How Tariffs Affect Trade Deficits*

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We study the positive (not normative) effect of a permanent import tariff on trade deficits. We consider a two-period trade model with general preferences and technology. We first develop an aggregation result showing one can work with induced preferences over aggregate imports and exports. This simplifies the analysis considerably. Our main result provides a sufficient statistic to evaluate the impact of tariffs around free trade: tariffs reduce trade deficits if the Engel curves for aggregate imports and exports are convex. Convexity is more likely when goods, at the micro-level, shift between being imported, non-traded, or exported. If this extensive margin is inactive and Engel curves are linear, then a permanent tariff is neutral.

1 Introduction

How do import tariffs affect trade imbalances? Setting welfare consequences aside, can a permanent increase in tariffs reduce an ongoing trade deficit?

The answer depends on whom you ask. Politicians and the general public often assume that tariffs, by discouraging imports, will narrow the trade deficit. The great trade policy disaster of the 1930s is a case in point. As Irwin (2011) convincingly argues, trade wars over that period were not primarily driven by lobbying and other forms of redistributive politics, but rather by countries' desire to correct trade imbalances via a rise in trade protection. The "reciprocal tariffs" put forward on April 2, 2025 by the Trump administration seem to derive from a similar belief that an increases in US tariffs can lower trade deficits by choking off imports.

Economists are quick to point out that this is only part of the story. Everything else being equal, tariffs may reduce imports, but why would exports be unaffected? Trade economists may note that import tariffs are equivalent to export taxes, an expression of Lerner symmetry (Costinot and Werning, 2019). Macroeconomists may add that, following textbook analyses, trade imbalances are fundamentally shaped by national savings

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and investment decisions that are orthogonal to trade policy. Tariffs affect the extent and nature of intratemporal trade, but the trade balance issue is one of intertemporal trade (Economic Expert Panel, 2025).[1]

Intuitions aside, formal analyses of the impact of tariffs on trade imbalances are scarce. It is fairly clear that temporary tariffs, which differentially affect the cost of living over time, may affect borrowing and lending. It is also fairly clear that if economic conditions were to vary over time, even a permanent tariff may have a different incidence on future costs of living, with implications for trade imbalances. Razin and Svensson (1983) have already made both points. A more subtle question, though, is whether *everything else being equal*, one might still expect a systematic effect of permanent tariffs on the trade deficit.

Our main finding is that there may indeed be such a systematic effect. In their influential work, Obstfeld and Rogoff (2000) have shown that permanent trade costs dampen both intratemporal and intertemporal trade. Our analysis elevates and extends their mechanism and applies it to the case of import tariffs. The key observation is that even if economic conditions are unchanged between today and tomorrow, the fact that a country is currently running a trade deficit implies that aggregate consumption will, in general, be different today and tomorrow. This opens the door for the non-neutrality of permanent tariffs on the trade deficit.

We consider a neoclassical trade model over two periods. We allow for an arbitrary number of goods, general preferences and general technology. Imports may be used as final goods or as inputs into production. The government levies a uniform tariff on all imports in both periods and rebates the revenue back to households.

The starting point of our analysis is a new aggregation result. We show that in each period, one can summarize all the relevant implications of our general trade model for trade deficits into a preference relation over aggregate imports and aggregate exports only. The existence of this aggregate preference relationship turns out to be key to simplify our analysis and to generate novel insights.

Using our new aggregation result, we derive two broad sets of insights about the relationship between tariffs and deficits. As a warm up, we first provide sufficient conditions for two extreme scenarios: neutral tariffs and autarky-inducing tariffs. The trade balance is locally unaffected by tariffs in an endowment economy with preferences that are appropriately homothetic when the equilibrium feature strictly positive imports and exports of all goods, with no non-tradables. These assumptions are strong and, it turns out, always violated for large enough tariffs. Indeed, we prove that large enough tariffs drive the

[1] Link to the Clark Center Economic Expert Panel Poll: https://www.kentclarkcenter.org/surveys/tariffs-reciprocal-and-retaliatory-2/

economy to autarky and thus, in the extreme, reduce trade deficits to zero.

As our main contribution, we then offer a general analysis of the impact of tariffs around free trade. We show that whether a tariff reduces a trade deficit depends on a single sufficient statistic: the slope of the Engel curve in the import vs export space. When this slope is is higher in the first period, then tariffs affect the first period more and tend to reduce a trade deficit. When there are no differences in preferences, technology, or prices, and the only difference across periods is the level of aggregate consumption, the slope of the Engel curve today versus tomorrow is determined by the curvature of the Engel curve, i.e. whether aggregate imports are a luxury good. If the Engel curve is linear, then they are not, and tariff neutrality holds. If the Engel curve is strictly convex, then they are, and tariffs reduce trade deficits.

This begs the question: What determines the curvature of the Engel curve? One possibility focuses entirely on the non-homotheticity of preferences over goods, as in Fajgelbaum and Khandelwal (2016). If consumers' preferences are such that imported goods have elasticities higher than one, then the Engel curve will tend to be convex. Interestingly, though, non-homothetic preferences over goods are not necessary. It is so because the relevant preferences over aggregate imports and exports also capture technological considerations. In particular, we show that the curvature of the Engel curve may capture an active extensive margin of trade. When consumers have CES preferences over goods and endowments are fixed, the Engel curve turns out to be linear if there is no action at the extensive margin, no shifting of goods between imported and non-traded, or between non-traded and exported (explaining our earlier neutrality result). In contrast, under the same assumptions, when goods do shift between these categories, then the Engel curve becomes strictly convex. With a fixed number of goods, Engel curves have kinks; with a continuum, they are smooth.

How large are the possible effects? We do not currently have estimates of the sufficient statistics that we uncovered to carry out directly the required calculations. We are optimistic that such statistics could be obtained in the future, as they boil down to how aggregate imports and exports respond to foreign interest rate shocks at different levels of the trade deficit. As an alternative, we conclude, for now, with preliminary simulations of the impact of tariffs on deficits in the context of a CES example.

Our results extend the applicability, and further shed light on the implications, of the mechanism in Obstfeld and Rogoff (2000). They explored the idea that trade costs may help explain a number of puzzles in international macro, including the Feldstein-Horioka puzzle. In the context of a two-good endowment economy, they derive two key results. First, they show that trade costs create a wedge between the real interest rates faced by borrowing and lending countries; second, they show that the magnitude of this wedge is larger when trade imbalances are larger as well. Based on these two observations, they argue that trade costs, by creating this wedge, may keep trade imbalances in a modest range, thereby explaining the high correlation between domestic savings and investment and offering a solution to the Feldstein-Horioka puzzle.[2] Although Obstfeld and Rogoff (2000) never formally study how trade costs or tariffs affect trade imbalances, the interest rate channel that they emphasize is at the heart of our analysis. As we show, the shape of the Engel curves encodes all the required information that is relevant for the interest rate channel. We discuss this connection in greater details in Section 3, including the fact that tariffs and trade costs are not identical due to their different wealth effects.[3]

Motivated by Obstfeld and Rogoff's original work, Eaton, Kortum and Neiman (2016) offer a quantitative exploration of the role of trade costs. At their preferred calibration, the find that very large changes in trade costs, going all the way to zero trade costs, can raise the US trade deficit to 20% of US GDP. effects on US deficits. Using a related model, Reyes-Heroles (2016) estimates the evolution of trade costs over time and quantifies their contribution to the observed changes in US deficits. He concludes that US trade deficits observed in the late 2000s could have been three times smaller absent his estimated changes in trade costs. Although we study a tariff, not iceberg trade costs, our analysis could be applied in that context with some adjustments. Our results suggest some caution in interpreting these prior quantitative findings, since these models had not been directly calibrated to any evidence on the curvature of Engel curves, nor to separating the action along the extensive margin from an intensive margin.

