Chapter 9: Obstacles to Trade, Enhanced Models of Selection, and the Impact of Policy Variation

This chapter uses structural models and data on the choice of occupation, source of funds, type of loan contract, and loan default to discern that moral hazard, limited commitment, transaction costs, and other obstacles to trade are salient features of the Thai financial landscape. This chapter tests one financial regime against another to judge which obstacle, or set of obstacles, is most important. There is regional variation. There is also evidence of incomplete markets over and above the impact of the obstacles. Policy variation and financial regime change and have rich implications in partial and general equilibrium settings for the distribution of gains and losses as well as for macro dynamics.

The model of occupational choice with heterogeneous talent is modified to make endogenous the choice of whether or not to borrow. This new model also allows investment, ROA, and insurance to be limited even for those with access to it. Various possible financial regimes are tested with the micro data in this partial equilibrium static context, taking interest rates and wages as given. Specifically, unobserved effort (moral hazard), the possibility of default (collateral/wealth backed loans), both problems together simultaneously, and exogenously limited regimes (savings only, borrowing and lending with bankruptcy but incomplete risk sharing) are each taken to the micro data on (predetermined) wealth and occupation transitions. The underlying parameters of preferences (risk aversion, work aversion), technology (marginal productivity of capital), and talent (relation to education and wealth) are estimated for each regime, and the best fit is determined from non-nested likelihood comparisons. That and auxiliary data on borrowing as a function of wealth are consistent in the prediction that either moral hazard alone or moral hazard with limited liability account well for the overall data. In the Central regional moral hazard is more clearly the dominant concern. The financial regime is less well characterized in the Northeast, and may include, for example, only liability.

Experiments with policy variation allow computation of the distribution of gains and losses to exogenous variation in (regulated on-lending) interest rates, losses (wealth transfers) to branch banks, enhanced collateral (larger borrowing limit), and movement from limited to more complete regimes (e.g., from moral hazard to full information, to be balanced by monitoring costs). The distribution of gains may be high for low wealth, talented households, as in the earlier occupation choice dual sector model, but orders of magnitude at estimated parameter values depend on the experiment and impediment to trade. For example, interest rate subsidies can yield surprisingly high gains for a few poor talented households if there is a moral hazard problem, and smaller gains for a larger group when limited liability is the problem. Still, this is not the ideal way to help the poor. In an expanded, general equilibrium context, one wants to move along a Pareto frontier, and this corresponds with direct lump sum wealth transfers, for example, allowing local banks to make losses with compensation via tax revenue from the government. To facilitate computation we imagine a small open economy in which wages and interest rates are fixed. The larger gains do come from changed access on the extensive margin, i.e., placement of funds which allow movement out of autarky, or out of savings only, into an imperfect financial regime.

One cannot distinguish in the data an information-constrained moral hazard regime from one with more limited insurance, that is, borrowing and lending with bankruptcy. A savings only regime fits the partial equilibrium, cross sectional data best in the distant past, suggesting that financial regimes have become more sophisticated over time. There is some success with the partial equilibrium lifecycle predictions; those who will eventually set up enterprises in the data save at higher rates and enter business at limited scales. Ironically, the observed wealth to occupation transition is shown in this context to be a downward-biased estimate of the gains to wealth transfers or the weakening of collateral constraints. But the models as they stand without transaction costs (or policy restrictions), do not do well with historical macro paths. That is, when limited only by wealth interacted with the constraints, these specific models with information problems or legal impediments tend to go to steady states (without growth) much too quickly.

Selection across formal and/or informal lenders can also help to quantify the importance of underlying impediments to trade and help to assess various policy options. Suppose households vary in underlying characteristics: productivity, potential scale of enterprise, wealth, and the availability of collateral. The first two are unobserved by the econometrician, as was talent earlier. Borrowing from formal lenders is at a relatively low interest rate but entails transaction costs and limited, asset backed loans. Borrowing from informal lenders is at a high interest rate but without enforcement problems. This structural model of selection is estimated via maximum likelihood methods. At estimated parameter values, transaction costs are low for the informal sector. Likewise, the most effective policies involve enhanced collateral or weakened default possibilities. This dominates placement of village funds (lower transaction costs) and interest subsides (which induce selection into business of less than talented people). But the transaction costs as estimated here are substantially lower than those which rationalized slow transition paths in the earlier endogenous financial access model.

Households can borrow as a group from the BAAC. Data on repayment rates in these joint liability groups allow, with explicit choice-based partial equilibrium static models, an assessment of the importance of obstacles to trade. There is another dimension of the moral hazard problem not mentioned earlier – joint liability partners may jointly select without outside knowledge the risk of their projects. This might be mitigated by monitoring of non-borrowing members, but there is an incentive constraint which equates the reduced likelihood of joint liability payment with the increased cost of effort. Another version of the limited commitment problem not mentioned earlier – borrowers in a joint liability group may play a strategic game in the decision of whether or not to repay, with the outcome determined as Nash equilibrium. Alternatively, there may be a cooperative, collusion solution against the lender. An additional information problem is adverse selection, that is, at given interest rates, households with safer projects decide not to borrow, depending on outside options. In all these models an increase in the (administered) interest rate lowers repayment rates while an increase in productivity (education) raises them. There are also data relevant to one model at a time: whether or not there is screening of customers, the cost of monitoring, and the magnitude of official penalties. But more often sign restrictions distinguish the models. The key dependent variable is a default rate, specifically whether the BAAC has raised its interest rate on borrowing groups. The right hand side variables include data on the magnitude of joint liability payments, correlation across project returns, cooperative behavior, loan size, and the prevalence of additional credit options. Again, the information models fit best in the Central region and overall, and more salient now, the strategic default model fits best in the Northeast.

Likewise, there are policy implications. Under certain conditions joint liability lending Pareto dominates individual lending, if the interest rate is allowed to clear the market. Enhanced penalties for default can be helpful, but if they are from the informal sector, households have to be willing to carry out ex ante threats to impose them. Likewise, cooperation among borrowers ex post can lower welfare. Lowered correlation can lower repayments in the moral hazard project selection model, a caution. Increased credit limits can also cause eventual decreases in repayment, as could the presence of additional outside lenders. This suggests a distinction between ex ante and ex post competition and the need for long term contracts and commitment.

Enhanced models of selection focus on insurance and the choice of individual versus joint liability loans. These offer tests of the presumption of moral hazard. The BAAC offers both individual as well as joint liability loans. Mutual loans and village funds allow borrowing with co-signers. We observe in the data borrower choices over these as a function of wealth and inequality. As wealth increases in the cross section, or with outsiders making larger losses, the prevalence of group loans first decreases and then increases, as in a model comparing relative performance versus group regimes. Likewise, disparity of wealth among potential joint liability borrowers increases the prevalence of joint liability loans, as in that model comparison. Note that wealth emerges again as the key state variable here, for choice of financial regime. Inequality and the distribution of wealth matter as well. Under some conditions the risk-sharing group has perfect arrangements internally but limited arrangements externally, as anticipated earlier in the discussion of sub-aggregation.

Related again is the adverse selection model and the decision of whether or not to borrow at all. One measure of a project/household type's risk conforms to the model's prediction—safe types are less inclined to borrow. Another measure that conforms is the cross sectional relationship with the correlation of project returns—high correlation across projects of members enhances joint liability loans. One Household and CDD (Community Development Department) covariates and the earlier supply side instruments are entered here as additional controls. While they are significant, the earlier results are robust. A conjecture is that the conclusions regarding adverse selection and participation on the extensive margin, as well as moral hazard and the selection into and across contracts on the intensive margin, will be robust to (exogenous) variation in financial infrastructure.

Finally, wealth, poverty, inequality, networks, and the organization of industry are co-determined with optimal financial contracts in these models. Imagine in a general equilibrium context that households can borrow or enter into insurance arrangements either as individuals or as groups. Production with a supervisor as in a credit cooperative may alleviate the moral hazard problem. All households are endowed with capital and may vary in talent for the worker or supervisory positions. Interest rates and wages (skill prices) are endogenous in a competitive equilibrium which determines the number and type of firms as well as borrowing/insurance arrangements. Supervision eliminating the moral hazard problem is costly in this model. Rather than exogenously remove the moral hazard problem, its magnitude is determined in equilibrium. Some moral hazard may or may not remain. The higher the economy is-wide capital, the scarcer labor is and the less likely it is to be used up in supervision. There can be limited commitment, the possibility of running off with financed capital. Likewise, there are welfare and impact implications for alleviating this--more inequality (or more transfers/taxes), higher firm values, lower interest rates or prices for capital, and increased wages. Moving along the Pareto frontier via wealth transfers from rich to poor makes individual production and teams with homogenous agents more likely.

Both the level of wealth and the distribution of wealth matter for equilibrium configurations. These would vary over time in a dynamic model with bequests, giving once again the dynamics of growth with inequality and poverty. More generally, networks are endogenous in these types of models. Transition from the relative performance to the group regime tends to occur when rewards exacerbate inequality, while transition from groups to relative performance tends to come with a decrease in utility promise. Thus financial contracts, networks, and industrial organization will evolve optimally in the economy; otherwise the economy will enter suboptimal states.

9.1 Distinguishing Obstacles to Trade from Occupation Choice

The prototypical moral hazard model assumes both consumption *c* and effort *z* enter into the contemporary utility function $U(c,z)$ according to

$$U(c,z)=\frac{c^{1-\gamma_1}}{1-\gamma_1}-\kappa\frac{z^{\gamma_2}}{\gamma_2} \tag{9.1.1}$$

Household risk aversion is increasing in γ_1; with $\kappa>0$ and $\gamma_2>0$, effort *z* is painful, giving the household a desire to shirk. Output of a project or business run by the household yields output $q=\theta$ with a probability of success which is increasing in effort *z* and also increasing in (positive) capital *k*, according to a Cobb Douglas specification,

$$P(q=\theta\,|\,z,k>0)=\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}} \tag{9.1.2}$$

Otherwise, $q=0$ and the project fails with the residual probability. Thus expected output has the usual features. Coefficient θ is meant to capture the underlying talent of the household, as in

$$\ln\theta=\delta_0+\delta_1\ln(A)+\delta_2\ln(1+S)+\eta\,. \tag{9.1.3}$$

where 1 is added to prevent the log of zero.

In this static specification talent is allowed to be correlated with wealth *A* and schooling *S*, as predetermined variables. In a larger more dynamic setting these would be endogenous.

The alternative to running a business is to be a wage worker, where $k=0$, with success of employment at wage w related again to effort,

$$P(q=w\,|\,z,\ k=0)=\frac{z}{1+z}\,. \tag{9.1.4}$$

As anticipated, the household decides on its occupation, as earlier, but here also on the level of finance. In sum, the choice problem is:

$$\max\left\{\begin{array}{l}\max\limits_{z} w\dfrac{z}{1+z}-\kappa\dfrac{z^{\gamma_2}}{\gamma_2}+rA \quad \text{if k= 0 (wage worker)} \\ \max\limits_{z,k} \theta\dfrac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}}-\kappa\dfrac{z^{\gamma_2}}{\gamma_2}+r\left(A-k\right) \quad \text{if k>0, k} \leq \text{A (entrepreneur, net saver)} \\ \max\limits_{z,k} \theta\dfrac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}}-\kappa\dfrac{z^{\gamma_2}}{\gamma_2}-r\left(A,\theta\right)\left(k-A\right)\dfrac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}} \quad \text{if k>0, k>A (entrepreneur, net borrower)}\end{array}\right\} \quad (9.1.5)$$

For clarity, we focus first on finance and introduce insurance subsequently. If the decision is to be a worker, then wealth A can be put into savings at gross interest rate $r=\left(1+\hat{r}\right)$. If the decision is to set up a firm, then there are two possibilities: self-finance with the excess put in savings accounts, or borrow at rate schedule $r(A,\theta)$, where repayment is possible only if the firm does not fail. The latter creates a potential divergence of incentives between borrower and lender. Lenders maximize profits, but free entry is imagined to push expected profits to zero for each wealth and talent group, on the assumption the lender has perfect information on wealth and talent. This yields:

$$r(A,\theta)\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}}=r, \ \ for \ \forall k>A, \forall\theta, \forall A. \qquad (9.1.6)$$

More generally, with risk aversion, let $c(q)$ denote the final consumption of the borrower. Then, the lender breaks-even. Constraint for each (z,k) firm unit rate for a household of wealth A:

$$\sum_{q}\left[q-\left(c\left(q\right)\right)\right]P\left(q\,|\,z,k\right)=r\left(k-A\right). \qquad (9.1.7)$$

Thus the cost of funds at exogenous outside interest rate *r* must be covered by an insurance premium and loan repayment, unless the institution takes a government subsidy (which can be analyzed by creating a gap of various sizes). The market will in effect maximize the utility of each household type $\left(A,\theta\right)$.

There are potential additional constraints on the problem. The lender may be concerned about repayment and only lend up to a proportion of wealth. Suppose, for example as in Banerjee (2001) (and the earlier discussion of informal credit) that the borrower is tempted to default but if he were to do so, he would be captured with a specified probability and forced to pay a penalty. This would yield $k \leq \lambda A$ as a possible constraint, as is assumed in Evans and Jovanovic (1989). Alternatively, effort *z* may be unobserved, in which case the borrower will choose *z* to solve its own sub-problem, with first-order condition

$$\left[\theta - r(A,\theta)(k - A)\right]\left[\frac{(1-\alpha)k^{\alpha}z^{-\alpha}}{\left(1+k^{\alpha}z^{1-\alpha}\right)^{2}}\right] - \kappa z^{\gamma_2 - 1} = 0 . \qquad (9.1.8)$$

The more general specification, allowing risk aversion, is the standard incentive compatibility constraint,

$$\sum_q U\left[c(q), z\right] P\left(q \mid z, k\right) \geq \sum_q U\left[c(q), z'\right] P\left(q \mid z', k\right) \text{ for all } z', k > 0 . \qquad (9.1.9)$$

In the analysis here we turn each constraint off and on, to discuss various financial regimes: limited liability/default alone, moral hazard alone, or both in combination.

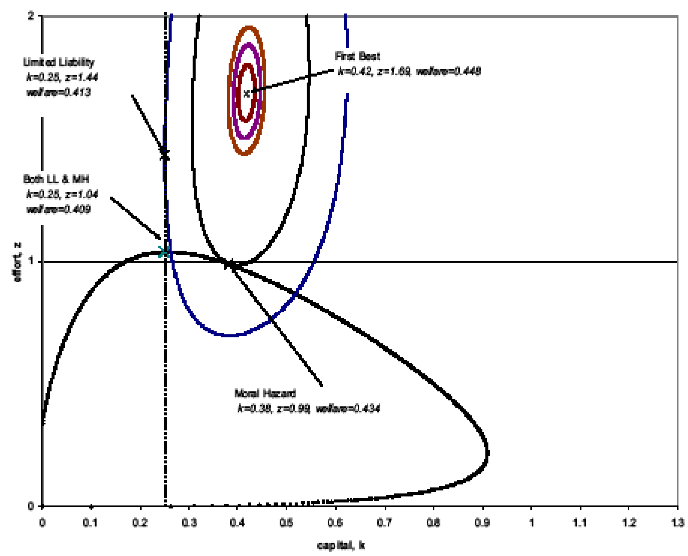


Figure 2: Assignments of Capital (k) and Effort (z) for the Entrepreneurs in the Risk Neutral Model Moral Hazard, Limited Liability and Both Moral Hazard and Limited Liability
assumptions: θ=2.56, A = 0.10, α=0.78, κ=0.08, γ2=1.00, r=1.10, λ = 2.50

[Figure 9.1.1. Configuration of Constraints. Source: Paulson, Townsend and Karaivanov (2006)]

The figure above indicates both the objective function and set of potential constraints for the case of risk neutrality, $\gamma_1 = 0$, at parameters close to those estimated in the data (see below). Utility for the household decreases as one moves away from the first best, the solution without constraints. This appears as a satiation or "bliss" point in the space of k, z combinations in the diagram. The moral hazard constraint 9.1.8 is a set of (k, z) pairs which satisfy the first-order condition and the solution to the moral hazard problem is the tangency of an indifference curve with that locus. Note the relatively low effort and high capitalization. The limited liability constraint set $k \leq \lambda A$ is on or to the left of the vertical line segment, and the limited liability solution is indicated as well; higher z and lower k. Potentially, both constraints are binding, where the two loci intersect, and in this case preferences do not matter. Increases and wealth A in the cross section move constraints northeast.

The risk neutral model described above includes special cases which have been studied in the literature. For example, Evans and Jovanovic (1989) can be derived by first eliminating a role for entrepreneurial effort by setting *z* to one and setting the disutility of effort, κ, to zero. Next, assume that output is a deterministic function of capital, *k*, so that $q = \theta k^{\alpha}$ and that loans must be fully repaid in the amount *rk*, no matter what. The maximum loan size is determined by the limited liability constraint, at equality, namely $k = \lambda A$ so maximum debt is $(\lambda - 1)A$, where $\lambda \geq 1$. Apart from the normalized probabilities, these assumptions deliver exactly the limited liability model of Evans and Jovanovic. The likelihood of becoming an entrepreneur is increasing in wealth and entrepreneurial talent. Holding wealth fixed, more talented entrepreneurs are more likely to be constrained. Entrepreneurial households who face a binding limited liability constraint will borrow and invest more when wealth increases.

We can also use our framework to generate the model of Aghion and Bolton (1996). Assume that capital k can be either 0 or 1. In other words, firms must be capitalized at *k=1*. Eliminate any role for entrepreneurial talent by setting θ equal to one, and assume that the income of wage workers is unaffected by effort, or equivalently assume that *z=1* for wage workers. Finally, assume that γ_2 is equal to two, so that the disutility of effort is quadratic. Apart from the normalized probabilities, these assumptions deliver exactly the model of Aghion and Bolton. As they stress, effort, *z*, which must be incentive compatible, will be a monotonically increasing function of wealth. As wealth increases, the probability of entrepreneurial success thus increases, which means that wealthier households will face lower interest rates. On the other end, low wealth households face such high interest rates that they may choose not to borrow, they become wage workers rather than entrepreneurs. Entrepreneurial households with wealth less than one must borrow an amount equal to $1 - A$ to finance their firm, which, as noted, must be capitalized at $k = 1$. These households are subject to a binding incentive compatibility constraint. In contrast to the limited liability models of Evans and Jovanovic (1989), when wealth increases for these constrained households, they will borrow *less* (and by construction continue to invest the same amount in their firms).

More generally, solutions to the various financial regimes can be determined numerically by a linear programming problem which assigns consumption $c \in C$, output $q \in Q$, effort $z \in Z$, and capital $k \in K$ according to probability $\pi(c,q,z,k)$. This formulation allows for risk aversion. The objective function for a household of talent θ, wealth A, and schooling S is

$$\max_{\pi(c,q,z,k)\geq 0} \sum_{c,q,z,k} \pi\left(c,q,z,k\right)U\left(c,z\right) \tag{9.1.10}$$

A constraint

$$\sum_{c} \pi(c,\overline{q},\overline{z},\overline{k}) = \tilde{p}(\overline{q} \mid \overline{z},\overline{k}) \sum_{c,q} \pi(c,q,\overline{z},\overline{k}) \text{ for all } \overline{q},\overline{z},\overline{k} \tag{9.1.11}$$

ensures that what appears to be an endogenous likelihood of output *q* is in fact consistent with the technology in nature, $\tilde{p}(\overline{q} \mid \overline{z},\overline{k})$; that is, if $k > 0$, $q = \theta'$ for firms, and $q = w$ for wage earners. Also,

$$\sum_{c,q,z,k} \pi(c,q,z,k)(c-q) = r \sum_{c,q,z,k} \pi(c,q,z,k)(A-k) \tag{9.1.12}$$

is again the zero profit or break-even constraint for the financial sector. Again,

$$\sum_{c,q} \pi(c,q,z,k)U(c,z) \geqq \sum_{c,q} \pi(c,q,z,k) \frac{\tilde{p}(q \mid z`,k)}{\tilde{p}(q \mid z,k)} U(c,z`) \text{ for all } k > 0, z, z' \tag{9.1.13}$$

is the moral hazard constraint where the household contemplates various alternative (lower) efforts *z'*. The limited liability constraint is imposed by varying the grid of possible capital assignments *k* given wealth *A* and specified parameter λ. Technological probability $\tilde{p}\left(q \mid z,k\right)$ comes from the earlier specification, that is, if positive, $q = \theta$ for firms and $q = 1$ for wage earners.

	Moral Hazard	Limited Liability	Both
1. Risk Aversion, Talent (Income)			
γ_1	0.0985	0.0982	0.1025
	(0.0125)	(0.0003)	(0.0046)
γ_2	2.1007	1.1713	2.4753
	(0.3216)	(0.0037)	(0.1797)
κ	0.1257	0.1079	0.1190
	(0.0227)	(0.0003)	(0.0062)
α	0.7775	0.6937	0.7208
	(0.0325)	(0.0165)	(0.0108)
λ	--	22.9885	20.8082
		(0.0727)	(1.4882)
Log Likelihood	-0.4038	-0.4706	-0.4683
2. Risk Neutral, Talent (Income)			
γ_2	1.5801	1.3475	1.5511
	(0.0243)	(0.0167)	(0.0171)
κ	0.0530	0.0675	0.0789
	(0.0009)	(0.0009)	(0.0008)
α	0.7700	0.6800	0.6902
	(0.0099)	(0.0273)	(0.0043)
λ	--	24.5000	28.3848
		(0.3307)	(0.3095)
Log Likelihood	-0.4104	-0.4608	-0.4372
3. Risk Aversion, Talent (% Entrepreneur)			
γ_1	1.0737	0.0668	0.7781
	(0.0123)	(0.0004)	(0.0035)
γ_2	1.0000	1.0000	1.0000
	(0.0192)	(0.0141)	(0.0105)
κ	0.0904	0.0722	0.1219
	(0.0001)	(0.0001)	(0.0016)
α	0.9780	0.9702	0.5062
	(0.0032)	(0.0003)	(0.0066)
λ		10.7281	1.9014
		(0.0305)	(0.0042)
Log Likelihood	-0.4590	-0.7514	-0.6064
4. Risk Aversion, Estimated Talent			
γ_1	0.5753	0.0957	0.1002
	(0.0175)	(0.0002)	(0.0005)
γ_2	1.0494	1.2314	1.0939
	(0.0171)	(0.0120)	(0.0061)
κ	1.2312	0.9889	1.0022
	(0.0649)	(0.0049)	(0.0065)
α	0.7931	0.2283	0.7985
	(0.0148)	(0.0030)	(0.0188)
δ_0	1.0175	0.8853	0.1002
	(0.0464)	(0.0108)	(0.0007)
δ_1	0.0604	0.0285	0.0503
	(0.0218)	(0.0002)	(0.0004)
δ_2	0.0516	-0.2226	0.3005
	(0.0053)	(0.0046)	(0.0018)
λ	--	21.0118	5.0088
		(0.2223)	(0.0970)
Log Likelihood	-0.3996	-0.4134	-0.4035

[Table 9.1.2. Whole Sample Structural Estimates, Bootstrap Standard errors in Parentheses. Parameter estimates allows various specification talent/income. Source: Paulson, Townsend and Karaivanov (2006)]

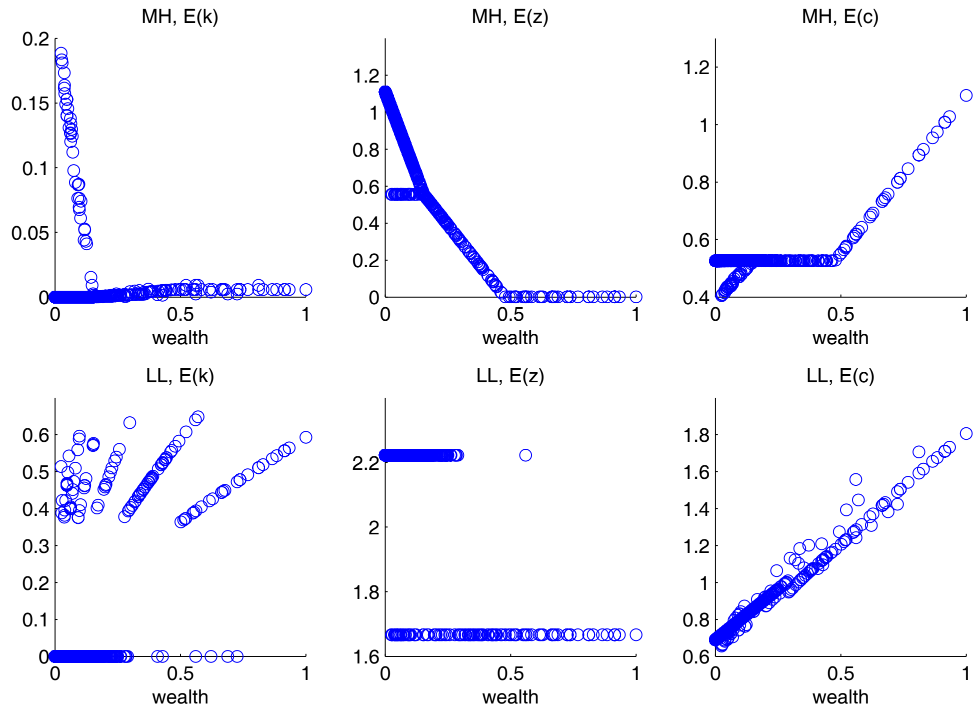
The various regimes are estimated with the Townsend Thai 1997 retrospective data, utilizing observed occupation transitions into business, initial wealth A, and schooling S as in the earlier occupation choice model. The underlying parameters are found as those that maximize the likelihood of the data, as created by solution to program. The likelihood function uses the optimal contract $\pi^*(c,q,z,k \mid \theta, A, S)$, computed by calling to the linear programming commercial library CPLEX. We obtain the probability of being an entrepreneur, namely those with positive k,

$$\pi^E(\theta, A, S) \equiv \sum_{c,q,z,k} \pi^*(c,q,z,k \mid \theta, A, S, k>0). \qquad (9.1.14)$$

The probability of being a worker is simply $1-\pi^E(\theta, A, S)$. Talent shock η is not seen by the econometrician, so a function $\bar{\pi}^E(A,S)$ is computed as integral, taking expectations over 9.1.14 using 9.1.3. We also compute only at 20 grid points for wealth A and separately estimate a relationship between wealth and schooling. To get the probability for all data points, we use a cubic spline interpolation of $\bar{\pi}^E(A,S)$. In sum, the probability of being an entrepreneur given wealth A_i and schooling S_i for household i in the data is denoted by $\bar{H}^E(A_i, S_i \mid \psi)$, where $\psi \equiv (\gamma_1, \gamma_2, \kappa, \alpha, \delta_0, \delta_1, \delta_2, \lambda)$ is the vector of model parameters: risk aversion, curvature in effort, work aversion, productivity, talent parameters $\delta_0, \delta_1, \delta_2$, and limited liability parameter λ, if estimated. Let E_i be a binary variable, which takes the value of 1 if agent *i* becomes an entrepreneur in the data, and 0 otherwise. The log-likelihood function $L(\psi)$ for n households is given by:

$$L(\psi) = \frac{1}{n}\sum_{i=1}^{n} E_i \ln H(A_i, S_i \mid \psi) + (1-E_i)\ln(1-H(A_i, S_i \mid \psi)). \qquad (9.1.15)$$

Various alternative assumptions are made about talent, to match either observed income differentials, as begged from the earlier discussion, or the average number of firms in the data (again something which the likelihoods may miss). See Table 9.1.2. For the most part the parameters make sense, with (modest) amounts of risk aversion, relatively high marginal productivities of capital, and so on. The parameter λ is occasionally in the range of two, as theory might suggest. Talent is estimated to increase with wealth and schooling.



[Figure 9.1.3. Assignments of capital, effort, and consumption and their variation with wealth. Legend: MH = moral hazard, LL = limited liability, $E(k)$, $E(z)$, $E(c)$ are expected utilities of capital, effort, and consumption. Source: Karaivanov (2005) unpublished research note]

Beneath these estimated parameters lie the assignments of capital, effort and consumption, and their variation with wealth, according to which constraint or constraints might be causing problems. For example with limited liability capital increases with wealth but the opposite is true under moral hazard.

A. Whole Sample

	MH v. LL	MH v. Both	LL v. Both	Best Overall Fit
Risk Aversion,	MH***	MH***	Both	MH
Talent (Income)	(0.0000)	(0.0001)	(0.8866)	
Risk Neutral,	MH***	MH**	Both***	MH
Talent (Income)	(0.0010)	(0.0252)	(0.0033)	
Risk Aversion,	MH***	MH***	Both***	MH
Talent (% Entrepreneur)	(0.0000)	(0.0000)	(0.0000)	
Risk Aversion, Estimated Talent	MH***	MH	Both***	MH or Both
	(0.0046)	(0.3402)	(0.0046)	

B. Northeast

	MH v. LL	MH v. Both	LL v. Both	Best Overall Fit
Risk Aversion,	MH***	MH*	Both***	MH
Talent (Income)	(0.0071)	(0.0519)	(0.0081)	
Risk Neutral,	MH***	MH***	Tie	MH
Talent (Income)	(0.0073)	(0.0073)	(0.1018)	
Risk Aversion,	MH***	MH***	Both***	MH
Talent (% Entrepreneur)	(0.0000)	(0.0012)	(0.0000)	
Risk Aversion, Estimated Talent	MH	Both	Both	MH, LL or
	(0.4213)	(0.3718)	(0.1846)	Both

C. Central

	MH v. LL	MH v. Both	LL v. Both	Best Overall Fit
Risk Aversion, Talent (Income)	MH***	MH***	Both	MH
	(0.0003)	(0.0008)	(0.1897)	
Risk Neutral,	MH***	MH**	Both**	MH
Talent (Income)	(0.0007)	(0.0263)	(0.0133)	
Risk Aversion,	MH***	MH***	Both***	MH
Talent (High)	(0.0000)	(0.0000)	(0.0027)	
Risk Aversion, Estimated Talent	MH***	MH**	Both	MH
	(0.0004)	(0.0426)	(0.1342)	

Note: MH = Moral Hazard, LL = Limited Liability, Both = Moral Hazard and Limited Liability. The abbreviation for model which best fits the data in the pairwise comparison is reported. The p-value for the Vuong tests are in parentheses. *** indicates significance at at least the one percent level, ** at at least the 5% level and * at at least the 10% level.

[Table 9.1.4. Comparison of Financial Regimes, Vuong Test Results. Source: Paulson, Townsend Karaivanov (2006)]

A Vuong (1989) likelihood ratio test for non-nested models provides the key measure of which financial regime is closest to the actual data. Moral hazard is by and large the financial regime which does best, both overall and without exception in the Central Region. Limited liability alone cannot explain the data overall, but it may act in combination with moral hazard, and may be of greater importance in the Northeast.

