Financial Conditions Targeting

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Abstract

We investigate how monetary policy should respond to financial noise—non-fundamental fluctuations in asset prices. We provide evidence that noisy financial flows significantly influence both financial conditions and macroeconomic activity in the U.S., with effects resembling standard demand shocks. We develop a macroeconomic model with financial noise and limits to arbitrage that replicates these empirical patterns. Our main result shows that Financial Conditions Index (FCI) targeting—announcing a soft, temporary FCI target and adjusting future policy rates to maintain actual conditions near this target—reduces both FCI volatility and output gap fluctuations. FCI targeting commits the central bank to greater FCI stabilization than under discretionary policy, which reduces FCI volatility and "recruits" arbitrageurs to insulate FCI and aggregate demand from noise. We extend policy counterfactual methods to incorporate endogenous risk reduction and estimate that FCI targeting could have meaningfully reduced output gap and FCI volatility without increasing interest rate volatility.

JEL Codes: E12, E32, E44, E52, G10
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1. Introduction

"Financial conditions have tightened significantly in recent months... We remain attentive to these developments because persistent changes in financial conditions can have implications for the path of monetary policy." (Chair Jerome Powell, Economic Club of New York Luncheon, October 19, 2023)

Monetary policy transmission occurs primarily through financial market conditions. Businesses and consumers react to market-based interest rates, stock prices, house prices, and currency exchange rates rather than to the policy rate alone. Financial Conditions Indices (FCI), which aggregate these asset classes based on their impact on aggregate spending, identify volatile asset prices, especially stock prices, as their main driver in the U.S. and most major economies (see, e.g., Hatzius et al. (2017)). These asset prices exhibit excess volatility that cannot be explained by rationally expected cashflows alone (see, e.g., Campbell (2014)). This excess volatility is partly driven by *noise shocks*—changes in asset demand or supply that are unrelated to economic fundamentals. Such noise shocks move asset prices in practice because sophisticated investors face constraints or risks that limit their ability to trade against mispricing (see De Long et al. (1990); Gabaix and Koijen (2021)). To quantify this mechanism, we estimate a mid-size vector-autoregression (VAR) with an external instrument for financial noise shocks. We find that noisy financial flows explain up to 40% of the contemporaneous variance of financial conditions and up to 25% of the variance of output gaps in the U.S. (see Section 2). This contribution to output variation is substantial—comparable to the high end of typical structural shocks like productivity, investment-specific, or monetary policy shocks (see Table 13 in Ramey (2016)). Given this large macroeconomic impact, how should monetary policy react to this financial noise?

Bernanke and Gertler (2000, 2001) examine this question within a New Keynesian model with exogenous asset bubbles, arguing that central banks should not target asset prices directly but should instead focus on stabilizing the inflation and output gaps that arise from asset price fluctuations. In contrast, we propose a model with *endogenous* financial volatility in which it is optimal for the central bank to target and partially stabilize financial conditions as an independent objective, *beyond* simply managing their effects on output and inflation. We further show that financial conditions targeting strictly dominates traditional interest rate forward guidance as a policy tool. Finally, using an extension of the policy counterfactual methods in Caravello et al. (2025), we find that financial conditions targeting would have significantly reduced the volatility of both the output gap and financial conditions in the U.S. over recent decades.

Our model builds on the "risk-centric" New Keynesian framework developed in Caballero and Simsek (2020, 2023), in which monetary policy transmits to macroeconomic activity primarily through financial market conditions rather than traditional interest rate channels. The key feature of this framework is that aggregate demand responds directly to the aggregate asset price (which serves as our Financial Conditions Index), capturing wealth effects (which stands in for broader mechanisms through which financial conditions influence spending decisions).

Monetary policy operates by adjusting the policy interest rate to influence this aggregate asset price, thereby steering aggregate demand and closing the output gap.

There are two key differences from Caballero and Simsek (2023). First, the aggregate asset price is not only influenced by the policy rate and standard financial forces but also by noise shocks. Specifically, households delegate their portfolio decisions to managers of three types: noise traders, inelastic funds, and risk-averse arbitrageurs. Noise traders create random market flows that must be absorbed by other investors. Inelastic funds cannot absorb these flows, since they passively invest according to the average optimal portfolio benchmark. Risk-averse arbitrageurs can absorb the noisy flows, but their limited capacity leaves substantial scope for noise to impact aggregate asset prices. Second, monetary policy reacts to aggregate noise shocks only with a delay, preventing it from fully managing financial conditions and aggregate demand.

Our main result demonstrates that an expanded monetary policy framework—where the central bank announces a soft and temporary FCI target and commits to setting the policy interest rate in the near term to keep the actual FCI close to the target—significantly improves welfare. The key mechanism is that FCI targeting "recruits" arbitrageurs for monetary policy, enabling them to absorb noise flows in real time. This mitigates the excess macroeconomic volatility that results from the delayed reaction of monetary policy to noise shocks.

This recruitment mechanism operates through an endogenous volatility feedback loop: noise has a greater impact on aggregate asset prices when return volatility is higher. This happens because higher return volatility makes arbitrageurs more reluctant to trade against noise. The larger price impact of noise leads to an endogenous increase in return volatility, which further amplifies the price impact of noise, and so on. This noise-driven volatility in aggregate asset prices affects macroeconomic activity and leads to "excessive" fluctuations in the output gap.

FCI targeting breaks this cycle by inducing the central bank to stabilize future returns beyond what is implied by its standard objectives. In our baseline model, this means the central bank dampens its own response to future macroeconomic data, reducing the return volatility associated with these responses, even though this might result in temporary output gaps. In an extension with costly interest rate adjustment, FCI targeting also implies that the central bank accelerates its own response to noise shocks, despite the adjustment costs. The common denominator is that FCI targeting induces a less volatile return—a more stable FCI—than implied by discretionary policy.

This lower return volatility is the key mechanism by which the policy recruits arbitrageurs: with lower volatility, arbitrageurs trade more aggressively to counter noisy financial flows and the market becomes more elastic with respect to these flows. This counterforce reverses the volatility feedback loop and thereby reduces the impact of noise on the FCI and output. Surprisingly, this reduction in the price impact of noise also implies that, in many instances, the policy objective can be achieved with lower policy rate volatility.

The downside of the volatility reduction mechanism is that it makes monetary policy less responsive to macroeconomic shocks, allowing these shocks to have a greater impact on the output gap. However, this trade-off is worthwhile. We show that starting from a discretionary benchmark, implementing some FCI targeting is always optimal. This is because the reduced flexibility with respect to macroeconomic shocks results in only a second-order loss, while the significant reduction in the impact of noise generates first-order gains in stabilizing the output gap. The intuition is that FCI targeting improves welfare by addressing a key time inconsistency in monetary policy: committing to stabilize *future* returns helps achieve *today's* policy objectives by reducing return volatility and recruiting arbitrageurs to trade more aggressively against noise.

FCI targeting is akin to providing *forward guidance* about the future path of the FCI, provided that such guidance is viewed as entailing some degree of commitment rather than mere communication. We show that FCI forward guidance is strictly superior to guidance on the policy interest rate. This is because FCI forward guidance naturally allows the policy to react to future noise shocks, whereas interest rate forward guidance reduces the flexibility of the central bank to react to these shocks. In other words, the optimal policy reduces data-dependency with respect to macroeconomic shocks while increasing data-dependency with respect to noise shocks. FCI targeting achieves both goals, while interest rate forward guidance only achieves the former.

Beyond the specific mechanisms we emphasize, FCI targeting could stabilize asset prices for a variety of additional reasons. For instance, as emphasized by Jeanne and Rose (2002), noise itself tends to decline when a stabilizing policy framework is in place, creating a virtuous cycle of reduced volatility. This stabilization is not limited to noise traders but extends to belief-driven fluctuations in asset prices, such as the anticipation of transformative events like an AI revolution. In an extension of our model, we demonstrate that when markets anticipate the Fed's commitment to maintaining stable financial conditions, belief shocks—whether justified or not—lead to smaller asset price fluctuations, thereby mitigating their broader impact on macroeconomic stability.

In the final part of the paper we conduct an empirical evaluation of FCI targeting by building upon the counterfactual policy evaluation methods described in McKay and Wolf (2023b) and Caravello et al. (2025). The key idea is to use estimated impulse responses to monetary policy *shocks* to approximate the effects of counterfactual monetary policy *rules*. This methodology identifies the correct counterfactual as long as the model is linear and monetary policy operates through current or expected policy interest rates. However, our model presents a complication: FCI targeting stabilizes the economy by reducing risk, and this risk reduction is a non-linear element. We adapt the methodology to account for this non-linearity and estimate counterfactuals for FCI targeting policies. This extension should prove useful in other settings where the effects of economic policy operate through second moments.

Our main empirical results examine a scenario in which the Fed minimizes a weighted average of output gaps, inflation gaps, interest rate changes, and the FCI deviations from a preannounced (optimized) FCI target. We show that this policy framework significantly reduces the impact of noise shocks on financial conditions, the output gap, and inflation compared to historical data. This stabilization is achieved with only a modest increase in the initial interest rate response (frontloading) while reducing overall interest rate volatility, as the framework encourages arbitrageurs to absorb more of the noise. With arbitrageurs playing a larger stabilizing role, the policy interest rate requires smaller total adjustments to these shocks. Beyond noise shocks, targeting financial conditions substantially reduces the volatility of macroeconomic and financial variables. Compared to historical data, the variance of the output gap and interest rates decreases by 26.3% and 0.4%, respectively, while the conditional variance of financial conditions decreases by 35.8%. These reductions outperform standard alternatives, such as a flexible dual mandate framework or interest rate forward guidance.

Literature review. Our paper connects two main literatures: one in macroeconomics and one in finance. On the macroeconomics side, our paper is part of an emerging literature on New Keynesian models with risk and asset prices (e.g., Caballero and Farhi (2018); Caballero and Simsek (2020, 2021, 2023, 2024); Pflueger et al. (2020); Kekre and Lenel (2022); Kekre et al. (2023); Beaudry et al. (2024); Adrian and Duarte (2018); Adrian et al. (2020)). Our main innovation is incorporating financial noise that interferes with monetary policy transmission, demonstrating the benefits of FCI targeting in such an environment.

On the finance side, our paper is related to a large literature that emphasizes asset price fluctuations driven by noise and limits to arbitrage (see Black (1986); Shleifer and Summers (1990); De Long et al. (1990) for early contributions). Noise refers to *nonfundamental* demand or supply by market participants that emerges from behavioral biases, institutional frictions, and segmented markets (see Gromb and Vayanos (2010)). Limits to arbitrage refers to the constraints faced by sophisticated investors in trading against such noise (see Shleifer and Vishny (1997)). This framework has been applied to explain asset price fluctuations in markets affecting financial conditions, including treasury bonds (Greenwood and Vayanos (2014); Vayanos and Vila (2021)), exchange rates (Gabaix and Maggiori (2015); Gourinchas et al. (2022); Greenwood et al. (2023)), and the aggregate stock market (Gabaix and Koijen (2021)). Our main contribution to this literature is to embed noise and limits-to-arbitrage into a macroeconomic model and show that these ingredients create a natural rationale for FCI targeting. We also provide empirical evidence that financial noise drives not only financial conditions but also macroeconomic activity.

Our model shares similarities with Jeanne and Rose (2002); Itskhoki and Mukhin (2021), who show that exchange rate stabilization can reduce volatility without significantly affecting other macroeconomic variables. Jeanne and Rose (2002) demonstrates that stabilization deters noise trader entry, creating a virtuous cycle, while Itskhoki and Mukhin (2021) shows it enables more aggressive trading against noise. While our mechanism resembles that of Itskhoki and Mukhin (2021), our framework differs crucially: noise-driven financial fluctuations have large *macroeconomic* effects (confirmed in Section 2), whereas exchange rate fluctuations have minimal aggregate impact in their model. This large macroeconomic impact is what justifies FCI targeting in our framework.

Our work relates to Woodford (2003), who shows that interest rate smoothing might be desirable even though it is not a social objective per se. Similarly, we show FCI targeting can be welfare-improving, but through a different mechanism: rather than affecting expectations of future interest rates, FCI targeting influences expectations of *future FCI volatility*, recruiting arbitrageurs to stabilize financial conditions.

Our paper also connects to the forward guidance literature (Campbell et al. (2012); Woodford (2013); Svensson (2014); Bassetto (2019); Caballero and Simsek (2025)). We show that forward guidance about the FCI—viewed as a soft commitment to an FCI target—can stabilize financial conditions and output gaps, complementing traditional interest rate guidance. Caballero and Simsek (2025) demonstrates that even without commitment, simply communicating the central bank's desired FCI level can activate the recruitment effect when markets are uncertain about the Fed's views, highlighting how delphic guidance (information provision) complements the odyssean guidance (commitment) analyzed here.

On the empirical side, our paper contributes to a growing literature examining Financial Condition Indices (FCIs) and their relationship with monetary policy (Hatzius et al., 2017; Hatzius and Stehn, 2018; Ajello et al., 2023a; Caballero et al., 2025). We make three contributions to this literature. First, we empirically document that financial noise affects both FCIs and macroeconomic activity. Second, we show the stabilization benefits of FCI targeting in noisy environments. Third, we provide an empirical evaluation of FCI targeting. Our approach also generates an FCI target that accounts for realistic policy constraints, including interest rate gradualism and other implementation frictions. This work complements (Caballero et al., 2025), who estimate an FCI analog of the natural rate of interest (r-star) using a parsimonious two-equation macroeconomic model following Laubach and Williams (2003).

Our empirical work builds on literature identifying macroeconomic effects of financial shocks (Gilchrist et al., 2009; Gilchrist and Zakrajšek, 2012; Christiano et al., 2014) and monetary policy transmission via financial markets (Gertler and Karadi, 2015; Caldara and Herbst, 2019). While previous work focuses primarily on credit spreads, we analyze the aggregate stock market and show that a plausibly exogenous variation in equity flows affects macroeconomic activity. We further show that equity market noise shocks are a significant driver of macroeconomic fluctuations.

Finally, our paper contributes to a recent literature on semi-structural policy counterfactuals (Hebden and Winkler, 2021; Barnichon and Mesters, 2023; Beraja, 2023; McKay and Wolf, 2023b; Caravello et al., 2025). Our methodological contribution demonstrates that for our class of models—where noise affects financial conditions proportionally to FCI variance—a simple departure from purely linear settings is sufficient to account for endogenous changes in risk levels in the counterfactuals. We use this approach to evaluate the efficacy of FCI targeting in stabilizing macroeconomic and financial fluctuations.

The rest of the paper is organized as follows. Section 2 presents facts on the macroeconomic effects of financial noise that motivate our theoretical analysis. Section 3 describes the model, characterizes the equilibrium with discretionary policy, and analyzes the destabilizing effects of noise. Section 4 presents our main theoretical results, demonstrating that FCI targeting can reduce financial volatility and improve macroeconomic stability. This section also compares the welfare effects of FCI targeting with interest rate forward guidance and discusses the robustness of FCI targeting in various model extensions. Section 5 extends recent methodology on counterfactual policy analysis to account for endogenous volatility in our model and uses it to estimate the effects of FCI targeting policies. Section 6 provides concluding remarks. The theory appendix A contains the derivations and various model extensions. The data appendix B presents the details of the empirical analysis and additional results. Finally, Appendix C contains details about the extensions to counterfactual methods.

2. The macroeconomic impact of financial noise

In this section, we explore the effects of stock market noise on financial conditions and macroeconomic activity. We focus on the stock market because it is the primary driver of FCI fluctuations in both the U.S. and other major economies. Our empirical analysis yields two key findings. First, financial noise shocks have effects similar to those of standard demand shocks. Second, these shocks are quantitatively important: they account for up to 40% of FCI forecast variance on impact and contribute up to 25% of output gap forecast variance over a two-year horizon. For reference, if we apply the procedure in Angeletos et al. (2020) to the residual variation (i.e., we choose a combination of all other shocks to maximize the share of output gap variance explained at business cycle frequencies), the resulting shock explains 31% of the variance of output gaps at a two-year horizon.

2.1. Data and methodology

We measure financial conditions with the Financial Conditions Impulse on Growth index (FCI-G) constructed by Ajello et al. (2023a). This index uses the Fed's macroeconomic models to predict how recent changes in seven financial variables will affect next year's GDP growth. The variables include the Dow Jones stock price index, the Zillow house price index, the broad dollar index, the federal funds rate, the 10-year Treasury yield, the 30-year fixed mortgage rate, and the triple-B corporate bond yield, with changes measured over the past three years. The index follows interest rate sign conventions, scaled so that a value of 1 indicates that current financial conditions will reduce next year's GDP growth by 1 percentage point. Figure 1 illustrates the index along with the contribution of each financial variable over 1990Q1-2024Q2. The stock market is the key driver of the index, followed by the exchange rate. The housing market is an important driver during the buildup of the GFC and the COVID-19 recovery.

We investigate the relationship between financial conditions and macroeconomic activity using a vector autoregression (VAR) that includes the FCI-G. Appendix B.1 contains a detailed discussion of our data construction. Our main sample is 1990Q1:2019Q4. We start in 1990 since the FCI-G index begins in this year and we stop before the Covid period to avoid outliers.[1] Given

[1]Appendix B.3.5 shows results when including the Covid period. The estimated IRFs remain unchanged, but

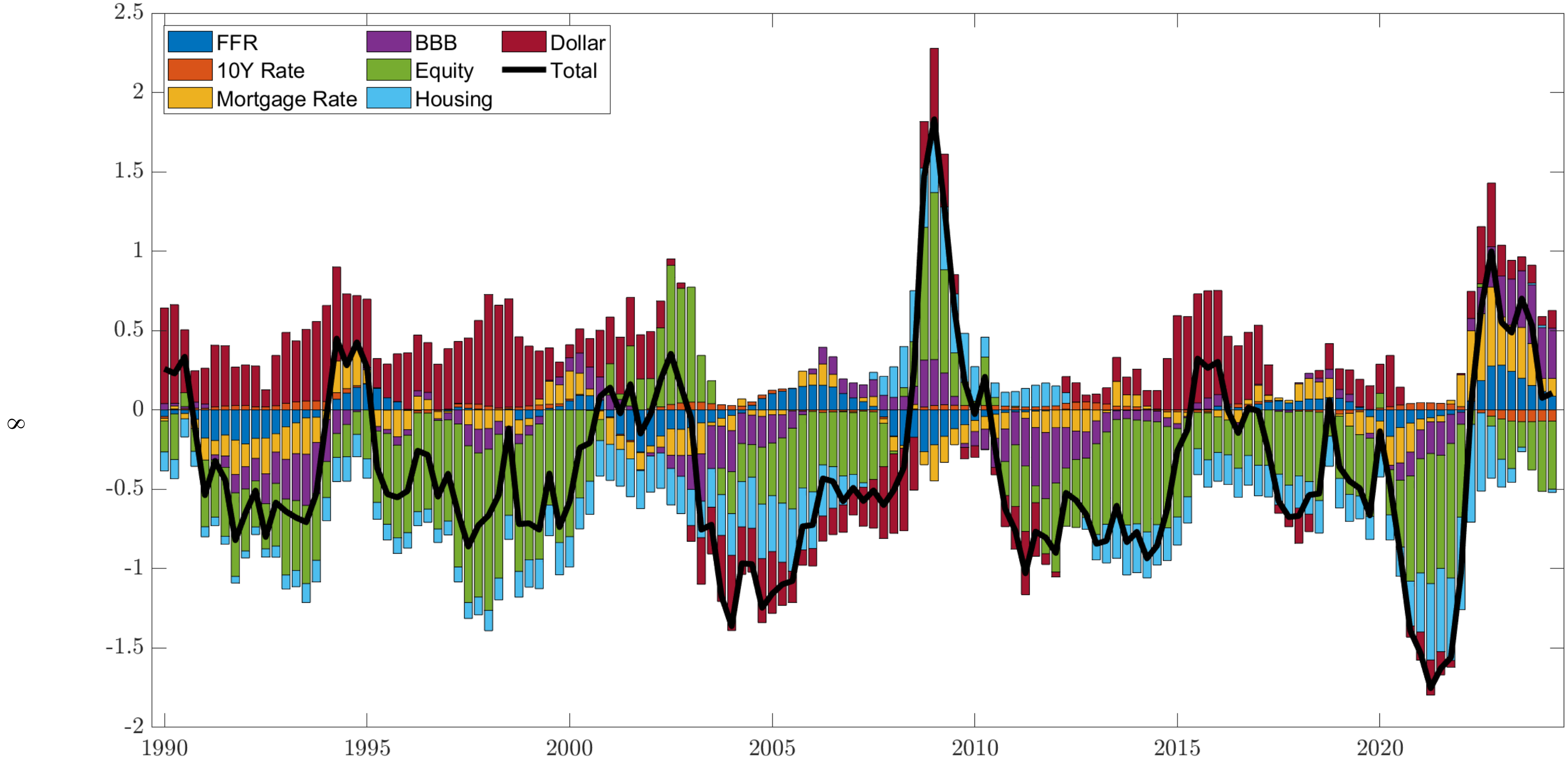

Figure 1: The FCI-G index (with a three-year lookback) and its drivers over 1990Q1-2024Q2. Positive values imply a decrease in GDP growth in the next year. Source: Ajello et al. (2023a)

that the counterfactual analysis requires invertibility, we include a relatively large number of variables. The estimated system contains the FCI, real GDP, the output gap measured as the log difference between real GDP and potential output estimates by the CBO, real investment, real consumption, annualized PCE inflation, the Excess Bond Premium of Gilchrist and Zakrajšek (2012), the 3-month nominal interest rate, hours per worker, the labor share, labor productivity, and TFP. These variables are standard in monetary and financial VAR specifications (Gilchrist and Zakrajšek, 2012; Gertler and Karadi, 2015). We include a constant and two lags. Given the large number of variables, we estimate a Bayesian VAR with a Minnesota prior. The tightness of the prior is chosen following Giannone et al. (2015). Appendix B.2 contains additional details on estimation, and Appendix B.3.1 reports additional estimation results.

We proxy for financial noise shocks using a Granular IV approach (Gabaix and Koijen, 2024). Following Gabaix and Koijen (2021), we construct the proxy using Flow of Funds data. Appendix B.1.2 reviews the construction details. The procedure works as follows: using flow-of-funds data, we measure the proportional changes in equity held by different sectors, Δq_{it}. Since these flows are endogenous, we residualize them using fixed effects, sector-specific trends, macro observables, and principal components to obtain residual flow shocks $\Delta \tilde{q}_{it}$ for each sector. These residuals can be interpreted as sector-specific financial noise shocks. We then construct our proxy for the financial flow shock Z_t^μ as an equity-share-weighted average:

$$Z_t^\mu = \sum_{i=1}^{I} S_{i,t-1} \Delta \tilde{q}_{it}. \tag{1}$$

Here, $S_{i,t-1}$ is the fraction of total equity held by sector i at time $t-1$. Since households represent a large share in the flow of funds data, this measure is primarily driven by residual changes in households' equity holdings. Gabaix and Koijen (2021) argue that this provides an appropriate measure of net flows into equities. The residualization step plays two important roles. First, it controls for confounding shocks that might affect equity valuations independently of changes in sectoral equity holdings. Second, it eliminates the mechanical constraint that aggregate flows sum to zero—the principal components absorb the sectors' response to price changes that ensure this identity holds. In the Appendix, we show that our results are robust to various choices regarding residualization.

As a final step, we further residualize the proxy shock with respect to lags of all variables in the VAR to construct the residualized proxy shock, $\tilde{Z}_t^\mu = Z_t^\mu - L\left[Z_t^\mu | \{Z_t^\mu, Y_\tau\}_{\tau<t}\right]$ (where $L(x|y)$ denotes the linear projection of variable x onto variables y). The red line in Figure 2 depicts this residualized proxy. The series shows substantial variation throughout the sample period. There is a large, negative shock around the GFC, but there are also other shocks of comparable magnitude in other periods, such as the early 1990s and mid-2000s.

Given our proxy for financial noise shocks and the macroeconomic observables, we assume

the variance contribution to output gaps decreases because the large Covid shock dominates and the noise shock plays a limited role post-Covid.

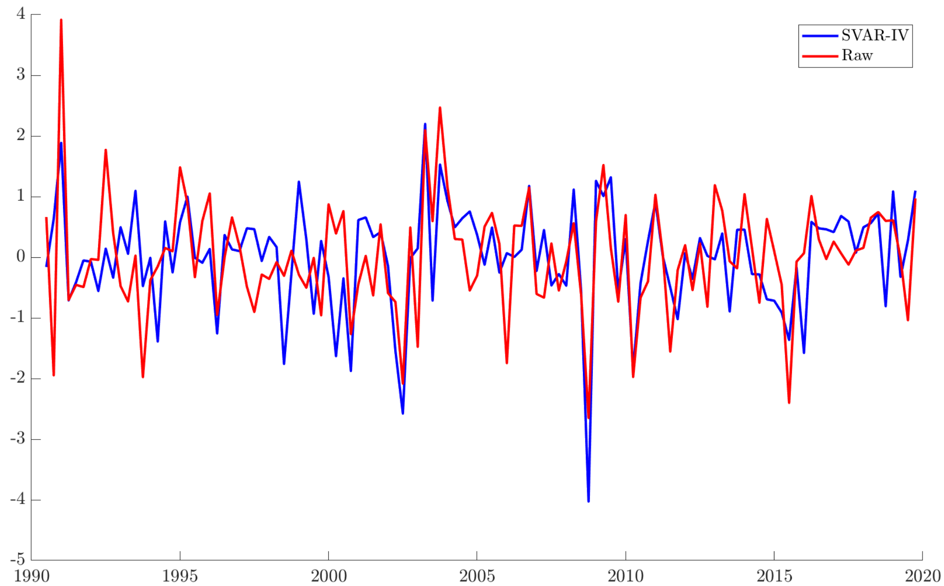

Figure 2: Red: raw shock, $\tilde{Z}_t^{\mu}$ as in (3). Blue: shock identified using SVAR-IV as in (5).

the data generating process can be characterized (in demeaned terms) as:

$$Y_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell} \tag{2}$$

$$\tilde{Z}_t^{\mu} = \alpha \varepsilon_t^{\mu} + v_t, \tag{3}$$

$$Y_t = \sum_{\ell=1}^{M} B_\ell Y_{t-\ell} + u_t, \tag{4}$$

where Y_t is the vector of macroeconomic variables, $\tilde{Z}_t^{\mu}$ is the residualized proxy shock, ε_t^{μ} is the structural financial noise shock, v_t is measurement error, and u_t is the vector of Wold innovations. We assume that ε_t^{μ} and v_t are white noise processes, independent of each other.

Eq. (2) is the structural vector moving average representation: Y_t depends on structural shocks ε_t that propagate according to Θ_ℓ. Eqs. (3) and (4) are measurement equations: (3) relates the measured proxy Z_t^{μ} to the structural shock of interest ε_t^{μ}, while (4) is a reduced-form Wold representation of the data. Within this framework, different identification assumptions can be used to recover impulse responses and variance decompositions from the data. We explain the details in the following subsections.

2.2. Causal effects of noise shocks

First, we use the proxy shock Z_t^{μ} to estimate the effect of noise shocks on macro and financial variables. Following Plagborg-Møller and Wolf (2021), we add the proxy to our baseline VAR

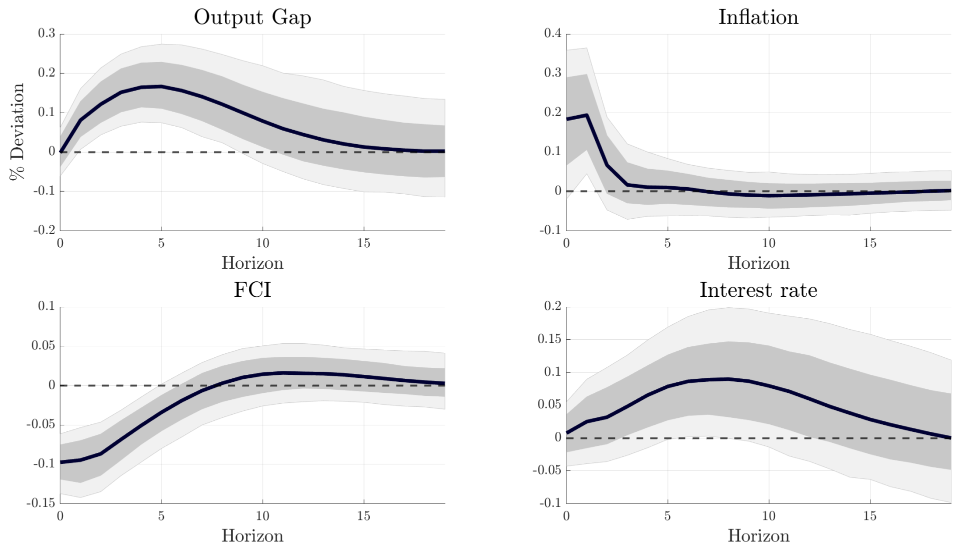

Figure 3: Impulse response to a financial noise shock. Shaded and light shaded grey bands indicate 68 and 90 credible sets respectively.

and use a recursive identification scheme, where the proxy is ordered first. Specifically, we construct the augmented vector $X_t = [Z_t^{\mu}, Y_t']'$, estimate a BVAR using X_t, and apply recursive identification.

Instrument Relevance. To evaluate instrument relevance, we perform an F-test of the relevance of the financial noise shock in explaining movements in the residuals of the FCI equation. The conventional F-statistic is 15.797, while the heteroskedasticity-robust F-statistic equals 11.101. Both values exceed the conventional threshold of 10.[2] We also tried the weak instrument test derived in Lewis and Mertens (2025). Using an absolute bias tolerance of $\tau = 0.1$ and at a 95% significance level, we reject the null of weak instruments. We conclude that the instrument is relevant.

Impulse Responses. Figure 3 depicts the impulse response of several macroeconomic outcomes of interest to an expansionary noise shock—an exogenous inflow into equity.[3] The shock lowers the FCI index on impact, implying looser financial conditions. This generates a positive output gap and inflation in the first few quarters.[4] The interest rate responds positively, but this response is insufficient to fully stabilize the shock. Overall, the effect of the financial noise shock resembles that of a textbook demand shock, which is only imperfectly stabilized by monetary

[2]We implement this test using the residuals of a VAR estimated by OLS, i.e without any Bayesian shrinkage.

[3]Appendix B.3.1 shows the impulse response to all variables included in the VAR.

[4]Some readers may find the large and fast inflation response puzzling. This is partly driven by our we focus on headline and not core inflation. Appendix B.3 shows that, if we modify this choice, the inflation response is much smaller. Importantly, the estimated responses of output gaps, FCI and nominal interest rates under this alternative specification are essentially unchanged.

policy.[5]

2.3. Importance of noise shocks for macroeconomic fluctuations

Next, we estimate the extent to which output fluctuations are driven by financial noise shocks. This magnitude is key for determining the potential volatility reductions from adopting FCI targeting, because the policy works by endogenously reducing the impact of noise shocks.

Forecast Variance Ratios. We present results under several alternative assumptions to estimate the contribution of the shock to forecast variance. The object of interest is the Forecast Variance Ratio (FVR). Following Plagborg-Møller and Wolf (2022), we define the FVR for variable i at horizon h as

$$FVR_{i,h} = 1 - \frac{\text{Var}\left(Y_{i,t+h} | \{Y_\tau\}_{\tau<t}, \{\varepsilon^\mu_\tau\}_{t\leq\tau<\infty}\right)}{\text{Var}\left(Y_{i,t+h} | \{Y_\tau\}_{\tau<t}\right)}.$$

his measures how much the forecast error (for variable i at horizon h) is reduced if we knew with certainty the realization of the shock at all future dates.

Within the DGPs described by Eqs. (3)-(4), there are several alternative assumptions about the relationship between $\tilde{Z}^\mu_t, \varepsilon^\mu_t$ and u_t, which yield different identified Forecast Variance Ratios.

First, the most common assumption is *invertibility.* Under this assumption, the structural shock satisfies

$$\varepsilon_{\mu,t} = q' u_t. \tag{5}$$

This assumes there exists a linear combination of the (contemporaneous) Wold residuals that spans the shock of interest. Since we have a proxy for this shock, we can use a SVAR-IV procedure (Mertens and Ravn, 2013) to identify q and, therefore, $\varepsilon_{\mu,t}$. Intuitively, we assume that the true structural shock can be recovered from the Wold residuals. Since we can estimate the pattern of comovements that the structural shock generates using $\tilde{Z}^\mu_t$, we can back out the correct structural shocks using the residuals and the proxy.[6] The blue line in Figure 2 depicts the identified shock under this assumption. The potential problem with this strategy is that if the invertibility assumption is violated, this could lead to overestimation of the identified shock's variance contribution.[7]

A second assumption that gives point identification of the FVR is to rule out measurement error, i.e., $v_t = 0$ for all t. In contrast to the invertibility case, if this assumption is violated, we would underestimate the shock's contribution: if $\tilde{Z}^\mu_t$ appears to have low correlation with Y_t, we

[5] Appendix B.3 shows that the estimated responses are robust to i) controlling for additional principal components when constructing the shock; ii) controlling for changes in consumer sentiment when constructing the shock; iii) using an SVAR-IV procedure; iv) using local projections. Also, in Appendix B.3.4 we show that the shock does not predict TFP, which suggests that it is not a proxy for news about future fundamentals.

[6] For additional implementation details see B.2.

[7] See, e.g., Plagborg-Møller and Wolf (2022) for an illustration of this bias when estimating the variance contribution of monetary and oil shocks.

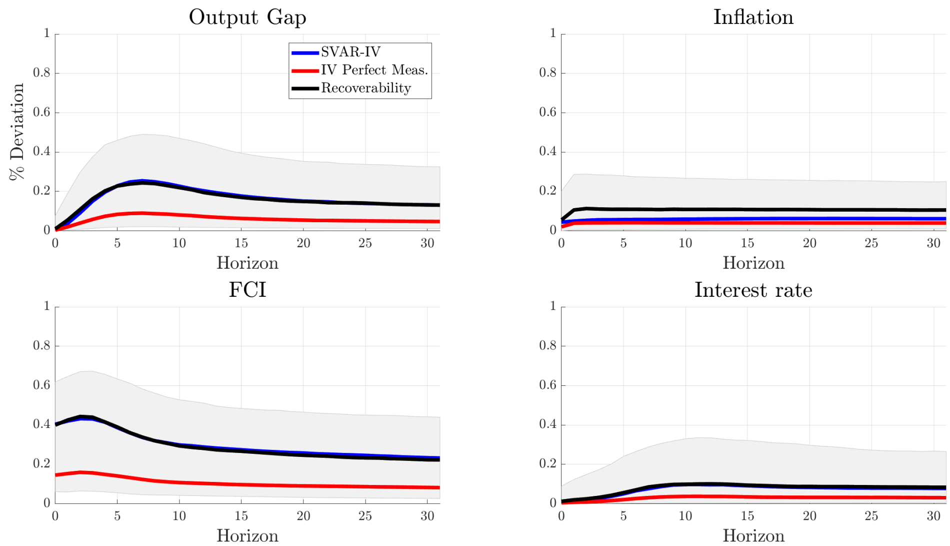

Figure 4: Identified Forecast Variance Ratios of for the noise shock. Blue: SVAR-IV, assuming invertibility. Red: lower bound, assumes perfect measurement of the shocks. Black: recoverability-based FVR. The three lines are the medians, horizon by horizon, of each object. Light grey areas are 90% credible regions of the identified set (Plagborg-Møller and Wolf, 2022).

would attribute this to the shock being unimportant rather than to measurement error. This assumption provides a lower bound for the true FVR of the shock (Plagborg-Møller and Wolf, 2022). The red line in Figure 2 corresponds to the shock identified under this no-measurement-error assumption. In practice, measurement error can naturally arise when we do not fully observe the shock. That is, the Gabaix and Koijen (2021) shock may be only one of many financial noise shocks. Thus, under this assumption we capture the importance of the directly measured noise, whereas under the invertibility assumption we aim to capture the full importance of noise shocks by imposing additional structure.

Finally, a third assumption that allows identification of the FVR is *recoverability* (Chahrour and Jurado, 2021; Plagborg-Møller and Wolf, 2022; Forni et al., 2023). Under this assumption, the structural shock satisfies:

$$\varepsilon_{\mu,t} = q'\left(L^{-1}\right) u_t,$$

where $q(L^{-1})$ is a lead polynomial, with L^{-1} denoting the lead operator. Under recoverability, $\varepsilon_{\mu,t}$ can be recovered from the data, but we may need future values of u_t to do so. This assumption is less stringent than invertibility. Plagborg-Møller and Wolf (2022) show how to compute an upper bound on the FVR under this assumption. However, this assumption still requires that we can recover the shock based on observables.

Figure 4 shows the FVRs identified under each of these three assumptions.[8] As we can see, the standard invertibility-based SVAR-IV produces variance ratios that are almost indistinguish-

[8]Appendix B.3.1 shows the analogous figure for all variables included in the VAR.

able from the recoverability-based estimates. Under either of these assumptions, the noise shock explains up to 25% of the forecast error in output gap at a 2-year horizon. The share of unconditional output gap variance explained by the shock is around 12%.[9] Additionally, this shock accounts for up to 40% of contemporaneous variation in FCI. The lower bound, obtained under the no-measurement-error assumption, shows smaller contributions, yet still indicates that a non-trivial share of output gap forecast variance at a 2-year horizon (9%) is driven by the shock. The shock also accounts for an important part of FCI fluctuations. Overall, even at the lower bound, the evidence indicates that financial noise is a meaningful driver of both FCI and output gap fluctuations.[10]

Assessing the Magnitude of the Explained Variance. In order to put the magnitude of the explained variance into perspective, we compare it to a variant of the Main Business Cycle shock constructed in Angeletos et al. (2020). In particular, we construct a shock explains the maximum variation of output gaps at business cycle frequencies, and that is orthogonal to the identified noise shock.[11] This shock does not have a structural interpretation. We see this as accounting exercise: given the relatively high number of variables of the VAR, how well can a *single* shock do in terms of explaining output gap variance, once we purge all the variation coming from the noise shock?

In the SVAR-IV specification, this "constrained" Main Business Cycle shock explains around 31% at the 8 quarters horizon. This is remarkable: the identified noise shock has a similar contribution to a shock that is constructed specifically to maximize variation at business cycle frequencies.[12] This underscores the relevance of financial noise shocks as drivers of an important part of macroeconomic fluctuations at business cycle frequencies.

Historical Decomposition. Finally, we present a historical decomposition exercise (detailed in Appendix B.3.2) to identify specific episodes when financial noise shocks were most influential. This analysis reveals that noise shocks were particularly important during the 1999-2008 period, explaining most FCI and output gap fluctuations in the run-up to the Great Financial Crisis. This pattern aligns with narratives attributing pre-crisis macroeconomic fluctuations to exuberance in financial markets driven by positive noise shocks. During the crisis itself, negative noise shocks partially explain the downturn, though other factors such as rising risk premiums and the zero lower bound constraint also played important roles. These findings complement our variance decomposition results by showing that the quantitatively significant effects of noise shocks documented above translated into economically meaningful episodes, particularly during

[9]In the Figure, we can proxy this contribution by looking at the FVRs at the longest horizon, since there is essentially no predictability left at that horizon.

[10]We also note that, for all sets of assumptions, the shock explains a modest amount of inflation and interest rate fluctuations.

[11]In particular, we follow the same procedure as in Angeletos et al. (2020) Supplementary Appendix K.

[12]Similar to the results in Angeletos et al. (2020), the "constrained" Main Business Cycle shock explains around 70% of the variance at shorter horizons.

periods of financial market turbulence.

3. A macroeconomic model with financial noise

In this section, we present a risk-centric macroeconomic model with the following key features: (i) financial noise drives the aggregate asset price (FCI) away from the central bank's desired level because of policy reaction lags; (ii) sophisticated investors (arbitrageurs) trade against this noise; and (iii) arbitrageurs face uncertainty about the future level of FCI. Within this framework, we demonstrate that noise shocks affect both financial conditions and macroeconomic activity, consistent with our motivating evidence. In Section 4, we use this model to investigate the effects of an explicit FCI targeting policy.

3.1. Environment

We relegate the details of the environment to Appendix A.2. Here, we summarize the real side of the economy and describe in more detail the financial market side.

Real economy. On the supply side, (the log of) potential output follows the process:

$$y_t^* = y_{t-1}^* + \varepsilon_{z,t}, \text{ where } \sigma_z^2 \equiv var\left(\varepsilon_{z,t}\right) \tag{6}$$

and $\varepsilon_{z,t}$ denotes an i.i.d. *aggregate supply shock*. Due to nominal price stickiness, (the log of) output, y_t, is determined by aggregate demand and can depart from potential output. For the baseline model, we assume firms' prices are fully sticky. Our main results are robust to allowing for partially flexible prices and a trade-off between inflation and output stabilization, as we show in Appendix A.6 and discuss in Section 4.6.1.

On the demand side, there are two types of households: hand-to-mouth agents and (asset holding) households. Hand-to-mouth agents do not play an important role beyond decoupling the labor supply decisions from household consumption behavior. They supply all of the labor and spend all of their income. Since their spending is driven by output, which is endogenous, they create a Keynesian multiplier effect but they do not drive aggregate demand.

Aggregate demand is driven by (asset holding) households. These households own the aggregate risky asset (the market portfolio): a claim on firms' share of output (νY_t). They have expected log utility and make portfolio allocation and consumption-savings decisions. They delegate the portfolio decision to the portfolio managers that we describe later. Their consumption rule is centered around the optimal rule with log utility, but it can deviate by an amount denoted by δ_t, which we refer to as an *aggregate demand shifter*. This is a modeling device to capture various factors that affect aggregate spending, e.g., a consumer sentiment shock, a fiscal policy shock, or a discount rate shock.

The upshot of these assumptions is the *output-asset price* relation

$$y_t = m + p_t + \delta_t, \tag{7}$$

where p_t denotes (the log of) the price of the market portfolio and m is a derived parameter that combines households' marginal propensity to consume and the multiplier. All else equal, higher aggregate asset prices raise spending and output through a consumption wealth effect. In practice, a higher price for aggregate assets such as stocks, bonds, and real estate raises spending for various reasons including wealth effects. Thus, we view p_t as the model counterpart to an FCI, and Eq. (7) as capturing the broader set of channels that links spending to asset prices.[13]

Naturally, a higher aggregate demand shifter δ_t also induces higher spending and output. We assume the aggregate demand shifter follows an AR(1) process

$$\delta_t = \varphi_\delta \delta_{t-1} + \varepsilon_{\delta,t}, \quad \text{where } \sigma_\delta^2 \equiv var\left(\varepsilon_{\delta,t}\right) \tag{8}$$

and $\varepsilon_{\delta,t}$ is an i.i.d. *aggregate demand shock*, which is independent from supply shocks.

