14.461: Technological Change, Lectures 12 and 13 Input-Output Linkages: Implications for Productivity and Volatility

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Motivation

- Most sectors use the output of other sectors in the economy as intermediate goods.
- This introduces interlinkages among sectors.
- Important for understanding several, potentially inter-related phenomena:
  - Inefficiency in one sector will have implications for productivity in others.
  - Shocks to a sector can have aggregate volatility implications.
  - Changes in sectoral composition can affect fundamental volatility in the economy.

Input-Output Linkages and Sectoral Misallocation

- Based on Jones (2010), consider the following (static) multi-sector model:
- Each of the N sectors produces with the following Cobb-Douglas technology:

$$Q_i = A_i \left(K_i^{\alpha_i} L_i^{1-\alpha_i} \right)^{1-w_i} d_{i1}^{w_{i1}} d_{i2}^{w_{i2}} \cdot \ldots \cdot d_{iN}^{w_{iN}} \tag{1}$$

  where:
  - $A_i \equiv A\eta_i$,
  - K_i and L_i are the quantities of physical and human capital used in sector i,
  - d_{ij}'s are intermediates (output of other sectors).
- Moreover, $w_i \equiv \sum_{j=1}^{N} w_{ij}$ and and $0 < \alpha_i < 1$, so the production function features constant returns to scale.

Network and Graph Interpretation

- This economy can be interpreted/represented as a network of interlinked sectors.
- Equivalently, it can be interpreted/represented as a directed weighted graph.
- In both cases, the key object is the matrix W, the matrix of w_{ij}'s.
- Its row sums are the in-degrees (how dependent is sector i on inputs from other sectors).
- Its column sums are the out-degrees (how important is sector i as input supplier to other sectors).

Sectoral Misallocation

- Each domestically produced good can be used for final consumption, c_j, or can be used as an intermediate good:

$$c_j + \sum_{i=1}^{N} d_{ij} = Q_j, \quad j = 1, \ldots, N. \tag{2}$$

- Suppose that there is a single final good, combining the output of different sectors is Cobb-Douglas:

$$Y = c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N}, \tag{3}$$

where $\sum_{i=1}^{N} \beta_i = 1$.

- This aggregate final good can itself be used in one of two ways, as consumption or exported to the rest of the world:

$$C + X = Y. \tag{4}$$

Sectoral Misallocation (continued)

- Finally, factors are supplied inelastically:

$$\sum_{i=1}^{N} K_i = K, \tag{5}$$

$$\sum_{i=1}^{N} L_i = L. \tag{6}$$

Equilibrium with Misallocation

- Why will there be "misallocation"?
- Jones assumes "sector specific wedges" causing sector-specific reductions in revenue in proportion to τ_i.
- Then equilibrium is defined as a competitive equilibrium given these distortions.

Definition of Equilibrium

A *competitive equilibrium with misallocation* in this environment is a collection of quantities C, Y, X, Q_i, K_i, L_i, c_i, d_{ij} and prices p_j, h, and r for $i = 1, \ldots, N$ and $j = 1, \ldots, N$ such that

1. $\{c_i\}$ solves the profit maximization problem of a representative firm in the perfectly competitive final goods market: $\max_{\{c_i\}} c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N} - \sum_{i=1}^N p_i c_i$ taking $\{p_i\}$ as given.
2. d_{ij}, K_i, L_i solve the profit maximization problem of a representative firm in sector i for $i = 1, \ldots, N$, i.e., maximize

$$(1 - \tau_i) p_i A_i \left(K_i^{\alpha_i} L_i^{1-\alpha_i} \right)^{1-w_i} d_{i1}^{w_{i1}} d_{i2}^{w_{i2}} \cdot \ldots \cdot d_{iN}^{w_{iN}} - \sum_{j=1}^{N} p_j d_{ij} - rK_i - hL_i.$$

3. Markets clear, i.e., $\sum_{i=1}^N K_i = K$, $\sum_{i=1}^N L_i = L$, and $c_j + \sum_{i=1}^N d_{ij} = Q_j$.

Equilibrium

- In the competitive equilibrium with misallocation, the solution for total production of the aggregate final good is

$$Y = A^{\tilde{\mu}} K^{\tilde{\alpha}} L^{1-\tilde{\alpha}} \epsilon,$$

  where
- $\mu' \equiv \beta' (I - W)^{-1}$ where β is the vector of β_i's and W is the matrix of w_{ij}
- $\tilde{\mu} \equiv \mu' \mathbf{1}$
- $\tilde{\alpha} \equiv \mu' (1 - w_i)$
- $\log \epsilon \equiv \omega + \mu' \bar{\eta}$ where $\bar{\eta}$ is the vector of $\log(\eta_i (1 - \tau_i))$'s and ω is a constant depending on the other parameters.

