On the Size of Stimulus Checks: How Much is Too Much?*

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Abstract

Stimulus checks have become an increasingly important policy tool in recent U.S. recessions. How large must checks be to close a given share of the output gap? How much is too much? In answering these questions, we recognize that households’ marginal propensity to spend (MPC) out of stimulus checks varies with their size. Lumpy purchases of consumer durables are central to this size-dependence. We augment a canonical model of durable spending by introducing a smooth adjustment hazard. This specification can accommodate a decreasing, flat, or increasing MPC. We discipline the adjustment hazard by matching the evidence on (i) the relative MPCs of durables and non-durables; (ii) the short-run price elasticity of durables; (iii) the size distribution of adjustments; and (iv) the conditional probability of adjustment since the last purchase. We find that the MPC remains elevated even for large stimulus checks. In a typical recession, the stimulus check that closes half the output gap is half as large compared to a canonical model of non-durables with the same MPC out of a small transfer, and three times larger compared to a canonical model of durables.

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1 Introduction

Stimulus checks have become an increasingly important policy tool in recent U.S. recessions. The average eligible household received a tax rebate of $300 in 2001 and $600 in 2008, and an economic impact payment of $2,000 in 2020–2021. The government relies on these stimulus checks to boost spending and narrow the output gap. Yet, we know surprisingly little about the appropriate size of stimulus checks in a recession. How large must the checks be to close a given share of the output gap? How much is too much?

To quantify the effect of stimulus checks, it is crucial to recognize that households’ marginal propensity to spend (MPC) varies with the size of the checks. This size-dependence in the MPC affects the “bang-for-the-buck” of each additional dollar. For instance, if households spend less of each additional dollar they receive (Kaplan and Violante, 2022), the government must send larger checks to offset this decline in the MPC to close the same share of the output gap.

There is a large degree of uncertainty regarding the size-dependence in the MPC, with a wide range of model-based predictions and empirical estimates. State-of-the-art models of non-durable spending predict that the MPC falls rapidly with the size of stimulus checks (Kaplan and Violante, 2014). The relevant quantity for the question of interest is total household spending, however, including durables. Indeed, durable spending represents a large share of the MPC of total spending (Souleles, 1999; Parker et al., 2013). We find that canonical models of lumpy durables purchases (e.g., Berger and Vavra, 2015) predict that the MPC increases steeply with the size of stimulus checks. Empirically, studies find that the MPC is decreasing (Coibion et al., 2020), essentially flat (Sahm et al., 2012), or even increasing (Fuster et al., 2021). Despite these mixed findings, all these studies point to a persistently high MPC for large stimulus checks, which neither decreases sharply nor shoots up as checks get larger.

In this paper, we augment a canonical incomplete-markets model of durable spending by introducing a flexible form of time-dependence in the adjustment process. This allows the model to accommodate a decreasing, flat, or increasing MPC. We discipline the model with micro-level moments, and use it to quantify the size of stimulus checks that close a given share of the output gap. We find that the MPC remains elevated even for large stimulus checks. In a typical recession, a $600 check closes roughly half the output gap. This amount is half as large compared to a canonical model of non-durable spending.

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1 We use the term “marginal propensity to spend” (MPC) to refer to the average spending response divided by the size of the check. The empirical counterpart of this object is what Kaplan and Violante (2014) refers to as the “rebate coefficient.” We also use the term MPC when analyzing total spending, i.e., including non-durables and durables. The response of durable spending is sometimes called the “marginal propensity to invest” (MPX) as in Laibson et al. (2022).
with the same MPC out of a small transfer, and three times larger compared to a canonical model of durable spending. This analysis also provides an upper bound for policymakers: the size of the check that fully closes the output gap. Checks exceeding that amount are “too much” in that they stimulate the economy beyond potential.\(^2\) Across different specifications of our model, we find that checks of $1,500 to $2,000 fully close the output gap in a typical recession.

The starting point of our analysis is a canonical model of durable purchases (Berger and Vavra, 2015) augmented with two realistic features. First, we allow for time-dependent adjustments in a flexible way. Households are subject to linearly additive taste shocks for adjustments (McFadden, 1973; Artuç et al., 2010) whose variance controls the degree of time-dependence in adjustment. This specification delivers a smoother adjustment hazard than the typical \((s, S)\) bands produced by the canonical model (where adjustment is purely state-dependent). In turn, this model can generate a decreasing, flat, or increasing MPC, depending on how steep the adjustment hazard is.

We discipline the shape of the hazard by matching four pieces of evidence that a purely state-dependent or time-dependent model cannot replicate jointly. In particular, our model (i) matches the evidence on the relative MPC on durables and non-durables; (ii) generates a realistic short-run price elasticity; (iii) replicates the size distribution of adjustments in the data; and (iv) matches the empirical probability of adjustment as a function of the time passed since the last adjustment, which is central to the response to shocks in fixed cost models (Alvarez et al., 2016b).

The second feature that we introduce is a down payment requirement.\(^3\) We distinguish between two financial assets: cash, which is liquid; and credit, which is illiquid. When purchasing a durable, households are required to make a down payment in cash, and can use credit to borrow the rest subject to an LTV constraint.\(^4\) They repay this debt at a constant rate which allows the model to remain tractable. This financial commitment turns out to be important to generate a realistic MPC of non-durables. The model is also able to produce a distribution of MPCs that is skewed, with many households whose MPC is above 1, which is consistent with the evidence of Misra and Surico (2014) and Lewis et al. (2019).

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\(^2\) In presence of distortionary taxation or inflation, the government would not fully close the output gap. That is, it would send smaller checks than the upper bound. How close the government gets to this bound depends in part on its preference for redistribution and insurance (McKay and Wolf, 2023).

\(^3\) Down payments are an important feature of durable goods purchases in practice (Argyle et al., 2020), and are key to understand the response of durable purchases to shocks (Jose Luengo-Prado, 2006).

\(^4\) This specification implies that households cannot continuously refinance. This is realistic for consumer durables (cars, furniture, etc.) that account for essentially all of the marginal spending on durables in response to stimulus checks.
We use our calibrated model to make two main contributions. First, we quantify the size-dependent response of spending to stimulus checks. In our preferred calibration, the MPC remains elevated even for large checks, which is in-line with the evidence of Sahm et al. (2012) and Fagereng et al. (2021). The extensive margin of durable adjustment plays an important role in this result. As stimulus checks become larger, a proportionately larger and larger share of households adjust its stock of durables, which is consistent with the survey evidence (Fuster et al., 2021). This effect mostly offsets the usual precautionary savings motive (Carroll and Kimball, 1996) at the intensive margin which contributes to a decreasing MPC.

Our second main contribution is to use our model to quantify the size of stimulus checks that close a given output gap. We embed our consumption, investment and saving problem into a general equilibrium model with sticky prices. We solve the model globally to capture the non-linearity that we are interested in.\textsuperscript{5} We find that the general equilibrium transfer multiplier remains elevated even after large stimulus checks, whereas it falls sharply in a model of non-durable spending and increases steeply in the canonical model of durables. As a result, the stimulus check that closes a given output gap is much smaller (larger) in our model compared to the one required in the canonical non-durables (durables) model. For instance, a $600 stimulus check closes roughly half the output gap during a typical recession (with a through of $−4\%$) in our model, while a check more than twice as large of $1,250 is needed in the non-durables model and a check of only $200 is required in the canonical durables model. Moreover, in our model, checks larger than $1500 are too much in that they stimulate output beyond potential in a typical recession.

Finally, we consider two model extensions that are known to be relevant for transfer multipliers. First, we study an open economy where part of the extra spending in durables “leaks” abroad through imports. The check that closes the output gap is $2000 — larger than our baseline $1500. Second, we study an economy where inflation can “lean against” the effect of stimulus checks. The check that closes the output gap depends on whether the zero lower bound on the interest rate binds or not, consistently with a large literature on fiscal multipliers (Farhi and Werning, 2016).

Methodologically, we contribute to a growing literature on durables demand in incomplete markets economies. Most notably, Berger and Vavra (2015) developed the canonical model that spearheaded this literature. McKay and Wieland (2021) find that this canonical model predicts an excessive elasticity of durable demand to interest rate changes. They address this shortcoming by augmenting the canonical model with operating costs, ex-

\textsuperscript{5} This sets our paper apart from other work on stimulus checks (Wolf, 2021; Wolf and McKay, 2022) or transfers in fiscal unions (Farhi and Werning, 2017; Beraja, 2023) which uses first order approximations in the aggregates.
ogenous adjustment shocks, and limited attention. Both models feature standard \((s, S)\)
adjustment bands, i.e., discontinuous adjustment hazard at the microeconomic level. In
contrast, we introduce a smoother adjustment hazard in the tradition of Caballero (1993)
and more recently Beraja et al. (2019) and Alvarez et al. (2020).

While the existing literature has used random fixed cost of adjustment as a device
to generate smooth hazards, we introduce a discrete choice problem with additive taste
shocks for adjustments à la McFadden (1973). This specification allows for purely time-
dependent adjustment (constant hazard), purely state-dependent adjustment (binary haz-
ard), and everything in between.\(^6\) An important body of work uses this form of dis-
crete choice to estimate the demand for durables both in static settings (Berry et al., 1995)
and dynamic ones (Chen et al., 2013; Gowrisankaran and Rysman, 2012). Some papers
in the heterogenous agent literature adopt taste shocks when studying discrete choices
(Iskhakov et al., 2017; Auclert et al., 2021). They do so for numerical reasons only; the
shocks have an arbitrary small variance and a zero mean. In contrast, we discipline both
the mean and the variance of these shocks using micro-data, and these moments are cen-
tral to the shape of the adjustment hazard and the size-dependence in the MPC.

Finally, our analysis is related to a literature that explores how behavioral frictions
affect the size-dependence in MPCs. Laibson et al. (2021) find that MPCs can remain
elevated for large shocks when households are present-biased. In an extension, they allow
for a durable good whose adjustment is frictionless. In contrast, non-convex adjustment
costs are key to our mechanism. Fuster et al. (2021) find that non-convex costs of attention
or re-optimization can generate an MPC that increases with income changes. Their model
allows for a single non-durable good, whereas durables are central to our analysis.

\section{Size-Dependence in the MPC}

Before turning to our structural model, Section 2.1 presents a reduced-form model of ag-
gregate demand to illustrate why size-dependence in the MPC is important for evalu-
ating a stimulus program. Then, in Section 2.2, we review what we know about this
size-dependence from canonical models of household spending as well as the evidence.
Finally, Section 2.3 describes the key determinants of the size-dependence in the MPC.

\footnote{This specification is rooted in the psychology literature (McFadden, 2001) and is used extensively in the context of consumption choices (Nevo, 2001), school choices (Agarwal and Somaini, 2020) and occupational choices (Artuç et al., 2010; Caliendo et al., 2019). Random fixed costs of adjustments do not have a clear empirical counterpart.}
2.1 Why is Size-dependence in the MPC Important?