Remark 1 (Policy transmits via FCI)**.** *Observe that the short-term interest rate does not enter into the output-asset price relation as a separate variable: it affects output only through its impact on the price of the market portfolio. This feature is driven by our assumptions (such as log utility) but it is supported by empirical evidence once we interpret p_t as the model counterpart to an FCI. In Appendix B.3, we use our empirical estimates from Section 2 to perform counterfactuals that show that broad financial conditions are a critical pathway for the transmission of monetary policy. In fact, we find that a monetary policy shock that significantly alters short-term interest rates* without *affecting financial conditions does not meaningfully impact the output gap or inflation.*

Financial markets. Households make a portfolio choice between two assets: the market portfolio and the risk-free asset (normalized to have zero net supply). The (log) return on the risk-free asset $r_t^f = \log R_t^f$ is set by the central bank as we describe later. The (log) return on the market portfolio, $r_{t+1} = \log R_{t+1}$, is approximately given by (see the appendix)

$$r_{t+1} = \rho - (1-\beta)\, m + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t,$$

where $\rho \equiv -\log \beta$ is the (log of) households' discount rate. After substituting the output asset price relation (7) in it, we can write the return as

$$r_{t+1} = \rho + p_{t+1} - p_t + (1-\beta)\, \delta_{t+1}. \tag{9}$$

[13]We caution that p_t is stated in price, rather than the usual rates convention of FCIs. Thus, it has the opposite sign-convention of the standard FCIs, where an increase in the index typically means tightening (see Section 2).

That is, the aggregate return is driven by the (log) price change and the aggregate demand shock (through its impact on cash flows).

Households delegate their portfolio choice to managers.[14] In each period, a fraction η of these managers are "noise traders" and their portfolio weight is $\omega_t^N = 1 + \frac{1}{\eta}\mu_t$. That is, they deviate from the optimal portfolio benchmark by an amount $\frac{1}{\eta}\mu_t$. We refer to μ_t as the aggregate noise—the total flow that needs to be absorbed by other investors (see Remark 2 for a discussion). We assume the aggregate noise μ_t follows an AR(1) process

$$\mu_t = \varphi_\mu \mu_{t-1} + \varepsilon_{\mu,t}, \qquad \text{where } \sigma_\mu^2 \equiv var\left(\varepsilon_{\mu,t}\right) \tag{10}$$

and $\varepsilon_{\mu,t}$ is an i.i.d. *financial noise shock*, which is independent from supply and demand shocks.

Among the remaining managers, a mass $1 - \eta - \alpha$ represent "inelastic funds" and their portfolio weight is $\omega_t^I = 1$. That is, they simply invest according to the average optimal portfolio benchmark. Finally, a mass α of managers are "arbitrageurs" (or elastic funds) who choose their portfolio weights optimally as we describe below. Combining the managers' positions, we obtain the market clearing condition

$$\alpha\omega_t^A + \eta\left(1 + \frac{\mu_t}{\eta}\right) + (1 - \eta - \alpha) = 1 \Longrightarrow \omega_t^A = 1 - \frac{\mu_t}{\alpha}. \tag{11}$$

In equilibrium, arbitrageurs must adjust their portfolio weight ω_t^A to absorb the aggregate noise.

The arbitrageurs choose their portfolio weight to maximize expected log assets-under-management, after observing the risk-free rate and the current noise μ_t:[15]

$$\max_{\omega_t^A} E_t\left[\log\left(\alpha W_t\left(R_t^f + \omega_t\left(R_{t+1} - R_t^f\right)\right)\right)\right].$$

In equilibrium, this implies a standard optimality condition $E_t\left[M_{t+1}\left(R_{t+1} - R_t^f\right)\right] = 0$, where $M_{t+1} = \frac{1}{R_t^f + \omega_t^A\left(R_{t+1} - R_t^f\right)}$. Assuming the market and the portfolio returns are log-normally distributed, we obtain the approximate optimality condition:

$$\omega_t^A \sigma_{t,r_{t+1}} = \frac{E_t\left[r_{t+1}\right] + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} - r_t^f}{\sigma_{t,r_{t+1}}}. \tag{12}$$

The arbitrageurs' demand for risk is equal to their (perceived) equilibrium Sharpe ratio. Combining this with (11), we derive the *financial market equilibrium condition:*

$$E_t\left[r_{t+1}\right] = r_t^f + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2 - \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}\mu_t. \tag{13}$$

[14]Managers do not consume themselves; they simply make a portfolio choice for households. For simplicity, each household invests with a very large random sample of managers. This ensures there is no portfolio and wealth heterogeneity across households, even though there is heterogeneity across managers.

[15]The equilibrium price fully reveals the noise, so this assumption is without loss of generality.

Substituting (9) into this condition, we further obtain a present discounted value relation that describes the equilibrium aggregate asset price:

$$p_t = \rho + E_t\left[p_{t+1}\right] + (1-\beta)\, E_t\left[\delta_{t+1}\right] - \left(r_t^f + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2\right) + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}\mu_t. \quad (14)$$

Eq. (14) shows that, all else equal, the effect of noise on the aggregate asset price *increases with return variance*. As market volatility rises, arbitrageurs become more reluctant to counteract the noise traders' flows and require a greater change in the price (and expected return) to absorb these flows. Moreover, the impact of noise is amplified when the mass of arbitrageurs α is smaller, leading to more inelastic aggregate asset demand. When combined with Eq. (7), Eq. (14) implies further that, all else equal, financial noise affects aggregate output proportionally to return variance—the distinguishing feature of our model.[16]

Remark 2 (Interpreting noise shocks)**.** *In practice, noisy flows emerge from various sources. Retail investors may engage in sentiment-driven trading based on excessive optimism or pessimism about assets. Institutional investors often face client inflows and outflows that require them to expand or reduce positions regardless of market conditions. Portfolio rebalancing by both retail and institutional investors can generate mechanical trading needs unrelated to asset fundamentals. While we remain agnostic about the specific sources of noisy flows, the key feature of our model is that these flows are relatively insensitive to return variance compared to arbitrageurs' portfolios (in the model, we make the stark assumption that noisy flows are completely insensitive to variance, though this is stronger than necessary for our results). This assumption is supported by extensive research on limits to arbitrage showing that return variance and risk management are primary constraints on arbitrage activities but play a smaller role in noise trader behavior (see, e.g., Shleifer and Vishny (1997)).*

3.2. Equilibrium with discretionary monetary policy

We next introduce standard *discretionary* monetary policy and characterize the resulting equilibrium. The central bank sets the nominal interest rate i_t^f. Since nominal prices are fully sticky (until Section 4.6.1), this is the same as the real interest rate, $r_t^f = i_t^f$, so we assume the central bank sets r_t^f. The central bank focuses on closing the output gap $\tilde{y}_t \equiv y_t - y_t^*$.

3.2.1. Benchmark without policy reaction lags

As a benchmark, we start with the (unrealistic) first-best scenario in which the central bank has no frictions in setting the interest rate. In this case, the central bank can set $\tilde{y}_t = 0$ in all

[16] In standard New Keynesian models with log utility, a log-normal Euler equation approximation yields $y_t = \rho + E_t\left[y_{t+1}\right] - \left(r_t^f + \frac{1}{2}\left(\sigma_{t,y_{t+1}}\right)^2\right)$. Comparing this with Eqs. (7) and (14) shows that financial noise creates a wedge in the Euler equation in proportion to the return variance.

periods and states. Using (7) , (9) and (14), along with the shock processes in (6) and (8), the equilibrium is

$$\begin{aligned} p_t &= p_t^* \equiv y_t^* - m - \delta_t, \qquad (15) \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1 - \beta\varphi_\delta)\,\delta_t + \frac{\sigma^2}{\alpha}\mu_t, \\ r_{t+1} &= \rho + \delta_t - \beta\delta_{t+1} + \varepsilon_{z,t+1}, \\ \text{where } \sigma^2 &\equiv var_t\,(r_{t+1}) = \sigma_z^2 + \beta^2\sigma_\delta^2. \end{aligned}$$

The first line defines p_t^* (pstar), which is the aggregate asset price that ensures output is equal to its potential. The second line describes the interest rate the central bank sets to achieve pstar (rstar). Note that noise affects the interest rate but it does not affect the aggregate asset price or output. The central bank fully adjusts the interest rate in response to noise to prevent noise-driven fluctuations in output. The third line describes the return conditional on $p_t = p_t^*$ (and $y_t = y_t^*$) at all times. The last line shows that the conditional return variance depends on supply and demand variance (macro-induced variance) but it *does not* depend on noise variance.

3.2.2. Equilibrium with policy reaction lags

Financial markets exhibit noise even over short horizons, while central banks typically adjust policy gradually, creating delayed responses to noise shocks as documented in Section 2. We model these policy lags through an informational assumption: the central bank sets r_t^f *before* observing the current-period noise μ_t. Consequently, the central bank cannot condition its policy decision on the current noise shock $\varepsilon_{\mu,t}$, although it can still respond to macroeconomic shocks $\varepsilon_{\delta,t}, \varepsilon_{z,t}$.

While this setup is particularly tractable and clearly demonstrates the benefits of FCI targeting, the informational friction is not essential for our results. In Section 4.4, we show that our main findings extend to an environment where the central bank faces adjustment costs rather than information constraints. A special case of that alternative model assumes the central bank chooses not to react to any current-period shocks $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$. The key commonality across these specifications is that the central bank faces constraints in responding to noise shocks, whether due to information limitations, adjustment costs, or policy choices.

Formally, the central bank sets the risk-free interest rate without commitment to solve:

$$G_t = \min_{r_t^f} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \tilde{y}_{t+h}^2 \right]. \qquad (16)$$

The central bank minimizes the expected discounted sum of squared log output gaps, which we term the output-gap loss. The expectation $\underline{E}_t\,[\cdot]$ is conditional on the central bank's information set at date t, which includes the realizations of macroeconomic shocks $\varepsilon_{\delta,t}, \varepsilon_{z,t}$, but excludes the current noise shock $\varepsilon_{\mu,t}$.

The first-best equilibrium in (15) is no longer feasible. Our main result in this section characterizes the second-best equilibrium with discretionary policy.

Proposition 1 (Equilibrium with Discretionary Policy)**.** *Suppose the planner sets policy according to* (16) *and the parameters satisfy* $\alpha^2 \geq 4\sigma_\mu^2 \left(\sigma_z^2 + \beta^2 \sigma_\delta^2\right)$*. Then, there is a (locally stable) equilibrium in which the asset price, output, and the interest rate are*

$$\begin{aligned} p_t &= p_t^* + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, \quad \text{where } p_t^* \equiv y_t^* - m - \delta_t, & (17) \\ y_t &= y_t^* + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, & (18) \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1 - \beta\varphi_\delta)\,\delta_t + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1}. & (19) \end{aligned}$$

The return is

$$r_{t+1} = \rho + \delta_t + \varepsilon_{z,t+1} - \beta\delta_{t+1} + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right), \tag{20}$$

and its variance $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smallest positive solution to the following fixed point problem:*

$$\sigma^2 = \sigma_{macro}^2 + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2, \quad \text{where } \sigma_{macro}^2 = \sigma_z^2 + \beta^2\sigma_\delta^2. \tag{21}$$

Greater noise variance σ_μ^2 *increases the return variance* σ^2*, and the output-gap loss* $G_t = \frac{\left(\frac{\sigma^2}{\alpha}\right)^2 \sigma_\mu^2}{1-\beta}$.

We relegate the proof of this Proposition to Appendix A.3 and discuss here the intuition for the equilibrium. Eq. (17) shows that, unlike in the benchmark case, the surprise component of noise $\varepsilon_{\mu,t}$ affects the aggregate asset price (cf. (15)). Eq. (18) shows that the noise-driven fluctuations in the aggregate asset price affect output through the output-asset price relation. Eq. (19) shows that the central bank adjusts the interest rate to insulate output from the predictable component of noise $E_{t-1}[\mu_t] = \varphi_\mu \mu_{t-1}$.

Eqs. $(20-21)$ characterize the equilibrium return and its variance. Note that the total return variance is greater than macro-induced variance because noise shocks are not fully stabilized by monetary policy. Crucially, the noise variance is *endogenous* and increasing with total return variance (see (14)): a greater variance allows noise shocks to have a greater impact, which then leads to even greater variance, and so on. Eq. (21) formalizes these feedbacks and shows that the equilibrium variance corresponds to the solution to a fixed point problem. This problem is a quadratic that has two positive solutions (under appropriate parametric restrictions). We focus on the smaller solution, as this solution is locally stable, whereas the larger solution is locally unstable.[17]

[17] The larger solution is locally unstable in the sense that a small increase (resp. decrease) in volatility would further increase (resp. descrease) the price impact, which would further increase (resp. decrease) the variance, and so on. In contrast, the smaller solution is robust to small fluctuations in volatility.

The last part of the result shows that greater noise variance raises the total variance, a channel amplified by the reinforcement feedbacks discussed above. This increased noise variance also worsens macroeconomic performance by increasing the output-gap loss. In other words, financial noise undermines monetary policy effectiveness, suggesting the value of an alternative policy framework that mitigates such noise impacts.

4. FCI targeting

In this section, we show that a framework where the central bank sets a (soft) FCI target for the upcoming period and strives to maintain the FCI near this target, in addition to focusing on its conventional objectives, enhances the central bank's ability to achieve its standard macroeconomic goals. This improvement arises from addressing a key time inconsistency in monetary policy. Under discretion, the central bank stabilizes financial conditions only to the extent implied by its current objectives. However, committing to stabilize financial conditions more aggressively in the future, even though this reduces future policy flexibility, generates first-order benefits today by enabling arbitrageurs to trade more aggressively against noise. FCI targeting provides the commitment device needed to achieve these benefits. Compared to the standard discretionary policy, this approach results in greater FCI stability and recruits the arbitrageurs to absorb aggregate noise, thereby lessening its impact on economic activity. Finally, we discusses the robustness of FCI targeting in various model extensions.

4.1. Equilibrium with FCI targeting

Formally, suppose the central bank solves the following *modified* problem:

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(p_{t+h} - \overline{p}_{t+h} \right)^2 \right] \right], \tag{22}$$

where $\overline{p}_{t+h}$ denotes an FCI target announced by the central bank in the previous period, $t+h-1$ (the initial target $\overline{p}_0$ is given). That is, in addition to minimizing the output gaps as usual, the central bank penalizes the deviations of the aggregate asset price from its pre-announced target. The parameter $\psi \geq 0$ captures the strength of the FCI targeting objective relative to the central bank's usual objectives. The standard model is a special case with $\psi = 0$.

While we change the central bank's *operational* objective function, it is important to note that the *true* objective function is unchanged and given by the output-gap loss in (16). That is, merely stabilizing asset prices does not improve welfare or the policy performance. Our goal is to analyze whether adopting an operational FCI targeting framework can improve the true policy performance. We focus on constrained improvements that result from solving the FCI targeting problem (22). Remark 3 discusses the full commitment solution and how it relates to the FCI targeting problem. Our next result characterizes the equilibrium with FCI targeting

Proposition 2 (Equilibrium with FCI Targeting)**.** *Suppose the planner follows the FCI targeting policy in* (22) *with* $\psi \geq 0$*, the parameters satisfy* $\alpha^2 \geq 4\sigma_\mu^2 \left(\sigma_z^2 + \beta^2 \sigma_\delta^2\right)$ *(and* $\beta > 1 - \beta$*), and the initial target satisfies* $\overline{p}_0 = E_{-1}[p_0^*]$*. Then, there is a (stable) equilibrium in which the planner announces the expected "pstar" for the next period as its target*

$$\overline{p}_{t+1} = \underline{E}_t \left[p_{t+1}^*\right] \;\; \text{where } p_{t+1}^* = y_{t+1}^* - m - \delta_{t+1}. \tag{23}$$

The equilibrium asset price, output, and interest rate are

$$\begin{aligned}
p_t &= \underline{E}_{t-1}[p_t^*] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, && (24)\\
y_t &= y_t^* + \frac{\psi}{1+\psi}\left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, && (25)\\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta\varphi_\delta\right)\delta_t + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1} + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right). && (26)
\end{aligned}$$

The equilibrium return is

$$r_{t+1} = E_t\left[r_{t+1}\right] + \frac{1}{1+\psi}\varepsilon_{z,t+1} - \left(\frac{1}{1+\psi} - (1-\beta)\right)\varepsilon_{\delta,t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}, \tag{27}$$

where the expected return $E_t\left[r_{t+1}\right]$ *is given by (A.37). The return variance* $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smaller positive solution to the following fixed point problem*

$$\begin{aligned}
&\sigma^2 = \sigma_{macro}^2(\psi) + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2, && (28)\\
&\text{where } \sigma_{macro}^2(\psi) = \sigma_z^2\left(\frac{1}{1+\psi}\right)^2 + \sigma_\delta^2\left(\frac{1}{1+\psi} - (1-\beta)\right)^2.
\end{aligned}$$

Let $\overline{\psi} = \arg\min_{\psi \geq 0} \sigma_{macro}^2(\psi)$*. Over the range* $\psi \in [0, \overline{\psi})$*, increasing* ψ *strictly reduces* σ^2 *as well as* $\sigma_{macro}^2(\psi)$ *and* $\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2$*. That is, stronger FCI targeting reduces the return variance and* ***both*** *of its components.*

We provide the equilibrium's intuition here and relegate the proof to Appendix A.3. Eq. (23) says that the central bank optimally announces its expected "pstar" as its target for the next period. Given this target, the central bank's optimal (interest rate) policy implies

$$\underline{E}_t\left[p_t\right] = \frac{1}{1+\psi}p_t^* + \frac{\psi}{1+\psi}\underline{E}_{t-1}\left[p_t^*\right]. \tag{29}$$

That is, the central bank's expected asset price for the current period (i.e., before observing the noise shock) is a weighted average of the current "pstar" and the last period's expected "pstar", which it had announced as a target. This implies Eq. (24), which says the asset price reflects the *surprises* in "pstar" but only *partially*: A positive supply shock $\varepsilon_{z,t} > 0$ raises the asset price but less than in the case with discretionary policy ($\psi = 0$); a positive demand shock $\varepsilon_{\delta,t} > 0$ decreases the asset price but less than with discretionary policy. This in turn implies Eq. (25), which says that the slow adjustment of asset prices to macroeconomic shocks affects output. A positive supply shock $\varepsilon_{z,t} > 0$ has a smaller effect on output than with discretionary policy, because the policy does not allow asset prices (and demand) to adjust to the higher supply immediately. Conversely, a positive demand shock $\varepsilon_{\delta,t} > 0$ has some effect on output, because the policy does not undo the effect of demand fully. Eq. (26) characterizes the policy interest rate that induces these outcomes. We discuss this policy rate response later in Section 4.3.

Eqs. $(27 - 28)$ describe the equilibrium return and its conditional variance. The total return variance is still determined as the solution to a fixed point problem. The difference is that the macro-induced variance is now endogenous to the degree of FCI targeting and typically lower than with discretionary policy. In particular, supply shocks always have a smaller impact on the return. Demand shocks also have a smaller impact as long as the FCI targeting is not too strong.[18] Consequently, there is a large range $\left[0, \overline{\psi}\right]$ (where $\overline{\psi} > \frac{\beta}{1-\beta}$) over which FCI targeting reduces the macro-induced variance $\sigma^2_{macro}(\psi)$. Importantly, over the same range, FCI targeting also reduces the total return variance σ^2. In fact, the reduction in total variance is greater than the reduction in macro-induced variance, because a lower variance enables the arbitrageurs to trade against noise more aggressively, reducing the noise-induced variance $\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_{\mu}$.

4.2. Macro-stabilization effects of FCI targeting

Proposition 2 demonstrates that a central bank adopting an FCI targeting policy mitigates market volatility. However, the central bank in our model is not concerned with market volatility *per se*. The question then arises: does FCI targeting aid the central bank in fulfilling its standard macro-stabilization goals? We next show that some FCI targeting always improves macroeconomic stabilization.

We evaluate the policy performance with the output-gap loss function G_t defined in (16). This function might depend on the current realizations of supply and demand shocks $\varepsilon_{z,t}, \varepsilon_{\delta,t}$. To obtain a welfare measure that averages across different shocks, we consider the expected output-gap loss :[19]

$$G^e(\psi) = E\left[G_t(\psi)\right] = E\left[\sum_{h=0}^{\infty} \beta^h \left(y_{t+h}(\psi) - y^*_{t+h}\right)^2\right]. \tag{30}$$

Here, $E[\cdot] = E\left[\underline{E}_t[\cdot]\right]$ denotes the unconditional distribution over all shocks. To evaluate this

[18]The coefficient, $\frac{1}{1+\psi} - (1-\beta)$, implies that when $\psi < \frac{\beta}{1-\beta}$ a positive demand shock decreases the return, although less than with discretionary policy. When $\psi > \frac{\beta}{1-\beta}$, a positive demand shock increases the return since its impact on output (cash flows) dominates its dampened effect on the aggregate asset price.

[19]Our main result qualitatively also holds with the current gap G_t; that is, some degree of FCI targeting improves welfare for *any* given realization of shocks $\varepsilon_{z,t}, \varepsilon_{\delta,t}$. However, the magnitude of welfare gains and the optimal degree of FCI targeting depends on $\varepsilon_{z,t}, \varepsilon_{\delta,t}$. The expected value $G^e(\psi)$ ensures that we evalute the welfare gains and the optimal FCI targeting by averaging across a variety of shocks.

expectation, observe that Eq. (25) implies the output gap is:

$$\tilde{y}_t = (\varepsilon_{\delta,t} - \varepsilon_{z,t}) \frac{\psi}{1+\psi} + \varepsilon_{\mu,t} \frac{\sigma^2}{\alpha}. \tag{31}$$

Using this expression, we calculate and decompose the expected output gap-loss as follows

$$\begin{aligned} G^e(\psi) &= G^e_{macro}(\psi) + G^e_{noise}(\psi), \qquad (32) \\ \text{where } G^e_{macro}(\psi) &= \frac{\left(\sigma_z^2 + \sigma_\delta^2\right)\left(\frac{\psi}{1+\psi}\right)^2}{1-\beta} \quad \text{and} \quad G^e_{noise}(\psi) = \frac{\sigma_\mu^2 \left(\frac{\sigma^2}{\alpha}\right)^2}{1-\beta}. \end{aligned}$$

$G^e_{macro}(\psi)$ and $G^e_{noise}(\psi)$ are the contributions of macroeconomic shocks and noise shocks to the expected output-gap loss, respectively. Our next result describes how FCI targeting affects $G^e(\psi)$ and its components.

Proposition 3 (Macrostabilization Effects of FCI Targeting)**.** *Consider the equilibrium in Proposition 2. Then, a small degree of FCI targeting reduces the expected output-gap loss*

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} < 0, \textit{ with } \frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = 0 \textit{ and } \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} < 0.$$

Therefore, $\psi^* = \arg\min_{\psi \geq 0} G^e(\psi) > 0$*; i.e., the output-gap loss minimizing policy features FCI targeting.*

For intuition, observe from Eqs. $(31-32)$ that FCI targeting has competing effects on output gaps. On the one hand, the policy creates new sources of output gaps as it does not fully allow output to adjust to supply surprises and it allows demand surprises to influence output (the terms $(\varepsilon_{\delta,t} - \varepsilon_{z,t}) \frac{\psi}{1+\psi}$). On the other hand, the policy reduces return variance σ^2, and this mitigates the asset price and output impact of noise surprises (the term $\varepsilon_{\mu,t}\frac{\sigma^2}{\alpha}$). However, the noise-reduction force always dominates for sufficiently low levels of ψ because $\frac{dG_{macro}(\psi)}{d\psi}|_{\psi=0} = 0$: starting from the baseline discretionary policy, allowing macroeconomic surprises to affect the output gap induces only a second-order increase in the output-gap loss. In contrast, Proposition 2 implies that $\frac{d\sigma^2}{d\psi}|_{\psi=0} < 0$ and thus $\frac{dG_{noise}(\psi)}{d\psi}|_{\psi=0} < 0$: increasing ψ induces a first-order reduction on return variance caused by noise $\sigma_\mu^2\left(\frac{\sigma^2}{\alpha}\right)^2$, and this "excess" variance affects output gaps and asset prices. Therefore, adopting an FCI targeting policy with positive ψ reduces the output-gap loss.

Numerical illustration. Figure 5 illustrates the effects of FCI targeting in a numerical example where noise accounts for half of total FCI variance absent targeting (see Appendix A.1 for the parameters). The left panel shows that FCI targeting substantially reduces return variance and its components. At the optimal targeting level, $\psi = \psi^*$ (vertical line), total variance de-

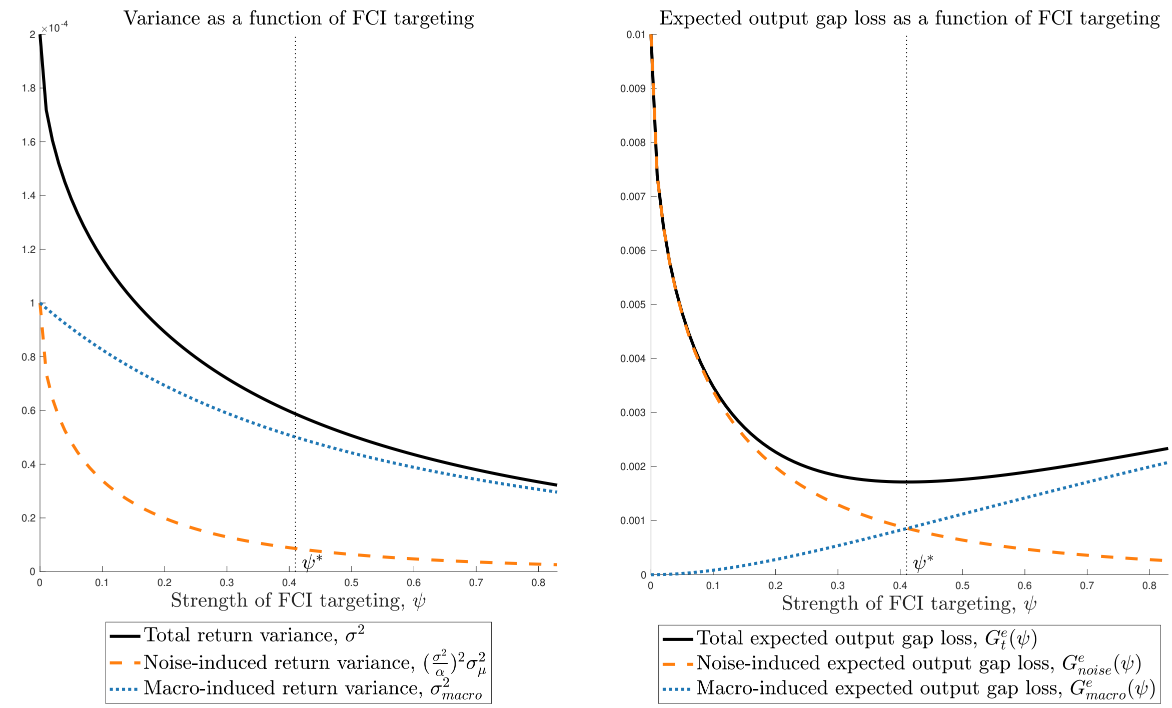

Figure 5: The left panel shows the effect of FCI targeting ψ on return variance (solid black line) and its components induced by noise shocks (dashed orange line) and by macroeconomic shocks (dotted blue line). The right panel shows the effect on the expected output gap loss (solid black line) and its components induced by noise shocks (dashed orange line) and by macroeconomic shocks (dotted blue line). The vertical lines illustrate the gap-minimizing level of FCI targeting. We use the parameters in Appendix A.1.

creases by approximately two-thirds, while noise-driven variance diminishes by more than ninety percent.

The right panel presents the effects on output-gap loss and its components. Starting from discretionary policy, FCI targeting substantially reduces both the noise component and total output-gap loss, while having only second-order effects on macro-induced output-gap loss. As targeting intensity increases, it continues reducing noise-induced losses but begins increasing macro-induced losses more rapidly. The optimal level, $\psi^* = 0.4$, corresponds to the central bank targeting an asset price that assigns roughly one-third weight to its pre-announced target and two-thirds to the current optimal level without targeting constraints. Although this represents relatively mild FCI targeting, it effectively eliminates nearly all noise-driven loss while achieving substantial overall welfare gains.

Remark 3 (FCI targeting vs the full commitment solution)**.** *Our analysis focuses on FCI targeting rather than the full commitment solution for both theoretical and practical reasons. In Appendix A.9, we show that the solution to a version of the problem with commitment shares the same principle underlying FCI targeting: the central bank commits to imperfectly stabilize future shocks in order to reduce variance and recruit arbitrageurs. Furthermore, the planner additionally conditions future output gaps on past noise shocks, adjusting them in the opposite direction to strengthen the recruitment effect. Therefore, under full commitment the central bank wants to exploit the recruitment effect even more than under FCI targeting. However, we focus on FCI targeting because policies conditioning on past shocks would be considerably more complex to communicate and implement. Additionally, the more extensive commitment required under the full solution may prove more onerous for the Fed to honor consistently, potentially undermining its credibility. This robustness of the recruitment mechanism across different commitment structures—and the substantial noise elimination already achieved under FCI targeting—reinforces the practical value of this approach.*

4.3. FCI targeting and interest rate volatility

One concern with FCI targeting is that it might require large movements in the policy interest rate to keep financial conditions close to the target. However, our model reveals that FCI targeting has competing effects on interest rate volatility and can, in fact, reduce it—even though reducing rate volatility is not an explicit policy goal.

In order to analyze the effects on interest rate volatility, we write Eq. (26) as

$$r_t^f = \underline{E}_{t-1}\left[r_t^f\right] + \frac{\psi}{1+\psi}\varepsilon_{z,t} + \left[1 - \beta\varphi_\delta - \frac{\psi}{1+\psi}\right]\varepsilon_{\delta,t} + \frac{\sigma^2}{\alpha}\varphi_\mu\varepsilon_{\mu,t-1}, \tag{33}$$

where $\underline{E}_{t-1}\left[r_t^f\right]$ is the expected interest rate in the previous period $t-1$. The remaining terms reflect the interest rate surprises induced by supply shocks, demand shocks, and financial noise shocks, respectively. On the one hand, FCI targeting typically increases the policy rate's

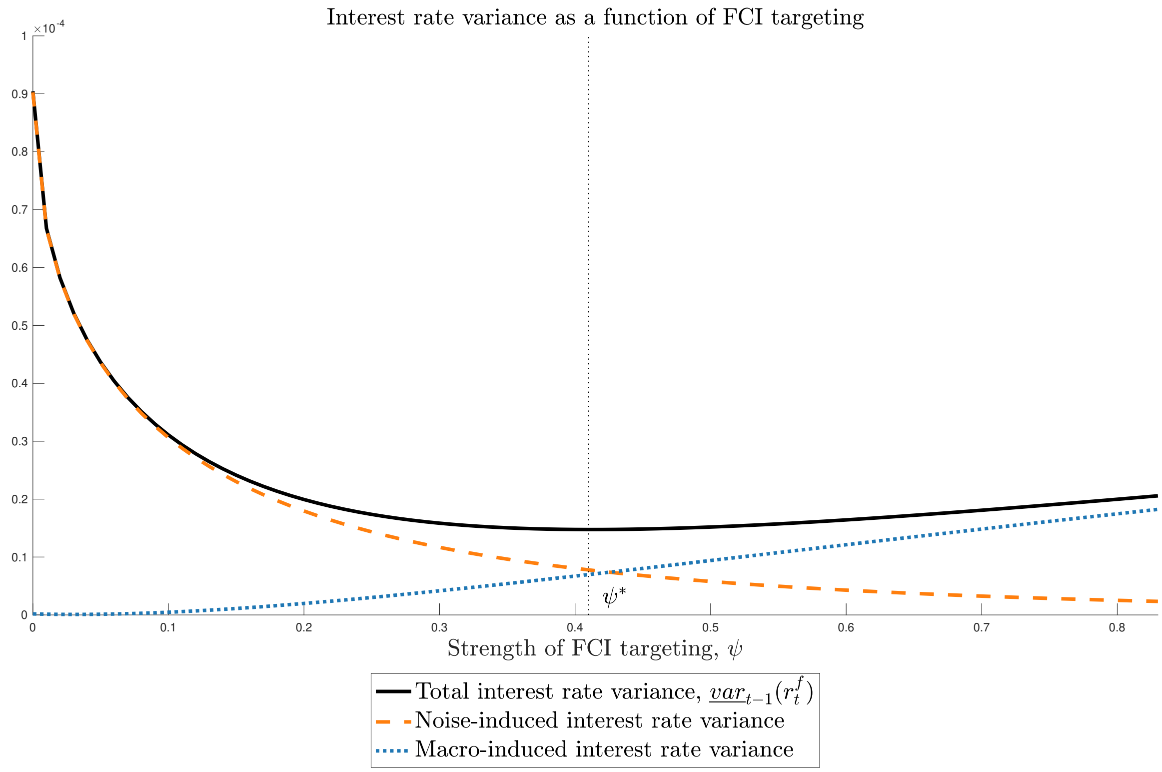

Figure 6: This figure shows the effect of FCI targeting ψ on the conditional interest rate variance (solid black line) and its components driven by noise shocks (dashed orange line) and by macroeconomic shocks (doted blue line). The line illustrates the gap-minimizing level of FCI targeting. We use the parameters in Appendix A.1.

sensitivity to supply and demand shocks—this captures the intuition that FCI targeting might require large movements in the policy rate to stabilize financial conditions.[20] On the other hand, FCI targeting diminishes the policy rate's sensitivity to financial noise shocks by reducing the return variance σ^2 and the noise's impact on asset prices. The overall effect hinges on the balance between this decreased sensitivity to financial noise and the generally increased sensitivity to macroeconomic shocks.

Figure 6 depicts the impact of FCI targeting on the conditional interest rate variance and its macro-induced and noise-induced components in the earlier numerical example. Stronger FCI targeting increases the macro-induced rate variance but significantly reduces the noise-induced rate variance. The reduction in the noise-induced variance is notably more substantial. As a result, FCI targeting overall *lowers* the total interest rate variance.

Why is the effect of FCI targeting on noise-induced interest rate variance more pronounced? In this example, under a discretionary policy ($\psi = 0$), financial noise shocks are the primary con-

[20]In our model, FCI targeting always raises the policy rate's responsiveness to supply shocks. It may also increase the sensitivity to demand shocks (although in the opposite direction)—this happens when φ_δ is high and ψ is not too low. In scenarios of persistent demand shocks, asset prices react in anticipation of future policy rate changes in response to the demand shift. Consequently, the central bank might need to adjust the current policy rate in the opposite direction to counteract these asset price movements.

tributors to interest rate volatility, despite macroeconomic shocks and noise shocks contributing equally to the return variance (cf. the left panel of Figure 5). Thus, as FCI targeting diminishes the price impact of noise, it also lessens the need for substantial rate adjustments.

The broader point is that, as arbitrageurs absorb the majority of the noise, the burden on the central bank to react to noise shocks is reduced. This can result in greater stability of the policy interest rate, especially if noise shocks are the primary driver of interest rate volatility.

4.4. FCI targeting with an interest rate adjustment constraint

In Appendix A.5, we explore the implications of FCI targeting when the central bank faces an interest rate adjustment constraint as opposed to an information constraint. Our main results continue to hold. In fact, FCI targeting generates potentially larger reductions in volatility because it induces the central bank to react to noise shocks more aggressively, in addition to mitigating the price effect of macroeconomic shocks. This decline in volatility improves the central bank's objectives by recruiting arbitrageurs to trade against noise.

Formally, suppose the central bank observes all shocks but its objective function is

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} \tilde{y}_t^2 + \frac{1}{\theta}\left(r_t^f - E_{t-1}\left[r_t^f\right]\right)^2 + \frac{\psi}{\theta}\left(p_t - \overline{p}_t\right)^2 + \beta E_t\left[G_{t+1}\right].$$

The case $\psi = 0$ is the discretionary benchmark without FCI targeting. In this benchmark, the central bank likes to be predictable and penalizes deviations from the rate path that was previously expected by the markets. When $\theta = 0$, the setup is similar to our earlier model with the difference that the central bank does not respond (by choice) to any current-period shocks $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$ rather than just the noise shock. When $\theta > 0$, the central bank responds to current-period shocks, but only partially, balancing the benefits of interest rate adjustment with the costs of deviating from the previously anticipated interest rate path. The parameter θ captures *the speed* at which the central bank is willing to react to new shocks. The case $\psi > 0$ captures FCI targeting. We have normalized the term on the FCI targeting term to simplify the expressions.

As before, the central bank's optimal target is $\overline{p}_t = E_{t-1}\left[p_t^*\right]$. Then, the optimality condition is given by an interest rate rule reminiscent of the Taylor rule

$$r_t^f = E_{t-1}\left[r_t^f\right] + \theta\tilde{y}_t + \psi\left(p_t - E_{t-1}\left[p_t^*\right]\right).$$

Absent FCI targeting ($\psi = 0$), the central bank adjusts the interest rate in response to output gaps; in particular, the central bank responds to current shocks by a limited amount, *only after* they have some effect on the output gap. FCI targeting expands the rule by making the central bank respond also to gaps between the FCI and the central bank's pre-announced FCI target.

To characterize the rest of the equilibrium, we assume there are no supply shocks, $\varepsilon_{z,t} = 0$. This leads to a particularly tractable analysis, although the insights apply more generally. In this case, Proposition 7 in the appendix shows the equilibrium satisfies

$$\begin{aligned}
\frac{\partial r_t^f}{\partial \varepsilon_{\mu,t}} &= \frac{\theta+\psi}{1+\theta+\psi}\frac{\sigma^2}{\alpha} \\
p_t &= p_t^* + \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t} + \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
y_t &= y^* + \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t} + \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
\sigma^2 &= \sigma^2_{macro}(\psi) + \frac{1}{(1+\theta+\psi)^2}\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_\mu .
\end{aligned}$$

In the appendix we describe $\sigma^2_{macro}(\psi)$ and show that is decreasing in ψ over the appropriate range.

The central bank's limited response to shocks allows noise to affect aggregate asset prices and output. Importantly, FCI targeting induces the central bank to respond to noise shocks more aggressively than implied by the interest rate adjustment costs. As before, FCI targeting also dampens the effect of demand shocks on asset prices (a higher ψ reduces $\frac{\partial p_t}{-\partial \varepsilon_{\delta,t}} = 1 - \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}$). Both of these forces reduce the endogenous return volatility, captured by the modified version of the fixed point equation (28) illustrated in the last line.

The appendix further shows that, as in the baseline model, some degree of FCI targeting always improves the central bank's *true* objective function. As before, committing to stabilize financial conditions *in the future period* helps recruit the arbitrageurs in *the current period* and mitigate the impact of noise. Moreover, since discretionary policy is already optimized to balance the costs of interest rate adjustment with the benefits of output gap minimization, small deviations from this policy induce second-order losses while generating first order gains via the recruitment effect.

In summary, modeling central bank lags with interest rate adjustment constraints reinforces the key findings from our baseline model. FCI targeting reduces the return volatility not only by slowing down the central bank's response to macroeconomic shocks, but also by accelerating its response to noise shocks. The common element is that FCI targeting keeps the aggregate asset price (FCI) closer to the pre-announced target than implied by the discretionary benchmark. By doing this, the central bank encourages arbitrageurs to trade against noise, recruiting them to mitigate the impact of noise on asset prices, output, and interest rates. This recruitment effect reverses the adverse return volatility feedback loop generated by noise shocks and improves macroeconomic stability.

4.5. FCI targeting vs interest rate forward guidance

In our model, FCI targeting functions similarly to issuing *forward guidance* about the future trajectory of the FCI, assuming that this type of guidance implies a soft degree of commitment. This similarity raises the question of whether (more conventional) *interest rate forward guidance*, interpreted as a soft commitment to a future interest rate, could yield similar advantages. Our analysis with an interest rate adjustment constraint already suggests the answer is likely negative; rigid interest rates—either due to gradualism or forward guidance—will allow noise to have a greater impact on the aggregate asset price and weaken the recruitment effect. We next verify this intuition and show that in our model *interest rate forward guidance is strictly worse than FCI targeting.*

We explore this question in Appendix A.4, where we analyze a version of the baseline model with information frictions (and no interest rate adjustment costs) in which the central bank targets the future interest rate rather than the future FCI. Specifically, suppose the central bank solves the following problem:

$$G_t^{r^f\text{-target}} = \min_{r_t^f, \overline{r}_{t+1}^f} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(r_{t+h}^f - \overline{r}_{t+h}^f \right)^2 \right] \right]. \tag{34}$$

In each period, the central bank sets the policy rate r_t^f and announces a target interest rate for the subsequent period $\overline{r}_{t+1}^f$. This problem leads to a similar equilibrium as in Proposition 2, with the key distinction that the central bank does not fully respond to *recent noise shocks* (in addition to the current noise shock). Consequently, this strategy leads to a less effective policy performance compared to a similar FCI targeting policy.

The solution is particularly tractable for the special case in which there are no supply shocks, $\varepsilon_{z,t} = 0$, and demand shocks are transitory, $\varphi_\delta = 0$. For this special case, Proposition 5 in the appendix shows that the equilibrium interest rate is

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\varphi_\mu^2 \mu_{t-1} + \frac{1}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} \right).$$

Compared to FCI targeting, the central bank underreacts to not only demand shocks but also to the persistent component of recent noise shocks $\varphi_\mu \varepsilon_{\mu,t-1}$ (cf. (25)). As a result, this predictable noise also generates volatility in asset prices and output. Moreover, current noise shocks have a greater effect on asset prices and output because financial markets anticipate that the future interest rate will underreact to these shocks. Specifically, the equilibrium asset price is

$$p_t = y_t^* - m - \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\frac{\psi}{1+\psi} \varphi_\mu \varepsilon_{\mu,t-1} + \left(1 + \frac{\psi}{1+\psi} \varphi_\mu \right) \varepsilon_{\mu,t} \right),$$

and output is described by a similar expression, detailed in the appendix.

Comparing these expressions with those in Proposition 2, it becomes evident that interest rate targeting results in larger output gaps and achieves a smaller reduction in return volatility than FCI targeting. In fact, interest rate targeting might even *increase* return volatility by amplifying the price impact of noise shocks for a given σ^2. Therefore, the interest rate targeting policy is strictly dominated by a comparable FCI targeting policy in this environment.

Intuitively, interest rate targeting reduces the flexibility of the central bank to control *the aggregate asset price (FCI)*. Given that output is driven by the aggregate asset price rather than the policy interest rate, this loss of control results in a larger output-gap loss.

4.6. Robustness of FCI targeting

Our baseline model is stylized. In this section, we show that the rationale for the benefits of FCI targeting holds in more complex economic environments—ranging from policy-side extensions like inflation-output tradeoffs to market-side extensions like time-varying arbitrageur beliefs and endogenous arbitrage activity.

4.6.1. FCI targeting with inflation and output trade-off

In Appendix A.6, we investigate the effects of FCI targeting when prices are partially flexible and the central bank faces a trade-off between stabilizing inflation and output. We find that our main results continue to hold in this more realistic setting.

We endogenize inflation via the standard New Keynesian Phillips Curve (NKPC)

$$\pi_t = \kappa \tilde{y}_t + \beta E_t \left[\pi_{t+1}\right] + u_t, \tag{35}$$

where π_t denotes (the log of) nominal price inflation and u_t denotes cost-push shocks that follow:

$$u_t = \varphi_u u_{t-1} + \varepsilon_{u,t}.$$

We also adjust the central bank's *true* objective function to incorporate the costs of inflation gaps. In particular, we consider the baseline model with information frictions (and no interest rate adjustment costs) but assume the central bank targets the real interest rate r_t^f to solve:[21]

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} E_{t-1} \left[\sum_{h=0}^{\infty} \beta^h \left[\tilde{y}_{t+h}^2 + \zeta \pi_{t+h}^2 + \psi \left(1 + \kappa^2 \zeta\right) \left(p_{t+h} - \overline{p}_{t+h}\right)^2 \right] \right].$$

The case $\psi = 0$ is the discretionary benchmark without FCI targeting. The parameter ζ denotes the relative welfare weight for the inflation gaps (we normalize the inflation target to zero). The case $\psi > 0$ captures FCI targeting. We have normalized the term on the FCI targeting term to simplify the expressions. The rest of the model is the same as in Sections 3 and 4.

