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ECONOMIC THEORY DISCUSSION PAPER

No. 191

PRICE,BUBBLES AND LEARNING

by

Stephen Morris
October 1993

ESRC Research Project on the
Economics of Missing Markets,
Learning and Games

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ECONOMIC THEORY DISCUSSION PAPERS

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RECENT TITLES IN THIS SERIES

Price Bubbles and Learning

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October 1993

Abstract

As traders learn about the true distribution of some asset's returns, a price bubble occurs as each trader anticipates the possibility of re-selling the asset to another trader before complete learning has occured. Reasonable 'ignorance' priors lead to large bubbles during the learning process. This phenomenon explains a paradox concerning the pricing of initial public offerings. The result casts light on the significance of the common prior assumption in economic models.

1. Introduction

A group of individuals may trade a risky asset whose returns are i.i.d. Over time, they learn the true distribution of the asset returns. But initially there is scope for disagreement on the true distribution. The traders are risk neutral and cannot sell the asset short. Therefore in equilibrium, the asset must always be held by the trader who values it the most, and the price must be equal to his valuation. How does the price vary as traders learn the true distribution? Does the price equal the true value, plus some noise which disappears as traders learn the true distribution?

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probability theory,

$$\sigma_i(n,t) = \frac{\int\limits_{\theta=0}^{1} \binom{t}{n} \theta^{n+1}(1-\theta)^{t-n}\pi_i(\theta)d\theta}{\int\limits_{\theta=0}^{1} \binom{t}{n} \theta^{n}(1-\theta)^{t-n}\pi_i(\theta)d\theta} \tag{2.1}$$

Let $\sigma_{MAX}(n,t)$ be the highest posterior probability of any individual,

$$\sigma_{MAX}(n,t) = \max_{i \in \{1,..I\}} \sigma_i(n,t)$$

Then equilibrium prices must satisfy

$$p(n,t,\delta) = \delta \begin{bmatrix} \sigma_{MAX}(n,t)\{1 + p(n+1,t+1,\delta)\} \\ +(1-\sigma_{MAX}(n,t))p(n,t+1,\delta) \end{bmatrix} \tag{2.2}$$

This condition states that the price of the asset after history (n,t) is equal to the highest expected discounted return (among all traders) of holding it to the next period. If the price of the risky asset was strictly higher than any trader's expected return from holding it to the next period, then no one will hold the asset and prices cannot be equilibrium prices. On the other hand, if the price was strictly lower than the highest expected return, then the trader with that highest expected return would want to hold infinite quantities, so markets would not clear.

A price scheme satisfying equation 2.2 can be explicitly calculated as follows. Set $p^0(n,t,\delta) = 0$ for all n, t, δ; define $p^k(n,t,\delta)$ recursively by

$$p^{k+1}(n,t,\delta) = \delta \begin{bmatrix} \sigma_{MAX}(n,t)\{1 + p^k(n+1,t+1,\delta)\} \\ +(1-\sigma_{MAX}(n,t))p^k(n,t+1,\delta) \end{bmatrix} \tag{2.3}$$

Now let $p^*(n,t,\delta) = \lim_{k\to\infty} p^k(n,t,\delta)$. To show the limit exists, I first show by induction that $p^k(n,t,\delta) \leq \frac{\delta}{1-\delta}$, for all k, n, t, δ: this is clearly true for $k = 0$; if it is true for k, then $p^{k+1}(n,t,\delta) \leq \delta\{1 + \frac{\delta}{1-\delta}\} = \frac{\delta}{1-\delta}$. Since $p^k(n,t,\delta)$ is non-decreasing in k, the limit exists. Since p^* is a fixed point of 2.3, it certainly satisfies equation 2.2.

It will be useful to study the normalized price of the risky asset in terms of the riskless asset. The value of the riskless asset in current dollars will always be

$\frac{\delta}{1-\delta}$, since the discounted (certain) return is $\delta + \delta^2 + \ldots$ Thus the normalized price of the risky asset is $q^*(n,t,\delta) = \frac{(1-\delta)p^*(n,t,\delta)}{\delta}$.