The size-dependence in the MPC is crucial for how stimulus checks affect aggregate demand. Consider an economy in a recession with an output gap $\Delta \equiv Y^n - Y$, where $Y^n$ refers to the natural level of output and $Y < Y^n$ its actual level. Following a Keynesian cross logic, the transfer $T$ that offsets this demand shortage solves

$$\Delta = \int_0^{\Delta + T} \text{MPC} (x) \, dx, \quad (2.1)$$

where $\text{MPC} (x)$ is the marginal propensity to spend of a household with cash-on-hand $x$.

The size-dependence in the MPC implies that (2.1) is not a standard linear Keynesian cross but a non-linear one. To understand this, it is useful to rewrite the expression as follows

$$\Delta = \frac{\text{MPC} (0)}{1 - \text{MPC} (0)} T + \int_0^{\Delta + T} \frac{d}{dx} \text{MPC} (x) \, \frac{\Delta + T - x}{1 - \text{MPC} (0)} \, dx. \quad (2.2)$$

The first term is the output stimulus achieved by transfer $T$ in an economy with a constant MPC, i.e., $\text{MPC} (0)$. This term captures the typical, linear Keynesian cross logic. The second term captures the non-linearity arising from size-dependence, i.e., $\frac{d}{dx} \text{MPC} (x)$. As Figure 2.1 shows, a smaller (larger) transfer is needed to close the same output gap in an economy with a constant MPC compared to an economy with a declining (increasing) MPC.
2.2 What Do We Know About the Size-Dependence in the MPC?

There is large degree of uncertainty regarding the size-dependence in the MPC, with model-based and empirical estimates covering a wide range of possibilities. As function of the size of stimulus checks, the MPC could be decreasing, steeply increasing or flat depending on the study and method used.\footnote{In all cases in this section and in the rest of the paper, the MPC is computed as a rebate coefficient, i.e., the average propensity to spend.}

Consider first a state-of-the-art model of non-durable spending (Kaplan and Violante, 2014). Figure 2.2 (blue curve) shows that the MPC rapidly declines with the size of a stimulus check. This decline is due to precautionary savings (Carroll and Kimball, 1996) and is even more pronounced in presence of illiquid financial assets (Kaplan and Violante, 2014).

For evaluating an economic stimulus program, however, policy-makers are interested on the impact of stimulus checks on total household spending, including not only non-durable goods and services but also purchases of durable goods. Indeed, durable spending is an important contributor to the MPC of total spending (Parker et al., 2013; Souleles, 1999; Havranek and Sokolova, 2020). Fuster et al. (2021) conjectures that durable purchases could become more responsive as stimulus checks become larger, both because durables are lumpy (Bertola and Caballero, 1990; Eberly, 1994) and can be financed by...
making a down payment (Attanasio et al., 2008). Thus, the MPC of total spending could increase with the size of stimulus checks, despite the fact that the MPC of non-durables declines. A canonical model with lumpy durables purchases and non-durables (similar to Berger and Vavra, 2015) confirms this view. Figure 2.2 shows that the MPC (in red) increases steeply initially as stimulus checks increase in size, and later declines for very large checks. This strong increase is due to a steep adjustment hazard of durable spending (see Section 2.3).

The evidence from micro-studies on the size-dependence in the MPC of total spending is far from conclusive as well. Coibion et al. (2020) finds that the MPC of total spending decreases with the size of stimulus checks. Sahm et al. (2012) also finds that the MPC decreases, albeit rather modestly. The survey evidence of Fuster et al. (2021) shows that the MPC slightly increases. Christelis et al. (2019) find that the MPC is mostly flat and remains elevated for large payments. The meta-study of Havranek and Sokolova (2020) concludes that estimates from studies with larger transfers correlate with lower MPCs.

2.3 What Determines the Size-Dependence in the MPC?

Durables differ from other forms of spending in that most of the adjustment takes place at the extensive margin. This margin is crucially governed by the shape of the adjustment hazard, i.e., the probability of adjustment, and so is the size-dependence in the MPC, as we argue next.

Following a transfer $T$, the total spending $X(T)$ on durables is

$$X(T) = \int \int S(a,d) \cdot x(a,d) \pi(a-T,d),$$

where $S(a,d)$ is the adjustment hazard and $x(a,d)$ is spending conditional on adjustment for a household with cash-on-hand $a$ and durable stock $d$; and $\pi$ is the associated distribution. The hazard captures the extensive margin of adjustment while the spending conditional on adjustment captures the intensive margin.

The left panel of Figure 2.3 illustrates these two objects as a function of cash-on-hand $a$ (fixing $d$). The hazard (in red) increases with cash-on-hand, as consumers are more likely to purchase durables when they are transitorily richer. The figure shows two such hazards. The first (solid red) is a very steep hazard. It resembles the discontinuous adjustment hazard associated with $(s,S)$ bands in canonical models of lumpy durable spending.

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8 The model and calibration are described in detail later in this paper.

9 This strong size-dependence is consistent with the large state-dependence produced by this class of models (Section 5.1). Indeed, a shock occurring in a boom is akin to a large shock starting from the steady state.
which are purely state-dependent. The second (dashed red) is a much flatter hazard. For instance, such hazard results from allowing for time-dependent adjustments, as in Alvarez et al. (2016a) and our model in the next section. Finally, the spending conditional on adjustment (in blue) is concave due to a standard precautionary savings motive. We also plot the distribution of cash-on-hand (in black).

Next, consider the effect of a transfer $T > 0$. The distribution of cash-on-hand shifts to the right (dashed black curve). Households are more likely to adjust their stock of durables (they move along the hazard) and they spend more conditional on adjustment. Both extensive and intensive margins contribute to an increase in total household spending. The right panel of Figure 2.3 illustrates the MPC (right panel) associated with transfers of different size. We repeat this experiment for the two hazards discussed above. In the first case (solid curve), the hazard is steep so that a large mass $d\pi$ of households chooses to adjust their durable stock. The MPC increases with the size of the transfer, as the extensive margin response is sufficiently strong to offset the decline in the MPC due to the intensive margin. In the second case (dashed curve), the hazard is much flatter. The MPC declines with the size of transfer, as the intensive margin dominates.

These examples clarify that the shape of the adjustment hazard is crucial for the size-dependence in the MPC.\footnote{Of course, the adjustment hazard also determines the shape of the distribution of idiosyncratic states, which matters for the size-dependence.} The canonical model of durables spending generates a steeply increasing MPC precisely because the adjustment hazard is so steep. A natural way of flattening the adjustment hazard is by moving away from a purely state-dependent model and allowing for time-dependent adjustments. Such hybrid model is flexible enough to
generate increasing, decreasing, or flat MPCs, as implied by the range of estimates in Figure 2.2. This discussion will inform our modelling choices in Section 3.

3 A Model With A Smooth Adjustment Hazard

We now introduce our model of household spending. The economy is populated by overlapping generations of households who die at rate $\xi \in (0, 1)$. Households consume non-durables and invest in durables, and they face uninsured earnings risk. Time is discrete, and there is no aggregate uncertainty. Periods are indexed by $t \geq 0$.

3.1 Goods and Preferences

Households use a homogenous good to consume $c_t \geq 0$, and invest in durables $d_t \geq 0$. Their utility is

$$U_t \equiv u(c_t, d_{t-1}) + \beta E_t[U_{t+1}], \quad (3.1)$$

for some discount factor $\beta \in (0, 1)$. We assume that inter- and intratemporal preferences are isoelastic

$$u(c, d) = \frac{1}{1 - \sigma} U(c, d)^{1-\sigma} \quad \text{and} \quad \sum_{x \in \{c, d\}} \left( \frac{\vartheta_x^{\frac{1}{\nu^1}} x}{U(c, d)} \right)^{\frac{\nu^1}{\nu^1 - \sigma}} = 1, \quad (3.2)$$

where $\sigma$ is the inverse elasticity of intertemporal substitution, $\nu$ is the elasticity of intratemporal substitution, and $\vartheta$ are consumption weights with $\sum_x \vartheta_x = 1$.

3.2 Durable Adjustment Hazard

We specify a flexible adjustment hazard that captures the time- and state-dependence in durable adjustment. Households are subject to linearly additive taste shocks for adjustment. These taste shocks are independent over time and distributed according to a logistic distribution whose mean and variance are controlled by $\kappa > 0$ and $\sigma > 0$, respectively.\footnote{Random adjustment costs (Dotsey et al., 1999; Alvarez et al., 2020) would also produce a smooth hazard — although their economic interpretation is somewhat unclear.}

The durable adjustment hazard is

$$S(x) = \frac{\exp \left( \frac{\nu^{\text{adjust} - \kappa}}{\eta} \right)}{\exp \left( \frac{\nu^{\text{adjust} - \kappa}}{\eta} \right) + \exp \left( \frac{\nu^{\text{non}}}{\eta} \right)}.$$
Figure 3.1: Adjustment hazard (fixing $d$)

Notes: The figure shows the adjustment probability conditional on the household not having adjusted so far.

where $V^{\text{adjust}}$ and $V^{\text{not}}$ denote the present discounted values of utility when adjusting and not adjusting, respectively.

As Figure 3.1 illustrates, the scale parameter $\eta$ controls the shape of the adjustment hazard while the location parameter $\kappa$ controls its level. In particular, the model reduces to a fully state-dependent model when $\eta \to 0$; and $\kappa$ controls the position of $(s, S)$ bands in this case. In this sense, $\kappa$ effectively governs the fixed cost of adjustment. Similarly, the model boils down to a fully time-dependent model when $\eta \to +\infty$; and $\kappa$ controls the probability of adjustment in this case.\textsuperscript{12} Note that this specification is flexible enough to accommodate an increasing, decreasing or constant MPC (Section 2.3). Finally, the stock of durables depreciates at rate $\delta$ and requires a mandatory maintenance rate $\iota$ so that $d_t = (1 - (1 - \iota) \delta) d_{t-1}$ when the household does not adjust.

3.3 Saving, Credit, and Downpayment

Households save in a liquid financial assets $m \geq 0$ (i.e., cash, deposits) with return $r^m$ and borrow using a partially illiquid asset $b \leq 0$ (credit) with return $r^b \geq r^m$. Households are required to make a down payment when they purchase a durable. Specifically, they are required to pay a share $\theta \in (0, 1)$ of the value of the stock they buy. That is,

$$b_t \geq - (1 - \theta) (1 - \delta) d_t,$$

\textsuperscript{12} In this limit, $\kappa = \log (1/\phi - 1) \eta$ induces a constant hazard $\phi \in (0, 1)$.}

11
We assume that the constraint (3.4) holds with equality. For numerical tractability, we also suppose that it remains binding at any point while the household holds a durable. This assumption allows us not to introduce credit as an additional state variable. In particular, we abstract from endogenous refinancing decisions (Berger et al., 2021; Laibson et al., 2021) between purchases. With this assumption, households make pre-determined credit repayments while they hold their stock, which mimicks the rule out thumb they appear to follow in practice (Argyle et al., 2020). Finally, the stock of assets held by those households who die is rebated to surviving generations in proportion to their own asset holdings. For the liquid asset $m$, this is equivalent to assuming that households trade annuities (Yaari, 1965; Blanchard, 1985). New generations start with zero assets.

