

Labor Economics, 14.661. Lectures 7-9: Investments in General and Specific Skills

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Introduction

- A large fraction of earnings growth in the US economy is due to growth of wages of workers along their careers.
- Ben-Porath: this is due to continued investment.
- What type of investment during the worker's career? *Training*.
- But training investments are somewhat different from those modeled by Ben-Porath, because both the decision to undertake the investments and the cost of investments are potentially shared by the worker and his current employer.
- First: incomplete contracts.

Introduction

- Important missing element so far: *incomplete contracts*
- For a variety of reasons, not all aspects of employment relations are regulated by contracts.
- Why are contracts incomplete?
- Two possibilities:
 - ① Certain events are not (easily) observable by outside parties, in particular by courts. Therefore, writing contracts that are contingent on these *nonverifiable* events might be impossible or too costly.
 - Critique: implementation schemes can be used for creating contingencies on nonverifiable events.
 - ② There is a multitude of future contingencies, and often even describing them is difficult.
 - Critiques: implementation schemes can overcome even these difficulties.
- Despite these critiques, most contracts in practice seem to be incomplete.

Incomplete Contracts in Labor

- For our purposes, it is important to investigate the implications of incomplete contracts for investments by workers and firms
 - even if the microfoundations are not always clear.
- This lecture:
 - potential underinvestment by firms
 - impact of organizational forms
- Later: implications for general and firm-specific training.

Investments without Binding Contracts

- A firm and a worker are matched together, and because of labor market frictions, they cannot switch partners, so wages are determined by bargaining.
- As long as it employs the worker, the total output of the firm is

$$f(k)$$

where k is the amount of physical capital of the firm.

- Standard assumptions on f : increasing, continuous and strictly concave.

Investments without Binding Contracts: Timing of Events

- The timing of events:
 - The firm decides how much to invest, at the cost rk .
 - The worker and the firm bargain over the wage, w . We assume that bargaining can be represented by the *Nash solution* with asymmetric bargaining powers. In this bargaining problem, if there is disagreement, the worker receives an outside wage, $\bar{w}$, and the firm produces nothing, so its payoff is $-rk$.
- Equilibrium concept: *Subgame Perfect Nash Equilibrium* → backward induction.
- Start with the asymmetric Nash solution to bargaining with the bargaining power of the worker equal to β .

Digression: Nash Bargaining

- The (asymmetric) Nash solution to bargaining between two players, 1 and 2, is given by maximizing

$$(\text{payoff}_1 - \text{outside option}_1)^\beta (\text{payoff}_2 - \text{outside option}_2)^{1-\beta}. \quad (1)$$

- Why?
- Nash's bargaining theorem considers the bargaining problem of choosing a point x from a set $X \subset \mathbb{R}^N$ for some $N \geq 1$ by two parties with utility functions $u_1(x)$ and $u_2(x)$, such that if they cannot agree, they will obtain respective *disagreement payoffs* d_1 and d_2 .

Digression: Nash Bargaining (continued)

Nash Bargaining theorem

- Suppose we impose the following four axioms on the problem and solution:
 - (1) $u_1(x)$ and $u_2(x)$ are Von Neumann-Morgenstern utility functions, in particular, unique up to positive linear transformations;
 - (2) Pareto optimality, the agreement point will be along the frontier;
 - (3) Independence of Irrelevant Alternatives; suppose $X' \subset X$ and the choice when bargaining over the set X is $x' \in X'$, then x' is also the solution when bargaining over X' ;
 - (4) Symmetry; identities of the players do not matter, only their utility functions.
- Then, there exists a unique bargaining solution that satisfies these four axioms. This unique solution is given by

$$x^{NS} = \arg \max_{x \in X} (u_1(x) - d_1)(u_2(x) - d_2).$$

Digression: Nash Bargaining (continued)

- If we relax the symmetry axiom, so that the identities of the players can matter (e.g., worker versus firm have different “bargaining powers”), then we obtain:

$$x^{NS} = \arg \max_{x \in X} (u_1(x) - d_1)^\beta (u_2(x) - d_2)^{1-\beta} \quad (2)$$

where $\beta \in [0, 1]$ is the bargaining power of player 1.

- Next note that if both utilities are linear and defined over their share of some pie, and the set $X \subset \mathbb{R}^2$ is given by $x_1 + x_2 \leq 1$, then the solution to (2) is given by

$$(1 - \beta)(x_1 - d_1) = \beta(x_2 - d_2),$$

with $x_1 = 1 - x_2$, which implies the linear sharing rule:

$$x_2 = (1 - \beta)(1 - d_1 - d_2) + d_2.$$

- Intuitively, player 2 receives a fraction $1 - \beta$ of the net surplus $1 - d_1 - d_2$ plus his outside option, d_2 .

Wage Determination without Binding Contracts

- In the context of the model here, the Nash bargaining solution amounts to choosing the wage, w , so as to maximize:

$$(f(k) - w)^{1-\beta} (w - \bar{w})^{\beta}.$$

- The cost of investment, rk , does not feature in this expression, since these investment costs are *sunk*.
 - In other words, the profits of the firm are $f(k) - w - rk$, while its outside option is $-rk$.
 - So the difference between payoff and outside option for the firm is simply $f(k) - w$.
- Therefore, the wage resulting from the Nash solution will be

$$w(k) = \beta f(k) + (1 - \beta) \bar{w}.$$

- This expression emphasizes the dependence of the equilibrium wage on the capital stock of the firm.
 - Contrast this with the equilibrium in a competitive labor market; with the ways depend on the capital stock?

Investments without Binding Contracts: Equilibrium

- Therefore, firm profits are:

$$\begin{aligned}\pi(k) &= f(k) - w(k) - rk \\ &= (1 - \beta)(f(k) - \bar{w}) - rk\end{aligned}$$

- Profit maximization:

$$(1 - \beta)f'(k^e) = r$$

- In comparison, the efficient level of investment that would have emerged in a competitive labor market, is given by

$$f'(k^*) = r$$

- The concavity of f immediately implies that $k^e < k^*$, thus there will be *underinvestment*.
- Why?

Investments without Binding Contracts: Underinvestment

- Because of bargaining, the firm is *not the full residual claimant* of the additional returns it generates by its investment.
- *Holdup problem*; once the firm invests a larger amount in physical capital, it is potentially “held up” by the worker.
- A fraction β all the returns are received by the worker, since the wage that the firm has to pay is increasing in its capital stock.
- What would happen if there were binding contracts?
- Sign a contract of the form $w(k)$ *before investment*.
- This wage schedule would satisfy $w'(k^*) = 0$ and encourage the efficient level of investment.
- This wage schedule would avoid the holdup problem.