Our crucial aggregation result combines elements of the perspectives put forward by Hicks (1936) and Meade (1952). It emphasizes preferences over exports and imports, as in Meade (1952), and further creates aggregate composites of the two, as in Hicks (1936). This approach allows us to study trade deficits in a tractable way. Despite the fact that we keep preferences and technologies general, our analysis is no more complex than in a simple two-good economy. Through the lens of our aggregation result, richer economies with a continuum of goods and active extensive margins of trade implicitly give rise to non-homothetic induced preferences in the space of aggregate exports and imports, which is what the impact of tariffs on deficits depends on.

[2] A similar emphasis on the relationship between trade costs and trade imbalances can be found in Dornbusch (1983). In his paper, it is the existence of non-tradable goods that create a wedge between domestic and world real interest rates whose magnitude varies with aggregate consumption.

[3] A related literature centered on wealth effects discusses the impact of terms-of-trade shocks, such as oil shocks, either temporary or permanent, on the current account. Classic references include Harberger (1950) and Laursen and Mezler (1950). Obstfeld (1982) and Svensson and Razin (1983) offer formal treatments of this issue in models with intertemporal utility maximization.

2 A Neoclassical Model of Trade Imbalances and Tariffs

To study the causal relationship between tariffs and trade imbalances, we start from a rich static neoclassical trade model and extend it to two periods. A representative agent with general preferences makes consumption choices. Production is handled by firms using a general technology, with any number of factors. Imports may be used as final goods or as inputs into production. The government levies a uniform tariff τ on all imports in both periods, and rebates the revenue back to consumers.

For simplicity, we abstract from terms of trade effects and consider a small open economy that takes international prices as given. We also abstract from capital investment decisions. An extension to endogenize investment may be of interest. Extensions to allow for agent heterogeneity, more than two periods and a large country that affect their terms of trade is of some interest, but in our view unlikely to provide substantial additional insights for the issue at hand.

2.1 Preferences, Technology, and Trade

Preferences. The representative agent has utility

$$U(C_1, C_2)$$

where U is increasing and concave in aggregate consumption (C_1, C_2). Some of our analysis applies without further restriction on U, but other results rely on U being homothetic, so that the marginal rate of substitution $U_1(C_1, C_2)/U_2(C_1, C_2)$ is a function of C_1/C_2. This is a common benchmark assumption for intertemporal decision problems. The CES specification $U = \frac{1}{1-\sigma}C_1^{1-\sigma} + \beta\frac{1}{1-\sigma}C_2^{1-\sigma}$ with $\sigma, \beta > 0$ ($U = \log C_1 + \beta \log C_2$ with $\sigma = 1$).

Aggregate consumption C_t is given by an aggregator

$$C_t = G_t(c_t)$$

where c_t is a finite or infinite dimensional vector representing all consumption goods, and $G_t(\cdot, t)$ is assumed to be increasing and concave. In some cases it is useful to specialize further and assume G_t is homogeneous of degree one, to capture the homotheticity of preferences.

Technology. Technology is described by an aggregate production set Y_t for $t = 1, 2$ determining feasibility by

$$(c_t, m_t, x_t) \in Y_t$$

where $m_t \geq 0$ represent imports and $x_t \geq 0$ represents exports. This formulation captures general production technologies, using any number of factors of production owned or hired by firms. Firms import m_t as inputs, produce c_t goods for domestic consumption and x_t goods for foreign consumption.[4]

The setup allows for general trade costs. Non-tradable goods are those for which technology dictates that $x_{it} = m_{it} = 0$. Iceberg trade costs are a special case where Y_t is given by requiring $y_i = c_i + \frac{1}{1+\delta_i} x_i - (1+\delta_i) m_i$ and $y \in \Omega$ where Ω represents a domestic net-production set.[5]

We implicitly assume all imports (and exports) are performed by firms, not directly by consumers. This is realistic and without loss of generality, any trade in final goods is handled by an importer firm.

Intertemporal Trade Balance. We take the world prices p^*_{mt} and p^*_{xt} for $t = 1, 2$ as well as a world interest rate R^* as given.[6] The intertemporal trade balance condition is then

$$D_1 + \frac{1}{R^*} D_2 = NFA \tag{1}$$

where NFA represents an inherited net foreign asset position, expressed in foreign goods, which we take as given. The trade deficit is

$$D_t = p^*_{mt} \cdot m_t - p^*_{xt} \cdot x_t.$$

Equilibrium with Tariffs. Our goal is to characterize how an equilibrium, and its trade balance, is affected by a change in the tariff rate τ. The next section develops an aggregation approach that allows us to characterize the equilibrium using only aggregate imports and exports. For completeness, the standard full conditions for an equilibrium are included in the appendix. Here we provide an informal discussion.

An equilibrium requires introducing one more object: a vector of domestic consumer prices. These prices are not necessarily equal to foreign prices due to trade costs, costs of production and retail, and perhaps most importantly, the possibility that some goods

[4]The three vectors c_t, m_t, and x_t may, in principle, have different dimensions. For instance, non-traded goods may be included in c_t, but not in m_t and x_t. Conversely, intermediate goods that are traded internationally may appear in m_t and x_t, but not in c_t. As is standard in general equilibrium theory, factors can be incorporated into the vector c_t as negative entries.

[5]The general technology constraint Y_t can accommodate still more general forms of trade costs. For example, trade may require hiring specific inputs such as transportation services, associated with specific elements in the vector of goods.

[6]It is standard to impose $p^*_{mt} = p^*_{xt} = p^*_t$. We allow $p^*_{mt} \neq p^*_{xt}$ more generally to capture the possibility of foreign tariffs or trade costs that are paid by foreigners. This implies that for a given good, the price received by a domestic exporter may differ from the price that a domestic importer would have to pay.

are simply not traded in equilibrium. In equilibrium, households choose consumption c subject to a standard budget constraint, taking as given consumer prices, firm profits and government transfers. Firms choose (c, m, x) to maximize profits, taking the tariff rate τ, consumer, import and exports prices as given. Government transfers T are equal to the revenue from tariffs. Finally, market clearing in international markets requires the intertemporal budget balance condition (1).

2.2 Static Equilibrium Conditions: Meade meets Hicks

In equilibrium, conditional on the vector of imports and exports, domestic consumption and production choices in a competitive market are are efficient. The tariff makes imports artificially more expensive than exports, so imports and exports will not be chosen efficient overall. However, since the tariff is uniform across imports it does not affect relative prices within imports nor relative prices within exports—equilibrium choices are efficient within each of these categories. Thus, the equilibrium is efficient conditional on aggregate imports and aggregate exports, even though it is inefficient in its choice of aggregate imports and aggregate exports.

Preferences Over Imports and Exports. The previous reasoning allows us to subsume both intra-period preferences G_t and technology Y_t by defining

$$\mathcal{C}_t(M_t, X_t) \equiv \max_{(c_t, m_t, x_t) \in Y_t} G_t(c_t),$$

subject to

$$p^*_{mt} \cdot m_t = M_t,$$

$$p^*_{xt} \cdot x_t = X_t.$$

Here $M_t \geq 0$ and $X_t \geq 0$ are scalars representing the international value of aggregate imports and aggregate exports, respectively. The trade deficit is simply

$$D_t = M_t - X_t.$$

All considerations that shape international trade in a given period t, either coming from preferences (G_t) or technology (Y_t), are encoded in the $\mathcal{C}_t$ preferences relation over (M, X), which is all we will need to know in order to study the impact of tariffs on trade deficits.[7]

[7]Since the existence of the $\mathcal{C}_t$ preferences derives from the efficiency of the competitive equilibrium, we conjecture that our analysis extends, without further qualifications, to monopolistically competitive environments in which the decentralized equilibrium is also efficient, as in Krugman (1980) or Melitz (2003).

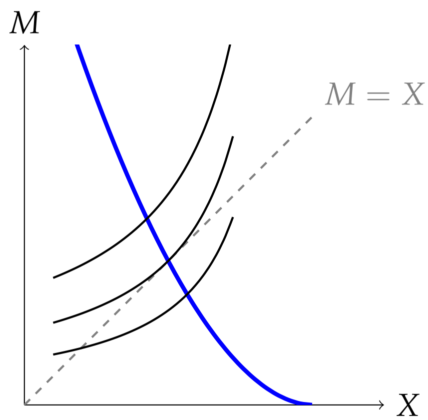


Figure 1: Indifference curves and an Engel curve (with $\tau = 0$) in (X, M).