Probit Estimates of Being a Net Borrower

	Whole Sample		Northeast		Central Region	
	dF/dx*	Z-statistic	dF/dx*	Z-statistic	dF/dx*	Z-statistic
Constrained (= 1 if constrained, 0 otherwise)*	0.0849	1.55	-0.0491	-0.48	0.1321	1.97
Wealth Six Years ago†	-0.0013	-0.24	0.1880	1.75	0.0007	0.12
Age of Head	-0.0115	-0.82	-0.0149	-0.58	-0.0116	-0.67
Age of Head Squared	0.0001	0.65	0.0001	0.47	0.0001	0.49
Years of Schooling – Head	0.0049	0.47	-0.0027	-0.16	0.0010	0.07
# of Adult Females in household	0.0494	1.37	0.1320	1.81	0.0268	0.62
# of Adult Males in household	-0.0701	-2.05	-0.1838	-2.64	-0.0334	-0.82
# of Children (< 18 years) in household	0.0344	1.47	0.1338	2.63	0.0059	0.21
Observed Frequency	0.5457		0.6066		0.5146	
Predicted Frequency at mean of X	0.5483		0.6367		0.5153	
Log Likelihood	-237.02		-70.50		-158.47	
Pseudo R-squared	4.70%		13.79%		4.28%	
Number of Observations	361		122		239	

Net savings is defined to be financial assets plus loans owned to household minus debt. Dummy variables are marked by an asterisk. †Wealth six years ago is made up of the value of household assets, agricultural assets and land. Number in table is estimated coefficient multiplied by 1,000,000. The sample excludes the top 1% of households by wealth. The estimates also include controls for past membership/patronage of various financial institutions and organizations.

Regression Estimates of Net Savings, Business Households

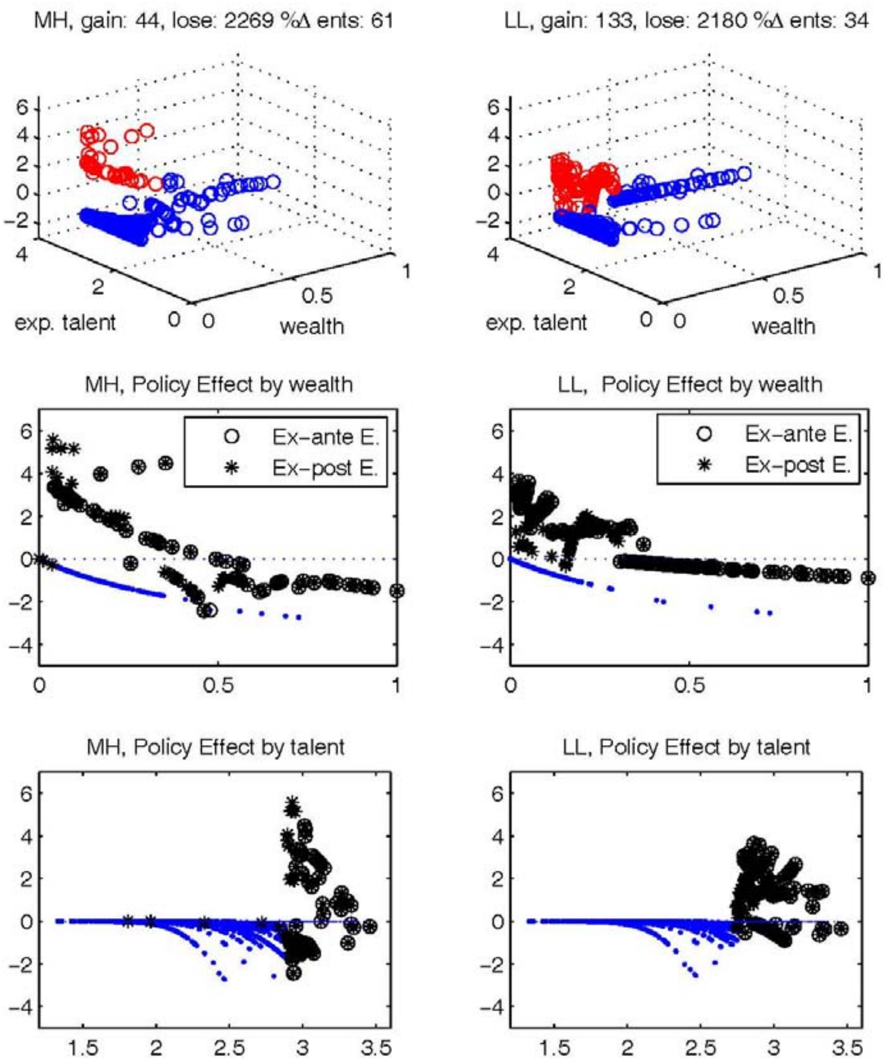
	Whole Sample		Northeast		Central Region	
	Coeff.	T-statistic	Coeff.	T-statistic	Coeff.	T-statistic
Wealth Six Years ago – Constrained Business†	0.048	4.32	-0.004	0.05	0.048	3.63
Wealth Six Years ago – Unconstrained Business†	0.012	1.42	0.383	3.31	0.012	1.19
Age of Head	9592.724	0.52	5639.596	0.29	15814.300	0.60
Age of Head Squared	-93.922	-0.56	-71.272	-0.41	-161.393	-0.68
Years of Schooling – Head	-23179.890	-1.67	-12283.410	-0.96	-28433.790	-1.35
# of Adult Females in household	-105875.200	-2.18	-133223.000	-2.59	-104812.200	-1.56
# of Adult Males in household	108636.700	2.37	60962.520	1.22	140117.500	2.22
# of Children (< 18 years) in household	37710.180	1.21	-60660.900	-1.68	64761.760	1.54
Constant	-234535.400	-0.48	121595.300	0.25	-461081.300	-0.65
Adjusted R-squared	7.86%		9.94%		8.71%	
Number of Observations	361		122		239	

Net savings is defined to be financial assets plus loans owned to household minus debt. Dummy variables are marked by an asterisk. †Wealth six years ago is made up of the value of household assets, agricultural assets and land. Number in table is estimated coefficient multiplied by 1,000,000. The sample excludes the top 1% of households by wealth.

[Table 9.1.5. Probit Estimates of Being a Net Borrower (upper) and Regression Estimates of Net Savings, Business Households (lower). Source: Paulson, Townsend and Karaivanov (2006)]

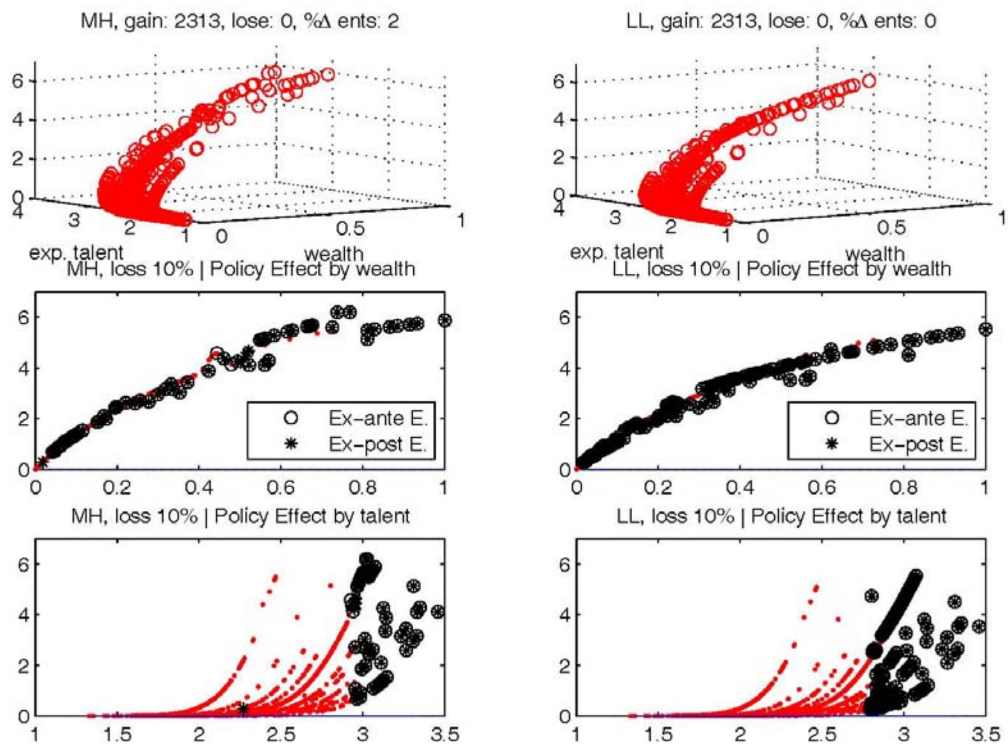
An auxiliary implication of the model is that credit should be increasing with wealth, and net savings decreasing with wealth, for constrained households in the limited liability regime. On the other hand, higher wealth would allow greater self-finance, which is a good thing in the moral hazard regime when the incentive constraint is binding – only constrained households should be borrowing. The evidence is consistent with moral hazard in the Central region. Net savings is increasing in non-financial wealth for constrained households overall and in the Central region (bottom, table 9.1.5) and being constrained is an indicator of borrowing there. These results are robust to demographic controls.

The variation in regional results begs an explanation. It may have something to do with the varying mix of lenders. In the Central region BAAC, commercial banks, and the informal sector all provide services and potentially compete with one another. Recall the earlier slides. In the Northeast, the BAAC is the dominant provider. It is as yet unclear, however, why information would be the major obstacle in the Central region and a willingness to default the major obstacle in the Northeast.



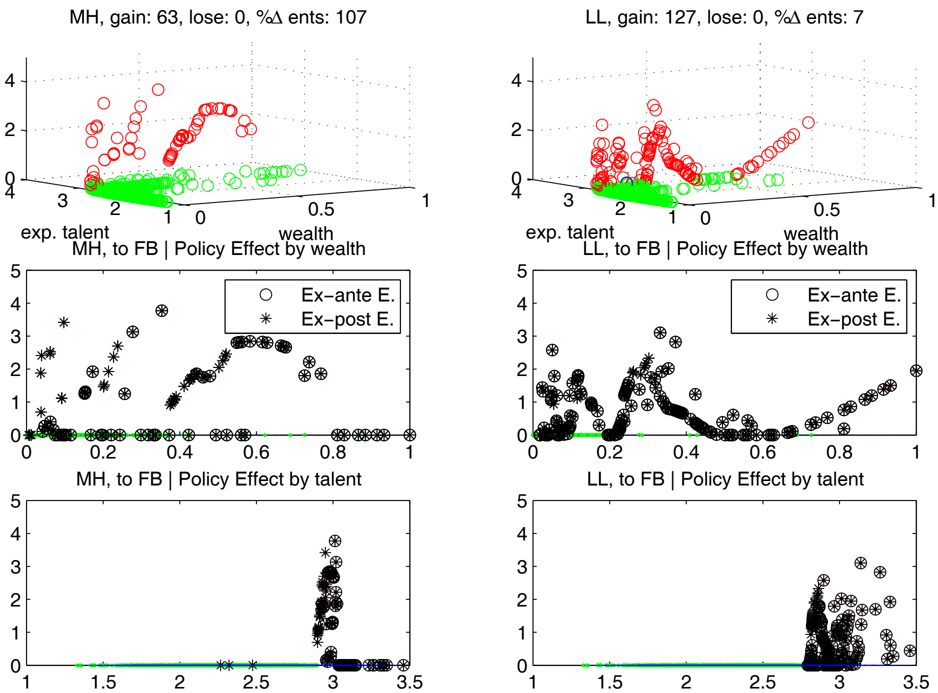
[Figure 9.1.6. Quantification of the gains and losses to various possible policy interventions. Note: MH= moral hazard, LL= limited liability, and r= 1.05 (down 1 consumption or supplements) Ex ante denotes someone who was an entrepreneur before the policy change, ex post E is someone who shifts into business as a result of policy change. Source: Karaivanov and Townsend, unpublished research note]

These structural models allow a quantification of the gains and losses to various possible policy interventions, moving from the contemporary situation at estimated parameter values for each potential regime. Suppose we were able to lower the borrowing/lending rate in this partial equilibrium setting from 1.10 to 1.05. This has a different effect depending on the regime. There is a tendency for talented poor borrowers to gain, but in the moral hazard regime the gains can be higher than under limited liability. Still, the number of winners overall is higher under limited liability. In both regimes, there are some high wealth losers. In both financial regimes, the number of entrepreneurs increases. Under the first best interest rate effects exist, but are not so dramatic.



[Figure 9.1.7. Consumption Supplements. Note: MH= moral hazard, LL= limited liability, and r= 1.05 (down 1 consumption or supplements) Ex ante denotes someone who was an entrepreneur before the policy change, ex post E is someone who shifts into business as a result of policy change. Source: Karaivanov and Townsend, unpublished research note]

Holding the interest rate fixed, we allow lump sum transfers in the sense of allowing banks to make losses (10%). This is potentially more efficient, and consumption equivalent gains are larger. Again the gains are larger for the moral hazard regime relative to limited liability. In neither regime is there any substantial change in the number of entrepreneurs.



[Figure 9.1.8. Consumption Supplements. Note: MH= moral hazard, LL= limited liability, and r= 1.05 (down 1 consumption or supplements) Ex ante denotes someone who was an entrepreneur before the policy change, ex post E is someone who shifts into business as a result of policy change. Source: Karaivanov and Townsend, unpublished research note]

Likewise eliminate moral hazard or limited liability. A move from moral hazard to full information again benefits the talented poor and is substantial enough that costly monitoring in terms of lost output might be justified. Of course in the model the talent is known by the bank, which is unrealistic. Changes in the limited liability parameter down (or up) are associated with gains (or losses) particularly for the talented poor as well. The fraction of households as firms can vary considerably, e.g., up 75%.

9.2 Exogenous Incomplete Markets

Various incomplete regimes can be considered; here we report on Karaivanov (2005). In somewhat similar notation let $p^e(z,k)$ denote the probability for an entrepreneur of successful outcome $q_h = \theta$, and the residual be the probability of failure, $q_l = 0$. $p^w(z)$ is the associated probability for a worker of getting paid a high wage, w_h, otherwise it is zero.

Under savings only the difference between wealth A and capital k is if any is put in the bank at interest rate r. Workers have nothing to finance and save A automatically. Thus the problem for entrepreneurs is:

$$\begin{aligned} &\max_{z,k} p^e(z,k)U(c_h,z)+(1-p^e(z,k))U(c_l,z) \\ &s.t. \quad c_h=\theta+r(A-k) \\ &\qquad c_l=r(A-k) \\ &\qquad 0\le k\le A \end{aligned} \tag{9.2.1}$$

and for wage earners it is:

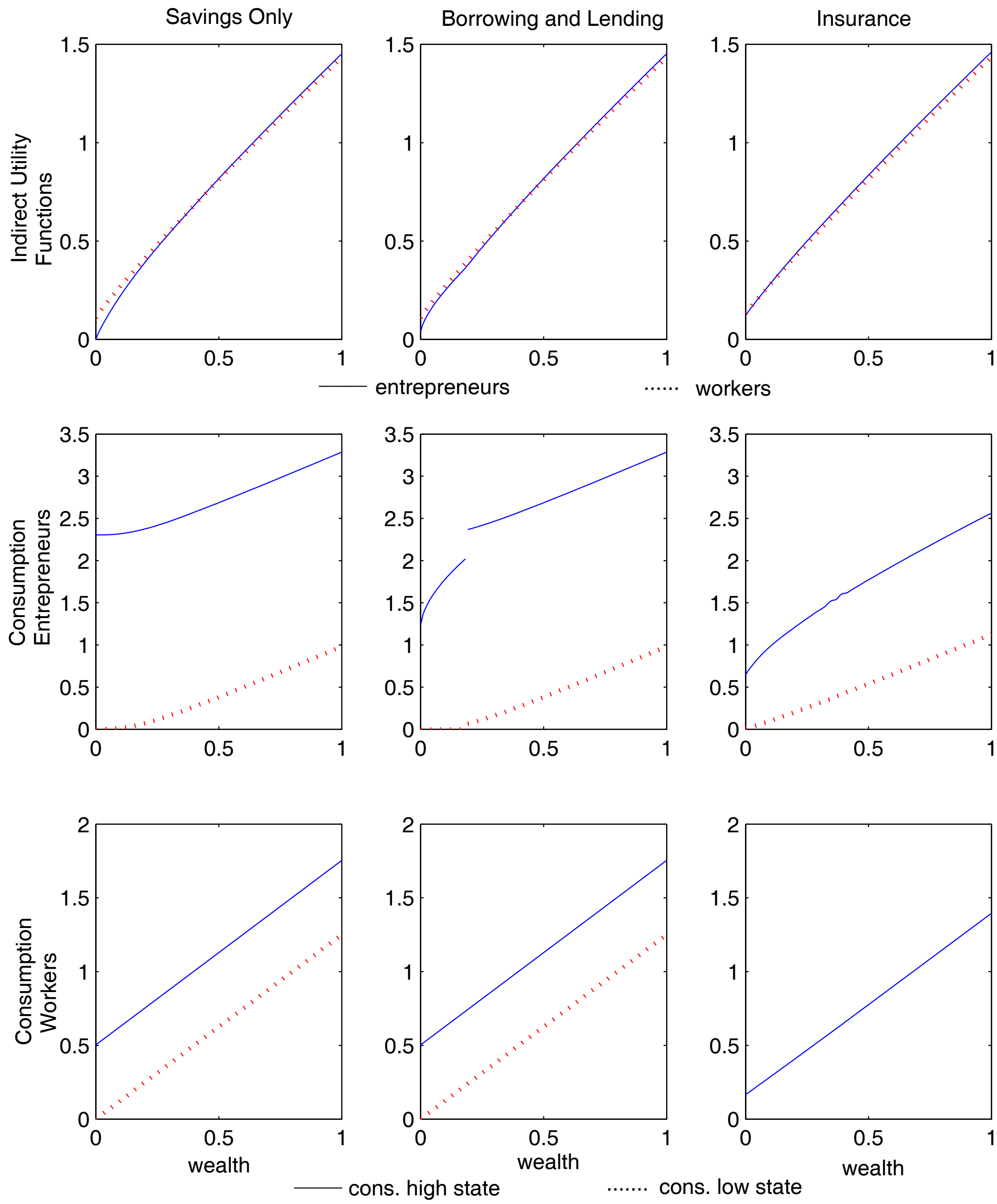
$$\begin{aligned} &\max_{z,k} p^w(z)U(c_h,z)+(1-p^w(z))U(c_l,z) \\ &s.t. \quad c_h=w_h+rA \\ &\qquad c_l=rA \end{aligned} \tag{9.2.2}$$

The borrowing/lending regime with bankruptcy is similar except that when investment k is greater than wealth A, the interest rate R is adjusted so that the bank breaks even: $R=\dfrac{r}{p^e\left(z(R),k(R)\right)}$

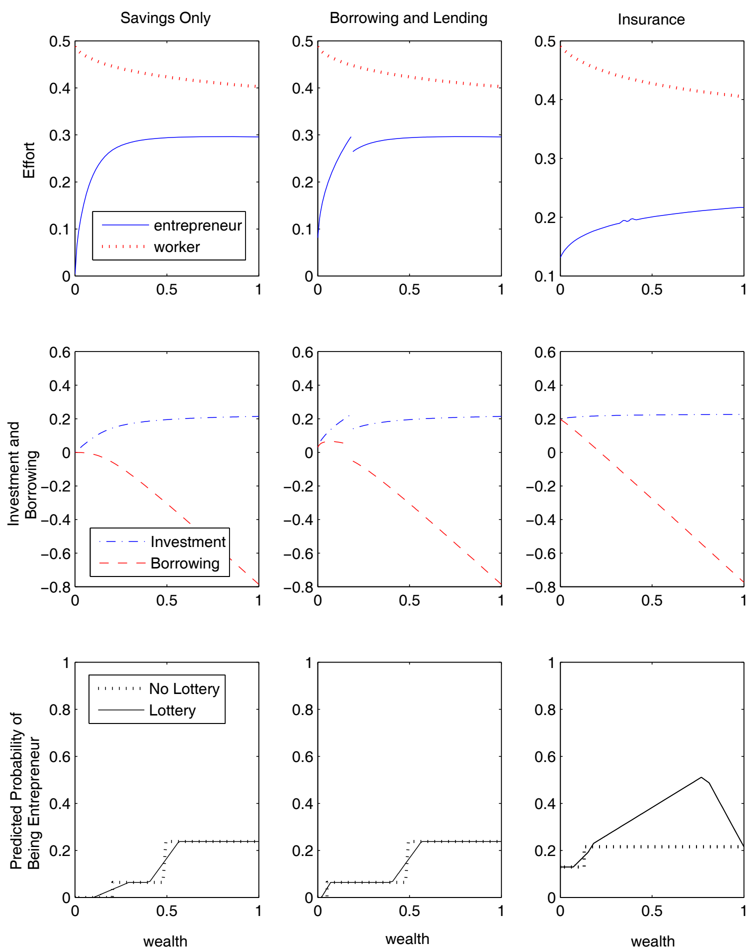
The borrower takes the rate $R=R(A,\theta)$ as given, but the modeler must solve for the rate as part of the equilibrium specification. This was considered before under risk neutrality, but if imposed under risk aversion, and contingencies are not added to the loan contract, then the financial regime is limited and exogenously incomplete. In sum, the problem is:

$$\begin{aligned} &\max_{k,z} p^e\left(z,k\right)U\left(c_h,z\right)+\left(1-p^e\left(z,k\right)\right)U\left(c_l,z\right) \\ &s.t. \quad c_h=\theta-R(k-A) \\ &\qquad c_l=0 \end{aligned} \tag{9.2.3}$$

An information-constrained moral hazard regime was considered previously, as in Paulson, Townsend and Karaivanov (PTK). For numerical examples we let preferences and technology be as in PTK and derive a likelihood which depends on parameters ϕ and on the financial regime. See Figures 9.2.1 and 9.2.2.



[Figure 9.2.1. Static Model Implications. Source: Karaivanov (2003).]



[Figure 9.2.2. Static Model Implications II. Source: Karaivanov (2003). Parameter values listed in Table 9.2.3. **Parameter,** $\gamma_1 = 0.1012, \gamma_2 = 1.5167, \lambda = 0.3934, \alpha = 0.8088,\ q = 1.0017, \theta = 2.3000, w = 0.5034,$ $s = 0.2500, r^* = 1.2500$,* The interest rate r is exogenously determined in the dynamic simulations]

Again consumption, the degree of insurance over success and failure states, effort, borrowing, investment, and the probability of being an entrepreneur all vary with wealth. See Figures 9.2.1 and 9.2.2. The profiles depend on the financial regime. Effort is generally higher in the incomplete regimes and rises quickly with wealth, as these provide less insurance and greater incentives to be diligent, even for the poor. Insurance, the difference in consumption between high and low states, increases as one moves from savings only to borrowing/lending only to the moral hazard insurance regime. Primarily, the increased insurance comes from a diminished consumption premium for success. Borrowing rises with wealth in the borrowing/lending-only regime at very low levels of wealth because investment is increasing with wealth there, but then at higher wealth, and as in the other models, borrowing declines with wealth (investment increases at a slow rate, and so savings increases and borrowing drops dramatically). The sharpest rise in the enterprise transition wealth diagram comes with the endogenously incomplete regime, so evidently this is a distinction which matters in the estimates with the data. Ex ante wealth lotteries allow for early gains in all regimes, and smooth out the enterprise wealth transition diagram.

N	% business	Stratification	Comparison Z-statistics					
			Contracts Without Lotteries			Contracts With Lotteries		
			SNL v BNL	SNL v INL	BNL v INL	SL v BL	SL v IL	BL v IL
2313	13.8%	Whole sample	-4.1471 **	-3.8197 **	0.9661	-2.5688 **	-1.9946 **	0.9102
1091	19.2%	Central	-1.5358	-1.6860 *	-0.6282	-1.7124 *	-0.7806	1.1583
1222	9.1%	Northeast	-1.1582	-1.2779	-0.1041	-0.6869	-2.0217 **	-0.2764
1157	12.6%	Wealth below median	-0.1354	-0.1906	-0.1849	0.0357	-0.9423	-1.0632
1156	15.1%	Wealth above median	-0.5959	0.3785	0.3952	-0.2858	0.5400	0.5503
455	9.0%	Education < 4 yrs	-0.6780	-0.5392	-0.1611	-0.1069	-0.0424	0.0920
1554	13.8%	Education = 4 yrs	-4.1975 **	-4.2849 **	0.9752	-2.1029 **	-1.7535 *	0.8791
304	21.1%	Education > 4 yrs	0.1908	0.3121	0.0463	0.3138	0.4651	0.2171
1927	12.8%	No formal credit	-3.9320 **	-3.2705 **	0.4407	-0.6726	0.3766	1.7073 *
386	19.2%	Formal credit	-0.5545	-0.7354	-0.1882	-0.1079	0.0152	0.1033
1388	16.1%	Any debt	-3.1712 **	-3.2053 **	0.3873	-1.8734 *	-1.2939	0.9408
925	10.4%	No debt	-0.6656	-0.2876	0.9157	-0.3331	-1.2774	-0.4402
231	15.1%	Northeast, BAAC	-1.1406	-0.5817	-0.2598	-0.2440	-1.2935	-0.5956
150	26.0%	Central, BAAC	-2.8807 **	-2.4783 **	-0.6522	0.5397	0.6003	-0.1715

[Table 9.2.3. Source: Karaivanov (2003) (S = saving, B = borrowing, I = endogenous incomplete insurance, NL = no lottery, L = lottery; negative value indicates second regime dominates in the comparison.)]

The savings only regime is typically rejected in the Townsend Thai data. See Table 9.2.3. But it is difficult to distinguish the limited borrowing/lending, *b-l* with bankruptcy regime from the endogenous insurance, moral hazard regime. In this sense, the data provides (weak) evidence that contemporary financial regimes may be incomplete. One exception may be revealing. Limiting attention to those who do not borrow formally (only borrow informally), the *b-l* limited regime fits best. This suggests that informal markets are not a good substitute for a more sophisticated formal financial market.

9.2.1 Welfare Experiments and the Dynamics of Financial Regimes

Historically, using the SES cross-section for the young in 1976, Karaivanov finds the more limited savings-only regimes fits best. See Table 9.2.1.1. This indicates more decisively the relatively low level or incompleteness of the formal and informal markets early on. As indicated, there are also gains from going from limited regimes such as savings only to more complete regimes, as if incomplete markets were filled in, in some way. But by far the largest gains come from moving from financial autarky to moral hazard or from financial autarky to limited liability regimes. If the move is from "savings only" to constrained borrowing/lending, then the range of gains is considerably less. These results suggest that access on the extensive margin may yet be the primary determinate of the distribution of welfare gains. Implicitly, obstacles to movement on the extensive margin are going to have to be large to rationalize the data.

Stratification	BL v SL	SL v IL	BL v IL
Whole sample	-0.2297*	0.7722	0.5865
Wealth below median	-0.4875	0.3914	0.1179
Wealth above median	-1.1841	0.3729	-0.7878

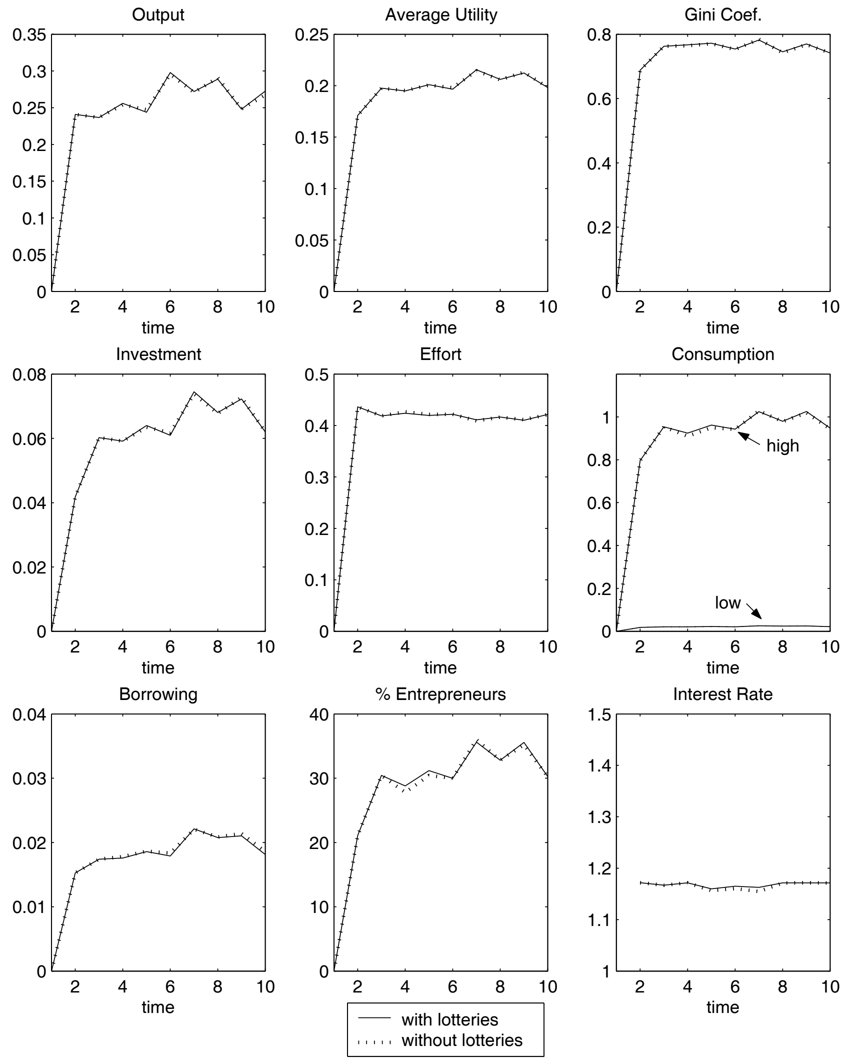
*The reported values are Z-statistics.

[Table 9.2.1.1. Model Comparisons, 1979 SES Young Household Data. Source: Karaivanov (2003)]

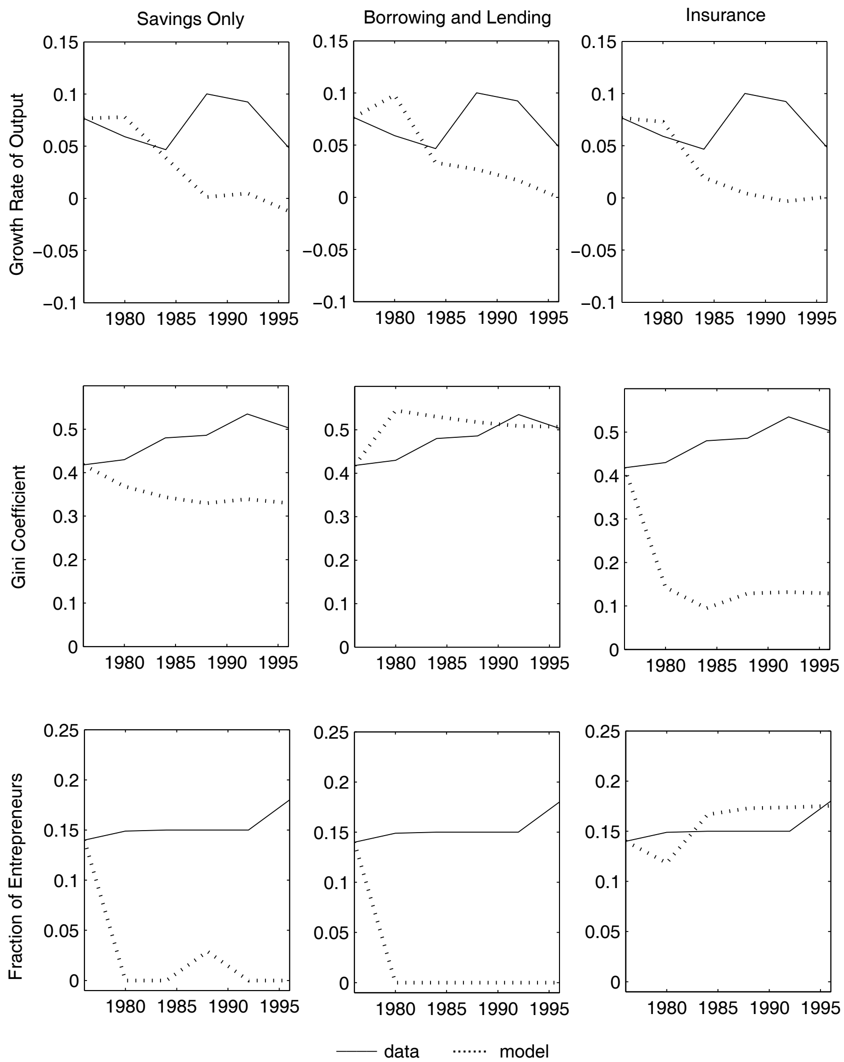
Likewise, these occupation choice financial regimes can be appended to a dynamic model and time paths examined. Given an initial distribution and parameters also from the SES, one can allow an end-of-period myopic savings rule, to link one period to the next, as in earlier occupation choice dual model. The interest rate and degree of financial deepening are endogenous too. But the dualism is attenuated. The credit market within each period is now not perfect for those with access, though access is now endogenous, i.e., autarky is self-determined.

