14.452 Economic Growth: Lecture 11, Directed Technological Change

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Introduction

- Thus far have focused on a single type of technological change (e.g., Hicks-neutral).
- But, technological change is often not neutral:
  1. Benefits some factors of production and some agents more than others. Distributional effects imply some groups will embrace new technologies and others oppose them.
  2. Limiting to only one type of technological change obscures the competing effects that determine the nature of technological change.
- *Directed technological change*: endogenize the direction and bias of new technologies that are developed and adopted.

Not Always Skill Biased

- Late 18th and early 19th *unskill-bias:*
  "First in firearms, then in clocks, pumps, locks, mechanical reapers, typewriters, sewing machines, and eventually in engines and bicycles, interchangeable parts technology proved superior and replaced the skilled artisans working with chisel and file." (Mokyr 1990, p. 137)
- Why was technological change unskilled-biased then and skilled-biased now?
- Recent changes in technology increasing automation and the capital share? Is that capital biased or labor bias?

Directed Technological Change: Basic Arguments I

- Two factors of production, say L and H (unskilled and skilled workers).
- Two types of technologies that can complement either one or the other factor.
- Whenever the profitability of H-augmenting technologies is greater than the L-augmenting technologies, more of the former type will be developed by profit-maximizing (research) firms.
- What determines the relative profitability of developing different technologies? It is more profitable to develop technologies...
  1. when the goods produced by these technologies command higher prices (*price effect*);
  2. that have a larger market (*market size effect*).

Equilibrium Relative Bias

- Potentially counteracting effects, but the market size effect will be more powerful often.
- Under fairly general conditions:
    - *Weak Equilibrium (Relative) Bias*: an increase in the relative supply of a factor always induces technological change that is biased in favor of this factor.
    - *Strong Equilibrium (Relative) Bias*: if the elasticity of substitution between factors is sufficiently large, an increase in the relative supply of a factor induces sufficiently strong technological change biased towards itself that the endogenous-technology relative demand curve of the economy becomes *upward-sloping*.

Equilibrium Relative Bias in More Detail

- Suppose the (inverse) relative demand curve:

$$w_H / w_L = D(H/L, A)$$

  where w_H / w_L is the relative price of the factors and A is technology.
- A is H-biased if D is increasing in A, so that a higher A increases the relative demand for the H factor.
- D is *always* decreasing in H/L.

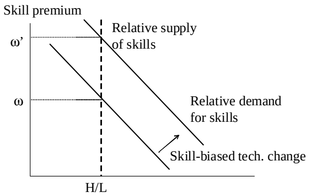


Figure: The effect of H-biased technological change

Weak and Strong Relative Bias

- Equilibrium bias: behavior of A as H/L changes,

$$A\left(H/L\right)$$

- Weak equilibrium bias:
  - $A\left(H/L\right)$ is increasing (nondecreasing) in H/L.
- Strong equilibrium bias:
  - $A\left(H/L\right)$ is sufficiently responsive to an increase in H/L that the total effect of the change in relative supply H/L is to increase w_H/w_L.
  - i.e., let the endogenous-technology relative demand curve be

$$w_H/w_L = D\left(H/L, A\left(H/L\right)\right) \equiv \tilde{D}\left(H/L\right)$$

$\rightarrow$*Strong equilibrium bias:* $\tilde{D}$ **increasing in** H/L.

Equilibrium Bias

- **Weak equilibrium bias** of technology: an increase in H/L, induces technological change biased towards H. For example, in the constant elasticity of substitution case we saw in the previous lecture, this means increasing A_H/A_L when $\sigma > 1$, and means reducing A_H/A_L when $\sigma < 1$.
- **Strong equilibrium bias:** an increase in H/L induces a sufficiently large change in the bias so that the relative marginal product of H relative to that of L increases following the change in factor supplies:

$$\frac{dMP_H/MP_L}{dH/L} > 0,$$

- The major difference is whether the relative marginal product of the two factors are evaluated at the initial relative supplies (weak bias) or at the new relative supplies (strong bias).

Baseline Model of Directed Technical Change I

- Framework: expanding varieties model with lab equipment specification of the innovation possibilities frontier (so none of the results here depend on technological externalities).
- Constant supply of L and H.

- Representative household with the standard CRRA preferences:

$$\int_0^\infty \exp\left(-\rho t\right) \frac{C\left(t\right)^{1-\theta}-1}{1-\theta} dt, \tag{1}$$

- Aggregate production function:

$$Y\left(t\right) = \left[\gamma_L Y_L\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_H Y_H\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where intermediate good $Y_L\left(t\right)$ is L-intensive, $Y_H\left(t\right)$ is H-intensive.

Baseline Model of Directed Technical Change II

- Resource constraint (define $Z(t) = Z_L(t) + Z_H(t)$):

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{3}$$

- Intermediate goods produced competitively with:

$$Y_L(t) = \frac{1}{1-\beta}\left(\int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu\right) L^\beta \tag{4}$$

and

$$Y_H(t) = \frac{1}{1-\beta}\left(\int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu\right) H^\beta, \tag{5}$$

where machines $x_L(\nu, t)$ and $x_H(\nu, t)$ are assumed to depreciate after use.

