14.452 Economic Growth: Lecture 10, Beyond Factor-Augmenting Technology: Factor Shares, Productivity and Inequality

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Introduction

- Thus far we have followed almost the entire macroeconomic and economic growth literatures, and focused on factor-augmenting technologies (Harrod neutral or skilled labor and unskilled labor augmenting).
- Does this matter?
- It is certainly convenient. But it is also very restrictive, and we will see that it has a range of (often unrecognized) implications.
- This lecture will develop one alternative building on "*task-based*" production.

Alternative: Task-Based Production

- An alternative that avoids these problems and makes progress in clarifying what might be going on is a task-based framework based on Zeira (1998), Acemoglu and Zilibotti (2000), Autor, Levy and Murnane (2003), Acemoglu and Autor (2011), Acemoglu and Restrepo (2018, 2019, 2020, 2022) and Acemoglu, Kong and Restrepo (2025).
- Main idea:
    - Give up the aggregate production function with factor-augmenting technological changes (what are they anyway?)
    - Consider the **allocation of tasks to factors** as the major economic choices affecting the demand for different types of factors.
- A more micro approach.
- But more importantly, very different comparative statics and implications.
- Let us first focus on just two factors, labor and capital, and then turn to a more general model with different types of labor to discuss

A First Model

- There is a unique final good (or alternatively a particular type of good in a specific sector) Y produced by combining a continuum of tasks $y(i)$, with $i \in [N-1, N]$.

$$Y = \left(\int_{N-1}^{N} y(i)^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}}, \lambda \in (0, \infty): \text{ elasticity of substitution.}$$

- Set the resulting ideal price index as numeraire.
- The range $N-1$ to N implies that the set of tasks is constant, but older tasks might be replaced by new (more complex and more productive) versions thereof.
- Namely, an increase in N adds a new task at the top while simultaneously replacing one at the bottom.
  - This structure is adopted for simplicity. Similar results with less clean algebra if integration is between $[0, N]$.

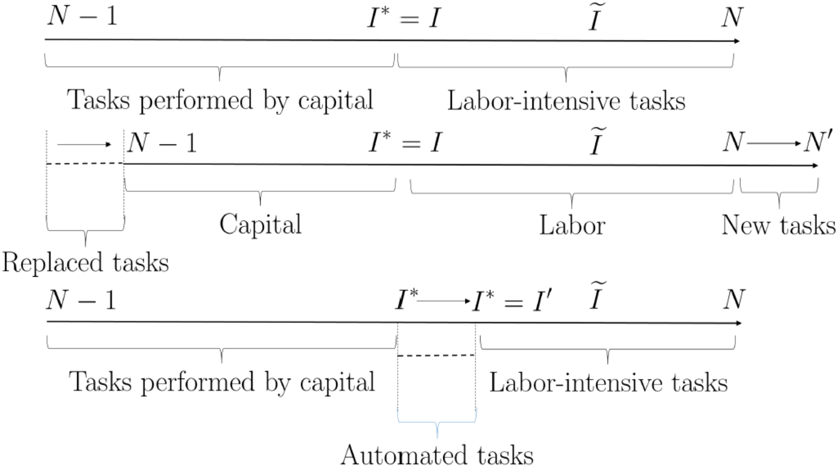
Task Production Function

- Tasks with $i \leq I$ are technologically **automated**, and can be produced with labor or capital.
- Tasks with $i > I$ are not technologically automated yet, and can only be produced with labor.

$$y(z) = \begin{cases} A^L \psi^L(z) l(z) + A^K \psi^K(z) k(z) & \text{if } z \in [N-1, I] \\ A^L \psi^L(z) l(z) & \text{if } z \in (I, N]. \end{cases}$$

- *Comparative advantage:* $\psi^L(z)/\psi^K(z)$ increasing in z — higher index tasks should be allocated to labor.
- *More importantly:* increases in I correspond to automation (at the extensive margin) and increases in N capture creation of new tasks
- Assumption that labor-intensive tasks use no capital can be relaxed.

