International Unions and Integration

Oğuzhan Çelebi* MIT

Elias Papaioannou† LBS

October 21, 2022

Abstract

We consider a model of international unions in which countries have heterogeneous preferences for integration and their integration decisions are strategic complements. We analyze equilibrium under several integration protocols that differ in the flexibility countries have in choosing how much to integrate. Unlike previous models with strategic substitutes, our setting is in line with the evolution of the European Union (EU), where enlargement and flexible integration coincide with enhanced integration and are often spearheaded by the "core" countries. Moreover, when non-members (candidates, exiting countries, and other nations) can partially integrate with the union, as in practice, restrictions on their integration determine the union's size and scope and are necessary for fostering cooperation. Motivated by Brexit and the rise of euroskepticism, we allow countries to leave the union and demonstrate how restrictions on the integration of leaving countries make the union robust to changes in members' preferences.

*MIT Department of Economics, 50 Memorial Drive, Cambridge, MA 02142. Email: ocelebi@mit.edu

†London Business School, Economics Department, Regent's Park. London NW1 4SA. United Kingdom. Email: eliasp@london.edu.

We are grateful to Daron Acemoglu, Glenn Ellison, Joel Flynn, Bård Harstad, Michal Kobielarz, David Myatt, Giacomo Ponzetto, Enrico Spolaore, Birger Wernerfelt, Michael Whinston, and seminar participants in LBS, MIT Theory Lunch, the Bank of Greece, and MIT Organizational Economics Lunch for helpful comments. First Draft: June 2021.

1. Introduction

International organizations play an instrumental role in the world economy, shaping national and international policies; their domains range from trade and banking to agriculture, industrial policy, product standardization, and climate change. While international organizations' objectives differ, a common goal is to foster cooperation and integration between members. The European Union (EU) is, arguably, the most prominent international organization, as its policy domains have expanded hugely after its establishment. Starting in 1951 as the European Coal and Steel Community among six countries, the EU has morphed into a complex international institution of 27 countries with a *single market for goods, services, capital and labor*, common trade policy with non-members, a common agriculture policy, legislative and regulatory harmonization in financial services, product markets standardization, among others.[1] Importantly, European integration deepened, covering increasingly wider policy domain, as the EU welcomed new members: the United Kingdom (UK) in the mid-1970s, countries from Southern Europe in the 1980s, Scandinavia in the mid-1990s, and Eastern Europe in the 2000s. BREXIT and the increased tensions with Hungary illustrate that the Union may be fragile. However, the 2010-15 economic crisis in the European periphery, the pandemic, and the war in Ukraine have led many policymakers, commentators, and the public to argue for deepening and expanding integration in new domains, like health and security.[2] Besides, EU's incremental approach towards integration and membership expansion serves as a model for regional integration worldwide.[3]

This paper aims to understand how international unions facilitate integration, with a particular focus on the EU. We contribute to the earlier literature on international unions that study a "public goods" setting where countries' actions are strategic substitutes in three main respects. First, our model stressing strategic complementarities is more tightly aligned with the main focus of international unions, which is fostering cooperation rather than coordinating infrastructure investments. Second, our framework can explain some stylized facts about the experience of the EU regarding countries' preferences over enlargement and

[1] See Spolaore (2013), Sapir (2011), and Eichengreen (2006) for extensive literature reviews on the functions and transformation of the EU, Alesina and Perotti (2004) for an evaluation of European integration and Gilbert (2020) for a historical overview.

[2] In an influential talk at the European Parliament on May 2, Mario Draghi, former Italian prime minister and President of the European Central Bank pleaded for "pragmatic federalism" and "acceleration of the integration process" (in defense, welfare state policies, and labor issues) as well as increased attention to enlargement towards Western Balkans. French President Emmanuel Macron' 2022 Presidential Elections campaign focused on deeper EU ties on defense, calling for Europe to achieve "strategic autonomy" from the US.

[3] For example, in 2021, the African Union established the African Continental Free Trade Area (AfCFTA), which aims to be the world's largest free trade area, connecting about $1.3bn$ people across 54 African countries.

flexibility of integration protocols. Third, extending the model by allowing non-members to integrate with the union, we gain insights into the relationship between the EU and non-members, candidates and exiting countries. Despite its simplicity, our setup sheds light, jointly, on some core issues of international unions: formation, enlargement, and relations and integration with non-member and exiting countries, which is an increasingly important topic after BREXIT and the rising Euroskepticism across Europe.

We develop a model of international integration where countries with heterogeneous preferences endogenously decide their *integration levels*, either forming a union with a common integration policy across members (*rigid union*), a union with a minimum threshold like the EU where further integration is possible (*flexible union*) or integrating independently without any explicit enforcement (*non-union integration*). Unlike the earlier literature, countries' actions represent integration and are strategic complements rather than strategic substitutes (*e.g.*, investments in a public good). Thus, as the union's size (number of countries) and depth (the extent of integration) increase, members have a stronger incentive to integrate more to reap up scale and market size effects. We believe this is more in line with the experience of the EU, its emphasis fostering a single market for goods, services, capital, and labor, standardization of regulations and safety protocols, financial sector legislative harmonization, and legal convergence in an increasingly larger domain. Besides, the EU budget is (still) small compared to the combined budget of the member countries (around 2% of the EU public spending) and classic public goods, like education, health, policing, and defense are provided mostly at the national level.

Not only our departure from a public investment setting is a more realistic representation of international unions, but with the introduction of strategic complementarities, our framework explains the experience of the EU where enlargement, deepening, and flexibility of integration have moved in tandem (*e.g.*, Eurozone and Schengen, which are not mandatory but allow for deeper integration). Our framework is also in line with some salient features of European integration that earlier theoretical explorations could not easily explain. First, flexible integration and enlargement to the periphery are often spearheaded by the "core" countries such as Germany, France and the Netherlands. Second, enlargement to periphery has been accompanied by higher, not lower, integration across the union. We show that both of these facts are in line with the predictions of our model. With strategic complementarities, non-mandatory higher integration from the "core" countries and enlargement to the periphery can increase the depth of the union; as such, "core" countries with strong preferences for integration may support the union's expansion. Theoretical explorations where countries actions are strategic substitutes, such as investments to a common public good, yield opposing results: a reduction of integration across the union after enlargement to periphery, which

creates opposition to union expansion by core countries, which do not square well with the EU evolution.

In addition, we introduce integration with non-members to the study of international unions, an issue that earlier works have abstracted from. Allowing non-member countries to partially integrate with the union is not only realistic, as the EU has many "enhance" integration agreements with candidate countries and non-members, but also allows studying exit.[4] This allows understanding the trade-offs of the relations between the EU and candidate countries (e.g., Kosovo, Serbia, Albania), other non-member nations (e.g., Switzerland, Norway), and exiting countries, like the United Kingdom. Our framework reveals that *restrictions* on the integration of non-members and exiting countries are chief determinants of the size and the scope of the union Besides, such restrictions on the integration of the union with non-members are necessary both for the union's effectiveness to foster integration and being robust against preference shocks.

1.1. Results Preview

Integration Methods/Protocols. We begin our analysis developing a model where countries' payoffs depend on their preferences for integration (types), their own and other countries' integration levels. We study three protocols: *rigid unions*, where all members integrate to the same level, set by majority voting; *flexible unions*, where, via majority voting, the union sets a lower bound for integration, but countries may integrate further; and *non-union integration*, where countries decide their actions without any constraints and enforcement mechanisms. Using tools from supermodular games and voting theory, we characterize the equilibrium policy, determined by the country with the median type, and equilibrium integration levels across the three protocols.

Comparison of Integration Methods. The theoretical framework uncovers the trade-offs between the three protocols. The distinction between a rigid union and non-union integration regards the enforcement power of the former and the flexibility of the latter. Depending on countries' preferences, each protocol may be strictly preferred to the other by every single country. Flexible union, however, balances the two countervailing forces; a majority of countries prefer it to the rigid union and non-union integration. The analysis of countries' preferences highlights the two main effects of a flexible union compared to rigid

[4] In our setting, non-members do not "invest" less in a common public good, bargaining how much of the benefits they will accrue. Much more realistically with the various accession protocols specified by the EU Treaty, the union and non-members choose to what extent (*e.g.*, in which domains) integration takes place (for example on trade, foreign investment, product standardization, etc).

union. First, while in rigid union the integration levels of all countries equal the union policy, under flexible union some countries integrate further, and therefore, fewer countries' integration directly depends on the union policy. This force pushes the median to pick a lower integration policy for the union. Second, due to complementarities, higher integration from countries with stronger preferences for integration leads *all* countries (including the median) to prefer higher integration. When the first effect dominates, flexible union policy is lower than rigid union policy and low-type countries prefer flexible arrangements. When the second effect dominates, flexibility results in a deeper integration, preferred by higher type members. The second possibility, absent from earlier studies (discussed below), appears to be in line with the evolution of the EU. Many countries of the "core", like Germany, France, Luxembourg, Belgium, and the Netherlands, have promoted flexible union, taking steps towards further integration in different domains (*e.g.,* monetary unification, banking union, integration of legal standards). At the same time, the deeper integration of the "core" has pushed more peripheral members, with ex-ante lower preferences for integration, to follow suit, integrating across more domains.

Formation and Enlargement. We then endogenize the composition of the union to study formation and enlargement. In every equilibrium of the formation game, all countries with types higher than the median become members. Turning to enlargement, we study a setting where (lower type) candidates apply for membership and (higher type) members vote over accession (as in practice of the EU). While enlargement results in a median country with a lower type (compared to the initial union), it is quite possible that the equilibrium policy *increases*, as integration is more beneficial when more countries integrate. While this pattern appears in line with the EU's history, where enlargement went handy with deeper integration, it does not emerge in public goods settings where countries' actions are substitutes. Therefore, our framework explains two chief regularities of the EU's history. First, countries with high -rather than low- integration preferences (such as Germany, France, and the Netherlands) have supported the EUs' various enlargement rounds. Second, post-enlargement, the EU has moved towards *more*, rather than less, integration.

Non-member Integration. We then extend our framework allowing non-member countries to integrate with the union to study the multiple-type agreements that international unions have with non-members, an issue that earlier research has abstracted from. Inspired by the EU, we consider a setting where non-members can integrate until an upper bound. We show that whenever the bound is not restrictive (*i.e.,* non-members are allowed to integrate as much as the union members), the union unravels, having no effect on fostering

integration. Therefore, restrictions on non-members are chief determinants of the size and depth of the union. Besides, the efficiency of the upper bound restriction depends on the incentives of the low-type members, who would prefer to integrate as non-members without such restrictions.

Enlargement, Exit, and Non-member Integration We then revisit enlargement, allowing for the integration of potential members outside the union. A new channel through which non-member integration restrictions affect the union composition emerges when we also introduce enlargement. When considering candidates from the periphery, higher type countries compare the benefits of integrating with more countries to the possibility of a lower union policy induced by the lower median country after enlargement. If non-member integration is not very restrictive, these countries have a stronger incentive to reject candidates, nudging them to integrate as non-members, as they can still reap up the benefits of integration without giving voting power. Thus, restrictions on the integration of non-members with the union are necessary for enlargement but integration must be restricted to satisfy incentive constraints of both the candidates and the initial members. Lastly, as an application of non-member integration and motivated by BREXIT, we study a setting where following a preference shock, members can exit the union while maintaining some degree of integration. We demonstrate that restrictions on the integration of former members with the union are *necessary* to make the union robust to preference shocks. Our theoretical result, thus, explains why exiting countries cannot "cherry-pick" the new integration with the union, as some proponents of BREXIT argued before the referendum. Besides, our set-up yields a natural exit protocol, in line with the Treaty of the European Union's article 50 and the heated negotiations between the EU and the UK after the June 2016 referendum. Moreover, countries preferring tighter exit restrictions are the ones with high preferences for integration, as these countries benefit and therefore are set to lose the most following an exit. This result is in line with the tough stance of core EU countries, like France, the Netherlands, and Germany on the negotiations with the UK, post the BREXIT referendum.

1.2. Related Literature

Our study relates to works examining the formation, functions, and enlargement of international unions, mostly focusing on the EU. Alesina, Angeloni, and Etro (2005) study the formation and enlargement of international unions, building on the optimal country size literature pioneered by Alesina and Spolaore (1997) and Bolton and Roland (1997) and the-

ories of federalism (Oates et al. (1972)).[5] As Alesina et al. (2005), we consider equally sized countries that solely differ on their preferences for integration. But, we modify, extend and reformulate their framework in multiple directions. First, we consider a setting where countries choose integration levels, which are strategic complements, rather than investments in a public good. This setting, we believe, is more realistic as many EU policies entail legislative-harmonization policies in product and capital markets, standardization, and common trade policies, subject to market size effects and scale; besides, the EU budget is small, and EU's role on standard public goods provision is small. Second, we allow non-member countries to integrate with the union, as it is the case.[6] Third, allowing jointly for non-member integration and enlargement provides insights on the arrangements and deals that the EU has with accession countries, like Turkey, and exiting from the union nations, like the UK.

Our paper also relates to works that explore theoretically certain aspects of the EU, and international unions, more generally. Harstad (2006) analyzes whether a union should allow members to form "inner clubs" to enhance cooperation. He shows that flexible cooperation is beneficial if country heterogeneity is large and externalities small. Berglof, Burkart, Friebel, and Paltseva (2008) also look at two-tier unions, showing in a framework where cooperation requires unanimity that the threat of an "inner club" by higher type members spurs contributions from lower type members, strengthening the union's cohesion. Although we take a different approach, our framework also yields that members prefer rigid union to more flexible, non-union integration when countries' preferences/types are similar [Proposition 2]. Kobielarz (2022) extends the Alesina et al. (2005)'s model to study exit allowing for transfers, adjustment of the union policy, and exit costs. Fiscal transfers can prevent inefficient exits; when transfers are unavailable, post-exit arrangements where the former member partially contributes to the public goods but receives limited spillovers may improve efficiency. Our results on exit are complementary, showing that similar economic mechanisms are present in a setting with different assumptions (strategic complements vs. substitutes) and interpretation (integration vs. investment in a public good).[7]

[5]See Bolton, Roland, and Spolaore (1996) and Oates (1999) for thorough literature reviews on size distribution of nations and fiscal federalism, respectively. Alesina and Spolaore (2005) offers a book length treatment on the theory of the size of nations.

[6]Thinking about integration with non-members is chief, as the EU has various trade and investment treaties with many countries. In addition, there are five candidates (Albania, Turkey, Serbia, Montenegro, and North Macedonia), currently integrating with the EU on various domains, trade, regulation of financial markets, legal (Copenhagen Criteria). And a few years ago, most current EU members implemented a plethora of reforms, converging to the EU, when there were candidates.

[7]In both models, a worse post-exit relationship helps prevent exit and a better post-exit relationship is welfare-improving. However, in our model, post-exit relationship is governed through a restriction on the integration level former member can choose (*i.e.,* a restriction on actions) which determines to what extent the exiting nations benefits from integration. In Kobielarz (2022), former member can invest as much as it wants, but receives an exogenously-determined and smaller share of spillovers after exit.

Our result on the necessity of restriction on non-member integration and characterization of efficient restrictions [Propositions 9 and 10] echo similar mechanisms in Bolton and Roland (1996, 1997), who build a model with two countries deciding whether to separate or not. There are efficiency gains from unification, but also political conflict costs from differences in income and preferences towards redistribution. Bolton and Roland (1997) conclusion is that the efficiency loss from independence due to trade and factor movements barriers cement the Union.[8] Abramson and Shayo (2022) consider a two country model with core and periphery in which countries have different endogenous identities that affect their preferences over policies. They study the interplay between international integration and identity politics and how the robustness of the union depends on the social identities of member countries.

Our paper builds on the large literature on the theory of clubs, initiated by Buchanan (1965). The closest is Roberts (1999), who develops a dynamic voting problem with an endogenous electorate under strategic complementarities (see also Roberts (2015)).[9] We take a less abstract viewpoint, as our objective is to characterize various forms of international integration, understand the trade-offs of their formation, expansion, and even shrinkage, allowing realistically for partial integration of the union with non-members.

Outline. Section 2 sets up our theoretical framework and characterizes equilibria under rigid union, flexible union, and integration without a union. Section 3 compares the equilibrium policies and integration levels under the different methods, exploring which policies are preferred by what types of countries. Section 4 endogenizes the union to analyze its formation and enlargement. Section 5 extends the model allowing for non-member integration. We then study enlargement and exit under this richer setup. In Section 6 we summarize and discuss avenues for future research.