Discussion

- Aggregate TFP, ϵ, depends on both sectoral TFPs and the underlying distortions, which is intuitive in light of the input-output linkages.
- Secondly, there is a multiplier determining the impact of distortions on aggregate output. In particular, the Metro multipliers is

$$\mu' \equiv \beta' (I - W)^{-1}.$$

- Here the matrix $(I - B)^{-1}$ is the Leontief inverse.
  - The typical element ℓ_{ij} of this matrix gives us the following information: a 1% increase in productivity in sector j raises output in sector i by ℓ_{ij}% — because of the indirect effects working to input-output linkages.

Discussion (continued)

- Multiplying this Leontief inverse matrix by the vector of value-added weights in β essentially amounts to adding up the effects of sector j on all the other sectors in the economy, weighting by their shares in aggregate value-added.
  - So the elements of this multiplier matrix show how a change in productivity in sector i affects overall value-added in the economy.
- Moreover, the elasticity of final output with respect to aggregate TFP is $\tilde{\mu} \equiv \mu' \mathbf{1}$.
  - Intuitively, this is obtained by adding up all the multipliers in μ because an increase in aggregate TFP affects all sectors through input-output linkages.

Further Intuition

- Consider the following simplification: $w_i \equiv \sum_{j=1}^{N} w_{ij} = \hat{w}$ for all i.
- Then
$$\frac{\partial \log Y}{\partial \log A} = \mu' \mathbf{1} = \beta' \left(I - W \right)^{-1} = \frac{1}{1 - \hat{w}}.$$
- This special case shows that the "sparseness" of the input-output matrix W is not important.
    - All that matters are the "out-degrees".
- Secondly, the common out-degree across sectors is all that matters for the multiplier with respect to aggregate TFP shock A.
- These results are also present in the general model — though naturally in a more complicated form.
- This result suggests a large amount of amplification of distortions.
    - But what happens when we look at "appropriately measured" TFP?

Distortions in the Symmetric Case

- Now consider the following special case:
  - $w_{ij} = \hat{w}/N$, $\beta_i = 1/N$, and $\alpha_i = \alpha$
  - $\log(1-\tau_i) \sim N(\theta, v^2)$ and let $1 - \bar{\tau} \equiv e^{\theta + \frac{1}{2}v^2}$ (which is the average distortion in this case).
- Then as $N \to \infty$, $\log C$ almost surely converges to

$$\text{Constant} + \frac{\hat{w}}{1-\hat{w}}(1-\bar{\tau}) + \log\left(1 - \hat{w}(1-\bar{\tau})\right) - \frac{1}{2}\frac{1}{1-\hat{w}}v^2.$$

- Therefore, what matters in this case is simply the dispersion of distortions.
- This is parallel to the dispersion of firm-level misallocations determining sectoral productivities in Hsieh and Klenow's accounting exercise.

Question

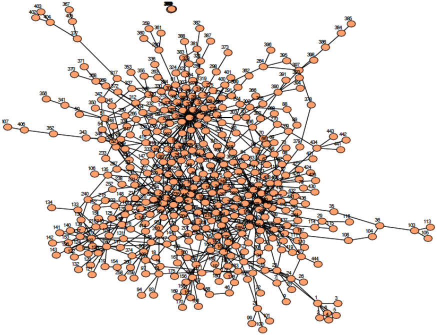
- Similar issues could be important in thinking about the origins of aggregate fluctuations.
- Aggregate shocks to productivity or demand (except for monetary policy shocks) seem less than fully compelling.
- Could they be the result of more microeconomic shocks, hitting disaggregated sectors?
- Conventional wisdom: No
    - "Diversification argument": firm-level or disaggregated sectoral shocks washed up at the rate $\sqrt{n}$ and for large n, they would be trivial.
- But intersectoral linkages introduce "network effects"
    - Shocks to some sectors may propagate to the rest of the economy and may even create "cascade effects".

Model

- Use the same structure as above, but with unrestricted interactions among sectors and for a sequence of economies.
- Results for rates of convergence of aggregate output to its mean.

