Regulation and Design of Financial Markets

Robert Townsend (MIT) and Juan Xandri (Princeton University)

December 10, 2018
Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98)).
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Introduction

Financial Intermediation in Endogenous Incomplete Markets:

- They play the role of creating assets for agents to invest in
- Non-convexities in aggregate production set, such as trading fix costs (Pesendorfer (95), Bisin (98))
- In general, financial intermediaries have a coordination purpose: obtain resources from consumers and invest them in the productive sector.
- If there are economies of scale in aggregate risk sharing, intermediaries help coordinate investment.
- In this paper, we study a (very simple) model of constrained efficient incomplete markets, (from min scale constraints)
- We also study optimal financial regulation = optimal arrangements for intermediation (contracts, institutions and rules of competition)
Introduction

- The paper is divided in two parts

1. An optimal mechanism design problem: Abstracting away from markets, choose optimal
   - span of assets (or technologies)
   - investment portfolio
   - “deposit” insurance for consumers

2. Implementation: Decentralize the optimal allocation with familiar institutions
   - Broker - Dealers (acting as commercial banks) with free entry
   - Firms (with free entry)

- However, this institutions must be regulated in order to implement the optimal mechanism.
- In our simple example, these are quite stark: shut down consumer ↔ firms channel.
Introduction

The paper is divided in two parts

1. **An optimal mechanism design problem:** Abstracting away from markets, choose optimal
   - span of assets (or technologies)
   - investment portfolio
   - “deposit” insurance for consumers

2. **Implementation:** Decentralize the optimal allocation with familiar institutions
   - Broker - Dealers (acting as commercial banks) with free entry
   - Firms (with free entry)

   However, this institutions must be regulated in order to implement the optimal mechanism.
   - In our simple example, these are quite stark: shut down consumer ↔ firms channel.
Introduction

- The paper is divided in two parts

1. **An optimal mechanism design problem:** Abstracting away from markets, choose optimal
   - span of assets (or technologies)
   - investment portfolio
   - “deposit” insurance for consumers

2. **Implementation:** Decentralize the optimal allocation with familiar institutions
   - Broker - Dealers (acting as commercial banks) with free entry
   - Firms (with free entry)

   - However, this institutions must be regulated in order to implement the optimal mechanism.
   - In our simple example, these are quite stark: shut down consumer ↔ firms channel.
1 Introduction

2 Mechanism Design Problem

3 Investment Program

4 Incentive Program

5 Decentralization

6 Failures of Implementation
Setup
Households

- Diamond-Dygvig model
  - Continuum set of households $I$, with measure 1.
  - One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.
  - At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption.
  - Consumers have no endowment in $t = 1, 2$.
  - Only derive utility from $c_1, c_2$.
- Private Information:
  - Ex-ante identical households.
  - At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space).
  - At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$. 
Setup

Households

• Diamond-Dygvig model

• Continuum set of households $I$, with measure 1.

• One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.

• At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption.

• Consumers have no endowment in $t = 1, 2$.

• Only derive utility from $c_1, c_2$

• Private Information:
  • Ex-ante identical households.
  • At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space)
  • At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$
Setup
Households

- Diamond-Dyvgig model
- Continuum set of households $I$, with measure 1.
- One consumption good, which is perishable. Households live for 3 periods $t = 0,1,2$.
- At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption
- Consumers have no endowment in $t = 1,2$.
- Only derive utility from $c_1, c_2$
- Private Information:
  - Ex-ante identical households.
  - At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space)
- At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0,1]$
Setup
Households

- Diamond-Dygvig model
- Continuum set of households \( I \), with measure 1.
- One consumption good, which is perishable. Households live for 3 periods \( t = 0, 1, 2 \).
- At \( t = 0 \), all households are identical, and receive an endowment of \( \omega > 0 \) units of \( t = 0 \) consumption
- Consumers have no endowment in \( t = 1, 2 \).
- Only derive utility from \( c_1, c_2 \)
- Private Information:
  - Ex-ante identical households.
  - At \( t = 1 \) a taste shock \( \theta \in \Theta \) is drawn from a distribution \( F(\theta) \) (compact, Banach space)
- At \( t = 1 \) there is also a publicly observed shock \( s \sim \text{Uniform}[0,1] \)
Diamond-Dygvig model

- Continuum set of households $I$, with measure 1.
- One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.
- At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption.
- Consumers have no endowment in $t = 1, 2$.
- Only derive utility from $c_1, c_2$.

Private Information:

- Ex-ante identical households.
- At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space).
- At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$. 
Setup
Households

- Diamond-Dygvig model
- Continuum set of households $I$, with measure 1.
- One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.
- At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption
- Consumers have no endowment in $t = 1, 2$.
- Only derive utility from $c_1, c_2$

Private Information:
- Ex-ante identical households.
- At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space)
- At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$
Setup

Households

- Diamond-Dygvig model
- Continuum set of households $I$, with measure 1.
- One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.
- At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption
- Consumers have no endowment in $t = 1, 2$.
- Only derive utility from $c_1, c_2$

Private Information:

- Ex-ante identical households.
- At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space)
- At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$
Setup

Households

- Diamond-Dygvig model
- Continuum set of households $I$, with measure 1.
- One consumption good, which is perishable. Households live for 3 periods $t = 0, 1, 2$.
- At $t = 0$, all households are identical, and receive an endowment of $\omega > 0$ units of $t = 0$ consumption.
- Consumers have no endowment in $t = 1, 2$.
- Only derive utility from $c_1, c_2$.
- **Private Information:**
  - Ex-ante identical households.
  - At $t = 1$ a taste shock $\theta \in \Theta$ is drawn from a distribution $F(\theta)$ (compact, Banach space).
  - At $t = 1$ there is also a publicly observed shock $s \sim \text{Uniform}[0, 1]$. 
Two types of securities:

- **Storage (short):** Safe, that pays only 1 unit of next period consumption.
- **Long assets, or productive technology,** that pay off in period 2 only.
- For each $\hat{s} \in [0,1]$ there is an asset $A_{\hat{s}}$ that pays a rate of return $r_{\hat{s}}(s)$

$$r_{\hat{s}}(s) = \begin{cases} 0 & \text{if } s \neq \hat{s} \\ R > 1 & \text{if } s = \hat{s} \end{cases}$$

- If invest $y$ in all long technologies equally, then gets $Ry$ w.p.1
- This would map directly to classical Diamond-Dygvig model
Setup

Minimum Scale

- **Constraint:** Minimum scale requirement for investment $y(s)$ in asset $s$:

  $$r_s(\hat{s} = s) = R \iff y(s) \geq M(s)$$

- This constraint is binding in the aggregate; i.e. there is not enough endowment to invest in all technologies

  $$\int_0^1 M(s) > \omega$$

- Extra assumptions:
  - $M(s)$ is weakly increasing (w.l.o.g)
  - Continuous
  - $M(s) = 0$ for all $s \in [0, \delta]$ for some $\delta > 0$
Setup
Minimum Scale

**Constraint:** Minimum scale requirement for investment $y(s)$ in asset $s$:

$$r_s(\hat{s} = s) = R \iff y(s) \geq M(s)$$

This constraint is binding in the aggregate; i.e. there is not enough endowment to invest in all technologies

$$\int_0^1 M(s) > \omega$$


- Extra assumptions:
  - $M(s)$ is weakly increasing (w.l.o.g)
  - Continuous
  - $M(s) = 0$ for all $s \in [0, \delta]$ for some $\delta > 0$
Timeline

- **t = 0:**
  - agents ex-ante identical, have $\omega > 0$ for investing
  - investments made in storage ($x$) and long technologies ($y(s)$)

