14.452 Economic Growth: Lecture 8, Endogenous Technological Change

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Introduction

- The key to understanding *technology* is that R&D and technology adoption are purposeful activities.
- This lecture, focus on technological change and R&D.
- The simplest models of endogenous technological change are those in which R&D expands the variety of inputs or machines used in production (Romer, 1990).
- Models with expanding input varieties:
  - research will lead to the creation of new varieties of inputs (machines) and a greater variety of inputs will increase the "division of labor"
  - *process innovation*.
- Alternative: *product innovation* (Grossman and Helpman (1991a,b)):
  - invention of new goods,
  - because of love-for-variety, "real" incomes increase

Key Insights

- Innovation as generating new blueprints or *ideas* for production.
- Three important features (Romer):
  1. Ideas and technologies *nonrival*—many firms can benefit from the same idea.
  2. Increasing returns to scale—constant returns to scale to capital, labor, material etc. and then ideas and blueprints are also produced.
  3. Costs of research and development paid as fixed costs upfront.
- We must consider models of *monopolistic competition*, where firms that innovate become monopolists and make profits.
- Throughout use the Dixit-Stiglitz constant elasticity structure.

The Lab Equipment Model with Input Varieties

- All that is required for research is investment in equipment or in laboratories
- That is, new machines and ideas are created using the final good.
  - rather than the employment of skilled or unskilled workers or scientists.
  - similar to Rebelo's AK economy.
  - useful benchmark, since it minimizes the extent of spillovers and externalities.

Demographics, Preferences, and Technology

- Infinite-horizon economy, continuous time.
- Representative household with preferences:

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt. \tag{1}$$

- L =total (constant) population of workers. Labor supplied inelastically.
- Representative household owns a balanced portfolio of all the firms in the economy.

Demographics, Preferences, and Technology I

- Unique consumption good, produced with aggregate production function:

$$Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu\right] L^{\beta}, \tag{2}$$

  where

  - $N(t)$=number of varieties of inputs (machines) at time t,
  - $x(\nu,t)$=amount of input (machine) type ν used at time t.

- The x's depreciate fully after use.
- They can be interpreted as generic inputs, intermediate goods, machines, or capital.
- Thus machines are *not* additional state variables.
- For given $N(t)$, which final good producers take as given, (2) exhibits constant returns to scale.

Demographics, Preferences, and Technology II

- Final good producers are competitive.
- The resource constraint of the economy at time t is

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{3}$$

  where $X(t)$ is investment on inputs at time t and $Z(t)$ is expenditure on R&D at time t.
- Once the blueprint of a particular input is invented, the research firm can create one unit of that machine at marginal cost equal to $\psi > 0$ units of the final good.

Innovation Possibilities Frontier and Patents I

- *Innovation possibilities frontier*:

$$\dot{N}(t) = \eta Z(t), \tag{4}$$

  where $\eta > 0$, and the economy starts with some $N(0) > 0$.
- There is free entry into research: any individual or firm can spend one unit of the final good at time t in order to generate a flow rate η of the blueprints of new machines.
- The firm that discovers these blueprints receives a *fully-enforced perpetual patent* on this machine.
- There is no aggregate uncertainty in the innovation process.
  - There will be uncertainty at the level of the individual firm, but with many different research labs undertaking such expenditure, at the aggregate level, equation (4) holds deterministically.

Innovation Possibilities Frontier and Patents II

- A firm that invents a new machine variety v is the sole supplier of that type of machine, and sets a profit-maximizing price of $p^x(\nu, t)$ at time t to maximize profits.
- Since machines depreciate after use, $p^x(\nu, t)$ can also be interpreted as a "rental price" or the user cost of this machine.

The Final Good Sector

- Maximization by final the producers:

$$\max_{[x(\nu,t)]_{l\nu\in[0,N(t)]},L} \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu\right] L^{\beta} \tag{5}$$
$$-\int_0^{N(t)} p^x(\nu,t)\, x(\nu,t) d\nu - w(t) L.$$

- Demand for machines:

$$x(\nu,t) = p^x(\nu,t)^{-1/\beta} L, \tag{6}$$

- Isoelastic demand for machines.
- Only depends on the user cost of the machine and on equilibrium labor supply but not on the interest rate, $r(t)$, the wage rate, $w(t)$, or the total measure of available machines, $N(t)$.

