# Cash Management in Village Thailand: Positive and Normative Implications

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#### **Outline**

- **❖**Baumol-Tobin and Miller-Orr
- **❖**Models
- **❖**Thai Data
- **\***Anomalies



#### **Existing Models of Cash Management: Baumol-Tobin and Miller-Orr**

- Let c be a given level of expenditure that agent has to spend within a year
- ❖Let *n* be total number of trips to withdraw cash at bank within a year
- **❖**Let *M* be average cash holding
- Let W be amount of withdrawal
- ❖ Let *R* be interest
- Let b be transaction cost per trip
- Given this setup, if cash holding is for transaction purpose, then its pattern would be sawtooth where each withdraw,

$$W = \frac{c}{n}$$

❖and the average cash holding will be  $M = \frac{W}{2} = \frac{c}{2n}$ 

$$M = \frac{W}{2} = \frac{c}{2n}$$

- \*To determine optimal cash holding: We compute the forgone interest and the transaction cost. The forgone interest would be  $RM = R \frac{c}{2n}$  while the transaction cost for all trips is bn. The total cost of cash management is thus  $R \frac{c}{2n} + bn$ .
- So  $n = \left(\frac{Rc}{2b}\right)^{1/2}$   $M = \left(\frac{cb}{2B}\right)^{1/2}$



#### Miller-Orr

- ❖Baumol-Tobin model should apply reasonably well to the household sector, particularly to salary-earning households.
- ❖Miller and Orr (1966) proposed a model of the demand for cash for firms.
- For many business firms, the typical pattern of cash management is not simple, regular and predictable.
- The cash balance fluctuates irregularly (and to some extent unpredictably) over time in both directions (up and down), building up when operating receipts exceed expenditures and falling off when the reverse is true.
- ❖If the build-up is at all prolonged, a point is eventually reached at which the owner/manager/financial officer decides that cash holdings are excessive, and transfers a sizable quantity of funds to some other source.
- ❖In the other direction, in the face of a prolonged net drain, a level will be reached at which the owner/manager/financial officer will do something to restore the cash balance to an "adequate working level".



#### Model

- The basic ingredients of the model are as follows:
  - Let e = c y be net expenditures. So e(t) > 0 means an expenditure paid in cash at time t and e(t) < 0 means an income received in cash.
  - $\triangleright$  We assume that the net expenditures in cash are iid through time and that during a period of length  $\triangle$  they are distributed as follows:

$$e(t) = \begin{cases} z_p \text{ with probability } \kappa_p \Delta \\ \Delta c + \sigma \Delta^{1/2} \text{ with probability } \frac{1 - (\kappa_p + \kappa_n) \Delta}{2} \\ \Delta c - \sigma \Delta^{1/2} \text{ with probability } \frac{1 - (\kappa_p + \kappa_n) \Delta}{2} \\ -z_n \text{ with probability } \kappa_n \Delta \end{cases}$$



#### Model (cont.)

- \*That is, net expenditures are the sum of two components, one is small recurrent net expenditures and one is infrequent lumpy ones. The small recurrent expenditures have mean c and variance  $\sigma^2$  per unit of time. We will take  $\kappa_i$  to be a small number, so most of the time or with probability  $1 (\kappa_p + \kappa_n)\Delta$ , there are only small recurrent expenditures. But when a large net expenditure occurs, which happens with small probability  $\kappa_i$  per unit of time, half of the time are purchases (or outflows of cash) and half of the time incomes (inflows of cash).
- As  $\Delta \to 0$ , the cumulative value of net expenditures is the sum of Brownian motion with drift c and volatility  $\sigma$ , and two independent Jump process, with Poisson arrival rates  $\kappa_p$   $\kappa_n$  and jump size  $z_p$  and  $-z_n$ .
- The evolution of cash will be as follows:

$$m(t + \Delta) = m(t) - e(t + \Delta) + w(t + \Delta) - d(t + \Delta)$$

where w is withdrawal and d is deposit which is action that households can take. In case of inaction, w = d = 0 and thus cash will either go up or go down, depending on whether net expenditure is negative or positive.



#### Model (cont.)

- Household wants to minimize the expected discounted value of the sum of two costs
  - > Flow opportunity cost
  - ➤ Adjustment cost
- \*We assume that costs are discounted at a real rate r per unit of time, and cash holdings have opportunity cost R per period. Given iid assumption, the state of the problem is given by cash holdings m. Let V(m) be the value function right before the agent makes the decision of whether or not to take action, i.e., withdraw or deposit. And assume that cash cannot be negative, or non-negativity constraint, household will thus be forced to take action when  $m \le 0$ .



