14.452 Economic Growth: Lecture 12, Search, Growth and Unemployment

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October-December 2025

Introduction

- In this lecture, I will introduce the basic search and matching framework due to Pissarides, together with Diamond and Mortensen, sometimes referred to as the DMP model.
- I will then use this model to discuss some linkages between growth and unemployment.
- The purpose is twofold:
  1. as an introduction to a model that has emerged as a workhorse framework for thinking about unemployment and frictional trade in labor markets (and sometimes beyond)
  2. to broaden the set of growth-related issues we are talking about in this course.

Motivation and Basic Setup

- The DMP model attempts to develop a tractable framework for the role of frictions in the labor market.
- The model is in continuous time with infinite horizon.
- The preference side is neoclassical. We can think of risk-neutral agents or different agents belonging to a single "family", so that idiosyncratic risks that emerge in the labor market do not matter.
- Here, let us take the risk-neutral setting and let r denote the pure discount rate of these risk-neutral households, which will also be the interest rate.
- The production side consists of many small producers, each of which can employ at most one worker to produce a unique final good.
  - It is also possible to have large firms in the setup, but for concreteness, I choose the one firm-one worker formulation.
- The main new element is that firms and workers come together in a frictional manner, which will be modeled via the matching function.

Matching Function

- The matching function specifies how unemployed workers looking for jobs and vacancies looking for workers generate new matches:

$$\text{Matches}_t = x(U_t, V_t)$$

where U_t denotes the stock of unemployed workers at time t and V_t denotes the stock of unfilled vacancies at time t.
- Because we are in continuous time, $x(U_t, V_t)$ is the flow rate of matches. I will follow Mortensen-Pissarides and assume that $x(U, V)$ exhibits constant returns to scale.
- In the baseline model all matches turn into employment, and hence new matches correspond to new hires. Therefore:

$$\begin{aligned} \text{Matches} &= xL = x(uL, vL) \\ &\Longrightarrow x = x(u, v) \end{aligned}$$

L is labor force, u is the unemployment rate (unemployed workers divided by force L) and v is vacancy rate (vacancies divided by L).

Evidence and Interpretation

- Existing aggregate evidence suggests that the assumption of x exhibiting CRS is reasonable.
- Intuitively, one might have expected "increasing returns" if the matching function corresponds to physical frictions
  - think of people trying to run into each other on an island.
- But the matching function is probably too reduced-form for this type of interpretation, and it may not be stable when there are changes in institutions, policies and parameters. But in this lecture I will follow the literature and take it is given.
- In practice, frictions due to differences in the supply and demand for specific types of skills.

Matching Rates and Job Creation

- Using the constant returns assumption, we can express everything as a function of the *tightness of the labor market.*

$$q(\theta) \equiv \frac{x}{v} = x\left(\frac{u}{v}, 1\right),$$

- Here $\theta \equiv v/u$ is the tightness of the labor market

$q(\theta)$: Poisson arrival rate of match for a vacancy
$\theta q(\theta)$:Poisson arrival rate of match for an unemployed worker

- Therefore, job creation is equal to

$$\text{Job creation} = u\theta q(\theta)L$$

Job Destruction

- What about job destruction?
- Let us start with the simplest model of job destruction, which is basically to treat it as "exogenous".
- Think of it as follows, firms are hit by adverse shocks, and then they decide whether to destroy or to continue.

$$\longrightarrow \text{Adverse Shock} \longrightarrow \text{destroy}$$
$$\longrightarrow \text{continue}$$

- Exogenous job destruction: Adverse shock $= -\infty$ at the flow rate s
- With such an adverse shock, there is essentially no choice—when the firm receives it it will have to destroy the job.

