Labor Economics, 14.661, Second Part, Problem Set 1

This problem set is due on or before the recitation on Friday, November 11.

Please answer the following questions:

Exercise 1 The economy lasts two periods. In period 1, an individual (parent) works, consumes c, saves s, decides how much education e to purchase on behalf of their offspring, and then dies at the end of the period. Utility of household i is given $U(c_i, \hat{c}_i)$, where $\hat{c}_i$ is the consumption of the offspring, and U is increasing in both of its arguments and jointly concave. There is heterogeneity among children, so the cost of education, $\theta_i e_i$, varies across i. In the second period, individuals receive a wage $w(e)$, where $w' > 0$ and $w'' < 0$ as usual.

1. Consider the case in which credit markets are perfect: households can borrow and lend at the same interest rate r. Characterize the household's decision problem. Show that the choice of education is independent of the form of the utility function.
2. Now assume a credit-market friction: households can lend at r, but cannot borrow going from period 1 to period 2. Write down the household's decision problem, including this new constraint. Show how the education and consumption decisions are no longer separable.
3. Now consider the following hypothetical situation: two households are identical, except that household A has a higher income and household B values offspring consumption more (i.e., $\frac{\partial U_B/\partial \hat{c}}{\partial U_B/\partial c} > \frac{\partial U_A/\partial \hat{c}}{\partial U_A/\partial c}$). How will their investments differ if credit markets are perfect? How will their investments differ if the households face a borrowing constraint as in (b)?
4. One of your colleagues just ran the following regression:

   $$\log(\text{Children's Income})_i \;=\; 0.35 \;\times\; \log(\text{Parents' Income})_i \;+\; \epsilon_i$$

   where i denotes a household "dynasty."

   Interpret this regression. Provide several theories that might explain this relationship and relate them to the model in part 1 above. Discuss how you might go about discriminating among these competing theories.
5. Upon including a variety of covariates (such as parents' education) into the equation, your colleague finds that the effect of parents' income drops by more than half. He claims that this constitutes evidence against the idea that poor parents cannot finance human capital investment due to credit-market imperfections. Outline an argument in support of this view. Then discuss problems with his conclusion.

Exercise 2 Recall the Ben-Porath model studied in the lectures and consider the following modification. First, assume that the horizon is finite. Second, suppose that $\phi'(0) < \infty$. Finally, suppose that

$$\phi'(h(0)) > \delta_h / (1 - \exp(-\delta_h T)),$$

where recall that δ_h is the rate of depreciation of human capital.

1. Provide the necessary conditions for an interior solution. Highlight how these necessary conditions should be modified to allow for corner solutions where $s(t)$ might take the value of 0 or 1.
2. Show that under these conditions the optimal path of human capital accumulation will involve an interval $[0, t')$ of full-time schooling with $s(t) = 1$ for all $t \in [0, t')$, where $t' > 0$, followed by another interval of on-the-job investment $s(t) \in (0, 1)$, and finally an interval of no human capital investment, i.e., $s(t) = 0$ for all $t \in (t'', T]$, where $t'' < T$. [Hint: suppose that the first part of the claim is not true, and show that in this case the necessary conditions must hold as equality. Combining the two necessary conditions, derive a first-order linear non-autonomous differential

equation for the costate variable $\lambda(t)$, and solve this differential equation with the boundary condition $\lambda(T) = 0$. Then, show that given the implied value for $\lambda(0)$ and the inequality above, the necessary conditions at $t = 0$ cannot be satisfied. Next use the assumption that $\phi' < \infty$ together with the fact that the costate variable $\lambda(t)$ is continuous and must satisfy $\lambda(T) = 0$ to prove that $s(t)$ must be equal to zero for some interval $[T - \xi, T]$. Finally, using these intermediate steps, conclude that $s(t)$ must take intermediate values before this final interval].

3. How do the earnings of the individual evolve over the life cycle?

4. How would you test the implications of this model?

Exercise 3 A worker of ability z faces the decision of how much education, e, to obtain. The wage schedule as a function of ability and education is

$$w(e; z) = \alpha_0 + \alpha_1 e z + \alpha_2 z$$

Education costs $c(e)$, where $c'(e) > 0$ and $c''(e) > 0$ for all e, and also $c'(0) > \alpha_0$.