Baseline Model of Directed Technical Change III

- Differences with baseline expanding product varieties model:
  1. These are production functions for intermediate goods rather than the final good.
  2. (4) and (5) use different types of machines–different ranges $\left[0, N_L\left(t\right)\right]$ and $\left[0, N_H\left(t\right)\right]$.
- All machines are supplied by monopolists that have a fully-enforced perpetual patent, at prices $p_L^x\left(\nu, t\right)$ for $\nu \in \left[0, N_L\left(t\right)\right]$ and $p_H^x\left(\nu, t\right)$ for $\nu \in \left[0, N_H\left(t\right)\right]$.
- Once invented, each machine can be produced at the fixed marginal cost ψ in terms of the final good.
- Normalize to $\psi \equiv 1 - \beta$.

Baseline Model of Directed Technical Change IV

- Total resources devoted to machine production at time t are

$$X(t) = (1-\beta)\left(\int_0^{N_L(t)} x_L(\nu,t)\,d\nu + \int_0^{N_H(t)} x_H(\nu,t)\,d\nu\right).$$

- Innovation possibilities frontier:

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t), \tag{6}$$

- Value of a monopolist that discovers one of these machines is:

$$V_f(\nu,t) = \int_t^{\infty} \exp\left[-\int_t^s r(s')\,ds'\right] \pi_f(\nu,s)ds, \tag{7}$$

where $\pi_f(\nu,t) \equiv p_f^x(\nu,t)x_f(\nu,t) - \psi x_f(\nu,t)$ for $f = L$ or H.

- Hamilton-Jacobi-Bellman version:

$$r(t)\,V_f(\nu,t) - \dot{V}_f(\nu,t) = \pi_f(\nu,t). \tag{8}$$

Baseline Model of Directed Technical Change V

- Normalize the price of the final good at every instant to 1, which is equivalent to setting the ideal price index of the two intermediates equal to one, i.e.,

$$\left[\gamma_L^{\varepsilon}\left(p_L\left(t\right)\right)^{1-\varepsilon}+\gamma_H^{\varepsilon}\left(p_H\left(t\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}=1 \text{ for all } t, \tag{9}$$

  where $p_L\left(t\right)$ is the price index of Y_L at time t and $p_H\left(t\right)$ is the price of Y_H.
- Denote factor prices by $w_L\left(t\right)$ and $w_H\left(t\right)$.

Equilibrium I

- Maximization problem of producers in the two sectors:

$$\max_{L,[x_L(\nu,t)]_{\nu\in[0,N_L(t)]}} p_L(t)\, Y_L(t) - w_L(t)\, L - \int_0^{N_L(t)} p_L^x(\nu,t)\, x_L(\nu,t)\, d\nu, \tag{10}$$

and

$$\max_{H,[x_H(\nu,t)]_{\nu\in[0,N_H(t)]}} p_H(t)\, Y_H(t) - w_H(t)\, H - \int_0^{N_H(t)} p_H^x(\nu,t)\, x_H(\nu,t)\, d\nu. \tag{11}$$

- Note the presence of $p_L(t)$ and $p_H(t)$, since these sectors produce intermediate goods.

Equilibrium II

- Thus, demand for machines in the two sectors:

$$x_L(\nu, t) = \left[\frac{p_L(t)}{p_L^x(\nu, t)}\right]^{1/\beta} L \quad \text{for all } \nu \in [0, N_L(t)] \text{ and all } t, \quad (12)$$

and

$$x_H(\nu, t) = \left[\frac{p_H(t)}{p_H^x(\nu, t)}\right]^{1/\beta} H \quad \text{for all } \nu \in [0, N_H(t)] \text{ and all } t. \quad (13)$$

- Maximization of the net present discounted value of profits implies a constant markup:

$$p_L^x(\nu, t) = p_H^x(\nu, t) = 1 \text{ for all } \nu \text{ and } t.$$

Equilibrium III

- Substituting into (12) and (13):

$$x_L(\nu, t) = p_L(t)^{1/\beta} L \quad \text{for all } \nu \text{ and all } t,$$

and

$$x_H(\nu, t) = p_H(t)^{1/\beta} H \quad \text{for all } \nu \text{ and all } t.$$

- Since these quantities do not depend on the identity of the machine profits are also independent of the machine type:

$$\pi_L(t) = \beta p_L(t)^{1/\beta} L \text{ and } \pi_H(t) = \beta p_H(t)^{1/\beta} H. \tag{14}$$

- Thus the values of monopolists only depend on which sector they are, $V_L(t)$ and $V_H(t)$.