Profit Maximization by Technology Monopolists I

- Consider the problem of a monopolist owning the blueprint of a machine of type ν invented at time t.
- Since the representative household holds a balanced portfolio of all the firms, no uncertainty in dividends and each monopolist's objective is to maximize expected profits.
- The monopolist chooses an investment plan starting from time t to maximize the discounted value of profits:

$$V(\nu, t) = \int_t^\infty \exp\left[-\int_t^s r\left(s'\right) ds'\right] \pi(\nu, s)\, ds \tag{7}$$

where

$$\pi(\nu, t) \equiv p^x(\nu, t) x(\nu, t) - \psi x(\nu, t)$$

denotes profits of the monopolist producing intermediate ν at time t, $x(\nu, t)$ and $p^x(\nu, t)$ are the profit-maximizing choices and $r(t)$ is the market interest rate at time t.

Profit Maximization by Technology Monopolists II

- For future reference, the discounted value of profits can also be written in the alternative Hamilton-Jacobi-Bellman form:

$$r(t)\,V(\nu,t)-\dot{V}(\nu,t)=\pi(\nu,t). \tag{8}$$

- This equation shows that the discounted value of profits may change because of two reasons:
  1. Profits change over time
  2. The market interest rate changes over time.

Characterization of Equilibrium I

- An allocation in this economy is defined by time paths of:
  - consumption levels, aggregate spending on machines, and aggregate R&D expenditure $\left[C\left(t\right), X\left(t\right), Z\left(t\right)\right]_{t=0}^{\infty}$,
  - available machine types, $\left[N\left(t\right)\right]_{t=0}^{\infty}$,
  - prices and quantities of each machine and the net present discounted value of profits from that machine, $\left[p^{x}\left(\nu, t\right), x\left(\nu, t\right), V\left(\nu, t\right)\right]_{\nu \in N(t), t=0}^{\infty}$, and
  - interest rates and wage rates, $\left[r\left(t\right), w\left(t\right)\right]_{t=0}^{\infty}$.
- An equilibrium is an allocation in which
  - all research firms choose $\left[p^{x}\left(\nu, t\right), x\left(\nu, t\right)\right]_{\nu \in [0, N(t)], t=0}^{\infty}$ to maximize profits,
  - $\left[N\left(t\right)\right]_{t=0}^{\infty}$ is determined by free entry,
  - $\left[r\left(t\right), w\left(t\right)\right]_{t=0}^{\infty}$, are consistent with market clearing, and
  - $\left[C\left(t\right), X\left(t\right), Z\left(t\right)\right]_{t=0}^{\infty}$ are consistent with consumer optimization.

Characterization of Equilibrium II

- Since (6) defines isoelastic demands, the solution to the maximization problem of any monopolist $\nu \in [0, N(t)]$ involves setting the same price in every period:

$$p^x(\nu, t) = \frac{\psi}{1-\beta} \text{ for all } \nu \text{ and } t. \tag{9}$$

- Normalize $\psi \equiv (1-\beta)$, so that

$$p^x(\nu, t) = p^x = 1 \text{ for all } \nu \text{ and } t.$$

- Profit-maximization also implies that each monopolist rents out the same quantity of machines in every period, equal to

$$x(\nu, t) = L \text{ for all } \nu \text{ and } t. \tag{10}$$

Characterization of Equilibrium III

- Monopoly profits:

$$\pi(\nu, t) = \beta L \text{ for all } \nu \text{ and } t. \tag{11}$$

- Substituting (6) and the machine prices into (2) yields:

$$Y(t) = \frac{1}{1-\beta} N(t) L. \tag{12}$$

- Even though the aggregate production function exhibits constant returns to scale from the viewpoint of final good firms (which take $N(t)$ as given), there are *increasing returns to scale* for the entire economy;
- An increase in $N(t)$ raises the productivity of labor and when $N(t)$ increases at a constant rate so will output per capita.

Characterization of Equilibrium IV

- Equilibrium wages:

$$w(t) = \frac{\beta}{1-\beta} N(t). \tag{13}$$

- Free entry

$$\begin{aligned} \eta V(\nu, t) &\leq 1,\ Z(\nu, t) \geq 0 \text{ and} \\ (\eta V(\nu, t) - 1) Z(\nu, t) &= 0,\ \text{for all } \nu \text{ and } t, \end{aligned} \tag{14}$$

  where $V(\nu, t)$ is given by (7).