[21] We assume the central bank sets the nominal interest rate i_t^f and show that (under appropriate assumptions) the central bank can still target the real interest rate r_t^f. Along the equilibrium path, the central bank implements a particular r_t^f by setting i_t^f after accounting for expected inflation and the inflation risk premium.

Proposition 8 in the appendix shows that the equilibrium satisfies:

$$\begin{aligned} p_t &= \underline{E}_{t-1}\left[p_t^o\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\ y_t &= y_t^* - Y_u u_t + \frac{\psi}{1+\psi} Y_u \varepsilon_{u,t} + \frac{\psi}{1+\psi}\left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\ \pi_t &= \Pi_u u_t + \frac{\psi}{1+\psi}\kappa Y_u \varepsilon_{u,t} + \frac{\psi}{1+\psi}\kappa\left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\kappa\varepsilon_{\mu,t}. \end{aligned}$$

The parameters $\Pi_u, Y_u > 0$ are derived coefficients [see $(A.63)$] and $p_t^o = y_t^* - m - \delta_t - Y_u u_t$ is the central bank's optimal asset price target absent noise. Absent FCI targeting ($\psi = 0$), cost-push shocks result in positive inflation gaps and negative output gaps, and they create a new source of asset price volatility. Importantly, noise shocks remain an important driver of output and (now) inflation gaps. FCI targeting ($\psi > 0$) mitigates the aggregate asset price reaction to cost-push shocks $\varepsilon_{u,t}$ as well as to supply and demand shocks. Therefore, FCI targeting still reduces the return volatility σ^2 and the impact of noise shocks $\varepsilon_{\mu,t}$.[22]

Finally, Proposition 9 in the appendix shows that, as in the baseline model, some degree of FCI targeting always improves the central bank's *true* objective function that excludes the FCI targeting term. Intuitively, while cost-push shocks induce nonzero gaps on average, discretionary policy is already optimized to minimize the (current-period) losses induced by these shocks. Therefore, small deviations from this policy generate only second-order losses, while still inducing first-order gains via the same noise-reduction mechanism as in our baseline model (see Proposition 3).

4.6.2. FCI targeting with time-varying arbitrageur beliefs

In Appendix A.7, we investigate FCI targeting when asset prices fluctuate due to both noisy flows and changes in arbitrageurs' beliefs. Our main results continue to hold in this setting. Moreover, belief-driven asset price fluctuations create an additional mechanism by which FCI targeting stabilizes the output gap by dampening belief shocks' impact on expected capital gains.

We capture belief shocks by allowing arbitrageurs to receive a signal about future productivity, $n_{z,t} = \varepsilon_{z,t+1} + e_{zt}$. Their posterior expectation for future productivity is then $E_t\left[y_{t+1}^*\right] = y_t^* + b_t$. Here, b_t denotes *the belief shock*, which is proportional to $n_{z,t}$ and has an ex-ante distribution $b_t \sim N\left(0, \sigma_b^2\right)$. When the news is good $b_t > 0$, arbitrageurs expect productivity and cash-flows to grow faster than usual, and vice versa when the news is bad $b_t < 0$. We remain agnostic about whether these beliefs are correct or biased—this distinction does not affect the equilibrium. We assume the central bank shares the same belief as arbitrageurs and it sets policy before observing current-period belief shock b_t as well as the current-period noisy flow shock $\varepsilon_{\mu,t}$.

[22]Note also that FCI targeting implies that cost-push shocks have a larger impact on inflation gaps and a smaller effect on output gaps.

In the appendix, we show that the equilibrium with discretion satisfies (see Eqs. $(A.91 - A.93)$)

$$\begin{aligned}
p_t &= \underline{E}_{t-1}[p_t^*] + \varepsilon_{z,t} - \varepsilon_{\delta,t} + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
y_t &= y_t^* + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
r_{t+1} &= E_t^A[r_{t+1}] + (\varepsilon_{z,t+1} - b_t) - \beta\varepsilon_{\delta,t+1} + b_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}.
\end{aligned}$$

Compared to the baseline model, the main difference is that the asset price and output are also influenced by the arbitrageurs' belief shocks b_t. When arbitrageurs become more optimistic about the future supply, the aggregate asset price and the *current* output both become higher. Therefore, belief shocks create another source of asset price and output gap fluctuations. Their impact does not depend on its variance, because, unlike noise shocks, belief shocks do not induce trade as they are common across investors. However, these belief-driven fluctuations influence the return process, increasing the return variance σ^2 both directly and indirectly by amplifying the price impact of noisy flows.

We then show that FCI targeting changes the equilibrium as follows (see Proposition 10)

$$\begin{aligned}
p_t &= \underline{E}_{t-1}[p_t^*] + \frac{1}{1+\psi}(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
y_t &= y_t^* + \frac{\psi}{1+\psi}(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t},
\end{aligned} \tag{36}$$

where the variance σ^2 is obtained by solving a fixed-point problem and is decreasing in ψ. As before, FCI targeting mitigates the effect of supply and demand shocks on the asset price (and raises their impact on output). Importantly, FCI targeting also *mitigates* the effect of belief shocks. Under FCI targeting, the central bank commits to limit the immediate response of *future* asset price to *future* supply, $p_{t+1} = \underline{E}_t[p_t^*] + \frac{1}{1+\psi}\varepsilon_{z,t+1} + ...$ The anticipation of this effect mitigates the impact of belief shocks. As asset prices become more stable, noise has a smaller impact on asset prices and output as in the main model.

Eq. (36) shows that in this setup FCI targeting ($\psi > 0$) is beneficial for two distinct reasons. As before, FCI targeting reduces variance σ^2 and mitigates the output gaps induced by noise shocks. In addition, FCI targeting also mitigates the output gaps induced by belief shocks. FCI targeting is still costly because it allows supply and demand shocks to induce some output gaps, but as before these costs are second order. It follows that FCI targeting is robust to allowing for belief-driven asset price fluctuations.

4.6.3. FCI targeting with endogenous arbitrageur supply

In Appendix A.8, we investigate whether our results are robust to endogenous arbitrage activity. This extension is relevant since FCI targeting affects arbitrageurs' equilibrium returns, potentially influencing their market participation. Our main results still hold. While FCI targeting reduces return variance and arbitrageurs' excess returns, inducing some exit, this exit effect cannot be strong enough to overturn the initial variance decline, leaving our results qualitatively unchanged.

We endogenize the fraction of arbitrageurs α via condition $(A.95)$

$$\frac{1}{2}\frac{1}{1-\beta}c\left(\alpha\right) = E\left[\sum_{t=0}^{\infty}\beta^{t}E_t\left[\log\left(R^{A}_{t+1}\right)\right] - E_t\left[\log\left(R_{t+1}\right)\right]\right].$$

Here, the left side captures the (normalized) cost of entry into arbitrage: we assume this supply curve is upward sloping. The right side captures the benefits from entry into arbitrage: the expected excess return generated by the arbitrageurs relative to the inelastic funds (that generate the return on the market portfolio). *In equilibrium*, the excess return in a period satisfies

$$E_t\left[\log R^{A}_{t+1}\right] - E_t\left[\log R_{t+1}\right] = \frac{1}{2}\left(\frac{\mu_t}{\alpha}\right)^2\sigma^2_{t,r_{t+1}}.$$

The excess return is increasing in noisy flows μ_t and in *the return variance*. When variance is higher, arbitrageurs require a higher Sharpe ratio to absorb noisy flows. A higher Sharpe ratio in turn raises the arbitrageurs' expected return.

Since FCI targeting reduces the return variance $\sigma^2_{t,r_{t+1}}$, it might also lower α and induce some arbitrageur exit. However, since this exit *is caused* by the decline in $\sigma^2_{t,r_{t+1}}$, it cannot be strong enough to undo the decline in $\sigma^2_{t,r_{t+1}}$. Appendix A.8 verifies this logic and shows that our results qualitatively still hold in this extended model with endogenous arbitrageur entry.

Beyond the specific mechanisms in our model, there are reasons to think that FCI targeting might actually *increase* arbitrage activity. With discretion, FCI-arbitrage is arguably difficult and requires a sophisticated understanding of macroeconomic conditions: in our model, arbitrageurs need to forecast $p^*_{t+1} = y^*_{t+1} - m - \delta_{t+1}$. In practice, only a handful of institutional investors have this type of knowledge. When the Fed announces its FCI target, it provides potentially valuable information about the FCI it expects to see, $\overline{p}_{t+1} = E_t\left[p^*_{t+1}\right]$. This might make FCI-arbitrage more accessible to a broader set of financial market players: in the above model, it might reduce the cost function $c\left(\alpha\right)$. While these effects are beyond our scope, this example illustrates that endogenizing arbitrage activity not only fails to eliminate but can even reinforce the beneficial effects of FCI targeting.

5. Policy Counterfactuals for the U.S.

In this section, we evaluate a counterfactual scenario in which U.S. monetary policy had adopted FCI targeting over the past few decades. Our findings indicate that FCI targeting would have delivered substantial improvements over historical outcomes, particularly through stabilizing the output gap and reducing financial market volatility. FCI targeting outperforms both a dual mandate-based optimal rule and interest rate-based forward guidance, achieving lower volatility across key macroeconomic and financial indicators. We also demonstrate the particular benefits of FCI targeting from the end of the dot-com bubble to the pre-GFC era, which was characterized by significant noise shocks.

5.1. Methodology

We adapt the methodology described in McKay and Wolf (2023b) and Caravello et al. (2025) to our problem. These papers combine estimated VARs with estimated impulse responses to *policy shocks* to approximate counterfactual *policy rules.* This approach generates accurate counterfactuals as long as the model is linear and monetary policy operates through current or expected policy interest rates—that is, if the source of changes in the interest rate path, whether from shocks or policy rules, is inconsequential. However, directly applying this methodology in our context presents a key challenge: our mechanism operates by reducing risk, and this risk reduction introduces a non-linear component. In the following discussion, we explain the necessary extensions to address this non-linearity.

Set-up and objects of interest. Our baseline set up is similar to Caravello et al. (2025). In particular, we observe data from a data generating process (DGP) of the form:

$$Y_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell} = \sum_{\ell=0}^{\infty} \Theta_{\mu,\ell} \varepsilon_{\mu,t-\ell} + \sum_{\ell=0}^{\infty} \Theta_{-\mu,\ell} \varepsilon_{-\mu,t-\ell}. \tag{37}$$

That is, a linear SVMA(∞), where Y_t is again a vector of macroeconomic aggregates, the shock vector ε_t is distributed as

$$\varepsilon_t \sim N(0, I), \tag{38}$$

and the $n_y \times n_\varepsilon$-dimensional matrices Θ_ℓ denote the impulse response of the vector of observables Y_t at horizon ℓ to a date-t vector of shocks ε_t. We partition the shock vector as $\varepsilon_t = (\varepsilon_{\mu,t}, \varepsilon'_{-\mu,t})'$ where $\varepsilon_{\mu,t}$ is the financial noise shock and $\varepsilon_{-\mu,t}$ stands for the rest of the structural macroeconomic shocks. Analogously, we partition the full impulse response matrices $\Theta_\ell = (\Theta_{\mu,\ell}, \Theta_{-\mu,\ell})$, where $\Theta_{\mu,\ell}$ is a $n_y \times 1$ column vector that collects the impulse response to the financial noise shock, and $\Theta_{-\mu,\ell}$ is a $n_y \times (n_\varepsilon - 1)$ matrix that collects the response to the rest of the shocks.

Next, define also the Wold representation of (37) as:

$$Y_t = \sum_{\ell=0}^{\infty} \Psi_\ell u_{t-\ell}, \tag{39}$$

where u_t are orthogonalized Wold innovations, $u_t = P\varepsilon_t$ for some orthogonal matrix P, and $\Psi_\ell = \Theta_\ell P'$.

We assume that the impulse responses Θ_ℓ can be obtained as the solution to a linear system of dynamic equations:

$$\mathcal{F}_w \boldsymbol{w} + \mathcal{F}_x \boldsymbol{x} + \mathcal{F}_z \boldsymbol{z} + \mathcal{F}_\mu(\sigma_r^2 \varepsilon_{\mu,0}) = \mathbf{0}, \tag{40}$$

$$\mathcal{H}_w \boldsymbol{w} + \mathcal{H}_x \boldsymbol{x} + \mathcal{H}_z \boldsymbol{z} + \mathcal{H}_\varepsilon \varepsilon_0 = \mathbf{0}, \tag{41}$$

$$\mathcal{A}_x \boldsymbol{x} + \mathcal{A}_z \boldsymbol{z} + \mathcal{A}_v v_0 = \mathbf{0}. \tag{42}$$

Here, $\boldsymbol{x} = (x_0, x_1, \dots)$ denotes the infinite sequence of variable x (analogously for $\boldsymbol{w}, \boldsymbol{z}$). As in McKay and Wolf (2023b), $\boldsymbol{x}$ collects all private sector variables, $\boldsymbol{z}$ is the path of the policy instrument (the policy interest rate), and $\boldsymbol{w}$ collects variables that are unobserved to the econometrician. $\Theta_{-\mu,\ell}$ includes the impulse response to ε_0 (macroeconomic shocks) and v_0 (policy shocks), and $\Theta_{\mu,\ell}$ collects the impulse responses to $\varepsilon_{\mu,0}$ (financial noise shocks).

As explained by Caravello et al. (2025), Eqs. $(40-42)$ embed common macroeconomic models that satisfy two key restrictions: (i) structural equations are linear, (ii) policy operates through current and expected policy interest rates—i.e., it is inconsequential if these policy rates change because of rule-based policy response captured by $\mathcal{A}_x\boldsymbol{x}$, or a policy shock captured by $\mathcal{A}_v v_0$.

Our main departure from the previous literature is the addition of equation (40), which represents the Financial Block of the model. The key restriction embedded in (40) is that the (endogenous) conditional variance of returns, σ_r^2, only affects the transmission of the financial noise shock, and it does so proportionally. In particular, we consider models in which the $\mathcal{F}, \mathcal{H}$ or $\mathcal{A}$ matrices do not depend on σ_r^2. This condition is satisfied in our model. This is because (i) the noise shock influences the remaining equilibrium variables through the aggregate asset price, (ii) the noise shock affects the equilibrium asset price in proportion to the conditional variance σ_r^2 (see Eq. (14)), and (iii) the model is conditionally homoskedastic, so the conditional variance of returns is constant. Although admittedly stringent, these assumptions allow us to depart from full linearity to study how an FCI targeting policy can reduce risk.

In this setup, our goal is to obtain three objects:

1. the counterfactual impulse responses for the noise shock, i.e., how would the economy react to the shock if policy had been different?

2. the counterfactual historical evolution between two dates t_1 and t_2, i.e., what would have been the realized path of variables in between those dates had the policy been different?

3. counterfactual second moments, i.e., what would have been the variance of the variables if the policy had been different?

To this end, we consider alternative policy rules, parameterized by $\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z, \tilde{\mathcal{A}}_v, \tilde{\mathcal{A}}_\varepsilon$, such that

$$\tilde{\mathcal{A}}_x \boldsymbol{x} + \tilde{\mathcal{A}}_z \boldsymbol{z} + \tilde{\mathcal{A}}_v v_0 + \tilde{\mathcal{A}}_\varepsilon \varepsilon_0 = \mathbf{0}. \tag{43}$$

In equilibrium, the counterfactual rule induces different impulse response matrices $\tilde{\Theta}_\ell$. Our first object of interest is the counterfactual impulse response corresponding to the noise shock. Our second object of interest is counterfactual second moments given by,

$$\tilde{\Gamma}_Y(\ell) = \sum_{m=0}^{\infty} \tilde{\Theta}_m \tilde{\Theta}'_{m+\ell}, \tag{44}$$

where $\tilde{\Gamma}_Y(\ell)$ is the counterfactual autocovariance function of vector Y_t. We can compute the counterfactual historical evolution as:

$$\tilde{Y}_t = \sum_{\ell=0}^{t-t_1} \tilde{\Theta}_\ell \varepsilon_{t-\ell} + \tilde{Y}_t^1, \quad \forall t \in [t_1, t_1+1, \ldots, t_2], \tag{45}$$

where $\tilde{Y}_t^1 = E_{t_1-1}[\tilde{Y}_t]$ is an initial conditions term.

Accounting for endogenous risk. If we followed Caravello et al. (2025) directly, coupling (43) with (40) and (41) would yield (a rotation of) counterfactual impulse responses $\tilde{\Theta}_\ell$, which can then be used to mechanically construct the counterfactual second moments using (44), and the counterfactual impulse response as a by product. However, in the present setting, this would yield an incorrect counterfactual, since this would not take into account the endogenous reaction of σ_r^2. In order to account for the endogenous reduction in risk, we use the following proposition.

Proposition 4. *Suppose that the SVMA(∞) process* (37) *is invertible; i.e., that*

$$\varepsilon_t \in \mathit{span}(\{Y_\tau\}_{-\infty<\tau\leq t}). \tag{46}$$

Then knowledge of: (i) the Wold representation of Y_t (i.e., the history of innovations $\{u_{t-\ell}\}_{\ell=0}^{\infty}$ together with $\Psi(L)$); (ii) policy causal effects Θ_ν; and iii) and identified time series for the financial noise shock, $\{\varepsilon_{\mu,t}\}$, suffices to construct the counterfactuals of interest— $\tilde{\Theta}_\ell$, $\tilde{\Gamma}_Y(\ell)$, and $\tilde{Y}_t$.

Appendix C.1 contains the proof. The essence of the proof begins by implementing the procedure described in Caravello et al. (2025), followed by rescaling the IRF of the financial noise shock by $\tilde{\sigma}_r^2/\sigma_r^2$ (where $\tilde{\sigma}_r^2$ is the counterfactual conditional variance, obtained via solving a quadratic analogous to that in the model section). This rescaling accounts for the endogenous variance reduction, and allows us to construct the counterfactuals of interest.

Implementation. We use the same data as in Section 2, with the augmented set of variables that includes labor market series. We use demeaned CBO output gap as our measure of output gap, and PCE inflation as our measure of inflation, the Financial Conditions Index built by Ajello et al. (2023a) to play the role of p_t, and the 3 month interest rate as a the policy rate.[23] Since the VAR includes a constant, all variables are demeaned to capture deviations from steady state. For our measured noise shock, we use the shock identified under SVAR-IV as the baseline.

We employ a fully semi-structural approach, using directly measurable impulse responses to approximate the counterfactual policy, as detailed in McKay and Wolf (2023b); Caravello et al. (2025). Although this is an approximation, Caravello et al. (2025) show in their applications for counterfactual second moments and counterfactual historical evolution, the approximation obtained with two shocks is quite good.

We obtain monetary policy impulse responses using the shocks provided by Romer and Romer (2004) and Aruoba and Drechsel (2025). We use a slightly modified specification.[24] We include both shocks in the VAR, and use a recursive identification scheme as suggested in Plagborg-Møller and Wolf (2021) and implemented in McKay and Wolf (2023b). In particular, the Aruoba and Drechsel (2025) shock is ordered first, then output gap, output, investment, consumption, inflation, then the Romer and Romer (2004) shock, and then the rest of the variables. Appendix B.3.6 depicts the estimated impulse responses to the variables of interest.

We take the Wold innovations and identified noise shock as given, but account for estimation uncertainty in the monetary IRFs.[25] Specifically, for each posterior draw, we compute the IRFs and construct the relevant counterfactual. We then report the distribution of these counterfactual outcomes as a means to assess the significance of estimation uncertainty. This is analogous to the procedure outlined in McKay and Wolf (2023b) or Caravello et al. (2025).

5.2. Evaluation of FCI Targeting

5.2.1. Description of the policy rules

We consider a central bank that minimizes a loss of the form:

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t \left[\pi_t^2 + \tilde{y}_t^2 + \lambda_{\Delta i}(i_t - i_{t-1})^2 + \psi(FCI_t - \overline{FCI}_t))^2 \right], \tag{47}$$

We truncate at $T = 20$ quarters. The benchmark policy rule has $\psi = 0$. This kind of loss is considered, for example, in the optimal control exercises reported by the Fed staff to FOMC ahead of their interest rate decision (Federal Reserve Tealbook, 2016). As in previous sections,

[23] See Caballero et al. (2025) for additional details on the mapping between p_t in the model and the measured FCI.

[24] We exclude hours, labor share, labor productivity and TFP, use the 1973Q1-2019Q4 sample, and use a diffuse prior such that the point estimate of the coefficients are given by OLS. This aligns us better with the conventions of the literature that estimates effects of monetary policy shocks. The obtained IRFs are completely standard.

[25] In particular, we use the medians of across posterior draws as point estimates of both objects.

the planner takes conditional volatility as given when choosing the interest rate. The main departure from the theoretical model is the inclusion of an interest rate smoothing term, $\lambda_{\Delta i}(i_t - i_{t-1})^2$, as in Woodford (2003). We refer to policies that arise from minimizing (47) as "Flexible Dual Mandate" (FDM). We set the degree of smoothing $\lambda_{\Delta i}$ to 2.7 to match the interest rate variance observed in the data. We compare this benchmark to the case with ψ^*, i.e.,the value of ψ that minimizes the (true) loss *omitting* the FCI term.

Construction of the optimal target. To align our analysis with the theoretical model, we consider policy responses that are determined with a one-period lag to the shock. We implement the timing restriction by assuming that, at time $t = 0$, the planner targets i_0 at the average rate, and then from $t = 1$ onwards it sets its policy optimally in order to minimize a quadratic loss as in McKay and Wolf (2023a). This is isomorphic to assuming an additional penalty on interest rate movements at time $t = 0$. As explained in Section 4.4, the exact form of the interest rate constraint does not affect the potential benefits of FCI targeting.

We also need to obtain the optimal FCI target. This is achieved by solving for the policies that minimize (47) as a function of the target. We then select this target to minimize (47) subject to the timing constraints.

In order to see how this works in more detail, consider the policy response to a non-policy shock $\boldsymbol{\varepsilon} = (\varepsilon, 0, 0, \cdots)'$, which is the building block of all counterfactuals presented in this section. Denote the sequences of output gap, inflation, nominal interest rate and FCI by $\tilde{\boldsymbol{y}}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{f}$ respectively. Then the optimal FCI target in response to a shock, $\bar{\boldsymbol{f}}$, is obtained by solving:

$$\begin{aligned} \min_{\nu, \bar{\boldsymbol{f}}} \quad & \tilde{\boldsymbol{y}}' \tilde{W}_{\tilde{y}} \tilde{\boldsymbol{y}} + \boldsymbol{\pi}' \tilde{W}_{\pi} \boldsymbol{\pi} + \lambda_{\Delta i} \boldsymbol{i}' \tilde{W}_i \boldsymbol{i} + \psi(\boldsymbol{f} - \bar{\boldsymbol{f}})' \tilde{W}_f (\boldsymbol{f} - \bar{\boldsymbol{f}}) \\ s.t. \quad & \boldsymbol{x}_i = \Theta_{x_i, v} \nu + \Theta_{x_i, \varepsilon} \boldsymbol{\varepsilon}, \quad \text{for } \boldsymbol{x}_i = \tilde{\boldsymbol{y}}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{f} & \text{(Implementation)} \\ & R_f \bar{\boldsymbol{f}} = 0 & \text{(Announcement Timing)} \end{aligned}$$

Several additional pieces of notation must be clarified. First, $\tilde{W}_{\tilde{y}}, \tilde{W}_{\pi}, \tilde{W}_i, \tilde{W}_f$ are matrices that encode discounting and timing restrictions. For example, under the assumption of a one period lag and discount factor β, $\tilde{W}_{\tilde{y}}$ is a diagonal matrix with $\text{diag}(\tilde{W}_{\tilde{y}}) = (0, \beta, \beta^2, \cdots)'$, where the first zero comes from the one period lag. Second, the constraints $\boldsymbol{x}_i = \Theta_{x_i,v}\nu + \Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}$ encode how the planner can affect outcomes via choosing policy paths. In particular, the $\Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}$ term is the path of the variable under the baseline rule, i.e., in the IRF estimated in sample. On top of this, the policymaker can commit to different policy paths by choosing ν appropriately.[26] Finally, the constraint $R_f \bar{\boldsymbol{f}} = 0$ encodes that the FCI path is announced one period in advance. Since the planner reacts to shocks with a one period lag, for a shock that hits at $t = 0$, any changes in $F\bar{C}I_t$ are announced at $t = 1$ and take effect in $t = 2$. Thus, R_f encodes that the

[26]Since we have only a subset of policy shocks in our empirical analysis (i.e only two as opposed to one for each time horizon), then ν is a two-dimensional vector, and the interpretation is that of a constrained optimal policy in which the planner can only alter baseline policy in the directions spanned by the two monetary policy shocks.

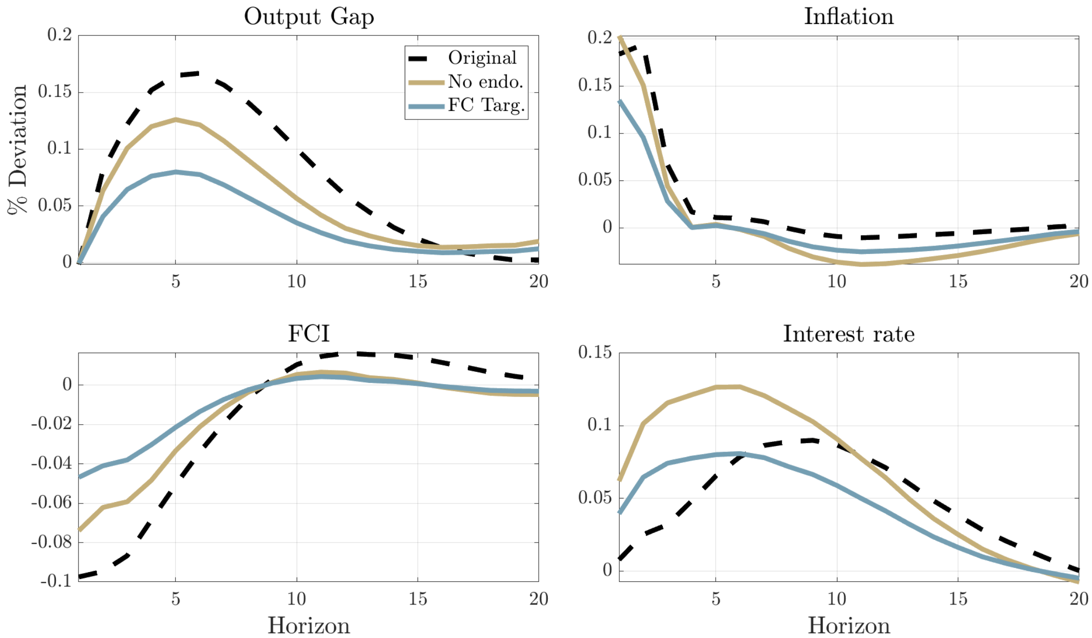

Figure 7: Counterfactual Impulse Responses for the noise shock under FCI Targeting. Beige: not accounting for endogenous risk reduction. Blue: accounting for endogenous risk reduction.

first two elements of $\bar{f}$ must be zero. Appendix C.2 expands on the details.

5.2.2. Results

Impulse Responses. Figure 7 shows the counterfactual impulse response to a noise shock under FCI targeting. As a preliminary step, the beige line shows the outcome if we did not account for endogenous risk reduction, essentially applying McKay and Wolf (2023b)'s methodology directly. In this scenario, monetary policy stabilizes the noise shock more effectively than observed in the historical data, with interest rates raised more aggressively and the noise shock having a smaller effect on the FCI, output gap, and inflation. Our main result is the blue line, which shows the accurate counterfactual that accounts for the endogenous reduction in FCI volatility under the FCI targeting policy. This scenario demonstrates even greater stabilization, with noise shocks having a further reduced impact on the FCI, output gap, and inflation. Observe also that interest rates are raised *less* than in the first scenario.

Compared to the data, the initial interest rate reaction is somewhat larger, but future interest rates increase *less* than in the data. The peak response of the interest rate is similar, but occurs earlier under FCI targeting. This is in line with results in Section 4: The Central Bank responds more on impact to noise shocks (Section 4.4), but the enhanced stabilization from FCI targeting makes the path of interest rates lower than in the data after a few quarters, since lagged noise terms are smaller (Section 4.3).

This enhanced stabilization without a significantly higher path of interest rates occurs because the FCI targeting policy effectively "recruits" arbitrageurs to help the Fed by trading more aggressively against noise. As arbitrageurs take on a larger stabilizing role, the interest rate does not need to react more.

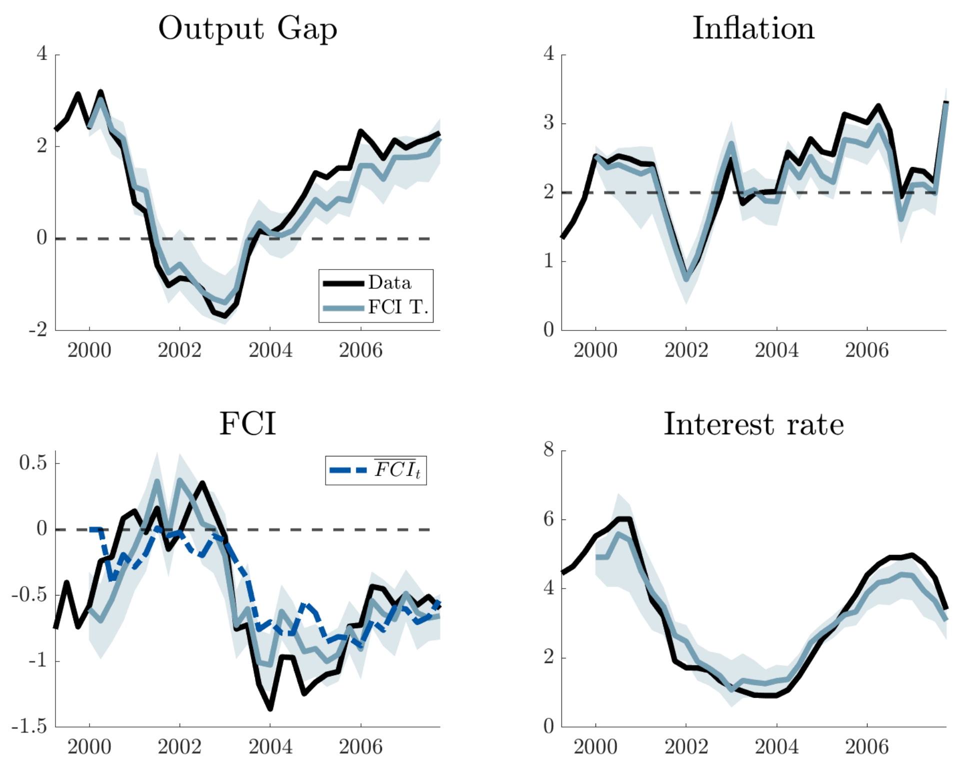

Figure 8: Counterfactual Historical Evolution for 2000Q1-2007Q4. Black: data. For output gap, this is demeaned CBO output gap. Inflation is in year-on-year terms. The rest of the variables are in levels. Blue: FCI targeting, i.e minimize 47 with $\psi = \psi^*$. Solid: median. Shaded area: 16 and 84 confidence bands. Dashed line in the FCI panel indicates the target $\overline{FCI}_t$.

Counterfactual historical evolution. To enhance understanding of how FCI targeting operates in practice, we demonstrate how FCI targeting would have impacted the realized paths of the output gap, inflation, FCI, and interest rates in the period leading up to the Global Financial Crisis. We choose this period because our historical decomposition in Appendix B.3.2 shows that the financial noise shock was a significant driver of the 2001 recession and the main driver of the later expansion. We consider the period 2000Q1-2007Q4, starting at the peak of the expansion preceding the 2001 recession and continuing until the start of the GFC. We use the version of FCI with the interest rate smoothing term discussed before, with the same values of $\lambda_{\Delta i}$ and ψ^*.

Figure 8 shows the results. First, the initial part of the recession appears unavoidable. At the start of the 2001 recession, the planner targets looser financial conditions in order to stabilize the

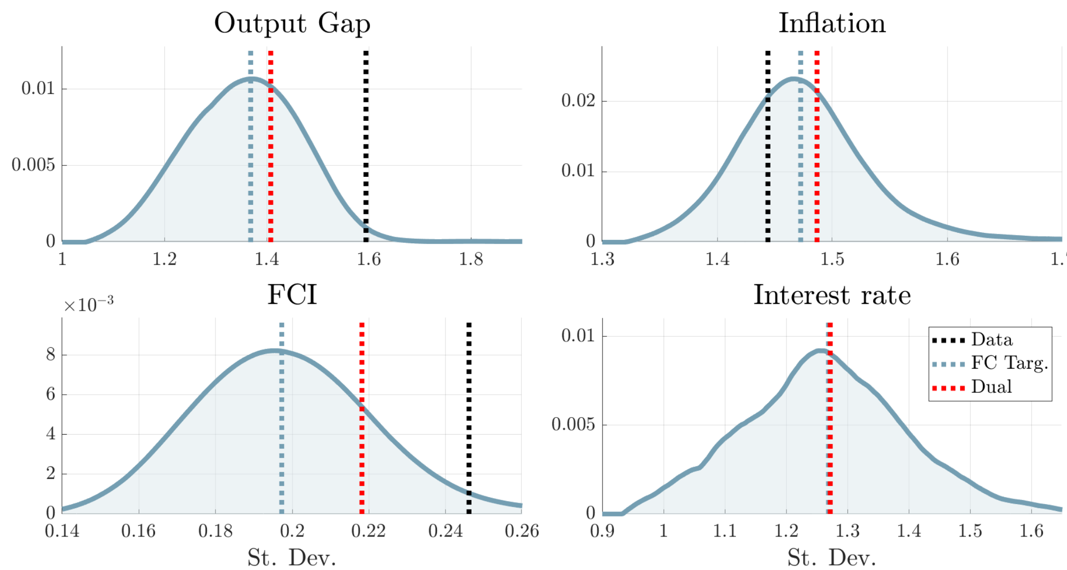

Figure 9: Counterfactual Standard Deviations. For Output Gap, Inflation and interest rates, this is the unconditional standard deviation, for FCI this is the conditional SD. Black dashed: data. Dashed lines the median. Red: Flexible Dual Mandate, i.e minimize (47) with $\psi = 0$. Blue: FCI targeting, i.e minimize (47) with $\psi = \psi^*$. Beige: FCI targeting, i.e minimize (47) with $\psi = \psi^*$, but without accounting for the endogenous reduction in risk when constructing the counterfactual. Solid Line: posterior density for the counterfactual with FCI targeting counterfactual.

drop in output gap. Around this period, FCI targeting makes financial conditions less restrictive than in the data by cutting interest rates.

Turning to the expansionary phase of the cycle stating in 2004Q1, the data shows that financial conditions became quite loose, a positive output gap opened up, and this was accompanied by above-target inflation. If policy had followed FCI targeting, looser financial conditions would have been counteracted by policy, both via higher interest rate and announcements of tighter FCI targets. This would have generated tighter financial conditions, which would have helped to achieve lower output gaps, and consequently, inflation closer to the 2% target. Overall, the adoption of FCI targeting would have smoothed both phases of the cycle, specially the pre-GFC boom.

Counterfactual macro and financial volatility. In order to get a sense of how much macro and financial volatility would have been reduced had policy followed FCI targeting, Figure 9 displays the counterfactual standard deviation for FDM (i.e., minimize (47) with $\psi = 0$) in red, and contrasts these with FCI targeting, shown in blue. Under FCI targeting, the volatilities of the output gap, FCI and the interest rate are reduced. Relative to the data, the variance of the output gap, and interest rates fall by 26.3%, and 0.4% respectively, and the conditional variance of the FCI falls by 35.8%. There is a small increase in the variance of inflation, of 4%. The small

Loss	Baseline			No i_t smoothing term		
	Median	10th Perc.	90th Perc.	Median	10th Perc.	90th Perc.
Data	4.97			4.63		
Dual Mandate ($\psi = 0$)	4.45	4.10	5.41	4.16	3.79	5.13
FCI T. ($\psi^* = 8.33$)	4.34	4.04	4.84	4.01	3.70	4.53

Table 1: Central bank loss function in the data, and median, 10th and 90th percentiles under the counterfactual policy rules. The policy rules always include the interest rate smoothing term. The first set of columns shows the baseline loss, $E[\mathcal{L}] = \sigma^2_{\tilde{y}} + \sigma^2_{\pi} + \lambda_{\Delta i}\sigma^2_{\Delta i}$. The second set of columns shows the values of $E[\tilde{\mathcal{L}}] = \sigma^2_{\tilde{y}} + \sigma^2_{\pi}$

effect on inflation variance relative to output gap variance is due to the small and delayed the response of inflation to monetary policy shocks, consistent with a flat Phillips curve. Conversely, the real effects of monetary policy are significant, so the variance gains come from output gaps.

When compared to FDM, the reductions are more modest, with the output gap and inflation decreasing by 5.5% and 1.9% respectively (when comparing medians), while the interest rate variance reduction is still 0.4% (recall that FDM is calibrated to fit the observed interest rate volatility). However, the decrease in financial conditions variance remains substantial, at approximately 18.2%. Thus, FCI targeting achieves somewhat better outcomes in terms of output gap and inflation with much lower financial volatility, and *without* larger swings in the interest rate as explained in section 4.3.

Table 1 presents the loss (47) with and without the interest rate smoothing term. As expected, the loss is lower under FCI targeting, and this is not driven by the interest rate smoothing term.

5.2.3. FCI targeting vs interest rate forward guidance

Following the discussion of Section 4.5, we provide a comparison of FCI targeting with interest rate forward guidance. In particular, we consider losses of the form:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \tilde{y}_t^2 + \psi_{if}(i_t^f - \bar{i}_t^f))^2 + \psi(FCI_t - \overline{FCI}_t))^2 \right], \tag{48}$$

and compare interest rate forward guidance ($\psi_{if} > 0, \psi = 0$) with FCI targeting ($\psi_{if} = 0, \psi > 0$). We pick ψ_{if} and ψ^* to minimize the pure dual mandate loss $\sum_{t=0}^{T} \beta^t [\pi_t^2 + \tilde{y}_t^2]$. We omit the interest rate smoothing term, in order to make the comparison between both "pure" regimes.

Figure 10 shows the results. While the inflation variance is roughly the same, the output gap variance is 6% lower under FCI targeting compared to interest rate forward guidance. As expected, FCI is less volatile under FCI targeting, while interest rates are less volatile under interest rate targeting. This analysis reinforces that FCI targeting delivers superior macroeconomic stabilization compared to standard interest rate forward guidance approaches.

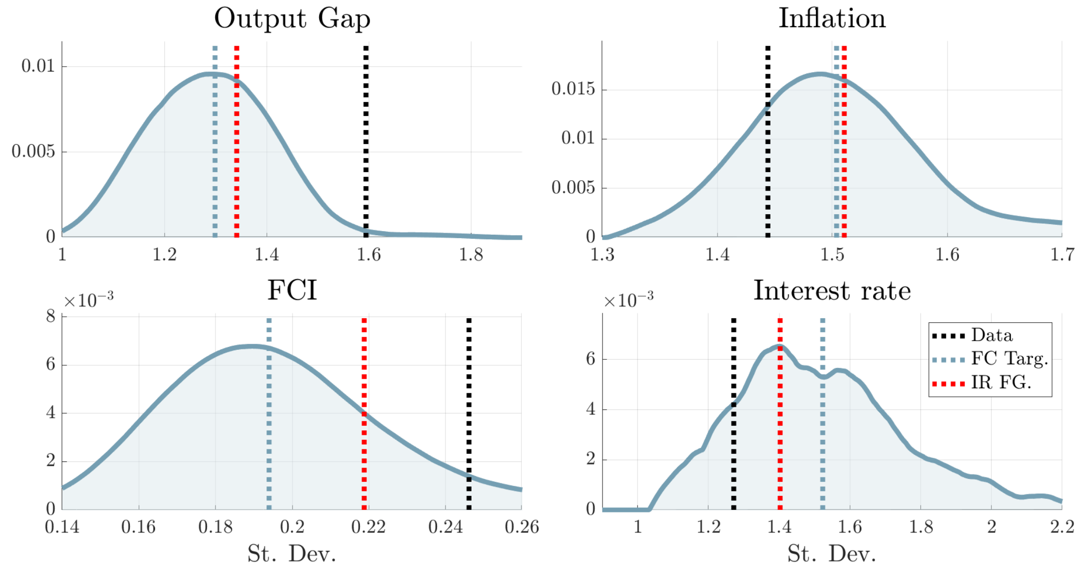

Figure 10: Counterfactual Standard Deviations. For Output Gap, Inflation and interest rates, this is the unconditional standard deviation, for FCI this is the conditional SD. Black dashed: data. Dashed lines the median. Red: Pure Dual Mandate with Forward Guidance in interest rates. Blue: FCI targeting, i.e minimize (48) with $\psi = \psi^*$. Solid Line: posterior density for the counterfactual with FCI targeting counterfactual.

6. Final Remarks

This paper theoretically and empirically investigates how monetary policy should respond to macroeconomic fluctuations driven by financial noise.

We first document the quantitative importance of financial noise using identified vector-autoregression models. Our findings reveal that financial noise shocks account for up to 40% of financial conditions variance and up to 25% of output gap variance in the U.S., establishing that noise-driven fluctuations represent a first-order macroeconomic phenomenon.

We then develop a macroeconomic model with financial noise and limits to arbitrage, demonstrating that it is optimal for the central bank to target and partially stabilize financial conditions as an independent objective, beyond simply managing their effects on output and inflation. Our main theoretical result shows that FCI targeting—whereby the central bank announces a soft FCI target and adjusts policy to keep actual conditions close to this target—reduces both FCI volatility and output gap fluctuations.

The welfare improvement occurs through an endogenous volatility feedback loop. FCI targeting commits the central bank to greater financial stabilization than under discretionary policy. While this commitment reduces future policy flexibility, it generates first-order benefits today by enabling arbitrageurs to trade more aggressively against noise. This "recruitment effect" creates a virtuous cycle: reduced FCI volatility enables more aggressive arbitrage, which further reduces noise impacts and volatility. The policy optimally rebalances monetary responses—

reducing sensitivity to macroeconomic data while potentially accelerating responses to noise shocks—without requiring real-time distinction between noise and fundamental movements. We further demonstrate that FCI targeting dominates interest rate forward guidance because it preserves monetary policy's flexibility to respond to post-announcement noise shocks, while forward guidance constrains such responses.

Finally, we extend policy counterfactual methods to incorporate our model's endogenous risk reduction and evaluate FCI targeting's potential effects in the U.S. Our estimates suggest that such a policy could have reduced the variance of financial conditions and output gaps by 35.8% and 26.3% respectively, without increasing interest rate volatility.

References

Adrian, T., Duarte, F., 2018. Financial vulnerability and monetary policy .

Adrian, T., Duarte, F., Liang, N., Zabczyk, P., 2020. Nkv: A new keynesian model with vulnerability, in: AEA Papers and Proceedings, American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203. pp. 470–476.

Ajello, A., Cavallo, M., Favara, G., Peterman, W.B., Schindler IV, J.W., Sinha, N.R., 2023a. A new index to measure us financial conditions. FEDS Notes. Washington: Board of Governors of the Federal Reserve System, June 30, 2023, https://doi.org/10.17016/2380-7172.3281 .