Investments without Binding Contracts: Recap

- Is the assumption of “incomplete contracts” reasonable?
 - There are two reasons for why binding contracts are generally not possible and instead contracts have to be “incomplete”:
- 1 Such contracts require the level investment, k , to be easily observable by outside parties, so that the terms of a contract that makes payments conditional on k are easily enforceable (notice the important emphasis here; there is no asymmetric information between the parties, but outside courts cannot observe what the firm and the worker observe; can there be no contracts that transmit this information to outside parties in order to make contracts conditional on this information?).
 - 2 We need to rule out renegotiation.

General Versus Specific Training

- Important distinction between two types of human capital in the context of training:
 - ① Firm-specific training: this provides a worker with *firm-specific skills*, that is, skills that will increase his or her productivity *only* with the current employer.
 - ② General training: this type of training will contribute to the worker's general human capital, increasing his productivity with a range of employers.
- Naturally, in practice actual training programs could (and often do) provide a combination of firm-specific and general skills.

Training in Competitive Markets

- Let us start with competitive labor markets.
- Then we will consider training investments in labor markets with *frictions*.

Model

- Consider the following stylized model (without discounting):
 - At time $t = 0$, there is an initial production of y_0 , and also the firm decides the level of training τ , incurring the cost $c(\tau)$. Assume that $c(0) = 0$, $c'(0) = 0$, $c'(\cdot) \geq 0$ and $c''(\cdot) > 0$.
 - At time $t = 1/2$, the firm makes a wage offer w to the worker, and other firms also compete for the worker's labor. The worker decides whether to quit and work for another firm (*competitive labor markets*: suppose that there are many identical firms who can use the general skills of the worker, and the worker does not incur any cost in the process of changing jobs).
 - At time $t = 1$, there is the second and final period of production, where output is equal to $y_1 + f(\tau)$, with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) < 0$.

First Best Training

- A social planner wishing to maximize net output would choose a positive level of training investment, $\tau^* > 0$, given by

$$c'(\tau^*) = f'(\tau^*).$$

- The fact that τ^* is strictly positive immediately follows from the fact that $c'(0) = 0$ and $f'(0) > 0$.

Pre-Becker Thinking

- There will be underinvestment in general training (e.g., Pigou).
- Because firms unwilling to invest in general skills.
The reasoning went along the following lines.
- Suppose the firm invests some amount $\tau > 0$. For this to be profitable for the firm, at time $t = 1$, it needs to pay the worker at most a wage of

$$w_1 < y_1 + f(\tau) - c(\tau)$$

to recoup its costs.

- But suppose that the firm was offering such a wage.
- Could this be an equilibrium?

Pre-Becker Thinking (continued)

- No, because there are other firms who have access to exactly the same technology, they would be willing to bid a wage of $w_1 + \varepsilon$ for this worker's labor services.
- Since there are no costs of changing employer, for ε small enough such that

$$w_1 + \varepsilon < y_1 + f(\tau),$$

a firm offering $w_1 + \varepsilon$ would both attract the worker by offering this higher wage and also make positive profits.

- This reasoning implies that in any competitive labor market:

$$w_1 = y_1 + f(\tau).$$

- But then, the firm cannot recoup any of its costs and would like to choose $\tau = 0$.
- Despite the fact that a social planner would choose a positive level of training investment, $\tau^* > 0$, the pre-Becker view was that this economy would fail to invest in training.

Pre-Becker Thinking: Is There a Mistake?

- The mistake in this reasoning was that it did not take into account the worker's incentives to invest in his own training.
- The worker is the *full residual claimant* of the increase in his own productivity, and in the competitive equilibrium of this economy without any credit market or contractual frictions, he would have the right incentives to invest in his training.
- Therefore, we have to look at *worker investments in general training*.

The Becker Model

- Let us now analyze the equilibrium when the worker can invest.
- First note that at $t = 1$, the worker will be paid $w_1 = y_1 + f(\tau)$.
- Next recall that τ^* is the efficient level of training given by $c'(\tau^*) = f'(\tau^*)$.
- In the unique subgame perfect equilibrium, in the first period the firm will offer the following package: training of τ^* and a wage of

$$w_0 = y_0 - c(\tau^*).$$

- Then, in the second period the worker will receive the wage of

$$w_1 = y_1 + f(\tau^*)$$

either from the current firm or from another firm.

The Becker Model (continued)

- To see why no other allocation could be an equilibrium, suppose that the firm offered (τ, w_0) , such that $\tau \neq \tau^*$.
- For the firm to break even we need that $w_0 \leq y_0 - c(\tau)$, but by the definition of τ^* , we have

$$y_0 - c(\tau^*) + y_1 + f(\tau^*) > y_0 - c(\tau) + y_1 + f(\tau) \geq w_0 + y_1 + f(\tau)$$

So the deviation of offering $(\tau^*, y_0 - c(\tau^*) - \varepsilon)$ for ε sufficiently small would attract the worker and make positive profits.

- Thus, the unique equilibrium is the one in which the firm offers training τ^* .
- Therefore, in this economy the efficient level of training will be achieved with firms bearing none of the cost of training, and workers financing training by taking a wage cut in the first period of employment (i.e, a wage $w_0 < y_0$).

The Becker Model: Applications

- There are a range of examples for which this model appears to provide a good description.
- These include
 - some of the historical apprenticeship programs where young individuals worked for very low wages and then “graduated” to become master craftsmen;
 - pilots who work for the Navy or the Air Force for low wages, and then obtain much higher wages working for private sector airlines;
 - securities brokers, often highly qualified individuals with MBA degrees, working at a pay level close to the minimum wage until they receive their professional certification;
 - or even academics taking an assistant professor job at Harvard despite the higher salaries in other departments.

The Becker Model: Theoretical Challenges

- Are the contracts necessary for the Becker solution to work reasonable?
- In particular, can the firm make a credible commitment to providing training in the amount of τ^* ?
- Such commitments are in general difficult, since outsiders cannot observe the exact nature of the “training activities” taking place inside the firm.
- For example, the firm could hire workers at a low wage pretending to offer them training, and then employ them as cheap labor.
- This implies that contracts between firms and workers concerning training investments are naturally *incomplete*.

Training with Incomplete Contracts

- To capture these issues let us make the timing of events regarding the provision of training somewhat more explicit.
 - At time $t = -1/2$, the firm makes a training-wage contract offer (τ', w_0) . Workers accept offers from firms.
 - At time $t = 0$, there is an initial production of y_0 , the firm pays w_0 , and also unilaterally decides the level of training τ , which could be different from the promised level of training τ' .
 - At time $t = 1/2$, wage offers are made, and the worker decides whether to quit and work for another firm.
 - At time $t = 1$, there is the second and final period of production, where output is equal to $y_1 + f(\tau)$.