#### Value Function

\* The value function will satisfy the following Bellman equation:

$$V\left(m\right) = \left\{ \begin{array}{l} b + \min_{m'} V\left(m'\right) \text{ if } m \leq 0 \\ b + \min_{m'} V\left(m'\right), \\ \Delta Rm + \frac{1}{1 + \Delta r} E\left[V\left(m - e\right)\right] \text{ if } m > 0 \end{array} \right\}$$

where

$$E[V(m-e)] = V(m+z_n) \kappa_n \Delta$$

$$+V(m-\Delta c + \sigma \Delta^{1/2}) \frac{1 - (\kappa_p + \kappa_n) \Delta}{2}$$

$$+V(m-z_p) \kappa_p \Delta$$

$$+V(m-\Delta c - \sigma \Delta^{1/2}) \frac{1 - (\kappa_p + \kappa_n) \Delta}{2}$$

i.e., if cash right before the agent makes the decision is negative, households must take an action, and when they do, that will incur transaction cost *b* and households will choose to have cash such that the value function is minimized; there is no discounting because it happens right away. But if cash right before the agent makes the decision is positive, household will choose to either take action or take no action, by comparing the value of adjusting with the value of inaction, and choose the one that gives lower cost.



#### **Policies**

$$V^* = \min_{m \ge 0} V(m)$$
 and  $m^* = \arg\min_{m \ge 0} V(m)$  (5)

In this case the optimal policy is described by two thresholds  $m^*$  and  $m^{**}$ , and the inaction set is the interval  $[0, m^{**}]$ . Given the value of cash after receiving the net expenditure shock, m - e', next period cash holding m' and the value of deposits, d and withdrawals, w, are given by:

$$m - e' > m^{**}$$
  $\Rightarrow m' = m^{*}, d = m - e' - m^{*}, w = 0$   
 $m - e' < 0$   $\Rightarrow m' = m^{*}, w = m^{*} - m + e', d = 0$   
 $m - e' \in [o, m^{**}] \Rightarrow m' = m - e', w = d = 0$ 



#### **Continuous Time**

\*As we let  $\Delta \downarrow 0$  we can write the following continuous time problem where the agent choses the stopping times  $\{\tau_i\}$  and the amounts to either withdraw or deposit at these times, subject to non-negativity of cash:

$$V(m_0) = \min_{\{\tau_i, w(\tau_i) \ge 0, d(\tau_i) \ge 0, i=1,2,...\}} \mathbb{E} \left[ \sum_{i=1}^{\infty} b \ e^{-r\tau_i} + \int_0^{\infty} e^{-rt} R \ m(t) dt \right]$$
(6)

subject to  $m(0) = m_0$  and the law of motion for cash after adjustment given by

$$m(t) = m(0) - ct + \sigma B(t) - z_p N_p(t) + z_n N_n(t) + \sum_{\tau_i \le t} [w(\tau_i) - d(\tau_i)] \ge 0 \text{ all } t \ge 0$$
 (7)

where B(t) is a standard Brownian motion, and  $N_p(t)$  and  $N_n(t)$  are the counters of the two independent Poisson process for with intensity  $\kappa_p$  and  $\kappa_n$  for cash outflows and inflows. The corresponding Bellman equation for m > 0 at point where V is twice differentiable and where inaction is optimal we have:



## Data



#### **Net Cash Variables**

- Net cash outflow is defined as exogenous cash outflow minus exogenous cash inflow.
- Net cash outflow can be defined two ways, depending on whether we consider only formal endogenous variables or both formal and informal ones
- ❖ With only formal endogenous variables, net cash contains the following variables that we treat as exogenous

Exogenous cash outflow	Exogenous cash inflow
c	у
assets purchase	assets sold new
repayment in borrowing new	borrowing
lending	repayment in lending
gift outflow	gift inflow
ROSCA outflow	ROSCA inflow

The actions that households choose are

Exogenous cash outflow	Exogenous cash inflow
$n_dD$	$n_{\rm w}W$



#### **Including Informal**

\*With both formal and informal endogenous variables, net cash contains the following variables that we treat as exogenous

Exogenous cash outflow	Exogenous cash inflow
C	у
assets purchase	assets sold ROSCA
ROSCA outflow	inflow

The actions that households choose are

Exogenous cash outflow	Exogenous cash inflow
$n_dD$	$n_w W$
repayment in borrowing	new borrowing
new lending	repayment in lending
gift outflow	gift inflow



#### **Treatment of the Data**

Basically, the idea is making sure that data is in cash. The following variables contain some data issue

Cash inflow	Cash outflow
у	$n_dD$
new borrowing	

\*Other variables such as consumption expenditure, gift, and lending are relatively easy to make sure that they are in cash.