Ajello, A., Cavallo, M., Favara, G., Peterman, W.B., Schindler IV, J.W., Sinha, N.R., 2023b. A new index to measure u.s. financial conditions. FEDS Notes 2023, 1. URL: `https://www.federalreserve.gov/econres/notes/feds-notes/a-new-index-to-measure-us-financial-conditions-20230630.html`.

Angeletos, G.M., Collard, F., Dellas, H., 2020. Business-cycle anatomy. American Economic Review 110, 3030–70. URL: `https://www.aeaweb.org/articles?id=10.1257/aer.20181174`, doi:`10.1257/aer.20181174`.

Aruoba, S.B., Drechsel, T., 2025. Identifying monetary policy shocks: A natural language approach. Forthcoming.

Barnichon, R., Mesters, G., 2023. A sufficient statistics approach for macro policy. American Economic Review 113, 2809–2845.

Bassetto, M., 2019. Forward guidance: communication, commitment, or both? Journal of Monetary Economics 108, 69–86.

Beaudry, P., Cavallino, P., Willems, T., 2024. Life-cycle Forces make Monetary Policy Transmission Wealth-centric. Technical Report. National Bureau of Economic Research.

Beraja, M., 2023. A semistructural methodology for policy counterfactuals. Journal of Political Economy 131, 190–201. URL: `https://doi.org/10.1086/720982`, doi:`10.1086/720982`, `arXiv:https://doi.org/10.1086/720982`.

Bernanke, B.S., Gertler, M., 2000. Monetary policy and asset price volatility. NBER working paper No. 7559 .

Bernanke, B.S., Gertler, M., 2001. Should central banks respond to movements in asset prices? American Economic Review 91, 253–257.

Black, F., 1986. Noise. The journal of finance 41, 528–543.

Caballero, R.J., Caravello, T.E., Simsek, A., 2025. Fci-star. NBER working paper No. 33952 .

Caballero, R.J., Farhi, E., 2018. The safety trap. The Review of Economic Studies 85, 223–274.

Caballero, R.J., Simsek, A., 2020. A risk-centric model of demand recessions and speculation. The Quarterly Journal of Economics 135, 1493–1566.

Caballero, R.J., Simsek, A., 2021. A model of endogenous risk intolerance and LSAPs: Asset prices and aggregate demand in a "COVID-19" shock. The Review of Financial Studies 34, 5522–5580.

Caballero, R.J., Simsek, A., 2023. A monetary policy asset pricing model. NBER working paper No. 30132 .

Caballero, R.J., Simsek, A., 2024. Monetary policy and asset price overshooting: A rationale for the wall/main street disconnect. The Journal of Finance 79, 1719–1753.

Caballero, R.J., Simsek, A., 2025. Fci-plot: Central bank communication through financial conditions. working paper .

Caldara, D., Herbst, E., 2019. Monetary policy, real activity, and credit spreads: Evidence from bayesian proxy svars. American Economic Journal: Macroeconomics 11, 157–92. URL: https://www.aeaweb.org/articles?id=10.1257/mac.20170294, doi:10.1257/mac.20170294.

Campbell, J.R., Evans, C.L., Fisher, J.D., Justiniano, A., Calomiris, C.W., Woodford, M., 2012. Macroeconomic effects of federal reserve forward guidance [with comments and discussion]. Brookings papers on economic activity , 1–80.

Campbell, J.Y., 2014. Empirical asset pricing: Eugene fama, lars peter hansen, and robert shiller. The Scandinavian Journal of Economics 116, 593–634.

Caravello, T.E., McKay, A., Wolf, C.K., 2025. Evaluating monetary policy counterfactuals: (when) do we need structural models? MIT FRB Minneapolis MIT & NBER URL: https://economics.mit.edu/sites/default/files/2024-02/mp_modelcnfctls_0.pdf, doi:10.3386/w30358.

Chahrour, R., Jurado, K., 2021. Recoverability and expectations-driven fluctuations. The Review of Economic Studies 89, 214–239. URL: https://doi.org/10.1093/restud/rdab010, doi:10.1093/restud/rdab010, arXiv:https://academic.oup.com/restud/article-pdf/89/1/214/42137430/rdab010.pdf.

Christiano, L.J., Motto, R., Rostagno, M., 2014. Risk shocks. American Economic Review 104, 27–65. URL: https://www.aeaweb.org/articles?id=10.1257/aer.104.1.27, doi:10.1257/aer.104.1.27.

Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: a new keynesian perspective. Journal of economic literature 37, 1661–1707.

De Long, J.B., Shleifer, A., Summers, L.H., Waldmann, R.J., 1990. Noise trader risk in financial markets. Journal of political Economy 98, 703–738.

Favara, G., Gilchrist, S., Lewis, K.F., Zakrajšek, E., 2016. Updating the recession risk and the excess bond premium. FEDS Notes .

Federal Reserve Tealbook, 2016. Report to the FOMC on Economic Conditions and Monetary Policy.

Fernald, J., 2014. A quarterly, utilization-adjusted series on total factor productivity URL: https://www.frbsf.org/economic-research/publications/working-papers/2012/19/. updated April 2014.

Forni, M., Gambetti, L., Ricco, G., 2023. External instrument svar analysis for noninvertible shocks. CEPR Discussion Paper No. 17886 URL: https://cepr.org/publications/dp17886.

Gabaix, X., Koijen, R.S., 2021. In search of the origins of financial fluctuations: The inelastic markets hypothesis. NBER working paper No. 28967 .

Gabaix, X., Koijen, R.S.J., 2024. Granular instrumental variables. Journal of Political Economy 132, 2274–2303. doi:10.1086/728743.

Gabaix, X., Maggiori, M., 2015. International liquidity and exchange rate dynamics. The Quarterly Journal of Economics 130, 1369–1420.

Gertler, M., Karadi, P., 2015. Monetary policy surprises, credit costs, and economic activity. American Economic Journal: Macroeconomics 7, 44–76. doi:10.1257/mac.20130329.

Giannone, D., Lenza, M., Primiceri, G.E., 2015. Prior selection for vector autoregressions. Review of Economic Studies 82, 899–940. doi:10.1093/restud/rdu039.

Gilchrist, S., Yankov, V., Zakrajsek, E., 2009. Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets. Journal of Monetary Economics 56, 471–493. doi:10.1016/j.jmoneco.2009.03.003.

Gilchrist, S., Zakrajšek, E., 2012. Credit spreads and business cycle fluctuations. American Economic Review 102, 1692–1720.

Gourinchas, P.O., Ray, W., Vayanos, D., 2022. A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers. NBER working paper No. 29875 .

Greenwood, R., Hanson, S., Stein, J.C., Sunderam, A., 2023. A quantity-driven theory of term premia and exchange rates. The Quarterly Journal of Economics 138, 2327–2389.

Greenwood, R., Vayanos, D., 2014. Bond supply and excess bond returns. The Review of Financial Studies 27, 663–713.

Gromb, D., Vayanos, D., 2010. Limits of arbitrage. Annu. Rev. Financ. Econ. 2, 251–275.

Hatzius, J., Stehn, S.J., 2018. The case for a financial conditions index. Goldman Sachs Global Economics Papers .

Hatzius, J., Stehn, S.J., Fawcett, N., Reichgott, K., 2017. Our new G10 financial conditions indices. Goldman Sachs Global Economics Analyst .

Hebden, J., Winkler, F., 2021. Impulse-based computation of policy counterfactuals. FEDS Working Paper.

Itskhoki, O., Mukhin, D., 2021. Mussa puzzle redux. NBER working paper No. 28950 .

Jeanne, O., Rose, A.K., 2002. Noise trading and exchange rate regimes. The Quarterly Journal of Economics 117, 537–569.

Kekre, R., Lenel, M., 2022. Monetary policy, redistribution, and risk premia. Econometrica 90, 2249–2282. doi:https://doi.org/10.3982/ECTA18014, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA18014.

Kekre, R., Lenel, M., Mainardi, F., 2023. Monetary policy, segmentation, and the term structure .

Laubach, T., Williams, J.C., 2003. Measuring the natural rate of interest. The Review of Economics and Statistics 85, 1063–1070.

Lewis, D.J., Mertens, K., 2025. A robust test for weak instruments for 2sls with multiple endogenous regressors. Review of Economic Studies URL: https://karelmertens.com/wp-content/uploads/2025/04/robust_weak_iv_with_multiple_endogenous_variables_april_2025_main.pdf. forthcoming, version dated April 2025.

Li, D., Plagborg-Møller, M., Wolf, C.K., 2024. Local projections vs. vars: Lessons from thousands of dgps. Journal of Econometrics , 105722URL: https://www.sciencedirect.com/science/article/pii/S030440762400068X, doi:https://doi.org/10.1016/j.jeconom.2024.105722.

McKay, A., Wolf, C.K., 2023a. Optimal policy rules in hank. FRB Minneapolis MIT & NBER URL: https://economics.mit.edu/sites/default/files/2023-03/hank_optpol.pdf.

McKay, A., Wolf, C.K., 2023b. What can time-series regressions tell us about policy counterfactuals? Econometrica 91, 1695–1725. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA21045, doi:https://doi.org/10.3982/ECTA21045, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA21045.

Mertens, K., Ravn, M.O., 2013. The dynamic effects of personal and corporate income tax changes in the united states. American Economic Review 103, 1212–1247. doi:10.1257/aer.103.4.1212.

Pflueger, C., Siriwardane, E., Sunderam, A., 2020. Financial market risk perceptions and the macroeconomy. The Quarterly Journal of Economics 135, 1443–1491.

Plagborg-Møller, M., Wolf, C.K., 2022. Instrumental variable identification of dynamic variance decompositions. Journal of Political Economy 130, 2164–2202. URL: https://doi.org/10.1086/720141, doi:10.1086/720141, arXiv:https://doi.org/10.1086/720141.

Plagborg-Møller, M., Wolf, C.K., 2021. Local projections and vars estimate the same impulse responses. Econometrica 89, 955–980. URL: https://www.nber.org/papers/w29939, doi:10.3386/w29939.

Ramey, V.A., 2016. Macroeconomic shocks and their propagation, in: Taylor, J.B., Uhlig, H. (Eds.), Handbook of Macroeconomics. Elsevier. volume 2A, pp. 71–162. URL: https://econweb.ucsd.edu/~vramey/research/Shocks_HOM_Ramey_published_corrected.pdf, doi:10.1016/bs.hesmac.2016.03.002.

Romer, C.D., Romer, D.H., 2004. A new measure of monetary shocks: Derivation and implications. American Economic Review 94, 1055–1084.

Shleifer, A., Summers, L.H., 1990. The noise trader approach to finance. Journal of Economic perspectives 4, 19–33.

Shleifer, A., Vishny, R.W., 1997. The limits of arbitrage. The Journal of finance 52, 35–55.

Svensson, L.E., 2014. Forward guidance. NBER working paper No. 20796 .

Vayanos, D., Vila, J.L., 2021. A preferred-habitat model of the term structure of interest rates. Econometrica 89, 77–112.

Woodford, M., 2003. Optimal interest-rate smoothing. The Review of Economic Studies 70, 861–886.

Woodford, M., 2013. Forward guidance by inflation-targeting central banks. CEPR Discussion Paper No. DP9722 .

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A. Theory Appendix

This appendix contains details related to the theoretical model. Section A.1 describes the parameters we use in the numerical examples that illustrate our findings. Section A.2 provides the microfoundations for the model. Section A.3 contains the proofs omitted from the main text. The remaining Sections A.4-A.6 provide the details of various extensions that we discuss in the main text.

A.1. Details of the numerical exercise

We next describe the parameters we use in the numerical examples that we use to illustrate the results in Sections 4.1-4.3. In our baseline model, the price impact of a unit change in asset demand (as a fraction of supply) is given by

$$\mathcal{I} \equiv \frac{dp_t}{d\varepsilon_{\mu,t}} = \frac{\sigma^2}{\alpha}.$$

Recent empirical analyses find that this price impact is large. For instance, Gabaix and Koijen (2021) suggest that for the stock market it could be as large as 5. To be conservative, we set the fraction of elastic funds α to target a price impact equal to one, $\mathcal{I} = \frac{\sigma^2}{\alpha} = 1$.

We set the rest of the parameters as follows

$$\begin{aligned} \sigma_\mu^2 &= \sigma_{macro}^2(0), \\ \sigma_\delta^2 &= \sigma_z^2 = \frac{\sigma_{macro}^2(0)}{1+\beta^2}, \\ \alpha &= \sigma_\mu^2 + \sigma_{macro}^2(0) = \sigma^2, \\ \text{with } \sigma_{macro}^2(0) &= (0.01)^2 \text{ and } \beta = 0.99 \\ \text{and } \varphi_\delta &= \varphi_\mu = 0.95. \end{aligned} \tag{A.1}$$

We equalize the noise variance to the macro-induced variance, and set the variances of demand and supply to be identical. We consider a quarterly calibration and set the discount rate to 1%. We set the macro-induced standard deviation to 1% to match (roughly) the standard deviation of quarterly output growth in the data.[27] Finally, in the last line (relevant only for the results in Section 4.3) we set the persistence of demand and noise shocks to match (roughly) the quarterly autocorrelation of the policy interest rate observed in the data.

A.2. Microfoundations of the model

In this section, we provide the microfoundations of the model that we summarize in Section 3.1 and use throughout the paper. The real side of the economy is the same as the baseline model in Caballero and Simsek (2023). The financial market side is different and allows for noise shocks.

The economy is set in discrete time $t \in \{0, 1, ..\}$. The model consists of four agent types: asset-holding households (households), hand-to-mouth agents, portfolio managers, and the central bank. The hand-to-mouth agents primarily serve to decouple labor supply decisions from household consumption behavior.

[27] In this calibration, the level of $\sigma_{macro}^2(0)$ does not change the optimal level of FCI targeting since all other variances scale with $\sigma_{macro}^2(0)$.

The asset-holding households are the main drivers of aggregate demand through their consumption and savings choices. The portfolio managers act on behalf of these households by making portfolio allocation decisions that determine asset prices in financial markets. The central bank conducts monetary policy.

A.2.1. Supply side

Hand-to-mouth agents provide all of the labor supply and spend all of their income (they do not save). Their problem is

$$\begin{aligned} &\max_{L_t} \log C_t^{HM} - \chi \frac{L_t^{1+\varphi}}{1+\varphi}, \\ &Q_t C_t^{HM} = W_t L_t + T_t. \end{aligned} \tag{A.2}$$

Here, φ denotes the Frisch elasticity of labor supply, Q_t denotes the nominal price for the final good, W_t denotes the nominal wage, and T_t denotes lump-sum transfers from the government (described subsequently). The optimality condition implies a standard labor supply equation

$$\frac{W_t}{Q_t} = \chi L_t^{\varphi} C_t^{HM}. \tag{A.3}$$

The rest of the supply side is similar to the standard New Keynesian Model. A competitive final goods producer combines the intermediate goods according to the CES technology,

$$Y_t = \left(\int_0^1 Y_t\left(\nu\right)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} d\nu \right)^{\varepsilon_t/(\varepsilon_t-1)} \qquad \text{where } Y_t\left(\nu\right) = Z_t L_t\left(\nu\right)^{1-\nu}. \tag{A.4}$$

Here, $\varepsilon_t > 1$ denotes the elasticity of substitution that determines the firm markups in equilibrium. We assume it is stochastic around a steady-state level $\varepsilon^* > 1$, which allows us to accommodate cost-push shocks. With these technologies, the demand for intermediate good firms satisfies,

$$Y_t\left(\nu\right) \leq \left(\frac{Q_t\left(\nu\right)}{Q_t}\right)^{-\varepsilon_t} Y_t, \tag{A.5}$$

$$\text{where } Q_t = \left(\int_0^1 Q_t\left(\nu\right)^{1-\varepsilon_t} d\nu \right)^{1/(1-\varepsilon_t)}. \tag{A.6}$$

$Q_t\left(\nu\right)$ denotes the nominal price set by the intermediate good firm ν and Q_t is the ideal price index.

The goods market clearing condition is:

$$Y_t = C_t^H + C_t^{HM}. \tag{A.7}$$

Here, C_t^H and C_t^{HM} denote consumption by the asset holding households and the hand-to-mouth agents, respectively.

The labor market clearing condition is

$$\int_0^1 L_t\left(\nu\right) d\nu = L_t. \tag{A.8}$$

Finally, to simplify the distribution of output across factors, we assume the government taxes part of the firms' profits lump-sum and redistributes to the hand-to-mouth agents to ensure they receive their production share of output. Specifically, each intermediate firm pays lump-sum taxes determined as follows:

$$T_t = (1-\nu) Q_t Y_t - W_t L_t. \tag{A.9}$$

This ensures that in equilibrium hand-to-mouth agents receive and spend their production share of output, $(1-\nu) Q_t Y_t$, and consume [see $(A.2)$]

$$C_t^{HM} = (1-\nu) Y_t. \tag{A.10}$$

Substituting this into the goods market clearing condition $(A.7)$, we further obtain

$$Y_t = \frac{C_t^H}{\nu}. \tag{A.11}$$

Hand-to-mouth agents create a Keynesian multiplier effect, but output is ultimately determined by (asset-holding) *households'* spending, C_t^H.

Flexible-price equilibrium. Consider a benchmark without nominal rigidities. In this benchmark, the intermediate good firm ν solves

$$\begin{aligned} \Pi &= \max_{Q,L} QY - W_t L - T_t, \\ &\quad \text{where } Y = Z_t L^{1-\nu} = \left(\frac{Q}{Q_t}\right)^{-\varepsilon_t} Y_t. \end{aligned} \tag{A.12}$$

The firm takes as given the aggregate price, wage, and output, Q_t, W_t, Y_t, and chooses its price, labor input, and output Q, L, Y.

The optimal price is

$$Q = \frac{\varepsilon_t}{\varepsilon_t - 1} W_t \frac{1}{(1-\nu) Z_t L^{-\nu}}. \tag{A.13}$$

The firm sets an optimal markup over the marginal cost, where the markup depends (inversely) on the elasticity of substitution and the marginal cost depends on the wage and (inversely) on the marginal product of labor.

In equilibrium, all firms choose the same prices and allocations, $Q_t = Q$ and $L_t = L$. Substituting this into $(A.13)$, we obtain a labor demand equation,

$$\frac{W_t}{Q_t} = \frac{\varepsilon_t - 1}{\varepsilon_t} (1-\nu) Z_t L_t^{-\nu}. \tag{A.14}$$

Combining this with the labor supply equation $(A.3)$, and substituting the hand-to-mouth consumption $(A.10)$, we obtain the equilibrium labor as the solution to,

$$\chi (L_t^*)^{\varphi} (1-\nu) Y_t^* = \frac{\varepsilon_t - 1}{\varepsilon_t} (1-\nu) Z_t (L_t^*)^{-\nu}.$$

In equilibrium, output is $Y_t^* = Z_t (L^*)^{1-\nu}$. Therefore, the flexible-price equilibrium conditions are :

$$\begin{aligned} \chi (L_t^*)^{1+\varphi} &= \frac{\varepsilon_t - 1}{\varepsilon_t}, \\ Y_t^* &= Z_t (L_t^*)^{1-\nu}. \end{aligned} \tag{A.15}$$

Potential output. Consider the flexible-price allocation in which the firms' markups are at their steady-state level, $\varepsilon_t = \varepsilon^*$, that is:

$$\begin{aligned} \chi \left(L^*\right)^{1+\varphi} &= \frac{\varepsilon^* - 1}{\varepsilon^*}, \\ Y_t^* &= Z_t \left(L^*\right)^{1-\nu}. \end{aligned} \tag{A.16}$$

We refer to L^* as the potential labor supply and $Y^* = Z_t \left(L^*\right)^{1-\nu}$ as the potential output. In the main text, we assume the central bank attempts to keep labor and output demand at these levels. In particular, the central bank attempts to stabilize the output fluctuations driven by shocks to ε_t (or markups), because these shocks are distortionary. This enables us to accommodate cost-push shocks that create a trade-off for the central bank for stabilizing inflation and output.[28]

Sticky prices and demand-driven output. We next describe the equilibrium with nominal rigidities. We start with the special case with full price stickiness and then extend the analysis to partially flexible prices. With fully sticky prices, intermediate good firms have a preset nominal price that remains fixed over time, $Q_t(\nu) = Q^*$. This implies the nominal price for the final good is also fixed and given by $Q_t = Q^*$. Then, each intermediate good firm ν at time t solves the following version of problem $(A.12)$,

$$\begin{aligned} \Pi &= \max_L Q^* Y - W_t L - T_t \\ &\quad \text{where } Y = AL^{1-\nu} \leq Y_t. \end{aligned} \tag{A.17}$$

Since the firm operates with a markup, for small aggregate demand shocks (which we assume) it optimally chooses to meet the demand for its goods, $Y = ZL^{1-\nu} = Y_t$. Therefore, each firm's output is determined by aggregate demand, which is driven by households' spending C_t^H according to $(A.11)$.

Partially flexible prices and the New Keynesian Phillips curve. We next allow for partially flexible prices. With partially flexible prices, each firm still optimally serves the demand and output is still determined by aggregate demand. However, inflation is also endogenous and reacts to output gaps as well as other (cost-push) shocks. We derive a Phillips curve that describes inflation.

We consider the setup in the textbook New Keynesian model in which in each period a randomly selected fraction, $1-\theta$, of firms reset their nominal prices. The firms that do not adjust their price in period t, set their labor input to meet the demand for their goods.

Consider the firms that adjust their price in period t. Let Q_t^{adj} denote the optimal price set by these

[28] The central bank does not attempt to stabilize the distortions generates by the *average* markup, because this would induce an average inflationary bias. In practice, these average distortions should ideally be corrected by other policy tools rather than monetary policy.

firms. We assume Q_t^{adj} solves the following version of problem $(A.12)$

$$\max_{Q_t^{adj}} \sum_{h=0}^{\infty} \theta^h E_t \left\{ M_{t,t+h} \left(Y_{t+h|t} Q_t^{adj} - W_{t+h} L_{t+h|t} - T_t \right) \right\}, \tag{A.18}$$

$$\text{where } Y_{t+h|t} = Z_{t+h} L_{t+h|t}^{1-\nu} = \left(\frac{Q_t^{adj}}{Q_{t+h}} \right)^{-\varepsilon_{t+h}} Y_{t+h}$$

$$\text{and } M_{t,t+h} = \beta^h \frac{1/P_{t+h}}{1/P_t} \frac{Q_t}{Q_{t+h}}.$$

The terms $L_{t+h|t}$ and $Y_{t+h|t}$ denote the input and the output of the firm (that resets its price in period t) in a future period $t+h$. The term $M_{t,t+h}$ is the stochastic discount factor (SDF) between periods t and $t+h$. Here, P_t denotes the end-of-period price of the market portfolio which we describe later in the appendix.[29]

The optimality condition for problem $(A.18)$ is

$$\sum_{h=0}^{\infty} \theta^h E_t \left\{ M_{t,t+h} Q_{t+h}^{\varepsilon_{t+h}} Y_{t+h} \left(Q_t^{adj} - \frac{\varepsilon_{t+h}}{\varepsilon_{t+h} - 1} \frac{W_{t+h}}{(1-\nu) Z_{t+h} L_{t+h|t}^{-\nu}} \right) \right\} = 0, \tag{A.19}$$

$$\text{where } L_{t+h|t} = \left(\frac{Q_t^{adj}}{Q_{t+h}} \right)^{\frac{-\varepsilon_{t+h}}{1-\nu}} \left(\frac{Y_{t+h}}{Z_{t+h}} \right)^{\frac{1}{1-\nu}}.$$

We next combine Eq. $(A.19)$ with the remaining equilibrium conditions to derive the New Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features real potential outcomes (with constant markups) and zero inflation, that is, $L_t = L^*, Y_t = Y_t^*, \frac{\varepsilon_t}{\varepsilon_t - 1} = \frac{\varepsilon^*}{\varepsilon^* - 1}, Q_t = Q^*$ for each t, where recall that L^* is given by $(A.16)$ and $Y_t^* = Z_t (L^*)^{1-\nu}$. Throughout, we use the notation $\tilde{x}_t = \log(X_t/X_t^*)$ to denote the log-linearized version of the corresponding variable X_t and we use $\tilde{\mu}_t = \frac{\varepsilon_t}{\varepsilon_t - 1} - \frac{\varepsilon^*}{\varepsilon^* - 1}$ to denote the deviation of the desired markup from its steady-state level level. We also let $W_t^{norm} = \frac{W_t}{Z_t Q_t}$ denote the normalized (productivity-adjusted) real wage.

We first log-linearize the labor-supply equilibrium condition $(A.3)$ and use $C_t^{HM} = (1-\nu) Y_t$ to obtain

$$\tilde{w}_t^{norm} = \varphi \tilde{l}_t + \tilde{y}_t. \tag{A.20}$$

Log-linearizing Eqs. $(A.4)$ and $(A.8)$, we also obtain

$$\tilde{y}_t = (1-\nu) \tilde{l}_t. \tag{A.21}$$

[29]Consistent with the financial market side of our model, we assume the SDF is determined by asset-holding households' wealth rather than their consumption. In equilibrium, asset-holding households' wealth is equal to the value of the market portfolio. The exact specification does not affect our analysis because we log-linearize the equation and the interaction of the SDF and prices, $M_{t,t+h} Q_t^{adj}$, generates second-order terms that drops out of the log linearization.

Finally, we log-linearize Eq. $(A.19)$ (and linearize for $\tilde{\mu}_t$) to obtain

$$\sum_{h=0}^{\infty} (\theta\beta)^h E_t \left\{ \tilde{q}_t^{adj} - \left(\tilde{w}_{t+h}^{norm} + \nu \tilde{l}_{t+h|t} + \tilde{q}_{t+h} \right) - \tilde{\mu}_{t+h} \right\} = 0, \tag{A.22}$$

$$\text{where } \tilde{l}_{t|t+h} = \frac{-\varepsilon^* \left(\tilde{q}_t^{adj} - \tilde{q}_{t+h} \right)}{1-\nu} + \tilde{l}_{t+h}.$$

The second line uses $\tilde{y}_t = (1-\nu)\,\tilde{l}_t$.

We next combine Eqs. $(A.20 - A.22)$ and rearrange terms to obtain a closed-form solution for the price set by adjusting firms

$$\begin{aligned} \tilde{q}_t^{adj} &= (1-\theta\beta) \sum_{h=0}^{\infty} (\theta\beta)^h E_t \left[\Theta \tilde{y}_{t+h} + \tilde{q}_{t+h} + \tilde{\mu}_{t+h} \right], \\ \text{where } \Theta &= \frac{1+\varphi}{1-\nu+\nu\varepsilon} \end{aligned}$$

Since the expression is recursive, we can also write it as a difference equation

$$\tilde{q}_t^{adj} = (1-\theta\beta)\left(\Theta \tilde{y}_t + \tilde{q}_t + \tilde{\mu}_t \right) + \theta\beta E_t \left[\tilde{q}_{t+1}^{adj} \right]. \tag{A.23}$$

Here, we have used the law of iterated expectations, $E_t[\cdot] = E_t[E_{t+1}[\cdot]]$.

Next, we consider the aggregate price index $(A.6)$

$$\begin{aligned} Q_t &= \left((1-\theta) \left(Q_t^{adj} \right)^{1-\varepsilon} + \int_{S_t} \left(Q_{t-1}(\nu) \right)^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)} \\ &= \left((1-\theta) \left(Q_t^{adj} \right)^{1-\varepsilon} + \theta Q_{t-1}^{1-\varepsilon} \right)^{1/(1-\varepsilon)}, \end{aligned}$$

where we have used the observation that a fraction θ of prices are the same as in the last period. The term, S_t, denotes the set of sticky firms in period t, and the second line follows from the assumption that adjusting terms are randomly selected. Log-linearizing the equation, we further obtain $\tilde{q}_t = (1-\theta)\,\tilde{q}_t^{adj} + \theta\tilde{q}_{t-1}$. After substituting inflation, $\pi_t = \tilde{q}_t - \tilde{q}_{t-1}$, this implies

$$\pi_t = (1-\theta)\left(\tilde{q}_t^{adj} - \tilde{q}_{t-1} \right). \tag{A.24}$$

Hence, inflation is proportional to the price change by adjusting firms.

Finally, note that Eq. $(A.23)$ can be written in terms of the price change of adjusting firms as

$$\tilde{q}_t^{adj} - \tilde{q}_{t-1} = (1-\theta\beta)\left(\Theta \tilde{y}_t + \tilde{\mu}_t \right) + \tilde{q}_t - \tilde{q}_{t-1} + \theta\beta E_t \left[\tilde{q}_{t+1}^{adj} - \tilde{q}_t \right].$$

Substituting $\pi_t = \tilde{q}_t - \tilde{q}_{t-1}$ and combining with Eq. $(A.24)$, we obtain the New Keynesian Phillips curve

(35) that we use in the main text

$$\begin{aligned}
\pi_t &= \kappa\tilde{y}_t + \beta E_t\left[\pi_{t+1}\right] + u_t, \\
\text{where } \kappa &= \frac{1-\theta}{\theta}\left(1-\theta\beta\right)\frac{1+\varphi}{1-\nu+\nu\varepsilon} \\
\text{and } u_t &= \frac{1-\theta}{\theta}\left(1-\theta\beta\right)\tilde{\mu}_t, \text{ where } \tilde{\mu}_t = \frac{\varepsilon_t}{\varepsilon_t - 1} - \frac{\varepsilon^*}{\varepsilon^* - 1}.
\end{aligned} \tag{A.25}$$

A.2.2. Demand side and financial markets

We next describe households' consumption-savings and portfolio allocation decisions. In equilibrium, together with monetary policy, these decisions determine aggregate demand, asset prices, and output.

Financial assets. There are two assets. There is a market portfolio, which is a claim on firms' profits νY_t (the firms' share of output). We let P_t denote the ex-dividend price of the market portfolio (which we also refer to as "the aggregate asset price" or "aggregate asset prices"). The gross return of the market portfolio is

$$R_{t+1} = \frac{\nu Y_{t+1} + P_{t+1}}{P_t}. \tag{A.26}$$

There is also a risk-free asset in zero net supply. Its gross return R_t^f is set by the central bank, as we describe in the main text.

Households' consumption-savings decisions. Households have standard preferences:

$$E_t\left[\sum_{h=0}^{\infty}\beta^{t+h}\log C_{t+h}^H\right], \tag{A.27}$$

along with the budget constraint

$$\begin{aligned}
W_{t+1} + C_{t+1}^H &= W_t\left(\left(1-\omega_t\right)R_t^f + \omega_t R_{t+1}\right) \\
&= D_{t+1} + K_{t+1}, \\
\text{where } D_{t+1} &= W_t\left[\left(1-\omega_t\right)\left(R_t^f - 1\right) + \omega_t\frac{\nu Y_{t+1}}{P_t}\right] \\
\text{and } K_{t+1} &= W_t\left[1-\omega_t + \omega_t\frac{P_{t+1}}{P_t}\right].
\end{aligned} \tag{A.28}$$

W_t denotes the end-of-period wealth and ω_t denotes the market portfolio weight in period t. The term $W_t\left(\left(1-\omega_t\right)R_t^f + \omega_t R_{t+1}\right)$ is the beginning-of-period wealth in period $t+1$. The second line breaks this term into a component that captures the interest and dividend income (D_{t+1}) and a residual component that captures the capital (K_{t+1}).

Households take their portfolio allocation as given (delegated to the portfolio managers) and make a consumption-savings decision. We assume their consumption follows the rule

$$C_t^H = \left(1-\beta\right)\left(D_t + K_t\exp\left(\delta_t\right)\right). \tag{A.29}$$

When $\delta_t = 0$, this is the optimal rule given the log preferences in $(A.27)$. When $\delta_t > 0$ (resp. $\delta_t < 0$), households spend more (resp. less) than the optimal rule. We refer to δ_t as an *aggregate demand shock* and view it as a modeling device to capture various factors that affect aggregate spending in practice, e.g., a consumer sentiment shock, a fiscal policy shock, or a discount rate shock. Having the demand shock multiply K_t rather than $D_t + K_t$ does not play an important role beyond simplifying the expressions.[30]

The portfolio managers (the market) and the portfolio allocation. Households delegate their portfolio choice to managers. The portfolio managers are infinitesimal and they do not consume themselves; they simply make a portfolio choice decision for households. For simplicity, each household invests with a continuum of managers randomly sampled from all managers. This assumption ensures that there is no portfolio heterogeneity across households and thus no individual wealth dynamics.

In each period, a fraction η of portfolio managers are "noise traders" and their portfolio weight is $\omega_t^N = 1+\frac{1}{\eta}\mu_t$. That is, they deviate from the optimal portfolio benchmark by an amount $\frac{1}{\eta}\mu_t$. We refer to μ_t as the aggregate noise—the total amount of flow that needs to be absorbed by other investors. Among the remaining managers, a mass $1-\eta-\alpha$ represent "inelastic funds" and their portfolio weight is $\omega_t^I = 1$. Finally, a mass α of managers are "arbitrageurs" (or elastic funds) who choose their portfolio weights to maximize expected log assets-under-management, after observing the risk-free rate $r_t^f = \log R_t^f$ and the current noise μ_t:[31]

$$\max_{\omega_t^A} E_t \left[\log\left(\alpha W_t \left(R_t^f + \omega_t \left(R_{t+1} - R_t^f\right)\right)\right)\right].$$

As we describe in the main text, the optimality condition is approximately given by (12)

$$\omega_t^A \sigma_{t,r_{t+1}} = \frac{E_t\left[r_{t+1}\right] + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} - r_t^f}{\sigma_{t,r_{t+1}}}.$$

Financial market clearing. Financial markets are in equilibrium when the households in the aggregate hold the market portfolio, both before and after the portfolio allocation:

$$W_t = P_t \quad \text{and} \quad \omega_t = \alpha\omega_t^A + \eta\left(1 + \frac{\mu_t}{\eta}\right) + (1-\eta-\alpha) = 1. \tag{A.30}$$

Output-asset price relation. We next derive the equilibrium condition (7) that we use in the main text. Combining Eqs. $(A.28)$ and $(A.30)$, we obtain $D_t = \nu Y_t, K_t = P_t$. In equilibrium, dividends are equal to the firms' share of output. Capital is equal to the (ex-dividend) value of the market portfolio. Substituting these observations into the consumption rule in $(A.29)$, we obtain

$$C_t^H = (1-\beta)\left(\nu Y_t + P_t \exp\left(\delta_t\right)\right).$$

[30] We could alternatively capture demand shocks as shocks to households' discount factor β in a fully optimizing framework. We prefer our approach where we view demand shocks as small consumption "mistakes" because doing so simplifies the analysis and gives us greater flexibility in specifying the process for δ_t.

[31] We assume arbitrageurs maximize log-wealth in line with the households' preferences in $(A.27)$. In the special case where households follow the optimal rule ($\sigma_\delta^2 = 0$), this problem results in portfolio allocations that maximize the households' utility. We formulate the portfolio problem in terms of wealth, rather than consumption, because we allow consumption to deviate from the optimal rule. In our setup, wealth is a more accurate representation of welfare, as it captures the *ideal* consumption a household could choose if she followed the optimal rule.

Substituting Eq. $(A.11)$ $(C_t^H = \nu Y_t)$ into this expression yields Eq. (7)

$$\begin{aligned} Y_t &= (1-\beta)\frac{1}{\nu\beta}P_t \exp(\delta_t) \\ \implies & y_t = m + p_t + \delta_t, \quad \text{where } m \equiv \log\left(\frac{1-\beta}{\nu\beta}\right). \end{aligned} \tag{A.31}$$

We refer to this as *the output-asset price relation.* In equilibrium, output depends on aggregate wealth, P_t, the MPC out of wealth, $1-\beta$, the demand shock, δ_t, and the Keynesian multiplier, $1/(\nu\beta)$. The second line describes the relation in logs and obtains the derived parameter m.

Financial market equilibrium condition. We next derive the equilibrium condition (13) that we use in the main text. Eq. $(A.30)$ implies $\omega_t^A = 1 - \frac{\mu_t}{\alpha}$. Substituting this into (12), we obtain (13)

$$E_t[r_{t+1}] = r_t^f + \frac{(\sigma_{t,r_{t+1}})^2}{2} - \mu_t \frac{(\sigma_{t,r_{t+1}})^2}{\alpha}.$$

In equilibrium, the expected return on the market portfolio depends on the risk premium, return variance, and noise. The impact of noise is increasing in the return variance and decreasing in the mass of arbitrageurs.

Campbell-Shiller approximation to the equilibrium return. We next derive the Campbell-Shiller approximation in (9). First note that Eq. $(A.26)$ implies

$$\begin{aligned} r_{t+1} &= \log\left(\frac{\nu Y_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t} + \frac{P_{t+1}}{P_t}\right) \\ &= \log\left(\frac{\nu Y_{t+1}}{P_{t+1}} + 1\right) + \log\left(\frac{P_{t+1}}{P_t}\right) \\ &= \log(1 + X_{t+1}) + p_{t+1} - p_t. \end{aligned} \tag{A.32}$$

Here, we have defined the dividend price ratio, $X_t = \nu Y_t / P_t$.

Setting the demand shifter to zero ($\delta_t = 0$) and output equal to its potential $Y = Y^*$, Eq. $(A.31)$ implies $Y^* = (1-\beta)\frac{1}{\alpha\beta}P^*$. This implies $X^* = \nu Y_t^* / P_t^* = \frac{1-\beta}{\beta}$.

Finally, log-linearize $(A.32)$ around $X_{t+1} = X^*$. Let $x_{t+1} = \log(X_{t+1}/X^*)$ denote the log deviation of the dividend price ratio from its steady-state level. Consider the term, $\log(1 + X_{t+1}) = \log(1 + X^* \exp(x_{t+1}))$. Using a Taylor approximation around $x_{t+1} = 0$, we obtain

$$\begin{aligned} \log(1 + X_{t+1}) &\approx \log(1 + X^*) + \frac{X^*}{1 + X^*}x_{t+1} \\ &\approx \log\left(\frac{1}{\beta}\right) + (1-\beta)\left(\log\left(\frac{\nu Y_{t+1}}{P_{t+1}}\right) - \log\left(\frac{1-\beta}{\beta}\right)\right). \end{aligned}$$

Substituting this into $(A.32)$ and collecting the constant terms, we obtain Eq. (9)

$$\begin{aligned} r_{t+1} &= \rho - (1-\beta)m + (1-\beta)y_{t+1} + \beta p_{t+1} - p_t \\ &= \rho + p_{t+1} + (1-\beta)\delta_{t+1} - p_t, \end{aligned}$$

where the second line substitutes the output asset price relation (7) to simplify the expression.

Present discounted value relation. Substituting Eq. (9) into the financial market equilibrium condition (13), we also obtain the present discounted value relation (14) that describes the equilibrium asset price

$$p_t = \rho + E_t[p_{t+1}] + (1-\beta) E_t[\delta_{t+1}] - \left(r_t^f + \frac{1}{2} \left(\sigma_{t,r_{t+1}}\right)^2 \right) + \mu_t \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{\alpha}.$$

A.3. Omitted proofs

This section contains the proofs omitted from the main text.

Proof of Proposition 1. To characterize the equilibrium, first observe that the central bank's problem is

$$G_t = \min_{r_t^f} \underline{E}_t \left[(y_t - y_t^*)^2 \right] + \beta \underline{E}_t [G_{t+1}].$$

The expected *future* gaps $\underline{E}_t[G_{t+1}]$ do not depend on the current policy rate r_t^f, because the model is forward looking without any endogenous state variables. Thus, the optimality condition is given by $\underline{E}_t \left[\frac{dy_t}{dr_t^f} \tilde{y}_t \right] = 0$. We conjecture (and verify) that in equilibrium $\frac{dy_t}{dr_t^f} = -1$. Consequently, the optimality condition implies

$$\underline{E}_t[\tilde{y}_t] = 0 \Longrightarrow \underline{E}_t[y_t] = \underline{E}_t[y_t^*] = y_t^*. \tag{A.33}$$

Since the central bank sets policy *before* observing the noise, it cannot ensure output is equal to its potential in every state, $y_t = y_t^*$. Instead, it does so in expectation. Combining this with Eq. (7), we also obtain

$$\underline{E}_t[m + p_t + \delta_t] = y_t^* \Longrightarrow \underline{E}_t[p_t] = p_t^* \equiv y_t^* - m - \delta_t. \tag{A.34}$$

That is, the central bank sets the asset price equal to "pstar" in expectation.

We next conjecture (and verify) that there is an equilibrium in which the return volatility σ^2 is constant and the aggregate asset price is given by (17). Substituting this into the output asset price relation (7), we obtain (18). Note that Eqs. $(17-18)$ satisfy the optimality conditions $(A.33 - A.34)$ since $\underline{E}_t[\varepsilon_{\mu,t}] = 0$. Substituting (17) into (9), we also obtain

$$\begin{aligned} r_{t+1} &= \rho + p_{t+1} + (1-\beta)\delta_{t+1} - p_t \\ &= \rho + p_{t+1}^* + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\delta_{t+1} - \left(p_t^* + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \right) \\ &= \rho + y_{t+1}^* - m - \delta_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\delta_{t+1} - \left(y_t^* - m - \delta_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \right) \\ &= \rho + \delta_t + \varepsilon_{z,t+1} - \beta\delta_{t+1} + \frac{\sigma^2}{\alpha}(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}). \end{aligned}$$

The third line substitutes for p_{t+1}^* and p_t^*, the fourth line uses (6) and simplifies the expressions. This proves (20). Combining this with (13), we also characterize the interest rate as

$$\begin{aligned} r_t^f &= E_t[r_{t+1}] - \frac{1}{2}\sigma^2 + \frac{\sigma^2}{\alpha}\mu_t \\ &= \rho + \delta_t - \beta\varphi_\delta\delta_t + \frac{\sigma^2}{\alpha}E_t[\mu_t - \varepsilon_{\mu,t}] \\ &= \rho + (1-\beta\varphi_\delta)\delta_t + \varphi_z z_t + \frac{\sigma^2}{\alpha}\varphi_\mu\mu_{t-1}. \end{aligned}$$

The second line substitutes the AR(1) process for δ_t and the last line substitutes the AR(1) process for μ_t. This proves (19).

We next characterize the return volatility corresponding to this equilibrium. Using (20), along with the observation that all shocks are conditionally independent, we obtain

$$\sigma^2 = var_t\left(r_{t+1}\right) = \sigma^2_{macro} + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma^2_{\mu}, \quad \text{where } \sigma^2_{macro} = \sigma^2_z + \beta^2\sigma^2_{\delta}.$$

In particular, the conditional volatility is a root of a quadratic, $P\left(\sigma^2\right) = 0$, given by

$$P\left(x\right) = \frac{\sigma^2_{\mu}}{\alpha^2}x^2 - x + \sigma^2_{macro}. \tag{A.35}$$

As long as the parameters satisfy $\alpha^2 > 4\sigma^2_{\mu}\sigma^2_{macro}$, which we assume, this polynomial has two positive roots. The larger root is unstable in the sense that small changes in volatility induce further changes in volatility that move the equilibrium away from this point. The smaller root corresponds to a stable equilibrium. This verifies that the equilibrium volatility is the smaller solution to the fixed point equation in (21). To assist with the calibrations, we observe that the smaller root is associated with a negative derivative for the polynomial,

$$P'\left(x\right) = 2\frac{\sigma^2_{\mu}}{\alpha^2}x - 1\bigg|_{x=\sigma^2} \leq 0.$$

This shows that a candidate solution that satisfies $P\left(\sigma^2\right) = 0$ is stable as long as it also satisfies $2\sigma^2_{\mu}\sigma^2 \leq \alpha^2$. In contrast, the larger root has $2\sigma^2_{\mu}\sigma^2 \geq \alpha^2$.