Equilibrium IV

- Combining these with (4) and (5), *derived* production functions for the two intermediate goods:

$$Y_L(t) = \frac{1}{1-\beta} p_L(t)^{\frac{1-\beta}{\beta}} N_L(t) L \tag{15}$$

and

$$Y_H(t) = \frac{1}{1-\beta} p_H(t)^{\frac{1-\beta}{\beta}} N_H(t) H. \tag{16}$$

Equilibrium V

- For the prices of the two intermediate goods, (2) imply

$$\begin{aligned} p(t) &\equiv \frac{p_H(t)}{p_L(t)} = \gamma \left(\frac{Y_H(t)}{Y_L(t)} \right)^{-\frac{1}{\varepsilon}} \\ &= \gamma \left(p(t)^{\frac{1-\beta}{\beta}} \frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{1}{\varepsilon}} \\ &= \gamma^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{\beta}{\sigma}}, \qquad (17) \end{aligned}$$

where $\gamma \equiv \gamma_H / \gamma_L$ and

$$\begin{aligned} \sigma &\equiv \varepsilon - (\varepsilon - 1)(1 - \beta) \\ &= 1 + (\varepsilon - 1)\beta. \end{aligned}$$

Equilibrium VI

- We can also calculate the relative factor prices:

$$\begin{aligned} \omega(t) &\equiv \frac{w_H(t)}{w_L(t)} \\ &= p(t)^{1/\beta} \frac{N_H(t)}{N_L(t)} \\ &= \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H(t)}{N_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L} \right)^{-\frac{1}{\sigma}}. \end{aligned} \tag{18}$$

- σ is the (derived) elasticity of substitution between the two factors, since it is exactly equal to

$$\sigma = -\left(\frac{d \log \omega(t)}{d \log (H/L)} \right)^{-1}.$$

Equilibrium VII

- Free entry conditions:

$$\eta_L V_L(t) \leq 1 \text{ and } \eta_L V_L(t) = 1 \text{ if } Z_L(t) > 0. \tag{19}$$

and

$$\eta_H V_H(t) \leq 1 \text{ and } \eta_H V_H(t) = 1 \text{ if } Z_H(t) > 0. \tag{20}$$

- Consumer side:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \tag{21}$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)(N_L(t)V_L(t) + N_H(t)V_H(t))\right] = 0, \tag{22}$$

where $N_L(t)V_L(t) + N_H(t)V_H(t)$ is the total value of corporate assets in this economy.

Balanced Growth Path I

- Consumption grows at the constant rate, g^*, and the relative price $p(t)$ is constant. From (9) this implies that $p_L(t)$ and $p_H(t)$ are also constant.
- Let V_L and V_H be the BGP net present discounted values of new innovations in the two sectors. Then (8) implies that

$$V_L = \frac{\beta p_L^{1/\beta} L}{r^*} \text{ and } V_H = \frac{\beta p_H^{1/\beta} H}{r^*}, \tag{23}$$

- Taking the ratio of these two expressions, we obtain

$$\frac{V_H}{V_L} = \left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}} \frac{H}{L}.$$

Balanced Growth Path II

- Note the two effects on the direction of technological change:
  1. The price effect: V_H / V_L is increasing in p_H / p_L. Tends to favor technologies complementing scarce factors.
  2. The market size effect: V_H / V_L is increasing in H / L. It encourages innovation for the more abundant factor.
- The above discussion is incomplete since prices are endogenous. Combining (23) together with (17):

$$\frac{V_H}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}. \tag{24}$$

- Note that an increase in H / L will increase V_H / V_L as long as $\sigma > 1$ and it will reduce it if $\sigma < 1$. Moreover,

$$\sigma \gtreqqless 1 \iff \varepsilon \gtreqqless 1.$$

- The two factors will be gross substitutes when the two intermediate goods are gross substitutes in the production of the final good.

Balanced Growth Path III

- Next, using the two free entry conditions (19) and (20) as equalities, we obtain the following BGP "technology market clearing" condition:

$$\eta_L V_L = \eta_H V_H. \tag{25}$$

- Combining this with (24), BGP ratio of relative technologies is

$$\left(\frac{N_H}{N_L}\right)^* = \eta^\sigma \gamma^\varepsilon \left(\frac{H}{L}\right)^{\sigma-1}, \tag{26}$$

  where $\eta \equiv \eta_H / \eta_L$.

- Note that relative productivities are determined by the innovation possibilities frontier and the relative supply of the two factors. In this sense, this model totally endogenizes technology.

Summary of Balanced Growth Path

Proposition Consider the directed technological change model described above. Suppose

$$\beta\left[\gamma_H^{\varepsilon}\left(\eta_H H\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} > \rho \quad (27)$$

$$\text{and} \quad (1-\theta)\,\beta\left[\gamma_H^{\varepsilon}\left(\eta_H H\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} < \rho.$$

Then there exists a unique BGP equilibrium in which the relative technologies are given by (26), and consumption and output grow at the rate

$$g^{*}=\frac{1}{\theta}\left(\beta\left[\gamma_H^{\varepsilon}\left(\eta_H H\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}-\rho\right). \quad (28)$$

Transitional Dynamics

- Differently from the baseline endogenous technological change models, there are now transitional dynamics (because there are two state variables).
- Nevertheless, transitional dynamics simple and intuitive:

Proposition Consider the directed technological change model described above. Starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (26), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. If $N_H(0)/N_L(0) > (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

- Summary: the dynamic equilibrium path always tends to the BGP and during transitional dynamics, there is only one type of innovation.