Training with Incomplete Contracts (continued)

- Now the subgame perfect equilibrium can be characterized as follows.
- At time $t = 1$, a worker of training τ will receive $w_1 = y_1 + f(\tau)$.
- Realizing this, at time $t = 0$, the firm would offer training $\tau = 0$, irrespective of its contract promise.
- Anticipating this wage offer, the worker will only accept a contract offer of the form (τ', w_0) , such that $w_0 \geq y_0$, and τ does not matter, since the worker knows that the firm is not committed to this promise.
- As a result, we are back to the outcome conjectured by Pigou, with no training investment by the firm.

Training with Credit Market Constraints

- A similar conclusion would also be reached if the firm could write a binding contract about training, but the worker were subject to credit constraints and

$$c(\tau^*) > y_0,$$

- This implies that the worker cannot take enough of a wage cut to finance his training.
- In the extreme case where $y_0 = 0$, we are again back to the Pigou outcome, where there is no training investment, despite the fact that it is socially optimal to invest in skills
 - which one of these problems, contractual incompleteness or credit market constraints, appears more important in the context of training?.

Training in Imperfect Labor Markets

- The general conclusion of both the Becker model with perfect (credit and labor) markets and the model with incomplete contracts (or severe credit constraints) is that there will be no firm-sponsored investment in general training.
- This conclusion follows from the common assumption of these two models, that the labor market is *competitive*, so the firm will never be able to recoup its training expenditures in general skills later during the employment relationship.
- Is this a reasonable prediction?
- The answer appears to be no.
- There are many instances in which firms bear a significant fraction (sometimes all) of the costs of general training investments.

Do Firms Pay for General Training

- Very common in the German apprenticeship system.
- Estimates of the net cost of apprenticeship programs to employers in Germany in the 1990s are quite large.
- Another interesting example comes from the recent growth sector of the US, the temporary help industry.
 - The temporary help firms provide workers to various employers on short-term contracts, and receive a fraction of the workers' wages as commission.
 - Most large temporary help firms offer (and pay for) such training to all willing individuals
- Management consulting firms hire highly paid MBAs and provide largely general training in the first year.
- Other evidence using regression also consistent, but not as clear-cut (because of unobservant originate the problems).

Basic Framework

- Consider the following two-period model.
- In period 1, the worker and/or the employer choose how much to invest in the worker's general human capital, τ .
- There is no production in the first period. In period 2, the worker either stays with the firm and produces output $y = f(\tau)$, where $f(\tau)$ is a strictly increasing and concave function.
- The worker is also paid a wage rate, $w(\tau)$ as a function of his skill level (training) τ , or he quits and obtains an outside wage.
- The cost of acquiring τ units of skill is again $c(\tau)$, which is again assumed to be continuous, differentiable, strictly increasing and convex, and to satisfy $c'(0) = 0$.
- There is no discounting, and all agents are risk-neutral.

Basic Framework (continued)

- Assume that all training is *technologically general* in the sense that $f(\tau)$ is the same in all firms.
- If a worker leaves his original firm, then he will earn $v(\tau)$ in the outside labor market.
- Suppose

$$v(\tau) < f(\tau).$$

That is, despite that fact that τ is general human capital, when the worker separates from the firm, he will get a lower wage than his marginal product in the current firm.

- The fact that $v(\tau) < f(\tau)$ implies that there is a surplus that the firm and the worker can share when they are together. Also note that $v(\tau) < f(\tau)$ is only possible in labor markets with frictions—otherwise, the worker would be paid his full marginal product, and $v(\tau) = f(\tau)$.

Wage Determination

- Let us suppose that this surplus will be divided by asymmetric Nash bargaining with worker bargaining power given by $\beta \in (0, 1)$.

$$w(\tau) = v(\tau) + \beta [f(\tau) - v(\tau)]. \quad (3)$$

- As usual, equilibrium wage rate $w(\tau)$ is independent of $c(\tau)$:
 - the level of training is chosen first, and then the worker and the firm bargain over the wage rate;
 - at this point the training costs are already sunk, so they do not feature in the bargaining calculations (bygones are bygones).

Training Investments

- Assume that τ is determined by the investments of the firm and the worker, who independently choose their contributions, c_w and c_f , and τ is given by

$$c(\tau) = c_w + c_f.$$

- Assume that \$1 investment by the worker costs \$ p where $p \geq 1$.
- When $p = 1$, the worker has access to perfect credit markets and when $p \rightarrow \infty$, the worker is severely constrained and cannot invest at all.

Timing of Events

- The worker and the firm simultaneously decide their contributions to training expenses, c_w and c_f .
The worker receives an amount of training τ such that $c(\tau) = c_w + c_f$.
- The firm and the worker bargain over the wage for the second period, $w(\tau)$, where the threat point of the worker is the outside wage, $v(\tau)$, and the threat point of the firm is not to produce.
- Production takes place.

Equilibrium

- Given this setup, the contributions to training expenses c_w and c_f will be determined noncooperatively.
- More specifically, the firm chooses c_f to maximize profits:

$$\pi(\tau) = f(\tau) - w(\tau) - c_f = (1 - \beta) [f(\tau) - v(\tau)] - c_f.$$

subject to $c(\tau) = c_w + c_f$.

- The worker chooses c_w to maximize utility:

$$u(\tau) = w(\tau) - pc_w = \beta f(\tau) + (1 - \beta)v(\tau) - pc_f$$

subject to the same constraint.

Equilibrium (continued)

- The first-order conditions are:

$$(1 - \beta) [f'(\tau) - v'(\tau)] - c'(\tau) = 0 \quad \text{if } c_f > 0 \quad (4)$$

$$v'(\tau) + \beta [f'(\tau) - v'(\tau)] - pc'(\tau) = 0 \quad \text{if } c_w > 0 \quad (5)$$

- Inspection of these equations implies that generically, one of them will hold as a strict inequality, therefore, one of the parties will bear the full cost of training.

Will Firms Invest in General Training?

- The result of no firm-sponsored investment in general training by the firm obtains when

$$f(\tau) = v(\tau),$$

which is the case of perfectly competitive labor markets. (4) then implies that $c_f = 0$, so when workers receive their full marginal product in the outside labor market, the firm will never pay for training.

- Moreover, as $p \rightarrow \infty$, so that the worker is severely credit constrained, there will be no investment in training.
- In all cases, the firm is not constrained, so one dollar of spending on training costs one dollar for the firm.