It remains to verify our conjecture that $\frac{dy_t}{dr^f_t} = -1$. Along the equilibrium path, output satisfies $y_t = m + p_t + \delta_t$, where the asset price satisfies (14)

$$p_t = \rho + E_t\left[p_{t+1}\right] + \left(1-\beta\right)E_t\left[\delta_{t+1}\right] - \left(r^f_t + \frac{1}{2}\sigma^2\right) + \frac{\sigma^2}{\alpha}\mu_t.$$

This shows $\frac{dy_t}{dr^f_t} = -1$ and completes the characterization of equilibrium.

We next establish the comparative statics with respect to the noise variance σ^2_{μ}. Observe that $P\left(x\right)$ in $(A.35)$ corresponds to an upward-sloping parabola with two positive roots. Observe also that increasing σ^2_{μ} increases $P\left(x\right)$ for each x and therefore lifts the parabola upward. Therefore, increasing σ^2_{μ} increases the smaller root (while reducing the larger root). Since the equilibrium volatility σ^2 corresponds to the smaller root, increasing σ^2_{μ} increases σ^2.

Finally, we characterize the expected output-gap loss $G_t = \underline{E}\left[\sum_{h=0}^{\infty}\beta^h\tilde{y}^2_{t+h}\right]$ along the equilibrium path. Note that the output gaps are given $\tilde{y}_{t+h} = \varepsilon_{\mu,t+h}\frac{\sigma^2}{\alpha}$. This implies $G_t = \frac{\sigma^2_{\mu}\left(\sigma^2\right)^2}{1-\beta}$. In particular, increasing σ^2_{μ} also increases G_t both directly by increasing noise and also indirectly by increasing the impact of noise. This completes the proof of the proposition. □

Proof of Proposition 2. To characterize the equilibrium, observe that the central bank's modified problem can be written as

$$G^{FCI}_t\left(\overline{p}_t\right) = \min_{r^f_t, \overline{p}_{t+1}} \underline{E}_t\left[\left(y_t - y^*_t\right)^2 + \psi\left(p_t - \overline{p}_t\right)^2\right] + \beta\underline{E}_t\left[G^{FCI}_{t+1}\left(\overline{p}_{t+1}\right)\right].$$

The expected future gaps $\underline{E}_t\left[G_{t+1}\left(\overline{p}_{t+1}\right)\right]$ depend on the announced target $\overline{p}_{t+1}$ but not on the current policy rate r_t^f (because the model is forward looking). Thus, the optimality condition for r_t^f is given by

$$\underline{E}_t\left[\frac{dy_t}{dr_t^f}\left(y_t-y_t^*\right)+\psi\frac{dp_t}{dr_t^f}\left(p_t-\overline{p}_t\right)\right]=0.$$

We conjecture (and verify) that in equilibrium $\frac{dy_t}{dr_t^f}=\frac{dp_t}{dr_t^f}=-1$. Therefore, the optimality condition implies

$$\underline{E}_t\left[y_t-y_t^*\right]+\psi\underline{E}_t\left[p_t-\overline{p}_t\right]=0.$$

Substituting $y_t=m+p_t+\delta_t$ and $y_t^*=p_t^*+m+\delta_t$, we obtain

$$\underline{E}_t\left[p_t\right]-p_t^*+\psi\left(\underline{E}_t\left[p_t\right]-\overline{p}_t\right)=0.$$

After rearranging, we obtain the optimality condition

$$\underline{E}_t\left[p_t\right]=\frac{1}{1+\psi}p_t^*+\frac{\psi}{1+\psi}\overline{p}_t=p_t^*+\frac{\psi}{1+\psi}\left(\overline{p}_t-p_t^*\right). \tag{A.36}$$

Under FCI targeting, the central bank's expected asset price is a weighted average of its pre-announced target $\overline{p}_t$ and the current "pstar" p_t^*.

We next conjecture an equilibrium in which $\sigma^2_{t,r_{t+1}}\equiv\sigma^2$ is constant over time, the central bank announces the expected "pstar" as its target $\overline{p}_t=\underline{E}_{t-1}\left[p_t^*\right]$, and the aggregate asset price satisfies

$$p_t=\frac{\psi}{1+\psi}\overline{p}_t+\frac{1}{1+\psi}p_t^*+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.$$

Taking the expectation of this expression and using $\underline{E}_t\left[\varepsilon_{\mu,t}\right]=0$, we obtain $(A.36)$. Hence, the conjectured allocation satisfies the optimality condition. Note also that this expression implies

$$\begin{aligned}
p_t &= \underline{E}_{t-1}\left[p_t^*\right]+\frac{1}{1+\psi}\left(p_t^*-\underline{E}_{t-1}\left[p_t^*\right]\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\\
&= \underline{E}_{t-1}\left[p_t^*\right]+\frac{1}{1+\psi}\left(y_t^*-\delta_t-\underline{E}_{t-1}\left[y_t^*-\delta_t\right]\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\\
&= \underline{E}_{t-1}\left[p_t^*\right]+\frac{1}{1+\psi}\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.
\end{aligned}$$

The first line substitutes the optimal target for period $t-1$ using (23), $\overline{p}_t=\underline{E}_{t-1}\left[p_t^*\right]$, the second line substitutes for p_t^*, and the last line uses the definition of supply and demand surprises, $y_t^*=\underline{E}_{t-1}\left[y_t^*\right]+\varepsilon_{z,t}$ and $\delta_t=\underline{E}_{t-1}\left[\delta_t\right]+\varepsilon_{\delta,t}$. This proves Eq. (24).

Substituting (24) into (7), we further obtain

$$\begin{aligned}
y_t &= m+\delta_t+E_{t-1}\left[y_t^*-\delta_t-m\right]+\frac{1}{1+\psi}\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}x\\
&= \varepsilon_{\delta,t}+y_t^*-\varepsilon_{z,t}+\frac{1}{1+\psi}\left(\varepsilon_{z,t}-\varepsilon_{\delta,t}\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\\
&= y_t^*+\frac{\psi}{1+\psi}\left(\varepsilon_{\delta,t}-\varepsilon_{z,t}\right)+\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}
\end{aligned}$$

The last line substitutes the definition of demand shocks $\varepsilon_{\delta,t} = \delta_t - E_{t-1}[\delta_t]$. This proves (25).

We substitute the aggregate asset price into (9) to characterize the equilibrium return,

$$\begin{aligned}
r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\delta_{t+1} - p_t \\
&= \rho + \underline{E}_t\left[p^*_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} \\
&\quad - \left(\underline{E}_{t-1}\left[p^*_t\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + E_t\left[y^*_{t-1} + \varepsilon_{z,t} + \varepsilon_{z,t+1} - \delta_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} \\
&\quad - \left(E_{t-1}\left[y^*_{t-1} + \varepsilon_{z,t} - \delta_t\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + \left(\varphi_\delta \delta_{t-1} + \frac{\varepsilon_{\delta,t}}{1+\psi}\right) - \left(\varphi_\delta \delta_t + \frac{\varepsilon_{\delta,t+1}}{1+\psi}\right) + (1-\beta)\left(\varphi_\delta \delta_t + \varepsilon_{\delta,t+1}\right) \\
&\quad + \frac{\varepsilon_{z,t+1}}{1+\psi} + \left(\varepsilon_{z,t} - \frac{1}{1+\psi}\varepsilon_{z,t}\right) + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right).
\end{aligned}$$

The third equation substitutes p^*_{t+1} and p^*_t. We also replace $\underline{E}[\cdot]$ with $E[\cdot]$ since the realization of noise does not affect the terms inside the expectation. The last equation substitutes the AR(1) process for δ_t and collects similar terms together. This proves (27) where the expected return is given by

$$E_t\left[r_{t+1}\right] = \rho + \varphi_\delta \delta_{t-1} + \frac{\varepsilon_{\delta,t}}{1+\psi} - \beta\varphi_\delta \delta_t + \frac{\psi \varepsilon_{z,t}}{1+\psi} - \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}. \tag{A.37}$$

We next combine the expression for the expected return with (13) to calculate the interest rate,

$$r^f_t = \rho - \frac{1}{2}\sigma^2 + \varphi_\delta \delta_{t-1} + \frac{\varepsilon_{\delta,t}}{1+\psi} - \beta\varphi_\delta \delta_t + \frac{\psi \varepsilon_{z,t}}{1+\psi} + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1},$$

where we substituted the AR(1) process for μ_t from (10). This proves (26).

We next use (27) to calculate the conditional return volatility as

$$\begin{aligned}
\sigma^2 &= var_t\left(r_{t+1}\right) = \sigma^2_{macro}(\psi) + \left(\frac{\sigma^2}{\alpha}\right)^2 \sigma^2_\mu \\
\text{where } \sigma^2_{macro}(\psi) &= \sigma^2_z \left(\frac{1}{1+\psi}\right)^2 + \sigma^2_\delta \left(\frac{1}{1+\psi} - (1-\beta)\right)^2.
\end{aligned}$$

In particular, the conditional volatility is a root of a quadratic, $P\left(\sigma^2;\psi\right) = 0$, given by

$$P(x;\psi) = \frac{\sigma^2_\mu}{\alpha^2}x^2 - x + \sigma^2_{macro}(\psi). \tag{A.38}$$

Note that $\sigma^2_{macro}(\psi)$ is convex, minimized at some $\overline{\psi} > 0$, and satisfies $\sigma^2_{macro}(0) = \sigma^2_z + \beta^2\sigma^2_\delta$ and $\lim_{\psi\to\infty} \sigma^2_{macro}(\psi) = (1-\beta)^2\sigma^2_\delta$. Since $\beta > 1-\beta$, this implies $\sigma^2_{macro}(0) \geq \sigma^2_{macro}(\psi)$ for each $\sigma^2_{macro}(\psi)$ Therefore, the assumed parametric condition $\alpha^2 > 4\sigma^2_\mu\left(\sigma^2_z + \beta^2\sigma^2_\delta\right)$ implies that $\alpha^2 > 4\sigma^2_\mu\sigma^2_{macro}(\psi)$ for each $\psi \geq 0$. Consequently, the polynomial in $(A.38)$ has two positive roots for each $\psi \geq 0$. The smaller root corresponds to the stable equilibrium. This proves (28).

Along the equilibrium path, output satisfies $y_t = m + p_t + \delta_t$ where the asset price satisfies (14)

$$p_t = \rho + E_t[p_{t+1}] + (1-\beta) E_t[\delta_{t+1}] - \left(r_t^f + \frac{1}{2}\sigma^2\right) + \frac{\sigma^2}{\alpha}\mu_t.$$

This verifies our conjectures $\frac{dy_t}{dr_t^f} = \frac{dp_t}{dr_t^f} = -1$.

It remains to verify our conjecture that it is optimal for the central bank to announce the target in (23). Fix period $t-1$ and consider the optimal choice of $\overline{p}_t$. This is chosen to minimize the objective function $\underline{E}_{t-1}\left[G_t^{FCI}(\overline{p}_t)\right]$ (since $\overline{p}_t$ does not affect the gaps in period $t-1$). To characterize this, note that Eq. $(A.36)$ applies for an arbitrary target $\overline{p}_t$,

$$\underline{E}_t[p_t] = p_t^* + \frac{\psi}{1+\psi}(\overline{p}_t - p_t^*) = \overline{p}_t + \frac{1}{1+\psi}(p_t^* - \overline{p}_t).$$

Combining this with $y_t = m + p_t + \delta_t$ and $y_t^* = p_t^* + m + \delta_t$, we also obtain the following expression for output that applies for an arbitrary target $\overline{p}_t$,

$$\underline{E}_t[y_t] = y_t^* + \frac{\psi}{1+\psi}(\overline{p}_t - p_t^*).$$

Substituting these expressions into the objective function, we obtain

$$\begin{aligned}
\underline{E}_{t-1}\left[G_t^{FCI}(\overline{p}_t)\right] &= \underline{E}_{t-1}\left[(y_t - y_t^*)^2 + \psi(p_t - \overline{p}_t)^2\right] + \beta\underline{E}_{t-1}\left[G_{t+1}^{FCI}(\overline{p}_{t+1})\right] \\
&= \left(\left(\frac{\psi}{1+\psi}\right)^2 + \psi\left(\frac{1}{1+\psi}\right)^2\right)\underline{E}_{t-1}\left[(\overline{p}_t - p_t^*)^2\right] + \beta\underline{E}_{t-1}\left[G_{t+1}^{FCI}(\overline{p}_{t+1})\right].
\end{aligned}$$

Taking the derivative with respect to $\overline{p}_t$ and observing that $G_{t+1}^{FCI}(\overline{p}_{t+1})$ does not depend on $\overline{p}_t$, we find $\overline{p}_t = \underline{E}_{t-1}[p_t^*]$. This verifies (23) and completes the characterization of equilibrium.

Next consider the comparative statics of return variance with respect to ψ. Recall that $x = \sigma^2$ corresponds to the smaller (positive) root of the polynomial $P(x;\psi)$ in $(A.38)$. This is an upward sloping parabola with two positive roots and the solution corresponds to the smaller root. Note that $P(0;\psi) = \sigma_{macro}^2(\psi)$. Note also that $\sigma_{macro}^2(\psi)$ is convex with a minimum that satisfies $\overline{\psi} > \frac{\beta}{1-\beta} > 0$. Therefore, increasing ψ over the range $\left[0,\overline{\psi}\right]$ shifts the parabola upward and reduces the smaller root. This proves that increasing ψ over the range $\left[0,\overline{\psi}\right]$ reduces both $\sigma_{macro}^2(\psi)$ and σ^2. Conversely, increasing ψ over the range $\left(\overline{\psi},\infty\right)$ increases both $\sigma_{macro}^2(\psi)$ and σ^2. □

Proof of Proposition 3. Note that Eq. (25) implies (31). After substituting this into (30) and calculating the variances, we further obtain (32). Differentiating with respect to ψ, we obtain $\frac{dG_{macro}^e(\psi)}{d\psi} = 0$ and

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} = \frac{dG_{noise}^e(\psi)}{d\psi}|_{\psi=0} = \frac{2}{1-\beta}\left(\sigma_\mu^2\sigma^2\frac{d\sigma^2}{d\psi}|_{\psi=0}\right) < 0.$$

The inequality follows since Proposition 2 shows that $\frac{d\sigma^2}{d\psi} < 0$ over the range $\psi \in \left[0,\overline{\psi}\right]$. This completes the proof. □

A.4. FCI targeting vs interest rate targeting

This section analyzes the extension we discuss in Section 4.5 where the central bank targets the future interest rate rather than the future FCI. We show that FCI targeting is strictly superior to interest rate targeting.

To capture interest rate targeting, consider the baseline model from Section 4 but suppose the central bank solves problem (34), which we replicate here:

$$G_t^{R\text{-target}} = \min_{r_t^f, \overline{r}_{t+1}^f} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^* \right)^2 + \psi \left(r_{t+h}^f - \overline{r}_{t+h}^f \right)^2 \right] \right],$$

where $\overline{r}_{t+h}^f$ denotes an interest rate target that the central bank announces in the previous period, $t+h-1$ (the initial target $\overline{r}_0^f$ is given). Similarly to FCI targeting, the central bank penalizes the deviations of *the interest rate* (rather than the FCI) from a pre-announced target. As before, the central bank's true objective function is unchanged and still given by (30).

The following result characterizes the equilibrium with interest rate targeting. We focus on the case in which there are no supply shocks, $\varepsilon_{z,t} = 0$, and demand shocks are transitory, $\varphi_\delta = 0$. This case makes the analysis tractable and directly comparable to FCI targeting, but the qualitative results also hold with supply shocks and more general processes for demand shocks.

Proposition 5 (Equilibrium with Interest Rate Targeting)**.** *Suppose the planner follows the interest rate targeting policy in* (34) *with* $\psi \geq 0$*, there are no supply shocks* $\varepsilon_{z,t} = 0$ *and demand shocks are transitory* $\varphi_\delta = 0$*, the parameters satisfy* $\alpha^2 > 4\sigma_\delta^2 \left(\frac{1}{1+\psi} - (1-\beta) \right)^2 \sigma_\mu^2 \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right)^2$*, and the initial target satisfies* $\overline{r}_0 = \underline{E}_{-1}\left[r_0^f \right]$*. There is a (stable) equilibrium in which the planner announces the expected interest rate for the next period as its target* $\overline{r}_{t+1}^f = \underline{E}_t \left[r_{t+1}^f \right]$*. The equilibrium, asset price, output, and interest rate are given by*

$$p_t = y_t^* - m - \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\frac{\psi}{1+\psi}\varphi_\mu \varepsilon_{\mu,t-1} + \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right) \varepsilon_{\mu,t} \right), \tag{A.39}$$

$$y_t = y_t^* + \frac{\psi \varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\frac{\psi}{1+\psi}\varphi_\mu \varepsilon_{\mu,t-1} + \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right) \varepsilon_{\mu,t} \right), \tag{A.40}$$

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + \frac{\varepsilon_{\delta,t}}{1+\psi} + \frac{\sigma^2}{\alpha} \left(\varphi_\mu^2 \mu_{t-1} + \frac{1}{1+\psi}\varphi_\mu \varepsilon_{\mu,t-1} \right). \tag{A.41}$$

The equilibrium return is

$$\begin{aligned} r_{t+1} &= E_t\left[r_{t+1}\right] - \varepsilon_{\delta,t+1} \left(\frac{1}{1+\psi} - (1-\beta) \right) + \frac{\sigma^2}{\alpha} \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right) \varepsilon_{\mu,t+1}, \\ \text{where } E_t\left[r_{t+1}\right] &= \rho + \frac{\varepsilon_{\delta,t}}{1+\psi} - \left[\varepsilon_{\mu,t} + \frac{\psi}{1+\psi}\varphi_\mu \varepsilon_{\mu,t-1} \right] \frac{\sigma^2}{\alpha}. \end{aligned} \tag{A.42}$$

The return variance $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smaller positive solution to the fixed point problem*

$$\begin{aligned} \sigma^2 &= \sigma_{macro}^2(\psi) + \frac{\left(\sigma^2\right)^2}{\alpha^2} \left(1 + \frac{\psi}{1+\psi}\varphi_\mu \right)^2 \sigma_\mu^2, \\ \text{where } \sigma_{macro}^2(\psi) &= \sigma_\delta^2 \left(\frac{1}{1+\psi} - (1-\beta) \right)^2 \end{aligned} \tag{A.43}$$

For a fixed ψ, the solution satisfies $\sigma^2 > \left(\sigma^{FCI}\right)^2$ where $\left(\sigma^{FCI}\right)^2$ is the equilibrium return variance with FCI targeting characterized in Proposition 2.

Comparing Eqs. $(A.40)$ and (25) shows that for a given return variance σ^2 interest rate targeting generates greater output gap volatility than FCI targeting. The last part of the result shows that interest rate targeting also induces greater return variance σ^2, which further increases output gap volatility. It follows that interest rate targeting is inferior to FCI targeting (it achieves higher expected squared output gaps). Intuitively, as we discuss in the main text, interest rate targeting stabilizes the incorrect financial variable and reduces the central bank's flexibility to respond to recent noise shocks, $\varphi_\mu \varepsilon_{\mu,t-1}$. This reduced flexibility implies that recent noise shocks affect asset prices and output gaps (captured by the term $\varphi_\mu \varepsilon_{\mu,t-1}$). Moreover, current noise shocks have a larger price impact, because financial markets anticipate that the central bank will not fully offset noise shocks (captured by the term $1 + \frac{\psi}{1+\psi}\varphi_\mu$).

Proof of Proposition 5. We conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future rate as its target $\overline{r}_t^f = \underline{E}_{t-1}\left[r_t^f\right]$, and the asset price and the interest rate satisfies

$$\begin{aligned} p_t &= y_t^* - m + D_p \varepsilon_{\delta,t} + \left(M_{p,1}\varphi_\mu \varepsilon_{\mu,t-1} + M_{p,0}\varepsilon_{\mu,t}\right)\frac{\sigma^2}{\alpha}, \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + D_r \varepsilon_{\delta,t} + \left(\varphi_\mu^2 \mu_{t-2} + M_{r,1}\varphi_\mu \varepsilon_{\mu,t-1}\right)\frac{\sigma^2}{\alpha} \end{aligned} \tag{A.44}$$

for undetermined coefficients $D_p, D_r, M_{p,1}, M_{p,0}, M_{r,1}$. Note that we allow the asset price and interest rate to react to the past period noise surprise as well as the current-period noise surprise $\varepsilon_{\mu,t}$. However, the interest rate cannot respond to the current noise surprise $\varepsilon_{\mu,t}$.We also conjecture that the interest rate will fully stabilize the current price impact of the noise shock from two periods before.

The optimality condition for r_t^f is given by

$$\underline{E}_t\left[\frac{dy_t}{dr_t^f}\left(y_t - y_t^*\right) + \psi\left(r_t^f - \overline{r}_t^f\right)\right] = 0.$$

As before, we conjecture (and verify later) that $\frac{dy_t}{dr_t^f} = -1$. Therefore, the optimality condition implies

$$\underline{E}_t\left[y_t\right] - y_t^* = \psi \underline{E}_t\left[r_t^f - \overline{r}_t^f\right].$$

Using the conjecture $\underline{E}_{t-1}\left[\overline{r}_t^f\right] = r_t^f$, observing that $\underline{E}_t\left[r_t^f\right] = r_t^f$, this further implies

$$\underline{E}_t\left[y_t\right] - y_t^* = \psi\left(r_t^f - \underline{E}_{t-1}\left[r_t^f\right]\right).$$

The pre-noise output is centered around y_t^* but it shifts with the information that shifts r_t^f between periods $t-1$ and t due to the policy pre-commitment. Substituting $y_t = m + p_t + \varepsilon_{\delta,t}$ (since demand shocks are i.i.d.), we further obtain

$$\underline{E}_t\left[p_t\right] = y_t^* - m - \varepsilon_{\delta,t} + \psi\left(r_t^f - \underline{E}_{t-1}\left[r_t^f\right]\right). \tag{A.45}$$

Combining this with $(A.44)$, we find

$$D_p\varepsilon_{\delta,t} + M_{p,1}\varphi_\mu\varepsilon_{\mu,t-1}\frac{\sigma^2}{\alpha} = -\varepsilon_{\delta,t} + \psi\left(D_r\varepsilon_{\delta,t} + M_{r,1}\varphi_\mu\varepsilon_{\mu,t-1}\frac{\sigma^2}{\alpha}\right).$$

The optimality condition holds for all shocks if the undetermined coefficients satisfy

$$\begin{aligned} D_p &= -1 + \psi D_r, \\ M_{p,1} &= \psi M_{r,1}. \end{aligned} \tag{A.46}$$

We next substitute the conjectured price into (9) to calculate the equilibrium return

$$\begin{aligned} r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\varepsilon_{\delta,t+1} - p_t \\ &= \rho + [D_p + 1 - \beta]\,\varepsilon_{\delta,t+1} - D_p\varepsilon_{\delta,t} \\ &\quad + \left[\left(M_{p,1}\varphi_\mu - M_{p,0}\right)\varepsilon_{\mu,t} + M_{p,0}\varepsilon_{\mu,t+1} - M_{p,1}\varphi_\mu\varepsilon_{\mu,t-1}\right]\frac{\sigma^2}{\alpha} \\ &= E_t\left[r_{t+1}\right] + \varepsilon_{\delta,t+1}\left[D_p + 1 - \beta\right] + \varepsilon_{\mu,t+1}M_{p,0}\frac{\sigma^2}{\alpha}, \end{aligned}$$

where the expected return is given by

$$E_t\left[r_{t+1}\right] = \rho - D_p\varepsilon_{\delta,t} + \left[\left(M_{p,1}\varphi_\mu - M_{p,0}\right)\varepsilon_{\mu,t} - M_{p,1}\varphi_\mu\varepsilon_{\mu,t-1}\right]\frac{\sigma^2}{\alpha}.$$

We combine this expression with (13) to calculate the interest rate,

$$\begin{aligned} r_t^f &= \rho - \frac{1}{2}\sigma^2 - D_p\varepsilon_{\delta,t} + \left[\mu_t + \left(M_{p,1}\varphi_\mu - M_{p,0}\right)\varepsilon_{\mu,t} - M_{p,1}\varphi_\mu\varepsilon_{\mu,t-1}\right]\frac{\sigma^2}{\alpha} \\ &= \rho - \frac{1}{2}\sigma^2 - D_p\varepsilon_{\delta,t} + \left[\varphi_\mu^2\mu_{t-1} + \left(1 + M_{p,1}\varphi_\mu - M_{p,0}\right)\varepsilon_{\mu,t} + \left(1 - M_{p,1}\right)\varphi_\mu\varepsilon_{\mu,t-1}\right]\frac{\sigma^2}{\alpha}. \end{aligned}$$

Here, the second line substitutes $\mu_t = \varphi_\mu^2\mu_{t-1} + \varphi_\mu\varepsilon_{\mu,t-1} + \varepsilon_{\mu,t}$ and collects terms. Comparing this with the conjectured interest rate in $(A.44)$, the undetermined coefficients must satisfy

$$\begin{aligned} D_r &= -D_p, \\ M_{r,1} &= 1 - M_{p,1}, \\ 1 + M_{p,1}\varphi_\mu - M_{p,0} &= 0. \end{aligned} \tag{A.47}$$

Combining Eqs. $(A.46)$ and $(A.47)$, we solve for the equilibrium coefficients

$$\begin{aligned} D_p &= -\frac{1}{1+\psi} \text{ and } D_r = \frac{1}{1+\psi}, \\ M_{p,1} &= \frac{\psi}{1+\psi} \text{ and } M_{r,1} = \frac{1}{1+\psi}, \\ M_{p,0} &= 1 + \varphi_\mu\frac{\psi}{1+\psi}. \end{aligned}$$

Substituting the solution into $(A.44)$ verifies that the equilibrium asset price and interest rate are given by $(A.39)$ and $(A.41)$. Combining the asset price expression with $y_t = m + p_t + \varepsilon_{\delta,t}$ verifies that output is given by $(A.40)$. Substituting the solution into the expression for the return verifies that the return is given by $(A.42)$.

Finally, observe that Eq. (27) implies that σ^2 solves the fixed point problem $(A.43)$. Under the assumed parametric condition, this problem has two positive roots. The smaller root corresponds to the stable equilibrium.

It remains to verify our conjectures that $\frac{dy_t}{dr_t^f} = -1$ and the central bank optimally announces the expected interest rate as its target $\underline{E}_{t-1}\left[\bar{r}_t^f\right] = r_t^f$. These follow from similar steps as in the proof of Proposition 2.

Finally, consider the comparative statics exercise. Note that σ^2 and $\left(\sigma^{FCI}\right)^2$ are the smaller root of the following two polynomials, respectively:

$$\begin{aligned} P\left(x\right) &= \left(1+\frac{\psi}{1+\psi}\varphi_{\mu}\right)^2 \frac{\sigma_{\mu}^2}{\alpha^2}x^2 - x + \sigma_{macro}^2\left(\psi\right), \\ P^{FCI}\left(x\right) &= \frac{\sigma_{\mu}^2}{\alpha^2}x^2 - x + \sigma_{macro}^2\left(\psi\right). \end{aligned}$$

Observe that $P\left(x\right) > P^{FCI}\left(x\right)$ for each $x > 0$. Since $P\left(\sigma^2\right) = 0$, this implies $P^{FCI}\left(\sigma^2\right) < 0$. This in turn implies $\left(\sigma^{FCI}\right)^2 < \sigma^2$ because $\left(\sigma^{FCI}\right)^2$ is the smaller positive root of $P^{FCI}\left(x\right)$. $\square$

A.5. FCI targeting with interest rate adjustment costs

This section considers a version of our model in which the central bank can react to all current shocks, including the noise shock $\varepsilon_{\mu,t}$, but it faces an additional interest rate adjustment cost. A special case of this model corresponds to a scenario in which the central bank does not react (by choice) to any current shock $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$ as opposed to only the noise shock. We show that FCI targeting still improves the central bank's objective and stabilizes macroeconomic outcomes. This shows that our results are not driven by our specific modeling choice for reaction lags.

Model with interest rate adjustment costs. We consider the baseline model in Section 3.1 with two changes. First, the central bank can react to all shocks. Second, the central bank's objective function is now given by

$$G_t = \min_{r_t^f} \tilde{y}_t^2 + \frac{1}{\theta}\left(r_t^f - E_{t-h}\left[r_t^f\right]\right)^2 + \beta E_t\left[G_{t+1}\right] \text{ with } h = 1. \tag{A.48}$$

This formulation says that the central bank likes to be predictable and penalizes deviations from the rate path that was previously anticipated by the markets. The parameter h captures the horizon for predictability. We focus on $h = 1$, so the central bank would like to be predictable for one period—the analysis can be extended to predictability at multiple horizons by adding more terms of this type. The parameter θ is an inverse measure of smoothing: it captures *the speed* at which the central bank is willing to react to shocks. For simplicity, we also exclude supply shocks $\varepsilon_{z,t} = 0$ so the potential output is constant $y_t = y^*$ (the analysis can be extended to supply shocks).

Note that when $\theta = 0$, the setup is a version of our baseline model in which the central bank does not respond to any current-period shocks $\varepsilon_{\mu,t}, \varepsilon_{\delta,t}, \varepsilon_{z,t}$. When $\theta > 0$, the central bank can respond to current shocks but by a limited amount. The setup also has similarities to the forward guidance policy that we analyzed in Section 4.5. Here, we make the forward guidance term an inherent policy objective, rather than an operational objective. As we will see, a desire to set predictable interest rates by itself can make FCI targeting useful.

Equilibrium with discretionary policy. First consider the equilibrium with discretion. Using the observation $\frac{d\tilde{y}}{dr_t^f} = -1$ (which holds in equilibrium) the optimality condition for $(A.48)$ implies the following interest rate rule

$$r_t^f = E_{t-1}\left[r_t^f\right] + \theta \tilde{y}_t.$$

The central bank adjusts the interest rate by a limited amount and allows some output gap to develop in response to current shocks. The following result characterizes the rest of the equilibrium (cf. Eqs. (15) and Proposition 1).

Proposition 6. *Suppose the planner sets policy to minimize* $(A.48)$. *Then, there is a (locally stable) equilibrium in which the asset price, output, and the interest rate are given by*

$$p_t = p_t^* + \frac{1}{1+\theta}\left(1 - \beta\varphi_\delta\right)\varepsilon_{\delta,t} + \frac{1}{1+\theta}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \quad \textit{where } p_t^* \equiv y^* - m - \delta_t, \tag{A.49}$$

$$y_t = y^* + \frac{1}{1+\theta}\left(1 - \beta\varphi_\delta\right)\varepsilon_{\delta,t} + \frac{1}{1+\theta}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \tag{A.50}$$

$$r_t^f = r_t^{f*} - \frac{1}{1+\theta}\left(1 - \beta\varphi_\delta\right)\varepsilon_{\delta,t} - \frac{1}{1+\theta}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \tag{A.51}$$

where $r_t^{f*} = \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\,\delta_t + \frac{\sigma^2}{\alpha}\mu_t$*. The return is given by*

$$r_{t+1} = E_t\left[r_{t+1}\right] + \left(\frac{1-\beta\varphi_\delta}{1+\theta} - \beta\right)\varepsilon_{\delta,t+1} + \frac{1}{1+\theta}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}, \tag{A.52}$$

and its variance $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smallest positive solution to the following fixed point problem:*

$$\sigma^2 = \left(\frac{1-\beta\varphi_\delta}{1+\theta} - \beta\right)^2\sigma_\delta^2 + \frac{1}{(1+\theta)^2}\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2. \tag{A.53}$$

The desire to smooth interest rates ($\theta < \infty$) dampens the central bank's reaction to demand and noise shocks. Therefore, demand and noise shocks both induce some output gaps. Note also that noise shocks affect asset returns and create a self-fulfilling volatility spiral as before.

Equilibrium with FCI targeting. Next suppose the central bank solves the following modified problem

$$G_t^{FCI} = \min_{r_t^f,\overline{p}_{t+1}} \tilde{y}_t^2 + \frac{1}{\theta}\left(r_t^f - E_{t-1}\left[r_t^f\right]\right)^2 + \frac{\psi}{\theta}\left(p_t - \overline{p}_t\right)^2 + \beta E_t\left[G_{t+1}\right]. \tag{A.54}$$

We have normalized the term on the FCI targeting term to simplify the expressions.

As before, the central bank's optimal target is given by $\overline{p}_t = E_{t-1}\left[p_t^*\right]$. Then, the optimality condition is given by

$$r_t^f = E_{t-1}\left[r_t^f\right] + \theta\tilde{y}_t + \psi\left(p_t - E_{t-1}\left[p_t^*\right]\right).$$

In this case, the interest rate adjustment to news depends on both the output gap and the price adjustment to news. The next result uses this condition to characterize thee equilibrium.

Proposition 7. *Suppose the planner sets policy to minimize (A.48). Then, there is a (locally stable) equilibrium in which the asset price, output, and the interest rate are given by*

$$p_t = p_t^* + \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t} + \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \quad \text{where } p_t^* \equiv y^* - m - \delta_t, \tag{A.55}$$

$$y_t = y^* + \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t} + \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \tag{A.56}$$

$$r_t^f = r_t^{f*} - \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t} - \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \tag{A.57}$$

where $r_t^{f*} = \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\,\delta_t + \frac{\sigma^2}{\alpha}\mu_t$*. The return is given by*

$$r_{t+1} = E_t\left[r_{t+1}\right] + \left(\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi} - \beta\right)\varepsilon_{\delta,t+1} + \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}, \tag{A.58}$$

and its variance $\sigma^2 = var_t\left(r_{t+1}\right)$ *is the smallest positive solution to the following fixed point problem:*

$$\sigma^2 = \sigma_{macro}^2(\psi) + \frac{1}{(1+\theta+\psi)^2}\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2 \text{ where } \sigma_{macro}^2(\psi) = \left(\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi} - \beta\right)^2\sigma_\delta^2. \tag{A.59}$$

Let $\overline{\psi} = \arg\min_{\psi\geq 0}\sigma_{macro}^2(\psi)$*. Over the range* $\psi \in [0,\overline{\psi})$*, increasing* ψ *strictly reduces* σ^2 *as well as* $\sigma_{macro}^2(\psi)$ *and* $\frac{1}{(1+\theta+\psi)^2}\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2$*.*

Compared to the previous case, we see that FCI targeting speeds up the response of the central bank to noise shocks; with respect to these shocks, it is as if the central bank's speed parameter is $\theta+\psi$ rather than θ. Thus, FCI targeting dampens the effect of noise shocks on both asset prices and output. On the other hand, as in our baseline model, FCI targeting dampens the effect of demand shocks on asset prices while increasing their impact on output ($\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}$ is increasing in ψ while $\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}-\beta$ is decreasing in ψ as long as $\psi\leq\overline{\psi}$). The intuition is the same as before: since demand shocks (typically) reduce asset prices via the current and anticipated future rate hikes, and since FCI targeting is concerned with stabilizing asset prices, a positive demand shock induces a smaller interest rate hike (and in fact, it might induce a temporary rate cut). So the policy allows the demand shock to have a larger impact on output to stabilize its effect on asset prices.

It follows that in this setting FCI targeting stabilizes asset prices for two separate reasons: It mitigates the asset price impact of demand shocks as in the baseline model, but it also mitigates the asset price impact of noise shocks—even though this requires more aggressive interest changes than what the central bank might desire. As before, these effects also trigger a virtuous cycle of volatility reduction. This endogenous decline in volatility further mitigates the impact of noise shocks on output.

How does FCI affect the central bank's objective function? In equilibrium, we have

$$\begin{aligned}\tilde{y}_t &= \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t}+\frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\\ r_t^f-E_{t-1}\left[r_t^f\right] &= \frac{(\theta+\psi)(1-\beta\varphi_\delta)-\psi}{1+\theta+\psi}\varepsilon_{\delta,t}+\frac{\theta+\psi}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.\end{aligned}$$

Therefore, we calculate the the central bank's expected per-period loss as

$$E\left[\tilde{y}_t^2+\frac{1}{\theta}\left(r_t^f-E_{t-1}\left[r_t^f\right]\right)^2\right]=\left\{\begin{array}{c}\left[\left(\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\right)^2+\left(\frac{(\theta+\psi)(1-\beta\varphi_\delta)-\psi}{1+\theta+\psi}\right)^2\right]\sigma_\delta^2\\ +\left[\left(\frac{1}{1+\theta+\psi}\right)^2+\left(\frac{\theta+\psi}{1+\theta+\psi}\right)^2\right]\left(\frac{\sigma^2}{\alpha}\right)^2\sigma_\mu^2\end{array}\right..$$

If σ^2 was constant, this term would be minimized at $\psi=0$: since a discretionary central bank already minimizes the objective term. However, we have $\frac{d\sigma^2}{d\psi}<0$ for $\psi\leq\overline{\psi}$. It follows that $\psi^*=\arg\min_{\psi\geq0}G_t^e(\psi)>0$, i.e., the expected gap loss minimizing policy features FCI targeting. Hence, FCI targeting improves the central bank's objective function even if we incorporate interest rate adjustment costs.

Note that in this case macroeconomic stability and the central bank's objective are related but not exactly the same. How does FCI targeting affect macroeconomic stability? The output gap is given by

$$\tilde{y}_t=\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t}+\frac{1}{1+\theta+\psi}\frac{\sigma^2(\psi)}{\alpha}\varepsilon_{\mu,t}.$$

As before, there are competing effects on the output gap. FCI targeting mitigates the output gap effect of noise shocks, but it also increases the output gap effect of demand shocks. Moreover, unlike in the baseline model, the output gap effects are not second order since $\frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}|_{\psi=0}=\frac{1-\beta\varphi_\delta}{1+\theta}>0$. However, as long as φ_δ is large, as in the data, we expect this term to be close to zero. Intuitively, when demand shocks are persistent, they require a smaller interest rate adjustment. Therefore, an interest-rate smoothing central bank still mostly stabilizes the effect of demand shocks. Introducing FCI targeting from this baseline increases the effect of demand shocks but the impact is likely to be quantitatively small. In contrast, the impact through noise shocks is likely to be large. We verify these observations in our counterfactual empirical analysis.

Proof of Propositions 1 and 2. We provide the proof for Proposition 2. Proposition 1 is the special case with $\psi = 0$. We conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future "pstar" as its target $\overline{p}_t = E_{t-1}[p_t^*]$, and the asset price, the output gap, and the interest rate satisfies

$$\begin{aligned}
p_t &= p_t^* + D_p (1-\beta\varphi_\delta) \varepsilon_{\delta,t} + M_p \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, \quad \text{where } p_t^* \equiv y^* - m - \delta_t, \\
y_t &= y^* + D_y (1-\beta\varphi_\delta) \varepsilon_{\delta,t} + M_y \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, \\
r_t^f &= r_t^{f*} - D_r (1-\beta\varphi_\delta) \varepsilon_{\delta,t} - M_r \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}. \\
\text{where } r_t^{f*} &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\delta_t + \frac{\sigma^2}{\alpha}\mu_t.
\end{aligned}$$

Here, (D, M)'s are undetermined coefficients that need to satisfy the optimality condition. We conjectured (and will verify) that $\frac{dp_t}{dr_t^f} = \frac{dy_t}{dr_t^f} = -1$. This observation along with the fact that $\left(p_t^*, y^*, r_t^{f*}\right)$ satisfy the equilibrium conditions absent interest adjustment costs imply that $D_p = D_y = D_r \equiv D$ and $M_p = M_y = M_r \equiv M$. We next solve for these common D and M coefficients to ensure the optimality condition.

The conjectured allocations imply

$$r_t^f - E_{t-1}\left[r_t^f\right] = (1-D)(1-\beta\varphi_\delta)\varepsilon_{\delta,t} + (1-M)\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.$$

We also have

$$\begin{aligned}
p_t - E_{t-1}[p_t^*] &= (-1 + D(1-\beta\varphi_\delta))\varepsilon_{\delta,t} + M\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
\tilde{y}_t &= D(1-\beta\varphi_\delta)\varepsilon_{\delta,t} + M\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.
\end{aligned}$$

Substituting into the optimality condition, we have

$$\begin{aligned}
& (1-D)(1-\beta\varphi_\delta)\varepsilon_{\delta,t} + (1-M)\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
= \; & \theta\left\{D(1-\beta\varphi_\delta)\varepsilon_{\delta,t} + M\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right\} + \psi\left\{\left(-\frac{1}{1-\beta\varphi_\delta} + D\right)(1-\beta\varphi_\delta)\varepsilon_{\delta,t} + M\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right\}.
\end{aligned}$$

We need this to hold for all shocks $\varepsilon_{\delta,t}, \varepsilon_{\mu,t}$, which implies

$$\begin{aligned}
1 - D &= (\theta+\psi) D - \frac{\psi}{1-\beta\varphi_\delta} \Longrightarrow D = \frac{1 + \frac{\psi}{1-\beta\varphi_\delta}}{1+\theta+\psi} \\
1 - M &= (\theta+\psi) M \Longrightarrow M = \frac{1}{1+\theta+\psi}.
\end{aligned}$$

The conjectured allocation implies Eqs. $(A.55 - A.57)$.

Next consider the equilibrium return and volatility. Substituting the conjectured price into (9), we calculate

$$
\begin{aligned}
r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\delta_{t+1} - p_t \\
&= E_t\left[r_{t+1}\right] - \beta\varepsilon_{\delta,t+1} + \frac{1+\psi-\beta\varphi_\delta}{1+\theta+\psi}\varepsilon_{\delta,t+1} + \frac{1}{1+\theta+\psi}\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}.
\end{aligned}
$$

This implies σ^2 solves the fixed point problem $(A.59)$. Under the assumed parametric condition, this problem has two positive roots for each $\psi \in [0, \overline{\psi})$. The smaller root corresponds to the stable equilibrium. The rest of the proof follows from similar steps as in the proof of Proposition 2. □

A.6. FCI targeting with inflation and output trade-off

This section analyzes the extension we discuss in Section 4.6.1 where prices are partially flexible and the central bank might face a trade-off between stabilizing inflation and output. In this case, cost-push shocks result in positive inflation and negative output gaps and create a new source of aggregate asset price volatility that further deters arbitrageurs. Moreover, noise shocks affect inflation gaps as well as output gaps. FCI targeting reduces the aggregate return volatility and enables arbitrageurs to absorb noise more effectively, reducing the impact of noise on inflation and output. Moreover, some degree of FCI targeting is still optimal and enables the central bank to achieve lower output gap and inflation losses. Intuitively, while cost-push shocks induce nonzero gaps on average, discretionary policy is already optimized to minimize the (current-period) losses induced by these shocks. Therefore, small deviations from this policy generate only second-order losses, while still inducing first-order gains via the noise-reduction mechanism.

Environment with inflation. Formally, consider the baseline model from Section 4 but suppose inflation is not necessarily zero and follows the New Keynesian Phillips Curve (NKPC) that we derived in Appendix A.2 (see $(A.25)$)

$$
\begin{aligned}
\pi_t &= \kappa\tilde{y}_t + \beta E_t\left[\pi_{t+1}\right] + u_t, \\
\text{where } u_t &= \varphi_u u_{t-1} + \varepsilon_{u,t} \text{ and } \sigma_u^2 \equiv var\left(\varepsilon_{u,t}\right).
\end{aligned}
$$

Here, $\pi_t \simeq \log\frac{Q_t}{Q_{t-1}}$ denotes inflation measured as the log change of the nominal price index Q_t. We assume the cost-push shocks u_t follow an AR(1) process that is independent from all other (supply, demand, and noise) shocks.