Directed Technological Change and Factor Prices

- In BGP, there is a positive relationship between H/L and N_H^*/N_L^* only when $\sigma > 1$.
- But this does not mean that depending on σ (or ε), changes in factor supplies may induce technological changes that are biased in favor or against the factor that is becoming more abundant.
- Why?
  - N_H^*/N_L^* refers to the ratio of factor-augmenting technologies, or to the ratio of *physical* productivities.
  - What matters for the bias of technology is *the value of marginal product* of factors, affected by relative prices.
  - The relationship between factor-augmenting and factor-biased technologies is reversed when σ is less than 1.
  - When $\sigma > 1$, an increase in N_H^*/N_L^* is relatively biased towards H, while when $\sigma < 1$, a *decrease* in N_H^*/N_L^* is relatively biased towards H.

Weak Equilibrium (Relative) Bias Result

Proposition Consider the directed technological change model described above. There is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

- The results reflect the strength of the market size effect: it always dominates the price effect.
- But it does not specify whether this induced effect will be strong enough to make the endogenous-technology relative demand curve for factors upward-sloping.

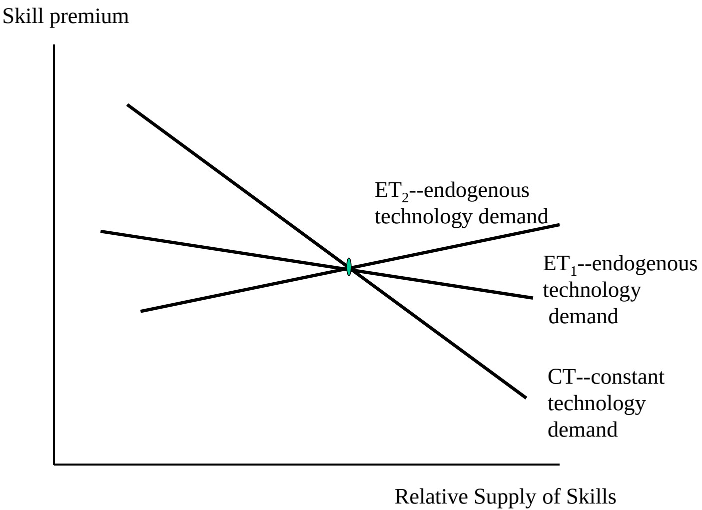
Strong Equilibrium (Relative) Bias Result

- Substitute for $(N_H/N_L)^*$ from (26) into the expression for the relative wage given technologies, (18), and obtain:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\sigma-1}\gamma^{\varepsilon}\left(\frac{H}{L}\right)^{\sigma-2}. \tag{29}$$

Proposition Consider the directed technological change model described above. Then if $\sigma > 2$, there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the factor H compared to factor L.

Relative Supply of Skills and Skill Premium



Discussion

- Analogous to Samuelson's *LeChatelier principle*: think of the endogenous-technology demand curve as adjusting the "factors of production" corresponding to technology.
- But, the effects here are caused by general equilibrium changes, not on partial equilibrium effects.
- Moreover ET_2, which applies when $\sigma > 2$ holds, is upward-sloping.
- A complementary intuition: importance of non-rivalry of ideas:
  - leads to an aggregate production function that exhibits increasing returns to scale (in all factors including technologies).
  - the market size effect can create sufficiently strong induced technological change to increase the relative marginal product and the relative price of the factor that has become more abundant.

Implications I

- Recall we have the following stylized facts:
  - Secular skill-biased technological change increasing the demand for skills throughout the 20th century.
  - Possible acceleration in skill-biased technological change over the past 25 years.
  - A range of important technologies biased against skill workers during the 19th century.
- The current model gives us a way to think about these issues.
  - The increase in the number of skilled workers should cause steady skill-biased technical change.
  - Acceleration in the increase in the number of skilled workers should induce an acceleration in skill-biased technological change.
  - Available evidence suggests that there were large increases in the number of unskilled workers during the late 18th and 19th centuries.

Implications II

- The framework also gives a potential interpretation for the dynamics of the college premium during the 1970s and 1980s.
  - It is reasonable that the equilibrium skill bias of technologies, N_H / N_L, is a sluggish variable.
  - Hence a rapid increase in the supply of skills would first reduce the skill premium as the economy would be moving along a constant technology (constant N_H / N_L).
  - After a while technology would start adjusting, and the economy would move back to the upward sloping relative demand curve, with a relatively sharp increase in the college premium.