### 3.4 Earnings

Households’ earnings $e_t \equiv y_t Y_t$ are the product of idiosyncratic productivity $y_t$ and aggregate income $Y_t$. The productivity $y_t$ follows an AR(1) process. We let

$$Y_t \equiv (1 - \tau) e_t + \left(1 + r_{t-1}^m\right) \frac{m_{t-1}}{1 - \chi} + r_{t-1}^b \frac{b_{t-1}}{1 - \chi} + T_t$$

(3.5)

denote cash-on-hand, where $\tau$ is a linear tax on earnings, and $T_t$ are lump sum transfers from the government.

### 4 Calibration

We parameterize the model using a mix of external and internal calibration. We assume that our single, composite durable good behaves as cars — in terms of frequency of adjust-

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13 Existing models of durables make no distinction between cash and credit (Jose Luengo-Prado, 2006; Berger and Vavra, 2015). They assume a single, liquid asset that is subject to loan-to-value constraint similar to (3.4). This presumes that households can refinance and prepay their debt continuously. Refinancing is virtually nonexistent for consumer durables, which we focus on. Auto loan prepayments are relatively rare too (Heitfield and Sabarwal, 2004). Our two-asset specification ensures that the effective supply of liquidity in the economy, i.e., the average distance to the borrowing constraint, matches conventional estimates in the literature (Kaplan et al., 2018). An even richer model could allow for refinancing.

14 In practice, the vast majority of down payments for cars do not exceed the minimum level required (Green et al., 2020).

15 Effectively, households repay a constant fraction of their loan in every period, as in Laibson et al. (2021). In practice, most car loans are repaid within 5 years and cars depreciate at roughly 20%.

16 It is worth noting that these assumptions imply that households in our model will use part of their stimulus checks to pay down debt (Shapiro and Slemrod, 2009; Graziani et al., 2016; Coibion et al., 2020) as the extra windfall allows households to make their pre-determined repayments.
ment, down payment, etc. — since they make up for most of the spending in consumer durables. We abstract from housing purchases since these are unlikely to be affected by stimulus checks of a realistic magnitude. Tables 4.1 and 4.2 summarize the parametrization.

4.1 External Calibration

External parameters are set to standard values in the literature. The inverse elasticity of intertemporal substitution is $\gamma = 2$, which is usual in the literature on durables (Berger and Vavra, 2015; Guerrieri and Lorenzoni, 2017). We choose an elasticity of substitution between durables and non-durables of $\nu \to 1$ to obtain a unitary long-run price elasticity for cars (Berry et al., 2004; Orchard et al., 2022).\footnote{The long-run price elasticity would be exactly 1 in our model with free adjustments ($\kappa = 0, \eta \to 0$). We obtain an elasticity of $-0.93$ in the full model with adjustment frictions.} We choose a quarterly depreciation rate $\delta = 5\%$. We set the down payment parameter to $\theta = 0.15$ which lies between the estimates of Adams et al. (2009) and Attanasio et al. (2008). The real return on the liquid asset is $r_{\text{m}} = -1.5\%$ per year and the borrowing spread is $r_{\text{b}} = 2.78\%$ as in Attanasio et al. (2022). We assume that idiosyncratic productivity follows an AR(1) process. We set the persistence of the income process $\rho = 0.977$ so as to obtain an annual persistence of 0.91 (Floden and Lindé, 2001). We set the standard deviation of the innovations $\sigma = 0.213$ to match an annual standard deviation of 0.92 in log-earnings (Auclert et al., 2018). We normalize the earnings process so that GDP is 1 at the stationary equilibrium. Households are subject to a linear tax on labor income of $\tau = 30\%$. Finally, we pick $\xi = 1/180$ so that the average life span (working age) is 45 years.

4.2 Internal Calibration

We calibrate five parameters internally: (i) the discount factor $\beta$; (ii) the relative weights on non-durables $\vartheta$; (iii) the maintenance rate $\iota$; (iv) the location parameter for preference shocks $\kappa$; and (v) the scale parameter for preference shocks $\eta$. We choose the discount factor to match an average stock of liquid asset holdings $m$ of 26\% of average annual income (Kaplan et al., 2018). We calibrate the relative weight on non-durables to target a ratio of durables to non-durable expenditures $x/c = 0.26$ based on CEX data.\footnote{We exclude housing from both durables and non-durables. We interpret durables as consumer durables (vehicles, appliances, computers, phones, etc.). Durable spending in the CEX consists of: household furnishings and equipment; vehicle purchases (net outlay); maintenance and repairs on vehicles; audio and visual equipment and services; and other entertainment supplies, equipment and services. Non-durable spending consists of total spending minus the categories above and housing.} We set the
Table 4.1: External calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target / Source</th>
</tr>
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<tr>
<td>$\xi$</td>
<td>Death Rate</td>
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<td>Average age (45 years)</td>
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<td>$\gamma$</td>
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<td>Berger and Vavra (2015)</td>
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<td>CES parameter</td>
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<td>Long-run price elasticity</td>
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</tr>
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<td>$\delta$</td>
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<td>NIPA</td>
</tr>
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<td><strong>Earnings process</strong></td>
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<tr>
<td>$\rho$</td>
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<td>Floden and Lindé (2001)</td>
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<td>Auclert et al. (2018)</td>
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<td>Kaplan and Violante (2014)</td>
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<td>Berger and Vavra (2015)</td>
</tr>
<tr>
<td>$r^m$</td>
<td>Return on cash</td>
<td>$-1.5%$</td>
<td>0.5% nominal return, 2% inflation</td>
</tr>
<tr>
<td>$r^b - r^m$</td>
<td>Borrowing spread</td>
<td>2.78%</td>
<td>Attanasio et al. (2022)</td>
</tr>
</tbody>
</table>
maintenance rate to obtain a ratio of maintenance spending to gross investment of 32.6%, consistent with the one for cars in the CEX. We choose the location parameter $\kappa$ to match an annual frequency of durable adjustment of 23.8% for vehicles in the PSID, which is in line with conventional estimates (Attanasio et al., 2022; McKay and Wieland, 2021). The rest of this section describes the calibration of the scale parameter $\eta$ given its importance in our analysis.

**Bounding the scale parameter.** The scale parameter $\eta$ controls the shape of the hazard (3.3). As such, it governs the propensity of households to adjust their stock of durables in response to shocks. Two empirical moments provide upper and lower bounds for this parameter.

The left panel of Figure 4.1 shows the MPC on durables and non-durables for different values of $\eta$. All other parameters are re-calibrated to match the moments described above. The MPC on durables declines monotonically in $\eta$: the more time-dependent the model, the lower the MPC on durables. The literature offers a wide range of estimates of the MPCs on durables and non-durables. For instance, Souleles et al. (2006) finds a low MPC on durables, while Parker et al. (2013) finds a rather high one. However, it is generally agreed that the MPC on durables is larger than the one on non-durables (see the meta analysis of Havranek and Sokolova, 2020). For this reason, 0.6 is effectively an upper bound for the scale parameter $\eta$. That is, the model cannot be too time-dependent to match the evidence on the relative MPC of durables.

The right panel shows the short-run elasticity of durable purchases after a one-quarter transitory increase in the price of durables by 1%. It is well-known that conventional models of durable spending produce an excessively high elasticity of durable demand to changes in the user cost (House, 2014). This effect is almost entirely driven by the extensive margin of adjustment (McKay and Wieland, 2022). Consistently, the fully state-dependent model with $(s, S)$ adjustments bands ($\eta \to 0$) predicts an implausibly high elasticity of $-90$. Introducing a smooth adjustment hazard is a parsimonious way to dampen this elasticity. There is much uncertainty about the precise elasticity in the empirical literature. Gowrisankaran and Rysman (2012) estimates a short-run elasticity of $-2.55$ for camcoders. Bachmann et al. (2021) finds an elasticity of $-11$ among households who were aware of a short-run decrease in the VAT in Germany. For this reason,

---

19 In Section 4.3, we describe how we estimate the empirical distribution $\pi_k$ of the duration $k$ between vehicle purchases. The frequency of adjustment is the inverse of the average duration $1/\sum_{k \geq 0} k \pi_k$.

20 McKay and Wieland (2022) dampen this elasticity by introducing a combination of low elasticity of intertemporal substitution, low elasticity of substitution between durables and non-durables, various operating costs, exogenous mandatory adjustments, and limited attention.
Figure 4.1: Bounding the scale parameter $\eta$

![Graph showing marginal propensities to spend and SR price elasticity of durable demand](image)

Notes: The left panel plots the MPC on durables and non-durables for various values of the scale parameter of preferences shocks $\eta$ in (3.3). Each MPC is computed as a rebate coefficient, i.e., the average propensity to spend. The right panel plots the short-run price elasticity of durable demand after a one-quarter increase in the price of durables by 1%. The dashed vertical line is our preferred estimate ($\eta = 0.4$).

$0.2$ is a plausible lower bound for the scale parameter $\eta$. That is, the model cannot be too state-dependent to match the evidence on the elasticity of durable purchases.

Overall, our preferred value for the scale parameter is $\eta = 0.4$ — the mid-point between the lower and upper bounds. It delivers an MPC on durables of 18% out of a $500$ windfall, which almost exactly matches the mean estimate in the meta analysis of Havranek and Sokolova (2020). The MPC on total spending is 33%, which is again very similar to the mean estimate in this study. We obtain a short-run price elasticity of durables of $-8.7$ in our preferred calibration, which is of the same order of magnitude as existing estimates. We will show that our results are robust to other choices of $\eta$ in the region $0.2 \leq \eta \leq 0.6$. Moreover, the next section shows that the the model with $\eta = 0.4$ matches well other important (untargeted) moments.

4.3 Untargeted Moments

Our calibrated model performs well along several untargeted dimensions. We start by inspecting two moments — the distribution of net investment in durables and the conditional probability of adjustment — which highlight the importance of allowing for a smooth adjustment hazard. We also examine the distribution of MPCs.