U.S. Input-output Structure

- Which one does the U.S. input-output structure resemble?



Model: Firms

- An economy $\mathcal{E}_n$ consisting of n sectors.
- The output of each sector is used by a subset of sectors as input (intermediate goods) for production.
- Cobb-Douglas production technologies:

$$x_i = z_i^{\alpha} \ell_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_{ij}},$$

where

  - ℓ_i: labor employed by sector i
  - $\alpha \in (0, 1]$: labor share
  - x_{ij} : amount of commodity j used in the production of good i
  - $w_{ij} \geq 0$: input share of sector j in sector i's production.
  - $\epsilon_i = \log(z_i) \sim F_i$: productivity shock to sector i.

Assumptions

Assumption

Constant return to scale: $\sum_{j=1}^{n} w_{ij} = 1$.

Assumption

Given a sequence of economies $\{\mathcal{E}_n\}_{n \in \mathbb{N}}$ *and for any sector* i

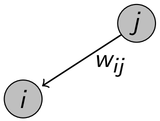
(a) $\mathbb{E}\epsilon_i = 0$, *and*

(b) $\mathrm{var}(\epsilon_i) = \sigma_i^2 \in (\underline{\sigma}^2, \bar{\sigma}^2)$, *where* $0 < \underline{\sigma} < \bar{\sigma}$ *are independent of* n.

Intersectoral Network

$$x_i = z_i^{\alpha} \ell_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_{ij}}$$

- Intersectoral network: weighted, directed graph



- Degree of sector j: share of j's output in the input supply of the economy

$$d_j = \sum_{i=1}^{n} w_{ij}$$

Firms

- Representative firm in sector i solves the problem:

$$\max_{\ell_i, x_i, \{x_{ij}\}_{j \in \mathcal{I}_n}} \quad p_i x_i - h l_i - \sum_{j=1}^{n} p_j x_{ij}$$

$$\text{subject to} \quad x_i = z_i^{\alpha} \ell_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_{ij}}.$$

  - h is the market wage
  - p_i is the market price of good i.

Consumers

- A continuum of identical consumers of mass one.
- Endowed with one unit of labor.
- Preferences:

$$u(c_1, c_2, \ldots, c_n) = A_n \prod_{i=1}^{n} (c_i)^{1/n}.$$

- Representative Consumer's problem:

$$\begin{aligned} \max_{\{c_i\}_{i \in \mathcal{I}_n}} \quad & u(c_1, \ldots, c_n) \\ \text{subject to} \quad & p_1 c_1 + \cdots + p_n c_n = h \end{aligned}$$

Competitive Equilibrium

Definition

In the competitive equilibrium of economy, the prices $(p_1, p_2, \ldots, p_n)$ and wage h are such that

(a) the representative consumer maximizes her utility,

(b) the representative firms in each sector maximize profits,

(c) labor and commodity markets clear.

$$c_i^* + \sum_{j=1}^{n} x_{ji}^* = x_i^* \qquad \forall i \in \mathcal{I}_n$$

$$\sum_{i=1}^{n} \ell_i^* = 1.$$

Competitive Equilibrium (continued)

Proposition

At the equilibrium, aggregate output (log real value added) is a convex combination of log sectoral shocks:

$$\log(\mathrm{GDP}) = v_n' \epsilon$$

where v_n is the influence vector given by

$$v_n \equiv \frac{\alpha}{n} \left[I - (1-\alpha) W_n' \right]^{-1} \mathbf{1}.$$

- v_n is also the sales vector

$$v_{in} = \frac{p_i x_i}{\sum_{j=1}^{n} p_j x_j}$$

- Bonacich centrality vector corresponding to the intersectoral network

Alternative Interpretations

- We could have alternatively consider a reduced-form model

$$\tilde{y} = \tilde{W}_n \tilde{y} + \tilde{\epsilon}.$$

- This could arise, for example, from:
  (a) Models in which ϵ_i's are not productivity shocks, but other shocks to sectoral or firm behavior.
  (b) Models in which units are firms rather than sectors (but then one needs to model "relationship-specific investments" and to some degree endogenize $\tilde{W}_n$).
  (c) Financial models with counterparty relationships between financial institutions. In this case, $w_{ij} > 0$ would correspond to firm i being a counterparty to firm j (i.e., holding some of firm j's debt or other liabilities on its balance sheet).
  (d) Models of "strategic complementarities".