Implications III

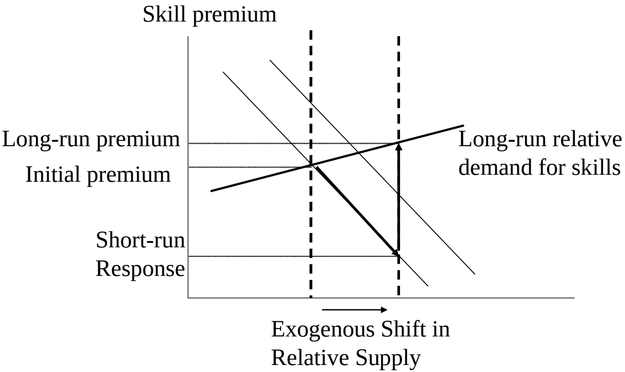


Figure: Dynamics of the skill premium in response to an exogenous increase in the relative supply of skills, with an upward-sloping endogenous-technology relative demand curve.

Implications IV

- If instead $\sigma < 2$, the long-run relative demand curve will be downward sloping, though again it will be shallower than the short-run relative demand curve.
- An increase in the relative supply of skills leads again to a decline in the college premium, and as technology starts adjusting the skill premium will increase.
- But it will end up below its initial level. To explain the larger increase in the college premium in the 1980s, in this case we would need some exogenous skill-biased technical change.

Implications V

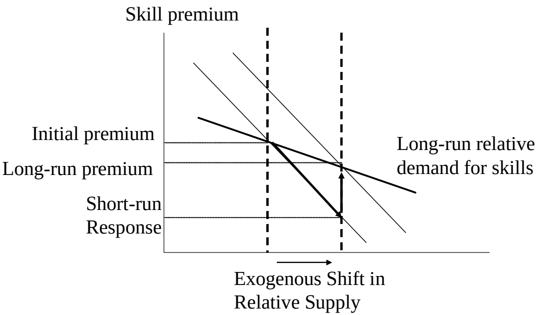


Figure: Dynamics of the skill premium in response to an increase in the relative supply of skills, with a downward-sloping endogenous-technology relative demand curve.

Implications VI

- Other remarks:
  - Upward-sloping relative demand curves arise only when $\sigma > 2$. Most estimates put the elasticity of substitution between 1.4 and 2. One would like to understand whether $\sigma > 2$ is a feature of the specific model discussed here
  - Results on induced technological change are not an artifact of the scale effect (exactly the same results apply when scale effects are removed, see below).

Directed Technological Change with Knowledge Spillovers

- The lab equipment specification of the innovation possibilities does not allow for *state dependence*.
- Assume that R&D is carried out by scientists and that there is a constant supply of scientists equal to S
- With only one sector, sustained endogenous growth requires $\dot{N}/N$ to be proportional to S.
- With two sectors, there is a variety of specifications with different degrees of state dependence, because productivity in each sector can depend on the state of knowledge in both sectors.
- A flexible formulation is

$$\begin{aligned} \dot{N}_L(t) &= \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} S_L(t) \qquad (30) \\ \text{and } \dot{N}_H(t) &= \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} S_H(t), \end{aligned}$$

where $\delta \leq 1$.

Directed Technological Change II

- Market clearing for scientists requires that

$$S_L(t) + S_H(t) \leq S. \tag{31}$$

- δ measures the degree of state-dependence:
  - $\delta = 0$. Results are unchanged. No state-dependence:

  $$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H / \eta_L$$

  irrespective of the levels of N_L and N_H.
  Both N_L and N_H create spillovers for current research in both sectors.
  - $\delta = 1$. Extreme amount of state-dependence:

  $$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H N_H / \eta_L N_L$$

  an increase in the stock of L-augmenting machines today makes future labor-complementary innovations cheaper, but has no effect on the cost of H-augmenting innovations.

Directed Technological Change III

- State dependence adds another layer of "increasing returns," this time not for the entire economy, but for specific technology lines.
- Free entry conditions:

$$\begin{aligned} \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} V_L(t) &\leq w_S(t) \quad (32) \\ \text{and } \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} V_L(t) &= w_S(t) \text{ if } S_L(t) > 0. \end{aligned}$$

and

$$\begin{aligned} \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} V_H(t) &\leq w_S(t) \quad (33) \\ \text{and } \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} V_H(t) &= w_S(t) \text{ if } S_H(t) > 0, \end{aligned}$$

where $w_S(t)$ denotes the wage of a scientist at time t.

Directed Technological Change IV

- When both of these free entry conditions hold, BGP technology market clearing implies

$$\eta_L N_L(t)^{\delta} \pi_L = \eta_H N_H(t)^{\delta} \pi_H, \tag{34}$$

- Combine condition (34) with equations (14) and (17), to obtain the equilibrium relative technology as:

$$\left(\frac{N_H}{N_L}\right)^{*} = \eta^{\frac{\sigma}{1-\delta\sigma}} \gamma^{\frac{\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\delta\sigma}}, \tag{35}$$

where $\gamma \equiv \gamma_H/\gamma_L$ and $\eta \equiv \eta_H/\eta_L$.

