

Belief Disagreement in Production Networks

William Chen*

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Abstract

This paper studies the macroeconomic impact of belief disagreement between firms in different sectors of a production network when firms choose labor quantities before sectoral TFP disturbances are realized. Belief disagreement can reduce aggregate output by causing firms and their suppliers to both cut labor demand. This result works through supply-chain spillovers between firms and their suppliers. A hypothetical vignette experiment run on U.S. small businesses provides empirical evidence that disagreement between firms and their suppliers can affect firms' own price and input decisions.

*Department of Economics, Massachusetts Institute of Technology. Email: wyc1@mit.edu.

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1 Introduction

Novel and complex shocks have precipitated many economic crises. The opacity of the balance sheets of shadow banks triggered the 2007-2008 financial crisis while the onset of COVID-19 entailed numerous unknowable uncertainties. When a shock is novel and complex, economic agents will likely disagree about the shock. Disagreements may exacerbate the shock’s effect on aggregate outcomes due to spillovers. The reaction of one set of agents may be perfectly reasonable under their beliefs but a significant mistake under the beliefs of another set of agents. If there are spillovers, then the reaction of the first set of agents can pose a significant problem to the second set.

This paper theoretically investigates these ideas within the environment of a production network in which firms have errors in their beliefs about sectoral TFP disturbances. Firms choose their labor inputs and commit to pay the market-clearing wage before they learn sectoral disturbances, so aggregate output becomes a function of both realized TFP disturbances and firms’ belief errors. In a benchmark CES vertical supply chain economy, belief errors affect aggregate output only when firms and their suppliers disagree upstream TFP disturbances. Output falls when downstream firms are more pessimistic. To test that disagreements with suppliers matter for business decisions, I run a hypothetical vignette experiment on U.S. small businesses and find supporting empirical evidence.

The model has two “stages”: in the “morning,” firms choose labor given their beliefs, and in the “afternoon,” all other decisions and prices are determined. I treat firms’ beliefs about sectoral TFP as primitives but assume that beliefs are common knowledge, firms agree to disagree, and firms are correctly specified about the economy’s structural relations. The model is an otherwise standard production network with elastic labor supply from a representative household. I characterize equilibrium in three steps. First, I characterize agents’ optimality conditions in an arbitrary temporary equilibrium. Second, I solve for

aggregate output, sectoral sales, and sectoral prices in the afternoon as a function of morning labor choices and realized TFP disturbances. Third, I show that belief errors induce labor wedges, which then determine morning labor choices. I apply this general solution to several example economies to study the determinants of the aggregate response to belief errors.

I first establish a neutrality result in which belief errors about TFP do not necessarily change aggregate output. When the production network is Cobb-Douglas and the income effect on labor supply equals one, aggregate output does not depend on belief errors. The intuition is that an income effect on labor supply of one ensures that wages adjust to offset endogenous movements in labor supply, and a Cobb-Douglas production network shuts down labor spillovers between sectors. Instead, prices adjust in equilibrium and prevents belief errors in one sector from distorting labor demand in other sectors. This paper relaxes the assumption of a Cobb-Douglas network and focuses on production networks composed of vertical CES supply chains.¹

In the CES supply chain network, belief errors affect aggregate output only when firms in different sectors disagree with each other. When labor and intermediate inputs are complementary, pessimism about upstream TFP by firms in downstream sectors causes their hiring to fall, but pessimism by firms in upstream sectors causes their hiring to *rise*. These changes in labor demand offset when firms are equally pessimistic throughout a supply chain. It is only when firms in downstream sectors are relatively more pessimistic that aggregate output falls. The reason is that downstream firms believes upstream firms should have chosen a different amount of labor, and vice-versa, so all firms decrease their labor demand instead of offsetting each other.

The sensitivity of aggregate output to belief errors depends on the extent of labor reallocation across sectors. The less willing the representative household is to reallocate

¹The appendix considers other example economies and, in particular, explores how belief errors affect aggregate output when the income effect on labor supply is not one. In this case, disagreement is not needed for belief errors to affect aggregate output.

labor, the smaller the impact that belief errors have on output. This result holds for arbitrary networks and income effects on labor supply. The reason is that a low willingness to reallocate labor implies a high willingness to accept wage reductions. If firms can count on large wage changes to offset the effects of TFP disturbances, then they do not worry about their labor decision.

To provide empirical evidence that belief disagreement between firms and their suppliers affects their decisions, I run a hypothetical vignette experiment on a sample of U.S. small businesses. I fielded this experiment in July 2025, a period during which firms faced substantial uncertainty about U.S. tariffs. Using this uncertainty as a motivating scenario, the experiment asked small businesses how they expected their prices, sales, and labor demand would respond if the suppliers of their main product or service expected randomly different tariff rates by the end of 2025 than the respondent’s own expectation. This experiment finds evidence consistent with this paper’s theoretical point that disagreements with suppliers cause changes in a downstream firm’s expected decisions. When suppliers expect higher tariff rates, downstream firms expect higher prices, higher unit costs, lower sales, and lower labor demand. The magnitude of the effects are economically meaningful. A 10pp increase in disagreement about the average tariff rate causes a respondent to raise their expected changes in prices by 1.9pp and in unit costs by 1.4pp as well as to lower their expected sales by 1pp and their expected labor demand by 0.4pp.

Related Literature. Information frictions are a common way to introduce errors in equilibrium beliefs into production networks. [Pellet and Tahbaz-Salehi \(2023\)](#) and [Bui et al. \(2024\)](#) focus on the noisy exogenous information case. [Nikolakoudis \(2024\)](#) considers learning from prices, [Jamilov et al. \(2024\)](#) studies endogenous attention about exogenous states, and [Chahrour et al. \(2021\)](#) examines the role of news media in belief formation. These papers show that specific information frictions alter the propagation of sectoral disturbances

and cause noise shocks to be sources of aggregate fluctuations. In contrast to these papers, I abstract from a specific model of belief formation, which enables me to characterize what types of beliefs in general are needed to generate aggregate downturns. I do restrict attention to beliefs which satisfy a form of rationality that I clarify below.

This paper also relates to the broader production networks literature. [Baqae and Farhi \(2019\)](#) and [Dew-Becker \(2023\)](#) show that nonlinear production networks matter for the aggregate effects of sectoral disturbances. When there are nonlinearities, sectoral disturbances change Domar weights and input-output multipliers. My paper shows that these same mechanisms have first-order effects when firms choose inputs subject to uncertainty. [Kopytov et al. \(2024\)](#) has a similar result but obtains it using endogenous input-output linkages in a Cobb-Douglas network instead of an exogenous nested-CES network. [Bigio and La'O \(2020\)](#) and [Baqae and Farhi \(2020b\)](#) study the impact of fluctuations in exogenous distortions in production networks. Labor wedges in my paper endogenously emerge from input choice given potentially incorrect beliefs and depend on the network structure.

Outline. The remainder of the paper is organized as follows. Section [2](#) presents the model, and Section [3](#) characterizes equilibrium. Section [4](#) studies the macroeconomic effects of belief errors, and Section [5](#) presents evidence from the hypothetical vignette experiment. Section [6](#) concludes.

2 Model

There is a representative household and n production sectors, indexed by $i \in \mathcal{N} \equiv \{1, \dots, n\}$. A representative firm in each sector exists and sells output either to the household for consumption or for use as an intermediate input. The household owns all firms. The economy is static, but it will be convenient to divide firms' actions into two stages, the "morning"

and the “afternoon.” In the morning, firms choose labor. In the afternoon, the household supplies labor, and the product and intermediate goods markets clear. Wages and goods prices are both state-contingent.

2.1 Household

The representative household has preferences

$$U(Y, \{h_i^s\}_{i \in \mathcal{N}}) = \frac{Y^{1-\gamma} - 1}{1-\gamma} - \sum_{i \in \mathcal{N}} \chi_i \frac{(h_i^s)^{1+\eta_i}}{1+\eta_i},$$

where Y denotes GDP and h_i^s denotes the quantity of labor supplied to sector i by the household. This specification of labor disutility is taken from [Bui et al. \(2024\)](#) and implies that each sector faces a labor supply curve which is isoelastic in the real wage with sector-specific inverse Frisch elasticity η_i . These preferences keep the model tractable while limiting the degree of labor reallocation across sectors through the parameters η_i . Labor is perfectly substitutable across sectors when $\eta_i = 0$ for all i .

The measure of aggregate output in this economy is GDP, which is a CES aggregate over sectoral goods

$$Y = \bar{Y} \left[\sum_{i \in \mathcal{N}} \beta_i \left(\frac{c_i}{\bar{c}_i} \right)^{\frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}}, \quad \sum_{i \in \mathcal{N}} \beta_i = 1.$$

where the barred variables are normalizing constants, c_i is consumption of good i , and p_i is the price of good i .² In the remainder of the paper, I will usually refer to aggregate output

²CES aggregators are not unitless. I choose this normalization so that β_i can be interpreted as good i 's steady-state budget share.

as GDP. I choose GDP as the numeraire, so the budget constraint can be written as

$$Y = \sum_{i \in \mathcal{N}} p_i c_i = \sum_{i \in \mathcal{N}} w_i h_i^s + \sum_{i \in \mathcal{N}} \pi_i,$$

where w_i are sector-specific wages and π_i are profits from sector i . Wages are sector-specific because labor is not perfectly substitutable across sectors unless $\eta_i = 0$ for all i .

2.2 Firms

Technology. The representative firm in sector i chooses labor h_i and intermediate inputs $\{x_{ij}\}_{j \in \mathcal{N}}$ to produce the sectoral good according to

$$y_i = \bar{y}_i z_i \left[\alpha_i \left(\frac{h_i}{\bar{h}_i} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} + \sum_{j \in \mathcal{N}} a_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}} \quad (1)$$

where z_i is a sectoral TFP disturbance, $\alpha_i, a_{ij} \in [0, 1]$, and $\alpha_i + \sum_{j \in \mathcal{N}} a_{ij} = 1$.³ Nominal profits are given by

$$\pi_i = p_i y_i - w_i h_i - \sum_{j \in \mathcal{N}} p_j x_{ij}.$$

Production under Uncertainty. In the morning, firms choose their labor inputs subject to uncertainty about the vector of TFP disturbances $\mathbf{z} \equiv (z_1, \dots, z_n)^\top$. Formally, firms solve the problem

$$\max_{h_i} E_i[Y^{-\gamma} \pi_i] \quad (2)$$

³My model could have an elasticity of substitution between intermediate inputs that differs from the elasticity of substitution between labor and intermediate inputs by assuming sector i purchases a composite intermediate from sector j , which uses no labor and simply aggregates intermediates from other sectors with elasticity of substitution $\theta_{yj} \neq \theta_{yi}$.

where $Y^{-\gamma}$ is the household's stochastic discount factor (SDF). The *subjective* expectation operator $E_i[\cdot]$ denotes sector i 's belief about a variable.

3 Equilibrium Definition and Characterization

Unless stated otherwise, proofs are deferred to Appendix A. Bolded variables usually denote vectors or matrices, and the log of a vector means $\log \mathbf{x} = (\log x_1, \log x_2, \dots, \log x_n)^\top$. Also, $\text{diag}(\mathbf{x})$ takes a vector as an input and returns a matrix with the vector along the diagonal.

In the remainder of the paper, $\mathbf{z}$ should be interpreted as the *realized* values of TFP in the afternoon. I denote firms' *beliefs* about these values using either hats or a subjective expectation operator.

3.1 Equilibrium Definition

I formally define the equilibrium concept as follows.

