14.461: Problem Set 3

Due: Friday October 24, 2014.

1. Assume a discrete time infinite horizon economy. In each period final output is produced under perfect competition, according to:

$$Y = \left[\gamma_c Y_c^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_d Y_d^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is the elasticity of substitution between clean and dirty inputs.

The environmental stock S evolves over time according to:

$$S_{t+1} = -\phi Y_{dt} + \delta S_t.$$

If S_t hits zero, then all agents receive a very large negative utility thereafter (i.e., there is an environmental disaster).

Clean and dirty inputs are produced as

$$Y_c = \int_0^1 \left(A_{c,i}^{1-\alpha} x_{c,i}^{\alpha} di\right) L_c^{1-\alpha} \qquad Y_d = \int_0^1 \left(A_{d,i}^{1-\alpha} x_{d,i}^{\alpha} di\right) L_d^{1-\alpha}$$

where $0 < \alpha < 1$, A's are productivity parameters, and x's are quantities of intermediates or machines, with the convention that there is a continuum of machine for each input. L is labor, supplied inelastically, with the market clearing condition

$$L_c + L_d \leq 1.$$

These intermediates are produced from the final good (one unit of the final good used for producing one unit of either intermediate of any quality). Let us also define

$$A_c = \int_0^1 A_{c,i} di \qquad A_d = \int_0^1 A_{d,i} di$$

Suppose that clean and dirty inputs are produced by monopolists. R&D (to improve the productivity of these inputs) is directed either towards dirty or clean inputs, and successful R&D directed towards sector j increases the productivity of a randomly chosen machine type from sector j by η. Suppose that R&D will take place only in whichever sector has higher (static) expected profits (which implies that A_c or A_d will grow at the rate η if all R&D resources are allocated to that sector).

(a) Derive the demand for clean and dirty inputs (i.e., determine their prices p_c and p_d as a function of Y_c and Y_d).

(b) Derive the demand for machines and the profit maximizing monopoly prices. Given these, compute the equilibrium values for Y_c, Y_d and Y (as a function of the initial values of A_c and A_d) and expected profits from R&D in the two sectors.

(c) Show that when $\varepsilon > 1$, A_d/A_c sufficiently high implies that all R&D inputs at that date and in the future will be allocated to the dirty sector.

(d) Now suppose final producers pay a tax τ on each unit of dirty output. Show that starting with ε sufficiently high, and $A_c < A_d$, a sufficiently high tax rate imposed for T periods (where T is sufficiently large but finite) will redirect R&D from dirty to clean inputs. Will there be R&D directed towards the dirty inputs in the long-run (note that T is finite, so there is no tax in the long run)?

(e) Now we will study what happens to long-run growth and pollution. First, suppose, differently from the models studied so far, that L_c and L_d are two exogenously supplied different inputs and remain at their initial values (i.e., there is no reallocation of labor between clean and dirty inputs). Show that this has no effect on the results in 3 and 4 above. Then show that in this case, even though a temporary tax as in 4 can redirect R&D from dirty to clean inputs, this does not avert an environmental disaster. [Hint: show that in this case, the production of the dirty input is proportional to $p_d^{1/\alpha} A_d$ and p_d will continue to increase along the equilibrium path].

(f) Now return to the actual model where labor can be reallocated between the two sectors, and show that in this case if ε is sufficiently high, the production of the dirt input is bounded when all R&D is directed to clean machines, and thus a temporary tax as in 4 is sufficient to prevent an environmental disaster. Provide an intuition for this result contrasting it with that in 5. What happens if ε is only slightly above 1?

(g) Explain (without doing the math) how the results might be different if the dirty sector used an exhaustible resource, such as oil?