Unfortunately, these various formal regimes converge to a steady state too fast relative to the slower transition in the data. The borrowing lending regime in figure 9.2.1.2, at estimated parameter values denoted earlier, is an illustrative simulation starting from a degenerate distribution of wealth.

More generally, we can simulate from the 1976 initial distribution of wealth.



[Figure 9.2.1.2. Dynamic Model Implications, Borrowing and Lending. (See Table 9.2.3 for parameter values.) Source: Karaivanov (2003)]



[Figure 9.2.1.3. Model Calibration, SES data (Best overall fit). Source: Karaivanov (2003)]

The savings rate is calibrated at about .25 as generating the best overall fit to historically observed paths. The micro parameters are as before. Again, the interest rate is endogenous, equalizing the supply and demand for funds. The moral hazard regime is most consistent with the observed path of entrepreneurs and the *b-l*-with-bankruptcy regime most consistent with the observed path of inequality. None of the models do well with output growth, as each predicts a growth slowdown that is not apparent in the data. We conclude from this that we have not yet hit on a compelling combination of obstacles.

9.3 Life Cycle and Occupation Choice with Limited Credit

More successful are life cycle predictions. As in Buera (2005), forward-looking behavior makes savings, hence wealth, endogenous. In his model savings, occupation transitions, the size and growth of firms, and ROA are related to each other and to talent. A typical household is imagined to maximize discounted expected utility:

$$\max_{c(t),\theta,a(t),k(t)\geq 0}\int_0^\infty e^{-\rho t}U\left(c\left(t\right)\right)dt \tag{9.3.1}$$

where

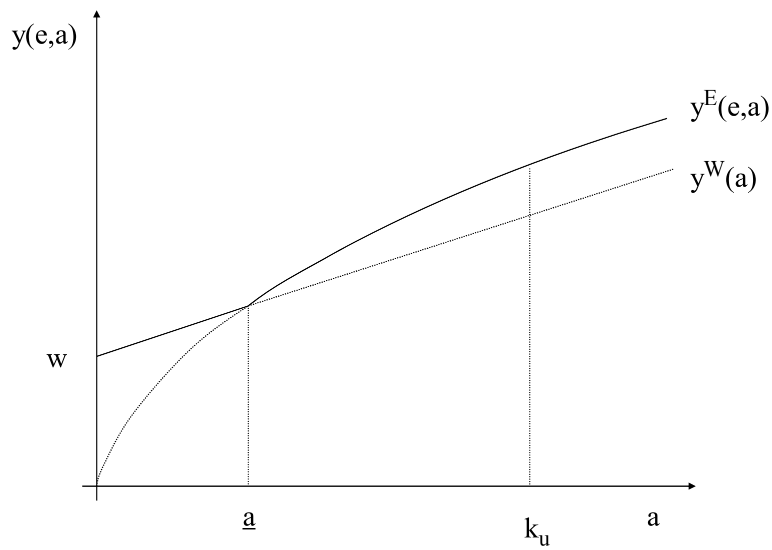
$$U\left(c\right)=\frac{c^{1-\gamma}}{1-\gamma}. \tag{9.3.2}$$

$$\dot{a}\left(t\right)=y\left(\theta,a\left(t\right)\right)-c\left(t\right) \tag{9.3.3}$$

where $p\leq r$ is the inter-temporal discount rate. To be determined is the choice of savings (change in wealth) at each data, with occupation choice contributing to earnings y, specifically,

$$y\left(\theta,a\left(t\right)\right)=\max\left\{y^e\left(\theta,a\left(t\right)\right),y^w\left(a\left(t\right)\right)\right\} \tag{9.3.4}$$

by choice of entrepreneurs, branch e or wage workers branch w, gives wealth $a\left(t\right)$ and given firm talent θ. A savings only prototype with exogenous interest *r* is easiest to envision, but one can allow credit (below $k\leq\lambda a$ with $\lambda>1$).



[Figure 9.3.1. Technologies available to households. Source: Buera (2009)]

The choice of occupation and earnings $y(\theta, a)$ is simply the choice at *any* given date which maximizes total income plus end-of-period wealth, much as in the occupation choice model discussed earlier.

$$y(\theta,a) = \begin{cases} w + ra & \text{if a } \in [0, \underline{a}(\theta)) \\ \max\limits_{k \le a} f(\theta,k) - r(k-a) & \text{if a } \in [\underline{a}(\theta), k_u(\theta)) \\ f(\theta, k_u(\theta)) + r(a - k_u(\theta)) & \text{if a } \in [k_u(\theta), \infty) \end{cases}$$

Again in the examples, Buera sets $k \le \lambda A$ with $\lambda = 1$. There is a wealth threshold. If wealth is less than $\underline{a}(\theta)$ then wage earnings dominate, though this depends on talent θ. Interestingly, this model carries a rate of return implication for firms. A household may find total end-of-period wealth greater as an entrepreneur but with limited wealth may be restricted in the scale k of the enterprise to something less than the efficient scale, $k_u(\theta)$, (which would obtain for a firm under perfect credit an interest rate *r*). See Figure 9.3.1. The range between $\underline{a}(\theta)$ and $k_u(\theta)$, and the degree of excess returns, depend on talent θ and the exact form of the production function *f*.

Suppose that the production is

$$f\left(\tilde{\theta},k\right)=A\tilde{\theta}^{1-\alpha}k^{a}.$$

Dividing by the wage w, profits relative to the wage thus define the relative gain to becoming an entrepreneur. Wealth to the wage becomes the current key state. It is convenient to normalize the ability to correspond to the net profits an entrepreneur would make if able to borrow at rate r and hence is unconstrained:

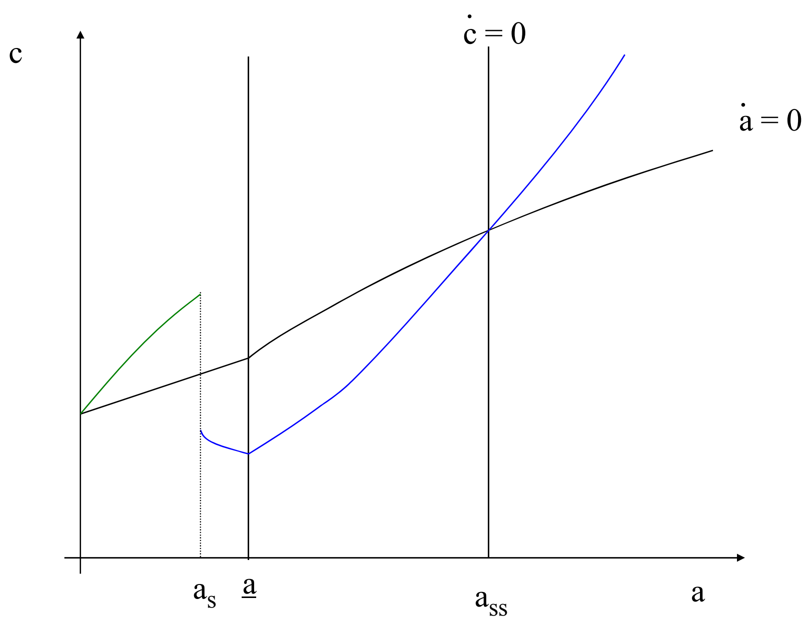
$$\theta=\max_k\left\{A\tilde{\theta}^{1-\alpha}k^{\alpha}-rk\right\}.$$

This gives the $f\left(\theta,k\right)$ of the earlier notation.

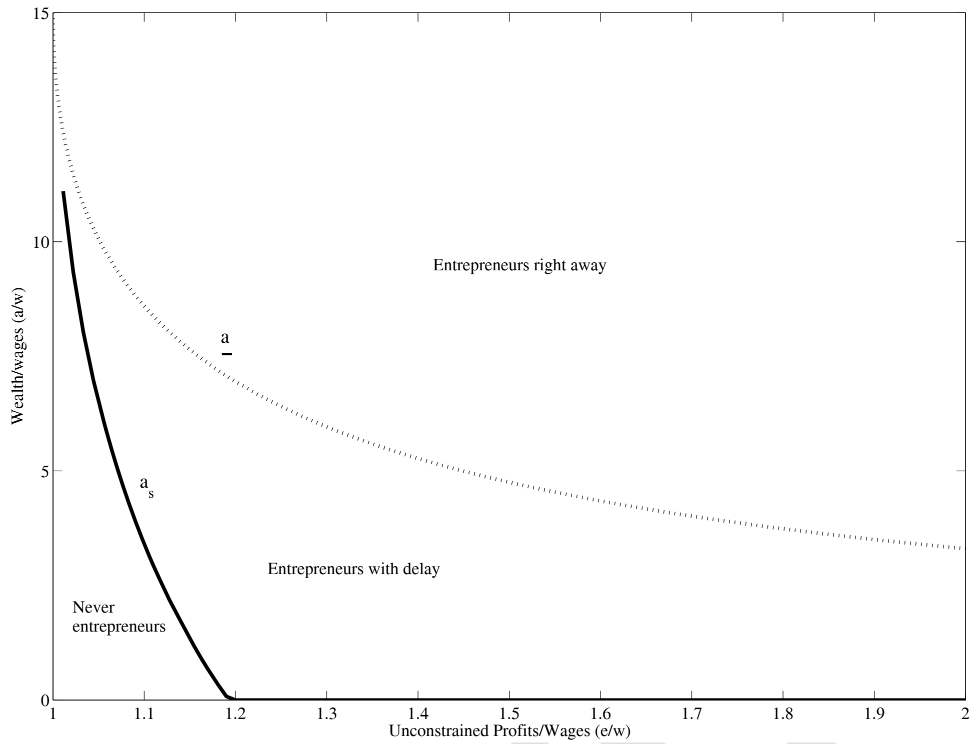
A standard Euler Equation describes the optimal consumption path,

$$-\frac{U''(c)c}{U'(c)}\frac{\dot{c}}{c}=\begin{cases} r-\rho & \text{if } a\in\left[0,\underline{a}(\theta)\right) \\ f_k\left(\theta,k\right)-\rho & \text{if } a\in\left[\underline{a}(\theta),k_u(\theta)\right), \\ r-\rho & \text{if } a\in\left[k_u(\theta),\infty\right) \end{cases}$$

The marginal rate of substitution should equal the marginal rate of transformation. Again, the law of motion for wealth is simply $\dot{a}=y\left(\theta,a\right)-c$.



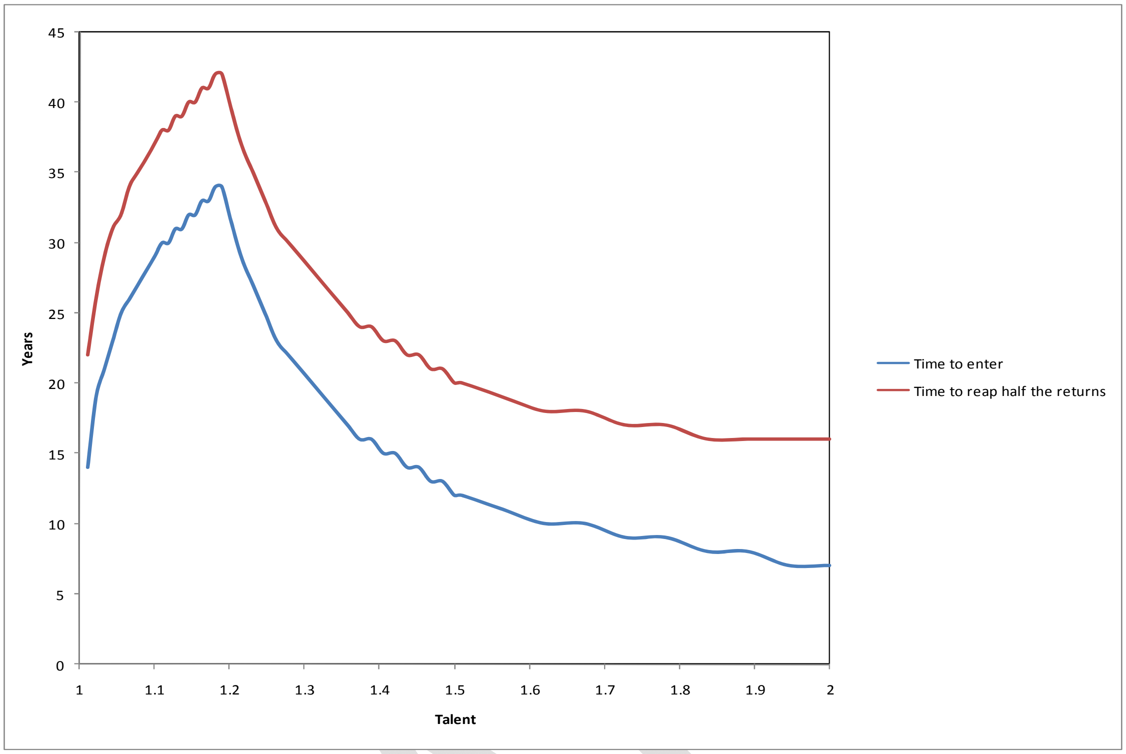
[Figure 9.3.2.a. Optimal Trajectories (Intermediate Ability). Source: Buera (2009)]



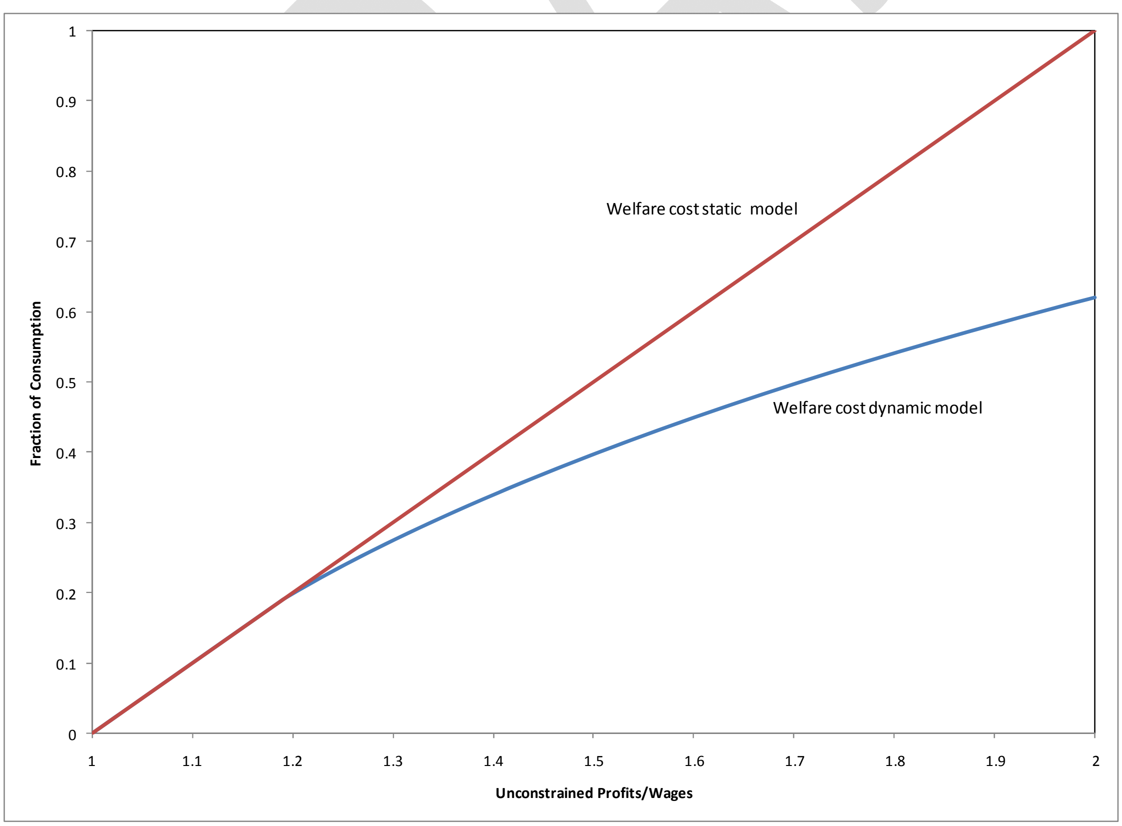
[Figure 9.3.2.b. Poverty Trap, α =.05. Source: Buera (2008)]

Buera describes the steady states in 9.3.2.a. If for given talent θ, wealth is less than yet another, lower critical value $a_s(\theta)$, the household will plan to stay a worker and converge to one possible steady state with zero wealth and consumption equal to the wage. Parameter $a_s(\theta)$ is a poverty trap threshold. For slightly higher wealth, the household will work toward another steady state $\left(a_{ss}, c_{ss}\right)$, that is, will plan to set up a firm eventually and save now at a relatively high rate. Once wealth is high enough, greater than $\underline{a}(\theta)$, that household will become an entrepreneur, though constrained and with a high ROA. Finally, the household reaches the position of being an unconstrained entrepreneur. See Figure 9.3.2.b.

The implication of higher savings rates for eventual will-be-entrepreneurs is borne out in the Townsend Thai data. Eleven years prior to the survey, the median wealth of households that started a business in the 5 years prior to the survey was only 58% of that of households who never started a business. However, 6 years prior to the survey, the median wealth of business households was 152% the median wealth of non-business households.



[Figure 9.3.3. Time to entry and time to reap half the unconstrained returns starting from the poverty trap threshold. Source: Buera (2008)]



[Figure 9.3.4. Consumption equivalent compensation for people who start with zero wealth. Source: Buera (2008)]

It may take time to become an entrepreneur. For example, at calibrated parameters that match the US data, the upper bound on the time to enter varies from zero to 18 years, depending on talent. This calculation assumes the household starts just above the "poverty trap," $a_s(\theta)$, the critical value of wealth below which they would decide to become a worker forever. Note that this poverty trap depends on talent, and is decreasing in talent, so as talent increases in the diagram 9.3.3, the starting point of wealth is lower. Initially this means it takes longer to reach the point of entry. On the other hand, higher talent raises the entrepreneur earnings profile in the Figure 9.3.1, making the switch to business at a given wealth earlier. This effect dominates after talent greater than 1.15. One can similarly plot the time to reach efficient scale. The difference between these two curves tells the analyst how many years to expect small but growing firms to be operating with high ROA.

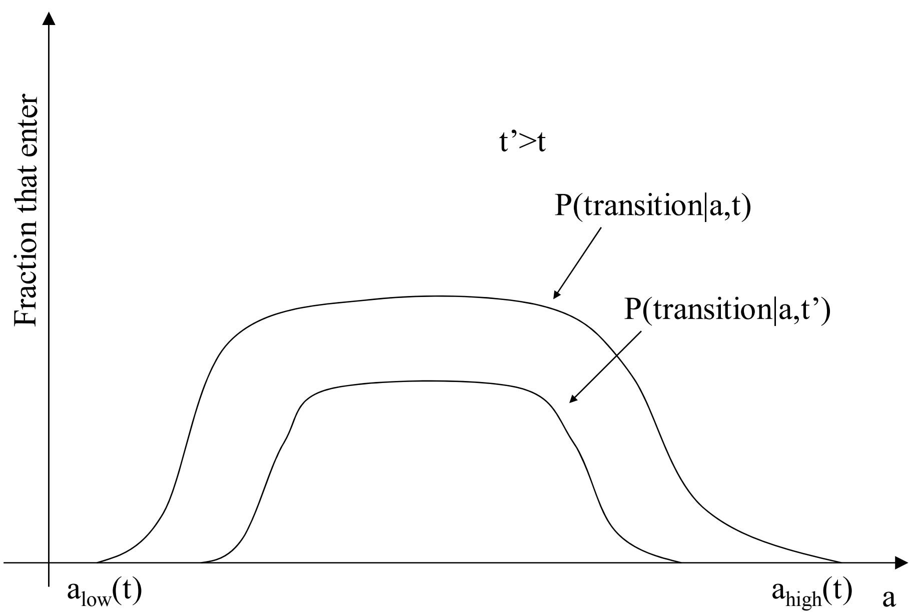
If it were possible to eliminate the credit constraint, there would be a welfare gain, for each wealth and talent combination. As earlier, this can be converted to an equivalent consumption gain. This depends on whether the model is static, or as here, dynamic. Endogenous savings does help talented households enter business but does not overcome the welfare loss to restricted credit markets. See figure 9.3.4. The diagram depicts the difference in welfare gain between the static and dynamic models for those households starting at the poverty trap, again featuring gains by talent.

	Zero Wealth	25th perc. wealth/ wage	median of wealth/ wage	75th perc. wealth/ wage
25th perc. entrants' ability	0.02	0.01	0.01	0.01
median entrants' ability	0.24	0.23	0.18	0.10
75th perc. entrants' ability	0.56	0.49	0.34	0.19

[Table 9.3.5. Distribution of Welfare Cost. Source: Buera (2002)]

The magnitude of the gain depends as earlier not only on ability but also on wealth. By restricting attention to the ability distribution for those planning to become entrepreneurs, the median household in ability and normalized wealth gains 18%. See Table 9.3.5. Less talented, wealthier households gain little. The distribution of gains is skewed toward low wealth, high talent households, reaching 56%.

Buera also makes a comparison between cross-sectional relations in wealth and the effect of wealth transfers. On the one hand he studies the relation between transitions into business in time interval Δ for agents observed at chronological date t (e.g $t = 0$) who have wealth a as observed in the cross section, as compared to the agents with higher wealth $\tilde{a}$ nearby. On the other hand he studies transition into business in time interval Δ for agents at date $t = 0$ who have wealth a in the cross section, as compared to what such a household would do if given a policy induced exogenous increment in wealth $\tilde{a}$, say via some transfer. The cross-sectional profile and its derivative under-estimates the impact of lump sum wealth transfers. This is due to a negative selection effect. At higher wealth in the cross section, the fraction that will make the transition may be going down because more talented agents have already entered business. See Figure 9.3.6. The larger point is that impact coefficients from a reduced form model may not provide reliable answers to policy questions.



[Figure 9.3.6. Transition into Entrepreneurship as a Function of Wealth and Age. Source: Buera (2009)]

9.4 Projects and Choice Among a Combination of Lenders

We turn, as in Giné, next to this issue of borrowing in setting up a business and in particular whether to borrow from the formal or the informal sector, or both in setting up a business. This will allow a further assessment of limited commitment with transaction costs.

Suppose a household is endowed with initial, predetermined wealth A, ability z, maximum scale of project K, and the fraction of capital η which as working capital cannot be used as collateral to secure loans. Only wealth A and collateral $(1-\eta)A$ are observed to the econometrician, but there is no uncertainty on the part of the household or lender.

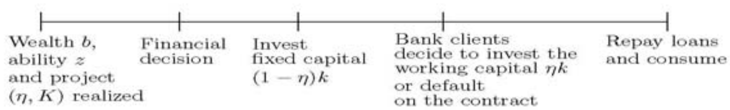


Figure 1: Time-line of the model

1

[Figure 9.4.1. Timeline of the model. Source: Giné (2005)]

The timeline of the model is depicted in figure 9.4.1 above. First wealth *b* and ability *z* are given as predetermined; then project characteristics, fraction of required working capital η and scale K are determined in the cross section as if drawn from a probability distribution. Then there is a financial decision to be described momentarily, namely how much to save and from whom to borrow. Then fixed capital $(1-\eta)k$ is supposed to be invested in the project. Commercial bank clients decide to invest the remainder working capital or to default on the contract. Otherwise, households repay loans and consume.

The technology of the household as entrepreneur is:

$$f(z,k,K,\eta) = zk + \hat{\delta}(1-\eta)k \qquad \text{s.t. } k \le K . \tag{9.4.2}$$

Here direct output is the sum of zk, a linear function of capital multiplied by talent, plus depreciated collateral capital, which can be sold. Parameter $\hat{\delta}$ is one minus the rate of depreciation.

In his paper, Giné refers to a constrained household as one which invests below maximum capacity, $k < K$. Essentially, with the linearity, if a project is undertaken and the household is not indifferent, then one would expect maximum capacity, with $k = K$. Otherwise some financial constraint must be limiting investment.

If the entrepreneur decides to self-finance he will obtain a net income of

$$\begin{gathered} Y_s\left(z,b,K,\eta\right) = \max_k zk + \delta k + (b-k)r_D \\ \text{s.t. } k \le b, \quad \text{k} \le K \end{gathered} \tag{9.4.3}$$

where r_D denotes the opportunity cost of using own funds, namely the deposit rate. We simplify notation by letting $\delta = \hat{\delta}(1-\eta)$. Since the technology is linear, we write the optimal choice of capital as

$$k_s\left(z,K,\eta\right) = \begin{cases} K \ \text{ if } z \ge r_D - \delta \quad \text{and } b \ge K, \\ b \ \text{ if } z \ge r_D - \delta \quad \text{and } b < K, \\ 0 \ \text{ if } z < r_D - \delta \end{cases}$$

In other words, the entrepreneur will invest K if it is profitable and there is enough wealth, and he will invest total wealth *b* if the maximum scale K is larger than his wealth b, and will not invest at all if the return on the investment is lower than the interest the bank pays for deposits.

If the entrepreneur goes to the bank (B), net income can be written as

$$\begin{gathered} Y_B\left(z,b,K,\eta\right) = \max_k zk - \left(k-b\right)r_B + \delta k - \Gamma_B \\ \text{s.t. } \ k \le K \end{gathered} \tag{9.4.4}$$

$$zk - \left(k-b\right)r_B + \delta k \ge \eta k . \tag{9.4.5}$$

The interest rate r_B denotes the cost of borrowing and the parameter Γ_B captures the fixed transaction cost of dealing with a bank. This cost parameter captures all expenses related to obtaining the loan: trips to the bank, bank fees, and due diligence to assess the repayment capacity of the borrower. By having the borrower pay Γ_B, the bank learns the borrower's characteristic (z, b, K, η).

A key constraint, 9.4.5, captures the enforcement disadvantage that banks face. Before producing, bank clients can "run away" with the working capital advanced, at the cost of losing all the previously deposited wealth as well as the fixed capital scrap value, both seized by the bank. Implicitly, we assume that although banks may fully observe their borrowers' actions, they have no legal mechanisms to prevent a borrower from "consuming" the working capital. The optimal choice of capital for the entrepreneur depends on whether or not the enforcement constraint is binding. If it binds, the maximum amount of capital that the bank is willing to lend is given by:

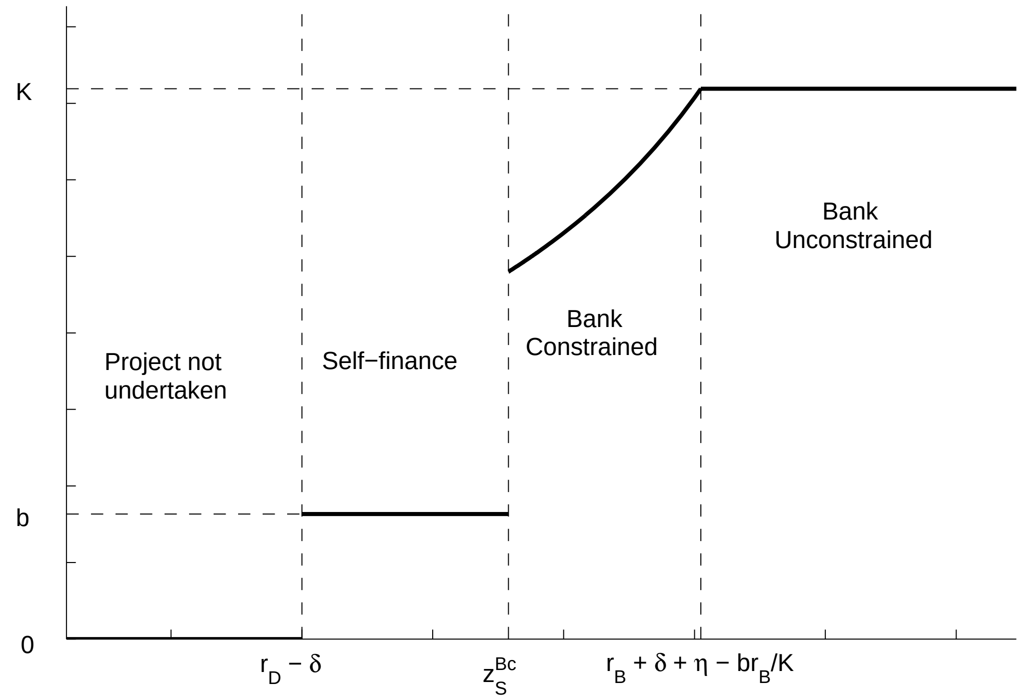
$$k^c = \frac{br_B}{\eta - (z + \delta - r_B)}. \tag{9.4.6}$$

The above expression is found using the enforcement constraint 9.4.5 at equality and solving for k. Notice that there will be less constraints if the project is more productive (ability *z* is high), the entrepreneur is richer, or he operates a technology with relatively more fixed assets (lower η). If the constraint does not bind, entrepreneurs earn net income,

$$Y_{BU} = (z + \delta - r_B)K + r_B b - \Gamma_B \tag{9.4.6}$$

whereas if it does bind,

$$Y_{Bc} = \eta k^c - \Gamma_B \tag{9.4.7}$$



[Figure 9.4.2. Optimal investment k. X-axis indicates ability z, Y-axis indicates investment k. Source: Giné (2005)]

Figure 9.4.2 plots the optimal investment *k* as a function of ability *z*. When the return on the investment $z+\delta$ is lower than the deposit rate r_D, it pays to keep the money in the bank. When ability is higher than the cutoff $r_D-\delta$ but lower than some level z_s^{Bc}, the agent self-finances, investing her total wealth b. The cutoff ability z_s^{Bc} is found by equating the net incomes from self-financing Y_S with that of resorting to a bank but being constrained, Y_{BC}. The gross capital expenses are larger than wealth *b* because the fixed cost Γ_B of transaction with the bank must be forgone. Notice that investment must be large enough so that returns cover fixed costs. In the segment above z_s^{BC}, investment is an increasing function of ability *z* at least until the capacity constraint K is reached. For higher ability values, the agent will be at capacity K.