## n<sub>d</sub>D or Actual Deposit

- The monthly survey does not have withdrawal and deposit per transaction. It does not have frequency of withdrawal and deposit and its average size per month. It has total amount of deposit, and withdrawal since the past interview (approximately 1 month).
- ❖But there is some confusion in recording the values. From the survey data, we often see that households make deposits and withdrawals at the same month, an observation which is not consistent with the idea that it is costly for households to adjust cash holding by going to the bank.
- This is because the survey team will record when money goes into savings accounts as a deposit, regardless of whether households made deposits by themselves. For instance, if households receive direct deposit, or money transfer from some organization, they will be treated as deposits in the survey.
- ❖But for our purpose, this distinction is crucial. So we have to fix this and make sure that deposit is in cash and it is done by household itself.



#### n<sub>d</sub>D or Actual Deposit (cont.)

#### **❖** We do the following

- Ask the team to either recall or visit the households for each deposit recorded in the data, whether it is made by household itself or by someone else
- ➤ The team would also record how reliable this retrospective answer is, scaling from 1 to 5.
- Because this relies on memory and it has been many years, we do not expect that it will completely fix the problem.
- About 80% of these retrospective answers score 4 or 5, and we only use these data and identify whether deposits are actually made by the households.



#### y or Income in Cash

- There are two types of income that households may receive as direct deposit but somehow they are recorded as cash in the survey
  - ➤ Salary from employer
  - > Revenue from selling milk to milk cooperatives for dairy farmers in Lopburi
- For salary from employer, we isolate wages received as direct deposit using variables in job form (to get information about type of worker and type of payment) and job module (to get payment each month)
- \*We focus on employees with monthly wages, or government workers with monthly wage/salary, then using this condition from the job form and match it at individual-job level to get monthly wage.
- Then we check it with deposit from savings module when source of deposit is "from salary or wages", and use the code from the Thai survey team that indicates whether household made deposit by itself or else.



#### y or Income in Cash (cont.)

- So the condition used finally are
  - ➤ Source of deposit is from salary or wages
  - > Employee with monthly wages or government worker, and
  - ➤ Household did not make deposit by itself (because in this case, employer made it)
- The match is not perfect since deposit is at household level and we ask for the most important source of deposit while wage is done at individual-job level. After subtraction, we come up with the new wage income that is supposed to be in cash.
- For households who raise dairy cows in Lopburi, from savings module, for each deposit of each account, we asked the most important source of deposit
- For selling milk to cooperative, enumerator will record this as "from selling agricultural product", but this code might include other things (like deposit from selling some other agricultural product besides milk)



#### y or Income in Cash (cont.)

- \*We merge this file with the one that contains livestock revenue, but livestock revenue might be something else (besides dairy cow)
- Then we merge it with the file from the Thai survey team which indicates whether household made deposit by itself or else.
- So the condition used finally are
  - ➤ Source of deposit is from selling agricultural product
  - ➤ Household did not make deposit by itself (because in this case, milk cooperative made it)
  - ➤ Having revenue from livestock from Lopburi (province that has milk cooperative)
- \*We checked the difference between revenue livestock and deposit for these households, about 85% match perfectly



#### **New Borrowing**

- For loan, we want to separate borrowing from formal sources like BAAC, commercial bank, village fund, and informal sources like friend, relative, neighbor, or moneylender.
- Because if new loan is from BAAC, commercial bank, village fund, they are supposed to go through savings account and thus we do not have to include that as cash inflow.
- In principle, while households still have it in savings account, that is not cash on hand yet, only when households withdraw from savings account, that will be counted as cash.
- The problem is that these loans are not always included in savings account (since some enumerators think that it is loan so it should not be treated as savings) so we cannot just use data from savings modules.



#### **New Borrowing (cont.)**

- We start from new loan form, and include these loans (that are supposed to go through savings account like BAAC, commercial bank, village fund) as withdrawals.
- We have to assume something about when they withdraw since we do not always have that information; we assume that households withdraw immediately based on what we saw from information that we have.



#### **Other Issues**

- There are new questions about "remaining amount after deposit", and "remaining amount after withdrawal".
- In theory, it does not make sense to have two different return points. We ask both questions just to see whether households behave differently regarding to deposit and withdrawal.
- About 95% of households answer the same number for both questions which is reassuring although this is from different sample.