Allocation of Tasks to Factors: Summary



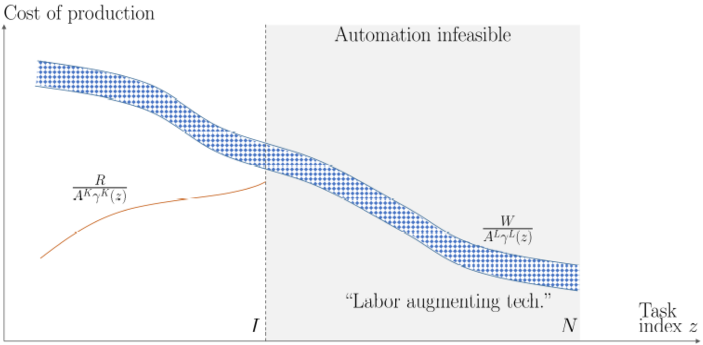
Factor Supplies

- At any point in time the stock of capital is fixed at K and capital is rented at a price r (determined endogenously).
- Suppose labor is inelastically supplied (easy to relax) and commands a wage W.
- Market clearing:

$$L = \int_{N-1}^{N} l(z)dz$$

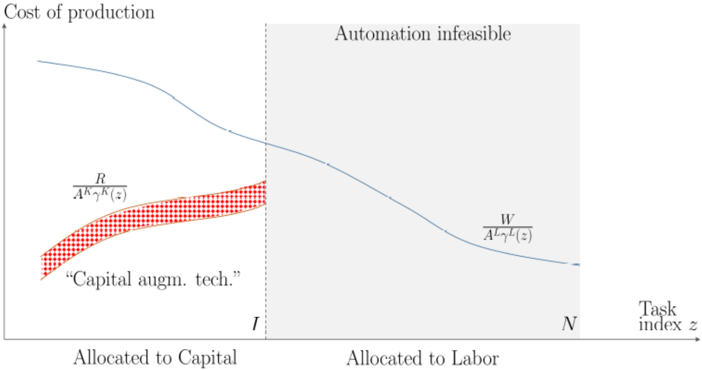
$$K = \int_{N-1}^{N} k(z)dz.$$

Effects of Different Technologies: Labor-Augmenting Technologies



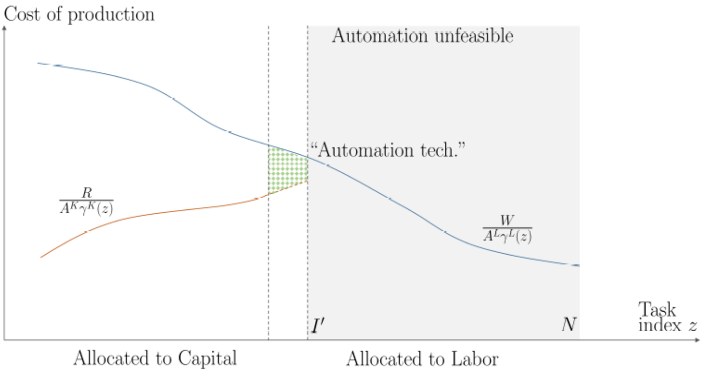
- Large productivity effects, but small distributional impacts because the allocation of tasks to factors doesn't change (much).

Effects of Different Technologies: Capital-Augmenting Technologies



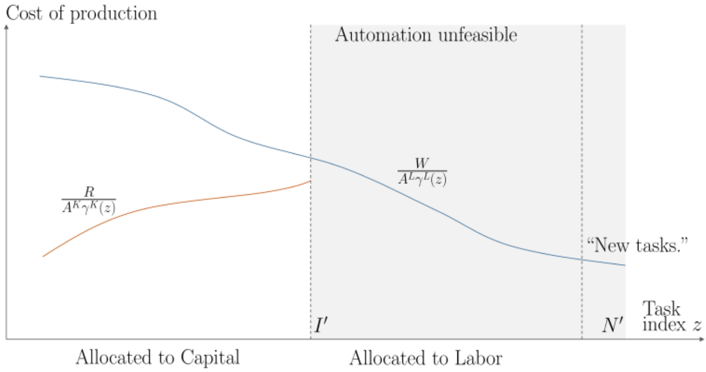
- Again, small distributional effects and large productivity gains.

Effects of Different Technologies: Automation



- Now large distributional effects and potentially small productivity gains.

Effects of Different Technologies: New Tasks



- Potentially large distributional effects and could be large or small productivity benefits.