- For relevant parameter values with positive entry and economic growth:

$$\eta V(\nu, t) = 1.$$

Characterization of Equilibrium V

- Finally, the representative household's problem is standard and implies the usual Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho) \tag{15}$$

and the transversality condition

$$\lim_{t \to \infty} \left[\exp \left(- \int_0^t r(s)\, ds \right) N(t) V(t) \right] = 0. \tag{16}$$

Equilibrium and Balanced Growth Path I

- We can now define an equilibrium more formally as time paths
  - $[C(t), X(t), Z(t), N(t)]_{t=0}^{\infty}$, such that (3), (15), (16) and (14) are satisfied;
  - $[p^x(\nu,t), x(\nu,t)]_{\nu \in N(t), t=0}^{\infty}$ that satisfy (9) and (10),
  - $[r(t), w(t)]_{t=0}^{\infty}$ such that (13) and (15) hold.
- We define a *balanced growth path (BGP)* as an equilibrium path where $C(t)$, $X(t)$, $Z(t)$ and $N(t)$ grow at a constant rate. Such an equilibrium can alternatively be referred to as a "steady state", since it is a steady state in transformed variables.

Balanced Growth Path I

- A balanced growth path (BGP) requires that consumption grows at a constant rate, say g_C. This is only possible from (15) if

$$r(t) = r^* \text{ for all } t$$

- Since profits at each date are given by (11) and since the interest rate is constant, $\dot{V}(t) = 0$ and

$$V^* = \frac{\beta L}{r^*}. \tag{17}$$

Balanced Growth Path II

- Let us next suppose that the (free entry) condition (14) holds as an equality, in which case we also have

$$\frac{\eta\beta L}{r^*} = 1$$

This equation pins down the steady-state interest rate, r^*, as:

$$r^* = \eta\beta L$$

- The consumer Euler equation, (15), then implies that the rate of growth of consumption must be given by

$$g_C^* = \frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r^* - \rho). \tag{18}$$

Balanced Growth Path III

- Note the current-value Hamiltonian for the consumer's maximization problem is concave, thus this condition, together with the transversality condition, characterizes the optimal consumption plans of the consumer.
- In BGP, consumption grows at the same rate as total output

$$g^* = g_C^*.$$

Therefore, given r^*, the long-run growth rate of the economy is:

$$g^* = \frac{1}{\theta} \left(\eta \beta L - \rho \right) \tag{19}$$

- Suppose that

$$\eta \beta L > \rho \text{ and } \left(1 - \theta\right) \eta \beta L < \rho, \tag{20}$$

which will ensure that $g^* > 0$ and that the transversality condition is satisfied.

Balanced Growth Path IV

Proposition Suppose that condition (20) holds. Then, in the above-described lab equipment expanding input variety model, there exists a unique balanced growth path in which technology, output and consumption all grow at the same rate, g^*, given by (19)..

- An important feature of this class models is the presence of the *scale effect*: the larger is L, the greater is the growth rate.

Transitional Dynamics I

- There are no transitional dynamics in this model.
- Substituting for profits in the value function for each monopolist, this gives
$$r\left(t\right)V(\nu,t)-\dot{V}(\nu,t)=\beta L.$$
- The key observation is that positive growth at any point implies that $\eta V(\nu,t)=1$ for all t. In other words, if $\eta V(\nu,t')=1$ for some t', then $\eta V(\nu,t)=1$ for all t.
- Now differentiating $\eta V(\nu,t)=1$ with respect to time yields $\dot{V}(\nu,t)=0$, which is only consistent with $r\left(t\right)=r^{*}$ for all t, thus
$$r\left(t\right)=\eta\beta L\text{ for all }t.$$

Transitional Dynamics II

Proposition Suppose that condition (20) holds. In the above-described lab equipment expanding input-variety model, with initial technology stock $N(0) > 0$, there is a unique equilibrium path in which technology, output and consumption always grow at the rate g^* as in (19).

- While the microfoundations here are very different from the neoclassical AK economy, the mathematical structure is very similar to the AK model (as most clearly illustrated by the derived equation for output, (12)).
- Consequently, as in the AK model, the economy always grows at a constant rate.
- But the economics is very different.

Social Planner Problem I

- Monopolistic competition implies that the competitive equilibrium is not necessarily Pareto optimal. The model exhibits a version of the *aggregate demand externalities*:
  1. There is a markup over the marginal cost of production of inputs.
  2. The number of inputs produced at any point in time may not be optimal.
- The first inefficiency is familiar from models of static monopoly, while the second emerges from the fact that in this economy the set of traded (Arrow-Debreu) commodities is endogenously determined.
- This relates to the issue of endogenously incomplete markets (there is no way to purchase an input that is not supplied in equilibrium).