Will Firms Invest in General Training? (continued)

- In contrast, suppose there are labor market imperfections, so that the outside wage is less than the productivity of the worker, that is

$$v(\tau) < f(\tau).$$

- Is this gap between marginal product and market wage enough to ensure firm-sponsored investments in training?

Will Firms Invest in General Training? (continued)

- *The answer is no.*
- Consider the case with no *wage compression*, that is the case in which a marginal increase in skills is valued appropriately in the outside market.
- Mathematically this corresponds to

$$v'(\tau) = f'(\tau) \text{ for all } \tau.$$

- Substituting for this in the first-order condition of the firm, (4), we immediately find that if $c_f > 0$, then $c'(\tau) = 0$.
- So in other words, there will be no firm contribution to training expenditures.

Wage Compression and Training

- Next consider the case in which there is wage compression, i.e.,

$$v'(\tau) < f'(\tau).$$

- Now it is clear that the firm may be willing to invest in the general training of the worker.
- The simplest way to see this is again to consider the case of severe credit constraints on the worker, that is, $p \rightarrow \infty$, so that the worker cannot invest in training.
- Then, $v'(0) < f'(0)$ is sufficient to induce the firm to invest in training.

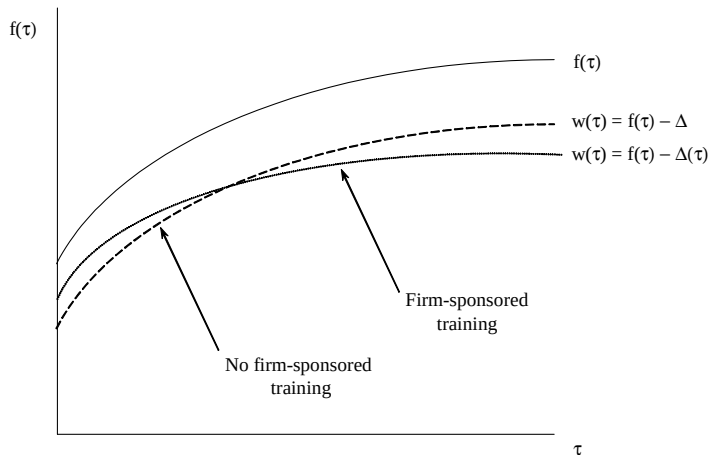
Wage Compression and Training (continued)

- Why does *wage compression* matter for firm-sponsored training?
- Wage compression in the outside market translates into wage compression inside the firm, i.e., it implies

$$w'(\tau) < f'(\tau).$$

- As a result, the firm makes greater profits from a more skilled (trained) worker, and has an incentive to increase the skills of the worker.

Wage Compression and Training (continued)



Who Will Invest in General Training?

- Suppose that

$$v(\tau) = af(\tau) - b.$$

- A decrease in a is equivalent to a decrease in the price of skill in the outside market, and would also tilt the wage function inside the firm, $w(\tau)$, decreasing the relative wages of more skilled workers because of bargaining between the firm and in the worker, with the outside wage $v(\tau)$ as the threat point of the worker.
- Starting from $a = 1$ and $p < \infty$, a point at which the worker makes all investments, a decrease in a leads to less investment in training from (5).
- This is simply an application of the Becker reasoning; without any wage compression, the worker is the one receiving all the benefits and bearing all the costs, and a decline in the returns to training will reduce his investments.

Who Will Invest in General Training?

- However, as a declines further, we will eventually reach the point where $\tau_w = \tau_f$.
- Now the firm starts paying for training, and a further decrease in a increases investment in general training (from (4)).
- Therefore, there is a U-shaped relation between the skill premium and training—starting from a compressed wage structure, a further decrease in the skill premium may increase training.
- Holding $f(\tau)$ constant a tilting up of the wage schedule, $w(\tau)$, reduces the profits from more skilled workers, and the firm has less interest in investing in skills.

General Equilibrium

- Above analysis: *partial equilibrium*, since the outside wage structure, $v(\tau)$, is taken as given.
- *General equilibrium*: endogenize $v(\tau)$

Basic Model of Adverse Selection and Training

- Simplified version of the model in Acemoglu and Pischke (1998).
- Suppose that fraction p of workers are high ability, and have productivity $f(\tau)$ in the second period if they receive training τ in the first period.
- The remaining $1 - p$ are low ability and produce nothing (in terms of the above model, we are setting $y = 0$).
- No one knows the worker's ability in the first period, but in the second period, the current employer learns this ability.
- Firms never observe the ability of the workers they have not employed, so outsiders will have to form beliefs about the worker's ability.

Timing of Events

- Firms make wage offers to workers. At this point, worker ability is unknown.
- Firms make training decisions, τ .
- Worker ability is revealed to the current employer and to the worker.
- Employers make second period wage offers to workers.
- Workers decide whether to quit.
- Outside firms compete for workers in the “secondhand” labor market. At this point, these firms observe neither worker ability nor whether the worker has quit or was laid off.
- Production takes place.

Equilibrium

- Since outside firms do not know worker ability when they make their bids, this is a (dynamic) game of incomplete information.
→ Perfect Bayesian Equilibrium
- First, note that all workers will leave their current employer if outside wages are higher.
- In addition, a fraction λ of workers, irrespective of ability, realize that they form a bad match with the current employer, and leave whatever the wage is.
- The important assumption here is that firms in the outside market observe neither worker ability nor whether a worker has quit or has been laid off.
- However, worker training is publicly observed.

Equilibrium (continued)

- These assumptions ensure that in the second period each worker obtains his expected productivity *conditional* on his training.
- That is, his wage will be independent of his own productivity, but will depend on the average productivity of the workers who are in the secondhand labor market.
- By Bayes's rule, the expected productivity of a worker of training τ , is

$$v(\tau) = \frac{\lambda p f(\tau)}{\lambda p + (1 - p)} \quad (6)$$

- Intuition?

Equilibrium (continued)

- Anticipating this outside wage, the initial employer has to pay each high ability worker $v(\tau)$ to keep him.
- This observation, combined with (6), immediately implies that there is wage compression in this world, in the sense that

$$v'(\tau) = \frac{\lambda p f'(\tau)}{\lambda p + (1 - p)} < f'(\tau),$$

so the adverse selection problem introduces wage compression, and via this channel, will lead to firm-sponsored training.

Equilibrium (continued)

- Now consider the previous stage of the game.
- Firm profits as a function of the training choice can be written as

$$\pi(\tau) = (1 - \lambda) p [f(\tau) - v(\tau)] - c(\tau).$$

- The first-order condition for the firm is

$$\begin{aligned} \pi'(\tau) &= (1 - \lambda) p [f'(\tau) - v'(\tau)] - c'(\tau) = 0 \\ &= \frac{(1 - \lambda) p (1 - p) f'(\tau)}{\lambda p + (1 - p)} - c'(\tau) = 0 \end{aligned} \quad (7)$$

- Main results follow from these conditions.