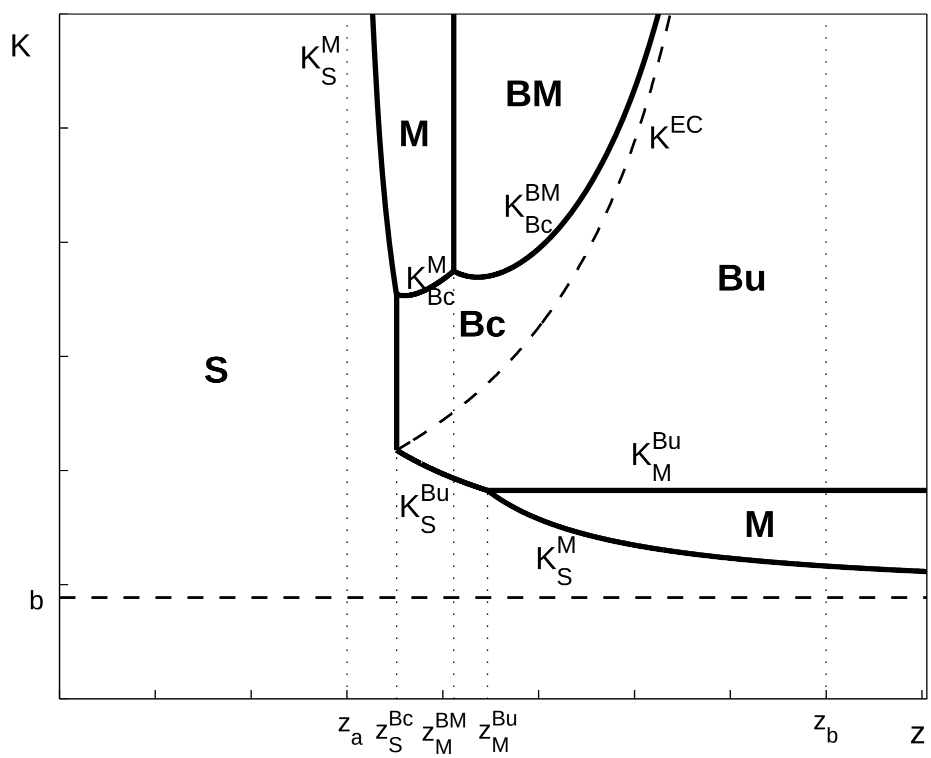
Now, suppose that the household resorts to a moneylender alone. The amount borrowed is denoted $k-b$ and income becomes:

$$Y_M\left(z,b,K,\eta\right)=\max_k zk-\left(k-b\right)r_M+\delta k-\Gamma_M \quad \text{s.t.} \quad k\le K \tag{9.4.9}$$

where r_M denotes the interest rate charged by the moneylender, and it is assumed that $r_M > r_B$. The moneylender is not subject to enforcement problems and will therefore advance the loan $k-b$ so that the entrepreneur operates the project at maximum capacity.

Finally, the entrepreneur may find it of interest to resort to both a bank and a moneylender (BM). This case will arise if the bank offers too little capital due to enforcement problems: the project may be intensive in working capital (high η) or the entrepreneur may not be talented enough to convince the bank that she will not default on the loan contract and run away with the working capital. Since the interest rate charged by the moneylender is higher than that of the bank, the agent borrows from the bank as much as the bank is willing to lend $l_B = k^c - b$ and will then turn to the moneylender to finance $l_m = K - k^c$, the remaining capital investment. Net income can be written as total revenues from investing the maximum scale minus loan repayments and fixed costs. More formally:

$$Y_{BM}(z,b,K,\eta) = zK - \left(k^c - b\right)r_B - \left(K - k^c\right)r_M + \delta K - \Gamma_B - \Gamma_M \qquad (9.4.10)$$



[Figure 9.4.3. Financial Choice Map. Source: Giné (2005) [Legend: solid, thick line mark the different financial choices, S, M, B, BM. X-axis indicates ability, z , Y-axis indicates investment, k . The horizontal dashed line indicates the level of wealth, b .]]

We can distinguish households in the data by their choices of lender. Figure 9.4.3 is illustrative. Low skill households will self-finance, as will those with low capacity. This is the region S in the figure. For given scale K, the higher is z, the more likely is the household to finance in some way. There are two distinct 'moneylender' M-only regions in Figure 9.4.3. When the scale K is relatively low and wealth is close to that scale, and talent is high, it is not economic for the household to borrow from the bank despite lower interest payments to the bank, due to the fixed bank transaction costs. When the scale K is high, but talent is relatively low, the amount that can be borrowed from the bank is low, due to the enforcement constraint 9.4.5, and the firm also turns to money lenders.

However, in this second M region, at the higher scale K, if talent z is a bit higher, then the household borrows both from the bank up to the maximum and then also from the moneylender, region *BM*. At yet even higher values for talent, or more modest scales, the household will borrow from the bank and be constrained, B_C. Finally, at highest talent and high scale, the household is unconstrained in borrowing from the bank, the region B_U.

The key observables to us are wealth b and the value of collateral assets. Realized project size K, talent z, transaction costs for the formal sector Γ_B, transaction costs for bank Γ_M, and the value of default $v = \eta k^c$ are the key unobserved variables. Note the latter is simply the working capital portion of a firm operating at constrained capital level k^c. Two different specifications for fixed costs and the value of default are considered: homogeneity, that is, uniformity across households versus heterogeneity, variety across households. Specifically the log of transaction costs and the log of the value of default are normally distributed with means possible functions of observables X_j for household *j* and a residual, orthogonal error term. The error terms are possibly correlated. Interest rates for the formal and informal sector are pinned down at their sample means, abstracting from the identity of individual providers, and $r_M > r_B > r_D$. Talent z and capacity K are drawn from a joint normal distribution with means M_S, M_K, variance δ_S, δ_K and covariation c. Thus, the model at fixed parameters delivers the likelihood of falling into the various regions of the figure. In a sense, the parameters moving in the likelihood estimation try to capture as much of the observed frequencies in the data as possible.

The scale K is estimated to have a mean of 3.9 million baht and high variance of 220 million baht. This is higher but comparable to mean household wealth. Recall the discussion of the size of business assets earlier in this manuscript. There is a negative correlation between scale K and talent z, i.e. the highest talent households are not running the largest businesses, as was also conjectured in the earlier discussion. The (common) fixed costs are estimated at between 311 to 685 baht, depending on the specification. This is a low number, roughly 2% of the average loan size—and much less than the fixed costs in the transaction costs model with endogenous financial deepening. Education, ties to the village committee, and the presence of a formal institution, each lower the cost of formal finance. Thus the new one million bath funds would help (but see below). Having savings with a formal institution and borrowing from the formal sector in the past also lowered the fixed formal costs (but not the informal). Transaction costs for the informal sector are virtually zero, 11 baht. Education and male household head increase the value of default (being more constrained in the sense of Evans and Jovanovic (1989)). Households in villages with more formal connections are less likely to be constrained, consistent with results earlier.

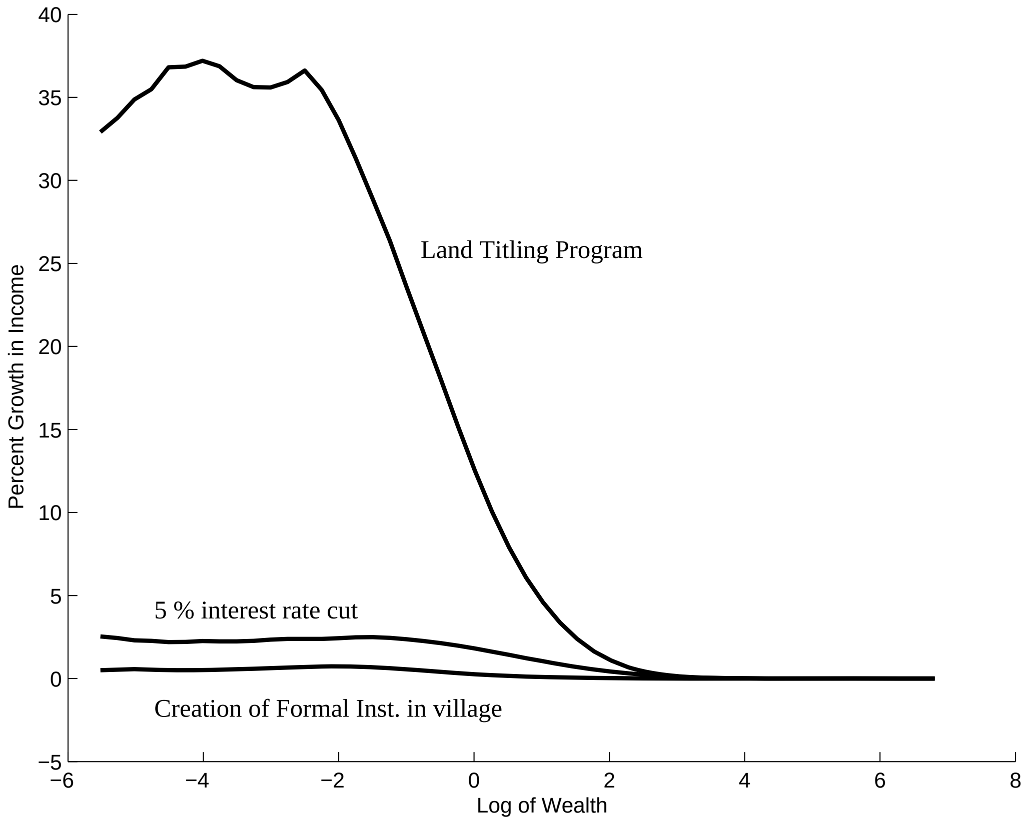
Both transaction costs and limited enforceability are needed. If transaction costs were zero (or equal) for banks and moneylenders, then the households would go first to the bank, and if constrained then to the moneylender. We would never observe households borrowing from a moneylender alone, region BM as in the data. If enforcement were perfect, then the region BM disappears also as the decision to go to the bank, or the moneylender, is determined by the trade-off between interest rates and transaction costs, depending on scale.

The model can rationalize some of the facts in the data presented earlier. High wealth households are likely to have high collateral, hence borrow from commercial banks. The data confirm that land and a few capital assets that serve as collateral in the data are more prevalent for the rich. Some high wealth households are not so talented, and will self-finance. Other things equal, self-finance increases with wealth. This makes wealth higher for those who self-finance relative to those who borrow from the informal sector. In this model, expected income is higher for those who borrow informally because they do not suffer the loss of transaction costs. This is one interpretation of the higher income/capital ratios for constrained borrowers. In fact the model offers some findings for who is likely to report being constrained, specifically those who borrow from both sources, as well as those who borrow exclusively from the formal sector.

	Common Default Value				Differentiated Default Value			
	Com. Cost		Dif. Cost		Com. Cost		Dif. Cost	
Variable	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Distribution								
μ_ζ	-0.191	0.0125	-0.221	0.0123	-0.139	0.0128	-0.165	0.0124
μ_κ	0.981	0.0199	1.012	0.0197	0.895	0.0226	0.916	0.0220
σ_ζ	0.969	0.0062	0.953	0.0061	0.966	0.0063	0.945	0.0059
σ_κ	1.561	0.0129	1.543	0.0127	1.610	0.0147	1.572	0.0140
ρ	-0.870	0.0038	-0.868	0.0038	-0.844	0.0052	-0.849	0.0049
Formal Access								
Constant (in Baht)	685.1	6.4	1,896.5	52.8	311.3	2.4	1,643.5	118.7
Formal Inst. in Village		—	0.369	0.0131		—	0.592	0.0243
Past mem. Formal Inst.		—	0.800	0.0516		—	0.669	0.0490
Past mem. Informal Inst.		—	1.741	0.2289		—	1.033	0.1912
Member of Village Com.		—	1.275	0.1092		—	0.844	0.0834
Education		—	1.030	0.0052		—	0.968	0.0082
Savings in Formal Inst.		—	0.260	0.0099		—	0.255	0.0128
Region		—	0.992	0.0222		—	1.022	0.0501
Informal Access								
Constant (in Baht)	0.1	6.4	9.0	5.6	4.8	2.7	10.8	3.0
Enforcement Constraint								
constant		—		—	0.898	0.0018	0.917	0.0129
Num. Formal Inst. in Vil.		—		—	0.985	0.0024	0.985	0.0031
Member of Vil. Com.		—		—	1.005	0.0076	0.993	0.0076
Sex of Head (Male)		—		—	1.085	0.0028	1.072	0.0133
Education		—		—	1.019	0.0003	1.024	0.0011
Region (Northeast)		—		—	1.006	0.0033	0.984	0.0069
Number of Obs.	2,270		2,270		2,270		2,270	
Likelihood	-51,647.04		-45,899.40		-43,284.36		-42,056.29	

[Table 9.4.4. Maximum likelihood estimates. Source: Giné (2005)]

Comparative static policy exercises are revealing. A cut in the interest rates formal banks charge leads to a decrease in self finance and a decrease in borrowing from money lender but an increase in bank and bank moneylender finance. See Table 9.4.3. There would be modest increases in investment and income, as in Table 9.4.2, and the distribution gains in terms of income growth are fairly uniform up to middle income borrowers, as in Figure 9.4.4. This policy is inefficient in that it attracts low talent borrowers to business. The creation of formal institutions in villages does not create this distortion. However, it does not alleviate enforcement problems either. Lower transactions costs accomplish little here. Thus more households would report themselves constrained. Those borrowing from a money lender decrease, as in the data, but those borrowing from both banks and money lender increases. The distribution of gains in terms of income growth is flat and relatively low on all dimensions for these two policies. See Figure 9.4.4. On the other hand, a land titling program changing at the estimated parameter values dramatically increases investment, and especially net income. Those borrowing from a bank increase dramatically, while self finance and other sources of finance, M and BM, all decline. The distribution of gains is high and peaked for relatively low wealth households.



[Figure 9.4.5. Percentage Income Growth from Different Policies. Note: Inst=institution. Source: Giné (2005)]

	Investment	Income
Relevance of Market Imperfections		
Limited Enforcement, No Transaction Costs	0.1	0.2
Perfect Enforcement, Transaction Costs	25.4	347.6
Perfect Enforcement, No transaction Costs	25.7	348.8
Policy Analysis		
5 percent cut in formal interest rate	1.4	1.2
Creation in formal institution in village	0.1	0.1
Land Titling Program	15.2	201.5

Note: For each household, 1,000 (z,K) pairs are simulated from the estimated distribution. Using each household's vector of characteristics and estimated parameters, the investment and income are computed under each scenario. Growth rates for each household and simulation are computed and the overall mean is reported.

[Table 9.4.6. Percentage Growth in Income and Investment. Source: Giné (2005)]

	S	B	M	BM
5 percent cut in formal interest rate	-0.92	2.92	-2.49	0.49
Creation of formal institution in village	-0.46	1.39	-2.38	1.45
Land Titling Program	-1.06	15.87	-2.32	-12.49

Note: Financial Choices are Self-finance (S), Bank (B), Moneylender (M) and Bank and Moneylender (BM).

Note: Table reports the percent changes in the predicted fraction of households making each financial choice for each policy considered relative to the benchmark estimation.

[Table 9.4.7. Percent Changes in Predicted Probabilities of Financial Choices. Source: Giné (2005)]

.

9.5 Distinguishing Obstacles from Repayment Data

We return to the issue of identifying the key obstacles to trade, allowing both limited commitment and moral hazard. But rather than focus on choice of occupation and wealth, or method of finance, we turn as in the work in Ahlin and Townsend (2004) that uses data on loan defaults from BAAC households in joint liability groups (with cosigned loans).

Entries with a ’‡’ are the result of our own extensions of the authors’ original models.

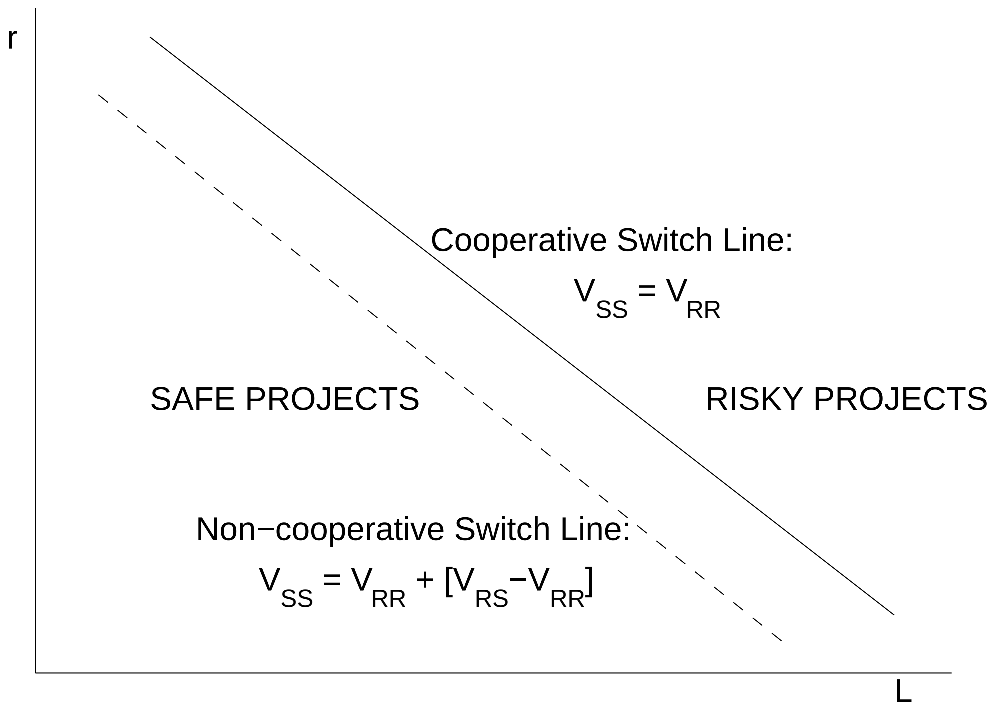
Variable	Effect on Repayment			
	Stiglitz	BBG	BC	Ghatak
interest rate r	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
loan size L	$\downarrow$	$\downarrow^{\ddagger}$	no pred.	$\nearrow\searrow^{\ddagger}$
liability payment q	$\downarrow^{4}$	$\uparrow$	no pred.	$\downarrow$
productivity H	$\uparrow^{\ddagger}$	$\uparrow^{\ddagger}$	$\uparrow^{\ddagger}$	$\uparrow^{\ddagger}$
screening	no pred.	no pred.	no pred.	$\uparrow$
positive correlation	$\uparrow^{\ddagger}$	no pred.	$\downarrow^{\ddagger 5}$	$\uparrow^{\ddagger}$
cost of monitoring	no pred.	$\downarrow$	no pred.	no pred.
cooperative behavior	$\uparrow^{\ddagger}$	$\downarrow^{\ddagger 6}$	$\downarrow^{\ddagger 7}$	—
outside credit options	$\downarrow^{\ddagger}$	no pred.	no pred.	no pred.
official penalties	no pred.	no pred.	$\uparrow$	no pred.
unofficial penalties	no pred.	no pred.	$\uparrow$	no pred.

[Table 9.5.1. Repayment Implications. List of possible control/policy experiments, change in interest rate, loan size, liability payment, encourage screening, cooperation, allowing more lenders/competition; increase of official penalties, facilitate monitoring. Source: Ahlin and Townsend (2004)]

The models considered are representative of the literature. In Stiglitz (1990) households choose among safe and risky projects, something not observed by the lender. In Banerjee, Besley, and Guinnane (‘BBG,’ 1994) the choice of project type is monitored at a cost by a second non-borrowing partner. Both these models have a moral hazard problem. In Besley and Coate (‘BC,’ 1995) the problem is repayment and strategic default. Repayment may not happen. In Ghatak (1999) there is an adverse selection problem.

The models have in common that an increase in the interest rate reduces repayment and increases the productivity of borrowers which leads to an increase in a project’s repayment. There are also implications specific to the individual models. Lowering monitoring costs increases repayment in BBG, higher official and/or unofficial penalties increase repayment in BC, and enhanced ability by the bank to screen outside customers raises repayment in Ghatak. More interesting are implications which distinguish the models. Cooperation raises repayment in Stiglitz and lowers it in two other models. The informal market may have this limit. Covariance in returns raises repayment in two of the information models but lowers it in BC. More generally, see Table 9.5.1 for a summary.

Fig. 1. Stiglitz model: Switch Line



[Figure 9.5.2. The Switch Line. Source: Townsend and Ahlin (2007 – Repayment Data)]
To the left of the solid line, safe projects are chosen, to the right risky ones. The dashed line is the Switch Line for groups acting non-cooperatively.

In the Stiglitz model of joint liability, each of two partners chooses the riskiness of projects, either safe S with probability of success p_s or risky R with probability of success p_R. Output upon success depends on risk type $i = R, S$ written $Y(p_i, L)$ where *L* is loan size. Output under failure is zero, and the borrower must default. Amount *rL* is repayment of principal and interest upon success - *r* is the gross rate inclusive of principal. If one borrower pays for another, the amount of payment is *qL*, and it is assumed that $q < r$. Utility $U(c)$ is concave in consumption *c*. Higher loan size *L* carries with it an implicit commitment for more effort and hence, via a separable negative term, disutility $W(L)$. In sum the expected utility to a borrower who chooses technology risk i while his partner chooses technology *j* (while loan size is *L* regardless):

$$V_{ij}(r,L,q) = p_i p_j U\left[Y(p_i,L) - rL\right] + p_i\left(1-p_j\right)U\left[Y(p_i,L) - rL - qL\right] - W(L), \quad i,j \in \{R,S\} \tag{9.5.1}$$

To illustrate the determination of the signs of derivatives of repayment with respect to cross group characteristics, restrict attention for the moment to three variables: interest rate *r*, loan size *L*, and joint liability payment *q* (all of which arguably the BAAC has under its control as policy variables). If the borrowing pair cooperate and choose the same level of risk, either *S* or *R*, then the observed probability of success, and the fractions of non-default in the population, would be determined by a simple indicator function of whether V_{SS} dominates V_{RR}:

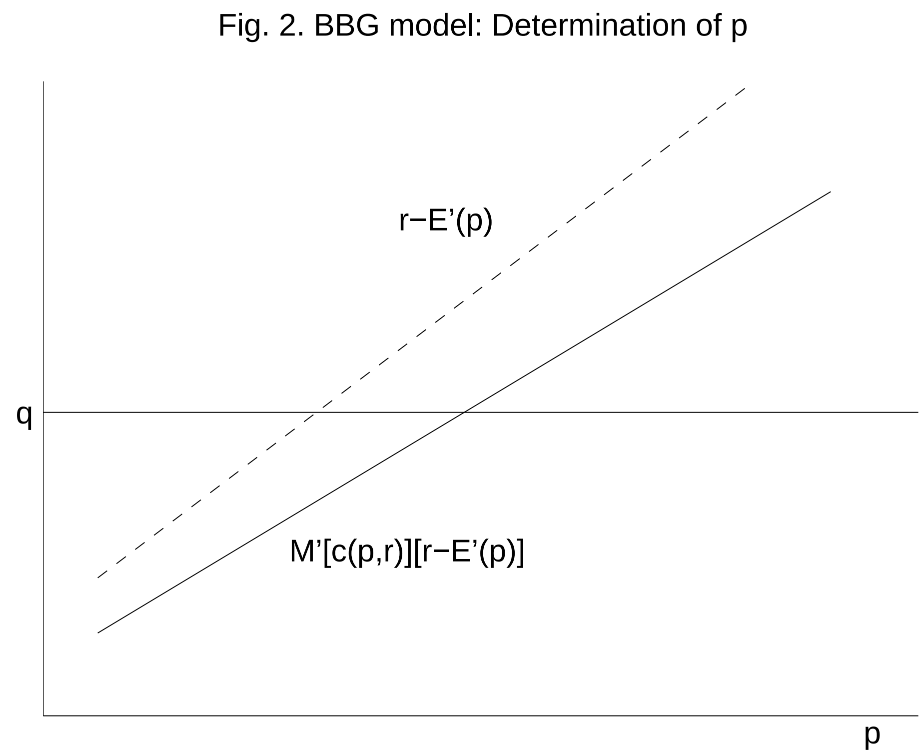
$$p = p_R + (p_S - p_R)1\left\{V_{SS}\left(r, L, q\right) \geq V_{RR}(r, L, q)\right\} \qquad (9.5.2)$$

Thus the derivative of $V_{RR} - V_{SS}$ with respect to covariates is the key to whether defaults should be more prevalent in the data. Consider the curves of indifference, as between *L* and *r* in Figure 9.5.2, with *q* held fixed. Increases in r and/or *L* push borrowers up into the region of risky project choice. Increases in *q* can be shown to move the entire line down. Non-cooperation among borrowers, as in a Nash equilibrium of project choice, also moves the line down. More comprehensively, see table 9.5.1 and the "Stiglitz" column; see Ahlin and Townsend (2004) for details of the derivation.

In BBG one cooperative saving member or partner is a monitor on a second borrowing partner. Both are risk neutral. The first partner guarantees the loan of the second. Thus there is a first-order condition for monitoring. This equates the gain from an incremental movement or derivative with respect to the probability of success *p*, namely the reduced probability of paying *q*, to the cost of securing *p*, namely the derivative of the monitoring costs $M(c)$ with respect to p where $M(c) = M(c(p, r))$. Here *c* is the penalty that can be inflicted on the borrower for deviation from *p*, and $c(p, r)$ is the critical *c* which ensures incentive compatible choice of project risk *p* when the interest rate is r. That is, let $E\left(p\right)$ denote the expected payoff when the risk is p. The incentive constraint is $E\left(p\right) - pr \geq E\left(\underline{p}\right) - \underline{p}r - c$, where $\underline{p}$ is the riskiest behavior which the borrower can resort to, and the penalty for deviation is c. This defines *c* as $c\left(p, r\right) \equiv E\left(\underline{p}\right) - E\left(p\right) + r\left(p - \underline{p}\right)$. In sum, the first-order monitoring equation is thus

$$q = M'\left[c\left(p, r\right)\right]c_p(p, r) \qquad (9.5.3)$$

Essentially, total differentiation of 9.5.3 determines how repayment rate *p* varies with key covariates. Illustrative in the p, q space of figure 9.5.3 is the intersection of the horizontal *q* line, the left-hand side of 9.5.3, with the up-sloping right-hand side of 9.5.3. The intersection determines project type, and the risk/probability of *p*. An increase in the interest rate r will shift the up-sloping line vertically upwards, so that *p* will go down. Under certain assumptions, cooperation also yields an alternative right-hand side schedule that is shifted up, reducing repayment. For further details of the derivation see Ahlin and Townsend (2004), and the column in table 9.5.1 labeled 'BBG.'



[Figure 9.5.3. Determination of p under costly monitoring (solid line) and costless operation (dashed line). Source: Ahlin and Townsend (2007) – Repayment Data]

In BC, borrowers can default, but penalties can be imposed, and these are increasing in project output. The two borrowers' returns are drawn independently from distribution $F(Y)$, with support $\left[0, Y_{\max}\right]$. Repayment decisions are then made non-cooperatively. Joint liability here implies that if the lender does not receive the full repayment amount of the group, $2r$, he imposes an *official penalty* of $C^0(Y_i)$ on each borrower $i \in \{1,2\}$. It is assumed that $C^0(Y)$ is continuous, strictly increasing, unbounded, and that $C^0(Y) < Y$ when $Y > 0$. In other words, the lender penalizes more severely when output is higher, but never as severely as outright confiscation. It is useful to define a cutoff output function $\underline{Y}(r)$, such that $C^0\left[\underline{Y}\left(r\right)\right] = r$ via the inverse function C:

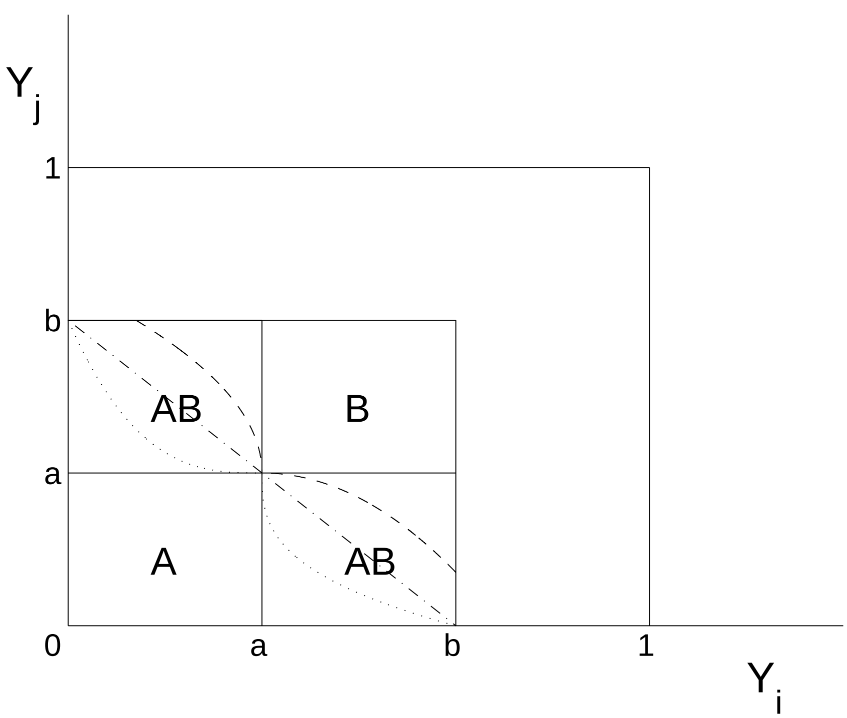
$$\underline{Y}(r) \equiv (C^0)^{-1}(r) \tag{9.5.4}$$

By construction, then when weighing repaying r against incurring penalties $C^0(Y)$, repayment is weakly more attractive if $Y \geq \underline{Y}(r)$ and default is more attractive if $Y < \underline{Y}(r)$. The properties of $C^0(Y)$ imply that $\underline{Y}(r)$ is strictly increasing in r.

Similarly, let $C_1^u\left(Y_i, \Lambda_j\right)$ denote the unofficial penalty on a delinquent borrower i who realized output Y_i and this default decreases his partner j's payoff by Λ_j, either through r if partner j pays i's loan, or the official penalty if partner j now defaults also.

The function $\underline{\hat{Y}}\left(r, Y_j\right)$ is defined to satisfy $r = C^0\left(\hat{Y}\right) + C^u\left[\hat{Y}, \Lambda\left(r, Y_j\right)\right]$. Thus, when Y is between $\underline{Y}(r)$ and $\underline{Y}(2r)$ the borrower finds it attractive to pay his own loan when the other is paying also, but to default if he is also liable for the others payment, *r*. This is the region where strategic default and the noncooperative game come into play. There is another region where each would default regardless by virtue of his personal loan itself, when $Y < \underline{Y}\left(r\right)$.

Fig. 3. BC model: Default Region



The Default Region. (Note that $a \equiv \underline{Y}\left(r\right)$, $b \equiv \underline{Y}\left(2r\right)$ and $Y_{\max}$ is normalized to one.) Default in the non-cooperative game occurs if joint output realizations fall in box A, or in boxes AB below the $\underline{\hat{Y}}$ curve (the dashed curve, for example). More generally, the only restriction on the curve through the lower AB box is that it start at (a, a) and strictly decrease to (b, z), for some $z > 0$. The curve through the upper AB box must be its reflection about the 45-degree line, due to borrower symmetry. Default under cooperation occurs below the dash-dotted line, for example, and under the non-cooperative game and severe unofficial penalties (including being positive even at $Y_i = 0$) below the dotted curve.