Equilibrium

- In equilibrium, all tasks below some threshold $I^* \leq I$ are automated (allocated to capital) and the rest are allocated to labor.
- We assume

$$I^* = I.$$

- This will be the case when the capital to labor ratio or the wage to rental rate ratio are intermediate — in particular, when

$$\frac{1-\Gamma(I,N)}{\Gamma(I,N)}\left(\frac{A^L}{A^K}\frac{\psi^L(I)}{\psi^K(I)}\right)^{\lambda} < \frac{K}{L} < \frac{1-\Gamma(I,N)}{\Gamma(I,N)}\left(\frac{A^L}{A^K}\frac{\psi^L(N)}{\psi^K(N-1)}\right)^{\lambda}.$$

  Or when

$$\frac{A^L}{A^K}\frac{\psi^L(I)}{\psi^K(I)} < \frac{W}{R} < \frac{A^L}{A^K}\frac{\psi^L(N)}{\psi^K(N-1)}.$$

- This is a major *simplification* because it implies no competition between capital and labor for marginal tasks. Will be relaxed later.

Equilibrium Production Function

- Under these assumptions, we have a "derived" constant elasticity of substitution expression for output:

$$Y = \Pi(I, N)\left(\Gamma(I, N)^{\frac{1}{\lambda}}(A^L L)^{\frac{\lambda-1}{\lambda}} + (1 - \Gamma(I, N))^{\frac{1}{\lambda}}(A^K K)^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}}.$$

- In particular, let us define the *task content of production* as

$$\Gamma(I, N) = \frac{\int_I^N \psi^L(z)^{\lambda-1}}{\int_{N-1}^I \psi^K(z)^{\lambda-1} dz + \int_I^N \psi^L(z)^{\lambda-1}}.$$

  The TFP term is then

$$\Pi(I, N) = \left(\int_{N-1}^I \psi^K(z)^{\lambda-1} dz + \int_I^N \psi^L(z)^{\lambda-1}\right)^{\frac{1}{\lambda-1}}.$$

- Major difference from factor-augmenting models: the distribution parameters of the CES are endogenous because of the *changes in the task content of production* as a result of technological change.

Equilibrium: The Labor Share

- The labor share, WL/Y, is given as

$$s^L = \frac{1}{1 + \frac{1-\Gamma(I,N)}{\Gamma(I,N)} \left(\frac{A^L}{W} \frac{R}{A^K} \right)^{1-\lambda}}.$$

- **Key result**: separation of the effects of factor-augmenting technological changes and the task content of reduction.
- The elasticity of substitution intermediates only the former.

Equilibrium: Technology and Labor Demand

- For a given level of factor utilization, L and K, labor demand from the sector can be written as

$$W^d(L, K; \theta) = \frac{Y(L, K; \theta)}{L} \times s^L(L, K; \theta).$$

- Labor demand $W^d(L, K; \theta)$ is decreasing in L and increasing in K as should be expected.

Comparative Statics with Factor-Augmenting Technologies

- The comparative statics of labor demand with respect to factor-augmenting technologies are identical to those in the standard model.

$$\begin{aligned}\frac{\partial W^d(L,K;\theta)}{\partial \ln A^L} =& s^L(L,K;\theta) && \text{(Productivity effect)}\\ &+\frac{\lambda-1}{\lambda}(1-s^L(L,K;\theta)) && \text{(Substitution effect)},\\ \frac{\partial W^d(L,K;\theta)}{\partial \ln A^K} =& (1-s^L(L,K;\theta)) && \text{(Productivity effect)}\\ &+\frac{1-\lambda}{\lambda}(1-s^L(L,K;\theta)) && \text{(Substitution effect)}.\end{aligned}$$

- *Productivity effect* always positive: lower cost/higher productivity mean more labor demand (higher wages and/or employment).
- *Substitution effect* positive or negative depending on whether $\lambda > 1$ or not (and whether labor or capital is becoming more productive).

Factor-Augmenting Technologies (continued)

- Overall impact on labor demand always positive from A^K and also positive from A^L so long as λ is not too low (greater than $1 - s^L$ suffices).
- Impact on the labor share depends on whether $\lambda > 1$.
- In particular, when $\lambda = 1$, the substitution effect is zero, and labor demand changes proportionately with factor-augmenting technologies.
- This implies the labor share is constant.
  - The labor share falls with A^K when $\lambda > 1$ and falls with A^L when $\lambda < 1$.
  - But when λ is close to 1, not much change in the labor share in response to factor-augmenting technologies.