Equilibrium (continued)

- First, $c'(0) = 0$ is sufficient to ensure that there is firm-sponsored training (that is, the solution to (7) is interior).
- There is underinvestment in training relative to the first-best which would have involved $pf'(\tau) = c'(\tau)$ (notice that the first-best already takes into account that only a fraction p of the workers will benefit from training). This is because of two reasons:
 - 1 a fraction λ of the high ability workers quit, and the firm does not get any profits from them;
 - 2 even for the workers who stay, the firm is forced to pay them a higher wage, because they have an outside option that improves with their training, i.e., $v'(\tau) > 0$. This reduces profits from training, since the firm has to pay higher wages to keep the trained workers.

Equilibrium (continued)

- The firm has *monopsony power* over the workers, enabling it to recover the costs of training. In particular, high ability workers who produce $f(\tau)$ are paid $v(\tau) < f(\tau)$.
- Monopsony power is not enough by itself. Wage compression is also essential for this result. To see this, suppose that we impose there is no wage compression, i.e., $v'(\tau) = f'(\tau)$, then inspection of the first line of (7) immediately implies that there will be zero training, $\tau = 0$.

Is Wage Compression Automatic?

- Let us modify the model so that high ability workers produce

$$\eta + f(\tau)$$

in the second period, while low ability workers produce $f(\tau)$.

- Training and ability are no longer complements.
- Both types of workers get exactly the same marginal increase in productivity (this contrasts with the previous specification where only high ability workers benefited from training, hence training and ability were highly complementary). Then

$$v(\tau) = \frac{\lambda p \eta}{\lambda p + (1 - p)} + f(\tau),$$

and hence

$$v'(\tau) = f'(\tau).$$

Thus no wage compression, and firm-sponsored training.

- Why?

Entry Wages in Training Firms

- What happens if equilibrium profits positive, i.e.,

$$\pi(\tau) = (1 - \lambda) p[f(\tau) - v(\tau)] - c(\tau) > 0$$

- If there is free entry at time $t = 0$, firms must make zero profits.
- Therefore, competition for workers implies that first-period wages

$$W = \pi(\tau) > 0.$$

- This is because once a worker accepts a job with a firm, the firm acquires monopsony power over this worker's labor services at time $t = 1$ to make positive profits.
- Competition then implies that these profits have to be transferred to the worker at time $t = 0$.
- The interesting result is that not only do firms pay for training, but they may also pay workers extra in order to attract them.

Evidence

- How can this model be tested?
- One way is to look for evidence of this type of adverse selection among highly trained workers.
- The fact that employers know more about their current employees may be a particularly good assumption for young workers, so a good area of application would be for apprentices in Germany.
- According to the model, workers who quit or are laid off should get lower wages than those who stay in their jobs, which is a prediction that follows simply from adverse selection.
- The more interesting implication here is that if the worker is separated from his firm for an exogenous reason that is clearly observable to the market, he should not be punished by the secondhand labor market.
- In fact, he's "freed" from the monopsony power of the firm, and he may get even higher wages than stayers (who are on average of higher ability, though subject to the monopsony power of their employer).

Evidence (continued)

- To see this, note that a worker who is exogenously separated from his firm will get to wage of

$$pf(\tau),$$

whereas stayers, who are still subject of the monopsony power of their employer, obtain the wage of

$$v(\tau)$$

as given by (6), which could be less than $pf(\tau)$.

- In the German context, workers who leave their apprenticeship firm to serve in the military provide a potential group of such exogenous separators.
- Interestingly, the evidence suggests that although these military quitters are on average lower ability than those who stay in the apprenticeship firm, the military quitters receive higher wages.

Mobility, Training and Wages

- The interaction between training and adverse selection also provides a different perspective in thinking about mobility patterns.
- Suppose now $\lambda = 0$, but workers now quit if

$$w(\tau) - v(\tau) < \theta$$

where θ is a worker-specific draw from a uniform distribution over $[0, 1]$.

- The variable θ can be interpreted as the disutility of work in the current job. This is the worker's private information.
- Therefore, the fraction of high ability workers who quit their initial employer will be

$$1 - w(\tau) + v(\tau).$$

- The outside wage is now

$$v(\tau) = \frac{p[1 - w(\tau) + v(\tau)]f(\tau)}{p[1 - w(\tau) + v(\tau)] + (1 - p)} \quad (8)$$

Mobility, Training and Wages (continued)

- Note that if $v(\tau)$ is high, many workers leave their employer because outside wages in the secondhand market are high.
- But also the right hand side of (8) is increasing in the fraction of quitters,

$$[1 - w(\tau) + v(\tau)],$$

so $v(\tau)$ will increase further.

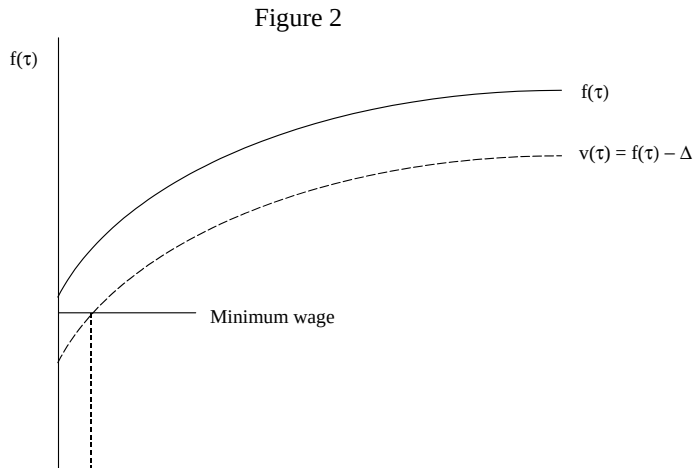
- This reflects the fact that with a higher quit rate, the secondhand market is not as adversely selected (it has a better composition).

Mobility, Training and Wages (continued)

- Another implication: multiple equilibria in this economy.
 - One equilibrium with a high quit rate, high wages for workers changing jobs, i.e. high $v(\tau)$, but low training.
 - Another equilibrium with low mobility, low wages for job changers, and high training.
- Multiple equilibria as a stylistic description of the differences between the U.S. and German labor markets.
 - In Germany, the turnover rate is much lower than in the U.S., and also there is much more training.
 - Also, in Germany workers who change jobs are much more severely penalized (on average, in Germany such workers experience a substantial wage loss, while they experience a wage gain in the U.S.).
- Which equilibrium is better?