[8]Other related papers that however focus on trade and redistribution rather than integration include Crémer and Palfrey (1996), who considers a model where districts composed of heterogeneous population decide between centralization and decentralization; Casella (2001), who explores coalition formation in a spatial club model; Alesina, Spolaore, and Wacziarg (2000), who adding border costs to Alesina and Spolaore (1997), show that globalization (lower border costs) yields a higher number of countries; Casella (2005), who studies the implications of a two-region model (core and periphery) with heterogeneous countries deciding on redistribution and Gancia, Ponzetto, and Ventura (2020), who study the effects on trade, income distribution, and welfare of economic unions differing in size and scope.

[9]Even though the payoff of members depends only on the club size and not actions, Roberts (1999) studies a setting similar to our rigid union. Our focus on international unions alongside with our core assumption of strategic complementarities can be viewed as a bridge between Alesina et al. (2005) and Roberts (1999), with the important addition of allowing for integration of non-members with the union and enlargement.

2. Theoretical Framework

This section develops the theoretical model that allows studying flexible unions, rigid unions, and non-union integration. We commence with the model set-up (Section 2.1). Then we define each integration method (Section 2.2), and characterize the equilibria (Section 2.3).

2.1. Model Set-Up

$U = \{1, 2, ..., |U|\}$ denotes the finite set of union members. Each country has a type $\gamma_i \in \mathbb{R}_+$, measuring the strength of its preference for integration, where $\gamma_1 < \gamma_2 < ... < \gamma_{|U|}$ and $\gamma = \{\gamma_i\}_{i \in U}$.[10] The action of country i is denoted by $t_i \in \mathbb{R}_+$; $t \in \mathbb{R}_+^{|U|}$ denotes the action profile. Country's utility is given by $u_i(t, \gamma_i) = u(t_i, t_{-i}, \gamma_i)$, where $u : \mathbb{R}_+^{|U|+1} \to \mathbb{R}_+$. The actions can be thought as investment/spending decisions on a public good, as in earlier research or perhaps more realistically as efforts towards common policies in goods and services trade, movement of labor across borders, product market standardization, homogenizing laws, regulations, and acts on banking and capital markets.

Assumption 1. *The utility function, u, satisfies the following conditions:*

(i) ***[Complementarity]*** *u is strictly increasing in γ_i, and satisfies strictly increasing differences in γ_i and t_i and γ_i and t_{-i}. u is increasing in t_{-i} and satisfies increasing differences in t_i and t_{-i}.*

(ii) ***[Costly integration]*** *For all γ_i and t_{-i}, $u(0, t_{-i}, \gamma_i) = 0$. There exists $t(\gamma_i)$ such that u is decreasing for all $t_i > t(\gamma_i)$.*

(iii) ***[Concavity]*** *u is strictly concave in t_i for any t_{-i}, γ.*

The first part of the assumption ensures that the benefit a country receives from integration is increasing in their own type (in line with the interpretation of the type as preference for integration) and the integration levels of other countries. It also introduces strategic complementarities: the payoff from increasing integration for a country is increasing in its own type and the integration levels of other countries. This is the key difference between our model and public goods models where countries' actions are interpreted as their investments to a public good (e.g., Alesina et al. (2005)). The second part makes sure cooperation is necessary for a country to reap the benefits of integration; for example, countries need to

[10] We do not take a stance of the origins of integration preferences. They could, for example, reflect the desire of elites to solidify post-World War II peace by bringing countries' policy-making closer, as in the early decades of the European integration project. They could also capture citizen's ideology, values, and beliefs.

amend domestic legislation to harmonize product standards, remove tariffs and trade barriers that protect local industries, change labor legislation promoting free-movement, and pass new laws on union domains. Moreover, costs eventually dominate the benefits, guaranteeing the existence of an equilibrium.[11] The third part assumes that given the integration levels of other countries, the benefit of integration is a strictly concave function of the integration choice.

2.2. Integration Protocols

We distinguish and analyze three methods/protocols of integration *(i) non-union integration*; *(ii) rigid union*; and *(iii) flexible union*. Below we define these protocols.

Non-union integration is the simplest and most flexible form of cooperation; countries independently and simultaneously choose their integration levels. There is no explicit bargaining, negotiation, or centralized enforcement. An action profile t^* is a *non-union integration equilibrium* if for all $i \in U$

$$t_i^* \in \arg\max_{t_i} u_i(t_i, t_{-i}^*, \gamma_i) \tag{1}$$

In a *rigid union*, couturiers decide on the common integration level by majority voting.[12] The *rigid union equilibrium policy* is an integration level $r^* \in \mathbb{R}_+$ that is preferred to any r' by a majority of countries, in other words, r^* satisfies the *Condorcet Criterion*.[13]

In a *flexible union*, which resembles the EU the most, there is a minimum threshold of integration for all members, but countries can take steps for deeper integration. The *flexible union equilibrium policy* is a lower bound b, determined by majority voting. Given b, countries may voluntarily choose any action in $[b, \infty)$. A vector of integration levels $T(b)$ is a flexible union equilibrium under threshold b if

$$T_i(b) \in \arg\max_{t_i \geq b} u_i(t_i, T_{-i}(b), \gamma_i) \quad \forall i. \tag{2}$$

There might be multiple equilibria under b. However, in Proposition 1, we show that there is a highest equilibrium, which is pareto dominant. Assuming the highest (lowest) equilibrium will be played, an integration level b^* denotes a flexible union equilibrium policy if it satisfies the *Condorcet Criterion*, where countries evaluate their payoffs in the highest (lowest) possible equilibrium under b. The lower bound b^* represents core issues for the

[11] Alternatively, we can bound the policy space, $t_i \in [0, 1]$, interpreting the maximum, $t_i = 1$, as becoming a single country. All results go through in this setting.

[12] When $|U|$ is even, it is possible that two integration levels get the same votes. In these cases, we break the tie in favor of the lower policy.

[13] If a country is indifferent between two policies, we assume that the country breaks ties in favor of the lower policy.

union. In the EU case, for example, a single market with common external tariffs and a union with free movement of goods, services, capital, and labor. Flexible union also allows further integration, like joining a currency area with common monetary policy. Further integration, while not mandated, allows some countries to integrate more than others in some domains, for example defense or welfare policies, or deepen their integration in core areas, for example setting up single regulatory agencies for product markets or banking. Nowadays, the EU is moving in this direction, as some countries are considering further harmonization in defense, immigration, banking, and social welfare.

2.3. Equilibrium

We start by characterizing equilibria under the three integration protocols, as this allows comparing the equilibrium policies and countries' preferences over these methods (Section 3). For the rest of the paper, we use m to denote the country with the median type, γ_m (if $|U|$ is even, $m = |U|/2$).

Proposition 1. *The following statements characterize the equilibria in each case:*

1. *In non-union integration, there are highest and lowest equilibria, $\overline{t}^*(\gamma)$ and $\underline{t}^*(\gamma)$, that are increasing in γ_i for all countries, i. $\overline{t}^*(\gamma)$ is the pareto dominant equilibrium.*
2. *In rigid union, the most preferred policy of the median country is the Condorcet winner: $r^* = \arg\max_r u_m(r, \ldots, r, \gamma_m)$*
3. *In flexible union, given a bound b, there are highest and lowest equilibria, $\overline{T}(b, \gamma)$ and $\underline{T}(b, \gamma)$. $\overline{T}(b, \gamma)$ is the pareto dominant equilibrium. Under extremal equilibrium selection, the preferred policy of the median is the Condorcet winner. That is, $b^* = \arg\max_b u(\overline{T}(b, \gamma), \gamma_m)$*

The characterization of extremal equilibria in non-union integration (case 1) is a direct consequence of increasing differences and follows from standard arguments in supermodular games (*e.g.*, Topkis (1979)).[14] In the rigid union (case 2), countries' preferences over the union policy satisfy single crossing: countries with higher preference prefer higher integration levels. The characterization of the Condorcet winner follows from the median voter theorem with single-crossing preferences (Gans and Smart, 1996). In the flexible union (case 3), the characterization of equilibrium (for a given b) is analogous to the non-union integration. However, in a flexible union, changes in b affect the endogenous equilibrium integration and increasing differences does not directly imply that the preferences of countries satisfy single

[14]See also Vives (1990) and Milgrom and Roberts (1990) for applications in oligopoly competition, arms' races, and search models.

crossing. However, we show that increasing differences still puts enough structure to the preferences and a majority votes for the policy the median prefers.

3. Comparison of Integration Methods

Using Proposition 1, we compare the three integration methods. We commence comparing rigid and flexible integration methods from the perspectives of countries with different types. Most importantly, Proposition 4 demonstrates how flexible protocols (i) can increase the union-wide integration and (ii) be preferred by the higher-type countries, explaining an important regularity in the adoption of flexible integration policies such as Schengen and Eurozone.

Rigid Union vs Non-Union Integration We first compare non-union integration to rigid union, starting with an example to illustrate the mechanisms at play.

Example 1. *There are five countries,* $U = \{1, 2, 3, 4, 5\}$*, with types* $\gamma_1 = 2.5$ $\gamma_2 = 2.499$*,* $\gamma_3 = 1$ $\gamma_2 = 0.95$ $\gamma_1 = 0.9$*. The utility function is:*

$$u(t_i, t_{-i}, \gamma_i) = \gamma_i \sum_{j \in U \setminus \{i\}} t_i t_j - \frac{t_i^3}{\gamma_i} \tag{3}$$

The following table gives integration levels and payoffs of countries under different protocols.[15]

	Integration Levels				Payoffs			
	{1,2}	3	4	5	{1,2}	3	4	5
Rigid Union	2.67	2.67	2.67	2.67	63.52	9.48	7.06	4.53
Non-union Integration	4.74	2.11	2.02	1.92	85.24	18.91	17.26	15.40
Flexible Union	5.49	3.01	3.01	3.01	133.10	23.97	19.98	15.82

Table 1: **Integration Levels and Payoffs under Different Integration Methods.**

In rigid union, the most preferred integration of the median, country 3*, is implemented; note that the median country has a low preference for integration. In non-union, all countries freely choose their integration levels. Table 1, rows (1) and (2) shows the equilibrium actions and payoffs. While all countries choose the same level of integration under rigid union, non-union integration allows higher type countries to contribute more and low type countries to*

[15] All values are rounded to 2 decimal places for all numerical examples.

contribute less. This is better for all countries since higher type countries enjoy the higher integration from each other, not possible under rigid policies, while lower type countries are also better off integrating less.

The example, therefore, reveals that how the flexibility of non-union integration can be beneficial to all countries. In contrast, non-union integration lacks the commitment power of a (rigid) union, which can increase integration across the union. As the following proposition shows, when countries have similar types (similar integration preferences), the commitment power of a rigid union dominates the flexibility of non-union integration. [See also Example 6 in the Appendix A.]

Proposition 2. *Assume u_i is continuously differentiable and strictly increasing in t_{-i}. Then for each γ_m, there exists an ϵ such that rigid union is preferred to non-union integration by all countries whenever $\gamma_i \in (\gamma_m - \epsilon, \gamma_m + \epsilon)$ for all $i \in U$.*

Flexible Union vs Non-Union Integration Next, we turn to the comparison of flexible union with non-union integration when $b = 0$, the two protocols coincide. However, when b is greater than the actions of the low-type countries, flexible integration increases the integration of these countries. As actions are complements, integration is higher under flexible union compared to non-union integration. Since the preferences of the median determine the equilibrium policy (Proposition 1), the median and all countries with higher types prefer the higher integration of the the flexible union. In Example 1, moving from non-union integration to flexible integration, the median country chooses an integration bound that is higher than its non-union integration level. This brings lower type countries with it, while also causing an increase in the integration of higher type countries due to complementarities. The following proposition shows that this is a general result.

Proposition 3. *All countries choose a (weakly) higher integration level under flexible union as compared to non-union integration and a majority of countries prefers flexible union to non-union integration.*

Flexible Union vs Rigid Union In a rigid union, the median country (effectively) chooses r^* by considering the benefits of a higher policy, the *direct* increase in the actions of others, and costs that come with increasing its action. Consider r^* as a potential flexible union policy. First, under a flexible union, some higher type countries may choose deeper integration and integrate more than r^*. As actions are complements, this, in turn, causes the median country to prefer a higher policy, pushing towards deeper integration. Second, in flexible integration, the equilibrium actions of higher type countries that integrate more

than r^* depend *indirectly* on the equilibrium policy. It is possible that higher type countries choose high actions regardless of flexible union policy determined by the median. When this indirect effect is weak, a higher policy has less of an effect on the equilibrium integration levels under flexible union compared to rigid union; this lowers the benefit of a higher policy and pushes the median country to pick a lower equilibrium policy. However, it is also possible that this indirect effect is strong and moves the preference of the median towards higher integration. This is what happens under the parametrization of Example 1 (see Table 1, row (3)); flexibility causes the median to prefer a higher union policy, raising the integration levels and payoffs for all countries. The second *indirect* channel may cause the equilibrium policy to be higher or lower under flexible union. Therefore, flexible union may result in either higher or lower union policy compared to rigid union. Both $b^* \geq r^*$ and $b^* < r^*$ are possible.

When $b^* \geq r^*$, all countries' actions are higher under flexible union. The median country can always choose r^* as the flexible union policy and therefore prefers flexible to rigid union. Moreover, when $b^* \geq r^*$, any country with higher than the median type also prefers the higher integration that comes with flexible union. Conversely, when $b^* < r^*$, all countries with lower type than the median prefer flexible union, since they have lower preferences for integration than the median and can integrate at b^* instead of r^*.

Proposition 4. *The comparison between b^* (flexible union minimum threshold) and r^* (common rigid-union integration) is ambiguous. A majority of countries prefers flexible union to rigid union.*

- *If $b^* \geq r^*$, then (at least) the median country and all countries with higher types prefer flexible union.*
- *If $b^* < r^*$, then (at least) median country and all countries with lower types prefer flexible union.*

Comparison. Strategic Substitutes vs Strategic Complements. It is instructive to compare our results with the case where countries' actions are strategic substitutes. First, with strategic substitutes, in a flexible union, higher type countries choose actions higher than the median. This causes the median country to choose a *lower* integration level, as effectively low-type countries and the median "free-rides" on the investments of higher-type members. Second, as in our strategic complements setting, the direct effect is weaker under a flexible union since fewer countries' decisions depend directly on the equilibrium policy. Third, under strategic substitutes, a higher policy level leads to *lower* actions from the higher type countries, further lowering the incentives of the median to choose a higher policy. All

three channels reduce the equilibrium policy and therefore, $b^* < r^*$; flexibility (always) causes low-type countries to free ride on the contributions of higher type countries. Hence, low, rather than high-type countries, prefer flexible union, while rigid union is preferred by the higher type countries.[16]

Conversely, when policies are complements, while a majority always prefers flexible to rigid union, it is not *ex-ante* clear which countries select more flexible arrangements. If integration is higher under flexible union, then countries with higher types prefer it, as they can (and will) integrate further while maintaining the benefits from the integration of low-type countries. Proposition 4, we believe, is consistent with the dynamics of the EU, as many countries of the "core" (like Germany and France) have promoted flexible union, allowing many (Eastern and Southern) European countries to join, while integrating themselves more (via monetary unification, for example). These developments cannot be easily explained by public goods models with strategic substitutes. In contrast, if the policy level is lower under flexible union, then countries with lower types prefer it. This (more direct) result - that echoes the Alesina et al. (2005) - follows from the fact that low integration type countries prefer a flexible union with a low bound, as they pay a cost from integrating to much-higher-than-desired level.

4. International Unions. Formation and Enlargement

In this section, we expand our theoretical model to study two major issues regarding international unions, formation (4.1) and enlargement (4.2).

4.1. Union Formation

Consider a finite set of countries, denoted by $N = \{1, 2, \ldots, |N|\}$. We study a union formation game where countries decide whether or not to form a union, vote over the union policy, and decide on their integration. $U \subseteq N$ denotes union members. We analyze the Subgame Perfect Equilibrium (SPE) of the following union formation game.

1. Countries decide to become a member or not.
2. Members decide the equilibrium policy b^* (r^*) of the flexible (rigid) union with majority voting.