[Figure 9.5.4. The Default Region. Source. Source: Ahlin and Townsend (2007) – Repayment Data]

The figure above describes the Default Region. (where $a \equiv \underline{Y}(r)$, $b \equiv \underline{Y}(2r)$, and $Y_{\max}$ is normalized to one.) Default in the non-cooperative game occurs if joint output realizations fall in box A, or in boxes AB below the dashed curve $\hat{\underline{Y}}(r, y)$ determining the critical value of output for Y_2 given Y_1. (The curve through the upper AB box must be its reflection above the 45-degree line, due to borrower symmetry.) In sum the repayment rate *p* is one minus the regions of default:

$$p = 1 - \left[F(\underline{Y}(r))\right]^2 - 2\int_{\underline{Y}(r)}^{\underline{Y}(2r)} F(\hat{\underline{Y}}(r, Y))dF(Y) \qquad (9.5.5)$$

Derivatives of the right hand side of 9.5.5 with respect to *r* determine the response of repayment rates to the interest rate, for example. Cooperation mitigates the use of unofficial penalties, changes the critical output levels, and delivers the changed regions of the graph. Other experiments are possible. Other implications from Ahlin and Townsend are marked in table 9.5.1, marked 'BC.'

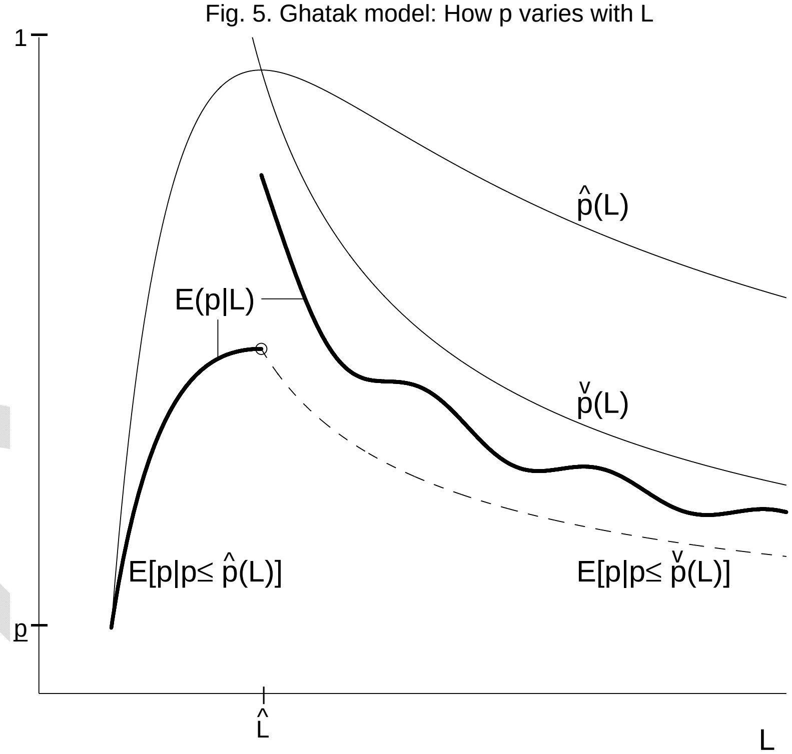
Under adverse selection, as in Ghatak (1999), agent types vary exogenously, that is, there are risky types and safe types, the riskiest type being $\underline{p} > 0$. Agents choose to borrow if the payoff to borrowing is greater than the supposed outside payoff $\underline{U}$. Some of the characteristics which determine $\underline{U}$ are observable. It can be established that agent types will choose to pair with one another homogenously, *p* to *p*. Intuitively, high *p* safe types are desirable partners, especially so for higher *p* who are likely not to default and thus may have to pay off the partner's loan. In Ghatak, the expected project return *E* is held constant, so ideally we need to control for this in the data. The marginal borrowing type, just indifferent to choosing to borrow at all is the type with $p = \hat{p}$ which solves

$$E(p) - pr - p(1-p)q = U. \qquad (9.5.6)$$

This is the selection equation. One can show that if $q \leq r$, the borrowing payoff, the left hand side of 9.5.6 is decreasing in *p*. Thus safe agents with type $p > \hat{p}$ prefer not to borrow, while risky agents with $p < \hat{p}$ prefer to borrow. The exclusion of safe agents from the borrowing pool stems from the inability of the bank to observe borrower risk; it can not vary the contract by risk-type.

The selection equation 9.5.6 determines repayment rate by considering the mix of households in the borrowing pool. We can show for example that positive correlation in project returns reduces the probability of off-diagonal, success-failure events $p(1-p)$, raises the payoff on the left-hand side of the selection equation, raises the cutoff $\hat{p}$, and so safer borrowers are drawn into the pool. This takes advantage of the fact that BAAC interest rates are administered, not designed to clear the market, and so r can be used as constant in the experiment. Other derivations are in Ahlin and Townsend (2004) and are summarized in table 9.5.1, marked 'Ghatak.' We now give one more example.

Fig. 5. Ghatak model: How p varies with L



[Figure 9.5.5. Source: Ahlin and Townsend (2007 – Repayment Data) NOTE: When L is small enough – below $\hat{L} = F'^{-1}([\underline{u} - EF(0)]/E)$ – then $\hat{p}(L) < \check{p}(L)$ and $E(p|L)$ follows $E[p|p \leq \hat{p}(L)]$. For $L \geq \hat{L}$, $p(L)$ and $E(p|L)$ is a convex combination of $\check{p}(L)$ and $E[p|p \leq \check{p}(L)]$

Suppose loan size is determined exogenously in random offers from the lender. Using separability assumptions, expected output is $EF(L)$, where L is loan size. We make typical assumptions on function

F, including strict concavity, Inada conditions, and that $F(0)=0$. Define $Z(p)\equiv pr+p(1-p)q$ as the (expected) unit borrowing cost of a type-*p* agent. The borrower payoff, under symmetric loan sizes, is then

$$EF(L)-prL-p(1-p)qL=EF(L)-Z(p)L \qquad (9.5.7)$$

Observing an agent borrowing *L* establishes two facts. First, it must be that $EF(L)-Z(p)L\geq\underline{U}$. Otherwise, the agent would choose the outside option. Rearranging,

$$Z(p)\leq[EF(L)-\underline{U}]/L \qquad (9.5.8)$$

Since $Z(p)$ is increasing in *p*, this implies that $p\in[\underline{p},\hat{p}(L)]$, where $\hat{p}(L)$ solves the selection equation 9.5.7 at equality. One can also show that the right-hand side of inequality 9.5.8 first increases, then decreases in *L*, implying that $\hat{p}(L)$ does the same; see Figure 9.5.4. Second, assuming the borrower can always borrow less (if not more) than the lender's offer, the borrower's payoff cannot be decreasing in loan size. Otherwise, the borrower could have refused some of the loan and increased his payoff. Applying this to payoff function 9.5.7 gives

$$EF'(L)-Z(p)\geq 0 \text{ or } Z(p)\leq EF'(L) \qquad (9.5.9)$$

This guarantees that $p\in[\underline{p},\breve{p}(L)]$, where $\breve{p}(L)$ solves relation 9.5.9 at equality. The larger *L*, the tighter the bound of inequality 9.5.9 and hence the lower $\breve{p}(L)$; again, see Figure 9.5.4. Intuitively, larger loans signal a lower (expected) cost of capital, which is true of more risky groups.

In sum, observing *L* tells us that $p\in[\underline{p},\min\{\hat{p}(L),\breve{p}(L)\}]$. Manipulating inequalities 9.5.8 and 9.5.9 makes it clear that the former bound is tighter, implying $\hat{p}(L)<\breve{p}(L)$, if

$$\Gamma(L)\equiv F(L)-LF'(L)<\underline{U}/E \qquad (9.5.10)$$

$\Gamma(L)$ is increasing from zero as L increases, so condition 9.5.10 holds when loan sizes are small enough, as in Figure 9.5.5. When it does, the marginal borrower is credit-constrained. Thus, a higher loan size means higher payoffs and borrowers being drawn from a larger, safer pool.

When the observed *L* is large enough so that the reverse of inequality 9.5.10 holds, $\breve{p}(L)\leq\hat{p}(L)$ and thus the group type is in $[\underline{p},\breve{p}(L)]$. However, the expected repayment rate is not simply $E[p\mid p\leq\breve{p}(L)]$, where the expectation is with respect to density of types denoted $g(p)$. The reason is that there is essentially a mass point at type $\breve{p}(L)$ corresponding to all groups of type $\breve{p}(L)$ who were offered more than *L*, but only accepted *L*, their optimal amount.

The expected repayment rate is a convex combination of $\breve{p}(L)$ and $E[p \mid p \leq \breve{p}(L)]$ as in Figure 9.5.5. Both of these terms are declining in L and approach $\underline{p}$, but the combination may be non-monotonic if the weights shift. In sum, one can anticipate from this model that defaults should be decreasing and then increasing in loan size.

Variables marked with an asterisk are taken or constructed from the household-level survey, HH. All others are from the group-level survey, BAAC.

Variables marked with a † might also be included under the next set of variables; variables marked with a ‡ might also be included under the previous set.

Variable	Description	Mean	(σ)
Productivity:			
AVGLAND	Average landholdings of group members (rai)	23.6	(15.7)
EDCATION	Index of group average education levels	3.1[a]	(0.32)
Screening:			
SCREEN	Do some want to join this group but cannot?	0.39	
KNOWTYPE	Do group members know the quality of each other's work?	0.94	
Covariance:			
COVARBTY*	Measure of coincidence of economically 'bad' years across villagers	0.28	(0.16)
HOMOCCUP†	Measure of occupational homogeneity within the group	0.87	(0.24)
Cost of Monitoring:			
LIVEHERE	Percent of group living in the same village	0.88	(0.22)
RELATPCT†	Percent of group members having a close relative in the group	0.58	(0.36)
Cooperation:			
SHAREREL	Measure of sharing among closely related group members	2.1	(1.6)
SHAREUNR	Measure of sharing among unrelated group members	1.5	(1.4)
BCOOPPCT*	Percent in tambon naming this village best in the tambon for "cooperation among villagers"	0.25	(0.11)
JOINTDCD	Number of decisions made collectively	0.37	(0.91)
Outside Credit Options:			
PCGMEM*‡	Percent in village claiming Production Credit Group membership	0.08	(0.16)
CBANKMEM*	Percent in village claiming to be clients of a commercial bank	0.28	(0.18)
Penalties for default:			
BINSTPCT*‡	Percent in tambon naming this village best in the tambon for "availability and quality of institutions"	0.27	(0.19)
SNCTIONS*	Percent of village loans where default is punishable by informal sanctions	0.10	(0.11)

[a] See text for the interpretation of this education index.

[Table 9.5.6. Independent Variables. Source: Ahlin and Townsend (2004)]

Variables marked with an asterisk are taken or constructed from the household-level survey, HH. All others are from the group-level survey, BAAC.

Variable	Description	Mean	(σ)
Control:			
LNYRSOLD	Number of years group has existed (Log)	11.4[a]	(8.5)
VARIBLTY*	Village average coefficient of variation for next year's expected income	0.30	(0.09)
WEALTH*	Village average wealth (million 97 Thai baht)	1.1	(2.1)
MEMBERS	Number of members in the group	12.3	(5.1)
Fundamentals:			
r	Average interest rate faced by the group	10.9	(2.0)
L	Average loan size borrowed by the group (thousand 97 Thai baht)	18.7	(18.3)
q	Percent landless in the group	0.06	(0.15)

[a]Here the mean and standard deviation are for age, not log of age.

[Table 9.5.7. Independent Variables and Controls. Source: Ahlin and Townsend (2004)]

Our measure of default is a binary dummy from the BAAC survey, which equals one if the BAAC has ever, in the history of the group, raised the interest rate as a penalty for late payment. Twenty seven percent of the groups responded affirmatively. This relatively high figure should not be taken as a mark against the BAAC lending program. Annual default rates are much lower, whereas this measures default over the entire history of the group (median group age is ten). Further, imposing an interest rate penalty is one of the first remedial actions in a dynamic process the BAAC uses with delinquent group-guaranteed borrowers, as discussed in section 2.1; repayment ultimately may have occurred. For the empirical tests, we recode this variable to let zero represent default and one represent repayment.

The key covariates for empirical work of all four theories are listed in table 9.5.6, grouped by the subject to which they are related: productivity, screening, covariance, cost of monitoring, cooperation, outside credit options, and penalties for default. The controls are the number of years the group has existed, the village average coefficient of variation for the next year's expected income, village average wealth, and number of members of the group. We proceed with three types of statistical tests.

First, for each independent, right hand side variable, the sample was partitioned into two subsamples as many ways as possible, satisfying the requirements 1) no subsample had less than fifteen groups and 2) each subgroup in one subsample had strictly higher values for the independent variable than each group in the other subsample. For each partition, a significant difference in the mean of the dependent variable across subsample was tested at the 90% level. The table lists the percentage of partitions for a given independent variable producing significant mean differences across the two subsamples, with the sign indicating a positive or negative relationship.

Variables that could be included in the next group are denoted by a †; in the previous group by a ‡.

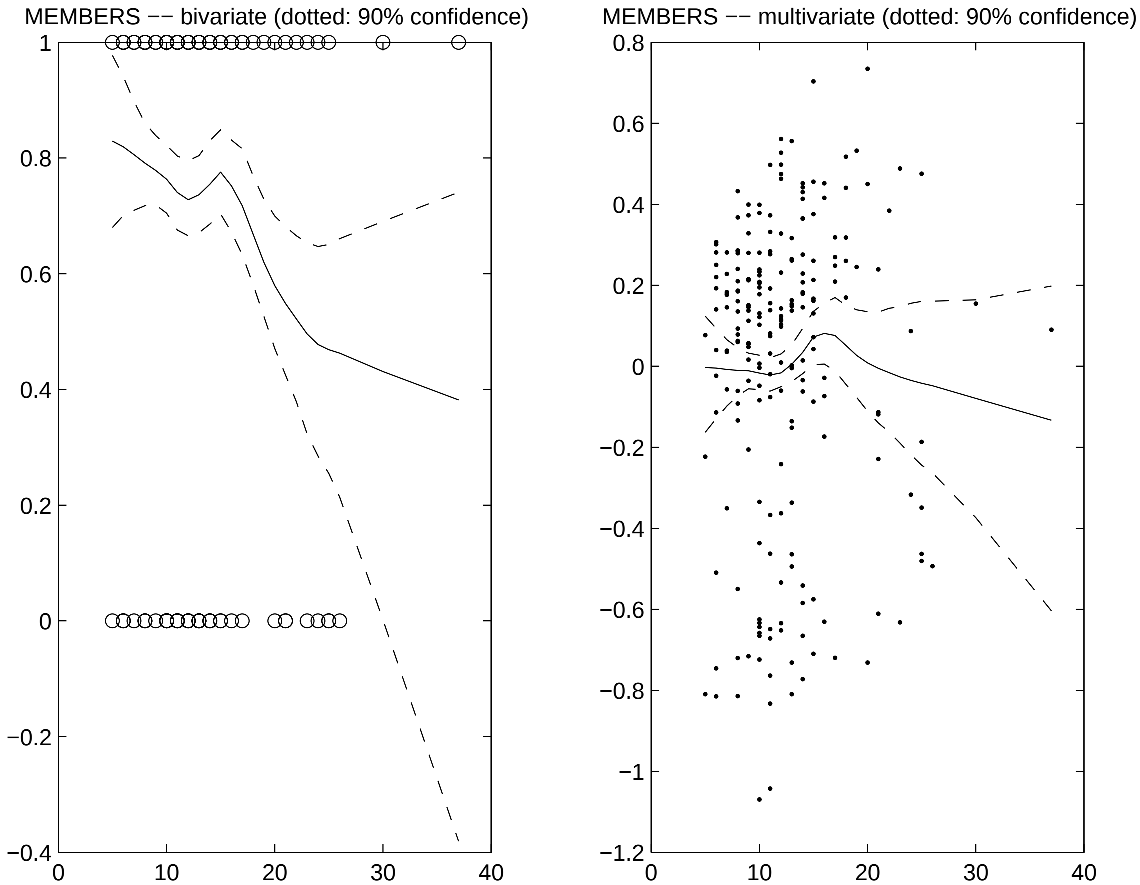
	Percent of tests significant at 90%, with sign						Total Tests		
	Northeast groups		Central groups		All groups		NE	CE	All
Control:	(+)	(−)	(+)	(−)	(+)	(−)			
LNYRSOLD	0%	83%	0%	90%	0%	85%	18	20	26
VARIBLTY	3%	3%	8%	0%	0%	0%	73	50	140
WEALTH	5%	0%	0%	49%	0%	12%	73	51	139
MEMBERS	0%	0%	0%	50%	0%	53%	9	14	17
Fundamentals:									
r	0%	5%	0%	0%	0%	28%	20	16	29
L	0%	3%	0%	0%	0%	9%	37	24	55
q	0%	0%	0%	95%	0%	96%	5	20	27
Productivity:									
AVGLAND	0%	0%	68%	0%	11%	0%	29	34	46
EDCATION	30%	0%	7%	0%	18%	0%	23	14	44
Screening:									
SCREEN	0%	0%	0%	0%	0%	0%	1	1	1
KNOWTYPE	—	—	—	—	0%	0%	0	0	1
Covariance:									
COVARBTY	0%	8%	38%	0%	10%	0%	49	32	71
HOMOCCUP†	0%	0%	0%	0%	0%	0%	11	28	46
Cost of Monitoring:									
LIVEHERE	0%	0%	20%	0%	38%	0%	13	30	42
RELATPCT†	0%	2%	0%	18%	0%	35%	44	39	57
Cooperation:									
SHAREREL	0%	0%	0%	25%	0%	0%	5	4	5
SHAREUNR	0%	0%	0%	67%	0%	50%	4	3	4
BCOOPPCT	0%	3%	0%	0%	0%	2%	60	38	108
JOINTDCD	33%	0%	—	—	100%	0%	3	0	3
Outside Credit Options:									
PCGMEM‡	0%	100%	0%	0%	0%	55%	9	6	11
CBANKMEM	0%	0%	0%	0%	0%	20%	13	9	20
Penalties for Default:									
BINSTPCT‡	0%	0%	8%	0%	0%	0%	63	37	108
SNCTIONS	55%	0%	0%	0%	0%	0%	33	25	51

[Table 9.5.8, Univariate Mean Comparisons Source: Ahlin and Townsend (2004)]

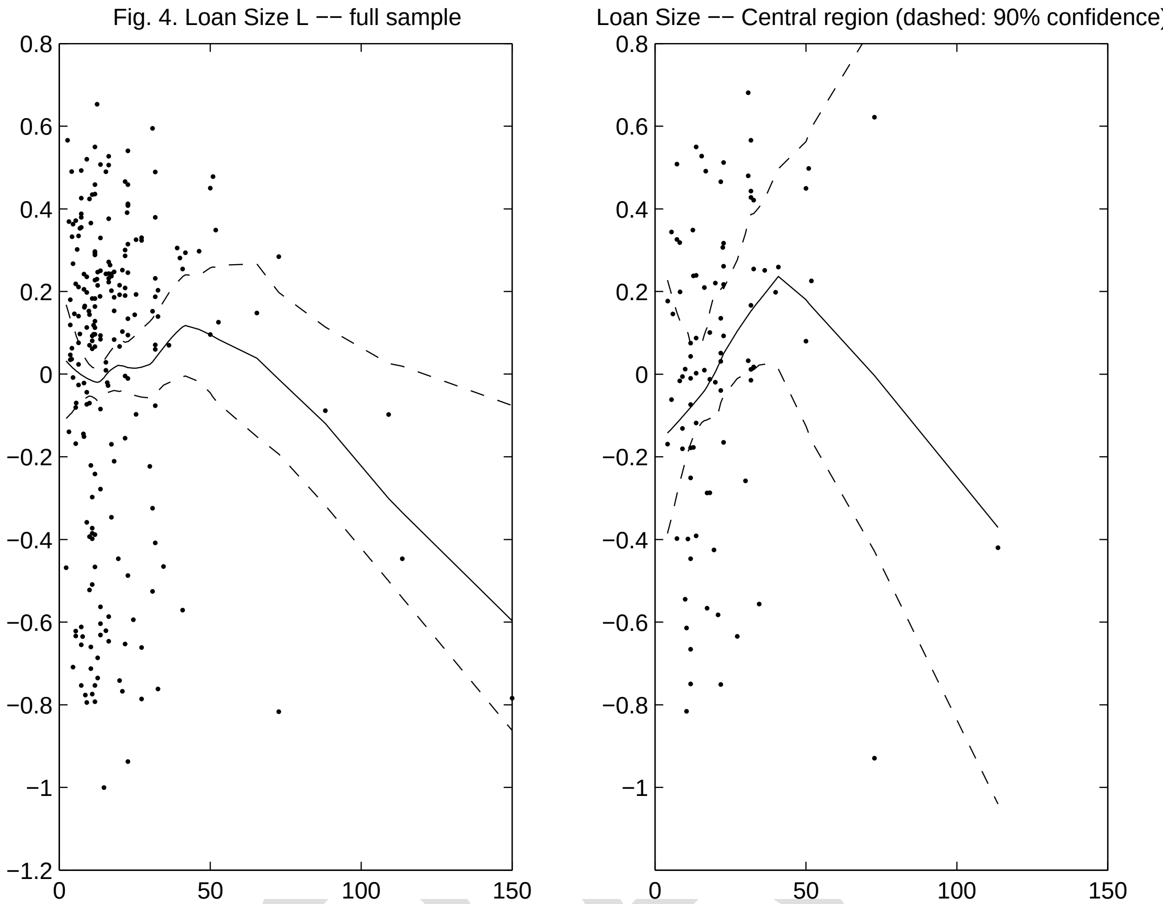
	Northeast groups	Central groups	All groups
	N=130	N=89	N = 219
Control:			
LNYRSOLD	-1.54 (.488)***	-1.61 (.701)***	-0.958 (.282)***
VARIBLTY	1.41 (4.40)	-10.1 (5.82)**	-3.47 (2.66)
WEALTH	1.00 (1.29)	0.125 (.117)	0.026 (.083)
MEMBERS	-0.014 (.089)	0.113 (.079)	0.034 (.047)
Fundamentals:			
r	-0.056 (.142)	-0.385 (.299)	-0.119 (.101)
L	20.4 (48.3)	187.4 (99.8)**	32.9 (30.8)
L^2	-0.319 (.409)	-2.39 (1.42)**	-0.463 (.335)
q	-4.69 (6.76)	-7.69 (3.31)***	-3.65 (1.51)***
Productivity:			
AVGLAND	-0.013 (.026)	-0.007 (.023)	-0.006 (.013)
EDCATION	1.88 (.935)***	0.949 (1.25)	1.28 (.698)**
Screening:			
SCREEN	-0.950 (.624)*	1.17 (.876)	-0.364 (.402)
KNOWTYPE	-1.38 (1.23)	2.24 (2.12)	-0.139 (.773)
Covariance:			
COVARBTY	1.66 (2.13)	1.82 (4.03)	2.05 (1.39)*
HOMOCCUP†	1.39 (1.45)	0.061 (1.69)	0.220 (.858)
Cost of Monitoring:			
LIVEHERE	-0.694 (1.84)	1.01 (1.26)	0.879 (.831)
RELATPCT†	-1.29 (.925)	-0.574 (1.21)	-0.590 (.573)
Cooperation:			
SHAREREL	0.491 (.417)	0.375 (.487)	0.382 (.250)*
SHAREUNR	-0.497 (.410)	-0.586 (.558)	-0.553 (.266)***
BCOOPPCT	-6.65 (3.53)**	-5.12 (5.97)	-2.30 (2.40)
JOINTDCD	0.317 (.358)	1.58 (.765)***	0.499 (.265)**
Outside Credit Options:			
PCGMEM‡	-6.56 (1.98)***	-2.98 (3.42)	-3.81 (1.18)***
CBANKMEM	-2.07 (2.39)	0.206 (2.34)	0.288 (1.21)
Penalties for Default:			
BINSTPCT‡	3.42 (1.91)**	5.24 (3.81)	2.10 (1.36)*
SNCTIONS	12.1 (4.34)***	-1.04 (3.55)	3.18 (1.95)*

[Table 9.5.9. Logit Results. Source: Ahlin and Townsend (2004)]

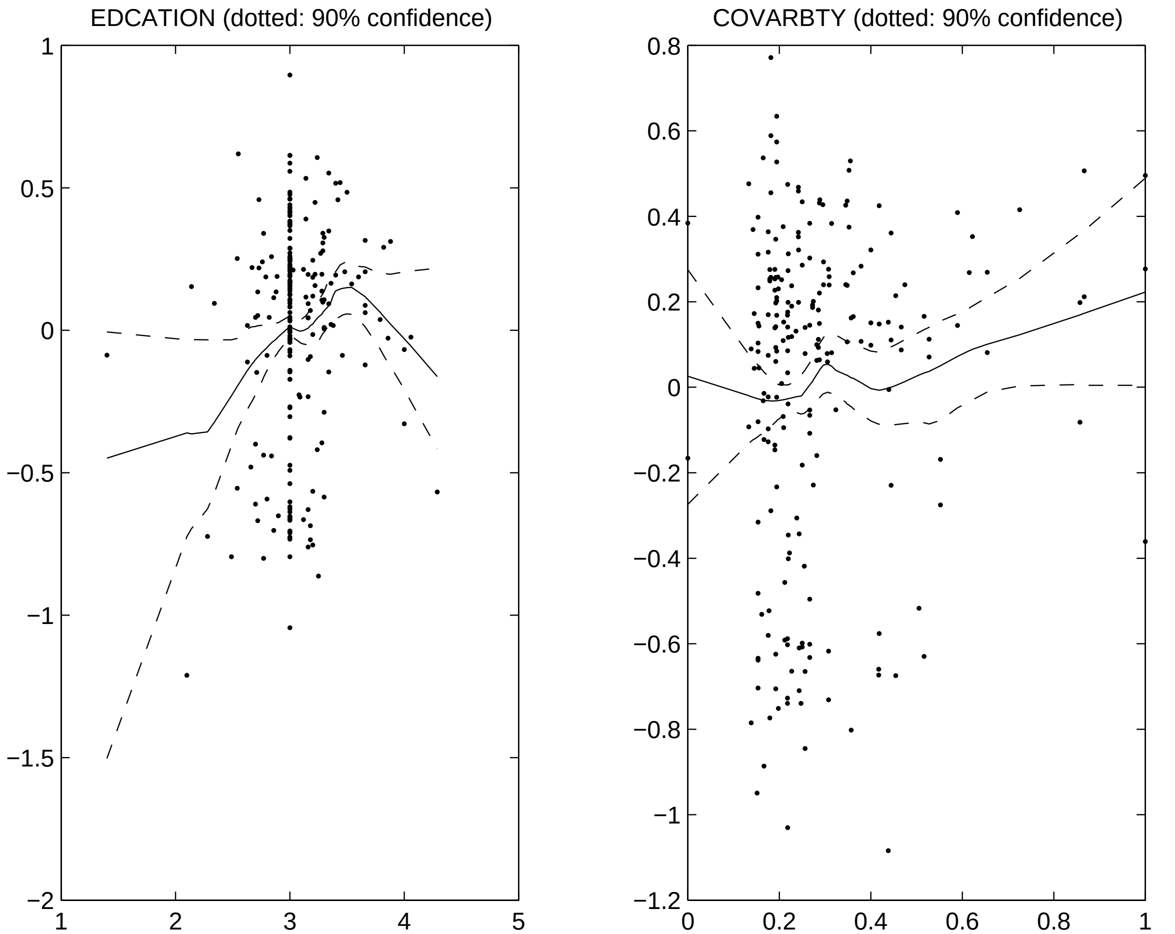
Second, a multivariate logit allows the inclusion of all covariates at the same time. The results are summarized in table 9.5.9. Standard errors are in parentheses; significance at 5, 10 and 15 percent denoted by ***, **, and * respectively.



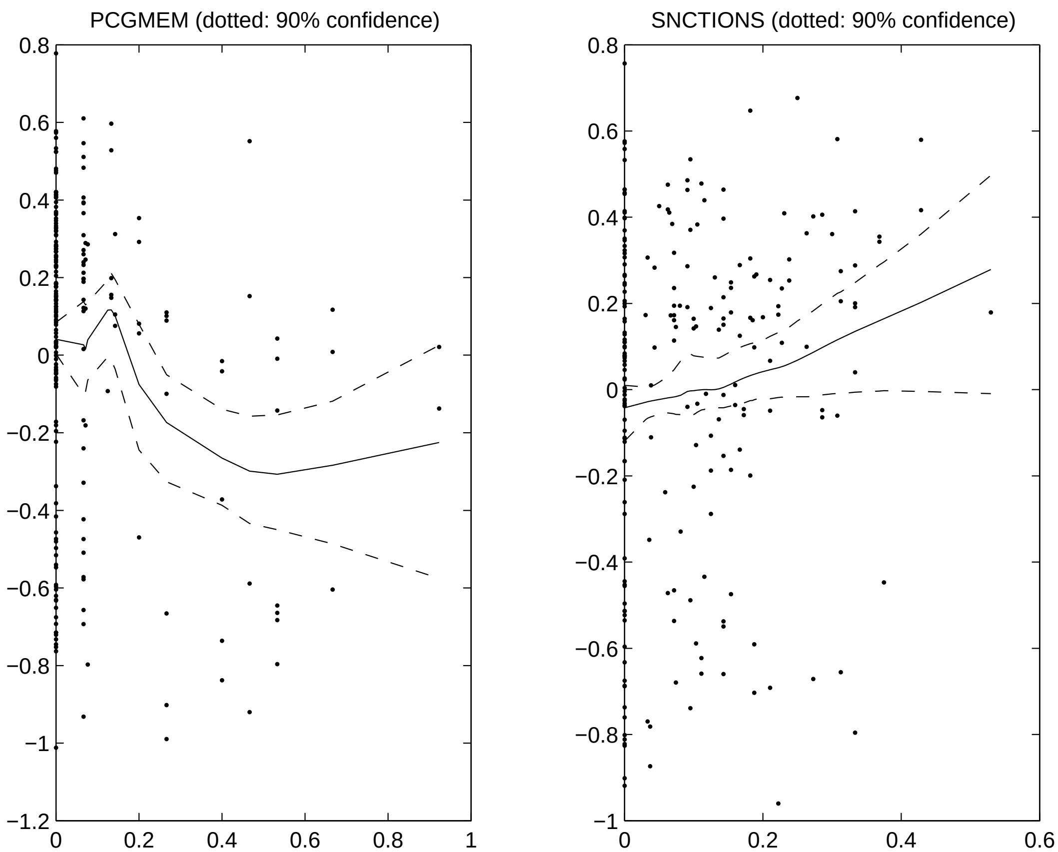
[Figure 9.5.10. Number of Members against Repayment in a bivariate locally linear regression and in a partially linear regression. Source: Ahlin and Townsend (2004)]



[Figure 9.5.11. Partially linear regressions – Loan size against Repayment for the whole sample and the central region only. Source: Ahlin and Townsend (2007) – Repayment Data]



[Figure 9.5.12 Partially linear regressions – Group Average Education, Village Covariability against Repayment. Source: Ahlin and Townsend (2004)]



[Figure 9.5.13 Partially linear regressions – PCG Prevalence, Village Sanctions against Repayment.. Source: Ahlin and Townsend (2004)]

Third, nonparametric regressions indicate the role of controls such as number of members and loans size, as well as potentially non-linear relations with the key covariates, such as education, covariance, outside credit, and sanctions. The left-hand sides of Figures 9.5.10 through 9.5.13 are sample univariate relations, and the right-hand sides with both dependent and right-hand sides purged of the other covariates.