Aggregate Volatility

- Aggregate output

$$\log(\mathrm{GDP}) = v_n' \epsilon$$

- Aggregate volatility

$$\sigma_{agg} = \sqrt{\sum_{i=1}^{n} v_{in}^2 \sigma_{in}^2}$$

- Rate of decay

$$\sigma_{agg} \sim \|v_n\|_2$$

- Rest of the talk:
  - How is $\|v_n\|_2$ related to the structural properties of the intersectoral network?

First-Order Interconnections

- Relate $\|v_n\|_2$ to the empirical degree distribution of the intersectoral network

Definition

Given an economy $\mathcal{E}_n$ with degrees, the coefficient of variation is

$$\mathrm{CV}_n \equiv \frac{1}{\bar{d}} \left[\frac{1}{n-1} \sum_{i=1}^{n} (d_i - \bar{d}) \right]^{1/2}$$

where $\bar{d} \equiv \frac{1}{n} \sum_{i=1}^{n} d_i$ is the average degree.

First-Order Interconnections and Aggregate Volatility

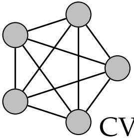
Theorem

For any sequence of economies, aggregate volatility satisfies

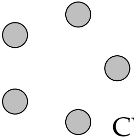
$$\sigma_{\text{agg}} = \Omega\left(\frac{1 + \text{CV}_n}{\sqrt{n}}\right).$$

$a_n = \Omega(b_n) \iff \liminf_{n\to\infty} a_n / b_n > 0.$

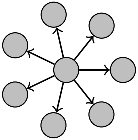
- High variability in the out-degrees implies slower rates of decay and thus, higher levels of aggregate volatility.



$\text{CV}_n = 0.$



$\text{CV}_n = 0.$



$\text{CV}_n \sim \sqrt{n}$

Power Law Degree Distributions

- An economy has a power law tail structure if, for large k,

$$P_n(k) \propto k^{-\beta}$$

  where $P_n(k)$ is the counter-cumulative distribution of the degrees.

- $\beta > 1$ is the scaling index of the power law (Pareto) distribution.

Corollary

For a sequence of economies $\{\mathcal{E}_n\}_{n \in \mathbb{N}}$ with a power law tail structure and scaling index $\beta \in (1, 2)$,

$$\sigma_{\text{agg}} = \Omega\left(n^{-\frac{\beta-1}{\beta}-\epsilon}\right),$$

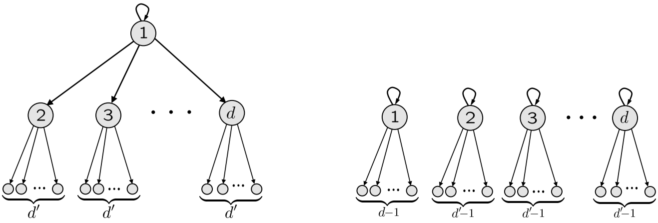
where $\epsilon > 0$ is arbitrary.

- A smaller β corresponds to higher aggregate fluctuations.

Higher-Order Interconnections and Cascades

- The degree distribution only captures first-order interconnections.
- Cascades are instead about higher-order interconnections.
- The degree distribution provides little information about higher-order interconnections.

Example: Two economies with identical degree distributions, but different levels of aggregate volatility



Second-Order Interconnections

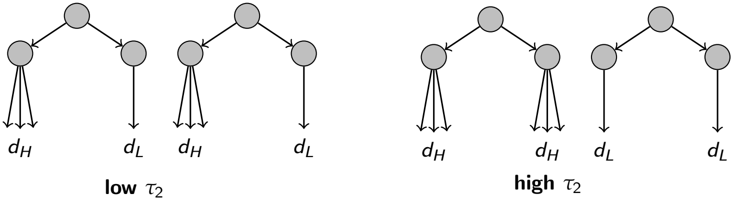
Definition

The second-order interconnectivity coefficient is defined as

$$\tau_2(W_n) \equiv \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i,j} w_{ji} w_{ki} d_j d_k,$$

where d_j is the degree of sector j.

- τ_2 takes higher values when high degree sectors share the same suppliers with other high-degree sectors $\rightarrow$ opening the way to cascades.