[16] Alesina et al. (2005) provide a formalization of this result (their Proposition 4), writing: *"For instance, a surprising result emerges if, in a rigid union with a uniform provision of public goods, countries are allowed individually to add extra expenditure. One may think that countries with strong preferences or public spending would support such a reform: in reality, these are the only countries that may oppose the reform and prefer the rigid union. The reason is that this reform would reduce the uniform provision chosen in political equilibrium at the union level so as to rely on the extra-provision of individual countries."*.

3. In flexible union, members choose their actions $t_i \in [b, \infty)$ and the payoffs are realized.

As in many settings with strategic complementarities, the union formation game features multiple equilibria. We start with an example to illustrate the multiplicity of equilibria and the mechanisms at play.

Example 2. *There are six countries,* $N = \{1, 2, 3, 4, 5, 6\}$*, with the following preferences over integration:* $\gamma_6 = 1.7$*,* $\gamma_5 = 1.6$*,* $\gamma_4 = 1.41$*,* $\gamma_3 = 1.4$*,* $\gamma_2 = 1.23$ *and* $\gamma_1 = 1.22$*. The utility function is:*

$$\hat{u}(t_i, t_{-i}, \gamma_i) = \sum_{j \neq i} (t_i t_j)^{\frac{\gamma_i}{2}} - t_i^2 \tag{4}$$

In the rigid union with k *members and equilibrium policy* r*, the payoff of each country is:*

$$u^k(r, \gamma_i) = (k-1) r^{\gamma_i} - r^2 \tag{5}$$

The following equation gives the equilibrium integration with k *members and median country type* γ_m*:*

$$r^*(k, \gamma_m) = \left(\frac{(k-1)\gamma_m}{2} \right)^{\frac{1}{2-\gamma_m}} \tag{6}$$

Table 2 gives the equilibrium policies and payoffs for different rigid unions.

Union	Equilibrium Policy	u_5	u_4	u_3	u_2	u_1
$\{5, 6\}$	0.56	> 0	0	0	0	0
$\{4, 5, 6\}$	3.28		-0.12	0	0	0
$\{3, 4, 5, 6\}$	3.56			> 0	0	0
$\{2, 3, 4, 5, 6\}$	5.80				1.14	0
$\{1, 3, 4, 5, 6\}$	5.80				0	0.53
$\{1, 2, 3, 4, 5, 6\}$	8.46				-2.45	-3.91

Table 2: **Equilibrium Integration Levels and (relevant) Payoffs, Example 2**.

We now explore which of these are indeed equilibria. We start by considering a two member union of countries $\{5, 6\}$*, row (1). The equilibrium policy is* 0.56*, giving both members positive payoff. To determine whether this is indeed an equilibrium, we check whether any of the remaining four countries prefers to deviate and join the union. In a three country union (with* 5 *as the median country), row (2), the equilibrium policy jumps to* 3.28*, due to the complementary nature of integration. However, for country* 4*, the payoff from joining is*

negative, -0.12*; the payoff is even lower for lower type countries. Therefore,* $U = \{5, 6\}$ *is an equilibrium union; there is no equilibrium with* 3 *members.*[17]

Next, we consider a four country union of $\{3, 4, 5, 6\}$*, row (3). The median country is* 4*; equilibrium policy is* 3.56*, giving all members positive payoff. Compared to a three member union of* $\{4, 5, 6\}$*, the equilibrium policy increases even though the median has lower type, due to the complementarity of countries' actions. To examine whether this is an equilibrium or not, we consider the incentives of the two non-members, low-type countries* 1 *and* 2*. In a potential five country union, the median is still country* 4*, but the equilibrium policy increases to* 5.80*, as the larger union and complementarity nudge for deeper integration. Moreover, country* 1 *and* 2 *obtain a positive payoff if they enter the* 5 *country union. Therefore, the four member union* $\{3, 4, 5, 6\}$ *is not an equilibrium.*

Finally, we explore whether the 5 *country unions are equilibria, considering countries' payoffs from the six country union. The median is country* 3*; and equilibrium policy is* 8.46*. This increase in integration level results in a negative payoff for the low-type countries* 1 *and* 2*, who do not join a six country union. Therefore, the six country union is not an equilibrium, while the two five country unions,* $\{1, 3, 4, 5, 6\}$ *and* $\{2, 3, 4, 5, 6\}$ *are. Moreover, in* $\{1, 3, 4, 5, 6\}$*,* 2 *is not a member although* $\gamma_2 > \gamma_1$*. Thus, countries with lower integration types may opt out even when a country with lower preferences for integration enters, as they anticipate that the entry of a new member will endogenously raise the equilibrium integration policy.*[18]

Nonetheless, we can partially characterize the equilibria exploring some properties. First, due to strategic complementarities, if we compare two unions with the same median, the larger union will have a higher integration policy. Second, if a country with above-median type joins a union, there are two effects, working on the same direction of deeper integration. To start with, the preferences for integration of the median country (weakly) increases, pushing the equilibrium policy towards higher integration. Besides, the union becomes larger, which also increases equilibrium integration, as countries' payoffs from integration rise.[19] Therefore, in any equilibrium union, all countries with types above the median country are members. The following proposition formalizes this result.

[17] This simple example reveals an additional result of our model: country 4 would prefer to join the union, if the equilibrium policy remained the same after joining in. However, due to complementarities, its entry pushes the high-type countries 5 and 6 to set a higher integration and prevents country 4 from joining.

[18] This shows that the equilibrium union may not be contiguous, which is the case when the actions are strategic substitutes; see Proposition 1 of Alesina et al. (2005).

[19] This result suggests that EU's 12 members incentive to integrate further increased, when the EU expanded in 1995, admitting Austria, Finland, and Sweden, which for mostly geo-political reasons were not members. The subsequent policies, for example, to integrate capital and product markets, for example with the Financial Services Action Plan (FSAP) are in line with the model's prediction.

Proposition 5. *If U is an equilibrium union with median country m and $\gamma_i \geq \gamma_m$, then $i \in U$.*

4.2. Enlargement

We now generalize the model to allow for enlargement. There are two types of countries: $I \subset N$ denotes the initial members, before potential enlargement; $C = N \setminus I$ denotes candidates. We study the following extensive-form game:

1. Candidate countries decide whether or not to apply for membership.
2. Each union member decides whether to admit or reject each candidate.
3. Equilibrium union U is the initial union plus the countries admitted unanimously.
4. Union members decide the equilibrium policy b^* (r^*) of the flexible (rigid) union with majority voting.
5. Members choose their actions $t_i \in [b, \infty)$ in flexible union and integrate at r^* in rigid union. Integration levels of non-members are 0. Payoffs are realized.

For the rigid union case, the payoff of a country depends on the equilibrium integration level r^*, the number of members $|U|$, and the country's type. Let r_S denote the integration vector where S countries integrate at r and others do not integrate. $u^r(r, \gamma, S) \equiv u(r_S, \gamma)$ denotes the payoff of a country with type γ in an S member union and union policy r. We focus on enlargement with unanimity, since this is the paradigm that the EU, and other international unions, has followed since inception. Before giving the propositions, we discuss a couple of examples to illustrate the main mechanisms.

Example 3. *There are four initial members, $I = \{2, 3, 4, 5\}$ and one candidate, $C = \{1\}$, with types: $\gamma_5 = 1.7$ $\gamma_4 = 1.6$, $\gamma_3 = 1.5$, $\gamma_2 = 1.38$ and $\gamma_1 = 1.37$. The utility function is $\hat{u}$ from Example 2.*

Union	Equilibrium Policy	u_2	u_1
$\{2, 3, 4, 5\}$	5.06	2.50	0
$\{1, 2, 3, 4, 5\}$	9.00	1.97	0.16

Table 3: **Equilibrium Integration Levels and (relevant) Payoffs, Example 3**

Without enlargement, the median country is 3 and the equilibrium policy is $r^* \approx 5.06$, giving country 2 a payoff of 2.5. If country 1 joins, the median country is unchanged. However, the equilibrium integration increases to $r^* = 9$. Even under this higher integration level,

1 *prefers to join. However, due to its low preference for integration, the payoff of country* 2 *drops to* 1.96 *and the candidacy of country* 1 *will be rejected by country* 2.

Example 4. *There are five members,* $I = \{2, 3, 4, 5, 6\}$*, and two candidates,* $C = \{1, 0\}$*. The countries' types are:* $\gamma_6 = 1.8$ $\gamma_5 = 1.6$ $\gamma_4 = 1.4$*,* $\gamma_3 = 1.24$ $\gamma_2 = 1.15$*, and* $\gamma_1 = 1.1$ $\gamma_0 = 1$*. The utility function is* $\hat{u}$ *from Example 2.*

Union	Equilibrium Policy	u_6	u_2	u_1	u_0
$\{2, 3, 4, 5, 6\}$	5.56	56.87	0.42	0	0
$\{1, 2, 3, 4, 5, 6\}$	4.43	53.27	4.89	6.07	0
$\{0, 1, 2, 3, 4, 5, 6\}$	5.63	103.00	16.02	8.45	2.07

Table 4: **Equilibrium Integration Levels and (relevant) Payoffs, Example 4**

Without enlargement, the median country is 4 *and the rigid union equilibrium policy* $r^* \approx 5.56$*, which gives country* 6 *a payoff of* 56.87*. First, suppose that country* 0 *is not a candidate. If country* 1 *joins, then country* 3 *becomes the decisive median, which reduces the equilibrium policy to* $r^* \approx 4.43$*. Even with more countries, the high-type country* 6 *is worse off (with a payoff of* 53.27*) due to the lower integration policy. Therefore, it rejects the candidacy of* 1*. Second, suppose that both* 0 *and* 1 *are candidates. If both countries are admitted, the median country type drops from* 1.4 *to* 1.24*. However, the equilibrium policy increases from* 5.56 *to* 5.63 *due to more countries and complementarities. Moreover, under the larger union, both the high-type country* 6 *and the low-type country* 2 *are better off; this implies that countries* 3*,* 4 *and* 5 *are also better off. Consequently, the union will admit the joint candidacy of* 0 *and* 1*. Thus, the larger union that includes all countries is an equilibrium.*[20] *However, no enlargement is another equilibrium that obtains when neither of* 1 *and* 0 *applies. This is an equilibrium as if either* 1 *or* 0 *deviates, their candidacy will be rejected, so there is no profitable deviation for both countries.*

So the equilibrium policy depends on the enlargement. As more countries integrate, the union's policy changes, reflecting two distinct mechanisms. First, as more countries integrate, all countries prefer higher integration; this mechanism pushes up the union's equilibrium policy. Second, the median country changes. If new members have lower preferences for integration (as is currently the case with the EU and accession countries in the Balkans), the median's lower preference for integration brings down the union's equilibrium policy. As

[20]This example shows that admitting at the same time more countries make balance the tradeoffs across countries with quite different preferences for integration, consistent with various EU enlargement rounds when the union admitted many countries (e.g., 2004 round in Eastern Europe, 1995 round with three countries).

a result, the equilibrium policy for the enlarged union is ambiguous, depending on the preferences (and number) of initial members and the candidates. These theoretical results stand in clear contrast with the outcome under strategic substitutes. In that setting, admitting lower type countries *always decreases* the equilibrium policy and all countries approve enlargement when the change in the median country is limited (Alesina et al. (2005) Proposition 2). In Example 3, the median country is the same under both union compositions, but country 2 rejects the enlargement. The discussion and examples highlight two potential roadblocks towards enlargement:

1. High-type members blocking entry as they do not want a reduced level of integration. This is the (direct) mechanism often discussed in media and policy circles when the EU expanded in the South in the 1980s, Eastern Europe and the Balkans in 2004 and 2007. Besides, many policymakers and commentators make this point, when discussing future EU enlargement with Albania, Serbia, Kosovo, North Macedonia, and Turkey, nowadays. [This mechanism is present in example 4.]
2. Low-type members blocking entry as they do not want increased integration after enlargement. This novel mechanism, present in example 3, is not much-discussed. Nonetheless, this mechanism is likely present, as many non-core EU countries with arguably low preferences for deeper integration, like Bulgaria and Hungary, are expressing concerns for its future expansion at the Balkans and the East.

Example 4 also demonstrates the possibility of equilibrium multiplicity, which is a direct consequence of strategic complementarity. While it is always challenging moving from the abstract model to the complex reality of European politics, we note that most enlargement rounds entailed many countries joining in at the same time. Spain and Portugal in 1986, Sweden, Finland, and Austria (in 1996), which during the Cold War (until 1990) maintained a"neutral position". During the Eastern Enlargement of 2004, the EU admitted eight Transition countries (the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovenia), plus Malta and Cyprus. And in 2007, the EU admitted jointly Bulgaria and Romania. Arguably all these countries were quite dissimilar to the existing members at the time, with low integration types in our framework. Yet the considerable increase in members, coupled with strategic complementarity, made even high integration members better off.

Efficiency and Enlargement A question regards the efficiency of the multiple equilibria. An equilibrium is an *initial union optimal equilibrium* if all initial members obtain (weakly) higher payoffs compared to any other equilibrium. To make a prediction about the union, motivated by the history of the EU, we assume that the initial members have higher integration preferences than candidates. This assumption appears reasonable, as the process of

European integration started with six "core" countries in a limited domain (coal and steel) and over time the union admitted new members, while integration deepened considerably. Moreover, in Proposition 5 we show that during the union formation phase all countries that have a higher type than the median country are union members, which offers another justification for this assumption. We keep this assumption for our results on enlargement (Propositions 6, 7, 8 and 11).

Assumption 2. *Initial union members are more numerous and have higher types than candidates. That is, (i)* $|C| < |I|$ *and (ii) if* $i \in I$ *and* $j \in C$*, then* $\gamma_i > \gamma_j$*.*

The following proposition shows that there is a set of payoff-equivalent initial union optimal equilibria, given assumption 2.

Proposition 6. *There is a set of initial union optimal equilibria. These equilibria have same number members, same integration levels for all initial members and are the equilibria with most members.*

Proposition 6 shows that the preferences of initial members over (equilibria of the) enlargement (game) are aligned. Therefore, it is reasonable to expect the initial members to agree on a union which is optimal, as the accession is an esoteric, complex, detailed and long process and the set of outcomes is a small set.

We next consider what determines enlargement in this context. As Example 4 demonstrates, enlargement has two main effects. First, integration increases due to complementarities. Second, the median country has a lower type, which reduces integration. Whether integration increases or decreases then depends on the difference between the median country in the initial union and the final union. If the type of the median country in the final union is high enough, then integration increases after enlargement that is preferred by the high-type countries. The following proposition formalizes this discussion.[21]

Proposition 7. *Consider rigid union and assume* u^r *is continuously differentiable and satisfies strictly increasing differences in integration level and union size. Let* γ_m *and* γ'_m *denote the types of the median countries under* I *and* $I \cup C$*. There exists a* $\hat{\gamma} < \gamma_m$ *such that integration increases after admission of* C *to the union if and only if* $\gamma'_m \geq \hat{\gamma}$*. Moreover, whenever integration increases after enlargement, initial members with above-median types prefer enlargement.*

[21] We prove Propositions 7 and 8 for the rigid union case as under flexible union, the effect of a higher union policy on the integration levels of higher type countries is not necessarily monotone in union size.

Comparison. Strategic Substitutes vs Strategic Complements. It is instructive to compare this result with Alesina et al. (2005) [their Proposition 2]. Under strategic substitutes, enlargement to the periphery is *always* accompanied by lower integration across the union and enlargement happens if and only if the change in the median is small enough. However, in our setting integration may increase even after enlargement towards the periphery due to complementarities. In our framework, the change in the median country does not determine enlargement, but determines whether the integration increases or not, which in turn determines whether candidates are admitted or not. We believe this is in line with EU's history where enlargement went handy with deeper integration, often supported by core countries. In particular, if the union policy after enlargement stays the same, then all countries are strictly better off, as they are integrating at the same level but now more countries are integrating. As utility function is continuous, a sufficient condition for enlargement is a small change in the union policy after enlargement:

Proposition 8. *Suppose that $u(t_i, t_{-i}, \gamma_i)$ is increasing in $t_j < t_i$ and continuous. If the union policy after enlargement is close enough to the union policy in the initial union, then all countries are in favor of enlargement.*

5. Non-member Integration

In this section, we allow non-members to integrate with the Union, as the EU and most other international unions, integrate on various policy areas with non-members. We start providing some motivation based on the evolution of European integration. Second, extending the model for non-member integration, we study countries' benefits from joining the union or staying out but with some (endogenous) degree of integration with the union. Third, we re-examine enlargement with outside integration to study in a more realistic setting the trade-offs of the ongoing debate on EU's potential expansion in the Balkans, Turkey, and elsewhere. Fourth, we leverage the model with non-member integration to study exit, a topical issue given UK's decision to leave the EU and the rise of euro-skepticism across the continent.

