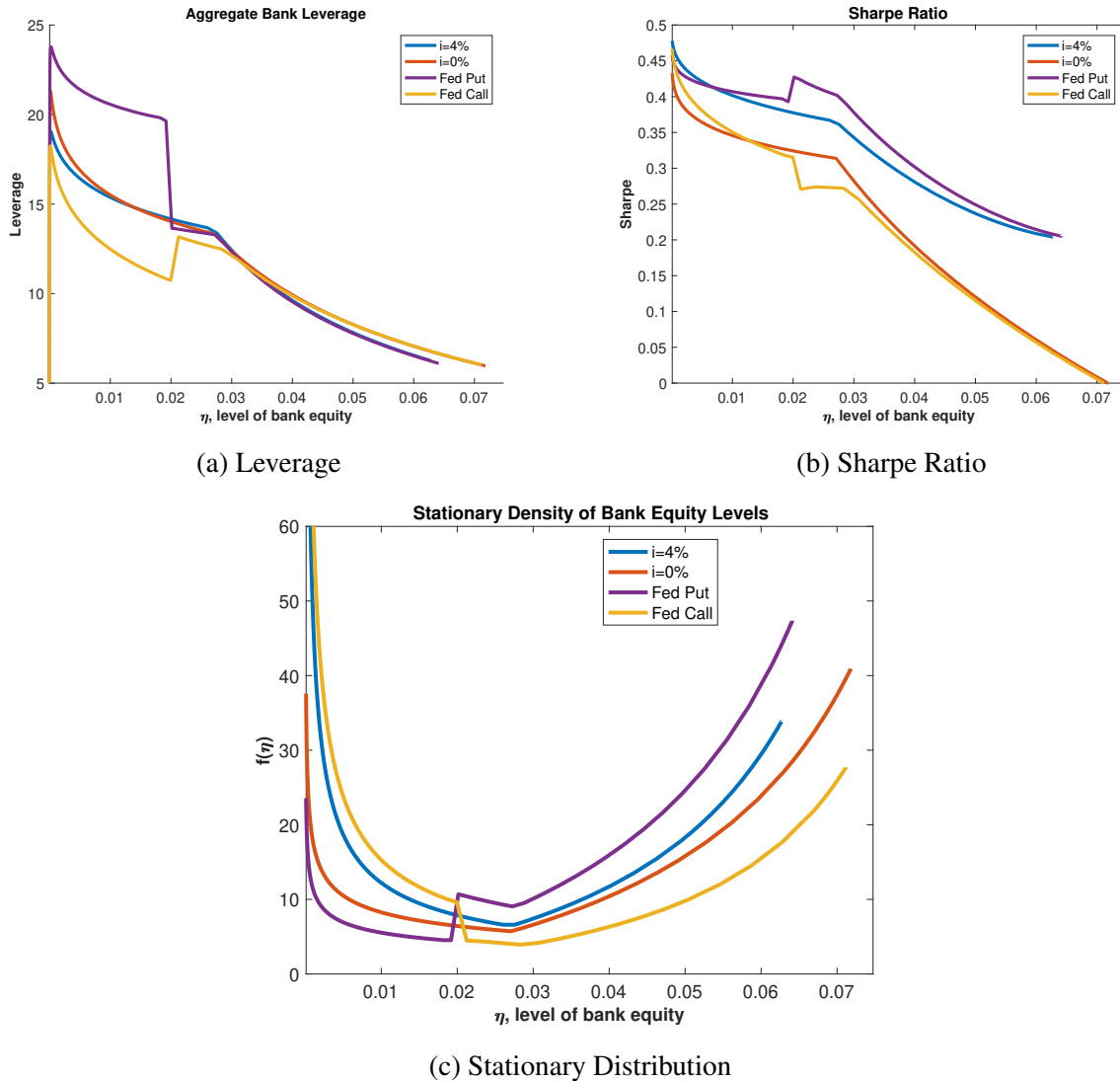


Figure 14: Leverage, Returns, and Stability

E Bank Misallocation

While it is outside the scope of the paper to microfound a channel for banks to misallocate capital (e.g. incentives to relax lending standards), we can study its impact in a reduced form way. To capture the idea that the efficient allocation of capital to banks is not 100% because banks tend to misallocate capital during booms, we suppose that bank productivity is an exogenous function of η . This assumption requires a small modification to the equilibrium system of differential equations; see the end of this section for details.