Dependent Variable = 1 if BAAC has never raised the interest rate as a penalty, 0 if it has.
Significance in the logit regression at 15, 10, and 5% denoted by one, two, and three arrows, respectively; significant mean differences at the 10% level in 20, 50, and 80% of the nonparametric, univariate tests denoted by one, two, and three arrows, respectively.
Variables that could be included in the next group are denoted by a †; in the previous group by a ‡.

Variable	Northeast (Nonparametric,Logit)	Central	Full Sample	Stiglitz (Theoretical Predictions)	BBG	BC	Ghatak
Control:							
LNYRSOLD	↓↓↓,↓↓↓	↓↓↓,↓↓↓	↓↓↓,↓↓↓				
VARIBLTY	∅,∅	∅,↓↓	∅,↓↓				
VILGWLTH	∅,∅	↓,∅	∅,↓				
MEMBERS	∅,∅	↓↓,∅	↓↓,∅				
Fundamentals:							
r	∅,∅	∅,∅	↓,↓	↓	↓	↓	↓
L	∅,∅	∅,↑↑↓↓	∅,↑↓	↓	↓		↗↘
q	∅,∅	↓↓↓,↓↓↓	↓↓↓,↓↓↓	↓	↑		↓
Productivity:				↑	↑	↑	↑
AVGLAND	∅,∅	↑↑,∅	∅,∅				
EDCATION	↑,↑↑↑	∅,∅	∅,↑↑↑				
Screening:							↑
SCREEN	∅,↓	∅,∅	∅,∅				
KNOWTYPE	∅,∅	∅,∅	∅,↓				
Covariance:				↑		↓	↑
COVARBTY	∅,∅	↑,∅	∅,↑↑				
HOMOCCUP†	∅,∅	∅,∅	∅,∅				
Ease of Monitoring:					↑		
LIVEHERE	∅,∅	↑,∅	↑,↑↑				
RELATPCT†	∅,∅	∅,∅	↓,↓↓				
Cooperation:				↑	↓	↓	
SHAREREL	∅,∅	↓,∅	∅,↑				
SHAREUNR	∅,∅	↓↓,∅	↓↓,↓↓↓				
BCOOPPCT	∅,↓↓	∅,∅	∅,∅				
JOINTDCD	↑,∅	∅,↑↑↑	↑↑↑,↑				
Outside Credit Options:				↓	↓		
PCGMEM‡	↓↓↓,↓↓↓	∅,∅	↓↓,↓↓↓				
CBANKMEM	∅,∅	∅,∅	↓,↓↓				
Penalties for Default:							
BINSTPCT‡	∅,↑↑	∅,∅	∅,↑↑			↑	
SNCTIONS	↑↑,↑↑↑	∅,∅	∅,↑↑			↑	

[Table 9.5.14. Summary of Results. Source: Based on Ahlin and Townsend (2004)]

The findings are with rare exception consistent with there being an information problem in the Central region, and overall, though this can be mixed on occasion with a default, strategic problem. The latter is clearly is more predominant in the Northeast.

Specifically, moving down the rows of table 9.5.14, the non-monotone derivative with respect to loan size in the adverse selection model of Ghatak is found is the Central regional and in the full sample. The negative sign with respect to the joint liability payment of the moral model of Stiglitz, and the model of Ghatak, is found in the Central regional and overall. The sign on screening is counter to the Ghatak model is the Northeast and overall. Covariance raises repayment as in the two information models in the Central region and overall. Ease of monitoring reducing moral hazard and raising repayment is found in the Central region and overall under one of the two variables. Cooperation lowers repayment in BBG and BC, and this is typical of most of the data, with the exception for cooperation in decision making, which has a positive sign in the moral hazard model of Stiglitz, and especially in the Central region and overall. Outside credit should lower repayment in the moral hazard models, and this is found overall but also in the Northeast. Sanctions for strategic default are especially effective in the Northeast.

9.6 Selection Into and Across Credit Contracts

We can test for the prevalence of moral hazard and adverse selection in models which focus on the method of borrowing, or whether to borrow at all. Here we follow Holmstrom and Milgrom (1990) closely. Output or public project yields are linearly related to private household effort and an array of potentially correlated but unobserved shocks. Specifically, there are two agents, or borrowers, indexed by *i*. Each produces output q_i as a function of his effort e_i and some random shock ε_i. One could think of output q_i as varying with loan size also, but the latter is regarded as fixed and dropped from the notation. Output is then an addition of effort and shock:

$$q_i = e_i + \varepsilon_i, \quad i = 1, 2 \tag{9.6.1}$$

The ε_i 's are distributed joint-normally with a means of zero and a variance-covariance matrix,

$$\sum \equiv \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \tag{9.6.2}$$

Thus, higher effort e_i makes higher output q_i more likely, but there is noise and the returns may be correlated. Note also that the projects may differ in risk σ_i^2 and we control for variance of output in some empirical specifications. Let ***q*** and ***e*** be the column vectors $[q_1, q_2]^T$ and $[e_1, e_2]^T$, respectively. Since only the q_i 's are publicly observed, borrower payoffs must be in terms of them. Attention is restricted to contracts giving agent *i* consumption c_i as a linear function of output:

$$c_i(\boldsymbol{q};\kappa_{i0},\boldsymbol{\kappa}_i)=\kappa_{i0}+\boldsymbol{\kappa}_i^T\boldsymbol{q},\quad i=1,2 \tag{9.6.3}$$

where the column vector $\boldsymbol{\kappa}_i=\begin{bmatrix}\kappa_{i1} & \kappa_{i2}\end{bmatrix}^T$. Further, $\boldsymbol{\kappa}$ is just the collection of all the compensation parameters; let $\boldsymbol{\kappa}\equiv\{\kappa_{ij}\}$, for $i=1,2$ and $j=0,1,2$.

In this model, we consider preferences in which the disutility of effort can be measured in consumption units by $C_i(e_i)$, a strictly convex function. We also assume that agents maximize expected utility, where the utility function over consumption is exponential with a coefficient of absolute risk aversion $r_i>0$. Then, given effort choices e_1 and e_2, the certainty equivalent (CE) for agent i of contract $\boldsymbol{\kappa}$ has an analytic form:

$$CE_i(\boldsymbol{e};\boldsymbol{\kappa})=\kappa_{i0}+\boldsymbol{\kappa}_i^T\boldsymbol{e}-C_i(e_i)-\left(\frac{1}{2}\right)r_i\boldsymbol{\kappa}_i^T\sum\boldsymbol{\kappa}_i,\quad i=1,2 \tag{9.6.4}$$

where $\boldsymbol{\kappa}_i^T\sum\boldsymbol{\kappa}_i=\kappa_{i1}^2\sigma_1^2+\kappa_{i2}^2\sigma_2^2+2\kappa_{i1}\kappa_{i2}\sigma_{12}$ is the variance of i's compensation. Note that diversity in cost of effort and risk aversion is allowed, though we cannot control for these in the data beyond using education and demographic and occupational variables. The lender is assumed risk neutral and thus has certainty equivalent utility,

$$CE_p(\boldsymbol{e};\boldsymbol{\kappa})=e_1+e_2-(\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)^T\boldsymbol{e}-\kappa_{10}-\kappa_{20} \tag{9.6.5}$$

Suppose first that the lender can deal with each borrower individually – the borrowers do not observe each others' actions or outcomes and cannot conduct side-contracts. Then the lender sets both contracts to maximize his payoff subject to the agents' participation constraints and incentive compatibility constraints. Since the model exhibits transferable utility, the optimal contract maximizes total surplus (the sum of all payoffs) subject to the incentive compatibility constraints only. Thus, at the optimum, $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ solve

$$\max_{\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2}\; e_1+e_2-C_1(e_1)-C_2(e_2)-\frac{1}{2}r_1\boldsymbol{\kappa}_1^T\sum\boldsymbol{\kappa}_1-\frac{1}{2}r_2\boldsymbol{\kappa}_2^T\sum\boldsymbol{\kappa}_2 \tag{9.6.6}$$

subject to the first-order conditions for household i's effort: $C_i'(e_i)=\kappa_{ii},\quad i=1,2$.

Total surplus equals expected output (the first and second terms in the maxim) minus the costs of effort (the third and fourth terms) and risk costs (the fifth and sixth terms). The optimal contract satisfies

$$\kappa_{ii}=\frac{1}{1+r_i\sigma_i^2(1-\rho^2)C_i''},\;\kappa_{ij}=-\kappa_{ii}\sigma_{12}/\sigma_j^2;\;\; i=1,2,\; j=1,2,\; j\neq i \tag{9.6.7}$$

where $\rho \equiv \sigma_{12} / \sigma_1 \sigma_2$ is the correlation coefficient for noise in project return ε_i. Note that the direct, own-production term κ_{ii} decreases in σ_i^2 and r_i as is natural with risk aversion. The cross term κ_{ij} varies inversely with the technological correlation σ_{12} and with the risk of the other borrower σ_j^2. The overall risk cost can be calculated:

$$\left(1-\rho^2\right) \frac{r_1 \kappa_{11}^2 \sigma_1^2 + r_2 \kappa_{22}^2 \sigma_2^2}{2} \qquad (9.6.8)$$

If $\rho = 1$ then relative performance evaluation works perfectly well: all deviations in effort are detectable, and the lender offers full insurance. The risk sharing occurs because the correlation between shocks mitigates the principal's lack of information about the agents' efforts.

Suppose next that the two borrowers *can cooperate*, as is presumably easier within a joint liability group. Specifically, they do observe each others' actions and can commit to transfers with each other conditional on observed actions and outcomes. The principal still sees output only. This allows the group to mutually reinsure each other and to coordinate to an agreed upon set of actions. The side contracts they can write will be of the form

$$\tau\left(\boldsymbol{e}, \boldsymbol{q}\right) = \gamma^T \boldsymbol{q} + \tau(e_1 + e_2), \qquad (9.6.9)$$

where $\tau(\boldsymbol{e}, \boldsymbol{q})$ gives the net transfer from agent 1 to agent 2 as a result of actions $\boldsymbol{e}$ and output realizations $\boldsymbol{q}$. The function $\tau(\boldsymbol{e}, \boldsymbol{q})$ allows the pair to enforce any set of actions as a Nash equilibrium. The mutual insurance agreements, for which coefficient γ denotes the vector $\left[\gamma_1 \ \gamma_2\right]^T$, are restricted to being linear in output, as above.

Holmstrom and Milgrom (1990) assume the pair will choose $\tau(\boldsymbol{e}, \boldsymbol{q})$ and γ to reach a Pareto optimal set of actions and transfers. Again, given transferable utility, which can be done within the group using $\tau(\cdot)$, this implies the pair will maximize joint surplus. Given the external borrowing contract with its incentive $\boldsymbol{\kappa}$, the two borrowers thus choose $\left(\boldsymbol{e}, \gamma\right)$ to maximize

$$\left(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2\right)^T \boldsymbol{e} - C_1(e_1) - C_2(e_2) - (r_1/2)\left(\boldsymbol{\kappa}_1 - \gamma\right)^T \sum \left(\boldsymbol{\kappa}_1 - \gamma\right) - (r_2/2)\left(\boldsymbol{\kappa}_2 + \gamma\right)^T \sum \left(\boldsymbol{\kappa}_2 + \gamma\right) \quad (9.6.10)$$

As before, the principal can be thought of as choosing $\left(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2\right)$ to maximize total surplus, constrained however by what the group is doing:

$$\max_{\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2} e_1 + e_2 - C_1(e_1) - C_2(e_2) - \left(r_1/2\right)(\boldsymbol{\kappa}_1 - \gamma)^T \sum (\boldsymbol{\kappa}_1 - \gamma) - \left(r_2/2\right)(\boldsymbol{\kappa}_2 + \gamma)^T \sum (\boldsymbol{\kappa}_2 + \gamma) \quad (9.6.11)$$

subject to: $(\boldsymbol{e},\gamma)$ maximizes 9.6.10 given $\boldsymbol{\kappa}$.

Holmstrom and Milgrom (1990) prove, as in Wilson (1968), that the lender's optimal design coincides with that for a single ("syndicate") borrower whose effort-cost function satisfies $C(e_1,e_2)=C_1(e_1)+C_2(e_2)$ and absolute risk aversion coefficient *r* satisfies $1/r=1/r_1+1/r_2$. In other words, the risk tolerance coefficient of the syndicate borrower is greater than the risk tolerance of each individual borrower. This corresponds to a lower total risk cost to the pair, due to internal risk sharing.

Now the principal is reduced from four degrees of freedom or parameters to two, since what matters is not $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ individually, but the sum $\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2$. So, without loss of generality, assume $\kappa_{12}=\kappa_{21}=0$. Given $(\kappa_{11},\kappa_{22})$, the total overall risk cost is

$$(1/2)r\left[\kappa_{11}^2\sigma_1^2+\kappa_{22}^2\sigma_2^2+2\kappa_{11}\kappa_{22}\rho\sigma_1\sigma_2\right] \tag{9.6.12}$$

This expression is increasing in ρ.

Here, as in the relative performance regime, actions will be chosen that equate κ_{ii} and $C_i'(e_i)$. Thus any pair of actions (e_1,e_2) must be implemented by the same contract parameters $(\kappa_{11},\kappa_{22})$ in both regimes. This enables us to determine easily which regime delivers higher total surplus when implementing a given set of actions. For a given set of actions, the only part of total surplus that varies by regime is the risk cost, given in expressions 9.6.8 and 9.6.12, respectively. These risk costs are easily compared, for a given set of actions, since $(\kappa_{11},\kappa_{22})$ are the same in both regimes.

It can be verified that at $\rho=0$ the risk cost of implementing any set of actions $\boldsymbol{e}$ is lower under the cooperative regime. It follows that at $\rho=0$, the cooperative regime gives higher total surplus than the individualistic regime. Similarly, at $\rho=1$ the risk cost of implementing any set of actions $\boldsymbol{e}$ is lower under the individualistic regime, and thus the individualistic relative performance regime gives higher total surplus. Further, as ρ increases, the cost of implementing every set of actions in the cooperative regime is increasing (see expression 9.6.12), which implies that maximized surplus under this regime is decreasing in ρ. The cost of implementing every set of actions in the individualistic regime is strictly decreasing in ρ (see expression 9.6.8), which implies that the maximized surplus under this regime is strictly increasing in ρ.

In summary, holding risk aversion and other parameters constant, the payoff is strictly increasing in ρ under relative performance and decreasing in ρ under cooperation. At $\rho = 0$, the cooperative regime dominates, while at $\rho = 1$, the relative performance regime dominates. This proves that there is a cutoff, $\overline{\rho} \in (0,1)$, above which the individualistic regime dominates and below which the cooperative regime does. The intuition is that when correlation is high, the scope for internal risk-sharing is low, while the lender is able to offer significant insurance through relative performance comparisons. When correlation is low there is great need for internal insurance and relative performance works poorly.

More generally, following Prescott and Townsend (2002), we need not assume constant absolute risk aversion, nor particular forms for production. Optimal contracts can be determined by the linear programming methods described earlier. That is, let the utility function of agent *i* be $U_i(c_i) + V_i(T_i - e_i)$ where T_1 is the total time endowment, c_i is consumption and e_i is effort. The principal or insurer is risk neutral. Let *c* denote the consumption row vector $c = (c_1, c_2)$, *q* the vector of outputs $q = (q_1, q_2)$, e_i denotes the vector of efforts over two projects $e_{i\bullet} = (e_{i1}, e_{i2})$, (though typically we imagine household i only works on his own project). Likewise let *a* denote the vector of efforts over the two projects $a = (a_1, a_2)$ (though typically these are agent specific efforts (e_1, e_2)). The technology of production is described by the probability that output vector is *q* given effort vector *a*, namely $p(q \mid a)$.

The programming problem for the determination of the relative performance regime searches over the policy probability $\pi(c, q, e_1, e_2)$. The optimal contract is found by maximizing surplus

$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\bullet},\boldsymbol{e}_{2\bullet}} \pi(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\bullet},\boldsymbol{e}_{2\bullet})(q_1 + q_2 - c_1 - c_2) \tag{9.6.13}$$

subject to promise keeping for the group at Pareto weight λ_i:

$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\bullet},\boldsymbol{e}_{2\bullet}} \pi(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\bullet},\boldsymbol{e}_{2\bullet}) \sum_i \lambda_i \left[U_i(c_i) + V_i(T_i - e_i)\right] \geq \overline{G}, \tag{9.6.14}$$

technological probability

$$\sum_{\boldsymbol{c}} \pi(\boldsymbol{c},\overline{\boldsymbol{q}},\overline{\boldsymbol{e}}_{1\bullet},\overline{\boldsymbol{e}}_{2\bullet}) = p(\overline{\boldsymbol{q}} \mid \overline{\boldsymbol{e}}_{1\bullet} + \overline{\boldsymbol{e}}_{2\bullet}) \sum_{\boldsymbol{c},\boldsymbol{q}} \Pi(\boldsymbol{c},\boldsymbol{q},\overline{\boldsymbol{e}}_{1\bullet},\overline{\boldsymbol{e}}_{2\bullet}), \quad \forall \overline{\boldsymbol{q}},\overline{\boldsymbol{e}}_{1\bullet},\overline{\boldsymbol{e}}_{2\bullet} \tag{9.6.15}$$

individual incentive constraints for agent 1

$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{2\bullet}} \pi(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\bullet},\boldsymbol{e}_{2\bullet})\left[U_1(c_1) + V_1(T_1 - e_1)\right] \geq$$

$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{2\bullet}} \pi(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\bullet},\boldsymbol{e}_{2\bullet}) \frac{p(\boldsymbol{q} \mid \hat{\boldsymbol{e}}_{1\bullet} + \boldsymbol{e}_{2\bullet})}{p(\boldsymbol{q} \mid \boldsymbol{e}_{1\bullet} + \boldsymbol{e}_{2\bullet})}\left[U_1(c_1) + V_1(T_1 - \hat{e}_1)\right], \quad \forall \boldsymbol{e}_{1\bullet},\hat{\boldsymbol{e}}_{1\bullet} \tag{9.6.16}$$

and individual incentive constraint for agent 2

$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{2\cdot}} \pi\left(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\cdot},\boldsymbol{e}_{2\cdot}\right)\left[U_2(c_2)+V_2\left(T_2-e_2\right)\right] \geq$$
$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{2\cdot}} \pi\left(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\cdot},\boldsymbol{e}_{2\cdot}\right)\frac{p\left(\boldsymbol{q}\,|\,\boldsymbol{e}_{1\cdot}+\hat{\boldsymbol{e}}_{2\cdot}\right)}{p\left(\boldsymbol{q}\,|\,\boldsymbol{e}_{1\cdot}+\boldsymbol{e}_{2\cdot}\right)}\left[U_2(c_2)+V_2\left(T_2-\hat{e}_2\right)\right],\ \forall \boldsymbol{e}_{2\cdot},\hat{\boldsymbol{e}}_{2\cdot} \tag{9.6.17}$$

Of course the contract must also satisfy probability measure constraints

$$\pi\left(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\cdot},\boldsymbol{e}_{2\cdot}\right)\geq 0, \quad \forall\ \boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\cdot},\boldsymbol{e}_{2\cdot} \tag{9.6.18}$$

for each

$$\sum_{\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\cdot},\boldsymbol{e}_{2\cdot}} \pi\left(\boldsymbol{c},\boldsymbol{q},\boldsymbol{e}_{1\cdot},\boldsymbol{e}_{2\cdot}\right)=1. \tag{9.6.19}$$

Again, this formulation of the problem maximizes the "surplus" of the principal subject to a reservation utility constraint. The first constraint ensures a given (λ_i -weighted) amount of utility for the pair. Note that as $\bar{G}$ increases, in effect the wealth of each borrower increases. Parameter $\bar{G}$ can be varied parametrically as in partial equilibrium comparative statics. Equivalently, one can maximize a weighted sum of utilities subject to a minimum surplus or "wealth" constraint for the outsider.

Again, a joint liability contract allows the two agents to enter into a risk-sharing group agreement with internal rules $c_i(c_g,\mu)$ where $\mu=\left(\mu_1,\mu_2\right)$ are the within-group Pareto weights, μ_i the Pareto weight of agent i, and consumption c_i of agent i as a function of group consumption c_g. We make a similar substitution for leisure/effort $e_i(a_g,\mu)$ where a_g is total effort a_1+a_2, or for ease of notation $e_i\left(\boldsymbol{a},\mu\right)$.

The programming problem for the determination of the group regime searches over policy probability $\pi\left(c_q,\boldsymbol{q},\boldsymbol{a},\mu\right)$. It should maximize surplus

$$\sum_{c_g,\boldsymbol{q},\boldsymbol{a},\mu} \pi\left(c_g,\boldsymbol{q},\boldsymbol{a},\mu\right)\left(q_1+q_2-c_g\right) \tag{9.6.20}$$

subject to reservation utility of the group, and technology constraint

$$\sum_{c_g,\boldsymbol{q},\boldsymbol{a},\mu} \pi\left(c_g,\boldsymbol{q},\boldsymbol{a},\mu\right)\sum_i \lambda_i\left[U_i\left(c_i\left(c_g,\mu\right)\right)+V_i\left(T_i-e_i\left(\boldsymbol{a},\mu\right)\right)\right]\geq \bar{G} \tag{9.6.21}$$

technological probability

$$\sum_{c_g} \pi\left(c_g, \bar{\boldsymbol{q}}, \bar{\boldsymbol{a}}, \bar{\mu}\right) = p\left(\bar{\boldsymbol{q}} \mid \bar{\boldsymbol{a}}\right) \sum_{c_g, \boldsymbol{q}} \Pi\left(c_g, \boldsymbol{q}, \bar{\boldsymbol{a}}, \bar{\mu}\right), \quad \forall \boldsymbol{q}, \bar{\boldsymbol{a}}, \bar{\mu}, \tag{9.6.22}$$

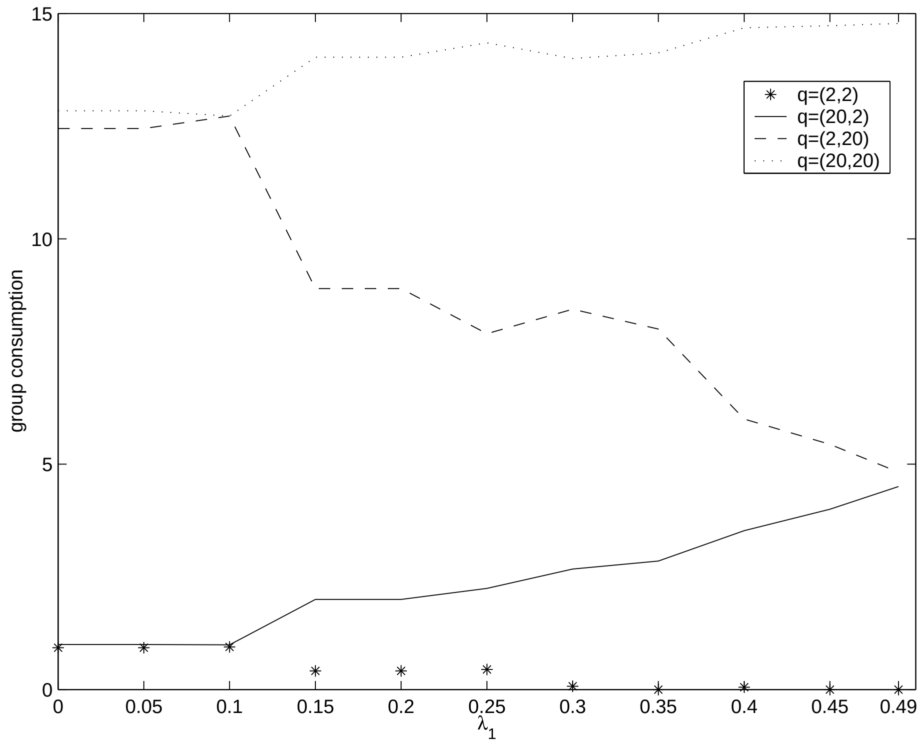
and group incentive constraint

$$\begin{aligned} &\sum_{c_g, \boldsymbol{q}} \pi\left(c_g, \boldsymbol{q}, \boldsymbol{a}, \mu\right) \sum_i \mu_i \left[U_i\left(c_i\left(c_g, \mu\right)\right) + V_i\left(T_i - e_i\left(\boldsymbol{a}, \mu\right)\right)\right] \geq \\ &\sum_{c_g, \boldsymbol{q}} \pi\left(c_g, \boldsymbol{q}, \boldsymbol{a}, \mu\right) \frac{p\left(\boldsymbol{q} \mid \hat{\boldsymbol{a}}\right)}{p\left(\boldsymbol{q} \mid \boldsymbol{a}\right)} \sum_i \mu_i \left[U_i\left(c_i\left(c_g, \mu\right)\right) + V_i\left(T_i - e_i\left(\hat{\boldsymbol{a}}, \mu\right)\right)\right] \ \forall \boldsymbol{a}, \hat{\boldsymbol{a}}, \mu. \end{aligned} \tag{9.6.23}$$

The last constraint is the group incentive to take action vector a over $\hat{\boldsymbol{a}}$ given internal weight μ. The degree of internal inequality μ is endogenous here and may differ from the objective weights λ because of this group incentive constraint. The difficulty of incentives is linked to the distribution of income. Typically, though, in the example solutions below we do not distinguish μ from λ.

For certain classes and technologies, we know that weights μ and λ must be equal. Assume that sets of feasible consumption and efforts are continua, just as we did in the analysis of internal group sharing rules, but retain the CRRA preference specification. These preferences aggregate in the sense of Gorman (1954). Then, varying the weights within the group will not affect the group's schedule of payments to the outsider in any way. It is as if the outsider were facing a single agent who has the choice of effort over the two technologies. The consumption and labor allocation to this "single agent" is determined as in the well-understood, classic, principal-agent model.

Suppose, in particular, that preferences do not aggregate. Specifically, individuals are required to work their own technologies. Suppose further, for example, that projects either succeed or fail. Then there are four possible outputs, reflecting all the different combinations of high or low outputs on the two technologies. Figure 9.6.1, from Prescott and Townsend (2002), describes which of the four lines corresponds to which output combination. Not shown in the Figure is the optimal labor assignment, but here for the full range of $\lambda_1 \in [0.0,\ 0.5]$ both agents are assigned the high labor effort.



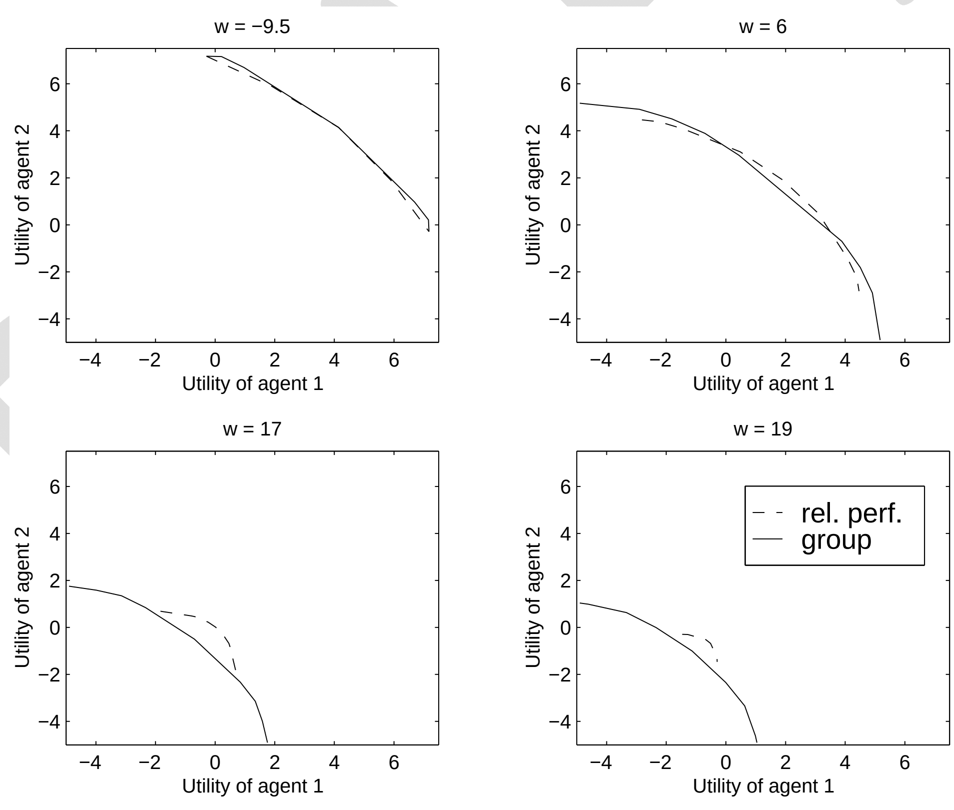
[Figure 9.6.1. Group consumption sharing rules for various λ_1. Source: Prescott and Townsend (2002)]

Starting from the left side of the graph, at $\lambda_1 = 0.0$, agent one receives no weight within the group. Not shown in the graph is individual consumption. Of course, at $\lambda_1 = 0.0$ agent one's consumption is zero for all outputs, while agent two consumes the entire group consumption c_g. Low consumption and high effort for agent one have little consequence for group utility because agent one has low weight. Essentially, agent one is a "serf." Internally, having agreed on the distribution of welfare implicit in λ, agent one abides by the agreement, which is to work hard and to consume little or nothing. That is, $c_g(q_1, q_2)$ lines for $\left(q_h, q_h\right) = (20,20)$) and $\left(q_l, q_h\right) = (2,20)$ nearly coincide in Figure 9.6.1 at $\lambda_1 = 0.0$ (any difference is due to numerical approximation). Coincident also are the lines for $\left(q_l, q_l\right) = (2,2)$ and $\left(q_h, q_l\right) = (20,2)$. Thus, group consumption does not depend on output from the technology utilized by agent one. The risk neutral principal provides full insurance on technology one because internal monitoring and perfect commitment take care of potential incentive problems for agent one.

In contrast, agent two, the so-called "lord" of the group, has the high λ_2 weight. At and near $\lambda_1 = 0.0$, and λ_2 near 1.0, the "mongrel consumer's" utility is nearly identical to that of agent two. Since the mongrel consumer cares (mostly) about the effort of agent two, the group must be given incentives to make him work hard. Thus, group consumption and agent two's consumption vary positively with the output of agent two on technology two.

This logic prevails more generally, as λ_1 increases toward the symmetric weight $\lambda_1 = 0.5$. Over this range, group consumption c_g depends primarily on output of agent two from technology two, namely q_2, as he is most inclined to shirk. Though output q_1, of agent one from technology one becomes increasingly important. In sum, for $0.0 < \lambda_1 < 0.5$, group consumption c_g is ordered with technology output:

$$c_g\left(q_1 = 2,\ q_2 = 2\right) < c_g\left(q_1 = 20,\ q_2 = 2\right) < c_g\left(q_1 = 2,\ q_2 = 20\right) < c_g\left(q_1 = 20,\ q_2 = 20\right).$$



[Figure 9.6.2. Slices of Pareto frontier. Source: Prescott and Townsend (2002)]

The dominance of one financial regime over another in this context is a function of the Pareto weight λ which determines where agents 1 and 2 lie on the utility possibilities frontier and the utility $\overline{W}$

of the principal (See Figure 9.6.2). (Variation $\overline{W}$ is achieved by varying parameter $\overline{G}$ in the program: the higher $\overline{W}$ is, the lower $\overline{G}$ is.) If the utilities U_1, U_2 of the agents lie on or close to the 45-degree line in utility space, that is, the agents are to be treated more or less equally, then the relative performance regime dominates the group insurance regime. Thus utility dispersion is a force for groups. Note also that as the utility of the principal is varied, groups emerge and disappear again.