Comparative Statics with Automation

- Very different results from automation:

$$\frac{\partial \ln W^d(L,K;\theta)}{\partial I} = \frac{\partial \ln Y(L,K;\theta)}{\partial I} + \qquad \text{(Productivity effect)}$$
$$\frac{1}{\lambda}\frac{1-s^L(L,K;\theta)}{1-\Gamma(I,N)}\frac{\partial \ln \Gamma(I,N)}{\partial I} \qquad \text{(Displacement effect)}.$$

- *The displacement effect* results from the direct displacement of labor as some tasks are automated, and is negative.
- **Immediate implication:** automation *always reduces* the labor share (regardless of the value of λ).
- The productivity effect is

$$\frac{\partial \ln Y(L,K;\theta)}{\partial I} = \frac{1}{\lambda-1}\left[\left(\frac{R}{A^K\psi^K(I)}\right)^{1-\lambda} - \left(\frac{W}{A^L\psi^L(I)}\right)^{1-\lambda}\right] > 0.$$

Comparative Statics with Automation (continued)

- When new automation technologies are **"so-so"** (meaning that the effective cost of producing with automation is approximately the same as the effective cost of producing with labor), then $\partial \ln Y(L, K; \theta)/\partial I \approx 0$ and the productivity effect is essentially zero.
- Then automation **reduces** labor demand — it reduces wages and/or employment.
- The same is possible even when automation increases productivity, but creates an even bigger displacement effect.

Comparative Statics with New Tasks

- Opposite results with the introduction of new tasks:

$$\frac{\partial \ln W^d(L,K;\theta)}{\partial N} = \frac{\partial \ln Y(L,K;\theta)}{\partial N} + \qquad \text{(Productivity effect)}$$

$$\frac{1}{\lambda}\frac{1-s^L}{1-\Gamma(I,N)}\frac{\partial \ln \Gamma(I,N)}{\partial N} \qquad \text{(Reinstatement effect)}$$

- *The reinstatement effect* is always positive, so labor demand always expands and does so more than proportionately with the productivity effect.
- Hence, new tasks always increase the labor share.

Incorporating Growth

- This setup can be embedded into a growth model and is in fact consistent with balanced growth if automation and new tasks take place at commensurate rates.
- Following Acemoglu and Restrepo (2018) suppose:

$$\psi^K(i) = \psi^K \text{ and } \psi^L(i) = e^{Bi} \text{ with } B > 0.$$

- Represent the path of technology as

$$n(t) = N(t) - I(t).$$

- Then if $n(t)$ asymptotically converges to a constant, there is **balanced growth**.
- *Intuition:* automation is balanced by the creation of new tasks (as we observe in the US data between 1947 and 1987).

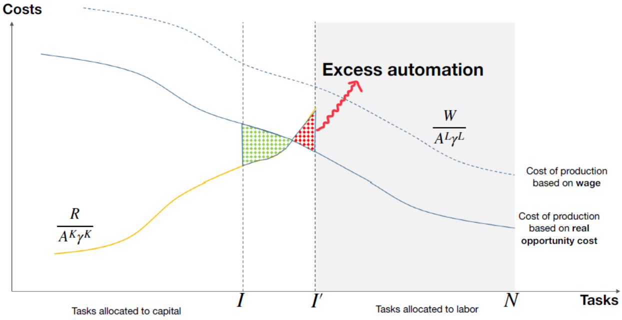
Why Balance?

- Why should we expect balance between automation and new tasks?
- Acemoglu and Restrepo (2018): *directed technological change.*
- Faster automation reduces the labor share significantly, and makes the creation of new tasks more profitable.
- Does this imply that the current decline in the labor share will be reversed?
- Not necessarily:
  - Large changes in technology can push the economy out of the basin of attraction of BGP (towards Leontief's "horse equilibrium").
  - Changes in the innovation possibilities frontier may lead to a different stable equilibrium, with the lower labor share.
  - Inefficiencies.

Inefficiencies in the Task-Based Framework

- New sources of inefficiencies—typically in the direction of **excessive automation:**
  - Labor market imperfections (increasing the wage above the social opportunity cost) can lead to inefficient automation
  - Automation may affect bargaining.
  - Other social effects from lower labor share, job loss etc.
  - Tax policy excessively favorable to capital and indirectly to automation—Acemoglu, Manera and Restrepo (2020), effectiv taxes on labor around 25%, while on capital they have fallen to around 5%, triggering excessive automation.