Table 1: Net Cash Consumption for Rural Thai Households

	$Median^a$	$Mean^a$	Std Dev $^a$
$\mathbb{E}[e]$	0149	-0.131	0.938
$\mathbb{V}ar[e]$	$4.00^{2}$	$5.08^{2}$	$6.41^{2}$
kurtosis[e]	20.54	30.74	25.87
skewness[e]	1.208	0.694	4.111
$\mathbb{C}orr[e,e']$	0.012	0.032	0.134
	0.005	0.000	0.000
$\kappa_p \Delta$	0.025	0.028	0.022
$z_p$	14.32	22.62	28.14
$\kappa_n \Delta$	0.016	0.018	0.016
$z_n$	13.37	23.17	66.88

289 households with active savings account. e: monthly net cash consumption, normalized by average monthly household consumption.  $^a$ : Mean, Median and Std Dev across households of corresponding statistic.  $z_p$ : average value of e if e > 2Std[e].  $\Delta \kappa_p$ : fraction of months for which e > 2Std[e].  $z_n$ : average value of |e| if e < 2Std[e].  $\Delta \kappa_n$ : fraction of months for which e < 2Std[e].

$$\mathbb{E}[e] = \Delta \left[ c + \kappa_p z_p - \kappa_n z_n \right] \tag{36}$$

$$\mathbb{E}[e^2] = (1 - \Delta(\kappa_n + \kappa_p)) \left(\Delta^2 c^2 + \Delta \sigma^2\right) + \kappa_p \Delta(z_p)^2 + \kappa_n \Delta(z_n)^2$$
(37)

• We use this simple statistical model to back up c and  $\sigma$  from equations (36) and (37) given  $\mathbb{E}[e]$ ,  $\mathbb{E}[e^2]$ ,  $\kappa_p$ ,  $z_p$ ,  $\kappa_n$ , and  $z_n$ .



#### **Cash Holding**

Table 2: Net Cash Consumption for Rural Thai Households by Occupation

Occupation			St	atistics				Implied	d Values
		$\mathbb{E}[e]$	$\mathbb{S}td[e]$	$\kappa_p \Delta$	$z_p$	$\kappa_n \Delta$	$z_n$	$\Delta c$	$\sqrt{\Delta} \ \sigma$
								$N_a$	M
All	mean median	-0.13 -0.015	5.08 3.99		22.62 14.32	0.018 0.017	23.16 13.37	See Marie 1	1.20 2.12

See Table 1 for definition of variables. The mean and median of the statistics across households in the occupation. The implied values of  $\Delta c$  and  $\sqrt{\Delta} \sigma$  are the solutions to equation (??) taking as given the mean values for  $\mathbb{E}[e], \mathbb{S}td[e], \Delta \kappa_p, \Delta \kappa_p, z_p$  and  $z_n$ . Given these values  $N_a$  is the expected number of adjustment per year, and M the average money holdings, the solution to the optimal policy using R = 0.05, r = 0.03 per year and b = 0.05/30, where 1 is the monthly average consumption. \* indicates a case where the implied solution for  $\sigma$  was negative, so it was set to  $\sigma = |c|$ .