- **t = 1:**
  - Aggregate shocks $s$ (publicly observed) and $\theta_i$ (private) are realized
  - Agents report types $\hat{\theta}_i$ and receive consumption $c_1(\hat{\theta}_i, s)$
  - Invest remainder $= x - \int c_1(\theta, s) dF(\theta)$ to storage technology again

- **t = 2**
  - Agents consume $c_2(\hat{\theta}_i, s)$
Planners Problem

**Planner Problem**: Choose optimal “consumption allocation” and “portfolio plan” to maximize consumers ex-ante expected utility

- **Consumption Allocation**: Functions $c_1(\theta, s), c_2(\theta, s)$
- **Portfolio Allocation**: Investment in short technology $x \geq 0$ and in long technologies $(y(s))_{s \in [0,1]}$
Planners Problem

- Feasibility:
  \[
  \int c_1 (\theta, s) \, dF (\theta) \leq x \text{ for all } s \in [0, 1] \quad (1)
  \]
  and
  \[
  \int c_2 (\theta, s) \, dF (\theta) \leq Ry (s) + \left( x - \int c_1 (\theta, s) \, dF (\theta) \right) \text{ for all } s \in [0, 1]
  \]

- Inada Condition: if \( \exists \hat{\Theta} \subseteq \Theta \) with \( \Pr (\hat{\Theta}) > 0 \) such that
  \[
  \frac{\partial U}{\partial c_1} (c_1, 0, \theta) = \infty \text{ for all } \theta \in \hat{\Theta}, c_1 \geq 0
  \]
  then (1) is not binding, and
  \[
  \int [c_1 (\theta, s) + c_2 (\theta, s)] \, dF (\theta) \leq x + Ry (s) \text{ for all } s \in [0, 1]
  \]
Planners Problem

- Feasibility:
  \[ \int c_1(\theta, s) \, dF(\theta) \leq x \text{ for all } s \in [0, 1] \quad (1) \]
  and
  \[ \int c_2(\theta, s) \, dF(\theta) \leq Ry(s) + \left( x - \int c_1(\theta, s) \, dF(\theta) \right) \text{ for all } s \in [0, 1] \]

- **Inada Condition:** if \( \exists \hat{\Theta} \subseteq \Theta \) with \( \Pr(\hat{\Theta}) > 0 \) such that

  \[ \frac{\partial U}{\partial c_1}(c_1, 0, \theta) = \infty \text{ for all } \theta \in \hat{\Theta}, c_1 \geq 0 \]

  then (1) is not binding, and

  \[ \int [c_1(\theta, s) + c_2(\theta, s)] \, dF(\theta) \leq x + Ry(s) \text{ for all } s \in [0, 1] \]
Planners Problem

\[ W^* = \max_{c_1(\theta,s), c_2(\theta,s), x, y(s)} \int_0^1 ds \int U(c_1(\theta,s), c_2(\theta,s)) dF(\theta) \]

1. Inter-temporal RC: for all \( s \in [0,1] \):
\[ \int [c_1(\theta,s) + c_2(\theta,s)] dF(\theta) \leq x + Ry(s) \quad (2) \]

2. IC constraints: for all \( s \in [0,1] \) and all \( \theta, \hat{\theta} \in \Theta \):
\[ U(c_1(\theta,s), c_2(\theta,s), \theta) \geq U(c_1(\hat{\theta},s), c_2(\hat{\theta},s), \theta) \quad (3) \]

3. Minimum scale constraints:
\[ x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \quad (4) \]

4. Portfolio Budget:
\[ x + \int_0^1 y(s) ds \leq \omega \quad (5) \]
Planners Problem

\[ W^* = \max_{c_1(\theta,s), c_2(\theta,s), x, y(s)} \int_0^1 ds \int U(c_1(\theta,s), c_2(\theta,s)) dF(\theta) \]

1. Inter-temporal RC: for all \( s \in [0,1] \):

\[ \int [c_1(\theta,s) + c_2(\theta,s)] dF(\theta) \leq x + Ry(s) \quad (2) \]

2. IC constraints: for all \( s \in [0,1] \) and all \( \theta, \hat{\theta} \in \Theta \):

\[ U(c_1(\theta,s), c_2(\theta,s), \theta) \geq U(c_1(\hat{\theta},s), c_2(\hat{\theta},s), \theta) \quad (3) \]

3. Minimum scale constraints:

\[ x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \quad (4) \]

4. Portfolio Budget:

\[ x + \int_0^1 y(s) \, ds \leq \omega \quad (5) \]
Planners Problem

\[ W^* = \max_{c_1(\theta,s), c_2(\theta,s), x, y(s)} \int_0^1 ds \int U(c_1(\theta,s), c_2(\theta,s)) \, dF(\theta) \]

1. Inter-temporal RC: for all \( s \in [0,1] \):

\[ \int [c_1(\theta,s) + c_2(\theta,s)] \, dF(\theta) \leq x + Ry(s) \quad (2) \]

2. IC constraints: for all \( s \in [0,1] \) and all \( \theta, \hat{\theta} \in \Theta \):

\[ U(c_1(\theta,s), c_2(\theta,s), \theta) \geq U(c_1(\hat{\theta},s), c_2(\hat{\theta},s), \theta) \quad (3) \]

3. Minimum scale constraints:

\[ x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \quad (4) \]

4. Portfolio Budget:

\[ x + \int_0^1 y(s) \, ds \leq \omega \quad (5) \]
Planners Problem

\[ W^* = \max_{c_1(\theta,s), c_2(\theta,s), x, y(s)} \int_0^1 ds \int U(c_1(\theta, s), c_2(\theta, s)) dF(\theta) \]

1. Inter-temporal RC: for all \( s \in [0,1] \):

\[ \int [c_1(\theta, s) + c_2(\theta, s)] dF(\theta) \leq x + Ry(s) \quad (2) \]

2. IC constraints: for all \( s \in [0,1] \) and all \( \theta, \hat{\theta} \in \Theta \):

\[ U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \quad (3) \]

3. Minimum scale constraints:

\[ x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \quad (4) \]

4. Portfolio Budget:

\[ x + \int_0^1 y(s) ds \leq \omega \quad (5) \]
Separate into two Programs

1. **Incentive program:** Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta)} \int U(c_1(\theta), c_2(\theta), \theta) \, dF(\theta)$$

$$\int [c_1(\theta) + c_2(\theta)] \, dF(\theta) \leq Y \quad (6)$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta \quad (7)$$

2. **Investment program:** Given $V(\cdot)$, choose investments:

$$W^* = \max_{x, (y(s))_{s \in [0,1]}} \int_0^1 V(x + Ry(s)) \, ds \quad (8)$$

$$x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \quad (9)$$

$$x + \int_0^1 y(s) \, ds = \omega \quad (10)$$
Separate into two Programs

1 **Incentive program:** Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta)} \int U(c_1(\theta), c_2(\theta), \theta) \, dF(\theta)$$

$$\int [c_1(\theta) + c_2(\theta)] \, dF(\theta) \leq Y \quad (6)$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta \quad (7)$$

2 **Investment program:** Given $V(\cdot)$, choose investments:

$$W^* = \max_{x, (y(s))_{s \in [0,1]}} \int_0^1 V(x + Ry(s)) \, ds \quad (8)$$

$$x \geq 0 \text{ and } y(s) \geq M(s) \text{ whenever } y(s) > 0 \quad (9)$$

$$x + \int_0^1 y(s) \, ds = \omega \quad (10)$$
Separate into two Problems

- **Incentive program:** Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta) \in \mathcal{C}} \int U(c_1(\theta), c_2(\theta), \theta) dF(\theta)$$

subject to

$$\int [c_1(\theta) + c_2(\theta)] dF(\theta) \leq Y$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta$$

- What is in $\mathcal{C}$?

