14.461: Technological Change, Lectures 5-7
Directed Technological Change and Applications

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Introduction

- Technological change is often not neutral:
  1. Benefits some factors of production and some agents more than others. Distributional effects imply some groups will embrace new technologies and others oppose them.
  2. Limiting to only one type of technological change obscures the competing effects that determine the nature of technological change.
- *Directed technological change*: endogenize the direction and bias of new technologies that are developed and adopted.

Skill-biased technological change

- As already discussed in the previous lecture, over the past 60 years, the U.S. relative supply of skills has increased, but:
    1. there has also been an increase in the college premium, and
    2. this might have been an acceleration in the late 1960s, and the skill premium increased very rapidly beginning in the late 1970s.
- Standard explanation: skill-biased technical change, and an acceleration that coincided with the changes in the relative supply of skills.
- But, late 18th and early 19th *unskill-bias:*
  "First in firearms, then in clocks, pumps, locks, mechanical reapers, typewriters, sewing machines, and eventually in engines and bicycles, interchangeable parts technology proved superior and replaced the skilled artisans working with chisel and file." (Mokyr 1990, p. 137)
- Why was technological change unskilled-biased then and skilled-biased now?

Wage push and capital-biased technological change

- First phase. Late 1960s and early 1970s: unemployment and share of labor in national income increased rapidly continental European countries.
- Second phase. 1980s: unemployment continued to increase, but the labor share declined, even below its initial level.
- Blanchard (1997):
    - Phase 1: wage-push by workers
    - Phase 2: *capital-biased* technological changes.

  Is there a connection between capital-biased technological changes in European economies and the wage push preceding it?
- Why is there balanced economic growth?
- What are the technological effects of globalization?

Directed Technological Change: Basic Arguments

- Two factors of production, say L and H (unskilled and skilled workers).
- Two types of technologies that can complement either one or the other factor.
- Whenever the profitability of H-augmenting technologies is greater than the L-augmenting technologies, more of the former type will be developed by profit-maximizing (research) firms.
- What determines the relative profitability of developing different technologies? It is more profitable to develop technologies...
  1. when the goods produced by these technologies command higher prices (*price effect*);
  2. that have a larger market (*market size effect*).

Equilibrium Relative Bias

- Potentially counteracting effects, but the market size effect will be more powerful often.
- Under fairly general conditions:
  - *Weak Equilibrium (Relative) Bias*: an increase in the relative supply of a factor always induces technological change that is biased in favor of this factor.
  - *Strong Equilibrium (Relative) Bias*: if the elasticity of substitution between factors is sufficiently large, an increase in the relative supply of a factor induces sufficiently strong technological change biased towards itself that the endogenous-technology relative demand curve of the economy becomes *upward-sloping*.

Equilibrium Relative Bias in More Detail I

- Suppose the (inverse) relative demand curve:

$$w_H / w_L = D(H/L, A)$$

  where w_H / w_L is the relative price of the factors and A is a technology term.
- A is H-biased if D is increasing in A, so that a higher A increases the relative demand for the H factor.
- D is *always* decreasing in H/L.
- Equilibrium bias: behavior of A as H/L changes,

$$A(H/L)$$

Equilibrium Relative Bias in More Detail II

- Weak equilibrium bias:
  - $A(H/L)$ is increasing (nondecreasing) in H/L.
- Strong equilibrium bias:
  - $A(H/L)$ is sufficiently responsive to an increase in H/L that the total effect of the change in relative supply H/L is to increase w_H/w_L.
  - i.e., let the endogenous-technology relative demand curve be

$$w_H/w_L = D(H/L, A(H/L)) \equiv \tilde{D}(H/L)$$

$\rightarrow$*Strong equilibrium bias:* $\tilde{D}$ **increasing in** H/L.

Factor-augmenting technological change

- Production side of the economy:

$$Y(t) = F(L(t), H(t), A(t)),$$

  where $\partial F / \partial A > 0$.
- Technological change is *L-augmenting* if

$$\frac{\partial F(L, H, A)}{\partial A} \equiv \frac{L}{A} \frac{\partial F(L, H, A)}{\partial L}.$$

- Equivalent to:
  - the production function taking the special form, $F(AL, H)$.
  - Harrod-neutral technological change when L corresponds to labor and H to capital.
- H-augmenting defined similarly, and corresponds to $F(L, AH)$.

Factor-biased technological change

- Technological change change is *L-biased*, if:

$$\frac{\partial \frac{\partial F(L,H,A)/\partial L}{\partial F(L,H,A)/\partial H}}{\partial A} \geq 0.$$

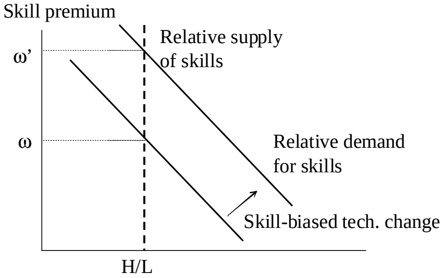


Figure: The effect of *H*-biased technological change on relative demand and relative factor prices.

Equilibrium Bias

- **Weak equilibrium bias** of technology: an increase in H/L, induces technological change biased towards H:

$$\frac{d\left(A_H(t)/A_L(t)\right)^{\frac{\sigma-1}{\sigma}}}{dH/L} \geq 0,$$

  so $A_H(t)/A_L(t)$ is biased towards the factor that has become more abundant.
- **Strong equilibrium bias:** an increase in H/L induces a sufficiently large change in the bias so that the relative marginal product of H relative to that of L increases following the change in factor supplies:

$$\frac{dMP_H/MP_L}{dH/L} > 0,$$

- The major difference is whether the relative marginal product of the two factors are evaluated at the initial relative supplies (weak bias) or at the new relative supplies (strong bias).

Evidence

- Various different pieces of evidence suggest that technology is "directed" to words activities with greater profitability.
- In the environmental context:
  - Evidence that technological change and technology adoption respond to profit incentives
  - Newell, Jaffe and Stavins (1999): energy prices on direction of technological change in air conditioning
  - Popp (2002): relates energy prices and energy saving innovation
- In the health-care sector:
  - Finkelstein (2004): government demand for vaccines leads to more clinical trials.
  - Acemoglu and Linn (2004): demographic changes increasing the demand for specific types of drugs increase FDA approvals and new molecular entities directed at these categories.

Market Size and Innovation: Market Size

- Market size for different drug categories driven by demographic changes:

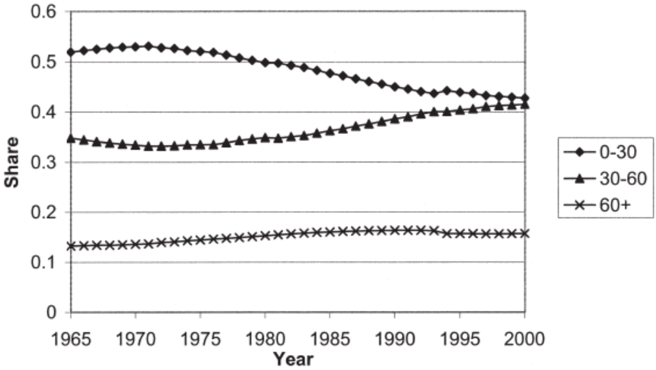


FIGURE I
Share of Population by Age Group from CPS, 1965–2000

Market Size and Innovation: Innovation Response

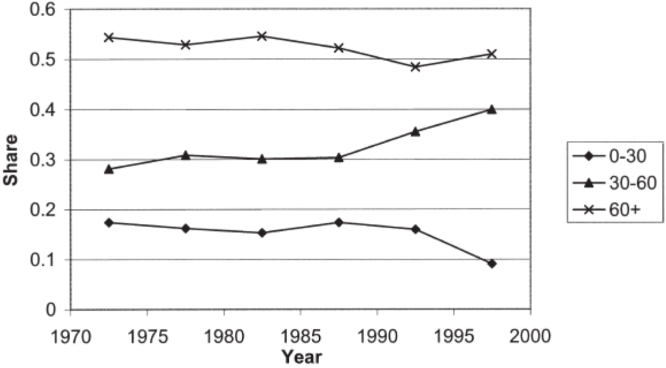


FIGURE III
Share of FDA Approvals by Age Group, 1970–2000

Market Size and Innovation: More Detailed Evidence

TABLE II
EFFECT OF CHANGES IN MARKET SIZE ON NEW DRUG APPROVALS

	(1)	(2)	(3)	(4)
Panel A: QML for Poisson model, dep var is count of drug approvals				
Market size	6.15	6.84	−2.22	
	(1.23)	(4.87)	(4.12)	
Lag market size		−0.61		
		(3.85)		
Lead market size			10.16	7.57
			(4.28)	(1.99)
Panel B: QML for Poisson model, dep var is count of nongeneric drug approvals				
Market size	3.82	6.72	2.91	
	(1.15)	(7.63)	(5.31)	
Lag market size		−2.49		
		(5.97)		
Lead market size			−1.77	1.73
			(6.94)	(2.02)
Panel C: QML for Poisson model, dep var is count of new molecular entities				
Market size	3.54	5.79	−1.38	
	(1.19)	(6.66)	(5.16)	
Lag market size		−1.99		
		(5.28)		
Lead market size			7.35	5.75
			(5.11)	(2.37)

Baseline Model of Directed Technical Change I

- Framework: expanding varieties model with lab equipment specification of the innovation possibilities frontier (so none of the results here depend on technological externalities).
- Constant supply of L and H.

- Representative household with the standard CRRA preferences:

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt, \tag{1}$$

- Aggregate production function:

$$Y(t) = \left[\gamma_L Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_H Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where intermediate good $Y_L(t)$ is L-intensive, $Y_H(t)$ is H-intensive.

Baseline Model of Directed Technical Change II

- Resource constraint (define $Z(t) = Z_L(t) + Z_H(t)$):

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{3}$$

- Intermediate goods produced competitively with:

$$Y_L(t) = \frac{1}{1-\beta}\left(\int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu\right) L^{\beta} \tag{4}$$

and

$$Y_H(t) = \frac{1}{1-\beta}\left(\int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu\right) H^{\beta}, \tag{5}$$

where machines $x_L(\nu, t)$ and $x_H(\nu, t)$ are assumed to depreciate after use.

Baseline Model of Directed Technical Change III

- Differences with baseline expanding product varieties model:
  1. These are production functions for intermediate goods rather than the final good.
  2. (4) and (5) use different types of machines–different ranges $[0, N_L(t)]$ and $[0, N_H(t)]$.
- All machines are supplied by monopolists that have a fully-enforced perpetual patent, at prices $p_L^x(\nu, t)$ for $\nu \in [0, N_L(t)]$ and $p_H^x(\nu, t)$ for $\nu \in [0, N_H(t)]$.
- Once invented, each machine can be produced at the fixed marginal cost ψ in terms of the final good.
- Normalize to $\psi \equiv 1 - \beta$.

Baseline Model of Directed Technical Change IV

- Innovation possibilities frontier:

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t), \tag{6}$$

- Value of a monopolist that discovers one of these machines is:

$$V_f(\nu, t) = \int_t^\infty \exp\left[-\int_t^s r(s')\, ds'\right] \pi_f(\nu, s) ds, \tag{7}$$

where $\pi_f(\nu, t) \equiv p_f^x(\nu, t) x_f(\nu, t) - \psi x_f(\nu, t)$ for $f = L$ or H.