Definition 1. A *temporary equilibrium with heterogeneous priors* is a vector of exogenous TFP disturbances $\mathbf{z}$ as well as a collection of endogenous variables $\{Y, \{w_i, c_i, p_i, h_i, y_i, \{x_{ij}\}_{j \in \mathcal{N}}, h_i\}_{i \in \mathcal{N}}\}$ and a collection of beliefs about those exogenous disturbances and endogenous variables for each sector such that

1. (Household Optimality) The representative household chooses consumption and labor supply in the afternoon to maximize utility subject to its budget constraint;
2. (Intermediate Inputs Optimality) Conditional on having chosen $\{h_i\}_{i \in \mathcal{N}}$ in the morning and after learning $\mathbf{z}$ in the afternoon, firms choose intermediate inputs to maximize their profits;

3. (Market Clearing) Sectoral goods prices and wages $\{p_i, w_i\}_{i \in \mathcal{N}}$ adjust in the afternoon to ensure all sectoral goods and labor markets clear.⁴ Specifically,

$$y_i = c_i + \sum_{j \in \mathcal{N}} x_{ji} \quad \forall i, \quad (3)$$

$$h_i^s = h_i \quad \forall i; \quad (4)$$

4. (Model-Consistent Beliefs) Firms' beliefs about endogenous variables are consistent with conditions 1–3;
5. (Heterogeneous Priors) Firms know each other's beliefs and agree to disagree.
6. (Labor Input Optimality) In the morning, firms choose their labor inputs to solve (2), taking as given their beliefs about exogenous disturbances.

Model-consistent beliefs just means that firms, when choosing labor, understand that the economy becomes a static production network with fixed sectoral labor endowments in the afternoon. Backward induction pins down firms' beliefs about endogenous variables in the afternoon as a function of the labor inputs chosen in the morning and the exogenous disturbances. This assumption is restrictive because it rules out some models of belief formation. However, many popular models of belief formation make some version of this assumption. For example, models with information frictions typically assume that all agents have the correct model of the economy but face uncertainty about exogenous disturbances and sometimes about each other's beliefs.

I further restrict attention to models in which higher-order beliefs are irrelevant. There is a representative firm in every sector, so higher-order uncertainty within a sector is not relevant. The heterogeneous priors restriction shuts down higher-order uncertainty across

⁴By Walras' law, the market for the aggregate consumption good will also clear.

sectors. While perhaps a strong assumption, it isolates the effect of dogmatic disagreement from the effect of disagreement simply because firms do not know each other's beliefs.

An equilibrium exists because the model is isomorphic to one in which labor wedges distort firms' labor choice. See [Bigio and La'O \(2020\)](#) for a proof of existence. Hereafter, whenever I write the “temporary equilibrium,” I am referring to my equilibrium concept, unless stated otherwise.

3.2 Log-Linearized Solution

This section characterizes the log-linearized solution to the temporary equilibrium around the deterministic and efficient steady state in which all firms have no uncertainty about TFP disturbances. In the special case where the network is Cobb-Douglas, the log-linear solution is global. I use bars over variables to denote their steady-state values. I also choose $\bar{y}_i$ so that $\bar{\mathbf{z}} = \mathbf{1}$.

Input-Output Objects. Define the steady-state input-output matrix and Leontief inverse matrix as

$$\Omega_{ij} = \frac{\bar{p}_j \bar{x}_{ij}}{\bar{p}_i \bar{y}_i}, \quad \Psi = (\mathbf{I} - \Omega)^{-1}.$$

The Domar weights and input-output multiplier are given by

$$\lambda_i = \sum_{j \in \mathcal{N}} \beta_j \Psi_{ji}, \quad \xi = \sum_{i \in \mathcal{N}} \lambda_i.$$

The Domar weights and labor cost shares satisfy

$$\sum_{i \in \mathcal{N}} \lambda_i \alpha_i = 1$$

because $\lambda_i \alpha_i$ denotes the labor share earned by sector i .

Equilibrium Conditions. I first state the log-linearized morning and afternoon equilibrium conditions derived from agents' optimality conditions and market-clearing.

Proposition 1. *To a first-order approximation of the temporary equilibrium around the deterministic steady state, changes in prices and allocations are characterized by the following system of equations*

$$d \log p_i + d \log c_i = d \log Y + (1 - \theta_c) d \log p_i, \quad (5)$$

$$d \log w_i = \gamma d \log Y + \eta_i d \log h_i, \quad (6)$$

$$d \log p_j + d \log x_{ij} = -(1 - \theta_{yi})(d \log z_i + d \log p_i - d \log p_j) + d \log s_i, \quad (7)$$

$$\begin{aligned} \left(1 + \alpha_i \frac{1 - \theta_{yi}}{\theta_{yi}}\right) d \log p_i &= \sum_{j \in \mathcal{N}} \Omega_{ij} d \log p_j + \frac{\alpha_i}{\theta_{yi}} (d \log s_i - d \log h_i) \\ &\quad - \left(1 + \alpha_i \frac{1 - \theta_{yi}}{\theta_{yi}}\right) d \log z_i, \end{aligned} \quad (8)$$

$$\begin{aligned} d \log h_i &= d \log s_i - \theta_{yi} d \log w_i \\ &\quad - (1 - \theta_{yi})(d \log z_i + d \log p_i) - d \log \tau_i, \end{aligned} \quad (9)$$

$$\begin{aligned} d \log \tau_i &= ((d \log s_i - d \log w_i) - (E_i[d \log s_i] - E_i[d \log w_i])) \\ &\quad - (1 - \theta_{yi})(d \log z_i + d \log p_i - d \log w_i) \\ &\quad + (1 - \theta_{yi})(E_i[d \log z_i] + E_i[d \log p_i] - E_i[d \log w_i]), \end{aligned} \quad (10)$$

$$\lambda_i d \log s_i = \beta_i (d \log p_i + d \log c_i) + \sum_{j \in \mathcal{N}} \Omega_{ji} \lambda_j (d \log p_i + d \log x_{ji}), \quad (11)$$

Solution for the Afternoon.

Proposition 2. *To a first-order approximation of the temporary equilibrium around the deterministic steady state, aggregate output, sales, and prices in the afternoon fluctuate ac-*

cording to

$$d \log Y = \sum_{i \in \mathcal{N}} \lambda_i d \log z_i + \sum_{i \in \mathcal{N}} \lambda_i \alpha_i d \log h_i, \quad (12)$$

$$d \log \mathbf{s} = \mathbf{S}_z d \log \mathbf{z} + \mathbf{S}_h d \log \mathbf{h} + \mathbf{S}_Y d \log Y, \quad (13)$$

$$d \log \mathbf{p} = \mathbf{P}_z d \log \mathbf{z} + \mathbf{P}_h d \log \mathbf{h} + \mathbf{P}_Y d \log Y. \quad (14)$$

where the formulas for the matrices $\mathbf{S}_z$, $\mathbf{S}_h$, $\mathbf{P}_z$, $\mathbf{P}_h$ and vectors $\mathbf{S}_Y$, $\mathbf{P}_Y$ are provided in the appendix.

Equation (12) is simply Hulten's Theorem when factor supplies are not fixed. Plugging (12) into (13) and (14) delivers expressions for sales and prices as functions of TFPs and labor demands. However, for characterizing the labor wedges, it is convenient to keep $d \log Y$. Proposition 2 holds in any temporary equilibrium, not just my equilibrium concept, since beliefs only affect the choice of $\mathbf{h}$ in the morning.

Solution for the Morning. To keep notation compact, define $\mathcal{E}_i[d \log x] \equiv E_i[d \log x] - d \log x$. The following proposition characterizes firms' morning labor demands in terms of labor wedges and provides the general formula for those wedges.

Proposition 3. *Consider a first-order approximation of the temporary equilibrium around the deterministic steady state. Equilibrium labor allocations depend on sectoral TFPs, GDP, and sectoral labor wedges according to*

$$d \log h_i = \sum_{j \in \mathcal{N}} \mathbf{H}_{z,ij} d \log z_j + \mathbf{H}_{Yi} d \log Y - \sum_{j \in \mathcal{N}} \mathbf{H}_{\tau,ij} d \log \tau_j. \quad (15)$$

If there are no errors in beliefs, then changes in GDP are given by

$$d \log Y^{(0)} = \frac{1}{1 - \sum_{i \in \mathcal{N}} \lambda_i \alpha_i \mathbf{H}_{Yi}} \sum_{i \in \mathcal{N}} \left(\lambda_i + \sum_{j \in \mathcal{N}} \lambda_j \alpha_j \mathbf{H}_{z,ji} \right) d \log z_i. \quad (16)$$

If there are errors in beliefs, then changes in GDP are given by

$$d \log Y = d \log Y^{(0)} - \frac{1}{1 - \sum_{i \in \mathcal{N}} \lambda_i \alpha_i \mathbf{H}_{Yi}} \sum_{i \in \mathcal{N}} \lambda_i \alpha_i \sum_{j \in \mathcal{N}} \mathbf{H}_{\tau,ij} d \log \tau_j, \quad (17)$$

where changes in sector i 's labor wedge satisfy the equations

$$d \log \tau_i = - \sum_{j \in \mathcal{N}} \mathbf{T}_{z,ij} \mathcal{E}_i[d \log z_j] - \mathbf{T}_{Yi} \mathcal{E}_i[d \log Y], \quad (18)$$

$$\mathcal{E}_i[d \log Y] = \boldsymbol{\lambda}^\top \mathcal{E}_i[d \log \mathbf{z}]. \quad (19)$$

Formulas for the matrices $\mathbf{H}_z, \mathbf{H}_\tau, \mathbf{T}$ and vector $\mathbf{H}_Y$ are in the appendix.

The optimality condition for a sector's labor demand (9) suggests that a sector is affected only by its own TFP and labor wedge. However, expenditure switching throughout the production network causes a firm's labor demand (15) to potentially depend on the TFPs and labor wedges of all sectors. The reason is that both the ratio of a sector's sales to its wage and the ratio of a sector's price to its wage depend directly on the vector of morning labor demands. By contrast, in the Cobb-Douglas case, the first ratio only depends on GDP, and the second ratio is irrelevant for a firm's labor demand.

Belief Spillovers Through the Network. In Proposition 3, while equation (15) shows that a sector's labor choice does potentially depend on other sectors' labor wedges, equations (18) and (19) show that a sector's labor wedge depends only on its own beliefs. This result simplifies the analysis, but it is a consequence of two modeling choices, not a necessary result.⁵

The first choice is the structure of the labor market. I assume that the firm pays a

⁵Removing the heterogeneous priors assumption does not necessarily cause a sector's labor wedge to depend on other sectors' beliefs. Bui et al. (2024) considers a production network with dispersed information and shows that a sector's total labor choice only depends on other sectors' beliefs when there are noise shocks that do not aggregate away (e.g. sectoral noise shocks).

state-contingent wage to the household, which supplies labor in the afternoon, and that the labor supply to one sector is not affected by the labor supplied to other sectors. If a firm needed to pay a non-contingent wage, then the household's beliefs in the morning would shift its labor supply to the firm, regardless of what the firm believes.⁶ Similarly, if a firm's labor supply was affected by labor supplied to other sectors, then its labor supply would be shifted by other sectors' beliefs. I shut down this possibility through my specification of labor disutility. An alternative specification like

$$\frac{H^{1+\eta}}{1+\eta} \equiv \frac{1}{1+\eta} \left[\bar{H} \left(\sum_{i \in \mathcal{N}} \chi_i \left(\frac{h_i}{\bar{h}_i} \right)^{\frac{\theta_h+1}{\theta_h}} \right)^{\frac{\theta_h}{\theta_h+1}} \right]^{1+\eta}$$

would generate belief spillovers through the labor market because a sector's labor supply now depends on an aggregate of sectoral labor supplies.⁷

The second choice is the structure of the market for the intermediate inputs. I assume that firms flexibly choose any intermediate inputs in the afternoon and thus all goods prices are also determined in the afternoon. There are a number of ways to change this assumption and thereby make a firm's decisions depend on other sectors' beliefs. For example, suppose a downstream sector is the only customer of an upstream supplier, and the market for this input must clear *in the morning*. The downstream sector's input choice depends on the market-clearing price, which is determined by not only the downstream sector's demand curve but also the upstream sector's supply curve. If the upstream sector has to choose some of its inputs in the morning, then its supply curve depends on the upstream sector's beliefs. Then the market-clearing price would depend on both the downstream sector's and the upstream sector's beliefs.