1. Hidden trades (Farhi, Golosov and Tsyvinski (2009))
3. Not in $\mathcal{C}$: incomplete contracts (i.e. not contingent on $Y \sim s$)
Separate into two Problems

- **Incentive program:** Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta) \in C} \int U(c_1(\theta), c_2(\theta), \theta) \, dF(\theta)$$

subject to

$$\int [c_1(\theta) + c_2(\theta)] \, dF(\theta) \leq Y$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta$$

- **What is in $C$?**

  1. Hidden trades (Farhi, Golosov and Tsyvinski (2009))
  3. Not in $C$: incomplete contracts (i.e. not contingent on $Y \sim s$)
Goal: characterize the optimal investment profile.

Steps:

1. Solve Investment Program (8) for general $V(\cdot)$, assuming $V(Y)$ to be strictly concave.
2. Find conditions on $U(c_1, c_2, \theta)$ and $C$ such that $V(Y)$ is in fact, strictly concave.
Goal: characterize the optimal investment profile.

Steps:

1. Solve Investment Program (8) for general $V(\cdot)$, assuming $V(Y)$ to be strictly concave.
2. Find conditions on $U(c_1, c_2, \theta)$ and $C$ such that $V(Y)$ is in fact, strictly concave.
Roadmap

- **Goal:** characterize the optimal investment profile.
- **Steps:**
  1. Solve Investment Program (8) for general $V(\cdot)$, assuming $V(Y)$ to be strictly concave
  2. Find conditions on $U(c_1, c_2, \theta)$ and $C$ such that $V(Y)$ is in fact, strictly concave.
1 Introduction

2 Mechanism Design Problem

3 Investment Program

4 Incentive Program

5 Decentralization

6 Failures of Implementation
Investment Program

Choose

- $x =$ storage from $t = 0$ to $t = 1$
- $y(s) =$ investment in technology $A_s$

to solve

$$W^* = \max_{x,(y(s))_{s \in [0,1]}} \int_0^1 V(x + Ry(s)) \, ds$$

subject to

$$y(s) \geq M(s) \text{ for all } s : y(s) > 0$$ (11)

$$x + \int_0^1 y(s) \, ds = \omega$$ (12)
Problem: Non-convex feasible set

Investment has two margins:
- Extensive: which technologies to fund
- Intensive: how much

We prove a series of conditions on the shape of the optimal investment profile

Result 1: If \( y^*(s) > M(s) \) and \( y^*(s') > M(s') \) \( \implies \) \( y^*(s) = y^*(s') \)

Result 2: \( \exists s^* \in (0,1) \) such that \( y^*(s) = y^* := M(s^*) \) for all \( s \leq s^* \)

Result 3: \( \exists \hat{s} \in (s^*, 1) \) such that \( y^*(s) = M(s) \) for all \( s \in [s^*, \hat{s}] \)

Result 4: \( y^*(s) = 0 \) for all \( s > \hat{s} \)
Investment Program

- **Problem**: Non-convex feasible set
- Investment has two margins:
  - Extensive: which technologies to fund
  - Intensive: how much
- We prove a series of conditions on the shape of the optimal investment profile

**Result 1**: If \( y^*(s) > M(s) \) and \( y^*(s') > M(s') \) \( \implies y^*(s) = y^*(s') \)

**Result 2**: \( \exists s^* \in (0, 1) \) such that \( y^*(s) = y^* := M(s^*) \) for all \( s \leq s^* \)

**Result 3**: \( \exists \hat{s} \in (s^*, 1) \) such that \( y^*(s) = M(s) \) for all \( s \in [s^*, \hat{s}] \)

**Result 4**: \( y^*(s) = 0 \) for all \( s > \hat{s} \)
Problem: Non-convex feasible set

Investment has two margins:
- Extensive: which technologies to fund
- Intensive: how much

We prove a series of conditions on the shape of the optimal investment profile

**Result 1:** If $y^*(s) > M(s)$ and $y^*(s') > M(s') \implies y^*(s) = y^*(s')$

**Result 2:** $\exists s^* \in (0,1)$ such that $y^*(s) = y^* := M(s^*)$ for all $s \leq s^*$

**Result 3:** $\exists \hat{s} \in (s^*, 1)$ such that $y^*(s) = M(s)$ for all $s \in [s^*, \hat{s}]$

**Result 4:** $y^*(s) = 0$ for all $s > \hat{s}$
**Problem**: Non-convex feasible set

Investment has two margins:
- Extensive: which technologies to fund
- Intensive: how much

We prove a series of conditions on the shape of the optimal investment profile

**Result 1**: If \( y^*(s) > M(s) \) and \( y^*(s') > M(s') \) \( \implies \) \( y^*(s) = y^*(s') \)

**Result 2**: \( \exists s^* \in (0, 1) \) such that \( y^*(s) = y^* := M(s^*) \) for all \( s \leq s^* \)

**Result 3**: \( \exists \hat{s} \in (s^*, 1) \) such that \( y^*(s) = M(s) \) for all \( s \in [s^*, \hat{s}] \)

**Result 4**: \( y^*(s) = 0 \) for all \( s > \hat{s} \)
**Problem:** Non-convex feasible set

Investment has two margins:
- Extensive: which technologies to fund
- Intensive: how much

We prove a series of conditions on the shape of the optimal investment profile

**Result 1:** If \( y^*(s) > M(s) \) and \( y^*(s') > M(s') \) \( \implies \) \( y^*(s) = y^*(s') \)

**Result 2:** \( \exists s^* \in (0, 1) \) such that \( y^*(s) = y^* := M(s^*) \) for all \( s \leq s^* \)

**Result 3:** \( \exists \hat{s} \in (s^*, 1) \) such that \( y^*(s) = M(s) \) for all \( s \in [s^*, \hat{s}] \)

**Result 4:** \( y^*(s) = 0 \) for all \( s > \hat{s} \)
Investment Program

\[ y^* = M(s^*) \]

Figure: Optimal \( y^*(s) \) schedule

\( s \in [0, 1] \)
Investment Program

- Find optimal cutoffs $s^*, \hat{s}$
- Storage investment is $x^* = \omega - s^*y^* - \int_{s^*}^{\hat{s}} M(s) \, ds$

Intensive/Extensive margin tradeoff:

- **Intensive margin**: Given a set of available assets→ optimal to invest the same amount in all of themselves
- **Extensive margin**: Lower investment in low MS assets to increase asset span

Pecuniary externality:

- A single agent cannot cover the minimum scale of any given asset.
- In a decentralized economy with no trading constraints, each agent takes asset span given
- Since she does not affect the set of available assets → invests the same in all
- In the aggregate, equivalent to a portfolio

$$y(s) = \begin{cases} \hat{y} & \text{for all } s \leq \hat{s} \\ 0 & \text{for all } s > \hat{s} \end{cases}$$
Investment Program

- Find optimal cutoffs $s^*, \hat{s}$
- Storage investment is $x^* = \omega - s^* y^* - \int_{s^*}^{\hat{s}} M(s) \, ds$

Intensive/Extensive margin tradeoff:

- **Intensive margin:** Given a set of available assets $\Rightarrow$ optimal to invest the same amount in all of themselves
- **Extensive margin:** Lower investment in low MS assets to increase asset span

Pecuniary externality:

- A single agent cannot cover the minimum scale of any given asset.
- In a decentralized economy with no trading constraints, each agent takes asset span given
- Since she does not affect the set of available assets $\Rightarrow$ invests the same in all
- In the aggregate, equivalent to a portfolio

$$y(s) = \begin{cases} 
\hat{y} & \text{for all } s \leq \hat{s} \\
0 & \text{for all } s > \hat{s}
\end{cases}$$
Investment Program

- Find optimal cutoffs $s^*, \hat{s}$
- Storage investment is $x^* = \omega - s^* \gamma - \int_{s^*}^{\hat{s}} M(s) \, ds$