- Hamilton-Jacobi-Bellman version:

$$r(t)\, V_f(\nu, t) - \dot{V}_f(\nu, t) = \pi_f(\nu, t). \tag{8}$$

- Set the ideal price index is numeraire:

$$\left[\gamma_L^\varepsilon (p_L(t))^{1-\varepsilon} + \gamma_H^\varepsilon (p_H(t))^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1 \text{ for all } t, \tag{9}$$

where $p_L(t)$ is the price index of Y_L at time t and $p_H(t)$ is the price of Y_H.

Equilibrium I

- Maximization problem of producers in the two sectors:

$$\max_{L,[x_L(\nu,t)]_{\nu\in[0,N_L(t)]}} p_L(t)\,Y_L(t) - w_L(t)\,L - \int_0^{N_L(t)} p_L^x(\nu,t)\,x_L(\nu,t)\,d\nu, \tag{10}$$

and

$$\max_{H,[x_H(\nu,t)]_{\nu\in[0,N_H(t)]}} p_H(t)\,Y_H(t) - w_H(t)\,H - \int_0^{N_H(t)} p_H^x(\nu,t)\,x_H(\nu,t)\,d\nu. \tag{11}$$

- Here $w_L(t)$ and $w_H(t)$ denote wages.
- Note the presence of $p_L(t)$ and $p_H(t)$, since these sectors produce intermediate goods.

Equilibrium II

- Thus, demand for machines in the two sectors:

$$x_L(\nu, t) = \left[\frac{p_L(t)}{p_L^x(\nu, t)}\right]^{1/\beta} L \quad \text{for all } \nu \in [0, N_L(t)] \text{ and all } t, \quad (12)$$

  and

$$x_H(\nu, t) = \left[\frac{p_H(t)}{p_H^x(\nu, t)}\right]^{1/\beta} H \quad \text{for all } \nu \in [0, N_H(t)] \text{ and all } t. \quad (13)$$

- Maximization of the net present discounted value of profits implies a constant markup:

$$p_L^x(\nu, t) = p_H^x(\nu, t) = 1 \text{ for all } \nu \text{ and } t.$$

Equilibrium III

- Substituting into (12) and (13):

$$x_L(\nu, t) = p_L(t)^{1/\beta} L \quad \text{for all } \nu \text{ and all } t,$$

and

$$x_H(\nu, t) = p_H(t)^{1/\beta} H \quad \text{for all } \nu \text{ and all } t.$$

- Since these quantities do not depend on the identity of the machine profits are also independent of the machine type:

$$\pi_L(t) = \beta p_L(t)^{1/\beta} L \text{ and } \pi_H(t) = \beta p_H(t)^{1/\beta} H. \qquad (14)$$

- Thus the values of monopolists only depend on which sector they are, $V_L(t)$ and $V_H(t)$.

Equilibrium IV

- Combining these with (4) and (5), *derived* production functions for the two intermediate goods:

$$Y_L(t) = \frac{1}{1-\beta} p_L(t)^{\frac{1-\beta}{\beta}} N_L(t) L \tag{15}$$

$$Y_H(t) = \frac{1}{1-\beta} p_H(t)^{\frac{1-\beta}{\beta}} N_H(t) H. \tag{16}$$

- For the prices of the two intermediate goods, (2) imply

$$\begin{aligned} p(t) &\equiv \frac{p_H(t)}{p_L(t)} = \gamma \left(\frac{Y_H(t)}{Y_L(t)} \right)^{-\frac{1}{\varepsilon}} = \gamma \left(p(t)^{\frac{1-\beta}{\beta}} \frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{1}{\varepsilon}} \\ &= \gamma^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{\beta}{\sigma}}, \end{aligned} \tag{17}$$

where $\gamma \equiv \gamma_H / \gamma_L$ and

$$\sigma \equiv 1 + (\varepsilon - 1) \beta.$$

Equilibrium VI

- We can also calculate the relative factor prices:

$$\begin{aligned}\omega(t) &\equiv \frac{w_H(t)}{w_L(t)} \\ &= p(t)^{1/\beta}\frac{N_H(t)}{N_L(t)} \\ &= \gamma^{\frac{\varepsilon}{\sigma}}\left(\frac{N_H(t)}{N_L(t)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \qquad (18)\end{aligned}$$

- σ is the (derived) elasticity of substitution between the two factors, since it is exactly equal to

$$\sigma = -\left(\frac{d\log\omega(t)}{d\log(H/L)}\right)^{-1}.$$

Equilibrium VII

- Free entry conditions:

$$\eta_L V_L(t) \leq 1 \text{ and } \eta_L V_L(t) = 1 \text{ if } Z_L(t) > 0. \tag{19}$$

and

$$\eta_H V_H(t) \leq 1 \text{ and } \eta_H V_H(t) = 1 \text{ if } Z_H(t) > 0. \tag{20}$$

- Consumer side:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \tag{21}$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)(N_L(t)\,V_L(t) + N_H(t)\,V_H(t))\right] = 0, \tag{22}$$

where $N_L(t)\,V_L(t) + N_H(t)\,V_H(t)$ is the total value of corporate assets in this economy.

Balanced Growth Path I

- Consumption grows at the constant rate, g^*, and the relative price $p(t)$ is constant. From (9) this implies that $p_L(t)$ and $p_H(t)$ are also constant.
- Let V_L and V_H be the BGP net present discounted values of new innovations in the two sectors. Then (8) implies that

$$V_L = \frac{\beta p_L^{1/\beta} L}{r^*} \text{ and } V_H = \frac{\beta p_H^{1/\beta} H}{r^*}, \tag{23}$$

- Taking the ratio of these two expressions, we obtain

$$\frac{V_H}{V_L} = \left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}} \frac{H}{L}.$$

Balanced Growth Path II

- Note the two effects on the direction of technological change:
  1. The price effect: V_H / V_L is increasing in p_H / p_L. Tends to favor technologies complementing scarce factors.
  2. The market size effect: V_H / V_L is increasing in H / L. It encourages innovation for the more abundant factor.
- The above discussion is incomplete since prices are endogenous. Combining (23) together with (17):

$$\frac{V_H}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}. \tag{24}$$

- Note that an increase in H / L will increase V_H / V_L as long as $\sigma > 1$ and it will reduce it if $\sigma < 1$. Moreover,

$$\sigma \gtreqless 1 \iff \varepsilon \gtreqless 1.$$

- The two factors will be gross substitutes when the two intermediate goods are gross substitutes in the production of the final good.

Balanced Growth Path III

- Next, using the two free entry conditions (19) and (20) as equalities, we obtain the following BGP "technology market clearing" condition:

$$\eta_L V_L = \eta_H V_H. \tag{25}$$

- Combining this with (24), BGP ratio of relative technologies is

$$\left(\frac{N_H}{N_L}\right)^* = \eta^\sigma \gamma^\varepsilon \left(\frac{H}{L}\right)^{\sigma-1}, \tag{26}$$

  where $\eta \equiv \eta_H / \eta_L$.

- Note that relative productivities are determined by the innovation possibilities frontier and the relative supply of the two factors. In this sense, this model totally endogenizes technology.

Summary of Balanced Growth Path

Proposition Consider the directed technological change model described above. Suppose

$$\beta\left[\gamma_H^{\varepsilon}\left(\eta_H H\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} > \rho$$

$$\text{and } (1-\theta)\,\beta\left[\gamma_H^{\varepsilon}\left(\eta_H H\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} < \rho.$$

Then there exists a unique BGP equilibrium in which the relative technologies are given by (26), and consumption and output grow at the rate

$$g^{*}=\frac{1}{\theta}\left(\beta\left[\gamma_H^{\varepsilon}\left(\eta_H H\right)^{\sigma-1}+\gamma_L^{\varepsilon}\left(\eta_L L\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}-\rho\right).$$

Transitional Dynamics

- Differently from the baseline endogenous technological change models, there are now transitional dynamics (because there are two state variables).
- Nevertheless, transitional dynamics simple and intuitive:

Proposition Consider the directed technological change model described above. Starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (26), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. If $N_H(0)/N_L(0) < (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

- Summary: the dynamic equilibrium path always tends to the BGP and during transitional dynamics, there is only one type of innovation.

Directed Technological Change and Factor Prices

- In BGP, there is a positive relationship between H/L and N_H^*/N_L^* only when $\sigma > 1$.
- But this does not mean that depending on σ (or ε), changes in factor supplies may induce technological changes that are biased in favor or against the factor that is becoming more abundant.
- Why?
  - N_H^*/N_L^* refers to the ratio of factor-augmenting technologies, or to the ratio of *physical* productivities.
  - What matters for the bias of technology is *the value of marginal product* of factors, affected by relative prices.
  - The relationship between factor-augmenting and factor-biased technologies is reversed when σ is less than 1.
  - When $\sigma > 1$, an increase in N_H^*/N_L^* is relatively biased towards H, while when $\sigma < 1$, a *decrease* in N_H^*/N_L^* is relatively biased towards H.

Weak Equilibrium (Relative) Bias Result

Proposition Consider the directed technological change model described above. There is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

- The results reflect the strength of the market size effect: it always dominates the price effect.
- But it does not specify whether this induced effect will be strong enough to make the endogenous-technology relative demand curve for factors upward-sloping.

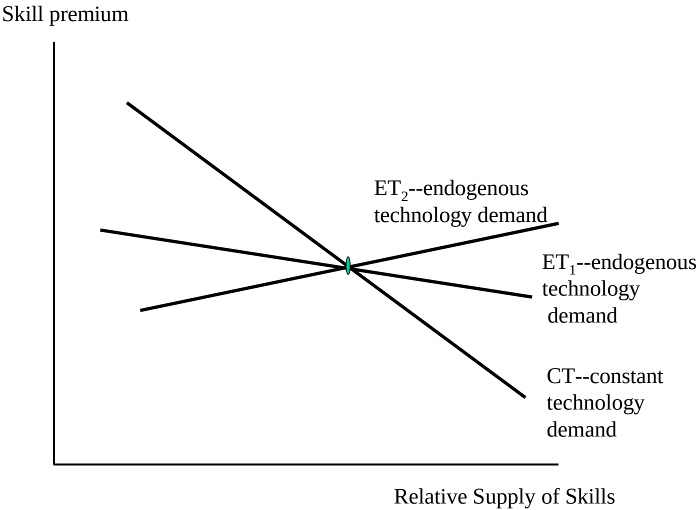
Strong Equilibrium (Relative) Bias Result

- Substitute for $(N_H/N_L)^*$ from (26) into the expression for the relative wage given technologies, (18), and obtain:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\sigma-1}\gamma^{\varepsilon}\left(\frac{H}{L}\right)^{\sigma-2}. \qquad (27)$$

Proposition Consider the directed technological change model described above. Then if $\sigma > 2$, there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the factor H compared to factor L.

Relative Supply of Skills and Skill Premium



Discussion

- Analogous to Samuelson's *LeChatelier principle*: think of the endogenous-technology demand curve as adjusting the "factors of production" corresponding to technology.
- But, the effects here are caused by general equilibrium changes, not on partial equilibrium effects.
- Moreover ET_2, which applies when $\sigma > 2$ holds, is upward-sloping.
- A complementary intuition: importance of non-rivalry of ideas:
  - leads to an aggregate production function that exhibits increasing returns to scale (in all factors including technologies).
  - the market size effect can create sufficiently strong induced technological change to increase the relative marginal product and the relative price of the factor that has become more abundant.

Implications I

- Recall we have the following stylized facts:
  - Secular skill-biased technological change increasing the demand for skills throughout the 20th century.
  - Possible acceleration in skill-biased technological change over the past 25 years.
  - A range of important technologies biased against skill workers during the 19th century.
- The current model gives us a way to think about these issues.
  - The increase in the number of skilled workers should cause steady skill-biased technical change.
  - Acceleration in the increase in the number of skilled workers should induce an acceleration in skill-biased technological change.
  - Available evidence suggests that there were large increases in the number of unskilled workers during the late 18th and 19th centuries.