Net investment. The left panel of Figure 4.2 plots the empirical distribution of net invest-
Table 4.2: Internal Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Non-durable</th>
<th>Canonical</th>
<th>Smooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal calibration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount Rate</td>
<td>0.961</td>
<td>0.915</td>
<td>0.912</td>
</tr>
<tr>
<td>( \vartheta )</td>
<td>Non-durable weight</td>
<td>1</td>
<td>0.690</td>
<td>0.659</td>
</tr>
<tr>
<td>( \iota )</td>
<td>Maintenance rate</td>
<td>0</td>
<td>0.281</td>
<td>0.283</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Location of pref. shifters</td>
<td>0</td>
<td>0.499</td>
<td>1.640</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Scale of pref. shifters</td>
<td>0</td>
<td>0</td>
<td>0.40</td>
</tr>
<tr>
<td>–</td>
<td>Down payment</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The table presents the internal calibration parameters along with their descriptions and values for three different models: non-durable, canonical, and smooth. The parameters include the discount rate (\( \beta \)), non-durable weight (\( \vartheta \)), maintenance rate (\( \iota \)), location of preference shifters (\( \kappa \)), scale of preference shifters (\( \eta \)), and down payment (\( – \)).

Net investment in vehicles by households who adjust their stock across two consecutive PSID waves \( w \) (in grey). To measure net investment, we restrict our sample to household heads (or reference persons) who are male, aged 21 or above, and appear in at least three PSID waves owning at least one vehicle. Households report the purchase year (Year\(_w\)) in wave \( w \) for their most recently bought vehicle (“#1”). A change in this variable across two consecutive waves indicates a new purchase. To limit measurement error (e.g., a household misreporting the purchase year for the same car), we further restrict the sample to households who report a purchase year that increases (weakly) over time. Since PSID waves are bi-annual, a purchase occurs (Purchase\(_w\) = 1) over the period covered by the current wave if Year\(_w\) > Year\(_{w-2}\). We measure net investment upon a purchase as \( \log (d_w) - \log (d_{w-2}) \) when Purchase\(_w\) = 1, where \( d_w \) is the value of the stock of vehicles net of liabilities reported by the household. Lastly, we standardize the distribution following Alvarez et al. (2016b). That is, we de-mean net investment and normalize it by its standard deviation. We trim the 1% of extreme values at the bottom and top of the distribution when standardizing.

The figure also plots the distribution of net investment in our model with a smooth adjustment hazard (in red) and in the canonical model (in blue). To ensure that the data and models are comparable, we discretize our model-simulated series into PSID waves and treat those identically to the actual data. We divide time into years, as our model is set up quarterly. A simulated household enters its first wave when they purchase durables in a wave that is not the one they were born in. That household leaves the PSID in the wave preceding its death. For each individual and wave, we compute Year\(_w\) as the year of the most recent purchase. Vehicle wealth is \( d_w = P_{T(w)}^d \theta d_{T(w)} \) in the model since households’ credit is given by (3.4) at any time, where \( T(w) \) is the last quarter in PSID wave \( w \).

Our calibrated model produces a bell-shaped distribution that resembles the one in the
Figure 4.2: Untargeted moments

Distribution of net investment

Data
Canonical
Smooth hazard

Conditional adjustment probability

Data
Model
Calvo

Notes: The left panel plots the distribution of net investment (standardized) across two consecutive PSID waves where the household adjusted. The grey curve is the data, while the red and blue bars are our calibrated model and the canonical version with \((s, S)\) bands (i.e., \(\eta \to 0\)), respectively. The right panel plots the adjustment probability conditional on a household not having adjusted so far.

Data. Crucially, our model matches well the tails of the distribution — an important moment in models with lumpy adjustment (Alvarez et al., 2016b). In contrast, the canonical model fails to reproduce the empirical distribution. There are too few negative adjustments and most adjustments are concentrated around the same value, as expected from a purely state-dependent model.

Probability of adjustment. The right panel of Figure 4.2 plots (in blue) the empirical probability that a household adjusts its stock of vehicles after a certain number of years conditional on not having adjusted so far (Alvarez et al., 2021). We construct this conditional probability using the purchase dates \(\text{Year}_{w}\) as follows. The duration between two consecutive purchases is given by \(\text{Duration}_{w} = \text{Year}_{w} - \text{Year}_{w-2}\) whenever a purchase occurs (\(\text{Purchase}_{w} = 1\)). We restrict attention to the first purchase by a given household.\(^{21}\) This yields an empirical probability distribution \(\pi_{k}\) over durations \(k = 1, 2, \ldots\) expressed in years. Following Alvarez et al. (2021), we compute the conditional probability of adjustment as

\[
\text{Prob}_{k} = \frac{\pi_{k}}{1 - \sum_{j<k} \pi_{j}}.
\]

\(^{21}\)The reason is that subsequent purchases, if observed in the PSID’s short time dimension, are more likely to be of shorter duration.
The figure compares the empirical probability to the one implied by our model with \( \eta = 0.4 \) (in red), and an alternative calibration with \( \eta \to +\infty \) (dashed) consistent with a purely time-dependent model. The conditional probability is flat in the purely time-dependent model. On the contrary, the data suggests that vehicle adjustments are fairly state-dependent. The model with \( \eta = 0.4 \) matches the empirical profile almost perfectly.\(^{22}\) Figure B.4 in Appendix B.2 reproduces the same plot for the canonical durables model \((\eta \to 0)\). The overall pattern is roughly similar, although the fit is slightly poorer as the horizon increases. Overall, this confirms that our calibrated model retains a substantial degree of state-dependence. This also means that the conditional probability of adjustment is only a partially informative moment. It allows us to rule out very large values of \( \eta \) (a strong time-dependence), as did the evidence on the relative MPC of durables in the left panel of Figure 4.1. But it does not allow us to discriminate between lower values of \( \eta \). Very low values of \( \eta \) are instead ruled out by the evidence on the price elasticity in the right panel of Figure 4.1 as well as the evidence on the distribution of net investments in the left panel of Figure 4.2.

**Distribution of MPC.** Finally, Figure B.1 in Appendix B.1 compares the distribution of the MPC on total spending produced by our model and one of purely non-durable spending.\(^{23}\) The distribution is bi-modal in the standard incomplete markets model. Most households behave as Ricardian agents, and hence have a low MPC. Some of them are near their borrowing constraint, and hence have a higher MPC (less than 0.5). In contrast, the distribution density of MPCs declines much less dramatically in our model of lumpy durable spending. The overall shape of the distribution is consistent with the evidence in Lewis et al. (2019) and Fuster et al. (2021). A non-negligible share of households displays an MPC close to (or above) 1, which is in-line with the findings of Misra and Surico (2014) and Jappelli and Pistaferri (2014). Lumpy adjustment and households’ ability to pay only a fraction of the price as a down payment make such high MPCs possible.

**Secondary market.** Households who adjust their stock of durables (upward or downward) first sell their existing stock. Part of households’ gross purchases is thus fulfilled effectively by old cars in the secondary market.\(^{24}\) In our calibrated model, used cars make 52% of

\(^{22}\) Note that the model matches the average probability, by construction. The reason is that we target the empirical frequency of adjustment in our calibration, which is computed using the empirical probability of adjustment. The model’s success lies in the fact that it matches the profile well.

\(^{23}\) For comparability, this model of non-durable spending is our full model (Section 3) specialized with no durables, i.e., with preference parameter \( \theta_c = 1 \). The discount factor is calibrated to match the same average stock of liquid asset holdings as in our full model (Section 4).

\(^{24}\) New and old durables are indistinguishable in our model. In particular, they have the same deprecia-
gross purchases. For comparison, used cars represent roughly 55% of total spending on cars in the US (DoT, 2023).\textsuperscript{25}

5 Size-Dependence in the MPC and Aggregate Stimulus

We now quantify the size-dependence in the MPC in our calibrated model (Section 5.1), and the effect of stimulus checks of varying size on aggregate output (Section 5.2).

5.1 Size-Dependence in the MPC

The left panel of Figure 5.1 plots the quarterly MPC following stimulus checks of varying size. We repeat the analysis starting from the stationary equilibrium, a ‘mild recession’ where average household income declines linearly over four quarters with a through of 4% and then recovers linearly over two years, and a ‘deep recession’ with a decline of 8% instead. The right panel of the figure compares the MPC across models starting from the stationary equilibrium.\textsuperscript{26}

We find that the MPC remains elevated even for large stimulus checks. This finding is consistent with the evidence of Sahm et al. (2012) and Fagereng et al. (2021). The MPC neither shoots up as in the canonical durables model, nor declines sharply as in the standard non-durables model (Section 2.2). Moreover, the MPC in our model is somewhat higher in deeper recessions, which is in-line with the evidence of Gross et al. (2020). In contrast, the canonical model of durable spending predicts a sharp decline in the MPC in deeper recessions (Figure 5.1 in Appendix B.2) through the mechanism put forth by Berger and Vavra (2015). Lastly, the size-dependence is mostly unchanged across recessions in our model — all lines in the left panel of the figure are essentially parallel.

The role of the extensive margin. Why does the spending response remain elevated even for large stimulus checks? The answer lies in the extensive margin of durable adjustment.

\textsuperscript{25} About 75% of car sales in the US involve a used car. However, used cars are cheaper than new ones in the data and hence account for a smaller share of total spending on cars. Modelling the second market explicitly by allowing for a quality ladder (Gavazza and Lanteri, 2021) is beyond the scope of the current paper.

\textsuperscript{26} Figure 5.1 reports quarterly MPCs. For reference, we plot the dynamic responses to contemporaneous and expected shocks of various sizes in Figures B.3 in Appendix B. Figure B.6 does the same for the canonical model.
Figure 5.1: Size-dependence in the MPC

Notes: The left panel plots the quarterly MPC as a function of the stimulus check. ‘Stationary’ corresponds to the stationary equilibrium. ‘Mild recession’ corresponds to a recession where average household income declines linearly over four quarters with a trough of 4% and then recovers linearly over two years. ‘Deep recession’ is the same with a decline of 8% instead. The right panel compares MPCs across models starting from a stationary equilibrium.

sizes. As the stimulus check becomes larger, a proportionately larger and larger share of households is pushed into adjustment. In other words, the extensive margin of adjustment exhibits itself an important size-dependence, which is consistent with the survey evidence from Fuster et al. (2021). This effect peaks at $600, which happens to coincide with the 2008 rebate for which Parker et al. (2013) document a large response of vehicles. At the intensive margin, households who were less likely to adjust initially buy smaller durables after receiving a stimulus check. The response at the extensive margin mostly offsets the one at the intensive margin. Overall, the MPC declines with the size of the stimulus check, albeit very slowly.

Aggregate conditions. We then explore how aggregate conditions affect the MPC and its size-dependence. The remaining curves in Figure 5.1 correspond to two different recessions. The first recession is mild: the average household income declines (linearly) by −4% over three quarters and then recovers linearly over two years. The second recession is deeper: average income decline by 8% instead of 4%. The MPCs are mostly unchanged across recessions. If anything, the MPC is somewhat higher in deeper recessions, which is in-line with the evidence of Gross et al. (2020). In contrast, the canonical model of durable spending predicts a sharp decline in the MPC in deeper recessions (Figure 5.1 in Appendix

27 Figures B.3 and B.6 in Appendix B plot the dynamic responses to various check sizes, both in our model and the canonical model.
5.2 How Much Bang-for-the-Buck?