Intensive/Extensive margin tradeoff:

- **Intensive margin:** Given a set of available assets $\Rightarrow$ optimal to invest the same amount in all of themselves
- **Extensive margin:** Lower investment in low MS assets to increase asset span

Pecuniary externality:

- A single agent cannot cover the minimum scale of any given asset.
- In a decentralized economy with no trading constraints, each agent takes asset span given
- Since she does not affect the set of available assets $\Rightarrow$ invests the same in all
- In the aggregate, equivalent to a portfolio

\[
y(s) = \begin{cases} 
\tilde{y} & \text{for all } s \leq \tilde{s} \\
0 & \text{for all } s > \tilde{s}
\end{cases}
\]
Investment Program

- It generates 3 type of states for output:

1. **Normal states** \((s < s^*)\): there, intertemporal output is constant and equal to \(Y = x^* + Ry^*\)

2. **“Crisis” states** \((s > \hat{s})\): no long technology gives return, so \(Y = x^*\)

3. **Boom states** \((s^* < s < \hat{s})\): output is variable, \(Y(s) = x^* + RM(s)\)

- When compared to previous case, output is more volatile
- But strictly welfare improving.
1. Introduction

2. Mechanism Design Problem

3. Investment Program

4. Incentive Program

5. Decentralization

6. Failures of Implementation
Incentive Program

- Given output $Y \geq 0$:

$$V(Y) := \max_{c_1(\theta), c_2(\theta)} \int U(c_1(\theta), c_2(\theta), \theta) \, dF(\theta)$$

subject to

$$\int [c_1(\theta) + c_2(\theta)] \, dF(\theta) \leq Y$$

$$U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \text{ for all } \theta, \hat{\theta} \in \Theta$$

- Gives optimum $(c_1(\theta, Y), c_2(\theta, Y))$. Optimal contract is

$$c^* = (c^*_1(\theta, s), c^*_2(\theta, s)) = (c_1(\theta, x + Ry(s)), c_2(\theta, x + Ry^*(s)))$$
Convexity of IC Contracts

\[ IC_Y = \left\{ c = (c_1, c_2) \in C : \begin{cases} U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \\ \mathbb{E}_\theta [c_1(\theta) + c_2(\theta)] \leq Y \end{cases} \right\} \]

where \( C \) is a potential set of extra constraints

- **Example:** access to secondary lending market (Golosov, Farhi and Tvinsky 2009) with price \( q = \frac{c_2}{c_1} \)
- Agents report type \( \hat{\theta} \) to maximize

\[ V(\hat{\theta}, \theta) = \max_{c_1, c_2} U(c_1, c_2, \theta) \]

s.t : \( c_1 + qc_2 \leq c_1(\hat{\theta}) + qc_2(\hat{\theta}) \)
Convexity of IC Contracts

\[ IC_Y = \left\{ c = (c_1, c_2) \in C : \begin{cases} U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \\ \mathbb{E}_\theta [c_1(\theta) + c_2(\theta)] \leq Y \end{cases} \right\} \]

where \( C \) is a potential set of extra constraints

- **Example:** access to secondary lending market (Golosov, Farhi and Tvinsky 2009) with price \( q = \frac{c_2}{c_1} \)

- Agents report type \( \hat{\theta} \) to maximize

\[ V(\hat{\theta}, \theta) = \max_{c_1, c_2} U(c_1, c_2, \theta) \]

s.t. \( c_1 + qc_2 \leq c_1(\hat{\theta}) + qc_2(\hat{\theta}) \)
Convexity of IC Contracts

\[ IC_Y = \left\{ c = (c_1, c_2) \in \mathcal{C} : \begin{cases} U(c_1(\theta, s), c_2(\theta, s), \theta) \geq U(c_1(\hat{\theta}, s), c_2(\hat{\theta}, s), \theta) \\ \mathbb{E}_\theta [c_1(\theta) + c_2(\theta)] \leq Y \end{cases} \right\} \]

where \( \mathcal{C} \) is a potential set of extra constraints

- **Example:** access to secondary lending market (Golosov, Farhi and Tvinsky 2009) with price \( q = \text{of } c_2/c_1 \)

- Agents report type \( \hat{\theta} \) to maximize

\[
V(\hat{\theta}, \theta) = \max_{c_1, c_2} U(c_1, c_2, \theta)
\]

s.t \( c_1 + qc_2 \leq c_1(\hat{\theta}) + qc_2(\hat{\theta}) \)
Convexity of IC Contracts

• **Result:** If $IC_Y$ is convex and $U(\cdot, \theta)$ is strictly concave $\implies V(Y)$ is strictly concave.

• **Problem:** In general $IC_Y$ set is not convex, particularly if extra constraints added.

**Theorem**

If $U(c_1, c_2, \theta) = g_1(\theta)u_1(c_1) + g_2(\theta)u_2(c_2)$ and $C$ has no extra constraints $\implies IC_Y$ is convex
Convexity of IC Contracts

- **Result:** If $I C_Y$ is convex and $U(\cdot, \theta)$ is strictly concave $\implies V(Y)$ is strictly concave.

- **Problem:** In general $I C_Y$ set is not convex, particularly if extra constraints added.

**Theorem**

If $U(c_1, c_2, \theta) = g_1(\theta)u_1(c_1) + g_2(\theta)u_2(c_2)$ and $C$ has no extra constraints $\implies I C_Y$ is convex
Convexity of IC Contracts

- **Result:** If $IC_Y$ is convex and $U(\cdot, \theta)$ is strictly concave $\implies V(Y)$ is strictly concave.

- **Problem:** In general $IC_Y$ set is not convex, particularly if extra constraints added.

**Theorem**

If $U(c_1, c_2, \theta) = g_1(\theta) u_1(c_1) + g_2(\theta) u_2(c_2)$ and $\mathcal{C}$ has no extra constraints $\implies IC_Y$ is convex
Convexity of IC Contracts with Re-trading Constraints

- With re-trading constraints, we need to add more constraints
- An equivalent formulation is to directly choose the price $q$ and Income $I$
- Let $\nu(q, I, \theta)$ be the indirect utility function for type $\theta$, with demand functions $c_1(q, I, \theta), c_2(q, I, \theta)$
- Incentive problem is

$$V(Y) = \max_{q,I} \int \nu(q, I, \theta) \, dF(\theta)$$

s.t

$$\int [c_1(q, I, \theta) + c_2(q, I, \theta)] \, dF(\theta) \leq Y$$

$$c_1(q, I, \theta) + qc_2(q, I, \theta) = I \text{ for all } \theta$$
Convexity of IC Contracts with Re-trading Constraints

- The preferences are Gorman\(^*\) if

\[ v(q, l, \theta) = g(a(q) + b(q, \theta) l) \]

for some function \( g(\cdot) \) strictly increasing and concave

- Preferences that satisfy this condition:
  - \( U = \theta \ln(c_1) + (1 - \theta) \ln(c_2) \)
  - \( U = (\alpha(\theta) c_1^\theta + \beta(\theta) c_2^\theta)^{\frac{1}{\theta}} \)
  - \( U = a(\theta) c_1 + b(\theta) g(c_2) \)

**Theorem**

*If* \( v(q, l, \theta) = g(a(q) + b(q, \theta) l) \) *then* \( V(Y) \) *is strictly concave as well.*

- We will assume that \( V(Y) \) is strictly concave for the rest of this presentation.
Convexity of IC Contracts with Re-trading Constraints

- The preferences are Gorman* if

\[ v(q, I, \theta) = g(a(q) + b(q, \theta) I) \]

for some function \( g(\cdot) \) strictly increasing and concave

- Preferences that satisfy this condition:
  - \( U = \theta \ln(c_1) + (1 - \theta) \ln(c_2) \)
  - \( U = (\alpha(\theta) c_1^\theta + \beta(\theta) c_2^\theta)^{\frac{1}{\theta}} \)
  - \( U = a(\theta) c_1 + b(\theta) g(c_2) \)

**Theorem**

If \( v(q, I, \theta) = g(a(q) + b(q, \theta) I) \) then \( V(Y) \) is strictly concave as well.