Implications II

- The framework also gives a potential interpretation for the dynamics of the college premium during the 1970s and 1980s.

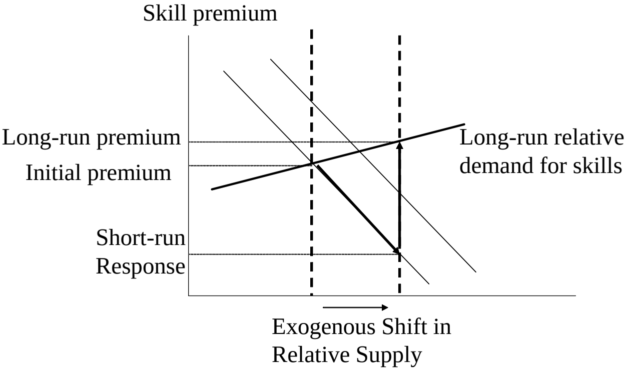


Figure: Dynamics of the skill premium in response to an exogenous increase in the relative supply of skills, with an upward sloping endogenous technology

Implications III

- If instead $\sigma < 2$, the long-run relative demand curve will be downward sloping, though again it will be shallower than the short-run relative demand curve.

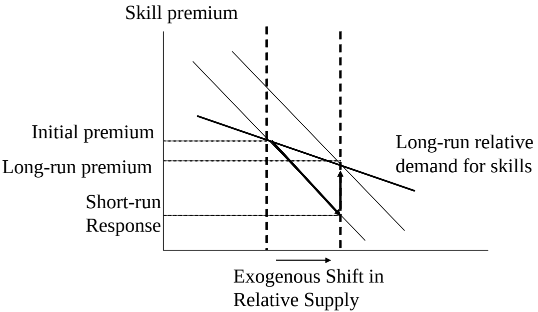


Figure: Dynamics of the skill premium in response to an increase in the relative supply of skills, with a downward-sloping endogenous-technology relative demand curve

Implications III

- Other remarks:
  - Upward-sloping relative demand curves arise only when $\sigma > 2$. Most estimates put the elasticity of substitution between 1.4 and 2. One would like to understand whether $\sigma > 2$ is a feature of the specific model discussed here
  - Results on induced technological change are not an artifact of the scale effect (exactly the same results apply when scale effects are removed, see below).

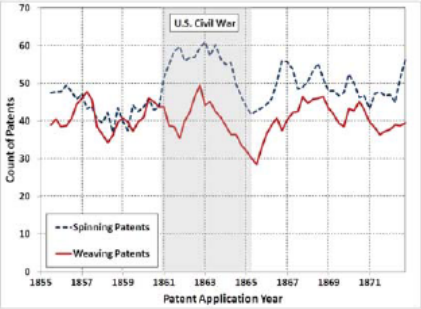
Evidence

- Hanlon (2014): evidence on factor-augmenting directed technological change and its impact on factor prices.
- Following the interruption to the British cotton textile industry caused by the US Civil War, the decrease in American cotton led to technological change directed to other types of cotton inputs.
- There was a flurry of new patents related to cotton spinning. These appear to be directed at Indian cotton which was relatively abundant but harder to prepare for spinning than American cotton.
- This looks like "factor-augmenting" technological change directed towards the more abundant input. Consistent with theory if the elasticity of substitution > 1, which Hanlon's estimates suggest.
- Hanlon also provides evidence of strong relative bias—relative Indian cotton prices actually increased despite this input's relative abundance.

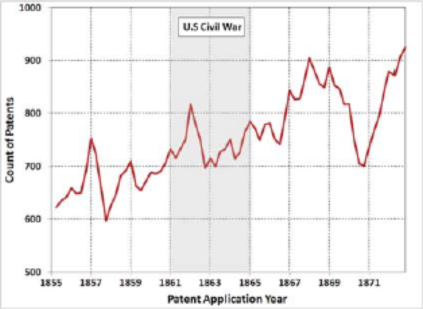
Evidence: Changes in Quantities

Evidence: Changes in Spinning Patents

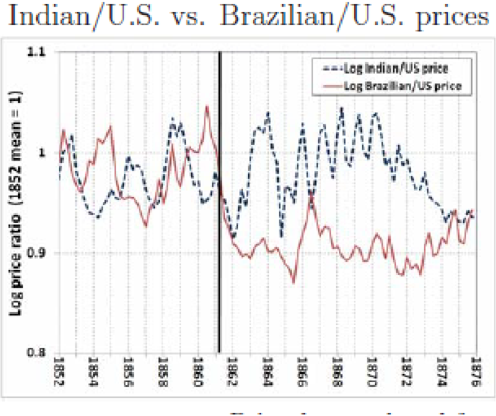
Spinning & Weaving Patents



All Other Technologies



Evidence: Changes in Input Prices



Directed Technological Change with Knowledge Spillovers

- The lab equipment specification of the innovation possibilities does not allow for *state dependence*.
- Assume that R&D is carried out by scientists and that there is a constant supply of scientists equal to S
- With only one sector, sustained endogenous growth requires $\dot{N}/N$ to be proportional to S.
- With two sectors, there is a variety of specifications with different degrees of state dependence, because productivity in each sector can depend on the state of knowledge in both sectors.
- A flexible formulation is

$$\begin{aligned} \dot{N}_L(t) &= \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} S_L(t) \quad (28) \\ \text{and } \dot{N}_H(t) &= \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} S_H(t), \end{aligned}$$

where $\delta \leq 1$.

Directed Technological Change with Knowledge Spillovers

- Market clearing for scientists requires that

$$S_L(t) + S_H(t) \leq S. \tag{29}$$

- δ measures the degree of state-dependence:
  - $\delta = 0$. Results are unchanged. No state-dependence:

    $$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H / \eta_L$$

    irrespective of the levels of N_L and N_H.
    Both N_L and N_H create spillovers for current research in both sectors.
  - $\delta = 1$. Extreme amount of state-dependence:

    $$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H N_H / \eta_L N_L$$

    an increase in the stock of L-augmenting machines today makes future labor-complementary innovations cheaper, but has no effect on the cost of H-augmenting innovations.
- So state dependence adds another layer of "increasing returns"

Directed Technological Change with Knowledge Spillovers

- Now assuming that both free entry conditions hold, BGP technology market clearing implies

$$\eta_L N_L(t)^{\delta} \pi_L = \eta_H N_H(t)^{\delta} \pi_H, \quad (30)$$

- Combine condition (30) with equations (14) and (17), to obtain the equilibrium relative technology as:

$$\left(\frac{N_H}{N_L}\right)^{*} = \eta^{\frac{\sigma}{1-\delta\sigma}} \gamma^{\frac{\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\delta\sigma}}, \quad (31)$$

where $\gamma \equiv \gamma_H / \gamma_L$ and $\eta \equiv \eta_H / \eta_L$.

Directed Technological Change with Knowledge Spillovers

- The relationship between the relative factor supplies and relative physical productivities now depends on δ.
- This is intuitive: as long as $\delta > 0$, an increase in N_H reduces the relative costs of H-augmenting innovations, so for technology market equilibrium to be restored, π_L needs to increase relative to π_H.
- Substituting (31) into the expression (18) for relative factor prices for given technologies, yields the following long-run (endogenous-technology) relationship:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \gamma^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}} . \tag{32}$$

- Growth is determined similarly.

Transitional Dynamics with Knowledge Spillovers

- Transitional dynamics now more complicated because of the spillovers.
- The dynamic equilibrium path does not always tend to the BGP because of the additional increasing returns to scale:
  - With a high degree of state dependence, when $N_H(0)$ is very high relative to $N_L(0)$, it may no longer be profitable for firms to undertake further R&D directed at labor-augmenting (L-augmenting) technologies.
  - Whether this is so or not depends on a comparison of the degree of state dependence, δ, and the elasticity of substitution, σ.

Summary of Transitional Dynamics

Proposition Suppose that

$$\sigma < 1/\delta.$$

Then, starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (31), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. $N_H(0)/N_L(0) < (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

If

$$\sigma > 1/\delta,$$

then starting with $N_H(0)/N_L(0) > (N_H/N_L)^*$, the economy tends to $N_H(t)/N_L(t) \to \infty$ as $t \to \infty$, and starting with $N_H(0)/N_L(0) < (N_H/N_L)^*$, it tends to $N_H(t)/N_L(t) \to 0$ as $t \to \infty$.

Equilibrium Relative Bias with Knowledge Spillovers I

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then there is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then if

$$\sigma > 2 - \delta,$$

there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the H factor compared to the L factor.

Equilibrium Relative Bias with Knowledge Spillovers II

- Intuitively, the additional increasing returns to scale coming from state dependence makes strong bias easier to obtain, because the induced technology effect is stronger.
- Note the elasticity of substitution between skilled and unskilled labor significantly less than 2 may be sufficient to generate strong equilibrium bias.
- How much lower than 2 the elasticity of substitution can be depends on the parameter δ. Unfortunately, this parameter is not easy to measure in practice.

Endogenous Labor-Augmenting Technological Change I

- Models of directed technological change create a natural reason for technology to be more labor augmenting than capital augmenting.
- Under most circumstances, the resulting equilibrium is not purely labor augmenting and as a result, a BGP fails to exist.
- But in one important special case, the model delivers long-run purely labor augmenting technological changes exactly as in the neoclassical growth model.
- Consider a two-factor model with H corresponding to capital, that is, $H(t) = K(t)$.
- Assume that there is no depreciation of capital.
- Note that in this case the price of the second factor, $K(t)$, is the same as the interest rate, $r(t)$.
- Empirical evidence suggests $\sigma < 1$ and is also economically plausible.

Endogenous Labor-Augmenting Technological Change II

- Recall that when $\sigma < 1$ labor-augmenting technological change corresponds to capital-biased technological change.
- Hence the questions are:
  1. Under what circumstances would the economy generate relatively capital-biased technological change?
  2. When will the equilibrium technology be sufficiently capital biased that it corresponds to Harrod-neutral technological change?
- To answer 1, note that what distinguishes capital from labor is the fact that it accumulates, so $K(t)/L$ increases in balanced growth.
- This implies that technological change should be *more labor-augmenting than capital augmenting.*

Proposition In the baseline model of directed technological change with $H(t) = K(t)$ as capital, if $K(t)/L$ is increasing over time and $\sigma < 1$, then $N_L(t)/N_K(t)$ will also increase over time.

Endogenous Labor-Augmenting Technological Change IV

- But the results are not easy to reconcile with purely-labor augmenting technological change. Suppose that capital accumulates at an exogenous rate, i.e.,

$$\frac{\dot{K}(t)}{K(t)} = s_K > 0. \tag{33}$$

Proposition Consider the baseline model of directed technological change with the knowledge spillovers specification and state dependence. Suppose that $\delta < 1$ and capital accumulates according to (33). Then there exists no BGP.

- Intuitively, even though technological change is more labor augmenting than capital augmenting, there is still capital-augmenting technological change in equilibrium.
- Moreover it can be proved that in any asymptotic equilibrium, $r(t)$ cannot be constant, thus consumption and output growth cannot be constant.

Endogenous Labor-Augmenting Technological Change V

- Special case that justifies the basic structure of the neoclassical growth model: extreme state dependence ($\delta = 1$).
- In this case:

$$\frac{r(t) K(t)}{w_L(t) L} = \eta^{-1}. \tag{34}$$

- Thus, directed technological change ensures that the share of capital is constant in national income—similar to Cobb-Douglas.
- In fact, from the equivalent of equation (18) for this case, we have that

$$\frac{rK}{w(t)L} = \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_K(t)}{N_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K}{L} \right)^{\frac{\sigma-1}{\sigma}},$$

which implies that $(N_L(t) L) / (N_K(t) K(t))$ is constant, thus $N_K(t)$ must also be constant.
- Therefore, technological change must be purely labor augmenting.

