14.461: Technological Change, Lecture 8
Innovation, Reallocation and Growth

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Motivation (I)

- Recent economic recession has reopened the debate on industrial policy.
- In October 2008, the US government bailed out GM and Chrysler. (Estimated cost, $82 Billion)
- Similar bailouts in Europe: Estimated cost €1.18 trillion in 2010, 9.6% of EU GDP.
- Many think that this was a success from a short-term perspective, because these interventions
  - protected employment, and
  - encouraged incumbents to undertake greater investments,

Motivation (II)

- More generally, what are the implications of "industrial policy" for R&D, reallocation, productivity growth, and welfare?
- Bailouts or support for incumbents could increase growth if there is insufficient entry or if they support incumbent R&D.
  - In fact, this is recently been articulated as an argument for industrial policy.
- They may reduce growth by
  - preventing the entry of more efficient firms and
  - slowing down the reallocation process.
- Reallocation potentially important, estimated sometimes to be responsible for up to 70-80% of US productivity growth.

Motivation (III)

- What's the right framework?
  1. endogenous technology and R&D choices,
  2. rich from dynamics to allow for realistic reallocation and matched the data (and for selection effects),
  3. different types of policies (subsidies to operation vs R&D),
  4. general equilibrium structure (for the reallocation aspect),
  5. exit for less productive firms/products (so that the role of subsidies that directly or indirectly prevent exit can be studied).
- Starting point: Klette and Kortum's (2004) model of micro innovation building up to macro structure.

Motivating Facts

- R&D intensity is independent of firm size.
- The size distribution of firms is highly skewed.
- Smaller firms have a lower probability of survival, but those that survive tend to grow faster than larger firms. Among larger firms, growth rates are unrelated to past growth or to firm size.
- Younger firms have a higher probability of exiting, but those that survive tend to grow faster than older firms.
- Gibrat's law holds approximately (but not exactly): firm growth rate roughly independent of size, though notable deviations from this at the top and the bottom.

Model I

- Representative household maximizes

$$U = \max \int_0^\infty e^{-\rho t} \log C_t dt$$

- All expenses are in terms of labor. Hence $C_t = Y_t$.
- The household owns all the firms including potential entrants. Therefore the total income is

$$Y_t = w_t L + r_t \mathcal{A}_t$$

where $\mathcal{A}$ is the total asset holdings and r_t is the rate of return on these assets.

Model II

- Final good production

$$\ln Y_t = \int_0^1 \ln y_{jt} dj$$

  - y_j : quantity of intermediate j

- A fixed mass L of labor

$$L_P + S_E + S_I = L$$

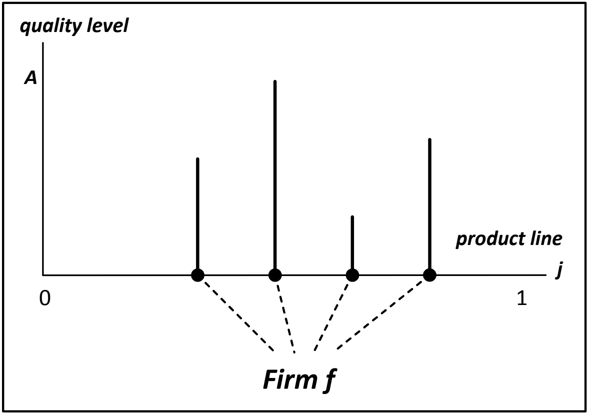
  - L_P : production
  - S_E : scientists working for outsiders
  - S_I : scientists working for incumbent firms.

- All workers receive w_t
- Normalize the price of the final good to 1.

Profits I

- A firm is defined as a **collection of product lines**.

FIGURE 3: EXAMPLE OF A FIRM



Profits II

- n will denote the number of product lines that the firm operates.
- Each intermediate is produced with a linear technology

$$y_{jt} = A_{jt} l_{jt}$$

- This implies that the marginal cost is

$$w_t / A_{jt}$$

  where w_t is the wage rate in the economy at time t.
- Innovations in each product line improves the productivity by $\lambda > 0$ such that

$$A_{jt+\Delta t} = \begin{cases} (1+\lambda) A_{jt} & \text{if successful innovation} \\ A_{jt} & \text{otherwise} \end{cases}$$

Profits III

- Bertrand competition $\Longrightarrow$ previous innovator will charge at least her marginal cost: $\frac{(1+\lambda)w_t}{A_{jt}}$.
- Hence the latest innovator will charge the marginal cost of the previous innovator

$$p_{jt} = \frac{(1+\lambda)\,w_t}{A_{jt}}.$$

- Recall that the expenditure on each variety is Y_t (since $P_t = 1$).
- Then the profit is

$$\begin{aligned} \pi_j &= y_j\,(p_j - MC_j) \\ &= \frac{A_{jt}\,Y_t}{(1+\lambda)\,w_t}\left(\frac{(1+\lambda)\,w_t}{A_{jt}} - \frac{w_t}{A_{jt}}\right) \\ &= \pi Y_t \end{aligned}$$

where $\pi \equiv \frac{\lambda}{1+\lambda}$.

Innovation Technology I

- Innovations are undirected across product lines.
- Innovation technology

$$X_i = \left(\frac{S_i}{\zeta}\right)^{1-\gamma} n^{\gamma}$$

where $\gamma < 1$, X_i is the innovation flow rate, S_i is the amount of R&D workers, n is the number of product lines to proxy for the firm specific (non-transferable, non-tradable) knowledge stock.