Steady State of the Flow Approach

- Accounting for changes in unemployment (in continuous time):

$$\dot{u} = \underbrace{s(1-u)}_{\substack{\text{flow into unemployment} \\ \text{or job destruction}}} - \underbrace{\theta q(\theta) u}_{\substack{\text{flow out of unemployment} \\ \text{or job creation}}}$$

- In steady state, we have $\dot{u} = 0$, and thus

$$\text{flow into unemployment} = \text{flow out of unemployment}$$

- With exogenous job destruction, this means:

$$s(1-u) = \theta q(\theta) u$$

- Therefore, steady state unemployment rate is:

$$u = \frac{s}{s + \theta q(\theta)}.$$

The Beveridge Curve

- This relationship is also referred to as the *Beveridge Curve*, or the U-V curve.
- It draws a downward sloping locus of unemployment-vacancy combinations in the U-V space that are consistent with flow into unemployment being equal with flow out of unemployment.
- Some authors interpret shifts of this relationship is reflecting structural changes in the labor market, but we will see that there are many factors that might actually shift at a generalized version of such relationship.

Production Side and Hiring

- Each firm can hire a single worker.
- A firm with a worker and capital stock K can produce

$$\begin{aligned} y &= F(K, A) \\ &= f(k) \end{aligned}$$

units of the final good, where A is a Harrod-neutral-style productivity term and F is constant returns to scale.

- To start with I take $A = 1$, so $k = K$ is simply capital per worker.
- Finally, firms need to post vacancies to hire workers and the cost of hiring a vacancy is assumed to be γ.

Investment and Reversibility

- We assume that the capital choices/investments of firms are completely reversible.
- This implies in particular that at any point in time the firm can sell its capital stock or add additional capital stock.
- Capital is also assumed to depreciate at the rate δ.
- Recall also that r is the interest rate.
- Therefore, putting all of these together, the equilibrium will have

$$f'(k) = r + \delta.$$

- Thus the neoclassical marginal product condition holds in this version of the search model as well (purposefully to make it as close to our baseline models as possible).

Bellman Equations

- We are going to solve for the equilibrium using a series of Bellman equations.
- Define:

J^V : PDV of a vacancy
J^F :PDV of a "job"
J^U :PDV of a searching worker
J^E :PDV of an employed worker

- Why is J^F not conditioned on k?
- Recall that by *assumption* we have perfectly reversible capital investments (why is this important?)

Value of Vacancies

- Perfect capital market gives the asset value for a vacancy (in steady state) as

$$rJ^V - \dot{J}^V = -\gamma + q(\theta)(J^F - J^V)$$

- Intuition in terms of asset values—as before.

Labor Demand and Job Creation

- Free Entry $\Longrightarrow$

$$J^V = \dot{J}^V \equiv 0$$

- If J^V were positive, more firms would enter.
- Why is $\dot{J}^V = 0$?
- Important implication: job creation can happen really "fast", except because of the frictions created by matching searching workers to searching vacancies.
- One could alternatively have $\gamma = \Gamma_0(V)$ or $\Gamma_1(\theta)$, but this would not make much difference for the analysis here.

Characterization of Equilibrium

- Free entry implies that

$$J^F = \frac{\gamma}{q(\theta)}$$

- Thus if $\dot{\theta} = 0$, then $\dot{J}^F = 0$ (as we will see).
- Asset value equation for the value of a field job:

$$r(J^F + k) - \dot{J}^F = f(k) - \delta k - w - s(J^F - J^V)$$

- Intuitively, the firm has two assets: the fact that it is matched with a worker, and its capital, k.
- So its asset value is $J^F + k$ (more generally, without the perfect reversability, we would have the more general $J^F(k)$).
- Its return is equal to production, $Af(k)$, and its costs are depreciation of capital and wages, δk and w.
- Finally, at the rate s, the relationship comes to an end and the firm loses J^F.

Wage Determination

- Can wages be equal to marginal cost of labor and value of marginal product of labor?
- No because of labor market frictions
- a worker with a firm is more valuable than an unemployed worker.
- How are wages determined?
- *Nash bargaining* over match specific surplus $J^E + J^F - J^U - J^V$
- Where is k?

Implications of Perfect Reversability

- Perfect Reversability implies that w does not depend on the firm's choice of capital

$\Longrightarrow$ equilibrium capital utilization $f'(k) = r + \delta$

- *Modified Golden Rule*

Equilibrium Job Creation

- Free entry together with the Bellman equation for filled jobs implies

$$f(k) - (r+\delta)k - w - \frac{(r+s)}{q(\theta)}\gamma = 0$$

- For unemployed workers

$$rJ^U - \dot{J}^U = z + \theta q(\theta)(J^E - J^U)$$

  where z is unemployment benefits.