- We will assume that \( V(Y) \) is strictly concave for the rest of this presentation.
The preferences are Gorman\(^*\) if

\[ v(q, l, \theta) = g(a(q) + b(q, \theta)l) \]

for some function \(g(\cdot)\) strictly increasing and concave

Preferences that satisfy this condition:

- \( U = \theta \ln(c_1) + (1 - \theta) \ln(c_2) \)
- \( U = (\alpha(\theta) c_1^\theta + \beta(\theta) c_2^\theta)^{\frac{1}{\theta}} \)
- \( U = a(\theta) c_1 + b(\theta) g(c_2) \)

**Theorem**

If \( v(q, l, \theta) = g(a(q) + b(q, \theta)l) \) then \( V(Y) \) is strictly concave as well.

We will assume that \( V(Y) \) is strictly concave for the rest of this presentation.
1 Introduction

2 Mechanism Design Problem

3 Investment Program

4 Incentive Program

5 Decentralization

6 Failures of Implementation
We propose a decentralization with three distinct sectors:

1. **Consumers**: Buy (lotteries) over deposit contracts
2. **Firms**: They manage short and long productive technologies (free entry)
3. **Broker-dealers** (or financial intermediaries): They sell contracts, and invest directly in firms

There is free entry in all sectors (anyone can run a firm or be a financial intermediary).

Endogenous markets:
- Study first equilibria for a given set of contracts and financial intermediaries
- Then determine set of contracts traded in equilibrium
Decentralization

- We propose a decentralization with three distinct sectors:

1. **Consumers**: Buy (lotteries) over deposit contracts
2. **Firms**: They manage short and long productive technologies (free entry)
3. **Broker-dealers** (or financial intermediaries): They sell contracts, and invest directly in firms

- There is free entry in all sectors (anyone can run a firm or be a financial intermediary)

- **Endogenous markets**:
  - Study first equilibria for a given set of contracts and financial intermediaries
  - Then determine set of contracts traded in equilibrium
Decentralization

- We propose a decentralization with three distinct sectors:

1. **Consumers:** Buy (lotteries) over deposit contracts
2. **Firms:** They manage short and long productive technologies (free entry)
3. **Broker-dealers** (or financial intermediaries): They sell contracts, and invest directly in firms

- There is free entry in all sectors (anyone can run a firm or be a financial intermediary)

- **Endogenous markets:**
  - Study first equilibria for a given set of contracts and financial intermediaries
  - Then determine set of contracts traded in equilibrium
1) Consumers

- Let $B$ be the (finite) set of Broker-Dealers (BD) active.
- A contract is $c = (c_1(\theta, s), c_2(\theta, s))_{(\theta, s) \in \Theta \times S}$ that is incentive-compatible.
- For $b \in B$, let $C_b$ be the (finite) set of contracts offered by $b$.
- Contracts are ex-post exclusive: each consumer can only use one contract ex-post.

**Competition:** BDs sell lotteries over contracts, at a price $P(b, c)$ per lottery unit.

**Budget constraint for consumers is**

$$
\sum_{b \in B, c \in C_b} P(b, c) \times \underbrace{q^d(b, c)}_{\text{lot. units bought}} \leq \omega
$$

where $q^d \in \Delta := \text{simplex over } \mathbb{R}^{\#\{(b, c): b \in B, c \in C_b\}}$ (we omit BD profits, which are 0 in eqm).
1) Consumers

- Let $B$ be the (finite) set of Broker-Dealers (BD) active.
- A **contract** is $c = (c_1(\theta, s), c_2(\theta, s))_{(\theta, s) \in \Theta \times S}$ that is incentive-compatible
- For $b \in B$, let $C_b$ be the (finite) set of contracts offered by $b$
- Contracts are ex-post exclusive: each consumer can only use one contract ex-post.
- **Competition**: BDs sell lotteries over contracts, at a price $P(b, c)$ per lottery unit
- Budget constraint for consumers is
  \[ \sum_{b \in B, c \in C_b} P(b, c) \times q^d(b, c) \leq \omega \]
  where $q^d \in \Delta := \text{simplex over } \mathbb{R}^{\# \{(b, c) : b \in B, c \in C_b \}}$ (we omit BD profits, which are 0 in eqm).
1) Consumers

- Let $B$ be the (finite) set of Broker-Dealers (BD) active.
- A **contract** is $c = (c_1(\theta, s), c_2(\theta, s)) \in \Theta \times S$ that is incentive-compatible.
- For $b \in B$, let $C_b$ be the (finite) set of contracts offered by $b$.
- Contracts are ex-post exclusive: each consumer can only use one contract ex-post.
- **Competition:** BDs sell lotteries over contracts, at a price $P(b, c)$ per lottery unit.
- Budget constraint for consumers is

$$\sum_{b \in B, c \in C_b} P(b, c) \times \underbrace{q^d(b, c)}_{= \text{lot. units bought}} \leq \omega$$

where $q^d \in \Delta := \text{simplex over } \mathbb{R}^{\# \{(b, c) : b \in B, c \in C_b\}}$ (we omit BD profits, which are 0 in eqm).
1) Consumers

The value of a contract for each consumer ex-ante is

\[ V(b, c) := \mathbb{E}_{\theta, s} \{ U(c_1(\theta, s), c_2(\theta, s), \theta) \} \]

Consumer problem is then

\[
\max_{q^d \in \Delta, b \in B, c \in C_B} \sum_{b \in B, c \in C_B} q^d(b, c) V(b, c) \quad \text{s.t.} \quad \sum_{b \in B, c \in C_B} P(b, c) q^d(b, c) \leq \omega
\]

Implicit trading constraints needed for consumers:

(a): Contract exclusivity (only trade ex-post with one BD)
(b): Cannot trade ex-post with other consumers
(c): Cannot trade directly with firms
2) Productive Sector

- **Technologies:** All productive assets $Y = \{A_s\}_{s \in [0,1]} \cup S$, where $S =$ storage technology.

- Firm $f$ has access to technology $Y_f = A_{\hat{s}}$ for some $\hat{s} \in [0,1]$ or $Y_f = S$

- Firms need to get financing to manage the asset.

- It offers to a potential financiers, a menu of payoffs $\rho(y, s)$ such that

  1. $\rho(y, s) = 0$ for all $s$ if $y < M(\hat{s})$
  2. $0 \leq \rho(y, s) \leq r_{\hat{s}}(s)y$

Trading constraint:

(d): Firms cannot have more than one source of financing
2) Productive Sector

- **Technologies**: All productive assets $Y = \{A_s\}_{s \in [0,1]} \cup S$, where $S =$ storage technology.

- Firm $f$ has access to technology $Y_f = A_{\hat{s}}$ for some $\hat{s} \in [0,1]$ or $Y_f = S$

- Firms need to get financing to manage the asset.

