Methodological Appendix

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"The Interaction of Public and Private Insurance:

Medicaid and the Long-Term Care Insurance Market"

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Our model considers a 65-year old individual who chooses an optimal consumption path to maximize expected discounted lifetime utility. The per-period utility function is defined on a monthly basis, with a maximum lifespan of 105, resulting in 480 periods denoted by t. In each month, the individual may be in one of five possible states of care, denoted by s: (1) at home receiving no care, (2) at home receiving paid home health care, (3) in residence at an assisted living facility, (4) in residence in a nursing home, or (5) dead. The cumulative probability of being in each state of care s at time t is denoted $Q_{t,s}$. Utility is a function of ordinary consumption $C_{s,t}$ as well as the consumption value (if any) derived from long-term care expenditures $F_{s,t}$. The individual discounts future utility at the monthly time preference rate ρ.

The general model also permits the consumption value of long-term care expenditures to vary depending on whether they are paid by Medicaid or by private insurance. We capture this difference in consumption value through the parameter α_s. In particular, if α_s=1, the assumption is that the consumption value of care is the same whether paid for by Medicaid or from private insurance. In contrast, α_s>1 would be consistent with a model in which private insurance allows one to purchase higher quality care, which thus provides higher consumption value. Although the baseline model assumes α_s=1, we discuss results for α_s>1 in section 6.2.

The consumer's utility function is therefore:

(A1) $$U\left(C_{s,t} + I^M_{s,t} \cdot F_{s,t} + \left(1 - I^M_{s,t}\right) \cdot \alpha_s \cdot F_{s,t}\right)$$

where $I_{s,t}^{M}$ is an indicator variable for whether or not the person is receiving Medicaid while in state s in period t. We assume that the utility function exhibits constant relative risk aversion, such that:

$$\text{(A2)} \quad U\left(C_{s,t} + I_{s,t}^{M} \cdot F_{s,t} + \left(1 - I_{s,t}^{M}\right) \cdot \alpha_s \cdot F_{s,t}\right) = \frac{\left(C_{s,t} + I_{s,t}^{M} \cdot F_{s,t} + \left(1 - I_{s,t}^{M}\right) \cdot \alpha_s \cdot F_{s,t}\right)^{(1-\gamma)} - 1}{1-\gamma}$$

The consumer's constrained dynamic optimization problem is therefore:

$$\underset{C_{s,t}}{Max} \sum_{t=1}^{480} \sum_{s=1}^{5} \frac{Q_{s,t}}{(1+\rho)^t} \cdot U\left(C_{s,t} + I_{s,t}^{M} \cdot F_{s,t} + \left(1 - I_{s,t}^{M}\right) \cdot \alpha_s \cdot F_{s,t}\right)$$

subject to

(Ai) W_0 is given

(Aii) $W_t \geq 0 \ \forall \ t$

$(Aiii)$ $W_{t+1} = [W_t + A_t + \min[B_{s,t}, X_{s,t}] - C_{s,t} - X_{s,t} - P_{s,t}](1+r) \quad \text{if } I_{s,t}^{M} = 0$

(Aiv) $W_{t+1} = [W_t - \max(W_t - \underline{W}, 0) + (\underline{C}_s - C_t)](1+r) \quad \text{if } I_{s,t}^{M} = 1$

where W_0 is pre-determined financial wealth at 65, A_t denotes annuity income, $B_{s,t}$ denotes the daily benefit cap on the private insurance payments, $X_{s,t}$ denotes long-term care expenditures, $P_{s,t}$ denotes the premium on the private insurance policy, and r is the monthly real rate of interest.

To be eligible for Medicaid (i.e. $I_{s,t}^{M} = 1$), the individual must:

(i) Be receiving care, i.e., $s \in \{2,3,4\}$

(ii) Meet the asset test, i.e., $W_t < \underline{W}$

(iii) Meet the income test: $A_t + \min[B_{s,t}, X_{s,t}] + r^*W_{t-1} - X_{s,t} < \underline{C}_s$

Where $\underline{W}$ is the asset eligibility threshold and $\underline{C}_s$ is the income eligibility threshold for care state *s*. Note that Medicaid eligibility at any given point in time is thus endogenous to consumption choices.

The solution to the constrained dynamic optimization problem (A1) involves the choice of a consumption plan at time 0, with the consumer's knowledge that he will be able to choose a new plan at time 1, and so on, until the final period. To solve this stochastic dynamic decision problem, we employ stochastic dynamic programming methods, as discussed in Blanchard & Fischer (1989) which reduce the multi-period problem to a sequence of simpler two-period decision problems. We begin by introducing a value function $V_{s,t}(W_t; A)$ for state *s* and time *t* that represents the present discounted value of expected utility evaluated along the optimal consumption path. This value depends on financial wealth (W_t), annuity income (A_t), and state of care (s) in which the individual finds himself, all at the start of period t.

The value function satisfies the recursive Bellman equation:

$$\text{(A3)} \qquad \underset{C_{s,t}}{Max} V_{s,t}\left(W_t; A\right) = \underset{C_{s,t}}{Max} U_s\left(C_{s,t} + I_{s,t}^{M} \cdot F_{s,t} + \left(1 - I_{s,t}^{M}\right) \cdot \alpha_s \cdot F_{s,t}\right) + \sum_{\sigma=1}^{5} \frac{q_{t+1}^{s,\sigma}}{\left(1+\rho\right)} V_{\sigma,t+1}\left(W_{t+1}; A\right)$$

where $q_{t+1}^{s,\sigma}$ the conditional probability that an individual who is in care state *s* at time *t* is in care state σ at time t+1.

We solve this problem using standard dynamic programming techniques (e.g. Stokey and Lucas, 1989). We begin by solving for the last period's problem at age 105, which produces a matrix of optimal consumption decisions, one for each combination of discrete value of wealth and state of care. We discretize wealth quite finely, down to $10 increments at low levels of wealth, and gradually rising at higher levels of wealth, but never exceeding 0.2% of starting wealth. (Thus for example, for the median household, for whom initial financial wealth is approximately $89,000, the maximum distance between two points on the financial wealth grid is $130.) In the final period of life, age 105, all remaining wealth is consumed, which maps into a value function matrix that is N_w x N_s, where N_w is the number of discrete wealth points evaluated on the grid (for a median wealth household, N_w is over 1,400) and N_s = 4 (assuming no bequest motives, only 4 of the 5 states of the world have value).

For each element in the state spaces, we continue to solve the model backwards, collecting separate decision rules and value functions for every month-by-care-state combination back to age 65. Given our discretization methods and the number of periods and states in the problem, a single set of parameters involves solving our model for approximately 3.5 million discrete points. This is implemented using a program written for Gauss.

References

Blanchard, Olivier and Stanley Fischer. 1989. "Lectures in Macroeconomics." MIT Press, Cambridge MA.

Stokey, Nancy and Robert Lucas. 1989. Recursive methods in economic dynamics. Harvard University Press, Cambridge MA.