Summary of Endogenous Labor-Augmenting Technological Change

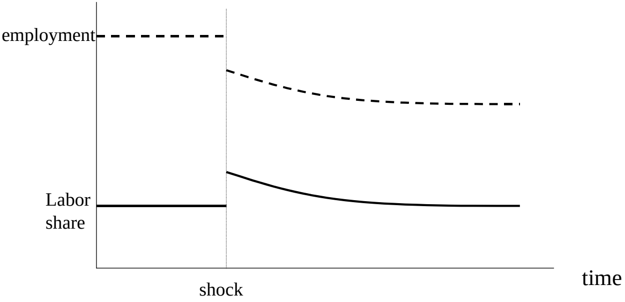
Proposition Consider the baseline model of directed technological change with the two factors corresponding to labor and capital. Suppose that the innovation possibilities frontier is given by the knowledge spillovers specification and **extreme state dependence**, i.e., $\delta = 1$ and that capital accumulates according to (33). Then there exists a constant growth path allocation in which there is only labor-augmenting technological change, the interest rate is constant and consumption and output grow at constant rates. Moreover, there cannot be any other constant growth path allocations.

Stability

- The constant growth path allocation with purely labor augmenting technological change is globally stable if $\sigma < 1$.
- Intuition:
  - If capital and labor were gross substitutes ($\sigma > 1$), the equilibrium would involve rapid accumulation of capital and capital-augmenting technological change, leading to an asymptotically increasing growth rate of consumption.
  - When capital and labor are gross complements ($\sigma < 1$), capital accumulation would increase the price of labor and profits from labor-augmenting technologies and thus encourage further labor-augmenting technological change.
  - $\sigma < 1$ forces the economy to strive towards a balanced allocation of effective capital and labor units.
  - Since capital accumulates at a constant rate, a balanced allocation implies that the productivity of labor should increase faster, and the economy should converge to an equilibrium path with purely labor-augmenting technological progress.

What Does Wage Push Do?

- What are the implications of wage push here? Why?



Conclusions

- Models of directed technological change enable us to investigate a range of new questions:
  - the sources of skill-biased technological change over the past 100 years,
  - the causes of acceleration in skill-biased technological change during more recent decades,
  - the causes of unskilled-biased technological developments during the 19th century,
  - the relationship between labor market institutions and the types of technologies that are developed and adopted,
  - why technological change in neoclassical-type models may be largely labor-augmenting.
- The implications of the class of models studied for the empirical questions mentioned above stem from the *weak equilibrium bias* and *strong equilibrium bias* results.

Introduction

- A classic question in the economics of technology: *does shortage of labor encourage innovation*?
- Related: *do high wages encourage innovation?*
- Answers vary.
- Neoclassical growth model: *No*, with technology embodied in capital and constant returns to scale, labor shortage and high wages always discourage technology adoption.
- Endogenous growth theory: *No*, it discourages innovation because of scale effects. True also in "semi-endogenous" growth models such as Jones (1995), Young (1999) or Howitt (1999).
- Ester Boserup: *No*, population pressure is a major factor in innovations.

Different Answers? (continued)

- John Hicks: *Yes,*
  "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive..." (*Theory of Wages*, p. 124).
- Habakkuk: *Yes*, in the context of 19th-century US-UK comparison

  "... it was scarcity of labor 'which laid the foundation for the future continuous progress of American industry, by obliging manufacturers to take every opportunity of installing new types of labor-saving machinery.' " (quoted from Pelling),

  "It seems obvious— it certainly seemed so to contemporaries— that the dearness and inelasticity of American, compared with British, labour gave the American entrepreneur ... a greater inducement than his British counterpart to replace labour by machines." (Habakkuk, 1962, p. 17).

Different Answers? (continued)

- Robert Allen: *Yes*, high British wages are the reason why the major technologies of the British Industrial Revolution got invented.

> *"... Nottingham, Leicester, Birmingham, Sheffield etc. must long ago have given up all hopes of foreign commerce, if they had not been constantly counteracting the advancing price of manual labor, by adopting every ingenious improvement the human mind could invent." (T. Bentley).*

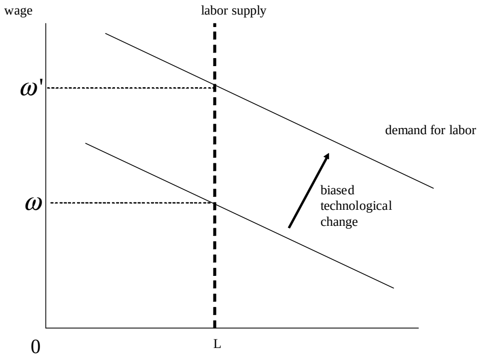
- Zeira; Hellwig-Irmen: *Yes,* high wages encourage switch to capital-intensive technologies.
- Alesina-Zeira and others: *Yes*, high wages may have encouraged adoption of certain capital-intensive technologies in Europe

General Results and Clarification

- Different answers mostly because "implicit assumptions" about how technology and factors interact. Other aspects of the not important.
- Let us clarify this.
- We will first establish the following two theorems:
  Theorem: Under some weak regularity conditions (to be explained below), a decrease in labor supply will change technology in a way that is *biased* against labor.
  Theorem: Under some weak regularity conditions, a decrease in labor supply will *decrease* wages *if and only if* the aggregate production possibilities set of the economy is *locally nonconvex.*

What Is (Absolute) Bias?

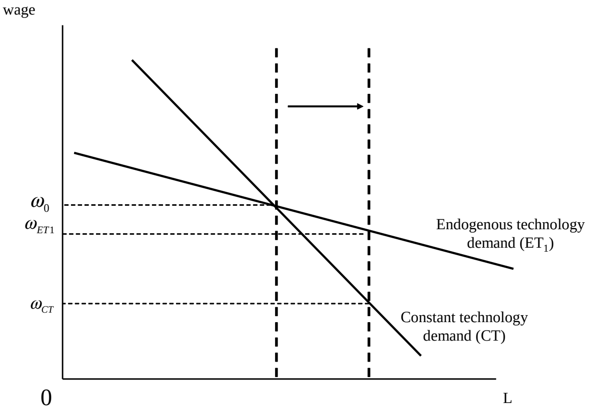
- Same as relative bias; but now “absolute,” i.e., shift of the usual demand curve.



Intuition For Bias

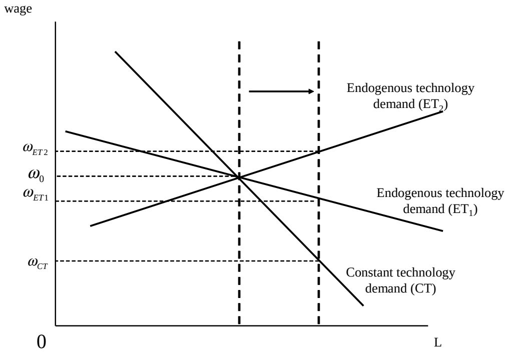
- An increase in employment (L), at the margin, increases the the value of technologies that are "complementary" to L.
  - Denote technology by θ.
  - Suppose that L and θ are complements, then an increase in L increases the incentives to improve θ, but then this *increases* the marginal contribution of L to output and thus wages$\rightarrow$*biased change.*
  - Suppose that L and θ are substitutes,then an increase in L reduces the incentives to improve θ, but then this *increases* the marginal contribution of L to output and thus wages$\rightarrow$*biased change*
- But this intuition also shows that an increase in L could lead to an increase or decrease in θ.
- Thus implications for "technological advances" unclear.

Induced (Absolute) Bias



Upward Sloping Demand Curves?

- Impossible in producer theory. But in general equilibrium, quite usual.



Labor Scarcity and Technological Advances

- The above discussion suggests that we should not look for an unambiguous relationship.
- Is there a simple unifying theme?
- Suppose that aggregate output can be represented as $F(L, Z, \theta)$, where Z is a vector of other inputs.
- Let us say that technological change is strongly labor saving if F exhibits *decreasing differences* in L and θ.
- Conversely, technological change is strongly labor complementary if F exhibits *increasing differences* in L and θ.
- **Answer:** labor scarcity will lead to technological advances if technology is strongly labor saving and will lead to technological regress if technology is strongly labor complementary.
- Intuitively, at the margin, labor and the relevant technologies need to be "substitutes".

Basic Framework

- Consider a static economy consisting of a unique final good and $N+1$ factors of production, $Z = (Z_1, ..., Z_N)$ and labor L.
- All agents' preferences are defined over the consumption of the final good.
- Suppose, for now, that all factors are supplied inelastically, with supplies denoted by $\bar{Z} \in \mathbb{R}_+^N$ and $\bar{L} \in \mathbb{R}_+$.
- The economy consists of a continuum of firms (final good producers) denoted by the set $\mathcal{F}$, each with an identical production function.
- Without loss of any generality let us normalize the measure of $\mathcal{F}$, $|\mathcal{F}|$, to 1.
- The price of the final good is also normalized to 1.

Economy D

- All markets are competitive and technology chosen by firms.
- Each firm $i \in \mathcal{F}$ has access to a production function

$$Y^i = G(L^i, Z^i, \theta^i), \tag{35}$$

- Here $L^i \in \mathbb{R}_+$ and $Z^i \in \mathbb{R}^N_+$.
- Most importantly, $\theta^i \in \Theta \subset \mathbb{R}^K$ is the measure of technology.
- Suppose that G is twice continuously differentiable in (L^i, Z^i, θ^i)—to be relaxed later.
- Thus factor prices are well defined and denote them by w_L and w_{Z_j} (vector w_Z).
- The cost of technology $\theta \in \Theta$ in terms of final goods is $C(\theta)$, convex and twice differentiable
  - but $C(\theta)$ could be increasing or decreasing.

Economy D (continued)

- Each final good producer maximizes profits, or in other words, solves:

$$\max_{L^i \in \mathcal{L}, Z^i \in \mathcal{Z}, \theta_i \in \Theta} \pi(L^i, Z^i, \theta^i) = G(L^i, Z^i, \theta^i) - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - C(\theta^i). \tag{36}$$

- Factor prices taken as given by the firm.
- Market clearing:

$$\int_{i \in \mathcal{F}} L^i di \leq \bar{L} \text{ and } \int_{i \in \mathcal{F}} Z_j^i di \leq Z_j \text{ for } j = 1, ..., N. \tag{37}$$

Definition

An equilibrium in Economy D is a set of decisions $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$ *and factor prices* (w_L, w_Z) *such that* $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$ *solve (36) given prices* (w_L, w_Z) *and (37) holds.*

Economy D (continued)

- Let us refer to any θ^i that is part of the set of equilibrium allocations, $\left\{L^i, Z^i, \theta^i\right\}_{i\in\mathcal{F}}$, as *equilibrium technology*.
- Also for future use, let us define the "net production function":

$$F(L^i, Z^i, \theta^i) \equiv G(L^i, Z^i, \theta^i) - C(\theta^i). \tag{38}$$

- For the competitive equilibrium to be well-defined, we introduce:

Assumption

Either $F(L^i, Z^i, \theta^i)$ is jointly strictly concave in (L^i, Z^i, θ^i) and increasing in (L^i, Z^i); or $F(L^i, Z^i, \theta^i)$ is increasing in (L^i, Z^i) and exhibits constant returns to scale in (L^i, Z^i, θ^i).