Finally, I require the following properties of beliefs.

Definition 1. *Priors are "common" if $\pi_i(\theta) = \pi_j(\theta)$ for all $i \in \{1,..I\}$, $\theta \in [0,1]$.*

Say that history (n',t') follows (n,t) if it is reachable from (n,t), i.e. $t' \geq t$ and $t' - t + n \geq n' \geq n$.

Definition 2. *Priors are "diverse" if for all i,n,t, there exist $j \neq i$, (n',t') following (n,t) such that $\sigma_i(n',t') < \sigma_j(n',t')$.*

We will see in the next section that this property is very weak.

Theorem 1. *(i) If priors are common, then $q^*(n,t) = \sigma_{MAX}(n,t) = \sigma_i(n,t)$ for all i, $n \leq t$; (ii) $q^*(n,t,\delta) \geq \sigma_{MAX}(n,t)$ for all $n \leq t$; (iii) if priors are diverse, $q^*(n,t) > \sigma_{MAX}(n,t)$ for all $n \leq t$; (iv) as $t \to \infty$, $q^*(\theta t,t,\delta) \to \theta$.*

Thus if all traders have the same prior (part i), the price of the risky asset in terms of the riskless asset after any history is simply the current common posterior probability of a dividend in future periods. If priors differ (part ii), then the price of the risky asset is at least as great as every trader's probability of a dividend. It will be strictly greater if some trader holding the asset anticipates the possibility of re-selling the asset at a premium over his valuation. This will always be true if priors are diverse (part iii). But through time traders' posteriors will converge to a common posterior $\frac{n}{t}$, so the price will converge to $\frac{n}{t}$ (part iv).

Proof. By the assumption that returns are i.i.d., trader i at time t has identical expectations of the return of the asset at times $t+1$ and $t+2$. Thus

$$\sigma_i(n,t) = \sigma_i(n,t)\sigma_i(n+1,t+1) + (1-\sigma_i(n,t))\sigma_i(n,t+1) \tag{2.4}$$

Now define

$$\alpha_i^k(n,t,\delta) = p^k(n,t,\delta) - \left\{\frac{\delta(1-\delta^k)}{1-\delta}\right\}\sigma_i(n,t) \tag{2.5}$$

and write $\alpha_i^*(n,t,\delta)$ for the (well-defined) $\lim_{k\to\infty} \alpha_i^k(n,t,\delta)$.

$$q^*(n,t,\delta) = \frac{(1-\delta)p^*(n,t,\delta)}{\delta} \begin{aligned} &= \tfrac{1-\delta}{\delta} \lim_{k\to\infty} \left\{\alpha_i^k(n,t,\delta) + \tfrac{\delta(1-\delta^k)}{1-\delta}\sigma_i(n,t)\right\} \\ &= \tfrac{1-\delta}{\delta}\alpha_i^*(n,t,\delta) + \sigma_i(n,t) \end{aligned} \tag{2.6}$$

By construction, $\alpha_i^0(n,t,\delta) = 0$. Now substituting equation 2.5 into 2.3 gives:

$$\alpha_i^{k+1}(n,t,\delta) = \delta \left[\begin{array}{c} \sigma_{MAX}(n,t) \left\{ \begin{array}{c} 1 + \alpha_i^k(n+1,t+1,\delta) \\ + \frac{\delta(1-\delta^k)}{1-\delta}\sigma_i(n+1,t+1) \end{array} \right\} \\ +(1-\sigma_{MAX}(n,t)) \left\{ \begin{array}{c} \alpha_i^k(n,t+1,\delta) \\ + \frac{\delta(1-\delta^k)}{1-\delta}\sigma_i(n,t+1) \end{array} \right\} \end{array} \right] - \left\{ \frac{\delta(1-\delta^{k+1})}{1-\delta} \right\} \sigma_i(n,t) \quad (2.7)$$