Social Planner Problem II

- Given $N(t)$, the social planner will choose

$$\max_{[x(\nu,t)]_{\nu\in[0,N(t)]},L} \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu\right] L^{\beta} - \int_0^{N(t)} \psi x(\nu,t) d\nu,$$

- Differs from the equilibrium profit maximization problem, (5), because the marginal cost of machine creation, ψ, is used as the cost of machines rather than the monopoly price, and the cost of labor is not subtracted.
- Recalling that $\psi \equiv 1-\beta$, the solution to this program involves

$$x^S(\nu,t) = (1-\beta)^{-1/\beta} L,$$

Social Planner Problem III

- The *net* output level (after investment costs are subtracted) is

$$Y^S(t) = \frac{(1-\beta)^{-(1-\beta)/\beta}}{1-\beta} N^S(t) L$$
$$= (1-\beta)^{-1/\beta} N^S(t) L,$$

- Therefore, the maximization problem of the social planner can be written as

$$\max \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} \exp(-\rho t)\, dt$$

subject to

$$\dot{N}(t) = \eta (1-\beta)^{-1/\beta} \beta N(t) L - \eta C(t) .$$

where $(1-\beta)^{-1/\beta} \beta N^S(t) L$ is net output.

Social Planner Problem IV

- In this problem, $N(t)$ is the state variable, and $C(t)$ is the control variable. The current-value Hamiltonian is:

$$\begin{aligned} \hat{H}(N, C, \mu) &= \frac{C(t)^{1-\theta} - 1}{1-\theta} \\ &\quad + \mu(t)\left[\eta(1-\beta)^{-1/\beta}\beta N(t) L - \eta C(t)\right]. \end{aligned}$$

- The conditions for a candidate Pareto optimal allocation are:

$$\begin{aligned} \hat{H}_C(N, C, \mu) &= C(t)^{-\theta} - \eta\mu(t) = 0 \\ \hat{H}_N(N, C, \mu) &= \mu(t)\eta(1-\beta)^{-1/\beta}\beta L \\ &= \rho\mu(t) - \dot{\mu}(t) \\ \lim_{t\to\infty}\left[\exp(-\rho t)\mu(t) N(t)\right] &= 0. \end{aligned}$$

Social Planner Problem V

- It can be verified easily that the current-value Hamiltonian of the social planner is (strictly) concave, thus these conditions are also sufficient for an optimal solution.
- Combining these conditions:

$$\frac{\dot{C}^S(t)}{C^S(t)} = \frac{1}{\theta}\left(\eta(1-\beta)^{-1/\beta}\beta L - \rho\right). \tag{21}$$

Comparison of Equilibrium and Pareto Optimum

- The comparison to the growth rate in the decentralized equilibrium, (19), boils down to that of

$$(1-\beta)^{-1/\beta}\beta \text{ to } \beta,$$

- The socially-planned economy will always grow faster than the decentralized economythe former is always greater since $(1-\beta)^{-1/\beta} > 1$ by virtue of the fact that $\beta \in (0,1)$.

Comparison

Proposition In the above-described expanding input variety model, the decentralized equilibrium is always Pareto suboptimal. Starting with any $N(0) > 0$, the Pareto optimal allocation involves a constant growth rate

$$g^S = \frac{1}{\theta}\left(\eta (1-\beta)^{-1/\beta}\beta L - \rho\right),$$

which is strictly greater than the equilibrium growth rate g^* given in (19).

Comparison

- Why is the equilibrium growing more slowly than the optimum allocation?
- Because the social planner values innovation more
- The social planner is able to use the machines more intensively after innovation, *pecuniary externality* resulting from the monopoly markups.
- Other models of endogenous technological progress we will study in this lecture incorporate technological spillovers and thus generate inefficiencies both because of the pecuniary externality isolated here and because of the standard technological spillovers.

Policies

- What kind of policies can increase equilibrium growth rate?

1. *Subsidies to Research:* the government can increase the growth rate of the economy, and this can be a Pareto improvement if taxation is not distortionary and there can be appropriate redistribution of resources so that all parties benefit.
2. *Subsidies to Capital Inputs:* inefficiencies also arise from the fact that the decentralized economy is not using as many units of the machines/capital inputs (because of the monopoly markup); so subsidies to capital inputs given to final good producers would also increase the growth rate.

- But note, the same policies can also be used to distort allocations.
- When we look at a the cross-section of countries, taxes on research and capital inputs more common than subsidies.