5.1. Motivation

Assuming that any country not a part of the union does not integrate was a useful starting point, as it features in much of earlier research (e.g., (Alesina et al., 2005; Roberts, 1999)), allowing comparability with earlier contributions. However, international unions

do allow for some integration with non-member countries. The EU has treaties with non-member countries covering an increasingly large set of domains. For example, Turkey and the EU have a *Customs Union*, an enhanced trade agreement allowing imports-exports to flow across the border freely. Norway, a non-member, has access to EU's single market, with some exceptions on agriculture, fishing, and food. Besides, candidate countries, like Serbia, Albania, Montenegro, the Republic of North Macedonia, and Turkey integrate with the EU on 35 domains, *accession chapters*, covering company law, public procurement, energy, taxation, financial services, consumer and health protection, among others. In each area of the *acquis*, candidate countries are *"required to adapt their administrative and institutional infrastructures and to bring their national legislation into line with EU legislation in these areas."*[22]

Integrating with outside countries is potentially beneficial for members and non-members for (at least) two reasons. First, given Proposition 5, when a union enlarges towards its periphery, the median country has a lower type. As non-member countries do not have voting rights, integrating with them as non-members increases total integration without moving the decisive median. Therefore, non-member integration has a potentially desirable feature from the viewpoint of heterogeneous members. This may explain why the EU core countries have historically favored expanding the number and depth of deals with non-member countries. For example, Germany has been historically a proponent of deepening ties with Turkey. Second, countries whose preferences are far from the union's policies may choose to integrate as non-members. Opting out of the union, but without totally excluded may be optimal, balancing the benefits of integration and (relatively) low preferences for integration. Reasonable examples include, we believe, Switzerland and Norway, which have been integrating with the EU in various domains, but without joining, as they prefer keeping national policy-making in some core areas.

5.2. Non-member Integration Restrictions

5.2.1. Set-up

Assumption 3. *$u(t_i, t_{-i}, \gamma_i) = \sum_{j \neq i} \tilde{u}(\min\{t_i, t_j\}, \gamma_i) - c(t_i, \gamma_i)$ where $\tilde{u}$ and c satisfy differentiability and supermodularity conditions.*[23]

The first part of Assumption 3 states that the payoff of integration is additively sepa-

[22] These areas are conceptually much closer to complementary policies rather than investments in a common public good.

[23] We assume that $\tilde{u}$ is concave, differentiable and satisfies increasing differences, strictly increasing in both arguments and $u(t_i, t_{-i}, 0) = 0$ for all t. c is strictly convex, differentiable, satisfies strictly decreasing differences and strictly increasing in t_i and strictly decreasing in γ_i.

rable across countries. The payoff of country i from integrating with country j depends on the integration levels of the two countries, t_i and t_j, while the total payoff of i is the sum of its integration with all countries. The additive separability of payoffs makes the analysis tractable, allowing to study the dynamic union formation and enlargement model with non-member integration, without sacrificing much generalizability. The second part of the assumption suggests that benefit of country i from integration with country j depends on $\min\{t_i, t_j\}$, the extent of common integration (aligned with our interpretation of t_i as the integration levels).

$N = \{1, 2, \ldots, |N|\}$ denotes the finite set of countries and $U \subseteq N$ union members. Besides, $\overline{o}$ denotes the maximum integration non-member countries can choose. In our analysis, we take $\overline{o}$ as given and study its effect on the size and the policies of an equilibrium union.[24] We modify the union formation game of Section 4.1 to allow for non-members to integrate and analyze the Subgame Perfect Equilibrium (SPE) of the following game.

1. Countries (in N) decide to become a member the union or not.
2. Members decide the equilibrium union policy b with majority voting.
3. Countries choose their actions.
    - If $i \in U$ [member country], then i chooses an integration level $t_i \in [b, \infty)$.
    - If $i \notin U$ [non-member], then i chooses an integration level $t_i \in [0, \overline{o}]$.

5.2.2. Implications

We start by analyzing the relationship between non-members and the union. A union U is *ineffective* if the integration and payoffs of all countries under non-union integration are weakly higher than their payoff in the union. An ineffective union does not increase integration, its main function. Moreover, whenever a union is ineffective, all members are indifferent between being members or integrating as non-members. The following proposition shows that restrictions on the degree of integration among non-members is necessary for a union to be effective.

Proposition 9. *Let U be an equilibrium union under policy level b^*. If $\overline{o} \geq b^*$, then U is ineffective.*

To prove Proposition 9, we first show that when non-member integration is unrestricted, countries with integration preferences below the median country do not have an incentive to join the union. Joining increases their integration to the union policy and these countries prefer to integrate as non-members, as they are allowed to choose their most preferred

[24] Proposition 9 shows why a non-trivial $\overline{o}$ is necessary in this setting.

integration, below the union policy. As a result, no union can include a country whose type is below the median. The only possible equilibria are two-country unions, where the lower type member chooses the union policy, which equals its non-union integration level. All such unions are ineffective, as they do not increase integration compared to non-union integration.[25] Proposition 9 shows that the policies towards non-members are important determinants of the union size and scope. However, when the non-member integration level is restricted (as in the EU), then lower type countries have an incentive to join the union, integrating more. The following example illustrates this.

Example 5. *There are four countries,* $N = \{1, 2, 3, 4, \}$*, with the following types:* $\gamma_4 = 1.6$*,* $\gamma_3 = 1.5$ $\gamma_2 = 1.2$ *and* $\gamma_1 = 1.1$*. The utility function is:*

$$u(t_i, t_{-i}, \gamma_i) = \sum_{j \neq i} (\min\{t_i, t_j\})^{\gamma_i} - t_i^2 \tag{7}$$

If all countries join, $U = \{4, 3, 2, 1\}$*, the equilibrium integration is* $r^* \approx 2.08$*. Let* u_1^{in} *denote the payoff of the low-integration type country* 1 *if it enters the union, where* $u_1^{in} \approx 2.38$*. If, however, three countries join,* $U = \{4, 3, 2\}$*, then equilibrium policy rises to* $r^* = 2.25$*. Country* 1 *solves the following problem to determine its non-member integration level* t*, if it stays out.*

$$\max_{t \leq \overline{o}} 3t^{\gamma_1} - t^2 \tag{8}$$

Plugging in $\gamma_1 = 1.1$*, this function is maximized at* $t \approx 1.74$ *and is increasing in* $[0, 1.74]$*. Therefore, country* 1 *chooses* $\min\{1.74, \overline{o}\}$ *as its integration level, obtaining a payoff of* 2.48*, which is greater than its member integration payoff,* $u_1^{in} \approx 2.38$*.*

Let o^* *denote the value that solves* $3\overline{o}^{1.1} - \overline{o}^2 = u_1^{in}$*; in other words,* o^* *is the non-member integration restriction that makes country* 1 *indifferent between joining and integrating as a non-member, where* $o^* \approx 1.40$*. Whenever* $\overline{o} > o^*$*, country* 1 *strictly prefers integrating as a non-member to joining. Thus the four-member union* $U = \{1, 2, 3, 4\}$ *is not an equilibrium. But, when non-member integration is restricted (for example,* $\overline{o} < 1.40$*), country* 1 *prefers to join and the four-member union becomes an equilibrium.*

The example illustrates the impact of imposing restrictions on non-member integration. A more restrictive non-member integration bound results in a larger union with less integration

[25] Our result on the necessity of non-member integration restriction echoes the discussion of Bolton and Roland (1997) on barriers of trade: *"An unpleasant implication of our analysis is that barriers to trade and factor movements between the European Union and neighboring non-Union states play a role in cementing the Union. In the absence of such barriers, a country would be less willing to join the Union if it can obtain most of the economic benefits of the Union by staying out and not paying the political costs in terms of loss of sovereignty."*

within members, while a less restrictive non-member integration bound allows for more integration with non-members, but results in a smaller union.

5.2.3. Efficient Non-member Integration Restrictions

Since the non-member integration restriction $\overline{o}$ is crucial for the effectiveness of the union, we analyze its efficient determination. Given a non-member integration restriction $\overline{o}$ and equilibrium union U, the *membership incentive constraint binds* if the member with the lowest type is indifferent between joining and integrating as a non-member. The following proposition characterizes the efficient levels of non-member integration.

Proposition 10. *Let U be a union where membership incentive constraint does not bind. Then there exists $\overline{o}' > \overline{o}$ such that U is an equilibrium union under $\overline{o}'$ and all countries are better off under $\overline{o}'$.*

Proposition 10 shows that it is without loss of optimality to restrict attention to bounds that make the lowest type non-member indifferent between becoming a member or integrating outside. Whenever membership incentive constraint does not bind, there is an inefficient restriction on the integration of non-members and allowing non-members to integrate more is beneficial for all countries, regardless of their membership status.

5.3. Enlargement with Non-member Integration

We now extend our analysis considering jointly enlargement and non-member integration. Formally, we study the SPE of the following game.

1. Candidate countries decide whether or not to apply for membership.
2. Each union member decides whether to admit or reject each candidate.
3. Equilibrium union U is the initial union plus the countries admitted unanimously.
4. Union members decide the equilibrium policy b^* with majority voting.
5. Countries choose their actions.
    - If $i \in U$ [member], then i chooses an integration level $t_i \in [b, \infty)$.
    - If $i \notin U$ [non-member], then i chooses an integration level $t_i \in [0, \overline{o}]$.
6. Payoffs are realized

Enlargement is similar to union formation for the incentives of candidates about joining. If non-member integration is not restricted, then candidates with lower types than the initial members, prefer integrating as non-members to joining the union. Thus, enlargement

does not happen if non-member integration is not restricted. However, in this (richer) setting, there is another mechanism limiting enlargement. As we discussed in Section 4.2, initial members may be resistant to admitting new members as they may be worse off after enlargement. When non-member integration is possible, a more permissive non-member integration restriction increases the payoffs of initial members from integrating with the candidates without admitting them to the union. For example, when the non-member integration bound is high, if high-type initial members believe that after enlargement the union policy will decrease, then they would be more willing to reject the candidates, keeping union policy high by not giving them voting power, while at the same time benefiting from integrating with them as non-members.[26]

Therefore, there are two bounds, $\overline{o}_C$ and $\overline{o}_I$ that depend on the preferences of candidates and initial members. The bound determined by the preferences of the candidates, $\overline{o}_C$, is similar to the union formation case; its assures that non-member integration is restricted so that candidates prefer to join the union. The bound determined by the preferences of the initial members, $\overline{o}_I$, makes sure that non-member integration is restricted so that the initial members have incentive to admit the candidates.

Proposition 11. *Consider an initial union I and possible enlargement $U = I \cup C$ with equilibrium policy b_U.*

1. *If U is not an equilibrium union under enlargement without non-member integration ($\overline{o} = 0$), then it is not an equilibrium under any $\overline{o}$.*
2. *If U is an equilibrium union under enlargement without non-member integration, then there exists two cut-offs $\overline{o}_I$ and $\overline{o}_C$ such that enlargement to U is an equilibrium if and only if $\overline{o} \leq \min\{\overline{o}_I, \overline{o}_C\}$. Moreover, $\min\{\overline{o}_I, \overline{o}_C\} < b_U$, that is, non-member integration must be restricted for enlargement.*

5.4. Exit

We leverage the flexibility of our framework with outside integration to study exit, an increasingly topical issue after BREXIT and the often adversarial relationship of the EU with Hungary. To study exit, we allow for a preference shock, as in many countries government's and people's view towards the EU and nationalism have changed over time.[27]

[26] Example 7 in Appendix A demonstrates how non-member integration restrictions may be necessary for high-type initial members to accept candidates.

[27] See Guriev and Papaioannou (2022) for an overview of works on populism, euro-skepticism, and attitudes against globalization, and Guiso, Herrera, and Morelli (2016) on the role of cultural differences on EU's institutional integration.

5.4.1. Set-up

There is a union, $U = \{1, \ldots, |U|\}$, consisting of members with types γ. With probability ϵ, a country gets a preference shock lowering its type. This can be related to an event that affects the preferences of the citizens, or election of a new leader who is against integration. After the preference shock, the country can decide to exit the union and integrate up to $\overline{e}$, which denotes the upper bound of integration for countries leaving the union. Moreover, if the country leaves, all remaining members receive a cost $\kappa \geq 0$. This cost represents the time and effort needed to negotiate the exit, legislative amendments and dealing with workers from the leaving country. Formally, we analyze the SPE of the following game:

1. Members vote over union policy b and decide their integration levels $t_i \geq b$.
2. With probability ϵ, a country gets a shock that reduces its type (preferences for integration) to γ_l.
3. The country facing the preference shock decides whether or not to exit the union and its integration level.
4. All members get an extra disutility of $-\kappa$ (where $\kappa \geq 0$) if a country exits. Payoffs are realized.

A union U is *robust* under $\overline{e}$ if the preference shock does not lead the country to leave the union.

5.4.2. Implications

The following proposition summarizes the core results linking union stability, the preference shock's size, and restrictions on integration with non-members.

Proposition 12. *Let U denote a union under $\overline{e}$ with $|U| > 2$.*

1. *There is a $\tilde{\gamma}$ such that if $\gamma_l < \tilde{\gamma}$, then U is not robust under any $\overline{e}$.*
2. *If $\gamma_l \geq \tilde{\gamma}$, there exists $e(\gamma_l)$ where U is robust under $\overline{e}$ if and only if $\overline{e} \leq e(\gamma_l)$, i.e., the exiting country is restricted to have an integration level below $e(\gamma_l)$.*
3. *The exit restriction that makes the union robust, $e(\gamma_l)$, is decreasing in γ_l.*

Proposition 12 shows that the union is robust when the upper bound on the integration of countries exiting the union is restricted to be under $e(\gamma_l)$. This result appears intuitive since the former member that gets the negative preference shock becomes similar to a potential candidate: the exiting (candidate) country compares the payoff under exit (non-member) integration levels to staying (becoming) a member. Therefore, harsher restrictions contribute to the robustness (size) of the union. If the "deal" the Union offers to exiting (candidate)

country is significantly worse to that of members, say restricting access to the single market and requiring licenses (passports) to conduct cross-border banking, then the union becomes more cohesive (part (2)). As the magnitude of the shock increases, more restrictive non-member integration policies are necessary to keep the union intact (part (3)). We coin the case where $\overline{e} \leq e(\gamma_l)$ as *exit restriction*, since in this scenario the policy is strong enough to deter exit, resulting in a robust union. Conversely, exit is not restricted when $\overline{e} > e(\gamma_l)$.

Next, we consider countries' preferences towards exit, analyzing when exit restrictions are pareto optimal and/or implemented in equilibrium with majority voting. A country *prefers exit restriction* if its expected utility under a robust union with exit restriction ($\overline{e} = e(\gamma_l)$) is greater than without. A robust union with exit restriction results in a larger union with deeper integration, while without restrictions there is exit after a preference shock. On the one hand, as higher integration from other countries is beneficial to a country and exit restrictions keep integration higher after a shock by preventing exit, they are beneficial to countries when they do not get the preference shock. On the other hand, when countries get the shock, exit restrictions are harmful since they prevent exiting and keeping a desired level of integration. As the benefit of more integration is higher for high-type countries, the following result emerges:

Proposition 13. *If country i prefers exit restriction and $\gamma_j > \gamma_i$, then country j also prefers exit restriction.*

Moreover, the following corollary characterizes when exit restriction is pareto-improving and is adopted under majority voting.

Corollary 1. *There is a cut-off country $k(\gamma_l, \kappa)$, such that all countries with higher types prefer restrictions on exit. Exit restriction is Pareto improving if the country with the lowest type prefers it. Exit restriction is adopted in majority voting if the median member prefers it.*

Intuitively, if the remaining countries face higher costs after a former member exit, then the benefit of a robust union is higher, while the cost of the restricted integration does not change. Therefore, with higher exit costs, countries prefer tighter exit policies. Conversely, if the preference shock is greater, the benefit of a robust union for the remaining countries do not change, while the cost of exit restriction is higher for the country that gets the preference shock. If the preference shock is greater, they prefer higher non-member integration following exit. The following proposition formalizes these points.