Innovation Technology II

- Alternatively, the cost of innovation:

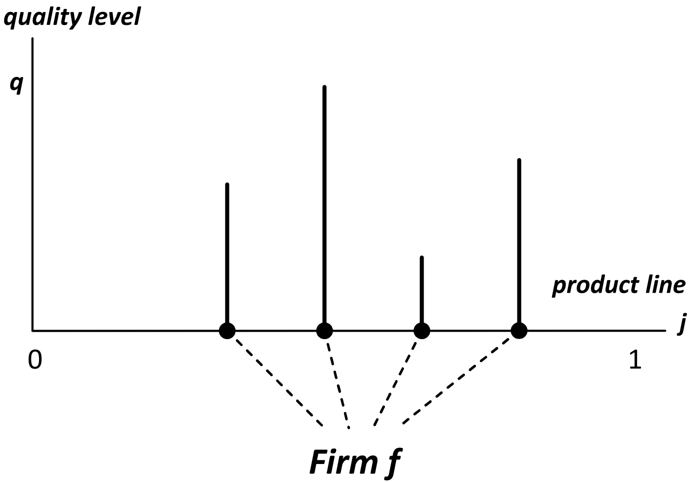
$$
\begin{aligned}
C(X, n) &= wS_i \\
&= \zeta wn \left[\frac{X_i}{n}\right]^{\frac{1}{1-\gamma}} \\
&= \zeta wn x_i^{\frac{1}{1-\gamma}}
\end{aligned}
$$

  where $x_i \equiv X_i / n$ is the innovation intensity (per product line).
- Let x denote the aggregate innovation rate in the economy.
- Innovation rate by entrants is x_e.
- Aggregate innovation rate is

$$\tau = x_i + x_e.$$

Innovation Technology III

- When a firm is successful in its current R&D investment, it innovates over a random product line $j' \in [0, 1]$.
  1. Then, the productivity in line j' increases from $A_{j'}$ to $(1 + \lambda)A_{j'}$.
  2. The firm becomes the new monopoly producer in line j' and thereby increases the number of its production lines to $n + 1$.
- At the same time, each of its n current production lines is subject to the *creative destruction* τ by new entrants and other incumbents.
- Therefore during a small time interval dt,
  1. the number of production units of a firm increases to $n + 1$ with probability $X_i dt$, and
  2. decreases to $n - 1$ with probability $n\tau dt$.
- A firm that loses all of its product lines exits the economy.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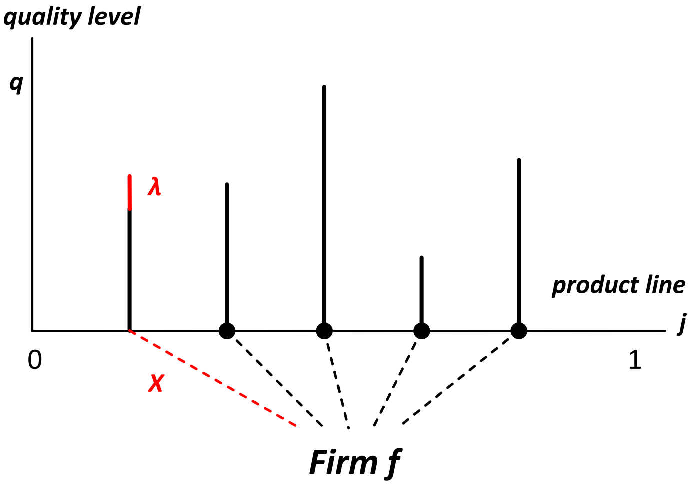
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Value Function I

- Relevant firm-level state variable: number of products in which the firm has the leading-edge technology, n.
- Then the value function of a firm as a function of n is

$$rV_t(n) - \dot{V}_t(n) = \max_{x_i \geq 0} \left\{ \begin{array}{c} n\pi_t - w_t \zeta n^{\frac{1}{1-\gamma}} \\ +nx_i \left[V_t(n+1) - V_t(n) \right] \\ +n\tau \left[V_t(n-1) - V_t(n) \right] \end{array} \right\}$$

- This can be rewritten as

$$\rho v = \pi - \tau v + \max_{x_i \geq 0} \left\{ x_i v - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}$$

where $v \equiv V_t(n)/nY_t$ is normalized per product value and $\omega \equiv w_t/Y_t$ is the labor share and constant in steady state.

Value Function II

- First-order condition of R&D choice gives:

$$x_i = \left(\frac{v}{\eta\zeta\omega}\right)^{\frac{1-\gamma}{\gamma}}. \tag{1}$$

- Or substituting it back:

$$v = \frac{\pi - \zeta\omega x_i^{\frac{1}{1-\gamma}}}{\rho + \tau - x_i}. \tag{2}$$

Value Function III

Proposition Per-product line value of a firm v can be expressed as a sum of production value v_P and R&D "innovation option" value v_R:

$$v = v_P + v_R$$

where

$$\begin{aligned} v_p &= \frac{\pi}{\rho + \tau} \\ v_R &= \frac{1}{(\rho + \tau)} \max_{x_i \geq 0} \left\{ x_i \left(v_R + v_P \right) - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}. \end{aligned}$$

Entry I

- A mass of potential entrants.
- In order to generate 1 unit of arrival, entrants must hire a team of ψ researchers, i.e., production function for entrant R&D is

$$x_e = \frac{S_E}{\psi}.$$

- The free-entry condition equates the value of a new entry $V_t(1)$ to the cost of innovation ψw_t such that

$$v = \omega\psi.$$

- Thus, together with (1) and (2) :

$$x_e = \frac{\pi}{\omega\psi} - (1-\gamma)\left(\frac{(1-\gamma)\,\psi}{\zeta}\right)^{\frac{1-\gamma}{\gamma}} - \rho \quad \text{and } x_i = \left(\frac{(1-\gamma)\,\psi}{\zeta}\right)^{\frac{1-\gamma}{\gamma}}.$$

Labor Market Clearing I

- Production workers

$$L_P = \frac{Y_t}{A_j p_j} = \frac{1}{(1+\lambda)\,\omega}$$

- Incumbent R&D workers

$$S_I = \zeta \left(\frac{(1-\gamma)\,\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}}$$