Re-arranging and substituting equation 2.4 gives

$$\alpha_i^{k+1}(n,t,\delta) = \delta \left[\begin{array}{c} \sigma_{MAX}(n,t)\,\alpha_i^k(n+1,t+1,\delta) + (1-\sigma_{MAX}(n,t))\alpha_i^k(n,t+1,\delta) \\ +[\sigma_{MAX}(n,t) - \sigma_i(n,t)] \left[1 + \frac{\delta(1-\delta^k)}{1-\delta} \left\{ \begin{array}{c} \sigma_i(n+1,t+1) \\ -\sigma_i(n,t+1) \end{array} \right\} \right] \end{array} \right] \quad (2.8)$$

Now if $\sigma_{MAX}(n,t) = \sigma_i(n,t)$ for all n,t, then $\alpha_i^k(n,t,\delta) = 0$, for all n,t,δ,k, so $\alpha_i^*(n,t,\delta) = 0$, for all n,t,δ, so (i) holds by equation 2.6; $\alpha_i^k(n,t,\delta)$ is non-decreasing in k, so $\alpha_i^*(n,t,\delta)$ is non-negative and (ii) holds by equation 2.6; diverse priors implies that, for all n,t, there exists (n',t'), following (n,t), such that $\sigma_i(n',t') < \sigma_{MAX}(n',t')$; thus $\alpha_i^k(n',t',\delta) > 0$, for all $k \geq 1$; this implies $\alpha_i^k(n,t,\delta) > 0$ for all $k \geq t'-t$; thus $\alpha_i^*(n,t,\delta) > 0$ and (iii) is true by equation 2.6; finally note that for any $\varepsilon > 0$, we can choose T such that $|\sigma_{MAX}(n,t) - \sigma_i(n,t)| < \varepsilon$ for all i, $n \leq t$, and $t \geq T$. Let $\alpha_{MAX}^k(T,\delta) = \max_{n \leq t, t \geq T, i \in \{1,..I\}} \alpha_i^k(n,t,\delta)$. By equation 2.8,

$$\alpha_{MAX}^{k+1}(T,\delta) \leq \delta\alpha_{MAX}^k(T,\delta) + \delta\varepsilon \left[1 + \frac{\delta(1-\delta^k)}{1-\delta} \right] \leq \delta\alpha_{MAX}^k(T,\delta) + \frac{\delta\varepsilon}{1-\delta} \quad (2.9)$$

Thus $\alpha_{MAX}^k(T,\delta) \leq \frac{\delta\varepsilon}{(1-\delta)^2}$ for all k, and thus (for all i) $q^*(\theta t,t,\delta) \to \sigma_i(\theta t,t)$ as $t \to \infty$ by equation 2.6. But (for all i) $\sigma_i(\theta t,t) \to \theta$ as $t \to \infty$, so (iv) holds □

When traders' priors differ, I do not have an analytic solution for q^*. However, it is simple enough to numerically calculate q^* using equation 2.3. This is done for various examples in the next section.

3. "Reasonable" Priors lead to Large Price Bubbles

A type i trader's valuation of the returns from the risky asset is $\sigma_i(n,t)$; this is what he would be prepared to pay if he was obliged to hold the asset forever. By a bubble, I will mean a situation where the price is strictly greater than any trader's valuation, i.e. $q^*(n,t,\delta) > \sigma_{MAX}(n,t)$. Theorem 1 showed that there is a bubble if traders' priors are diverse. In this section, I want to ask how reasonable is the diversity condition, and how big are the bubbles generated by such diversity.

Imagine a situation where the risky asset is being traded for the first time, so that traders do not have a history of past returns on the basis of which to form beliefs about θ. They must form some kind of "ignorance priors" about θ. I want to consider traders who are as reasonable and conservative as possible in the light of their ignorance, but I want to argue this will not pin down the exact prior they use.