Now, imagine a variety of local economies k which vary in the preferences $U_{ki}(c_i) + V_{ki}(T_{ki} - e_i)$, technologies $p\left(q \mid e_{1} + e_{2}\right)$, and Pareto weights $\left(\lambda_{k1}, \lambda_{k1}\right)$ of local residents. Then contract regimes could vary across these local economies k, especially with the degree of local inequality. Indeed, with a continuum of economies of each type, we can let α_k be the relative number of economies of type k, and let ρ_k be the relative Pareto weight of local economy k. One can then write down a mechanism design problem similar to the earlier programs, but here for the larger single economy. We would maximize the ρ_k weighted sums of type k utilities subject to a single economy-wide resource constraint, that the surplus when added across all small economies be no less than zero. In effect, the utility $\overline{W}$ of a single risk neutral principal would be at least zero.

The solution, when reinterpreted, does allow interactions among the local economies. In principle, transfers *across* economies of *different* types are allowed, though the natural benchmark would be no transfers across types. Key is the provision of insurance for local fluctuations among local economies of the same type. When there are nontrivial lotteries for an economy of a given type, then the extended model predicts coexistence of regimes, as observed in the data. Note the nonconvexities in the outer envelope of the utilities frontier in Figure 9.6.2.

There is in fact a decentralization of the large economies in which each local economy (with its own level of inequality) interacts in larger, economy-wide markets. The advantage of that decentralization is that it sets the surplus of each economy type to zero endogenously. But also as in the second welfare theorem, there are possible wealth reallocations across economy types as ρ_k is varied. There would of course be an overall restriction, zero net resource use across all economies, so that intermediary losses in subsidies to one group are financed by taxes on other groups.

Later, we shall discuss making the local Pareto weights $(\lambda_{k1}, \lambda_{k1})$ endogenous, as well. They will be determined naturally enough by the endogenous value of individual endowments evaluated at equilibrium prices. These connections underlie our interchangeable use of Pareto weights and wealth, that is asymmetric Pareto weights are equivalent with high wealth dispersion in the local economy, and low weight ρ_k in the local economy with low average wealth.

To test these credit selection models, Ahlin and Townsend (2005) use the data from the BAAC and households instruments of the Townsend Thai survey. As described earlier, some households have entered into group guaranteed loans and others into individual loans. We also broaden the categories to include loans from village funds. We have these data for each loan and each individual. The dependent variables that we use are dummies reflecting whether the household has taken out a group-guaranteed loan from a lending institution in the past year. There are two versions of this variable. The first, BAGPLOAN, restricts attention to group-guaranteed loans from the BAAC. As noted, this government institution is the primary institutional lender in rural Thailand: for example, 64% of institutional loans in our sample are from the BAAC. The BAAC offers both individual loans, which must be guaranteed by some form of collateral, usually land, and joint liability loans. To receive the latter, one must form or join an official BAAC-registered borrowing group and enter into a joint liability arrangement. BAGPLOAN equals one if the household has had an outstanding loan from the BAAC in the past year and lists the collateral for this loan as either none, a single guarantor, or multiple guarantors. About 23% of the household sample has such a loan.

The second version of the dependent variable is GRUPLOAN, which incorporates group guaranteed loans from the BAAC and other institutions. These others are typically smaller institutions such as agricultural cooperatives and often village-based ones such as production credit groups (PCGs), but they also include commercial banks. Using this broader definition increases the proportion of the sample that qualifies as having a group-guaranteed loan to about 30%. However, the institutions incorporated are diverse in size and practice, which makes isolating contracts that are clearly group contracts more imprecise. We report specifications using both BAGPLOAN and GRUPLOAN. Two analogous variables measure whether the household has an individual loan contract from a lending institution. BAIDLOAN and INDLOAN correspond to BAGPLOAN and GRUPLOAN, respectively, in the lenders they cover. The criterion for a loan counting as an individual loan are that the collateral used was land, savings, current or future crops, and other collateral such as house or boat. BAIDLOAN is positive for about 13% of the population, INDLOAN for about 22%. Neither of these will be used directly in regressions, but will at times be used to limit the sample to only those households having secured either an individual or a group loan.

Let $\overline{WEALTH}$ denote average village wealth. To measure wealth dispersion, labeled WLTHDSPR in the logits, we use the following function of household wealth and village average wealth.

$$WLTHDSPR = \left[1 - \frac{WEALTH}{\overline{WEALTH}}\right]^{1/2}$$

This is similar to a simple distance function, $\left[\overline{WEALTH} - WEALTH\right]$. The differences are that we divide by $\overline{WEALTH}$, which makes it a scale-free measure, and we take a square root which dampens the effect of *WEALTH*'s long right tail. To check robustness relative to functional form, we will also use a nonparametric regression technique on

$$\frac{WEALTH}{\overline{WEALTH}}$$

itself. Inequality matters for prediction of whether we should see joint liability or individual loan contracts, assuming there are no policy restrictions.

Standard errors in parentheses; significance at 5, 10, and 15% denoted by ***, **, and *, respectively.

	BAGPLOAN			GRUPLOAN		
WEALTH	-.417 (.169)***	-.418 (.169)***	-.418 (.169)***	-.174 (.096)**	-.173 (.097)**	-.175 (.097)**
WEALTHSQ	3.24E-8 (1.73E-8)**	3.28E-8 (1.72E-8)**	3.26E-8 (1.73E-8)**	1.03E-8 (4.91E-9)***	1.03E-8 (4.98E-9)***	1.02E-8 (4.94E-9)***
WLTHDSPR	.696 (.253)***	.686 (.254)***	.695 (.254)***	.505 (.217)***	.500 (.217)***	.509 (.217)***
TITLE	.213 (.164)	.219 (.165)	.216 (.165)	-.053 (.090)	-.043 (.090)	-.046 (.090)
TITLESQ	-3.14E-8 (1.70E-8)**	-3.21E-8 (1.70E-8)**	-3.17E-8 (1.70E-8)**	-8.14E-9 (4.63E-9)**	-8.19E-9 (4.71E-9)**	-8.10E-9 (4.66E-9)**
PROBHI	.023 (.353)	.017 (.354)	.029 (.353)	-.139 (.308)	-.124 (.308)	-.123 (.308)
SAMEBEST	.176 (.401)			.549 (.350)*		
SAMEWRST		-.190 (.407)			-.059 (.367)	
SAMEITHR			.009 (.513)			.482 (.471)
INCOME	.312 (.877)	.335 (.882)	.310 (.876)	.075 (.738)	.082 (.735)	.037 (.727)
EXINCOME	-.732 (1.10)	-.771 (1.11)	-.734 (1.10)	-.090 (.943)	-.102 (.938)	-.042 (.928)
NRTHEAST	-.368 (.214)**	-.344 (.213)*	-.355 (.214)**	-.223 (.176)	-.191 (.176)	-.222 (.177)
AGRYES	-.538 (.522)	-.537 (.522)	-.530 (.522)	.003 (.354)	.040 (.353)	.021 (.353)
AGRNO	-.804 (.531)*	-.802 (.532)*	-.798 (.531)*	-.415 (.361)	-.387 (.360)	-.403 (.361)
OWNSBSNS	-.116 (.225)	-.125 (.226)	-.117 (.226)	-.512 (.189)***	-.521 (.189)***	-.508 (.189)***
LANDOWND	-7.28E-3 (4.12E-3)**	-7.21E-3 (4.12E-3)**	-7.25E-3 (4.11E-3)**	-6.21E-3 (3.45E-3)**	-6.22E-3 (3.45E-3)**	-6.27E-3 (3.44E-3)**
EDYEARH	.030 (.042)	.029 (.042)	.030 (.042)	.010 (.033)	.007 (.033)	.010 (.033)
MALEH	.437 (.243)**	.446 (.243)**	.445 (.243)**	.628 (.208)***	.644 (.208)***	.632 (.208)***
N	573	573	573	736	736	736

[Table 9.6.3. Restricted samples – these sample are restricted only to those borrowers having either a group-guaranteed loan or an individual loan. Source: Ahlin and Townsend (2007 – Selection)]

The multivariate logit examines the prevalence of group guaranteed, joint liability loans with wealth, wealth squared, wealth spread, titled land with a linear and quadratic term, and some other controls (expected income, Northeast, in agriculture, land owned, education, male head). We also examine the implication of correlation in project returns by including various measures one at a time: the fractions of the household among village respondents for whom the best year in the last five was the same year, the fraction for whom the worst year was the same, and a measure which counts identical responses across both good and bad years.

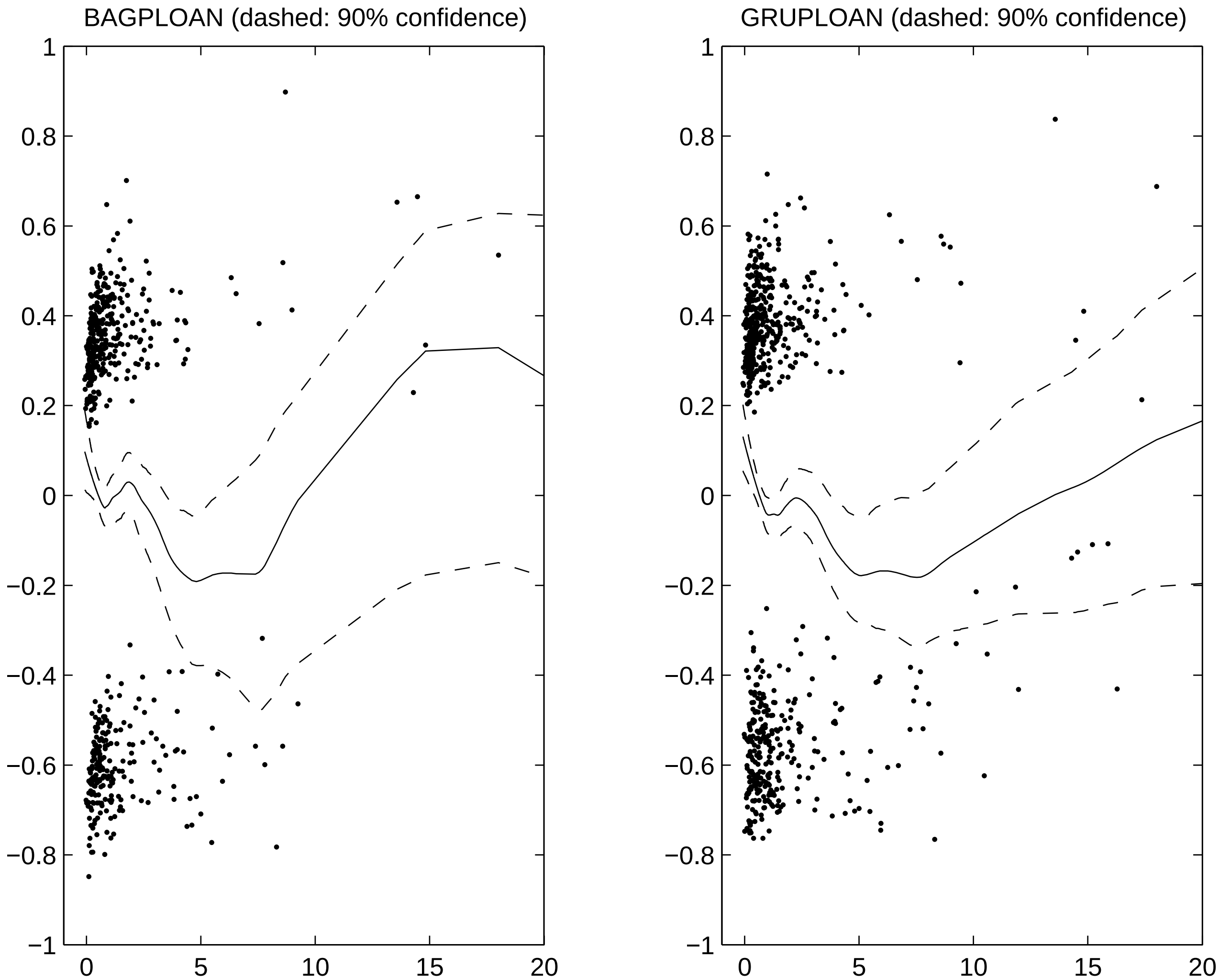
The best indicator we have of the wealth of the potential group is the wealth of the household itself. One can see from non-parametric regression Figure 9.6.5 and the logits in Table 9.6.3, both with multivariate controls that an individual is more likely to be in a joint liability group as a U-shaped function of wealth. The initial negatively sloped part is especially prominent.

There may be other explanations for such a relationship. Chief among these could be a story revolving around collateralizable wealth: since individual loans require collateral, we would expect them to be more prevalent at higher wealth levels, at the expense of group loans. This story, however, does not immediately make clear why the relationship should turn up again to deliver a U-shape. Further, we separate out and control for the part of wealth that is most commonly used and accepted as collateral, TITLE and TITLESQ, and still find the U-shaped relationship between total wealth and being in a group contract. Thus the collateral story does not seem to be driving the results.

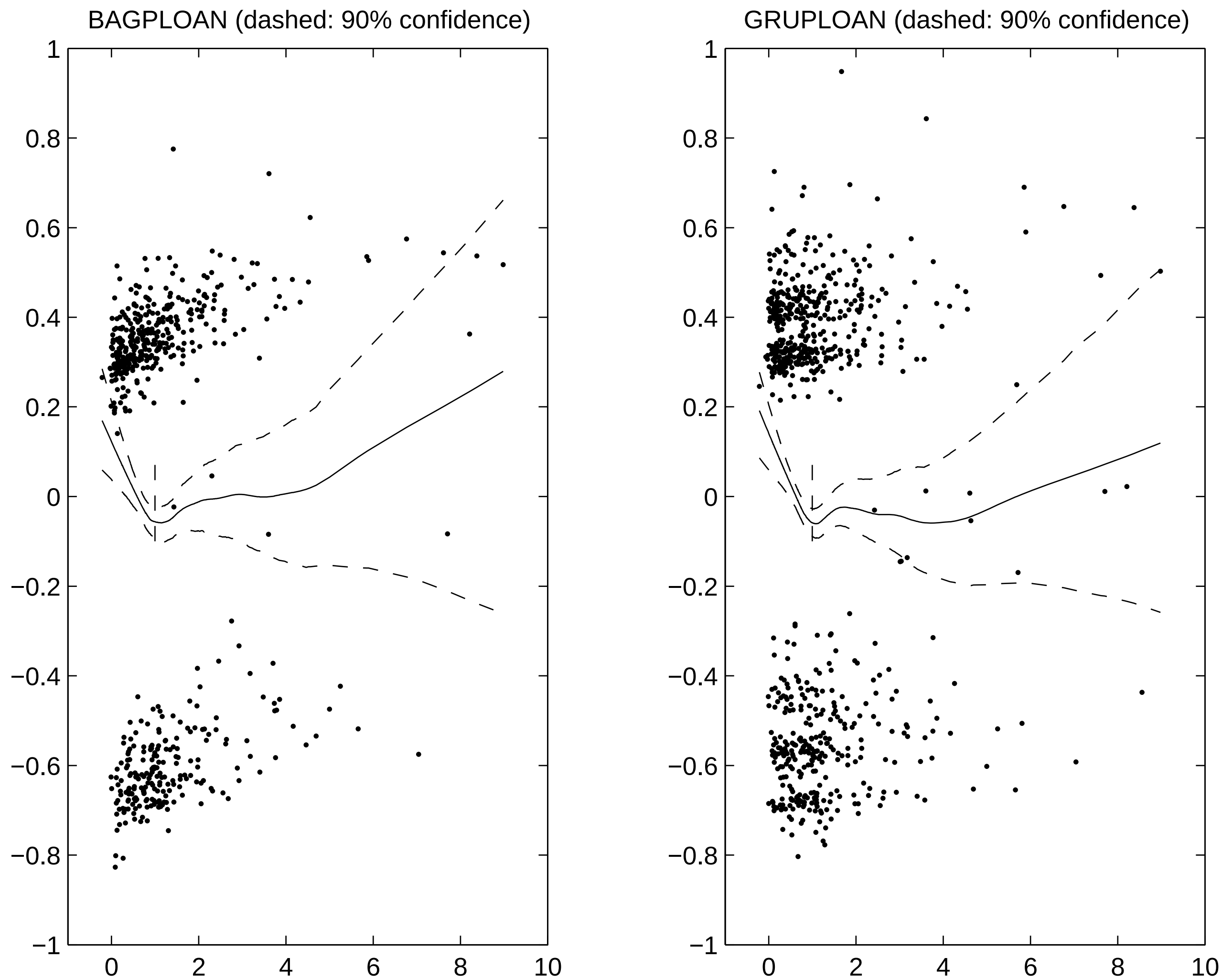
One might also think that poor households borrow in groups, moderately wealthy households borrow as individuals, and the wealthiest households take out both kinds of loans since their demand for credit is higher. This would produce a U-shape relationship between wealth and having a group loan. But note that this explanation is ruled out since the sample includes households with one or the other kind of loan, but not both.

We do not have group-level wealth, only wealth of households. Another approach would be to proxy group wealth for a given household as the average between that household's wealth and the average household wealth level in the village (or among villagers who borrow). The inverted-U shape loses significance in the majority of specifications under this approach. This need not be due to a deficiency of the model, but rather an imperfect proxy for group wealth.

A strong and statistically significant result is that group borrowing is more likely the higher the spread in wealth. The non-parametric regressions in Figure 9.6.4 using the simple measure $\frac{WEALTH}{\overline{WEALTH}}$ show that prevalence of the group reaches its minimum almost precisely when individual wealth equals the village mean. Here the declining portion is most significant, within tight standard error bands, though the upturn is evident locally as well.



[Figure 9.6.4. Wealth as a predictor of having a group loan. Source: Ahlin and Townsend (2007 – Selection)]



[Figure 9.6.5. Relative wealth – household wealth divided by the village mean – as a predictor of having a group loan. Source: Ahlin and Townsend (2007 – Selection)]

The theory was not designed to capture supply side variation. But households who borrow from the BAAC have made a broader decision as to whether to borrow at all, in turn a function of the availability or ease of use of BAAC facilities. Though we do not expand in this section the selection model, we conduct a robustness check. We include in the logits the variables we had used earlier as supply side instruments, such as headmen responses on the use of the BAAC and other institutions in the village, time to the district center, various GIS averages, as well as a batch of CDD village level characteristics. Many of the co-variate's controls and (former) instruments are significant. Remarkably though, the featured significant variables, wealth and wealth spread remain. See Table 9.6.2.

Variable	Original •	New ••
WEALTH	-416845.45***	-462334.08***
WEALTHSQ	.0324**	.0345*
WLTHDSPR	.6962***	.6609***
TITLE	213194.59	313053.16**
TITLESQ	-.0314**	-.0336*
PROBHI	0.0233	-.1441
% OF OTHER RESPS.	0.1755	0.4329
INCOME	0.3122	0.6668
EXINCOME	-.7318	-1.1167
NRTHEAST	-.3677**	-.5586**
AGRYES	-.5382	-.6078
AGRNO	-.8035*	-.7778
OWNSBSNS	-.1163	-0.1166
LANDOWND	-.0073**	-.0063
EDYEARH	0.03	0.0148
MALEH	.4371**	0.3906
AGRI ONLY		-1.3951
CDD MULTIPLE Occ.		-1.3545
ASSEMBLY HALL CDD		.7180***
ECON STATUS CDD		-.4465**
HELP FROM GOVT		-.4435**
TIME TO DIST. CENTER		-.0095
USE OF BAAC		-.4268
COMMUNITY BANK		0.1961
HAS RICE BANK		-.3657
HAS BUFFALO BANK		-.0811
COTTAGE IND.		-30.1780***
PREDIC. lb 5		-.3723
PREDIC. Rb 5		0.9221**
PREDIC. vf 5		0.004
CONSTANT	0.8436	2.6715***
N	573	501

Legend: * p<.15; ** p<.10; *** p<.05

• : Original variables (Table 9.6.3); •• : New variables (including CDD)

[Table 9.6.6. Selection with Supply-Side Variation – Moral Hazard. Source: Puentes and Townsend (2006), unpublished research note]

Moral hazard is not the only possible information problem. The adverse selection model of Ghatak was described earlier. The key selection equation as in 9.5.6 earlier is:

$$E(p) - pr - p(1-p)q = \underline{U}.$$

The key variable to measure is the risk-types of the borrower. We do so using subjective income assessments, taking Ghatak (1999) quite literally. Specifically, each household was asked what their income would be in the coming year if it were a good year (Hi), what their income would be if it was a bad year (Lo), and what they expected their income to be (Ex). We assume the income distribution is binomial over the high and low states, as in Ghatak (1999). The probability of success, $PROBHI$, is then calculated to be

$$PROBHI = \frac{Ex - Lo}{Hi - Lo}$$

using the fact that $PROBHI * Hi + (1 - PROBHI) * Lo = Ex$.

The full sample, including of course those not borrowing, and those borrowing with individual collateral, is used to determine whether borrowing in a group is a function of risk, or others variables such as the correlation of returns. (See Table 9.6.7) Under the theory, low risk, high p types should not be borrowing. Strikingly, in multivariate logits with a binary one, zero variable for the use of joint liability, the higher is the probability of success p, the less likely is a household to borrow under joint liability (we also include individual loans in the dependent variable as a robustness check so that the alternative is not borrowing at all). Risk type appears as an important variable in virtually all specifications, at varying levels of significance. It seems evident that there is an adverse selection problem in the provision of insurance.

Standard errors in parentheses; significance at 5, 10, and 15% denoted by ***, **, and *, respectively.

	BAGPLOAN			GRUPLOAN		
WEALTH	-.077 (.078)	-.077 (.078)	-.077 (.078)	.016 (.067)	.016 (.068)	.015 (.068)
WEALTHSQ	1.08E-9 (2.81E-9)	1.04E-9 (2.84E-9)	1.08E-9 (2.81E-9)	-1.37E-9 (2.85E-9)	-1.48E-9 (2.96E-9)	-1.43E-9 (2.91E-9)
WLTHDSPR	.119 (.176)	.130 (.176)	.123 (.176)	-.098 (.164)	-.078 (.165)	-.084 (.165)
TITLE	.017 (.110)	.014 (.110)	.014 (.110)	-.125 (.071)**	-.124 (.071)**	-.127 (.071)**
TITLESQ	-3.30E-9 (6.06E-9)	-3.09E-9 (6.06E-9)	-3.16E-9 (6.04E-9)	2.21E-9 (2.80E-9)	2.30E-9 (2.90E-9)	2.29E-9 (2.84E-9)
PROBHI	-.460 (.235)***	-.431 (.236)**	-.448 (.235)**	-.386 (.218)**	-.344 (.219)*	-.353 (.218)*
SAMEBEST	-.008 (.269)			.370 (.251)*		
SAMEWRST		.351 (.289)			.408 (.272)*	
SAMEITHR			.331 (.367)			.784 (.346)***
INCOME	.611 (.559)	.605 (.562)	.603 (.560)	.049 (.508)	.042 (.513)	.035 (.511)
EXINCOME	-.578 (.672)	-.565 (.674)	-.563 (.672)	.066 (.591)	.075 (.596)	.087 (.594)
NRTHEAST	.618 (.133)***	.603 (.133)***	.600 (.134)***	.317 (.122)***	.326 (.121)***	.301 (.122)***
AGRYES	1.03 (.244)***	1.03 (.244)***	1.02 (.244)***	1.16 (.214)***	1.17 (.213)***	1.15 (.214)***
AGRNO	.761 (.251)***	.753 (.251)***	.752 (.252)***	.737 (.221)***	.735 (.221)***	.725 (.221)***
OWNBSNS	.280 (.152)**	.294 (.153)**	.287 (.153)**	.079 (.141)	.093 (.142)	.093 (.142)
LANDOWND	4.21E-3 (2.68E-3)*	4.13E-3 (2.67E-3)*	4.19E-3 (2.67E-3)*	4.17E-3 (2.36E-3)**	4.01E-3 (2.36E-3)**	4.09E-3 (2.35E-3)**
EDYEARH	.037 (.024)*	.038 (.024)*	.038 (.024)*	.056 (.023)***	.057 (.023)***	.057 (.023)***
MALEH	.510 (.168)***	.508 (.168)***	.504 (.168)***	.468 (.152)***	.476 (.152)***	.467 (.152)***
N	1666	1666	1666	1602	1602	1602

[Table 9.6.7. Full Samples - The full sample is used in each of these regressions. Source: Ahlin and Townsend (2007) – Selection]

Note that we have not identified adverse selection vis-à-vis moral hazard. A moral hazard interpretation of the data could be that risk was endogenous to having a group loan; that is, having the loan was causing the borrower to operate with more risk, rather than vice versa. In order to identify a pure adverse selection effect, we make use of the fact that some of the loans in our data have already been repaid (usually within the last few months). Under adverse selection, those who took a loan but have already repaid it would still be forecasting low probabilities of success, since they are inherently more risky. Under moral hazard, the incentives for risk-taking from having a limited liability loan vanish when the loan is repaid, so those who have already repaid their loan should look no different from those who never had a loan. We run the logits again after eliminating all households with a group loan that has not yet been repaid. Unfortunately, this leaves less than 20% of the households who had group loans. In all specifications, the coefficient actually increases noticeably in magnitude (i.e. becomes more negative), but so do the standard errors. In the three specifications using BAGPLOAN, PROBHI remains significant, twice at the 10% level and once at the 15% level. In the specifications using GRUPLOAN, the estimates drop just beyond conventional significance levels (15-30%). This may be expected from the significant drop in sample size. We interpret these results as suggestive that adverse selection specifically is occurring in this credit market.

Correlation in returns also enters with the predicted, positive sign. It is interesting to distinguish here the contrasting results across the two selection models. Correlation of returns is helpful in drawing in safer borrowers in the joint liability adverse selection model of Ghatak. But correlation of returns should have made relative performance more prevalent than group loans in the regime comparison model of HM. This is true even though we act as if those borrowing under individual liability (or relative performance) were not borrowing at all. Apparently, the correlation force is greater on the extensive margin.

Evident also from the regression is the significance of many covariates which might well help to determine the reservation, alternative utility $\underline{U}$. Households were asked whether or not they engage in agricultural activity, and if so, whether or not they would like to expand their operations. From these questions we derive two dummy variables, AGRYES and AGRNO. AGRYES (AGRNO) equals one for the 46% (33%) of households that engage in agricultural activity and would (would not) like to expand their activity. The remaining 21% of households for whom neither equals one are those who do not engage in agricultural activity. It is crucial to control for occupation, since the BAAC and several other institutional lenders targeted agricultural activities exclusively at the time of the survey. One further proxy for desirability of a loan is the dummy variable OWNSBSNS, which equals one for the 21% of households that own a business. Consistent with the discussion, these are much more likely to appear when the choice is on the extensive margin then when examining, as earlier, how to borrow, that is individual versus joint liability.

Variable	Original •	New ••
WEALTH	-76698.667	-127602.89
WEALTHSQ	0.0011	0.0008
WLTHDSPR	0.119	0.1218
TITLE	17421.048	45550.29
TITLESQ	-.0033	-.0026
PROBHI	-.4599***	-.5131***
% OF OTHER RESPS.	-.0085	0.221
INCOME	0.6113	1.589**
EXINCOME	-.5781	-1.6910**
NRTHEAST	0.6175***	.4896***
AGRYES	1.033***	.8278***
AGRNO	.7609***	.5311***
OWNSBSNS	.2804**	0.1852
LANDOWND	.0042*	.0072***
EDYEARH	.0370*	0.0266
MALEH	.5102***	.4783***
AGRI ONLY		-.3080
CDD MULTIPLE Occ.		-.0931
ASSEMBLY HALL CDD		.4841***
ECON STATUS CDD		-.2419*
HELP FROM GOVT		0.1553
TIME TO DIST. CENTER		-.0013
USE OF BAAC		.7030***
COMMUNITY BANK		.3306**
HAS RICE BANK		0.0217
HAS BUFFALO BANK		0.1382
STATE ELECTRICITY		0.2402
COTTAGE IND.		-19.4008**
PREDIC. lb 5		0.0402
PREDIC. Rb 5		-.0804
PREDIC. vf 5		-.1815
CONSTANT	-2.8114***	-3.2150***
N	1666	1430

Effect Still Here

Legend: * p<.15; ** p<.10; *** p<.05

• : Original variables (Table 9.6.3); •• : New variables (including CDD)

[Table 9.6.8. Selection with supply side variation – adverse selection. Source: Puentes and Townsend (2004), unpublished research note]

Including CDD controls as in the earlier analysis does not alter these conclusions, indeed, it enhances them in some instances. See Table 9.6.8. Adverse selection remains even when there is plausible variation on the supply side. The risk type variable remains negative and significant. Likewise, the former instruments which determined membership in financial institutions are significant here in determining the prevalence of joint liability loans. Some variables such as expected income, required by the theory, are now significant also. On the other hand, the correlation result is slightly weakened.

9.7 Endogenous Industrial Organization

Likewise there are general equilibrium forces which make the organization of industry and financial contracts endogenous. Suppose households are not stuck in villages but rather can migrate and either work alone, as in single proprietorships, or pair with households in worker supervisory relationships as in an industrial group.

There is a continuum of agents of measure one. All agents have the same preferences over consumption $c \in C$, effort $a \in A$, and job $j \in J = \{w_1, w_2, s\}$. Consumption is bounded below by zero. Job $j = w_1$ means that the agent is the first worker, job $j = w_2$ means that he is the second worker, and job $j = s$ means that he is the supervisor. If there is only one worker in the firm, we drop the subscript and refer to the worker by $j = w$. If these are only a worker or supervisor, we let $j = w_1, s$. We express the utility function over consumption, effort, and job as $U(c, a, j)$. It is strictly increasing in c and decreasing in *a*.

There is a finite number *I* of agent types in the economy. We denote an agent's type by $i \in \{1, ..., I\}$. For each type i the number of agents is a positive fraction $\alpha_i > 0$ of the population. Types only differ in their non-negative endowment of capital κ_i . The total endowment of capital is $\kappa = \sum_i \alpha_i \kappa_i$. This capital is divisible, and it is the fundamental ingredient into creating the capital input k .

There is a production technology f freely available to all agents that produces output $q \in Q$ as a stochastic function of workers' efforts and capital input $k \in K$. As earlier in the moral-hazard model of occupation choice, we assume that Q can only take on a finite number of values.

A self-employment firm consists of one unsupervised agent, who is treated as a worker for utility purposes. With no supervisor, the effort of the single working agent is private information. A self-employment contract is an *n*-tuple $\left(c\left(q\right), a, k\right)$.

An incentive compatible self-employment contract is one that satisfies the constraints that the actual action a be the same as the one recommended in the contract, that is,

$$\sum_q f(q \mid a,k)U(c(q),a,w) \geq \sum_q f(q \mid \hat{a},k)U(c(q),\hat{a},w), \quad \forall \hat{a} \in A \tag{9.7.1}$$

The utility an agent receives from choosing one such contract is his expected utility,

$$u(b_1) = \sum_q f(q \mid a,k)U(c(q),a,w) \tag{9.7.2}$$

.