- Entrant R&D workers

$$S_E = \frac{\pi}{\omega} - \zeta \left(\frac{(1-\gamma)\,\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi\rho$$

Labor Market Clearing II

- Therefore labor market clearing determines the normalized wage rate

$$
\begin{aligned}
L &= \frac{1}{(1+\lambda)\omega} + \zeta \left(\frac{(1-\gamma)\,\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} \\
&\quad + \frac{\pi}{\omega} - \zeta \left(\frac{(1-\gamma)\,\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi\rho \\
&\Longrightarrow \\
\omega &= \frac{1}{L+\rho\psi}
\end{aligned}
$$

Equilibrium Growth I

- Recall the final good production function

$$\begin{aligned}
\ln Y_t &= \int_0^1 \ln y_{jt} dj \\
&= \int_0^1 \ln A_{jt} l_{jt} dj \\
&= \ln \frac{Y_t}{(1+\lambda) w_t} + \int_0^1 \ln A_{jt} dj \\
&= \ln \frac{L + \rho\psi}{1+\lambda} + \int_0^1 \ln A_{jt} dj
\end{aligned}$$

Equilibrium Growth II

- Define

$$
\begin{aligned}
Q_t &\equiv \exp\left(\int_0^1 \ln A_{jt}\,dj\right) \\
&\implies \\
\ln Q_t &\equiv \int_0^1 \ln A_{jt}\,dj
\end{aligned}
$$

- Thus

$$
g = \frac{\dot{C}_t}{C_t} = \frac{\dot{Q}_t}{Q_t}
$$

Equilibrium Growth III

- Moreover

$$\begin{aligned} \ln Q_{t+\Delta t} &= \int_0^1 \left[\tau \Delta t \ln(1+\lambda) A_{jt} + (1-\tau\Delta t) \ln A_{jt}\right] dj + o(\Delta t) \\ &= \tau \Delta t \ln(1+\lambda) + \ln Q_t + o(\Delta t) \\ &\Longleftrightarrow \\ g &= \tau \ln(1+\lambda) \end{aligned}$$

- Hence

$$g = \left[\left(\frac{\lambda}{1+\lambda}\right)\frac{L}{\psi} + \frac{1-\gamma}{\gamma}\left(\frac{(1-\gamma)\,\psi}{\zeta}\right)^{\frac{1-\gamma}{\gamma}} - \frac{\rho}{1+\lambda}\right]\ln(1+\lambda)$$

Moments

- Consider a firm with n product lines. The "approximate" growth rate is

$$
\begin{aligned}
n_{t+\Delta t} &= n_t + nx_i\Delta t - n\tau\Delta t \\
&\Longrightarrow \\
\frac{\dot{n}_t}{n_t} &= x_i - \tau
\end{aligned}
$$

- R&D spending/intensity

$$
\frac{R\&D}{Sales} = \frac{\zeta w n x_i^{\frac{1}{1-\gamma}}}{n} = \zeta w x_i^{\frac{1}{1-\gamma}}
$$

- Both of these are independent of firm size (consistent with "Gibrat's law").

Firm Size Distribution

- Firm size distribution: fraction of firms with n leading-edge products, μ_n, given by:

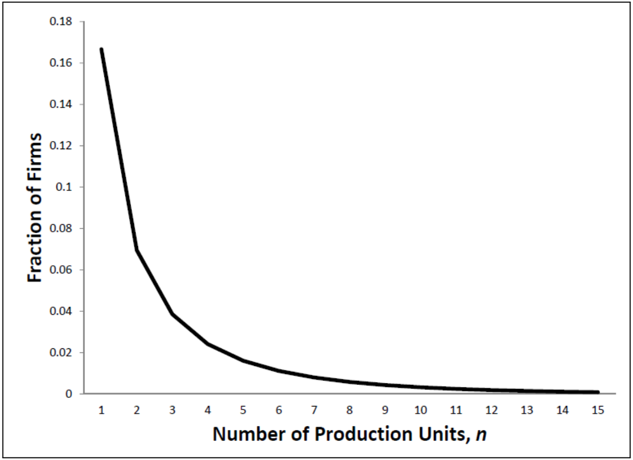
$$
\begin{array}{lrcl}
 & \underline{\text{Outflow}} & & \underline{\text{Inflow}} \\
\text{entry\&exit:} & \mu_1 \tau & = & x_e \\
\text{1-product:} & (x_i + \tau)\,\mu_1 & = & \mu_2 2\tau + x_e \\
n\text{-product:} & (x_i + \tau)\, n\mu_n & = & \mu_{n+1}\,(n+1)\,\tau + \mu_{n-1}\,(n-1)\,x_i
\end{array}
$$

- This implies the following simple firm size distribution:

$$
\begin{array}{rcl}
\mu_1 & = & x_e / \tau \\
\mu_2 & = & \dfrac{x_e}{2\tau^2} x_i \\
\mu_3 & = & \dfrac{x_e x_i}{3\tau^3} \\
\dots & = & \dots \\
\mu_n & = & \dfrac{x_e x_i}{n\tau^n}
\end{array}
$$

Firm Size Distribution

Figure 4: Firm Size Distribution

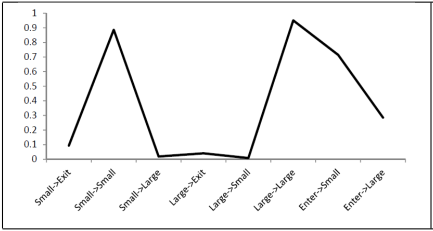


What's Missing?