⁶The wage is determined in the morning in [Bui et al. \(2024\)](#). However, the household's beliefs are not part of the solution because the household takes on a family structure in which workers are sent to each information island in a sector and the workers inherit the beliefs of the island.

⁷This specification of labor disutility is used by [Horvath \(2000\)](#).

4 The Macroeconomic Impact of Belief Errors

While Proposition 3 gives the general solution to my model, its equations do not provide much insight about the precise channels which determine the impact of belief errors on aggregate output. To explain what these channels are and how they shape the response of GDP to belief errors, I study special cases of the general model in the following subsections.

4.1 A Neutrality Result

I first study an economy in which belief errors have zero impact on GDP. This neutrality result clarifies the necessary ingredients for belief errors to affect aggregate output.

Proposition 4. *If $\gamma = 1$ so that the income effect on labor supply equals one and the production network is Cobb-Douglas ($\theta_c = 1$ and $\theta_{yi} = 1$ for all $i \in \mathcal{N}$), then $d \log Y = d \log Y^{(0)}$.*

This result holds not only locally but also globally. For intuition, plug the household's optimal labor supply equation into sector i 's labor demand equation to obtain

$$(1 + \theta_{yi}\eta_i)d \log h_i = E_i[d \log s_i] - (1 - \theta_{yi})(E_i[d \log z_i] + E_i[d \log p_i]) - \theta_{yi}\gamma E_i[d \log Y].$$

Labor demand may fluctuate through two channels. First, sales and prices may have asymmetric exposure to sectoral TFP disturbances due to expenditure switching. Second, labor demand can be cyclical in equilibrium.⁸ This paper focuses on the first channel since disagreement about TFP disturbances ultimately affects GDP because firms have asymmetric exposure. Appendix B studies the second channel in a purely horizontal economy.

⁸The parameter γ controls the income effect on the household's labor supply and thus the cyclicalities of wages to GDP. Because I log-linearize around a deterministic steady state, γ does not represent risk aversion in this paper.

A Cobb-Douglas production network shuts down the first channel because it implies that $d \log s_i = d \log Y$ and $\theta_{yi} = 1$, hence

$$(1 + \eta_i)d \log h_i = (1 - \gamma)E_i[d \log Y].$$

In other words, sectoral sales comove perfectly with GDP and movements in a sector's own prices or TFP become irrelevant for labor demand.⁹

Labor demand may still deviate from steady state if labor demand is cyclical. In a Cobb-Douglas network, labor demand is cyclical if the cyclicality of sales does not match the cyclicality of wages. Intuitively, an economic downturn implies not only lower sales but also lower wage costs, which insures firms against the downturn and mitigates the decline in labor demand. When $\gamma = 1$, this insurance is perfect.

4.2 CES Supply Chains

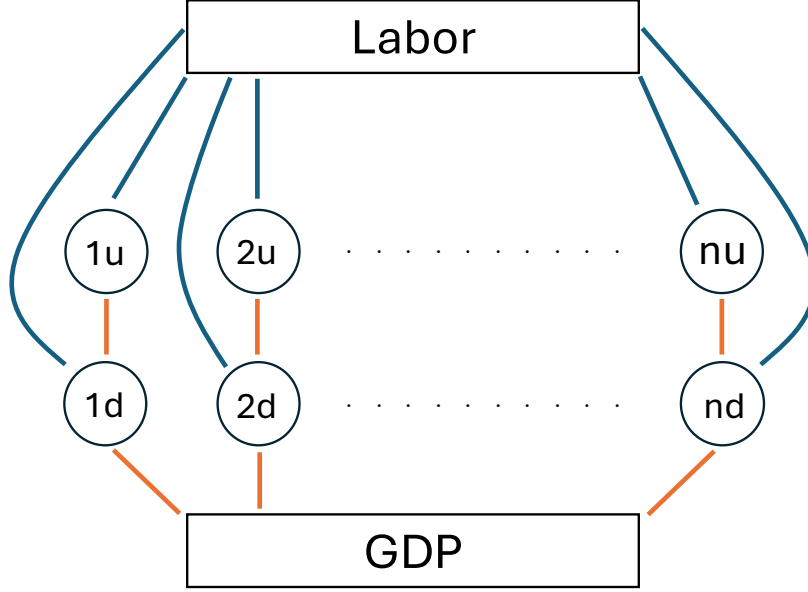
This section considers this paper's primary example economy, in which there are n horizontal "sectors," and each sector comprises a downstream and upstream industry. Figure 1 illustrates this network. The downstream industry sells to the household while the upstream industry sells output only to the downstream industry. To improve readability, I make an abuse of notation and redefine $\mathcal{N} = \{1d, 1u, \dots, nd, nu\}$ so that id and iu denote, respectively, the downstream and upstream industries of sector i . To keep expressions interpretable, I assume that the upstream industries only use labor and the downstream industry only uses the output of its own sector's upstream industry as an intermediate. I also maintain

⁹This neutrality result may not hold if firms have belief errors about other disturbances, such as changes in monopoly distortions.

$d \log \mathbf{z} = \boldsymbol{\eta} = \mathbf{0}$. Then the Domar weights and input-output multiplier are given by

$$\boldsymbol{\lambda} = \left((1 - \alpha_{1d})\beta_1, \beta_1, \dots, (1 - \alpha_{nd})\beta_n, \beta_n \right)^\top, \quad \xi = 1 + \sum_{i=1}^n (1 - \alpha_{id})\beta_i.$$

Figure 1: CES Supply Chain



The log labor wedges in the upstream and downstream industries are

$$d \log \tau_{iu} = (1 - \gamma)\mathcal{E}_{iu}[d \log Y] + \alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} \mathcal{E}_{iu}[d \log z_{iu}],$$

$$d \log \tau_{id} = \theta_{y,id}(1 - \gamma)\mathcal{E}_{id}[d \log Y] - (1 - \alpha_{id})(1 - \theta_{y,id})\mathcal{E}_{id}[d \log z_{iu}].$$

The log labor wedge in this case depends on beliefs about upstream TFP and aggregate output. Downstream TFP is irrelevant for labor allocations because it does not shift sales or prices except through movements in GDP. Fixing the belief error about upstream TFP, the signs are opposite for the two labor wedges. When $\theta_{y,id} < 1$, optimism by upstream causes its labor wedge to increase while optimism by downstream causes its labor wedge to

decrease. The intuition is that, when downstream has complementarity over its inputs, high upstream TFP causes the downstream industry to switch expenditures from intermediate inputs toward labor. This expenditure switching reduces sales for upstream, holding GDP constant, so upstream wants to reduce labor. In contrast, intermediate inputs are relatively cheap for downstream, so downstream can expand hiring. Thus, if upstream and downstream have common belief errors, then the two labor wedges move in opposite directions.

Set $\gamma = 1$ to shut down the cyclical channel. Then GDP fluctuates according to

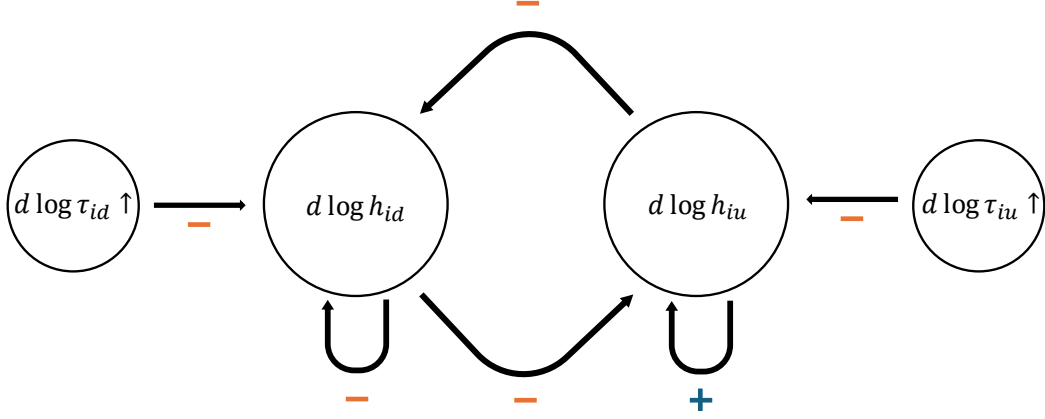
$$d \log Y = - \sum_{i=1}^n \lambda_{id} (1 - \alpha_{id}) \alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} (\mathcal{E}_{iu}[d \log z_{iu}] - \mathcal{E}_{id}[d \log z_{iu}]). \quad (20)$$

Changes in GDP occur if and only if downstream and upstream disagree about upstream's TFP. The intuition for this result is the same intuition for the behavior of labor wedges. When both industries agree about upstream's TFP, the labor wedges move in opposite directions, so a shift in upstream's labor demand is offset by the shift in downstream's labor demand. It is only when the industries disagree about upstream's TFP that the labor wedges fail to offset each other.

Inspecting the Result. To clarify why disagreement directly matters for GDP fluctuations, I discuss some of the calculations which lead to (20).

Within each sector of the CES supply chain economy, labor choices have spillovers between industries, and disagreement affects aggregate output through these spillovers. Figure 2 illustrates these spillovers in sector i when downstream has complementary input demand. Suppose $d \log \tau_{id}$ initially increases. By Figure 2, downstream's labor demand $d \log h_{id}$ falls in response. Then the scale of downstream's operation falls, which decreases demand for intermediate inputs. Upstream responds by reducing its labor demand $d \log h_{iu}$, hence the negative spillover from downstream's labor demand to upstream's labor demand in Figure 2. The price of upstream's output rises, which generates a negative spillover back to down-

Figure 2: Labor Spillovers ($\theta_{y,id} < 1$)



stream.¹⁰ The reason is that downstream anticipates spending more on intermediate inputs and thus switches expenditures away from labor. A similar logic applies to an initial increase in $d \log \tau_{iu}$. Without disagreement, the changes in the labor wedges and the resulting spillovers would offset each other so that aggregate output ultimately does not change.

I solve the matrix system of equations underlying Figure 2 to obtain expressions for labor demand in each industry as a function of the labor wedges.

$$\begin{pmatrix} d \log h_{iu} \\ d \log h_{id} \end{pmatrix} = \begin{pmatrix} -1 + \alpha_{id}(1 - \theta_{y,id}) & -\alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} \\ -(1 - \alpha_{id})(1 - \theta_{y,id}) & -1 - \alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} \end{pmatrix} \begin{pmatrix} d \log \tau_{iu} \\ d \log \tau_{id} \end{pmatrix}.$$

With complementary input demand, both industries reduce their labor demand when upstream's labor wedge increases, although the decline in upstream's demand is partially mitigated by the change in the price of its output.

¹⁰There is also a feedback loop within each industry because changes in labor allocations directly affect goods prices.

The impact on GDP equals the sum of labor changes, weighted by each industry's labor share of GDP. To keep notation compact, define disagreement in belief errors within sector i by $D_i \equiv \mathcal{E}_{iu}[d \log z_{iu}] - \mathcal{E}_{id}[d \log z_{iu}]$. Also, suppose only sector i' makes a belief error. Then the change in GDP is

$$\begin{aligned}
d \log Y &= \underbrace{\lambda_{i'd}(1 - \alpha_{i'd})}_{=\lambda_{i'u}} \underbrace{\alpha_{i'u}}_{=1} d \log h_{i'u} + \lambda_{i'd} \alpha_{i'd} d \log h_{i'd} \\
&= \underbrace{\lambda_{i'd} - \alpha_{i'd}(1 - \alpha_{i'd}) \frac{1 - \theta_{y,i'd}}{\theta_{y,i'd}} (D_{i'} + \theta_{y,i'd} \mathcal{E}_{i'd}[d \log z_{i'u}] - \alpha_{i'd}(1 - \theta_{y,i'd}) D_{i'})}_{=(1 - \alpha_{i'd}) d \log h_{i'u}} \\
&\quad + \underbrace{\lambda_{i'd} - \alpha_{i'd}(1 - \alpha_{i'd}) \frac{1 - \theta_{y,i'd}}{\theta_{y,i'd}} (-\theta_{y,i'd} \mathcal{E}_{i'd}[d \log z_{i'u}] + \alpha_{i'd}(1 - \theta_{y,i'd}) D_{i'})}_{=\alpha_{i'd} d \log h_{i'd}}
\end{aligned}$$

The second equality shows the intuition described previously. When the downstream industry has a belief error about upstream TFP ($\mathcal{E}_{i'd}[\log z_{i'u}] \neq 0$), both industries adjust their labor in exactly the correct amounts such that GDP is unchanged. When the two industries disagree, upstream and downstream further adjust their labor responses. GDP ultimately changes because the upstream industry's labor demand reacts more to disagreement than the downstream industry's labor demand.