Suppose that type 1 traders believe, after history (n,t) that there is a probability $\frac{n}{t}$ that a dividend will be paid next period, i.e. $\sigma_1(n,t) = \frac{n}{t}$ for $t \geq 1$; suppose also that before any observations, they believe that the probability of a dividend in the first period is $\frac{1}{2}$, i.e. $\sigma_1(0,0) = \frac{1}{2}$. What could more reasonable than this frequentist behavior? This "unbiased" posterior is essentially derivable from a prior $\pi_1(\theta)$ proportional to $\frac{1}{\theta(1-\theta)}$.[3]

Type 2 traders feel that their ignorance should be reflected in a uniform prior on the interval $[0,1]$, $\pi_2(\theta) = 1$. This apparently reasonable procedure implies that their posteriors are weighted closer to $\frac{1}{2}$: in particular, type 2 traders' posterior probability of a dividend in the next period, after history (n,t), is $\frac{n+1}{t+2}$, i.e. $\sigma_2(n,t) = \frac{n+1}{t+2}$. This also seems a thoroughly reasonable posterior. It is perhaps worth emphasizing that, despite many attempts, there is no philosophical (or other) agreement on how to assign priors in the face of ignorance. The "principle of insufficient reason" might be thought to support the uniform prior of type 2 traders, but this prior is not uniform on reasonable transformations of the parameter space.[4]

[3]This prior is improper (it has an infinite integral). But for $n \notin \{0,t\}$, $\frac{n}{t}$ is the posterior obtaining by the usual conditioning rule. For all (n,t) and $\varepsilon > 0$, $\frac{n+\varepsilon}{t+2\varepsilon}$ is the posterior for the (proper) prior proportional to $\theta^{\varepsilon-1}(1-\theta)^{\varepsilon-1}$; $\sigma_1(n,t)$ is the limit of that posterior as $\varepsilon \to 0$. For more on improper priors and posteriors under the binomial distribution, see Hartigan (1983), pages 76-78.

[4]Jeffreys (1946) proposed a procedure for constructing a prior which satisfies certain statistical properties which can be thought of as reflecting ignorance. In our example, the Jeffreys prior implies posteriors $\frac{n+\frac{1}{2}}{t+1}$. But there is no compelling argument why this prior should be

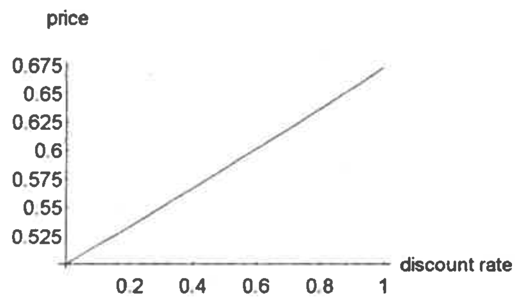

FIGURE 1.

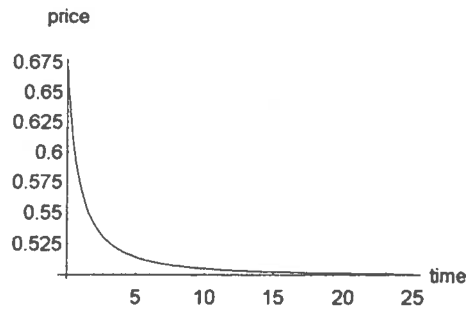

FIGURE 2.

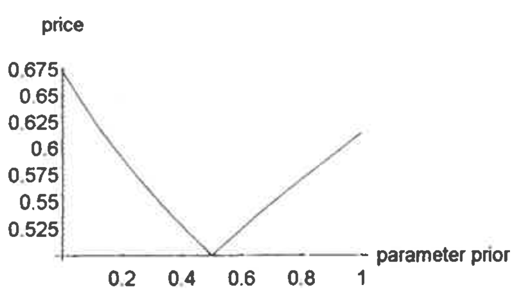

FIGURE 3.

I first show that these beliefs are diverse in the sense of definition 2. Observe that $\sigma_{MAX}(n,t) = \sigma_1(n,t) > \sigma_2(n,t)$ if $n > \frac{t}{2}$, while $\sigma_{MAX}(n,t) = \sigma_2(n,t) > \sigma_1(n,t)$ if $n < \frac{t}{2}$. So after any history (n,t), $\sigma_1(n+t+1, 2t+1) > \sigma_2(n+t+1, 2t+1)$ (since $\frac{n+t+1}{2t+1} > \frac{1}{2}$) and $\sigma_2(n, 2t+1) > \sigma_1(n, 2t+1)$ (since $\frac{n}{2t+1} < \frac{1}{2}$). Intuitively, type 1 traders are always more optimistic after good histories, type 2 traders are always more optimistic after bad histories, and no matter how much they have learned about θ, they always attach positive probability to switching from a good history to a bad history, or vice versa.