- It offers to a potential financiers, a menu of payoffs $\rho(y, s)$ such that

  1. $\rho(y, s) = 0$ for all $s$ if $y < M(\hat{s})$

  2. $0 \leq \rho(y, s) \leq r_{\hat{s}}(s)y$

Trading constraint:

(d): Firms cannot have more than one source of financing
2) Productive Sector

- **Technologies:** All productive assets $Y = \{A_s\}_{s \in [0,1]} \cup S$, where $S =$ storage technology.
- Firm $f$ has access to technology $Y_f = A_{\hat{s}}$ for some $\hat{s} \in [0,1]$ or $Y_f = S$
- Firms need to get financing to manage the asset.
- It offers to a potential financiers, a menu of payoffs $\rho(y,s)$ such that

1. $\rho(y,s) = 0$ for all $s$ if $y < M(\hat{s})$
2. $0 \leq \rho(y,s) \leq r_{\hat{s}}(s) y$

Trading constraint:

(d): Firms cannot have more than one source of financing
2) Productive Sector

- Because agents are atomistic, this implies that they can only ask one BD
- Firm profits are then $= r(\hat{s}) y - \rho(y, s)$
- **Assumption:** Free entry in productive sector
- This pushes profits to zero, so in equilibrium

$$\rho(y, s) = r(\hat{s}) y \times 1\{y \geq M(\hat{s})\}$$

- **Question:** Why not just consider them as Arrow-Debreu securities, and study classical GE?
3) Broker Dealers

- Each BD has an (exogenous, for now) set of available contracts $C_b$
- BD chooses:
  1. Its supply of each contract lotteries, $q^s(b,c)$
  2. Its investments in firms to fund the contracts expenditures.

- In period 1 it has to pay out
  
  $$e_1(s) = \sum_{c \in C_b} q^s(b,c) \mathbb{E}_\theta [c_1(\theta,s)]$$

  which can only be financed by an investment in a firm running short tech.
- In period 2, it has to pay out
  
  $$e_2(s) = \sum_{c \in C_b} q^s(b,c) \mathbb{E}_\theta [c_2(\theta,s)]$$

  which is financed with (a) storage and (b) long technologies
3) Broker Dealers

- Each BD has an (exogenous, for now) set of available contracts $C_b$
- BD chooses:
  1. Its supply of each contract lotteries, $q^s(b, c)$
  2. Its investments in firms to fund the contracts expenditures.

In period 1 it has to pay out

$$e_1(s) = \sum_{c \in C_b} q^s(b, c) \mathbb{E}_\theta [c_1(\theta, s)]$$

which can only be financed by an investment in a firm running short tech.

In period 2, it has to pay out

$$e_2(s) = \sum_{c \in C_b} q^s(b, c) \mathbb{E}_\theta [c_2(\theta, s)]$$

which is financed with (a) storage and (b) long technologies.
3) Broker Dealers

- Each BD has an (exogenous, for now) set of available contracts $C_b$
- BD chooses:
  1. Its supply of each contract lotteries, $q^s(b,c)$
  2. Its investments in firms to fund the contracts expenditures.

In period 1 it has to pay out

$$e_1(s) = \sum_{c \in C_b} q^s(b,c) \mathbb{E}_\theta [c_1(\theta,s)]$$

which can only be financed by an investment in a firm running short tech.

In period 2, it has to pay out

$$e_2(s) = \sum_{c \in C_b} q^s(b,c) \mathbb{E}_\theta [c_2(\theta,s)]$$

which is financed with (a) storage and (b) long technologies.
3) Broker Dealers

- Each BD has an (exogenous, for now) set of available contracts $C_b$

\[
\pi_b = \max_{q^s, x_b, y_b(s)} \sum_{c \in C_b} P(b, c) q^s(b, c) - \left( x_b + \int_0^1 y_b(s) \, ds \right)
\]

subject to

\[
\begin{align*}
  e_1(s) &\leq x \\
  e_2(s) &\leq x - e_1(s) + \rho(y(s), s)
\end{align*}
\]

for all $s \in [0, 1]$

- Free-entry into BS industry
Households

Invests (lends) wealth endowments to

Financial Sector

Invests in projects managed by

Productive Sector

Pays dividends to

Gives Insurance Contracts to

(1) – Households cannot trade with Productive sector
Households (lends) wealth endowments to Financial Sector, which pays dividends to Productive Sector. Financial Sector invests in projects managed by Productive Sector, and Productive Sector gives insurance contracts to Households. (2) Households cannot trade among themselves.
Quasi-equilibrium

- Given \( B \) and \( C = \{ C_b \}_{b \in B} \), a quasi-equilibrium is a tuple \( z \) consisting of:
  - Consumer demand \( q^d \)
  - BS supply \( q^s \) and investment decisions \( (x_b, (y_b(s))_{s \in [0,1]}) \)
  - Prices \( P(b,s) \) and Payoffs \( \rho(y,s) \), such that:

1. \( q^d \) solves consumer problem given \( P(b,s) \)
2. \( q^s \) and investment decisions \( (x_b, (y_b(s))_{s \in [0,1]}) \) max \( \pi_b \) given prices \( P(b,s) \) and \( \rho(y,s) \)
3. Free-entry: \( \rho(y,s) = r(s)y \times 1 \{ y \geq M(s) \} \)
4. Resource constraints:

\[
\begin{align*}
q^d(b,c) &= q^s(b,c) \\
E_\theta [c_1(\theta,s)] &\leq \sum_{b \in B} x_b \\
E_\theta [c_1(\theta,s) + c_2(\theta,s)] &\leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) \\
\sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s) \, ds &= \omega
\end{align*}
\]

\( \forall b \in B, c \in C_b \) 
\( \forall s \in [0,1] \)
Quasi-equilibrium

- Given $B$ and $C = \{ C_b \}_{b \in B}$, a quasi-equilibrium is a tuple $z$ consisting of
  - Consumer demand $q^d$
  - BS supply $q^s$ and investment decisions $\left( x_b, (y_b(s))_{s \in [0,1]} \right)$
  - Prices $P(b,s)$ and Payoffs $\rho(y,s)$, such that:

1. $q^d$ solves consumer problem given $P(b,s)$
2. $q^s$ and investment decisions $\left( x_b, (y_b(s))_{s \in [0,1]} \right)$ max $\pi_b$ given prices $P(b,s)$ and $\rho(y,s)$
3. Free-entry: $\rho(y,s) = r\hat{s}(s)y \times 1 \{ y \geq M(\hat{s}) \}$
4. Resource constraints:

$$
\begin{aligned}
q^d(b,c) &= q^s(b,c) & \forall b \in B, c \in C_b \\
\mathbb{E}_\theta [c_1(\theta,s)] &\leq \sum_{b \in B} x_b & \forall s \in [0,1] \\
\mathbb{E}_\theta [c_1(\theta,s) + c_2(\theta,s)] &\leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) & \forall s \in [0,1] \\
\sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s) ds &= \omega
\end{aligned}
$$
Quasi-equilibrium

- Given $B$ and $C = \{C_b\}_{b \in B}$, a quasi-equilibrium is a tuple $z$ consisting of
  - Consumer demand $q^d$
  - BS supply $q^s$ and investment decisions $\left(x_b, (y_b(s))_{s \in [0,1]}\right)$
  - Prices $P(b, s)$ and Payoffs $\rho(y, s)$, such that:

1. $q^d$ solves consumer problem given $P(b, s)$
2. $q^s$ and investment decisions $\left(x_b, (y_b(s))_{s \in [0,1]}\right)$ max $\pi_b$ given prices $P(b, s)$ and $\rho(y, s)$
3. Free-entry: $\rho(y, s) = r_s(s)y \times 1\{y \geq M(s)\}$
4. Resource constraints:

$$\begin{cases} 
q^d(b, c) = q^s(b, c) & \forall b \in B, c \in C_b \\
\mathbb{E}_\theta [c_1(\theta, s)] \leq \sum_{b \in B} x_b & \forall s \in [0,1] \\
\mathbb{E}_\theta [c_1(\theta, s) + c_2(\theta, s)] \leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) & \forall s \in [0,1] \\
\sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s) \, ds = \omega
\end{cases}$$
Quasi-equilibrium

Given \( B \) and \( C = \{ C_b \}_{b \in B} \), a quasi-equilibrium is a tuple \( z \) consisting of