Figure 1 provides a graphical illustration of the aggregate preferences, with three indifference curves and an Engel curve. The latter is defined as the loci of points (M, X) where $\mathcal{C}_{tM}/\mathcal{C}_{tX}$ is constant. Its shape will play a key role below.

Imports and Exports in Equilibrium. In the rest of our analysis, we will make extensive use of the expenditure function associated with $\mathcal{C}_t$,

$$e_t(C, \tau) \equiv \min_{M, X \geq 0} \{(1+\tau)M - X\}$$
$$s.t : \mathcal{C}_t(M, X) \geq C.$$

The static equilibrium conditions require import and exports to solve this problem. We let $M_t(C, \tau)$ and $X_t(C, \tau)$ denote its solution. We can then define the deficit function

$$\mathcal{D}_t(C, \tau) \equiv M_t(C, \tau) - X_t(C, \tau).$$

Note that if tariffs are positive, the deficit function differs from the expenditure function, with $\mathcal{D}_t(C, \tau) = e_t(C, \tau) - \tau M_t(C, \tau)$, a reflection of the distortionary effect of tariff.

Historical Note. The notion that preferences and domestic production can be combined, exploiting domestic efficiency, follows and extends a perspective introduced by Meade (1952) and further formalized by Dixit and Norman (1980). However, because of the questions they were studying, they worked with the vectors of net imports $x - m$. In contrast, we do not net out and keep imports and exports separate. We do so because the import tariff and other general trade costs may create different prices for the same good depending on whether it is imported or exported. In addition, we aggregate the import vector m to the scalar M and the exports vector x to the scalar X. This reflects our interest in the impact of a uniform tariff τ that affects the price of (all) imports relative to (all) exports, which allows us to apply what Deaton and Muellbauer (1980) refer to as the composite commodity theorem due to Hicks (1936). Thus, our analysis extends and combines elements of Meade and Hicks.

2.3 Dynamic Equilibrium Conditions

Using the expenditure function associated with the static equilibrium conditions, we can express the household problem as

$$\max_{C_1,C_2} U(C_1,C_2)$$

$$e_1(C_1,\tau)+\frac{1}{R^*}e_2(C_2,\tau)=NFA+T,$$

where the lump-sum transfer $T=\tau M_1(C_1,\tau)+\frac{1}{R^*}\tau M_2(C_2,\tau))$ is taken as given. The dynamic equilibrium conditions are then

$$MRS(C_1,C_2)=R(C_1,C_2,\tau),$$
$$\mathcal{D}_1(C_1,\tau)+\frac{1}{R^*}\mathcal{D}_2(C_2,\tau)=NFA.$$

where $MRS(C_1,C_2)\equiv U_1(C_1,C_2)/U_2(C_1,C_2)$ denotes the marginal rate of substitution between aggregate consumption in the two periods and

$$R(C_1,C_2,\tau)\equiv R^*\frac{e_{1C}(C_1,\tau)}{e_{2C}(C_2,\tau)}$$

denotes their relative marginal cost, i.e. the domestic real interest rate, with the convention $e_{tC}\equiv\partial e_t/\partial C$.

Given the focus of our analysis, it is convenient to change variables and rearrange the previous system directly as a function of the trade deficits in the two periods. Let $\mathcal{D}_t^{-1}(D,\tau)$ denote the inverse of the deficit function, i.e. the level of consumption C solving $\mathcal{D}_t(C,\tau)=D$, and $\mathcal{D}^{-1}(D_1,D_2,\tau)=(\mathcal{D}_1^{-1}(D_1,\tau),\mathcal{D}_2^{-1}(D_2,\tau))$ denote the associated vector. We can then express the equilibrium trade deficits (D_1,D_2) as the solution to a system of two equations.

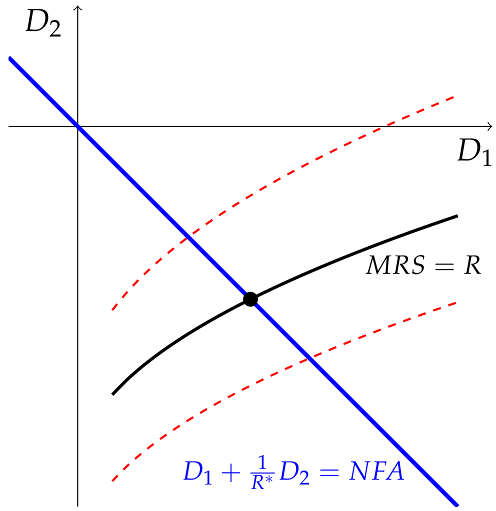


Figure 2: Equilibrium conditions (2)–(3).

Proposition 1. *Aggregate deficits D_1 and D_2 solve*

$$MRS(\mathcal{D}^{-1}(D_1, D_2, \tau)) = R(\mathcal{D}^{-1}(D_1, D_2, \tau), \tau), \tag{2}$$

$$D_1 + \frac{1}{R^*}D_2 = NFA. \tag{3}$$

Proposition 1 encapsulates our aggregate approach, formally showing how the aggregate preferences $\mathcal{C}_t$—and the associated functions e_t, $\mathcal{D}_t$, and R— shape the causal relationship between tariffs and trade imbalances in a general neoclassical model. We illustrate this relationship in Figure 2.

Whether or not tariffs reduce trade deficit boils down to whether the $MRS = R$ locus shifts up or not in this figure. Such a shift may happen for two reasons. First, a change in τ may affect the domestic real interest rate, as captured by the partial derivative $\partial R/\partial\tau$. Second, a tariff, because it is distortionary, may also raise the level of the deficit, i.e. the transfer from the rest of the world required, to achieve a given level of consumption, as captured by the partial derivatives $\partial\mathcal{D}_t^{-1}/\partial\tau$. We now use equations (2) and (3) to explore the conditions under which a rise in tariff may reduce trade deficits.

3 So, Do Tariffs Reduce Trade Deficits?

Throughout the rest of our analysis, we let $(D_1(\tau), D_2(\tau))$ denote the solution to (2)-(3). The goal is to characterize the monotonicity of $D_t(\tau)$ with respect to τ.

3.1 Warming Up

We start with two extreme results that illustrate the range of effects that a permanent increase in tariffs may have on trade imbalances, from fully neutral to entirely closing the trade deficit.

Exact Local Neutrality. For our first result, we provide sufficient conditions under which marginal tariff increases are fully neutral. We need the following assumptions.

Assumption 1. *Homothetic preferences: $U(C_1, C_2)$ and $G_1(c) = G_2(c) = G(c)$ homogenous of degree one.*

Assumption 2. *Fixed endowments: y_1 and y_2.*

Assumption 3. *Iceberg trade costs: $c_{it} = y_{it} + \frac{1}{1+\delta_i} m_{it} - (1 + \delta_i) x_{it}$.*

Assumption 4. *Stationary prices: $p^*_{m1} = p^*_{m2}$ and $p^*_{x1} = p^*_{x2}$.*

These assumptions nest as a special case an Armington model in which G is a CES utility over two goods, the Home good and the Foreign good, and where Home is only endowed with its own good, as in Obstfeld and Rogoff (2000).

Proposition 2 (Exact Local Neutrality)**.** *Suppose Assumptions 1-4 hold. For a given tariff τ, suppose further that the equilibrium has each good i either strictly imported ($m_{it} > 0$) or strictly exported ($x_{it} > 0$) and that the sets of goods imported and exported are the same across periods $t = 1, 2$. Then, locally, tariffs do not affect trade deficits: $D'_t(\tau) = 0$.*

Proposition 2 resonates well with the broad intuition that unlike temporary tariffs, permanent tariffs may have no effect on a country's incentives to borrow and save.[8] It should already be clear, though, that exact neutrality requires quite a bit more than just the same tariff in the two periods. We will explain in details why in the next section. But before doing so, we turn to the other extreme scenario in which tariffs choke off all trade.