Thus by theorem 1 $q^*(n,t,\delta) > \max\left\{\frac{n}{t}, \frac{n+1}{t+2}\right\}$ for all $n, t \geq 1$ and $q^*(0,0,\delta) > \frac{1}{2}$. Thus at time 0, each type of trader values the asset at $\frac{1}{2}$. But each attaches positive probability to being able to re-sell it at a higher price to the other type after some history. I have calculated $q^*(n,t,\delta)$ numerically[5], and, in figure 1, I plot $q^*(0,0,\delta)$ as δ varies from 0 to 1. As $\delta \to 0$, the price converges to $\frac{1}{2}$ because the option to re-sell is worthless: traders care only about the next period returns. As δ increases, the option to re-sell becomes valuable, and $q^*(0,0,\delta) \to 0.674$ as $\delta \to 1$. These apparently reasonable priors lead to a price bubble of 35%.

Figure 1 illustrated the comparative statics of varying discount rates. What happens to the price through time? Suppose $\theta = \frac{1}{2}$. Prices are defined for particular realizations, but we can ask what happens along a path where dividends are in fact paid in exactly half the time periods. Theorem 1 part (iv) showed that through time, the price must converge from $q^*(0,0,\delta)$ down to $\frac{1}{2}$. This is shown in figure 2 for $\delta = 1$: $q^*(\frac{t}{2}, t, 1)$ is plotted as a function of t.[6]

We can also solve for what happens as we vary traders' priors. Suppose as before that type 2 traders have posterior $\frac{n+1}{t+2}$. But suppose now that type 1 traders have posterior $\sigma_1(n,t) = \frac{n(1-\gamma)+\gamma}{t(1-\gamma)+2\gamma}$. This posterior is derived from a prior $\pi_1(\theta)$ proportional to $(\theta(1-\theta))^{\frac{2\gamma-1}{1-\gamma}}$. Figure 3 plots $q^*(0,0,1)$ as γ (a suppressed argument of q^*) varies from 0 to 1.

Note that when $\gamma = 0$, type 1 traders have a posterior of $\frac{n}{t}$, so once again the initial price is 0.674. When $\gamma = \frac{1}{2}$, type 1 traders also have a uniform prior and (by part (i) of theorem 1) the price must equal $\frac{1}{2}$. Thus the price falls between

used rather than either of the two priors discussed above.

[5]More precisely, q^6 was calculated analytically by Mathematica software (giving a fifty page expression!); q^6 solves equilibrium conditions with errors uniformly below 0.001.

[6]Note that $q^*(n,t,\delta)$ is only defined for $\delta \in (0,1)$. By $q^*(n,t,1)$, I actually mean $\lim_{\delta \to 1} q^*(n,t,\delta)$. Also note that while we can calculate $q^*(\frac{t}{2}, t, \delta)$ for any value of t, it is only economically meaningful for even (and thus integer) values of t.

$\gamma = 0$ and $\gamma = \frac{1}{2}$ as traders' priors converge. But when $\gamma = 1$, type 2 traders believe that $\theta = \frac{1}{2}$ independent of history (they put unit mass on $\frac{1}{2}$), and the price is 0.604. Thus as γ increases from $\frac{1}{2}$ to 1, the price increases as traders' priors diverge again.

4. Initial Public Offerings

There are two empirical puzzles associated with initial public offerings [Ritter (1991)]. Offer prices tend to be significantly lower than initial market prices (the "under pricing" anomaly). Initial market prices tend to be high relative to long run prices (the "hot issue" anomaly). The model presented in section 2 provides one explanation for the latter phenomenon. When is it the case that an asset is traded in a situation where traders must form beliefs in the absence of historical data on the performance of returns? An initial public offering is an obvious example. While there may exist plentiful information on which to form an assessment, the lack of historical data creates scope for different traders to have different beliefs on the basis of the same information.