From the model

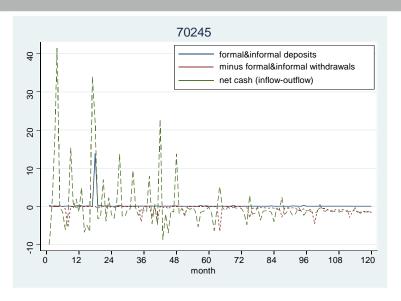


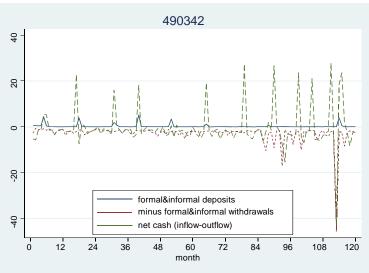
#### **Measuring Cash**

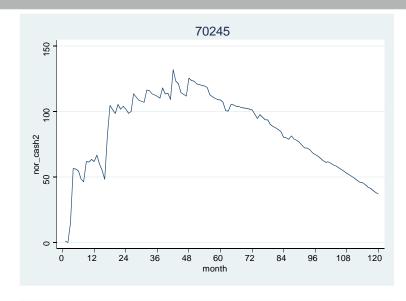
- \*All transactions in the monthly data are recorded as in cash or in kind.
- \*We do not know initial cash balance (decided not to ask). But we do see measured cash transactions, so we guess initial stock is zero (most conservative estimate). And if balance goes negative in some month, we add to initial stock so balance would be positive.
- We still see trends in the data and it seems cash is a store of value for year to year, even life cycle. The is an even bigger Anomaly!
- To study transaction demand, we "detrend" as best we can. Cash consumption number is adjusted so that on average, net cash of household inflow of those who have inflow more than outflow now will be zero (or as close as possible).
- ❖ Then, use this adjusted cash consumption to compute new *net\_cash*, and the statistics are based on active user only.

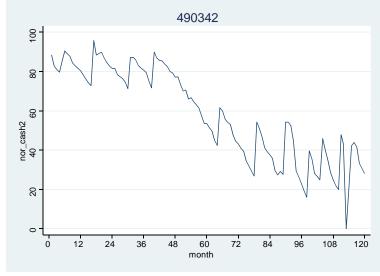


## **Illustrative Examples - 1**



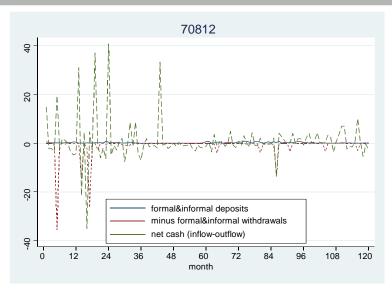


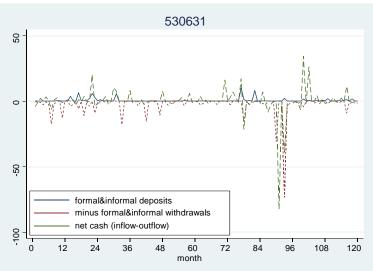


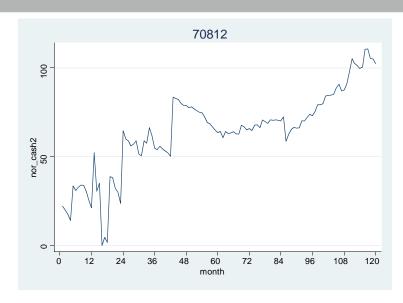


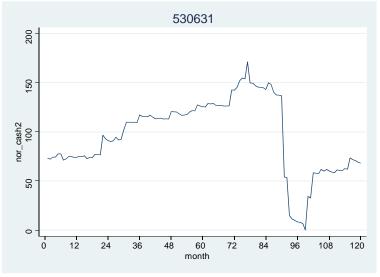


## **Illustrative Examples - 2**











## Money Holding, Frequency and Size of Adjustments: Data vs. Model at Plausible Values

	Money in Terms of Monthly Consumption							
	mean	sd	p25	p50	p75	N		
Total	45	37	21	32	57	531		

Table 3: Empirical Frequency and Size of Adjustments

Occupation	Statistics	Frequency	of Adjustments	Avg. Size of Adjustment		
111111111111111111111111111111111111111		Deposits	Withdrawals	Deposits	Withdrawals	
All	mean	15.6	10.3	0.267	0.869	
	median	16.2	8.7	0.172	0.569	

Frequency of Adjustment is number per year. Size of Adjustments is in terms of average household monthly consumption. Adjustment is the sum of formal (i.e. at financial intermediaries) and informal adjustments (to other households). Adjustments on different accounts are regarded as different adjustment.

Table 4: Model Frequency and Size of Adjustments

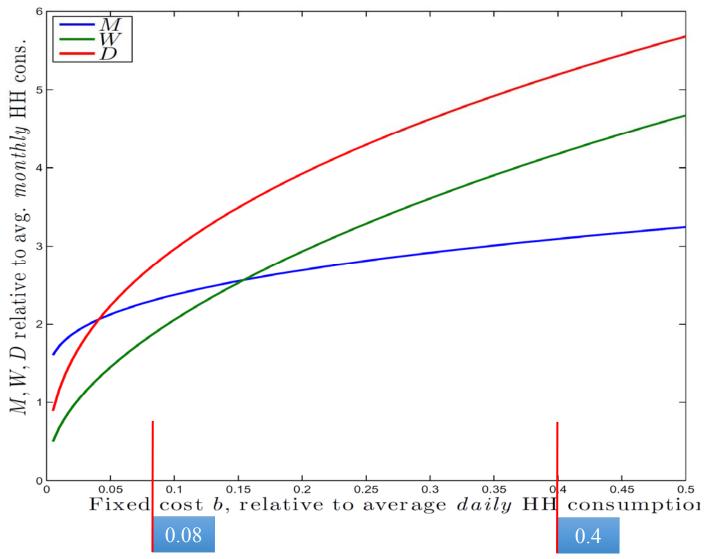
Occupation	b/R	Frequency	of Adjustments	Avg. Size	of Adjustment
		Deposits	Withdrawals	Deposits	Withdrawals
All	0.4	6.95	9.21	2.23	1.45
	0.08	17.9	27.7	1.17	0.676