Autarky. Our next result provides conditions under which a large enough tariff leads to zero aggregate imports, zero aggregate exports, or both. In the special case where both imports and exports are zero, the economy is under autarky and tariffs fully close the deficit. This occurs when $NFA = 0$. Otherwise the economy runs persistent deficits, if $NFA > 0$, or persistent surpluses, if $NFA < 0$, as the next proposition demonstrates.

[8] Of course, even a permanent tariff may affect a country's incentives to borrow and save if economic conditions differ in periods $t = 1$ and $t = 2$. In Proposition 2 we, therefore, require preferences, trade costs, and prices to be the same in the two periods (though, interestingly, the stationarity of endowments can be dispensed with). Razin and Svensson (1983) show how neutrality breaks down when the same goods are exported and imported in both periods, but the environment is not stationary.

Proposition 3 (Intratemporal and Intertemporal Autarky)**.** *Suppose $\mathcal{C}_t$ has bounded derivatives and all aggregate commodities, C_t, M_t,and $-X_t$, are normal. Then there exists a $\hat{\tau}$ such that for all $\tau \geq \hat{\tau}$: (i) if $NFA = 0$ then $M_t = X_t = 0$ and $D_t = 0$; (ii) if $NFA > 0$ then $X_t = 0$ and $D_t > 0$; and (iii) if $NFA < 0$ then $M_t = 0$ and $D_t < 0$.*

We view the two technical conditions imposed in Proposition 3 as very mild. We assume the derivatives of $\mathcal{C}$ are finite, even at $M = 0$ or $X = 0$ for simplicity, to avoid the need for a limit $\tau \to \infty$ argument. Economically, this represents the realistic assumption of finite choke prices for supply and demand.[9] We require the normality of all aggregate commodities to establish the uniqueness of

Note that Proposition 2 and 3 are not in contradiction with each other: for high enough tariffs the equilibrium is no longer interior, so Proposition 2 cannot be applied. Indeed, when Proposition 3 is applied under Assumptions 1–4 hold, it proves that there exists a $\tau < \bar{\tau}$ for which some goods have $m_i = 0$ or $x_i = 0$ and such that the trade imbalance is falling $|D_1'(\tau)| < 0$ (if $|D_1(\tau)| \neq 0$).

3.2 A Sufficient Statistic: Slope of the Engel Curve!

To get a deeper understanding of the relationship between tariffs and deficits, we now return to the general model of Section 2 and to the system of equations (2)–(3) that determine the effect of tariff on deficits. As previously discussed, tariffs have two types of effects: an interest rate channel, via $\partial R/\partial\tau$, and a distortion channel, via $\partial \mathcal{D}_t^{-1}/\partial\tau$. For our main result, we zoom in on the first of these two channels by considering a small change in tariff around free trade, which implies that the distortionary effect of the tariff is second-order.

Interest Rate Channel. Consider the domestic real interest rate,

$$R(C_1, C_2, \tau) = R^* \frac{e_{1C}(C_1, \tau)}{e_{2C}(C_2, \tau)}.$$

How would a change in the tariff τ affect the real interest rate, holding fixed aggregate consumption in the two periods? To answer this question, we can take logs and differentiate the previous expression,

$$\frac{\partial \ln R(C_1, C_2, \tau)}{\partial \ln \tau} = \frac{e_{1C\tau}(C_1, \tau)}{e_{1C}(C_1, \tau)} - \frac{e_{2C\tau}(C_2, \tau)}{e_{2C}(C_2, \tau)} \tag{4}$$

[9]Note, also, that G_t may still satisfy the Inada condition $G_{tc_i} \to \infty$ as $c_{ti} \to 0$, since $m_i = 0$ does not necessarily imply $c_i = 0$ if good i can be produced domestically.

In any given period, the first and cross-derivatives of the expenditure function satisfy

$$\begin{aligned} e_C(C,\tau) &= (1+\tau)M_C(C,\tau) - X_C(C,\tau) \geq 0 \\ e_\tau(C,\tau) &= M(C,\tau) \geq 0, \\ e_{C\tau}(C,\tau) &= M_C(C,\tau) \geq 0, \end{aligned}$$

where we have dropped the subscript t for notational convenience. It follows that

$$\frac{e_{C\tau}(C,\tau)}{e_C(C,\tau)} = \frac{M_C(C,\tau)}{(1+\tau)M_C(C,\tau) - X_C(C,\tau)} = \frac{1}{(1+\tau) - X_C(C,\tau)/M_C(C,\tau)} \tag{5}$$

is a decreasing function of $M_C(C,\tau)/X_C(C,\tau)$, which is the slope of the Engel curve described in Figure 1,

$$\frac{dM}{dX} = \frac{M_C(C,\tau)}{X_C(C,\tau)}.$$

Put together, equations (4) and (5) imply that an increase in tariff shifts up the real interest rate, $\partial \ln R/\partial \ln \tau \geq 0$, if and only if the slope of the Engel curve, dM/dX is lower at $t=1$ than at $t=2$.

No Distortion Channel. Would a change in tariff τ have any other effect? In general, the answer is yes. Tariffs may also shift the deficit function, $\mathcal{D}_t(C,\tau)$, and in turn, its inverse $\mathcal{D}_t^{-1}(D,\tau)$. To see this, note that

$$\mathcal{D}_{t\tau}(C,\tau) = e_{t\tau}(C,\tau) - M_t(C,\tau) - \tau M_{t\tau}(C,\tau) = -\tau M_{t\tau}(C,\tau) \leq 0.$$

This captures the usual welfare loss due to a fiscal externality measured as a change in the area of the "Harberger triangle" under the demand curve for imports. For $\tau > 0$ we have $\mathcal{D}_{t\tau} < 0$ a negative wealth effect. At $\tau = 0$, however, this is not the case and $\mathcal{D}_{t\tau} = 0$. This then implies that the shift in $MRS = R$ locus in Figure (2) is entirely driven by the shift in the interest rate, $\partial R/\partial \tau$. This leads to our next proposition.

Proposition 4 (Sufficient Statistic)**.** *Starting from free trade ($\tau = 0$), an increase in tariffs reduces the deficit in period* 1*,*

$$D_1'(\tau) < 0,$$

if and only if the Engel curve is steeper in this period,

$$\left|\frac{dM_1}{dX_1}\right| > \left|\frac{dM_2}{dX_2}\right|. \tag{6}$$

Intuitively, inequality (6) represents a higher reliance, at the margin, on imports in the first period. A tariff then raises the cost of consumption more in the first period, creating a substitution effect away from C_1 towards C_2, i.e. an incentive to save, which reduces the trade deficit. Note that this may happen even though preferences, technology, and, in turn, Meade preferences over imports and exports, $\mathcal{C}_t$, are invariant over time. This can happen merely because the country is running a trade deficit in one period and a trade surplus in another, which opens the door for a different incidence of the same tariff τ in the two periods, as long as the Engel curve is not linear.

A naive intuition may be that what matters is the direct incidence of the tariff on an aggregate consumption price index. Our result shows that this intuition, however, is generally lacking. First, the use of a consumption price index presumes homotheticity of preferences, but our result does not invoke such an assumption. Thus, our result allows for inferior, superior or luxury goods. Second, in general, imports may be used in production as inputs, rather than directly consumed as final goods. Third and most surprising, according to our sufficient statistic, non-traded goods simply do not enter the picture. In most economies the share of non-traded goods is large relative to imports, for example. But neither the share nor the marginal consumption, nor the impact on price, of non-traded goods matters directly. For all these reasons, a naive intuition based on the share of imports in total consumption is misleading. According to our result, only the relative expansion of imports to exports matters.

3.3 What Shape for the Engel Curve?

The shape of the Engel curve described in Figure 1 is sufficient to evaluate the impact of a small change in tariff. But this begs the question: In practice, what shape do we expect Engel curves to take?