This explanation for the hot issue anomaly was presented some years ago by Miller (1977, page 1156). It is worth quoting in detail:

> The prices of new issues, as of all securities, are set not by the appraisal of the typical investor, but by the small number who think highly enough of the investment merits of the new issue to include it in their portfolio. The divergence of opinion about a new issue are greatest when the stock is issued. Frequently the company has not started operations, or there is uncertainty about the success of new products or the profitability of a major business expansion. Over time, this uncertainty is reduced.... With the passage of time, and the reduction of uncertainty, the appraisal of the top x percent of the investors is likely to decline even if the average assessment is not changed. This would explain the poor performance of a group of new issues when compared to a group of stocks about which the uncertainty does not decrease over time.[7]

[7]Miller also suggests that this argument offers a partial but non-strategic explanation for the under-pricing anomaly: "Incidently, if underwriters ignore the above arguments and price new issues on the basis of their own best estimates of the prices of comparable seasoned securities, they will typically underprice new issues. The mean of their appraisals will resemble the mean appraisal of the typical investor, and this will be below the appraisals of the most optimistic investors who actually constitute the market for the security. This may be a partial explanation for the underpricing of new issues by underwriters." But this implies that underwriters do not act strategically to exploit the heterogeneity of beliefs in the market.

The model of section 2 can be seen as a formalization of Miller's argument. However, the examples of section 3 make clear that the result does not require that traders have different prior valuations of the asset. It is enough that their "ignorance" priors over unknown parameters do not have the same shape.

Ritter (1991) provides further evidence for Miller's hypothesis which has been formalized here. Ritter finds that initial public offerings under perform the market by an average of 17% in their first three years. But this average varies with the age of the company - i.e. the number of years between its founding and the initial public offering. Controlling for industry, the initial overpricing of initial public offerings decreases monotonically from 34% for firms that are less than one year old to 4% for firms which are more than twenty years old. It is more plausible that traders have different beliefs on the basis of the same public information when the firm going public also has a shorter record under private ownership.

5. The Common Prior Assumption in Economic Theory

When Miller proposed his explanation of the "hot issue" anomaly, economists were less explicit about the origin of different (posterior) beliefs than they are today. Now we make a clear distinction between differences in posterior beliefs which are explained by differential information, and those which are unexplained by private information and thus represent a contradiction of the common prior assumption. The differences in beliefs in sections 2 and 3 cannot be interpreted as differences in information. If they were, then a "no trade" theorem along the lines of Milgrom and Stokey (1982) would guarantee no trade and no price bubbles.

I will argue that the example of section 3 helps explain why and how economists might - selectively - allow differences in prior beliefs to be used to understand economic phenomena[8]. I will do so by discussing in turn each of three strands of argument which are made in support of the common prior assumption, and arguing why they are not compelling at least in this and certain other contexts.

[8]Morris (1993) contains a more detailed discussion of the role of the common prior assumption in economic theory.

5.1. "Anything can happen when priors differ"

This puzzling opinion is highly prevalent in the economics folklore. Of course allowing differences in prior beliefs introduces another degree of freedom into modelling. Allowing differences in utility functions and information also introduces extra degrees of freedom. "Anything can happen" in many economic models under some appropriate assumptions about the heterogeneity of utility functions and information. Nonetheless, we remain interested in explaining phenomena in terms of heterogeneity of utility functions and information, even though we do not necessarily model the source of that heterogeneity. Heterogeneity of prior beliefs seems to be a victim of some kind of double standard.

In any case, heterogeneity of prior beliefs is just a particular form of heterogeneity of (ex ante) utility functions. Morris (1992) showed how the model of Harrison and Kreps which was the basis for this paper can be extended to allow for risk aversion and many goods in a full general equilibrium model (it is essentially the same model with differences in utility functions and endowments playing the role of differences of prior beliefs). This model is then similar in spirit to many models (including models of money) where short sales constraints drive a wedge in first order conditions to create phenomena which can be interpreted as bubbles: see, for example, Santos and Woodford (1992). The increased generality of allowing arbitrary utility functions is admirable. But if we think (as is surely the case for initial public offerings) that different priors are the primary source of the ex ante gains from trade, why not exploit the linear structure that focussing on beliefs allows?