Directed Technological Change V

- The relationship between the relative factor supplies and relative physical productivities now depends on δ.
- This is intuitive: as long as $\delta > 0$, an increase in N_H reduces the relative costs of H-augmenting innovations, so for technology market equilibrium to be restored, π_L needs to fall relative to π_H.
- Substituting (35) into the expression (18) for relative factor prices for given technologies, yields the following long-run (endogenous-technology) relationship:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \gamma^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}} . \tag{36}$$

Directed Technological Change VI

- The growth rate is determined by the number of scientists. In BGP we need $\dot{N}_L(t)/N_L(t) = \dot{N}_H(t)/N_H(t)$, or

$$\eta_H N_H(t)^{\delta-1} S_H(t) = \eta_L N_L(t)^{\delta-1} S_L(t).$$

- Combining with (31) and (35), BGP allocation of researchers between the two different types of technologies:

$$\eta^{\frac{1-\sigma}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{-\frac{\varepsilon(1-\delta)}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{-\frac{(\sigma-1)(1-\delta)}{1-\delta\sigma}} = \frac{S_L^*}{S-S_L^*}, \qquad (37)$$

- Notice that given H/L, the BGP researcher allocations, S_L^* and S_H^*, are uniquely determined.

Balanced Growth Path with Knowledge Spillovers

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Suppose that

$$(1-\theta)\frac{\eta_L \eta_H (N_H/N_L)^{(\delta-1)/2}}{\eta_H (N_H/N_L)^{(\delta-1)} + \eta_L} S < \rho,$$

where N_H/N_L is given by (35). Then there exists a unique BGP equilibrium in which the relative technologies are given by (35), and consumption and output grow at the rate

$$g^* = \frac{\eta_L \eta_H (N_H/N_L)^{(\delta-1)/2}}{\eta_H (N_H/N_L)^{(\delta-1)} + \eta_L} S. \tag{38}$$

Transitional Dynamics with Knowledge Spillovers

- Transitional dynamics now more complicated because of the spillovers.
- The dynamic equilibrium path does not always tend to the BGP because of the additional increasing returns to scale:
  - With a high degree of state dependence, when $N_H(0)$ is very high relative to $N_L(0)$, it may no longer be profitable for firms to undertake further R&D directed at labor-augmenting (L-augmenting) technologies.
  - Whether this is so or not depends on a comparison of the degree of state dependence, δ, and the elasticity of substitution, σ.
- It can be shown that now stability requires $\sigma < 1/\delta$, and conversely, if $\sigma > 1/\delta$, we go to one of the "corners".

Equilibrium Relative Bias with Knowledge Spillovers I

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then there is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then if

$$\sigma > 2 - \delta,$$

there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the H factor compared to the L factor.

Equilibrium Relative Bias with Knowledge Spillovers II

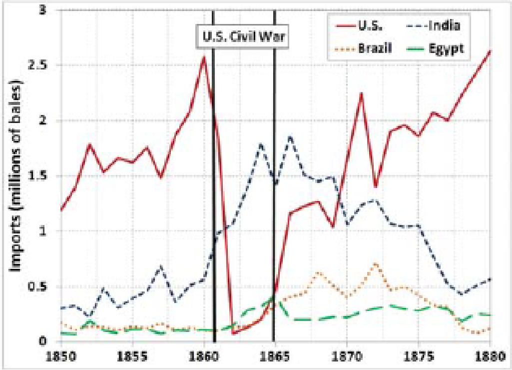
- Intuitively, the additional increasing returns to scale coming from state dependence makes strong bias easier to obtain, because the induced technology effect is stronger.
- Note the elasticity of substitution between skilled and unskilled labor significantly less than 2 may be sufficient to generate strong equilibrium bias.
- How much lower than 2 the elasticity of substitution can be depends on the parameter δ. Unfortunately, this parameter is not easy to measure in practice.

Evidence

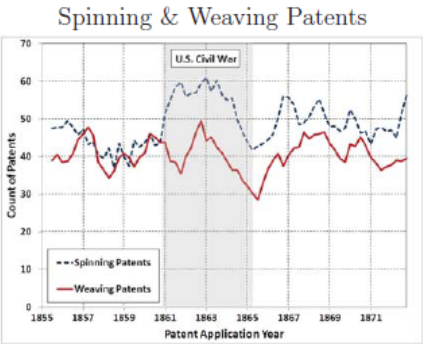
- Hanlon (2014): evidence on factor-augmenting directed technological change and its impact on factor prices.
- Following the interruption to the British cotton textile industry caused by the US Civil War, the decrease in American cotton led to technological change directed to other types of cotton inputs.
- There was a flurry of new patents related to cotton spinning. These appear to be directed at Indian cotton which was relatively abundant but harder to prepare for spinning than American cotton.
- This looks like "factor-augmenting" technological change directed towards the more abundant input. Consistent with theory if the elasticity of substitution > 1, which Hanlon's estimates suggest.
- Hanlon also provides evidence of strong relative bias—relative Indian cotton prices actually increased despite this input's relative abundance.