Excessive Automation in a Diagram



- Red area, subtracting from TFP can be larger than the green area—TFP losses from excessive automation.
- Red area is also a measure of inefficiency.

Task-Based Approach to Inequality

- Once again, the task-based approach provides richer perspectives.
- Exposition here follows Acemoglu and Restrepo (2022) and Acemoglu, Kong and Restrepo (2025).
- Many sectors, many factors, many tasks.
- Also relax the assumption that there is no competition between factors for marginal tasks.

Formal Model

- Extend the task-based model from the previous lecture to include several groups of workers (to discuss inequality).
- Also for simplicity, ignore new tasks.
- Start with a single sector model again, where output is given as:

$$y = \left(\frac{1}{M} \int_{\mathcal{T}} (M \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda-1}} .$$

- Here λ is the elasticity of substitution between tasks and $\mathcal{T}$ is the set of tasks required for this product.

Formal Model (continued)

- Task services are provided either by capital or one of G types of labor, according to production functions:

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g \in \mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x).$$

- Here, $\ell_g(x)$ is the amount of labor of type g allocated to task x, while $k(x)$ is the amount of task-specific capital produced for and assigned to this task.
- The A_k and A_g represent standard factor-augmenting technologies, which make factors uniformly more productive at all tasks.
- Task-specific productivity functions, $\psi_k(x)$ and $\{\psi_g(x)\}_{g \in \mathcal{G}}$, provide information on comparative advantage.
- If capital or some other factor cannot perform a task, its productivity is zero.

Formal Model (continued)

- Let us suppose that capital is produced from final good with constant marginal cost $1/q(x)$.
- Then a competitive equilibrium maximizes net output:

$$c = y - \int_{\mathcal{T}} (k(x)/q(x)) \cdot dx.$$

- Labor of all types is supplied inelastically, and we denote the total supply of labor of type g by ℓ_g.

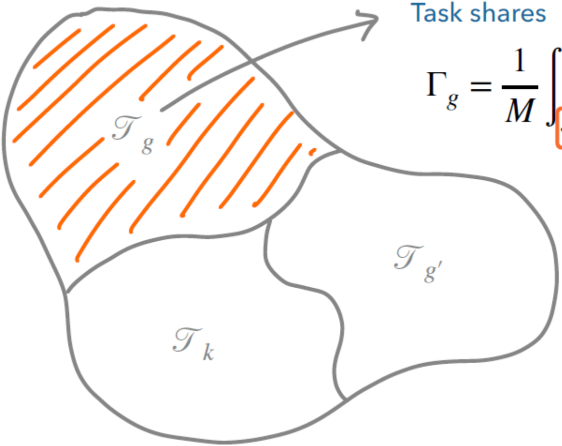
Equilibrium

- A unique equilibrium exists and in equilibrium, output is given by

$$y = \left(\Gamma_k^{\frac{1}{\lambda}} \cdot (A_k \cdot k)^{\frac{\lambda-1}{\lambda}} + \sum_g \Gamma_g{}^{\frac{1}{\lambda}} \cdot (A_g \cdot \ell_g)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}} .$$

- Here, k is the total amount of capital stock used in production (produced out of the final good).
- Γ_k and Γ_g's are *task shares*, which are endogenous and encapsulate the most important economic interactions.
- This production function looks like a standard CES. It is not.
- Task shares are endogenous, rather than parameters.
- The elasticity of substitution is not λ, but an endogenous quantity $\sigma \geq \lambda$.
- More importantly, most of the technology affects will work through changes in the task shares.

Tasks Shares

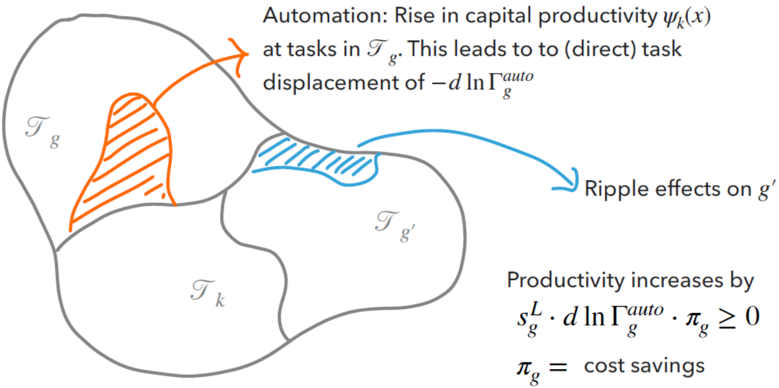


Task shares

$$\Gamma_g = \frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda - 1} \cdot dx$$