- Employed workers:

$$rJ^E - \dot{J}^E = w + s(J^U - J^E)$$

- Reversibility again: w independent of k.

Steady-State Values For Workers

- Solving these equations in steady state, we obtain the value of an unemployed worker as:

$$rJ^U = \frac{(r+s)z + \theta q(\theta) w}{r+s+\theta q(\theta)}.$$

- Similarly, we could obtainthe value of an employee worker

$$rJ^E = \frac{sz + [r + \theta q(\theta)]\, w}{r+s+\theta q(\theta)},$$

and then use these two equations and Nash bargaining.

- Would that be right? What is the wage in these equations representing?

Nash Bargaining

- Here is the right way to do Nash bargaining: take the surplus of pair i:

$$\begin{aligned} rJ_i^F &= f(k) - (r+\delta)k - w_i - sJ_i^F \\ rJ_i^E &= w_i - s(J_i^E - J^U). \end{aligned}$$

- Why is it important to do this for pair i (rather than use the equilibrium expressions above)?
- The Nash solution will solve

$$\begin{aligned} & \max (J_i^E - J^U)^\beta (J_i^F - J^V)^{1-\beta} \\ \beta &= \text{bargaining power of the worker} \end{aligned}$$

- Since we have linear utility, thus "transferable utility", this implies

$$J_i^E - J^U = \beta(J_i^F + J_i^E - J^V - J^U)$$

Nash Bargaining

- Using the expressions for the value functions

$$w = (1-\beta)z + \beta\left[f(k) - (r+\delta)k + \theta\gamma\right]$$

- Here

$$f(k) - (r+\delta)k + \theta\gamma$$

is the *quasi-rent* created by a match that the firm and workers share.

- Why is the term $\theta\gamma$ there?

Digression: Irreversible Capital Investments

- Much more realistic, but typically not adopted in the literature (why not?)
- Suppose k is not perfectly reversible then suppose that the worker captures a fraction β all the output in bargaining.
- Then the wage depends on the capital stock of the firm, as in the holdup models discussed before.

$$\begin{aligned} w(k) &= \beta f(k) \\ f'(k) &= \frac{r+\delta}{1-\beta} \text{ ; capital accumulation is distorted} \end{aligned}$$

Steady State Equilibrium Redux

- Steady State Equilibrium is given by four equations
  1. The Beveridge curve:
  $$u = \frac{s}{s + \theta q(\theta)}$$
  2. Job creation leads zero profits:
  $$f(k) - (r + \delta)k - w - \frac{(r + s)}{q(\theta)}\gamma = 0$$
  3. Wage determination:
  $$w = (1 - \beta)z + \beta\left[f(k) - (r + \delta)k + \theta\gamma\right]$$
  4. Modified golden rule:
  $$f'(k) = r + \delta$$

Steady State Equilibrium (continued)

- These four equations define a block recursive system

$$(4) + r \longrightarrow k$$

$$k + r + (2) + (3) \longrightarrow \theta, w$$

$$\theta + (1) \longrightarrow u$$

Steady State Equilibrium (continued)

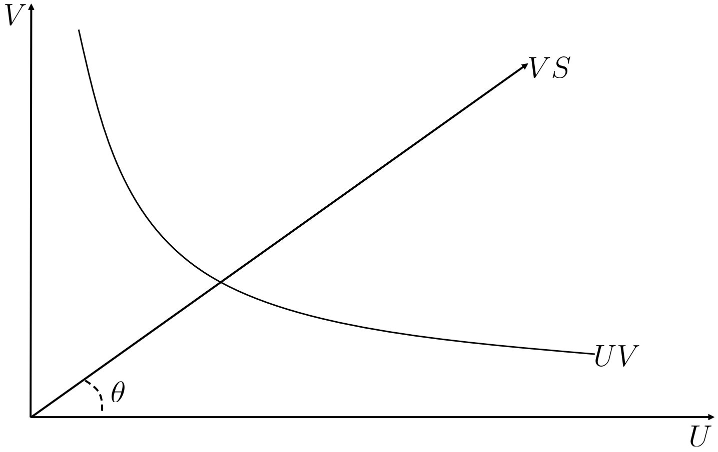
- Alternatively, combining three of these equations we obtain the zero-profit locus, the VS curve.