Measurement. Although the focus of our analysis is theoretical, it is important to note that the slope of the Engel curve emphasized in Proposition 4 is not an esoteric object. To the contrary, it is the answer to the following empirical question: if the domestic trade deficit increases exogenously by ΔD, say, because of a small decrease in the foreign interest rate R^*, what are the associated changes in imports and exports, ΔM and ΔX? The ratio of these two numbers is equal to the slope of the Engel curve, evaluated around the original deficit level D.[10] If the response of imports relative to exports, $\Delta M/\Delta X$, differs

[10]Formally, imports, exports, and deficits are only a function of the tariff τ and aggregate consumption C. So for a fixed level of the tariff τ, the change in the deficit ΔD must reflect the change in aggregate consumption ΔC caused by the change in foreign interest rate.

with the level of the deficit, D, then Proposition 4 implies that a small increase in tariff has a first-order effect on the trade deficit.

Theory. We are not aware of empirical work that has tried to estimate directly the previous responses of imports and exports. We think that such work would be valuable, and hope that our theoretical analysis may serve as motivation. For now, in order to get a better understanding of what Engel curves may actually look like, we return to theory and the simple endowment economy used for our exact neutrality result.

In Proposition 2, we have focused on the case where the extensive margin of trade was inactive. Our next result shows that whether or not the extensive margin is active, in the sense that changes in C affect the split between exports, non-traded goods and imports is critical for the shape of the Engel curve.

Proposition 5. *Suppose that Assumptions 1–4 hold, with G CES. Then the Engel curves of $C_t(M, X)$ are convex for all $(M, X) > 0$ and strictly convex if the extensive margin is active.*

The broad intuition is as follows. By definition, both imports and exports must be non-negative. When aggregate consumption C goes up, we move along an Engel curve increasing imports M decreasing exports X. At the micro level, various imports and exports adjust. Due to the homotheticity of preferences, absent non-negativity constraints, these adjustments are linear, implying a linear Engel curve.[11] However, this is no longer the case when non-negativity constraints bind. In particular, as consumption C_t rises, the non-negativity constraint for exports becomes binding for a greater number of goods, going from $x_{it} > 0$ to $x_{it} = 0$, blunting the adjustment on the aggregate export X margin. Likewise, as consumption C_t rises, a greater number of goods that were not previous imported start being imported, going from $m_{it} = 0$ to $m_{it} > 0$, facilitating the adjustment along the aggregate import margin. Both these extensive margins tend to make Engel curves more convex.

A corollary of Propositions 4 and 5 is that around free trade, a tariff must decrease the trade deficit whenever the extensive margin of trade is active. The fact it was not in Proposition 2 was therefore critical for exact neutrality result. Figure 3 explains why.

In this simple economy, when the extensive margin of trade is inactive, the Engel curve that connects (M_1, X_1) to (M_2, X_2) is linear. So the incidence of a tariff on the cost

[11] The fact that we start from an endowment economy with homothetic preferences implies that Meade preferences absent non-negativity constraints and defined over the entire vector of goods, are quasi-homothetic in $m - x$. Absent non-negativity constraints, Engel curves would therefore be linear. The CES assumption guarantees that quasi-homotheticity is preserved when non-negativity constraints are binding for a subset of non-traded goods.

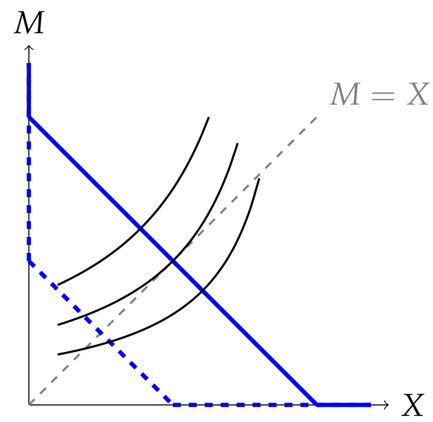


Figure 3: Convex Engel Curves and Non-Neutrality.

of living, or more precisely the marginal cost of aggregate consumption, is the same in the two periods, leading to no effect on the trade deficit. In contrast, when the extensive margin of trade is active, in the extreme because imports or exports go to zero in one period, the Engel curve that connects (M_1, X_1) to (M_2, X_2) is strictly convex, tariffs must increase the cost of living more in periods of deficit, incentivizing people to save and the deficit to shrink.

3.4 An Example with CES Utility

We end this section by showing that a case where we can extend the conclusions of Proposition to any $\tau > 0$. We adopt Assumptions 1–4 but impose additionally that both G_t and $U(C_1, C_2)$ are CES with identical elasticity of substitution. Then utility is additive

$$\int \theta u(c_i) dF + \beta \int \theta u(c_i) dF,$$

with

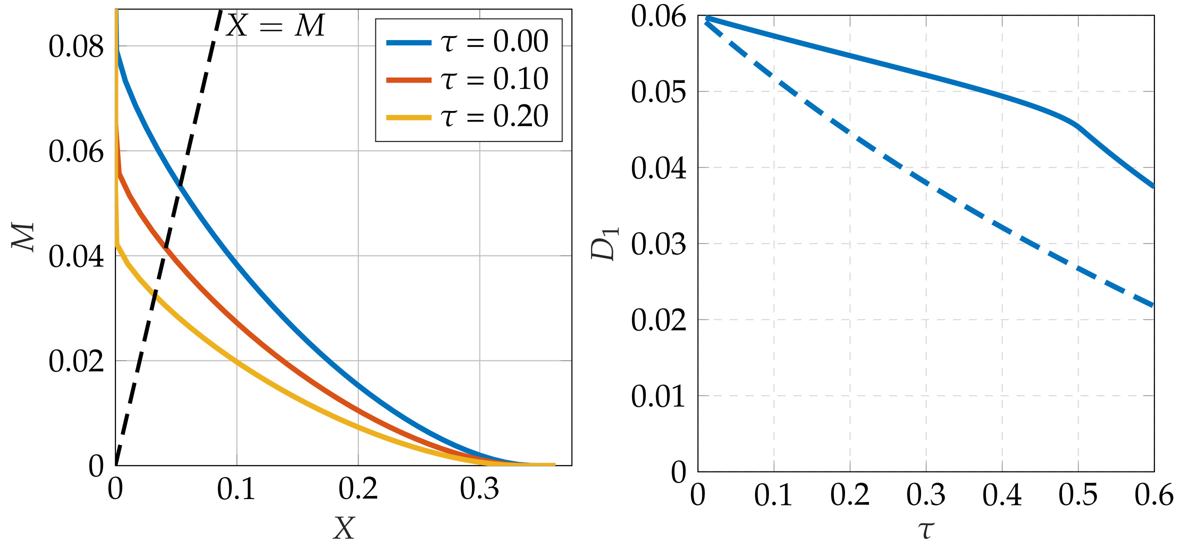
$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

$\sigma > 0$. We then have the following result.

Proposition 6. *Consider the additive economy described above. If preferences and prices are stationary, but (i) $\beta R^* < 1$ and/or (ii) $y_1 \leq y_2$ then*

$$D_1(\tau) > D_2(\tau),$$
$$D_1'(\tau) < 0.$$

for all $\tau < \bar{\tau}$ with $\bar{\tau}$ defined as in Proposition 3.



(a) Engel Curves

(b) Deficit and Tariff

Figure 4: Permanent Change in Tariff in a CES Example

Notes: Figure 4a plots the Engel curves in (M, X) space for different values of τ at the baseline parameters. Figure 4b plots for the deficit D_t, expressed as a share of GDP, as a function of the tariff τ, both at the baseline parameters (solid line) and at the alternative parameters (dashed line).