Three Industry Supply Chain. A longer supply chain creates greater scope for belief disagreement to affect aggregate output. Suppose the supply chain now has three industries: downstream, midstream, and upstream. Maintaining $\gamma = 1$, the labor wedges become

$$\begin{aligned}
d \log \tau_{iu} &= \frac{1 - \theta_{y,id}}{\theta_{y,id}} \alpha_{id} \mathcal{E}_{iu}[d \log z_{im}] + \left(\alpha_{im} \frac{1 - \theta_{y,im}}{\theta_{y,im}} + \alpha_{id}(1 - \alpha_{im}) \frac{1 - \theta_{y,id}}{\theta_{y,id}} \right) \mathcal{E}_{iu}[d \log z_{iu}] \\
d \log \tau_{im} &= \theta_{y,im} \frac{1 - \theta_{y,id}}{\theta_{y,id}} \alpha_{id} \mathcal{E}_{im}[d \log z_{im}] - \theta_{y,im}(1 - \alpha_{im}) \left(\frac{1 - \theta_{y,im}}{\theta_{y,im}} - \alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} \right) \mathcal{E}_{im}[d \log z_{iu}] \\
d \log \tau_{id} &= -(1 - \alpha_{id})(1 - \theta_{y,id}) \mathcal{E}_{id}[d \log z_{im}] - (1 - \theta_{y,id})(1 - \alpha_{im} - \alpha_{id}(1 - \alpha_{im})) \mathcal{E}_{id}[d \log z_{iu}]
\end{aligned}$$

Belief errors about midstream's and upstream's TFP disturbances generate labor wedges, and there is asymmetric exposure across industries, which is determined by whether the disturbance affects an industry as a customer or a supplier. For example, downstream's exposure to $d \log z_{im}$ is the opposite of the exposure of other industries because midstream's TFP affects downstream as a customer and the other industries as suppliers.

Changes in GDP are given by

$$d \log Y = - \sum_{i=1}^n \lambda_{id} (1 - \alpha_{id}) \left[\alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} ((1 - \alpha_{im}) D_{im}^{ud} + \alpha_{im} D_{im}^{md}) \right. \\ \left. + (1 - \alpha_{im}) \left(\alpha_{im} \frac{1 - \theta_{y,im}}{\theta_{y,im}} D_{iu}^{um} + \alpha_{id} \frac{1 - \theta_{y,id}}{\theta_{y,id}} ((1 - \alpha_{im}) D_{iu}^{um} + D_{iu}^{md}) \right) \right] \quad (21)$$

where $D_{ia}^{bc} \equiv \mathcal{E}_{ib}[d \log z_{ia}] - \mathcal{E}_{ic}[d \log z_{ia}]$. For example, D_{iu}^{um} is the disagreement between upstream and midstream about $d \log z_{iu}$, upstream's TFP disturbance. Suppose upstream and midstream have the same beliefs about $d \log z_{im}$ but disagree with downstream so that $D_{im}^{ud} = D_{im}^{md} \neq 0$. Then the first line of (21) coincides with (20). Thus, the addition of a third industry does not significantly change the impact of disagreement about the TFP of downstream's immediate supplier, $d \log z_{im}$. The only difference is that there are now two supplier industries exposed to $d \log z_{im}$ instead of one, and their disagreement is weighted by their labor share of sales from downstream.

The second line of (21) shows that the impact of disagreement about upstream's TFP adds to the impact from disagreement about $d \log z_{im}$. Suppose input demand is complementary throughout the network ($\theta_{y,id} < 1$ and $\theta_{y,im} < 1$). Then positive D_{iu}^{bc} , i.e. disagreement between two industries about upstream's TFP, does not offset the response of GDP to positive disagreement about midstream's TFP, and vice-versa. Positive disagreement about either industry's TFP reduces GDP. Thus, when a production network features complementary demand throughout the supply chain, recessions are larger if downstream industries are more pessimistic than upstream industries.

4.3 Frictions in Labor Reallocation

I model frictions in labor reallocation across sectors using the sector-specific inverse Frisch elasticities. Let $\underline{\eta} = \min_{i \in \mathcal{N}} \eta_i$. A high $\underline{\eta}$ implies a relatively inelastic labor supply to every sector. For example, consider a two-sector economy in which the household supplies h_1 and h_2 . The household would accept a significant wage reduction from sector 1 and would require a significant wage increase from sector 2 before reallocating labor from sector 1 to sector 2. The impact of $\underline{\eta}$ on outcomes is summarized in the following proposition.

Proposition 5. *Fix firms' belief errors, and assume χ changes to ensure that the steady state remains the same as $\underline{\eta}$ changes. Then belief errors have a vanishing effect on GDP as $\underline{\eta} \rightarrow \infty$.*

When labor is not mobile across sectors, firms can count on wages dropping significantly if they ever have a reason to reduce hiring. Absent wage rigidities, firms know that in the limit as $\underline{\eta} \rightarrow \infty$ wages will adjust to offset whatever occurs in the afternoon, so they do not need to reduce hiring. High $\underline{\eta}$ also reduces the importance of spillovers between sectors through wages. Each sector's wage depends more and more on the sector's own demand as $\underline{\eta}$ increases and relatively less on GDP and thus labor demand in other sectors.

5 Survey Evidence: Disagreement About Tariffs

This section provides empirical support for the idea that disagreement between firms and their suppliers affects their decisions using a hypothetical vignette experiment.

5.1 Setting and Data

In early 2025, U.S. trade policy drastically changed. In March, the U.S. sharply increased tariffs on Canada, Mexico, and China and then paused those increases. In April, the U.S.

imposed a minimum 10% tariff on most countries and announced higher “reciprocal” tariffs that were then paused for 90 days. Over the next two months, the U.S. negotiated a deal with the U.K. that set a 10% minimum tariff while the U.S. and China agreed to keep negotiating until August, and a trade court questioned the legality of many of the tariff changes. In early July, the U.S. struck a deal with Vietnam that set a 20% minimum tariff and extended the 90-day pause for many other countries to August 1. I fielded the vignette experiment over July 16–23, 2025, with most responses collected over July 16–21. Respondents therefore faced substantial uncertainty about tariff rates on most trading partners, including China. While there were August deadlines coming up that could potentially resolve uncertainty, it is reasonable to assume that firms expected tariffs to continue to change after August given the frequent shifts in tariff policy over the first half of 2025.

Respondents for the vignette experiment were drawn from Dynata, a consumer and B2B survey research company. Survey respondents are invited by an email that does not describe the content of the survey. To ensure that respondents know their firm’s expectations and have decision-making authority, I require respondents to be an owner, a C-suite officer, or an executive, director, or manager in accounting, finance, procurement, or purchasing.¹¹ A respondent’s firm must also have a supplier, which can be domestic or foreign, and I impose quotas by employment size and sector. Finally, there is an attention check in the middle of the survey. The final sample contains 312 respondents, most of whom are at firms with fewer than 500 employees, spread across sectors and U.S. Census regions.

Survey Versus Model. The survey does not cleanly map onto the theoretical model by design. Theoretically, TFP disturbances are useful objects to study, but in reality, firms likely do not think about the world in terms of TFP disturbances. To make hypothetical disagreement understandable and realistic to respondents, the survey asks about tariffs instead.

¹¹I ask respondents to select their seniority level, which includes an option for entry-level workers, and their department within the firm to mask which types of respondents I want.

While the model could be altered to model tariffs instead of TFP disturbances, as tariffs are not equivalent to TFP shocks, such a change would complicate the theoretical analysis without changing the main point that disagreement between firms and their suppliers can affect aggregate output.

5.2 Survey Design

This section summarizes the survey. The precise survey is available on request. The survey has three parts. First, I elicit firm characteristics and prior beliefs. Second, I randomly assign the respondent a treatment. Third, I elicit posterior beliefs and some additional responses. To economize on language, the remainder of the paper will use respondent to refer to the actual person who responded to the survey as well as the firm at which a respondent works or that a respondent owns.

Before Treatment. Before assigning treatment, the survey elicits the import share of the unit cost of the respondent’s main product or service and the regions from which the firm imports. The survey then elicits the firm’s expected average tariff rate by the end of 2025 on imports from China, X_C , and from other countries, X_N , as well as the highest and lowest rates the firm considers possible. Finally, the survey elicits the firm’s expected percent changes in the price, unit cost, and sales of its main product or service over the next 6 and 12 months, the expected change in its number of employees over the same horizons, and expected average weekly hours per employee over the next 6 months.¹²

Treatment. I draw a level of disagreement ε uniformly from $\{-30, -25, \dots, 25, 30\}$ and set the hypothetical expected tariff rates by a respondent’s suppliers to $X_C + \varepsilon$ and $X_N + \varepsilon$,

¹²I ask about the number of employees instead of the percent change because some respondents may run or work at firms with fewer than 10 employees. These types of firms do not think about percent changes in the number of employees.

rounded to the nearest multiple of five when $\varepsilon \neq 0$. The survey redraws ε until both rates are nonnegative since some respondents may expect low tariff rates.¹³ In addition to ε , respondents are also randomized into “confident supplier” or “uncertain supplier” scenarios. Respondents then read the following hypothetical scenario:

Consider this hypothetical scenario. Suppose your firm learns that the suppliers for your firm’s main product or service generally expect average tariff rates by the end of 2025 to be $X_C + \varepsilon\%$ on imports from China and $X_N + \varepsilon\%$ on imports from other countries. [CONFIDENT:] Your suppliers are very confident in their forecasts and are making business decisions based on them. [UNCERTAIN:] However, your suppliers are uncertain and think a wide range of tariff rates are possible. Assume your suppliers are making business decisions based on these forecasts.

To minimize cognitive effort, the scenario then reminds respondents of the rates X_C and X_N that they reported earlier and states whether their suppliers’ expected rates are higher than, the same as, or lower than their own.

After Treatment. I elicit the firm’s expected tariff rates and expected firm-level outcomes again to collect their posterior beliefs.

5.3 What the Experiment Identifies

The treatment ε randomizes the *ex-ante* disagreement between the respondent and its suppliers. Ex-ante disagreement can move a respondent’s plans through two channels. The first is information provision: the respondent learns something from its suppliers’ beliefs and revises its own tariff expectations. The second is *ex-post* disagreement: having learned its

¹³If disagreement linearly affects beliefs, then this choice is inconsequential.

suppliers’ beliefs and revised its own, the respondent continues to disagree with its suppliers and expects their decisions to affect it. The second channel is the one the model studies. In the model, firms know each other’s beliefs and agree to disagree, and disagreement matters because the labor choice of firms in upstream sectors spills over onto the costs of firms in downstream sectors.

The reduced-form effect of ex-ante disagreement combines both channels. Separating them would require a second instrument. The randomized uncertainty of suppliers in theory could be an instrument to disentangle belief revision from ex-post disagreement because it should change how much respondents learn from their suppliers. However, in this sample, the uncertainty of suppliers does not have a large effect on belief revisions. I therefore report reduced-form estimates and OLS estimates which decompose ex-ante disagreement into ex-post disagreement and belief revision.