Proposition 14. *$k(\gamma_l, \kappa)$ is decreasing in κ (i.e. more countries prefer exit restriction under higher c) and increasing in γ_l (fewer countries prefer exit restriction if shock is greater).*

6. Conclusion

We develop a model of international integration where countries with heterogeneous preferences decide to integrate, either joining a rigid or a flexible international union or integrating without the commitment of the union. Inspired by the EU's focus on fostering a single market for goods, services, capital and labor, legislative regulatory harmonization policies in capital markets, the ongoing banking union, and product market standardization, we model countries' actions as strategic complements, rather than contributions in a public good. First, we study the trade-offs of each integration method, comparing equilibrium policies. Second, we examine union formation and enlargement. Unlike previous models that focus on public goods games where actions are strategic substitutes, our model can explain how flexible integration and enlargement to the periphery are spearheaded by the "core" countries and how these changes are accompanied by higher integration across the union. Our model also incorporates non-member integration to the study of international unions, allowing us to think about the incentives, pros and cons of union's tendency to have special arrangements with non-members. We show that restrictions on the integration of non-members are necessary for a union to be effective and determine the size and scope of the union. Besides, modeling non-member integration allows thinking about exit, a topical issue given BREXIT and the rise of anti-EU parties across the continent. We demonstrate that placing restrictions on the post-exit integration level with leaving countries, even severe, are necessary to maintain the robustness of the union. Our framework, therefore, implies that euroskeptic politicians pushing for their countries' exit from the EU but at the same time arguing that the leaving nations will fully maintain the integration benefits (with some"special deal") are unrealistic.

Our analysis and conceptual framework offer a useful baseline to think about the trade-offs of international integration with unions. Our setting can be extended in further directions to study some first-order issues. First, integration can be modeled in a two dimensional space, where countries have different preferences over different domains. One domain may be subject to strategic complementarities, reflecting trade and financial integration, while the second domain may represent investments in a public good, like defense. Second, one could explore how in a flexible union, different tiers with further lower bounds may increase or slow down integration.[28] Third, extending the model to include countries with different sizes may yield additional insights on enlargement and relations with non-members. Fourth, adding dynamics will shed light on union stability. Fifth, allowing for investments (by the union) on countries' preferences for integration may be useful studying how integration

[28]Berglof et al. (2008) analyzes how threat clubs within clubs increase low-type members' contributions.

may promote its own constituency and opponents. Sixth, one could consider alternative voting rules for enlargement or determination of integration protocols, such as qualified majority. Finally, allowing for within-country heterogeneity on preferences for integration and integration promoting winners and losers may bring insights on euro-skepticism and how to tackle it.

References

Abramson, B., Shayo, M., 2022. International integration and social identity. Journal of International Economics 137, 103577.

Alesina, A., Angeloni, I., Etro, F., 2005. International unions. American Economic Review 95, 602–615.

Alesina, A., Perotti, R., 2004. The european union: a politically incorrect view. Journal of Economic Perspectives 18, 27–48.

Alesina, A., Spolaore, E., 1997. On the number and size of nations. The Quarterly Journal of Economics 112, 1027–1056.

Alesina, A., Spolaore, E., 2005. The size of nations. Mit Press.

Alesina, A., Spolaore, E., Wacziarg, R., 2000. Economic integration and political disintegration. american economic Review 90, 1276–1296.

Berglof, E., Burkart, M., Friebel, G., Paltseva, E., 2008. Widening and deepening: reforming the european union. American Economic Review 98, 133–37.

Bolton, P., Roland, G., 1996. Distributional conflicts, factor mobility, and political integration. The American Economic Review 86, 99–104.

Bolton, P., Roland, G., 1997. The breakup of nations: a political economy analysis. The Quarterly Journal of Economics 112, 1057–1090.

Bolton, P., Roland, G., Spolaore, E., 1996. Economic theories of the break-up and integration of nations. European Economic Review 3, 697–705.

Buchanan, J. M., 1965. An economic theory of clubs. Economica 32, 1–14.

Casella, A., 2001. The role of market size in the formation of jurisdictions. The Review of Economic Studies 68, 83–108.

Casella, A., 2005. Redistribution policy: A european model. Journal of Public Economics 89, 1305–1331.

Crémer, J., Palfrey, T. R., 1996. In or out?: Centralization by majority vote. European Economic Review 40, 43–60.

Eichengreen, B., 2006. The parallel-currency approach to asian monetary integration. American Economic Review 96, 432–436.

Gancia, G., Ponzetto, G. A., Ventura, J., 2020. A theory of economic unions. Journal of Monetary Economics 109, 107–127.

Gans, J. S., Smart, M., 1996. Majority voting with single-crossing preferences. Journal of public Economics 59, 219–237.

Gilbert, M., 2020. European integration: A political history. Rowman & Littlefield Publishers.

Guiso, L., Herrera, H., Morelli, M., 2016. Cultural differences and institutional integration. Journal of International Economics 99, S97–S113.

Guriev, S., Papaioannou, E., 2022. The political economy of populism. Journal of Economic Literature 60, 753–832.

Harstad, B., 2006. Flexible integration? mandatory and minimum participation rules. The Scandinavian Journal of Economics 108, 683–702.

Kobielarz, M. L., 2022. A theory of international unions with exits. Working Paper .

Milgrom, P., Roberts, J., 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities. Econometrica: Journal of the Econometric Society pp. 1255–1277.

Oates, W. E., 1999. An essay on fiscal federalism. Journal of economic literature 37, 1120–1149.

Oates, W. E., et al., 1972. Fiscal federalism. Books .

Roberts, K., 2015. Dynamic voting in clubs. Research in Economics 69, 320–335.

Roberts, K. W., 1999. Dynamic voting in clubs. LSE STICERD Research Paper .

Sapir, A., 2011. European integration at the crossroads: A review essay on the 50th anniversary of bela balassa's theory of economic integration. Journal of Economic Literature 49, 1200–1229.

Spolaore, E., 2013. What is european integration really about? a political guide for economists. Journal of Economic Perspectives 27, 125–44.

Topkis, D. M., 1979. Equilibrium points in nonzero-sum n-person submodular games. Siam Journal on control and optimization 17, 773–787.

Topkis, D. M., 1998. Supermodularity and complementarity. Princeton university press.

Vives, X., 1990. Nash equilibrium with strategic complementarities. Journal of Mathematical Economics 19, 305–321.

Appendices

A. Omitted Examples

Example 6. *There are two countries* 1 *and* 2 *where* $\gamma_1 = 1$ *and* $\gamma_2 = 1.5$

$$u_i(t_i, t_j, \gamma_i) = \gamma_i t_i t_j - \frac{1}{3} t_i^3 \tag{9}$$

Non-union integration admits a unique equilibrium, where $t_1 \approx 1.14$ *and* $t_2 \approx 1.31$ *with payoffs* $u_1(t^*, \gamma_1) = 1$ *and* $u_2(t^*, \gamma_2) = 1.5$. *The rigid union equilibrium policy is* $r^* = 2$ *and countries' payoffs are* $u_1(2, 2, \gamma_1) \approx 1.33$ *and* $u_2(2, 2, \gamma_2) \approx 3.33$. *In this example, the enforcement power of the union allows countries to increase their integration to higher levels, making both countries better off compared to non-union integration. This example illustrates that the enforcement power of unions plays an important role in fostering integration and moving from non-union integration (without commitment and enforcement) to rigid union may be Pareto-improving for all members.*

Example 7. *There is an initial union of nine countries,* $I = \{3, \ldots, 11\}$ *and there are two candidates,* $C = \{1, 2\}$. *The types of countries are:* $\gamma_{11} = 1.93$, $\gamma_i \in [1.8, 1.93]$ *for* $i \in \{8, 9, 10\}$, $\gamma_7 = 1.8$ $\gamma_6 = 1.6$ $\gamma_i = [1.5, 1.6]$ *for* $i \in \{3, 4, 5\}$, $\gamma_2 = 1.2$ *and* $\gamma_1 = 1$. *All countries have the following the utility function:*

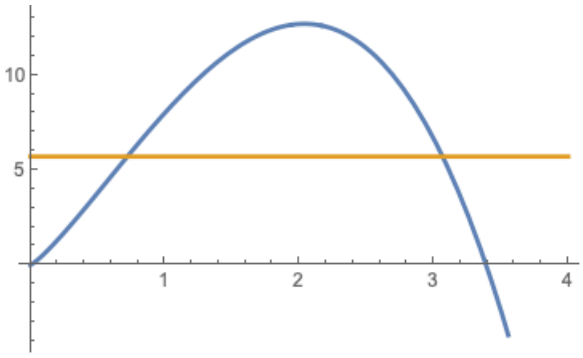

Fig. 1. **Example 8. Joining or Integrating as Non-Members. Country** 2. The horizontal axis denotes non-member integration level and the vertical axis denotes the integration payoff of members (Orange Line) and non-members integration payoff (Blue Line) for Country 2.

$$u(t_i, t_{-i}, \gamma_i) = \sum_{j \neq i} (\min\{t_i, t_j\})^{\gamma_i} - t_i^3 \tag{10}$$

In the initial union, the median country is 7 *and the equilibrium policy is* $r^*(I) \approx 3.7$. *We now consider when country* 2 *can join. If the country joins, country* 6 *becomes the decisive median; the new equilibrium policy falls,* $r^*(I \cup \{2\}) \approx 3$. *Although the union has added a member, the median country prefers a lower integration. We check the preferences of country* 2 *if it becomes a member or integrates without joining. On the one hand (net of integration with country* 1*, which is the same in both cases), joining gives country* 2 *a payoff of* $9 \times 3^{\gamma_2} - 3^3 \approx 5.7$. *On the other hand, integrating as an non-member at the cutoff level o gives a payoff of* $9(\overline{o}^{\gamma_2}) - \overline{o}^3$. *Figure 1 plots the payoffs (in the vertical axis) against the non-member integration level (horizontal line) for country* 2 *if it joins the union (orange line) and integrates at the union policy,* $r^*(I \cup \{2\}) = 3$*, and integrates as a non-member (blue line). When non-member integration is restricted below* 0.7 *(the point where the two curves intersect), country* 2 *prefers becoming a member since it results in a higher payoff, while if non-member integration restriction is weaker, then it prefers to integrate as a non-member. This echoes Example 5, illustrating how non-member integration is important for equilibrium union.*

Next, examine the policy trade-off for country 1 *plotting in Figure 2 its preferences joining the union and integrating as a non-member. Joining the union entails a negative payoff, due to the country's low type. Nonetheless, the country is willing to integrate as much as* 1.7 *as a non-member. However, given the* 0.7 *bound that the union imposes to make sure country* 2 *has incentive to join the union, country* 1 *cannot benefit fully from non-member integration. Moreover, the initial members also lose out from this restriction as they beneficial integration*

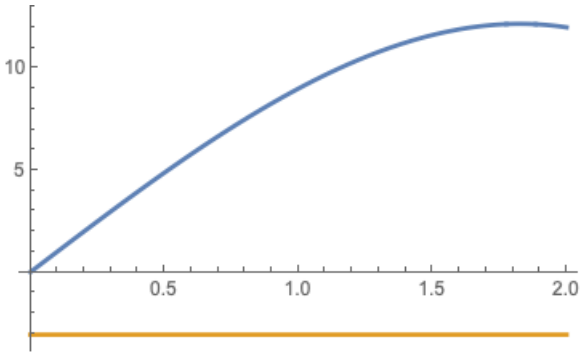

Fig. 2. **Example 8. Joining or Integrating as Non-Members. Country 1.** Horizontal axis denotes non-member integration level and vertical axis denotes membership payoff (Orange Line) and non-member integration payoff (Blue Line) for Country 1.

with country 1 *is prevented. However, the country is still better off as it can integrate partly with the union; and union members also benefit from country* 1 *partial integration as a non-member.*

Lastly, we explore the incentives of the high-type country 11 *on whether to allow country* 2 *to join. Country* 11 *compares a lower integration level but with a larger union against a higher integration level under a smaller union. In particular, the comparison entails the smaller union payoff (when* 2 *integrates at non-member integration bound* $\overline{o}$*)* $8\times(3.7)^{\gamma_{11}}+\overline{o}^{\gamma_{11}}-(3.7)^3$ *with the payoff if country* 2 *joins,* $9\times 3^{\gamma_1}-3^3\approx 49.5$*, plotted in Figure 3. Whenever the union allows country* 2 *to integrate more than cutoff* 0.4*, country* 11 *prefers country* 2 *to integrate as a non-member and therefore, rejects the candidacy of country* 2*. As a result, a non-member integration bound at* 0.4 *is necessary for unanimous acceptance of country* 2*'s candidacy. Note that at that bound, the incentive constraint of country* 2 *is slack. Country* 2 *would prefer to join the union even if non-members are allowed to integrate more than* 0.4*. Moreover,* 1 *wants to integrate more than* 0.4*. Thus, under enlargement, the efficiency of the non-member integration policies is not determined only by the incentives of the marginal members, but also depend on the incentives of the higher type countries who might block enlargement in favor of higher non-member integration.*

When we did not consider enlargement, the incentives of the marginal members were the main determinants of union size; and the integration bound for non-members, $\overline{o}$*, was critical. But the example above shows that with enlargement, the incentives of the initial members are also important. Higher type countries may prefer to exclude lower type members joining in, so as to keep the equilibrium integration policy of the union high. Allowing for non-member integration, if anything, boosts this, as high-type countries have an even stronger incentive to reject candidates when non-member integration bound is high, since they can still benefit*

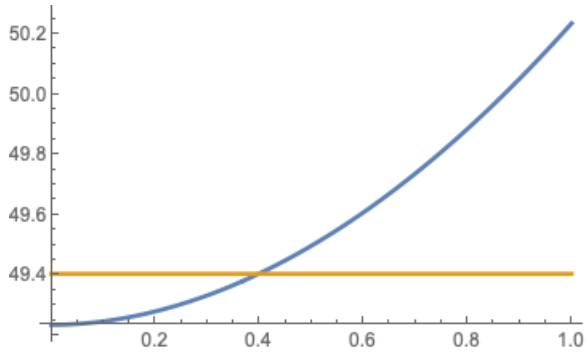

Fig. 3. **Example 8. Trade-off of High-Type Member Country** Horizontal axis denotes non-member integration level of country 2 and vertical axis denotes the payoff of country 11 when 2 becomes a member (Orange Line) and integrates as a non-member (Blue Line).

from their integration. Therefore, a low non-member integration bound might be important to convince such countries to allow enlargement. This is an additional channel that may cause inefficiency, as the integration of non-members may be restricted not only for the incentives of marginal members, but for the high-type countries.

B. Proofs

B.1. Proof of Proposition 1

B.1.1. Part 1

For any country with type γ_i, there is a $t(\gamma_i)$ such that $u'(t_i, t_{-i}, \gamma) < 0$ for all $t_i > t(\gamma_i)$, hence all integration levels above $t(\gamma_i)$ is strictly dominated for country i. Let $t_{max} = \max_i t(\gamma_i)$. Then we can restricting attention to $[0, t_{max}]^N$ as the strategy space eliminates only strictly dominated strategies and does not change the set of equilibria. Moreover, $[0, t_{max}]^N$ is compact, $u(t_i, t_{-i}, \gamma)$ is continuous in t_i and t_{-i} and u satisfies increasing differences in t_i and t_{-i}. Thus this is a supermodular game, and first part of the result follows from Theorem 4.2.1 in Topkis (1998).

To prove the comparative statics, note that $u(t, \gamma)$ has increasing differences in γ_i, t_i and t_{-i}. Then second part of the proposition follows from Theorem 4.2.2 in Topkis (1998). Finally, it is easy to see that $\overline{t}^*(\gamma)$ is the pareto dominant equilibrium. Let $\hat{t}(\gamma)$ denote another equilibrium. For all i, we have $u(\hat{t}_i(\gamma), \hat{t}_{-i}(\gamma), \gamma_i) \leq u(\hat{t}_i(\gamma), \overline{t}^*_{-i}(\gamma), \gamma_i) \leq u(\overline{t}^*_i(\gamma), \overline{t}^*_{-i}(\gamma), \gamma_i)$, where the first inequality follows from the fact that $\overline{t}(\gamma)$ is the highest equilibrium (i.e., $\overline{t}^*_{-i}(\gamma) \geq \hat{t}_{-i}(\gamma), \gamma_i)$ and second follows from the fact that $\overline{t}^*_i(\gamma)$ is a best response to $\overline{t}^*_{-i}(\gamma)$.