We adjust the financial market side of the model to allow for a *nominal* interest rate (which is what the Fed sets) in addition to the real interest rate. There is a *nominal* risk-free asset with nominal rate denoted by $\exp\left(i_t^f\right)$, in addition to the real risk-free asset with real rate $\exp\left(r_t^f\right)$, and the market portfolio with real return R_{t+1}. Both risk-free assets are in zero net supply. There are three sets of investors as in Section 3: noise traders, arbitrageurs, and inelastic funds. Noise traders and arbitrageurs are the same as before; in particular, they do not trade the nominal bonds. Likewise, inelastic funds are constrained to hold the average market portfolio weight $\omega_t^I = 1$. These assumptions ensure that we still have the financial market equilibrium condition in (13)

$$
E_t\left[r_{t+1}\right] + \frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2 = r_t^f + \left(\sigma_{t,r_{t+1}}\right)^2\left(1-\frac{\mu_t}{\alpha}\right).
$$

There is a second financial equilibrium condition that describes the relationship between the nominal and the real rates. To derive this condition, we assume for simplicity that *only* the inelastic funds can trade the nominal bond in exchange for the real bond. They maximize the expected wealth under management similar to arbitrageurs. In equilibrium, their optimization problem implies

$$E_t\left[M_{t+1}^I\left(\frac{\exp\left(i_t^f\right)}{Q_{t+1}/Q_t}-\exp\left(r_t^f\right)\right)\right]=0,\text{ where } M_{t+1}^I=\frac{1}{R_{t+1}}.$$

Assuming R_{t+1} and inflation $\frac{Q_{t+1}}{Q_t}$ are (approximately) log-normally distributed, we obtain

$$i_t^f=r_t^f+\left[E_t\left[\pi_{t+1}\right]-\frac{1}{2}\sigma_t^2\left(\pi_{t+1}\right)\right]-cov_t\left(\pi_{t+1},r_{t+1}\right). \tag{A.60}$$

This equation is like the Fisher equation except that it also accounts for inflation risk. The nominal interest rate is equal to the real rate plus the expected inflation (adjusted for a Jensen's term) and an inflation risk premium. The latter depends on the covariance between the inflation and the real return, $-cov_t\left(\pi_{t+1},r_{t+1}\right)$. Our assumption that nominal bonds are traded only by the inelastic funds ensures that *the current noise* μ_t does not affect the wedge between the nominal and the real rate (future noise can still affect the wedge via the covariance term). Thus, even though the Fed decides before observing μ_t, it can effectively still target a particular real interest rate r_t^f by setting the nominal rate i_t^f according to $(A.60)$. In the rest of this appendix, we will assume the Fed "sets" the real interest rate r_t^f and verify that the implied nominal rate i_t^f does not depend on μ_t.

Finally, we modify the central bank's (true) objective function to capture the costs of inflation:

$$G_t=\underline{E}_t\left[\sum_{h=0}^{\infty}\beta^h\left[\tilde{y}_{t+h}^2+\zeta\pi_{t+h}^2\right]\right]. \tag{A.61}$$

We normalize the inflation target to zero. The parameter ζ captures the cost of inflation gaps relative to output gaps. The rest of the environment is the same as in Sections 3 and 4. The baseline model is the special case with $\kappa=u_t=0$.

Equilibrium with discretionary policy. We first characterize the discretionary equilibrium. Suppose the central bank (effectively) sets r_t^f to maximize $(A.61)$ subject to the equilibrium conditions and taking its future actions as given. The solution is as in the textbook New Keynesian model (see Clarida et al. (1999)) with the difference that noise also affects the equilibrium outcomes. In particular, the central bank may no longer target a zero output gap on average. Its optimality condition is given by:

$$\underline{E}_t\left[\tilde{y}_t\right]=-\kappa\zeta\underline{E}_t\left[\pi_t\right]. \tag{A.62}$$

With a positive cost-push shock, the central bank targets a negative average output gap to stabilize inflation. The output gap is more negative when it has a greater impact on inflation (higher κ) and when the central bank puts a greater weight on inflation (high ζ). To solve for the equilibrium, we conjecture that the (pre-noise) output and inflation gaps are linear functions of the cost-push shock

$$\underline{E}_t\left[\pi_t\right]=\Pi_u u_t\text{ and }\underline{E}_t\left[\tilde{y}_t\right]=-Y_u u_t.$$

Combining this conjecture with the NKPC (and the AR(1) process for the cost-push shocks), we obtain the closed-form solutions

$$\Pi_u = \frac{1}{1+\kappa^2\zeta-\beta\varphi_u} \text{ and } Y_u = \frac{\kappa\zeta}{1+\kappa^2\zeta-\beta\varphi_u}. \tag{A.63}$$

The rest of the equilibrium is similar to Section 3.2 and is given by:

$$\begin{aligned}
p_t &= p_t^o + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \quad \text{ where } p_t^o \equiv y_t^* - m - \delta_t - Y_u u_t, && \text{(A.64)}\\
y_t &= y_t^* - Y_u u_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t},\\
\pi_t &= \Pi_u u_t + \kappa\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t},\\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\,\delta_t + (1-\varphi_u)\,Y_u u_t + \frac{\sigma^2}{\alpha}\varphi_\mu\mu_{t-1}.
\end{aligned}$$

p_t^o is the central bank's optimal asset price target, which is different from p_t^* due to cost-push shocks. Noise creates additional gaps from central bank's targets and its impact depends on σ^2, which is the smaller solution to:

$$\sigma^2 = \sigma_{macro}^2 + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2, \text{ where } \sigma_{macro}^2 = \sigma_z^2 + Y_u^2\sigma_u^2 + \beta^2\sigma_\delta^2.$$

In this case, the impact of noise is higher because cost-push shocks create a new source of asset price and return volatility.

Equilibrium with FCI targeting. We next consider the equilibrium with FCI targeting. In particular, suppose the central bank instead solves

$$G_t^{FCI} = \min_{r_t^f,\overline{p}_{t+1}} E_{t-1}\left[\sum_{h=0}^{\infty}\beta^h\left[\left(y_{t+h}-y_{t+h}^*\right)^2 + \zeta\pi_{t+h}^2 + \psi\left(1+\kappa^2\zeta\right)\left(p_{t+h}-\overline{p}_{t+h}\right)^2\right]\right]. \tag{A.65}$$

Here, the term $1+\kappa^2\zeta$ is a normalizing factor for the FCI targeting objective that helps to simplify the expression. The next result characterizes the equilibrium.

Proposition 8 (Equilibrium with Inflation and FCI Targeting)**.** *Consider the setup with inflation described above and suppose the planner follows the FCI targeting policy in* (A.65) *with* $\psi \geq 0$*. Let* Π_u, Y_u *denote the coefficients in* (A.63)*, and suppose the parameters satisfy* $\alpha^2 \geq 4\sigma_\mu^2\left(\sigma_z^2 + Y_u^2\sigma_u^2 + \beta^2\sigma_\delta^2\right)$ *(and* $\beta > 1-\beta$*) and the initial target satisfies* $\overline{p}_0 = E_{-1}\left[p_0^o\right]$*. Then, there is a (stable) equilibrium in which the planner announces as its target the expected optimal asset price for the next period*

$$\overline{p}_{t+1} = \underline{E}_t\left[p_{t+1}^o\right] \text{ where } p_{t+1}^o = y_{t+1}^* - m - \delta_{t+1} - Y_u u_{t+1}. \tag{A.66}$$

The equilibrium asset price, output, inflation, and real and nominal interest rates are

$$\begin{aligned}
p_t &= \underline{E}_{t-1}\left[p_t^o\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, &(A.67)\\
y_t &= y_t^* - Y_u u_t - \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, &(A.68)\\
\pi_t &= \Pi_u u_t - \frac{\psi}{1+\psi}\kappa\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\kappa\varepsilon_{\mu,t}, &(A.69)\\
r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta\varphi_\delta\right)\delta_t + Y_u\left(1 - \varphi_u\right)u_t + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - \varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1}. &(A.70)
\end{aligned}$$

The equilibrium return is

$$r_{t+1} = E_t\left[r_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - Y_u\varepsilon_{u,t+1}\right) - \left(\frac{1}{1+\psi} - (1-\beta)\right)\varepsilon_{\delta,t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}, \quad (A.71)$$

where $E_t\left[r_{t+1}\right]$ is given by $(A.37)$. The return variance $\sigma^2 = var_t\left(r_{t+1}\right)$ is the smaller positive solution to the following fixed point problem

$$\begin{aligned}
\sigma^2 &= \sigma^2_{macro}(\psi) + \frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2, &(A.72)\\
\text{where } \sigma^2_{macro}(\psi) &= \left(\sigma_z^2 + Y_u^2\sigma_u^2\right)\left(\frac{1}{1+\psi}\right)^2 + \sigma_\delta^2\left(\frac{1}{1+\psi} - (1-\beta)\right)^2.
\end{aligned}$$

Let $\overline{\psi} = \arg\min_{\psi\geq 0}\sigma^2_{macro}(\psi)$. Over the range $\psi \in [0, \overline{\psi})$, increasing ψ strictly reduces σ^2 as well as $\sigma^2_{macro}(\psi)$ and $\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2$. The equilibrium nominal interest rate i_t^f is given by $(A.78)$ and it does not depend on the current noise shock μ_t.

We relegate the proof of this result to the end of the theory appendix. The equilibrium with FCI targeting has a similar structure as before, with the difference that FCI targeting mitigates the policy response to cost-push shocks u_t as well as to supply and demand shocks (cf. Proposition 2). Consequently, cost-push shocks have a greater effect on inflation than with discretion. Moreover, since supply and demand shocks affect the output gaps, they also affect inflation unlike the case with discretion (cf. $(A.64)$). On the other hand, FCI targeting exerts a stabilizing influence on inflation, by mitigating the return volatility and the impact of noise on inflation as well as on output gaps.

Macro-stabilization effects of FCI targeting. We next explore the macro-stabilization effects of FCI targeting more systematically. As before, we evaluate the policy performance with the true loss function G_t in $(A.61)$. This function might depend on the current supply, demand, and cost-push shocks as well as the expected level of the cost-push shock, $\varepsilon_{z,t}, \varepsilon_{\delta,t}, \varepsilon_{u,t}, \varphi_u u_{t-1}$. To evaluate performance across a variety of shocks, we consider the unconditional expectation of this function given by

$$G^e(\psi) = E\left[G_t(\psi)\right] = E\left[\sum_{h=0}^{\infty}\beta^h\left[\tilde{y}^2_{t+h}(\psi) + \zeta\pi^2_{t+h}(\psi)\right]\right]. \quad (A.73)$$

Using Eqs. $(A.68)$ and $(A.69)$, output and inflation gaps are given by:

$$\begin{aligned}
\tilde{y}_t &= -Y_u\left(\varphi_u u_{t-1} + \varepsilon_{u,t}\right) + \frac{\psi}{1+\psi} Y_u \varepsilon_{u,t} - \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\
\pi_t &= \Pi_u\left(\varphi_u u_{t-1} + \varepsilon_{u,t}\right) + \frac{\psi}{1+\psi}\kappa Y_u \varepsilon_{u,t} - \frac{\psi}{1+\psi}\kappa\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\kappa\varepsilon_{\mu,t}.
\end{aligned}$$

We substitute $\tilde{y}_t$ and π_t into $(A.73)$ to calculate and decompose $G^e(\psi)$ into two components:

$$G^e(\psi) = G^e_{macro}(\psi) + G^e_{noise}(\psi). \tag{A.74}$$

$G^e_{noise}(\psi)$ is the expected loss driven by noise shocks, which is given by a similar expression as before (cf. (32))

$$(1-\beta)\, G^e_{noise}(\psi) = \sigma_\mu^2\left(\frac{\sigma^2}{\alpha}\right)^2\left(1+\zeta\kappa^2\right). \tag{A.75}$$

$G^e_{macro}(\psi)$ is the expected loss driven by macroeconomic shocks, which is given by

$$\begin{aligned}
(1-\beta)\, G^e_{macro}(\psi) &= \left(Y_u^2 + \zeta\Pi_u^2\right)\frac{\varphi_u^2\sigma_u^2}{1-\varphi_u^2} \\
&\quad + \left[\left(Y_u - \frac{\psi}{1+\psi}Y_u\right)^2 + \zeta\left(\Pi_u + \frac{\psi}{1+\psi}\kappa Y_u\right)^2\right]\sigma_u^2 \\
&\quad + \left[\left(\frac{\psi}{1+\psi}\right)^2\left(\sigma_z^2 + \sigma_\delta^2\right)\right]\left(1+\zeta\kappa^2\right).
\end{aligned} \tag{A.76}$$

The first line uses the observation that the *unconditional* distribution of $\varphi_u u_{t-1}$ is given by $N\left(0, \frac{\varphi_u^2\sigma_u^2}{1-\varphi_u^2}\right)$ to evaluate the losses driven by the conditionally expected level of the cost-push shock $\varphi_u u_{t-1}$. The second line evaluates the losses driven by the surprise component of cost-push shocks $\varepsilon_{u,t}$. The last line evaluates the losses driven by the supply and demand shocks. Our next result describes how FCI targeting affects $G^e(\psi)$ and its components.

Proposition 9 (Macrostabilization Effects of FCI Targeting with Inflation)**.** *Consider the equilibrium in Proposition 2. Then, a small degree of FCI targeting reduces the output-gap loss*

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} < 0, \text{ with } \frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = 0 \text{ and } \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} < 0.$$

Thus, $\psi^* = \arg\min_{\psi\geq 0} G_t(\psi) > 0$, i.e., the gap loss minimizing policy features FCI targeting.

Proof of Proposition 9. We differentiate Eq. $(A.76)$ with respect to ψ to obtain

$$\frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = \frac{2\sigma_u^2}{1-\beta}\left[-Y_u^2 + \zeta\kappa Y_u\Pi_u\right] = 0,$$

where we have used the observation that the coefficients satisfy $Y_u = \zeta\kappa\Pi_u$ in view of the central bank's optimality condition [see $(A.63)$ and $(A.62)$]. It follows that

$$\frac{dG^e(\psi)}{d\psi}|_{\psi=0} = \frac{dG^e_{noise}(\psi)}{d\psi}|_{\psi=0} = \frac{2\left(1+\zeta\kappa^2\right)}{1-\beta}\left(\sigma_\mu^2\sigma^2\frac{d\sigma^2}{d\psi}|_{\psi=0}\right) < 0.$$

The inequality follows since Proposition 8 shows that $\frac{d\sigma^2}{d\psi} < 0$ over the range $\psi \in [0, \overline{\psi}]$. $\square$

In this case, unlike in the baseline model without inflation, $G^e_{macro}(0)$ is not necessarily zero: even absent FCI targeting, macroeconomic (cost-push) shocks induce some gap losses. Nonetheless, it is still the case that small degrees of FCI targeting has a second-order effect on these losses, $\frac{dG^e_{macro}(\psi)}{d\psi}|_{\psi=0} = 0$. Intuitively, while cost-push shocks create nonzero gaps on average, discretionary policy is already optimized to minimize the (current-period) losses induced by the cost-push shocks, u_t, captured by the condition $Y_u = \zeta\kappa\Pi_u$ [see $(A.62)$ and $(A.63)$]. Thus, small deviations from this policy generate only second-order losses, while still inducing first-order gains by reducing the impact of noise on inflation and output.

Proof of Proposition 8. The central bank's modified problem is given by

$$G^{FCI}_t(\overline{p}_t) = \min_{r^f_t, \overline{p}_{t+1}} \underline{E}_t\left[(y_t - y^*_t)^2 + \zeta\pi^2_t + \psi\left(1+\kappa^2\zeta\right)(p_t - \overline{p}_t)^2\right] + \beta\underline{E}_t\left[G^{FCI}_{t+1}(\overline{p}_{t+1})\right].$$

The optimality condition for r^f_t is given by

$$\underline{E}_t\left[\frac{dy_t}{dr^f_t}(y_t - y^*_t) + \zeta\frac{d\pi_t}{dr^f_t}\pi_t + \psi\left(1+\kappa^2\zeta\right)\frac{dp_t}{dr^f_t}(p_t - \overline{p}_t)\right] = 0.$$

We conjecture (and verify) that in equilibrium $\frac{dy_t}{dr^f_t} = \frac{dp_t}{dr^f_t} = -1$ and $\frac{d\pi_t}{dr^f_t} = \kappa$. Therefore, the optimality condition implies

$$\underline{E}_t[y_t - y^*_t] + \kappa\zeta\underline{E}_t[\pi_t] + \psi\left(1+\kappa^2\zeta\right)\underline{E}_t[p_t - \overline{p}_t] = 0. \tag{A.77}$$

We next conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future asset price target $\overline{p}_t = E_{t-1}[p^o_t]$, the expected next-period inflation is the same as in the case with discretion $E_t[\pi_{t+1}] = \Pi_u\varphi_u u_t$ [see $(A.64)$], and the equilibrium asset price is given by

$$p_t = \underline{E}_{t-1}[p^o_t] + P_z\varepsilon_{z,t} - P_\delta\varepsilon_{\delta,t} - P_u Y_u\varepsilon_{u,t} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t},$$

for appropriate coefficients P_δ, P_z, P_u that describes the central bank's response new information. Substituting this conjecture into the output and asset price relation and using $p^o_t = y^*_t - m - \delta_t - Y_u u_t$ we obtain

$$y_t = y^*_t - Y_u u_t - (1-P_z)\varepsilon_{z,t} + (1-P_\delta)\varepsilon_{\delta,t} + (1-P_u)Y_u\varepsilon_{u,t} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t},$$

where we have used $y^*_t = \underline{E}_{t-1}[y^*_t] + z_t, \delta_t = \underline{E}_{t-1}[\delta_t] + \varepsilon_{\delta,t}$ and $u_t = \underline{E}_{t-1}[u_t] + \varepsilon_{u,t}$. Substituting this into the NKPC and using $E_t[\pi_{t+1}] = \Pi_u\varphi_u u_t$, we further obtain

$$\begin{aligned}
\pi_t &= -\kappa Y_u u_t + \beta\Pi_u\varphi_u u_t \\
&\quad -\kappa(1-P_z)\varepsilon_{z,t} + \kappa(1-P_\delta)\varepsilon_{\delta,t} + \kappa(1-P_u)Y_u\varepsilon_{u,t} + \kappa\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\
&= \Pi_u u_t - \kappa(1-P_z)\varepsilon_{z,t} + \kappa(1-P_\delta)\varepsilon_{\delta,t} + \kappa(1-P_u)Y_u\varepsilon_{u,t} + \kappa\frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.
\end{aligned}$$

Here, we have used $-\kappa Y_u + \beta\varphi_u\Pi_u = \Pi_u$ which holds from the definition of Y_u, Π_u (see $(A.63)$). Substituting these expressions into the optimality condition $(A.77)$, and using $Y_u = \kappa\zeta\Pi_u$, we obtain

$$\left[\begin{array}{c} \left(1+\kappa^2\zeta\right)\left(-\left(1-P_z\right)\varepsilon_{z,t}+\left(1-P_\delta\right)\varepsilon_{\delta,t}+\left(1-P_u\right)Y_u\varepsilon_{u,t}\right) \\ +\psi\left(1+\kappa^2\zeta\right)\left(P_z\varepsilon_{z,t}-P_\delta\varepsilon_{\delta,t}-P_uY_u\varepsilon_{u,t}\right) \end{array}\right]=0.$$

Solving for the undetermined coefficients, we obtain

$$P_z = P_\delta = P_u = \frac{1}{1+\psi}.$$

This proves Eqs. $(A.67 - A.69)$. We verify that the solution for inflation satisfies the conjecture for expected inflation since $E_t\left[\pi_{t+1}\right] = \Pi_u E_t\left[u_{t+1}\right] = \Pi_u\varphi_u u_t$.

We next substitute the aggregate asset price into (9) to characterize the equilibrium return,

$$\begin{aligned} r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\delta_{t+1} - p_t \\ &= \rho + \underline{E}_t\left[p^o_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1} - Y_u\varepsilon_{u,t+1}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} + (1-\beta)\,\delta_{t+1} \\ &\quad - \left(\underline{E}_{t-1}\left[p^o_t\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - Y_u\varepsilon_{u,t}\right) + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\ &= E_t\left[r_{t+1}\right] + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - Y_u\varepsilon_{u,t+1}\right) - \left(\frac{1}{1+\psi} - (1-\beta)\right)\varepsilon_{\delta,t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} \end{aligned}$$

where

$$E_t\left[r_{t+1}\right] = \rho + (1-\beta)\,\varphi_\delta\delta_t + Y_u\left(1-\varphi_u\right)u_t + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t} - \varepsilon_{u,t}\right) - \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.$$

This proves Eq. $(A.71)$. Combining this with (13) proves $(A.70)$.

Eq. (27) implies the conditional return volatility is the solution to the following quadratic

$$\begin{aligned} \sigma^2 &= var_t\left(r_{t+1}\right) = \sigma^2_{macro}(\psi) + \left(\frac{\sigma^2}{\alpha}\right)^2\sigma^2_\mu, \\ \text{where } \sigma^2_{macro}(\psi) &= \left(\sigma^2_z + Y^2_u\sigma^2_u\right)\left(\frac{1}{1+\psi}\right)^2 + \sigma^2_\delta\left(\frac{1}{1+\psi} - (1-\beta)\right)^2. \end{aligned}$$

Under the assumed parametric condition, this quadratic has two positive roots for each $\psi \geq 0$. The smaller root corresponds to the stable equilibrium. This proves $(A.72)$.

We verify the conjectures $\frac{dy_t}{dr^f_t} = \frac{dp_t}{dr^f_t} = -1$ and $\overline{p}_t = \underline{E}_{t-1}\left[p^o_t\right]$ as in the proof of Proposition 2. To verify the conjecture $\frac{d\pi_t}{dr^f_t} = \kappa$, observe that along the equilibrium path inflation satisfies the NKPC

$$\pi_t = \kappa\tilde{y}_t + \beta E_t\left[\pi_{t+1}\right] + u_t,$$

where the expected inflation $E_t\left[\pi_{t+1}\right] = \Pi_u\varphi_u u_t$ is exogenous to the current policy rate. Therefore, we have $\frac{d\pi_t}{dr^f_t} = \frac{dy_t}{dr^f_t} = \kappa$, verifying the remaining conjecture.

Finally, we characterize the equilibrium nominal interest rate i^f_t. Combining Eqs. $(A.60)$ with $(A.69)$

and $(A.71)$, we have

$$\begin{aligned}
i_t^f &= r_t^f + \left[E_t\left[\pi_{t+1}\right] - \frac{1}{2}\sigma_t^2\left(\pi_{t+1}\right)\right] - cov_t\left(\pi_{t+1}, r_{t+1}\right), \qquad \text{(A.78)}\\
\text{where } E_t\left[\pi_{t+1}\right] &= \Pi_u \varphi_u u_t\\
\sigma_t^2\left(\pi_{t+1}\right) &= \left(\Pi_u + \kappa Y_u \frac{\psi}{1+\psi}\right)^2 \sigma_u^2 + \left(\frac{\psi}{1+\psi}\right)^2 \kappa^2\left(\sigma_z^2 + \sigma_\delta^2\right) + \left(\frac{\sigma^2}{\alpha}\right)^2 \kappa^2 \sigma_\mu^2\\
-cov_t\left(\pi_{t+1}, r_{t+1}\right) &= \left(\Pi_u + \kappa Y_u \frac{\psi}{1+\psi}\right) \frac{1}{1+\psi} Y_u\\
&\quad + \frac{\psi}{1+\psi}\kappa\left[\frac{1}{1+\psi}\sigma_z^2 + \left(\frac{1}{1+\psi} - (1-\beta)\right)\sigma_\delta^2\right] - \left(\frac{\sigma^2}{\alpha}\right)^2 \kappa \sigma_\mu^2.
\end{aligned}$$

Note that i_t^f does not depend on the current noise shock μ_t (although it depends on the variance of the future noise shocks σ_μ^2). This verifies that the central bank can implement the equilibrium by setting the nominal rate i_t^f under its information set and completes the proof. □

A.7. FCI targeting with time-varying beliefs

In this appendix, we consider a version of our model in which the aggregate asset price can fluctuate due to the market's (arbitrageurs') average beliefs about future productivity in addition to noise shocks. We accommodate cases in which these beliefs are informative about future productivity (driven by news) as well as cases they are not informative. For either case, we show that some FCI targeting is still optimal. In fact, belief-driven asset price fluctuations create an additional mechanism by which FCI targeting stabilizes the aggregate asset price and output gaps.

Model with time-varying beliefs. We consider the baseline model in Section 3.1 with two changes. First, we modify Eq. (6) slightly so that:

$$y_{t+1}^* = y_t^* + \varepsilon_{z,t+1} \text{ where } \varepsilon_{z,t+1} \sim N\left(0, \overline{\sigma}_z^2\right). \qquad \text{(A.79)}$$

We assume $\varepsilon_{z,t}$ is i.i.d. Normally distributed and we denote the variance with $\overline{\sigma}_z^2$ (we reserve the notation σ_z^2 for the variance of $\varepsilon_{z,t+1}$ conditional on news). Second, we assume the arbitrageurs receive a signal (news) that *they believe* is informative about future productivity

$$n_{z,t} =^A \varepsilon_{z,t+1} + e_{zt} \text{ where } e_{zt} \sim N\left(0, \tilde{\sigma}_z^2\right). \qquad \text{(A.80)}$$

Here, e_{zt} is i.i.d. Normally distributed and the notation $=^A$ describes arbitrageurs' beliefs. For simplicity, we assume the central bank has the same belief as the arbitrageurs (we could relax this assumption with minor adjustments, since the central bank's beliefs about future productivity does not play a central role).[32]

We remain agnostic about whether these beliefs are correct. For concreteness, suppose $n_{z,t}$ is actually (in the data) drawn according to the relation $n_{z,t} = \eta_t + \varepsilon_{z,t+1} + e_{zt}$ where $\eta_t \sim N\left(\overline{\eta}_t, \sigma_\eta^2\right)$. When

[32]We remain agnostic about the households' beliefs. The households' belief does not affect the equilibrium in our model as they do not make a portfolio choice decision and their consumption is driven by asset prices and demand shocks, which we take as exogenous (in practice, households' beliefs can be one driver of demand shocks—we leave this extension for future work).

$\overline{\eta}_t = \sigma_\eta^2 = 0$, the arbitrageurs' beliefs are correct. When $\overline{\eta}_t = 0$ but $\sigma_\eta^2 > 0$, the arbitrageurs have unbiased beliefs but they are overconfident (the signal is less informative than what they think). When $\overline{\eta}_t \neq 0$, arbitrageurs' beliefs are also biased. As we will see, the parameters $\overline{\eta}_t$ and σ_η^2 do *not* affect the equilibrium. Therefore, our model accommodates these possibilities.

The rest of the model is the same as in Section 3.1. In particular, demand shocks and noisy flow shocks follow the same processes as before (see (8) and (10)) and agents do not have any signal about these processes.

To characterize the equilibrium, observe that Eqs. $(A.79)$ and $(A.80)$ imply that the arbitrageurs' posterior belief is given by

$$\begin{aligned} \varepsilon_{z,t+1} &\sim {}^A N\left(b_t, \sigma_z^2\right) \text{ where} \\ b_t &= \frac{1/\tilde{\sigma}_z^2}{1/\tilde{\sigma}_z^2 + 1/\overline{\sigma}_z^2} n_{zt} \quad \text{and} \quad \sigma_z^2 = \frac{1}{1/\tilde{\sigma}_\delta^2 + 1/\overline{\sigma}_\delta^2}. \end{aligned} \tag{A.81}$$

Observe also that ex-ante (before observing the news) agents believe arbitrageurs' posterior belief will be distributed according to

$$b_t \sim^A N\left(0, \sigma_b^2\right) \text{ where } \sigma_b^2 = \overline{\sigma}_z^2 - \sigma_z^2. \tag{A.82}$$

The ex-ante mean of beliefs is zero as the news is unpredictable, and the ex-ante variance of beliefs is equal to the variance reduction induced by (perceived) news.

Arbitrageurs choose their portfolio allocations to maximize expected log assets-under-management under this belief. This implies that Eqs. (9) and (14) in the main text still hold but with beliefs driven by (perceived) news about future productivity. In particular, the equilibrium asset price satisfies

$$p_t = \rho + E_t^A\left[p_{t+1}\right] + (1-\beta) E_t^A\left[\delta_{t+1}\right] - \left(r_t^f + \frac{1}{2} var_t^A\left[r_{t+1}\right]\right) + \frac{var_t^A\left[r_{t+1}\right]}{\alpha}\mu_t.$$

Benchmark equilibrium without policy reaction lags. As before, we start by describing the benchmark equilibrium without reaction lags:

$$\begin{aligned} p_t &= p_t^* \equiv y_t^* - m - \delta_t, \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta\varphi_\delta\right)\delta_t + b_t + \frac{\sigma^2}{\alpha}\mu_t, \\ r_{t+1} &= \rho + \delta_t - \beta\delta_{t+1} + \varepsilon_{z,t+1} - b_t, \\ \text{where } \sigma^2 &\equiv var_t^A\left(r_{t+1}\right) = \sigma_z^2 + \beta^2\sigma_\delta^2. \end{aligned}$$

These equations are similar to their counterparts in (15). The aggregate asset price depends on neither noisy flows *nor* beliefs about future productivity. Both factors are absorbed by the interest rate, $b_t + \frac{\sigma^2}{\alpha}\mu_t$. The "pstar" that ensures zero output gaps is determined by the *current (near-term)* supply and demand. Therefore, the central bank needs to adjust the interest rate to insulate economic activity from asset price fluctuations driven by beliefs about future productivity. Observe also that the return expression is slightly different and depends on the productivity surprise, $\varepsilon_{z,t+1} - b_t$. Since we have redefined $\sigma_z^2 = var^A\left(\varepsilon_{z,t+1} - b_t\right)$, the expression for total variance is unchanged.

Equilibrium with policy reaction lags and FCI targeting. We next consider the main setup in which the central bank sets r_t^f before observing the current-period noisy flows μ_t *and* the current period beliefs of the arbitrageurs b_t. We allow for FCI targeting: that is, the central bank solves the same problem as before

$$G_t^{FCI} = \min_{r_t^f, \overline{p}_{t+1}} \underline{E}_t^A \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^*\right)^2 + \psi \left(p_{t+h} - \overline{p}_{t+h}\right)^2 \right] \right]. \tag{A.83}$$

Discretionary policy is the special case with $\psi = 0$. The following result generalizes Proposition 2 to this setting.

Proposition 10 (Equilibrium with FCI Targeting and Time-Varying Beliefs)**.** *Suppose the planner follows the FCI targeting policy in* (22) *with* $\psi \geq 0$*, the parameters satisfy* $\alpha^2 \geq 4\sigma_\mu^2 \left(\sigma_z^2 + \sigma_b^2 + \beta^2 \sigma_\delta^2\right)$ *(and* $\beta > 1 - \beta$*), and the initial target satisfies* $\overline{p}_0 = E_{-1}^A \left[p_0^*\right]$*. Then, there is a (stable) equilibrium in which the planner announces the expected "pstar" for the next period as its target*

$$\overline{p}_{t+1} = \underline{E}_t^A \left[p_{t+1}^*\right] \text{ where } p_{t+1}^* = y_{t+1}^* - m - \delta_{t+1}. \tag{A.84}$$

The equilibrium asset price, output, and interest rate are

$$\begin{aligned} p_t &= \underline{E}_{t-1}^A \left[p_t^*\right] + \frac{1}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, & \text{(A.85)} \\ y_t &= y_t^* + \frac{\psi}{1+\psi} \left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) + \frac{1}{1+\psi} b_t + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}, & \text{(A.86)} \\ r_t^f &= \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta\varphi_\delta\right) \delta_t + \frac{\sigma^2}{\alpha} \varphi_\mu \mu_{t-1} + \frac{\psi}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right). & \text{(A.87)} \end{aligned}$$

The equilibrium return is

$$r_{t+1} = E_t^A \left[r_{t+1}\right] + \begin{bmatrix} \frac{1}{1+\psi} \left(\varepsilon_{z,t+1} - b_t\right) \\ -\left(\frac{1}{1+\psi} - (1-\beta)\right) \varepsilon_{\delta,t+1} \end{bmatrix} + \frac{1}{1+\psi} b_{t+1} + \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t+1} \tag{A.88}$$

where the expected return is

$$E_t^A \left[r_{t+1}\right] = \rho + \delta_t \left(1 - \beta\varphi_\delta\right) + \frac{\psi}{1+\psi} \left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) - \frac{\sigma^2}{\alpha} \varepsilon_{\mu,t}. \tag{A.89}$$

The return variance $\sigma^2 = var_t^A \left(r_{t+1}\right)$ *is the smaller positive solution to the following fixed point problem*

$$\begin{aligned} &\sigma^2 = \sigma_{macro}^2 (\psi) + \frac{\left(\sigma^2\right)^2}{\alpha^2} \sigma_\mu^2, & \text{(A.90)} \\ &\text{where } \sigma_{macro}^2 (\psi) = \left(\frac{1}{1+\psi}\right)^2 \left(\sigma_z^2 + \sigma_b^2\right) + \left(\frac{1}{1+\psi} - (1-\beta)\right)^2 \sigma_\delta^2. \end{aligned}$$

Let $\overline{\psi} = \arg\min_{\psi \geq 0} \sigma_{macro}^2 (\psi)$*. Over the range* $\psi \in [0, \overline{\psi})$*, increasing* ψ *strictly reduces* σ^2 *as well as* $\sigma_{macro}^2 (\psi)$ *and* $\frac{\left(\sigma^2\right)^2}{\alpha^2} \sigma_\mu^2$*.*

First consider the case with discretionary policy $\psi = 0$. For this case, the result shows the equilibrium is similar to its counterpart in Proposition 1 with the main difference that the asset price and output are also influenced by the arbitrageurs' belief shocks

$$p_t = \underline{E}^A_{t-1}[p^*_t] + \varepsilon_{z,t} - \varepsilon_{\delta,t} + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \quad \text{(A.91)}$$

$$y_t = y^*_t + b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \quad \text{(A.92)}$$

Consequently, these belief shocks also generate return variance, in addition to the previous drivers of return variance

$$r_{t+1} = E^A_t[r_{t+1}] + (\varepsilon_{z,t+1} - b_t) - \beta\varepsilon_{\delta,t+1} + b_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1}. \quad \text{(A.93)}$$

The rest of the equilibrium is as before. In particular, the return variance is the solution to a fixed point problem. Observe that the "macro-induced" variance $\sigma^2_{macro}(\psi)$ is driven by not only supply and demand shocks but also the belief shocks about future supply.

Next consider the case with FCI targeting $\psi > 0$. In this case, the asset price and output are given by

$$\begin{aligned} p_t &= \underline{E}^A_{t-1}[p^*_t] + \frac{1}{1+\psi}(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t} \\ y_t &= y^*_t + \frac{\psi}{1+\psi}(\varepsilon_{\delta,t} - \varepsilon_{z,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}. \end{aligned}$$

As before, the supply and demand shocks have a smaller effect on the asset price (and some effect on output), because the central bank is partially committed to the previously announced FCI target. Importantly, belief shocks about future supply have a smaller effect on the asset price and output. This is because FCI targeting implies the central bank will not allow the *future* asset price to adjust to the *future* supply immediately, $p_{t+1} = \underline{E}^A_t[p^*_t] + \frac{1}{1+\psi}\varepsilon_{z,t+1} + ...$ This reduces the impact of arbitrageurs' beliefs about future supply on the aggregate asset price and output. It follows that FCI targeting reduces the macro-induced variance by reducing the impact of belief shocks as well as supply and demand shocks. This in turn reduces the noise-induced variance as in the main text.

In this setup FCI targeting ($\psi > 0$) is beneficial for two distinct reasons. To see this, consider the expression for the output. As before, FCI targeting reduces variance σ^2 and mitigates the impact of noise shocks on output gaps. Moreover, FCI targeting also reduces the effect of belief shocks on the output gap through the mechanism we described. FCI targeting also creates a new source of gaps through supply and demand shocks, but as before these costs are second order in ψ. In contrast, the benefits through the belief shocks is first order in ψ, similar to the benefits through the noise shocks. It follows that FCI targeting is robust to allowing for belief-driven fluctuations in asset prices. In fact, these fluctuations create an additional channel by which FCI targeting stabilizes the aggregate asset price and output gaps.

Proof of Proposition 10. We conjecture and verify an equilibrium in which the return volatility σ^2 is constant, the central bank announces the expected future "pstar" as its target $\overline{p}_t = \underline{E}^A_{t-1}[p^*_t]$, and the asset price and the interest rate satisfies $(A.85)$ and $(A.87)$, which can be written as

$$\begin{aligned} p_t &= y^*_{t-1} - m - \varphi_\delta\delta_{t-1} + \frac{1}{1+\psi}(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}, \\ r^f_t &= \rho - \frac{1}{2}\sigma^2 + (1-\beta\varphi_\delta)\delta_t + \frac{\psi}{1+\psi}(\varepsilon_{z,t} - \varepsilon_{\delta,t}) + \frac{\sigma^2}{\alpha}\varphi_\mu\mu_{t-1} \end{aligned}$$

Following similar steps as in the proof of Proposition 2, the central bank's optimality condition satisfies

$$\underline{E}_t\left[p_t\right] = \frac{1}{1+\psi}p_t^* + \frac{\psi}{1+\psi}\overline{p}_t.$$

Substituting $\overline{p}_t = \underline{E}_{t-1}\left[p_t^*\right], p_t^* = y_t^* - m - \delta_t$ and the AR(1) process for δ_t, we obtain

$$\begin{aligned}
\underline{E}_t\left[p_t\right] &= \frac{1}{1+\psi}\left(y_t^* - m - \delta_t\right) + \frac{\psi}{1+\psi}\underline{E}_{t-1}\left[y_{t-1}^* + \varepsilon_{z,t} - m - \delta_t\right] \\
&= y_{t-1}^* - m - \varphi_\delta \delta_{t-1} + \frac{\psi}{1+\psi}\varepsilon_{z,t} - \frac{1}{1+\psi}\varepsilon_{\delta,t} \\
&= y_t^* - m - \varphi_\delta \delta_{t-1} + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right).
\end{aligned}$$

This verifies that the conjectured price satisfies the optimality condition.

We next substitute the conjectured price into (9) to calculate the equilibrium return

$$\begin{aligned}
r_{t+1} &= \rho + p_{t+1} + (1-\beta)\,\delta_{t+1} - p_t \\
&= \rho + y_t^* - \varphi_\delta \delta_t + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} - \varepsilon_{\delta,t+1}\right) + \frac{1}{1+\psi}b_{t+1} + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t+1} \\
&\quad + (1-\beta)\,\delta_{t+1} - \left(y_{t-1}^* - \varphi_\delta \delta_{t-1} + \frac{1}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{1}{1+\psi}b_t + \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}\right) \\
&= \rho + \varepsilon_{z,t} + \frac{1}{1+\psi}\left(\varepsilon_{z,t+1} + b_{t+1} - \left(\varepsilon_{z,t} + b_t\right)\right) + \frac{\sigma^2}{\alpha}\left(\varepsilon_{\mu,t+1} - \varepsilon_{\mu,t}\right) \\
&\quad + \delta_t\left(1 - \beta\varphi_\delta\right) - \frac{\psi}{1+\psi}\varepsilon_{\delta,t} + \left(1 - \beta - \frac{1}{1+\psi}\right)\varepsilon_{\delta,t+1} \\
&= E_t^A\left[r_{t+1}\right] + \left(\varepsilon_{z,t+1} - b_t + b_{t+1}\right)\frac{1}{1+\psi} + \varepsilon_{\delta,t+1}\left(1 - \beta - \frac{1}{1+\psi}\right) + \varepsilon_{\mu,t+1}\frac{\sigma^2}{\alpha},
\end{aligned}$$

where the expected return is given by $(A.37)$

$$E_t^A\left[r_{t+1}\right] = \rho + \delta_t\left(1 - \beta\varphi_\delta\right) + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) - \frac{\sigma^2}{\alpha}\varepsilon_{\mu,t}.$$

We combine this expression with (13) and substitute $\mu_t - \varepsilon_{\mu,t} = \varphi_\mu \mu_{t-1}$ to calculate the interest rate

$$r_t^f = \rho - \frac{1}{2}\sigma^2 + \left(1 - \beta\varphi_\delta\right)\delta_t + \frac{\psi}{1+\psi}\left(\varepsilon_{z,t} - \varepsilon_{\delta,t}\right) + \frac{\sigma^2}{\alpha}\varphi_\mu \mu_{t-1}.$$

This verifies the expression for the interest rate.

Combining the conjecture for the price with $y_t = m + p_t + \delta_t$ verifies that output satisfies $(A.86)$. Substituting the solution into the expression for the return, we also find that the return satisfies $(A.88)$.

Finally, observe that Eq. $(A.88)$ implies that σ^2 solves the fixed point problem $(A.90)$. Under the assumed parametric condition, this problem has two positive roots for each $\psi \in [0, \overline{\psi})$. The smaller root corresponds to the stable equilibrium. The rest of the proof follows from similar steps as in the proof of Proposition 2. □

A.8. FCI targeting with endogenous arbitrageur supply

In the main text, we took the fraction of arbitrageurs [redacted] that trade the market portfolio as an exogenous constant. Could endogenizing arbitrage activity overturn the beneficial effects of FCI targeting? In this appendix, we address this question by developing a version of the model in which α is endogenous and driven by arbitrageurs' expected excess returns. We show that while FCI targeting reduces these returns (by reducing volatility) and induces some arbitrageur exit, our main results qualitatively still hold as long as the supply of arbitrage capital is not completely elastic.

Model with endogenous arbitrage activity. Consider the baseline model in Section 3.1 with one change: the fraction η of portfolio managers are noise traders as before whereas the remaining fraction $1-\eta$ are *potential* arbitrageurs indexed by subscript $j \in [0, 1-\eta]$. Potential arbitrageurs make a one-time decision (for simplicity) before time 0 whether to become an arbitrageur or to become an inelastic fund. We assume each fund manager is compensated by a fraction of the expected log returns they generate for households. Therefore, the expected lifetime benefit to becoming an entrepreneur is equal to the present discounted sum of expected *excess* log returns:

$$u^A = E\left[\sum_{t=0}^{\infty} \beta^t E_t\left[\log\left(R_{t+1}^A\right)\right] - E_t\left[\log\left(R_{t+1}\right)\right]\right]. \tag{A.94}$$

Here, R_{t+1}^A denotes the arbitrageurs' return and R_{t+1} is the inelastic fund's return—which is simply the return on the market portfolio. The entry cost is heterogeneous across agents and given by $\frac{1}{2}\frac{1}{1-\beta}c(j)$ for potential arbitrageur j (normalizing terms will simplify the expressions). We assume $c(j)$ is a strictly increasing function with a boundary conditions $\lim_{j\to 1-\eta} c(j) = \infty$. The equilibrium fraction of arbitrageurs $\alpha \in (0, 1-\eta)$ is then the solution to

$$\frac{1}{2}\frac{1}{1-\beta}c(\alpha) = u^A. \tag{A.95}$$

In equilibrium, agents $j \in [0, \alpha]$ become arbitrageurs as their entry costs are sufficiently low, whereas agents $j \in (\alpha, 1-\eta]$ become inelastic funds. We assume the benefits and costs are both infinitesimal (as arbitrageurs are infinitesimal) so they do not affect the resource constraints. The rest of the model is unchanged. For notational simplicity, we assume noise shocks are i.i.d. ($\rho_\mu = 0$).