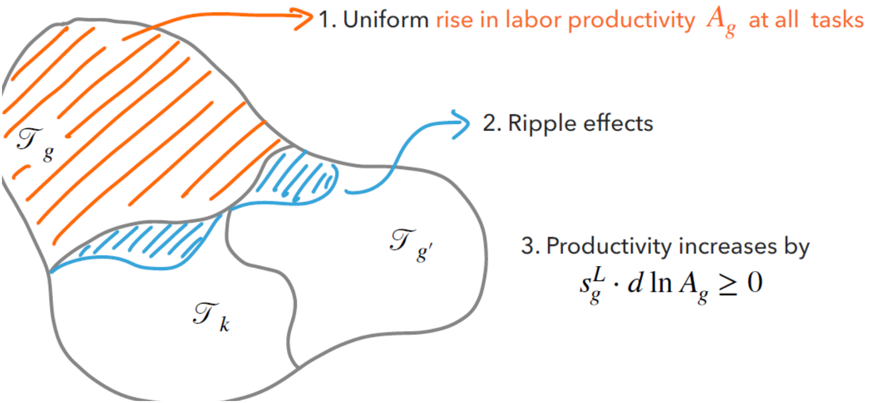
Set of tasks allocated to g

Automation, Displacement and Ripple Effects



- These ripple effects were ruled out by assumption until now. Will be studied now.

Factor-Augmenting Technology: No Displacement



Effects of Automation

- Wage and productivity effects of automation are given as

$$d \ln w_g = \frac{1}{\lambda} d \ln y + \frac{\lambda - 1}{\lambda} d \ln \tilde{A}_g - \frac{1}{\lambda} d \ln \Gamma_g^{\text{auto}}$$
$$+ \text{ Endogenous Substitution Effects}_g + \text{Ripple Effects}_g ,$$
$$d \ln \text{tfp} = \sum_{g \in \mathcal{G}} s_g^L \cdot d \ln \tilde{A}_g + s^K \cdot d \ln \tilde{A}_k + \sum_{g \in \mathcal{G}} s_g^L \cdot d \ln \Gamma_g^{\text{auto}} \cdot \pi_g .$$

- The direct, task-displacement effect of automation on labor type g:

$$d \ln \Gamma_g^{\text{auto}} = \frac{\frac{1}{M} \int_{\mathcal{D}_g} \psi_g(x)^{\lambda - 1} dx}{\frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda - 1} dx} .$$

- π_g is cost-savings from direct displacement (impacting labor type g). Moreover, $d \ln \tilde{A}_g = d \ln A_g + d \ln \Gamma_g^{\text{deep}}$ and $d \ln \tilde{A}_k = d \ln A_k + d \ln \Gamma_k^{\text{deep}}$ are composite terms that capture factor-augmenting technological change technological changes.

Macroeconomic Elasticities

- How do we understand these endogenous substitution effects? From competition for marginal tasks.
- Specifically, in the task framework, for $g' \neq g$, the macroeconomic elasticity of substitution between groups g and g' is

$$\sigma_{gg'} = \underbrace{\lambda}_{\text{substitution between tasks}} + \underbrace{\frac{1}{s^y_{g'}} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w_{g'}}}_{\text{substitution within marginal tasks}}$$

- Macro elasticities which involve reassignment of tasks between groups are different than micro-elasticities.
- To understand these more fully, we need to study the propagation matrix.

The Propagation Matrix

- From $w_g = (y/\ell_g)^{1/\lambda} \cdot A_g^{(\lambda-1)/\lambda} \cdot \Gamma_g(w)^{1/\lambda}$, a direct shock z_g to group g's demand yields

$$d \ln w_g = z_g + \frac{1}{\lambda} \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot d \ln w$$

which implies a total impact on wages

$$d \ln w = \left(\mathbb{I} - \frac{1}{\lambda} \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \right)^{-1} \cdot z = \Theta \cdot z.$$

- The propagation matrix (in the form of the Leontief inverse) captures the full general equilibrium effect to a first order and is isomorphic to macro elasticities.
- Let the matrix of elasticities of substitution be denoted by $\Sigma = \{\sigma_{gg'}\}_{g,g' \in \mathbb{G}}$. Then

$$\Theta = \text{diag}\left(\frac{1}{s^y}\right) \cdot (\lambda - \Sigma)^{-1}.$$