Economy D (continued)

Proposition

Suppose Assumption 1 holds. Then any equilibrium technology θ in Economy D is a solution to

$$\max_{\theta' \in \Theta} F(\bar{L}, \bar{Z}, \theta'), \tag{39}$$

and any solution to this problem is an equilibrium technology.

- Therefore, to analyze equilibrium technology choices, we can simply focus on a simple maximization problem.
- Moreover, the equilibrium is a Pareto optimum (and vice versa).
- Equilibria factor prices given by marginal products, in particular:

$$w_L = \frac{\partial G(\bar{L}, \bar{Z}, \theta)}{\partial L} = \frac{\partial F(\bar{L}, \bar{Z}, \theta)}{\partial L}.$$

Economy M

- Let us next consider a more usual environment for models of technological progress (similar to, but more general than Romer, 1990, Aghion-Howitt, 1992, Grossman-Helpman, 1991).
- The final good sector is competitive with production function

$$Y^i = \alpha^{-\alpha}(1-\alpha)^{-1} G(L^i, Z^i, \theta^i)^{\alpha} q(\theta^i)^{1-\alpha}. \quad (40)$$

- Now $G(L^i, Z^i, \theta^i)$ is a subcomponent of the production function, which depends on θ^i, the technology used by the firm.
- Assumption 2 now applies to this subcomponent.
- The subcomponent G needs to be combined with an intermediate good embodying technology θ^i, denoted by $q(\theta^i)$—conditioned on θ^i to emphasize that it embodies technology θ^i.
- This intermediate good is supplied by the monopolist.
- The term $\alpha^{-\alpha}(1-\alpha)^{-1}$ for normalization.

Economy M (continued)

- The monopolist can create technology θ at cost $C(\theta)$ from the technology menu.
- Suppose that $C(\theta)$ is convex, but for now, it could be increasing or decreasing in θ;
  - There is as yet no sense that the higher θ corresponds to "better technology".
- Once θ is created, the technology monopolist can produce the intermediate good embodying technology θ at constant per unit cost normalized to $1-\alpha$ unit of the final good (this is also a convenient normalization).
- It can then set a (linear) price per unit of the intermediate good of type θ, denoted by χ.
- This economy can be easily generalize to a "oligopolistic" one.

Economy M (continued)

- Each final good producer takes the available technology, θ, and the price of the intermediate good embodying this technology, χ, as given and maximizes

$$\max_{\substack{L^i \in \mathcal{L}, Z^i \in \mathcal{Z}, \\ q(\theta) \geq 0}} \pi(L^i, Z^i, q(\theta) \quad | \quad \theta, \chi) = \frac{1}{(1-\alpha)\,\alpha^{-\alpha}} G(L^i, Z^i, \theta)^{\alpha} q(\theta)^{1-\alpha}$$

$$-w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - \chi q(\theta), \tag{41}$$

- This problem gives the following simple inverse demand for intermediates of type θ:

$$q^i\left(\theta, \chi, L^i, Z^i\right) = \alpha^{-1} G(L^i, Z^i, \theta) \chi^{-1/\alpha}. \tag{42}$$

Economy M (continued)

- The problem of the monopolist is then to maximize its profits:

$$\max_{\theta,\chi,\left[q^i(\theta,\chi,L^i,Z^i)\right]_{i\in\mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i\in\mathcal{F}} q^i\left(\theta,\chi,L^i,Z^i\right) di - C(\theta) \tag{43}$$

subject to (42).

Definition

An equilibrium in Economy M is a set of firm decisions $\left\{L^i, Z^i, q^i\left(\theta,\chi,L^i,Z^i\right)\right\}_{i\in\mathcal{F}}$*, technology choice* θ*, and factor prices* (w_L, w_Z) *such that* $\left\{L^i, Z^i, q^i\left(\theta,\chi,L^i,Z^i\right)\right\}_{i\in\mathcal{F}}$ *solve (41) given* (w_L, w_Z) *and technology* θ*, (37) holds, and the technology choice and pricing decision for the monopolist,* (θ,χ)*, maximize (43) subject to (42).*

- Equilibrium easy to characterize because (42) defines a constant elasticity demand curve.

Economy M (continued)

- Profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and is equal to $\chi = 1$.
- Consequently, $q^i(\theta) = q^i(\theta, \chi = 1, \bar{L}, \bar{Z}) = \alpha^{-1} G(\bar{L}, \bar{Z}, \theta)$ for all $i \in \mathcal{F}$.
- Substituting this into (43), the profits and the maximization problem of the monopolist can be expressed as

$$\max_{\theta \in \Theta} \Pi(\theta) = G(\bar{L}, \bar{Z}, \theta) - C(\theta). \tag{44}$$

- Assumption 1 is no longer necessary. Instead only concavity in (L, Z) is imposed:

Assumption

Either $G(L^i, Z^i, \theta^i)$ is jointly strictly concave and increasing in (L^i, Z^i); or $G(L^i, Z^i, \theta^i)$ is increasing and exhibits constant returns to scale in (L^i, Z^i).

Economy M (continued)

Proposition

Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy M is a solution to

$$\max_{\theta' \in \Theta} F(\bar{L}, \bar{Z}, \theta') \equiv G(\bar{L}, \bar{Z}, \theta') - C\left(\theta'\right)$$

and any solution to this problem is an equilibrium technology.

Economy M (continued)

- Relative to Economies D and C, the presence of the monopoly markup implies greater distortions in this economy.
- But equilibrium technology is still a solution to a problem identical to that in Economy D or C, that of maximizing

$$F(\bar{L},\bar{Z},\theta) \equiv G(\bar{L},\bar{Z},\theta) - C(\theta).$$

- Aggregate (net) output in the economy can be computed as

$$Y(\bar{L},\bar{Z},\theta) \equiv \frac{2-\alpha}{1-\alpha} G(\bar{L},\bar{Z},\theta) - C(\theta).$$

- Note that if $C'(\theta) > 0$, then $\partial F(\bar{L},\bar{Z},\theta^*)/\partial\theta = 0$ implies $\partial Y(\bar{L},\bar{Z},\theta^*)/\partial\theta > 0$ (since $(2-\alpha)/(1-\alpha) > 1$).
- Factor prices slightly different, but no effect on comparative statics:

$$w_L = \frac{1}{1-\alpha}\frac{\partial G(\bar{L},\bar{Z},\theta)}{\partial L} = \frac{1}{1-\alpha}\frac{\partial F(\bar{L},\bar{Z},\theta)}{\partial L} = \frac{1}{2-\alpha}\frac{\partial Y(\bar{L},\bar{Z},\theta)}{\partial L}.$$

Equilibrium Bias: Definitions

- For simplicity, let us suppose that all of the functions introduced above are twice differentiable.

Definition

An increase in technology θ_j for $j = 1, ..., K$ is absolutely biased *towards factor L at $\bar{L}, \bar{Z}$ if $\partial w_L / \partial \theta_j \geq 0$.*

- Note the definition at *current* factor proportions.

Definition

Denote the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ by $\theta^* (\bar{L}, \bar{Z})$ and assume that $\partial \theta_j^* / \partial L$ exists at $(\bar{L}, \bar{Z})$ for all $j = 1, ..., K$. Then there is weak absolute equilibrium bias *at $(\bar{L}, \bar{Z})$ if*

$$\sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \geq 0. \tag{45}$$

Main Result on Weak Bias

Theorem

(Weak Absolute Equilibrium Bias) *Let the equilibrium technology at factor supplies* $(\bar{L}, \bar{Z})$ *be* $\theta^*(\bar{L}, \bar{Z})$ *and assume that* $\theta^*(\bar{L}, \bar{Z})$ *is in the interior of* Θ *and that* $\partial\theta_j^*/\partial L$ *exists at* $(\bar{L}, \bar{Z})$ *for all* $j = 1, ..., K$. *Then, there is weak absolute equilibrium bias at all* $(\bar{L}, \bar{Z})$, *i.e.,*

$$\sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \geq 0 \text{ for all } (\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}, \tag{46}$$

with strict inequality if $\partial\theta_j^*/\partial L \neq 0$ *for some* $j = 1, ..., K$.

Why Is This True?

- The result is very intuitive.
- Consider the case where $\theta \in \Theta \subset \mathbb{R}$ (the general case is similar with more notation).
- In equilibrium we have $\partial F / \partial \theta = 0$ and $\partial^2 F / \partial \theta^2 \leq 0$.
- Then from the Implicit Function Theorem

$$\frac{\partial \theta^*}{\partial L} = -\frac{\partial^2 F / \partial \theta \partial L}{\partial^2 F / \partial \theta^2} = -\frac{\partial w_L / \partial \theta}{\partial^2 F / \partial \theta^2}, \tag{47}$$

- And therefore,

$$\frac{\partial w_L}{\partial \theta} \frac{\partial \theta^*}{\partial L} = -\frac{\left(\partial w_L / \partial \theta\right)^2}{\partial^2 F / \partial \theta^2} \geq 0. \tag{48}$$

- Moreover, if $\partial \theta^* / \partial L \neq 0$, then from (47), $\partial w_L / \partial \theta \neq 0$, so (48) holds with strict inequality.

Intuition

- Similarity to the LeChatelier principle
  - but in *general equilibrium*, which is important as we will see.
- More detailed intuition:
  - Suppose that L and θ are complements (i.e., $\partial^2 F/\partial\theta\partial L \geq 0$), then an increase in L increases the incentives to improve θ, but then this raises the marginal contribution of to L output and thus wages$\longrightarrow$*biased change.*
  - Suppose that L and θ are substitutes (i.e., $\partial^2 F/\partial\theta\partial L < 0$), then an increase in L reduces the incentives to improve θ, but then this increases the marginal contribution of L to output and thus wages$\longrightarrow$*biased change*

Equilibrium Bias: Further Results

- The main result above is "local" in the sense that it is true only for small changes.
- Interestingly, it may not be true for large changes, because technological change that is biased *towards* labor at certain factor proportions may be biased *against* labor at certain other factor proportions.
- We thus need to ensure that such "reversals" not happen.
- These will be "supermodularity" type conditions.

Equilibrium Bias: Further Results (continued)

- Let us define:

Definition

Let θ^* be the equilibrium technology choice in an economy with factor supplies $(\bar{L}, \bar{Z})$. Then there is global absolute equilibrium *bias if for any $\bar{L}' \geq \bar{L}$*

$$w_L\left(\tilde{L}, \bar{Z}, \theta^*\left(\bar{L}', \bar{Z}\right)\right) \geq w_L\left(\tilde{L}, \bar{Z}, \theta^*\left(\bar{L}, \bar{Z}\right)\right) \text{ for all } \tilde{L}, \bar{Z}.$$

- Note: two notions of "globality" in this definition:
  - Large changes
  - Statement about factor prices at all intermediate factor proportions.

Global Results

Theorem

(Global Equilibrium Bias) *Suppose that* Θ *is a lattice, let* $\theta^*(\bar{L}, \bar{Z})$ *be the equilibrium technology at factor proportions* $(\bar{L}, \bar{Z})$*, and suppose that* $F(Z, L, \theta)$ *is continuously differentiable in* Z*, supermodular in* θ *on* Θ *for all* Z *and* L*, and exhibits strictly increasing differences in* (Z, θ)*, then there is global absolute equilibrium bias, i.e., for any* $\bar{L}' \geq \bar{L}$*,*

$$\theta^*(\bar{L}', \bar{Z}) \geq \theta^*(\bar{L}, \bar{Z}) \text{ for all } \bar{Z}\text{, and}$$

$$w_L(\tilde{L}, \bar{Z}, \theta^*(\bar{L}', \bar{Z})) \geq w_L(\tilde{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z})) \text{ for all } \tilde{L} \text{ and } \bar{Z}, \tag{49}$$

with strict inequality if $\theta^*(\bar{L}', \bar{Z}) \neq \theta^*(\bar{L}, \bar{Z})$*.*

- This result follows from Topkis's Monotonicity Theorem.