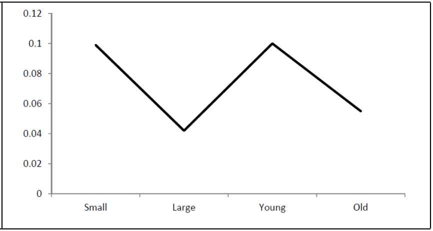
- A nice and tractable model, but:
  - *no reallocation* (all firms that previous in equilibrium are equally good at using all factors of production);
  - *no endogenous exit* of less productive firms;
  - limited heterogeneity (see next slide).
- All of these together imply very little room for endogenous selection which could be impacted by policy.
- We now consider a model that extended this framework to introduce these features.

Why Heterogeneity Matters

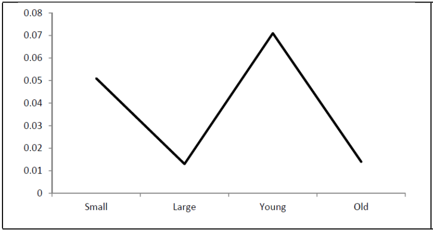
1A: Transition Rates



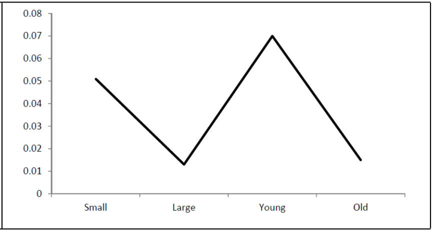
1B: R&D Intensity



1C: Sales Growth



1D: Employment Growth



Baseline Model: Preferences

- Simplified model (abstracting from heterogeneity and non-R&D growth).
- Infinite-horizon economy in continuous time.
- Representative household:

$$U = \int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

- Inelastic labor supply, no occupational choice:
    - Unskilled for production: measure 1, earns w^u
    - Skilled for R&D: measure L, earns w^s.
- Hence the budget constraint is

$$C(t) + \dot{A}(t) \leq w^u(t) + w^s(t) \cdot L + r(t) \cdot A(t)$$

- Closed economy and no investment, resource constraint:

$$Y(t) = C(t).$$

Final Good Technology

- Unique final good Y :

$$Y = \left(\int_{\mathcal{N}} y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

- $\mathcal{N} \subset [0, 1]$ is the set of *active* product lines.
- The measure of $\mathcal{N}$ is less than 1 due to
  1. exogenous destructive shock
  2. obsolescence

Intermediate Good Technology

- As usual, each intermediate good is produced by a **monopolist**:

$$y_{j,f} = q_{j,f} l_{j,f},$$

$q_{j,f}$: worker productivity, $l_{j,f}$: number of workers.

- Marginal cost :

$$MC_{j,f} = \frac{w^u}{q_{j,f}}.$$

- Fixed cost of production, ϕ in terms of skilled labor.
- Total cost

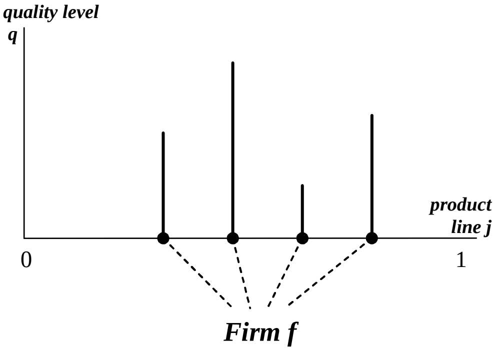
$$TC_{j,f}(y_{j,f}) = w^s \phi + w^u \frac{y_{j,f}}{q_{j,f}}.$$

Definition of a Firm

- A firm is defined as a collection of product qualities as in Klette-Kortum

$$\text{Firm } f = \mathcal{Q}_f \equiv \left\{ q_f^1, q_f^2, ..., q_f^n \right\}.$$

$n_f \equiv \left| \mathcal{Q}_f \right|$: is the number of product lines of firm f.



Relative Quality

- Define *aggregate quality* as

$$Q \equiv \left(\int_{\mathcal{N}} q_j^{\varepsilon - 1} dj \right)^{\frac{1}{\varepsilon - 1}}.$$

- In equilibrium,

$$Y = C = Q,$$

- Define *relative quality*:

$$\hat{q}_j \equiv \frac{q_j}{w^u}.$$

R&D and Innovation

- Innovations follow a "controlled" Poisson Process

$$X_f = n_f^{\gamma} h_f^{1-\gamma}.$$

X_f : flow rate of innovation
n_f : number of product lines.
h_f : number of researchers (here taken to be regular workers allocated to research).

- This can be rewritten as *per product* innovation at the rate

$$x_f \equiv \frac{X_f}{n_f} = \left(\frac{h_f}{n_f}\right)^{1-\gamma}.$$

- Cost of R&D as a function of per product innovation rate x_f:

$$w^s G(x_f) \equiv w^s n_f x_f^{\frac{1}{1-\gamma}}.$$

Innovation by Existing Firms

- Innovations are again *undirected* across product lines.
- Upon an innovation:
  1. firm f acquires another product line j
  2. if technology in j is active:
$$q(j, t + \Delta t) = (1 + \lambda) q(j, t).$$
  3. if technology in j is not active, i.e., $j \notin \mathcal{N}$, a new technology is drawn from the steady-state distribution of relative quality, $F(\hat{q})$.

Entry and Exit

- A set of potential entrants invest in R&D.
- Exit happens in three ways:
  1. **Creative destruction**. Firm f will lose each of its products at the rate $\tau > 0$ which will be determined endogenously in the economy.
  2. **Obsolescence**. Relative quality decreases due to the increase in the wage rate, at some point leading to exit.
  3. **Exogenous destructive shock** at the rate φ.