5.4 Disagreement About Tariffs

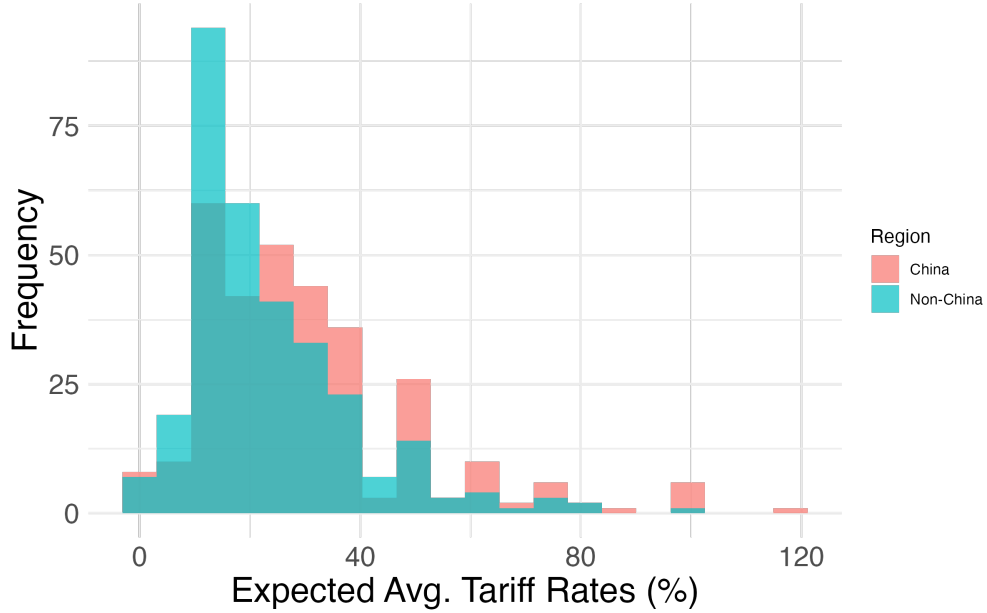
Firms expect a broad range of average tariff rates. Figure 3 plots the distribution of firms’ expected average tariff rates by the end of 2025. The distribution is roughly consistent with [Andrade et al. \(2025\)](#), which uses Morning Consult’s SMB survey as a data source. Expected rates are primarily between 10 and 40 percent, and the distribution has a long right tail. Since there is substantial dispersion in expected tariffs, the hypothetical scenario of disagreement about tariff rates is likely plausible to respondents.

5.5 Disagreement Changes Posterior Beliefs

I estimate the causal effect of ex-ante disagreement on a firm’s posterior beliefs. The regression is

$$Y_i = \sum_{g \in \{\text{Low, High}\}} \beta_g \text{Disagree}_i \times \mathbb{1}\{\text{ImpShr}_i = g\} + \gamma' X_i + u_i, \quad (22)$$

Figure 3: Respondents' expected average tariff rates by the end of 2025.



where Disagree_i is the ex-ante disagreement in percentage points (suppliers' expected tariff rate minus the respondent's) and ImpShr_i is a binary variable that indicates whether imports account for at least 20 percent of the respondent's unit cost ("High") or less ("Low"). Firms in the low group have little exposure to tariffs, so I expect disagreement to matter mainly for the high group. The dependent variable Y_i is a posterior expectation while the controls X_i are the respondent's prior expected tariff rates, the import-share group, the corresponding prior expectation, and a fixed effect for the randomly assigned supplier-uncertainty scenario. I include these controls to improve precision. Following [Coibion et al. \(2023\)](#) and [Coibion et al. \(2024\)](#), I drop influential observations identified by a jackknife when Y_i is continuous.¹⁴ I re-estimate the regression dropping one observation at a time and drop an observation if it moves β by more than $2/\sqrt{N}$ standard deviations, a threshold based on [Belsley et al. \(2005\)](#). Standard errors are heteroskedasticity robust. The results are robust to adding additional controls and fixed effects.

¹⁴Their regressions are IV regressions in which the first stage uses Huber robust regressions and the second stage applies jackknife. Since my analysis does not have a first stage, I only perform the jackknife procedure.

Prices, Unit Costs, and Sales. Relatively higher expected average tariff rates by suppliers (positive disagreement) cause a respondent to expect higher prices, higher unit costs, and lower sales over the next 6-12 months. Table 1 reports the results for expected percent changes over the next 6 months (columns 1–3) and the next 12 months (columns 4–6). At both horizons, higher expected tariff rates by suppliers cause respondents with high import shares to revise their expectations about price changes and unit costs upward. At the 12-month horizon, a 10 percentage point increase in disagreement leads to 1.9 percentage point higher prices and 1.4 percentage point higher unit costs. Sales also respond for firms with high import share, decreasing by about 1 percent. Since prices are revised upward, the decrease in sales likely corresponds to a contraction in output sold. One perhaps puzzling estimate is that, at the 6-month horizon, low import share respondents expect sales to rise when their suppliers expect higher tariff rates, and this effect is significant at the 5% level. This effect could be driven by domestic suppliers which anticipate high tariff rates to be good news, and the respondent expects positive effects on their suppliers to benefit their own sales.

Input Choices. This vignette experiment also supports the theoretical prediction that disagreement causes firms to adjust their labor demand. Table 2 reports the effects on expected average hours worked by employees over the next 6 months. Disagreement lowers a firm’s posterior labor demand. The magnitude is also reasonably large. If a firm’s suppliers expect 10pp higher tariff rates, then the firm would reduce expected hours demanded by about 0.4pp and raise the probability that the firm cuts hours demanded by 2.7pp.

Disagreement or Information Provision? While I cannot identify the causal effect of ex-post disagreement, I can still provide descriptive evidence about its effects on a respondent’s posterior beliefs. I define ex-post disagreement ExPostDisagree_i as the suppliers’ expected tariff rate minus the respondent’s posterior expected rate. Since firms can revise

Table 1: Disagreement raises expected prices and unit costs but reduces expected sales

	6 Months			12 Months		
	Price	Unit Cost	Sales	Price	Unit Cost	Sales
Disagree $\times$ High ImpShr	0.111*** (0.029)	0.097*** (0.026)	-0.011 (0.029)	0.187*** (0.035)	0.136*** (0.035)	-0.109*** (0.039)
Disagree $\times$ Low ImpShr	0.063 (0.050)	-0.057 (0.055)	0.095** (0.042)	0.113* (0.060)	0.017 (0.066)	-0.003 (0.060)
Observations	286	273	287	291	282	283

Notes: This table reports the causal effect of ex-ante disagreement on a respondent's posterior expected percent change in the price, unit cost, and sales of its main product or service over the next 6 months (columns 1-3) or 12 months (columns 4-6), in percentage points. Coefficients are the effect of a 1 percentage point increase in Disagree. Low (High) ImpShr denotes firms for which imports are less than 20 percent (at least 20 percent) of unit cost. Controls include the respondent's prior belief about the dependent variable, prior expected tariff rates, import-share group, and supplier uncertainty scenario. I remove influential observations using jackknife. Heteroskedasticity-robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 2: Disagreement reduces posterior labor demand

	Log Hours	Reduce Hours
Disagree $\times$ High ImpShr	-0.043** (0.020)	0.27** (0.13)
Disagree $\times$ Low ImpShr	0 (0.026)	0.063 (0.30)
Observations	278	310

Notes: This table reports the causal effect of ex-ante disagreement on a respondent's posterior expected labor demand over the next 6 months. Column 1 reports the effect on log hours while column 2 reports a linear probability model where the dependent variable is whether the firm cuts average hours demanded. I scale the dependent variable so that the reported coefficients are the effect of a 1 percentage point increase in Disagree on a percentage point change in posterior hours demanded or probability that posterior hours are cut. High (Low) ImpShr denotes firms for which imports are at least 20 percent (less than 20 percent) of unit cost. Controls include the respondent's prior belief about the dependent variable, prior expected tariff rates, import-share group, and supplier uncertainty scenario. I remove influential observations using jackknife for the log hours specification. Heteroskedasticity-robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

their expectations for the tariff rate on China and non-China countries separately, I must choose the relevant posterior belief for a respondent. For firms that import only from China or only from other countries, the choice is obvious. If firms import from both but revise only one of the two tariff rates, I use the revised rate. Otherwise, I take an equal-weighted average across the two revised rates. The residual difference between Disagree_i and ExPostDisagree_i is the belief revision. Using this decomposition of Disagree_i , I re-run (22) for the 12-month posterior beliefs and report the results in Table 3.

Table 3: Effects of disagreement appear to be driven by ex-post disagreement

	Price	Unit Cost	Sales
$\text{ExPostDisagree} \times \text{High ImpShr}$	0.126*** (0.036)	0.127*** (0.036)	-0.086** (0.038)
$\text{ExPostDisagree} \times \text{Low ImpShr}$	0.076 (0.059)	-0.146* (0.077)	0.021 (0.060)
Observations	289	274	285

Notes: This table reports the (descriptive) effect of ex-post disagreement on a respondent's posterior expected percent change in the price, unit cost, and sales of its main product or service over the next 12 months in percentage points. Coefficients are the effect of a 1 percentage point increase in ExPostDisagree . Low (High) ImpShr denotes firms for which imports are less than 20 percent (at least 20 percent) of unit cost. Controls include the respondent's prior belief about the dependent variable, prior expected tariff rates, import-share group, supplier uncertainty scenario, and belief revision about average tariff rates. I remove influential observations using jackknife. Heteroskedasticity-robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

I find that, for high-import-share firms, ex-post disagreement is associated with statistically significant effects on expected 12-month price, unit cost, and sales changes, holding fixed how much the firm revised its own tariff expectations. The magnitudes are also similar to what I obtain in Table 1. Thus, disagreement does not appear to affect posterior beliefs purely through belief revision.

6 Conclusion

How does the aggregate impact of a novel and complex shock depend on belief disagreement between economic agents when there are spillovers? This paper shows that disagreement directly affects aggregate output when an economy’s production network is organized into vertical CES supply chains because CES production and the vertical supply chain create spillovers between firms and their suppliers. To provide empirical support for this theoretical result, I run a hypothetical vignette experiment on U.S. small businesses and find that disagreement can causally change their expected decisions.

I have focused on the case in which agents perfectly know the model but face uncertainty about exogenous states. In a sense, this approach is a minimal departure from complete information rational expectations because I alter agents’ beliefs about exogenous disturbances and not the structural relations of the model. I defer analysis of the interaction between misspecified beliefs and production networks to future research.

References

- ANDRADE, P., A. M. DIETRICH, J. LEER, X. LIN, R. S. SCHOENLE, J. TANG, AND E. ZAKRAJŠEK (2025): “Who Will Pay for Tariffs? Businesses’ Expectations about Costs and Prices,” Tech. rep., Federal Reserve Bank of Boston.
- BAQAEE, D. AND E. FARHI (2020a): “Nonlinear Production Networks with an Application to the Covid-19 Crisis,” Working paper, National Bureau of Economic Research.
- BAQAEE, D. R. AND E. FARHI (2019): “The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten’s Theorem,” *Econometrica*, 87, 1155–1203.
- (2020b): “Productivity and Misallocation in General Equilibrium,” *Quarterly Journal of Economics*, 135, 105–163.
- BELSLEY, D. A., E. KUH, AND R. E. WELSCH (2005): *Regression diagnostics: Identifying influential data and sources of collinearity*, John Wiley & Sons.
- BIGIO, S. AND J. LA’O (2020): “Distortions in Production Networks,” *Quarterly Journal of Economics*, 135, 2187–2253.
- BUI, H., Z. HUO, A. A. LEVCHENKO, AND N. PANDALAI-NAYAR (2024): “Noisy Global Value Chains,” Working paper.
- CHAHROUR, R., K. NIMARK, AND S. PITSCHNER (2021): “Sectoral Media Focus and Aggregate Fluctuations,” *American Economic Review*, 111, 3872–3922.
- COIBION, O., D. GEORGARAKOS, Y. GORODNICHENKO, G. KENNY, AND M. WEBER (2024): “The Effect of Macroeconomic Uncertainty on Household Spending,” *American Economic Review*, 114, 645–77.
- COIBION, O., D. GEORGARAKOS, Y. GORODNICHENKO, AND M. VAN ROOIJ (2023): “How Does Consumption Respond to News about Inflation? Field Evidence from a Randomized Control Trial,” *American Economic Journal: Macroeconomics*, 15, 109–52.
- DEW-BECKER, I. (2023): “Tail Risk in Production Networks,” *Econometrica*, 91, 2089–2123.
- HORVATH, M. (2000): “Sectoral Shocks and Aggregate Fluctuations,” *Journal of Monetary Economics*, 45, 69–106.
- JAMILOV, R., A. KOHLHAS, O. TALAVERA, AND M. ZHANG (2024): “Granular Sentiments,” Working paper.
- KOPYTOV, A., B. MISHRA, K. NIMARK, AND M. TASCHEREAU-DUMOUCHEL (2024): “Endogenous Production Networks Under Supply Chain Uncertainty,” *Econometrica*, 92, 1621–1659.