Definitions

Definition

Denote the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ *by* $\theta^* (\bar{L}, \bar{Z})$ *and suppose that* $\partial\theta_j^* / \partial Z$ *exists at* $(\bar{L}, \bar{Z})$ *for all* $j = 1, ..., K$. *Then there is* strong absolute equilibrium bias *at* $(\bar{L}, \bar{Z})$ *if*

$$\frac{dw_L}{dL} = \frac{\partial w_L}{\partial L} + \sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} > 0.$$

- In this definition, dw_L / dL denotes the total derivative, while $\partial w_L / \partial L$ denotes the partial derivative holding $\theta = \theta^* (\bar{L}, \bar{Z})$.
- Recall also that if F is jointly concave in (L, θ) at $(L, \theta^* (\bar{L}, \bar{Z}))$, its Hessian with respect to (L, θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, is negative semi-definite at this point (though negative semi-definiteness is not sufficient for local joint concavity).

Main Result

Theorem

(Nonconvexity and Strong Bias) *Suppose that* Θ *is a convex subset of* $\mathbb{R}^K$, F *is twice continuously differentiable in* (L, θ), *let* $\theta^* (\bar{L}, \bar{Z})$ *be the equilibrium technology at factor supplies* $(\bar{L}, \bar{Z})$ *and assume that* θ^* *is in the interior of* Θ *and that* $\partial\theta_j^* (\bar{L}, \bar{Z}) / \partial L$ *exists at* $(\bar{L}, \bar{Z})$ *for all* $j = 1, ..., K$. *Then there is strong absolute equilibrium bias at* $(\bar{L}, \bar{Z})$ *if and only if* $F(L, Z, \theta)$*'s Hessian in* (L, θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, *is not negative semi-definite at* $(\bar{L}, \bar{Z})$.

Corollary: There cannot be strong bias in a fully competitive economy such as Economy D.

- This is because competitive equilibrium exists only when the production possibilities set is locally convex.

Why Is This True?

- Again, for simplicity, take the case where $\Theta \subset \mathbb{R}$.
- The fact that θ^* is the equilibrium technology implies that $\partial F / \partial \theta = 0$ and that $\partial^2 F / \partial \theta^2 \leq 0$.
- Moreover, we still have

$$\frac{\partial \theta^*}{\partial L} = -\frac{\partial^2 F / \partial \theta \partial L}{\partial^2 F / \partial \theta^2} = -\frac{\partial w_L / \partial \theta}{\partial^2 F / \partial \theta^2}.$$

- Substituting this into the definition for dw_L / dL and recalling that $\partial w_L / \partial L = \partial^2 F / \partial L^2$, we have the condition for strong absolute equilibrium bias as

$$\begin{aligned} \frac{dw_L}{dL} &= \frac{\partial w_L}{\partial L} + \frac{\partial w_L}{\partial \theta} \frac{\partial \theta^*}{\partial L}, \\ &= \frac{\partial^2 F}{\partial L^2} - \frac{\left(\partial^2 F / \partial \theta \partial L\right)^2}{\partial^2 F / \partial \theta^2} > 0. \end{aligned}$$

Why Is This True?

- From Assumption 1 or 2, F is concave in Z, so $\partial^2 F/\partial L^2 \leq 0$, and from the fact that θ^* is an equilibrium and $\partial\theta^*/\partial L$ exists, we also have $\partial^2 F/\partial\theta^2 < 0$.
- Then the fact that F's Hessian, $\nabla^2 F_{(L,\theta)(L,\theta)}$, is not negative semi-definite at $(\bar{L}, \bar{Z})$ implies that

$$\frac{\partial^2 F}{\partial L^2} \times \frac{\partial^2 F}{\partial\theta^2} < \left(\frac{\partial^2 F}{\partial L \partial L\theta}\right)^2, \tag{50}$$

- Since at the optimal technology choice, $\partial^2 F/\partial\theta^2 < 0$, this immediately yields $dw_L/dZ > 0$, establishing strong absolute bias at $(\bar{L}, \bar{Z})$ as claimed in the theorem.
- Conversely, if $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite at $(\bar{L}, \bar{Z})$, then (50) does not hold and this together with $\partial^2 F/\partial\theta^2 < 0$ implies that $dw_L/dL \leq 0$.

Intuition

- Induced bias can be strong enough to overwhelm the standard "substitution effect" leading to downward sloping demand curves.
- Why is "local nonconvexity" sufficient?
- If there is local nonconvexity, then we are not at a global maximum but at a **saddle point.**
  - with technology and factor demands chosen by different firms/agents;
  - note that this is all that equilibrium requires.
- Then there exists a direction in which output can be increased locally.
- A change in L induces technology firms to move θ in that direction, and locally this increases the marginal contribution of L to all put (and thus wages).

Labor Scarcity and Technology

- Let us now turn to the effect of labor scarcity on "technological advances".
- Results so far silent on this, since either an "increase" or a "decrease" in θ may correspond to technology advances.
- Let us focus on labor scarcity for simplicity, but the results apply to "wage push" provided that equilibrium labor demand downward is sloping (more on this below).

Definitions

- Suppose that $C(\theta)$ strictly increasing in θ everywhere, so that higher θ corresponds to *technological advances*.

Assumption

(Supermodularity) *$G(L, Z, \theta)$ [$Y(L, Z, \theta)$] is supermodular in θ on Θ for all L and Z.*

Definition

Technological progress is strongly labor saving *at $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$ if there exist neighborhoods $\mathcal{B}_\Theta$, $\mathcal{B}_L$ and $\mathcal{B}_Z$ of $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$ such that $Y(L, Z, \theta)$ exhibits decreasing differences in (L, θ) on $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\Theta$.*
Technological progress is strongly labor complementary *at $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$ if there exist neighborhoods $\mathcal{B}_\Theta$, $\mathcal{B}_L$ and $\mathcal{B}_Z$ of $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$ such that $Y(L, Z, \theta)$ exhibits increasing differences in (L, θ) on $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\Theta$.*

- $Y(L, Z, \theta)$ is increasing in θ since $C(\theta)$ is strictly increasing.

Main Result

Theorem

Suppose that Y is supermodular in θ. Then labor scarcity starting from $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$ will induce technological advances if technology is strongly labor saving at $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$.
Conversely, labor scarcity will discourage technological advances if technology is strongly labor complementary.

- The result from Topkis's Monotonicity Theorem.
- Throughout, important ingredient is that in Economies M, O or E, the equilibrium condition $\partial F\left(\bar{L}, \bar{Z}, \theta^{*}\right) / \partial\theta = 0$ implies

$$\partial Y\left(\bar{L}, \bar{Z}, \theta^{*}\right) / \partial\theta > 0.$$

Interpretation

- But this result is not possible in Economy D.
  - Because by construction in this economy,

$$\partial F\left(\bar{L}, \bar{Z}, \theta^*\right)/\partial\theta = \partial Y\left(\bar{L}, \bar{Z}, \theta^*\right)/\partial\theta = 0,$$

  so there cannot be "local technology advances" starting in equilibrium.

- Note also that even when technologies strongly labor saving, this does not imply that an exogenous increase in wages will lead to a Pareto improvement.
  - But it is also possible to construct examples, in Economies M, and O, where this is the case.

Implications

- What does this theorem imply?
  1. Wage push and labor scarcity can easily induce technological advances
  2. But there is no guarantee that they will and the opposite is equally (or more) likely.
- We need to understand what the condition "strongly labor saving" entails.
- It turns out that technology is generally *not* strongly labor saving with Cobb-Douglas or CES production functions (in labor and capital or other factors of production).
- But this is partly a shortcoming of these production functions. With other approaches to production structure, the situation is different, in particular, when machines "replace labor".

Static Model: Production

- Acemoglu and Restrepo (2016), model of race between machine and man.
- There is a unique final good Y produced by combining a continuum of tasks $y(i)$, with $i \in [N-1, N]$.

$$Y = \left(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \sigma \in (0, \infty): \text{ elasticity of substitution.}$$

- Set the resulting ideal price index as numeraire.
- The range $N-1$ to N implies that the set of tasks is constant, but older tasks might be replaced by new (more complex and more productive) versions thereof.
- Namely, an increase in N adds a new task at the top while simultaneously replacing one at the bottom.

Static Model: Intermediates

- Each task is produced by combining capital or labor with an intermediate good $q(i)$ embodying technology.
- In preparation for the model with endogenous technology, we assume that each intermediate is supplied by a monopolist, which can produce one unit of intermediate at the cost of $\mu\psi$ units of the final good (where $\mu \in (0,1)$).
- There is also fringe of competitive imitators that can copy the technology for each intermediate and produce it at the cost of ψ units of the final good.
- We assume that μ is such that the unconstrained monopoly price is greater than ψ.
- This implies that in the pricing game between the monopolist and the fringe, there will be a limit price equilibrium, and each unit of every intermediate will be sold at a constant price ψ.

Static Model: Task Production Function

- Tasks with $i \leq I$ are technologically **automated**, and can be produced with labor or capital according to

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(k(i) + \gamma(i)l(i)\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}$$

- Tasks with $i > I$ are not technologically automated yet, and can only be produced with labor:

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(\gamma(i)l(i)\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}.$$

- We assume $\gamma(i)$ is strictly increasing, so labor has a comparative advantage in more complex tasks (in fact, it is more productive in these tasks), and normalize $B \equiv (1-\eta)^{\zeta/(1-\zeta)}$.
- The assumption that labor-intensive tasks use no capital can be easily relaxed.

Static Model: Factor Supplies

- In the static model, we take capital to be fixed at K and rented at a price r (determined endogenously).
- Total labor used is given by

$$L^s\left(\frac{W}{rK}\right),$$

where L^s is a weakly increasing function, and W is the wage rate.

  - This is a reduced form for many different models of labor supply and quasi-labor supply behavior.

Equilibrium: Task Prices

- Let $c^u(\cdot)$ be the unit cost of production for a task as a function of the price of the relevant factor.
- Then, equilibrium task prices will be given by

$$p(i) = \begin{cases} c^u\left(\min\left\{r, \frac{W}{\gamma(i)}\right\}\right) \equiv \left[\psi^{1-\zeta} + (\frac{1-\eta}{\eta})^{\zeta} \min\left\{r, \frac{W}{\gamma(i)}\right\}^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \text{if } i \leq I, \\ c^u\left(\frac{W}{\gamma(i)}\right) \equiv \left[\psi^{1-\zeta} + (\frac{1-\eta}{\eta})^{\zeta} \left(\frac{W}{\gamma(i)}\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \text{if } i > I, \end{cases} \tag{51}$$

- This expression shows that, for the static model, the CES structure is largely irrelevant. We could simply work with the cost functions. The CES functional form is important for the dynamic extensions.