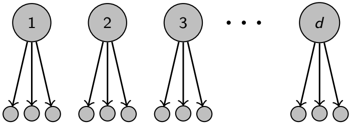


Second-Order Interconnections and Cascades

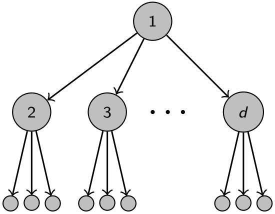
Theorem

Given a sequence of economies, the aggregate volatility satisfies

$$\sigma_{\text{agg}} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\text{CV}}{\sqrt{n}} + \frac{\sqrt{\tau_2(W_n)}}{n}\right)$$



$\tau_2 = 0$



$\tau_2 \sim n^2$

Power Law Distribution of Second-Order Degrees

- Second-order degrees:

$$q_i \equiv \sum_{j=1}^{n} d_j w_{ji}.$$

Corollary

If the second-order degrees of a sequence of economies have a power law tail with shape parameter $\zeta \in (1, 2)$, then aggregate volatility satisfies

$$\sigma_{\text{agg}} = \Omega\left(n^{-\frac{\zeta-1}{\zeta}-\epsilon}\right),$$

for any $\epsilon > 0$.

- If both the first-order and second-order degrees have power law tails:

$$\sigma_{\text{agg}} = \Omega\left(n^{-\frac{\beta-1}{\beta}} + n^{-\frac{\zeta-1}{\zeta}}\right)$$

- Dominant term: $\min\{\beta, \zeta\}$.

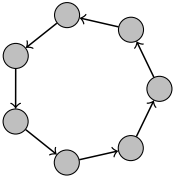
When The Diversification Argument Applies

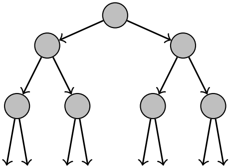
Definition

A sequence of economies is balanced if $\max_i d_i < c$ for some positive constant c and all n.

Theorem

For any sequence of balanced economies, $\sigma_{agg} \sim 1/\sqrt{n}$.



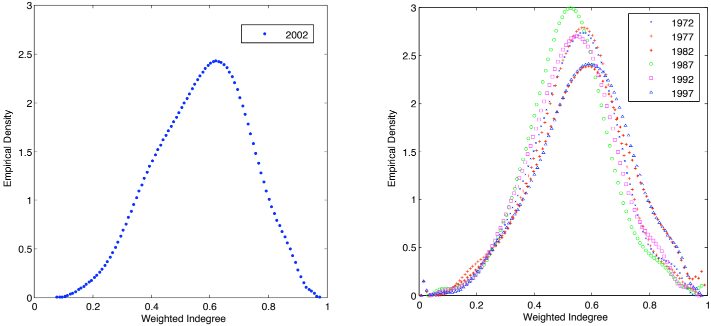


Application: The U.S. Intersectoral Network

- The U.S. input-output matrix (not disaggregated enough, but still useful).
- 1972–2002 commodity-by-commodity direct requirements table.
  (Bureau of Economic Analysis)
- This gives us the equivalent of our W_n matrix.
- Includes sectors
  - Semi-conductor and related device manufacturing, Wholesale trade, Retail trade, Real estate, Truck transportation, Advertising and related services.

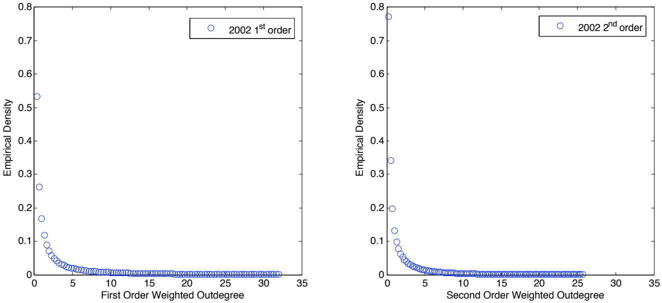
Intermediate Input shares

- Empirical densities of intermediate input shares
- Concentrated around the mean



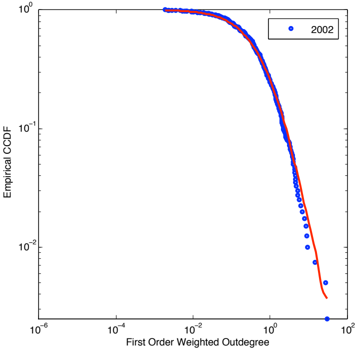
Outdegrees

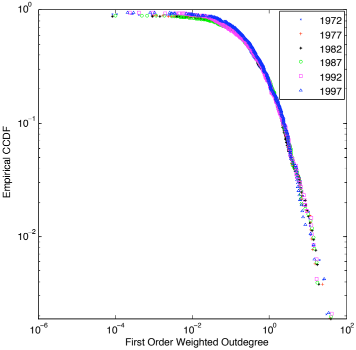
- Empirical densities of first- and second-order degrees
- Skewed with heavy right tails (unlike the indegrees)