- Consumer demand \( q^d \)
- BS supply \( q^s \) and investment decisions \( \left( x_b, (y_b(s))_{s \in [0,1]} \right) \)
- Prices \( P(b,s) \) and Payoffs \( \rho(y,s) \), such that:

1. \( q^d \) solves consumer problem given \( P(b,s) \)
2. \( q^s \) and investment decisions \( \left( x_b, (y_b(s))_{s \in [0,1]} \right) \) max \( \pi_b \) given prices \( P(b,s) \) and \( \rho(y,s) \)
3. Free-entry: \( \rho(y,s) = r\hat{s}(s)y \times 1 \{ y \geq M(\hat{s}) \} \)
4. Resource constraints:

\[
\begin{aligned}
q^d(b,c) &= q^s(b,c) & \forall b \in B, c \in C_b \\
\mathbb{E}_\theta [c_1(\theta,s)] &\leq \sum_{b \in B} x_b & \forall s \in [0,1] \\
\mathbb{E}_\theta [c_1(\theta,s) + c_2(\theta,s)] &\leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) & \forall s \in [0,1] \\
\sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s) \, ds &= \omega
\end{aligned}
\]
Quasi-equilibrium

Given $B$ and $C = \{C_b\}_{b \in B}$, a quasi-equilibrium is a tuple $z$ consisting of

- Consumer demand $q^d$
- BS supply $q^s$ and investment decisions $\left(x_b, (y_b(s))_{s \in [0,1]}\right)$
- Prices $P(b,s)$ and Payoffs $\rho(y,s)$, such that:

1. $q^d$ solves consumer problem given $P(b,s)$
2. $q^s$ and investment decisions $\left(x_b, (y_b(s))_{s \in [0,1]}\right)$ max $\pi_b$ given prices $P(b,s)$ and $\rho(y,s)$
3. Free-entry: $\rho(y,s) = r\hat{s}(s)y \times 1 \{y \geq M(\hat{s})\}$
4. Resource constraints:

$$\begin{align*}
q^d(b,c) &= q^s(b,c) & \forall b \in B, c \in C_b \\
E_{\theta} [c_1(\theta,s)] &\leq \sum_{b \in B} x_b & \forall s \in [0,1] \\
E_{\theta} [c_1(\theta,s) + c_2(\theta,s)] &\leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) & \forall s \in [0,1] \\
\sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s) \, ds &= \omega
\end{align*}$$
Given $B$ and $C = \{C_b\}_{b \in B}$, a quasi-equilibrium is a tuple $z$ consisting of

- Consumer demand $q^d$
- BS supply $q^s$ and investment decisions $(x_b, (y_b(s))_{s \in [0,1]})$
- Prices $P(b,s)$ and Payoffs $\rho(y,s)$, such that:

1. $q^d$ solves consumer problem given $P(b,s)$
2. $q^s$ and investment decisions $(x_b, (y_b(s))_{s \in [0,1]})$ maximize $\pi_b$ given prices $P(b,s)$ and $\rho(y,s)$
3. Free-entry: $\rho(y,s) = r\hat{s}(s)y \times 1\{y \geq M(\hat{s})\}$
4. Resource constraints:

\[
\begin{cases}
q^d(b,c) = q^s(b,c) & \forall b \in B, c \in C_b \\
\mathbb{E}_\theta [c_1(\theta,s)] \leq \sum_{b \in B} x_b & \forall s \in [0,1] \\
\mathbb{E}_\theta [c_1(\theta,s) + c_2(\theta,s)] \leq \sum_{b \in B} x_b + R \sum_{b \in B} y_b(s) & \forall s \in [0,1] \\
\sum_{b \in B} x_b + \sum_{b \in B} \int_0^1 y_b(s) ds = \omega
\end{cases}
\]
Quasi-equilibrium

- We write \( z \in \mathbb{Q}(B, \{C_b\}_{b \in B}) \) as the set of all quasi-equilibria.

Lemma

There always exist a Quasi-Equilibrium, where only one contract \( (\hat{b}, \hat{c}) \) is traded in eqm (i.e. \( q^d(\hat{b}, \hat{c}) = q^s(\hat{b}, \hat{c}) = 1 \)), and

\[
P(\hat{b}, \hat{c}) = x_{\hat{b}} + \int_0^1 y_{\hat{b}}(s) \, ds
\]

Proof.

Constructive, and straightforward

- So, in any quasi-equilibrium, only one best-ex ante contract is traded
- If two BD \( b_1, b_2 \) are selling the same contract \( \hat{c} \), then there are two equilibria
Quasi-equilibrium

- We write $z \in Q\left(B, \{C_b\}_{b \in B}\right)$ as the set of all quasi-equilibria.

Lemma

There always exist a Quasi-Equilibrium, where only one contract $(\hat{b}, \hat{c}) \in \text{argmax}_{b \in B, c \in C_b} V(b, c)$ is traded in eqm (i.e. $q^d(\hat{b}, \hat{c}) = q^s(\hat{b}, \hat{c}) = 1$), and

$$P(\hat{b}, \hat{c}) = x_{\hat{b}} + \int_0^1 y_{\hat{b}}(s) \, ds$$

Proof.

Constructive, and straightforward

- So, in any quasi-equilibrium, only one best-ex ante contract is traded
- If two BD $b_1, b_2$ are selling the same contract $\hat{c}$, then there are two equilibria
Quasi-equilibrium

- We write \( z \in Q(B, \{C_b\}_{b \in B}) \) as the set of all quasi-equilibria.

Lemma

There always exist a Quasi-Equilibrium, where only one contract \( (\hat{b}, \hat{c}) \in \arg\max_{b \in B, c \in C_b} V(b, c) \) is traded in eqm (i.e. \( q^d(\hat{b}, \hat{c}) = q^s(\hat{b}, \hat{c}) = 1 \)), and

\[
P(\hat{b}, \hat{c}) = x_{\hat{b}} + \int_0^1 y_{\hat{b}}(s) \, ds
\]

Proof.

Constructive, and straightforward

- So, in any quasi-equilibrium, only one best-ex ante contract is traded
- If two BD \( b_1, b_2 \) are selling the same contract \( \hat{c} \), then there are two equilibria
Full Competitive Equilibrium (Makowski 1980)

- So far, we took the technology of each BD as given (i.e. the contracts they propose)
- Equivalent to taking the commodity space as exogenous.
- What contracts should be chosen to be introduced by banks?
Full Competitive Equilibrium (Makowski 1980)

Definition (Profitable Deviation)

Take a set of BD $B$ with contracts $C_b$, and let $z$ be a quasi-equilibrium. We say $c' \not\in \bigcup_{b \in B} \{C_b\}$ is a profitable deviation if

1. Is incentive compatible
2. $\exists z' \in Q(B, \{C'_b\})$ where $C'_b = C_b$ for all but $\hat{b}$, where $C'_{\hat{b}} = \{c'\} \cup C_{\hat{b}}$, such that

$$\pi_{\hat{b}}(z') > \pi_{\hat{b}}(z)$$

Definition (FCE)

A Full Competitive Equilibrium (FCE) is a family of contracts $\{C_b\}_{b \in B}$ and $z$ such that (1) $z \in Q(B, \{C_b\}_{b \in B})$ and (2) there are no profitable deviations
Weak FCE

- **Problem:** in all quasi-equilibria we have \( \pi_b(z) = 0 \)

Definition (Weak Profitable Deviation)

Take a set of BD \( B \) with contracts \( C_b \), and let \( z \) be a quasi-equilibrium. We say \( c' \not\in \bigcup_{b \in B} \{ C_b \} \) is a weak profitable deviation if