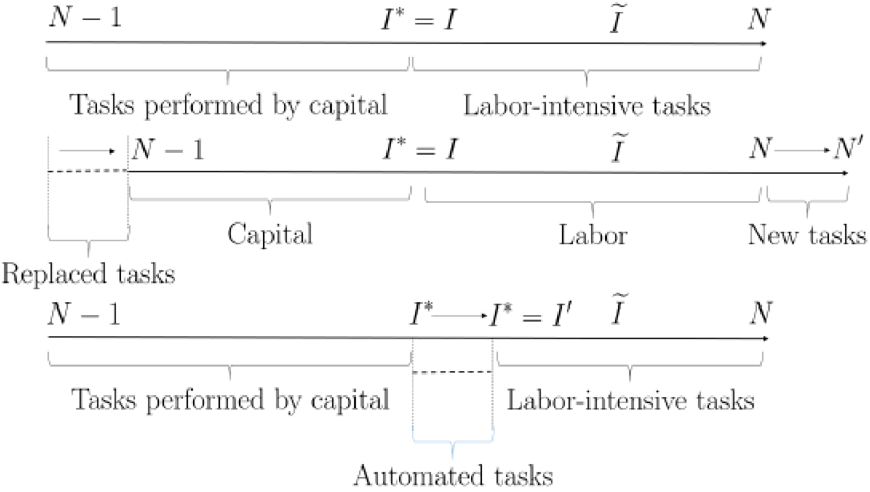
Equilibrium: Factor Prices

- Given factor prices, firms are indifferent between using capital and labor in task $\widetilde{I}$:

$$\frac{W}{r} = \gamma(\widetilde{I}). \tag{52}$$

- Tasks with $i \leq I^* \equiv \min\{I, \widetilde{I}\}$ will be automated and produced with capital, and tasks with $i > I^*$ will be produced with labor.

Equilibrium: Task Space



Equilibrium: Market Clearing

- Factor demands in capital- and labor-intensive tasks are

$$k(i) = Yc^u(r)^{\zeta-\sigma}r^{-\zeta} \text{ if } i \leq I^*$$

and

$$l(i) = Y\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta} \text{ if } i > I^*.$$

- Thus, factor market clearing conditions can be written as

$$Y(\min\{I,\widetilde{I}\} - N + 1)c^u(r)^{\zeta-\sigma}r^{-\zeta}di = K, \tag{53}$$

and

$$Y\int_{\min\{I,\widetilde{I}\}}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}W^{-\zeta}di = L^s\left(\frac{W}{rK}\right). \tag{54}$$

Equilibrium in the Static Model

Assumption 1: $\left(\frac{\gamma(N-1)}{\gamma(N)}\right)^{2+\sigma+\eta} > |\sigma - \zeta|$, $\zeta \to 1$, or $\eta \to 0$.

Proposition (Equilibrium in the static model)

Suppose Assumption 1 holds. Then for any range of tasks $[N-1, N]$, automation $I \in (N-1, N]$, and capital K, there exists a unique equilibrium characterized by factor prices, W and r, and threshold tasks, $\widetilde{I}$ and I^*, such that: (i) $\widetilde{I}$ is determined by equation (52) and $I^* = \min\{I, \widetilde{I}\}$; (ii) all tasks $i \leq I^*$ are produced using capital and all tasks $i > I^*$ are produced using labor; (iii) capital and labor market clearing conditions, equations (53) and (54), are satisfied; and (iii) factor prices satisfy:

$$(I^* - N + 1) c^u(r)^{1-\sigma} + \int_{I^*}^{N} c^u\left(\frac{W}{\gamma(i)}\right)^{1-\sigma} di = 1. \qquad (55)$$

Diagrammatic Representation

- Let $\omega \equiv \frac{W}{rK}$.

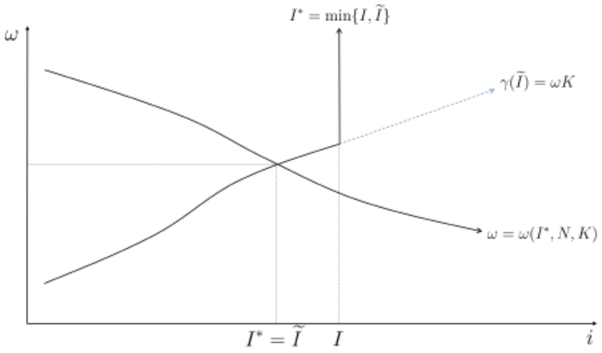


Figure: Graphical representation of the equilibrium. Case with $I^* < I$.

Diagrammatic Representation

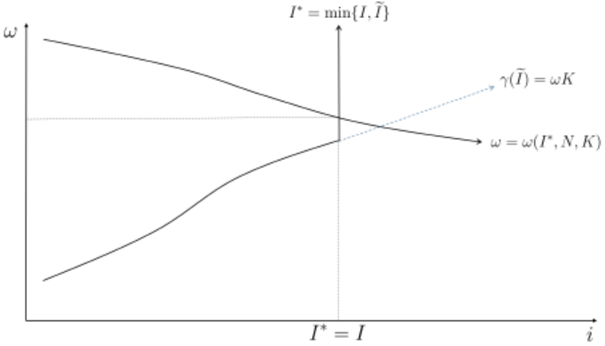


Figure: Graphical representation of the equilibrium. Case with $I^* = I$.

Comparative Statics

Proposition (Comparative statics if $I^* = I < \widetilde{I}$)

Suppose Assumption role 1 holds. Then

$$\frac{d\ln(W/r)}{dI} = \frac{d\ln\omega}{dI} = \frac{1}{\omega}\frac{\partial\omega}{\partial I^*} < 0, \; \frac{d\ln(W/r)}{dN} = \frac{d\ln\omega}{dN} = \frac{1}{\omega}\frac{\partial\omega}{\partial N} > 0$$

and

$$\frac{d\ln(W/r)}{d\ln K} = \frac{d\ln\omega}{d\ln K} + 1 = \frac{1+\varepsilon_L}{\sigma_{SR}+\varepsilon_L} > 0,$$

where, $\sigma_{SR} \in (0,\infty)$ is the short-run elasticity of substitution between labor and capital, which is a weighted average of σ and ζ, and $\varepsilon_L = \frac{\partial \ln L^s(\omega)}{\partial \ln \omega}$ is the elasticity of the quasi-labor supply.

- *Moreover, $\frac{d\ln W}{dI}, \frac{d\ln r}{dN} < 0$ if σ_{SR} is sufficiently large. Otherwise $\frac{d\ln W}{dI}, \frac{d\ln r}{dN} \geq 0$.*

Comparative Statics (continued)

Proposition (Comparative statics if $I^* = \widetilde{I} < I$)

Let $\varepsilon_\gamma \equiv \frac{\partial \ln \gamma(I)}{\partial I} > 0$. *Then:*

$$\frac{d\ln(W/r)}{dI} = \frac{d\ln\omega}{dI} = 0, \; \frac{d\ln(W/r)}{dN} = \frac{d\ln\omega}{dN} = \frac{\frac{1}{\omega}\frac{\partial\omega}{\partial N}}{1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}} > 0$$

and

$$\frac{d\ln(W/r)}{d\ln K} = \frac{d\ln\omega}{d\ln K} + 1 = \left(\frac{1+\varepsilon_L}{\sigma_{SR}+\varepsilon_L}\right)\frac{1}{1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}} > 0.$$

Here, the medium run elasticity of substitution $\sigma_{MR} \in (0,\infty)$ *is given by*

$$\sigma_{MR} = (\sigma_{SR} + \varepsilon_L)\left(1 - \frac{1}{\omega}\frac{\partial\omega}{\partial I^*}\frac{1}{\varepsilon_\gamma}\right) - \varepsilon_L > \sigma_{SR}.$$

- *Moreover,* $\frac{d\ln r}{dN} < 0$ *if* σ_{MR} *is sufficiently large, and* $\frac{d\ln r}{dN} \geq 0$ *otherwise.*

Comparative Statics: Interpretation

- Novel element: all comparative statics driven by the changes in the allocation of tasks to factors.
- In particular, new result relative to the standard factor-augmenting technology framework: technological advances can reduce factor prices (here wages for automation and the rental rate on capital for new tasks).
- This is related to Acemoglu and Autor (2011): technologies change the range of tasks performed by factors, creating "strong price effects".
- Note also that when $\widetilde{I} < I$, the elasticity of substitution between capital and labor is σ_{MR} rather than σ_{SR} because of the endogenous changes in the set of tasks produced by capital (in response to changes in factor prices).

Special Cases

- Two special cases further highlight the workings of the model: $\eta \longrightarrow 0$ (so intermediates determine the equilibrium allocation of tasks to factors, but do not get revenue) or $\zeta \longrightarrow 1$ (where their revenues become a constant fraction of total revenue).

- In this case,

$$\ln \omega = \left(\frac{1}{\hat{\sigma}} - 1 \right) \ln K + \frac{1}{\hat{\sigma}} \ln \left(\frac{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di}{I^* - N + 1} \right), \text{ and}$$

$$Y = \left[(I^* - N + 1)^{\frac{1}{\hat{\sigma}}} K^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} + \left(\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di \right)^{\frac{1}{\hat{\sigma}}} L^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} \right]^{\frac{\hat{\sigma}}{\hat{\sigma}-1}},$$

where $\hat{\sigma} \equiv \eta + (1 - \eta)\sigma$ (and thus when $\eta \longrightarrow 0$, $\hat{\sigma} = \sigma$), highlighting the role of the two different types of technologies in changing the role of the two factors in the (derived) aggregate production function.

Dynamic Model: Preferences and Resource Constraint

- We now move to a dynamic model with capital accumulation and endogenous technological change.
- A representative household economy with preferences over consumption

$$\int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

and the resource constraint:

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t) - \psi\mu \int_{N-1}^{N} q(i,t) di.$$

- $r(t)$ is the rental rate of capital and depreciation is δ.

The Structure of Balanced Growth Path

- We assume the specific form for the comparative advantage schedule:

$$\gamma(i) = e^{Ai}, \text{ with } A > 0.$$

- This implies that labor is more productive in new more complex tasks and will build growth through quality improvements.
- Assumption 1 takes the form:

Assumption 1′: One of the following three conditions are satisfied:

1. $e^{-(2+2\sigma+\eta)A} > |\sigma - \zeta|$, or
2. $\zeta \to 1$, or
3. $\eta \to 0$.

- We start by assuming exogenous technological change, and define

$$n(t) \equiv N(t) - I(t)$$

The Structure of Balanced Growth Path (continued)

- Assume $I^* = I$ (as we will guarantee later).
- Normalize the variables $y(t) \equiv Y(t)/\gamma(I(t))$, $k(t) \equiv K(t)/\gamma(I(t))$, $c(t) \equiv C(t)/\gamma(I(t))$, and $w(t) \equiv W(t)/\gamma(I(t))$.
- The market clearing conditions become:

$$y(t)(1-n(t))c^u(r(t))^{\zeta-\sigma}r(t)^{-\zeta} = k(t),$$

and

$$y(t)\int_0^{n(t)} \gamma(i)^{\zeta-1}c^u\left(\frac{w(t)}{\gamma(i)}\right)^{\zeta-\sigma} w^{-\zeta}di = L^s\left(\frac{w(t)}{r(t)k(t)}\right).$$

- Additionally, the ideal price index condition becomes

$$(1-n(t))c^u(r(t))^{1-\sigma}di + \int_0^{n(t)} c^u\left(\frac{w(t)}{\gamma(i)}\right)^{1-\sigma} di = 1.$$

- These uniquely determine the rate of return on capital, $r(t) = r^E(n(t), k(t))$, wages $w^E(n(t), k(t))$, and net output, $f^E(n(t), k(t))$.