Minimum Wages and Training

- The effect of minimum wages on training (Δ constant mobility cost):



Minimum Wages and Training (continued)

- Comparative static result: higher minimum wages can increase training (as long as $\Delta > 0$ and sufficiently large).
- In the standard Becker model with competitive labor markets, minimum wages always reduce training (why?).
- Empirical evidence: mixed, but more of a positive effect.

Introduction

- General skills rewarded by the market, even if not fully in imperfect labor markets.
- What about specific skills?
- By definition, only one employer values the skills.
- How will the worker be rewarded for possessing and investing in the skills?
 - bargaining
- Since bargaining will be ex post, *holdup problems*.

Interpretation

- The empirical investigation of the importance of firm-specific skills and rents is a difficult and challenging area.
- There are two important conceptual issues:
 - 1 We can imagine a world in which firm-specific skills are important, but there may be no relationship between tenure and wages. This is because productivity increases due to firm-specific skills do not necessarily translate into wage increases.
 - 2 An empirical relationship between tenure and wages does not establish that there are imported firm-specific effects. This might result because of backloaded compensation or because of *selection*.

Wage-Tenure Relationship

- The first type of evidence is from regression analyses of the relationship between wages and tenure exploiting within job wage growth.
- Here the idea is that by looking at how wages grow within a job (as long as the worker does not change jobs), and comparing this to the experience premium, we will get an estimate of the tenure premium.
- Imagine the following empirical model

$$\ln w_{it} = \beta_1 X_{it} + \beta_2 T_{it} + \varepsilon_{it} \quad (9)$$

where X_{it} this total labor market experience of individual i , and T_{it} is his tenure in the current job.

- Alternatively, wage growth within the job is

$$\Delta \ln w_{it} = \beta_1 + \beta_2 + \Delta \varepsilon_{it}.$$

- If we knew the experience premium, β_1 , we could then immediately compute the tenure premium β_2 .
- The problem is that we do not know the experience premium.
- Topel suggests that we can get an upper bound for the experience premium by looking at the relationship between entry-level wages and labor market experience (that is, wages in jobs with tenure= 0).
- This is an upper bound to the extent that workers do not randomly change jobs.

Wage-Tenure Relationship (continued)

- Therefore, whenever $T_{it} = 0$, the disturbance term ε_{it} in (9) is likely to be positively selected.
- According to this reasoning, we can obtain a *lower bound* estimate of β_2 , $\hat{\beta}_2$, using a two-step procedure—first estimate the rate of within-job wage growth, $\hat{\beta}_1$, and then subtract from this the estimate of the experience premium obtained from entry-level jobs
 - can you see reasons why this will lead to an upwardly biased estimate of the importance of tenure rather than a lower bound on tenure affects as Topel claims?.

Wage-Tenure Relationship (continued)

- Using this procedure Topel estimates relatively high rates of return to tenure.
- For example, his main estimates imply that ten years of tenure increase wages by about 25 percent, over and above the experience premium.
- It is possible, however, that this procedure might generate tenure premium estimates that are upward biased.
 - For example, this would be the case if the return to tenure or experience is higher among high-ability workers, and those are underrepresented among the job-changers.
 - Alternatively, returns to experience may be non-constant, and they may be higher in jobs to which workers are a better match.
- If this is the case, returns experience for new jobs will understate the average returns to experience for jobs in which workers choose to stay.

Wage Consequences of Separation

- The second type of evidence comes from the wage changes of workers resulting from job displacement.
- A number of papers, most notably Jacobson, LaLonde and Sullivan, find that displaced workers experience substantial drop in earnings.
- They typically find big drops in wages when workers separate from their firms.
- The question is how to interpret this

Wage Consequences of Separation: Evidence

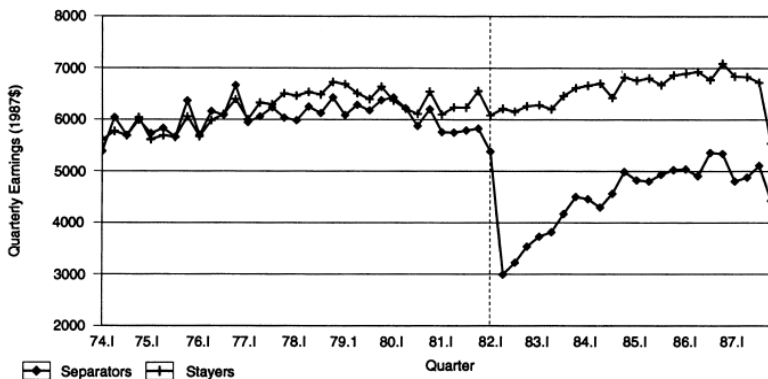


FIGURE 1. QUARTERLY EARNINGS (1987 DOLLARS) OF HIGH-ATTACHMENT WORKERS SEPARATING IN QUARTER 1982:1 AND WORKERS STAYING THROUGH QUARTER 1986:4

Wage Consequences of Separation: Evidence

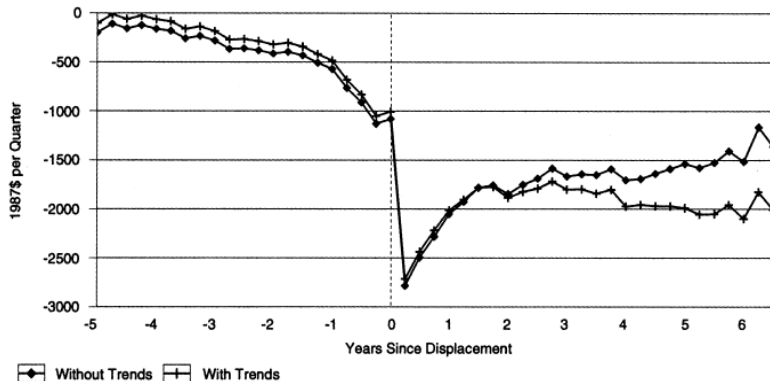


FIGURE 2. EARNINGS LOSSES FOR SEPARATORS IN MASS-LAYOFF SAMPLE

Wage Consequences of Separation: Interpretation

- Part of this is due to non-employment following displacement, but even after three years a typical displaced worker is earning about \$1500 less (1987 dollars).
- Econometrically, this evidence is simpler to interpret than the tenure-premium estimates. Economically, the interpretation is somewhat more difficult than the tenure estimates, since it may simply reflect the loss of high-rent (e.g. union) jobs.
- Overall, these two pieces of evidence together are consistent with the view that there are important firm-specific skills/expertises that are accumulated on the job.

What Are Firm-Specific Skills?