1. Characterize the education decision for a given worker.

2. What is the "structural" return to education? (That is, what is the benefit of an additional unit e of schooling for a given worker?).

3. Derive the equilibrium observed return to schooling. Show that it can be decomposed into a "structural" and a "selection" component.

4. Explain whether/why this example is different from the one we saw in class in which observed returns to schooling depended on how we took the perspective of a "high z" type or a "low z" type.

Exercise 4 In this problem, you are asked to work through a model that combines signaling with productive aspects of schooling. There are two types of agents: "high" and "low" ability. Education (e) is continuous and observed, but individual ability (and output) is not. The labor productivity for the "low" type is $y_l(e) = \alpha_0$ and the cost of education is $c_l(e) = 3e^2/2$. For the "high" type, output and education costs are $y_h(e) = \alpha_1 + \alpha_2 e$ and $c_h(e) = e^2$, respectively. Let $\alpha_1 = \alpha_0$ for now.

1. Define Perfect Bayesian Equilibria (PBE) of this game (Hint: be specific about the actions of workers of different types and the actions of firms—which are wage offers as functions of publicly observable objects—at different points in time).

2. Solve for the PBE corresponding to the "Riley Equilibrium" (most efficient separating equilibrium) of this game. In particular, show that high type workers do not have an incentive to deviate from your proposed equilibrium strategies. Does the high types' investment in education differ from what would have obtained in the perfect-information case? Why or why not?

3. Suppose now that $c_l(e) = 10e^2$. Does the high types' investment in education differ from what would have obtained in the perfect-information case? Why or why not?

4. Suppose again that $c_l(e) = 3e^2/2$ and furthermore suppose that there is a compulsory schooling requirement of $\underline{e}$, where $0 < \underline{e} < \alpha_2/4$. Characterize the Riley Equilibrium. Does the high type invest in education more or less in this case than in (2)? Explain why.

5. Characterize the equilibrium if $y_l(e) = y_h(e) = \alpha_1 + \alpha_2 e$; $c_l = 3e^2/2$; and $c_h = e^2$.

6. Does a separating equilibrium exist when $y_l(e) = \alpha_1' + \alpha_2 e$; $y_h(e) = \alpha_1 + \alpha_2 e$; $c_l = e^2$; and $c_h = e^2$ with $\alpha_1' < \alpha_1$?

7. Compute the observed return to schooling in part (1).

8. How does the observed return to schooling change if $\alpha_1 - \alpha_0$ increases (starting from zero)? Explain the intuition for both the forces that tend to increase and decrease observed returns to schooling in this case.

Exercise 5 Consider the following three-period economy with a unique final good. There are two types of workers, high ability and low ability, with respective fractions λ and $1 - \lambda$ in the population. Denote worker ability by $a \in \{0, 1\}$, with $a = 0$ corresponding to low ability. Worker ability is private information. At $t = 1$, workers, knowing their ability level, choose a level of schooling $e \in \{0, 1\}$. The cost of education to both types (in terms of the final good) is c. At $t = 2$, a large number of risk-neutral homogeneous firms compete for workers and at this point, worker ability is not observable to firms, but worker education is. Suppose that each firm can only hire one worker, and the production function of all firms is such that at time $t = 2$, each produces

$$y_2 + v_2 ae$$

where a and e refer to the worker that is hired.

At $t = 3$, worker ability becomes public information (for example, because the output of each firm in period 2 is publicly observed). Firms again compete for workers, now with production function for $t = 3$:

$$y_3 + v_3 ae.$$

There is no discounting between periods, and workers maximize the net consumption of the unique final good of this economy (i.e., sum of their wage income minus cost of education).

1. Define Perfect Bayesian Equilibria (PBE) of this game.

2. Suppose that
$$v_2 < c < v_2 + v_3.$$
Under this assumption, characterize a separating PBE, where high ability workers choose education ($e = 1$) and low ability workers do not ($e = 0$).

3. How does this equilibrium differ from the standard separating equilibrium of the Spence signaling model (with the single-crossing assumption)? Interpret and compare the two models. In your view, which one is a better approximation to the signaling role of college education in reality?

4. Now show that if, in addition, $c > v_3$, there exists a pooling equilibrium in which no worker obtains education. Explain whether or not you find this equilibrium satisfactory. Why or why not? If not, how would you eliminate this pooling equilibrium by strengthening the equilibrium concept?