For parsimony, we specify the bank productivity function as

$$a_b(\eta) = \begin{cases} \bar{a}_b & \text{if } \eta \leq \eta_{mh} \\ \underline{a}_b & \text{otherwise.} \end{cases} \quad (63)$$

While empirically unrealistic, this simple productivity function generates the desired feature that the statically optimal allocation of capital to banks is less than 100%. Figure 15 plots the equilibrium level of output (solid line) and allocation of capital to banks ψ (dashed line) when the monetary policy rule is Calibration and (63) is parameterized by $\bar{a}_b = 0.127$, $\underline{a}_b = 0.95\bar{a}_b$, and $\eta_{mh} = 0.05$. This drop in productivity is akin to a 25% decrease in the productivity gap between banks and households, which a priori seems like a meaningful reduction in production efficiency. Under these parameters, output is the highest when the share of capital allocated to banks is between approximately 75% and 87%. This region of the state space is the shaded area in Figure 15.

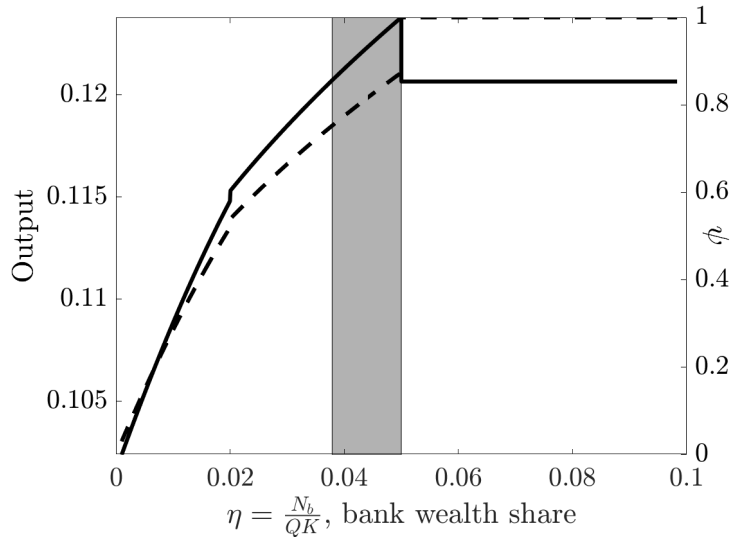


Figure 15: Output and ψ . The solid line is the level of output while the dashed line is the allocation of capital to banks ψ . The left axis corresponds to output while the right axis corresponds to ψ . The shaded area highlights where the level of output is higher with $\psi < 1$ than with $\psi = 1$.

We then calculate the optimal monetary policy rule in the usual class of piecewise linear rules (18). Figure 16 plots the optimal interest rate rule and welfare costs from inflation when there is

no bank misallocation (black) and when there is bank misallocation (blue). Thus, the black line is the OptNoMaP equilibrium in the main text while the blue line corresponds to the equilibrium in which bank productivity follows (63).