Multiple Sectors

- Now extend this set up to multiple sectors, each with a similar structure to the single-sector economy above:

$$y_i = A_i \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_i} (M_i \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda-1}} .$$

- In addition, A_i is a Hicks-neutral productivity term. $\mathcal{T}_{gi}$ denotes the set of tasks in industry i allocated to workers of type g, and $\mathcal{T}_{ki}$ denotes those allocated to capital.
- Industry-level task shares, Γ_{gi} and Γ_{ki} are now

$$\Gamma_{gi}(\mathbf{w}, \Psi) = \frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} \cdot dx; \; \Gamma_{ki}(\mathbf{w}, \Psi) = \frac{1}{M_i} \int_{\mathcal{T}_{ki}} (\psi_k(x) \cdot q(x))^{\lambda}$$

- Finally, household budgets are allocated across sectors with expenditure shares $s_i^Y(\mathbf{p})$, where $\mathbf{p} = (p_1, p_2, \ldots, p_I)$ is the vector of industry prices.

Equilibrium

- A unique competitive equilibrium exists again and has a similar characterization. In particular, wage effects of technology are given as:

$$\begin{aligned} d \ln w_g \quad = \quad & \frac{1}{\lambda} d \ln y + \frac{1}{\lambda} \sum_{i \in \mathcal{I}} \omega_g^i \cdot d \ln \zeta_i + \frac{\lambda - 1}{\lambda} d \ln \tilde{A}_g \\ & - \frac{1}{\lambda} \sum_{i \in \mathcal{I}} \omega_g^i \cdot d \ln \Gamma_{gi}^{\text{auto}} + \text{ Substitution/Ripple Effects}_g . \end{aligned}$$

- The new term is $\frac{1}{\lambda} \sum_{i \in \mathcal{I}} \omega_g^i \cdot d \ln \zeta_i$, which represents the consequences of sectoral reallocation following from technology changes (ω_g^i is the share of labor income of group g earned in industry i, capturing the importance of the sector for the labor demand for group g.
- In addition, the direct task displacement effect now combines displacements across sectors.

Measurement: Additional Assumptions

- Let us simplify measurement by imposing the following two assumptions:

Assumption

(parallel automation) *Only routine tasks can be automated and, within an industry, different groups of workers are displaced from their routine tasks at a common rate.*

- Let us also ignore ripple effects to start with.

Measurement: Task Displacement

- Under these assumptions, (direct) task displacement can be measured as

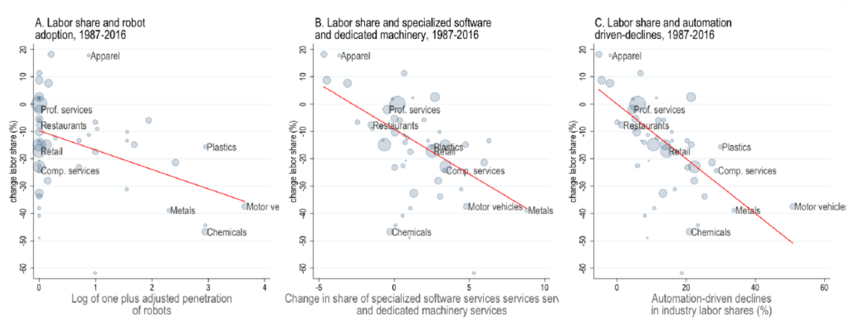
$$\text{Task displacement}_g^{\text{direct}} = \sum_{i \in \mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot \left(-d \ln s_i^{L,\text{auto}} \right).$$

- Note everything is filtered through industry labor share changes (induced by automation), denoted by $d \ln s_i^{L,\text{auto}}$, and the term $\omega_{gi}^R / \omega_i^R$ represents the specialization of group g in industry i (formally, share of labor income in routine tasks in this industry going to demographic group g).
- This task displacement measure can be interpreted as fraction of tasks in which a group has a comparative advantage that are then lost automation.