Static Equilibrium

- Drop the time subscripts.
- Isoelastic demands imply the following monopoly price and quantity

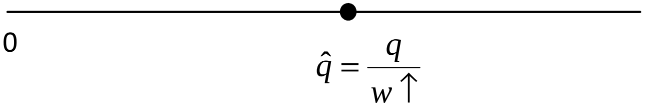
$$p_{j,f}^* = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{1}{\hat{q}_j} \text{ and } c_j^* = \left(\frac{\varepsilon - 1}{\varepsilon} \hat{q}_j\right)^{\varepsilon} Y$$

- Gross equilibrium (before fixed costs) profits from a product with relative quality $\hat{q}_j$ are:

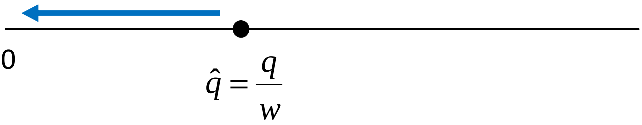
$$\pi\left(\hat{q}_{j,f}\right) = \hat{q}_j^{\varepsilon-1} \left(\frac{(\varepsilon - 1)^{\varepsilon-1}}{\varepsilon^{\varepsilon}}\right) Y.$$

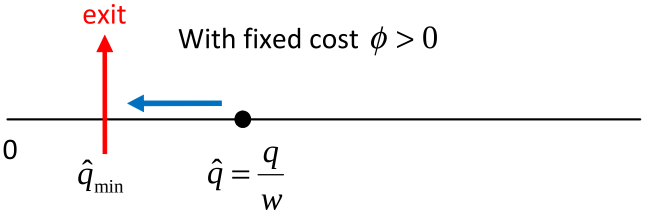

0
$\hat{q} = \frac{q}{w}$

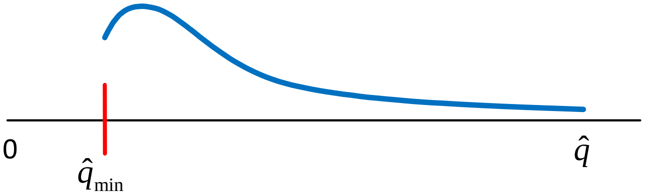

0
$\hat{q} = \frac{q}{w \uparrow}$


0
$\hat{q} = \frac{q}{w\uparrow}$

Without a fixed cost




exit
With fixed cost $\phi > 0$
0
$\hat{q}_{\min}$
$\hat{q} = \frac{q}{w}$


0
$\hat{q}_{\min}$
$\hat{q}$

Dynamic Equilibrium

- In equilibrium,

$$Y = C = Q$$

and

$$w^u = \frac{\varepsilon - 1}{\varepsilon} Q.$$

- Let us also define *normalized values* as

$$\tilde{V} \equiv \frac{V}{Y}, \ \tilde{\pi}\left(\hat{q}_{j,f}\right) = \frac{\pi\left(\hat{q}_{j,f}\right)}{Y}, \ \tilde{w}^u \equiv \frac{w^u}{Y} \text{ and } \tilde{w}^s \equiv \frac{w^s}{Y}.$$

Dynamic Equilibrium (continued)

$$r^* \tilde{V}\left(\hat{\mathcal{Q}}_f\right) = \left[\begin{array}{c} \sum_{\hat{q}_{j,f} \in \hat{\mathcal{Q}}_f} \left\{ \begin{array}{c} \tilde{\pi}\left(\hat{q}_{jf}\right) - \tilde{w}^s \phi_j \\ + \ \dot{\tilde{V}} \\ +\tau\left[\tilde{V}\left(\hat{\mathcal{Q}}_f \backslash \left\{\hat{q}_{jf}\right\}\right) - \tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right] \end{array} \right\} + \\ \left|\hat{\mathcal{Q}}_f\right| \max_{x_f} \left\{ \begin{array}{c} -\tilde{w} G\left(x_f\right) \\ +x_f\left[\mathbb{E}_{\hat{q}} \tilde{V}\left(\hat{\mathcal{Q}}_f \cup (1+\lambda)\, \hat{q}_{j',f}\right) - \tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right] \end{array} \right\} \\ +\varphi\left[0 - \tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right] \end{array} \right.$$

τ: creative destruction rate in the economy.

Dynamic Equilibrium (continued)

$$r^{*}\tilde{V}\left(\hat{\mathcal{Q}}_f\right)=\left[\begin{array}{c}\sum_{\hat{q}_{j,f}\in\hat{\mathcal{Q}}_f}\left\{\begin{array}{c}\tilde{\pi}\left(\hat{q}_{jf}\right)-\tilde{w}^{s}\phi_j\\+\frac{\partial\tilde{V}}{\partial\hat{q}_{jf}}\frac{\partial\hat{q}_{jf}}{\partial w^{u}(t)}\frac{\partial w^{u}(t)}{\partial t}\\+\tau\left[\tilde{V}\left(\hat{\mathcal{Q}}_f\setminus\left\{\hat{q}_{jf}\right\}\right)-\tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right]\end{array}\right\}+\\\left|\hat{\mathcal{Q}}_f\right|\max_{x_f}\left\{\begin{array}{c}-\tilde{w}G\left(x_f\right)\\+x_f\left[\mathbb{E}_{\hat{q}}\tilde{V}\left(\hat{\mathcal{Q}}_f\cup(1+\lambda)\,\hat{q}_{j',f}\right)-\tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right]\end{array}\right\}\\+\varphi\left[0-\tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right]\end{array}\right.$$

τ: creative destruction rate in the economy.