B.1.2. Part 2

To simplify notation, let t^r denote the integration level vector where all countries choose integration level r while (r', t^r_{-i}) denotes the vector where all countries other than i choose integration level r and i chooses r'.

Lemma 1. *Let $r < r'$ be two integration levels. There is a cut-off country $n(r, r') \in U$ such that if $i < n(r, r')$ then i prefers r while if $i \geq n(r, r')$, then i prefers r'.*

Proof. Let $\gamma_i > \gamma_j$ and $r < r'$. We will show that if j prefers $t^{r'}$ to t^r, then so does i. If j prefers $t^{r'}$ to t^r, we have $u(t^{r'}, \gamma_j) - u(t^r, \gamma_j) \geq 0$. Note that by increasing differences, we have

$$u(t^{r'}, \gamma_i) - u(r', t^r_{-i}, \gamma_i) \geq u(t^{r'}, \gamma_j) - u(r', t^r_{-j}, \gamma_j) \tag{11}$$

$$u(r', t^r_{-i}, \gamma_i) - u(t^r, \gamma_i) \geq u(r', t^r_{-j}, \gamma_j) - u(t^r, \gamma_j) \tag{12}$$

Summing these to equations, we obtain $u(t^{r'}, \gamma_i) - u(t^r, \gamma_i) \geq 0$ and i prefers r' to r. Let $n(r, r')$ be the country with lowest type that prefers r' to r. Then all countries k with $\gamma_k > \gamma_{n(r,r')}$ also prefer r' to r. From the definition of $n(r, r')$, all countries with $\gamma_k < \gamma_{n(r,r')}$ prefer r to r', which proves the result. □

Let $m = \lfloor N/2 \rfloor$ denote the median country and $m' = m + 1$. Let r^* denote the most preferred integration level of m and let r' denote the most preferred integration level of m'.[29]

First, as $\gamma_{m'} > \gamma_m$ implies that $r' \geq r^*$. Let $\tilde{r} \neq r^*$ be any integration level. Note that if $\tilde{r} < r^*$, then $n(\tilde{r}, r^*) \leq m$, thus more than half of the countries prefer r^* to $\tilde{r}$. If $\tilde{r} > r'$, then $n(r', \tilde{r}) \geq m'$, thus again more than half of the countries prefer r to $\tilde{r}$. If $\tilde{r} \in [r^*, r']$, then $n(r^*, \tilde{r}) \geq m'$. If $|U|$ is odd, then $m > |U| - m$ and more than half of the countries prefer r^* to $\tilde{r}$. If $|U|$ is even and $n(r^*, \tilde{r}) > m'$, then more than half of the countries prefer r^* to $\tilde{r}$. If $|U|$ is even and $n(r^*, \tilde{r}) = m'$, then both policies get same amount of votes. As we break the tie in favor of the lower policy, r^* is selected. Thus r^* is the Condorcet winner.

B.1.3. Part 3

The characterization of equilibria under a given b follows from restricting the action space in the proof of Part 1 to $[b, t_{max}]^N$. To prove the fact that the most preferred policy of the median country is the Condorcet winner, we show that the structure of preferences of countries under rigid union is still true under flexible union. For notational simplicity, we suppress γ in $\overline{T}(b, \gamma)$. We first show that the equilibrium integration level $\overline{T}(b)$ is increasing in b.

[29] Whenever a country is indifferent between two policies, it breaks the tie in favor of highest policy, thus $r^* = \max\{\arg\max_{r'} u_m(t^{r'}, \gamma)\}$ and $r' = \max\{\arg\max_{r'} u'_m(t^{r'}, \gamma)\}$

Lemma 2. *$\overline{T}(b)$ is increasing in b.*

Proof. Define t_{max} as in proof of Part 1 and let $BR_i(t_{-i}, \gamma_i, b) = \arg\max_{t'_i \in [b, t_{max}]} u_i(t'_i, t_{-i}, \gamma)$. Let $\overline{BR}_i(t, \gamma_i, b) = \max BR_i(t_{-i}, \gamma_i, b)$ and let $\overline{BR}(t, b)$ denote the profile of best response strategies obtained from $\overline{BR}_i(t, \gamma_i, b)$ for all $i \in N$. Starting from $\bar{t}_0 = (t_{max}, ..., t_{max})$ and repeatedly applying best response correspondences, we obtain following sequence:

$$\overline{T}_k(b) \equiv \overline{BR}^{k-1}(\bar{t}_{k-1}, b) \tag{13}$$

Note that $\overline{BR}(t, b)$ is isotone in t and $\lim_{k\to\infty} \overline{BR}^k \to \overline{T}^*(b)$. As t_{max} does not depend on b and $\overline{BR}(t, b)$ is isotone, for $\tilde{b} \leq b$, we have, for all k, $\overline{T}_k(b) \geq \overline{T}_k(\tilde{b})$. Thus we have:

$$\overline{T}^*(t^b) = \lim_{k\to\infty} \overline{BR}_i^k(b) \geq \lim_{k\to\infty} \overline{BR}_i^k(\tilde{b}) = \overline{T}^*(\tilde{b}) \tag{14}$$

which yields the result. □

Next, we prove following the lemma, which shows that the most preferred integration bound of the median of the median country is the Condorcet winner in flexible union and proves the proposition:

Lemma 3. *The most preferred integration bound of the median country, b, is the Condorcet winner.*

Proof. Let $b' \neq b$. We will show that b wins against b' in majority voting. First, let $b' < b$ denote an alternative policy level that is lower than b. From Lemma 2, $\overline{T}(b) \geq \overline{T}(b')$. From the definition of b, median country votes for b and not for b'. Let j be another country such that $\gamma_j > \gamma_m$. There are two cases: either $\overline{T}_j(b') \geq b$ or $\overline{T}_j(b') < b$. If $\overline{T}_j(b') \geq b$, then $\overline{T}_j(b')$ is still available to choose under the bound b. As $\overline{T}(b) \geq \overline{T}(b')$, j prefers b in this case.

If $\overline{T}_j(b') < b$, then we will consider two cases, first is $\overline{T}_j(b) > b$ and the second is $\overline{T}_j(b) = b$. In the first case, we have that

$$u(\overline{T}_j(b), \overline{T}_{-j}(b), \gamma_j) > u(\overline{T}_j(b'), \overline{T}_{-j}(b), \gamma_j) \geq u(\overline{T}_j(b'), \overline{T}_{-j}(b'), \gamma_j) \tag{15}$$

where first inequality holds from strict concavity of u in its first argument and second follows since $b' < b$, $\overline{T}_{-j}$ is increasing in b and u is increasing in $\overline{T}_{-j}$. In the second case, note that $\overline{T}_j(b) = \overline{T}_m(b) = b$ and therefore $\overline{T}_{-j}(b) = \overline{T}_{-m}(b)$. Therefore,

$$
\begin{aligned}
u(b, \overline{T}_{-j}(b), \gamma_j) - u(\overline{T}_j(b'), \overline{T}_{-j}(b'), \gamma_j) \geq \\
u(b, \overline{T}_{-j}(b), \gamma_j) - u(\overline{T}_m(b'), \overline{T}_{-j}(b'), \gamma_j) \geq \\
u(b, \overline{T}_{-j}(b), \gamma_j) - u(\overline{T}_m(b'), \overline{T}_{-m}(b'), \gamma_j) \geq \\
u(b, \overline{T}_{-m}(b), \gamma_j) - u(\overline{T}_m(b'), \overline{T}_{-m}(b'), \gamma_j) \geq \\
u(b, \overline{T}_{-m}(b), \gamma_m) - u(\overline{T}_m(b'), \overline{T}_{-m}(b'), \gamma_m) \geq 0
\end{aligned} \tag{16}
$$

where the first inequality holds from optimality of $\overline{T}_j(b')$, second holds as $u(\overline{T}_m(b'), \overline{T}_{-j}(b'), \gamma_j) = u(\overline{T}_m(b'), \overline{T}_{-m}(b'), \gamma_j)$, third holds as $\overline{T}_j(b) = \overline{T}_m(b)$ fourth holds as $T(b) \geq T(b')$, $\gamma_j > \gamma_m$ and increasing differences and fifth holds as the median prefers b to b'.

Next, assume that $b' > b$. Moreover, let $\hat{t}$ denote the smallest $t \geq b'$ such that $\overline{T}_m(\hat{t}) = \hat{t}$ (of course, it is possible that $\hat{t} = b'$). Such $\hat{t}$ exists as the series defined by $t_0 = b'$ and $t_k = \overline{T}_m(t_{k-1})$ is increasing and bounded. Moreover, we have:

$$
u(\overline{T}_m(t_k), \overline{T}_{-m}(t_k), \gamma_m) \geq u(\overline{T}_m(t_{k-1}), \overline{T}_{-m}(t_{k-1}), \gamma_m) \text{ for all } k \tag{17}
$$

Thus $u(\overline{T}_m(\hat{t}), \overline{T}_{-m}(\hat{t}), \gamma_m) \geq (\overline{T}_m(b'), \overline{T}_{-m}(b'), \gamma_m)$. Let $\gamma_j < \gamma_m$. From optimality of b for the median country and $\hat{t} > b$, we know that:

$$
u(\overline{T}(b), \gamma_m) - u(\hat{t}, \overline{T}_{-m}(\hat{t}), \gamma_m) > 0 \tag{18}
$$

Moreover, as $\overline{T}_m(\hat{t}) = \hat{t}$ and $\gamma_j < \gamma_m$, we have $\overline{T}_j(\hat{t}) = \hat{t}$. Thus $\overline{T}_{-m}(\hat{t}) = \overline{T}_{-j}(\hat{t})$ and we have

$$
u(b, \overline{T}_{-j}(b), \gamma_m) - u(\hat{t}, \overline{T}_{-j}(\hat{t}), \gamma_m) > 0 \tag{19}
$$

As $\hat{t}$ is the smallest $t \geq b'$ such that $\overline{T}_m(\hat{t}) = \hat{t}$, we have:

$$
u(b, \overline{T}_{-j}(b), \gamma_m) - u(b', \overline{T}_{-j}(\hat{t}), \gamma_m) > 0 \tag{20}
$$

As $b' > b$ and $\overline{T}_{-j}(\hat{t}) \geq \overline{T}_{-j}(b)$, from $\gamma_j < \gamma_m$ and increasing differences we have:

$$
u(b, \overline{T}_{-j}(b), \gamma_j) - u(b', \overline{T}_{-j}(\hat{t}), \gamma_j) > 0 \tag{21}
$$

As $\hat{t} \geq b'$, we have $\overline{T}_{-j}(\hat{t}) \geq \overline{T}_{-j}(b')$, thus:

$$
u(b, \overline{T}_{-j}(b), \gamma_j) - u(b', \overline{T}_{-j}(b'), \gamma_j) > 0 \tag{22}
$$

This shows that j prefers b to b', which proves the result. □

B.2. Proof of Proposition 2

Assume that $\gamma_i = \gamma_m$ for all i, denote this type profile by $\tilde{\gamma}$. Let t^* denote the integration levels under non-union integration equilibrium. Note that $t_i^* = t_j^*$ for all j. Since u_i is differentiable, we have that $\frac{\partial u_i(t_i, t^*_{-i}, \gamma)}{\partial t_i} = 0$. Define $\hat{u}(x, \gamma_m) = u(x, x, \ldots, x, \gamma_m)$. Since u is strictly increasing in t_{-i}, $\frac{\partial \hat{u}(t^*, \gamma)}{\partial t^*} > 0$, which means that there exists ϵ such that when $r = t^* + \epsilon$, $u_i(r, r, \ldots, r, \gamma) > u_i(t^*, t^*, \ldots, t^*, \gamma)$. Since r^* is the most preferred integration level of all countries with homogeneous types, $u_i(r^*, r^*, \ldots, r^*, \gamma) \geq u_i(r, r, \ldots, r, \gamma)$ and the utility under rigid union equilibrium policy r^* is higher compared to non-union equilibrium. Let U_R denote the payoff of a country under rigid union equilibrium and U_N denote the payoff under non-union integration. We have showed that $U_R > U_N$

Lemma 4. *For each $\delta > 0$, there exists ϵ such that $|U_R - U_R^i(\gamma')| < \epsilon$.*

Proof. Now, let γ' denote a set of types such that $\gamma_i' \in (\gamma_m - \epsilon, \gamma_m + \epsilon)$. Let $U_R^i(\gamma')$ denote the payoff of i under rigid union equilibrium. First, note that the rigid union equilibrium is r^* under γ' since the median country has the same time. The result then follows from the continuity of u_i in γ. □

Let $U_N^i(\gamma')$ denote the non-union integration payoff of country i at γ'. Next, we prove the following lemma

Lemma 5. *For each δ, there exists ϵ such that $U_N^i(\gamma') + \delta < U_N$ whenever $\gamma_i \in [\gamma_m - \epsilon, \gamma_m + \epsilon]$*

Proof. We first prove the following claim.

Claim 1. *For each $\delta > 0$, there exists ϵ such that $|t^*(\gamma) - t^*| < \epsilon$ whenever $\gamma_i \in [\gamma_m, \gamma_m + \epsilon]$.*

Proof. If this result is not true, then there exists a sequence of type profiles γ^n such that $\gamma^n \to \tilde{\gamma}$ but $\lim_{n\to\infty} t^*(\gamma_n) \geq t^* + \epsilon$. However, this contradicts upper-hemi continuity of the nash equilibrium (which is satisfied due to continuity of the utility function and compactness of the action space) correspondence as larges equilibrium under $\tilde{\gamma}$ is strictly smaller than $\lim_{n\to\infty} t^*(\gamma_n)$, which is a contradiction. □

The lemma then follows from the above claim as the utility is continuous in t and γ and increasing in γ. □

Finally, taking $\delta = |U_R > U_N|/3$ in lemmas 4 and 5, we obtain the ϵ that proves the result.

B.3. Proof of Proposition 3

First, note that flexible union with a trivial bound ($b = 0$) is same as non-union integration.

Lemma 6. $\overline{T}(0, \gamma) = \bar{t}^*(\gamma)$

Proof. Follows immediately from the definition of equilibria in both cases. □

Let b denote the equilibrium policy for the flexible union. To simplify notatin, for the rest of the proof, we write t^* instead of $\bar{t}^*$. Next lemma shows that countries that choose higher levels than flexible integration level prefer flexible union to non-union integration.

Lemma 7. *Let $t_i^*(\gamma) \geq b$. Then i prefers flexible union with bound b to non-union integration.*

Proof. Since $\overline{T}(b, \gamma)$ is increasing in b, we have that $\overline{T}_{-i}(b, \gamma) \geq \bar{t}_{-i}^*(\gamma)$. Therefore, $u(t_i^*, \overline{T}_{-i}(t_i^*, \gamma), \gamma_i) \geq u(t_i^*, t_{-i}^*(\gamma), \gamma_i)$. Since i can choose t_i^* in flexible union with $b \leq t_i^*$, the result follows. □

Corollary 2. *The median country prefers flexible union to non-union integration.*

Proof. Follows from letting $b = t_m^*$ in Lemma 7. □

Next, we show that all countries with types higher than the median also prefer flexible integration. To simplify notation, we suppress γ in $\overline{T}(b, \gamma)$.