What does this size-dependence imply for the response of aggregate output to stimulus checks? To get a sense of the magnitudes, we feed our size-dependent MPCs into the static non-linear Keynesian cross (2.1). This exercise serves as an intermediate step to our fully dynamic general equilibrium analysis (Section 6). We focus on three scenarios: (i) the MPC declines slowly as in our calibrated model; (ii) the MPC increases steeply as in the canonical durables model; and (iii) the MPC declines sharply as in the non-durables model of Kaplan and Violante (2014).

We plot in Figure 5.2 the response of output in these three scenarios starting from a mild recession where the output gap is 4%. The responses are very similar for small stimulus checks, e.g. the MPC out of $100 is comparable across models. However, they start to diverge as the checks become larger and the size-dependence kicks in. A stimulus check of roughly $600 closes half the output gap when using the MPCs implied by our calibrated model (blue curve). To achieve the same outcome, a much smaller check of $200 is needed for the canonical durables model (red curve), whereas a much larger check of $1500 is needed for the non-durables model (green curve).

In our model, stimulus checks still deliver a large “bang-for-the-buck” even for large
amounts, whereas they become substantially less and less powerful in non-durables models (Kaplan and Violante, 2014). For instance, closing the full output gap requires sending a $1500 check whereas a check roughly three times larger is needed in the non-durables model.

6 Stimulus Checks in General Equilibrium

We now embed our spending model into a dynamic general equilibrium environment. This model accounts for three effects that are important to quantify the bang-for-buck of stimulus checks. First, the spending response to income checks is persistent, and future income changes further raise today’s spending. That is, we account for intertemporal MPCs (Auclert et al., 2018). Second, a share of the additional spending following stimulus checks “leaks” abroad through international imports. Finally, sticky prices and a responsive monetary policy can “lean against” the effect of stimulus checks. We first close the model by specifying the source of aggregate disturbances and fiscal policy. Numerical details are provided in Appendix A.5.

6.1 Environment

Workers import part of their consumption of non-durables and investment in durables. A representative firm produces the final good used for consumption and investment. Workers claim both labor income and profits in proportion to their idiosyncratic productivity (Auclert et al., 2018). The economy experiences firm investment shocks — the main driver of US business cycle fluctuations (Justiniano and Primiceri, 2008; Auclert et al., 2020).28

**Open economy.** Households purchase domestic and foreign goods.29 Households’ consumption $c_t$ and investment $x_t$ are given by

$$c_t = \left[ \sum_{j \in \{H,F\}}^{H} \left( \alpha_c^j \right)^{\frac{1}{\rho}} \left( c_t^j \right)^{\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}} \quad \text{and} \quad x_t = \left[ \sum_{j \in \{H,F\}}^{H} \left( \alpha_d^j \right)^{\frac{1}{\rho}} \left( x_t^j \right)^{\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}},$$  \hspace{1cm} (6.1)

where $c_t^H$ and $c_t^F$ are the consumptions of the domestic and foreign goods, respectively, and the weights $\alpha_c^H + \alpha_c^F = 1$ governs the spending share on domestic and foreign goods.

28 An alternative approach would be to perturbate households’ demand directly (e.g., a discount factor shock). An advantage of introducing investment shocks is that we are able to compute efficiently the sequence of shocks that produce a given recession of interest (Appendix A.5).

29 For instance, a fourth of durable expenditure is spent on foreign goods in the US.
The terms $x^H_t$, $x^F_t$ and $\alpha^d_t$, $\alpha^F_t$ are defined similarly for investment in durables. We set the elasticity of substitution to $\rho = 2$ in our benchmark calibration. This value lies between the short-run and long-run estimates of Boehm et al. (2023). In the following, we let $P^c_t$ and $P^d_t$ denote the price of the consumption baskets (6.1) expressed in terms of the domestic good. The country can run a current account deficit (i.e., borrow from the rest of the world) to finance its imports. The demands from the rest of the world still take the form (6.1). Total consumption of non-durables $c^*_t$ and investment in durables $x^*_t$ in the rest of the world are constant and chosen so there are no net imports at the steady state.

Firms. A representative firm produces the domestic final good using labor with a technology $Y_t = aN_t$ with $a > 0$ and rebates its revenues to workers. A separate investment firm undertakes productive projects. Fluctuations in investment will act as aggregate demand disturbances. This firm invests in capital using the final good and produces using the linear technology $AK_{t-1}$ where $K_t$ is the stock of capital and $A > 0$ is the productivity of capital. It solves

$$\max_{\{I_t, K_t\}} \sum_t Q_t \{AK_{t-1} - I_t\} \quad \text{s.t.} \quad K_t = \left\{1 - \delta^K + \Phi \left(\frac{I_t}{K_{t-1}}\right) + z_t\right\}K_{t-1}, \quad (6.2)$$

and $K_t \geq 0$ with initial condition $K_{-1} = K$, where $K \equiv 1$ is the steady state level of capital, $Q_t \equiv \prod_{s \leq t} (1 + \pi_{s-1})(1 + r_{s-1})^{-1}$ is the firm’s stochastic discount factor, $I_t$ is investment, $\delta^K$ is the depreciation rate of capital, and $\Phi \left( x \right)$ is the adjustment cost which is increasing and concave. The depreciation shocks $\{z_t\}$ are the source of aggregate fluctuations in our economy, as in Brunnermeier and Sannikov (2014).

Dividends. The investment firm smooths dividends (Leary and Michaely, 2011). The fact that investment shocks are partially debt-financed ensures that they generate movements in aggregate demand. Specifically, we assume that the firm adopts a constant dividend $\text{Div}_t = \text{Div}$ over the first 15 years, and the lets the dividend mean-revert linearly to its steady state value over the next 5 years. The constant Div is chosen to ensure that the firm repays its debt in the long-run.

Price setting. We allow for sticky prices and active monetary policy. We do not microfound 30 This value also falls between the short-run estimate of Auer et al. (2021) using border prices, and the long-run estimates of Caliendo and Parro (2015). 31 This specification with a linear technology and concave adjustment costs is common in the asset pricing literature (Jermann, 1998; Brunnermeier and Sannikov, 2014). 32 Absent dividend smoothing, investment raises output but not incomes when output is demand-determined, as is evident from (6.7)–(6.8) below.
the price setting process since it is not the focus of the paper. Instead, we assume that the domestic price follows an ad-hoc Phillips curve

$$\pi_t = \kappa \hat{Y}_t + \beta \pi_{t+1}, \quad (6.3)$$

where $\pi_t$ is inflation in the domestic price, $\hat{Y}_t$ is the real (log) output gap, and $\kappa > 0$ is the slope of the Phillips curve. The price of the foreign good is fixed throughout. The nominal exchange rate is pinned down by purchasing power parity in the long-run, and uncovered interest rate parity during the transition (Appendix A.3). Domestic and foreign prices are normalized to 1 at the initial stationary equilibrium. Monetary policy follows a standard rule

$$r_t^m = \max \left\{ r_t^m + \phi_{\Pi} \pi_t^{\text{CPI}} + \phi_y \hat{Y}_t, 0 \right\}, \quad (6.4)$$

where $r_t^m$ is the steady state return, $\pi_t^{\text{CPI}}$ is CPI inflation, and $\phi_{\Pi}$ and $\phi_y$ parametrize the rule. We set the slope of the Phillips curve to $\kappa = 0.1$ and the Taylor coefficients of $\phi_{\Pi} = 1.5$ and $\phi_y = 0$ as in Auclert et al. (2021).

**Fiscal policy.** The government finances stimulus checks and government spending by taxing households’ income. It also claims the households’ net payments on the illiquid asset. The government’s flow budget constraint is

$$B_t^g = \frac{1 + r_t}{1 + \pi_t} B_{t-1}^g + \tau_t E_t + \Sigma_t - t_t - G_t, \quad (6.5)$$

where $B_t^g$ is the government’s real asset holdings, $\tau_t$ is still the linear tax on labor income, $E_t$ is households’ total real income, $t_t$ is real stimulus checks to households, $G_t$ is the government consumption of non-durables, and

$$\Sigma_t \equiv (1 - \theta) (1 - \delta) \left\{ \left( 1 + r_{t-1}^b \right) P_t^d D_{t-1} - P_{t+1}^d D_t \right\} \quad (6.6)$$

is the net payments on the illiquid asset with $D_t \equiv \int d'(x) \times \pi_{t-1} (dx)$ denoting aggregate durable holdings. As in our baseline calibration, the government maintains a constant ratio of debt to output at the stationary equilibrium, and taxes income at a constant rate

---

33 Note that we allow for relative price movements between the durable and non-durables baskets (6.1). In a recession, the price of the durable basket falls proportionately less than the one of the non-durable basket since the import share is higher for the former. This is consistent with the evidence: the CPI for durables is typically slightly countercyclical relative the one on non-durables (BLS data).

34 The CPI index is $P_t^{\text{CPI}} = \omega_c P_t^c + \omega_d P_t^d$, where $\omega_c$ and $\omega_d$ are spending shares at the steady state. In turn, CPI inflation is $\pi_t^{\text{CPI}} \equiv P_t^{\text{CPI}} / P_{t-1}^{\text{CPI}} - 1$.

35 An alternative would be to introduce a separate financial sector.
Its spending $G > 0$ on domestic goods balances the budget (6.5). In period $t = 0$, the government sends a one-time nominal stimulus check $T_0 > 0$ to every household, or $t_0 \equiv T_0/P_0^H$ in real terms. It borrows ($\Delta B_1 < 0$) to finance these checks. In subsequent periods $t > 0$, the government maintains a constant spending $G_t = G > 0$ and raises its tax rate $\tau_t$ by a constant amount over 15 years and then lets it mean-revert to its steady state value. This adjustment ensures that the government eventually repays the additional debt $\Delta B_1 < 0$ it contracted.

**Resource constraint.** The aggregate resource constraint is

$$\int \left[ P^c_t c_t (x) + P^d_t x_t (x) \right] d\mu_{t-1} + G + Z_t + TB_t \left( e_t, P^c_t, P^d_t \right) = Y_t$$

(6.7)

in each period $t \geq 0$, where $c_t (x)$ and $x_t (x)$ denote the consumption and investment choices of a household with state $x$, $Z_t \equiv I_t - AK_{t-1}$ capture the effective investment shocks, $\mu_{t-1}$ is the distribution of states, and the trade balance $TB_t$ is given by (A.29) in Appendix A.3. Households’ real net income before interest rate payments is

$$E_t^{\text{net}} (x) = (1 - \tau_t) y (Y_t + \text{Div}_t),$$

(6.8)

where $y$ still captures idiosyncratic income shocks, and real dividends $\text{Div}_t$ smooth profits $\pi_t = -Z_t$ over time. Note that (6.7)–(6.8) define a non-linear Keynesian cross where spending determines incomes and incomes feed back into spending.