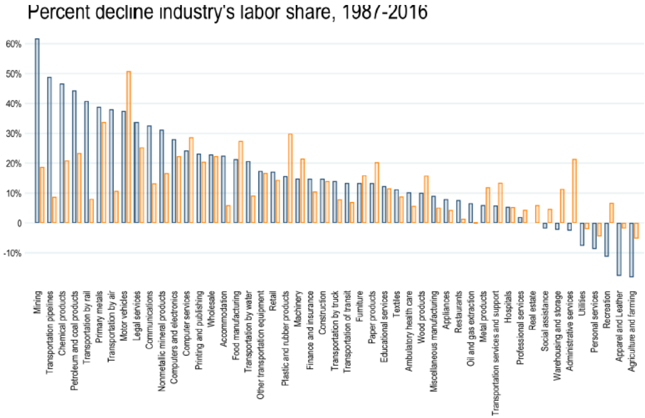
Measurement: Practical Issues

- Now task displacement at group level can be measured using standard Census/ACS type data sets, which include information on demographics, employment, wages, industry and occupation, combined with estimates of $d \ln s_i^{L,\text{auto}}$, which can be obtained by regressing industry-level labor share changes on proxies for the adoption of automation technologies.
- In practice, use advances in industry-level robotics technology; measures of specialized software investment at the industry level; and measures of specialized equipment at the industry level.

Industry Labor Share Changes

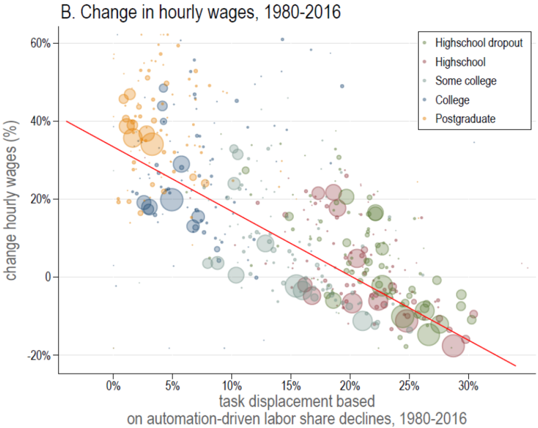


Overall and Automation-Induced Industry Labor Share Declines



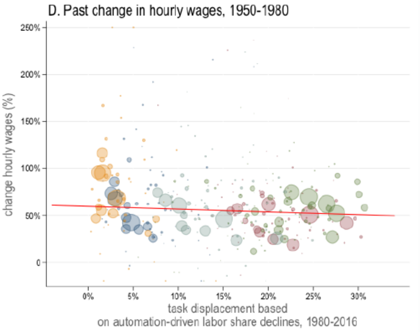
- Blue: labor share decline. Orange: automation-driven decline ($R^2 = 50\,\%$)

Main Result


B. Change in hourly wages, 1980-2016
Highschool dropout
Highschool
Some college
College
Postgraduate
change hourly wages (%)
60%
40%
20%
0
-20%
0%
5%
10%
15%
20%
25%
30%
task displacement based
on automation-driven labor share declines, 1980-2016

Quantitative Magnitudes and Pre-Trends

- Task displacement (due to automation and offshoring) explains between 50% and 70% of all between-group wage inequality changes in the US since 1980.
- Most of this (about 90%) due to automation.
- No similar trends before 1980, before the onset of digital and robotics and automation technologies.



Robustness

- Very robust across various specifications. For example, to flexibly controlling for
  - other types of skill-biased technological change (which appear unimportant).
  - other types of capital deepening and technological change (not themselves important and no similar effects).
  - import competition from China (turns out to be much less important than automation for inequality)
  - various labor market institutional features (some effects, but less important)
  - product market concentration and markups (no similar effects, though some impacts through compositional changes).
- Similar patterns across regions.

What is Left Out?

- General equilibrium effects — in particular ripple effects and induced composition effects — also need to be estimated.
- The extent of productivity effects need to be estimated separately.
- For a general methodology for doing so, see paper.
- When these are incorporated, quantitative magnitudes are similar, but in a more nuanced way.
  - Direct effects become somewhat less important but ripple effects are present and quantitatively significant.
  - Ripple effects "democratize" wage losses, as the direct effects of automation are spread to indirectly-affected groups.
  - There are positive productivity effects from automation, but not large enough to eliminate wage losses for many demographic groups.
  - Thus, large distributional effects from automation coupled with small productivity gains.
- Why all of this since 1980? Perhaps because more automation and less focus on new tasks today.

Conclusion

- Task-based models to aggregate production and growth thus also useful for studying inequality.
- Much to be done both theoretically and empirically.