Finally, consider theorem 1. Heterogeneity of prior beliefs, far from leading to "anything happening", leads to a unique set of prices[9]. Those prices have remarkable qualitative properties that are do not depend on the particular priors.

5.2. "Rationality entails the common prior assumption"

There is a widespread intuition that differences in beliefs between rational people must be a consequence of private information. This intuition conflicts with the usual economists' notion of rationality as consistency [Savage (1954)], and other attempts to formalize the intuition mathematically or philosophically have met with little success[10]. On a more practical level, I presented in section 3 the thought experiment of imagining traders forming priors about the returns of an as yet

[9] Section 6 discusses in detail the sense in which the prices are unique.

[10] See Morris (1993) section 3 for a more detailed discussion.

unobserved asset. It was extremely hard to conceive of any criteria - rational or otherwise - that might require traders to have the same prior. A number of different priors seem entirely reasonable.

An alternative way of presenting the "rationality implies common priors" argument can be told in the context of initial public offerings. The explanation of the over-pricing anomoly has something of a "winner's curse" flavor. It is tempting to argue that any trader holding the asset should want to revise downwards his valuation of the asset in the light of others' willingness to sell to him at that price. It is tempting, in other words, to interpret the different priors as different information.

No doubt many apparent differences in prior beliefs are explainable by different information at some level. But consider how implausible this argument is in the particular context of the example of section 3. Two traders with different priors over the value of θ must each conclude that the other knows something (about some true underlying distribution generating distributions over θ) that he does not. This would make sense only if we believed that there was some stationary distribution from which the value of initial public offerings was being drawn. But if we believe that each initial public offering contains something genuinely new (not predictable from past data), then genuine differences in priors are surely reasonable.

5.3. "Learning implies the common prior assumption"

We are justified in assuming common priors, the argument goes, because past experience will have removed differences in beliefs unexplained by differences of information. The common prior assumption is then justified when learning has finished, so that everyone has learned the true underlying data generating process. But presumably we live in a world where rational learning is still taking place. One reason why this is true is that there are new types of events whose distribution cannot be predicted from past experience. In the economy, there may be some data generating processes which have been learned. Initial public offerings are presumably a situation where learning has not been completed.

Indeed, the argument that learning justifies the common prior assumption can be turned around. Suppose we want to test the idea that learning has (typically) led to a world in which all differences in posteriors are explained by information. Then consider those (rare) situations where there has not been a chance for complete learning to occur. Presumably we should expect to find distinctive

behavior in those situations reflecting the heterogeneity of priors. In that sense, the over-pricing of initial public offerings is an indirect confirmation that learning does lead to a world which is as if the common prior assumption holds. But it takes a significant amount of time.

6. Other Equilibrium Prices

The purpose of this section is to identify possible equilibrium prices, other than those of section 2, but to argue that they are in some sense unreasonable. The argument of this section is essentially a special case of results in Harrison and Kreps (1978). In section 2, I assumed that prices do not depend on the historical pattern of dividends, only on their number. Here I want to allow for "sunspot" behavior, where the apparently irrelevant pattern of dividends effects prices.

A history of risky asset realizations is a vector $h^t = \{d_1, d_2, ..d_t\} \in H^t = \{0,1\}^t$. Write $n(h^t)$ for the number of dividends paid during history h^t, i.e. $n(h^t)$ is the number of elements of $\{s \in \{1,..,t\} | h^t_s = 1\}$. Write $\bar{p}(h^t, \delta)$ for the price of the risky asset after history h^t (in current dollars). As before, prices $\bar{p}$ are equilibrium prices if they are non-negative, no type of trader would strictly prefer to hold more of the asset and some trader is indifferent between holding the asset and not holding it. Now we have:

Theorem 1. *Prices $\bar{p} \geq 0$ occur is some competitive equilibrium if and only if:-*

$$\bar{p}(h^t, \delta) = \delta \underset{i \in \{1,..,I\}}{Max} \left[\sigma_i(n,t) \{1 + \bar{p}((h^t,1),\delta)\} + (1 - \sigma_i(n,t))\bar{p}((h^t,0),\delta)\right] \quad (6.1)$$