**Calibration.** As in our baseline calibration (Section 4), the interest rate is $r = -1.5\%$ at the stationary equilibrium, aggregate income is $E_t \equiv 1$, the government maintains a constant ratio of debt to output $-B/Y = 104\%$, and income taxes are $\tau_t = \tau = 30\%$. Households import 23% of their durable spending at the steady state, and 19% of their non-durable spending (Hale et al., 2019). The investment adjustment cost is $\Phi (x) = 1/\kappa \left( \sqrt{1 + 2\kappa x} - 1 \right)$ with $\kappa \equiv 2$, following Brunnermeier and Sannikov (2014). We choose the productivity $A$ so there is no long-run growth. We normalize the labor supply $N \equiv 1$ and the capital stock $K \equiv 1$ at the steady state. We normalize the labor productivity $a$ so that real income is 1 at the steady state. In our benchmark calibration, the government slowly repays the debt it contracted to finance the stimulus checks by raising tax rate $\tau_t$ uniformly over 15 years, and then let it decay to its long-run value $\tau = 30\%$ over 5 years.
6.2 Closing the Output Gap in HANK

We now quantify the effect of stimulus checks during a demand-driven recession. We suppose that the economy experiences a persistent investment shock \( \{ z_t \} \) such that output contracts over three quarters (by -4%) and then recovers linearly over the next two years.\(^\text{36}\) Starting from this point, we are interested in the general equilibrium response of output following stimulus checks of various sizes. Compared to Figure 5.2, our economy captures three new effects: intertemporal feedbacks, international leakages, and endogenous prices. To assess the contribution of each component, we introduce them one at a time.

**Intertemporal feedbacks.** We plot the output gap as a function of the stimulus checks in Figure 6.1. We first consider a closed economy \(( \alpha^d = \alpha^c = 0)\) with rigid prices \((\kappa = 0)\). Compared to Figure 5.2 in Section 5.2, our model captures the full, intertemporal response to stimulus checks in general equilibrium (black line). As the checks grow larger, the government still enjoys a relatively large bang-for-buck. A $1,400 check (or 8.5% of the average quarterly income) closes the output gap in a typical recession.\(^\text{37}\)

**International leakages.** The blue line plots the response of the output gap when we account for imports. The bang-for-the-buck is smaller, as part of the additional spending is directed towards foreign goods (i.e., it “leaks” abroad) and does not raise domestic incomes. A $2,000 stimulus check now fully closes the output gap in a typical recession, compared to $1,400 in the closed economy.

**Sticky prices and responsive monetary policy.** Finally, price and interest rate movements affect the response to stimulus checks. We plot the response of the output gap in an open economy when allowing for a Phillips curve \((6.3)\) and active monetary policy with an occasionally binding zero lower bound \((6.4)\). The response is muted for small checks. The zero lower bound binds during the recession and deflation increases the interest rate. This increase in the user cost decreases the MPC on durables in recessions. For larger checks, the response is substantially stronger compared to the open economy with fully rigid prices. The inflation generated by the checks now lowers the real interest rate, amplifying

\(^\text{36}\) We solve the model globally to capture the non-linearities in spending and allow for an occasionally-binding zero lower bound for the interest rate. Therefore, the usual linear perturbation methods (Auclert et al., 2021) that allow to back out a sequence of shocks do not apply. We propose an efficient approach to compute the sequence of investment shocks \( \{ z_t \} \) that induces a specific sequence of output \( \{ Y_t \} \) in our non-linear model (Appendix A.5).

\(^\text{37}\) We assume an average quarterly income of $16,500 as in Kaplan et al. (2018).
Figure 6.1: Closing the output gap

Notes: The black line corresponds to a closed economy with rigid prices. The blue line corresponds to an open economy with rigid prices. Finally, the orange line corresponds to an open economy with sticky prices, active monetary policy, and an occasionally binding zero lower bound.

their stimulus. Overall, the response is similar to the closed economy with rigid prices. A check of roughly $1400 again fully closes the output gap.

6.3 The Timing of Stimulus Checks

[TBA]

7 Conclusions

We augment a canonical incomplete-markets model of durable spending by introducing a smooth adjustment hazard. This gives the model enough flexibility to generate a marginal propensity to spend (MPC) that is decreasing, flat, or increasing. We discipline the adjustment hazard by matching evidence on (i) the relative MPCs of durables and non-durables; (ii) the short-run price elasticity of durables; (iii) the size distribution of adjustments; and (iv) the conditional probability of adjustment since the last purchase. We use the model to quantify the size of stimulus checks that close a given share of the output gap.

We find that the MPC remains elevated even for large stimulus checks. A $600 check closes roughly half the output gap in a typical US recession. This check is half as large than

38 It is well-known that fiscal and transfer multipliers can be much larger when the zero lower bound binds (Farhi and Werning (2016)).
in a canonical model of non-durable spending with the same MPC out of a small transfer, and three times larger than in a canonical model of durables. A relevant upper bound for policymakers is the check that fully closes the output gap. Larger checks are “too much” in that they stimulate output beyond potential. Across alternative model specifications, we find that checks beyond the $1,500 to $2,000 range are too much in a typical recession.

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A Quantitative Appendix

In this appendix, we discuss the numerical solution of the income fluctuations problem. Section A.1 states the problem recursively for the full model. The intermediate model is a hybrid between the two. Section A.2 discuss the numerical implementation.

A.1 Households’ Problem

We now state the household’s problem recursively. The household is indexed by three idiosyncratic states: their holdings of durables \((d)\); their holdings of liquid asset \((m)\); and their idiosyncratic income \((y)\). In the following, we let \(x \equiv (d, z, y)\) to save on notation. We state the general problem where the price of the consumption \((P^c)\) and investment \((P^d)\) goods might not be equal to anticipate the open economy case (Section 6).

Continuation values. We follow the steps below to compute recursively a sequence a continuation values \(\{V_t(\cdot)\}\).\(^{39}\)

1. Consumption-saving. The household chooses how much to consume and save in liquid asset

\[
W_t^C(x) \equiv \max_{c,m'} u(c, d) + \beta \int V_{t+1}(d, m', y') \Gamma(dy'; y) \tag{A.1}
\]

\text{s.t. } P_t^c + m' \leq m \quad \text{and} \quad m' \geq 0,

2. Durable adjustment. The household chooses how much durables to purchase

\[
W_t^D(m, y) \equiv \max_{d', m'} W_t^C(d', m', y) \tag{A.2}
\]

\text{s.t. } \left[ P_t^d - (1 - \theta) P_{t+1}^d (1 - \delta) \right] d' + m' \leq m

3. Discrete choice. Finally, the household chooses whether to adjust her stock of durables.

The value associated to the discrete choice problem is\(^{40}\)

\[
V_t(x) \equiv \eta \log \left( \sum_{h \in \{D, C\}} \exp \left( \frac{W_t^{*, h}(x)}{\eta} \right) \right) \tag{A.3}
\]

\(^{39}\) The terminal condition for \(V_{t+1}(\cdot)\) is the stationary value when \(T_t = 0\) in each period \(t\).

\(^{40}\) See Artuç et al. (2010) for the derivation.
where

\[ W_{t}^{*,D}(x) \equiv W_{t}^{D}(\mathcal{V}(x; T_{t}) + \theta P_{t}^{d}(1 - \delta) \frac{d}{1 - \chi}, y) - \kappa \]  \hspace{1cm} (A.4)

and

\[ W_{t}^{*,C}(x) \equiv W_{t}^{C}(\mathcal{V}(x; T_{t}) - \Delta_{t} \frac{d}{1 - \chi} - i \delta P_{t}^{d} \frac{d}{1 - \chi}, y) \]  \hspace{1cm} (A.5)

and

\[ \mathcal{Y}(x; T_{t}) \equiv \left(1 - \tau^{t}\right)y + (1 + r_{t-1}^{m}) \frac{m}{1 - \chi} - r_{t-1}^{b} (1 - \theta) P_{t}^{d}(1 - \delta) \frac{d}{1 - \chi} + T_{t} \]  \hspace{1cm} (A.6)

is cash-on-hand, and

\[ \Delta_{t} \equiv (1 - \theta)(1 - \delta) \times \left\{ P_{t}^{d} - P_{t+1}^{d}(1 - (1 - i) \delta) \right\} \]  \hspace{1cm} (A.7)

captures the debt payment on the principal. The associated adjustment hazard is

\[ S_{t}(x) \equiv \exp\left(\frac{W_{t}^{*,D}(x)}{\eta}\right) \sum_{h' \in \{D,C\}} \exp\left(\frac{W_{t}^{*,h'}(x)}{\eta}\right) \]  \hspace{1cm} (A.8)

In the fully state-dependent limit \( \eta \to 0 \), the value (A.3) and hazard (A.8) become

\[ V_{t}(x) \equiv \max_{h \in \{D,C\}} \left\{ W_{t}^{*,h}(x) \right\} \quad \text{and} \quad S_{t}(x) = \begin{cases} 1 & \text{if } W_{t}^{*,D}(x) > W_{t}^{*,C}(x) \\ 0 & \text{otherwise} \end{cases} \]  \hspace{1cm} (A.9)

### A.2 Numerical Implementation

We now describe how we solve numerically for the value functions defined above, and how we iterate on the associated policy functions to obtain aggregate quantities.

**Value functions.** We proceed as follows

1. **Guess.** Fix an initial guess for \( \mathcal{V}(x) \equiv \int V(d, z, y') \Gamma(dy'; y) \). Let \( \mathcal{V}_{z}(\cdot) \equiv \partial_{z} \mathcal{V}(\cdot) \) for the durable and liquid assets \( z \in \{d, m\} \).

2. **Consumption-saving.** Fix the (terminal) states / policies \((d, y)\). Consider sequentially the two cases described below.
(a) **Borrowing constraint not binding.** If the household’s borrowing constraint \( m' \geq 0 \) is not binding, a necessary condition for an optimum is

\[
 u_c (c,d) = \beta P_t^c V_m (d,z',y),
\]  

(A.10)

together with the budget constraint \( c = m - m' \). These conditions are not sufficient, however, since the problem is typically non-convex.\(^{41,42}\) To recover policy functions, i.e., maps \( z \mapsto (c,m') \), we proceed as follows. We first obtain maps \( m' \mapsto (c,m) \) using the endogenous grid method (EGM) of Carroll (2006). The (generalized) inverse of this map (as a function of \( m \)) might contain several points since the problem is non-convex. These points define a set of candidates, together with \( m' = 0 \) (and the upper bound of the grid for \( m \)). The optimum is found by comparing the values of the objective in (A.1) associated to each candidate. We recover the policy functions using an approach similar Druedahl and Jørgensen (2017). Fix some \( m \) on the grid of interest. Find the couples \( (m'_0,m'_1) \) such that the couple \( (m_0,m_1) \) recovered by EGM are such that \( m_0 \leq m \leq m_1 \). Then, interpolate linearly the value of \( m' \) at \( m \) using \( (m_0,m_1) \) and \( (m'_0,m'_1) \) and compare the value of the objective for this value of \( m' \). The policy function \( m \mapsto m' \) is the one that provides the highest value, and \( m \mapsto c \) is recovered using the budget constraint. Whenever this policy violates the borrowing constraint \( m' \geq 0 \), consider instead the next case. Otherwise, proceed to Step 3.