Franchise and R&D Option Values

Lemma *The normalized value can be written as the sum of franchise values:*

$$\tilde{V}\left(\hat{\mathcal{Q}}_f\right) = \sum_{\hat{q}\in\hat{\mathcal{Q}}_f} \Upsilon\left(\hat{q}\right),$$

where the franchise value of a product of relative quality $\hat{q}$ is the solution to the differential equation (iff $\hat{q} \geq \hat{q}_{\min}$):

$$r\Upsilon\left(\hat{q}\right) - \frac{\partial \Upsilon\left(\hat{q}\right)}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u\left(t\right)} \frac{\partial w^u\left(t\right)}{\partial t} = \tilde{\pi}\left(\hat{q}\right) - \tilde{w}^u\phi + \Omega - \left(\tau + \varphi\right)\Upsilon\left(\hat{q}\right),$$

where Ω is the R&D option value of holding a product line,

$$\Omega \equiv \max_{x_f \geq 0}\left\{-\tilde{w}^s G\left(x_f\right) + x_f\left(\mathbb{E}_{\hat{q}}\tilde{V}\left(\hat{\mathcal{Q}}_f \cup \left(1+\lambda\right)\hat{q}_{j'f}\right) - \tilde{V}\left(\hat{\mathcal{Q}}_f\right)\right)\right\},$$

Moreover, exit follows a cut-off rule: $\hat{q}_{\min} \equiv \pi^{-1}\left(\tilde{w}^s\phi - \Omega\right).$

Equilibrium Value Functions and R&D

Proposition

Equilibrium normalized value functions are:

$$\begin{aligned} Y(\hat{q}) &= \frac{\tilde{\pi}(\hat{q})}{r+\tau+\varphi+g(\varepsilon-1)}\left[1-\left(\frac{\hat{q}_{\min}}{\hat{q}}\right)^{\frac{r+\tau+\varphi+g(\varepsilon-1)}{g}}\right] \\ &\quad + \frac{\Omega-\tilde{w}^s\phi}{r+\tau+\varphi}\left[1-\left(\frac{\hat{q}_{\min}}{\hat{q}}\right)^{\frac{r+\tau+\varphi}{g}}\right], \end{aligned}$$

and equilibrium R&D is

$$x^*(\hat{q}) = x^* = \left[\frac{(1-\gamma)\,\mathbb{E}_{\hat{q}} Y(\hat{q})}{\tilde{w}^s}\right]^{\frac{1-\gamma}{\gamma}}.$$

Entry

- Entry by outsiders can now be determined by the free entry condition:

$$\max_{x^{entry}\geq 0}\left\{-w^s\phi + x^{entry}\mathbb{E}V^{entry}(\hat{q},\theta) - w^s G\left(x^{entry},\theta^E\right)\right\} = 0$$

  where $G\left(x^{entry},\theta^E\right)$, as specified above, gives a number of skilled workers necessary for a firm to achieve an innovation rate of x^{entry} (with productivity parameter θ^E).

- $X^{entry} \equiv mx^{entry}$ is the total entry rate where
  - m is the equilibrium measure of entrants, and
  - x^{entry} innvation rate per entrant.

Labor Market Clearing

- Unskilled labor market clearing:

$$1 = \int_{\mathcal{N}(t)} l_j \left(w^u\right) dj.$$

- Skilled labor market clearing

$$L^s = \int_{\mathcal{N}(t)} \left[\phi + h\left(w^s\right)\right] dj + m \left[\phi + G\left(x^{entry}, \theta^E\right)\right].$$

Transition Equations

- Finally, we need to keep track of the distribution of relative quality $\longrightarrow$ stationary equilibrium distribution of relative quality F.
- This can be done by writing transition equations describing the density of relative quality.
- These are more complicated than in Klette-Kortum because there is no strict Gibraltar's law anymore.

Preferences and Technology in the General Model

- Same preferences.
- Introduce managerial quality affecting the rate of innovation of each firm.
- Some firms start as more innovative than others, over time some of them lose their innovativeness.
  - Young firms are potentially more innovative but also have a higher rate of failure.
- Introduce non-R&D growth (so as not to potentially exaggerate the role of R&D and capture potential advantages of incumbents).

Definition of a Firm

- A firm is again defined as a pair of technology set and "management quality" θ:

$$\text{Firm } f \equiv \left(\mathcal{Q}_f, \theta_f\right),$$

where

$$\mathcal{Q}_f \equiv \left\{q_f^1, q_f^2, ..., q_f^n\right\}.$$

- $n_f \equiv \left|\mathcal{Q}_f\right|$: is the number of product lines owned by firm f.

R&D and Innovation

- Innovations follow a controlled Poisson Process.
- Flow rate of innovation for leader and follower given by

$$X_f = (n_f \theta_f)^\gamma \, h_f^{1-\gamma}.$$

n_f : number of product lines.
θ_f : firm type (management quality).
h_f : number of researchers.

Innovation Realizations

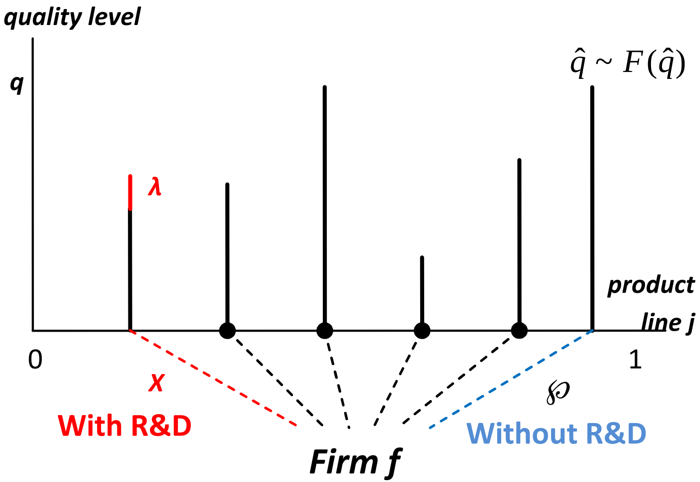
With R&D

- Innovations are *undirected* within the industry.
- After a successful innovation, innovation is realized in a random product line j. Then:
  1. firm f acquires product line j
  2. technology in line j improves

$$q\left(j, t + \Delta t\right) = \left(1 + \lambda\right) q\left(j, t\right).$$

Without R&D

- Firms receive a product line for free at the rate ϱ .


quality level
q
$\hat{q} \sim F(\hat{q})$
λ
product
line j
0
1
X
℘
With R&D
Firm f
Without R&D

Entry and Exit

- There is a measure of potential entrants.
- Successful innovators enter the market.
- At the time of initial entry, each firm draws a management quality θ :

$$\begin{aligned} \Pr\left(\theta = \theta^{H}\right) &= \alpha \\ \Pr\left(\theta = \theta^{L}\right) &= 1 - \alpha, \end{aligned}$$

where $\alpha \in (0, 1)$ and $\theta^{H} > \theta^{L} > 0$.