- 1 Firm-specific skills can be thought to result mostly from firm-specific training investments made by workers and firms. Here it is important to distinguish between firms' and workers' investments, since they will have different incentives.
- 2 Firm-specific skills simply reflect what the worker learns on-the-job without making any investments. In other words, they are simply unintentional byproducts of working on the job.
- 3 Firm-specific skills may reflect "matching".
- 4 There may be no technologically firm-specific skills. Instead, all skills are technologically general but some are transformed into *de facto* firm-specific skills because of market imperfections (why is this different from 1?).

Investment in Firm-Specific Skills

- Problem with general training investments was that part of the costs had to be borne by the firm,
- But then the worker is fully or partly the residual claimant.
- With firm-specific skills, the problem is reversed.

Model

- At time $t = 0$, the worker decides how much to invest in firm-specific skills, denoted by s , at the cost $\gamma(s)$. $\gamma(s)$ is strictly increasing and convex, with $\gamma'(0) = 0$.
- At time $t = 1$, the firm makes a wage offer to the worker.
- The worker decides whether to accept this wage offer and work for this firm, or take another job.
- Production takes place and wages are paid.

Model (continued)

- Let the productivity of the worker be

$$y_1 + f(s),$$

where y_1 is also what he would produce with another firm.

- Since s is specific skills, it does not affect the worker's productivity in other firms.
- First best:

$$\gamma'(s^*) = f'(s^*)$$

with $s^* > 0$.

Equilibrium

- By backward induction again, starting in the last period.
- The worker will accept any wage offer $w_1 \geq y_1$, since this is what he can get in an outside firm.
- Knowing this, the firm simply offers $w_1 = y_1$.
- In the previous period, realizing that his wage is independent of his specific skills, the worker makes no investment in specific skills.

Market Failure?

- By investing in his firm-specific skills, the worker is increasing the firm's profits.
- Therefore, the firm would like to encourage the worker to invest.
- However, given the timing of the game, wages are determined by a take-it-leave-it offer by the firm after the investment.
- Therefore, *holdup* problem.
- Since firm-specific skills difficult to verify, contractual solutions are imperfect.

Worker Power and Investment

- How can we improve the worker's investment incentives?
- General solution: make worker's earnings conditional on specific skills.
- One imperfect but realistic solution: increase the bargaining power of the worker.
 - For example, the firm may purposefully give access to some important assets to the worker
 - The firm may change its organizational form in order to make a credible commitment not to hold up the worker.
- Alternative: the firm may develop a reputation for not holding up workers who have invested in firm-specific human capital.

Worker Power and Investment (continued)

- Suppose that the worker wage as a function of firm-specific skills is

$$w_1(s) = y_1 + \beta f(s)$$

- Now at time $t = 0$, the worker maximizes

$$y_1 + \beta f(s) - \gamma(s),$$

- Solution:

$$\beta f'(\hat{s}) = \gamma'(\hat{s}) \quad (10)$$

- Here $\hat{s}$ is strictly positive, so giving the worker bargaining power has improved investment incentives.
- However, $\hat{s}$ is also strictly less than the first-best investment level s^* .

Worker Power and Investment (continued)

- What about firm profits?

$$\pi_{\beta} = (1 - \beta) f(\hat{s}) .$$

- If the firm could choose (or manipulate) β without constraints, then it would set $\bar{\beta}$ such that

$$\frac{\partial \pi_{\beta}}{\partial \beta} = 0 = -f(\hat{s}(\bar{\beta})) + (1 - \bar{\beta}) f'(\hat{s}(\bar{\beta})) \frac{d\hat{s}(\bar{\beta})}{d\beta}$$

where $\hat{s}(\beta)$ and $d\hat{s}/d\beta$ are given by the first-order condition of the worker, (10).

- The firm would certainly choose $\bar{\beta} < 1$, since with $\bar{\beta} = 1$, we could never have $\partial \pi_{\beta} / \partial \beta = 0$.

Worker Power and Investment (continued)

- But $\beta = 1$ would maximize firm-specific skills.
- The reason why the firm would not choose the structure of organization that achieves the best investment outcomes is that it cares about its own profits, not total income or surplus.
- Also, firms “selling the firm” to worker unlikely in practice.

Promotions

- An alternative arrangement to encourage workers to invest in firm-specific skills is to design a promotion scheme.
- Consider the following setup.
- Suppose that there are two investment levels, $s = 0$, and $s = 1$ which costs c .
- Suppose also that at time $t = 1$, there are two tasks in the firm, difficult and easy, D and E.
- Assume outputs in these two tasks as a function of the skill level are

$$y_D(0) < y_E(0) < y_E(1) < y_D(1)$$

- Therefore, skills are more useful in the difficult task, and without skills the difficult task is not very productive.

Promotions (continued)

- Moreover, suppose that

$$y_D(1) - y_E(1) > c$$

- Therefore, the productivity gain of assigning a skilled worker to the difficult task is greater than the cost of the worker obtaining skills.
- In this situation, the firm can induce firm-specific investments in skills if it can commit to a wage structure attached to promotions.
- In particular, suppose that the firm commits to a wage of w_D for the difficult task and w_E for the easy task.
- Notice that the wages do not depend on whether the worker has undertaken the investment, so we are assuming some degree of commitment on the side of the firm, but not modifying the crucial incompleteness of contracts assumption.

Promotions (continued)

- Now imagine the firm chooses the wage structures such that

$$y_D(1) - y_E(1) > w_D - w_E > c, \quad (11)$$

and then ex post decides whether the worker will be promoted.

- Again by backward induction, we have to look at the decisions in the final period of the game.
- When it comes to the promotion decision, and the worker is unskilled, the firm will naturally choose to allocate him to the easy task (his productivity is higher in the easy task and his wage is lower).
- If the worker is skilled, and the firm allocates him to the easy task, his profits are $y_E(1) - w_E$.
- If it allocates him to the difficult task, his profits are $y_D(1) - w_D$.

Promotions (continued)

- The wage structure in (11) ensures that profits from allocating him to the difficult task are higher.
- Therefore, with this wage structure the firm has made a credible commitment to pay the worker a higher wage if he becomes skilled, because it will find it profitable to promote the worker.
- Next, going to the investment stage, the worker realizes that when he does not invest he will receive w_E , and when he invests, he will get the higher wage w_D .
- Since, again by (11), $w_D - w_E > c$, the worker will find it profitable to undertake the investment.

Labor Market Learning and Mobility

- Important idea: firm-specific skills are (at least in part) a manifestation of the quality of the match between a worker and his job.
- Moreover, jobs are “experience goods,” meaning that workers can only find out whether they are a good match to a job (and to a firm) by working in that firm and job.
- Therefore, this type of learning does not take place immediately.
- These ideas captured by learning and matching models→ a range of interesting predictions about labor market mobility and wage patterns.