Intuitively, this result states that if either the country is impatient, the world interest rate is low, or the country has lower output today than tomorrow, then the country will spend more in the present. (If NFA=0 this implies it runs a trade deficit.) Under these conditions, a higher tariff reduces the trade deficit. Note that this result is consistent with Proposition 5, showing that Engel curves are convex, since preferences are assumed CES. The additivity of utility allows us to go beyond a neighborhood of free trade as in Proposition 4 and apply the result to any tariff level $\tau > 0$ that does not induce autarky, $\tau < \bar{\tau}$.

Figure 4 offers a numerical illustration of the relationship between tariffs and deficit in this environment. For our simulations, we set the discount factor to $\beta = 0.87$, the world interest rate to $R^* = 1$, and the intertemporal elasticity of substitution to $\mu = 1$. We normalize all world prices to unity: $p_m^* = p_x^* = 1$. We assume that international to iceberg trade costs $\delta = 0.37$, which implies a share of imports to GDP equal to 15% in the free trade equilibrium ($\tau = 0$). We set the elasticity of substitution between goods to $\sigma = 2.4$. Finally, we assume that Home's comparative advantage follows from variation in its endowment across goods $i \in [0, 1]$, with $y_i = [1 + e^{\frac{2}{\gamma}\left(i - \frac{1}{2}\right)}]^{-1}$. The parameter γ captures the importance of extensive margin considerations. As γ goes to zero, the model converges to a standard Armington model with a domestic good (with positive endowment) and a foreign good (with zero endowment). In our simulations, we set γ to target a trade elasticity of 2, with the trade elasticity defined as the elasticity of the relative demand for imports (holding aggregate consumption fixed).

Figure 4a displays the Engel curves associated with different values of τ. As previously discussed, Engel curves are convex, consistent with Proposition 6. In this CES example, going from free trade to a 60% tariff cuts the deficit from 6% of GDP to about 4%, as can be seen from the solid line in Figure 4b. Results, however, are very sensitive to the importance of extensive margin considerations. Raising γ from 0.05 to 0.5, and lowering the value of σ to 1.25 in order to target the same trade elasticity, leads to a steeper relationship between tariffs and deficits, especially around free trade, as can be seen from the dashed line in Figure 4b. Much remains to be done empirically to estimate the shape of Engel curves and credibly quantify these effects.

4 Concluding Remarks

We have used a flexible trade model to study the effect of tariffs on the trade balance and to isolate the relevant sufficient statistics. In particular, we find that the response is controlled by the aggregate Engel curve for imports and exports.

Except in special cases, economic theory predicts that tariffs do affect trade imbalances and are likely to reduce them. Our results provide a step forward in understanding the underlying mechanism and determining the magnitudes, with extensive margin considerations playing a central role.

We conclude with a word of caution. We started this paper by noting that our focus was a positive rather than a normative one. We are interested here in whether tariffs can affect trade deficits, not whether tariffs should be used to affect them. Through the lens of our model, the answer to the second question is easy: they should not.

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A Equilibrium Definition

This appendix lays out the full set of equilibrium conditions without using our Meade-Hicks aggregation trick.

Household face a domestic budget constraint

$$p_{c1} \cdot c_1 + \frac{1}{R} p_{c2} \cdot c_2 = \Pi_1 + T_1 + \frac{1}{R}(\Pi_2 + T_2)$$

where $\{p_{ct}\}$ are domestic consumer prices, R_H a domestic interest rate, Π represents profits from firms and T represents a transfer from the government. The domestic interest rate could be normalized to unity without loss of generality, letting the level of p_{c2} relative to p_{c1}.

Each period t, firm profits are given by

$$\Pi_t = p_{ct} \cdot c_t + p^*_{xt} \cdot x_t - (1+\tau) p^*_{mt} \cdot m_t.$$

Given τ, an equilibrium is $\{(c_t, m_t, x_t), p_{ct}\}_{t=1,2}$ such that

1. Consumption (c_1, c_2) maximizes utility subject to the budget constraint taking p_{ct},Π_t and T as given.

2. Each period, (c_t, m_t, x_t) maximizes profits Π_t taking p_{ct}, p^*_{mt}, p^*_{xt} and τ as given.

3. Transfers satisfy
$$T_t = \tau p^*_{mt} \cdot m_t.$$

B Proof of Proposition 1

We draw on the full equilibrium definition from Appendix A. We proceed in two steps. First, we establish that for any given C_t, the equilibrium in period t can be found using the aggregates and $\mathcal{C}_t$, as argued in Section 2.2. Second, we establish that we can find the aggregates C_1 and C_2 by focusing on the intertemporal problem in Section (2.3). To save notation, we fix τ, p^*_{mt} and p^*_{xt} and drop these variables from the arguments of all functions.

Static equilibrium conditions. For a given price p_c, the firm problem in period t is

$$\begin{aligned} \tilde{e}_t(p_c) \equiv & \min_{c,m,x} (1+\tau) p^*_{mt} \cdot m - p^*_{xt} \cdot x - p_c \cdot c \\ & s.t. :(c, m, x) \in Y_t. \end{aligned}$$

The dual of the household problem can be written as

$$\hat{e}_t(p_c, C) \equiv \min_c p_c \cdot c$$
$$s.t : G_t(c) = C.$$

For a given C_t, a static equilibrium at t corresponds to (p_{ct}, c_t, m_t, x_t) that solve both the firm and household problems. A static equilibrium thus also solves

$$e_t(C) \equiv \min_{c,m,x} (1+\tau) p_{mt} \cdot m - p_{xt} \cdot x$$
$$s.t : (c, m, x) \in Y_t,$$
$$G_t(c) = C.$$

At the equilibrium price p_{ct}, note that

$$e_t(C_t) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C_t).$$

The minimization problem $e_t(C)$ is identical to minimizing $(1+\tau)M - X$ subject to $\mathcal{C}_t(M, X) = C$. We can then find m, x from the maximization problem defining the aggregate preferences $\mathcal{C}_t$. This establishes that for any given C_t the equilibrium in period t can be found using the aggregates and $\mathcal{C}_t$.

Dynamic equilibrium conditions. Since preferences are separable over time, we can use two-stage budgeting to express the intertemporal household problem as

$$\max_{C_1, C_2} U(C_1, C_2)$$

$$\hat{e}_1(p_{c1}, C_1) + \frac{1}{R^*} \hat{e}_2(p_{c2}, C_2) = NFA + T,$$

where $\hat{e}_t(p_c, C)$ denotes the static expenditure function of the household and the transfer T is taken as given. This gives the first-order condition

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = R^* \frac{\hat{e}_{1C}(C_1, p_{c1})}{\hat{e}_{2C}(C_2, p_{c2})}.$$

To establish that this matches the first-order condition (2) in Section 2.3, i.e. equation it is sufficient to show that

$$e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t).$$

We already argued above that for $C = C_t$,

$$e_t(C_t) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C_t).$$

Now we want to show that for $C \neq C_t$,

$$e_t(C) \geq \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C). \tag{7}$$

This then implies that $e_t(\cdot)$ is an upper envelope for $\hat{e}_t(p_{ct}, \cdot)$ and, in turn, that

$$e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t),$$

as desired.

To establish (7) we first rewrite

$$\begin{aligned} e_t(C) = \min_{c_d, c_s, m, x} & (1+\tau) p^*_{mt} \cdot m - p^*_{xt} \cdot x \\ s.t : & (c_s, m, x) \in Y_t, \\ & G_t(c_d) = C, \\ & c_d = c_s. \end{aligned}$$

Then for any p_{ct}, we note that

$$\begin{aligned} e_t(C) \geq \min_{c_d, c_s, m, x} & (1+\tau) p_{mt} \cdot m - p_{xt} \cdot x + p_{ct} \cdot (c_d - c_s) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C) \\ s.t. : & (c_s, m, x) \in Y_t, \\ & G_t(c_d) = C, \end{aligned}$$

where the inequality derives from the fact the the right-hand side is a relaxed version of the original expenditure problem and the equality derives from the fact that c_d and c_s only appear in the household and firm's problems, respectively. This completes the proof.