First-Order Degrees

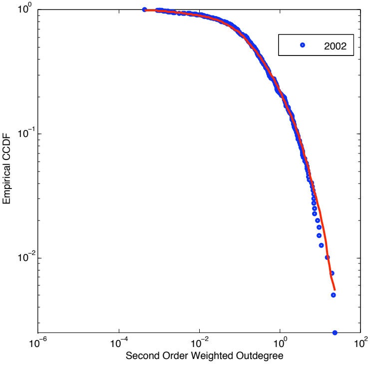
- Empirical counter-cumulative distribution of first-order degrees
- Linear tail in the log-log scale $\longrightarrow$ power law tail

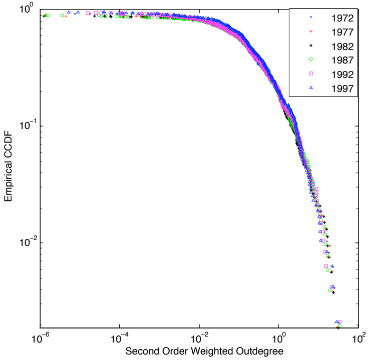




Second-Order Degrees

- Empirical counter-cumulative distribution of second-order degrees
- Linear tail in the log-log scale $\longrightarrow$ power law tail





Shape Parameter Estimates

	1972	1977	1982	1987	1992	1997	2002
$\widehat{\beta}$	1.38 (0.20; 97)	1.38 (0.19; 105)	1.35 (0.18; 106)	1.37 (0.19; 102)	1.32 (0.19; 95)	1.43 (0.21; 95)	1.46 (0.23; 83)
$\widehat{\zeta}$	1.14 (0.16; 97)	1.15 (0.16; 105)	1.10 (0.15; 106)	1.14 (0.16; 102)	1.15 (0.17; 95)	1.27 (0.18; 95)	1.30 (0.20; 83)
n	483	524	529	510	476	474	417

Table: OLS estimates of β and ζ. The numbers in parenthesis denote the associated standard errors and the number of observations corresponding to the 20% largest sectors.

- Averaging across years: $\hat{\beta} = 1.38$, $\hat{\zeta} = 1.18$

Implied Behavior of Aggregate Volatility

- $\hat{\zeta} < \hat{\beta}$: second-order effects dominate first-order effects.
- Average (annual) standard deviation of total factor productivity across 459 four-digit (SIC) manufacturing industries between 1958 and 2005 is 0.058. (NBER Manufacturing Productivity Database)
- Since manufacturing is about 20% of the economy, for the entire economy this corresponds to $5 \times 459 = 2295$ sectors at a comparable level of disaggregation.
- Had the structure been balanced: $\sigma_{\text{agg}} = 0.058/\sqrt{2295} \simeq 0.001$.
- But from the lower bound from the second-order degree distribution:

$$\sigma_{\text{agg}} \sim \sigma / n^{0.15} \simeq 0.018$$

The Limiting Distribution

- Is aggregate volatility the right metric for measuring aggregate fluctuations?

Theorem

Consider a sequence of economies with i.i.d. shocks.

(1) *If* $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$*, then* $\frac{1}{\|v_n\|_2} y_n \xrightarrow{d} \mathcal{N}(0, \sigma^2)$.

(2) *If* $\frac{\|v_n\|_\infty}{\|v_n\|_2} \longrightarrow 0$ *(with* F_i*'s arbitrary), then* $\frac{1}{\|v_n\|_2} y_n \xrightarrow{d} \mathcal{N}(0, \sigma^2)$.

(3) *Else, the asymptotic distribution of* $\frac{1}{\|v_n\|_2} y_n$*, when it exists, is non-normal and has finite variance* σ^2.

- Not only the scaling factor, but also the asymptotic distribution depends on the influence vector.

Finite Economies

- So far results focusing on the case where n grows large.
- Similar insights are applicable to economies of finite size n — though with somewhat less sharp results.
- Define *regular* network as those were $d_i = d$ for all i.
- Measure of aggregate volatility same as before.
- Suppose also that all sectors face shocks with the same variance, σ^2.