Parameter values r = 0.02, R = 0.05 per year and b = 0.05/30 and b = 0.01/30. The remaining parameter values can be fond on Table 2.



#### Cash as a Function of Fixed Cost

Figure 1: Avg. cash balances M and agv. size of withdrawals W and deposits D





### Frequency as a Function of Fixed Cost

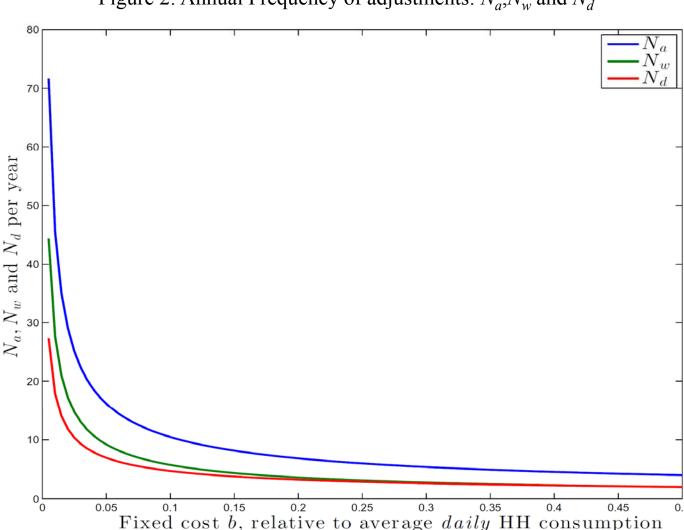


Figure 2: Annual Frequency of adjustments:  $N_a$ ,  $N_w$  and  $N_d$ 



# Cash and Adjustments According to Miller-Orr

- **\*** We use Miller-Orr model to make the point that Thai households seem to be holding too much cash and/or doing too many transactions.
- From Miller and Orr, we can combine both the optimal cash holdings and number of adjustments to get:
- $M^2 n = \left(\frac{4}{3}\right)^2 \frac{\sigma^2}{2}$ 
  - $\triangleright$  Where  $\sigma$  is the variance of the net cash expenditures per unit of time
- $2 \log M + \log n = 2 \log(4/3) \log(2) + 2 \log \sigma$
- $\bullet \log M = \log \sigma 1/2 \log n + \log(4/3) \log(2)$
- What are the units of M, n and  $\sigma$ ?
- $\bullet$  n = number of adjustments per unit of time, say days or years (we will use months)
- $\sigma^2$  = variance of the net cash flows per unit of time, say days or years (we will use months)
- $\star$  M = is a stock, independent of the time units
- ❖ But we have normalized *M* by the average monthly household consumption
- We measure  $\sigma^2$  as the variance of the monthly net cash expenditure
- \* We can rewrite the equation in levels:
- $M = \sigma n^{-\frac{1}{2}} \frac{2}{3}$
- $\star$   $M = \cosh / \text{ average monthly consumption}$
- $\sigma^2$  = variance of ( net cash consumption / average monthly real consumption)
- Using the numbers in Table 1 and 3 we have that
- $\sigma^2$  per month is  $\sigma^2$ , so  $\sigma = \sigma^2$ , and number of adjustment is about 25 per year.