The Effects of Competition I

- Recall that the monopoly price is:

$$p^x = \frac{\psi}{1-\beta}.$$

- Imagine, instead, that a fringe of competitive firms can copy the innovation of any monopolist.
    - But instead of a marginal cost ψ, the fringe has marginal cost of $\gamma\psi$ with $\gamma > 1$.
- If $\gamma > 1/(1-\beta)$, no threat from the fringe.
- If $\gamma < 1/(1-\beta)$, the fringe would forced the monopolist to set a "*limit price*",

$$p^x = \gamma\psi. \tag{22}$$

The Effects of Competition II

- Why? If $p^x > \gamma\psi$, the fringe could undercut the price of the monopolist, take over to market and make positive profits.
  If $p^x < \gamma\psi$, the monopolist could increase price and make more profits.
  Thus, there is a unique equilibrium price given by (22).
- Profits under the limit price:

$$\text{profits per unit } = (\gamma - 1)\,\psi = (\gamma - 1)\,(1-\beta) < \beta,$$

- Therefore, growth with competition:

$$\hat{g} = \frac{1}{\theta}\left(\eta\gamma^{-1/\beta}\,(\gamma - 1)\,(1-\beta)^{-(1-\beta)/\beta}\,L - \rho\right) < g^*.$$

Growth with Knowledge Spillovers I

- In the lab equipment model, growth resulted from the use of final output for R&D. This is similar to the endogenous growth model of Rebelo (1991), since the accumulation equation is linear in accumulable factors. In equilibrium, output took a linear form in the stock of knowledge (new machines), thus a AN form instead of Rebelo's AK form.
- An alternative is to have "scarce factors" used in R&D: we have scientists as the key creators of R&D.
- With this alternative, there cannot be endogenous growth unless there are knowledge spillovers from past R&D, making the scarce factors used in R&D more and more productive over time.

Innovation Possibilities Frontier I

- Innovation possibilities frontier in this case:

$$\dot{N}(t) = \eta N(t) L_R(t) \tag{23}$$

  where $L_R(t)$ is labor allocated to R&D at time t.
- The term $N(t)$ on the right-hand side captures spillovers from the stock of existing ideas.
- Notice that (23) imposes that these spillovers are proportional or linear. This linearity will be the source of endogenous growth in the current model.
- In (23), $L_R(t)$ comes out of the regular labor force.The cost of workers to the research sector is given by the wage rate in final good sector.

Characterization of Equilibrium I

- Most of equilibrium characterization very similar.
- Labor market clearing:

$$L_R(t) + L_E(t) \leq L.$$

- Aggregate output of the economy:

$$Y(t) = \frac{1}{1-\beta} N(t) L_E(t), \tag{24}$$

  and profits of monopolists from selling their machines is

$$\pi(t) = \beta L_E(t). \tag{25}$$

- The net present discounted value of a monopolist (for a blueprint ν) is still given by $V(\nu, t)$ as in (7) or (8), with the flow profits given by (25).

Characterization of Equilibrium II

- The free entry condition is no longer the same. Instead, (23) implies:

$$\eta N(t) V(\nu, t) = w(t), \tag{26}$$

  where $N(t)$ is on the left-hand side because it parameterizes the productivity of an R&D worker, while the flow cost of undertaking research is hiring workers for R&D, thus is equal to the wage rate $w(t)$.

- The equilibrium wage rate must be the same as before:

$$w(t) = \beta N(t) / (1 - \beta)$$

- Balanced growth again requires that the interest rate must be constant at some level r^*.

Characterization of Equilibrium III

- Using these observations together with the free entry condition, we obtain:

$$\eta N(t)\frac{\beta L_E(t)}{r^*}=\frac{\beta}{1-\beta}N(t). \tag{27}$$

Hence the BGP equilibrium interest rate must be

$$r^*=(1-\beta)\eta L_E^*,$$

where $L_E^*=L-L_R^*$. The fact that the number of workers in production must be constant in BGP follows from (27).