(b) **Borrowing constraint binding.** If the household’s borrowing constraint \( m' \geq 0 \) is binding, holdings of the liquid asset are \( m' = 0 \) and non-durable consumption equals cash-on-hand \( c = m \).

Using the resulting policy function \( m' (\cdot) \), compute the value \( W_C (x) \) using (A.1),

\(^{41}\) The reason is that the continuation value involves the upper envelopes (A.3) and (A.9). Random fixed adjustment costs for durables can make continuation value smooth (i.e., no kinks) but not necessarily concave.

\(^{42}\) Condition (A.10) is still necessary for an optimum. To see this, consider a simplified version of the problem of interest: \( \max_x f (c) + G (-c) \) with \( f (\cdot) \) and \( G (\cdot) \) smooth except for a convex kink in \( G (\cdot) \) at \( \bar{c} \in \mathbb{R} \). Suppose (by contradiction) that the optimizer is \( c^* = \bar{c} \). Then, \( f' (\bar{c}) \geq G'_+ (-\bar{c}) \) and \( f' (\bar{c}) \leq G'_- (-\bar{c}) \). However, \( G'_+ (-\bar{c}) > G'_- (-\bar{c}) \) since \( G (\cdot) \) admits a convex kink at \( \bar{c} \). This leads to the desired contradiction. Therefore, the optimizer cannot be the point where the kink occurs. The argument generalizes immediately to multiple kinks and multiple financial assets.
and the marginal values
\[
\partial_d W^C(x) = u_d(m - m'(\cdot), d) + \beta Y_d(d, m'(\cdot), y) \tag{A.11}
\]
\[
\partial_m W^C(x) = 1/P_t u_c(m - m'(\cdot), d) \tag{A.12}
\]
for the durable, the liquid asset and the illiquid asset.

3. **Durable adjustment.** A necessary condition for an optimum is\(^{43}\)
\[
\partial_d W^R(d', m', y) - \left[ P_t^d - (1 - \theta) P_{t+1}^d \right] \partial_m W^R(d', m', y) = 0 \tag{A.13}
\]
where
\[
m' = m - \left[ P_t^d - (1 - \theta) P_{t+1}^d \right] d' \tag{A.14}
\]
Again, (A.13) is typically not sufficient for an optimum. We thus define a set of candidates \(d'\) that satisfy either (A.13) or \(d' = \bar{d}\) where \(\bar{d}\) is the upper bound of our numerical grid for durables. We compute the value (A.2) associated to these candidates. The policy function for \(d'\) is the one that provides the highest value. We compute the value \(W^D(x)\) using (A.2), and the marginal value
\[
\partial_m W^D(z, y) = \partial_m W^C(d'(\cdot), z'(\cdot), y), \tag{A.15}
\]
and proceed to Step 4.

4. **Continuation values.** Compute the values (A.4)–(A.5) and the marginal values
\[
\partial_d W^*_{t,D}(x) = \frac{1}{1 - \chi} \left\{ -r_{t-1}^b (1 - \theta) P_t^d (1 - \delta) \right. \tag{A.16} \\
+ \theta P_t^d (1 - \delta) \left. \right\} \partial_z W^D(\cdot) \\
\partial_d W^*_{t,C}(x) = \frac{1}{1 - \chi} (1 - (1 - \delta) \partial_d W^C(\cdot) + \frac{1}{1 - \chi} \left\{ -r_{t-1}^b (1 - \theta) P_t^d (1 - \delta) \right. \tag{A.17} \\
- \Delta_t - \nu \delta P_t^d \left. \right\} \partial_z W^C(\cdot)
\]

\(^{43}\)The solution is necessarily interior in this case, i.e., \(d' = 0\) cannot be optimal.
for the durable asset, with $\Delta_t$ defined by (A.7), and
\[
\partial_m W_{t}^{*,k} (x) = \frac{1}{1 - \chi} (1 + r_{t-1}^m) \partial_m W_{t}^{k} (\cdot) \quad \text{for each choice } k \in \{C, D\} \tag{A.18}
\]
for the liquid asset.

5. **Discrete choice.** Compute the value (A.3) and the marginal values\(^{44}\)
\[
\partial_x V (x) = \sum_{k \in \{D,C\}} S (x) \partial_x W^x (x) \tag{A.19}
\]
for the durable and liquid asset $x \in \{d, m\}$, where $S (x)$ is the adjustment hazard (A.8).

6. **Update.** Update the expected utility $V (x) \equiv \int V (d, m, y') \Gamma (dy'; y)$ and the marginal utilities $V_\times (x) \equiv \int \partial_x V (d, m, y') \Gamma (dy'; y)$ for the durable and liquid asset $x \in \{d, m\}$. Finally, repeat Step 1 until convergence.

**Computational details.** We use 100 points for the grids for the durable and liquid assets. We discretize the income process $\Gamma (y'; y)$ on a 7-point grid using the method of Rouwenhorst (1995). To iterate on the distribution, we use the policy functions computed above together, together with the income process $\Gamma$ and we randomly assign households between adjustment (D) and no adjustment (C) according the adjustment hazard (A.8). For numerical reasons, we endow new generations with a small stock of durables ($d \equiv 10^{-4}$). In our partial equilibrium exercises, we start with a random sample of 15,000 households and simulate them for 3,200 periods, including 400 burn out periods. In our general equilibrium exercises, we randomly draw 250,000 households from the stationary distribution we obtained in partial equilibrium and simulate them for 125 periods.

### A.3 Sticky Prices, Trade Balance and Exchange Rate

In Section 6, we allow for sticky prices and a non-zero trade balance.

**Price indices.** We define the price of non-durables and durables relative to the price of the domestic good
\[
P^c_t \equiv \left[ \alpha_c + (1 - \alpha_c) (e_t)^{1-\rho} \right]^{1/\rho} \quad \text{and} \quad P^d_t \equiv \left[ \alpha_d + (1 - \alpha_d) (e_t)^{1-\rho} \right]^{1/\rho} \tag{A.20}
\]
\(^{44}\) We omit the arguments on the right-hand side of (A.19) for concision.
where $e_t$ is the real exchange rate. The level of domestic price is
\[ p_{\text{dom}}^{t} = \prod_{s=0}^{t} (1 + \pi_s) , \tag{A.21} \]
where $\pi_t$ is inflation in the domestic price given by the Phillips curve (6.3). The CPI price index is

\[ \text{CPI}_t \equiv \left\{ \omega^c P^c_t + (1 - \omega^c) P^d_t \right\} p_{\text{dom}}^{t} , \tag{A.22} \]

where $\omega^c$ is the spending share on non-durables at the stationary equilibrium. In turn, the monetary policy (6.4) depends on CPI inflation $\pi_{\text{CPI}}^t \equiv \text{CPI}_t / \text{CPI}_{t-1} - 1$ into account.

**Budget constraints.** It is convenient to express all the budget constraints in real terms.\(^{45}\) We now use $m$ to denote real cash-on-hand (in terms of domestic goods). The budget constraints in (A.1)–(A.2) now become
\[ p^c_t c + m' \leq \tilde{m}^c , \tag{A.23} \]
and
\[ \left[ p^d_t - (1 - \theta) p^d_{t+1} (1 + \pi_t) (1 - \delta) \right] d' + m' \leq \tilde{m}^d , \tag{A.24} \]

In turn, $\tilde{m}^c$ and $\tilde{m}^d$ are the real cash-on-hand after the adjustment decision (A.4)–(A.5). That is,
\[ \tilde{m}^c \equiv \mathcal{Y} (x; T_t) + \theta P^d_t (1 - \delta) \frac{d}{1 - \chi} \tag{A.25} \]
or
\[ \tilde{m}^d \equiv \mathcal{Y} (x; T_t) - \Delta_t \frac{d}{1 - \chi} - \nu \delta P^d_t \frac{d}{1 - \chi} \tag{A.26} \]
depending on the option, where
\[ \mathcal{Y}_t (x; T_t) \equiv (1 - \tau_t) y (Y_t + \text{Div}_t) + \frac{(1 + r^m_t)}{1 + \pi_{t-1}} \frac{m}{1 - \chi} - r^b_t (1 - \theta) P^d_t (1 - \delta) \frac{d}{1 - \chi} + t_t \tag{A.27} \]
where $Y_t$ is real output, $\text{Div}_t$ is real dividends, $t_t$ are real stimulus checks, and
\[ \Delta_t \equiv (1 - \theta) (1 - \delta) \times \left\{ P^d_t - P^d_{t+1} (1 + \pi_t) (1 - (1 - t) \delta) \right\} \tag{A.28} \]

\(^{45}\)We originally opted for a nominal formulation in Appendix A.1 to simplify the exposition.
The envelope conditions (A.11)–(A.12), (A.15) and (A.16)–(A.18) are modified accordingly.

**Trade balance.** The real trade balance is

\[
TB_t \equiv -e_t \left\{ (1 - \alpha_c) \left( \frac{e_t}{P_{c,t}} \right)^{-\rho} c_t + (1 - \alpha_d) \left( \frac{e_t}{P_{d,t}} \right)^{-\rho} x_t \right\} \\
+ \left\{ (1 - \alpha_c) \left( \frac{1}{e_t} \right)^{-\rho} c^* + (1 - \alpha_d) \left( \frac{1}{e_t} \right)^{-\rho} x^* \right\},
\]

where \( P_{c,t}^{c,*} \) and \( P_{d,t}^{d,*} \) are the counterpart of the price indices (A.20) in the foreign country. Consumption \( c^* \) and investment \( x^* \) in the rest of the world are constant and equal to the steady state levels at home, i.e., \( c^* = c \) and \( x^* = x \), so there is no net imports initially.

**Exchange rate.** The nominal exchange rate satisfies uncovered interest parity

\[
\frac{E_{t+1}}{E_t} = \frac{1 + r_t}{1 + r^*}
\]

where \( r^* \) is the foreign interest rate, which is constant and equal to the steady state level at home, i.e., \( r^* = r \). The terminal condition is \( \lim_{t \to +\infty} E_t = \lim_{t \to +\infty} P_{H,t} \) by purchasing power parity. Finally, the real exchange rate is \( e_t \equiv E_t / \prod_{s=0}^{t} (1 + \pi_s) \) since the foreign price is constant and equal to 1.