- NIKOLAKOUDIS, G. (2024): “Incomplete Information in Production Networks,” Working paper.
- PELLET, T. AND A. TAHBAZ-SALEHI (2023): “Rigid Production Networks,” *Journal of Monetary Economics*, 137, 86–102.
- ROJAS-BERNAL, A. AND A. KARIMIRAD (2023): “Nonlinearities in Production Network Economies with Distortions,” Working paper.
- RUBBO, E. (2023): “Networks, Phillips Curves, and Monetary Policy,” *Econometrica*, 91, 1417–1455.

Appendices

A Proofs

A.1 Proposition 1

A.1.1 Nonlinear Optimality Conditions

Household Problem. The household's Lagrangian takes the form

$$\mathcal{L} = u \left(\bar{Y} \left[\sum_{i \in \mathcal{N}} \beta_i \left(\frac{c_i}{\bar{c}_i} \right)^{\frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}} \right) - \sum_{i \in \mathcal{N}} \chi_i \frac{(h_i^s)^{1+\eta_i}}{1+\eta_i} + M \left(\sum_{i \in \mathcal{N}} (w_i h_i^s + \pi_i) - \sum_{i \in \mathcal{N}} p_i c_i \right),$$

where $u(Y) = \frac{Y^{1-\gamma}}{1-\gamma}$ and M is the multiplier on the household's budget constraint. The household's controls are $\{c_i, h_i^s\}_{i \in \mathcal{N}}$.

The FOC w.r.t. c_i is

$$u'(Y) \beta_i \left(\frac{\bar{Y}}{\bar{c}_i} \right)^{\frac{\theta_c - 1}{\theta_c}} \left(\frac{Y}{c_i} \right)^{\frac{1}{\theta_c}} = M p_i.$$

Since a CES aggregator is homogeneous of degree one, and GDP is the numeraire, it can be shown that¹⁵

$$\beta_i \left(\frac{\bar{Y}}{\bar{c}_i} \right)^{\frac{\theta_c - 1}{\theta_c}} \left(\frac{Y}{c_i} \right)^{\frac{1}{\theta_c}} = p_i,$$

which implies $M = u'(Y)$ and re-arranges to

$$p_i c_i = \beta_i^{\theta_c} \left(\frac{\bar{Y}}{\bar{c}_i} \right)^{\theta_c - 1} p_i^{1-\theta_c} Y. \quad (23)$$

¹⁵See the proof of Lemma 1 in the supplementary appendix of [Rubbo \(2023\)](#).

To obtain a formula for the consumption expenditure share β_i , divide both sides by Y and evaluate at the steady state to obtain

$$\beta_i = \frac{\bar{p}_i \bar{c}_i}{\bar{Y}}.$$

The FOC w.r.t. h_i^s is

$$w_i = \chi_i (h_i^s)^{\eta_i} Y^\gamma. \quad (24)$$

Firm Problem in the Afternoon. In the afternoon, sector i chooses intermediate inputs to minimize its costs, given some required output y_i , and then determines its optimal scale.

The cost minimization problem for firms is

$$\min_{\{x_{ij}\}_{j \in \mathcal{N}}} \sum_{j \in \mathcal{N}} p_j x_{ij}$$

subject to the production function

$$y_i = z_i \bar{y}_i \left[a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} + \sum_{j \in \mathcal{N}} a_{ij} x_{ij}^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}}.$$

The FOC w.r.t. x_{ij} then is

$$0 = p_j - amc_i^{1/\theta_{yi}} (z_i \bar{y}_i)^{\frac{\theta_{yi}-1}{\theta_{yi}}} y_i^{1/\theta_{yi}} a_{ij} x_{ij}^{-1/\theta_{yi}}$$

Re-arrange to

$$x_{ij} = \left[p_j^{-1} (z_i \bar{y}_i)^{\frac{\theta_{yi}-1}{\theta_{yi}}} (amc_i y_i)^{1/\theta_{yi}} a_{ij} \right]^{\theta_{yi}}$$

Plug this expression for x_{ij} into the production function.

$$y_i = z_i \bar{y}_i \left[a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} + \sum_{j \in \mathcal{N}} a_{ij} \left((z_i \bar{y}_i)^{\theta_{yi}-1} a_{ij}^{\frac{\theta_{yi}}{\theta_{yi}-1}} \frac{amc_i y_i}{p_j^{\frac{\theta_{yi}}{\theta_{yi}-1}}} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}}$$

To get an expression for $amc_i y_i$, re-arrange to

$$amc_i y_i = (z_i \bar{y}_i)^{-(\theta_{yi}-1)} \left(\sum_{j \in \mathcal{N}} \frac{a_{ij}^{\theta_{yi}}}{p_j^{\theta_{yi}-1}} \right)^{-\frac{\theta_{yi}}{\theta_{yi}-1}} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}}$$

This expression concludes the inner minimization problem.

Now consider the firm's optimal scale. Write expenditures on intermediate inputs as

$$p_j x_{ij} = \frac{a_{ij}^{\theta_{yi}}}{p_j^{\theta_{yi}-1}} \left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{\theta_{yi}}{\theta_{yi}-1}} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}}.$$

Using this expression and some algebra, profits become

$$\pi_i = p_i y_i - w_i h_i - \left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{1}{\theta_{yi}-1}} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}}$$

The FOC of profits w.r.t. y_i yields a relationship between the firm's price and its marginal cost

$$p_i = \left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{1}{\theta_{yi}-1}} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{1}{\theta_{yi}-1}} (z_i \bar{y}_i)^{-\frac{\theta_{yi}-1}{\theta_{yi}}} y_i^{-\frac{1}{\theta_{yi}}}. \quad (25)$$

This expression can be re-arranged to write

$$\left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{\theta_{yi}}{\theta_{yi}-1}} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}} = (z_i \bar{y}_i)^{\theta_{yi}-1} p_i^{\theta_{yi}} y_i. \quad (26)$$

Then expenditures on intermediate goods take the simpler form

$$p_j x_{ij} = (z_i \bar{y}_i)^{\theta_{yi}-1} \left(\frac{p_i}{p_j} \right)^{\theta_{yi}-1} a_{ij}^{\theta_{yi}} p_i y_i. \quad (27)$$

Firm Problem in the Morning. The firm solves

$$\max_{h_i} E_i \left[Y^{-\gamma} \left(p_i y_i - w_i h_i - \sum_{j \in \mathcal{N}} p_j x_{ij} \right) \right].$$

Since firms are sequentially rational, they know their optimality conditions in the afternoon, and they also know that those conditions hold for every value of $\mathbf{z}$. As a result, they know

$$\sum_{j \in \mathcal{N}} p_j x_{ij} = \left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{1}{\theta_{yi}-1}} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}},$$

hence the firm's FOC w.r.t. h_i is

$$\begin{aligned} 0 &= E_i \left[Y^{-\gamma} \left(p_i \frac{\partial y_i}{\partial h_i} \right) \right] - E_i [Y^{-\gamma} w_i] \\ &\quad - E_i \left[Y^{-\gamma} \left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{1}{\theta_{yi}-1}} \frac{\partial}{\partial y_i} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}} \frac{\partial y_i}{\partial h_i} \right] \\ &\quad - E_i \left[Y^{-\gamma} \left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{1}{\theta_{yi}-1}} \frac{\partial}{\partial h_i} \left[\left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{\frac{\theta_{yi}}{\theta_{yi}-1}} \right] \end{aligned}$$

The sum of the first and third terms equals zero because they compose the firm's FOC w.r.t. y_i in the afternoon. Using (26), the remaining two terms become

$$\begin{aligned} E_i[Y^{-\gamma}w_i] &= E_i \left[Y^{-\gamma} \left[\left(\sum_{k \in \mathcal{N}} \frac{a_{ik}^{\theta_{yi}}}{p_k^{\theta_{yi}-1}} \right)^{-\frac{1}{\theta_{yi}-1}} \left(\frac{y_i}{(z_i \bar{y}_i)} \right)^{\frac{\theta_{yi}-1}{\theta_{yi}}} - a_{hi} h_i^{\frac{\theta_{yi}-1}{\theta_{yi}}} \right]^{-\frac{1}{\theta_{yi}-1}} a_{hi} h_i^{-1/\theta_{yi}} \right] \\ &= E_i \left[Y^{-\gamma} (z_i \bar{y}_i)^{\frac{\theta_{yi}-1}{\theta_{yi}}} p_i y_i^{\frac{1}{\theta_{yi}}} a_{hi} h_i^{-1/\theta_{yi}} \right]. \end{aligned}$$

Since sector i knows its own labor choice,

$$h_i^{\frac{1}{\theta_{yi}}} = a_{hi} \frac{E_i[Y^{-\gamma} (z_i p_i \bar{y}_i)^{\frac{\theta_{yi}-1}{\theta_{yi}}} (p_i y_i)^{\frac{1}{\theta_{yi}}}] }{E_i[Y^{-\gamma} w_i]}. \quad (28)$$

A.1.2 Log-Linearization

Household Problem. The household's optimality conditions are log-linear, so it is straightforward to obtain (5) and (6).

Firm Problem in the Afternoon. The two optimality conditions are firms' expenditures on intermediate inputs (27) and the relationship between sectoral prices and marginal costs (25). The first condition is log-linear, so its log-linearization is straightforward. After using properties of CES aggregators and some algebra, the second condition log-linearizes to

$$d \log p_i = \frac{1}{1 - \alpha_i} \sum_{j \in \mathcal{N}} \Omega_{ij} d \log p_j + \frac{1}{\theta_{yi}} \frac{\alpha_i}{1 - \alpha_i} (d \log y_i - d \log h_i) - \left(1 + \frac{1}{\theta_{yi}} \frac{\alpha_i}{1 - \alpha_i} \right) d \log z_i$$

Multiply both sides by $1 - \alpha_i$ and re-arrange to get

$$\begin{aligned} & \left(1 + \frac{1 - \theta_{yi}}{\theta_{yi}} \alpha_i \right) d \log p_i \\ &= \sum_{j \in \mathcal{N}} \Omega_{ij} d \log p_j + \frac{1}{\theta_{yi}} \alpha_i (d \log s_i - d \log h_i) - \left(1 + \frac{1 - \theta_{yi}}{\theta_{yi}} \alpha_i \right) d \log z_i \end{aligned}$$

Firm Problem in the Morning. The firm's labor demand condition (28) is log-linear. Equation (9) is obtained after some algebra and using the definition of the log-linearized labor wedge (10).

Market-Clearing for Sectoral Expenditures. Using the fact that $\bar{p}_i \bar{x}_{ji} = \Omega_{ji} \bar{p}_j \bar{q}_j$ and dividing both sides by $\bar{Y}$, the market-clearing equation for sectoral expenditures log-linearizes to

$$\lambda_i d \log s_i = \beta_i (d \log p_i + d \log c_i) + \sum_{j \in \mathcal{N}} \Omega_{ji} \lambda_j (d \log p_i + d \log x_{ji}),$$

which is (11).

A.2 Proposition 2

Hulten's Theorem for GDP holds because sectoral labor choices are determined in the morning. When the afternoon arrives, the economy is isomorphic to a static production network model in which there are multiple factors with endogenous supply. See Baqaee and Farhi (2020a) or Rojas-Bernal and Karimirad (2023) for the proof of Hulten's Theorem in such a setting.