The Structure of Balanced Growth Path (continued)

- Define a BGP as an equilibrium in which W, K and Y grow at a constant rate, g, and the interest rate, r, is constant.
- Thus, in a BGP the normalized variables converge to fixed values, and necessarily $\dot{I} = \dot{N} = \Delta$, so that $n(t)$ is constant.
- Their behavior outside the steady state is determined by the Euler equation

$$\frac{\dot{c(t)}}{c(t)} = \frac{1}{\theta}(r^E(n(t), k(t)) - \delta - \rho) - A\Delta, \tag{56}$$

and resource constraint

$$\dot{k}(t) = f^E(k(t), n(t)) - c(t) - (\delta + A\Delta)k(t). \tag{57}$$

Dynamic Equilibrium: Summary

Proposition (Dynamic equilibrium with exogenous technological change)

Suppose Assumption $1'$ holds and technology evolves exogenously. There exists a threshold $\overline{\rho}$ such that, for $\rho > \overline{\rho}$:

1. *There exists $\overline{n}$ such that, for $n(t) < \overline{n}$, we have $I^* < I$, while for $n(t) \geq \overline{n}$, $I^*(t) = I(t)$.*
2. *Suppose that $\lim_{t\to\infty} n(t) = n \in [\overline{n}, 1]$. Then a unique balanced growth path exists if and only if asymptotically $\dot{N} = \dot{I} = \Delta$. In this balanced growth path $I^*(t) = I(t)$. Y, C, K and w grow at a constant rate $A\Delta$ and r is constant.*
3. *Suppose that $\lim_{t\to\infty} n(t) = n < \overline{n}$. Then there exists a unique balance growth path, and it features $I^*(t) < I(t)$.*
4. *Given such paths for technology, the dynamic equilibrium is unique starting from any initial condition and converges to the balanced growth path, and new tasks are immediately utilized with labor.*

Dynamic Equilibrium: Diagrammatic Representation

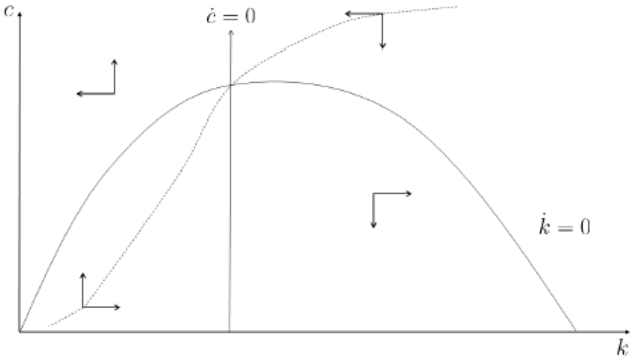


Figure: BGP and dynamics for our model with exogenous technological change and $n(t) \rightarrow n$.

Directed Technology: Innovation Possibilities Frontier

- We now endogenize technology by assuming that it can be developed by firms using scientists, which are in inelastic supply, S.
- $S_I(t) \geq 0$ of these scientists are hired by monopolists at a competitive wage w^S for automation, and $S_N(t) \geq 0$ of them are hired for creating new tasks. The market clearing condition for scientists is

$$S_I(t) + S_N(t) \leq S.$$

- Advances in automation and creation of new tasks follow the next two differential equations

$$\dot{I}(t) = \kappa_I S_I(t), \tag{58}$$

and

$$\dot{N}(t) = \kappa_N S_N(t), \tag{59}$$

where κ_I and κ_N are positive constants.

Dynamic Model: Profits and Value Functions (continued)

- The flow profits from automation, which naturally replaces a task previously performed by labor (i.e., $i > I(t)$), can be written as

$$\pi_I(t,i) = Y(t)(1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}c^u\left(r(t)\right)^{\zeta-\sigma}.$$

- Flow profits of producing such task with an intermediate technology that only allows the use of labor, is given by

$$\pi_N(t,i) = Y(t)(1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}c^u\left(\frac{W(t)}{\gamma(i)}\right)^{\zeta-\sigma}.$$

Dynamic Model: Profits and Value Functions (continued)

- A different director technology assumption: assume a patent structure in which new entrants must get the license to build on the previous innovation (no "creative destruction of profits").
- They make a take-it-or-leave it offer to this previous patentees, and will pay them, in equilibrium, the full continuation value.
- Therefore, values from automation and new labor-intensive tasks depend on differences in costs:

$$V_I(t) = cst \int_t^{\infty} e^{-\int_t^{\tau}(r(s)-\delta)ds} Y(\tau) \left(c^u \left(r(\tau)\right)^{\zeta-\sigma} - c^u \left(\frac{W(\tau)}{\gamma(I(t))} \right)^{\zeta-\sigma} \right) d\tau, \tag{60}$$

$$V_N(t) = cst \int_t^{\infty} e^{-\int_t^{\tau}(r(s)-\delta)ds} Y(\tau) \left(c^u \left(\frac{W(\tau)}{\gamma(N(t))} \right)^{\zeta-\sigma} - c^u \left(r(\tau)\right)^{\zeta-\sigma} \right) d\tau. \tag{61}$$

- Observe that these values are positive only when $\sigma > \zeta$ (intermediates and labor or capital are gross complements).

Dynamic Model: Equilibrium conditions

- A dynamic equilibrium with endogenous technology is determined by:
  - The evolution of the state variables is given by

$$\begin{aligned} \dot{k}(t) &= f^E(k(t), n(t)) - c(t) - (\delta + A\kappa_I S_I(t))k(t) \\ \dot{n}(t) &= \kappa_N (S - S_I(t)) - \kappa_I S_I(t). \end{aligned}$$

  - Consumption satisfies the Euler equation

$$\dot{c}(t) = c(t)\left(\frac{1}{\theta}(r^E(k(t), n(t)) - \delta - \rho) - A\kappa_I S_I(t)\right).$$

  - The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 \text{ if } \kappa_I V_I(t) < \kappa_N V_N(t) \\ \in [0, S] \text{ if } \kappa_I V_I(t) = \kappa_N V_N(t) \\ S \text{ if } \kappa_I V_I(t) > \kappa_N V_N(t) \end{cases}.$$

    (In the paper, more general with a potentially smooth response).
  - A transversality condition for the household holds.

Dynamic Equilibrium: Summary

Proposition (Directed technological change)

Suppose that $\sigma > \zeta$ and $\rho > \bar{\rho}$. There exists $\bar{S}$ such that, for $S < \bar{S}$:

1. *There exists $\bar{\kappa}$ such that for $\frac{\kappa_I}{\kappa_N} > \bar{\kappa}$ there is a balanced growth path. In this balanced growth path, Y, C, K and W grow at the constant rate $g = A\frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$, and r, the labor share and employment are constant. Along this path we have $I^*(t) = I(t)$ and $N(t) - I(t) = n^D$, with n^D determined endogenously from the condition $\kappa_N V_N = \kappa_I V_I$, and satisfying $n^D \in (\bar{n}, 1)$.*
2. *In addition, there exists $\bar{\bar{\rho}} \geq \bar{\rho}$, such that if $\rho > \bar{\bar{\rho}}$, the balanced growth path is unique.*
3. *Suppose $\rho > \bar{\bar{\rho}}$. Then, when $\theta = 0$, the dynamic equilibrium is globally asymptotically (saddle-path) stable. For $\theta > 0$, this dynamic equilibrium is unique and is locally asymptotically (saddle-path) stable.*

Determination of the Balanced Growth Path

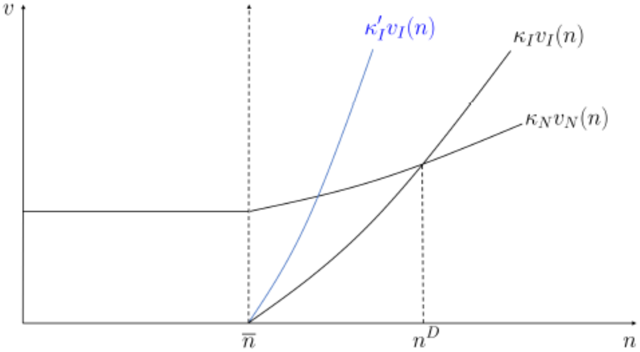


Figure: Determination of n^D in steady state.

NB. Note that $n > \overline{n}$ is no longer imposed. This is guaranteed by $\rho > \overline{\rho}$ and $S < \overline{S}$.

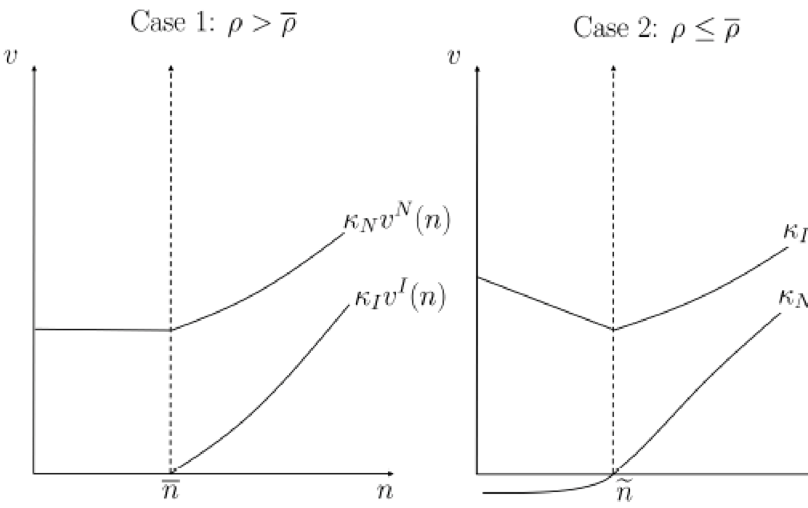
The Race between Machine and Man

- Consider an increase in I away from the balance growth path. This reduces the labor share and total employment.
- But it also reduces the wage per effective unit of labor $W/\gamma(I)$ relative to interest rates.
- Profit-making incentives create forces for **self-correction**, i.e., for the economy to revert back to the same balanced growth path, employment and factor shares.
- The role of $\sigma > \zeta$ is important: an increase in I reduces $W/\gamma(I)$ and this *increases* $V_N(t)$ relative to $V_I(t)$, providing incentives for monopolists to introduce new more complex (more labor-intensive) tasks, instead of automating tasks in which the production cost with labor has fallen.
- But this effect needs to be stronger than the productivity effect. Our assumptions ensure that it is.

Other Potential Equilibrium Paths

- In the proposition we imposed $\frac{\kappa_I}{\kappa_N} > \overline{\kappa}$. What happens if $\frac{\kappa_I}{\kappa_N} < \overline{\kappa}$?
  - In this case, the economy admits a "Schumpeterian" long-run equilibrium, in which there is economic growth driven by continuous improvement of labor productivity coming from the introduction of new tasks, and automation remains unprofitable.
  - This is like an "$A(t)L$" economy.
- We also imposed $\rho > \overline{\rho}$? What happens when $\rho \leq \overline{\rho}$?
  - In this case, we obtain an "AK" economy.
  - In particular, now capital is very cheap, and automation is profitable, while the use of new tasks with labor is not.
  - In this case, the economy grows by capital deepening (by automation).

Other Types of Potential Equilibrium Paths



Interpretation and Additional Effects

- The new threshold $\overline{\overline{\rho}} \geq \overline{\rho}$ is also related to the productivity effect:
  - When $\rho < \overline{\overline{\rho}}$, the productivity effect resulting from from automation increases wages (in the balance growth path) so much that it actually further stimulates the incentives for automation. This can lead to multiple balanced growth paths.
  - Thus $\rho > \overline{\overline{\rho}}$ ensures that the balance growth path is unique.
- Why did we impose $S < \overline{S}$?
  - When $S > \overline{S}$, there is excessively rapid economic growth, and thus rapid growth of wages. This implies that even new tasks that can now be produced much more cheaply using labor than capital are not necessarily profitable, because they will have to pay much higher wages in the near future.
  - The condition $S < \overline{S}$ ensures that the comparison of current costs is sufficient to guarantee the profitability of introducing new tasks.

Conclusion

- Different approaches to the competition between capital and labor possible.
- Very much work in progress.