- Exit happens in three ways as in the baseline model.

Maturity Shock

- Over time, high-type firms become low-type at the rate $\nu > 0$:

$$\theta^H \longrightarrow \theta^L.$$

- Convenient to capture the possibility of once-innovative firms now being inefficient (and the use of skilled labor).

Equilibrium

- Equilibrium definition and characterization similar to before (with more involved value functions and stationary transition equations).

Data: LBD, Census of Manufacturing and NSF R&D Data

- Sample from combined databases from 1987 to 1997.
- Longitudinal Business Database (LBD)
  - Annual business registry of the US from 1976 onwards.
  - Universe of establishments, so entry/exit can be modeled.
- Census of Manufacturers (CM)
  - Detailed data on inputs and outputs every five years.
- NSF R&D Survey.
  - Firm-level survey of R&D expenditure, scientists, etc.
  - Surveys with certainty firms conducting $1m or more of R&D.
- USPTO patent data matched to CM.
- Focus on "continuously innovative firms":
  - I.e., either R&D expenditures or patenting in the five-year window surrounding observation conditional on existence.

Data Features and Estimation

- 17,055 observations from 9835 firms.
- Accounts for 98% of industrial R&D.
- Relative to the universal CM, our sample contains over 40% of employment and 65% of sales.
- "Important" small firms also included:
  - of the new entrants or very small firms that later grew to have more than 10,000 employees or more than $1 billion of sales in 1997, we capture, respectively, 94% at 80%.
- We use Simulated Method of Moments on this dataset to estimate the paremeters the parameters of the model.

Creating Moments from the Data

- We target 21 moments to estimate 12 parameters.
- Some of the moments are:
  - Firm entry/exit into/from the economy by age and size.
  - Firm size distribution.
  - Firm growth by age and size.
  - R&D intensity (R&D/Sales) by age and size.
  - Share of entrant firms.

RESULTS

Table 1. Parameter Estimates

#	**Parameter**	**Description**	**Value**
1.	ε	CES	1.701
2.	ϕ	Fixed cost of operation	0.032
3.	L^S	Measure of high-skilled workers	0.078
4.	θ^H	Innovative capacity of high-type firms	0.216
5.	θ^L	Innovative capacity of low-type firms	0.070
6.	θ^E	Innovative capacity of entrants	0.202
7.	α	Probability of being high-type entrant	0.428
8.	ν	Transition rate from high-type to low-type	0.095
9.	λ	Innovation step size	0.148
10.	γ	Innovation elasticity wrt knowledge stock	0.637
11.	φ	Exogenous destruction rate	0.016
12.	ϱ	Non-R&D innovation arrival rate	0.012

TABLE 2. MOMENT MATCHING

#	Moments	model	data		#	Moments	model	data
1.	Firm Exit (small)	0.086	0.093		12.	Sales Gr. (small)	0.115	0.051
2.	Firm Exit (large)	0.060	0.041		13.	Sales Gr. (large)	-0.004	0.013
3.	Firm Exit (young)	0.078	0.102		14.	Sales Gr. (young)	0.070	0.071
4.	Firm Exit (old)	0.068	0.050		15.	Sales Gr. (old)	0.030	0.014
5.	Trans. large-small	0.024	0.008		16.	R&D/Sales (small)	0.097	0.099
6.	Trans. small-large	0.019	0.019		17.	R&D/Sales (large)	0.047	0.042
7.	Prob. small	0.539	0.715		18.	R&D/Sales (young)	0.083	0.100
8.	Emp. Gr. (small)	0.063	0.051		19.	R&D/Sales (old)	0.061	0.055
9.	Emp. Gr. (large)	-0.007	0.013		20.	5-year Ent. Share	0.363	0.393
10.	Emp. Gr. (young)	0.040	0.070		21.	Aggregate growth	0.022	0.022
11.	Emp. Gr. (old)	0.010	0.015					


2A: Transition Rates
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Small->Exit
Small->Small
Small->Large
Large->Exit
Large->Small
Large->Large
Enter->Small
Enter->Large
Data
Model
2B: R&D Intensity
0.12
0.1
0.08
0.06
0.04
0.02
0
Small
Large
Young
Old
Data
Model
2C: Sales Growth
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
Small
Large
Young
Old
Data
Model
2D: Employment Growth
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
Small
Large
Young
Old
Data
Model

Non-Targeted Moments

Table 3: Non-targeted Moments

Moments	**Model**	**Data**
Corr(exit prob, R&D intensity)	0.04	0.05
Exit prob of low-R&D-intensive firms	0.36	0.32
Exit prob of high-R&D-intensive firms	0.37	0.34
Corr(R&D growth, emp growth)	0.48	0.19
Share firm growth due to R&D	0.77	0.73
Ratio of top 7.2% to bottom 92.8% income	13.4	9.3

Comparison to Micro Estimates

- Estimates of the elasticity of patents (innovation) to R&D expenditures (e.g., Griliches, 1990):
  - [0.3, 0.6]
  - This corresponds to $1-\gamma$, so a range of $[0.4, 0.7]$ for γ.
  - Our estimate is in the middle of this range.
- Use IV estimates from R&D tax credits.
  - US spending about $2 billion with large cross-state over-time variation.
  - Literature estimates:

$$\log(R\&D_{i,t}) = \alpha_i + \beta_t + \gamma \log(R\&D\_Cost\_of\_Capital_{i,t})$$

  - Bloom, Griffith and Van Reenen (2002) find -1.088 (0.024) on a cross-country panel. Similar estimates from Hall (1993), Baily and Lawrence (1995) and Mumuneas and Nadiri (1996).
  - In the model, $\ln R\&D = \frac{\gamma-1}{\gamma} \ln\left(c_{R\&D}\right)$ +constant.
  - So approximately $\gamma \approx 0.5$, close to our estimate of $\gamma = 0.637$.