Matrix Equation for Sectoral Sales. Take the market-clearing equation for sectoral sales (11) and plug in expenditure demand on consumption (5) and intermediate inputs to get

$$\begin{aligned} \lambda_i d \log s_i &= \beta_i (d \log Y + (1 - \theta_c) d \log p_i) \\ &\quad + \sum_{j \in \mathcal{N}} \Omega_{ji} \lambda_j (d \log s_j - (1 - \theta_{yj}) (d \log z_j + d \log p_j - d \log p_i)). \end{aligned}$$

In matrix form,

$$\begin{aligned}
& (\mathbf{I} - \mathbf{\Omega}^\top) \text{diag}(\boldsymbol{\lambda}) d \log \mathbf{s} \\
&= \boldsymbol{\beta} d \log Y + \mathbf{\Omega}^\top \text{diag}(\boldsymbol{\lambda}) \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) d \log \mathbf{z} \\
&+ (\mathbf{\Omega}^\top \text{diag}(\boldsymbol{\lambda} \odot (\boldsymbol{\theta}_q - \mathbf{1})) - \text{diag}(\mathbf{\Omega}^\top (\boldsymbol{\lambda} \odot (\boldsymbol{\theta}_q - \mathbf{1})))) + (1 - \theta_c) \text{diag}(\boldsymbol{\beta}) d \log \mathbf{p}.
\end{aligned}$$

Define the matrices

$$\begin{aligned}
\mathbf{A}_z &= \mathbf{\Psi}^\top \mathbf{\Omega}^\top \text{diag}(\boldsymbol{\lambda} \odot (\boldsymbol{\theta}_q - \mathbf{1})) \\
\mathbf{A}_p &= \mathbf{A}_z - \mathbf{\Psi}^\top \text{diag}(\mathbf{\Omega}^\top (\boldsymbol{\lambda} \odot (\boldsymbol{\theta}_q - \mathbf{1}))) + \mathbf{\Psi}^\top (1 - \theta_c) \text{diag}(\boldsymbol{\beta}).
\end{aligned}$$

Then the sectoral sales equation becomes

$$\text{diag}(\boldsymbol{\lambda}) d \log \mathbf{s} = \mathbf{\Psi}^\top \boldsymbol{\beta} d \log Y + \mathbf{A}_z d \log \mathbf{z} + \mathbf{A}_p d \log \mathbf{p}. \quad (29)$$

Matrix Equation for Prices. Now consider the relationship between prices and marginal costs (8) in matrix form:

$$\begin{aligned}
& (\text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q) - \mathbf{\Omega}) d \log \mathbf{p} \\
&= \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q) (d \log \mathbf{s} - d \log \mathbf{h}) - \text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q) d \log \mathbf{z}.
\end{aligned}$$

Define the matrix

$$\mathbf{\Psi}_\theta = (\text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q) - \mathbf{\Omega})^{-1},$$

which resembles the Leontief inverse matrix but adjusts for the fact that labor allocations are decided before wages are paid. Then prices solve

$$\begin{aligned} d \log \mathbf{p} &= \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q)(d \log \mathbf{s} - d \log \mathbf{h}) \\ &\quad - \mathbf{\Psi}_\theta \text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q) d \log \mathbf{z} \end{aligned} \tag{30}$$

Remainder of Proof. Plug (30) into (29).

$$\begin{aligned} \text{diag}(\boldsymbol{\lambda}) d \log \mathbf{s} &= \mathbf{\Psi}^\top \boldsymbol{\beta} d \log Y + \mathbf{A}_z d \log \mathbf{z} + \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q)(d \log \mathbf{s} - d \log \mathbf{h}) \\ &\quad - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q) d \log \mathbf{z}. \end{aligned}$$

Re-arrange to get

$$\begin{aligned} &(\mathbf{I} - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash (\boldsymbol{\theta}_q \odot \boldsymbol{\lambda}))) \text{diag}(\boldsymbol{\lambda}) d \log \mathbf{s} \\ &= \mathbf{\Psi}^\top \boldsymbol{\beta} d \log Y - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q) d \log \mathbf{h} \\ &\quad + (\mathbf{A}_z - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q)) d \log \mathbf{z} \end{aligned}$$

Solving for $d \log \mathbf{s}$ delivers (13) and plugging the result into the matrix equation for prices (30) obtains (14).

For reference, the formulas for the matrices are as follows.

$$\begin{aligned} \mathbf{\Psi}_\theta &= (\text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q) - \boldsymbol{\Omega})^{-1} \\ \mathbf{A}_z &= \mathbf{\Psi}^\top \boldsymbol{\Omega}^\top \text{diag}(\boldsymbol{\lambda} \odot (\boldsymbol{\theta}_q - \mathbf{1})) \\ \mathbf{A}_p &= \mathbf{A}_z - \mathbf{\Psi}^\top \text{diag}(\boldsymbol{\Omega}^\top (\boldsymbol{\lambda} \odot (\boldsymbol{\theta}_q - \mathbf{1}))) + \mathbf{\Psi}^\top (1 - \theta_c) \text{diag}(\boldsymbol{\beta}) \\ \mathbf{S}_z &= \text{diag}(\boldsymbol{\lambda})^{-1} (\mathbf{I} - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash (\boldsymbol{\theta}_q \odot \boldsymbol{\lambda})))^{-1} (\mathbf{A}_z - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q)) \\ \mathbf{S}_h &= -\text{diag}(\boldsymbol{\lambda})^{-1} (\mathbf{I} - \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash (\boldsymbol{\theta}_q \odot \boldsymbol{\lambda})))^{-1} \mathbf{A}_p \mathbf{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q) \end{aligned}$$

$$\mathbf{S}_Y = \text{diag}(\boldsymbol{\lambda})^{-1}(\mathbf{I} - \mathbf{A}_p \boldsymbol{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash (\boldsymbol{\theta}_q \odot \boldsymbol{\lambda}))^{-1} \boldsymbol{\Psi}^\top \boldsymbol{\beta}$$

$$\mathbf{P}_z = \boldsymbol{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q) \mathbf{S}_z - \boldsymbol{\Psi}_\theta \text{diag}(\mathbf{1} + \boldsymbol{\alpha} \odot (\mathbf{1} - \boldsymbol{\theta}_q) \oslash \boldsymbol{\theta}_q)$$

$$\mathbf{P}_h = \boldsymbol{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q)(\mathbf{S}_h - \mathbf{I})$$

$$\mathbf{P}_Y = \boldsymbol{\Psi}_\theta \text{diag}(\boldsymbol{\alpha} \oslash \boldsymbol{\theta}_q) \mathbf{S}_Y.$$

A.3 Proposition 3

Labor Wedges. Plug (6), (13), and (14) into (10) to get

$$\begin{aligned} d \log \tau_i &= -\mathbf{e}_i^\top (\mathbf{S}_z + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})(\mathbf{I} + \mathbf{P}_z)) \mathcal{E}_i[d \log \mathbf{z}] \\ &\quad - \mathbf{e}_i^\top (\mathbf{S}_Y + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_Y - \boldsymbol{\theta}_q \gamma) \mathcal{E}_i[d \log Y] \\ &\quad - \mathbf{e}_i^\top (\mathbf{S}_h + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_h - \text{diag}(\boldsymbol{\theta}_q \odot \boldsymbol{\eta})) \mathcal{E}_i[d \log \mathbf{h}] \end{aligned}$$

This calculation is valid because beliefs are model-consistent.

Define

$$\begin{aligned} \mathbf{T}_z &= \mathbf{S}_z + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})(\mathbf{I} + \mathbf{P}_z), \\ \mathbf{T}_Y &= \mathbf{S}_Y + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_Y - \boldsymbol{\theta}_q \gamma. \end{aligned}$$

Under the heterogeneous priors assumption, $\mathcal{E}_i[d \log \mathbf{h}] = \mathbf{0}$ for all i . Then sector i 's log labor wedge can be written as

$$d \log \tau_i \equiv -\mathbf{T}_{z,i} \mathcal{E}_i[d \log \mathbf{z}] - \mathbf{T}_{Y,i} \mathcal{E}_i[d \log Y],$$

where the notation $\mathbf{T}_{z,i}$ means the i th row of $\mathbf{T}_z$.

By Hulten's theorem,

$$d \log Y = \boldsymbol{\lambda}^\top d \log \mathbf{z} + \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) d \log \mathbf{h}.$$

Take expectations of both sides to get

$$E_i[d \log Y] = \boldsymbol{\lambda}^\top E_i[d \log \mathbf{z}] + \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) E_i[d \log \mathbf{h}].$$

Then the expectation error is simply

$$\mathcal{E}_i[d \log Y] = \boldsymbol{\lambda}^\top \mathcal{E}_i[d \log \mathbf{z}] + \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathcal{E}_i[d \log \mathbf{h}]$$

Under the heterogeneous priors assumption, $\mathcal{E}_i[d \log \mathbf{h}] = \mathbf{0}$, hence

$$d \log \tau_i = -(\mathbf{T}_{z,i} + \mathbf{T}_{Y,i} \boldsymbol{\lambda}^\top) \mathcal{E}_i[d \log \mathbf{z}].$$

Labor Demand in Terms of Labor Wedges. Plug (13), (14), and (6) into (9) to get

$$\begin{aligned} & (\mathbf{I} + \text{diag}(\boldsymbol{\theta}_q \odot \boldsymbol{\eta}) - \mathbf{S}_h - \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_h) d \log \mathbf{h} \\ &= (\mathbf{S}_z + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})(\mathbf{I} + \mathbf{P}_z)) d \log \mathbf{z} + ((\mathbf{S}_Y + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_Y) - \gamma \boldsymbol{\theta}_q) d \log Y - d \log \boldsymbol{\tau}. \end{aligned}$$

Define the matrices $\mathbf{H}_\tau$, $\mathbf{H}_z$, and $\mathbf{H}_Y$

$$\mathbf{H}_\tau = (\mathbf{I} + \text{diag}(\boldsymbol{\theta}_q \odot \boldsymbol{\eta}) - \mathbf{S}_h - \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_h)^{-1}$$

$$\mathbf{H}_z = \mathbf{H}_\tau (\mathbf{S}_z + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})(\mathbf{I} + \mathbf{P}_z))$$

$$\mathbf{H}_Y = \mathbf{H}_\tau ((\mathbf{S}_Y + \text{diag}(\boldsymbol{\theta}_q - \mathbf{1}) \mathbf{P}_Y) - \gamma \boldsymbol{\theta}_q)$$

so that

$$d \log \mathbf{h} = \mathbf{H}_z d \log \mathbf{z} + \mathbf{H}_Y d \log Y - \mathbf{H}_\tau d \log \boldsymbol{\tau}.$$

Aggregation. Plug the expression for labor demand in terms of labor wedges into $d \log Y$ from Hulten's Theorem and re-arrange to

$$(1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y) d \log Y = \boldsymbol{\lambda}^\top (\mathbf{I} + \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_z) d \log \mathbf{z} - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_\tau d \log \boldsymbol{\tau}.$$

Define

$$d \log Y^{(0)} = \frac{1}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \boldsymbol{\lambda}^\top (\mathbf{I} + \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_z) d \log \mathbf{z}.$$

Then, writing the matrix algebra as sums,

$$\begin{aligned} d \log Y &= d \log Y^{(0)} - \frac{1}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \sum_{j \in \mathcal{N}} \lambda_j \alpha_j \sum_{i \in \mathcal{N}} \mathbf{H}_{\tau,ji} d \log \tau_i \\ &= d \log Y^{(0)} + \frac{1}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \sum_{j \in \mathcal{N}} \lambda_j \alpha_j \sum_{i \in \mathcal{N}} \mathbf{H}_{\tau,ji} (\mathbf{T}_{z,i} + \mathbf{T}_{Yi} \boldsymbol{\lambda}^\top) \mathcal{E}_i[d \log \mathbf{z}] \\ &= d \log Y^{(0)} + \frac{1}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \sum_{i \in \mathcal{N}} (\mathbf{T}_{z,i} + \mathbf{T}_{Yi} \boldsymbol{\lambda}^\top) \mathcal{E}_i[d \log \mathbf{z}] \sum_{j \in \mathcal{N}} \lambda_j \alpha_j \mathbf{H}_{\tau,ji} \\ &= d \log Y^{(0)} + \frac{1}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \sum_{i \in \mathcal{N}} \underbrace{\left(\sum_{j \in \mathcal{N}} \lambda_j \alpha_j \mathbf{H}_{\tau,ji} \right)}_{=\boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_\tau \mathbf{e}_i} (\mathbf{T}_{z,i} + \mathbf{T}_{Yi} \boldsymbol{\lambda}^\top) \mathcal{E}_i[d \log \mathbf{z}]. \end{aligned}$$

A.4 Proposition 4

Consider a Cobb-Douglas network with $\gamma = 1$. Plugging intermediate expenditure demand (7) into the sales market-clearing equation (11) from Proposition 1 implies that $d \log s_i = d \log Y$. Then the sales to wage ratio is $d \log s_i - d \log w_i = -\eta_i d \log h_i$. Because beliefs are

common knowledge, $E_i[d \log h_i] = d \log h_i$, hence $d \log \tau_i = 0$ and $d \log Y = d \log Y^{(0)}$.