A Simple Model of Market Learning and Mobility

- Each worker is infinitely lived in discrete time and maximizes the expected discounted value of income, with a discount factor $\beta < 1$.
- There is no ex ante heterogeneity among the workers.
- But worker-job matches are random.
- The worker may be a good match for a job (or a firm) or a bad match.
- Original model by Jovanovic, with normal distributions.
- Here a simplified version.

A Simple Model of Market Learning and Mobility (continued)

- Let the (population) probability that the worker is a good match be $\mu_0 \in (0, 1)$.
- A worker in any given job can generate one of two levels of output, high, y_h , and low $y_l < y_h$:

good match $\rightarrow$ y_h with probability p
 y_l with probability $1 - p$

and

bad match $\rightarrow$ y_h with probability q
 y_l with probability $1 - q$

with

$$p > q.$$

A Simple Model of Market Learning and Mobility (continued)

- Let us assume that all learning is symmetric (as in the career concerns model).
- Therefore, the firm and the worker will share the same *posterior probability* that the worker is a good match to the job.
- Denoted is posterior by μ .

A Simple Model of Market Learning and Mobility (continued)

- Jovanovic assumes that workers always receive their full marginal product in each job.
- This is a problematic assumption, since match-specific quality is also firm specific, thus there is no reason for the worker to receive this entire firm-specific surplus.
- As in the models with firm-specific investments, the more natural assumption would be to have some type of wage bargaining.
- Let us assume the simplest bargaining structure in which a firm will pay the worker a fraction $\phi \in (0, 1]$ of his expected productivity at that point.
- In particular, the wage of a worker whose posterior of a good match is μ will be

$$w(\mu) = \phi [\mu (py_h + (1 - p) y_l) + (1 - \mu) (qy_h + (1 - q) y_l)] .$$

- Different from the Nash bargaining solution.

Equilibrium

- Consider a worker with belief μ .
- If this worker produces output y_h , then Bayes's rule implies that his posterior (belief) next period should be

$$\mu'_h(\mu) \equiv \frac{\mu p}{\mu p + (1 - \mu) q} > \mu,$$

- Similarly, following an output realization of y_l , the belief of the worker will be

$$\mu'_l(\mu) \equiv \frac{\mu (1 - p)}{\mu (1 - p) + (1 - \mu) (1 - q)} < \mu.$$

- Finally, let us also assume that every time a worker changes jobs, he has to incur a training or mobility cost equal to $\gamma \geq 0$.

Equilibrium (continued)

- Under these assumptions, we can write the net present discounted value of a worker with belief μ recursively using simple dynamic programming arguments.
- In particular, this is

$$\begin{aligned} V(\mu) = & w(\mu) + \beta[(\mu p + (1 - \mu) q) V(\mu'_h(\mu)) \\ & + (\mu(1 - p) + (1 - \mu)(1 - q)) \times \\ & \max \{ V(\mu'_l(\mu)) ; V(\mu_0) - \gamma \} . \end{aligned}$$

- Intuition?

Equilibrium (continued)

- An immediate result from dynamic programming is that if the instantaneous reward function, here $w(\mu)$, is strictly increasing in the state variable, which here is the belief μ .
- Therefore, the value function $V(\mu)$ is strictly increasing.
- This implies that there will exist some cutoff level of belief μ^* such that workers will stay in their job as long as

$$\mu \geq \mu^*,$$

and they will quit if $\mu < \mu^*$.

- Let $\bar{\mu} = \inf \{\mu: \mu'(\mu) > \mu^*\}$. Then a worker with beliefs $\mu > \bar{\mu}$ will not quit irrespective of the realization of output.
- Workers with beliefs $\mu \in [\mu^*, \bar{\mu}]$ will quit the job if he generates low output.

Results

- Provided that $\mu_0 \in (0, 1)$, μ will never converge to 0 or 1 in finite time. Therefore, a worker who generates high output will have higher wages in the following period, and a worker who generates low output will have lower wages in the following period. Thus, in this model worker wages will move with past performance.
- It can be easily proved that if $\gamma = 0$, then $\mu^* = \mu_0$. This implies that when γ is equal to 0 or is very small, a worker who starts a job and generates low output will quit immediately. Therefore, as long as γ is not very high, there will be a high likelihood of separation in new jobs.
- Next consider a worker who has been in a job for a long time. Such workers will on average have high values of μ , since they have never experienced (on this job) a belief less than μ^* . This implies that the average value of their beliefs must be high. Therefore, workers with long tenure are unlikely to quit or separate from their job.

Results (continued)

- With the same argument, workers who have been in a job for a long time will have high average μ and thus high wages. This implies that in equilibrium there will be a tenure premium.
- Moreover, because Bayesian updating immediately implies that the gaps between $\mu'_h(\mu)$ and μ and between $\mu'_l(\mu)$ and μ are lowest when μ is close to 1 (and symmetrically when it is close to 0, but workers are never in jobs where their beliefs are close to 0), workers with long tenure will not experience large wage changes. In contrast, workers at the beginning of their tenure will have higher wage variability.

Results (continued)

- What will happen to wages when workers quit?
 - If $\gamma = 0$, wages will necessarily fall when workers quit (since before they quit $\mu > \mu_0$, whereas in the new job $\mu = \mu_0$).
 - If, on the other hand, γ is non-infinitesimal, workers will experience a wage gain when they change jobs, since in this case $\mu^* < \mu_0$ because they are staying in their current job until this job is sufficiently unlikely to be a good match.
 - This last prediction is also consistent with the data, where on average workers who change jobs experience an increase in wages.
 - But is it driven by a reasonable mechanism?

Further Thoughts

- Missing ingredients: differential learning opportunities in different jobs.
- Learning about general skills.
- With these features, some jobs may play the role of “stepping stones” because they reveal information about the skills and productivity of the worker in a range on other jobs.

Other Problems

- Reduced-form wage equation problematic.
- Alternative: Bertrand competition among firms.
- Clearly the worker will start working for the firm where the prior of a good match is greatest.
- Bertrand competition implies that this firm will pay the worker his value at the next best job.
- Once the worker receives bad news and decides to quit, then he will switch to the job that was previously his next best option.
- But this implies that his wage, which will now be determined by the third best option (which may in fact be his initial employer) is necessarily smaller, thus job changes will always be associated with wage *declines*.
- Again: learning about general skills and job heterogeneity
 - so that workers quit not only because they have received bad news in their current job but also because they have learned about their ability and can therefore go and work for “higher-quality” jobs.