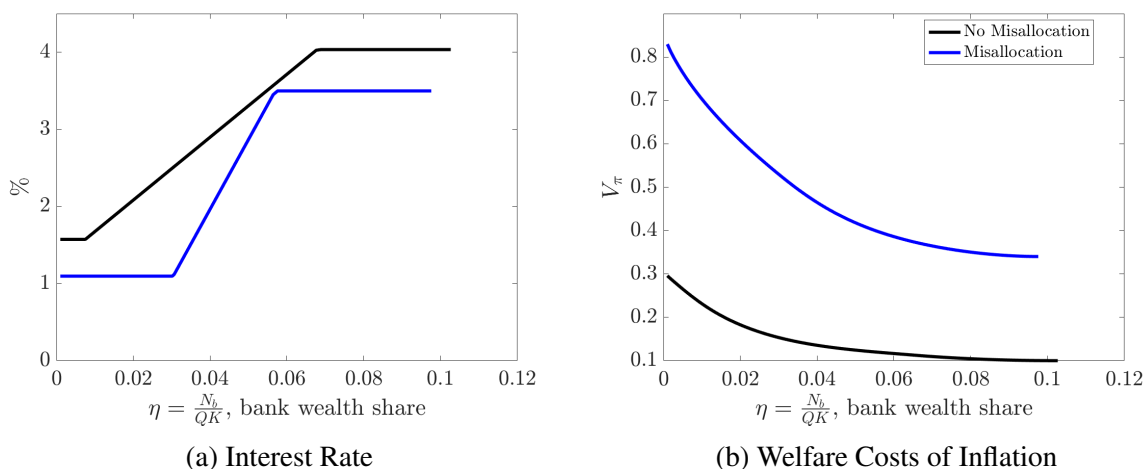


Figure 16: Nominal interest rate and welfare costs of inflation. The black line plots outcomes under OptNoMaP and no misallocation by banks while the blue line plots outcomes under the optimal monetary policy *with* bank misallocation.

The exogenous drop in bank productivity in (63) causes the optimal interest rate rule to become more accommodative in two ways. First, the strikes become closer together. It is optimal to delay raising rates for longer (η^{Put} increases) and to set $i = i^{LAW}$ sooner (η^{LAW} falls). The latter change results from the fact that the exogenous drop in bank productivity incentivizes banks to hold the entire capital stock for lower η . Second, the level of rates is everywhere lower. On the one hand, the risk-free interest rate has fallen due to banks' lower productivity, which lowers capital growth, so inflation costs make it optimal to set lower nominal rates. On the other hand, Figure 16(b) shows that the optimal policy rule with bank misallocation globally incurs *more* inflation costs. These higher costs result from the policy rule cutting rates more quickly and to a lower level when the economy is in distress or a crisis and not from setting i^{LAW} too high. In other words, the optimal rule deviates even more from inflation targeting in a crisis when bank misallocation is present.

Thus, the presence of bank misallocation, at least in the simple form in this analysis, does not overturn our results. On the contrary, it becomes optimal to implement a more aggressive

Put. The welfare losses from inflation targeting increase to -0.89% from -0.59% when there is no bank misallocation. The economy also spends more time in distress because output is highest when $\psi < 100\%$. With bank misallocation, the probability of distress increases by 4.39 percentage points, but the probability of crisis falls by 0.27 percentage points. Thus, the optimal rule achieves a higher duration of time in distress without increasing the likelihood of crisis.

Numerical Algorithm Allowing bank productivity to be an exogenous function of η changes the differential equation for Q . When $\psi = 1$ and leverage constraints are slack, we need to account for the fact that a_b depends directly on η . Re-compute the derivative as follows.

$$Q = \frac{a_h + (a_b(\eta) - a_h)\psi + \frac{1-\varepsilon\delta}{\varepsilon}}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}$$

$$\Rightarrow Q' = \frac{a'_b(\eta)}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}} - \frac{rQ(\theta'\eta + (\theta - 1))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}$$

When leverage constraints are binding,

$$Q = \frac{a_h + (a_b(\eta) - a_h)\eta(L(\eta) + 1) + \frac{1-\varepsilon\delta}{\varepsilon}}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}$$

$$\Rightarrow Q' = \frac{a'_b(\eta)\eta(L(\eta) + 1) + (a_b(\eta) - a_h)(L(\eta) + 1 + \eta L'(\eta))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}} - \frac{rQ(\theta'\eta + (\theta - 1))}{r(1 + (\theta - 1)\eta) + \frac{1}{\varepsilon}}.$$