To characterize the equilibrium, first consider the arbitrageurs' return, $E_t\left[\log R_{t+1}^A\right]$. Up to a log-Normal approximation, we have

$$\begin{aligned} E_t\left[\log R_{t+1}^A\right] &= \max_{\omega_t^A} r_t^f + \omega_t^A\left(E_t\left[r_{t+1}\right] + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} - r_t^f\right) - \frac{1}{2}\left(\omega_t^A\right)^2\left(\sigma_{t,r_{t+1}}\right)^2 \\ &= r_t^f + \frac{1}{2}\left(\frac{E_t\left[r_{t+1}\right] + \frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2} - r_t^f}{\sigma_{t,r_{t+1}}}\right)^2. \end{aligned}$$

Here, the second line substitutes the optimal portfolio weight in (12). Arbitrageurs' *optimal* expected return is increasing in the Sharpe ratio. Next note that in equilibrium the Sharpe ratio satisfies (see (11) and (12))

$$\frac{E_t\left[r_{t+1}\right]+\frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2}-r_t^f}{\sigma_{t,r_{t+1}}}=\underbrace{\left(1-\frac{\mu_t}{\alpha}\right)\sigma_{t,r_{t+1}}}_{\text{arbitrageur risk}}.$$

Combining these expression, we calculate

$$E_t\left[\log R_{t+1}^A\right]=r_t^f+\frac{1}{2}\left(1-\frac{\mu_t}{\alpha}\right)^2\sigma_{t,r_{t+1}}^2.$$

Next consider the inelastic funds' return, which can be written as

$$\begin{aligned}E_t\left[\log R_{t+1}\right] &= r_t^f+\left(E_t\left[r_{t+1}\right]+\frac{\left(\sigma_{t,r_{t+1}}\right)^2}{2}-r_t^f\right)-\frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2\\ &= r_t^f+\left(1-\frac{\mu_t}{\alpha}\right)\sigma_{t,r_{t+1}}^2-\frac{1}{2}\left(\sigma_{t,r_{t+1}}\right)^2.\end{aligned}$$

Here, the second line substitutes for the equilibrium Sharpe ratio as before.

Combining these observations, we calculate the per-period excess log return as

$$E_t\left[\log R_{t+1}^A\right]-E_t\left[\log R_{t+1}\right]=\frac{1}{2}\left(\frac{\mu_t}{\alpha}\right)^2\sigma_{t,r_{t+1}}^2. \tag{A.96}$$

The excess return is increasing in the amount of noise μ_t and in *the return variance*. With higher variance, arbitrageurs require a greater Sharpe ratio to absorb noisy flows. A greater Sharpe ratio in turn induces arbitrageurs to earn a greater expected excess return.

Note also that when the variance is constant over time, $\sigma_{t,r_{t+1}}^2=\sigma^2$, we can combine $(A.96)$ with $(A.94)$ and $(A.95)$ and use the simplifying assumption that μ_t is i.i.d. to obtain the equilibrium version of the entry condition

$$\frac{1}{2}\frac{1}{1-\beta}c\left(\alpha\right)=\frac{1}{2}\frac{1}{1-\beta}\frac{\sigma_\mu^2}{\alpha^2}\sigma^2\Longrightarrow c\left(\alpha\right)=\frac{\sigma_\mu^2}{\alpha^2}\sigma^2. \tag{A.97}$$

All else equal, there is greater entry when the return variance is larger and the noise variance is larger—as this creates more frequent trading opportunities.

Equilibrium with discretion. Next consider the equilibrium with discretion. For a given α, Proposition 1 still applies. In particular, there is an equilibrium with constant variance σ^2 that is the smaller root of the quadratic (21)

$$\sigma^2=\sigma_{macro}^2+\frac{\left(\sigma^2\right)^2}{\alpha^2}\sigma_\mu^2,\quad \text{where } \sigma_{macro}^2=\sigma_z^2+\beta^2\sigma_\delta^2.$$

We also require this solution to satisfy $(A.97)$. Substituting for σ^2 in terms of α, we obtain

$$\alpha^2c\left(\alpha\right)\left(1-c\left(\alpha\right)\right)=\sigma_{macro}^2\sigma_\mu^2 \text{ and } \sigma^2=\frac{\sigma_{macro}^2}{1-c\left(\alpha\right)}.$$

Greater exogenous variance, σ_{macro}^2 and σ_μ^2, results in greater entry α. It also results in greater endogenous variance σ^2, despite greater entry. Intuitively, the endogenous arbitrageur entry mitigates but does not overturn the endogenous increase in variance.

Equilibrium with FCI targeting. Next consider the equilibrium with FCI targeting. For a given ψ and α, Proposition 2 still holds. Following the same steps as above, we obtain

$$\begin{aligned}
\alpha^2 c(\alpha)(1-c(\alpha)) &= \sigma^2_{macro}(\psi)\,\sigma^2_\mu \text{ and } \sigma^2 = \frac{\sigma^2_{macro}(\psi)}{1-c(\alpha)} \\
\text{where } \sigma^2_{macro}(\psi) &= \sigma^2_z\left(\frac{1}{1+\psi}\right)^2 + \sigma^2_\delta\left(\frac{1}{1+\psi} - (1-\beta)\right)^2.
\end{aligned}$$

Recall that over the range $\psi \in [0, \overline{\psi})$, FCI targeting reduces $\sigma^2_{macro}(\psi)$. Therefore, FCI targeting also reduces the equilibrium values of α and σ^2. In particular, FCI targeting still reduces the equilibrium variance, albeit less than in the main text where α is exogenous. Intuitively, the decline in the return variance reduces arbitrageurs' excess returns and causes some exit. However, as long as the supply curve is strictly increasing (not fully elastic), this exit mitigates but does not overturn the initial decline in variance.

Next consider the effects of FCI targeting on macroeconomic stability and the central bank's objective. The output gap is still given by (31)

$$\tilde{y}_t = (\varepsilon_{\delta,t} - \varepsilon_{z,t})\frac{\psi}{1+\psi} + \varepsilon_{\mu,t}\frac{\sigma^2}{\alpha}.$$

The noise impact satisfies, $\frac{\sigma^2}{\alpha} = \frac{\alpha c(\alpha)}{\sigma^2_\mu}$, which is strictly decreasing in FCI targeting ψ. It follows that Proposition 3 still holds: that is, FCI targeting generates first order benefits by reducing noise-driven output gaps while inducing only second order costs.

A.9. Comparison of FCI targeting with the full commitment solution

In the main text, we focus on a constrained set of commitment policies in which the central bank solves the FCI targeting problem (22). We next investigate the full commitment solution and compare it with the FCI targeting solution. In a tractable special case, we show that FCI targeting is the solution to a constrained version of the full commitment problem where the central bank can condition future output gaps *only* on future shocks.

Full (linear) commitment solution in a tractable case. For analytical tractability, we focus on a version of the baseline model in which there is noise shock only at date 0, that is, we set $\sigma^2_{\mu,t} = \varepsilon_{\mu,t} = 0$ for $t \geq 1$. To simplify the notation, we also abstract away from demand shocks $\sigma^2_\delta = 0$. In this case, Eqs. (9) and (7) imply $r_{t+1} = \rho + y_{t+1} - y_t$. This in turn implies that the return variance is given by $\left(\sigma_{t,r_{t+1}}\right)^2 = \left(\sigma_{0,y_1}\right)^2$ and Eq. (14) can be simplified as

$$y_t = \rho + E_t[y_{t+1}] - \left(r^f_t + \frac{1}{2}\left(\sigma_{0,y_1}\right)^2\right) + \frac{\left(\sigma_{0,y_1}\right)^2}{\alpha}\mu_t. \tag{A.98}$$

Next recall that in the baseline model the central bank sets r^f_t before observing $\varepsilon_{\mu,t}$. Since $\varepsilon_{\mu,t} = 0$ for $t \geq 1$, for these dates the central bank can implement any y_t by appropriately setting r^f_t. At date 0, the central bank is subject to additional implementability constraints. Taking the expectations of $(A.98)$ under the central bank's information set, and eliminating the policy interest rate, we write the implementability constraints as

$$y_0 - \underline{E}_0[y_0] = E_0[y_1] - \underline{E}_0[y_1] + \begin{pmatrix} -\frac{1}{2}\left(\left(\sigma_{0,y_1}\right)^2 - \underline{E}_0\left[\left(\sigma_{0,y_1}\right)^2\right]\right) \\ +\frac{\left(\sigma_{0,y_1}\right)^2}{\alpha}\mu_0 - \underline{E}_0\left[\frac{\sigma_{0,y_1}^2}{\alpha}\mu_0\right] \end{pmatrix} \tag{A.99}$$

Here, recall that $\underline{E}[\cdot]$ is the expectation before observing $\varepsilon_{\mu,0}$. The central bank can implement a process for y_0 that satisfies $(A.99)$ by setting the interest rate r_0^f appropriately. Conversely, if a process y_0 does not satisfy $(A.99)$, it cannot be implemented.

We can then write the central bank's full commitment problem at date 0 as follows:

$$\begin{aligned} G_0 \quad = \quad & \min_{\{y_t = Y(\varepsilon_{\mu,0},\varepsilon_{z,1},..,\varepsilon_{z,t})\}_t} \underline{E}_0\left[\sum_{h=0}^{\infty}\beta^h\left(y_h - y_t^*\right)^2\right] \\ & \text{s.t. } y_t^* = y_{t-1}^* + \varepsilon_{z,t} \\ & \text{and } (A.99). \end{aligned} \tag{A.100}$$

The central bank sets the output after every history to minimize expected quadratic output gaps subject to an implementability constraint at date 0.

Since the constraint is not affected by allocations for dates $t \geq 2$, it is easy to check the central bank sets the output gaps at these dates to zero

$$Y\left(\varepsilon_{\mu,0}, \varepsilon_{z,1}, .., \varepsilon_{z,t}\right) = y_t^* \text{ for each } t \geq 2.$$

At date 1, the central bank might want to set non-zero gaps to relax the implementability constraint at date 0.

To simplify the analysis further, we assume the central bank is constrained to set policies that are a linear function of the realized shocks

$$Y\left(\varepsilon_{\mu,0}, \varepsilon_{z,1}\right) = y_0^* - \lambda_1 - \lambda_{\mu,0}\varepsilon_{\mu,0} + \left(1 - \lambda_{z,1}\right)\varepsilon_{z,1}.$$

Here, $\lambda_1, \lambda_{\mu,0}, \lambda_{z,1}$ capture the deviations of the proposed linear policy from the unconstrained (zero-gap) solution. This linearity assumption implies $\underline{E}_0\left[\left(\sigma_{0,y_1}\right)^2\right] = \left(\sigma_{0,y_1}\right)^2$. This in turn simplifies the implementability constraint $(A.99)$ as follows:

$$y_0 - \underline{E}_0[y_0] = E_0[y_1] - \underline{E}_0[y_1] + \frac{\left(\sigma_{0,y_1}\right)^2}{\alpha}\varepsilon_{\mu,0} \text{ with } \left(\sigma_{0,y_1}\right)^2 = \left(1 - \lambda_{z,1}\right)^2\sigma_z^2. \tag{A.101}$$

Here, we have used $\mu_0 - \underline{E}_0[\mu_0] = \varepsilon_{\mu,0}$. The implementability constraint now says that the impact of current noise on current output is driven by the future variance (as in the main text) and by the sensitivity of future output to the past noise (since expectation of future output affects current output).

Substituting these observations into $(A.100)$, we write the central bank's problem as

$$\begin{aligned} & \min_{\{y_0\},\lambda_1,\lambda_{\mu,0},\lambda_{z,1}} \underline{E}_0\left[\left(y_0 - y_0^*\right)^2\right] + \beta\underline{E}_0\left[\left(-\lambda_1 - \lambda_{\mu,0}\varepsilon_{\mu,0} - \lambda_{z,1}\varepsilon_{z,1}\right)^2\right] \\ & \text{s.t. } y_0 = \underline{E}_0[y_0] + \left(-\lambda_{\mu,0} + \frac{\left(1 - \lambda_{z,1}\right)^2\sigma_z^2}{\alpha}\right)\varepsilon_{\mu,0}. \end{aligned}$$

The central bank sets $\lambda_1 = 0$ and implements $\underline{E}_0 [y_0] = y_0^*$, centering both current and future output gaps around zero. The problem then becomes

$$\min_{\lambda_{\mu,0},\lambda_{z,1}} \left(-\lambda_{\mu,0} + \frac{(1-\lambda_{z,1})^2 \sigma_z^2}{\alpha}\right)^2 \sigma_{\mu,0}^2 + \beta \left(\lambda_{\mu,0}^2 \sigma_{\mu,0}^2 + \lambda_{z,1}^2 \sigma_z^2\right) \tag{A.102}$$

The central bank commits to generating future gaps in response to noise and productivity shocks (via $\lambda_{\mu,0}$ and $\lambda_{z,1}$) to mitigate the current gaps that result from noise shocks.

The optimality conditions for $\lambda_{z,1}$ and $\lambda_{\mu,0}$ are given by

$$\lambda_{z,1} = \frac{2\left(-\lambda_{\mu,0} + \frac{(1-\lambda_{z,1})^2\sigma_z^2}{\alpha}\right)\frac{1-\lambda_{z,1}}{\alpha}\sigma_{\mu,0}^2}{\beta} \tag{A.103}$$

$$\lambda_{\mu,0} = \frac{(1-\lambda_{z,1})^2 \sigma_z^2}{\alpha}\frac{1}{1+\beta}. \tag{A.104}$$

Combining them, we find that the loading on the supply shock $\lambda_{z,1} \in (0,1)$ is the unique solution to the following equation

$$\lambda_{z,1} = \frac{2}{1+\beta}\frac{(1-\lambda_{z,1})^3}{\alpha^2}\sigma_z^2\sigma_{\mu,0}^2. \tag{A.105}$$

The loading on the past noise shock $\lambda_{\mu,0} > 0$ is then found from $(A.104)$.

It follows that the full linear commitment solution has two features. First, as in the main text, the central bank reduces future variance by mitigating the impact of supply shocks on future output (and asset prices): specifically, $\frac{dy_1}{d\varepsilon_{z,1}} = 1 - \lambda_{z,1} < 1$. Second, and unlike in the main text, the central bank also makes future output (and asset prices) respond to past demand shocks, $\frac{dy_1}{d\lambda_{\mu,0}} = -\lambda_{\mu,0} < 0$. Intuitively, after realization of a positive noise shock, the central bank promises to induce negative future gaps to mitigate the impact of noise on current output.

Comparison with the FCI targeting solution. Next consider the FCI targeting problem for the same setup: suppose the planner minimizes (22)

$$\min_{r_t^f, \overline{p}_{t+1}} \underline{E}_t \left[\sum_{h=0}^{\infty} \beta^h \left[\left(y_{t+h} - y_{t+h}^*\right)^2 + \psi_t \left(p_{t+h} - \overline{p}_{t+h}\right)^2\right]\right].$$

We allow the FCI targeting parameter ψ_t to change over time because the model is non-stationary. Since there are no future noise shocks, it is optimal to set $\psi_t = 0$ and implement $y_t = y_t^*$ for $t \geq 2$. We also assume $\psi_0 = 0$ so there is no inherited commitment from the past. The FCI targeting problem for a given ψ_1 is then given by

$$\min_{r_0^f, \overline{p}_1} \underline{E}_0 \left[(y_0 - y_0^*)^2 + \beta\left((y_1 - y_1^*)^2 + \psi_1 (p_1 - \overline{p}_1)^2\right)\right].$$

Following the same steps as in the proof of Proposition 2, the solution is given by

$$\begin{aligned}
\overline{p}_1 &= E_0\left[p_1^*\right] \text{ and } p_1 = p_0^* + \frac{1}{1+\psi_1}\varepsilon_{z,1} \\
y_1 &= y_1^* - \frac{\psi_1}{1+\psi_1}\varepsilon_{z,1} = y_0^* + \frac{1}{1+\psi_1}\varepsilon_{z,1} \\
y_0 &= y_0^* + \frac{\sigma_1^2}{\alpha}\varepsilon_{\mu,0} \text{ where } \sigma_1^2 = \sigma_z^2\left(\frac{1}{1+\psi_1}\right)^2.
\end{aligned}$$

Now consider the choice of the optimal FCI target ψ_1. Recall that the planner does not care about FCI targeting per se. Thus, the optimal target minimizes $\underline{E}_0\left[(y_0 - y_0^*)^2 + \beta\,(y_1 - y_1^*)^2\right]$. After substituting the solution, we write the planning problem as

$$\min_{\psi_1}\left(\frac{\sigma_1^2}{\alpha}\right)^2\sigma_\mu^2 + \beta\left(\frac{\psi_1}{1+\psi_1}\right)^2\sigma_z^2 \text{ where } \sigma_1^2 = \left(\frac{1}{1+\psi_1}\right)^2\sigma_z^2$$

After applying a change of variables $\lambda_{z,1} = \frac{\psi_1}{1+\psi_1}$, this problem becomes

$$\min_{\lambda_{z,1}=\frac{\psi_1}{1+\psi_1}}\left(\frac{(1-\lambda_{z,1})^2\,\sigma_z^2}{\alpha}\right)^2\sigma_\mu^2 + \beta\lambda_{z,1}^2\sigma_z^2. \tag{A.106}$$

Comparing this problem with $(A.102)$ reveals that FCI targeting is equivalent to the earlier commitment problem with the additional constraint $\lambda_{\mu,0} = 0$. Using the optimality condition, the solution is given by the $\lambda_{z,1} \in (0,1)$ that solves (cf. $(A.103 - A.105)$)

$$\lambda_{z,1} = \frac{2}{\beta}\frac{(1-\lambda_{z,1})^3\,\sigma_z^2}{\alpha^2}\sigma_{\mu,0}^2.$$

In sum, this analysis shows that, in a tractable special case, the FCI targeting solution corresponds to the solution to a commitment problem where the central bank can commit to future output gaps *only* as a function of future shocks. The central bank uses this commitment to reduce the future variance and to recruit the arbitrageurs as we have emphasized in the main text. The full commitment solution has the additional feature that the central bank conditions future output gaps also on *past* noise shocks. The central bank adjusts these gaps in the *opposite* direction of past noise shocks so as to strengthen the recruitment effect.

In the main text, we focus on FCI targeting because policies conditioning on past shocks would be considerably more complex to communicate and implement. Additionally, the more extensive commitment required under the full solution may prove more onerous for the Fed to honor consistently, potentially undermining its credibility. This robustness of the recruitment mechanism across different commitment structures—and the substantial noise elimination already achieved under FCI targeting—reinforces the practical value of this approach.

B. Empirical Appendix

B.1. Data

B.1.1. Macroeconomic data

We download the following data from FRED (FRED series name in parenthesis): nominal potential GDP (NGDPPOT), nominal GDP (GDP), nominal investment (GPDI), nominal personal consumption expenditures (PCEC), GDP deflator (GDPDEF), PCE price index (PCEPI), PCE price index excluding food and energy (PCEPILFE), the Chicago Fed National Financial Conditions Index (NFCI), the 3-month yield (TB3), labor productivity (OPHNFB), labor share (PRS85006173), weekly hours (PRS85006023), employment (CE16OV), population (CNP16OV) and consumer sentiment (UMCSENT). We obtain the updated series for the Excess Bond Premium from Favara et al. (2016). In order to compute real variables, we divide the nominal variables by the GDP deflator. For the new FCI index from Ajello et al. (2023b), we use the baseline construction, that allows shocks to have effects up to 3 years. Using the 1-year version does not alter the results. We compute hours per worker as weekly hours times employment divided over population. Inflation is computed as 400 times the log-difference in the PCE price index. Real variables are computed as nominal variables over the GDP deflator. For example, real GDP is computed as nominal GDP over the GDP deflator, real potential output is computed as nominal potential GDP over the GDP deflator, and so on. The output gap is computed as the log difference between real GDP and real potential GDP. Except from inflation, the FCI index, the EBP, and the nominal interest rate, all variables are transformed by taking logarithms and multiplying by 100.

Since the FCI index is available from 1990 onwards, we use the Chicago Fed FCI (NFCI) for the sample period 1973-1990 when computing the IRFs of monetary policy shocks. Ajello et al. (2023b) show that their index is similar in sample to the Chicago Fed FCI, and estimated IRFs are similar if we use that FCI for the full sample. We use the utilization adjusted TFP series from Fernald (2014), available in his website. For the local projections in B.3.4, we use additional financial variables. For earning and stock prices, we use the real values reported in the Shiller database. We convert to quarterly by taking the value at the end of the quarter.

B.1.2. Construction of the financial noise shock

In order to construct the shock, we follow Gabaix and Koijen (2021) closely. We use quarterly data from the Flow of Funds.[33] We use the same sample as the VAR specification. We use unadjusted flows (FU), and for the levels we use unadjusted market values when available (LM), and otherwise the estimated level. We collect data on flows for the following sectors: 15 (households), 21 (state and local governments), 22 (state and local retirement funds), 26 (rest of the world), 34 (federal retirement funds), 51 (property and casualty insurance), 54 (life insurance companies), 55 (closed end funds), 56 (ETFs), 57 (private pension funds), 65 (mutual funds), 66 (securities brokers and dealers), and 76 (us chartered deposit institutions). As in Gabaix and Koijen (2021), we use data on three asset classes: 30611 (treasury securities), 30630 (corporate and foreign bonds), 30641 (corporate equities). For returns data, we use ex-dividend returns on the CRSP value-weighted market portfolio. For GDP growth, we use the log difference of real GDP obtained from FRED. We adjust the data on flows for foreign holdings following Appendix C.1.3 in Gabaix and Koijen (2021).

[33]Raw data is downloaded from https://www.federalreserve.gov/datadownload/Build.aspx?rel=Z1.

We follow the same notation and conventions as Appendix C.1.2 in Gabaix and Koijen (2021). We construct a measure of the proportional change of the quantity of equity in sector i between $t-1$ and t ($\Delta q^{\mathcal{E}}_{it}$) as follows: In the FoF, equity flows are defined by $\Delta F^{\mathcal{E}}_{it} = W^{\mathcal{E}}_{it} - W^{\mathcal{E}}_{i,t-1} R^X_t$. We assume the securities are adjusted at the end of the period, so $\Delta F^{\mathcal{E}}_{it} = (\Delta Q^{\mathcal{E}}_{it}) P^{\mathcal{E}}_t$, where Q_{it} is the amount of equities held by sector i at time t, and $P^{\mathcal{E}}_t$ is the price of each share. The relative flow in equities is $\Delta f^{\mathcal{E}}_{it} = \dfrac{\Delta F^{\mathcal{E}}_{it}}{W^{\mathcal{E}}_{i,t-1}}$. The proportional change in quantity of equity is $\Delta q^{\mathcal{E}}_{it} = \Delta f^{\mathcal{E}}_{it} (R^X_t)^{-1} = \dfrac{\Delta Q^{\mathcal{E}}_{it}}{Q^{\mathcal{E}}_{i,t-1}}$. We assume that total wealth is given by the sum of holdings of the three asset classes we consider.

With our measure of $\Delta q^{\mathcal{E}}_{it}$ in hand, we proceed exactly as in Appendix B of Gabaix and Koijen (2021) in order to construct the financial flow shock series. We briefly expand on each of the steps below:

1. Construct precision weights $\tilde{E}_i = \min\{k \dfrac{\sigma_i^{-2}}{\sum_{j=1}^N \sigma_j^{-2}}, 1.5/N\}$ where k is chosen so that the weights add up to 1.

2. Run the panel regression:
$$\Delta q_{it} = \alpha_i + \beta_t + \gamma_i \Delta y_t + \delta_i t + \Delta \check{q}_{it} \tag{B.1}$$
using $\tilde{E}_i$ as weights. Here Δy_t is quarter-on-quarter real GDP growth. In an alternative specification, we also control for changes in the consumer sentiment index ($Sent_t$), i.e we add $\Delta Sent_t$ as an additional control. We implement the weighting scheme by multiplying each observation by $\tilde{E}_i^{1/2}$ and then running OLS, using dummies to capture the fixed effects. Denote the residuals of this transformed regression as $\tilde{E}_i^{1/2} \Delta \check{q}_{it}$

3. We run PCA on $\tilde{E}_i^{1/2} \Delta \check{q}_{it}$. In our baseline specification, we control for aggregate factors by removing the first N principal components (ordered in terms of share of variance explained) from $\tilde{E}_i^{1/2} \Delta \check{q}_{it}$. That is, we construct:
$$\Delta \tilde{q}_{it} = \tilde{E}_i^{1/2} \Delta \check{q}_{it} - \sum_{n=1}^{N} \lambda_{i,n} \eta^{PC}_{t,n} \tag{B.2}$$
where $\eta^{PC}_{t,n}$ is principal component n at time t, and $\lambda_{i,n}$ is the loading of sector i on that principal component. Our baseline uses $N = 2$. Our results are essentially unchanged if we use $N = 3$ or $N = 4$ instead.

4. Finally, we construct the financial flow shock as:
$$Z^{\mu}_t = \sum_{i=1}^{I} S_{i,t-1} \Delta \tilde{q}_{it}$$
where $S_{i,t-1} = \frac{W^{\mathcal{E}}_{it}}{\sum_{j=1}^{I} W^{\mathcal{E}}_{jt}}$ is the share of total equity held by sector i at time $t-1$.

B.2. Bayesian VAR implementation details

For a VAR with M lags, let $\check{y} = [Y_{M+1}, ..., Y_{T+M}]'$ be a matrix where each row is a date and each column is a macro variable. Let $\check{x} = [x_{M+1}, ..., x_{T+M}]$ be a matrix that stacks controls for all equations: each x_t contains a constant and lagged macro variables. In this context, we can write the system to be estimated as:

$$\check{y} = \check{x}B + u \tag{B.3}$$

where $u \sim N(\mathbf{0}, \Sigma \otimes I_T)$. Letting $\beta = vec(B)$, assume the following priors:

$$\beta|\Sigma \sim N(vec(b), \Sigma \otimes \Omega(\xi)) \tag{B.4}$$

$$\Sigma \sim IW(\Psi(\xi), d) \tag{B.5}$$

where ξ is a vector of hyperparameters. It is well known that the posterior distribution of β and Σ is conjugate with:

$$\beta \mid \Sigma, y \sim N(\hat{\beta}, \hat{V}), \tag{B.6}$$

$$\Sigma \mid Y \sim IW\left(\Psi + \hat{u}'\hat{u} + (\hat{B} - b)'\Omega^{-1}(\hat{B}) - b), T + d\right) \tag{B.7}$$

where:

$$\begin{aligned}
\hat{\beta} &= \text{vec}(\hat{B}), \\
\hat{B} &= \left(x'x + (\Omega)^{-1}\right)^{-1}\left(x'y + (\Omega)^{-1}b\right), \\
\hat{V} &\equiv \Sigma \otimes \left(x'x + (\Omega)^{-1}\right)^{-1} \\
\hat{u} &= \check{y} - \check{x}\hat{B}.
\end{aligned}$$

We parametrize the hyperpriors exactly as in Giannone et al. (2015): we set b according to the Minnesota prior (all variables but the proxy are assumed random walks a priori), $d = n + 2$ where n is the number of variables in $\check{y}$, $\Psi = diag(\psi)$ where ψ is a vector of hyperpriors, and Ω is a diagonal matrix with i, i-th entry equal to $\lambda^2/(s_j^2\psi_j)$, where s_j is the lag order of the control and ψ_j is the prior variance of the shock in the equation of the variable that is being lagged.[34] The constant assumed to have a diffuse prior. Thus, $\xi = (\lambda, \psi')'$. The key hyperparameter is λ, that controls the tightness of the prior in β, and thus controls the degree of shrinkage used in estimation. We select the hyperparameters to maximize the marginal likelihood. Given those hyperparameters, we can then obtain the posterior distribution of β and Σ in closed form using (B.6) and (B.7). We generate draws directly from these, and for each draw we can compute any objects of interest (impulse responses, Wold decompositions, Forecast variance ratios, forecasts, etc.)

In section 2, we estimate identify the shock using two different procedures: i) add the proxy first in the VAR and use recursive identification, ii) an SVAR-IV procedure. These two differ exclusively on what the matrix $\check{x}$ includes, and what rotation of Σ is utilized. In the first case, as mentioned in the main text, we let $X_t = [Z_t^\mu, Y_t']'$ and estimate a standard BVAR in X_t. Thus, $\check{y}$ contains all macro variables plus the proxy, and $\check{x}$ includes a constant, two lags of each macro variable and two lags of the proxy.

For the SVAR-IV case, we estimate the following restricted system:

$$\begin{pmatrix} Z_t^\mu \\ Y_t \end{pmatrix} = \begin{pmatrix} c^z \\ c^y \end{pmatrix} + \sum_{\ell=1}^{N} \begin{pmatrix} 0 & B_\ell^{zy,\ell} \\ 0 & B_\ell^{yy,\ell} \end{pmatrix} \begin{pmatrix} Z_{t-\ell}^\mu \\ Y_{t-\ell} \end{pmatrix} + \begin{pmatrix} u_t^z \\ u_t^y \end{pmatrix} \tag{B.8}$$

[34] In this part only ψ denotes the hyperprior to be consistent with the original notation of Giannone et al. (2015).

	Unrestricted	Restricted (SVAR-IV)
λ	0.2202	0.2206
Log Marg. Lik.	-518.1520	-519.0676

Table 2: Estimated prior tightness (λ) and marginal likelihood. The restricted specification sets the coefficients of lagged values of the proxy to zero.

that is, we impose that the coefficients of lagged Z_t^μ are 0 in all equations. Thus, $\check{y}$ is the same as above, but $\check{x}$ does no include any lags of Z_t^μ.

For the Y_t equations, this is required by the assumption of invertibility: indeed, as shown in Plagborg-Møller and Wolf (2022), the shock is invertible if and only if it does not Granger cause macro outcomes. For the Z_t^μ, this is an assumption just made out of convinience: this way, all equations have the same regressors and thus we can use exactly the same formulas as above. We note that the constructed Z_t^μ is serially uncorrelated and that the marginal likelihoods of the restricted and unrestricted BVARs are quite close, thus this restriction does not seem to be relevant.

Finally, for any given draw of β, Σ, we can estimate impulse responses to noise shock as follows: for the unrestricted case, we just use a Cholesky decomposition. For the restricted case we can create a C matrix such that $\varepsilon = Cu$ using the fact that $\varepsilon_t^u = \frac{Cov(u_t^z, u_t^y)'[Var(u_t^y)]^{-1}}{Cov(u_t^z, u_t^y)'[Var(u_t^y)]^{-1}Cov(u_t^z, u_t^y)}\hat{u}_t^y$ for the first row, and then filling out the rest by orthogonalizing recursively. We then recover the rotation of interest by simply inverting C.

B.3. Additional Empirical results

B.3.1. Additional Estimation Results

Table 2 shows the estimated prior tightness (λ) as well as the log marginal likelihood. The degree of tightness is similar to the ones in Giannone et al. (2015). Both specifications yield very similar results in terms of fit, indicating that lagged z_t do not meaningfully improve fit and thus have their coefficients shrunk to 0. Figures 11 and 12 show impulse responses and variance decompositions for all macro variables included in the VARs.

B.3.2. Historical decomposition

In the main text, we focus on estimating the impulse response to the noise shock and the contribution of noise to macroeconomic fluctuations. In this appendix, we analyze the historical episodes in which noise shocks played a prominent role. To this end, we perform the following exercise: we take the estimated VAR, and feed in only the identified noise shock, setting all other innovations as well as constants to zero. More specifically, we create alternative time series $\check{Y}_t$ using:

$$\check{Y}_t = \sum_{\ell=1}^{L} \hat{B}_\ell \check{Y}_{t-\ell} + \hat{p}\hat{\varepsilon}_t^\mu, \quad \check{Y}_j = 0 \text{ for } j < 0. \tag{B.9}$$

Where $\{\hat{B}_\ell\}$ are the estimated VAR coefficients, $\hat{p}$ is the vector of estimated contemporaneous effect of the structural shock on each VAR equation, and $\hat{\varepsilon}_t^\mu$ is the estimated time series for the shock. We do this for two identifying assumptions: (i) SVAR-IV, (ii) no measurement error. We omit recoverability

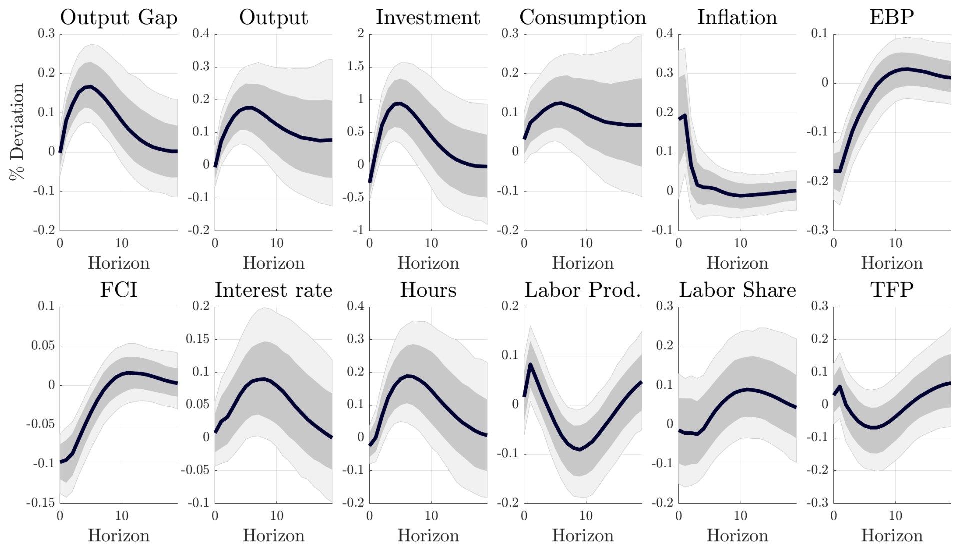

Figure 11: Impulse response to a financial noise shock for all variables. Unrestricted VAR with recursive identification. Shaded and light shaded grey bands indicate 68 and 90 credible sets respectively.

due to similarity with SVAR-IV. We emphasize that this lacks a direct counterfactual interpretation, but it is a useful accounting device to see when the shock is important in the sample.

Figure 13 shows the results of this exercise. The dashed line is the raw data, and the solid lines show the alternative time series $\check{Y}_t$ based on the SVAR-IV identified shock (in blue), and the shock identified under the perfect measurement assumption (in red). The alternative series based on the SVAR-IV assumption track the data on FCI quite well for the 1999-2008 period. Before and after, there is some relation but the gap between the data and the alternative series is wider. Something similar happens with the output gap: the blue line tracks the data very well during the 1999-2008 period, just before the GFC. During the GFC, the noise shock explains some of the drop, but it is far from explaining the full depth and slow recovery from the recession. Similar patterns are observed with interest rates and inflation. Overall, the SVAR-identified shock explains most of the FCI and output fluctuations in the 1999-2008 period, less so before and after. This is consistent with the narrative that attributes macro fluctuations preceding the GFC to exuberance in financial markets, associated with positive noise shocks. When the GFC happened, this is partially amplified by negative noise shocks, but other factors (such as rising risk premium and the binding ZLB) also must have played a role, since financial noise shocks alone cannot explain the data after the GFC. Focusing on the shock identified under the perfect measurement assumption, the overall patterns point to the same direction, but magnitudes are smaller. This may reflect measurement error, in particular the fact that the Gabaix and Koijen (2021) shock is only a subset of all financial noise shocks.

B.3.3. Robustness for impulse response estimation

Figures 14 and 15 show the estimated IRFs when we control for 3 and 4 Principal components (respectively) in equation (B.2). As we can see, results are virtually identical, which is strong evidence that the procedure followed adequately controls for aggregate factors in this setting (Gabaix and Koijen, 2024).

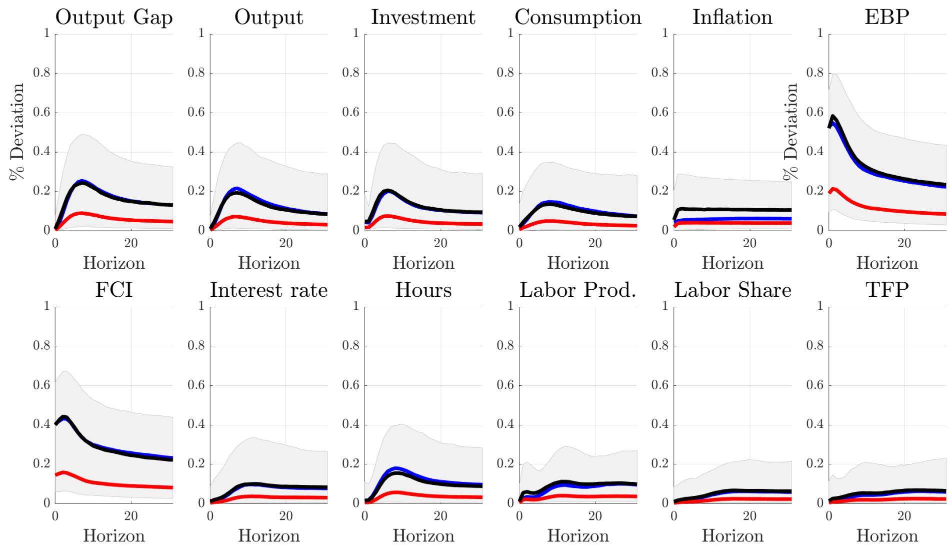

Figure 12: Identified Forecast Variance Ratios of for the noise shock. Blue: SVAR-IV, assuming invertibility. Red: lower bound, assumes perfect measurement of the shocks. Black: recoverability-based FVR. The three lines are the medians, horizon by horizon, of each object. Light grey areas are 90% credible regions of the identified set, (Plagborg-Møller and Wolf, 2022)

Figure 16 shows the results when we further residualize by changes in consumer sentiment in the construction of the shock. Figure 17 shows the results when we use core PCE instead of headline to estimate the VAR coefficients. The inflation response is much smaller, but the rest of the IRFs are essentially unchanged. Figure 18 depicts the IRF estimated using an SVAR-IV procedure. Figure 19 shows the results of estimating the IRFs with local projections. In both cases, results are similar to the baseline.

B.3.4. Robustness: drivers of the financial noise shock

As it is well known, news shocks about future productivity or earnings may drive financial trade and look similar to financial noise shocks. In order to test for this competing interpretation, we look at the responses of TFP, stock prices and price-to-earnings ratios (all in logs) to the identified shock. Given that some of this shocks may take several years to unfold, we use local projections since VARs are biased at longer horizons (Li et al., 2024). As controls, we use two lags of TFP, stock prices, price-to-earnings ratios, FCI, output gap, inflation and interest rates. We use the same controls for all equations.

Figure 20 shows the results. The estimated path for FCI after the shock is essentially the same as the one estimated with the main VAR specification. The shock induces an increase in stock prices, that mean reverts back to zero in the following years. This is consistent with the shock being financial noise, that generates a transitory increase in stock prices that then reverts. TFP basically does not react to the shock in short horizons, and then reacts *negatively* after two years. This is consistent with a financial noise shock that is unrelated to TFP, and inconsistent with the view that these shocks are generated by positive news about future TFP. Finally, the price-to-earnings ratio has an initially negative reaction, which quickly reverts to zero. This is consistent with the boom in aggregate demand generated by the shock, that increases earnings in the first few quarters. Notice that if the shock was generated by news about future earnings, then the price-to-earnings ratio should increase in the short run, since prices will

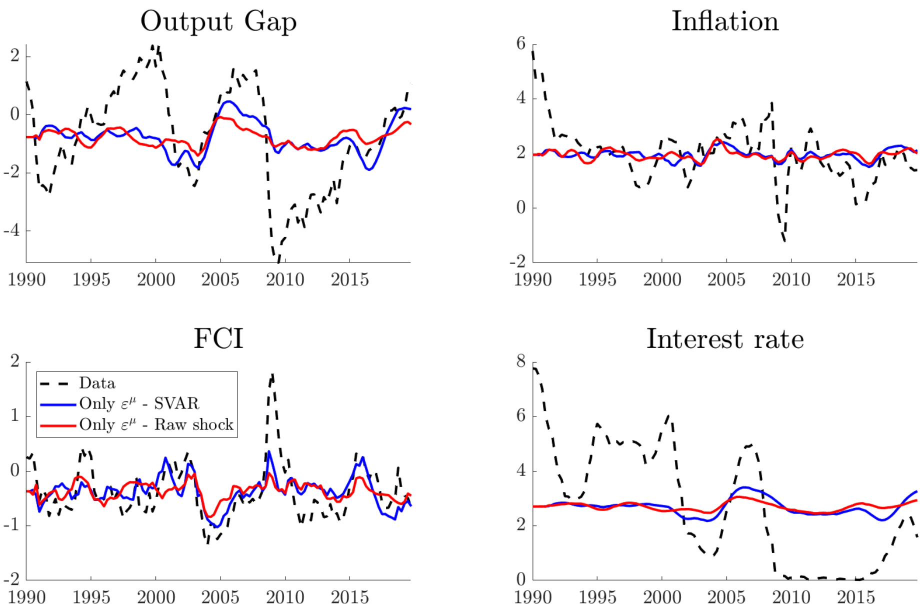

Figure 13: Alternative time series constructed by setting to zero all shocks other than the identified noise shock, following (B.9). Red: raw shock, $\tilde{Z}_t^\mu$ as in (3). Blue: shock identified using SVAR-IV as in (5). Note: inflation is reported in year-on-year terms.

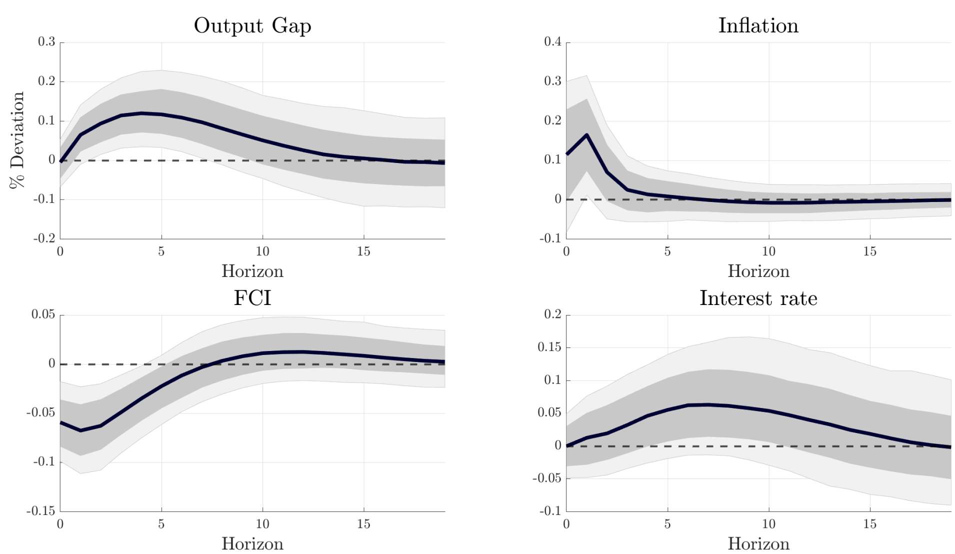

Figure 14: Impulse response to a financial noise shock, where the shock is identified controlling for 3 Principal Components in (B.2). Shaded and light shaded grey bands indicate 68 and 90 credible sets respectively.