Lemma 8. *If $\gamma_i > \gamma_m$, then i prefers flexible union to flexible integration.*

Proof. There are two cases, either $t_i^* \geq b$ or $\bar{t}_i^* < b$. First case is immediate from Lemma 7. To prove the second case, assume $t_i^* < b$. Note that due to increasing differences in γ_i and t_i, $t_i^* \geq t_m^*$. There are two cases, $\overline{T}_i(b) > b$ and $\overline{T}_i(b) = b$. If $\overline{T}_i(b) > b$:

$$\begin{aligned} u(\overline{T}_i(b), \overline{T}_{-i}(b), \gamma_i) &> u(b, \overline{T}_{-i}(b), \gamma_i) \\ &> u(t_i^*, \overline{T}_{-i}(b), \gamma_i) \\ &\geq u(t_i^*, t_{-i}^*, \gamma_i) \end{aligned} \tag{23}$$

where first inequality follows from the optimality of $\overline{T}_i(b)$, second from the strict concavity of u in its first argument and third from the fact that $\overline{T}_{-i}(b) \geq t_{-i}^*$. If $\overline{T}_i(b) = b$, first note that

$$u(b, \overline{T}_{-m}(b), \gamma_m) - u(t_m^*, t_{-m}^*, \gamma_m) \geq 0 \tag{24}$$

Then $\overline{T}_i(b) = b$ implies that $\overline{T}_{-m}(b) = \overline{T}_{-i}(b)$. Moreover, note that

$$u(t_m^*, t_{-m}^*, \gamma_m) \geq u(t_i^*, t_{-m}^*, \gamma_m) \geq u(t_i^*, t_{-i}^*, \gamma_m) \tag{25}$$

Combining these, we obtain:

$$u(b, \overline{T}_{-i}(b), \gamma_m) - u(t_i^*, t_{-i}^*, \gamma_m) \geq 0 \tag{26}$$

As $b > t_{-i}^*$ and $\overline{T}_{-i}(b) \geq t_i^*$, by increasing differences we get:

$$u(b, \overline{T}_{-i}(b), \gamma_i) - u(t_i^*, t_{-i}^*, \gamma_i) \geq 0 \tag{27}$$

which proves the result. □

This proves that median country and all countries with higher types than the median prefer flexible union to non-union integration. To finish the proof, we show that for any two countries i and j with $\gamma_i < \gamma_j < \gamma_m$, if i prefers flexible union to non-union integration, then so does j.

Lemma 9. *$\overline{T}_i(b) = b$ for all i with $\gamma_i \leq \gamma_m$.*

Proof. If $\gamma_i \leq \gamma_m$ and $t_i(b) > b$, then this is a contradiction to optimality of b. □

Lemma 10. *$b \geq t_m^*$*

Proof. Follows from Lemma 2, which shows $\overline{T}$ is increasing in b and the fact that $t_m^* = \overline{T}_m(0)$. □

Assume that $\gamma_i < \gamma_j < \gamma_m$ and i prefers flexible union to non-union integration. Then we have that

$$\begin{aligned} & u(b, \overline{T}_{-i}(b), \gamma_i) - u(t_i^*, t_{-i}^*, \gamma_i) \geq 0 \\ \implies & u(b, \overline{T}_{-j}(b), \gamma_i) - u(t_j^*, t_{-i}^*, \gamma_i) \geq 0 \\ \implies & u(b, \overline{T}_{-j}(b), \gamma_i) - u(t_j^*, t_{-j}^*, \gamma_i) \geq 0 \\ \implies & u(b, \overline{T}_{-j}(b), \gamma_j) - u(t_j^*, t_{-j}^*, \gamma_j) \geq 0 \end{aligned} \tag{28}$$

First line follows from the fact that $\overline{T}_i(b) = \overline{T}_j(b) = b$ and from optimality of t_i^* for country i. Second line follows that $t_{-i}^* \geq t_{-j}^*$ as $t_j^* \geq t_i^*$. Third line follows from increasing differences and proves the result.

B.4. Proof of Proposition 4

We first prove the following Lemma. Let t^x denote the integration levels under a rigid union with policy x.

Lemma 11. *For any x, $u(\overline{T}(x), \gamma_i) \geq u(t^x, \gamma_i)$ for all γ_i, i.e. all countries prefer a flexible union with policy x to a rigid union with policy x.*

Proof. First, note that $\overline{T}_{-i}(x) \geq t^x_{-i}$. Then

$$u(\overline{T}(x), \gamma_i) \geq u(t^x_i, \overline{T}_{-i}(x), \gamma_i) \geq u(t^x, \gamma_i) \tag{29}$$

which proves the result. □

Let r denote the rigid union equilibrium policy level and t^r denote the policy vector. From Lemma 11, $u(\overline{T}(x), \gamma_i) \geq u(t^x, \gamma_i)$ so median country prefers flexible union to rigid union.

Next, let b denote the equilibrium policy level under flexible union. There are three cases, $b = r$, $b > r$ and $b < r$. First case directly follows from Lemma 11. In the second case, let $\gamma_i > \gamma_m$. There are two sub-cases, either $\overline{T}_i(b) > b$ or $\overline{T}_i(b) = b$. In the first sub-case, note that

$$\begin{aligned} u(\overline{T}_i(b), \overline{T}_{-i}(b), \gamma_i) &\geq u(b, \overline{T}_{-i}(b), \gamma_i) \\ &> u(r, \overline{T}_{-i}(b), \gamma_i) \\ &> u(r, t^r_{-i}, \gamma_i) \end{aligned} \tag{30}$$

where first inequality follows from the optimality of $\overline{T}_i(b)$, second from the strict concavity of u in its first argument and third from the fact that $\overline{T}_{-i}(b) \geq t^r_{-i}$ (which is implied by $b > r$). In the second sub-case,

$$u(b, \overline{T}_{-m}(b), \gamma_m) > u(r, \overline{T}_{-m}(b), \gamma_m) \geq -u(r, t^r_{-m}, \gamma_m) > 0 \tag{31}$$

where first inequality follows from strict concavity of u and second from $\overline{T}_{-m}(b) \geq t^r_{-m}$. Note that $\overline{T}_i(b) = b$ implies that $\overline{T}_{-m}(b) = \overline{T}_{-i}(b)$. Therefore

$$u(b, \overline{T}_{-i}(b), \gamma_m) - u(r, t^r_{-i}, \gamma_m) > 0 \tag{32}$$

Since $b > r$ and $\overline{T}_{-i}(b) \geq t^r_{-i}$, by increasing differences we have

$$u(b, \overline{T}_{-i}(b), \gamma_i) - u(r, t^r_{-i}, \gamma_i) > 0 \tag{33}$$

which proves the result.

Third, let $r > b$ and $\gamma_i < \gamma_m$. Since m prefers flexible union and $r > b$,

$$u(b, T_{-m}(b), \gamma_m) > u(r, T_{-m}(r), \gamma_m) \tag{34}$$

As otherwise r would be the Condorcet winner under flexible union. From Lemma 9, $\overline{T}_i(b) = \overline{T}_m(b) = b$. Therefore, $\overline{T}_{-i}(b) = \overline{T}_{-m}(b)$. Then above equation and by increasing differences imply

$$u(b, T_{-i}(b), \gamma_i) > u(r, T_{-i}(r), \gamma_i) \geq u(r, t^r, \gamma_i) \tag{35}$$

which proves the result.

B.5. Proof of Proposition 5

Assume for a contradiction U is an equilibrium with median m, $\gamma_i > \gamma_m$ and $i \notin U$. Consider $\tilde{U} = U \bigcup \{i\}$ and let $\tilde{m}$ denote the median country at $\tilde{U}$. Since $\gamma_i > \gamma_m$, $\gamma_i \geq \gamma_{\tilde{m}}$.

We first consider the rigid union case. Let r denote the rigid union equilibrium policy under $\tilde{U}$. Clearly, $u(t^r, \gamma_{\tilde{m}}) > 0$. Then $u(t^r, \gamma_i) > 0$ since $u(\cdot, \gamma)$ is increasing in γ.

Next, consider the flexible union case. Let b denote the flexible union equilibrium policy under $\tilde{U}$. Note that $\overline{T}_{\tilde{m}}(b) = b$ and $\overline{T}_i(b) \geq b$. If $\overline{T}_i(b) = b$, then

$$u(b, \overline{T}_{-i}(b), \gamma_i) = u(b, \overline{T}_{\tilde{m}}(b), \gamma_i) \geq u(b, \overline{T}_{\tilde{m}}(b), \gamma_{\tilde{m}}) > 0 \tag{36}$$

where first equality holds by $\overline{T}_i(b) = b$, second inequality holds by increasing differences and $\gamma_i \geq \gamma_{\tilde{m}}$ and the final inequality holds from the fact that $\tilde{m}$ is the median country under $\tilde{U}$.

Finally, assume that $\overline{T}_i(b) > b$. Then from strict concavity of $u(t_i, \cdot)$

$$u(\overline{T}_i(b), \overline{T}_{-i}(b), \gamma_i) > u(b, \overline{T}_{-i}(b), \gamma_i) \tag{37}$$

Moreover, $u(0, \overline{T}_i(b), \gamma_i) = 0$ and $0 < b < \overline{T}_i(b)$. Then the result follows from strict concavity of of $u(t_i, \cdot)$.

B.6. Proof of Proposition 6

To prove the first part, for a contradiction assume that $U' \subset U$, U' and U are equilibria and U gives a payoff that is strictly larger than its payoff under U' to some initial member i. Note that a profitable deviation for country i is rejecting all countries in $U \setminus U'$. Under

that deviation, the outcome would be U', which is a contradiction to our assumption U is an equilibrium.

Next, from Assumption 2 and $|I| > |C|$, it is guaranteed that the median country will be an initial union member. If $|U| = |U'|$, then the integration profile and thus the payoff of all initial members are same. Therefore, if U and U' are both equilibria and $|U| > |U'|$, then all initial members prefer U to U'. Thus, there exists a set of initial union optimal equilibria and all these equilibria has the same size, integration profile and initial member payoffs.

B.7. Proof of Proposition 7

Given I, γ_m and the integration level in the initial union r_I, we have that

$$\frac{\partial u^r(r, \gamma_m, |I|)}{\partial r}\Big|_{r=r_I} = 0 \tag{38}$$

as otherwise, the median country would either decrease or increase r_I. As u^r satisfies strictly increasing differences in integration level and union size,

$$\frac{\partial u^r(r, \gamma_m, |U|)}{\partial r}\Big|_{r=r_I} > 0 \tag{39}$$

Moreover, from definition of r_I, we have

$$u^r(r_I, \gamma_m, |I|) \geq u^r(r', \gamma_m, |I|), \quad \forall r' \leq r_I \tag{40}$$

As u^r satisfies strictly increasing differences in integration level and union size and $|U| > |I|$,

$$u^r(r_I, \gamma_m, |U|) > u^r(r', \gamma_m, |U|), \quad \forall r' \leq r_I \tag{41}$$

As u^r is continuous,

$$u^r(r_I, \gamma, |U|) > u^r(r', \gamma, |U|), \quad \forall r' \leq r_I \tag{42}$$

whenever γ is close to γ_m. Let

$$\hat{\gamma} = \inf_{\gamma < \gamma_m} : \{\gamma : u^r(r_I, \gamma, |U|) \geq u^r(r', \gamma, |U|), \quad \forall r' \leq r_I\} \tag{43}$$

As u^r satisfies increasing differences, for all $\gamma'_m < \hat{\gamma}_1$, there exists $r' < r_I$ such that r' is strictly preferred to r_I by the new median country. Conversely, note that whenever $\gamma'_m > \gamma$, then again by increasing differences, r_I is more preferred to all $r' < r_I$ by the new median

country. This proves that $r_U \geq r_I$, where r_U is the union policy under U.

Finally, whenever $r_U \geq r_I$, for all i such that $\gamma_i \geq \gamma_m$, we have the following

$$\begin{aligned} u^r(r_U, \gamma_i, |U|) - u^r(r_I, \gamma_i, |I|) &\geq u^r(r_U, \gamma_i, |U|) - u^r(r_I, \gamma_i, |U|) \\ &> u^r(r_U, \gamma'_m, |U|) - u^r(r_I, \gamma'_m, |U|) \\ &> 0 \end{aligned} \tag{44}$$

where first inequality holds $u^r(r_I, \gamma_i, |I|) \leq u^r(r_I, \gamma_i, |U|)$, second inequality holds by strictly increasing differences in integration and type and third holds as r_U is chosen as the integration level under U. This shows that all above median initial union members are in favor of enlargement and finishes the proof.

B.8. Proof of Proposition 8

First, let r_I and r_U denote the equilibrium policies under initial and larger unions. Note that if $r_I = r_U$, then all countries strictly prefer the larger union as all non-member strictly increase their integration levels. As u is continuous, there exists ϵ such that all countries strictly prefer whenever $|r_U - r_I| < \epsilon$.

B.9. Proof of Proposition 9

We start by characterizing the non-union integration equilibrium. The following lemma shows that the integration level of a country is monotone in its type in any equilibrium.

Lemma 12. *Let t^* denote a non-union integration equilibrium and $\gamma_j > \gamma_k$. Then $t^*_j \geq t^*_k$.*

Proof. Assume for a contradiction $t^*_j < t^*_k$. As $\tilde{u}$ (and therefore u) is strictly concave in first argument and t^*_k is chosen by country k, we have that

$$u(t^*_k, t^*_j, t^*_{-jk}, \gamma_k) - u(t^*_j, t^*_j, t^*_{-jk}, \gamma_k) > 0 \tag{45}$$

As $\gamma_k < \gamma_j$ and $t^*_k > t^*_j$, this implies that

$$u(t^*_k, t^*_j, t^*_{-jk}, \gamma_j) - u(t^*_j, t^*_j, t^*_{-jk}, \gamma_j) > 0 \tag{46}$$

Next, as $t^*_k > t^*_j$, by increasing differences, we have

$$u(t^*_k, t^*_k, t^*_{-jk}, \gamma_j) - u(t^*_j, t^*_k, t^*_{-jk}, \gamma_j) > 0 \tag{47}$$

Which contradicts the choice of t_j^* by country j. □

Lemma 12 suggests an iterative procedure to characterize the equilibrium integration levels in the highest equilibrium. Note that when all higher type countries have a (weakly) higher integration level, the utility of country 1 is given by

$$u(t_1, t_{-1}, \gamma_1 | t_{-1} \geq t_1) = (|U| - 1)\tilde{u}(t_1, \gamma_1) - c(t_1, \gamma_1) \tag{48}$$

Let t_1 denote the solution to the following equation:

$$(|U| - 1)\tilde{u}'(t_1, \gamma_1) - c'(t_1, \gamma_1) = 0 \tag{49}$$

As $\tilde{u}$ is strictly concave and c is strictly convex, there is a unique t_1 that satisfies the equation above. And therefore, for any $t > t_1$ and t_{-1}, $u(t_1, t_{-1}, \gamma_1) > u(t, t_{-1}, \gamma_1)$, thus t_1 is the highest integration level country 1 can have in any equilibrium. Moreover, whenever $t_j \geq t_1$ for all j, t_1 is a best response to t_{-1}.

For any $i > 1$, define t_i inductively by:

$$t_i = \max\{t, t_{i-1}\} \quad \text{where} \quad t \text{ solves } (|U| - i)\tilde{u}'(t, \gamma_i) - c'(t, \gamma_i) = 0 \tag{50}$$

Note that t_i denotes the highest integration level country i can have in any equilibrium, as in any equilibrium $t_j \leq t_i$ whenever $j < i$.

Lemma 13. *The integration levels obtained by the iterative procedure above is an (thus, the highest) equilibrium of non-union integration.*

Proof. Let t denote the integration levels obtained by this procedure. Assume for a contradiction t is not an equilibrium. Then there exists i and t_i' such that $u(t_i', t_{-i}, \gamma_i) - u(t_i, t_{-i}, \gamma_i) > 0$. There are two cases, $t_i > t_{i-1}$ and $t_i = t_{i-1}$. The first case is a contradiction as from strict concavity of $\tilde{u}$, i is already choosing the unique best response to t_{-i} and such a t_i' cannot exist. In the second case, it must be that $t_i' < t_i$ as otherwise this will contradict the choice of t_i during the iterative procedure. Let j denote the lowest type country with $t_j = t_i$. Note that this also means that $t_{-i} = t_{-j}$. Then from strict concavity of $\tilde{u}$,

$$u(t_j, t_{-j}, \gamma_j) - u(t_i', t_{-j}, \gamma_j) > 0 \tag{51}$$

which implies, by increasing differences, $t_{-i} = t_{-j}$ and $t_i' < t_i$.

$$u(t_i, t_{-i}, \gamma_i) - u(t_i', t_{-j}, \gamma_i) > 0 \tag{52}$$

which contradicts that t_i' is optimal. □

We will now characterize the equilibrium policy and integration levels, denoted by h, for a given union U with median country m. To this end, divide countries to four sets

- $\underline{N} = \{\underline{n}_1, \ldots, \underline{n}_{|\underline{N}|}\}$, non-members with types below m
- $\overline{N} = \{\overline{n}_1, \ldots, \overline{n}_{|\overline{N}|}\}$, non-members with types above m
- $\underline{U} = \{\underline{u}_1, \ldots, \underline{u}_{|\underline{U}|}\}$, members with types below m
- $\overline{U} = \{\overline{u}_1, \ldots, \overline{u}_{|\overline{U}|}\}$, members with types (weakly) above m.