### A.4 Firm’s Problem

**Investment.** The firm’s investment problem is

\[
\max_{\{I_t, K_t\}} \sum_t Q_t \{ AK_{t-1} - I_t \} \quad (A.31)
\]

s.t. \( K_t \leq \left\{ 1 - \delta^K + \Phi \left( I_t / K_{t-1} \right) + z_t \right\} K_{t-1} \quad \text{and} \quad K_t \geq 0 \)

with the initial condition \( K_{-1} \equiv K \) where \( K \) is steady state capital. At optimum,

\[
\frac{1}{\Phi' (x_t) \left( 1 + \pi_{t+1} \right)} = A + \frac{1}{\Phi' (x_{t+1} + z_{t+1})} \left\{ 1 - \delta^K + \Phi \left( x_{t+1} \right) - x_{t+1} \Phi' \left( x_{t+1} \right) \right\} \quad (A.32)
\]

46 This terminal condition uses the fact that the foreign price is normalized to 1. We work with a finite horizon in our simulation and assume that \( E_t = P_{H,t} \) after 20 years.
with terminal condition $\lim_{T \to +\infty} x_{T+1} = \Phi^{-1}(\delta^K)$, where $x_t \equiv I_t/K_{t-1}$ and where we have used the definition of the firm’s stochastic discount factor $Q_{t+1}/Q_t \equiv (1 + \pi_{t+1}) / (1 + r_t)$. This initial value problem (solving for $x_0$) can be solved using a standard shooting algorithm. The sequence of capital can then be constructed recursively using the law of motion of capital

\[
\frac{K_t}{K_{t-1}} = 1 - \delta^K + \Phi(x_t) + z_t, \tag{A.33}
\]

with initial condition $K_{-1} \equiv K$.

**Dividends.** The firm’s dividends are $\text{Div}_t = \overline{\text{Div}} + \Psi_t \hat{\text{Div}}$, where $\overline{\text{Div}}$ is the increase in dividends relative to steady state, and $\{\Psi_t\}$ was defined in Appendix A.3. The increase $\hat{\text{Div}}$ ensures that $\sum_t Q_t \text{Div}_t = \sum_t Q_t \Pi_t$ where $\Pi_t$ is real profits. Therefore,

\[
\text{Div}_t = \overline{\text{Div}} + \Psi_t \frac{\sum_s Q_s \{\Pi_s - \overline{\text{Div}}\}}{\sum_s Q_s \Psi_s} \tag{A.34}
\]

### A.5 Investment Shocks

In Section 6, we consider recessions of various magnitudes induced by firm investment shocks. We are interested in constructing a sequence of investment shocks $\{z_t\}$ that produces a particular recession, i.e., a path of equilibrium outputs $\{Y_t\}$. In this appendix, we show that this sequence of shocks can be constructed in a straightforward way despite the non-linearities inherent to aggregate demand in our economy. In the following, we let $C_t(\{E_t\})$, $X_t(\{E_t\})$ and $\text{TB}_t(\{E_t\})$ denote total (real) demands for non-durables and durables and the trade balance given pre-tax incomes $\{E_t\}$.

**Lemma 1.** Consider a sequence of real outputs $\{Y_t\}$ with $Y_t \to 1$ as $t \to +\infty$. There exists a (unique) sequence of investment shocks $\{z_t\}$ that induces $\{Y_t\}$ in equilibrium. It can be constructed in four steps. First, fix an initial guess for dividends, e.g., $\text{Div}_t = \overline{\text{Div}}$ for every period $t$. A sequence of net investment $\{Z_t\}$ is backed out residually from the resource constraint

\[
Z_t \equiv Y_t - P_t^c C_t(\{Y_t + \text{Div}_t\}) - P_t^d X_t (\{Y_t + \text{Div}_t\}) - G - \text{TB}_t (\{Y_t + \text{Div}_t\}), \tag{A.35}
\]

Second, dividends, tax and prices are updated: dividends $\text{Div}_t$ are given by (A.34) with $\Pi_t \equiv -Z_t$; taxes are backed out from the government’s budget constraint (6.5)–(6.6); and prices are computed using the pricing equations (6.3) and (A.20). Updating the guess for dividends and iterating on the first step after until converge allows to recover the equilibrium sequence of net investment $\{Z_t\}$.

Expression (A.32) defines a unique map $x_t \mapsto x_{t+1}$ since the right-hand side of (A.32) is increasing in $x \geq 0$ when the expression itself is positive, given our choice $\Phi(x) = 1/\kappa (\sqrt{1 + 2\kappa x} - 1)$ with $\kappa \equiv 2$. 

47
Third, the sequence of investment rates \( \{x_t\} \) is obtained from the firm’s Euler equation by solving the second-order difference equation

\[
\frac{1}{\Phi'(x_t)} \left( \frac{1 + r_t}{1 + \pi_{t+1}} \right) = A + \frac{1}{\Phi'(x_{t+1})} \left\{ \frac{Z_{t+2} x_{t+1} - A}{Z_{t+1} x_{t+2} - A} - x_{t+1} \Phi'(x_{t+1}) \right\}
\]  
(A.36)

with initial condition \( x_0 \equiv A + Z_0^*/K \) where \( K \) is steady state capital, and terminal condition \( \lim_t x_{T+1} = \Phi^{-1}(\delta^K) \). This initial value problem (solving for \( x_1 \)) can again be solved using a standard shooting algorithm.\(^{48}\) Finally, the investment shocks \( \{z_t\} \) are backed out residually from the law of motion of capital

\[
\frac{Z_{t+1}}{Z_t} \frac{x_t - A}{x_{t+1} - A} = 1 - \delta^K + \Phi(x_t) + z_t
\]  
(A.37)

Proof. Combining aggregate spending (6.7), the trade balance (??), and households’ aggregate income (6.8), the sequence of investment \( \{I_t\} \) induces incomes \( \{Y_t\} \) in equilibrium if and only if

\[
(1 - \alpha_c) C_t (\{Y_t + Div_t\}) + (1 - \alpha_d) X_t (\{\{Y_t + Div_t\}\}) + G + Z_t = Y_t
\]  
(A.38)

where

\[
Z_t \equiv I_t - AK_{t-1}
\]  
(A.39)

is the effective investment shock. Combining the firm’s Euler equation for investment (A.32) and the law of motion of capital (A.33),

\[
\frac{1}{\Phi'(x_t)} \left( \frac{1 + r_t}{1 + \pi_t} \right) = A + \frac{1}{\Phi'(x_{t+1})} \left\{ \frac{K_{t+1}}{K_t} - x_{t+1} \Phi'(x_{t+1}) \right\}
\]  
(A.40)

Expression (A.36) is obtained using the definition (A.39) and the Euler equation (A.40). Similarly, expression (A.35) is obtained using the same definition and the law of motion of capital (A.33). The rest of proof follows immediately. \( \square \)

A.6 Computational Details

Computation. We use 125-point grids for the financial and durable assets. We discretize the income process on a 7-point grid using the method of Rouwenhorst (1995). Given

\(^{48}\)Note that the initial value problem consists here of solving for \( x_1 \) — not \( x_0 \) contrary to the problem (A.32). The reason is that \( x_0 \equiv I_0/K_{-1} \) is predetermined in period \( t = 0 \) given \( Z_0^* \).
the non-convexities inherent to our model, we use a stochastic simulation.\footnote{A non-stochastic simulation (e.g., Young, 2010) would produce a different stationary distribution in presence of non-convexities.} When computing our stationary moments (Section 4), we simulate 15,000 households over 3,000 quarters with a burn of 400 quarters. In our general equilibrium experiments, we sample 200,000 households from this stationary distribution and simulate them over 125 quarters.

\textit{Legacy assets}. Households claim the assets of the old generations who died. They claim these legacy assets in proportion to their own asset holdings, as is apparent from (A.4)–(A.6). We simulate a finite sample, however. Therefore, there might be a (small) discrepancy between the effective amount of legacy assets, and the amount claimed by surviving households. The government reconciles these two quantities by sending transfers to the households. For instance,

\[(1 + r_{t-1}) \int \frac{m}{1 - \chi} d\pi_{t-1} = (1 + r_{t-1}) \int m'_{t-1}(x) d\pi_{t-2} + T'_m\]

for financial assets, where \(\pi_{t-1}\) is the distribution of idiosyncratic states in period \(t\), \(m'_{t}(x)\) is the amount of bonds chosen by a household with idiosyncratic state \(x\) in period \(t\), and \(T'_m\) is the effective transfer from the government. We introduce similar transfers for durables (\(T'_d\)) and income (\(T'_y\)) — since households’ aggregate productivity \(\int y d\pi_{t-1}\) is not necessarily 1 in every period. These transfers are added to the government’s budget constraint (6.5).

\section{Additional Quantitative Results}

\subsection{Our Model}

B.1 Our Model
Figure B.1: Distribution of MPCs

Notes: This figure plots the distribution of MPC in our full model, and in a model of non-durable spending (our model specialized with $\vartheta_c = 0$).

Figure B.2: Extensive margin

Notes: This figure plots the marginal propensity to adjust (i.e., the increase in the average hazard $S$ normalized by the size of the check) as a function of the stimulus check. ‘Stationary’ corresponds to the stationary equilibrium. ‘Mild recession’ corresponds to a recession where average household income declines linearly over four quarters with a trough of 4% and then recovers linearly over two years. ‘Deep recession’ is the same with a decline of 8% instead.
**Figure B.3: Intertemporal MPCs**

![Graph showing Intertemporal MPCs](image)

Notes: This figure plots the spending response over time to transitory income shocks in quarters 1 and 6.

### B.2 Canonical Model

**Figure B.4: Conditional adjustment probability**

![Graph showing Conditional adjustment probability](image)

Notes: This figure plots the marginal propensity to adjust (i.e., the increase in the average hazard $S$ normalized by the size of the check) as a function of the stimulus check. ‘Stationary’ corresponds to the stationary equilibrium. ‘Mild recession’ corresponds to a recession where average household income declines linearly over four quarters with a trough of 4% and then recovers linearly over two years. ‘Deep recession’ is the same with a decline of 8% instead.
**Figure B.5: State-dependent MPCs**

![Figure B.5](image1)

*Notes:* The figure plots the MPC as a function of the stimulus check. ‘Stationary’ corresponds to the stationary equilibrium. ‘Mild recession’ corresponds to a recession where average household income declines linearly over four quarters with a trough of 4% and then recovers linearly over two years. ‘Deep recession’ is the same with a decline of 8% instead.

**Figure B.6: Intertemporal MPCs**

![Figure B.6](image2)

*Notes:* This figure plots the spending response over time to transitory income shocks in quarter 1 and 6.