Baseline Results

Table 4. Baseline Model

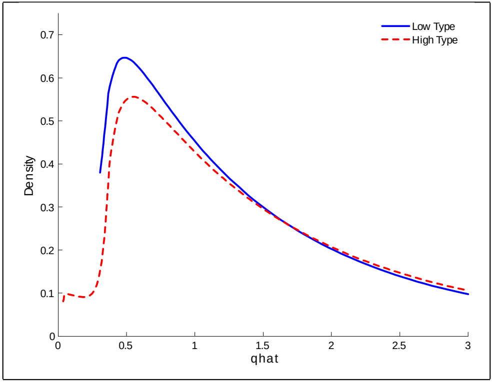
x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

Note: All numbers except wage ratio and welfare are in percentage terms.

g : growth rate
x^{out} : entry rate
x^{low} : low-type innv rate
x^{high} : high-type innv rate
Φ^{low} : fraction of low p. lines
Φ^{high} : fraction of high p. lines
$\hat{q}_{l,\min}$: low-type cutoff quality
$\hat{q}_{h,\min}$: high-type cutoff quality
wel : welfare in cons equiv.

Relative Quality Distribution

Figure 3



- Explains why very little obsolescence of high-type products.

Policy Analysis: Subsidy to Incumbent R&D

Table 4. Baseline Model

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Use 1% and 5% of GDP, resp., to subsidize incumbents R&D:

Table 5A. Incumbent R&D Subsidy ($s_i = 15\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	3.05	10.56	68.1	70.74	24.96	13.40	0.00	2.23	99.86

Table 5B. Incumbent R&D Subsidy ($s_i = 39\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	3.61	13.04	49.8	69.58	25.97	13.15	0.00	2.16	98.48

Policy Analysis: Subsidy to the Operation of Incumbents

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Use 1% of GDP to subsidize operation costs of incumbents:

TABLE 6. OPERATION SUBSIDY ($s_o = 6\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.59	73.7	71.30	24.52	11.74	0.00	2.22	99.82

- Now an important negative selection effect.

Policy Analysis: Entry Subsidy and Selection

Table 4. Baseline Model

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Use 1% of GDP to subsidize entry:

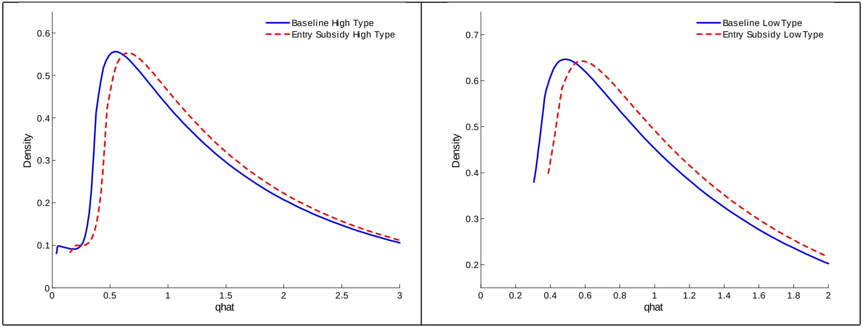
Table 7. Entry Subsidy ($s_e = 5\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.73	9.30	75.3	71.16	24.41	15.91	0.00	2.26	100.15

Understanding the Selection Effect

Figure 4. Policy effect on Productivity Distributions

A. high type B. Low Type



Social Planner's Allocation

Table 4. Baseline Model

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- What would the social planner do (taking equilibrium markups as given)?

Table 8. Social Planner

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.55	10.47	80.9	54.06	27.76	118.6	1.02	3.80	106.5

Optimal Policy (I)

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Optimal mix of incumbent R&D subsidy, operation subsidy and entry subsidy:

TABLE 9. OPTIMAL POLICY ANALYSIS AND WELFARE

INCUMBENT & ENTRY POLICIES ($s_i = 17\%, s_o = -246\%, s_e = 6\%$)									
x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	3.04	10.21	75.5	62.19	25.53	96.28	55.88	3.12	104.6

Optimal Policy (II)

Table 4. Baseline Model

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Optimal mix of incumbent R&D subsidy and operation subsidy:

Table 9. Optimal Policy Analysis and Welfare

Incumbent Policies ($s_i = 12\%, s_o = -264\%$)									
x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
8.46	3.04	10.21	75.3	62.31	25.53	91.38	54.85	3.11	104.6

Summing up

- Industrial policy directed at incumbents has negative effects on innovation and productivity growth—though small.
- Subsidy to entrants has small positive effects.
- But not because R&D incentives are right in the laissez-faire equilibrium.
- The social planner can greatly improve over the equilibrium.
- Similar gains can also be achieved by using taxes on the continued operation of incumbents (plus small R&D subsidies).
    - This is useful for encouraging the exit of inefficient incumbents who are trapping skilled labor that can be more productively used by entrants and high-type incumbents.

Robustness

- These results are qualitatively and in fact quantitatively quite robust.
- The remain largely unchanged if:
  - $\gamma = 0.5$.
  - $\varrho = 0$.
  - entry margin much less elastic.