The equilibrium prices identified in section 2 certainly satisfied this property. But there may exist other equilibrium prices. This difference equation does not rule out many kinds of apparently odd behavior. For example, it is possible that prices $\bar{p}$ satisfying equation 6.1 have [1] prices depending on the order of dividends i.e. $\bar{p}(h^t, \delta) \neq \bar{p}(g^t, \delta)$, for some $g^t, h^t \in H^t$ and $n(h^t) = n(g^t)$; [2] prices decreasing in the number of dividends i.e. $\bar{p}(h^t, \delta) < \bar{p}(g^t, \delta)$, for some $g^t, h^t \in H^t$ and $n(h^t) > n(g^t)$; [3] prices higher than the value of the maximum conceivable payment of dividends, i.e. $\bar{p}(h^t, \delta) > \frac{\delta}{1-\delta}$, for all $h^t \in H^t$. But all these properties depend on the infinite horizon and thus entail "Ponzi schemes". Equilibrium prices p^* of section 2 are the limit (as $T \to \infty$) of the unique prices which would obtain in a T period truncation of the economy. Any other equilibrium prices satisfying equation 6.1 can be written in the form $\bar{p}(h^t, \delta) = p^*(n(h^t), t, \delta) + r(h^t, \delta)$, where $r \geq 0$ is a Ponzi scheme satisfying

$$r(h^t, \delta) = \delta[\sigma_{i(h^t)}(n,t)\, r((h^t,1),\delta) + (1 - \sigma_{i(h^t)}(n,t)) r((h^t,0),\delta)] \quad (6.2)$$

where $i(h^t) \in \underset{i \in \{1,..,I\}}{ArgMax} \{\sigma_i(n,t)\{1 + \bar{p}((h^t,1),\delta)\} + (1 - \sigma_i(n,t))\bar{p}((h^t,0),\delta)\}$.
This property is captured formally in the following lemma:-

Lemma 1. *Any non-negative prices $\bar{p}$ satisfying equation 6.1 also satisfy*

$$\bar{p}(h^t, \delta) \geq p^*(n(h^t), t, \delta)$$

Proof. We can describe a more general iterative process to generate a solution to equation 6.1. Define a mapping f on price functions such that

$$[f(\bar{p})](h^t, \delta) = \delta \underset{i \in \{1,..,I\}}{Max} \left[\sigma_i(n,t) \{1 + \bar{p}((h^t,1),\delta)\} + (1 - \sigma_i(n,t))\bar{p}((h^t,0),\delta)\right] \quad (6.3)$$

Thus $\bar{p}$ satisfies 6.1 if and only if it is a fixed point of f. Observe that f is increasing: if $\bar{p}(h^t, \delta) \geq \bar{p}'(h^t, \delta)$, for all $h^t \in H^t$, $t \geq 0$, then $[f(\bar{p})](h^t, \delta) \geq [f(\bar{p}')](h^t, \delta)$, for all $h^t \in H^t$, $t \geq 0$. Now define sequence of prices $\bar{p}^k$ inductively by setting $\bar{p}^0(h^t, \delta) = 0$, for all $h^t \in H^t$ and $\delta \in (0,1)$, and $\bar{p}^{k+1} = f(\bar{p}^k)$. Now if we let $\bar{p}^* = \lim_{k \to \infty} \bar{p}^k$, it is easy to check that $\bar{p}^*$ satisfies equation 6.1 and $\bar{p}^*(h^t, \delta) = p^*(n(h^t), t, \delta)$. Now consider some arbitrary non-negative prices $\bar{p}$ satisfying 6.1. Since $\bar{p} \geq \bar{p}^0$ and $f(\bar{p}) = \bar{p}$, we have by induction and monotonicity of f, $\bar{p}(h^t, \delta) \geq \bar{p}^k(h^t, \delta)$ for all k and so $\bar{p}(h^t, \delta) \geq \bar{p}^*(h^t, \delta) = p^*(n(h^t), t, \delta)$ □

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