  $(2), (3), (4) \Longrightarrow \qquad$ the VS curve

$$(1-\beta)\left[f(k)-(r+\delta)k-z\right]-\frac{r+s+\beta\theta q(\theta)}{q(\theta)}\gamma=0.$$

- Combine this relationship with the Beveridge curve to obtain the full equilibrium.
- Therefore, the equilibrium looks very similar to the intersection of "quasi-labor demand" and "quasi-labor supply".

Steady State Equilibrium in a Diagram



Steady state comparative statics

Comparative Statics of the Steady State

- From the figure, we obtain the following comparative statics:

parameter	interpretation	U	V	θ	w
$s \uparrow$	separation rate increases	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$r \uparrow$	interest rate increases	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\gamma \uparrow$	hiring costs increase	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\beta \uparrow$	worker bargaining power increases	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$z \uparrow$	unemployment benefits increase	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

- I come back to the effects of technology below.

Dynamics

- It can be verified that in this basic model there are no dynamics in θ.
- This follows essentially from the observations so far—essentially vacancies are forward-looking and can jump.
- But there will still be dynamics of unemployment because job creation is slow.

Incorporating Growth

- Let us now incorporate growth into this basic model.
- The simplest way of doing that is to now assume that labor productivity A grows exponentially at the rate g:

$$A_t = e^{gt}$$

I suppress the subscript in what follows to simplify notation.

- Now we have $k \equiv K/A$, and the total output of a firm with effective capital-labor ratio k is $Af(k)$ due to constant returns to scale.
- Suppose also that unemployment benefits and costs of vacancies also scale with A, so that they are

$$\begin{aligned} z(A) &= zA \\ \gamma(A) &= \gamma A \end{aligned}$$

(We could also assume that these are proportional to the equilibrium wage because they use labor or they are proportional to labor income, but this would be equivalent, as we will see).

Conjecture

- Let us conjecture a “linear equilibrium” where the main endogenous objects grow linearly in A:

$$
\begin{aligned}
w(A) &= wA \\
J^V(A) &= J^V A \\
J^F(A) &= J^F A \\
J^E(A) &= J^E A \\
J^U(A) &= J^U A.
\end{aligned}
$$

Solving for Steady-State Equilibrium

- Then we can simplify the value functions. For example, using the fact that the total capital stock of the firm is Ak, we have:

$$r(J^F(A)+Ak)-\dot{J}^F(A)-\dot{A}k = Af(k)-\delta Ak - w(A) - s(J^F(A)-J^V($$

- Now using the linear forms above, the fact that, with linearity, in steady state we have $\dot{J}^F(A)/J^F(A) = \dot{A}/A = g$ and simplifying by dividing everything by A, we have

$$(r-g)(J^F+k) = f(k)-\delta k - w - s(J^F-J^V)$$

  which is equivalent to before, except that now the effective discount rate becomes $r-g$.
- The same applies to all of the other value functions.
- The first-order condition of the firm for capital stock can be obtained from this equation and coincides with the modified golden rule again:

$$f'(k) = r-g+\delta$$

Recap of the Steady-State Equilibrium

- 1. The Beveridge curve:

$$u = \frac{s}{s + \theta q(\theta)}$$

  2. Job creation leads zero profits:

$$f(k) - (r - g + \delta)k - w - \frac{(r - g + s)}{q(\theta)}\gamma = 0$$

  3. Wage determination:

$$w = (1 - \beta)z + \beta\left[f(k) - (r - g + \delta)k + \theta\gamma\right]$$

  4. Modified golden rule:

$$f'(k) = r - g + \delta$$

- Or combining the last two expressions we again have the VS curve:

$$(1 - \beta)\left[f(k) - (r - g + \delta)k - z\right] - \frac{r - g + s + \beta\theta q(\theta)}{q(\theta)}\gamma = 0$$

Capitalization Effect

- The capitalization effect corresponds to the impact of a higher growth rate, g, on job creation.
- Higher g implies more job creation because firms anticipate that their jobs will bring greater incomes in the future due to the higher growth rate of in A
- The capitalization effect implies that higher growth should be associated with lower unemployment—due to more job creation.