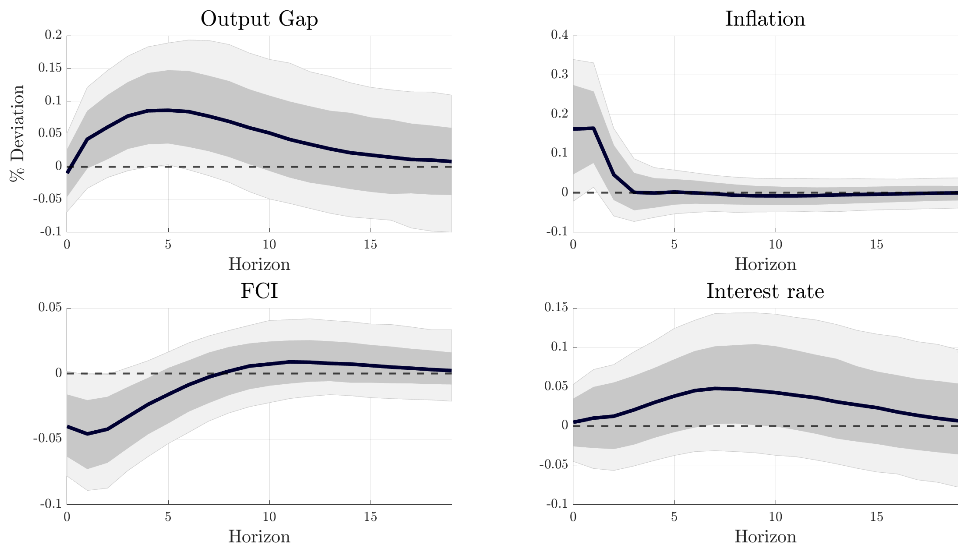

Figure 15: Impulse response to a financial noise shock, where the shock is identified controlling for 4 Principal Components in (B.2). Shaded and light shaded grey bands indicate 68 and 90 percent credible sets respectively.

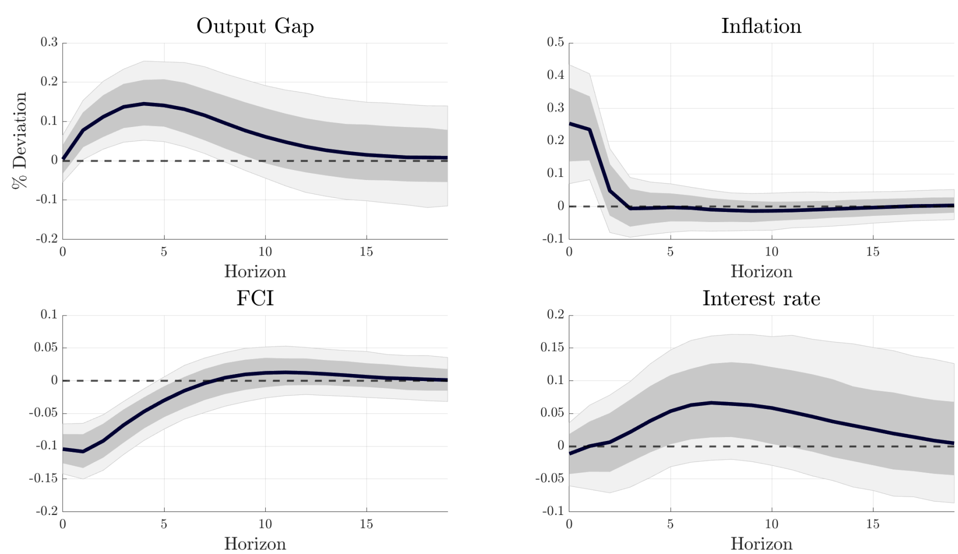

Figure 16: Impulse response to a financial noise shock, where we also residualize by changes in consumer sentiment when constructing the shock. Shaded and light shaded grey bands indicate 68 and 90 percent credible sets respectively.

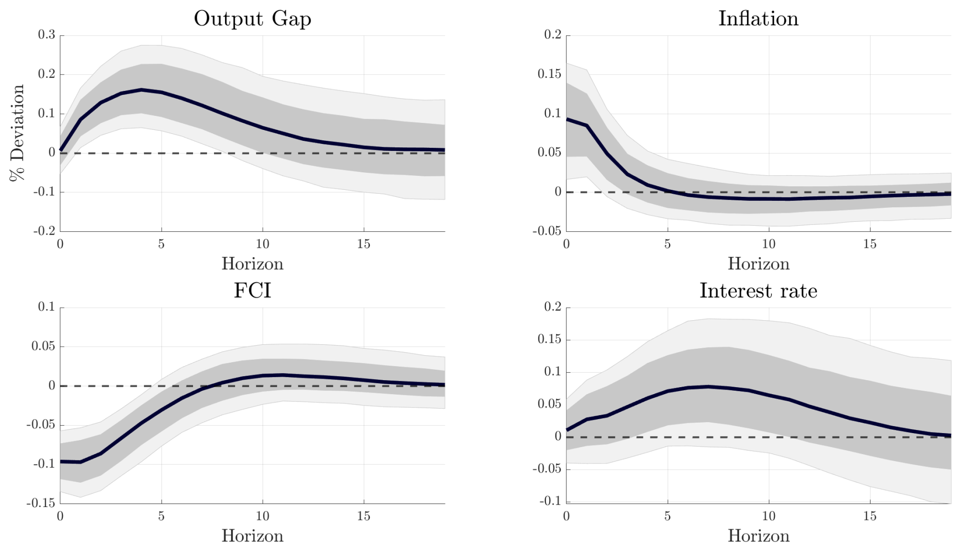

Figure 17: Impulse response to a financial noise shock, using core PCE instead of headline. Shaded and light shaded grey bands indicate 68 and 90 percent credible sets respectively.

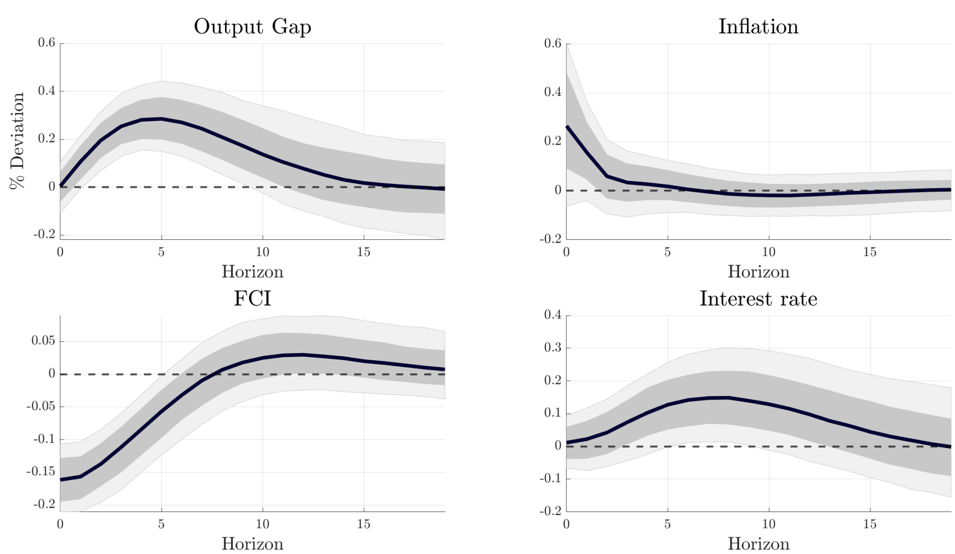

Figure 18: Impulse response to a financial noise shock, where the shock is identified using an SVAR-IV procedure. Shaded and light shaded grey bands indicate 68 and 90 percent credible sets respectively.

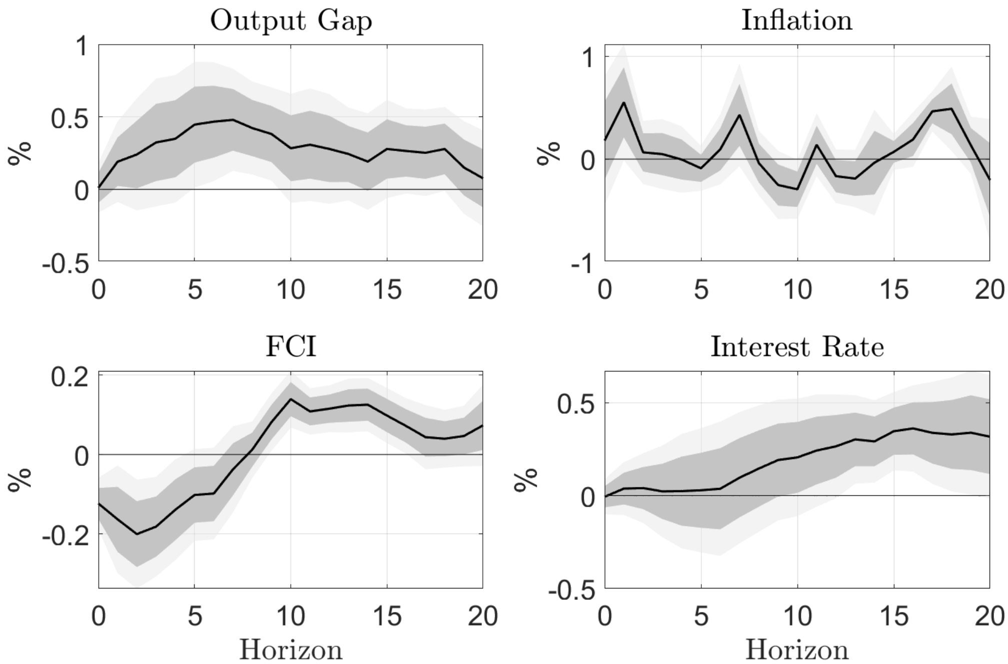

Figure 19: Impulse response to a financial noise shock, estimated via local projections. The regerssions include two lags of the outcome variable as well as FCI. Shaded and light shaded grey bands indicate 68 and 90 percent heteroskedasticity-robust confidence intervals respectively.

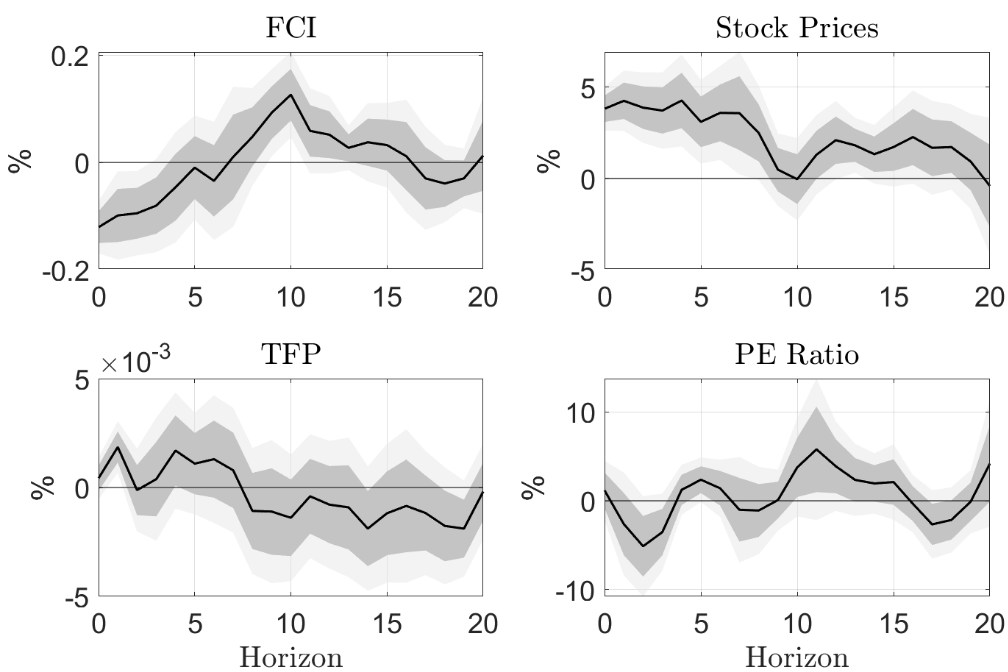

Figure 20: Impulse response to a financial noise shock for FCI, S&P 500, TFP and Price-to-earnings ratio (all in logs)

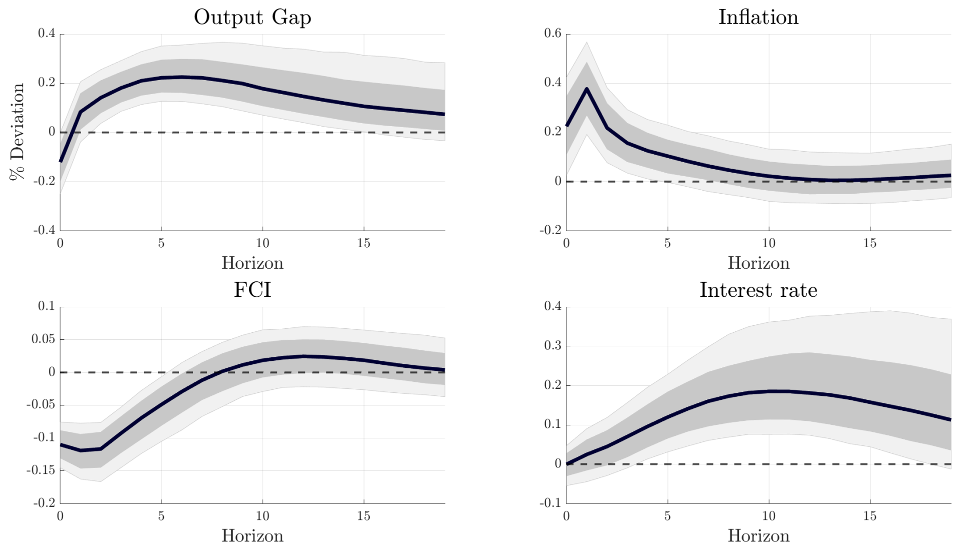

Figure 21: Impulse response to a financial noise shock, sample: 1990Q3-2024Q2. Shaded and light shaded grey bands indicate 68 and 90 credible sets respectively.

lead earnings. Empirically, the opposite happens. Overall, this additional empirical evidence supports the notion that the shock is related to financial noise, as opposed to some news about future fundamentals.

B.3.5. Robustness: extending the sample to include Covid

In order to assess the stability of our estimates to including the recent Covid period, we extend the sample to 2024Q2. Figure 21 shows the estimated impulse response. The overall patterns are unchanged: upon the shock, there are a delayed responses in the output gap and policy rates, and a faster response in inflation and FCI. There is a mild initially *negative* response. Figure 22 depicts the forecast variance ratios. The importance for output gaps is somewhat lower. This outcome is expected: the large COVID shock increases the total forecast variance in the sample. However, since this shock is orthogonal to financial noise shocks, the estimated proportional significance of the latter is reduced.

B.3.6. Monetary Policy Shocks IRFs

Figures 23 and 24 contain the impulse-response to monetary policy shocks identified by Aruoba and Drechsel (2025) and Romer and Romer (2004) respectively. The responses are standard. The Aruoba and Drechsel (2025) shock generates a small, marginally significant inflation response. The time pattern of the response of FCI is different in both specifications, with FCI spiking more strongly for the Romer and Romer (2004) IRF. Using two shocks that generate different paths allows for extra degrees of freedom when implementing the counterfactuals.

B.3.7. Counterfactual propagation of monetary policy shocks

One of the key observation of risk-centric models (Caballero and Simsek (2020)) is that monetary policy affects the economy via asset prices. Given our setting, we can test this claim empirically using the tools developed in McKay and Wolf (2023b).

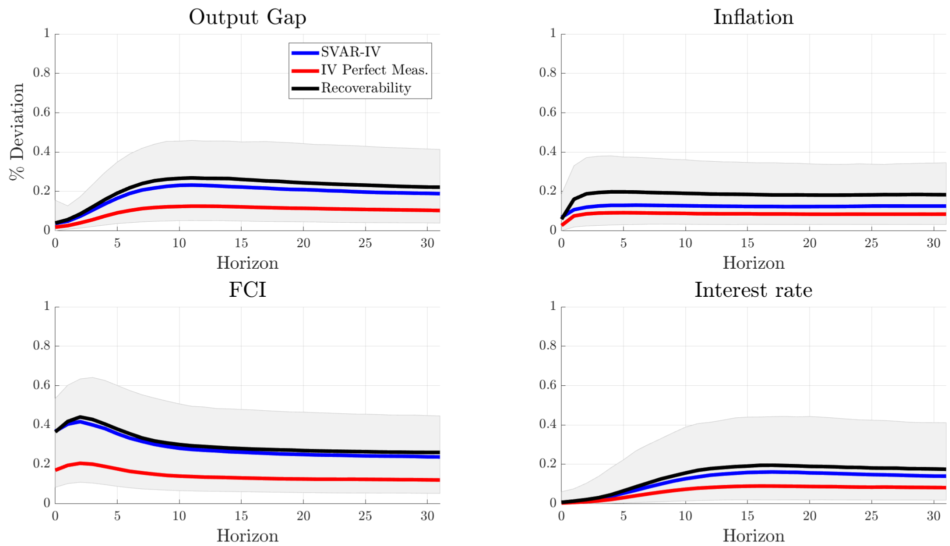

Figure 22: Identified Forecast Variance Ratios of for the noise shock, sample: 1990Q3-2024Q2. Blue: SVAR-IV, assuming invertibility. Red: lower bound, assumes perfect measurement of the shocks. Black: recoverability-based FVR. The three lines are the medians, horizon by horizon, of each object. Light grey areas are 90% credible regions of the identified set, (Plagborg-Møller and Wolf, 2022)

Specifically, we examine the following counterfactual question: how would a monetary policy shock propagate if financial conditions were unresponsive to monetary policy? To answer that question, we take the impulse-response of the Aruoba and Drechsel (2025) monetary policy shock, and use the identified response to a financial flow shock to approximately enforce $FCI_t = 0$ on impact and in expectation. Importantly, although the methodology is the same as in McKay and Wolf (2023b), this is not a *policy* counterfactual. Instead, we are asking how a given policy shock would have propagated under a different mapping between monetary policy and financial conditions.[35]

Figure 25 shows the results. As we can see, the approximation is good for the first 12 quarters, but we still get some delayed response of financial conditions at longer horizons approximation error. Crucially, the path of interest rates is basically unchanged. Turning to output gap, the response is essentially zero at all horizons. Regarding inflation, there is a *positive* initial response attributable in part to a price-puzzle-type response in the original monetary impulse-response. Then the path for inflation is reverts back to the original IRF, which features a very small, delayed response of inflation. Overall, the result is consistent with the key tenet of the risk-centric view of monetary policy: monetary policy affects the economy via financial conditions. Our results indicate that in a counterfactual economy where short-term interest rates and broader Financial Conditions are disconnected, monetary policy shocks would have essentially no impact in output gaps or inflation.

[35]The assumptions required for this to yield the correct counterfactual are analogous to the ones in McKay and Wolf (2023b): we need that financial conditions enter in the rest of the private sector equations and in the monetary policy rule only through its expected values.

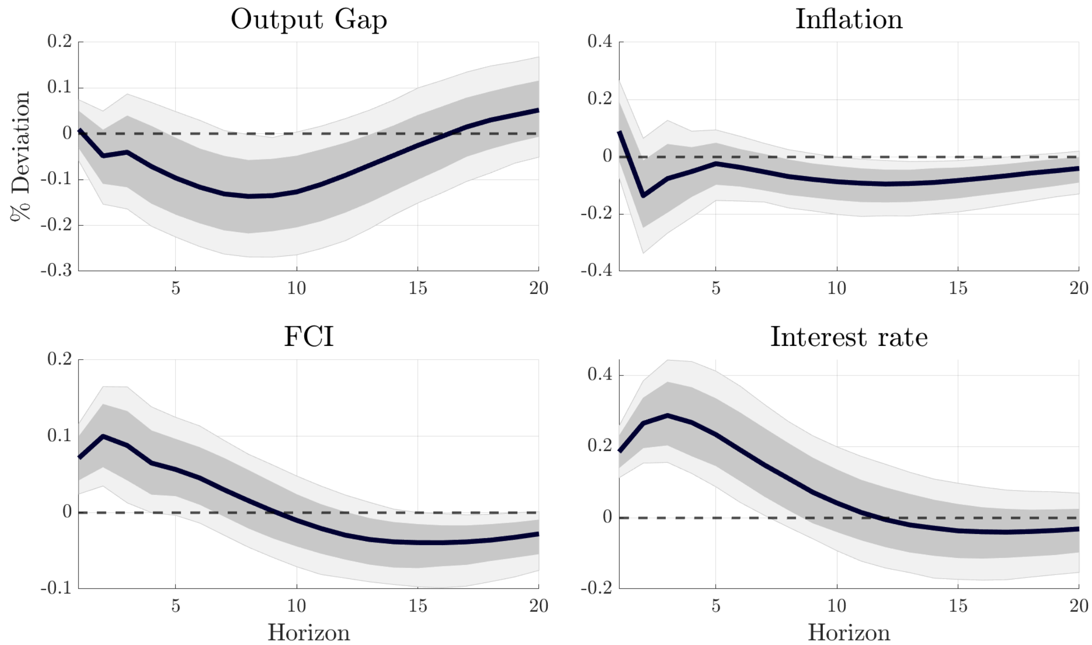

Figure 23: Impulse response to the Aruoba and Drechsel (2025) monetary policy shock. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

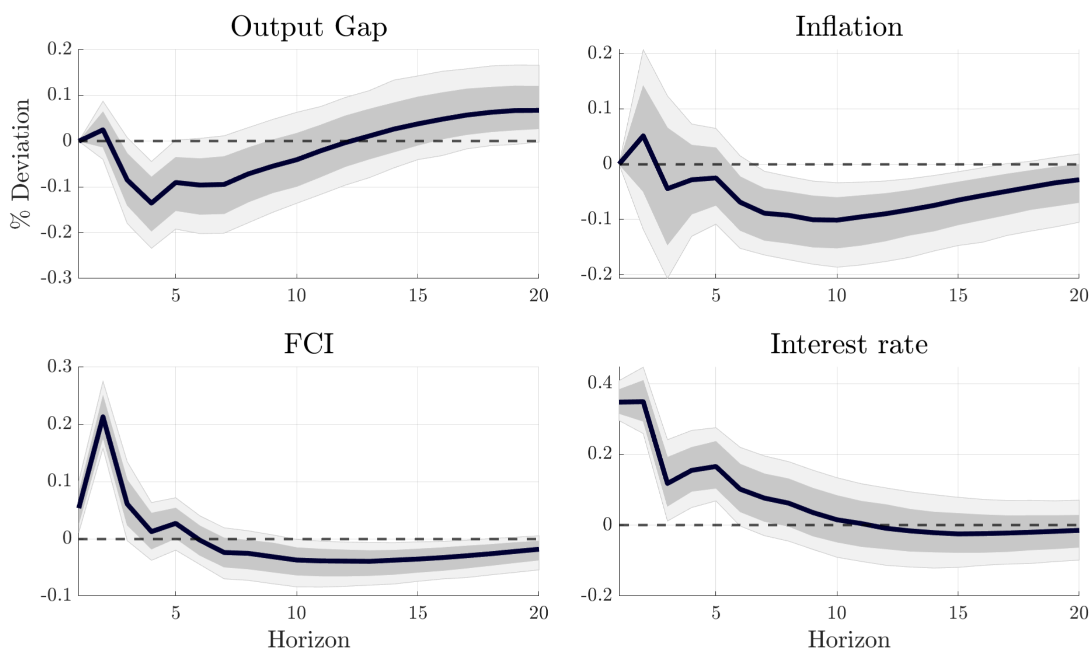

Figure 24: Impulse response to the Romer and Romer (2004) monetary policy shock. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

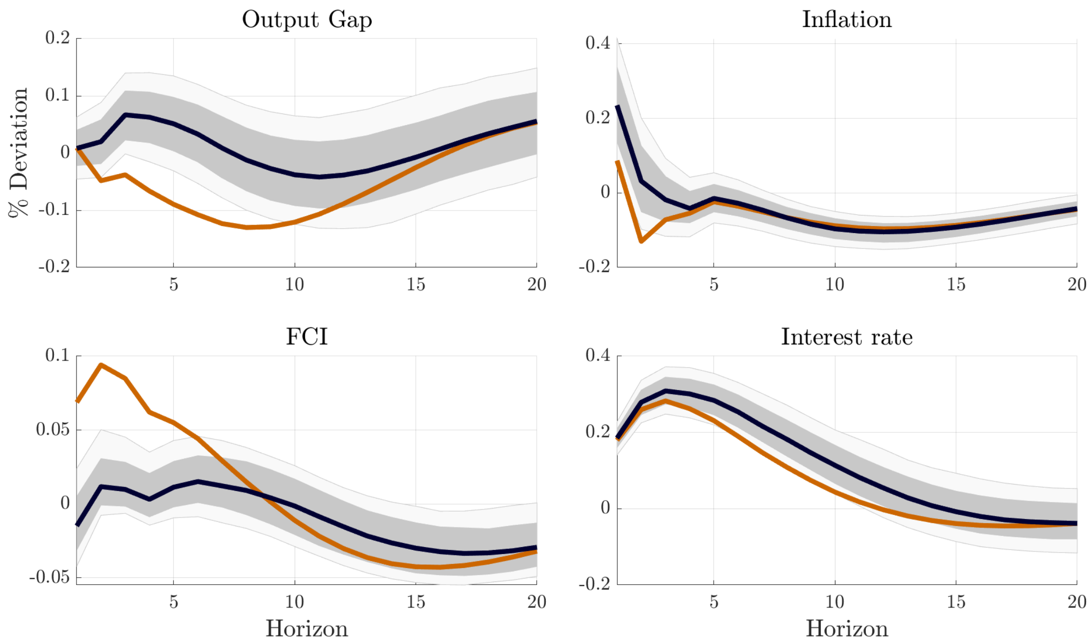

Figure 25: Counterfactual impulse response to a monetary policy shock, identified following Aruoba and Drechsel (2025). The orange line is the original point estimate, black line is the best approximation to the counterfactual impulse response. Shaded and light shaded grey bands indicate 68 and 90 confidence sets respectively.

C. Policy Counterfactuals

C.1. Proof of Proposition 4

Without loss of generality, assume that the noise shock is ordered first in the ε_t vector. Using the time series of the shock, we can partition the rotation matrix P' in two. The first column (known) $(P^{'})_{\bullet,1}$, and the rest of the matrix $(P^{'}_{\bullet,-1}$, which does not need to be fully identified. The first column identifies the financial flow shock, which is the only one whose transmission is affected by risk. This column is identifiable up to scale by regressing the Wold residuals on ε_t^{μ}. For the remaining macroeconomic shocks, we simply identify a rotation of these shocks, as argued in Caravello et al. (2025), which suffices for our purposes.

We then apply the same procedure as in Caravello et al. (2025). This yields the correct counterfactual for (a rotation of) $\tilde{\Theta}_{-\mu,\ell}$. We only have left to construct the correct $\tilde{\Theta}_{\mu,\ell}$. Applying McKay and Wolf (2023b) to $\Theta_{\mu,\ell}$ yields $\hat{\Theta}_{\mu,\ell}$, which is the solution to a unit-size shock for the impulse response system that satisfies:

$$\begin{aligned} \mathcal{F}_w\hat{\Theta}_{\mu,w} + \mathcal{F}_x\hat{\Theta}_{\mu,x} + \mathcal{F}_z\hat{\Theta}_{\mu,z} + \mathcal{F}_\mu(\sigma_r^2 \times 1) &= \mathbf{0}, \\ \mathcal{H}_w\hat{\Theta}_{\mu,w} + \mathcal{H}_x\hat{\Theta}_{\mu,x} + \mathcal{H}_z\hat{\Theta}_{\mu,z} &= \mathbf{0}, \\ \tilde{\mathcal{A}}_x\hat{\Theta}_{\mu,x} + \tilde{\mathcal{A}}_z\hat{\Theta}_{\mu,z} = \mathbf{0}. \end{aligned}$$

However, the true counterfactual solves that system with $\tilde{\sigma}_r^2$ instead of σ_r^2. By linearity of the solution, if we knew $\tilde{\sigma}_r^2$, we can obtain the true counterfactual as $\tilde{\Theta}_{\mu,\ell} = \hat{\Theta}_{\mu,\ell}\frac{\tilde{\sigma}_r^2}{\sigma_r^2}$. Finally, in order to obtain $\tilde{\sigma}_r^2$, note that the true conditional volatility satisfies:

$$\begin{aligned}
\sigma_r^2 &= \left(\theta_{r,\mu,0}/\sigma_r^2\right)^2 \sigma_r^4 + \Theta_{r,-\mu,0}\Theta'_{r,-\mu,0} \\
&= \left(\theta_{r,\mu,0}/\sigma_r^2\right)^2 \sigma^4 + \Psi_{r,0}P_{\bullet,-1}P'_{\bullet,-1}\Psi'_{r,0}
\end{aligned}$$

where $\theta_{r,\mu,0}$ is the response on impact of returns to the financial noise shock, $\Theta_{r,-\mu,0}$ is a $1\times(n_\varepsilon-1)$ row vector that contains all the responses to structural shocks other than ε_μ, $\Psi_{r,-\mu,0}$ is the analogous object for Wold innovations, and $P_{\bullet,-1}$ is a $n_y\times(n_y-1)$ matrix obtained by taking P and deleting the first column, which corresponds to the financial noise shock. Note, therefore, that the original volatility is the root of a quadratic of the form $P(x) = ax^2 - x + c$ where $a = \left(\theta_{r,\mu,0}/\sigma_r^2\right)^2$, and $c = \Psi_{r,0}P_{\bullet,-1}P'_{\bullet,-1}\Psi'_{r,0}$, and that c is the same for any rotation of the Wold shocks since $P_{\bullet,-1}P'_{\bullet,-1}$ always equals a matrix that has a zero in the (1,1) element, ones along the rest of the diagonal and zeros everywhere else, given that P is orthogonal and because of our identification assumption on $\varepsilon_{\mu,t}$, the first column and row of P are equal to the n_y vector $(1,0,\dots)$.[36]

In order to find the counterfactual conditional variance, we solve the quadratic:

$$\tilde{a}x^2 - x + \tilde{c} = 0$$

where $\tilde{a} = \left(\tilde{\theta}_{r,\mu,0}/\sigma_r^2\right)^2$ and $\tilde{c} = \tilde{\Psi}_{r,0}P_{\bullet,-1}P'_{\bullet,-1}\tilde{\Psi}'_{r,0}$ can be both constructed from the initial step.

Given that, we obtain the correct $\tilde{\Theta}_{\mu,\ell}$, and a rotation of the correct $\Theta_{-\mu,\ell}$ as in Caravello et al. (2025). This yields the counterfactual impulse response of interest. With the full $\tilde{\Theta}_\ell$ matrix, we can proceed as in Caravello et al. (2025) to obtain the counterfactuals second moments, our second object of interest. Note that, for the counterfactual historical evolution, the treatment of the initial condition is the same as in Caravello et al. (2025): since the policy change is unanticipated, whatever extra volatility the reaction to the initial condition generates is unanticipated, and moving forward future conditional volatility is not affected by this term.

C.2. Policy with a time-varying target set one period in advance

The building block of our counterfactuals is the counterfactual response to a particular shock. Intuitively, once we know how to obtain this, we can collect the response to multiple shocks to obtain a full counterfactual.

Consider the response to a shock. We assume the policy minimizes a quadratic loss. Define $\lambda_i\tilde{W}_i$ as a matrix that collects proper discount factors (in $\tilde{W}_i$) and weights (in λ_i) for variable i. For example, using $\tilde{W}_i$ with terms β^t along the diagonal defines the standard loss as in McKay and Wolf (2023a). Putting a zero in the first element of such matrix means that the planner ignores that variable in the first period. As explained, we account for the transmission lags by having a planner that targets i_0 at the natural rate in the first period (no reaction), and then optimal policy from then onwards. Let $\boldsymbol{y} = (\tilde{y}_0, \tilde{y}_1, \dots)$ denote the sequence of output gaps, $\boldsymbol{\pi}$ denote the sequence of inflation, $\boldsymbol{i}$ denote the sequence of interest rates, and $\boldsymbol{f}$ denote the sequence of FCI.

[36] In our implementation, we pick the first shock to correspond to ε_μ, and then build the rest of the rotation recursively by imposing that the shock ε_n has to be orthogonal to the past $n-1$ shocks.

The problem of the central bank can be written in two steps. First, an "operational" central bank, who picks policy to minimize its loss subject to an FCI target. Second, a "long run" central bank who optimally chooses the target for the future periods.

First, the operational central bank solves:[37]

$$\begin{aligned} &\min_{\boldsymbol{v}} \frac{1}{2} \sum_i \lambda_i [\boldsymbol{x}_i' \tilde{W}_i \boldsymbol{x}_i + 2c_i' x_i] + \lambda_f (\boldsymbol{f} - \bar{\boldsymbol{f}})' \tilde{W}_f (\boldsymbol{f} - \bar{\boldsymbol{f}}) \\ s.t. \quad & \boldsymbol{x}_i = \Theta_{x_i,v} \boldsymbol{v} + \Theta_{x_i,\varepsilon} \boldsymbol{\varepsilon}, \\ & \boldsymbol{f} = \Theta_{f,v} \boldsymbol{v} + \Theta_{f,\varepsilon} \boldsymbol{\varepsilon}. \end{aligned}$$

The first order condition is:

$$\sum_i \lambda_i \Theta'_{x_i,v} [\tilde{W}_i \boldsymbol{x}_i + c_i] + \lambda_f \Theta'_{f,v} \tilde{W}_f (\boldsymbol{f} - \bar{\boldsymbol{f}}) = 0, \tag{C.1}$$

and the shock that solves this is:

$$\begin{aligned} \tilde{v}^*(\bar{\boldsymbol{f}}) &= -\underbrace{\left(\sum_i \lambda_i \Theta'_{x_i,v} \tilde{W}_i \Theta_{x_i,v} + \lambda_f \Theta'_{f,v} \tilde{W}_i \Theta_{f,v} \right)^{-1}}_{A^{-1}} \left(\sum_i \lambda_i \Theta'_{x_i,v} [\tilde{W}_i \Theta_{x_i,\varepsilon} \boldsymbol{\varepsilon} + c_i] + \lambda_f \Theta'_{f,v} \tilde{W}_f (\Theta_{f,\varepsilon} \boldsymbol{\varepsilon} - \bar{\boldsymbol{f}}) \right) \\ \tilde{v}^*(\bar{\boldsymbol{f}}) &= -A^{-1} \left((\sum_i \lambda_i \Theta'_{x_i,v} \tilde{W}_i \Theta_{x_i,v}) \underbrace{(\sum_i \lambda_i \Theta'_{x_i,v} \tilde{W}_i \Theta_{x_i,v})^{-1} (\sum_i \lambda_i \Theta'_{x_i,v} [\tilde{W}_i \Theta_{x_i,\varepsilon} \boldsymbol{\varepsilon} + c_i])}_{-v^{**}} + \lambda_f \Theta'_{f,v} \tilde{W}_f (\Theta_{f,\varepsilon} \boldsymbol{\varepsilon} - \bar{\boldsymbol{f}}) \right) \\ \tilde{v}^*(\bar{\boldsymbol{f}}) &= -A^{-1} \left(-A v^{**} + \lambda_f \Theta'_{f,v} \tilde{W}_f \Theta_{f,v} v^{**} + \lambda_{fci} \Theta'_{f,v} \tilde{W}_f (\Theta_{f,\varepsilon} \boldsymbol{\varepsilon} - \bar{\boldsymbol{f}}) \right) \\ \tilde{v}^*(\bar{\boldsymbol{f}}) &= \tilde{v}^{**} - \left(\sum_i \lambda_i \Theta'_{x_i,v} \tilde{W}_1 \Theta_{x_i,v} + \lambda_f \Theta'_{f,v} \tilde{W}_f \Theta_{f,v} \right)^{-1} \lambda_{fci} \Theta'_{f,v} \tilde{W}_f (f^{**} - \bar{\boldsymbol{f}}) \end{aligned}$$

where $\tilde{v}^{**} = -\left(\sum_i \lambda_i \Theta'_{x_i,v} \tilde{W}_i \Theta_{x_i,v} \right)^{-1} \left(\sum_i \lambda_i \Theta'_{x_i,v} [\tilde{W}_i \Theta_{x_i,\varepsilon} \boldsymbol{\varepsilon} + c_i] \right)$ is the shock that would solve the pure dual mandate problem, and $f^{**} = \Theta_{f,\varepsilon} \boldsymbol{\varepsilon} + \Theta_{f,v} v^{**}$ is the value of f^{**} that a pure dual mandate central bank would choose. From now on, denote $\Theta_{\bar{f}} = \left(\sum_i \lambda_i \Theta'_{x_i,v} \tilde{W}_i \Theta_{x_i,v} + \lambda_f \Theta'_{f,v} \tilde{W}_f \Theta_{fci,v} \right)^{-1} \lambda_f \Theta'_{f,v} \tilde{W}_f$.

Secondly, we have the "long-run" central bank, who chooses $\bar{\boldsymbol{f}}$ in order to minimize the loss, conditional on their timing constraints:

$$\begin{aligned} &\min_{\bar{\boldsymbol{f}}} \frac{1}{2} \sum_i \lambda_i [\boldsymbol{x}_i' \tilde{W}_i \boldsymbol{x}_i + 2c_i' x_i] + \lambda_f (\boldsymbol{f} - \bar{\boldsymbol{f}})' \tilde{W}_f (\boldsymbol{f} - \bar{\boldsymbol{f}}) \\ s.t. \quad & \boldsymbol{x}_i = \Theta_{x_i,v} (\tilde{v}^{**} - \Theta_{\bar{f}} (f^{**} - \bar{f})) + \Theta_{x_i,\varepsilon} \boldsymbol{\varepsilon}, \\ & \boldsymbol{f} = \Theta_{f,v} (\tilde{v}^{**} - \Theta_{\bar{f}} (f^{**} - \bar{f})) + \Theta_{f,\varepsilon} \boldsymbol{\varepsilon}, \\ & R \bar{\boldsymbol{f}} = 0, \end{aligned}$$

where R is a $N \times T$ matrix that incorporates timing restrictions, in this case that $\bar{\boldsymbol{f}}$ has to be equal to

[37] In our application, the $c_i' x_i$ term only shows up in order to account for the initial conditions when we start the historical episode counterfactual. After the initial period, it plays no role.

zero in the first $N+1$ periods.[38] Forming a Lagrangian with vector of multipliers $\gamma' = (\gamma_1, \gamma_2, \dots)$, the first order condition is:

$$R'\gamma + \sum_i \lambda_i \Theta'_{\bar{f}} \Theta'_{x,v} [\tilde{W}_i \left(\Theta_{x_i,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{x_i,\varepsilon}\boldsymbol{\varepsilon}\right) + c_i] +$$
$$\lambda_f \Theta'_{\bar{f}} \Theta'_{f,v} \tilde{W}_f (\Theta_{f,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{f}) - \lambda_f \tilde{W}_f (\Theta_{f,v}(\tilde{v}^{**} - \Theta_{\bar{f}}(f^{**} - \bar{f})) + \Theta_{f,\varepsilon}\boldsymbol{\varepsilon} - \bar{f}) - \bar{f}) = 0$$

and the constraint. Working with the first equation:

$$\begin{aligned} R'\gamma + \Theta'_{\bar{f}} \left(\sum_i \lambda_i \Theta'_{x,v} [\tilde{W}_i (\boldsymbol{x}_i^{**} - \Theta_{x_i,v}\Theta_{\bar{f}}(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}})) + c_i] + \lambda_f \Theta'_{f,v} \tilde{W}_f (I - \Theta_{f,v}\Theta_{\bar{f}})((\boldsymbol{f}^{**} - \bar{\boldsymbol{f}})) \right) + \\ -\lambda_f \tilde{W}_f (I - \Theta_{f,v}\Theta_{\bar{f}})(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) = 0 \\ R'\gamma + \Theta'_{\bar{f}} \left(\sum_i \lambda_i \Theta'_{x,v} [\tilde{W}_i \boldsymbol{x}_i^{**} + c_i] \right) - \Theta'_{\bar{f}} (\sum_i \lambda_i \Theta'_{x,v} \tilde{W}_i \Theta_{x_i,v} + \lambda_f \Theta'_{f,v} \tilde{W}_f \Theta_{f,v}) \Theta_{\bar{f}} (\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) + \\ \lambda_f \Theta'_{\bar{f}} \Theta'_{f,v} \tilde{W}_f (\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) - \lambda_f \tilde{W}_f (I - \Theta_{f,v}\Theta_{\bar{f}})(\boldsymbol{f}^{**} - \bar{\boldsymbol{f}}) = 0 \end{aligned}$$

Define the following matrices:

$$\begin{aligned} A_1 &= -\left(\Theta'_{\bar{f}} (\sum_i \lambda_i \Theta'_{x,v} \tilde{W}_i \Theta_{x_i,v} + \lambda_f \Theta'_{f,v} \tilde{W}_f \Theta_{f,v}) \Theta_{\bar{f}} + \lambda_f (\tilde{W}_f - \tilde{W}_f \Theta_{f,v}\Theta_{\bar{f}} - \Theta'_{\bar{f}} \Theta'_{f,v} \tilde{W}_f) \right), \\ A_2 &= \Theta'_{\bar{f}} \left(\sum_i \lambda_i \Theta'_{x,v} [\tilde{W}_i \boldsymbol{x}_i^{**} + c_i] \right), \end{aligned}$$

then the equations can be written more compactly as:

$$-A_1(\bar{\boldsymbol{f}} - \boldsymbol{f}^{**}) + A_2 + R'\gamma \quad = \quad 0, \tag{C.2}$$
$$A_1(\bar{\boldsymbol{f}} - \boldsymbol{f}^{**}) \quad = \quad (A_2 + R'\gamma). \tag{C.3}$$

If the matrix A_1 is invertible, we can solve for $\bar{f}$ as:

$$\bar{\boldsymbol{f}} = \boldsymbol{f}^{**} + A_1^{-1}[A_2 + R'\gamma],$$

and then using the constraint:

$$0 = R\bar{\boldsymbol{f}} \quad = \quad R\boldsymbol{f}^{**} + RA_1^{-1}[A_2 + R'\gamma] \tag{C.4}$$
$$\gamma \quad = \quad -\left[RA_1^{-1}R'\right]^{-1}[R\boldsymbol{f}^{**} + RA_1^{-1}A_2] \tag{C.5}$$

which fully characterizes the target.

If transmission lags are included, then A_1 is not invertible. In particular, if the Central Bank reacts with a lag of N periods, the first N rows and columns are zeros. Furthermore, the first $N \times N$ submatrix of A_2 is also full of zeros. Thus, this implies that we can solve this by setting $\gamma_1, \dots, \gamma_N = 0$, and then

[38]Take, for example, $N = 1$. In the first period, the target is preset at the SS value of 0. The target in the second period is chosen in the first period *before* observing any shock, thus it also equals zero.

in find γ_{N+1} by deleting the rows and columns of zeros of A_1, A_2 and the first N equations in R. We obtain:

$$A_{1,(N+1:\bullet,N+1:\bullet)}(\bar{\boldsymbol{f}} - \boldsymbol{f}^{**}) = (A_{2,(N+1:\bullet)} + R'_{\bullet,N+1}\gamma_{N+1})$$

and then we can proceed as before to find γ. With γ, the elements $N+1, \dots$ of $\bar{f}$ are uniquely determined by C.3, and the first N elements are zeros thanks to the constraint.