2. Time is continuous and the economy consists of a homogeneous set the workers with total supply of labor normalized to 1 (and supplied inelastically). The final good at time t is produced from intermediates as

$$Y(t) = \left(\int_{\mathcal{N}(t)} y_j(t)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\mathcal{N}(t) \subset [0,1]$ is the set of active intermediates at time t and $\varepsilon > 1$. Suppose the economy is open so that the interest rate is fixed at r^*. At any point in time some firm has access to the highest quality production for each intermediate. Total cost of producing y_j units of this intermediate at time t is then

$$TC_j(y_j, t) = w(t)\left[\phi + \frac{y_j}{q_j}\right],$$

where q_j is the quality, ϕ is the fixed cost of operation and $w(t)$ is the wage rate at time t. A large set of potential entrants can hire labor to improve over the quality of one of the products (randomly chosen from $[0,1]$), and these entrants are the only source of new research this economy. If an entrant hires h workers, it achieves successful innovation at the flow rate ηh. If the chosen product following the successful innovation is active (i.e., in $\mathcal{N}(t)$), then the quality of this product increases by a factor of $\lambda > 1$. If the product is inactive, then the quality of the innovation is equal to mean quality of all active products. Assume that λ is large enough so that the firm with the highest quality can charge the unconstrained monopoly price.

(a) Focus on a stationary equilibrium in which output and the wage rate grow at the rate g^* and aggregate innovation takes place at the flow rate ηh^*. Define $\hat{q}(t) \equiv q/Q(t)$ (where $Q(t) = \left(\int_{\mathcal{N}(t)} y_j(t)^{\varepsilon-1} dj\right)^{\frac{1}{\varepsilon-1}}$) as relative quality. Write profits as a function of relative quality and show that there exists some threshold $\hat{q}_{\min}$ such that if $\hat{q}(t) < \hat{q}_{\min}$, it is not profitable to produce intermediate with relative quality $\hat{q}(t)$. What is $\hat{q}_{\min}$?

(b) Show that the value of having a product of relative quality $\hat{q} > \hat{q}_{\min}$ at time t can be determined as the solution to the following differential equation:

$$(r^* + \eta h^*) V(\hat{q}) - g^* \hat{q} V'(\hat{q}) = \Pi \hat{q}^{\varepsilon-1} - \tilde{w}\phi \text{ for } \hat{q} > \hat{q}_{\min},$$

where Π is a constant (scaling profits) and $\tilde{w} \equiv w(t)/Y(t)$ is the wage to GDP ratio which is constant in a stationary equilibrium. Provide an intuition for this differential equation. What accounts for the second term on the left hand side? Explain why the presence of this term implies that there exists some $T < \infty$ such that a product will remain active at most for an interval of time T (and also explain why it may be replaced before this time).

(c) Can you solve this differential equation for the value function?

(d) Write labor market clearing condition. (Hint: total labor demand from entrants and all active products must sum to 1).

(e) Derive the free entry condition that determines h^*.

(f) Can you write an equation that determines the growth rate g^*.

(g) Explain intuitively the conditions under which model will generate a large fraction of firms endogenously stopping operation at each point in time.

(h) What are the pros and cons of this model relative to the basic Schumpeterian model? How would you generalize this model (please be explicit about what are the features in the data or that are interesting conceptually that the model in its current form cannot deal with and motivates your generalization)?

3. Consider the "Growing like China" model covered in the lecture. Suppose that there is no moral hazard problem between managers and owners of firms.

   (a) Characterize the equilibrium and explain how the results are different from those in the paper.

   (b) How would you modify the model in the paper to make it a better representation of the process of structural change in China?

4. Consider a competitive economy consisting of two sectors. Sector 1's production function is

$$x_1 = A_1 l_1$$

   and sector 2's production function is

$$x_2 = A_2 l_2^{\alpha} x_{21}^{1-\alpha},$$

   where l_j is labor demand by sector j, and x_{21} is the amount of sector 1's output used as intermediates for sector 2's production.

   Suppose also that the consumer side can be represented by demand relationships:

$$c_j = B_j p_j^{-\beta},$$

   for $j = 1, 2$, where c_j and p_j denote, respectively, consumption and price of sector j.

   (a) Show that when $\beta = 1$, a decrease in A_1 reduces output in sector 2, but a decrease in A_2 has no impact on output in sector 1. Explain the intuition for these results carefully.

   (b) Show that when $\beta = 1$, a decrease in B_1 has no impact on output in sector 2, but a decrease in B_2 reduces output in sector 1. Explain the intuition and contrast it with a part (a).

   (c) Now suppose that $\beta > 1$, show that a decrease in A_2 reduces output in sector 1. What happens when $\beta < 1$?

   (d) What can you conclude from this simple example about the impact of input-output linkages on volatility and propagation of shocks in a general economy?