- Now using the Euler equation of the representative household, (15), for all t:

$$\begin{aligned}\frac{\dot{C}(t)}{C(t)} &= \frac{1}{\theta}\left((1-\beta)\eta L_E^*-\rho\right) &(28)\\ &\equiv g^*.\end{aligned}$$

Characterization of Equilibrium IV

- To complete the characterization of the BGP equilibrium, we need to determine L_E^*. In BGP, (23) implies that the rate of technological progress satisfies

$$\frac{\dot{N}(t)}{N(t)} = \eta L_R^* = \eta (L - L_E^*)$$

This implies that the BGP level of employment is

$$L_E^* = \frac{\theta \eta L + \rho}{(1-\beta)\eta + \theta \eta}. \tag{29}$$

Summary of Equilibrium in the Model with Knowledge Spillovers

Proposition Consider the above-described expanding input-variety model with knowledge spillovers and suppose that

$$(1-\theta)(1-\beta)\,\eta L_E^* < \rho < (1-\beta)\,\eta L_E^*, \tag{30}$$

where L_E^* is the number of workers employed in production in BGP, given by (29).Then there exists a unique balanced growth path in which technology, output and consumption grow at the same rate, $g^* > 0$, given by (28) starting from any initial level of technology stock $N(0) > 0$.

- As in the lab equipment model, the equilibrium allocation is Pareto suboptimal.

Growth without Scale Effects: Motivation

- The models so far feature a scale effect.
- A larger population $L \Longrightarrow$ higher interest rate and a higher growth rate.
- Potentially problematic for three reasons:

1. Larger countries do not necessarily grow faster.
2. The population of most nations has not been constant. If we have population growth as in the standard neoclassical growth model, e.g., $L(t) = \exp(nt) L(0)$, these models would not feature balanced growth, rather, the growth rate of the economy would be increasing over time.
3. In the data, the total amount of resources devoted to R&D appears to increase steadily, but there is no associated increase in the aggregate growth rate.

Knowledge Spillovers Model with two Differences

- Differences:

  1. Population growth at exponential rate n, $\dot{L}(t) = nL(t)$.
     Representative household, also growing at the rate n, with preferences:

$$\int_0^\infty \exp\left(-(\rho - n)t\right) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt, \tag{31}$$

  2. R&D sector only admits limited knowledge spillovers and (23) is replaced by

$$\dot{N}(t) = \eta N(t)^{\phi} L_R(t) \tag{32}$$

     where $\phi < 1$ and $L_R(t)$ is labor allocated to R&D activities at time t.
     Labor market clearing requires

$$L_E(t) + L_R(t) = L(t), \tag{33}$$

Growth without Scale Effects I

- Aggregate output and profits are given by (24) and (25) as in the previous section. An equilibrium is also defined similarly.
- Focus on the BGP. Free entry with equality:

$$\eta N(t)^{\phi} \frac{\beta L_E(t)}{r^* - n} = w(t). \tag{34}$$

- As before, the equilibrium wage is determined by the production side, (13), as

$$w(t) = \beta N(t) / (1 - \beta).$$

  Thus,

$$\eta N(t)^{\phi - 1} \frac{(1 - \beta) L_E(t)}{r^* - n} = 1.$$

Growth without Scale Effects II

- Differentiating this condition with respect to time, we obtain

$$(\phi - 1)\frac{\dot{N}(t)}{N(t)} + \frac{\dot{L}_E(t)}{L_E(t)} = 0.$$

- Since in BGP, the fraction of workers allocated to research is constant, we must have

$$\dot{L}_E(t) / L_E(t) = n$$

- Thus,

$$g_N^* \equiv \frac{\dot{N}(t)}{N(t)} = \frac{n}{1-\phi}. \tag{35}$$

$$\begin{aligned} g_C^* &= g_N^* \\ &= \frac{n}{1-\phi}. \end{aligned} \tag{36}$$

Summary of Equilibrium without Scale Effects

Proposition In the above-described expanding input-variety model with limited knowledge spillovers as given by (32), starting from any initial level of technology stock $N(0) > 0$, there exists a unique balanced growth path in which, technology and consumption per capita grow at the rate g_N^* as given by (35), and output grows at rate $g_N^* + n$.

- Sustained equilibrium growth of per capita income is possible with growing population.
- Instead of the linear (proportional) spillovers, only a limited amount of spillovers.
- Without population growth, these spillovers would affect the level of output, but not sufficient to sustain long-run growth.
- Population growth increases the market size for new technologies steadily and generates growth from these limited spillovers.

Discussion

- "Growth without scale effects"?
- There are two senses in which there are still scale effects:
  1. A faster rate of population growth translates into a higher equilibrium growth rate.
  2. A larger population size leads to higher output per capita.
- Empirical evidence?
- "Semi-endogenous growth" models, because growth is determined only by population growth and technology, and does not respond to policies.
  - Extensions to allow for the impact of policy and growth possible (though under somewhat restrictive assumptions).