14.452: Problem Set 5

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For Review Only. Not to Be Handed in.

Question 1: Consider the following endogenous growth model. Population at time t is $L(t)$ and grows at the constant rate n (i.e., $\dot{L}(t) = nL(t)$). All agents have preferences given by

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

where C is consumption defined over the final good of the economy. This good is produced as

$$Y(t) = \left[\int_0^{N(t)} y(\nu, t)^\beta \, d\nu \right]^{1/\beta},$$

where $y(\nu, t)$ is the amount of intermediate good ν used in production at time t and $N(t)$ denotes the number of intermediate goods available at time t. The production function of each intermediate is

$$y(\nu, t) = l(\nu, t)$$

where $l(\nu, t)$ is labor allocated to this good at time t. New goods are produced by allocating workers to the R&D process, with the production function

$$\dot{N}(t) = \eta N^\phi(t) L_R(t)$$

where $\phi \leq 1$ and $L_R(t)$ is labor allocated to R&D at time t. So labor market clearing requires $\int_0^{N(t)} l(\nu, t) \, dv + L_R(t) = L(t)$. Risk-neutral firms hire workers for R&D. A firm who discovers a new good becomes the monopoly supplier, with a perfectly and indefinitely enforced patent.

1. Characterize the BGP in the case where $\phi = 1$ and $n = 0$, and show that there are no transitional dynamics. Why is this? Why does the long-run growth rate depend on θ? Why does the growth rate depend on L? Do you find this plausible?

2. Now suppose that $\phi = 1$ and $n > 0$. What happens? Interpret.

3. Now characterize the BGP when $\phi < 1$ and $n > 0$. Does the growth rate depend on L? Does it depend on n? Why? Do you think that the configuration $\phi < 1$ and $n > 0$ is more plausible than the one with $\phi = 1$ and $n = 0$?

Question 2: Consider a model of direct technological change with two sectors, and the final good produced from these two sectors according to the technology:

$$Y(t) = F[Y_L(t), Y_H(t)]$$

where F is linear homogeneous, differentiable and increasing in both of its arguments. Sector $j = L, H$ has the production function

$$Y_j(t) = \frac{1}{1-\beta}\left(\int_0^{N_j(t)} x_j(\nu, t)^{1-\beta}\, d\nu\right) Q_j^{\beta},$$

with $Q_L = L$ and $Q_H = H$, and x's corresponding quantities of machines that depreciate fully after use. The innovation possibilities frontier of the economy takes the form

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t),$$

and the resource constraint is $C(t) + X(t) + Z_L(t) + Z_H(t) \leq Y(t)$. The household side is represented by a representative household with CRRA preferences defined over the final good, $Y(t)$. Throughout, the total supplies of the two factors, L and H, are taken as constants.

1. Solve the maximization problem of sectors L and H and show that in equilibrium

   $$Y_j(t) = \frac{1}{1-\beta} p_j(t)^{\frac{1-\beta}{\beta}} N_j(t) Q_j,$$

   where $p_j(t)$ is the price of the output of sector j at time t.

2. Define a Balanced Growth Path (BGP) equilibrium as an allocation in which the final good grows at a constant rate and $p_L(t)$ and $p_H(t)$ are constant. Derive a condition for the BGP as a function of L, H, p_L and p_H. Derive relative wage ratio w_H/w_L as a function of L, H, p_L and p_H. Then solve out for p_H/p_L as a function of H/L (Hint: first derive the relationship $p_H/p_L = (Y_H/Y_L)^{-1/\varepsilon}$ where ε is the local elasticity of substitution (of the function F) between Y_H and Y_L, defined as $\varepsilon = \partial \ln(Y_H/Y_L)/\partial \ln(p_H/p_L)$).

3. What is the effect of an increase in H/L on w_H/w_L? Can an increase in H/L raise w_H/w_L? (If not, why not? If yes, under what conditions?). Can you define the notion of induced bias of technology and show that an increase in H/L always makes technology further bias towards factor H? (Hint: first establish this result assuming that the local elasticity of substitution, ε, is constant, and then argue that even if it is not constant, the same result applies).

4. Suppose now that final good producers face a tax rate of τ when purchasing Y_H (i.e., instead of p_H, they pay $(1+\tau)\,p_H$), the proceeds of which are rebated lump-sum to the representative household. Taking N_L and N_H as given (and constant), find the relation between τ and p_H/p_L. Now, starting from a BGP with $\tau = 0$, consider a permanent increase to $\tau > 0$. Show that this will at first change p_H/p_L but then in finite time p_H/p_L will return to its initial level (to its BGP level with $\tau = 0$). Is taxing sector H inducing further biased towards factor H? Explain and provide the intuition.

Question 3: Consider a world economy that consists of J countries, indexed by $j = 1, .., J$. Each country admits a representative household with preferences at time 0 given by

$$\int_0^\infty \exp(-\rho t)\,\frac{C_j(t)^{1-\theta} - 1}{1-\theta}\,dt.$$

Population in each country is constant and given by L_j. Labor is supplied inelastically. The unique final good in each country is produced with the production function

$$Y_j(t) = \frac{1}{1-\beta}\left[\int_0^{N_j(t)} x_j(\nu,t)^{1-\beta}\,d\nu\right] L_j^\beta,$$

where $\beta \in (0,1)$, $x_j(\nu,t)$ denotes intermediate goods of type ν used in final good production at time t, and $N_j(t)$ is the number of intermediate good types available in country j at time t. Once a particular type of intermediate good is invented, it can be produced by using $\psi \equiv 1-\beta$ units of final good. The innovation possibilities frontier of country j is

$$\dot{N}_j(t) = \eta_j \left(\frac{N(t)}{N_j(t)}\right)^\phi Z_j(t), \tag{1}$$

where $\eta_j > 0$, $\phi > 0$, $Z_j(t)$ is total amount of R&D spending, and

$$N(t) = \Phi(N_1(t), ..., N_J(t)) \tag{2}$$

represents the average technology in the world, where Φ is a linearly homogeneous function (in its J arguments). The resource constraint in each country is $C_j(t) + X_j(t) + Z_j(t) \le Y_j(t)$, where $X_j(t)$ is spending on intermediate goods. There is free entry into research and a firm that invents a new intermediate good type receives a perpetual patent and becomes the monopolist producer of that good. Each country starts with $N_j(0) > 0$ intermediate goods at time $t = 0$.

1. Define a world equilibrium.

2. Consider a BGP equilibrium in which the growth rate in each country is the same and constant at g. Characterize the BGP equilibrium output and monopolist profits in each country, and determine the free entry condition. Use the free-entry condition to solve for the relative level of output $v_j \equiv$

$\frac{N_j(t)}{N(t)}$ in each country j, as a function of the world growth rate g. Show that countries with a greater η_j and L_j have greater relative levels of output on the BGP.

3. Is the allocation characterize in 2 the world equilibrium? Is it asymptotically stable? [Hint: you can answer these questions without doing any algebra, just giving the high-level idea.] Why is it that countries with different levels of η_j and L_j differ in terms of their relative incomes but not their growth rates?

4. Suppose that $\Phi(N_1(t), ..., N_J(t)) = \max\{N_1(t), ..., N_J(t)\}$. Derive the world growth rate. Now suppose that $J = 2$ and both countries are identical. How does this change the equilibrium? Is the world equilibrium "symmetric" (meaning the two countries investing same amount in R&D and achieving the same level of income)? Now suppose that the level of R&D in each country is determined by a country-welfare maximizing social planner (so that we have a game between two country social planners). Argue (without providing algebraic details) whether you expect the world equilibrium in this modified to still be symmetric.