Destruction Effect

- However, faster growth can also lead to more job destruction.
- In practice, this can take several different forms.
- One is through "creative destruction"—growth is brought by new firms that destroy old ones. This can be included in extensions of the search model, but I will not do so for simplicity.
- Another one would be through imperfect adaptation to new technologies. Suppose that the growth in A is growth through a series of discrete improvements generated by a Poisson process:
  - At each instant, there is a flow rate of arrival rate, ψ, of a new improvement which brings a discrete increase of v in productivity. Then

$$g = \psi v.$$

  - Suppose, however, that each existing job will fail to adapt to the new technology with probability ζ.
  - Then the separation rate becomes

$$s + \psi\zeta = s + g\zeta/v.$$

Value Functions with Endogenous Job Destruction

- 1. The Beveridge curve:

$$u = \frac{s + g\zeta/\upsilon}{s + g\zeta/\upsilon + \theta q(\theta)}$$

  2. Job creation leads zero profits:

$$f(k) - (r - g + \delta)k - w - \frac{(r - g + s + g\zeta/\upsilon)}{q(\theta)}\gamma = 0$$

  3. Wage determination:

$$w = (1 - \beta)z + \beta\left[f(k) - (r - g + \delta)k + \theta\gamma\right]$$

  4. Modified golden rule:

$$f'(k) = r - g + \delta$$

- Or combining these expression we again have:

$$(1 - \beta)\left[f(k) - (r - g + \delta)k - z\right] - \frac{r - g + s + g\zeta/\upsilon + \beta\theta q(\theta)}{q(\theta)}\gamma = 0$$

Growth and Unemployment

- The capitalization effect is now more nuanced because faster growth also leads to faster destruction (why does this not affect the cost of capital? What would happen if it did?).
- Under the sufficient condition that $\zeta/v < 1$, the capitalization effect still pushes for lower unemployment in higher-growth economies.
- But the job destruction effect now pushes for higher unemployment, and the overall relationship between growth and unemployment is ambiguous.
- Different types of growth can have different effects on unemployment:
  - An increase in v only creates the capitalization effect.
  - An increase in ψ creates both effects.
  - An increase in ζ only creates the disruption effect.

What is Missing?

- Matching of heterogeneous skills to heterogeneous jobs—this might become important in growing versus stagnant economies.
- More variegated effects of technology on labor markets, once we take into account that technology can take richer forms than just Harrod-neutral or Hicks-neutral.
  - It can automate work, create new tasks or simply increase the productivity of labor in existing tasks (recall previous lectures).
  - Each of these will have different effects on the labor market equilibrium.
- Human capital investments and growth.
- The effects of unemployment on growth—through human capital investments or aggregate demand channels.
- Political economy of employment—facilitating versus slowing down creative destruction.

Efficiency?

- Is the search equilibrium efficient?
- Clearly, it is inefficient relative to a first-best alternative, e.g., a social planner that can avoid the matching frictions.
- Instead look at "surplus-maximization" subject to search constraints (why not constrained Pareto optimality?)
- To do this, I return to the model without technological change and growth.
- This question is important and interesting, both because one of the insightful aspects of studying frictional models is to think more systematically about inefficiencies, and also because it will provide insights on policy debates related to unemployment being too high or too low.

Search Externalities

- There are two major externalities

$$
\begin{array}{lll}
\theta \uparrow & \Longrightarrow & \text{workers find jobs more easily} \\
& \hookrightarrow & \text{thick-market externality} \\
& \Longrightarrow & \text{firms find workers more slowly} \\
& \hookrightarrow & \text{congestion externality}
\end{array}
$$

- Why are these externalities?
- Pecuniary or nonpecuniary?
- Why should we care about the junior externalities?