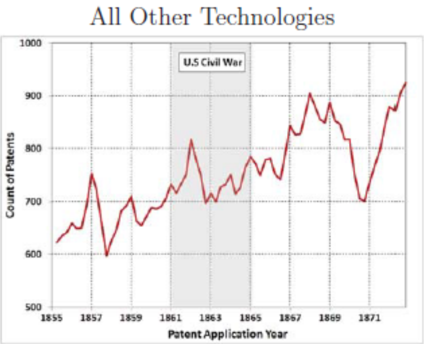
Evidence: Changes in Quantities

Cotton imports by supplier 1850-1880

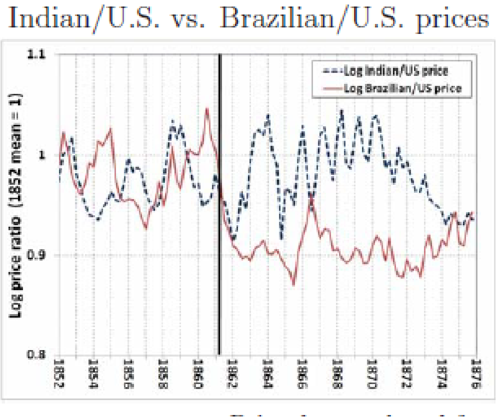


Evidence: Changes in Spinning Patents





Evidence: Changes in Input Prices



Endogenous Labor-Augmenting Technological Change I

- Models of directed technological change create a natural reason for technology to be more labor augmenting than capital augmenting.
- Under most circumstances, the resulting equilibrium is not purely labor augmenting and as a result, a BGP fails to exist.
- But in one important special case, the model delivers microfoundations for the neoclassical growth model.
- Consider a two-factor model with H corresponding to capital, that is, $H(t) = K(t)$, and assume no depreciation.
- Empirical evidence: in this case, $\sigma < 1$, so that labor-augmenting technological change corresponds to capital-biased technological change.
- Hence the questions are:
  1. Under what circumstances would the economy generate relatively capital-biased technological change?
  2. When will the equilibrium technology be sufficiently capital biased that it corresponds to Harrod-neutral technological change?

Endogenous Labor-Augmenting Technological Change II

- To answer 1, note that what distinguishes capital from labor is the fact that it accumulates.
- The neoclassical growth model with technological change experiences continuous capital-deepening as $K(t)/L$ increases.
- This implies that technological change should be *more labor-augmenting than capital augmenting.*

Proposition In the baseline model of directed technological change with $H(t) = K(t)$ as capital, if $K(t)/L$ is increasing over time and $\sigma < 1$, then $N_L(t)/N_K(t)$ will also increase over time.

Endogenous Labor-Augmenting Technological Change III

- But the results are not easy to reconcile with purely-labor augmenting technological change. Suppose that capital accumulates at an exogenous rate, i.e.,

$$\frac{\dot{K}(t)}{K(t)} = s_K > 0. \tag{39}$$

Proposition Consider the baseline model of directed technological change with the knowledge spillovers specification and state dependence. Suppose that $\delta < 1$ and capital accumulates according to (39). Then there exists no BGP.

- Intuitively, even though technological change is more labor augmenting than capital augmenting, there is still capital-augmenting technological change in equilibrium.
- Moreover it can be proved that in any asymptotic equilibrium, $r(t)$ cannot be constant, thus consumption and output growth cannot be constant.

Endogenous Labor-Augmenting Technological Change IV

- However, one special case works:

Proposition Consider the baseline model of directed technological change with the two factors corresponding to labor and capital. Suppose that the innovation possibilities frontier is given by the knowledge spillovers specification and **extreme state dependence**, i.e., $\delta = 1$ and that capital accumulates according to (39). Then there exists a constant growth path allocation in which there is only labor-augmenting technological change, the interest rate is constant and consumption and output grow at constant rates. Moreover, there cannot be any other constant growth path allocations.

- Stable when $\sigma < 1$: capital-labor complementarity forces the economy to strive towards a balance between effective capital and labor.
  - Since capital accumulates at a constant rate, a balanced allocation implies that the productivity of labor should increase faster—hence purely labor-augmenting technological progress.

Alternative Approach

- What we outlined in the previous lecture is an alternative approach, where instead of looking at factor-augmenting technologies, we consider automation technologies and new tasks.
- Directed technological change and determines the speed at which **new automation technologies** and **new tasks** are introduced.
  - Recall the balanced growth resulted in the basic task framework depending on $n_t = N_t - I_t$.
  - Direction of technological change determines how quickly N_t increases relative to I_t
  - Both technologies contribute to long-run growth, and if n_t asymptotes to a constant, there will be stable factor shares in the long run.
  - However, this is not guaranteed, and there can also be equilibria leading to only capital being used in the long run (asymptotic automation of all or most tasks).
  - See Acemoglu and Restrepo (2018).