C Proof of Proposition 2

Under the stated conditions, preferences $\mathcal{C}_t$ over aggregate imports and exports are quasi-homothetic. This implies that the associated expenditure function $e_t(C, \tau)$ is locally linear in C,

$$e_t(C, \tau) = \alpha_t(\tau) + \beta(\tau) C. \tag{8}$$

So are aggregate imports and exports,

$$M_t(C,\tau) = \alpha_{Mt} + \beta_M(\tau)C, \tag{9}$$
$$X_t(C,\tau) = \alpha_{Xt} + \beta_X(\tau)C, \tag{10}$$

with $\alpha_t(\tau) = (1+\tau)\alpha_{Mt} - \alpha_{Xt}$ and $\beta(\tau) = (1+\tau)\beta_M(\tau) - \beta_X(\tau)$. Using equations (9) and (10), one can compute the deficit function $D_t(C,\tau)$ and then solve for its inverse,

$$\mathcal{D}_t^{-1}(D,\tau) = \frac{D - (\alpha_{Mt} - \alpha_{Xt})}{\beta_M(\tau) - \beta_X(\tau)}.$$

Since $U(C_1, C_2)$ is homothetic, the previous expression implies

$$MRS(\mathcal{D}^{-1}(D_1, D_2, \tau)) = MRS(\frac{D_1 - (\alpha_{M1} - \alpha_{X1})}{D_2 - (\alpha_{M2} - \alpha_{X2})}, 1).$$

From equation (8), we already know that

$$R(\mathcal{D}^{-1}(D_1, D_2, \tau), \tau) = R^* \frac{e_{1C}(\mathcal{D}_1^{-1}(D_1,\tau),\tau)}{e_{2C}(\mathcal{D}_2^{-1}(D_2,\tau),\tau)} = R^*.$$

It follows that the system of equations that characterize deficits as a function of tariffs,

$$MRS(\mathcal{D}^{-1}(D_1, D_2, \tau)) = R(\mathcal{D}^{-1}(D_1, D_2, \tau), \tau),$$
$$D_1 + \frac{1}{R^*} D_2 = NFA,$$

is locally independent of τ. Hence tariffs must be locally neutral: $D_t'(\tau) = 0$.

D Proof of Proposition 3

Assumptions. We focus on a stationary environment such that the preferences over aggregate imports and exports do not vary over time:

$$\mathcal{C}_t(M, X) = \mathcal{C}(M, X) \text{ for } t = 1, 2.$$

We assume that M and $-X$ are normal goods.

Assumption (Normality of M and $-X$)**.** *The utility function $\mathcal{C}(M, X)$ is concave in $(M, -X)$ and has decreasing interior Engel curves:*

$$\frac{\mathcal{C}_M(M, X)}{\mathcal{C}_X(M, X)} = 1 + \tau$$

defines a downward sloping relation between M and X for $M, X > 0$.

We also introduce the required normality assumptions for aggregate consumption over time.

Assumption (Normality of C_1 and C_2)**.** *The utility function U is concave and has increasing Engel curves, so that for any $R > 0$ the condition*

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = R$$

defines a strictly upward sloping relation between C_1 and C_2 for $C_1, C_2 > 0$.

Let $\phi(e, \tau)$ denotes the aggregate consumption level associated with expenditure e given a tariff τ,

$$\phi(e, \tau) = \max_{m,x \geq 0} \mathcal{C}(M, X) \qquad (1+\tau)M - X = e,$$

and let $V(e_1, e_2)$ denote the indirect utility function,

$$V(e_1, e_2) \equiv U(\phi(e_1, \tau), \phi(e_2, \tau)).$$

The lemma below shows that normality for (C_1, C_2) in U implies normality for (e_1, e_2) in V.

Lemma. *If C_1 and C_2 are normal and $\phi(e, \tau)$ is concave in e then the solution to*

$$\max_{e_1, e_2} V(e_1, e_2)$$
$$s.t.{:}e + \frac{1}{R^*}e' = T$$

has both e_1 and e_2 strictly increasing in T.

With these two assumptions we can prove our autarky result.

Proof of autarky result ($NFA = 0$**).** Choose τ large enough so that

$$\frac{\mathcal{C}_M(0,0)}{1+\tau} + \mathcal{C}_X(0,0) \leq 0. \tag{11}$$

Since M and $-X$ are normal, the static optimum as a function of expenditure e,

$$\phi(e, \tau) = \max_{M,X \geq 0} \mathcal{C}(M, X) \qquad (1+\tau)M - X = e$$

is obtained at the corners: $X = 0$ for $e \geq 0$ or $M = 0$ for $e \leq 0$. Thus,

$$\phi(e,\tau) = \begin{cases} \mathcal{C}(\frac{e}{1+\tau},0) & e \geq 0 \\ \mathcal{C}(0,-e) & e \leq 0 \end{cases}$$

Furthermore, one can verify that $\phi(e,\tau)$ is a concave function of e. We denote the right derivative by $\phi_{e+}(e,\tau)$ and the left derivative by $\phi_{e-}(e,\tau)$. For $e \neq 0$ they coincide, but at $e = 0$ we have $\phi_{e+}(e,\tau) = \frac{1}{1+\tau}\mathcal{C}_M(\frac{e}{1+\tau},0) > 0$ and $\phi_{e-}(e,\tau) = -\mathcal{C}_X(0,-e) > 0$. Note that $\phi_{e-}(0,\tau) = -\mathcal{C}_X(0,0) \geq \frac{1}{1+\tau}\mathcal{C}_M(0,0) = \phi_{e-}(0,\tau)$ so there is a concave kink at $e = 0$.

Agents solve the intertemporal problem

$$\max_{e_1,e_2} U(\phi(e_1,\tau),\phi(e_2,\tau))$$
$$\text{s.t.:} e_1 + \frac{1}{R^*}e_2 = T$$

taking T as given. In equilibrium

$$T = \left(M_1 + \frac{1}{R^*}M_2\right)\tau.$$

For any value of T the problem is strictly convex so the optimum is unique.

The first order conditions are

$$U_1(\phi(e_1,\tau),\phi(e_2,\tau))\phi_{e+}(e_1,\tau) - U_2(\phi(e_1,\tau),\phi(e_2,\tau))R^*\phi_{e-}(e_2,\tau) \leq 0,$$

$$-U_1(\phi(e_1,\tau),\phi(e_2,\tau))\phi_{e-}(e_1,\tau) + U_2(\phi(e_1,\tau),\phi(e_2,\tau))R^*\phi_{e+}(e_2,\tau) \leq 0.$$

The first inequality insures that it is not optimal to increase e_1; the second condition ensures it is not optimal to lower e_1. Note that when $e_1 \neq 0$ we have $\phi_{e-}(e_1,\tau) = \phi_{e+}(e_1,\tau)$ so the two conditions are equivalent to

$$U_1(\phi(e_1,\tau),\phi(e_2,\tau))\phi_e(e_1,\tau) - U_2(\phi(e_1,\tau),\phi(e_2,\tau))R^*\phi_e(e_2,\tau) = 0.$$

We first verify that $e_1 = e_2 = 0$ is an equilibrium with $T = 0$.

$$U_1(\phi(0,\tau),\phi(0,\tau))\phi_{e+}(0,\tau) - U_2(\phi(0,\tau),\phi(0,\tau))R^*\phi_{e-}(0,\tau) \leq 0$$

$$-U_1(\phi(0,\tau),\phi(0,\tau))\phi_{e-}(0,\tau) + U_2(\phi(0,\tau),\phi(0,\tau))R^*\phi_{e+}(0,\tau) \leq 0$$

Using that

$$\phi_{e+}(0,\tau) = \frac{1}{1+\tau}\mathcal{C}_M(0,0),$$
$$\phi_{e-}(0,\tau) = -\mathcal{C}_X(0,0),$$

this is equivalent to

$$\frac{1}{1+\tau}\mathcal{C}_M(0,0) + \beta R^* \mathcal{C}_X(0,0) \leq 0, \tag{12}$$
$$\beta R^* \frac{1}{1+\tau}\mathcal{C}_M(0,0) + C_X(0,0) \leq 0, \tag{13}$$

where we have defined

$$\beta \equiv \frac{U_2(\phi(0,\tau),\phi(0,\tau))}{U_1(\phi(0,\tau),\phi(0,\tau))}.$$

There are two cases to consider. If $\beta R^* \leq 1$ let $\hat{\tau}$ denote the lowest value of τ for which condition (12) is satisfied. Then for any $\tau \geq \hat{\tau}$ conditions (11)-(13) are verified. If $\beta R^* > 1$ let $\hat{\tau}$ denote the lowest value of τ for which condition (13) is satisfied. Then for any $\tau \geq \hat{\tau}$ conditions (11)-(13) are verified. We conclude that $e_1 = e_2 = 0$ is a solution to the agent's intertemporal problem for $T = 0$. It follows that $M_1 = X_1 = 0$ and $M_2 = X_2 = 0$ is an equilibrium.