1. Is incentive compatible
2. \( \exists z' \in Q(B, \{ C'_b \}) \) where \( C'_b = C_b \) for all but \( \hat{b} \), where \( C'_\hat{b} = \{ c' \} \cup C_{\hat{b}} \), such that
   - \( \pi_{\hat{b}}(z') \geq \pi_{\hat{b}}(z) \)
   - \( \sum_{c \in C_{\hat{b}}} q_s(\hat{b}, c') > \sum_{c \in C'_b} q_s(\hat{b}, c') \)

Definition (Weak FCE)

A Weak FCE is a family of contracts \( \{ C_b \}_{b \in B} \) and \( z \) such that (1) \( z \in Q(B, \{ C_b \} \) and (2) there are no weak profitable deviations
Weak FCE Implementation

Lemma

Take \( \{ C_b \}_{b \in B} \) and \( z \in Q(B, \{ C_b \}_{b \in B}) \). If \( c^* \notin \bigcup_{b \in B} C_b \implies \) \( (\{ C_b \}_{b \in B}, z) \) is not a weak FCE

- Proof is trivial: Based on Lemma 3 we know that in any \( z \), the best contract for consumers corners all the market
- \( c^* \) is the best feasible contract, so any BD selling it may get all market in an equilibrium.

Theorem (Full Implementation)

In any weak FCE, the first best contract \( c^* \) (with corresponding optimal investment allocation) is implemented.
Weak FCE Implementation

Lemma

Take $\{C_b\}_{b \in B}$ and $z \in Q(B, \{C_b\}_{b \in B})$. If $c^* \notin \bigcup_{b \in B} C_b \implies (\{C_b\}_{b \in B}, z)$ is not a weak FCE.

- Proof is trivial: Based on Lemma 3 we know that in any $z$, the best contract for consumers corners all the market.
- $c^*$ is the best feasible contract, so any BD selling it may get all market in an equilibrium.

Theorem (Full Implementation)

In any weak FCE, the first best contract $c^*$ (with corresponding optimal investment allocation) is implemented.
Weak FCE Implementation

Lemma

Take $\{C_b\}_{b \in B}$ and $z \in Q(B, \{C_b\}_{b \in B})$. If $c^* \notin \bigcup_{b \in B} C_b \implies (\{C_b\}_{b \in B}, z)$ is not a weak FCE

- Proof is trivial: Based on Lemma 3 we know that in any $z$, the best contract for consumers corners all the market
- $c^*$ is the best feasible contract, so any BD selling it may get all market in an equilibrium.

Theorem (Full Implementation)

In any weak FCE, the first best contract $c^*$ (with corresponding optimal investment allocation) is implemented.
1 Introduction

2 Mechanism Design Problem

3 Investment Program

4 Incentive Program

5 Decentralization

6 Failures of Implementation
Failures of Implementation

**Failure 1:** Hidden trades, Hidden savings

- This reduces the set of incentive compatible mechanisms, and therefore changes $c^*$
- However, if $V(Y)$ is still st. concave $\implies$ Shape of optimal portfolio still the same
- Market design still implements the second best allocation
Failures of Implementation

**Failure 2:** Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing
  - In this case, consumers would benefit from directly investing in already “open” firms
  - ... but they would invest the same amount in all of them.
  - No agent would invest in BDs: they would wait for BD to cover MS
  - In ex-ante problem, this translates into a reservation utility for each agent
  - In constrained optimal mechanism, a BD would give the same utility as direct financing:
    \[
    y^*(s) = \begin{cases} 
    y^* = M(s^*) & \text{for all } s \leq s^* \\
    0 & \text{for all } s > s^* 
    \end{cases}
    \]
  - Same reason behind Acemoglu and Zilibotti (1997)
Failures of Implementation

**Failure 2:** Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing
- In this case, consumers would benefit from directly investing in already “open” firms
  - ... but they would invest the same amount in all of them.
  - No agent would invest in BDs: they would wait for BD to cover MS
  - In ex-ante problem, this translates into a reservation utility for each agent
  - In constrained optimal mechanism, a BD would give the same utility as direct financing:

\[
y^*(s) = \begin{cases} 
  y^* = M(s^*) & \text{for all } s \leq s^* \\
  0 & \text{for all } s > s^*
\end{cases}
\]

- Same reason behind Acemoglu and Zilibotti (1997)
Failures of Implementation

**Failure 2:** Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing.
- In this case, consumers would benefit from directly investing in already “open” firms.
- ... but they would invest the same amount in all of them.
- No agent would invest in BDs: they would wait for BD to cover MS.
- In ex-ante problem, this translates into a reservation utility for each agent.
- In constrained optimal mechanism, a BD would give the same utility as direct financing:

\[
y^*(s) = \begin{cases} 
y^* = M(s^*) & \text{for all } s \leq s^* \\
0 & \text{for all } s > s^*
\end{cases}
\]

- Same reason behind Acemoglu and Zilibotti (1997)
Failures of Implementation

**Failure 2: Consumers may directly invest in firms**

- Firms, once they cover their minimum scale, can now ask for more external financing.
- In this case, consumers would benefit from directly investing in already “open” firms.
- ... but they would invest the same amount in all of them.
- No agent would invest in BDs: they would wait for BD to cover MS.
- In ex-ante problem, this translates into a reservation utility for each agent.
- In constrained optimal mechanism, a BD would give the same utility as direct financing:

\[
 y^*(s) = \begin{cases} 
 y^* = M(s^*) & \text{for all } s \leq s^* \\
 0 & \text{for all } s > s^* 
\end{cases}
\]

- Same reason behind Acemoglu and Zilibotti (1997)
Failures of Implementation

**Failure 2:** Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing.
- In this case, consumers would benefit from directly investing in already “open” firms.
- ... but they would invest the same amount in all of them.
- No agent would invest in BDs: they would wait for BD to cover MS.
- In ex-ante problem, this translates into a reservation utility for each agent.

- In constrained optimal mechanism, a BD would give the same utility as direct financing:

\[ y^*(s) = \begin{cases} 
  y^* = M(s^*) & \text{for all } s \leq s^* \\
  0 & \text{for all } s > s^*
\end{cases} \]

- Same reason behind Acemoglu and Zilibotti (1997)
Failures of Implementation

Failure 2: Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing
- In this case, consumers would benefit from directly investing in already “open” firms
- … but they would invest the same amount in all of them.
- No agent would invest in BDs: they would wait for BD to cover MS
- In ex-ante problem, this translates into a reservation utility for each agent
- In constrained optimal mechanism, a BD would give the same utility as direct financing:

\[
y^*(s) = \begin{cases} 
y^* = M(s^*) & \text{for all } s \leq s^* \\
0 & \text{for all } s > s^* \end{cases}
\]

- Same reason behind Acemoglu and Zilibotti (1997)
Failures of Implementation

**Failure 2:** Consumers may directly invest in firms

- Firms, once they cover their minimum scale, can now ask for more external financing.
- In this case, consumers would benefit from directly investing in already “open” firms.
- ... but they would invest the same amount in all of them.
- No agent would invest in BDs: they would wait for BD to cover MS.
- In ex-ante problem, this translates into a reservation utility for each agent.
- In constrained optimal mechanism, a BD would give the same utility as direct financing:

\[
y^*(s) = \begin{cases} 
y^* = M(s^*) & \text{for all } s \leq s^* \\
0 & \text{for all } s > s^* \end{cases}
\]

- Same reason behind Acemoglu and Zilibotti (1997).
Failures of Implementation

**Failure 3:** Contracts are incomplete

- What if state $s$ is not (perfectly) contractible? (Allen and Gale (2004), etc)
- In this case, separation between incentive and investment programs is not possible.
- Typically, investment in storage would be larger.
- However, non-smooth investment in long technologies would still be optimal (conjecture!)

**Failure 4:** Please tell me us!