Application: Energy Transition

- One place where issues of direction of technology are critical is in the energy transition—transitioning to a zero or low carbon economy.
- Although one could imagine this could happen by reducing energy consumption overall, it is actually impossible to achieve it just by reducing energy, given the increase in energy needs around the world and the political opposition to it.
- Much more realistic is to do it by developing new, low-carbon energy sources, such as renewables.
- This is a classic direction of technology problem.

Choice Between Two Competing Technologies

- We have already covered enough of the general theory to be able to analyze this case easily.
- Suppose, for simplicity, we are in the growth with knowledge spillovers case, with research being carried out with scientists.
- Then let me specialized the framework into additional ways:
  1. Suppose there is extreme *state dependence*, $\delta = 1$, so that the two stocks of knowledge for clean and dirty technologies are given as:
  $$\begin{aligned} \dot{N}_F(t) &= \eta_F N_F(t) S_F(t) \\ \text{and } \dot{N}_G(t) &= \eta_G N_G(t) S_G(t), \end{aligned}$$
  where F denotes fossil fuels and G is green technology. The justification is that, in general, improvements in solar panels have limited benefits for efficient coal extraction, and internal combustion engine techs do not help much with electric or hydrogen vehicles.
  2. Suppose that the two types of energy are highly substitutable, and definitely $\sigma > 1$. In most tasks the two types of energy are highly substitutable.

Decentralized Equilibrium

- We know from our analysis above that the case where $\sigma > 1/\delta$, the interior BGP is unstable, and we will instead go to one of the corner BGPs—either with just fossil fuels or with just green technology.
- This makes a lot of sense, given that they are highly substitutable and there are limited spillovers between the two technologies.
- Which one of these two asymptotic equilibria/BGPs will we end up in?
- The answer is typically whichever one starts sufficiently ahead, and of course in the case of energy, fossil fuels started much ahead.
- And hence, in the decentralized equilibrium (without any policy intervention), the economy will start innovating and investing in fossil fuels and will stay there, and renewables will never get off the ground.

An Alternative Path

- But an alternative equilibrium in which the economy heavily invests in renewables and heads towards the pure renewables BGP is also possible.
- In fact, this is, arguably, the right understanding of what the energy transition should be about.
- This also shows that the economy may end up in an inefficient path (with much more pollution), highlighting that in models of directed technological change with substitutable technologies, the equilibrium can generally involve the choice of the wrong technology or wrong technology paradigm.
- The question then is: what types of interventions can effectively and efficiently switch the economy from the fossil fuel path to the alternative, green path?

Policy in the Exogenous Climate Change Model

- Before discussing policy in the directed technology model, let us review the main insights in the workhorse model that economists use, due to William Nordhaus's work in the 1990s.
- With exogenous technological change, the main policy lessons are:
    - Optimal policy is just Pigovian taxes (only one externality from emissions, and you deal with that with a carbon tax).
    - The carbon tax does not and should not try to get the economy to zero emissions (because that would mean zero energy consumption which is very costly)
    - The path of optimal policy heavily depends on the discount factor (because it is all about trading off reductions in current consumption due to high carbon taxes versus high climate damages in the future).
    - For discount factors implied by short-run interest rates, the optimal path takes a "ramp-up" pattern: carbon taxes start low and increase gradually (because, with high discounting, you don't want to reduce consumption today given, but you want to commit to reducing it in the future).

Policy with Directed Technological Change

- Policy insights from this directed technological change model are the complete opposite of the Nordhaus lessons:
  - Optimal policy should use at least two instruments—a carbon tax **plus** a clean innovation subsidy (because you do not want to rely just on a carbon tax to redirect technological change, and by redirecting technological change you save future distortions from carbon taxes).
  - Optimal policy should try to get the economy to zero emissions (that is the alternative desirable path).
  - Optimal policy should be immediate and non-gradual (because every year of delay means fossil fuels will pull further ahead of green technologies and catching up will be all the more difficult).
  - The form of optimal policy is not very sensitive to the discount factor, provided that the optimal allocation does involve in energy transition.
- You can make up your own mind on which ones of these appear more plausible.

Conclusions

- The bias of technological change is potentially important for the distributional consequences of the introduction of new technologies (i.e., who will be the losers and winners?).
- Models of directed technological change point to some general results (weak and strong bias), and enable us to investigate a range of new questions:
  - the sources of skill-biased technological change over the past 100 years,
  - the causes of acceleration in skill-biased technological change during more recent decades,
  - the causes of unskilled-biased technological developments during the 19th century,
  - the relationship between labor market institutions and the types of technologies that are developed and adopted,
  - why technological change in neoclassical-type models may be largely labor-augmenting.
  - why the economy may end up in the wrong technologies (for example in the case of energy) and how could get out of this.