# Cash and Adjustments According to Miller-Orr (cont.)

- \* Using that observed number of adjustments, we have:
  - $M = 5\left(\frac{25}{12}\right)^{-\frac{1}{2}} \frac{2}{3}$ , or approximately 2.309
  - This is much smaller than in the data.
  - > In the data we think that M is larger than 24, i.e. two years of household consumption.
- Alternatively suppose we want to get M = 24 using  $\sigma = 5$ . We find the value of b/R to obtain that number.
- We use that in Miller and Orr we have that the optimal decision rules are:
- $M = \frac{4}{3} \left( \frac{3}{4} \sigma^2 \frac{b}{R} \right)^{\frac{1}{3}}$ , or b/R = approximately 311
- For instance suppose that the cost b is one day of consumption, so that b = 1/30 and that annual interest rates are 3 percent, so that monthly rates are 0.03/12. Then we have
  - $\rightarrow$  *b/R* = approximately 13
- On the other hand, if interest rates would have been 0.001 per year (10 basis points) we have
- b/R = 1/30 / (0.001/12) = 400
- Note that with b/R = 311, so that we have M = 24
- ❖ But there will be "too few" transactions, then using
- $n = \sigma^2/2 [(3/4) \sigma^2 (b/R)]^{(-2/3)}$
- or
- $n = 25/2 [(3/4)(25)(311)]^{(-2/3)}$  approximately 0.04 per month or less than once every two years!
- ❖ That is, once every 2.22 years



#### **Currency: National Level Comparison**

- Currency in circulation per capita as of 31 December 2012: 17,628 baht
  - > nominal GDP: 11363.0 billion baht
  - > population: 64.5 million
- So nominal GDP per capita is about 176,170 baht (11363000/64.5), and per month it would be 14,680 (176170/12) baht. If consumption is about 55%, then monthly consumption is about 8,074 baht per person (14680\*0.55). So per capita stock of cash to monthly consumption is about 2.1833 (17628/8074).
- ❖ By 2010 population census, the number of individuals in a household is 65.5/20.3 =3.22. So 2.1833 per person scale up to per household
  - > we multiply to arrive at 7.03
- ❖ Of course, this is still a lot lower by a factor of 5-7 than the average or median number in our monthly survey; but it is higher than what we seem to get with out calculations coming from the standard model.
- \* SES, cash holdings at beginning (and end) of last month as percent of consumption expenditures
- Lots of zeros and non-responses
  - ▶ Bangkok = 15%
  - $\rightarrow$  Urban = 31-32%
  - ightharpoonup Rural = 36-43%
- So, say rural is perhaps 3 times higher than Bangkok with 1/7<sup>th</sup> of population, with algebra we might get a hard number, like 8.0
- \* We are off by a factor of 4 in the monthly data



# Comparison of Consumption in SES: We Do Not Appear to Mismeasure

- Monthly survey
  - ➤ Average monthly household consumption

by categories	of: CWT (C	hangwat)					
CWT	mean	sd	min	p25	p50	p75	max
   16-Lop Buri	11706.52	9678.398	 150	6016	8983	14626.5	78042
Chachoengsao	15173.52	9669.423	283	9149	12869	18606	58699
Buri Ram	7732.019	7613.458	645	3640	5635	9534	83316
Si Sa Ket	7826.542	6641.805	157	3908	5847	9600	47371
   Total	11263.92	10423.31	 0	5108	 8606	 14205	319061

at	mean	sd	min	p25	p50	p75	max
7	15630.74	28731.37	1379.925	7283.819	10049.66		323591.2
27	6369.197	4615.734	1240.64	3699.946	5255.231	7142.436	30007.67
19	10036.64	8913.054	471.3846	4965.662	7865.282	11090.56	66140.45
53	3136.89	2085.184		1761.991		3679.682	14274.67
+- al		16373.21		3309.462		10049.66	323591.2



## A Model That Matches National, Rural or Even Monthly Data (But At Implausible Parameter Values)

- \*We set parameters as in Thai data as before in the benchmark
- Then to match the frequency of transactions we allow free withdrawals (or deposits) at the observed frequency, 16 per year
- $\clubsuit$  Finally we raise the cost b for all trips, quite high



#### **Modify Benchmark Model**

- \* The case where discount rates are the same
  - > compares cases that have free adjustment opportunities with cases without free adjustment opportunities.
- **.** Cash balances as well as the frequency of total adjustment are the same, as long as  $\kappa_f + \kappa_n + \kappa_p$  stays constants.
- Only difference is on
  - > the average size of deposits and withdrawals
  - > and potentially on the ratio of the number of deposits to withdrawals.
- The mechanism of free adjustment opportunities and the one of large net cash purchases are substitutes to explain the size of cash balances and frequency of adjustments.
- **\diamond** Consider the following two setting of the parameters:  $\theta$  and  $\theta'$  with  $\theta_0$  and  $\theta'_0$  and with:

$$\kappa_f = 0$$
,  $\kappa_f + \kappa_n + \kappa_p = \kappa_f' + \kappa_n' + \kappa_p'$  and  $r = r'$ 