Within each set, countries are ordered according to their types, therefore $\overline{u}_1$ is the median country. First, note that, the highest integration level any country in $\underline{N}$ can have is already corresponds to the values we have characterized for non-union integration. Moreover, these values are best response if all other countries have weakly higher integration levels. Therefore, set $h_i = t_i$ for these countries, where t is the integration level in non-union integration. To determine the equilibrium policy, there are two cases. First case is when

$$(|U| - 1)\tilde{u}'(\overline{o}, \gamma_m) - c'(\overline{o}, \gamma_m) \geq 0 \tag{53}$$

This means that even if all non-member integrate below $\overline{o}$ due to restrictions, the median still sets an integration level above $\overline{o}$, which is given by

$$b^* = t \text{ where } (|U| - 1)\tilde{u}'(t, \gamma_m) - c'(t, \gamma_m) = 0 \tag{54}$$

Then b^* is the equilibrium policy in U in the first case. From definition of b^*, and the fact that the types of countries in $\overline{N}$ are higher than m, their best response is to choose $\overline{o}$ whenever all countries in U has higher integration levels than $\overline{o}$. Therefore, set $h_i = \overline{o}$ for these countries. Next, since all countries in $\underline{U}$ has lower types than m, from definition of b^*, they would never choose a higher integration level. Therefore, $h_i = b^*$ for these countries, which is their best response in $[b, \infty)$. Finally, the highest integration level any country $\overline{u}_i$ in $\overline{U}$ (where $1 < i < |\overline{U}|$) can have is given by the solution to the equation (which is defined inductively, starting with $\overline{u}_2$, setting $\overline{u}_1 = b$) $h_{\overline{u}_i} = \max\{h_{\overline{u}_{i-1}}, t\}$ where t solves

$$(|\overline{U}| - i)\tilde{u}'(t, \gamma_{u_i}) - c'(t, \gamma_m) = 0 \tag{55}$$

Moreover, $h_{\overline{u}_i}$ is best response whenever all higher type countries in $\overline{U}$ has weakly higher integration levels, which is satisfies at h. Therefore, h is the highest equilibrium in the first case. Second case is when

$$(|U| - 1)\tilde{u}'(\overline{o}, \gamma_m) - c'(\overline{o}, \gamma_m) < 0 \tag{56}$$

In this case, the equilibrium policy is given by b^* that solves:

$$(|U| + |\overline{N}| - 1)\tilde{u}'(t, \gamma_m) - c'(t, \gamma_m) = 0 \tag{57}$$

Given b^*, all countries in $\overline{N}$ choose weakly higher integration levels than b^*, since they have higher types than the median and therefore, choice of b^* would be optimal. We do not need to further characterize the integration levels of the countries in $U \cup \overline{N}$, but the existence of an equilibrium where all these countries choose integration levels higher than b^* follows from Topkis Theorem with action spaces $[b^*, \infty]$ for members and $[b^*, \overline{o}]$ for non-members. Now, given the characterization of the equilibria, we first show that any equilibrium union with 2 member does not increase integration compared to non-union integration.

Lemma 14. *If U is an equilibrium union with $|U| = 2$, the union do not increase integration compared to non-union integration.*

Proof. Let γ_m denote the lower type country in the union, which is setting the policy and b the union policy. For a contradiction, assume $b > t_m$. All countries with types lower than m are non-members and free to choose any integration level in $[0, \overline{o}]$. Moreover, from the construction above, we know that none would choose an integration level higher than their non-union integration level, therefore all countries with lower types than m are not integrating more than the non-union integration level. Moreover, even if all other countries integrate more than m, from definition of t_m, the median would prefer setting t_m as the union policy rather than $b > t_m$, which is a contradiction. Therefore, $b \leq t_m$. Finally, for a contradiction assume there exist a non-empty set of countries, N^*, (which can be a singleton) that increase its integration level under U compared to non-union integration. Let j denote the country with the lowest type country in that set. We have showed that $\gamma_j > \gamma_m$. However, in non-union integration, j was already choosing the highest integration level it can choose in any equilibrium where all countries with higher types choose weakly higher integration levels. Since all countries with lower types choose a lower integration level under U, j chooses an integration level weakly smaller than its non-union integration, which is a contradiction. Therefore, N^* is empty, which proves the result. □

Finally the following lemma proves the result.

Lemma 15. *If U is an equilibrium union and $|U| > 2$, then $b^* > \overline{o}$.*

Proof. For a contradiction, assume $b^* \leq \overline{o}$. Let j denote the lowest type member. We will show that j is strictly better of by integrating as a non-member, which is a contradiction. First, note that if j does not enter the union, all countries with lower types integrate at the

same level. Moreover, j receives first best pay-off (conditional on all lower type countries integrating at their non-union integration levels) by not entering. To see why, note that from the definition of t_j (Equation 50), t_j is a best response when all countries with higher types than j has weakly higher integration levels, regardless of those levels.

Next, from the definition of b^* and $\gamma_j < \gamma_m$, we know that

$$(|U| - 1)\tilde{u}'(b^*, \gamma_j) - c'(b^*, \gamma_j) < 0 \tag{58}$$

Therefore, for small enough ϵ, j's utility increases if j integrates at $b^* - \epsilon$ instead of b^*, which means that it receives a payoff strictly below first best pay-off (conditional on all lower type countries integrating at their non-union integration levels), which proves the result. □

B.10. Proof of Proposition 10

Assume U is an equilibrium with equilibrium policy $b > \overline{o}$ and median country m.

Lemma 16. *If $\gamma_i > \gamma_m$, then $i \in U$.*

Proof. For a contradiction, assume $i \notin U$. Then given the optimality of b and $\gamma_i > \gamma_m$, i prefers to integrate more. Moreover, in the case i joins and we have $\tilde{U} = U \cup i$, the median country has (weakly) higher type compared to U, but still a (weakly) lower type than i. Thus, all countries have higher integration levels under $\tilde{U}$ compared to U and i can choose a higher integration level. Therefore i is better off by joining the union. □

Lemma 17. *Let $b > \overline{o}$ and membership incentive constraint does not bind. Then U is still equilibrium for $o' > \overline{o}$ if membership incentive constraint is still satisfied under o' (including the case it binds).*

Proof. First, note that $o' < b$ since membership incentive constraints are satisfied. Moreover, all members prefer to stay members under o' as membership constraints are satisfied and equilibrium policy is same as the equilibrium policy under o. Let $i \notin U$. From Lemma 16, $\gamma_i < \gamma_m$. Let $\hat{t}$ denote the equilibrium under o'. First, if $\hat{t}_i < \overline{o}$, then i would not join since i is getting the highest payoff i can get when all members are integrating more than i. If $\hat{t}_i = o'$ and i gains strictly by joining, then this would also be the case under $\overline{o}$, which is a contradiction that U is an equilibrium under $\overline{o}$. □

Note that all non-members weakly increase their integration under o' compared to $\overline{o}$, which proves the result.

B.11. Proof of Proposition 11

First, take an arbitrary country $i \in C$. We first compare the utility of i as a member in U and as a non-member when the union is $\tilde{U} = U \setminus \{i\}$, under a given $\overline{o}$. Let b_U and $b_{\tilde{U}}$ denote the respective equilibrium policies under U and $\tilde{U}$. From definitions of b_U and $b_{\tilde{U}}$ and the fact that initial members have higher types and are more numerous than the candidates, both values are determined by an initial member with higher type than i as the median country. If i becomes a member, then its utility is given by

$$u^i_{join} = (|U| - 1)\tilde{u}(b_U, \gamma_i) - c(b_U, \gamma_i) \tag{59}$$

Note that this value is not affected by $\overline{o}$. If i decides not to join, then there are two changes. First, the union policy is $b_{\tilde{U}}$, and the integration of i is restricted at $\overline{o}$.

Therefore, given the integration levels of all countries under the union U, the utility of i is maximized at some $t_i < b_U$. From the construction of the equilibrium in the proof of Proposition B.9, changes in $\overline{o}$ do not affect $b_{\tilde{U}}$ (and equilibrium integration levels of any other countries) as i has lower type than the median country at $\tilde{U}$. Let t^*_i denote the most preferred integration level of i under the union $\tilde{U}$. As u is strictly concave in t_i given the integration of other countries, the utility of i is constant in $\overline{o}$ for $\overline{o} \geq t^*_i$ and is strictly decreasing in $\overline{o}$ for $\overline{o} < t^*_i$ and is given by

$$u^i_{non-member}(\overline{o}) = \max_{t_i \leq \overline{o}} (|U| - 1)\tilde{u}(t_i, \gamma_i) - c(t_i, \gamma_i) \tag{60}$$

Thus, if $u^i_{join} < u^i_{non-member}(0)$, then i would not join to the union under any $\overline{o}$. If this holds for such an $i \in C$, then U is not an equilibrium union.

Conversely, if $u^i_{join} \geq u^i_{non-member}(0)$ for all i, then there exists a $\overline{o}_i$ such that whenever $\overline{o} \leq \overline{o}_i$, it is a best response for i to join to the union, while if $\overline{o} > \overline{o}_i$, the there is no equilibrium where U is the equilibrium union. Finally, letting $\overline{o}_C = \min_{i \in C} \overline{o}_i$ (which is given by the candidate with the lowest type) yields the a cut-off such that all candidates would prefer to join to U if non-member integration is restricted below $\overline{o}_C$. Finally, for a contradiction assume that $\overline{o}_C \geq b_U$. Take the lowest type candidate country, j. Let t^*_j denote the best response of j under U. As j has a lower type than the median at U, we have that $t^*_j < b_U$. Moreover, if j does not join, the new median will be higher type and will choose $b_{\tilde{U}} \geq t^*_j$, which means that j will be better off by not joining. Thus $\overline{o}_C < b_U$.

We now consider the incentives of non-members. Consider U and $\tilde{U}$, where $I \subseteq \tilde{U} \subset U$ and some initial union member i. Let p^i_U denote country i's utility under U and $p^i_{\tilde{U}}(\overline{o})$ denote country i's utility under $\tilde{U}$ and non-member integration bound $\overline{o}$. Suppose that i rejects candidates in $U \setminus \tilde{U}$ when $\overline{o} = 0$. Let t^*_j denote the chosen integration levels of

country $j \in U \setminus \tilde{U}$ under the union $\tilde{U}$ and define $\bar{t} = \max_{j \in U \setminus \tilde{U}} t_j^*$. As all such j has lower type than the median country in $\tilde{U}$, $\bar{t} \leq b_{\tilde{U}}$ where $b_{\tilde{U}}$ is the union policy under $\tilde{U}$. Therefore, the union policy and integration levels of all countries are same for all $\overline{o} \geq \bar{t}$.

As the initial member i has higher type than all $j \in U \setminus \tilde{U}$, $p^i_{\tilde{U}}(\overline{o})$ is increasing in $\overline{o}$ for $\overline{o} \leq \bar{t}$. Moreover, since the union policy and integration levels of all countries are same for all $\overline{o} \geq \bar{t}$, $p^i_{\tilde{U}}(\overline{o})$ is constant in $\overline{o}$ for all $\overline{o} \geq \bar{t}$. Since p^i_U does not depend on $\overline{o}$, if i rejects candidates in $U \setminus \tilde{U}$ when $\overline{o} = 0$, i also rejects them under all $\overline{o}$, finishing the first part of the result.

To finish the proof, given initial union member i, smaller union $\tilde{U}$, where $I \subseteq \tilde{U} \subset U$ and corresponding $\bar{t}$, the utility $p^i_{\tilde{U}}(\overline{o})$ is increasing in $\overline{o}$. Suppose that i prefers enlargement to U under $\overline{o} = 0$ and let $\overline{o}_i$ as the supremum over $\overline{o}$ such that $p^i_U \geq p^i_{\tilde{U}}(\overline{o})$ for all $\tilde{U}$ with $I \subseteq \tilde{U} \subset U$. Thus, whenever $\overline{o} \geq \overline{o}_i$, i would prefer to reject some subset of candidates, while otherwise i votes in favor of enlargement. Letting $\overline{o}_I = \sup_{i \in I} \overline{o}_i$ then yields the result.

B.12. Proof of Proposition 12

To prove part 1, note that $\tilde{u}(t, 0) = 0$ and $\tilde{u}$ is continuous. Therefore, when γ_l is low enough, the country with the preference shock prefers no integration and exits under any $\overline{e}$. Moreover, since the initial union is stable, there exists $\overline{e}^*$ such that if γ_l is equal to the type of the lowest member, the union will be robust. Existence of a such $\tilde{\gamma}$ follows from the fact that $\tilde{u}$ is continuous and increasing in γ.[30].

To prove part 2, we first show the following lemma.

Lemma 18. *If U is robust under $\overline{e}$, then U is robust under all $\overline{e}' < \overline{e}$.*

Proof. Assume i prefers to stay in the union after a shock under $\overline{e}$. Then the union is stable under $\overline{e}$ and $\overline{e}' < \overline{e}$. Then the result is immediate from the fact that any payoff that an exiting country can attain under $\overline{e}'$ is attainable under $\overline{e}$ and staying in the union is preferred under $\overline{e}$. □

Part 2 follows from Lemma 18.

Lemma 19. *If i prefers to stay in the union after a shock under γ_l, then i prefers to stay in the union after a shock under $\gamma_l' > \gamma_l$*

Proof. Let t and k, denote the integration level profiles under γ_l if i stays in the union and exits, while t' and k', denote the integration level profiles under γ_l' if i stays in the union and

[30] $\tilde{\gamma}$ does not depend on which country gets the shock, since the shocked country does not care about the integration level of countries who stay members and exists by Lemma 19

exits. If $t_i' > b$, then the result is immediate from increasing differences. If $t_i' = b$, then by increasing differences, $t_i = b$, which also implies that $t = t'$. Since, $k'_{-i} = k_{-i}$, we have,

$$\begin{aligned} u_i(t_i', t'_{-i}, \gamma_l') - u_i(k_i', k'_{-i}, \gamma_l') &= u_i(b, t_{-i}, \gamma_l') - u_i(k_i', k'_{-i}, \gamma_l') \\ &\geq u_i(b, t_{-i}, \gamma_l) - u_i(k_i', k'_{-i}, \gamma_l) \\ &\geq u_i(b, t_{-i}, \gamma_l) - u_i(k_i, k'_{-i}, \gamma_l) \\ &\geq 0 \end{aligned} \tag{61}$$

where first line substitutes $t_i' = b$ and $t = t'$, second line holds due to increasing differences, third line holds due to the optimality of k_i under γ_l and final inequality is from the assumption that i prefers to stay in the union after a shock under γ_l. □

From Lemma 19, we know that for all $\gamma' > \tilde{\gamma}$, the country prefers to stay as a member at $\overline{e} = 0$. For each γ_l, let $e(\gamma_l)$ denote the highest exit restriction that the shocked country prefers to stay as a member, which exsits by Lemma 18. From Lemma 19, this country prefers to stay in for all $\overline{e} \leq e(\gamma_l)$ and exit for all $\overline{e} > e(\gamma_l)$, which finishes the proof of part 3.

B.13. Proof of Proposition 13

Let t denote the integration levels of countries, while t_i^e denote the integration level i chooses if i gets the shock and exits the union. Since after getting a shock all countries have type γ_l, $t^e \equiv t_i^e$ for all $i \in U$. Next, note that since $\gamma_l < \gamma_m$, if i gets the preference shock and does not exit, then i chooses to integrate at b.

Therefore, the following equation characterizes the difference between the utility level i obtains under exit restrictions and without exit restrictions:

$$\epsilon(|U| - 1)\left(u(b, \gamma_l) - u(t^e, \gamma_l)\right) + \sum_{j \neq i} \epsilon\left[u(b, \gamma_i) - u(t^e, \gamma_i) + \kappa\right] \tag{62}$$

First term is equal for all i. The second term is increasing in γ_i since u satisfies increasing differences and $b > t^e$, which proves the result.

B.14. Proof of Proposition 14

The result follows from the fact that Equation 62 is increasing in κ and decreasing in γ_l.