Proposition

All regular graphs achieve the lowest possible aggregate volatility, $\sigma_{agg} = \sigma / \sqrt{n}$.

- This results follow simply from the fact that for all regular graphs (for any n), $\|v_n\|_2 = \sqrt{n}$.
- Implication: complete graph and cycles are again equally "robots".

Finite Economies (continued)

- The highest level of volatility, on the other hand, generated by the network.
- In particular, in this case, $\|v_n\|_2 = 1$, and thus $\sigma_{\text{agg}} = \sigma$.
- If we impose uniform bound on the degree of any sector (say k), then the highest volatility is reached by network structures that have high second-order (and higher-order) interconnectivity coefficients.
  - E.g., sector 1 has degree k, and is connected by another sector set of sectors each with degrees of k, etc.

Further Empirical Directions

- Sectoral linkages in fact introduce a lot of empirical structure.
- Consider the above model and suppose that there are no aggregate shocks. Then the only reason why there should be correlation across sectors is because of input-output linkages.
- Using this idea, one could estimate the importance of sectoral shocks and aggregate shocks and also whether the "overidentification" structure implied by sectoral shocks holds in the data.
- One step in this direction of is Foerster, Sartre and Watson (2011), but much to do along these lines (also using more economics and economic structure implied by models).

Back to Basics

- Carvalho and Gabaix (2013) observe that changes in sectoral and firm-size distribution can impact "fundamental" volatility in the economy.
- With the same reasoning as before (see also Gabaix (2011) and Hulten (1978)),

$$\log(GDP) = v'\epsilon,$$

where v and ϵ are n-dimensional vectors (where n is the number of firms are sectors in the economy).

- Now if the n elements of ϵ are independent, aggregate volatility can be written as

$$\sigma_{agg} = \sqrt{\sum_{i=1}^{n} v_i^2 \sigma_i^2},$$

where σ_i^2 is the variance of the ith firm or sector, and v_i is its sale to GDP ratio:

$$v_i = \frac{S_i}{GDP_i}.$$

Fundamental Volatility

- Carvalho and Gabaix define this object computed at time t (which can be defined even when sectoral shocks are not independent) as the economy's *fundamental volatility* at time t:

$$\sigma_{Ft} = \sqrt{\sum_{i=1}^{n} v_{it}^2 \sigma_i^2},$$

where σ_i is taken to be time-invariant.

- This object can be easily computed from available data (Carvalho and Gabaix do it using a sectoral breakdown at the level of 88 sectors).

Fundamental and Actual Volatility

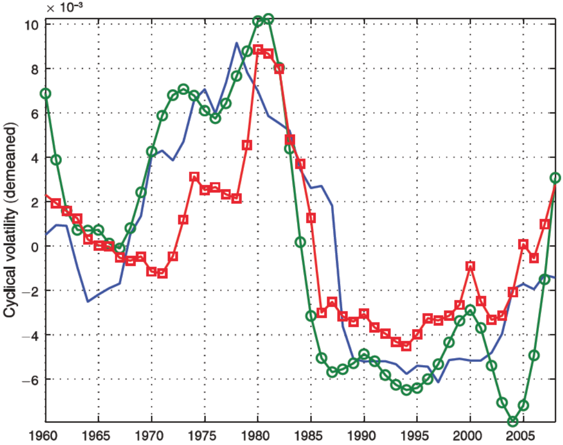


FIGURE 1. FUNDAMENTAL VOLATILITY AND GDP VOLATILITY

Notes: The squared line gives the fundamental volatility ($4.5\sigma_{Ft}$, demeaned). The solid and circle lines are annualized (and demeaned) estimates of GDP volatility, using respectively a rolling-window estimate and an HP trend of instantaneous volatility.

Fundamental and Actual Volatility (continued)

- Regression of actual volatility (computed from residuals at annual frequency or from a regression) on fundamental volatility show that much of the variation in actual annual volatility is explained by annual fundamental volatility (between 43 and 60%).
- Moreover, there does not seem to be a trend break in actual volatility once we control for fundamental volatility.
- This implies that the great moderation and the recent increase in aggregate volatility are due to changes in sectoral composition of output.
    - Great moderation driven by the declining share of highly volatile heavy manufacturing industries.
    - Greater aggregate volatility more recently due to the increasing share of finance.