 $\diamond$  Assume that for  $\theta$  the optimal policy is such that large net cash expenditure shocks trigger an adjustment, i.e. that

$$\min\left\{z_n\,,\,z_p\right\} > m^{**}\left(\theta\right)\,.$$

\* Then, the optimal policy thresholds are the same for  $\theta$  and  $\theta'$  and the value functions differ from a constant, i.e.:

$$m^*(\theta') = m^*(\theta)$$
,  $m^{**}(\theta') = m^{**}(\theta)$ ,  $V(m; \theta') = V(m; \theta) - b\left(\frac{\kappa_f}{r}\right)$  for all  $m$ 

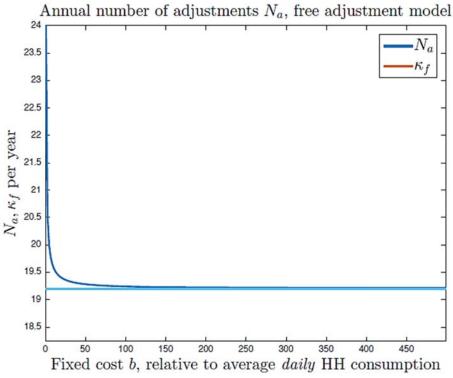
❖ Moreover, the distribution of cash holdings, the average cash balances, and the average number of adjustments per unit of time (i.e. sum of deposits and withdrawals) are the same for the two set of parameters, i.e.:

$$f(m;\theta') = f(m;\theta)$$
 for all  $m$ ,  $M(\theta') = M(\theta)$ , and  $N_a(\theta') = N_a(\theta)$ 



#### **Predicted Model Behavior**

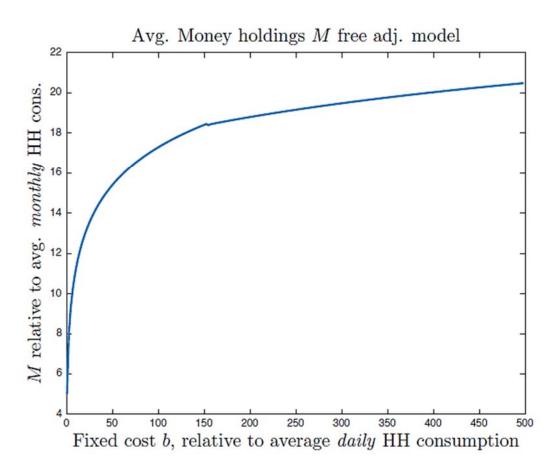
- ❖ In the limit households wait for free withdrawals which come at random times.
- ❖ When they do the withdraw the return point m\*
- ❖ If money drifts up and up, there will be an upper bound for deposits
  - > this happens rarely,
  - > quite expensive when it happens
- ❖ Overall average money balances in steady state will in fact be m\* if all shocks are symmetric
  - > as intuition suggests and numerical calculations confirm





#### **Predicted Money**

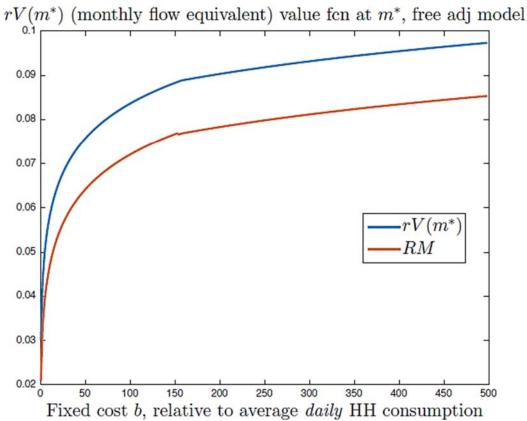
- $\bullet$  To get to M = 8, b > 1 day
- To get to M = 30, b is  $\approx 2$  years





#### **Welfare Costs**

- ❖ The costs, discounted expected present value
  - $\triangleright$  The current value function as a function of current money holdings, in particular  $m^*$ , can be calculated
    - Accounting costs at 8%
    - o Behavioral model at 9.5% of consumption
  - > If off by a factor of 4
    - $\circ$  Costs are 2 2.5% of monthly consumption





#### **Conclusion: High Cost of Petty Cash**

#### **❖**Model

➤ We do not believe existing class of models in the literature can rationalize observed patterns at plausible parameter values

#### **❖**Data

- Ideally have payment or diary
- ➤ Match with administrative data on savings accounts, loans, formal financial transactions

#### **❖** Normative

- ➤ Introduce cash management training, services, diaries, mobile money
  - Better recording
  - Less expensive transactions