Contracts use resources. Expected net consumption of a $b_1 = \left[\left(q\right), a, k\right]$ self-employment firm is

$$r_{c-q}(b_1) = \sum_q f(q \mid a,k)(c(q)-q), \tag{9.7.3}$$

and its usage of the capital input is

$$r_k(b_1) = k \tag{9.7.4}$$

Each $b_1 = (c(q), a, k)$ indexes a different self-employment contract or firm. Thus, a self-employment firm is indexed by all of its characteristics: the capital input k it uses, the effort a of its member, and the schedule $c(q)$ used to determine final consumption. If capital, effort, or consumption can take on a continuum of values, then there is an infinite number of possible types of self-employment firms, a set B_1.

The second type of organization we consider is a two-agent firm with a worker who operates the technology and a supervisor who monitors him. We assume that the supervision process makes the worker's effort public, though it easy enough to relax this assumption so the supervisor sees only a correlated signal, as in Holmström (1979). A supervisor-worker contract b_2 is an n-tuple $(c_w(q), c_s(q), a_w, a_s, k)$.

The monitoring technology requires that supervisory effort equals worker effort so only contracts that satisfy $a_w = a_s$ are feasible. There are no incentive constraints in this type of firm. Again, we define each agent's utility directly in terms of the contract, though now utility also depends on the job. Utility is

$$u(b_2, j) = \sum_q f(q \mid a_w, k) U(c_j(q), a_j, j), \;\; j = w, s \qquad (9.7.5)$$

Consumption resource usage is

$$r_{c-q}(b_2) = \sum_q f(q \mid a_w, k)(c_w(q) + c_s(q) - q) \qquad (9.7.6)$$

and the usage of the capital input is

$$r_k(b_2) = k \qquad (9.7.7)$$

In this two-agent worker-supervisor firm, the object b_2 is a contract that specifies joint usage of the capital input, coordinates efforts, and gives each member's output-dependent consumption. It is joint consumption and production features that make these contracts club goods. B_2 is the set of all b_2 firms.

For simplicity, here we limit consideration to these two types of firms. Still, agents can be in only one firm at a time, so the commodity space needs to respect this feature. One method for handling this problem is to let agents choose indicator functions over the types of firms, the b_1, b_2, and in the case of the multi-agent firms, their job j as well.

To decentralize, let $p(b_1)$ denote the price of a unit of a b_1 self-employment firm, $p(b_2, j)$ the price of a unit of a b_2 firm jointly with the decision to be the worker $j = w$ or the supervisor $j = s$. Let $x_i(b_i)$ denote type-i's purchase of probability of being assigned to a b_1 firm and let $x_i(b_2, j)$ be the purchase of job j in a type-b_2 firm.

The problem for a type-i consumer is then

$$\max \sum_{b_1} x_i(b_i) u(b_1) + \sum_{b_2, j} x_i(b_2, j) u(b_2, j) \qquad (9.7.8)$$

subject to $x_i \in X$, the space of lotteries, and the budget constraint

$$\sum_{b_1} x_i(b_i)\,p(b_1) + \sum_{b_2,j} x_i(b_2, j)\,p(b_2, j) \le p_k \kappa_i \tag{9.7.9}$$

where the capital endowment, κ_i, is sold for income at price p_k. This is the key wealth variable. Price p_k can be taken as the measure and set at unity)

The intermediation sector carries out several activities. It converts the capital stock endowments into the capital input, it supplies the capital input to firms, it intermediates state-contingent consumption across agents, and it staffs the firms. In performing these activities, the intermediation sector is creating firms. There are constant returns to scale in these activities, so it does not matter how many profit-maximizing entities there are, and profits will be zero in equilibrium.

For convenience we will refer to a single such representative intermediary and to distinguish it from the firms of our theory, we will refer to this entity as the production sector. Denote $\delta(b_i)$ as the number of b_1 firms produced. The first constraint is a resource constraint on the capital input.

$$\sum_{l=1,2} \sum_{b_l} \delta(b_l) r_k(b_l) + y_k \le 0 \tag{9.7.10}$$

so that capital input used in creating the firms is less than or equal to the capital endowment purchased. In creating the firms, the production sector also provides insurance via firms' compensation schedules to individuals. It collects consumption from some firms and transfers it to others. These transfers, analogous to premia and indemnities, need to sum to zero. The net consumption resource constraint is

$$\sum_{l=1,2} \sum_{b_l} \delta(b_l) r_{(c-q)}(b_l) \le 0\,. \tag{9.7.11}$$

In creating two-agent firms – the b_2 - the production sector staffs positions in them. Each of these firms requires two members. Let $y(b_2, w_1)$ be the number of b_2 firms with a worker and let $y(b_2, s)$ be the number of b_2 firms with a supervisor. Thus to fully staff a firm,

$$\forall b_2,\ \delta(b_2) = y(b_2, w_1) = y(b_2, s) \tag{9.7.12}$$

Given prices, the production sector's maximization problem is

$$\max_{y(b_1),y(b_2,j),\delta(b_1)} \sum_{b_1} p(b_1) y(b_1) + \sum_{b_2,j} p(b_2, j) y(b_2, j) + p_k y_\kappa \tag{9.7.13}$$

subject to $y \in Y$.

A competitive equilibrium in this economy is a (x^*, y^*, p^*) such that:

1) $\forall i,\ x_i^*$ solves the Consumer's Problem;

2) y^* solves the Production Sector's Problem;

3) Markets clear.

Both Welfare theorems hold for this economy: Competitive equilibria are Pareto optimal and Pareto optimum can be supported as competitive equilibria. Thus, as is standard, a Pareto program will be useful for analyzing prices and for developing an algorithm for computing competitive equilibria. Let λ_i denote the Pareto weight on type-i agents. Using the market clearing constraints to substitute out for production y, the Pareto program is

$$\max_{x_i>0,\delta\geq 0} \sum_i \lambda_i \alpha_i \left(\sum_{b_1} x_i(b_1)u(b_1) + \sum_{b_2,j} x_i(b_2,j)u(b_2,j) \right) \tag{9.7.14}$$

subject to the probability measure constraints,

$$\forall i,\ \sum_{b_1} x_i(b_i) + \sum_{b_2} x_i(b_2,j) = 1, \tag{9.7.15}$$

The club or matching constraints are

$$\forall b_1,\ \delta(b_1) = \sum_i \alpha_i x_i(b_1), \tag{9.7.16}$$

$$\forall b_2,\ \delta(b_2) = \sum_i \alpha_i x_i(b_2,w) = \sum_i \alpha_i x_i(b_2,s), \tag{9.7.17}$$

The resource constraint on net consumption

$$\sum_{l=1,2} \sum_{b_l} \delta(b_l) r_{(c-q)}(b_l) \leq 0 \tag{9.7.18}$$

and the resource constraints on capital, where again κ is the aggregate capital endowment,

$$\sum_{l=1,2} \sum_{b_l} \delta(b_l) r_k(b_l) \leq \kappa. \tag{9.7.19}$$

A key result here is that a monotonic relationship between the λ_i in the Pareto program and the k_i in the competitive equilibria.

By framing the formation of firms as an activity in the production set, prices of firms follow naturally. Let $\mu_{(c-q)}$ be the shadow price of consumption and let μ_k be the shadow price of the capital input. At a competitive equilibrium, these are the prices of consumption and capital. These μ_k and the associated interest rates are endogenous. The price of firms are determined by their costs,

$$p(b_1) = \mu_{(c-q)} \sum_q f(q \mid a, k)(c(q) - q) + \mu_k k, \tag{9.7.20}$$

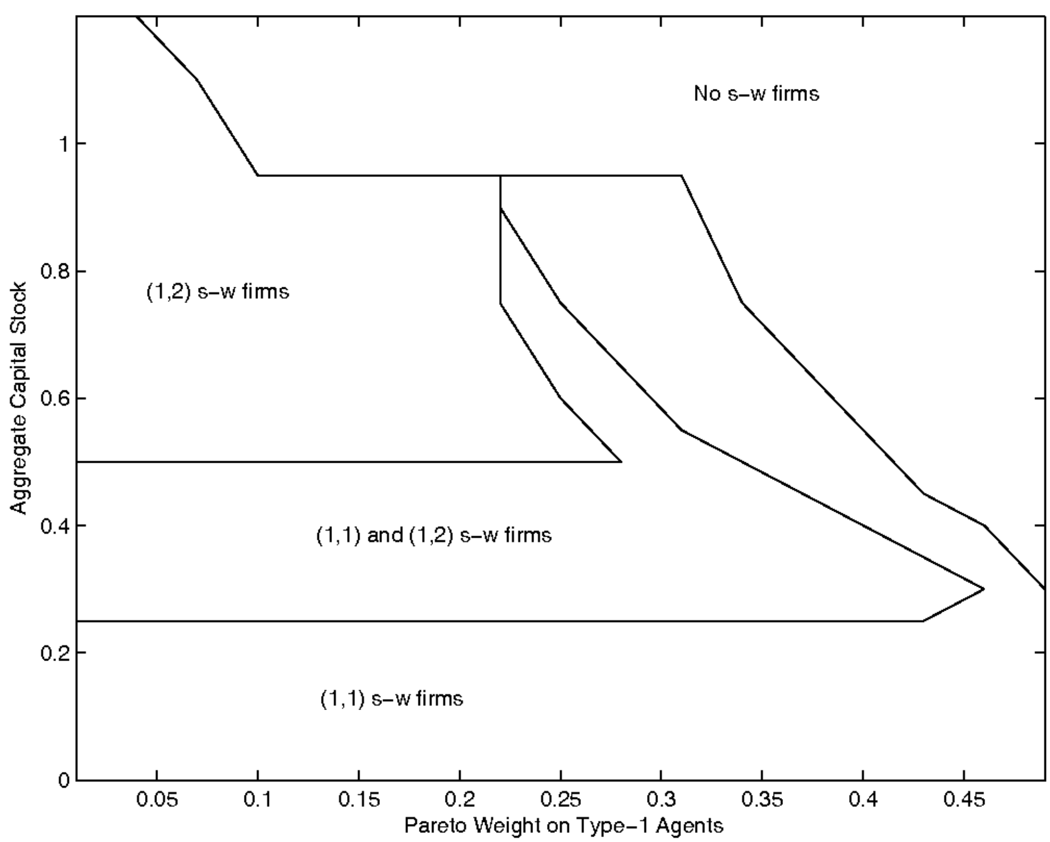
$$p(b_2, w) + p(b_2, s) = \mu_{(c-q)} \sum_q f(q \mid a_w, k)(c_w(q) + c_s(q) - q) + \mu_k k, \tag{9.7.21}$$

with an inequality when a firm does not exist, that is, prices do not cover cost.

The wage, or price of jobs, are also endogenous to the equilibrium. Let $\Delta U \equiv u(b_2, s) - u(b_2, w)$, and $\Delta P \equiv p(b_2, s) - p(b_2, w)$. Suppose, for example, an agent type-i is indifferent to purchasing an additional unit of probability of being a worker or a supervisor, then

$$\lambda_i \Delta U = \Delta P. \tag{9.7.22}$$

Since agent type i is indifferent on the margin, the price differential is precisely the utility differential, scaled by the Pareto weight (again, the inverse of his marginal utility of wealth).



[Figure 9.7.1. The occurrence of supervisor-worker firms as a function of the wealth level and the wealth distribution. Source: Prescott and Townsend (2006)]

Figure 9.7.1 describes the occurrence of supervisor-worker firms as a function of the wealth level and the wealth distribution (as measured by the Pareto weights). A (1,1) supervisor-worker firm means that both members are type-1 agents. A (1,2) supervisor-worker firm means a type-1 agent is the worker and a type-2 agent is the supervisor. Note that in the figure the existence of a supervisor-worker firm does not preclude the existence of a self-employment firm.

The particular optimum that will prevail depends on the Pareto weights λ_i and on the amount of economy-wide capital κ, or equivalently the distribution of wealth, the k_i. As an example, suppose there are only two types – the rich and the poor. The figure describes parameter values for which supervisor-worker firms occur in equilibrium. For high aggregate capital levels and relatively equal Pareto weights (the upper right-hand corner of Figure 1) all firms are single proprietorships.

As the aggregate capital level declines and the Pareto weights become more unequal, supervisor-worker firms begin to appear. The composition of these two-agent firms varies with the parameters. At high capital levels but relatively unequal Pareto weights (the upper left-hand side of Figure 1) all the supervisor-worker firms consist of a type-1 as the worker and a type-2 as the supervisor. Capital is plentiful and labor is scarce in these economies, so it is worthwhile to use the high Pareto weight (type-2) agents as a supervisor. At low capital levels but relatively equal Pareto weights (the lower right-hand side of Figure 9.7.1), the program assigns the low Pareto weight (type-1) agents to supervise their fellow type-1 workers. For the remaining parameter values both types of supervisor-worker firms are observed in equilibrium.

Some forms of limited commitment can be incorporated. Let $d \in D = \{0,1\}$, where $d = 0$ means the agent stays in the firm and does not run off with the capital. Conversely, $d = 1$ means the agent runs off with the capital. If an agent runs off with the capital, he converts it into consumption at some exogenous rate $\tilde{r}$ with no effort supplied. Define the value of default as $V(k, d = 1) \equiv U(\tilde{r}k, 0, j)$. The utility from staying $d = 0$ is unchanged from before. Thus a self-employment contract under limited commitment when $d = 0$ is an n-tuple $(c(q), a, d, k)$ such that:

$$\sum_q f(q \mid a, k) U(c(q), a, w) \geq V(k, d = 1) \qquad (9.7.23)$$

Heterogeneity in agents' abilities can also be incorporated into our framework.

This framework allows us to better conceptualize policy advice and the meaning of previous policy experiments. First, any given equilibrium is unlikely to deliver the first best, or if it does, it is only by diverting considerable resources to monitoring. Otherwise, from capitalization, ROA, and induced effort can all be expected to vary in an observed cross section. Second, movements along the Pareto frontier are associated with redistribution of wealth k_i, as if varying the right-hand side of the household's budget constraint with lump sum taxes and subsidies. Note that overall the intermediation sector still makes zero profits and satisfies overall resource and other constraints. Third, prices such as interest rates and job premia can vary as one moves along the frontier. Without the general equilibrium it is hard to allow for this. Fourth, removal in some way of the limited commitment constraint, as if going from one economy to the other, also has these indirect effects. In computed examples, industrial organization, job assignment, inequality, and the price of firms and capital all move with the distribution of wealth, and with whether or not default is allowed.

If at the end of the period households had myopic savings rates, then one can imagine the evolution of the economy over time. There would be a tendency for economy-wide wealth to increase as in the neo-classical growth model, and the path of inequality would be determined as well.

9.8 Dynamic Village Networks

The possibilities for dynamics with inequality and organization become clear with explicit examples. For this purpose we return to the discussion of individual, relative performance regimes versus risk-sharing, joint liability groups as in section 9.6, but imagine there are two periods. The second period is as described earlier, static problem and the financial organization results are the same. In somewhat simplified notation, let $\pi^r(c,q,e)$ denote the probability of the relative performance regime with allocation (c,q,e) and $\pi^g(c,q,e,\mu)$ those of the group regime. The maximum surplus that can be obtained in the second period given a utility pair (w_1,w_2) is thus given by Program 1:

$$\begin{aligned} S^2(w_1,w_2) = \max_{\pi^r,\pi^g} & \sum_{c,q,e} \pi^r(c,q,e)\left[q_1+q_2-c_1-c_2\right] \\ & + \sum_{c,q,e,\mu} \pi^g(c,q,e,\mu)\left[q_1+q_2-c_1-c_2\right] \end{aligned} \tag{9.8.1}$$

$$w_i = \sum_{c,q,e} \pi^r(c,q,e)\left[U(c_i)+V(e_i)\right] + \sum_{c,q,e,\mu} \pi^g(c,q,e,\mu)\left[U(c_i)+V(e_i)\right], \tag{9.8.2}$$

for $i = 1, 2$

$$\sum_{c,q,e} \pi^r(c,q,e) + \sum_{c,q,e,\mu} \pi^g(c,q,e,\mu) = 1;\ \pi^r, \pi^g > 0. \tag{9.8.3}$$

Here π^g satisfies technological and group constraints like 9.6.22 and 9.6.23 for all values of μ in *M* and π^r satisfies technological and incentive constraints 9.6.15 through 9.6.17 earlier. The surplus is maximized conditional on the incentive and technological constraints and on the utility level of individuals - implied by (9.8.2). Equation (9.8.3) implies that the choice of the planner is a probability distribution.

The dynamic problem is solved in the first period. The planner makes all the contingent plans at this period. The surplus function $S^2(w_1, w_2)$ is essential because the planner can use the first and the second period to achieve desired initial utility pairs for the individuals, and the surplus function defines the effect on the objective function of assigning utility pairs (w_1, w_2) in the second period. The planner can assign to each individual a utility level in the second period belonging to the set *W*. The possible outcomes in the first period are consumption, output and effort vectors in the first period; the type of organization that they are part of in the first period; and again the utility pairs for the second period. The distribution of effort, consumption, output and the type of organization in the second period is implied by the choices of w_1 and w_2 as noted above.

The incentive constraints for the relative performance regime in the first period are

$$\sum_{c,q,e_2,w} \pi^r(c,q,e_1,e_2,w)\left[U(c_1)+V(e_1)+\beta w_1\right] \geq \sum_{c,q,e_2,w} \pi^r(c,q,e_1,e_2,w)\frac{p(q\,|\,\hat{e}_1,e_2)}{p(q\,|\,e_1,e_2)}\left[U(c_1)+V(\hat{e}_1)+\beta w_1\right], \tag{9.8.4}$$

and

$$\sum_{c,q,e_2,w} \pi^r(c,q,e_1,e_2,w)\left[U(c_2)+V(e_2)+\beta w_2\right] \geq \sum_{c,q,e_2,w} \pi^r(c,q,e_1,e_2,w)\frac{p(q\,|\,e_1,\hat{e}_2)}{p(q\,|\,e_1,e_2)}\left[U(c_2)+V(\hat{e}_2)+\beta w_2\right], \tag{9.8.5}$$

$\forall e_2, \hat{e}_2,$ where $\pi^r(c,q,e,w)$ is the probability that individuals are assigned to the relative performance regime; and the vector of consumption, outputs, efforts and promised utilities is (c,q,e,w). Notice that these conditions are similar to the static problem above, but here the distribution of promised utilities in the second period can also be used as an incentive tool. The technological constraint is similar to (9.8.3):

$$\sum_{c,w} \pi^\tau(c,\tilde{q},\tilde{e},w) = p(\tilde{q}\,|\,\tilde{e}) \sum_{c,q,w} \pi^\tau(c,q,\tilde{e},w), \quad \forall \tilde{q},\tilde{e}. \tag{9.8.6}$$

The incentive constraint for a group in the first period with Pareto weights (μ_1,μ_2) is:

$$\begin{aligned} &\sum_{c,q,w} \pi^g(c,q,e,w,\mu) \sum_i \mu_i \left[U(c_i)+V(e_i)+\beta w_i\right] \\ &\geq \sum_{c,q,w} \pi^g(c,q,e,w,\mu) \frac{p(q\,|\,\hat{e})}{p(q\,|\,e)} \sum_i \mu_i \left[U(c_i)+V(\hat{e}_i)+\beta w_i\right], \end{aligned} \tag{9.8.7}$$

$\forall e, \hat{e},$ where $\pi^g(c,q,e,w,\mu)$ represents the probability that individuals are assigned to a group and the vector of consumption, outputs, efforts and promised utilities for the second period is (c,q,e,w). The technological constraints analogous to (9.8.5); are:

$$\begin{aligned} &\sum_{c,w} \pi^g(c,\tilde{q},\tilde{e},w,\tilde{\mu}) = p(\tilde{q}\,|\,\tilde{e}) \sum_{c,q,w} \pi^g(c,q,\tilde{e},w,\tilde{\mu}), \ \forall \tilde{q},\tilde{e}. \sum_{c,w} \pi^g(c,\tilde{q},\tilde{e},w,\tilde{\mu}) \\ &= p(\tilde{q}\,|\,\tilde{e}) \sum_{c,q,w} \pi^g(c,q,\tilde{e},w,\tilde{\mu}), \quad \forall \tilde{q},\tilde{e}. \end{aligned} \tag{9.8.8}$$

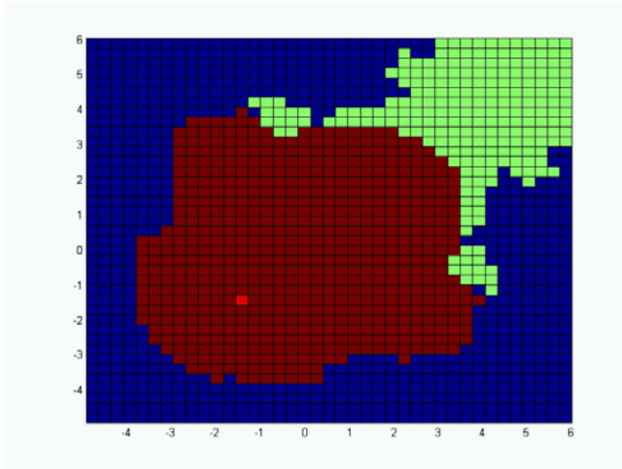
The first period Pareto problem is thus:

Program 2

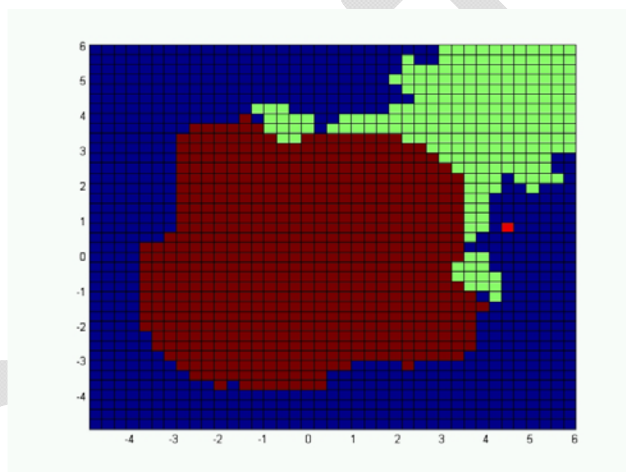
$$\begin{aligned} S^1(u_1,u_2) = \max_{\pi^\tau,\pi^g} &\sum_{c,q,e,w} \pi^r(c,q,e,w)\left[q_1+q_2-c_1-c_2+\beta S^2(w)\right] \\ &+ \sum_{c,q,e,w,\mu} \pi^g(c,q,e,w,\mu)\left[q_1+q_2-c_1-c_2+\beta S^2(w)\right] \end{aligned} \tag{9.8.9}$$

st.

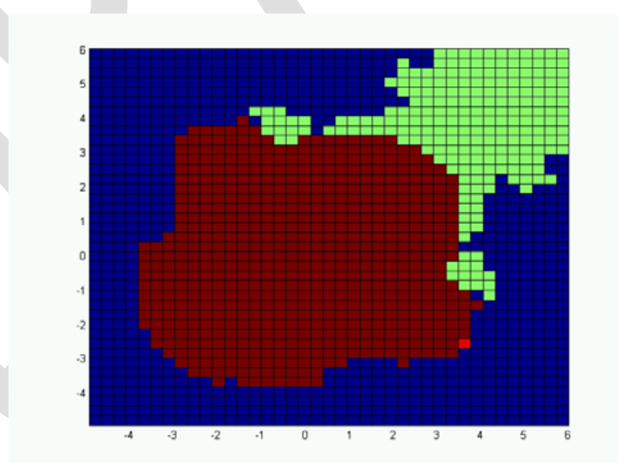
$$\begin{aligned} &\sum_{c,q,e,w} \pi^r(c,q,e,w,)\left[U(c_i)+V(e_i)+\beta w_i\right] \\ &\geq \sum_{c,q,e,w,\mu} \pi^g(c,q,e,w,\mu)\left[U(c_i)+V(e_i)+\beta w_i\right] \geq u_i \end{aligned} \tag{9.8.10}$$


6
5
4
3
2
1
0
-1
-2
-3
-4
-4 -3 -2 -1 0 1 2 3 4 5 6

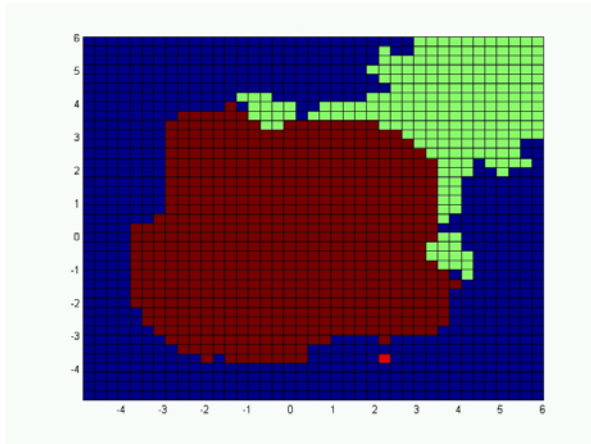
a)


6
5
4
3
2
1
0
-1
-2
-3
-4
-4 -3 -2 -1 0 1 2 3 4 5 6

b)

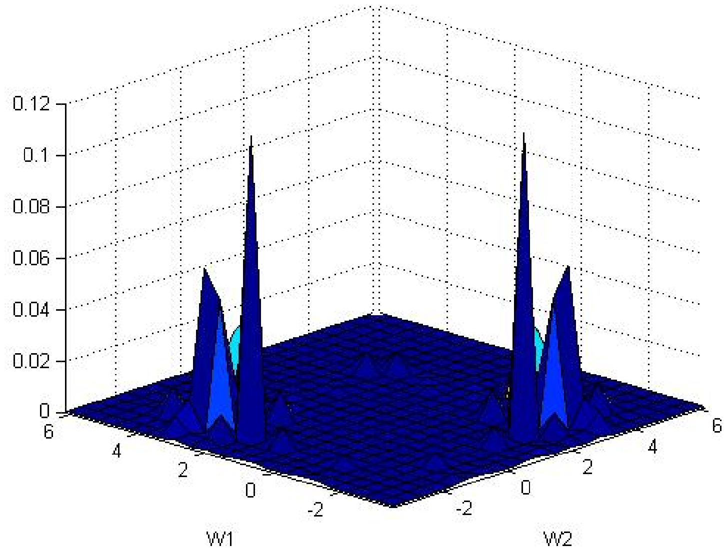

6
5
4
3
2
1
0
-1
-2
-3
-4
-4 -3 -2 -1 0 1 2 3 4 5 6

c)

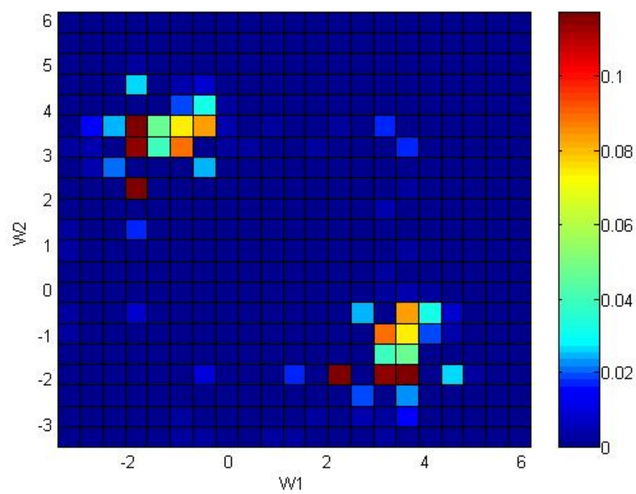
d) 

[Figure 9.8.1 (a)-(d). In (a), u = (1.5738, -1.5738). In (b), u = (4.3169, 0.6703). In (c), u = (3.4754, -2.6959). In (c), u = (2.0728, -3.8179). Utility pairs studied, transition from $T-1$ to T. Source: With help from Madeira. Research note 2007]

A numerical example makes clear that entry into joint liability or risk sharing groups, and exit from them, can depend on the history of performance as well as assigned wealth or utilities. This is summarized in the Figures, and the paths of transition in various starting points. If initially two households are similar in wealth or promises, then relative performance is optimal, as earlier. But if outcomes in the current period are not similar, if for example one project succeeds while another fails, then the two households will be treated differently not only in current compensation, as earlier, but in next periods promises/wealth. Heterogeneous wealth is a force for groups, so one thus sees a group forming over time. Likewise, those in a group in the current period which does well should be rewarded not just in terms of higher consumption but higher utility next period. The latter can send them down the U-shaped group prevalent profile, and thus exit them for joint liability. The steady state distribution of utility pairs for the infinite horizon problem is depicted earlier in the Figures. The point is that joint liability, within village networks, industrial conglomerates and other types of groups are endogenous and we would expect to see their evolution over time. Alternatively if there were policy restrictions this could lead to welfare losses and inefficiency.

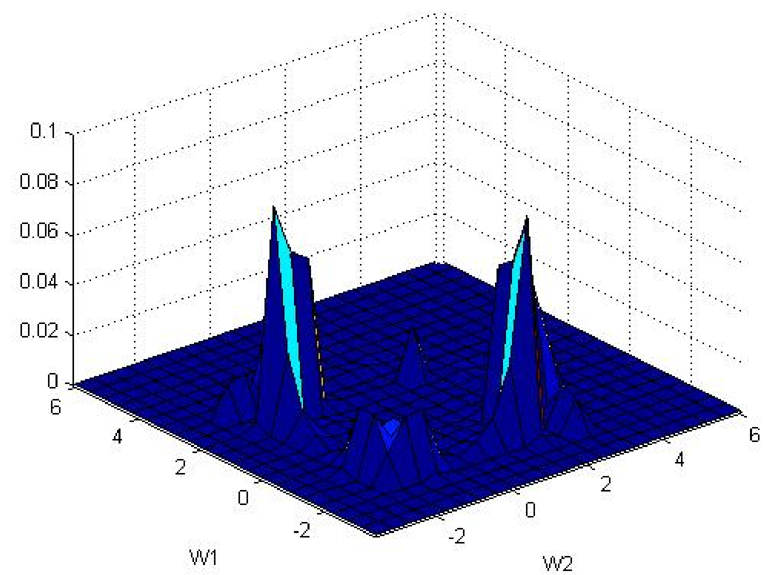


a)

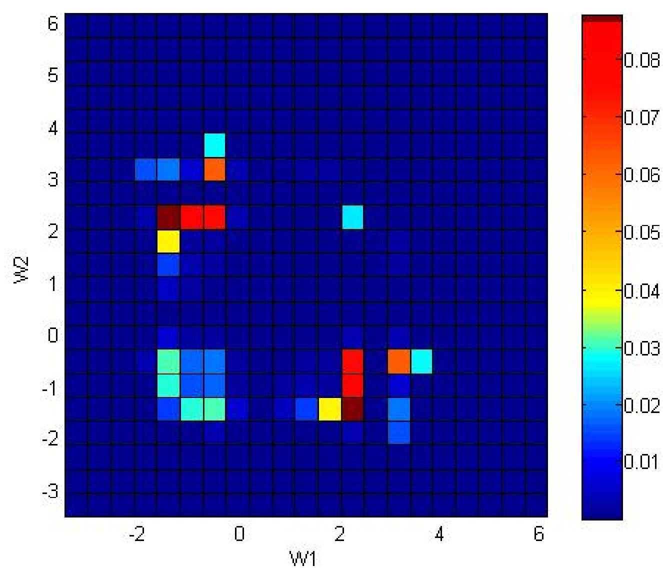


b)

[Figure 9.8.2 (a)-(b). Distribution of Pairs switching from groups to RP. Source: With help from Madeira. Research note 2007.]



a)



b)

[Figure 9.8.3 (a)-(b). Distribution of Pairs Switching from RP to groups. Source: With help from Madeira. Research note 2007.]