To establish that this is the unique equilibrium, we need to rule out $T > 0$. We proceed by contradiction. Suppose $T > 0$. Since C_1 and C_2 are normal, the previous lemma implies $e_1 \geq 0$ and $e_2 \geq 0$. But then

$$T = e_1 + \frac{1}{R^*}e_2 = (1+\tau)M_1 + \frac{1}{R^*}(1+\tau)M_2 > \tau M_1 + \frac{1}{R^*}\tau M_2 = T.$$

This concludes our proof for $NFA = 0$.

Proof of autarky result ($NFA \neq 0$). The argument is similar. The only difference is that for τ large the economy will not converge to $M_1 = X_1 = 0$ and $M_2 = X_2 = 0$, but instead to whatever level of aggregate imports or exports is consistent with the level of NFA.

Formally, if $NFA > 0$, define (M_1^a, M_2^a) that solves

$$\max_{M_1,M_2} U(\mathcal{C}(M_1,0),\mathcal{C}(M_2,0))$$
$$\text{s.t.:} M_1 + \frac{1}{R^*}M_2 = NFA.$$

One can then check the first-order conditions (11)-(13) around $M_1 = M_1^a$, $X_1 = 0$, $M_2 = M_2^a$, and $X_2 = 0$ to establish that for τ large enough this is the unique equilibrium.

Likewise, if $NFA < 0$, define (X_1^a, X_2^a) that solves

$$\max_{X_1,X_2} U(\mathcal{C}(0,X_1),\mathcal{C}(0,X_2))$$
$$\text{s.t.:} -X_1 - \frac{1}{R^*}X_2 = NFA.$$

The same approach as before shows that for τ large enough, the unique equilibrium is $M_1 = 0$, $X_1 = X_1^a$, $M_2 = 0$, and $X_2 = X_2^a$.

E Proof of Proposition 5

This appendix proves that the Engel curves of $\mathcal{C}(M,X)$ are convex for the CES case where

$$\mathcal{C}(M,X) = u^{-1}\left(\max_{m,x} \int \theta u(y + \frac{m(y,\theta,p_m,p_x)}{p_m} - \frac{x(y,\theta,p_m,p_x)}{p_x})dF\right)$$

$$\int x(y,\theta,p_m,p_x)dF = X$$
$$\int m(y,\theta,p_m,p_x)dF = M$$

and $u(c) = c^{1-\sigma}/(1-\sigma)$. The cdf F is taken over the characteristics (y,θ,p_m,p_x) of individuals goods, where y denotes the endowment of a given good, θ denotes the level of demand, p_m denotes its import price, and p_x denotes its export price.

Assuming $p_x \leq p_m$ we can ignore the possibility that both $m, x > 0$. The first order conditions are then

$$\frac{\theta}{p_m}u'\left(y + \frac{m(y,\theta,p_m,p_x)}{p_m}\right) \leq \mu_m$$
$$\frac{\theta}{p_x}u'\left(y - \frac{x(y,\theta,p_m,p_x)}{p_x}\right) \geq \mu_x$$

with the usual complementary slackness.

With $u(c) = c^{1-\sigma}/(1-\sigma)$, we obtain

$$m(y,\theta,p_m,p_x) = \left\langle 0, \mu_m^{-1/\sigma}\left((1+\tau)\frac{p_m}{\theta}\right)^{-1/\sigma} - y\right\rangle$$
$$x(y,\theta,p_m,p_x) = \left\langle 0, y - \mu^{-1/\sigma}\left(\frac{p_x}{\theta}\right)^{-1/\sigma}\right\rangle$$

where for $i = m, x$ we define

$$\tilde{\mu}_i \equiv \mu_i^{-1/\sigma}$$
$$\tilde{z}_i \equiv p_i^{1-1/\sigma}\theta^{1/\sigma}$$
$$\tilde{y}_i \equiv p_i y$$

Then

$$M(\tilde{\mu}_m) = \int_{\tilde{y}_m/\tilde{z}_m \leq \tilde{\mu}_m} (\tilde{z}_m\tilde{\mu}_m - \tilde{y}_m)dF \tag{14}$$
$$X(\tilde{\mu}_x) = \int_{\tilde{y}_x/\tilde{z}_x \geq \tilde{\mu}_x} (\tilde{y}_x - \tilde{z}_x\mu_x)dF \tag{15}$$

Note that

$$M'(\tilde{\mu}_m) = \int_{\tilde{y}_m/\tilde{z}_m \leq \tilde{\mu}_m} \tilde{z}_m dF = \bar{z}_m \geq 0 \tag{16}$$
$$X'(\tilde{\mu}_x) = -\int_{\tilde{y}_x/\tilde{z}_x \geq \tilde{\mu}_x} \tilde{z}_x dF = \bar{z}_x \leq 0 \tag{17}$$

So that $M''(\tilde{\mu}_m) \geq 0$ and $X''(\tilde{\mu}_x) \leq 0$, with strict inequality if there is an active extensive margin.

To see that the Engel curves of $\mathcal{C}(M, X)$ are convex, take $\mu_m = \mu_x(1+\tau)$ so that $\tilde{\mu}_m = \tilde{\mu}_x(1+\tau)^{-1/\sigma}$ then

$$\frac{dM}{dX} = \frac{M'(\tilde{\mu}_x(1+\tau)^{-1/\sigma})}{X'(\tilde{\mu}_x)}$$

Then it follows from our previous calculations that this ratio is negative and decreasing in $\tilde{\mu}_x$.

F Proof of Proposition 6

This section provides conditions in the additive case for a trade deficit and for a rise in tariffs to reduce the trade deficit.

We draw on the the characterization from E. In particular, using (14)-(15) we can write the intertemporal trade balance in as a function of the two multipliers $(\tilde{\mu}_m, \tilde{\mu}_x)$:

$$M_1(\tilde{\mu}_m) - X_1(\tilde{\mu}_x) + M_2(\tilde{\mu}_m) - X_2(\tilde{\mu}_x) = NFA$$

Since $\tilde{\mu}_m = \tilde{\mu}_x(1+\tau)^{-1/\sigma}$. An increase in the tariff amounts to a decrease in $\tilde{\mu}_m$ and an increase in $\tilde{\mu}_x$ with $\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} < 0$ solving

$$M_1'(\tilde{\mu}_m) - X_1'(\tilde{\mu}_x)\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} + M_2'(\tilde{\mu}_m) - X_2'(\tilde{\mu}_x)\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} = 0$$

Then

$$\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} = -\frac{\bar{z}_{1m} + \bar{z}_{2m}}{\bar{z}_{1x} + \bar{z}_{2x}}$$

It then follows that

$$dTB = \bar{z}_{1m} - \bar{z}_{1x}\frac{\bar{z}_{1m} + \bar{z}_{2m}}{\bar{z}_{1x} + \bar{z}_{2x}} < 0$$

Since $\bar{z}_{ti} > 0$ this holds if and only if

$$\frac{\bar{z}_{2m}}{\bar{z}_{2x}} > \frac{\bar{z}_{1m}}{\bar{z}_{1x}}.$$