Efficiency of Search Equilibrium

- The question of efficiency boils down to whether these two externalities cancel each other or whether one of them dominates.
- To analyze this question more systematically, consider a social planner subject to the same constraints, intending to maximize "total surplus", in other words, pursuing a utilitarian objective.
- First ignore discounting, i.e., $r \longrightarrow 0$, and letting the value of a match be y (e.g., $y = f(k) - (r + \delta)k$), we have that the planner's problem can be written as

$$\max_{u,\theta} SS = (1-u)y + uz - u\theta\gamma.$$

s.t.

$$u = \frac{s}{s + \theta q(\theta)}.$$

where we assumed that z corresponds to the utility of leisure rather than unemployment benefits (how would this be different if z were unemployment benefits?)

Efficiency of Search Equilibrium

- Why is $r = 0$ useful?
- It turns this from a dynamic into a static optimization problem.
- Form the Lagrangian:

$$\mathcal{L} = (1-u)y + uz - u\theta\gamma + \lambda \left[u - \frac{s}{s + \theta q(\theta)} \right]$$

- The first-order conditions with respect to u and θ are straightforward:

$$(y - z) + \theta\gamma = \lambda$$

$$u\gamma = \lambda s \frac{\theta q'(\theta) + q(\theta)}{(s + \theta q(\theta))^2}.$$

- What is λ?

Efficiency of Search Equilibrium (continued)

- The constraint will clearly binding (why?)
- Then substitute for u from the Beveridge curve, and obtain:

$$\lambda = \frac{\gamma \left(s + \theta q \left(\theta\right)\right)}{\theta q' \left(\theta\right) + q \left(\theta\right)}$$

- Now substitute this into the first condition to obtain

$$\left(\theta q' \left(\theta\right) + q \left(\theta\right)\right) [y - z] + \left(\theta q' \left(\theta\right) + q \left(\theta\right) \theta\right) \gamma - \gamma \left(s + \theta q \left(\theta\right)\right) = 0$$

- Simplifying and dividing through by $q\left(\theta\right)$, we obtain

$$(1 - \eta(\theta))[y - z] - \frac{s + \eta(\theta)\theta q(\theta)}{q(\theta)}\gamma = 0.$$

where

$$\eta\left(\theta\right) = -\frac{\theta q' \left(\theta\right)}{q\left(\theta\right)} = \frac{\frac{\partial M(U,V)}{\partial U} U}{M\left(U, V\right)}$$

is the elasticity of the matching function respect to unemployment.

Comparison to Equilibrium

- Recall that in equilibrium (with $r = 0$) we have

$$(1-\beta)(y-z) - \frac{s + \beta\theta q(\theta)}{q(\theta)}\gamma = 0.$$

- Comparing these two conditions we find that efficiency obtains if and only if *the Hosios condition*

$$\beta = \eta(\theta)$$

  is satisfied
- In other words, efficiency requires the bargaining power of the worker to be equal to the elasticity of the matching function with respect to unemployment.
- This is only possible if the matching function is constant returns to scale.
- What happens if not?
- Intuition?

Efficiency with Discounting

- Exactly the same result holds when we have discounting, i.e., $r > 0$
- In this case, the objective function is

$$SS^* = \int_0^\infty e^{-rt} \left[Ny - zN - \gamma\theta(L - N) \right] dt$$

and will be maximized subject to

$$\dot{N} = q(\theta)\theta(L - N) - sN$$

- Simple optimal control problem.

Efficiency with Discounting (continued)

- Solution (once again with some rearrangement):

$$(1-\eta(\theta))[y-z]-\frac{r+s+\eta(\theta)q(\theta)\theta}{q(\theta)}\gamma=0$$

- Compared to the equilibrium where

$$(1-\beta)[y-z]-\frac{r+s+\beta q(\theta)\theta}{q(\theta)}\gamma=0$$

Efficiency with Discounting

- Again, $\eta(\theta) = \beta$ would decentralize the constrained efficient allocation.
- Does the surplus maximizing allocation to zero unemployment?
- Why not?
- What is the social value unemployment?