A.5 Derivation of Prices for CES Supply Chain Economy

It is straightforward to compute the changes in labor wedges and GDP. To determine prices, some extra work is needed.

$$\begin{aligned} d \log \mathbf{p} &= \mathbf{P}_z d \log \mathbf{z} + \mathbf{P}_h d \log \mathbf{h} + \mathbf{P}_Y d \log \mathbf{Y} \\ &= (\mathbf{P}_z + \mathbf{P}_h \mathbf{H}_z) d \log \mathbf{z} - \mathbf{P}_h \mathbf{H}_\tau d \log \boldsymbol{\tau} + (\mathbf{P}_H \mathbf{H}_Y + \mathbf{P}_Y) d \log Y \end{aligned}$$

When $\gamma = 1$, $\mathbf{H}_Y = \mathbf{0}$. It follows that

$$\begin{aligned} d \log \mathbf{p} &= \left(\mathbf{P}_z + \mathbf{P}_h \mathbf{H}_z + \frac{\mathbf{P}_Y \boldsymbol{\lambda}^\top (\mathbf{I} + \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_z)}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \right) d \log \mathbf{z} \\ &\quad - \left(\mathbf{P}_h \mathbf{H}_\tau + \frac{\mathbf{P}_Y \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_\tau}{1 - \boldsymbol{\lambda}^\top \text{diag}(\boldsymbol{\alpha}) \mathbf{H}_Y} \right) d \log \boldsymbol{\tau} \end{aligned}$$

Suppose only sector i' makes belief errors and $d \log \mathbf{z} = \mathbf{0}$. Then prices in this sector satisfy

$$\begin{pmatrix} p_{i'u} \\ p_{i'd} \end{pmatrix} = \begin{pmatrix} 1 - \boldsymbol{\lambda}_{i'd} + \alpha_{i'd} \boldsymbol{\lambda}_{i'd} & -\frac{\alpha_{i'd} \boldsymbol{\lambda}_{i'd}}{\theta_{y,i'd}} \\ 1 - \boldsymbol{\lambda}_{i'd} - \alpha_{i'd} (1 - \boldsymbol{\lambda}_{i'd}) & \frac{\alpha_{i'd} (1 - \boldsymbol{\lambda}_{i'd})}{\theta_{y,i'd}} \end{pmatrix} \begin{pmatrix} d \log \tau_{i'u} \\ d \log \tau_{i'd} \end{pmatrix}.$$

Upstream's price is then given by

$$\begin{aligned} p_{i'u} &= \alpha_{i'd} \frac{1 - \theta_{y,i'd}}{\theta_{y,i'd}} ((1 - \boldsymbol{\lambda}_{i'd} (1 - \alpha_{i'd})) \mathcal{E}_{i'u}[d \log z_{i'u}] + \boldsymbol{\lambda}_{i'd} (1 - \alpha_{i'd}) \mathcal{E}_{i'd}[d \log z_{i'u}]) \\ &= \alpha_{i'd} \frac{1 - \theta_{y,i'd}}{\theta_{y,i'd}} (\mathcal{E}_{i'u}[d \log z_{i'u}] - \underbrace{\boldsymbol{\lambda}_{i'd} (1 - \alpha_{i'd})}_{=\boldsymbol{\lambda}_{i'u}} D_{i'}). \end{aligned}$$

When there is zero disagreement and input demand is complementary, optimism by upstream

causes upstream's price to rise. This result is intuitive: optimism by upstream causes upstream to reduce hiring, which causes the equilibrium price to go up. Disagreement causes upstream's price to rise by less or even fall. The more pessimistic downstream is relative to upstream, the less downstream will demand intermediate goods. While upstream is under-producing, downstream is also under-demanding, which leads to lower prices overall.

Now consider the behavior of prices under each industry's belief in sector i' . The matrix loading on $d \log \mathbf{z}$ is

$$\begin{pmatrix} -(1 - \lambda_{i'u}) & \lambda_{i'd} \\ -(1 - \alpha_{i'd})(1 - \lambda_{i'd}) & -(1 - \lambda_{i'd}) \end{pmatrix}$$

Then

$$\begin{aligned} E_{i'u}[d \log p_{i'u}] &= -(1 - \lambda_{i'u})E_{i'u}[d \log z_{i'u}] - \alpha_{i'd} \frac{1 - \theta_{y,i'd}}{\theta_{y,i'd}} \lambda_{i'd} (1 - \alpha_{i'd}) D_{i'} \\ E_{i'd}[d \log p_{i'u}] &= -(1 - \lambda_{i'u})E_{i'd}[d \log z_{i'u}] + \alpha_{i'd} \frac{1 - \theta_{y,i'd}}{\theta_{y,i'd}} (1 - \lambda_{i'd} (1 - \alpha_{i'd})) D_{i'} \end{aligned}$$

As expected, each industry's expected price varies inversely with its TFP belief. Optimism leads to lower expected prices. Disagreement also changes the sign. When downstream is more pessimistic than upstream ($D_{i'} > 0$) and input demand is complementary, upstream expects prices to be lower while downstream expects prices to be higher. For upstream, this is because relative optimism causes expenditure switching that makes the price fall even further (due to low demand). For downstream, this is because relative pessimism causes expenditure switching that makes the price rise even further.

A.6 Proposition 5

By Appendix A.3, which proves Proposition 3,

$$d \log \mathbf{h} = \mathbf{H}_z d \log \mathbf{z} + \mathbf{H}_Y d \log Y - \mathbf{H}_\tau d \log \tau.$$

All the matrices on the RHS depend on $\mathbf{H}_\tau$, which in the case of a uniform inverse Frisch elasticity is given by

$$\mathbf{H}_\tau = (\mathbf{I} + \text{diag}(\boldsymbol{\eta}) \text{diag}(\boldsymbol{\theta}_q) - \mathbf{S}_h - \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})\mathbf{P}_h)^{-1}.$$

Since $\underline{\eta} = \min_{i \in \mathcal{N}} \eta_i$,

$$\mathbf{I} + \text{diag}(\boldsymbol{\eta}) \text{diag}(\boldsymbol{\theta}_q) - \mathbf{S}_h - \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})\mathbf{P}_h \geq \mathbf{I} + \text{diag}(\underline{\eta}\boldsymbol{\theta}_q) - \mathbf{S}_h - \text{diag}(\boldsymbol{\theta}_q - \mathbf{1})\mathbf{P}_h \rightarrow \infty$$

as $\underline{\eta} \rightarrow \infty$, hence $\mathbf{H}_\tau \rightarrow \mathbf{0}$ and thus $d \log \mathbf{h} \rightarrow \mathbf{0}$. Note that $d \log Y \neq 0$ because sectoral TFPs can still deviate from steady state.

B Cyclicalilty of Labor Demand

This appendix section breaks Proposition 4 by setting $\gamma \neq 1$ to study the cyclicalilty of labor demand. Otherwise, the economy is purely horizontal so that all sectors hire only labor, and the household's demand is Cobb-Douglas. Also set $\eta_i = 0$ for all i to simplify expressions. By Proposition 3, sector i 's belief error about GDP is

$$\mathcal{E}_i[d \log Y] = \sum_{j \in \mathcal{N}} \lambda_j \mathcal{E}_i[d \log z_j],$$

and sector i 's log labor wedge is

$$d \log \tau_i = \underbrace{-(1 - \gamma) \mathcal{E}_i[d \log Y]}_{\text{cyclicalty of labor demand}} = -(1 - \gamma) \sum_{j \in \mathcal{N}} \lambda_j \mathcal{E}_i[d \log z_j].$$

Set $d \log \mathbf{z} = \mathbf{0}$ to simplify expressions. Then the labor wedges aggregate to

$$d \log Y = \sum_{i \in \mathcal{N}} \lambda_i \frac{1 - \gamma}{\gamma} \underbrace{\sum_{j \in \mathcal{N}} \lambda_j \mathcal{E}_i[d \log z_j]}_{= \mathcal{E}_i[d \log Y]}. \quad (31)$$

Changes in GDP result from changes in labor demand since the realized TFPs are fixed to steady state. Further, each sector's change in labor only depends on its own beliefs about $d \log Y$, holding labor allocations fixed.¹⁶ The income effect on labor supply determines whether optimism about GDP raises or reduces labor demand.

To build further intuition, suppose only sector i' has incorrect beliefs about TFPs. Then (31) simplifies to

$$d \log Y = \frac{1 - \gamma}{\gamma} \lambda_{i'} \sum_{j \in \mathcal{N}} \lambda_j \mathcal{E}_{i'}[d \log z_j].$$

GDP moves only because sector i' changes its labor demand relative to steady state. The fact that other sectors are correct about sectoral TFPs does not matter for sector i' . The impact on GDP is proportional to the size of the sector and depends on which TFPs about which sector i' has incorrect beliefs. It is common in the literature to assume that a sector is more informed about its own TFP than the TFPs of other sectors. A stark version is setting

¹⁶By Hulten's Theorem, the change in GDP, holding labor allocations fixed, equals the Domar-weighted sum of changes in sectoral TFPs.

$\mathcal{E}_{i'}[d \log z_{i'}] = 0$. Then

$$d \log Y = \frac{1 - \gamma}{\gamma} \lambda_{i'} (1 - \lambda_{i'}) \sum_{j \neq i'} \frac{\lambda_j}{1 - \lambda_{i'}} \mathcal{E}_{i'}[d \log z_j].$$

Changes in GDP are the largest when sector i' is intermediate in size and thus $\lambda_{i'}(1 - \lambda_{i'})$ is large.¹⁷ With $\lambda_{i'} \approx 1/2$, not only is sector i' 's labor demand significant, but errors in i' 's beliefs about the TFPs of other sectors also lead to errors about GDP changes.

The sign of $1 - \gamma$ determines whether pessimism or optimism induces a recession. Consider $\gamma < 1$ so that sales are more cyclical than wages. Then pessimistic belief errors are recessionary, and the downturn is more severe when large sectors (by Domar weight) are more pessimistic about the TFPs of large sectors. Conversely, suppose $\gamma > 1$ so that sales are less cyclical than wages. Then a firm receives so much insurance from wage declines that broad pessimism about TFP disturbances actually *increases* GDP.

Belief heterogeneity matters only to the extent that it determines the magnitude of the double sum in (31). It is not required for the double sum to be nonzero. The key force is whether sectors are on average pessimistic/optimistic (depending on the sign of $1 - \gamma$) about the economy. Belief errors generate larger changes in GDP when the errors are about TFPs of large sectors.

¹⁷The normalized weights $\{\frac{\lambda_j}{1 - \lambda_{i'}}\}_{j \neq i'}$ sum to one, so the weighted average of belief errors by sector i' about other sectors is invariant to $\lambda_{i'}$, the Domar weight of sector i' .